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An Enquiry Into the Predictive
Value of Grammar School Entrance
Examinations.

by

Dryden Rutter, M.Sc.

Thesis Submitted for Degree of M.Ed.

The University,

Durham.

March 1949.

Preface.

My sixteen years teaching experience has been varied. I have taught pupils in the primary school, before they have taken the grammar school entrance examination, in the grammar school after they have been successful, and in the senior elementary school after they have been rejected. Like many of my colleagues, I have often had misgivings on both the methods and the results of selection. I have found that the grammar school staffs are usually less pleased with the results of selection than those of the primary school. There seems to be a tendency in the grammar school to judge pupils, on entry, from the standard of the "A" pupil and, if I may venture a criticism, grammar school teaching is too often biased in favour of these "A" pupils. This, no doubt, is due to the fact that all roads in the grammar school lead to the school certificate examination. The new examination system may give the grammar schools freedom for development in spheres other than academic.

The object of this treatise is to investigate, in some detail, the methods of selection used in this county over a number of years. Unfortunately, I have been able to deal with only the selected pupils, as information on the unselected pupils, for the years in question, is unavailable. Tracing the scholarship results as far back as 1934, and the school certificate results back to 1939 has been a tedious operation. However, I am greatly indebted

to the Director of Education and the Headmaster of the school for permission to use the results. I am also very grateful to the Northern Universities Joint Matriculation Board who so readily gave me permission to use their school certificate results. My debt to Professor Peel cannot be overestimated, for without his ready guidance, the work would have probably foundered on some statistical reef.

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March 1949.

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An Enquiry Into the Prediction Value
of Grammar School Entrance Examinations.

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Chapter One.

Introduction: The Problem of Selection.

One of the most difficult and controversial problems confronting educationalists today is that of deciding what type of secondary education is best suited to the individual child. Since the demand for grammar school education far exceeds the supply, the problem has resolved itself into one of deciding which children are best fitted for grammar school education. As a result, the entrance examination for these schools has become highly competitive. Various methods of selection have been, and are being tried, all, having the same aim, that of choosing those pupils who would benefit most from an academic education. To estimate the success of any method of selection, we must follow the careers of the pupils selected through the grammar school. However, no matter what criteria we use as a measure of success in the grammar school, we must not expect too high a comparison with the entrance examination. The latter, in far too many cases, occupies only an hour or two, and in such a short time the effect of extraneous factors can be considerable. Many authorities rectify this, to a certain extent, by a system of transference between the types of secondary education. Schemes of transference are worthy of much more consideration than many authorities are giving them.

Several authorities have inaugurated systems of selection using qualifying examinations incorporated with a record of the pupils' progress in the primary school and the head teachers' estimate of fitness for entry into the grammar school. An interesting method is that used by the Borough of Walsall since 1944. (Ref. 1).

All candidates are given two intelligence tests with a three weeks interval, the average result being taken. The grammar school places are then shared out, each primary school being awarded the number of places corresponding to the number of its pupils gaining the minimum I.Q. necessary to fill the grammar schools. The places are then awarded to the individual pupils on the combined result of the intelligence tests, the school report and the primary school examinations. The intelligence tests are weighted in the ratio 3 : 2 against the combination of the other two. This method of selection avoids the extremely difficult task of comparing assessments of head teachers.

A similar scheme involving primary school ratings has been recently introduced in Middlesex. The primary school ratings are on a fifteen point scale and are based on the average mark of the pupils of each particular school in a qualifying examination. Awards are made on the total of marks in the qualifying examination and the "grading marks" of the primary school.

Experiment in the method of selection is a very welcome feature. However, the value of any method of selection is best measured by a comparison of the achievement of the pupils in the grammar school with their achievement in the qualifying tests. It is the aim of this work to make such a comparison. There have been several investigations of a similar nature, many of which have made valuable suggestions for the improvement of the technique of selecting pupils for the grammar school.

A.D. Amos (Ref. 2) came to the conclusion that the best prediction of secondary school success was from a combination

of intelligence tests and ordinary school examinations. J.W. Collier (Ref. 3) compared entrance results consisting of attainment tests in English and Arithmetic and an Intelligence Test (M.H.T.3) with marks gained in the secondary school. His results showed that the Intelligence Test was the best predictor, the Arithmetic Test was the worst, while the best prediction was obtained by a combination of all three. He also suggested a weighting in favour of the Intelligence Test and the English Test. A.G. Hughes (Ref.4) supported the use of a "carefully constructed intelligence test" in addition to the English and the Arithmetic examinations.

C.W. Valentine (Ref. 5) investigated the progress of several pupils in the secondary school. He compared their order of merit in the entrance examination with their order of merit at various stages of the secondary school. He found practically no relation between the order in the entrance examination and the school certificate examination. A better prediction of school certificate order was given by the order of merit after one year in the secondary school, a result which was to be expected. Valentine was an early advocate of the transference of pupils unsuited to the grammar school, to make way for those better fitted for this type of education.

T.E. Stubbins (Ref. 6) conducted a follow-up of two groups of 180 boys entering grammar schools in 1930 and 1931, and taking the school certificate examination five years later in 1935 and 1936 respectively. The entrance examination consisted of attainment tests in English and Arithmetic, an Intelligence Test and an estimate mark supplied by the primary school headteacher. For the pupils concerned, he found that the English mark in the

the entrance examination was the most reliable predictor of school certificate marks: the headteachers' estimate was of practically no predictive value; the Intelligence Test was the best predictor of Mathematics and Physics, while the Arithmetic marks were of little value, except when used with the Intelligence Test to predict school certificate Mathematics.

Probably the first large scale investigation into the problem of selection was carried out in 1937 in the West Riding of Yorkshire (Ref. 7). The object was to find the relative value of two types of examination, the first consisting of a Moray House Intelligence Test with ordinary English and Arithmetic Tests, and the second consisting of three Moray House tests in Intelligence, English and Arithmetic. The results in these tests was compared with the order of merit as estimated (before the tests) by each elementary school headteacher. The order of merit was reversed and treated as ordinary marks. For the unselected pupils the multiple correlations of each examination with the headteachers' ranking were almost equal. For the ~~un~~selected pupils the multiple correlation of the standardised tests was slightly better than that of the ordinary tests. In each case the Arithmetic had the highest predictive value, with the English the least. Amongst other recommendations, the report suggested that the entrance examination results should be considered along with the elementary school headteachers' ranking (made before the examination) plus a grading of the pupils on a five point scale.

The West Riding provided material for a similar enquiry by W.G. Emmett (Ref. 8). He compared the entrance results of about 1,000 pupils in 1933 and 1934, with their order of merit

estimated by the grammar school headteachers two years later. A further comparison of about 750 of these was made with their estimated order of merit after three years in the grammar school. The entrance examination consisted of attainment tests in English and Arithmetic with a Moray House Intelligence Test. These were compared with the grammar school headteachers' ranks, as before, the ranks were reversed and treated as ordinary marks. His results showed that the best single measure of later performance was the Intelligence Test. Considering the three entrance examinations as a team for prediction purposes, the Intelligence Test was almost equal to the English and Arithmetic papers in their best weighted combination.

On a still larger scale W.W. McClelland (Ref. 9) conducted an enquiry with the object of finding which test or combination of tests was the best predictor of secondary school success. His tests consisted of Intelligence Tests, standardised scholastic tests, ordinary school examinations and teachers' estimates concerning pupils entering the secondary schools in Dundee in 1936. The results of the tests were compared with the secondary school teachers' estimate of success after three years in the secondary school. The investigation indicated that the best battery for predicting secondary school success consisted of the I.Q., the combined English and Arithmetic in the qualifying examination, together with the teachers' estimate in English and Arithmetic scaled on the actual mark gained in these two. It also showed that the total English and Arithmetic mark gained in the qualifying examination had a higher predictive value than the I.Q.

This investigation concerns pupils entering a grammar school from 1934 to 1943, the year 1937 excepted. The actual marks gained by the pupils in the various examinations were compared with the actual marks gained by them in three separate school certificate subjects five years later. Previous investigations have used a single criterion of success in the grammar school, usually an estimated order of merit. In this case the criterion is complex, consisting of three assessments. Using Peel's method of calculating prediction for a complex criterion (Ref. 10) the predictive value of the entrance examination for certain arbitrary weightings of the tests was calculated. In addition, the maximum prediction of the entrance examination and the weightings of tests and criteria associated with it were obtained.

Chapter Two.

Method and Scope of the Enquiry.

The school concerned in the enquiry is a large mixed grammar school in an urban area, drawing pupils from a district where the predominant industry is coal-mining. The annual intake is about ninety six being approximately 10% of the age group. The number of pupils in the area going to private grammar schools has always been negligible. There is no technical education and the competition for entry to the grammar school has always been very keen. The age group consists of those pupils who are eleven but not twelve on 1st August of the year in which they sit. The character of the entrance examination has changed considerably over the period of the experiment, and since the object of the investigation is to compare the predictive values, a detailed statement of the methods of selection is warranted.

1934, 1935, 1936.

All pupils in the age group were given a qualifying examination in English and Arithmetic. About one-third of these took the competitive examination, which again consisted of unstandardised papers in English and Arithmetic. In each case the English and Arithmetic were weighted in the ratio 3 : 2 and in the Competitive Examination there was an age-allowance of 1% of the actual marks for each complete month of age below 12 years. Final selection was made after an interview which involved a number of pupils equal to 150%

of the places available. The number of pupils whose entry was decided by the interview in any year, was small.

1938 and 1939.

All pupils were given a qualifying examination in English and Arithmetic, weighted in the ratio 3 : 2. Approximately 40% proceeded to the Competitive Examination, which contained in addition to the unstandardised papers in English and Arithmetic, an unstandardised Intelligence Test. The weights of the three tests was almost equal. In 1938, an age-allowance of 2% of the possible marks was given in the Qualifying Examination, but in the Competitive Examination this was reduced to 1% for each month of age below 12. In 1939, the age-allowance was the same for both examinations, 1% of the possible marks for each complete month of age below 12 years of age. There was no interview in either of these years.

1940, 1941, 1942, 1943.

Entry into the grammar schools for these years was decided on a single standardised Moray House Intelligence Test.

The year 1937 is omitted owing to the fact that the entrance results for that year are not available. The method of selection for that year was similar to that for 1938 and 1939. It has always been the practice in this county to provide a supplementary examination for those pupils who are unable to take the usual examinations through illness.

In the grammar school, all pupils enter for the school certificate examination after five years; no pupil is kept out of

the examination, even though his estimated chance of passing is negligible. The school certificate examination is that of the Northern Universities Joint Matriculation Board, who very kindly permitted the use of their examination results. Only those pupils who completed five years in the school and took the school certificate examination in English Language, English Literature, Mathematics and Geography were considered. All were taking the examination for the first time. This reduced the number of pupils in the experiment considerably as premature withdrawals, transfers to other grammar schools and pupils who missed one or more of the examinations were omitted. English Language, English Literature, Mathematics and Geography were used as all pupils in the school take these subjects in the school certificate examination. The following table shows the number of pupils in the investigation :-

Year	Boys	Girls	Total
1934	18	26	44
1935	26	23	49
1936	25	21	46
1938	24	31	55
1939	27	26	53
1940	29	24	53
1941	36	32	68
1942	24	26	50
1943	32	22	54
Total	241	231	472

The marks gained in the various parts of the entrance examination, without age allowance, were compared with the marks

gained in the School Certificate Examination in English, Mathematics and Geography. The English mark used was the average of that for the Language and Literature papers. Pearson Product- Moment Coefficients of Correlation were calculated. It should be noted that these correlation coefficients apply to the sample of selected pupils, which constituted only about 10% of the total population. Higher correlation coefficients would have been obtained if it had been possible to correct these to include the unselected pupils. As information regarding the unselected pupils was not available this could not be done, a fact which should be kept in mind when valuing the results.



Entrance 1934 - School Certificate 1939.

n 44 For significance at 5% level .297, at 1% level .384.

	Entrance		School Certificate		
	Arith. w_1	English w_2	Mathematics u_1	English u_2	Geography u_3
w_1		.2866	.4762	.2115	.3402
w_2	.2866		.4145	.4241	.3377
u_1	.4762	.4145		.4454	.6761
u_2	.2115	.4241	.4454		.6215
u_3	.3402	.3377	.6761	.6215	

Entrance 1935 - School Certificate 1940.

n 49. For significance at 5% level .281, and 1% level .364

	Entrance		School Certificate		
	Arith. w_1	English w_2	Mathematics u_1	English u_2	Geography u_3
w_1		.01948	.2131	.4786	.2830
w_2	.01948		.5243	.2642	.3367
u_1	.2131	.5243		.6286	.7178
u_2	.4786	.2642	.6286		.5348
u_3	.2830	.3367	.7178	.5348	

Entrance 1936 - School Certificate 1941.

n 46. For significance at 5% level .291, at 1% level .376.

	Entrance		School Certificate		
	Arith. w_1	English w_2	Mathematics u_1	English u_2	Geography u_3
w_1		.2342	.3666	.3406	.2801
w_2	.2342		.1957	.4501	.2987
u_1	.3666	.1957		.5880	.5905
u_2	.3406	.4501	.5880		.5840
u_3	.2801	.2987	.5905	.5840	

Entrance 1938 - School Certificate 1943.

n 55. For significance at 5% level .265, at 1% level .345.

	Entrance			School Certificate		
	Arith. w_1	English w_2	Intell. w_3	Maths. u_1	English u_2	Geography u_3
w_1		.00537	.3196	.4751	.1356	.3318
w_2	.00537		.3362	.2545	.4008	.1972
w_3	.3196	.3362		.5303	.5060	.5493
u_1	.4751	.2545	.5303		.5129	.7132
u_2	.1356	.4008	.5060	.5129		.3142
u_3	.3318	.1972	.5493	.7132	.3142	

Entrance 1939 - School Certificate 1944.

n 53. For significance of correlation coefficient at the 5% level .270, at the 1% level .352.

	Entrance			School Certificate		
	Arith. w_1	English w_2	Intell. w_3	Maths. u_1	English u_2	Geography u_3
w_1		.08696	.02021	.5428	.3624	.1320
w_2	.08696		.4943	.2242	.3840	.2130
w_3	.02021	.4943		.3717	.5193	.3818
u_1	.5428	.2242	.3717		.5411	.4505
u_2	.3624	.3840	.5193	.5411		.3006
u_3	.1320	.2130	.3818	.4505	.3006	

Entrance 1940 - School Certificate 1945.

n 53. For significance at 5% level .270, at 1% level .352.

	Entrance Intell.	School Certificate		
		Maths.	English	Geography
Intell		.5552	.6251	.4842
S.C. Maths.	.5552		.7939	.7260
S.C. Eng.	.6251	.7939		.5370
S.C. Geog.	.4842	.7260	.5370	

Entrance 1941 - School Certificate 1946.

n 68. For significance at 5% level .238, at 1% level .310.

	Entrance Intell.	School Certificate		
		Maths.	English	Geography
Intell.		.4184	.4141	.2379
S.C. Maths.	.4184		.5357	.6619
S.C. Eng.	.4141	.5357		.4898
S.C. Geog.	.2379	.6619	.4898	

Entrance 1942 - School Certificate 1947.

n 50. For significance at 5% level .279, at 1% level .361.

	Entrance Intell.	School Certificate		
		Maths.	English	Geography.
Intell.		.4289	.6282	.4761
S.C. Maths.	.4289		.5200	.6127
S.C. Eng.	.6282	.5200		.7021
S.C. Geog.	.4761	.6127	.7021	

Entrance 1943 - School Certificate 1948.

n 54. For significance at 5% level, .266, at 1% level .348.

	Entrance Intell.	School Certificate		
		Maths.	English	Geography.
Intell.		.5171	.6874	.4422
S.C. Maths.	.5171		.5321	.7416
S.C. Eng.	.6874	.5321		.5368
S.C. Geog.	.4422	.7416	.5368	

Chapter Three.

Calculation of Regression Coefficients and Prediction Values.

In this chapter will be found the results of calculations based on the previous batteries of correlation coefficients. There are three main groups of calculations, those for 1934, 1935 and 1936 with two selection tests; those for 1938 and 1939 with three selection tests and those for 1940 to 1943 where there was a single test of selection. Regression coefficients were calculated by Aitken's method of pivotal condensation.

Where the criterion was complex, Peel's method of calculating the prediction was used.

Each battery was considered in the form

	u'	w'
u	R_{aa}	R_{ab}
w	R_{ba}	R_{bb}

where u denotes the criteria (three school certificate subjects) and w denotes the tests of entry into the grammar school. The value of $R_{ab}R_{bb}^{-1}$ was first calculated by Aitken's method, giving the test weights in terms of the criteria. The maximum prediction was then calculated when the criteria was weighted in a given way. (The ratios 1 : 1 : 1 and 1 : 2 : 1 were used). This is given by

$$r \equiv \sqrt{\frac{u' R_{ab} R_{bb}^{-1} R_{ba} u}{u' R_{aa} u}}$$

Now this is the only maximum prediction obtainable with the given weightings of criteria, and is not the maximum prediction possible in the Hotelling sense. The latter is the highest possible correlation between the batteries of tests and criteria. This value and the weights of tests and criteria associated with it, were

found by solving the equation :-

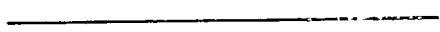
$$\begin{vmatrix} R_{ab} & R_{bb}^{-1} & R_{ba} \\ & & \end{vmatrix} - \lambda \cdot R_{aa} = 0$$

This is a symmetrical determinantal equation in λ of the same degree as the number of tests in the criterion, its largest positive root being the square of the maximum correlation.

As a check on the working, the position of the tests and the criteria in the battery were interchanged and new regression coefficients were obtained, giving the criteria test weights in terms of the entrance tests. The maximum prediction was calculated from these, together with the weights associated with it. These values agreed with those obtained previously. This was done for each of the years 1934 to 1939.

The reversability of the process can be particularly helpful when the number of tests in the criterion is bigger than the number of predicting tests. Interchanging reduces the degree of the equation to be solved to give maximum correlation between the two batteries. Thus for 1934, 1935 and 1936, reversing involved the solution of a quadratic instead of a cubic, which was necessary by the direct method.

At this stage the results are stated, but detailed working for the years 1936 and 1938 is given in the appendix. The calculations were performed to five or six figures but are recorded to four. Five or six figure working is necessary in calculating the maximum prediction for 1938 and 1939, as these are obtained by solving a determinantal cubic equation.



Entrance 1934 - School Certificate 1939.

The battery of correlation coefficients was as follows :

	School Certificate			Entrance	
	Maths. u_1	English u_2	Geography u_3	Arithmetic w_1	English w_2
u_1		.4454	.6761	.4762	.4145
u_2	.4454		.6215	.2115	.4241
u_3	.6761	.6215		.3402	.3377
w_1	.4762	.2115	.3402		.2866
w_2	.4145	.4241	.3377	.2866	

From this we have the following test weights :

$$w_1 = .3894 u_1 + .0980 u_2 + .2652 u_3 \dots\dots\dots (1)$$

$$w_2 = .3029 u_1 + .3960 u_2 + .2617 u_3$$

If the assessments, Mathematics, English and Geography are weighted in the ratio 1 : 1 : 1 we have from (1), $w_1 : w_2 \cong 3 : 4$, and the maximum prediction with these weights is .5418.

Now if the assessments are weighted in the ratio 1 : 2 : 1, we find $w_1 : w_2 \cong 2 : 3$, and the maximum prediction is reduced to .4350.

Maximum (Hotelling) prediction was .5803 with $u_1 : u_2 : u_3 \cong 14 : 6 : -1$, and $w_1 : w_2 \cong 10 : 11$.

Reversing the criteria and the tests in the calculation the following regression coefficients were obtained :-

$$u_1 = .3274 w_1 + .4543 w_2$$

$$u_2 = .3356 w_1 - .0185 w_2 \dots\dots\dots (2)$$

$$u_3 = -.0922 w_1 + .0445 w_2$$

These gave the same Hotelling maximum prediction with the same weights as the direct method, using equations (1).

Entrance 1935 - School Certificate 1940.

The battery of correlation coefficients was as follows :-

	School Certificate			Entrance	
	Maths. u_1	English u_2	Geography u_3	Arithmetic w_1	English w_2
u_1		.6286	.7178	.2131	.5243
u_2	.6286		.5348	.4786	.2642
u_3	.7178	.5348		.2830	.3367
w_1	.2131	.4786	.2830		.01948
w_2	.5243	.2642	.3367	.01948	

From this we have the following weights :-

$$w_1 = .2030 u_1 + .4736 u_2 + .2765 u_3 \dots\dots\dots (1)$$

$$w_2 = .5203 u_1 + .2550 u_2 + .3313 u_3$$

If the assessments, Mathematics, English and Geography are weighted in the ratio 1 : 1 : 1, we have from (1), $w_1 : w_2 = 6 : 7$, and the maximum prediction with these weights is .5669.

Now if the assessments are weighted in the ratio 1 ; 2 ; 1, we find that $w_1 : w_2 = 1 : 1$, and the maximum prediction, with these weights is .5727.

Maximum (Hotelling) prediction is .5781 with $u_1 : u_2 : u_3 = 17 : 13 : 3$ and $w_1 : w_2 = 4 : 5$.

Reversing the criteria and the tests the following regression coefficients were obtained :-

$$u_1 = -.2549 w_1 + .6331 w_2$$

$$u_2 = .5457 w_1 - .0991 w_2 \dots\dots\dots (2)$$

$$u_3 = .1741 w_1 - .0647 w_2$$

These gave the same Hotelling maximum prediction with the same weights as those obtained using equations (1).

Entrance 1936 - School Certificate 1941.

The following battery of correlation coefficients was obtained :-

	School Certificate			Entrance	
	Maths. u_1	English u_2	Geography u_3	Arithmetic w_1	English w_2
u_1		.5880	.5905	.3666	.1957
u_2	.5880		.5840	.3406	.4501
u_3	.5905	.5840		.2801	.2987
w_1	.3666	.3406	.2801		.2342
w_2	.1957	.4501	.2987	.2342	

From this we have the following test weights :-

$$\begin{aligned}
 w_1 &= .3394 u_1 + .2488 u_2 + .2223 u_3 \\
 w_2 &= .1162 u_1 + .3918 u_2 + .2466 u_3
 \end{aligned}
 \dots\dots\dots (1)$$

Weighting the assessments, Mathematics, English and Geography in the ratio 1 : 1 : 1, we have from (1), $w_1 : w_2 = 1 : 1$, and the maximum prediction with these weights is .4816.

Weighting the assessments in the ratio 1 : 2 : 1, we find that $w_1 : w_2 = 11 : 12$, and the maximum prediction, with these weights is .4974.

Maximum (Hotelling) prediction is .5177 with $u_1 : u_2 : u_3 = 1 : 30 : 7$ and $w_1 : w_2 = 2 : 3$.

Reversing the positions of the criteria and the tests the following regression coefficients were obtained :-

$$\begin{aligned}
 u_1 &= .2412 w_1 - .1467 w_2 \\
 u_2 &= .1794 w_1 + .4725 w_2 \\
 u_3 &= .0325 w_1 + .1094 w_2
 \end{aligned}
 \dots\dots\dots (2)$$

These gave the same Hotelling maximum prediction with the same weights as those obtained using equations (1).

Entrance 1938 - School Certificate 1943.

The battery of correlation coefficients was as follows :-

	School Certificate			Entrance		
	Maths. u_1	English u_2	Geography u_3	Arith. w_1	English w_2	Intell. w_3
u_1		.5129	.7132	.4751	.2545	.5303
u_2	.5129		.3142	.1356	.4008	.5060
u_3	.7132	.3142		.3318	.1972	.5493
w_1	.4751 .2545	.1356	.3318		.0054	.3196
w_2	.2545	.4008	.1972	.0054		.3362
w_3	.5303	.5060	.5493	.3196	.3362	

From this we have the following test weights :-

$$w_1 = .3548 u_1 + .0005 u_2 + .1779 u_3$$

$$w_2 = .1267 u_1 + .2601 u_2 + .0346 u_3 \dots\dots\dots (1)$$

$$w_3 = .3743 u_1 + .4184 u_2 + .4808 u_3$$

Weighting the assessments, Mathematics, English, Geography in the ratio 1 : 1 : 1, from (1) we have $w_1 : w_2 : w_3 = 5 : 4 : 12$, and the maximum prediction, with these weights is .6884.

Weighting the assessments in the ratio 1 : 2 : 1, we have $w_1 : w_2 : w_3 = 4 : 5 : 12\frac{1}{2}$, and the maximum prediction with these weights is .6803.

Maximum (Hotelling) prediction is .6892 with $u_1 : u_2 : u_3 = 3 : 4 : 4$, and $w_1 : w_2 : w_3 = 1 : 1 : 3$.

Reversing the criteria and the tests, the following regression coefficients were obtained :-

$$u_1 = .5728 w_1 + .0132 w_2 + .0795 w_3$$

$$u_2 = -.1488 w_1 + .3717 w_2 + .3445 w_3 \dots\dots\dots (2)$$

$$u_3 = -.0300 w_1 + .0710 w_2 + .3844 w_3$$

The same maximum prediction with the same weights was obtained from these as from equations (1).

Entrance 1939 - School Certificate 1944.

The following battery of correlation coefficients was obtained :-

	School Certificate			Entrance		Intell.
	Maths. u_1	English u_2	Geography u_3	Arithmetic w_1	English w_2	
u_1		.5411	.4505	.5428	.2242	.3717
u_2	.5411		.3006	.3624	.3840	.5193
u_3	.4505	.3006		.1320	.2130	.3818
w_1	.5428	.3624	.1320		.0870	.0202
w_2	.2242	.3840	.2130	.0870		.4943
w_3	.3717	.5193	.3818	.0202	.4943	

From this we have the following test weights :-

$$w_1 = .5356 u_1 + .3418 u_2 + .1229 u_3$$

$$w_2 = -.0010 u_1 + .1336 u_2 + .0197 u_3 \dots\dots\dots (1)$$

$$w_3 = .3614 u_1 + .4463 u_2 + .3695 u_3$$

Weighting the assessments in the ratio 1 : 1 : 1, we have from (1),

$w_1 : w_2 : w_3 = 7 : 1 : 8$, and the maximum prediction, with these weights is .6902.

Weighting the assessments in the ratio 1 : 2 : 1, we have

$w_1 : w_2 : w_3 = 5 : 1 : 6$, and the maximum prediction, with these weights is .7073.

Maximum (Hotelling) prediction is .7217, with $u_1 : u_2 : u_3 = 6 : 6 : 1$

and $w_1 : w_2 : w_3 = 7 : 1 : 6$.

Reversing the criteria and the tests, the following regression coefficients were obtained :-

$$u_1 = .5509 w_1 + .0235 w_2 + .0313 w_3$$

$$u_2 = .1091 w_1 + .3622 w_2 + .4308 w_3 \dots\dots\dots (2)$$

$$u_3 = -.1490 w_1 + .1147 w_2 + .2382 w_3$$

The same maximum prediction with the same weights was obtained from these as from equations (1)

Entry 1940 to 1943 - School Certificate 1945 to 1948.

The batteries of correlation coefficients were as follows :-

	School Certificate			Moray House Intell. Test.
	Maths.	English	Geography	
1940				
Maths.	1	³ .7949	.7260	.5552
English	.7939	1	.5370	.6251
Geography	.7260	.5370	1	.4842
M.H. Intell.	.5552	.6251	.4842	1
1941				
Maths.	1	.5357	.6619	.4184
English	.5357	1	.4898	.4141
Geography	.6619	.4898	1	.2379
M.H. Intell.	.4184	.4141	.2379	1
1942				
Maths.	1	.5200	.6127	.4289
English	.5200	1	.7021	.6282
Geography	.6127	.7021	1	.4761
M.H. Intell.	.4289	.6282	.4761	1
1943				
Maths.	1	.5321	.7416	.5171
English	.5321	1	.5368	.6874
Geography	.7416	.5368	1	.4422
M.H. Intell.	.5171	.6874	.4422	1

The calculations based on these batteries followed similar lines to those given by G.H. Thomson (Ref. 11) except that the position of tests and criteria is reversed. Regression

coefficients and their standard errors were calculated, the latter being obtained from the reciprocal matrix of correlations of tests (in this case School Certificate Mathematics, English and Geography).

The results are summarised in the following table :-

	School Cert. Maths.	School Cert. English	School Cert. Geog.	Prediction
1940				
Regr. Co.	- .01483	.52145	.21494	.6495
St. Err.	.2201	.1789	.1587	
1941				
Regr. Co.	.3541	.2923	-.13998	.486
St. Err.	.1508	.1297	.1461	
1942				
Regr. Co.	.13985	.5549	.0008	.6395
St. Err.	.1450	.1610	.1740	
1943				
Regr. Co.	.2459	.5859	-.05465	.7111
St. Err.	.1525	.1213	.1531	

Chapter Four.

Observations on the results.

The following table gives a summary of the prediction values and the approximate weights associated with them.

	Selection			Maths.	Criteria		Pred.
	Arith.	Eng.	Intell.		Eng.	Geog.	
1934	3	4		1	1	1	.5418
	2	3		1	2	1	.4350
	10	11		14	6	-1	.5795
1935	6	7		1	1	1	.5669
	1	1		1	2	1	.5727
	4	5		17	13	3	.5781
1936	1	1		1	1	1	.4816
	11	12		1	2	1	.4974
	2	3		1	30	7	.5177
1938	5	4	12	1	1	1	.6884
	4	5	12	1	2	1	.6803
	1	1	3	3	4	4	.6892
1939	7	1	8	1	1	1	.6902
	5	1	6	1	2	1	.7073
	7 5	1	6	6	6	1	.7217
1940				-1	35	14	.6495
1941				6	5	-2	.486
1942				1	4	0	.6395
1943				5	12	-1	.7111

Viewing these results as a whole, we conclude for the results investigated :

- (1) The best prediction of grammar school success is given by a battery of unstandardised attainment tests in Arithmetic and English with an unstandardised Intelligence Test.
- (2) A single Moray House Intelligence Test gave a prediction almost as good as the above battery.
- (3) Both the above were considerably better for prediction purposes than unstandardised attainment tests in Arithmetic and English.

The results for the year 1941 need some explanation. This was the second year that this type of test was used, and it was found that in the county as a whole (approximately 12,000 pupils) the results of the test were better than those for 1940. The reason suggested at the time was the effect of concentrated training which was given in certain schools for this type of test. Such training could have some influence on the results when entry was decided by a single Moray House Test. Another contributory factor to the low prediction for that year is the low correlation of school certificate Geography, for which staffing difficulties are held responsible.

For the years 1934, 1935, 1936, the Arithmetic and English tests are of almost equal value, maximum prediction being obtained by a weighting slightly in favour of the English. However, taking the three years as a whole, very little indication of possible success in the school certificate examination is given by either the Arithmetic or the English.

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In 1938 the Entrance Arithmetic gave a fairly good prediction of School Certificate Mathematics, but was of little value in predicting School Certificate English or Geography. In a similar way the Entrance English gave a much better prediction of School Certificate English than School Certificate Mathematics or Geography. The Entrance Intelligence Test, however, gave a better prediction of all three than either the Arithmetic or English.

In 1939, the predictive values of the various entrance tests was roughly the same as those for 1938, except that the Entrance Arithmetic was better than the Entrance Intelligence Test for predicting School Certificate Mathematics.

For the years 1940 to 1943 there was a high degree of correlation between the school certificate subjects and between the entrance tests and the latter. The School Certificate English showed the closest relation to the Entrance Intelligence Test. The multiple correlation of the school certificate subjects, as a team, with the Intelligence test was high, reaching .7111 in 1943. The regression coefficients of Mathematics and Geography were lower than those for English, in some cases being negative. This is due to the high correlations between the tests. Such a tendency is often characteristic of regression coefficients obtained from data in which the ^{school subjects} tests correlate highly with each other.

Since 1943 the entrance examination has undergone still further changes. In 1944 the Qualifying Examination was a Moray House Intelligence Test, while the Competitive Examination consisted of two standardised Attainment Tests in Arithmetic and English (Moray

House Tests). These pupils will take the School Certificate Examination in July 1949. It will be interesting to compare the predictive value of these tests with those of preceding years. This will be done in September or October when the school certificate results are available.

The following table shows how the method of selection for grammar schools has varied in this county over the last fifteen years.

Year	Qualifying Exam.	Competitive Exam.
1934 1935 1936	Unstandardised Tests in Arithmetic and English	Unstandardised Tests in Arithmetic and English with an interview
1937 1938 1939	Unstandardised Tests in Arithmetic and English	Unstandardised Tests in Arith. and English with an unstand. Intell. Test.
1940 1941 1942 1943	None	Standardised Moray House Intelligence Test
1944	Standardised Moray House Intell. Test	Standardised Moray House Tests in Arith. and Eng.
1945 1946	Unstandardised tests in Arith. and Eng. with a Stand. Moray House Intelligence Test	Standardised Moray House Tests in Arithmetic and English.
1947 1948 1949	Unstandardised tests in Arithmetic and English	Standardised Moray House Tests in Arithmetic, English and Intelligence.

The Qualifying Examination has in the fifteen years completed the circle, returning to unstandardised attainment tests in Arithmetic and English. During this time, the Competitive

examination has developed from two unstandardised tests to three standardised tests. Such changes provide abundant material for investigation into the merits of the respective methods of selection. Mine has been an individual experiment and I feel that much valuable information on the problem of selection for secondary grammar education could be obtained from a team research. This should, if possible, consider the unslected pupils as W.G. Emmett did in his enquiry.

Appendix A.

Detailed Working for 1936 - 1941.

Consider the battery of correlation coefficients in the following forms :-

	School Certificate			Entrance	
	Maths. u_1	English u_2	Geography u_3	Arith. w_1	English w_2
u_0	1	.5880	.5905	.3666	.1957
u_2	.5880	1	.5840	.3406	.4501
u_3	.5905	.5840	1	.2801	.2987
w_1	.3666	.3406	.2801	1	.2342
w_2	.1957	.4501	.2987	.2342	1

	u'	w'
u	R_{aa}	R_{ab}
w	R_{ba}	R_{bb}

The regression coefficients are obtained by evaluating $R_{a.b}$, R_{bb}^{-1} , using Aitken's method of pivotal condensation. From these we calculate the maximum prediction with the school certificate subjects equally weighted, and the maximum prediction when the School Certificate English has double the weight of either Mathematics or Geography. Lastly, we calculate maximum prediction in the Hotelling sense. The method is that given by Peel for the calculation of the prediction of a complex criterion.

To calculate $R_{ab} R_{bb}^{-1}$

1	.2342	-1		.2342
.2342	1		-1	.2342
.3666	.1957			.5623
.3406	.4501			.7907
.2801	.2987			.5788
	.94515	.2342	-1	.17935
	1	.247791	-1.058033	.189758
	.109842	.3666		.476442
	.370331	.3406		.710931
	.233101	.2801		.513201
		.339382	.116216	.455598
		.248835	.391822	.640657
		.222340	.246629	.468969

We then have the test weights in terms of the criteria weights.

$$\begin{aligned}
 w_1 &= .339382 u_1 + .248835 u_2 + .222340 u_3 \dots\dots\dots (1) \\
 w_2 &= .116216 u_1 + .391822 u_2 + .246629 u_3
 \end{aligned}$$

The maximum prediction with arbitrary weightings of the three criteria is given by

$$r \cong \sqrt{\frac{u' R_{ab} R_{bb}^{-1} R_{ba} u}{u' R_{aa} u}} \dots\dots\dots (2)$$

First let us weight the three school certificate subjects equally, then $u_1 : u_2 : u_3 \cong 1 : 1 : 1$, and from (1) we have

$$w_1 = .810557 \quad \text{and} \quad w_2 \cong .754667 \dots\dots\dots (3)$$

Then

$$u' R_{ab} R_{bb}^{-1} R_{ba} u = \begin{bmatrix} .81056 & .75467 \end{bmatrix} \begin{bmatrix} .3666 & .3406 & .2801 \\ .1957 & .4501 & .2987 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} .81056 & .75467 \end{bmatrix} \begin{bmatrix} .9873 \\ .9445 \end{bmatrix} = 1.513052 \dots (4)$$

Also

$$u' R_{aa} u = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & .5880 & .5905 \\ .5880 & 1 & .5840 \\ .5905 & .5840 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2.1785 \\ 2.1720 \\ 2.1745 \end{bmatrix}$$

$$= 6.5250 \dots (5)$$

Hence the maximum prediction is

$$\sqrt{\frac{1.5131}{6.5250}} = .4816$$

Now let us weight the three criteria in the ratio 1 : 2 : 1 .

From (1) we have

$$w_1 = 1.05939 \quad \text{and} \quad w_2 = 1.14649$$

Then

$$u' R_{ab} R_{bb}^{-1} R_{ba} u = \begin{bmatrix} 1.05939 & 1.14649 \end{bmatrix} \begin{bmatrix} .3666 & .3406 & .2801 \\ .1957 & .4501 & .2987 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$= 3.00566 \dots (6)$$

Also

$$u' R_{aa} u = \begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & .5880 & .5905 \\ .5880 & 1 & .5840 \\ .5905 & .5840 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = 11.869 \dots (7)$$

The maximum prediction with these weights is

$$\sqrt{\frac{3.0060}{11.869}} = .4974$$

Interchanging the tests and the criteria is equivalent to calculating $R_{ba} R_{aa}^{-1}$.

1	.5880	.5905	-1			1.1785
.5880	1	.5840		-1		1.1720
.5905	.5840	1			-1	1.1745
.3666	.3406	.2801				.9873
.1957	.4501	.2987				.9445
	.654256	.236786	.5880	-1		.479042
	1	.361916	.898731	-1.528454		.732193
	.236786	.651310	.5905		-1	.478596
	.125039	.063623	.3666			.555262
	.335028	.183139	.1957			.713868
		.565613	.377693	.361917	-1	.305223
		1	.667759	.639866	-1.767993	.539632
		.018369	.254224	.191116		.463709
		.061887	-.1054	.512075		.468562
			.241958	.179362	.032476	.453796
			-.146726	.472476	.109416	.435166

This gives us the criteria weights in terms of the test weights.

$$\begin{aligned}
 u_1 &= .241958 w_1 - .146726 w_2 \\
 u_2 &= .179362 w_1 + .472476 w_2 \dots\dots\dots (8) \\
 u_3 &= .032476 w_1 + .109416 w_2
 \end{aligned}$$

Maximum prediction, in the Hotelling sense, is obtained by solving

$$\left| R_{ba} R_{aa}^{-1} R_{ab} - \lambda R_{bb} \right| = 0 \dots\dots\dots (9)$$

The maximum prediction is the square root of the largest root of this equation (9).

$$R_{ba} R_{aa}^{-1} R_{ab} = \begin{bmatrix} .241958 & .179362 & .032476 \\ -.146726 & .47276 & .109416 \end{bmatrix} \begin{bmatrix} .3666 & .1957 \\ .3406 & .4501 \\ .2801 & .2987 \end{bmatrix}$$

$$\equiv \begin{bmatrix} .158889 & .137782 \\ .137782 & .216630 \end{bmatrix}$$

Thus we have to solve the quadratic :-

$$\begin{vmatrix} .15889 - \lambda & .13778 - .2342 \lambda \\ .13778 - .2342 \lambda & .21663 - \lambda \end{vmatrix} = 0 \dots\dots\dots (10)$$

The larger root of equation (10) is .2681, and the maximum prediction is the square root of this, .5178.

Substituting the value $\lambda = .2681$ in equation (10) we have

$$\begin{vmatrix} -.10921 & .07499 \\ .07499 & -.05147 \end{vmatrix} \dots\dots\dots (11)$$

From which we get $w_1 : w_2 = .05147 : .07499 = 2 : 3$ (Approx.)

Using these ratios for w_1 and w_2 in equations (8) we find that

$$u_1 : u_2 : u_3 = .014484 : .44667 : .098782 = 1 : 30 : 7 \text{ (approx.)}$$

Now if we used these ratios $u_1 : u_2 : u_3 = 1 : 30 : 7$ in equations (1) we find

$$w_1 = 9.3607 \text{ and } w_2 = 13.5971$$

These are in the approximate ratio 2 : 3 . Calculating the maximum prediction in the usual way we obtain .5177. Thus both the maximum prediction and the weights associated with it can be obtained from either equations (1) or equations (8).

If we had obtained the maximum prediction by using

equations (1), we would have had to solve

$$\begin{vmatrix} R_{ab} & R_{bb}^{-1} & R_{ba} \\ & & \lambda R_{aa} \end{vmatrix} = 0$$

This would be a symmetrical determinantal cubic equation and its largest root would be the same as the larger root of the quadratic (10).

Appendix B.

Detailed working for 1938 - 1943.

Consider the battery of correlation coefficients in the following form :-

	School Certificate				Entrance	
	Maths. u_1	English u_2	Geography u_3	Arith. w_1	English w_2	Intell. w_3
u_1	1	.5129	.7132	.4751	.2545	.5303
u_2	.5129	1	.3142	.1356	.4008	.5060
u_3	.7132	.3142	1	.3318	.1972	.5493
w_1	.4751	.1356	.3318	1	.0054	.3196
w_2	.2545	.4008	.1972	.0054	1	.3362
w_3	.5303	.5060	.5493	.3196	.3362	1

	u'	w'
u	R_{aa}	R_{ab}
w	R_{ba}	R_{bb}

The calculation of regression coefficients and the prediction values, with the weights associated with them, follows in the same way as those for Appendix A.

Calculation of $R_{ab} R_{bb}^{-1}$.

1	.0054	.3196	-1			.3250
.0054	1	.3362		-1		.3416
.3196	.3362	1			-1	.6558
.4751	.2545	.5303				1.2599
.1356	.4008	.5060				1.0424
.3318	.1972	.5493				1.0783
	.99997	.33447	.0054	-1		.33984
	1	.33448	.0054	-1.00003		.33985
	.33447	.89786	.3196		-1	.55193
	.25193	.37846	.4751			1.10549
	.40007	.46266	.1356			.99833
	.19541	.44326	.3318			.97047
		.78598	.31779	.33448	-1	.43826
		1	.40435	.42555	-1.27229	.55760
		.29419	.47374	.25194		1.01987
		.32884	.13344	.40008		.86237
		.37790	.33074	.19542		.90406
			.35478	.12675	.37430	.85583
			.00047	.26014	.41839	.67900
			.17794	.03460	.48080	.69334

Thus the test weights are given by :-

$$\begin{aligned}
 w_1 &= .35478 u_1 + .00047 u_2 + .17794 u_3 \\
 w_2 &= .12675 u_1 + .26014 u_2 + .03460 u_3 \dots\dots\dots (1) \\
 w_3 &= .37430 u_1 + .41839 u_2 + .48080 u_3
 \end{aligned}$$

Weighting School Certificate Mathematics, English and Geography

equally, we have $u_1 : u_2 : u_3 = 1 : 1 : 1$, and from (1)

$$w_1 = .53319, w_2 = .42149, w_3 = 1.27348.$$

Then $u' R_{ab} R_{bb}^{-1} R_{ba} u = \begin{bmatrix} .53319 & .42149 & 1.27348 \end{bmatrix} \begin{bmatrix} .4751 & .1356 & .3318 \\ .2545 & .4008 & .1972 \\ .5303 & .5060 & .5493 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$$= 2.88108$$

Also $u' R_{aa} u = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & .5129 & .7132 \\ .5129 & 1 & .3142 \\ .7132 & .3142 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 6.0806$

Hence the maximum prediction with the school subjects equally

weighted is $\sqrt{\frac{2.88108}{6.0806}} = .6884$

Weighting the school subjects in the ratio 1 : 2 : 1, we have from

equation (1), $w_1 = .53366, w_2 = .68163, w_3 = 1.69187.$

Then $u' R_{aa} R_{bb}^{-1} R_{ba} u = \begin{bmatrix} .53366 & .68163 & 1.69187 \end{bmatrix} \begin{bmatrix} .4751 & .1356 & .3318 \\ .2545 & .4008 & .1972 \\ .5303 & .5060 & .5493 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

$$= 4.96831$$

Also $u' R_{aa} u = \begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & .5129 & .7132 \\ .5129 & 1 & .3142 \\ .7132 & .3142 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = 10.7348$

Hence the maximum prediction with the weight of the English doubled

is $\sqrt{\frac{4.9683}{10.7348}} = .6803$

To find the Hotelling maximum prediction we evaluate :

$$R_{ab} R_{bb}^{-1} R_{ba} = \begin{bmatrix} .35478 & .12675 & .374295 \\ .00047 & .26014 & .41839 \\ .17994 & .03460 & .48080 \end{bmatrix} \begin{bmatrix} .4751 & .1356 & .3318 \\ .2545 & .4008 & .1972 \\ .5303 & .5060 & .5493 \end{bmatrix}$$

$$= \begin{bmatrix} .39930 & .28830 & .34831 \\ .28830 & .31603 & .28128 \\ .34831 & .28128 & .33000 \end{bmatrix}$$

The maximum prediction is then the square root of the largest root of the following cubic equation :-

$$\begin{vmatrix} .39930 - \lambda & .28830 - .5129 \lambda & .34831 - .7132 \lambda \\ .28830 - .5129 \lambda & .31603 - \lambda & .28128 - .3142 \lambda \\ .34831 - .7132 \lambda & .28128 - .3142 \lambda & .33000 - \lambda \end{vmatrix} = 0 \dots (2)$$

The largest positive root of this equation is .4750, and hence the maximum prediction is .6892.

Substituting the value $\lambda = .4750$ in equation (2), and working to four figures, we obtain the determinant :

$$\begin{vmatrix} -.0757 & .0447 & .0095 \\ .0447 & -.1590 & .1321 \\ .0095 & .1321 & -.1450 \end{vmatrix}$$

The ratio of the minors of this determinant will give the ratio of the criteria weights for maximum prediction. Hence, we have

$$u_1 : u_2 : u_3 = 5574 : 7745 : 7430 \approx 3 : 4 : 4 \quad (\text{approx}) \dots (3)$$

Using the ratios in (3), we obtain from equations (1)

$$w_1 : w_2 : w_3 = 3303 : 2978 : 8899 \approx 1 : 1 : 3 \quad (\text{approx}) \dots (4)$$

We now reverse the positions of the criteria and the entrance tests in the calculation, and proceed in a similar way.

Calculation of $R_{ba} R_{aa}^{-1}$.

1	.5129	.7132	-1			1.2261
.5129	1	.3142		-1		.8271
.7132	.3142	1			-1	1.0274
.4751	.1356	.3318				.9425
.2545	.4008	.1972				.8525
.5303	.5060	.5493				1.5856
	.73693	-.0516	.5129	-1		.19823
	1	-.07002	.69600	-1.35700		.26900
	-.0516	.49135	.7132		-1	.15295
	-.10808	-.00704	.4751			.35998
	.27027	.01569	.2545			.54046
	.23401	.17109	.5303			.93540
		.48773	.74911	-.07002	-1	.16683
		1	1.53591	-.14356	-2.05030	.34204
		-.01461	.55032	-.14666		.38905
		.03461	.06640	.36675		.46776
		.18748	.36743	.31754		.87245
			.57276	-.14876	-.02995	.39405
			.01323	.37171	.07097	.45592
			.07949	.34446	.38438	.80833

Hence the school certificate test weights are given by :-

$$\begin{aligned}
 u_1 &= .57276 w_1 + .01323 w_2 + .07949 w_3 \\
 u_2 &= -.14876 w_1 + .37171 w_2 + .34446 w_3 \dots\dots\dots (5) \\
 u_3 &= -.02995 w_1 + .07097 w_2 + .38438 w_3
 \end{aligned}$$

Maximum prediction in the Hotelling sense is found by first evaluating

$$\begin{aligned}
 & \begin{bmatrix} .57276 & .14876 & .02995 \\ .01323 & .37171 & .07097 \\ .07949 & .34446 & .38438 \end{bmatrix} \begin{bmatrix} .4751 & .2545 & .5303 \\ .1356 & .4008 & .5060 \\ .3318 & .1972 & .5493 \end{bmatrix} \\
 & \equiv \begin{bmatrix} .24201 & .08024 & .21201 \\ .08024 & .16635 & .23409 \\ .21201 & .23409 & .42759 \end{bmatrix}
 \end{aligned}$$

The maximum prediction is the square root of the largest root of the equation :-

$$\begin{vmatrix} .24201 - \lambda & .08024 - .0054 \lambda & .21201 - .3196 \lambda \\ .08024 - .0054 \lambda & .16635 - \lambda & .23409 - .3362 \lambda \\ .21201 - .3196 \lambda & .23409 - .3362 \lambda & .42759 - \lambda \end{vmatrix} = 0$$

.475 was found to be a root of this equation. Substituting this value for λ we obtain the following determinant :-

$$\begin{vmatrix} -.23299 & .07757 & .06020 \\ .07757 - .30856 & .074393 & \\ .06020 & .07439 & -.047412 \end{vmatrix}$$

The ratios of the minors of this determinant will give the ratio of the entrance tests for maximum prediction. Thus :-

$$w_1 : w_2 : w_3 \equiv 909959 : 815639 : 2435193 \equiv 1 : 1 : 3 \text{ (approx.)}$$

Using these ratios in equations (5) we find that u_1, u_2, u_3 are in the approximate ratios 3, 4, 4, which is in agreement with equations (3) and (4).

We thus arrive at the same Hotelling maximum prediction with the same weights by either the direct or reverse process.

Appendix C.

Solving the Cubic.

Equation (2) in Appendix B was :-

$$\begin{vmatrix} .3993 - \lambda & .28830 - .5129 \lambda & .34831 - .7132 \lambda \\ .28830 - .5129 \lambda & .31603 - \lambda & .28128 - .3142 \lambda \\ .34831 - .7132 \lambda & .28128 - .3142 \lambda & .33000 - \lambda \end{vmatrix} = 0 \dots (1)$$

This reduces to,

$$-.3594 \lambda^3 + .2363 \lambda^2 - .03279 \lambda + .00077 = 0$$

i.e. $\lambda^3 - .6575 \lambda^2 + .09124 \lambda - .00214 = 0$

Substituting $x + .2192 = \lambda$

we have $x^3 - .05286 x - .00320 = 0 \dots (2)$

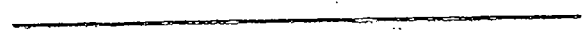
Since $4 \cdot (.05286)^3 > 27 \cdot (.0032)^2$

all the roots are real, and the largest root is given by :-

$$x = \sqrt[3]{\frac{2}{3} \cdot \sqrt{.05286}} \cos \frac{\theta}{3}$$

where $\cos \theta = \frac{3 \sqrt[3]{.0032}}{2 \cdot (.05286)^{3/2}}$

From which we get the root of equation (2) is .2558, and hence the largest root of equation (1) is .4750.



12. Whittaker Calculus of Observations.
13. Lindquist Statistical Analysis in Educational Research.
14. Aitken Determinants and Matrices.
15. Davies and Jones The Selection of Children for Secondary Education.
16. Brereton The Case for Examinations.