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U N I V E R S I T Y O F D U R H A M

A CRITICAL APPRAISAL OF PLATE GIRDER DESIGN

BY

OSCAR BATACHARIA

A Thesis submitted for the Degree of
Master Of Science.

ENGINEERING SCIENCE DEPARTMENT,

OCTOBER 1977.



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SYNOPSIS

This thesis describes the design of the Alexandra-Langton swing bridge, at Liverpool, and examines the design in the light of the most recently published work on this field. The text includes a brief historical background of the development of plate girder bridges. A critical study has been made to understand the evolution of the plate girder design formulas, recommended in B.S. 153.

This thesis also covers the most recently published work on limit state theory. Simple analytical methods are used to predict the plastic moments and the inelastic behaviour of the plate girder and the orthotropic steel-deck unit.

A comparison has been made to examine the savings of steel if limit state method is applied instead of elastic method.

Symbols Used in Elastic analysis

A	area.
A_0	actual width of closed rib.
A_e	effective width of deck plate at top rib.
a	greater clear dimension of the web in a panel.
B	width of flange plate.
b	lesser dimension of the web in a panel.
C	Warping Constant.
C_s	critical stress.
D	Overall depth of Plate Girder.
d	clear depth of web plate.
E	Young's modulus.
E_0	Actual rib spacing.
E_e	Effective rib spacing.
f	stress.
f_s	average shear stress.
f_{sc}	critical shear stress.
f_x	longitudinal stress in web.
f_y	yield stress.
G	modulus of rigidity.
h	distance between flange centroid.
I	2 nd moment of area.
K	appropriate torsional constant.
L	span length.
M	bending moment.
P	applied force.
p_q	permissible shear stress.
R	Radius of curvature.
r	Radius of gyration.

- λ_r Slenderness ratio.
 S Floor beam spacing.
 S_e effective span of floor beam.
 S_f Shear force.
 T thickness of flange plate.
 t thickness of web plate.
 u coefficient.
 y vertical distance from the neutral axis to the point under consideration.
 Z modulus of a section = $\frac{I}{y}$.
 δ deflection.

Symbols Used in Limit State analysis

d	clear depth of the web plate
t	thickness of the web plate
M	bending moment
B	Width of flange plate
T	thickness of flange plate
D	Overall depth of the plate girder
A_f	area of single flange plate
A_w	area of the web plate
r'	A_f/A_w ratio
σ	normal stress
σ_{cr}	critical stress
σ_f	flange stress
σ_y	yield stress
σ_{yw}	yield stress - web
σ_p	Plastic bending stress
σ_b	bending stress
σ_r	residual stress
$\tau_{cr}(V)$	St. Venant torsion
$\tau_{cr}(W)$	Warping torsion
ϵ_y	yield strain = σ_y/E
ϵ_f	flange strain
ν	Poisson's ratio = 0.3.
l	buckling length
I	2 nd moment of area
K	torsional constant
E	modulus of Elasticity
Γ	warping constant
G	shear modulus
r	radius of gyration
Z	section modulus

λ	$d/t \cdot \sqrt{\frac{12(1-\nu^2) \cdot \sigma_{yw}}{\sqrt{3} \pi^2 E \cdot K}}$
λ'	standard slenderness ratio
β	d/t ratio
V	resultant shear force
V_u	ultimate shear force
V_p	plastic shear force
a	transverse stiffener spacing
L	a/d ratio
τ_y	yield shear stress
τ	applied shear stress
τ_{cr}	critical shear stress
τ_u	ultimate shear stress
τ_a	average shear stress
τ_{al}	allowable shear stress
c	position of plastic hinge - see fig. 6.2.11.
s	width of yield band - see fig. 6.2.8(c).
F_s	shear force in stiffeners
A_s	area of a pair of stiffeners - see fig. 6.2.10(a)
A_s'	area of a single stiffener - see fig. 6.2.10(b)
τ_{yw}	tensile yield shear stress of the web plate
Z_{pf}	plastic modulus of flange plate
N	load factor

1.

INTRODUCTION

1.1. Scope:

The aim of this research project is to analyse the structural mechanism of a plate girder swing bridge, which has been built and at present is in operation at Alexandra-Langton dock at Liverpool. It is also the intention to analyse the super-structure of the bridge on the basis of Elastic (in accordance with B.S.153) as well as ultimate load theory, and compare the results.

1.2. Aesthetics:

Aesthetics may influence the design of a bridge and this includes character and scale of natural and artificial surroundings. A bridge should merge or be complementary to its natural surroundings as much as possible. These considerations affect the line, level, shape, size and colour of the proposed bridge. Dominance of artificial surroundings by structure is often desirable and may have the effect of giving unity to an otherwise haphazard pattern.

1.3. Historical Background:

It is very difficult to find the origin of the construction of bridges. Probably, man invented the first bridge when he started to move from one place to another, and nature might be the first one to provide a bridge in form of a tree fallen across a stream. Probably the Egyptians were the first bridge engineers, during the period of their reign, and concurrently Mesopotamians took over. During those days men learned how to build bridges of stone, then wood, because bridges were necessary to conquer wars and grow vines etc. The actual birth of the science began during the Greek supremacy. Although the Egyptians had developed some mathematics, Greek scholars such as Pythagoras, Aristotle and Archimedes gave impetus

to the knowledge of Science. Later, this leadership in knowledge of Science was passed on to the Great Romans. Romans were more nearly the engineers, rather than Scientists. Many of their superb ancient stone arches which were built with dressed stones without mortar, centuries ago are still in existence. The military engineers accompanying the conquering Roman armies were adept at pile bent bridges and timber arches. Caesar's bridge over the Rhine, which was destroyed during the world war II, was the unique example of that period.

During the "Medieval" period, (approx. 600 AD) the Arabic System of numbers was developed and during that time the St. Benezet Bridge at France, & the old London Bridge were built.

The period of 1450 to 1850 can be thought of as the beginning of the Scientific focus upon the unanswered questions of nature. During this time the great men such as Da Vinci, Palladio, Galileo, Hooke, Newton, Bernoulli, Euler, Coulomb and many more, contributed plenty towards the Scientific developments of today's world.

In 1776 Coulomb published the first correct analysis for the fibre stresses in a beam of a rectangular cross section subjected to a bending moment. Coulomb was soon followed by others, such as Navier, Claperyron, Saint Venant etc.

Maxwell developed his method for Statically indeterminate structures in the mid-nineteenth century. During the same period Mohr and Castigliano, developed a new approach to the statically indeterminate structures. This is important to note that, until the analysis of statically indeterminate structures was known, it was not at all possible to analyse the behaviour of a continuous beam system. The famous German engineer Muller-Breslan has further developed the previous work of Maxwell and Castigliano.

In 1840, the first all-iron bridge was built. The material used for this bridge was Cast Iron and Wrought Iron and was built across the Erie Canal at Frankfort, New York. It was a highway girder bridge with a span of 77 feet. Up to the late 1850's, metal bridges were built with cast iron compression members and wrought iron tension members. The manufacturing of steel was invented in 1856, but it was not until 1869, the first all steel bridge was erected across the Mississippi River at St. Louis. By this time, beams and girders as bridge members came into wide use. During the early part of the 20th Century, rolled beams were standardized, which gave some indications of the availability of steel sections. In the late 30's, wide flange shapes became easily available, so, the highway stringer bridges were erected with simply supported, wide flange beams on spans up to about 110 ft. Riveted plate girders were also used during this period for the through-bridges, span up to about 150 ft.

During 1950s engineers were more equipped to build bridges up to 300 ft span, by taking advantage of welding techniques and composite construction. In the 1960's spans up to three times as long, became economically feasible with the use of High-Strength Steel and box girders or Orthotropic-plate Construction or Stayed girders.

The ever increasing cost of material and labour since World War II, made the welded plate girder construction more competitive. The added advantages of this construction are clean appearance, easy to erect and easier maintenance. A simple I- Section is the ideal for welded constructions. The material can be disposed more effectively than riveted construction, as flange angles are not required, tension flanges need not be increased in area to compensate for rivet holes and more rigid stiffening of the web can be provided with higher sections of the stiffeners. Plate girders with parallel flanges of constant size and unstiffened web of uniform thickness, are the cheapest in cost per ton. But as

the span increases, the depth of the plate girder increases, therefore the web has to be stiffened by vertical and / or horizontal ribs to prevent buckling.

1.4. Tender:

Tenders were invited at the end of 1972 by Mersey Docks and Harbour Board, Liverpool for the replacement of an old Pratt-Truss type bridge, which was constructed sometime during the year 1903-04, across a canal stemming from the river Mersey. The purpose of this bridge was to provide access for the transporter to carry goods from the cargo brought by the container ships. Although the bridge was in working condition, some restrictions were made for the modern heavy vehicles, as in those days bridges were not quite so scientifically designed to bear heavy loads.

The Cleveland Bridge Engineering Co. Ltd., of Darlington, was one of the tenderers to submit a Plate Girder bridge to the board and it was accepted. During the tender period, due to lack of time, and just to give an indication to the estimators how the bridge would look, and what might be the total tonnages of steelwork required, no detail calculations were prepared. Then after obtaining the contract, the writer went through every detail of the calculations for the superstructures and submitted his results to the Board for their comments.

1.5. The New Bridge Structure:

The basic structure of the new bridge is quite straightforward. The two main plate girders run East-West across the Canal and these girders are connected by cross girders (@ 11'-6" centres). The deck plate is stiffened with steel troughs. Walkways are provided at either side of the bridge, which are cantilevered out from the main girders.

When the bridge is open to traffic it is supported by two nose bearings, two intermediate bearings and two slip

blocks at the tail. One of the nose bearings is designed in such a way, so that it prevents the bridge from turning. Both of the slip blocks can be hydraulically retracted. In this position the structure is lifted clear of the centre pivot. When it is desired to open the lane to shipping, a pair of hydraulic tail jacks lift sufficiently so that the slip blocks can be withdrawn. The jacks are then lowered to allow the bridge to rest on the centre pivot. The tail end is weighted so that the nose of the structure lifts clear of the nose bearings while the tail end drops until it is resting upon the two trail rollers. The bridge is then ready to be swung clear by means of two wire ropes, one end of each being anchored at the slewing drum. The ropes are led around guide sheaves and anchored to adjustable links which are attached to the slewing cylinder bedplate.

When the bridge approaches the fully open or fully closed position, Cam operated hydraulic valves, which are fitted to the slewing mechanism, ensure that the bridge decelerates to a creep speed. When it is swung back to its original position, the bridge is again lifted by the hydraulic tail jacks, so that the tail slip blocks can be returned. The jacks are then lowered to allow the bridge to rest upon the nose bearings, the intermediate bearings and the slip blocks, so that it is again held clear of the central pivot. All the operating equipment used was designed, supplied and fitted by Mactaggart Scott Co. Ltd., of Midlothian, Scotland.

1.6. The Design Requirements:

The Mersey Docks & Harbour Board required the bridge to be designed as per B.S.153, in imperial units, and the requirement included the following features:-

1. All steel to be B.S.4360 grade 43A (i.e. mild steel).
2. Overall length 158' - 9", width 25' - 0" with 20' - 0" Carriageway and 2' - 6" Kerb at each side of Carriageway.

3. 3' - 6" wide walkway at each side running throughout the length of the bridge.

Thus the total weight calculated after the final design was 193.38 tons. The site connections were made of high strength friction grip bolts as per B.S.3139 and all welding to B.S.1719 class 2 Lincoln multiweld. Electrodes for manual metal arc butt welds were to B.S.1719 class 6 ESAB/OK unitrode. Electrodes and flux for submerged arc butt welding were to 3.25 mm. diameter Lincoln L60 wire armco F80 flux.

According to Mersey Docks Harbour Board's requirements, before fabrication all steel had been automatically grit blasted with chilled iron to 2nd. quality B.S.4232, maximum profile 100 microns. All laminations or surface defects exposed by blasting had been chipped off and ground level before priming. All loose shop rust and dust had been removed by wire brushing and vacuum cleaning the surface. Within 4 hours of blasting, one coat protection "EPILUX 66P" blast primer first coat pink 4E650 had been applied by airless spray to 25 microns equal to a spreading rate of 6 sq m/litre and allowed to harden over night. After priming, one coat protection high built "EPOXY MICACEOUS" dark grey HB58 had been applied to 75 microns equal to a spreading rate of 6 sq m/litre and allowed to harden overnight. All areas local to H.S.F.G. butt connections had been left free from paint, oil, drift, grease, rust etc.

1.7. Operating System:

The road barriers and traffic signal installation comprises four (in number) road traffic barriers, each with folding skirt and four sets of pole mounted visual and audible warnings. The latter consists of twin red flashing lamps, an amber lamp and gong. Control of all barriers and warning systems is available at the desk in the control house, and individual sides operation is available from a console forming

an integral part of the South East and South West barrier's pedestals. All this equipment was supplied by Messrs. Godwin Warren Engineering Ltd.

Navigation signals are two sets of pole mounted navigation lights provided on the roof of the control house. Each set consists of a red and a green lamp, one set facing North and the other facing Southwards. Control of the lamps is from the control desk, interlocks ensuring that the bridge must be fully open, to allow either the North or the South bound passage open green lamps to be operated. Switching arrangements prevent both North and Southbound passage open lamps from operating simultaneously.

The control desk is situated on the upper floor of the control house and includes all controls and indicating lamps associated with road barriers, bridge and navigation lights operation. Road barrier's controls are supplied by a 240 volts 50 HZ line/neutral supply from the barriers relay panel and comprises:-

- 1 No. Red control available lamp.
- 1 No. Green East Barrier's down lamp.
- 1 No. Green West Barrier's down lamp.
- 1 No. Stop push button.
- 1 No. East Barrier's "UP" push button.
- 1 No. East Barrier's "DOWN" push button.
- 1 No. West Barrier's "UP" push button.
- 1 No. West Barrier's "DOWN" push button.
- 2 No. Local/Remote key operated selector switches.

Normal operation from the desk is achieved with "Local" selected. If "Remote" operation from the barriers is required, both the key switches should be turned to the "Remote" position, the keys removed and these then used to operate the barriers control units On/Off switches, and both switches must be either "Local" or "Remote" for control purposes. With "Local" selected, the key cannot be removed.

A 240/24 v, 150 VA supply with primary and secondary fuses is provided for navigation lights operation. Relays and indicating lamps associated with navigation lights are operated from the primary supply voltage. Bridge control relays etc. are operated at 240 volts 50 HE, power being derived from the hydraulic power pack motor starter.

An On/Off switch is also fitted to the desk which when off, isolates all controls with power on and all road barriers down, operation of the "Open" push button indicates the pump motor starter of the hydraulic power pack, provided the starter is set for "Remote" Control. When the pump is running, a signal is provided at the desk to energise a relay, which in turn causes solenoid valves to initiate the bridge open sequence. Bridge movements and blocks and rams interlocking is purely hydraulic with pressure switches introduced at various positions, to provide indication and a measure of electrical interlocking. The rams at the tail of the bridge extended to the fully raised position and when both reach this position, the pressure switch operates to illuminate the "Rams Up" lamp. Hydraulic interlocking then causes the blocks to retract and when both withdraw, the pressure switch closes to indicate "Block Out". The rams then lower to operate and slewing commences.

At this time the "Bridge moving" lamp illuminates via contacts on the bridge open and bridge closed relays, both of which are de-energised since the bridge is neither open or closed.

As the bridge approaches the open position, a limit switch operates and at the fully open position another switch operates, to energise the bridge open relay to stop the pumps, de-energise the solenoid valves and thus stop bridge movements. Simultaneously, the "Bridge open" lamp illuminates, and the "Moving" lamp is extinguished.

The bridge "Close" sequence is initiated by depressing the "Close" push button to start the power pack which when running energises two solenoid valves, slewing commences and "Bridge moving" lamp operates. At the "Closed position", one pressure switch operates and the "Bridge closed" lamp illuminates, simultaneously the "Moving" lamp is extinguished. Automatic Rams and Blocks operation takes place and with "Rams Down" and Blocks In", the power pack is stopped. In the situation with "Bridge Closed", Blocks In" and "Rams Down", all three relays energise and allow road barriers to be raised. Operation of the bridge stop push button causes immediate de-energising of solenoid valves, dependant on whether "Open" or "Closed" has been selected. However, should the emergency stop be depressed, the solenoids and pumps are stopped immediately. This push button has a latch feature and remains locked in the depressed condition until reset.

All the foregoing pressure switches are sited on the hydraulic control unit. Also mounted on the control unit are interlock switches which ensure that when a lever is inserted to provide manual operation of any control valve, the solenoid operated valves which provide the automatic sequence are to be de-energised. The pump, however, continues to run.

Maintenance is required to all mechanical items of the operational mechanism, viz:-

1. Apply grease to balance roller, tail roller, top guide sheave assembly and slewing gear cross head assembly.
2. Keep the wire ropes covered with grease.
3. Examine all moving parts regularly for formation of corrosion and rust.
4. Keep the rope adjusting screw well covered with grease.
5. Keep the rollers of the roller actuated sequence valves well covered with grease.
6. Ensure that no water has been collected on the part of the centre pivot.

2.1. The Report of the Bridge Stress Committee 1928:

The report of the bridge stress committee 1928 was based on strain measurements on members of 52 railroad bridges of various spans, under the influence of moving loads, such as trains and locomotives of different types, and due to the imposition of a bridge oscillator to give pulsations for purposes of comparison. During this period Prof. C.E.Inglis also carried out the theoretical investigations on bridge oscillations and established a Vector method for computing deflections and stresses in a bridge subjected to vertical alternating forces.

This theory allows a reliable estimate for the greatest impact on bridges, due to locomotives. This report states that the main reason for the dynamic effect of a moving load on a railway bridge being greater than the static effect of the same load, is the hammer blow. The effect of the hammer blow is intensified in all bridges except the short span bridges by resonance arising from synchronism between the natural frequency of vibration of the loaded bridge and the frequency of the application of the blow.

In a long bridge, the recurring impulses due to hammer blow may cause a larger cumulative effect, especially when there is close proximity between their period and free period of vibration of the structure. The successive period of impulses are limited by the length of span. The damping of the oscillations are set up by the imperfect elasticity of the structure, i.e. ballast of the permanent way etc., friction in the support at the piers, dissipation of the energy by its transmission through the piers to the neighbouring ground and the friction in the spring suspension of the locomotive. For practical design purposes the whole

allowances for impact is set in the form of an equivalent uniformly distributed load, which should be added to the weight arising from the live load to be carried.

The equivalent load is based on the principle of simple harmonic motion in the periodic oscillations and such an imaginary static load of sinusoidal distribution is imposed that will hold the bridge in its maximum oscillatory form of curvature. Then a uniformly distributed load is considered which is equivalent to the sinusoidal load to give the same maximum bending moment at the centre of the span. This bending moment is slightly higher at $\frac{1}{4}$ points of the span but with approximately equal shear force. From the recommendation of this report, research work was carried out and finally B.S.153 was revised.

2.2. M.O.T. LOADING CONDITIONS:

The standard loading issued by the Ministry of Transport in 1922 suggested that the bridge should be loaded with such standard trains or part of standard trains so that it would produce the maximum stress in any bridge member, provided that in any line of trains, there should not be more than one engine per 75ft. of the span of the bridge and each standard train shall occupy a width of 10ft. Where the width of the carriageway exceeds a multiple of 10ft., such excess should be loaded with a fraction of the axle loads of a standard train.

2.3. British Standard 153:

B.S.153 is the specification for steel girder bridges and is available in the following parts:-

- Part 1 deals with material.
- Part 2 deals with workmanship.
- Part 3A deals with loading.

Part 3b & 4 deals with stresses and details of Construction.
Part 5 deals with Erection.

All these parts were first published in 1922 and 1923. Part 3a shows the detail of 15 units of standard loading which was recommended for highway bridges in this country. This was agreed in principle with the standard loading train issued by the Ministry of Transport in 1922 and was supplemented by the M.O.T. standard loading curves in 1932.

Part 3b was revised in 1933 where as part 4 and 5 were ammended in 1937. For various reasons these rules remained unchanged until 1958, although in 1948, a code of practice for simply supported steel bridges was jointly issued by I.C.E. and I struct.E.

The 1972 version of B.S.153 is only the metricated revision including some amendments, but again, it covers simply supported steel girder bridges, of up to 100m span. To design over 100m span or continous bridges, extrapolation from B.S.153 is used. In the 1960's before the Milford Haven bridge failure, a new all-embracing bridge code was conceived. Later the Merrison design rules acted as an interim code.

A new code will be written shortly, with major changes in design practice. It will be published in 10 parts to make future revisions easier for different materials used. An ultimate load approach rather than present elastic method will be used in this new code. The view is widely expressed that Merrison rules are too complex to follow. It is felt that Merrison's change to ultimate load design is academic and engineers may loose track of their terminology during application. Particularly, the Merrison rules do not mention whether the loads are working loads or ultimate loads. The design methods in the new code are expected to be simple. The welding stresses in the new code are also expected to be better understood.

However, in this project, the present version of B.S.153 has been utilized. B.S.153 part 3A has been divided into two types of loading systems:-

- a) H.A. type loading, covers majority of the requirements and consists of:-
 - i) An equivalent uniformly distributed load known as EUDL, based on a particular train of road vehicles. The distributive effect varies according to the span. Short spans have to support heavy axle loads without any sizeable distributive effect which occurs in longer spans, and the EUDL for shorter spans is higher than for longer spans.
 - ii) A knife edge load, which is known as KEL in short, is also applied and represents the excess load of a heavy axle in a vehicle which is not capable of being evenly distributed as an EUDL.

EUDL and KEL are used together, to give the adverse loading condition in the member, which is to be designed. In the case of a simply supported member the KEL should be placed at mid-point, to give the required condition for bending moment, but must be put at the support for the maximum shear condition. If the bending moment is required at the quarter point of a span, then KEL should be placed at that point. The critical position for placing the KEL is easily determined from the influence line diagrams as being the point of maximum ordinate. B.S.153 part 3A, Table 1 gives the relevant values for EUDL and KEL. (Refer appendix A, loading condition C)

- b) H B type loading - This loading represents the unit loading from an abnormal vehicle. It is usual practice to specify that the bridge must be checked after a preliminary design to HA loading, to ensure that the bridge is capable of supporting the specified number

of units of abnormal loading. M.O.T. memorandum No.771 specifies the number of units of HB loading applicable to various classes of roads, eg. class I and II roads should be checked for $37\frac{1}{2}$ units of HB loading, (refer to Appendix A, loading condition E)

2.4. Wind Loading:

Natural wind is never even, steady or uniform. The variations are due to the presence of eddies and velocity gradients. The effect of wind pressure on a structure depends on the shape and size of the structure. The amount of wind pressure depends on the site, where the structure is to be placed. Horne⁽³⁾ stated that due to the varying nature of the eddies in natural wind, it is very difficult to assess the action of the wind. So, it is not very safe to take a velocity for the whole structure which may be significantly less than the maximum velocity. The nature of a wind varies greatly with the height, i.e. the mean velocity of wind increases with height (see fig. 2.3.1).

The vertical gradient of wind speed depends on the mean wind speed and on the vertical temperature gradient. The wind speed also depends on the natural obstacles, such as, mountains etc. The duration of the highest wind gusts is of the order of 1-10 seconds and Horne⁽³⁾ suggests that it is very important to assess the behaviour of structures under loads acting for such periods.

The pressure and velocity of wind are connected by the formula $p = kv^2$, where k is a constant; for the pressure on a flat plate placed normal to the wind k is 0.0031. For a wide area of wind flow, a flat plate 1ft.square is relatively small plate. When the free wind stream is horizontal, then the only energy it possesses is Kinetic energy equal to $\frac{1}{2} \rho V^2$. Insertion of the plate into the wind brings one particular stream line to rest and according to Bernoulli's

theorem (in a fluid with steady stream line flow, the sum of the Kinetic energy, the pressure and the potential energy, is a constant); the pressure on the plate must equal the Kinetic energy in the unimpeded stream before it was brought to rest.

If w = weight of air in lb. per c.ft. = 0.0765

v = Velocity of wind in miles per hour

V = Velocity of wind in ft. per sec. = $v \times 5280 \div (60 \times 60) \left\{ \frac{22}{15} \right\} v$.

g = 32.2 ft per sec²

$$\text{then } p = \frac{1}{2} w \text{ lb/ft}^3 \times (V \text{ ft/sec})^2 \times \frac{\text{sec}^2}{g \text{ ft}} = \frac{1}{2} \frac{wV^2}{g} \text{ lb/ft}^2$$

$$\text{or } p = \frac{1}{2} \left[\left(\frac{0.0765}{32.2} \right) \left\{ \left(\frac{22}{15} \right) v \right\}^2 \right] = 0.00511 v^2 \left(\frac{1}{2} \right)^* \text{ or, } p = 0.00256 v^2$$

In this formula no allowance was made for viscosity and if it was assumed that all the air in the stream line impinged upon the plate and was duly brought to rest and also all this occurred without interference with the surrounding air stream lines. Obviously, later, by experiments⁽³⁾, it was found that the theoretical coefficient ($\frac{1}{2}$)* in the above equation, should be replaced by the experimental coefficient 0.6 which is a more correct value of the pressure.

$$\text{Hence } p = 0.6 w v^2 \div g = 0.0031 v^2$$

According to B.S. 449 (use of structural steel in building)

$$p = \frac{v^2}{600} \sqrt{1 + 0.06(h-s)}$$

Where p = uniform wind pressure in lb. per sq.ft.

v = velocity of wind.

h = height in ft. above ground level.

s = height in ft. above ground level of the area assumed to be sheltered by some obstacle and s shall never exceed $\frac{1}{2}h$ in the formula, while $(h-s)$ shall not be less than 10 ft.

Previously, it was the custom to consider the wind as exerting pressure only on surfaces directly fronting the wind.

If a surface was inclined to the path of the wind, then the normal pressure on the inclined surface (p_n) was assumed to be yp where y was less than unity and this was given by Duchemin & Hutton formulas. No allowance was made for suction or negative pressure on the leeward side of a structure.

B.S.153 clause 12 in part 3A deals with wind pressure effects. It states that, wind load should be treated as a moving load, acting at the centroids of the exposed area. For maximum lateral effect on unloaded structures, the wind pressure will be taken as 30 lbf/ft^2 , which corresponds to a wind speed of 90 miles per hour, and it should be considered, as though, acting horizontally and normal to the sides of the bridge on a total exposed area (windward girder). At the leeward girder, when it is a plate girder, $n/16$ fraction (should not exceed unity) of the net exposed area in normal projected elevation, should be taken into account, where n is the ratio of the distance, centre to centre between the windward and outermost leeward girder, to the depth of the windward girder. On loaded structures, allowance shall be made for screening effect, based on the projected areas of the structure on the live load or of the live load on the structure, or the live loads on each other. On highway bridges, a wind pressure of 15 lbf/ft^2 corresponding to 63 miles per hour shall be taken as acting horizontally and normal to the sides of the bridge on the exposed area of the super-structure.

The intensity of the wind pressure is not always constant over a large area, for somewhere in the path of the wind, "Pockets" of wind will be found, which travel at a higher velocity than the surrounding wind and therefore, exert a higher pressure. On an average it would appear that the mean velocity is about 75% of the maximum gust velocity. The duration of this gust velocity is quite small, somewhere about one or two seconds. It has become a custom these days

to make use of models of bridges and make wind - tunnel experiments. These experiments definitely give more accurate results for the wind pressure. Mersey Docks & Harbour Board advised that a 20 lbf/ft² of wind pressure to be taken into consideration when designing the bridge, (see appendix A Part 1 para.A4.2). Normally, wind pressure values are taken from B.S.153 and/or CP3, unless a specific value has been given by the clients.

In the following chapters, it is intended to apply the various loading systems, at worst possible combinations, as described in B.S.153 part 3A and calculate the critical stresses. An attempt has also been made to understand the implication of the formulas suggested in B.S.153 to calculate the stresses and their basic principles.

Approx. hourly mean velocities at varying heights in terms of V

$V =$ approx. hourly mean velocity of the wind at a height of 10 metres above the ground.
This chart was published by the Meteorological Office, in 1918



Fig. 2.3.1

3. ANALYSIS OF MAIN GIRDERS: ELASTIC THEORY:

3.1. Introduction

Structural analysis is concerned with the forces acting on the structure and deformation due to these forces.-The main part of the structural analysis is to appreciate the principle of Virtual work and its application. Deformations may arise for various reasons. Applied loads will cause internal forces in a structures and these forces will deform the structure. Whatever, the cause may be, three basic conditions should always be considered, when full analysis is carried out:-

- a. Statical equilibrium; the external loads and internal forces must satisfy all the conditions of equilibrium
- b. Geometrical compatibility of deformation: the deformed members of a structure must continue to fit together, so that the deformations should be geometrically compatible.
- c. The characteristics of the members: within the elastic range of the member, the quantitative relationships between stress and strain must be utilized and these relationships are referred to as the member characteristics.

In short, the deformations are required to be compatible and the internal forces must satisfy the requirements of equilibrium with the loads, and the deformation of each member must be accurately related to the internal forces and other strain producing factors in accordance with the member characteristics.

The following assumptions are the basic requirements to derive the formula for the moment of resistance, which is indeed, the first step towards the design of a structural

member and the theory is called Beam Theory:-

1. The beam section should have one axis symmetrical, in the plane of which bending moment is applied.
2. The bending is assumed to be simple or circular, which produces an equal and opposite couples and shear is absent.
3. Section of beam which is plane before bending remains plane after bending.
4. The stress in any fibre is proportional to strain.
5. Young's modulus (E) is constant.

Therefore, the basic theory is

$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R} \quad \text{which is universally known to all engineers.}$$

From the above expression it may be written as $M = f \frac{I}{y}$.

But by definition $\frac{I}{y}$ = modulus of a section which is Z .
Therefore, $f = \frac{M}{Z}$ which means that to find the maximum stress, f to a member, it is essential to know the bending moment M , then the 2nd moment of area I and the distance (vertical) from the neutral axis to the point under consideration, y .

In this chapter only the elastic theory will be considered. So, in keeping with Hooke's law, the elongation will be doubled, if the force is doubled and the steel will return to its original unstrained form on the withdrawal of the force. If the force be increased to such an extent that the resulting stress exceeds the yield stress, then the member will pass the elastic limit. But, this does not ever happen, if the structure is designed correctly, because, a factor of safety is introduced, which is the ultimate or breaking stress divided by the working stress. Although the working load on a tension member might be increased from three to four times before it collapsed, the actual safety

factor is much less than this; because when the load is increased to about 1.7 times its original value, permanent distortion takes place and the member ceases to have any further useful life. In average the factor of safety is about 2. Two important load values occur when a structure is gradually over-loaded to destruction:-

- i. The load which causes the structure to have no further useful life.
- ii. The load which causes complete collapse.

The load which causes the structure to have no further useful life, is divided by the load the structure normally should carry, gives a number which is called the load factor. The ultimate load divided by the working load is sometimes termed factor of safety. But it is not correct, because this term is definitely reserved for stress ratio and not load ratio. So the correct phrase is load factor. Load factors and factors of safety have a common aim, as both express the margins of safety in the structures and both terms depend upon the economy, since a large load factor and factor of safety means misuse of material.

The ultimate aim of design is to obtain the working stresses in a structural member, due to applied loads. Basically, the working stress is defined to be the applied force or the force induced in the member due to the applied force, divided by the cross-sectional area of the member. Working stress of a member depends on the following factors:-

- a) The variable quality of the member (if not homogeneous material), and the standard of workmanship during fabrication and erection.
- b) Whether the load is gradually or suddenly applied.
- c) Duration and cycle of application of the load at maximum intensity.

- d) Whether the structure is a temporary one or permanent one.
- e) degree of accuracy of calculation of the forces.
- f) Corrosion & future maintenance.
- g) Various possible combinations of loads and the degree of probability of all loading conditions at worse, occurring simultaneously, (i.e. dead load + live load + impact + wind load etc.).

So, to maintain a guard against some uncertainties due to the above points, the factor of safety or the load factor is always used. In the following chapters it will be noticed clearly that, allowances have been made to take precautions against these uncertainties.

3.2. Scope

The main purpose of the stress analysis is to produce the lightest girders compatible with economy and easy maintenance. To obtain this, it is necessary that, both the net tension and compression flanges are working at the maximum permissible stresses and the depth of the girder should be such that the overall weight of the whole girder is minimum.

Once the spans are known, it is possible to calculate the bending moments (B.M.), shear forces (Sf) etc., by using B.S.153 Part 3A (Loads). (See appendix A part 1). To determine the stresses, the following items are to be decided first (See appendix A Part 2):-

- a) Approximate depth of the girder.
- b) Area of the web required.
- c) Should the web be stiffened.
- d) The approximate flange areas.
- e) D/T ratio

Where D = the overall depth of girder (at the point of maximum B.M).

and T = effective thickness of Compression flange.

f) Slenderness ratio; l/r .

Where l = the effective length of the compression member. (See B.S.153 Part 4, clauses 5, 33 & 34).

Where r = the appropriate radius of gyration.

The advantages of a welded plate girder construction are:-

- a) it is economical, largely because of the use of a thin web plate.
- b) There are no complications in the welding process.
- c) It is easy to erect.
- d) maintenance costs are low.
- e) neat appearance.

3.3. Working Stress in flanges:

Roberts and Kerensky⁽⁴⁾ stated that there is no risk of failure by instability of the compression flanges, if the ratio of unsupported length to least radius gyration (l/r_y value) does not exceed 90 for a symmetrical I section. Kerensky, Flint & Brown⁽⁵⁾ conducted a few experiments to prove the above statement, and they also said that depth by web thickness (d/t) and l/r_y values not exceeding 90, develop a plastic moment of almost maximum value. Therefore for these girders the allowable fibre stresses need not be sharply reduced from the maximum basic values given by yield/factor of safety. (A very deep girder will buckle laterally at a much lower stress than the flange of a rolled steel joist of the same width.)

B.S.153 part 3B clause 28a recommends that the tensile and compressive bending stresses, f_{bt} and f_{bc} should not exceed the appropriate basic permissible stresses (F_{bt} and F_{bc}) in table 3 (which is 9.5 ton f/in^2) and where the flanges have equal moment of inertia about the y-y axis, critical

$$\text{stress, } C_s = \frac{170000}{(1/r_y)^2} \sqrt{\left[1 + \frac{1}{20} \left(\frac{1/r_y}{D}\right)^2\right]} \text{ tonf/in}^2$$

Timoshenko⁽⁶⁾ derived the critical values of equal terminal couples, applied to a member in the plane of its maximum bending stiffness and sufficient to cause overall lateral buckling. He assumed that the beam was supported at its ends and no restraint is afforded to bending actions, but with rotation of the end sections about its longitudinal axis rigidly prevented. He expressed critical stress, which is the maximum fibre stress at the instant of buckling as,

$$C_s = \frac{\pi}{Z_x L} \left[\frac{E I_y G K}{r} \left(1 + \frac{\pi^2 C}{G K L^2}\right) \right] \text{ ----- 1.}$$

- where
- Z_x = modulus of section about x - axis
 - L = span
 - E = Youngs modulus.
 - I_y = 2nd moment of area of the whole section about y-axis
 - G = modulus of rigidty.
 - K = appropriate torsion constant
 - $r = \frac{I_x - I_y}{I_x}$
 - I_x = 2nd moment of area of the whole section about x-axis.
 - $C = \text{warping constant} = \frac{E I_f h^2}{2}$ for I sections.
 - I_f = 2nd moment of area of the compression flange only about y-y axis of the girder.
 - h = distance between flange centroids.

For symmetrical I section (symmetrical about x - axis, initially undeformed along its length and of homogeneous material), equation 1 reduces to

$$C_s = \frac{\pi^2 E I_y h}{2 Z_x L^2} \sqrt{\left[\frac{L}{r} \left\{ 1 + \frac{4 G K L^2}{\pi^2 E I_y h^2} \right\} \right]} \text{ ----- 2}$$

The second term inside the root represents the contribution

of the torsional rigidity of the member. This is a governing factor for shallow beams, but for deep plate girders, this is negligible, because the differential flange bending provides the major resistance to torque.

The recommended formula for critical stress in B.S.153 part 3B clause 28 b (ii)A, has been derived by the introduction of certain approximate geometric properties. These properties evaluated for symmetrical sections, provide a lower limit to the critical stress. The properties are listed below:-

$$I_x = 1.1 \frac{BTD^2}{2}$$

Where B = flange breadth
 T = effective thickness of flange
 D = overall depth of the section

$$I_y = \frac{B^3T}{6}$$

$$I_f = \frac{B^3T}{12}$$

$$K = 0.9BT^3$$

$$r = 1.0$$

$$h = D$$

$$B = 4.2 \times \text{radius of gyration about } y\text{-axis (i.e., } r_y)$$

$$E = 2.5G = 13\,000 \text{ tonf/in}^2$$

Substitution the above values to equation (2),

$$C_s = \frac{170\,000}{(\frac{1}{r_y})^2} \sqrt{\left[1 + \frac{1}{20} \left(\frac{1}{r_y} \cdot \frac{T}{D}\right)^2\right]} \text{ ----- 3.}$$

and this equation (3) has been used in calculating the critical stress (See appendix A Part II para.5.3.C.)

3.4. Working Shear Stress in Solid Web Plate:

B.S.153 Part 3B clause 29a recommends that the calculated average shear stress, f_q , on the effective sectional area of the web should not exceed the given value in table 3, which is 6.0 tonf/in² for steel grade 43A (mild steel) or the permissible shear stress p_q should be calculated as:-

$$p_q = 6 \left[1.3 - \frac{b/t}{250 \left\{ 1 + \frac{1}{2} (b/a)^2 \right\}} \right] \text{ tonf/in}^2.$$

where a = the greater clear dimension of the web in a panel, not greater than 270 x thickness of the web, t .
where b = the lesser dimension of the web in a panel, not greater than 180 t .

Among the above two values, whichever is less should be used and f_q must not exceed the lesser p_q value (see appendix A part II para A5.3 C (ii)).

This formula has been based on the theory of buckling of thin plate as described by Timoshenko⁽⁶⁾. According to Kerensky and others⁽⁵⁾, an absolutely flat plate, which is subjected to shearing forces, will remain flat until the critical stress has reached. Once it has reached the critical stress, then it will buckle out of its original plane. Initially, this buckle will be very small, but it will be more visible, if the load is increased. To demonstrate the above fact, it may be shown that the web plate acts as a bar which has both ends built in. These are reactive moments that prevent the ends from rotating during buckling. These end moments and the axial compressive forces are equivalent to forces P applied, as shown in the figure. Inflection points are located where the line of action of P intersects the deflection curve, because at these points the bending moments are Zero. It may be noted that in this particular case, the inflection

points, and the mid-point of the span divide the bar into four equal regions (see fig 3.4)

The figure 1 indicates Kerensky's theoretical and experimental maximum deflections for square simply supported plates. In his figure curve A shows the values for the absolutely flat plates and curve B give the values for the plates which were initially buckled. From curve A it may be observed that the maximum web deflection at a load, which is twice the critical value, is approximately equal to the plate thickness.

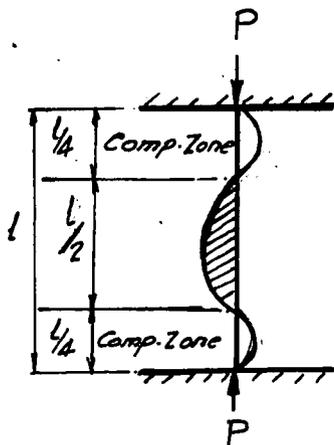


Fig 3.4

When load is applied, the web plate buckles; but before the plate buckles, the stresses in it are due to the shear only and after the plate has buckled, bending stresses are also introduced, and in addition, the middle plane of the plate is deformed and extended, introducing membrane tensile stresses, which react on the compressive zone, minimizing their influence.

This introduction of the membrane tensile stresses, is the difference between the buckling of the web plates in girders and buckling of the columns. Thus, according to Kerensky, Euler load for thin sheets $P = \frac{4\pi^2 EI}{l^2}$ is misleading, when it is applied to calculate the critical loads on web plates. The intensity of the bending stresses is greater near the centre of the plate where buckles are larger and the shear stress is almost uniform.

According to the Huber-Von-Mises-Hencky theory, if these stresses are combined together, then the maximum of

$$\frac{\sqrt{(f_x^2 + f_y^2 - f_x f_y + 3f_s^2)}}{\sqrt{3}f_s}$$

in relation to

Bergman's f_s/f_{sc} ratio, where
$$\frac{\sqrt{(f_x^2 + f_y^2 - f_x f_y + 3f_s^2)}}{\sqrt{3}f_s}$$

represents the equivalent axial stress or $\sqrt{3}$ x apparent shear stress. (see fig 2).

(f_x = longitudinal stress in web; f_y = yield stress,)
 (f_s = average shear stress; f_{sc} = critical shear stress)

Using the value of the curve in fig. 2, an approximation of the critical stress given by Timoshenko is

$$f_{sc} = \left(256 \div \frac{b}{t}\right)^2 \left\{1 + \frac{3}{4} \left(\frac{b}{a}\right)^2\right\}.$$

Figs. 3 & 4 are the original work of Timoshenko. Kerensky and others⁽⁵⁾ developed Timoshenko's idea, by substituting different d/t and $\frac{b}{d}$ ratios.

According to Timoshenko, critical stress, within the elastic limit, $f_{sc} = \frac{\pi^2 D}{d^2 t} \left(\frac{b}{d} + \frac{d}{b}\right)^2$

where

D = flexural stiffness of the plate. For a square plate

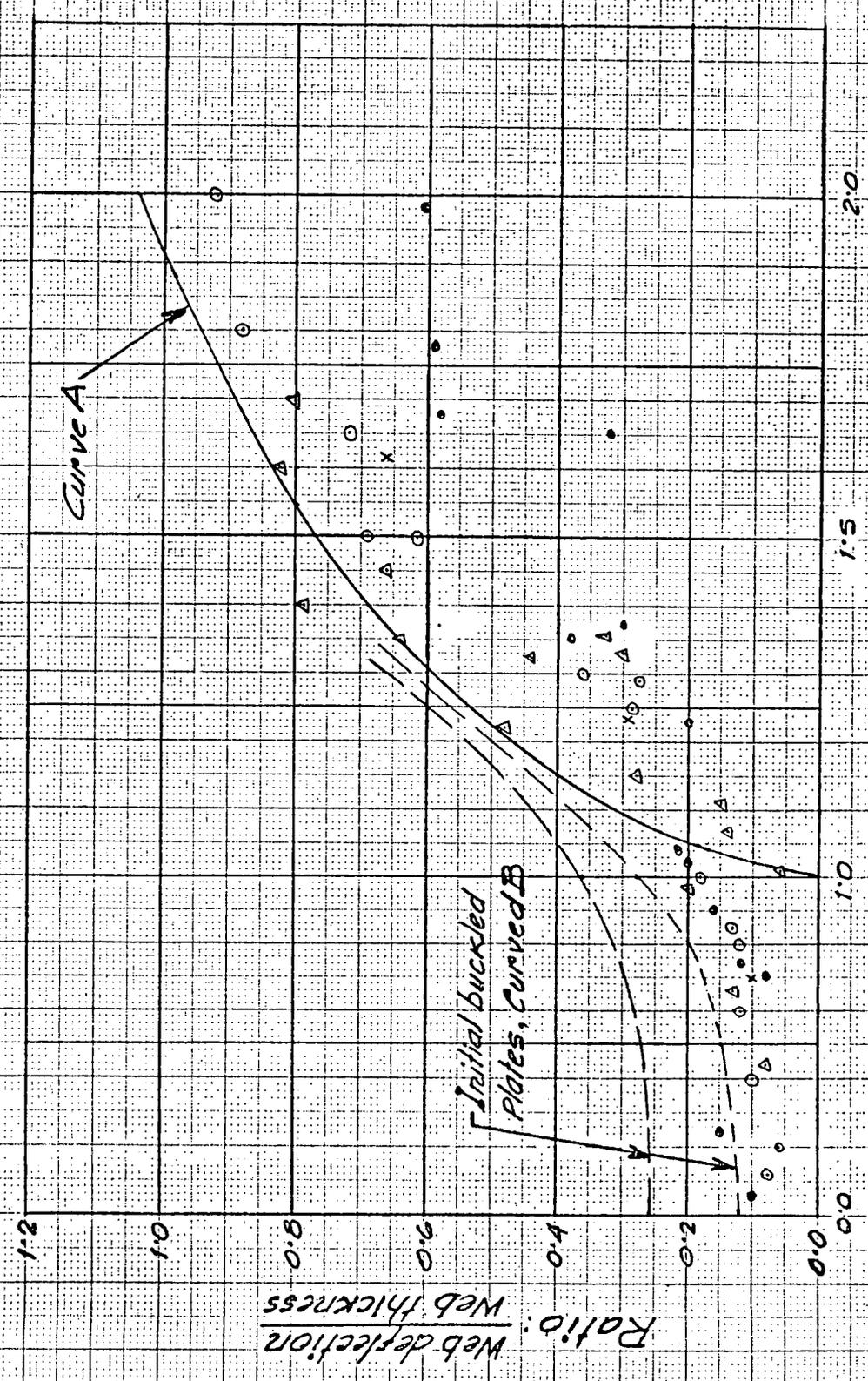
$$f_{sc} = \frac{4\pi^2 D}{d^2 t} = \frac{\pi^2 E}{3(1-\nu^2)} \frac{t^2}{d^2}, \quad \text{where } \nu = \text{Poissons ratio,}$$

and this value can be used also for long rectangular plate, buckling into many waves. When $E = 30 \times 10^6 \text{ lbf/in}^2$ and $\nu = 0.3$, according to Timoshenko, f_{sc} is a function of the ratio $\frac{d}{t}$ and this is represented by the curves of the figs. 3 and 4. These curves can be used for obtaining f_{sc} values within the elastic region. If the material has a

sharply defined yield point and follows Hooke's law up to that point, where the compressive stress reaches the yield point of the material then the horizontal lines, together with the curves, determine the values of the f_{sc} for any value of the ratio d/t .

Now, from Kerensky and others⁽⁵⁾ modified figures (figs 3 & 4), Curves A give shear stresses, which for all values of d/t up to 240, exceed the critical stress by an amount which will cause an increase in apparent shear stress up to yield value and this yield will not occur over the whole plate, but only at the crest of the buckle. The equation for the permissible shear stress, given in B.S.153 part 3B clause 29a, has been obtained by dividing curves A by the required load factor of 1.45 and these approximate to the straightline relationship.(fig 4).

Behaviour of initially buckled plates (Curve B) compared to that of initially flat plates (Curve A).



- Key
- Wastlund & Bergman
 - Sparks & Brown
 - △ Moore
 - × Seydel

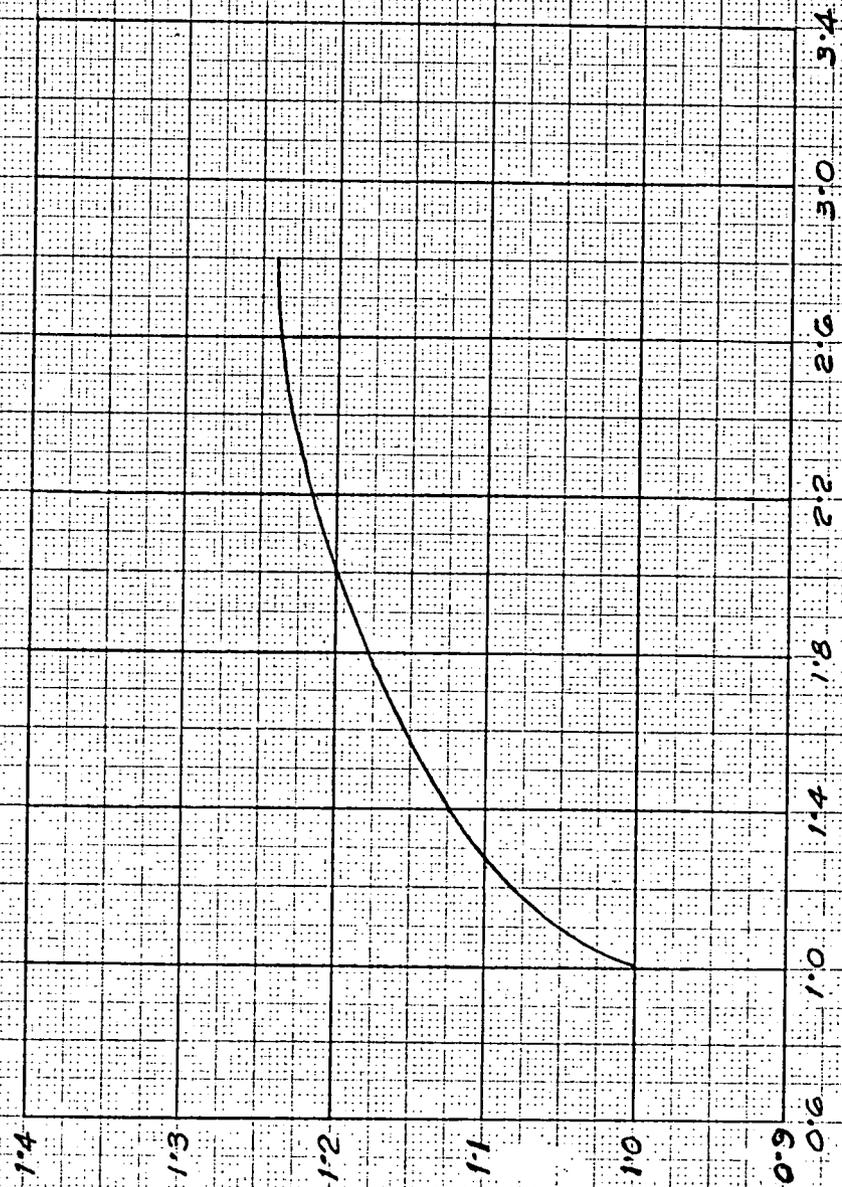
Ratio: $\frac{\text{Applied Shear}}{\text{Critical Shear}}$

Ratio: $\frac{\text{Web deflection}}{\text{Web thickness}}$

Fig. 1

The increase of apparent shear stress with increase of ratio of applied shear stress to critical shear.

$$\text{Ratio: Apparent Shear} = \frac{\sqrt{f_x^2 + f_y^2 + 3f_s^2 - f_x f_y}}{\sqrt{3}f_s}$$



$$\text{Ratio: Applied Shear} = \frac{f_s}{f_{sc}}$$

Fig 2

Timoshenko's critical shear stresses and the corresponding crippling stresses for various values of d/b and t for various values of b and d .

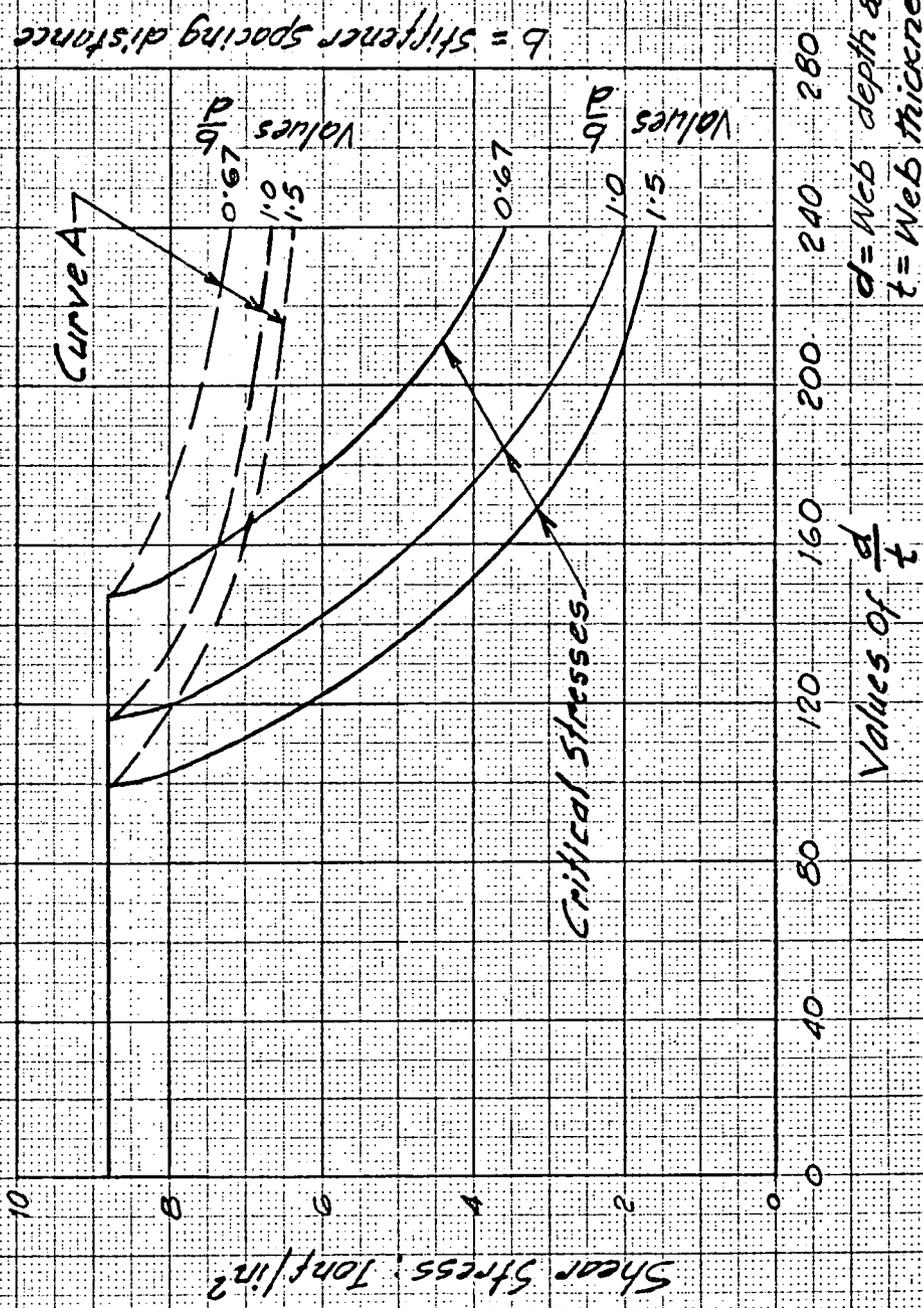


Fig. 3

Shear failure stresses (curves A) & allowable shear stresses
for various values of d/f & p/d .

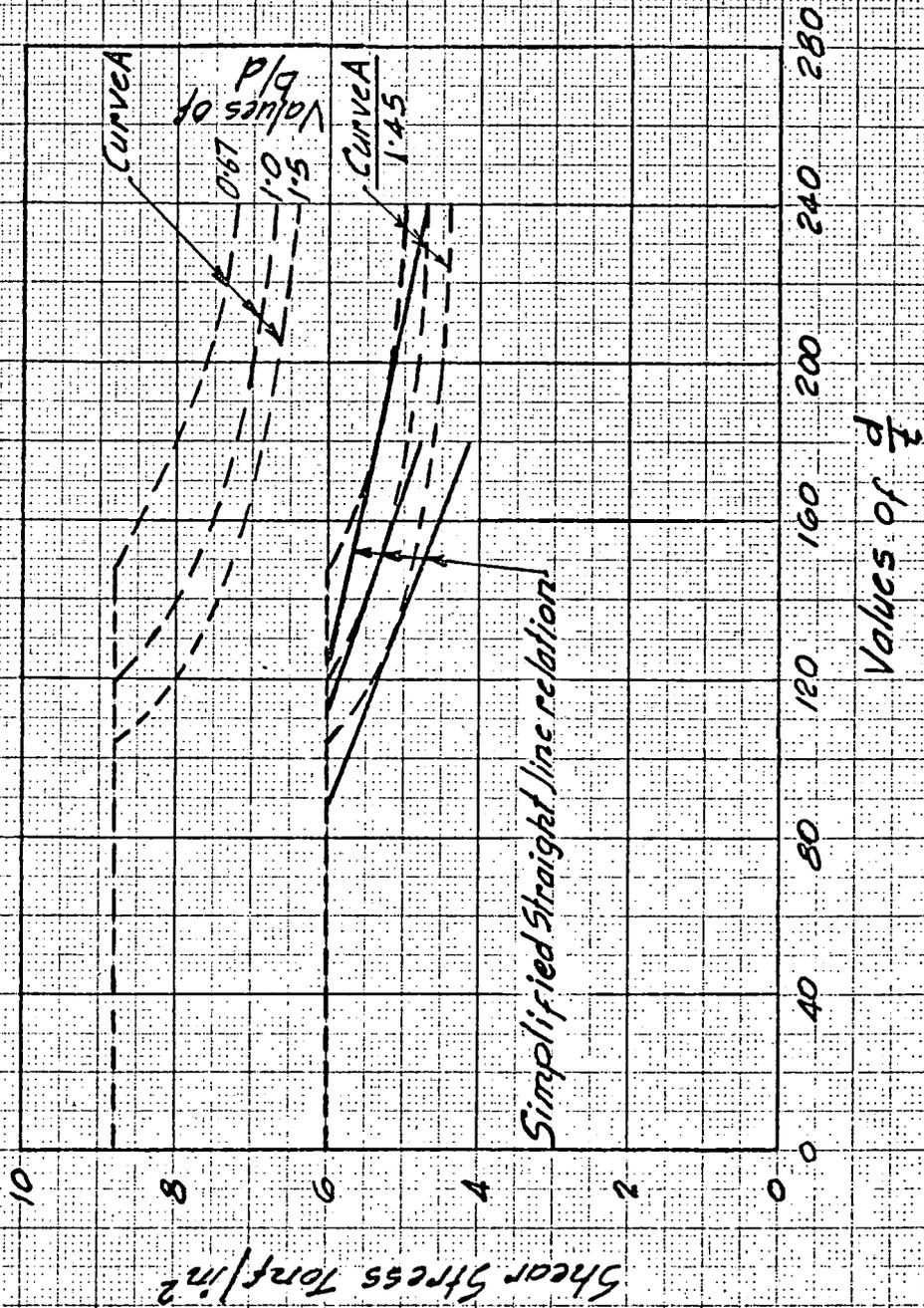


Fig 4

3.5. Web buckling and reinforcement:

It is necessary to employ both vertical and horizontal stiffeners in a deep plate girder with a high value of depth to thickness ratio (d/t value). Timoshenko⁽⁶⁾ said that the critical shear stress value for a simply supported plate is a function of $(t/d)^2$ and therefore it decreases rapidly with the increases in d/t ratio. Thus, a web plate subjected to shear stresses will buckle before it yields, if the critical stress is reached before the shear stress. Sparkes & Heyman have shown by experiments that for mild steel, web plates with d/t ratio 90, usually yield before buckling. Kerensky & others⁽⁵⁾ stated that, by using horizontal stiffeners, the overall d/t ratio can be increased up to 300 with one stiffener and up to 400 by using two stiffeners. B.S.153 Part 4 clause 26a (ii) recommends that the spacing of the vertical stiffeners should never exceed 180 t of the smaller clear panel dimension and 270 t of the greater clear panel dimension. These limitations are to improve the ultimate load-carrying capacity of the girder and also for stiffening the web for fabrication and transport purposes.

Timoshenko said, for an initially plane plate, the 2nd moment of area of a pair of stiffeners about centre of the web (I) should be $I = 0.3d^4 \frac{t^3}{b^2}$. But Moore said that, no plate can be initially plane at all and therefore, for practical purposes, $I = \frac{4}{3} d^3 \frac{t^3}{b^2}$, if the maximum stiffener deflection is to remain small in comparison with web. Kerensky & others⁽⁵⁾ did some ultimate load tests on girders and found that Moore's I value is adequate. But B.S.153 part 4 clause 27b (i) recommends that, the value of I should not be less than $1.5 d^3 \frac{t^3}{b^2}$ (see appendix A part II para. A5.9). This value is about 3.35 times greater than that of Timoshenko's theoretical

value for long panels and 10 times greater for short panels.

For a given web thickness, the higher the shear, the closer the spacing and the heavier the stiffeners. B.S.153 part 4 clause 27 b (ii) recommends that the horizontal stiffeners should be used in addition to the vertical stiffeners and one stiffener should be placed at a distance from the compression flange equal to $\frac{2}{5}$ of the distance from the compression flange to the neutral axis when the thickness of the web is less than $\frac{d^2}{200}$ for steel grade 43A, where d^2 is, twice the clear distance from the compression flange to the neutral axis (see appendix A Part II para A 5.8). For economic reasons the $\frac{d}{t}$ ratio should be as high as it is permitted and since a limited amount of experimental data is available, the above rule is based on theoretical analysis with a suitable factor of safety. The horizontal stiffeners must withstand axial loads due to bending moments in addition to restraining the web plate from buckling and so, the maximum inertia required is dependant on the ratio of the area of the stiffener to area of the web plate (i.e. $\frac{A_{stiff}}{dt}$ value, see fig 5).

Kerensky & others⁽⁵⁾ stated that the horizontal stiffeners are not expected to carry any increases in axial load, when loads in excess of the theoretical critical values are reached. Therefore, the theoretical values are increased by a factor and the minimum rigidity value is given by $I = 4 bt^3$ (see B.S.153 part 4 clause 27 (ii)). The same statement has been justified by Kerensky & others⁽⁵⁾ on fig. 5. For further increases in $\frac{d}{t}$ ratio, an additional horizontal stiffener should be placed at neutral axis of the girder, when web thickness is less than $\frac{d^2}{250}$ for grade 43^A steel. This stiffener acts to limit the panel

dimensions and does not carry any direct load and therefore it can be smaller than the upper horizontal stiffener. This stiffener should have a 2nd moment of area (I) not less than $d_2^3 t^3$ (see B.S.153 part 4 clause 27 ii).

The buckling of a plate due to shearing action, does not cause failure of a panel immediately, but produces a change in the manner in which any additional shear load is carried by the plate. Rockey's⁽⁹⁾ investigation showed that after buckling has occurred, the rigidity of the plate normal to the principal compressive stresses is increased slowly, while the tensile stresses increase rapidly. These principal tensile stresses, which act along the length of the waves, exert lateral and direct loads upon the flanges. So, it is important that the flanges should have enough rigidity to carry these loads. Rockey carried out some experiments on stiffened plate girders and his findings are:-

1. The web plate started to deflect as soon as the loads were applied and these deflections were due to the presence of initial deformations, but these were considerably less than the thickness of the web plate and therefore negligible.
2. For welded plate girders, the flexural rigidity of the flange about an axis passing through the centroid of the flange and normal to the plane of the web plate is low and such a flange is not capable of carrying loads without severe deflections. These deflections will increase the depth of the web buckles and will cause increase in bending stresses in the web. Subsequently, this will reduce the ultimate load carrying capacity of the panel.
3. The post-buckled behaviour of web plate subjected to

shear has shown that the buckle formation and more particularly the depth of the buckles, is affected by the flexural rigidity of the flanges.

It is apparent now that, in deep plate girder where $\frac{d}{t}$ ratio is high, both vertical and horizontal stiffeners are necessary and the most effective position for a single horizontal stiffener is at mid-depth of the web. Rockey has examined the influence of changes in the size of this central horizontal stiffener and of the vertical stiffeners, upon the buckling stress. Consequently, he has suggested a new formula, superseding Timoshenko's theoretical formula and supported by the experimental research work of Budiansky, Conner, Kuhn, Petterson etc.

Timoshenko and others, $f_{sc} = \frac{K\pi^2 E t^2}{12(1-\nu^2)d^2}$ where K is the critical shear stress coefficient whose magnitude will depend upon the support which the web receives from the flanges and the stiffeners. Rockey⁽¹⁰⁾ said that, for a web, reinforced by the vertical stiffeners of zero torsional rigidity, Stein and Fralich have provided theoretical values of K for different values of stiffener rigidity. He also said that, Kleeman has developed Stein & Fralich work to allow for the effect of the torsional rigidity of stiffeners. Later Rockey established that for single sided stiffeners, the relationships between the buckling coefficient, the non-dimensional parameter, r and the aspect ratio (b/d) were

$$K = K_u + A \sqrt[3]{r}$$

Where $K = K_{Lv}$; K_{Lv} is the limiting value of K, for a web reinforced by vertical stiffeners only.

K_u = critical shear stress coefficient of the unstiffened plate.

$$r = \frac{EI}{d} = \frac{\text{stiffness rigidity}}{\text{rigidity of strip of web plate equal to clear depth, } d}$$

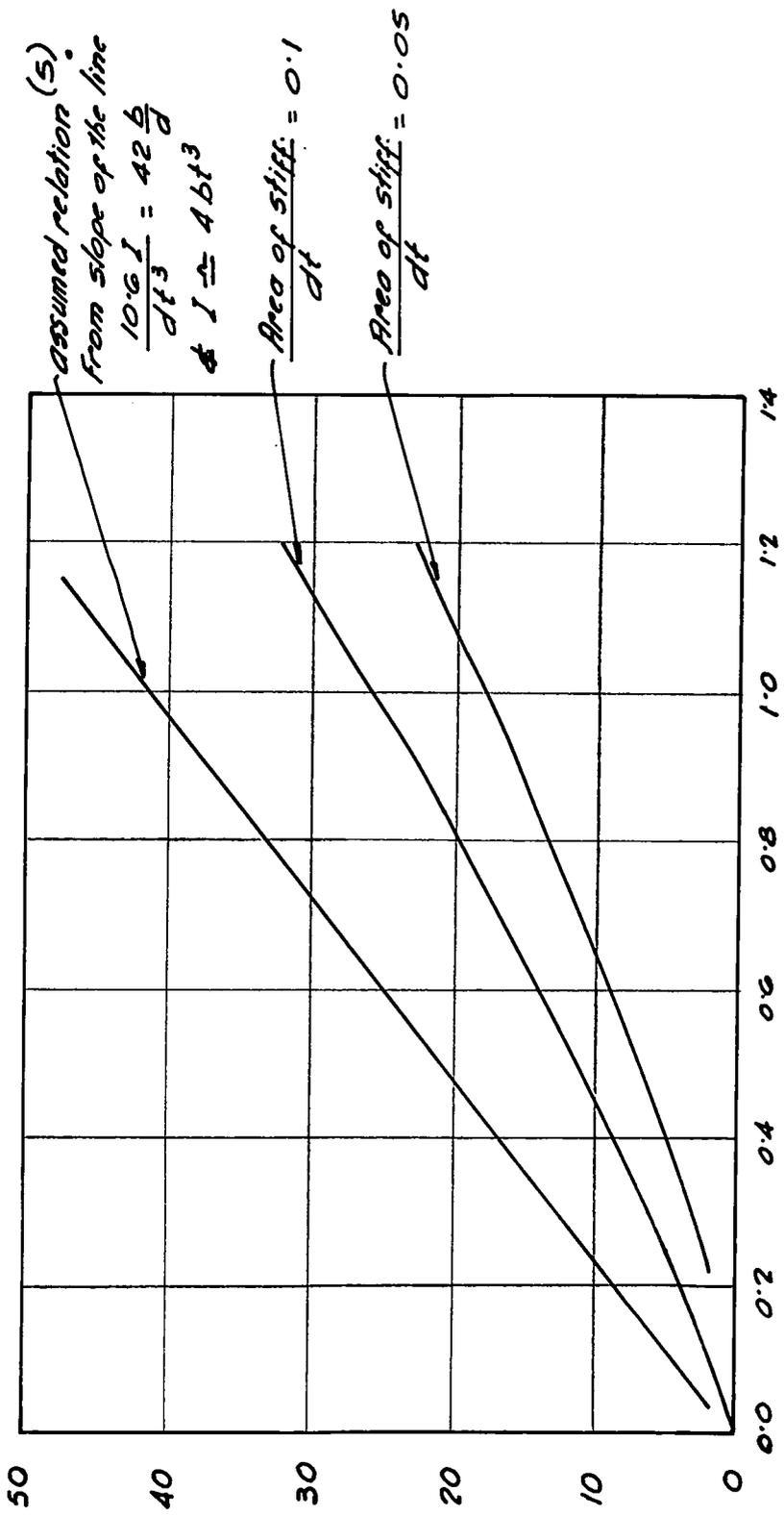
but $r \geq r_{LV}$ where r_{LV} = the limiting value of
for vertical stiffener.

A = a constant.

$$r_{LV} = 21.5 \left(\frac{d}{b}\right) - 7.5 \left(\frac{b}{a}\right)$$

$$K_{LV} = 8.0 + 5.7 \left(\frac{d}{b}\right)^2$$

But, no similar relationships, theoretical or experimental, have been obtained for the case of a web subjected to shear and reinforced by both vertical and central horizontal stiffeners. Therefore, webs with central horizontal stiffeners must still be designed according to previous practice, such as B.S.153 part 4 clause 27 (ii), where it is stated that for horizontal stiffeners, the 2nd moment of area(I), should not be less than $4S_1t^3$ where S_1 is the actual distance between stiffeners (see appendix A part III para. A5.8).



Ratio: $\frac{\text{Distance between vertical stiffeners}}{\text{Clear depth of web}}$

The required theoretical & proposed (s) stiffress of the horizontal stiffener (@ $\frac{1}{3}$ depth from compression flange) relative to stiffness of the web plate, subjected to bending for different spacing of vertical stiffeners.

Fig. 5.

3.6. Restraints on stability of the compression flange:

The stability of a beam member is usually ensured by judicious connection of restraints along its length. In through-bridge girders, intermittent restraint is provided by stiffeners attached to the transverse back beams. If the restraints exceeds a minimum stiffness, then the girder may be forced to buckle with nodes at these supports. In such case, it is advised in B.S.153 part 4 clause 34, that the compression chord should be treated as a strut supported by a number of deflectional springs. Seide and others⁽¹⁴⁾ sought to define the minimum stiffness requirements, and found that the buckling occurs with a half-wave length, equal to the spacing, (a), between supports; with more than three intermediate supports within the span of the girder, the stiffness of each of these supports at flange level should be defined as:-

$$\delta = \frac{a^3}{40EI_c} \text{-----} 1$$

where, δ = stiffness of supports to the through-bridge girder. i.e. lateral deflection per unit load of the compression flange at the frame nearest mid span of the girder, taken as the horizontal deflection of the stiffener at the point of its inter-section with the centroid of the compression flange, under the action of unit horizontal force applied to the frame.

a = distance between frames (or supports)

E = Young's modulus

Ic = maximum moment of inertia of compression flange about y-y axis of the girder.

B.S.153 part 4 clause 21(b) suggests that in the case of very rigid frames where δ is less than $\frac{a^3}{40EI_c}$, the horizontal force F shall be obtained by putting $\delta = \frac{a^3}{EI}$ and the effective length, $l = a$.

Kerensky and others said that when less than three supports are used throughout the length of the girder, the above definition of stiffness is conservative. According to B.S153, the permissible compressive stress is 9 tonf/in.² (from table 3) and this value may be used in deep plate girder provided that the critical bending stress

$$f_{bc} = \frac{\pi^2 E}{12} \left(\frac{B}{a}\right)^2 = 47.7\mu \quad \text{--- 2}$$

where $\mu > 1$

where B = width of flange plate

and μ = a numerical coefficient.

The above expression has been derived by Kerensky and others⁽⁵⁾. They also said that if μ is greater than 1, the support stiffness may be reduced without altering the permissible compressive stress. The curve plotted in Fig. 6 may be used to estimate the modified stiffness. Once the value of μ is known, from this curve, the value of $\frac{a^3}{\delta EI_c}$ may be derived.

Again the curve can be used to obtain the effective length for buckling over a half-wave length, greater than the spacing, a , in which the stiffeners are deformed during buckling.

Now the critical stress equation may be written as

$$f_{bc} = \pi^2 E \left(\frac{r}{l}\right)^2 = \frac{\pi^2 E}{\mu} \left(\frac{r}{a}\right)^2 \quad \text{--- 3}$$

Where μ is > 1

For a given value of $\frac{a^3}{\delta EI_c}$, μ may be obtained from the curve in fig.6 and therefore $l = a\sqrt{\mu}$.

Timoshenko⁽⁶⁾ dealt with the problem and he states that $l = 2.2\sqrt[4]{EI_c \delta a}$.

Kerensky and others⁽⁵⁾ said that Timoshenko's expression is accurate when the half-wave-length contains several stiffeners and is in error as $l \rightarrow 0$. The numerical constant has been modified and B.S.153 suggests that

$$l = 2.5 \sqrt[4]{E I_c S a} . \quad (\text{see appendix A part II para.5.3b})$$

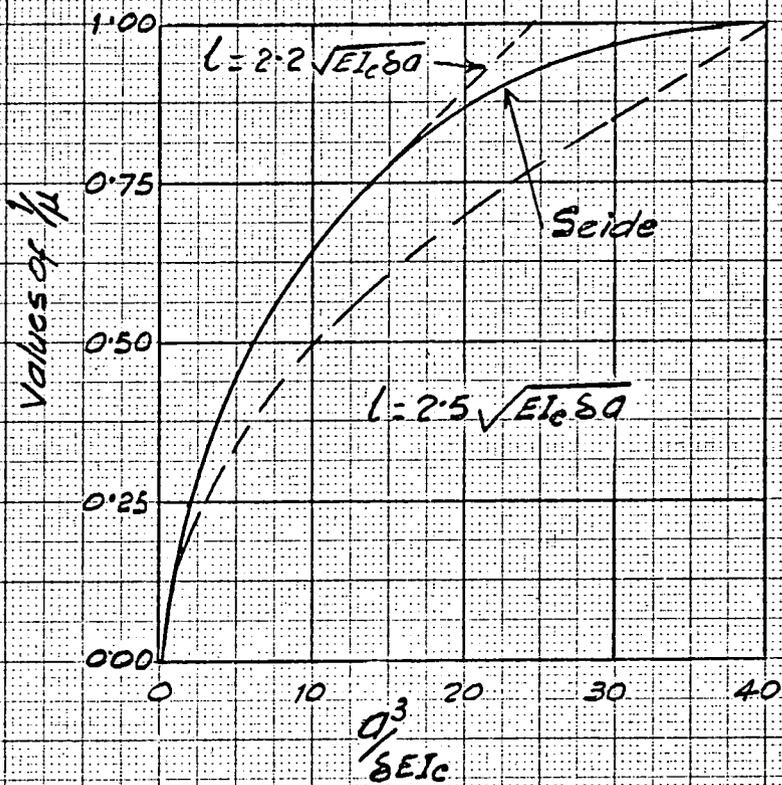


Fig. 6 - Influence of support stiffness on the stability of compression chords of through-bridge (Kernesky & Others⁽⁵⁾).

3.7. Welding

The evolution of welding practice has followed the outcome of research and experience. Up to 1930 riveted structures were popular. Although welding was in existence during that time, it was used for limited purposes. Those days, for plate girders, flanges were made up of multiple of thin plates, graduated in width and then fillet welds were applied along their edges. For long flanges, plates were butt welded, but in addition to the weld, fish plates (cover plates) used to be riveted at joints. In early days, fillet welds used in longitudinal shear were considered to be safer than butt welds. Research and developments increased the confidence in welding and use of butt welds for main joints and the substitution of single thickness plates for multiple-plate flanges took place. The allowable stresses in butt-welded connections varied from country to country, and it was between 80 and 100 per cent of the parent-metal permissible tensile stress and from 89 to 100 per cent of the parent-metal permissible compressive stress. The extensive use of thick single-flange plates, led to the rolling of special flange plates, in Germany during 1930's. This was intended to facilitate the welding fabrication and to reduce the stress in the web-to-flange zone and this also made x-ray inspection much simpler. In U.S.A., the allowance for fatigue of the welded road bridges was set out in 1947 by the American Welding Society and had the greater significance in U.S.A., than in U.K. as the design loading systems in these two countries are not the same.

Research and development work on steel and electrode

have improved the method of design and fabrication. The present knowledge of the behaviour of welded bridges has increased the permissible stresses in connections.

As mentioned above, in early days, the permissible stresses in weld had been taken far less than the permissible stresses in parent-metal, not only that, in addition to the welds, reinforcement plates used to be fixed to take precautions against failure. Now it is known that the strength of a dynamically loaded structure is not improved by the addition of the extra material and this extra material introduces more stresses and consequently the joint becomes more susceptible to failure. Today welding is essentially a ready means for achieving fully monolithic construction. It is economical if combined with sound fabrication practice. The inherent rigidity of welded joints between cross girders and main girders may be turned to good account in designing for the distribution of live load over a greater width of deck than if hinged ends to the cross girders were assumed in design. This will increase the economy in material of the main girders on bridges where the lateral spacing of the girders is comparatively close.

To design a steel structure, the first preference is given to mild steel, because it is cheap when compared with H.Y.Steel, easily available and satisfactory for welding. In general, the carbon content in steel should be limited to 0.25%., if welding is to be easy and trouble free. Steel containing more than 0.25% of carbon may be weldable, but pre-heating is necessary to prevent cracks in welding.

B.S.4360 (old B.S.15) does not limit the carbon content and does not insist upon a chemical analysis other than a

limitation of sulphur and phosphorus and it is vital to have a constant manganese content (approximately 0.5 to 0.65./.). The carbon content in mild steel depends on the thickness of the plate (e.g. for $\frac{1}{4}$ " thickness, approx. 0.16./. and for $1\frac{1}{2}$ " thickness approx. 0.24./.). On thicker material, the carbon content will increase more, to maintain the ultimate tensile strength of the steel within the specified limits. Care must be taken, by obtaining the mills chemical analysis, to ensure that the carbon content does not exceed 0.25./..

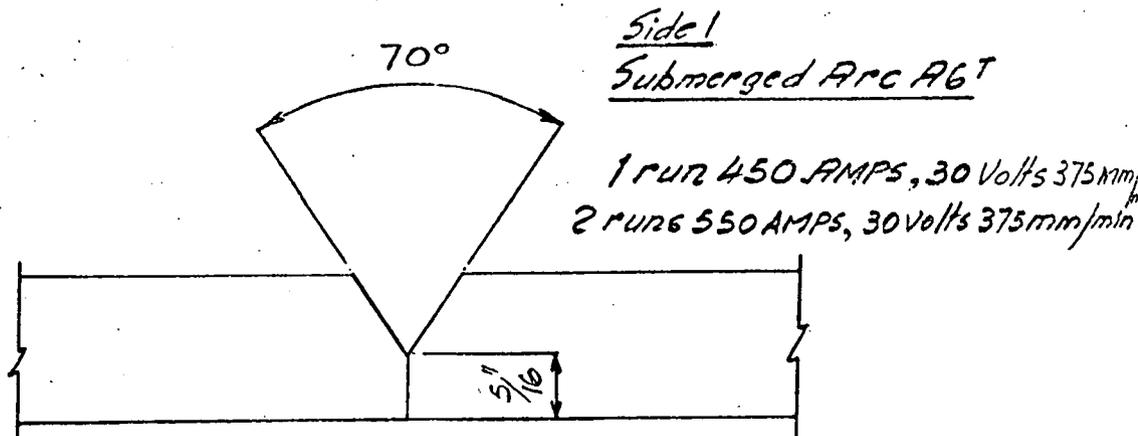
B.S.639 specifies which electrodes should be used. It recommends that for manual process of welding mild steel, the electrodes should be covered for metal arc welding. The parts 1 & 2 of the specification, apply to normal penetration electrodes and part 1 & 3 apply to deep penetration electrodes. In an automatic process of arc welding, the deposited metal must have mechanical properties equal to those obtained by the deposition of electrodes mentioned in B.S.639.

A fillet weld is defined as being any fusion weld approximately triangular in transverse cross section, which is not a butt weld, but including a weld at a corner joint. A fillet weld, as deposited, should not be less than the specified dimensions which should be clearly indicated as throat thickness and/or leg length as appropriate. For concave fillet welds, the actual throat thickness should not be less than 0.7 times the specified leg length. For convex fillet welds, the actual throat thickness should not be more than 0.9 times the leg length. The effective length of a fillet weld should not be less than four times the size of the weld.

A butt weld is defined as a weld in which the metal lies substantially within the extension of the planes of the surfaces of the parts joined or within the extension of the planes of the smaller of two parts of differing sizes.

The size of a butt weld shall be specified by the effective throat thickness of the weld, the effective throat thickness of a complete penetration butt weld being taken as the thickness of the thinner part joined; and the effective throat thickness of an incomplete-penetration butt weld is taken as the minimum depth of the weld metal, excluding reinforcement, common to the parts joined.

B.S.153 part 4 clause 59 recommends only that the design of welds in steel should be in accordance with B.S.1856. So, the B.S.1856, which contains the general requirements for the metal-arc welding for mild steel, has to be followed; but this does not provide with any empirical formulas to work out the required weld size. The current design is based on the conventional evaluation of stress in welds as it is explained in B.C.S.A. publication No.6 , 1952, (The use of welding in steel building structure) and a specific text book reference (See appendix A part 11 para. A6.).

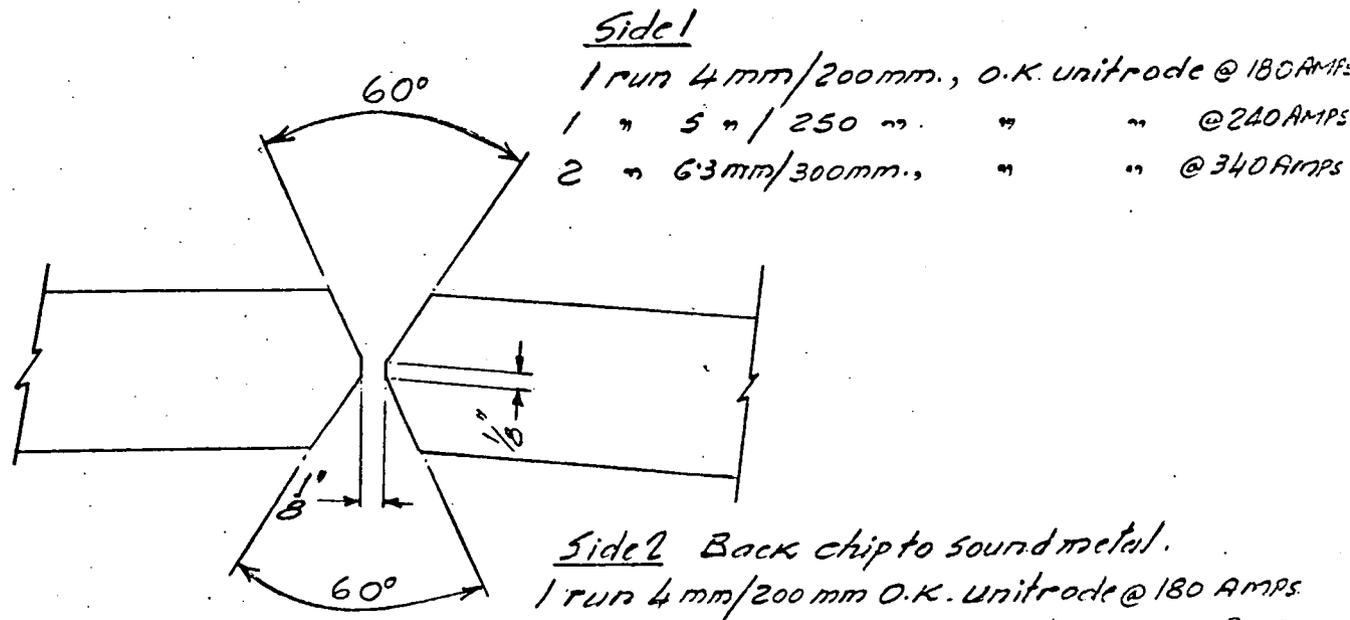


Side 1
Submerged Arc A6T

1 run 450 AMPS, 30 Volts 375 mm/min.
2 runs 550 AMPS, 30 Volts 375 mm/min.

Side 2 Automatic A6CR1R
Weld detail - Web plate Gouge to sound metal.

2 runs 550 AMPS, 30 Volts
375 mm/min.



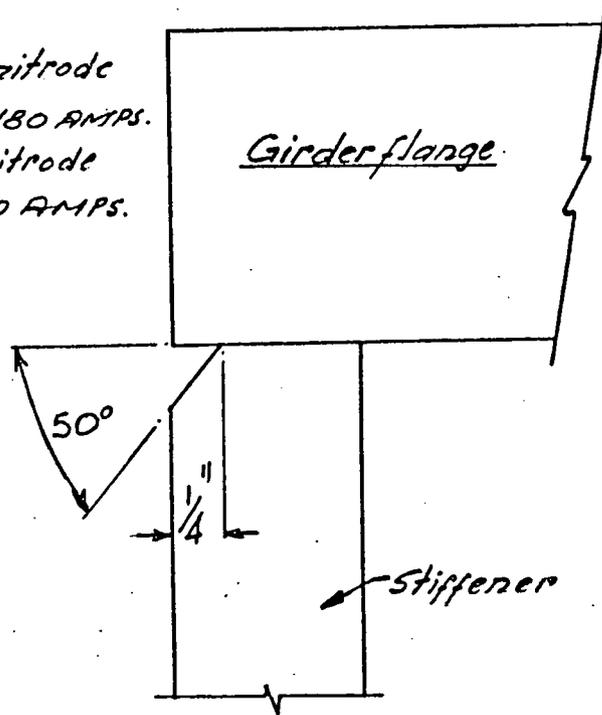
Side 1

1 run 4 mm / 200 mm., O.K. unitrode @ 180 AMPS
1 " 5 " / 250 " " " @ 240 AMPS
2 " 6.3 mm / 300 mm., " " @ 340 AMPS

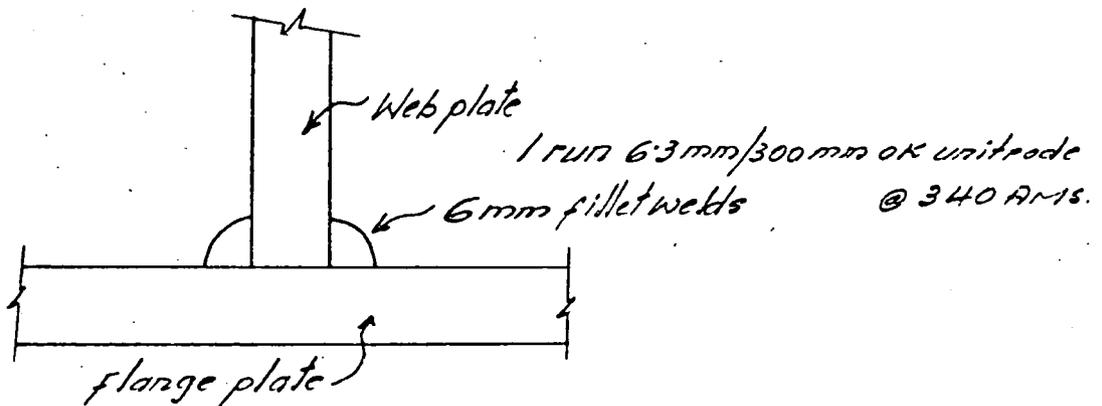
Side 2 Back chip to sound metal.
1 run 4 mm / 200 mm O.K. Unitrode @ 180 AMPS
1 " 5 " / 250 " " " @ 240 AMPS.
2 runs 6.3 mm / 300 mm., OK unitrode
@ 340 AMPS.

Weld detail - Top & Bottom flange plates

1 run 4mm/200mm OK unitrode
@ 180 AMPS.
1 run 5mm/250mm OK unitrode
@ 240 AMPS.



Weld detail - Stiff. to Girder flange



Typical Weld detail

Flange plate to web plate

4.1. Introduction:

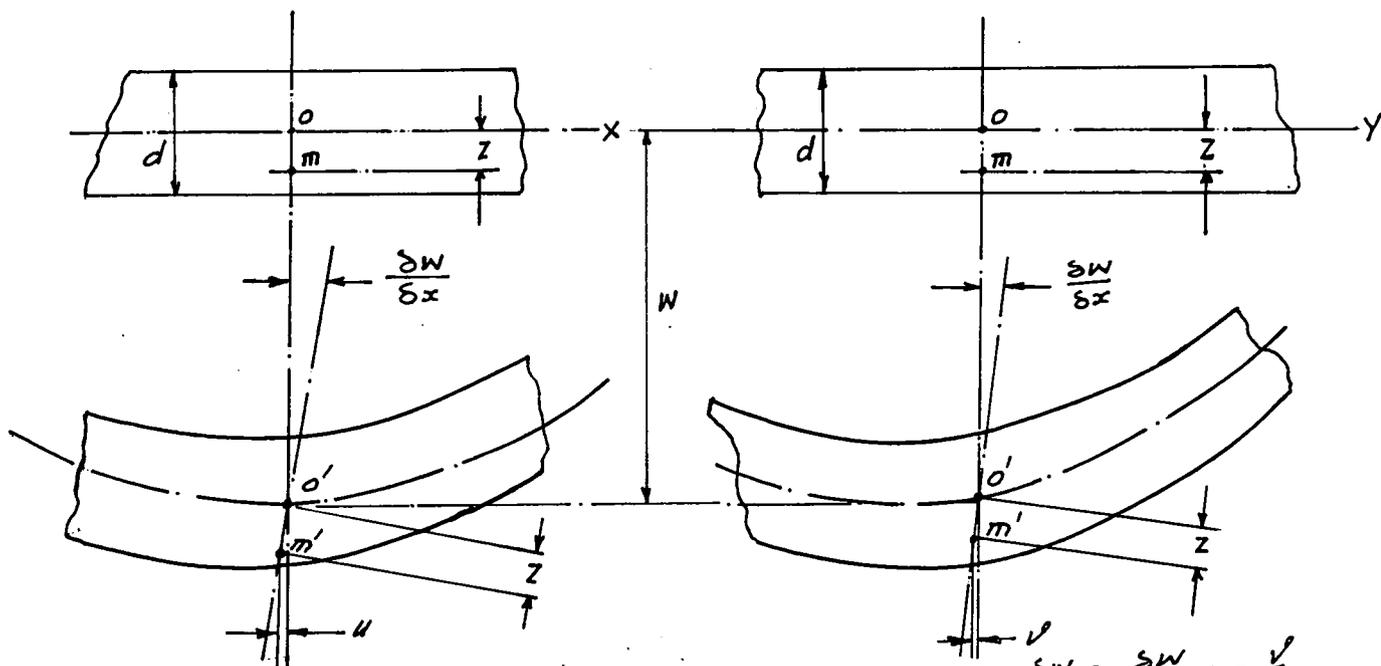
In bridge construction, the magnitude of loads (both moving and concentrated), are steadily increasing. To obtain a reasonable long servicable bridge, engineers have sought for lighter structure, which means a decrease in self weight (dead load) of the bridge members. Steel decks have been used in bridge construction since the turn of this century. Progress in the design of steel decks was achieved just before the world war II. The initial work towards the analysis of the steel deck or the grid problems was done by Faltus and Leonhardt. After the war Dašek in Czechoslovakia proceeded further into the clarification of the load distribution methods. In 1930's, The American Institute of Steel Construction (A.I.S.C.) introduced a system of steel plate deck system, which is known as "Battle deck floor", in an attempt to reduce the dead load of highway bridges. The function of the deck plate was to transmit the local wheel loads transversely to the stringers as a part of their top flanges, with an assigned effective width. The deck plate did not participate in the floor beam stresses and also did not contribute much towards the rigidity and strength of the main carrying members of the bridge.

In 1946, Guyon derived a solution for orthotropic plates of negligible torsional rigidity. He showed that any variation of load can be handled, if the coefficients of lateral distribution are employed. Later Massonnet derived valid relations from the principles given by Guyon and Massonnet's formulations included also the effects of

torsion. Working from these basic principles, a number of researches carried out numerous experiments, which contributed to the further developments of the orthotropic deck system. Bares^v has systematically compared all these theoretical and experimental data and made some conclusive statements which extended the range of application of the orthotropic deck system.

The analytical approach to the problem of the anisotropic plate is based on some simple assumptions. Bares^v-Massonnet⁽¹⁷⁾ stated that Poisson-Kirchoff's assumptions may be used and they are:-

1. The material of the plate is perfectly elastic (the stress-strain relationship obeys the Hooke's law).
2. The material of the plate is homogeneous.
3. The thickness of the plate is small, compared with its other dimensions.
4. The points which are normal to the middle plane of the plate, initially, should remain normal to middle plane of the plate, even after bending, which means:



For small angles $\tan \frac{\Delta W}{\Delta x} \cong \frac{\Delta W}{\Delta x} = -\frac{u}{z}$ and $\tan \frac{\Delta W}{\Delta y} \cong \frac{\Delta W}{\Delta y} = -\frac{v}{z}$
 therefore $u = -z \frac{\Delta W}{\Delta x}$ & $v = -z \frac{\Delta W}{\Delta y}$

5. Normal stresses in the direction transverse to the plane of the plate are negligible and the thickness of the plate does not change during or after bending.
6. The deflections of the plate are smaller than its thickness.
7. The directions of all external forces are perpendicular to the plane of the plate.

4.2. Description:

In orthotropic plate construction, a steel plate deck is used instead of the more common reinforced concrete slab deck (composite construction). The plate is topped with a wearing surface, which may be concrete or some other lightweight wearing surface such as bitumen bound base surface. The deck plate serves the function of distributing loads to the carrying members. As the deck provides a large area, orthotropic plate deck may be very efficient in resisting bending. Dowling⁽¹⁶⁾ and others have shown by experiments that the stiffened steel deck has a large reserve of strength under wheel loading. This capacity is provided by the membrane strength of the deck plate. Dowling also said that the compressive and tensile forces are developed through the shear connection between deck and the main girder webs and their distribution across the width of the deck is dependant upon the deck's shear rigidity, the geometry of the bridge and the applied load. After doing a number of experiments, Dowling concluded that, the finite element method employing rectangular shell elements may be used to analyse three dimensional overall and local elastic behaviour of an orthotropic steel deck, subjected to both lateral and inplane bending. The inplane loading in the deck caused by its composite action with the main girders may have a significant effect on the maximum

stringer deflections and stresses under design lateral load, particularly when the stringers are initially deformed.

The A.I.S.C. have stated that the stresses in any member of a loaded orthotropic steel deck is due to the combined effects of the various functions performed by the deck in the bridge structure. The main bridge system, with the steel plate deck and longitudinal ribs acting as a part of the main carrying members of the bridge.

The stiffened steel plate deck consists of the longitudinal ribs, transverse floor beams and the deck plate as the common upper flange, acting as the bridge floor. The steel plate deck transmits the live load distribution to the main girders as a simple beam. The participation of the deck in the main girder stresses causes tension in the areas of the negative moments of the main girders and compression in the area of the positive moments. Due to the ample longitudinal and transverse stiffening of the deck plate by the ribs and floor beams, the factor of safety against buckling of the deck in compressive stress areas is quite high. The longitudinal ribs act as continuous beams supported by the floor beams which transmit their load to the main girders. The transverse floor beams deflect proportionately to the loads they carry and thus provide elastic supports for the longitudinal ribs. The floor beams connected to the main girders are taken as simply supported beams. The assumption⁽¹⁸⁾ is that, the ribs and the floor beams carrying locally applied loads are to act as purely flexural members, i.e. free from axial forces, conforming to the usual first-order theory of bending, disregarding the effects of the deflections on the stresses.

The bending moments in an orthotropic steel deck depends

on the following factors:-

- a) Loading
- b) floor beam spacings.
- c) main girder spacings.
- d) the magnitudes and the ratio of the three characteristic stiffnesses of the substitute orthotropic plate used to represent the actual system, which are the flexural stiffnesses in the X & Y axis direction and the effective torsional stiffness.

The main function of the deck plate is to directly support the traffic load and to transmit it to the longitudinal ribs and it should possess an adequate capacity to carry the traffic load and also should have some reserve capacity to support any excess loading due to occasional heavy vehicle or future increase in any further loading. It should also be capable of resisting the effects of the pulsating and alternating stresses occurring at critical points of the deck plate under the effects of the passing wheel loads.

The design of the orthotropic deck system in this project has been carried out by following A.I.S.C's recommendations, (see appendix B) in their design manual for orthotropic steel plate deck bridges (18).

5.1. Introduction:

The ultimate value of the plastic theory is closely connected with the physical behaviour of materials beyond the elastic limit.

Plasticity may be defined as the property of a material which enables it to be deformed continuously and permanently without rupture during the application of stresses exceeding those necessary to cause yielding of the material. Thus, permanent distortion occurs under stress and this distortion can be built up to large amounts. So, the final distortion does not depend upon the final state of stress alone, but upon the series of stresses from the very beginning.

Baker⁽²⁰⁾ pointed out that, as compared with elastic design methods, more rational and economical designs can be achieved by the use of plastic methods. In an elastic design method, the stresses due to working load should not exceed permissible working stresses, which are laid down in the British standard specification, viz B.S.153, 449 etc. The working loads are supposed to be the loads which will be applied to the structure, in normal usage, during the lifetime of the structure and the working permissible stresses are intended to ensure an adequate margin of safety to accommodate the unpredictable over-loads, defective materials etc.

Neal⁽²¹⁾ stated that, as the loads increase, yielding spreads quite rapidly through the most highly stressed section. When this section has yielded fully, it is transmitting a bending moment which for a beam is about 1.15

times greater than the bending moment which was developed at first yield. At this point, according to plastic method, a hinging action occurs at this section, the hinge rotation taking place while the bending moment transmitted across the hinge, remains constant. When this hinge action operates, the longitudinal fibres of the beam are extending or contracting while the stress in them remains constant at the yield value, so that each fibre may be flowing in a completely plastic manner. The fundamental hypothesis of the plastic theory is that, a plastic hinge can undergo rotation of any magnitude, provided that the bending moment remains constant at the fully plastic value. Baker and others have summarised their theories and experimental findings in many publications. Beedle⁽²²⁾ at Lehigh University, published recently, his findings on plastic design. B.S.153 in its present form does not mention any consideration of plastic design. In the following chapters, it is intended to investigate the effect of plastic method of design of the welded plate girder bridge, which has already been analysed elastically.

The history of plastic behaviour of any material had been observed many years ago. In 1914, Kazinczy of Hungary, was probably the first to carry out tests on fixed-end beams and came to a conclusion that, the failure took place when yielding had occurred at three cross-sections, at which hinging actions occurred. Thus, the concept of the plastic hinge was established. Kist of Holland in 1917 also published papers, about his findings in plastic behaviour of ductile materials. In 1926, Grüning in Germany, took interest in this new approach and published a book on this topic. This book contains general results, concerning the failure conditions of pin-jointed trusses, but the contents of the book were theoretical and no experimental confirmation was available. Several investigators

carried out experiments on simple and continuous beams, but Maier-Leibnitz may be put as the path-finder for the plastic analysis of continuous beams during 1928 and 1929. Bleich in 1932 published an article in Berlin, which was devoted to a review of the plastic methods for beams and portal frames. Girkmann also published a paper, about a year earlier than Bleich, which suggested an approximate method of designing multi-storey and multi-bay rectangular frames. In 1936 Maier-Leibnitz, collected all the information regarding the up-to-date experiments and developments which was also published.

Interest in the plastic theory was stimulated in the U.S.A., after Van-den Brock published his paper in 1940, and he named the plastic theory as "limit design". Baker was the first to realise that, the plastic theory might be the simple and rational method for the design of complex frames. A study of the behaviour of frame members which failed after they have reached yield point was made and by 1956, Baker described the work in "The Steel Skeleton", vol.2. Horne reviewed Merchant, Wood and others work in 1961, which dealt with stability of frames. Recently, the use of plastic methods of design has been common practice in the U.K. (B.S.449 and the revised edition of B.S.153, which is not published at the time this thesis is written) and the U.S.A.

In the following chapters a study has been made to investigate the modes of failure of the following:-

1. Main girders.
 - a) due to bending.
 - b) due to shear.
 - c) due to combination of bending and shear.
 - d) deflection.
2. Simple plastic failure of cross girders with centre "hinge".
3. Plastic failure locally in deck.

5.2. Main Girders - Design Considerations:

a) In bending - In a symmetrical plate girder, subjected to bending, the static carrying capacity is reached, when the compression flange fails. The compression flange may be considered as an isolated column. When buckling is concerned, a column has three degrees of freedom⁽²⁷⁾: i) Vertically, ii) laterally, and/or iii) torsionally. Fig.5.2.1(b) shows all these direction of movements.

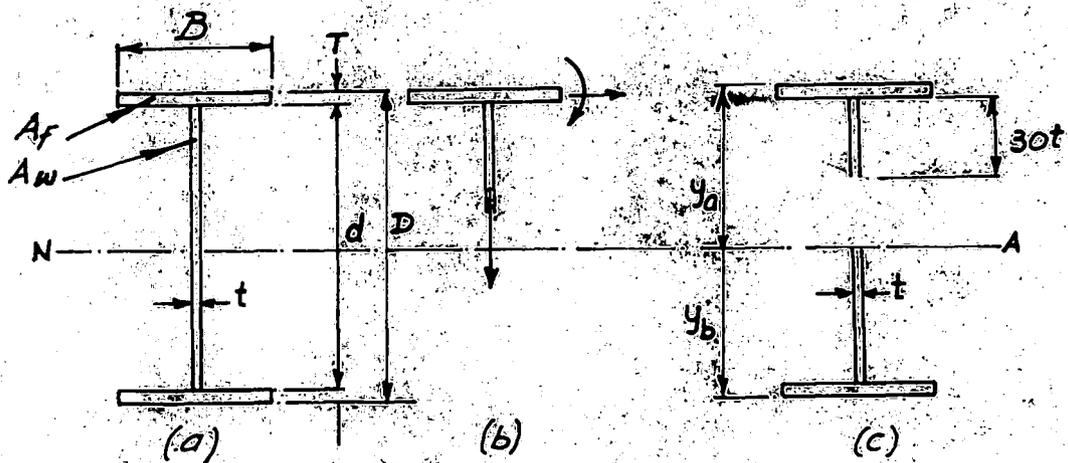
i) Vertical buckling of the flange: In bending, the top flange of a plate girder is in compression and if it is rigid in all directions then it will be the only carrier of the compressive force; this, together with the bottom flange which is in tension, will provide the resistance to the bending moment. In a plate girder the web is braced with the flanges continuously, and since the required bracing stiffness is small, the danger of compression flange failure in vertical direction is limited to high slenderness ratio. When the girder is in process of bending, the curvature increases a uniform compressive stress on the upper and lower edges of the web. If the web is subjected to bear this kind of stress, then the failure will occur similar to that of an Euler column under a stress intensity of

$$\sigma = \sigma_{cr} = \frac{\pi^2 E}{12(1-\nu^2)} \frac{t^2}{d^2} \quad \text{--- 1}$$

Basler suggested that to prevent vertical buckling, the applied force should be smaller than the resisting force. That means,

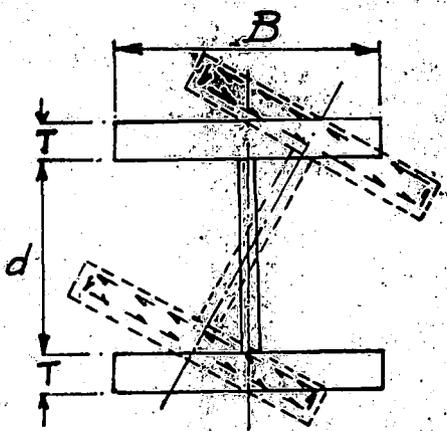
$$\frac{d}{t} < \frac{\pi^2 E}{24(1-\nu^2)} \frac{A_w}{A_f} \frac{1}{\sigma_f E_f} \quad \text{--- 2}$$

This d/t ratio limit depends on $\frac{A_w}{A_f}$ ratio and in general the $\frac{A_w}{A_f}$ ratio is not below 0.5.



(a) Dimensions. (b) Buckling modes of compression flange. (c) Effective girder cross-section.

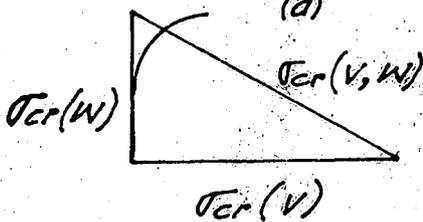
Fig. 5.2.1



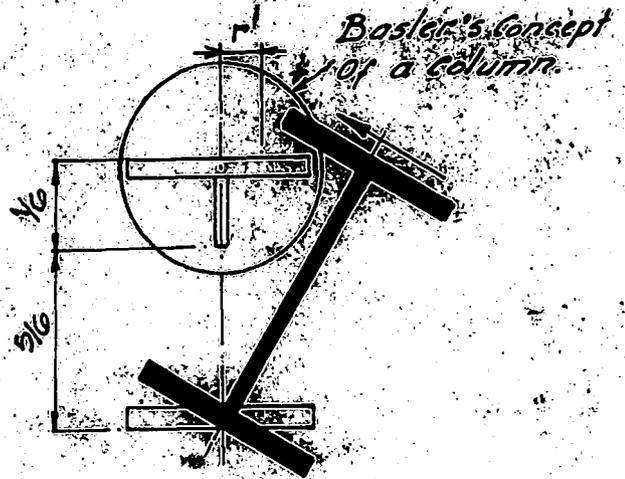
St. Venant torsion

$$\tau_{cr}(V) = \frac{0.65E}{(d/BT)}$$

(a)



(c) Shear Stress flow

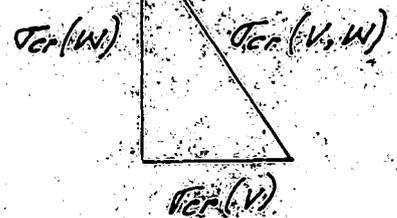


Basler's concept for a column

Warping torsion

$$\tau_{cr}(W) = \frac{TEI}{(I_p)^2}$$

(b)



(d) Lateral torsion buckling of beam.

Lateral Buckling

Fig. 5.2.2

For an upper limit of d/t ratio, equation 2 may be used in practical design, provided $\frac{A_w}{A_f}$ ratio has been taken as 0.5. In the case where every flange fiber reaches yield stress before failure, then the flange force $A_f \sigma_y$ should be obtained. The flange strain must be greater than the yield strain, because, an elimination of residual stress (σ_r), requires a strain $\epsilon_f = \frac{(\sigma_y + \sigma_r)}{E}$. Now substituting the known numerical values,

equation 2 may be written as

$$\frac{d}{t} < \frac{0.48E}{\sqrt{\sigma_y(\sigma_y + \sigma_r)}} ; \text{ but for mild steel } \sigma_y = 15.25 \text{ tonf/in}^2,$$

$\sigma_r = 7.5 \text{ tonf/in}^2$ and $E = 13000 \text{ tonf/in}^2$, the d/t ratio becomes less than 360.

ii) Lateral buckling: de Vries⁽²⁸⁾ stated that, the lateral buckling of a plate girder depends on the parameter of l^d/BT

Basler⁽²⁷⁾ suggested that, to obtain lateral buckling stress two formulas should be used; each one applicable when the other becomes unnecessarily restrictive. Timoshenko⁽⁶⁾ stated that the resistance of an I - beam against lateral buckling consists of two parts: St.Venant torsion and warping torsion. These are referred to as pure torsion and flange bending torsion. The St.Venant part is due to twisting of each component plate where the angle of twist gives rise to a shear stress flow. The warping contribution is due to lateral bending of the flange plates. The St. Venant torsion equation is

$$\sigma_{cr}(V) = \frac{0.65E}{ld/BT} \quad \text{--- 3}$$

and the warping torsion,

$$\sigma_{cr}(W) = \frac{\pi^2 E}{(l/r)^2} \quad \text{--- 4}$$

Basler's concept of the equation 4 is that, of a column whose effective cross-section is composed of the compression flange and $1/6^{\text{th}}$ of the web, (see fig 5.2.2).

In order to reflect these two parts, the expression for the critical bending moment is re-written by Basler as

$$M_{cr} = \sqrt{\frac{\pi^2 E I_y G K}{l^2} + \frac{\pi^4 E^2 I_y \Gamma}{l^4}} \quad \text{----- 5}$$

Where $K = \frac{1}{3} (B T^3 + d t^3 + B T^3) = \frac{1}{3} A_f (2 T^2 + \frac{A_w}{A_f} t^2)$

$$\Gamma = \frac{1}{4} I_y d^2 = \frac{1}{6} A_f d^2 (\frac{1}{2} B)^2 = \frac{1}{24} A_f d^2 B^2$$

$$G = \frac{E}{2(1+\nu)}$$

$$I_y = \frac{2}{3} A_f (\frac{B}{2})^2 = \frac{1}{6} A_f B^2$$

Therefore, the equation 5 can now be re-written as

$$M_{cr} = \frac{\pi}{l} \left\{ (\sqrt{E I_y G K}) \left(\sqrt{1 + \frac{\pi^2 E \Gamma}{l^2 G K}} \right) \right\} \quad \text{----- 6}$$

Since $\sigma = \frac{M}{Z}$

Where $Z = d A_f (1 + \frac{1}{6} \frac{A_w}{A_f})$

Therefore,

$$\sigma_{cr} = \frac{M_{cr}}{Z} \left\{ \frac{\frac{\pi^2 E^2}{18(1+\nu)}}{\frac{l^2 d^2}{B^2 T^2}} \right\} \left\{ \frac{1 + \frac{t^2}{272} \frac{A_w}{A_f}}{(1 + \frac{1}{6} \frac{A_w}{A_f})^2} \right\} + \left\{ \frac{\pi^4 E^2}{l^4} \right\} \left\{ \frac{B^4}{(3 + \frac{1}{2} \frac{A_w}{A_f})^2} \right\} \quad \text{----- 7}$$

(1)

(2)

(3)

(4)

In equation 7

$$\text{term } (1) = \left\{ \sigma_{cr}(V) \right\}^2 \quad , \text{ because the numerator becomes } (0.65E)^2 \text{ if } V = 0.3$$

$$\text{term } (2) = 1, \text{ if the flange thickness, } T \approx 1.2 \times t \text{ and if } A_w \text{ is negligible in compared with } A_f, \text{ then } \frac{A_w}{A_f} \approx 0.$$

$$\text{term } (4) = \text{ the radius of gyration, of the equivalent column composed of the compression flange and } 1/6^{\text{th}} \text{ of the web is defined as } r^2 = \frac{I}{A}$$

$$\text{but } A = A_f + \frac{1}{6} A_w = BT \left(1 + \frac{1}{6} \frac{A_w}{A_f} \right)$$

$$\text{Hence } r^2 = \frac{B^2}{\left(3 + \frac{1}{2} \frac{A_w}{A_f} \right)}$$

$$\text{term } (4) = r^4$$

$$\text{Therefore the term } (3) \times (4) = \left\{ \sigma_{cr}(W) \right\}^2$$

$$\text{Hence } \sigma_{cr}(V, W) = \sqrt{\left\{ \sigma_{cr}(V) \right\}^2 + \left\{ \sigma_{cr}(W) \right\}^2} \quad \text{--- --- } \sigma$$

Thus, it is proved by Basler, that the relation between these three stresses is the same as that existing between the sides of a right angled triangle and the length of the hypotenuse represents the correct critical stress; therefore, a conservative estimate of the lateral buckling stress may be obtained, if one of the approximation formulae is used alone. When either of the two values of $\sigma_{cr}(V)$ & $\sigma_{cr}(W)$ is predominant, the other can be neglected, since the length of the hypotenuse is only slightly more than the larger leg of the triangle. In plate girders, the warping torsion is generally the governing factor and the design should be based on the column concept⁽²⁷⁾, as stated above.

Fig. 5.2.3 shows the critical buckling stress of a plate girder, plotted against the slenderness ratio of a column whose effective cross-section is composed of the compressive flange and $1/6$ th of the web. By assuming T/t ratio of 3 and with various slenderness ratios, the stress resulting from the formula $\sigma_{cr}(v)$ and the exact stress $\sigma_{cr}(v, w)$ have been plotted in the same co-ordinate system. In this figure, it will be noticed that the exact lateral torsional buckling stress $\sigma_{cr}(v, w)$, considering both warping and St. Venant torsion, exceed simple column prediction only a little for a web slenderness ratio $\beta = 200$ and even less for higher values of β . Because the phenomenon of lateral buckling, if St. Venant torsion is neglected, is simply one of the lateral buckling of the compression flange, the buckling curves in the inelastic range must be those of weak axis buckling of wide flange type columns. Basler concluded that, for welded girders, the critical stress in the inelastic range must be significantly different. He suggested that, the basic column curve shown in fig 5.2.4 should be used for lateral buckling of plate girders analysed with the warping torsion concept:

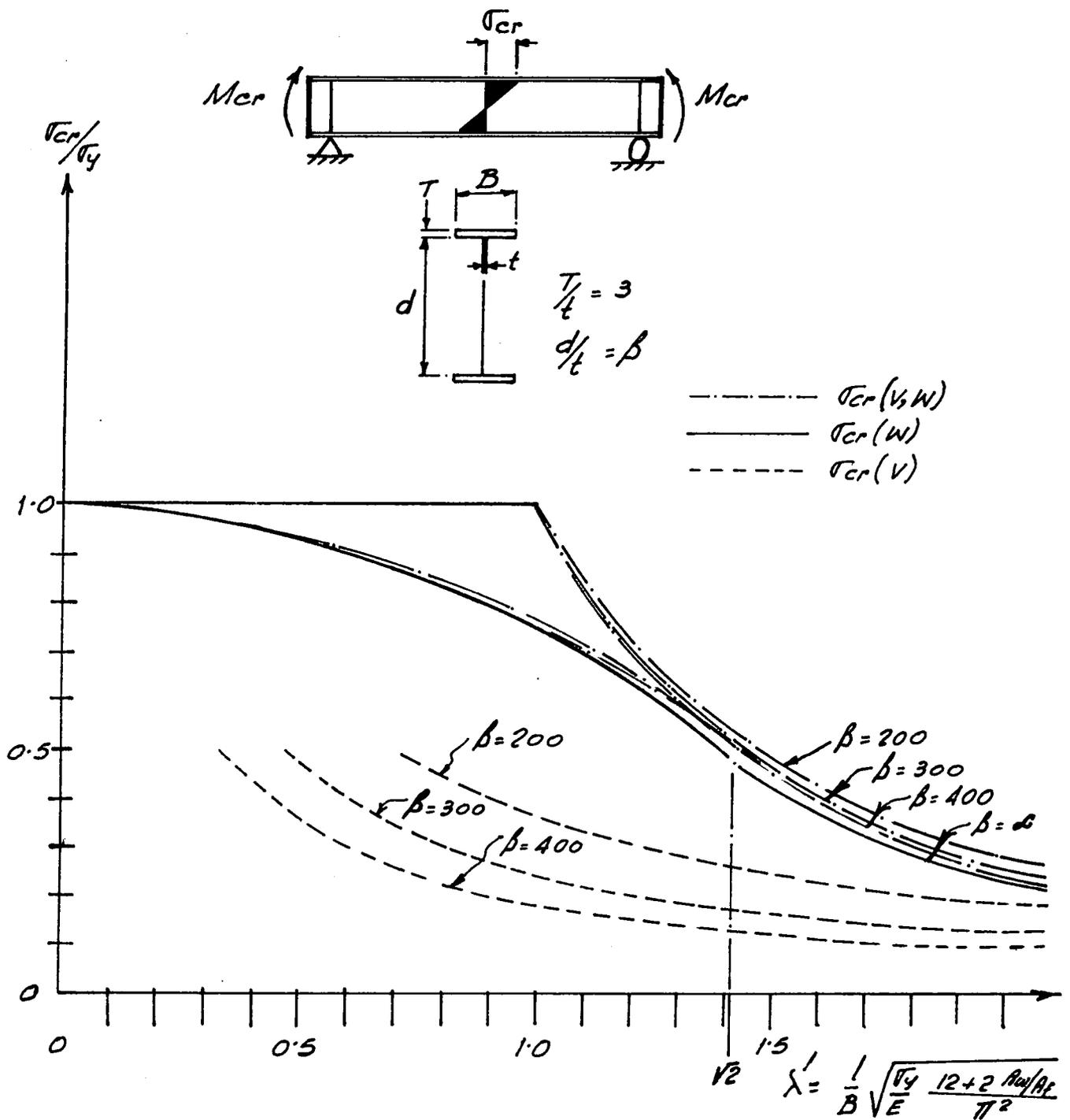
$$\frac{\sigma_{cr}}{\sigma_y} = 1 - \frac{(\lambda')^2}{4} \quad \text{--- --- 9}$$

$$\text{for } 0 < \lambda' < \sqrt{2}, \quad \frac{\sigma_{cr}}{\sigma_y} = \frac{1}{(\lambda)^2} \quad \text{--- --- 10}$$

for $\lambda' > \sqrt{2}$ with

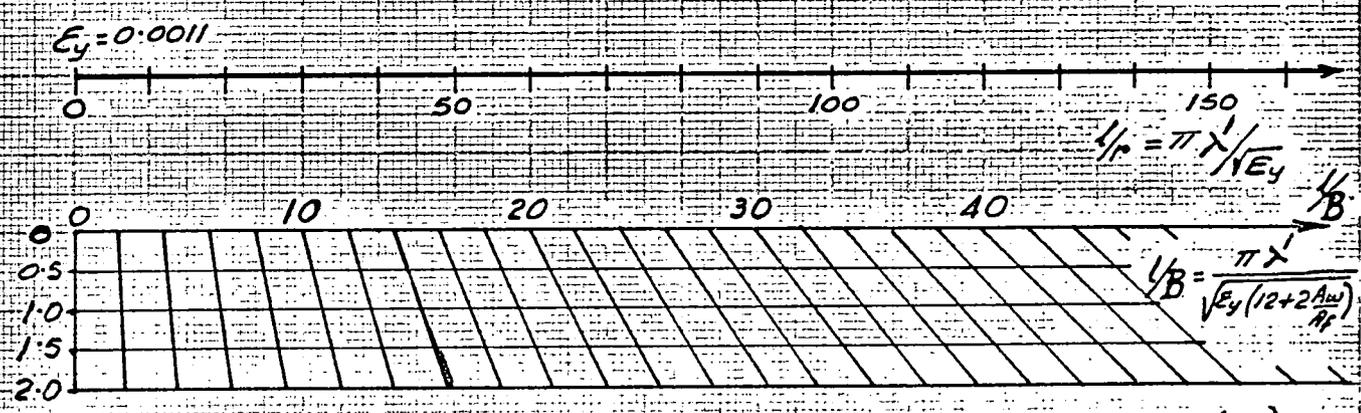
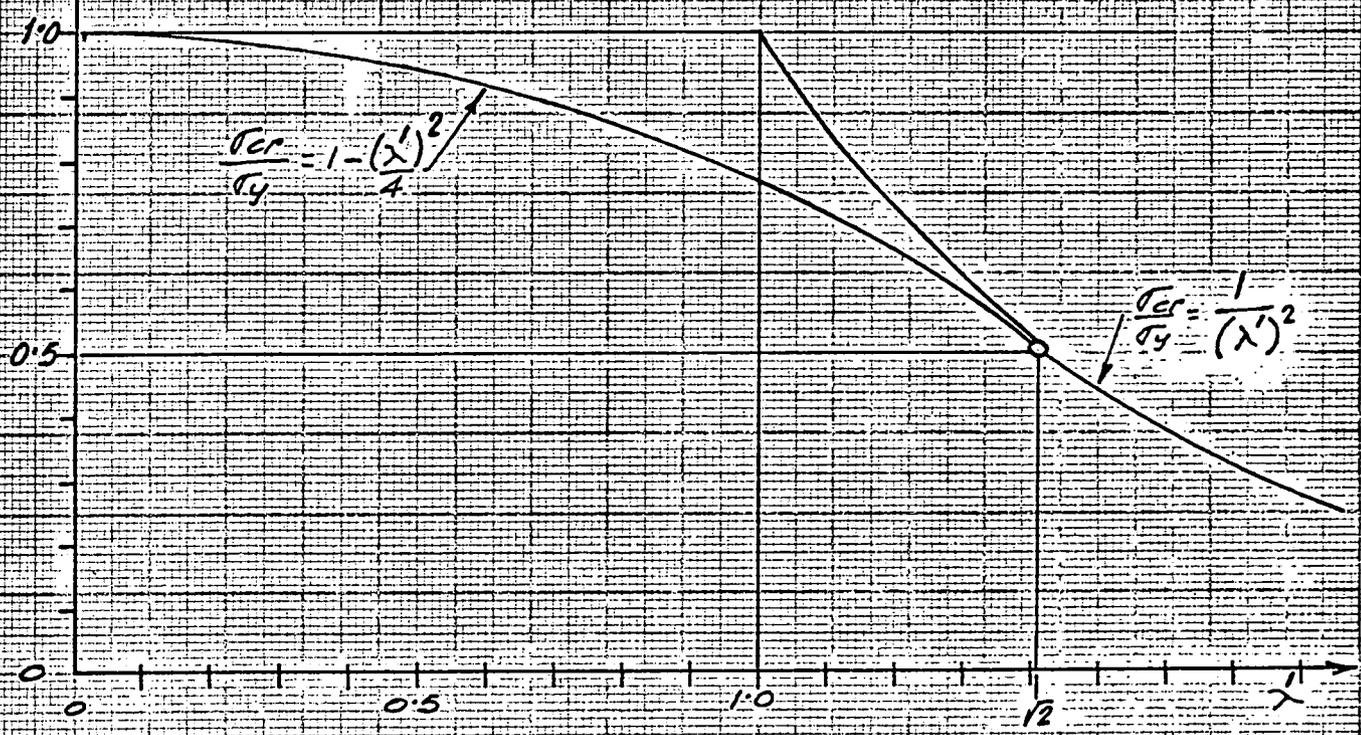
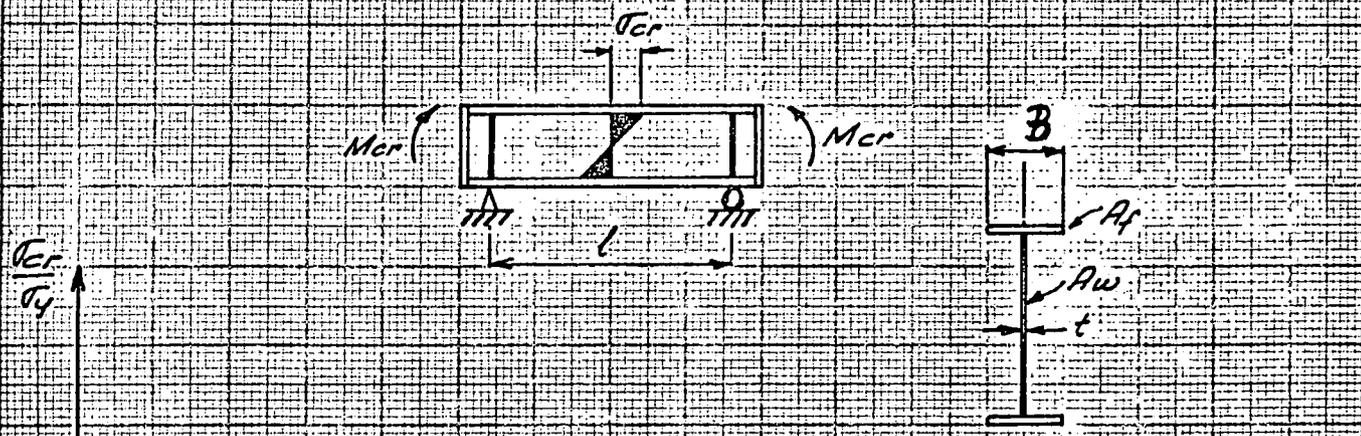
$$\lambda' = \frac{l}{r} \sqrt{\frac{E_y}{\pi^2}} = l \sqrt{\frac{E_y}{\pi^2} \frac{A_f + \frac{1}{6} A_w}{I_f}} \quad \text{----- 11}$$

In fig. 5.2.4, first the standard slenderness ratio λ' , which makes the plot independant of yield stress, is used for the abscissa scale. Next the scale for the parameter $\frac{l}{r}$, valid only for $E_y = \frac{\sigma_y}{E} \approx 0.0011$ as shown.



Lateral Buckling Stresses for Girders With Slender Webs ⁽²⁷⁾

Fig 5.2.3



Lateral Buckling Curve, $V_{cr}(W)$ - Basler⁽²⁷⁾

Fig. 5-2.4.

Finally, the established parameter, buckling length to flange width is presented. This lowest abscissa scale is applicable only in the case, when the compression flange is rectangular in shape. The equation 6, for the general lateral buckling, has been based on the assumption⁽²⁷⁾ that, the shape of the girder cross-section is preserved at the instant of buckling. This assumption is invalid for plate girders with high slenderness ratio and also when the transverse stiffeners are not fitted to the tension flange. However, this uncertainty of the validity of the assumption, concerns only the St. Venant torsion part, because it is dependant on an underformed cross-section whose component plates are assumed to be forced through the same angle of twist. If the joint between the tension flange and web is pinned, the torsional constant K in equation 6 will be reduced to half the value for a rigid joint, consequently, the value of critical stress $\sigma_{cr}(V)$ will be reduced to 30%, but value of $\sigma_{cr}(W)$ will not be affected at all. As stated above, the warping torsion in a plate girder is the predominant contribution to lateral stability, a deformation of profile shape has got very little effect on the resulting buckling stress.

iii) Torsional buckling of the flange plate: Vertical-buckling of the flange plate is independent of any other effects, because it occurs in a direction of symmetry. But the torsional buckling of the flange plate should be treated with the lateral buckling, otherwise the estimate of the stresses due to the buckling will become very conservative. If all restraint on the flange from the web is neglected, then the situation reduces to buckling of a long, hinged plate under pure edge compression at its ends. Therefore, the only parameter on which the flange plate buckling stress depends is the ratio of outstanding width to plate thickness, i.e. $B/2t$ ratio.

In fig. 5.2.5 , the critical flange plate stress is plotted⁽²⁷⁾ as a function of this parameter and λ' . In the inelastic range the curve is obtained by assuming that,

- a) the onset of strain-hardening of flange plate is at $\lambda' = 0.45$.
- b) the compressive residual stress of $\frac{\sigma_y}{2}$ ($\lambda' = \sqrt{2}$) exists.
- c) the transition curve is tangent to the curves at these two points.

Then the expressions of the buckling curve are⁽²⁷⁾

$$\frac{\sigma_{cr}}{\sigma_y} = 1 - 0.53 (\lambda' - 0.45)^{1.36} \quad \text{----- 12}$$

for $0.45 < \lambda' < \sqrt{2}$,

$$\frac{\sigma_{cr}}{\sigma_y} = \frac{1}{(\lambda')^2} \quad \text{----- 13}$$

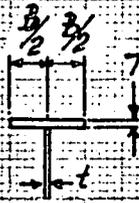
for $\lambda' > \sqrt{2}$ with

$$\lambda' = \frac{B}{2T} \sqrt{\frac{12(1-\nu^2)E_y}{\pi^2 K}} \quad \text{----- 14}$$

Where plate buckling coefficient $K = 0.425$. again assuming yield strain $\epsilon_y = 0.0011$, the critical stress can be expressed in the terms of the plate slenderness ratio $B/2T$ as shown on the second abscissa scale in fig.5.2.5. In order to eliminate torsional buckling as a primary cause of failure, the critical stress of the flange plate given by equation 12, should exceed that of lateral buckling given by equation 9, resulting in

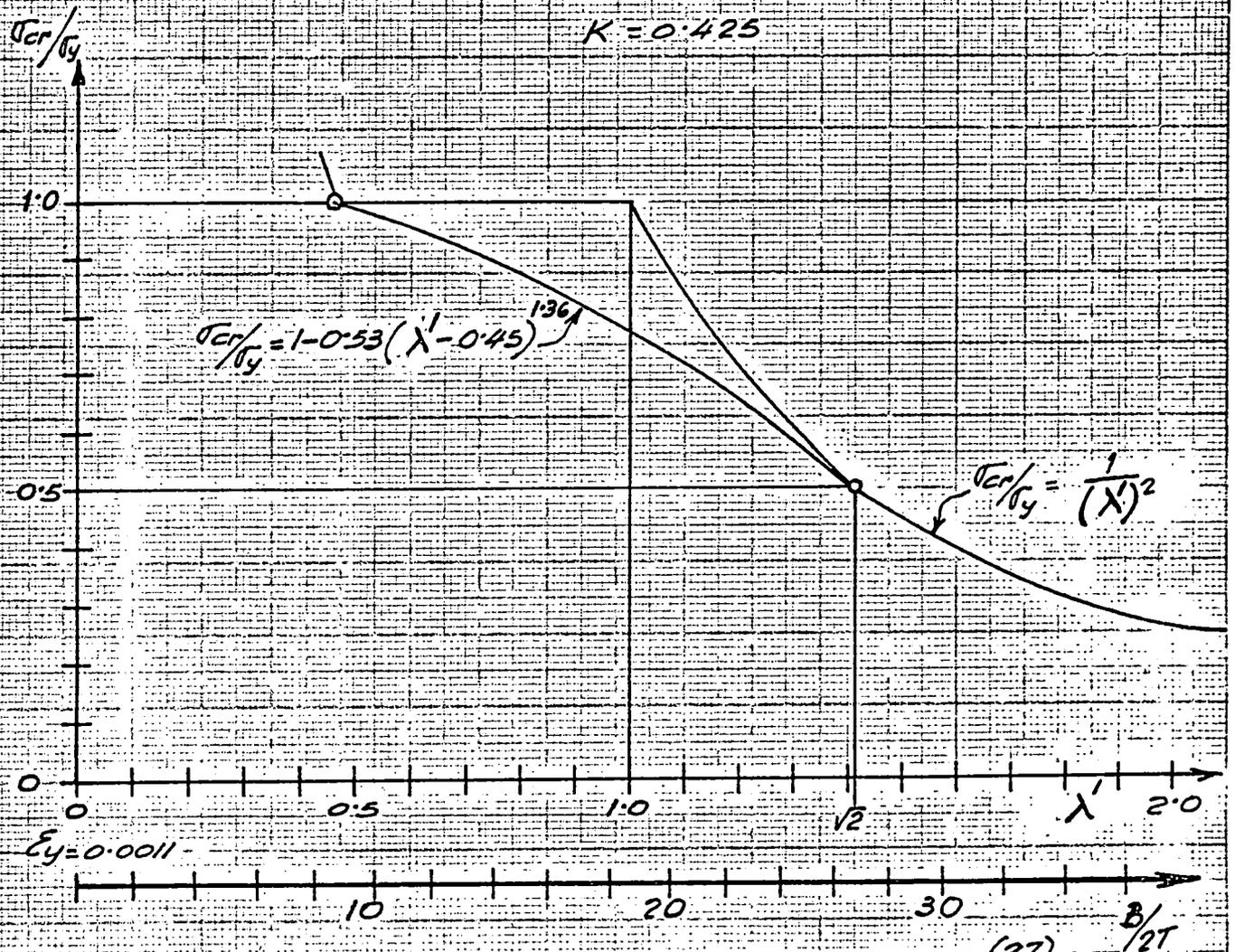
$$\frac{1}{2B} > \frac{2.9}{\sqrt{1 + \frac{1}{6} \frac{A_w}{A_f}}} \cdot \frac{B}{2T} \quad \text{----- 15}$$

for plastic range, $B/2T < 26$. This correlation between $B/2T$ & l/B has been plotted in fig 5.2.6.



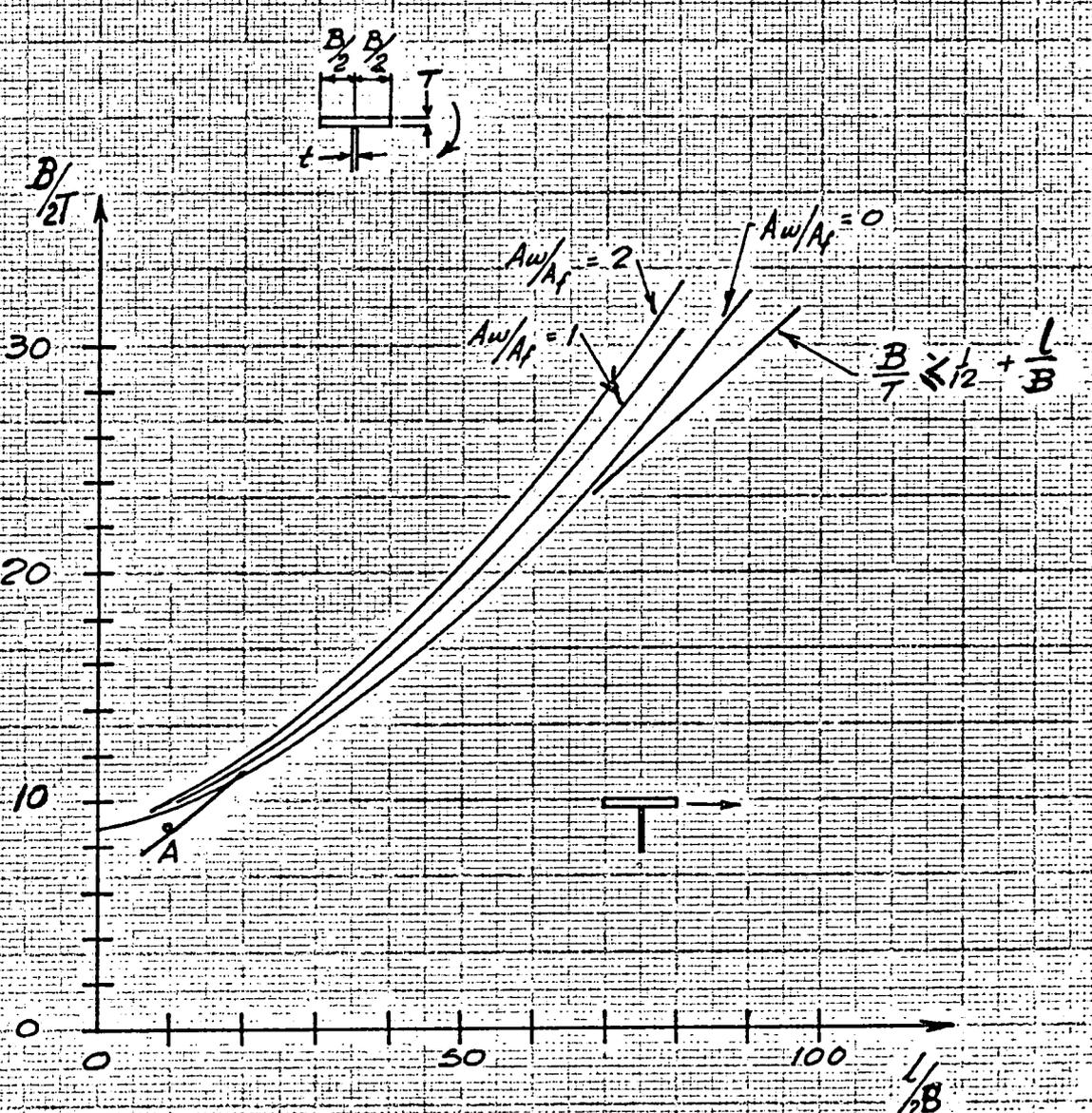
$$\lambda' = \frac{B}{2T} \sqrt{\frac{12(1-\nu^2)E_y}{\pi^2 K}}$$

$$K = 0.425$$



Torsional Buckling Of the Flange Plate (27)

Fig. 5 . 2 . 5



Boundary Between Lateral & Torsional Buckling of Flange. ⁽²⁷⁾

Fig. 5.2.6.

Design Considerations

- A. Ostapenko & others⁽²⁹⁾ stated that the ultimate strength of a plate girder under pure bending and having the large portion of the web in compression may be assumed to be controlled by the failure of the compression flange column. Ostapenko's equation for the ultimate moment

$$M_u = \frac{I}{y_a} \cdot \sigma_f \left\{ \frac{\sigma_{yw}}{\sigma_f} \left[\frac{I_w}{I} - 0.002 \frac{y_a t}{A_f} \left(\frac{y_a}{t} - 2.85 \sqrt{\frac{E}{\sigma_{yw}}} \right) + \left(1 - \frac{I_w}{I} \right) \right] \right\} \text{ ---- 16}$$

where I_w = 2nd moment of area web

σ_{yw} = yield stress, web

σ_f = buckling stress of the compression flange

limitation on equation 16 are the following⁽²⁹⁾; -

if $\sigma_{yw} > \sigma_f$ then use $\sigma_{yw} = \sigma_f$.

and if $\left(\frac{y_a}{t} - 2.85 \sqrt{\frac{E}{\sigma_{yw}}} \right) < 0$. ---- 17

then use $\left(\frac{y_a}{t} - 2.85 \sqrt{\frac{E}{\sigma_{yw}}} \right) = 0$. ---- 18

The buckling stress of the compression flange is calculated as follows⁽²⁹⁾:-

1. Lateral buckling:

Check $\frac{D}{T} \leq 12 + \frac{1}{D}$

$$\sigma_f = \left(1 - \frac{(\lambda'')^2}{4} \right) \sigma_y \text{ for } 0 < \lambda'' < \sqrt{2} \text{ ---- 19}$$

$$\text{or } \sigma_f = \frac{1}{(\lambda'')^2} \sigma_y \text{ ---- 20}$$

for $\lambda'' \geq \sqrt{2}$.

where l = unbraced length of the compression flange
(i.e. distance of vertical stiffener centres)

$$\text{and } \lambda'' = l \sqrt{\frac{\sigma_y}{E\pi^2} \cdot \frac{A_f + (\frac{1}{3}) Y_0 t}{I_f}} \quad \text{----- 21}$$

where I_f = 2nd moment of area of the compression flange
about the vertical axis.

2. Local torsional buckling:

check $\frac{B}{T} > 12 + \frac{l}{B}$

$$\sigma_f = [1 - 0.53 (\lambda''' - 0.45)^{1.36}] \sigma_y \quad \text{----- 22}$$

the equation 22 is for $0.45 \leq \lambda''' < \sqrt{2}$

$$\text{or } \sigma_f = \frac{1}{(\lambda''')^2} \sigma_y \quad \text{for } \lambda''' \geq \sqrt{2} \quad \text{----- 23}$$

$$\text{where } \lambda''' = \frac{B}{2T} \sqrt{\frac{12(1-\nu^2)\sigma_y}{0.425\pi^2 E}} \quad \text{----- 24}$$

B. Basler (27) stated that the strength prediction of a girder cross section, subjected to pure bending is not so difficult, because only three possible types of compression flange buckling have to be considered; but some difficulties in specifying admissible compressive flange stresses may appear. This is due to the presence of the following parameters, which influence the result:-

$\frac{l}{r}$, $\frac{B}{2T}$, $\frac{A_w}{A_f}$ and $\frac{d}{t}$ ratios.

As it has been seen before $\frac{l}{r}$ and $\frac{B}{2T}$ control lateral and torsional buckling of the compression flange and $\frac{A_w}{A_f}$ and $\frac{d}{t}$ influence the ultimate bending moment.

A slender web burdens the flanges with the stresses which the web cannot resist. This leads to an increase of the compressive flange stresses above the nominally calculated values.

b. In shear:- Basler⁽³⁰⁾ stated that the ultimate shear force V_u of a transversely stiffened plate girder, depends on the following variables:

- i) the stiffener spacing, a .
- ii) the clear depth of web plate between flanges, d .
- iii) web plate thickness
- iv) the material properties: yield stress and young's modulus (yield stress, $\sigma_y = 15.25 \text{ tonf/in}^2$ & $E = 13000 \text{ tonf/in}^2$).

Since plastic shear force, V_p has the dimension of a force, then it must be possible to express the ultimate shear force, V_u in the form of $V_u = V_p \cdot f(a, d, t, \sigma_y, E)$, where f is a function and is nondimensional. It is also possible to express E_y in terms of σ_y/E . All the above variables can only occur in ratios and let $\alpha = \frac{a}{d}$ & $\beta = \frac{d}{t}$. Thus, the ultimate shear load may be re-written in the form of $V_u = V_p \cdot f(\alpha, \beta, E_y)$.

From Von Mises' yield condition for plane stress, the shear yield stress $\tau_y = \frac{\sigma_y}{\sqrt{3}}$ (see fig 5.2.7) and the full plastic shear force is reached when yielding occurs throughout the web depth. Hence $V_p = \tau_y dt = \frac{1}{\sqrt{3}} \sigma_y dt$ ----- 25
From fig. 5.2.8 (a) it may be noticed that, a field of uniform tension stresses σ_t flows through the web's cross-section and the resulting shear force, V is dependent on the inclination ϕ of the tension stresses. Fig 5.2.8 (b) proves ⁽³⁰⁾ that the V is maximum when $\phi = 45^\circ$ (see fig 5.2.8).

$$\therefore V_{max} = \frac{1}{2} \sigma_t dt.$$

Thus a tension field, which is a membrane stress field, is dependent on the boundaries of the plate. These boundaries may be restricted by means of stiffeners, as

shown in fig 5.2.8 (c). A thin-web plate girder, which is subjected to shear, will reach a stage at which the compressive stress σ_2 (fig 5.2.7 a) ceases to increase, because the web deflects. When the shear force is increased yielding initiates along the tension diagonal and a further increase of the applied shear force causes a wider portion of the web to yield. As stated above, that the increase in field width increases by virtue of decrease in the inclination of the tension stress with respect to the girder axis. So, an optimum value of the tension field contribution ΔV_T to the shear force V_T is reached.

$$\text{Therefore } \frac{d'}{d\phi} (\Delta V_T) = \frac{d'}{d\phi} (\sigma_y s t \sin \phi) = 0$$

$$\text{or } \sigma_y t \left[\frac{d's(\phi)}{d\phi} \sin \phi + s \cos \phi \right] = 0$$

with $S(\phi) = d \cos \phi - a \sin \phi$, this reduces to

$$\text{or, } d \tan^2 \phi + 2 a \tan \phi - d = 0$$

$$\tan \phi = -a \pm \frac{\sqrt{a^2 + d^2}}{d}$$

$$\text{or, } \tan \phi = \sqrt{1 - \alpha^2} - \alpha \quad \text{----- 26}$$

$$\text{and } \sin \phi = \left[\frac{1}{2} - \frac{\alpha}{2\sqrt{1 + \alpha^2}} \right]^{\frac{1}{2}} \quad \text{----- 27}$$

$$\text{and } \cos \phi = \left[2\sqrt{1 + \alpha^2} (\sqrt{1 + \alpha^2} - \alpha) \right]^{-\frac{1}{2}} \quad \text{----- 28}$$

From the above equations, Basler⁽³⁰⁾ pointed out that the tension strip inclination is less than the inclination of the panel diagonal and the strip width a little wider than half the girder depth. Basler also stated that in plate girders with slender webs, the ultimate shear load is:-

$$V_u = V_T + V_T \quad \text{----- 29}$$

If any post buckling benefit is contributed by the tension field action, then

$$V_T = V_{cr} = \tau_{cr} dt = V_p \frac{\tau_{cr}}{\tau_y} \text{ ----- 30}$$

From equations 29 and 30 the ultimate shear load

$$V_u = V_p \left[\frac{\tau_{cr}}{\tau_y} + \frac{\sqrt{3}}{2} \cdot \frac{\sigma_E}{\sigma_y} \cdot \frac{1}{\sqrt{(1+L^2)}} \right] \text{ ----- 31}$$

Where, $\frac{\sigma_E}{\sigma_y} = \sqrt{1 + \left(\frac{\tau_{cr}}{\tau_y}\right)^2 \left\{ \left[\frac{3}{2} \sin 2\phi \right]^2 - 3 \right\}} - \frac{3}{2} \frac{\tau_{cr}}{\tau_y} \sin 2\phi$

for limiting case of $\phi = 45^\circ$

$$\sigma_1 = \tau_{cr} + \sigma_E$$

$$\& \quad \sigma_2 = -\tau_{cr}$$

then, $\frac{\sigma_E}{\sigma_y} = 1 - \frac{\tau_{cr}}{\tau_y} \text{ ----- 32}$

using equations 31 and 32

$$\frac{V_u}{V_p} = \frac{\tau_{cr}}{\tau_y} + \frac{\sqrt{3}}{2} \cdot \frac{1 - \frac{\tau_{cr}}{\tau_y}}{\sqrt{(1+L^2)}} \text{ ----- 33}$$

Where V_u = ultimate shear force

" V_p = plastic shear force

" τ_y = yield stress = $\frac{\sigma_y}{\sqrt{3}}$ where $\sigma_y = -\tau_{cr} \sin 2\phi$

" L = a/d

$$\& \quad \tau_{cr} = \kappa(L) \frac{\pi^2 E}{12(1-\nu^2)} \cdot \left(\frac{t}{a}\right)^2$$

when $L \geq 1$, $\kappa = 5.34 + \frac{4}{L^2}$

and when $L \leq 1$, $\kappa = 4 + \frac{5.34}{L^2}$; this value for κ is only for shear buckling coefficient for pin-ended rectangular plates.

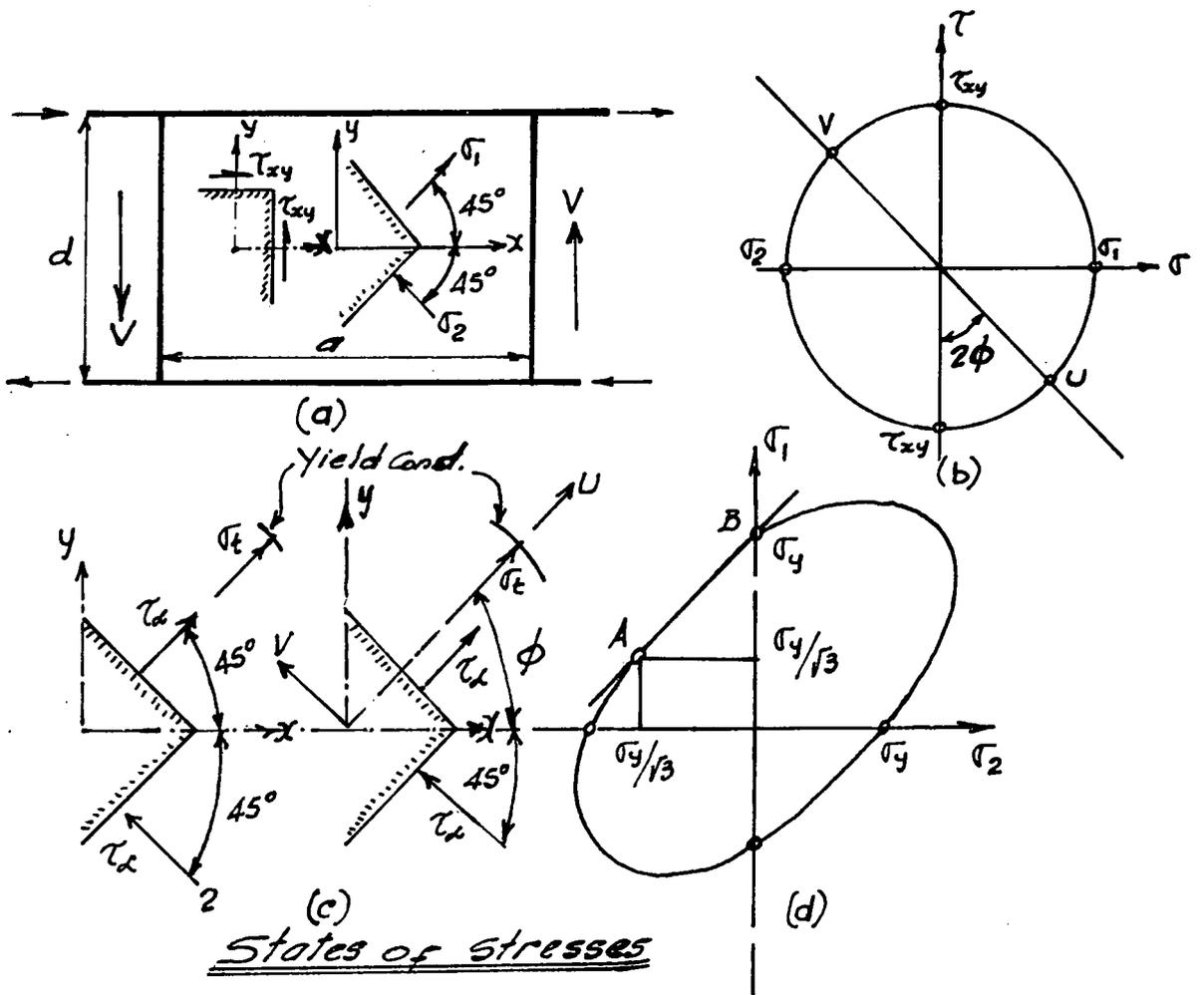


Fig. 5.2.7

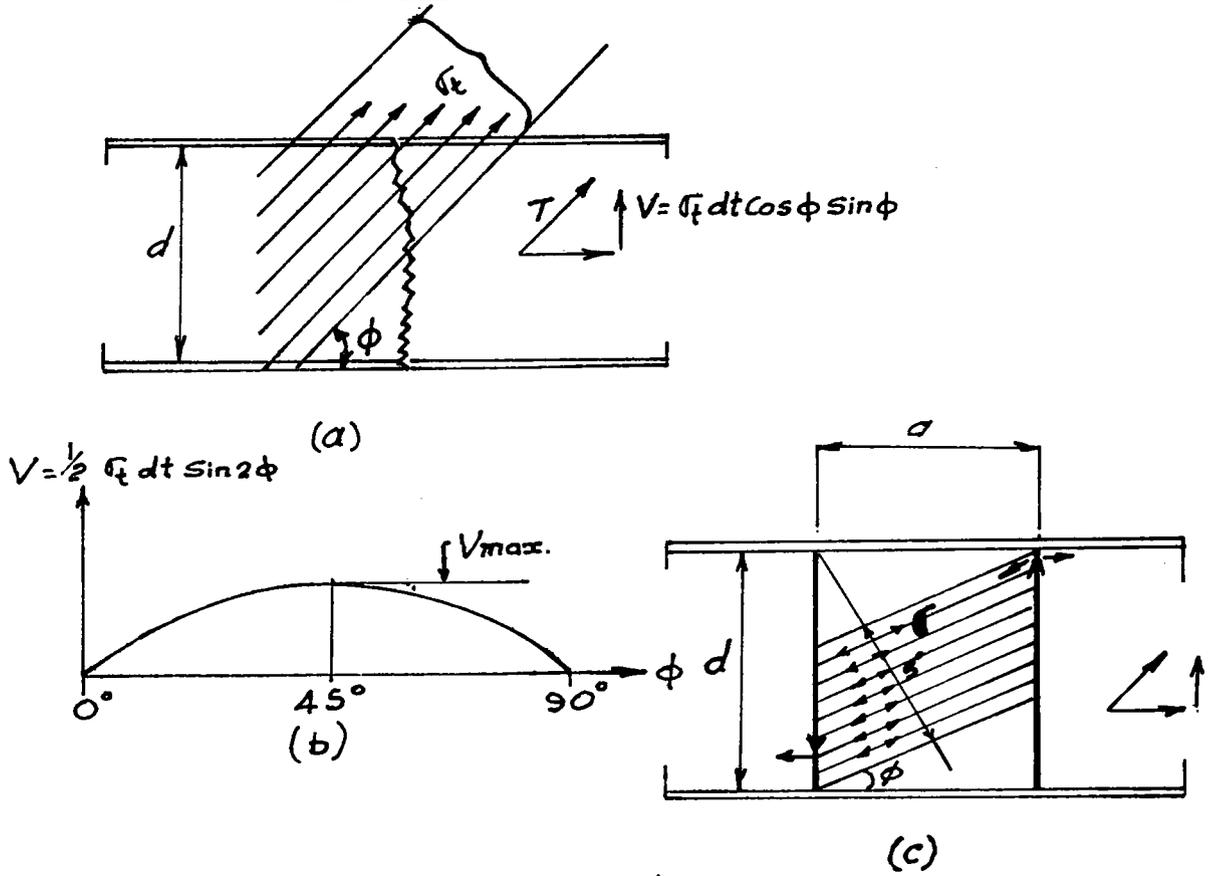


Fig. 5.2.8

An alternate way Basler has expressed the following values for the critical shear stress:-

$$\text{for } L < 1 \quad \& \quad K = 5.34 + 4L^2$$

$$\tau_{cr} = \frac{K \pi^2 E}{12(1-\nu^2)} \cdot \left(\frac{t}{a}\right)^2 \quad \text{----- 34}$$

$$\text{for } L > 1 \quad \& \quad K = 5.34 + \frac{4}{L^2}$$

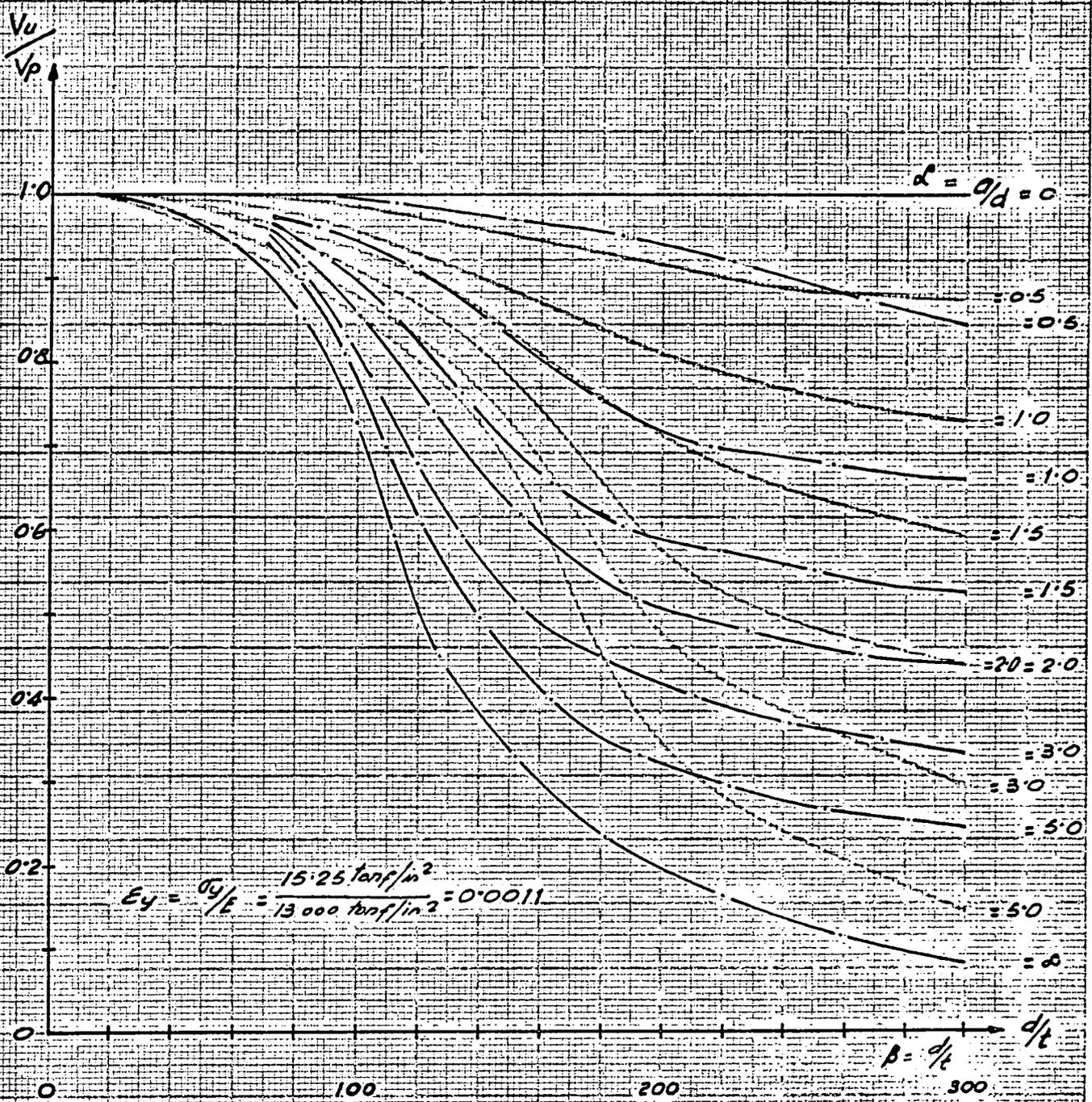
$$\tau_{cr} = \frac{K \pi^2 E}{12(1-\nu^2)} \cdot \left(\frac{t}{d}\right)^2 \quad \text{----- 35}$$

Fig. 5.2.9 shows the dependance of ultimate shear force on a/d and d/t ratio.

It is obvious that it is necessary to provide the load bearing stiffeners at the point of a plate girder where point loads are applied, it is also necessary to provide intermediate stiffeners to preserve the shape of the plate girders cross-section and to ensure post-buckling strength. To preserve the shape of the cross-section, a certain minimum number of stiffeners are required at certain cross-centre and to ensure the post-buckling strength, the stiffeners should have a certain minimum cross-sectional area, (A_s). To determine these two factors, the girder's carrying capacity may be divided in two parts:-

- i) Simple beam action up to $\tau = \tau_{cr}$ and
- ii) Tension field action up to yielding in the web.

Basler stated that, the shear stress corresponding to simple beam action, causes no axial load in the stiffener in any respect, the stiffener should only be rigid enough to force, at its location, a nodal line in the lateral deflection mode of the web. In tension field, the stiffener must resist the vertical components of the diagonal stresses out of the web at one end and transfer them to its other end, i.e. it should have the capacity to sustain compression.



Basler's equation $\frac{V_u}{V_p} = \frac{T_{cr}}{T_y} + \frac{\sqrt{3}}{2} \frac{1 - \frac{T_{cr}}{T_y}}{\sqrt{1 + L^2}}$

Rockey's equation 48. in conjunction with equation 46

Dependence of the Ultimate Shear Force on L & β

Fig. 5.2.9

According to Basler, force in the stiffener.

$$F_s = \sigma_E \cdot t \cdot d \left(\frac{L}{2} - \frac{L^2}{2\sqrt{1+L^2}} \right) \text{ ----- } 36$$

for maximum shear force the value of σ_E may be taken from the equation 32

When $L < 1$

$$F_s = dt\sigma_y \left(1 - \frac{\tau_{cr}}{\tau_y}\right) \frac{L}{2} \left(1 - \frac{L}{\sqrt{1+L^2}}\right) \text{ ----- } 37$$

when $L \geq 1$ & $E_y \beta^2 \geq 10.5 + \frac{7.8}{L^2}$,

$$F_s = d^2 \sigma_y \left[\frac{1}{2\beta} - \left(4.2 + \frac{3.1}{L^2}\right) \frac{1}{E_y \beta^3} \right] \left(L - \frac{L^2}{\sqrt{1+L^2}}\right) \text{ ----- } 38$$

The stiffener force F_s can be resisted by the actual area of the stiffeners A_s , because when the tension field is already formed, the part of the web at this stage is unrestricted to yielding and no additional stresses can be borne by the web. In case of stiffeners used in pairs, (see fig.5.2.10 a), the area required is, $A_s = \frac{F_s}{\sigma_u}$. But if local buckling is to be avoided then,

$$A_s \geq 0.015 \sqrt{E_y} (d)^2 \text{ ----- } 39$$

In the case where a single plate (one sided) stiffener is used (see 5.2.10 b) then,

$$A'_s = \frac{F_s}{0.25 \sigma_y} \text{ ----- } 40$$

But if F_s is increased and the stiffener is proportioned in such a way that the unrestricted yielding is possible prior to stiffener buckling, then

$$A'_s = \frac{F_s}{0.414 \sigma_y} \text{ ----- } 40 a.$$

when $E_y = 0.0011$, Basler said that the maximum value of $F_s = 5.0 \times 10^{-4} \sigma_y d^2$, which gives

yielding along the loading edge: $A'_s = 0.0020 d^2$
 and yielding all over the cross-section: $A'_s = 0.0012 d^2$ } ----- 41

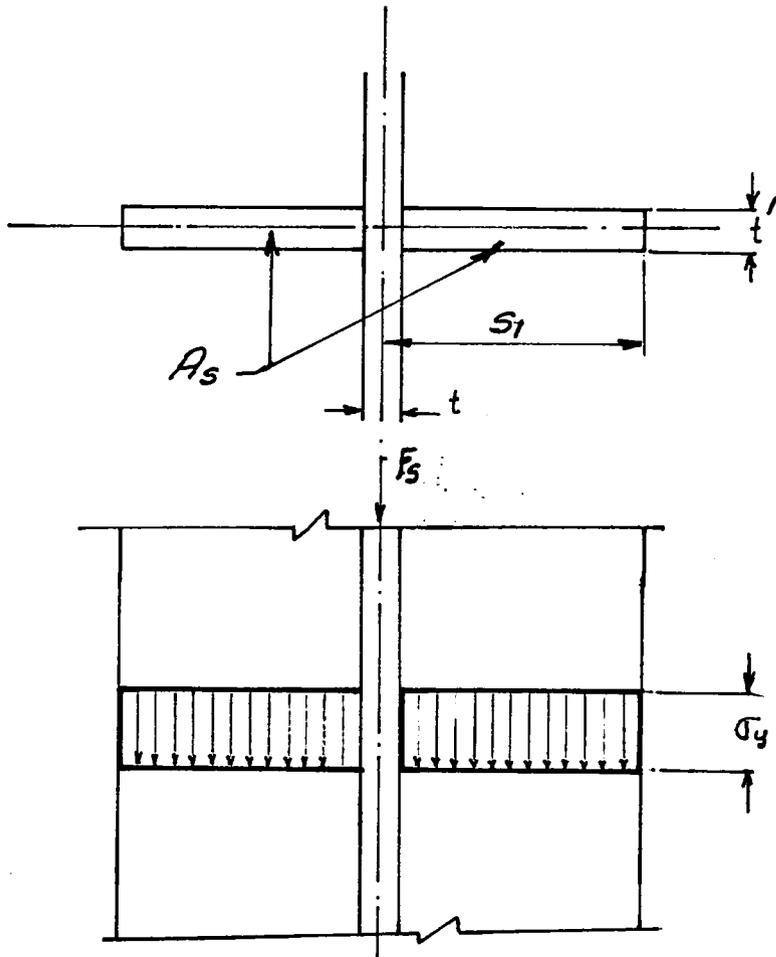
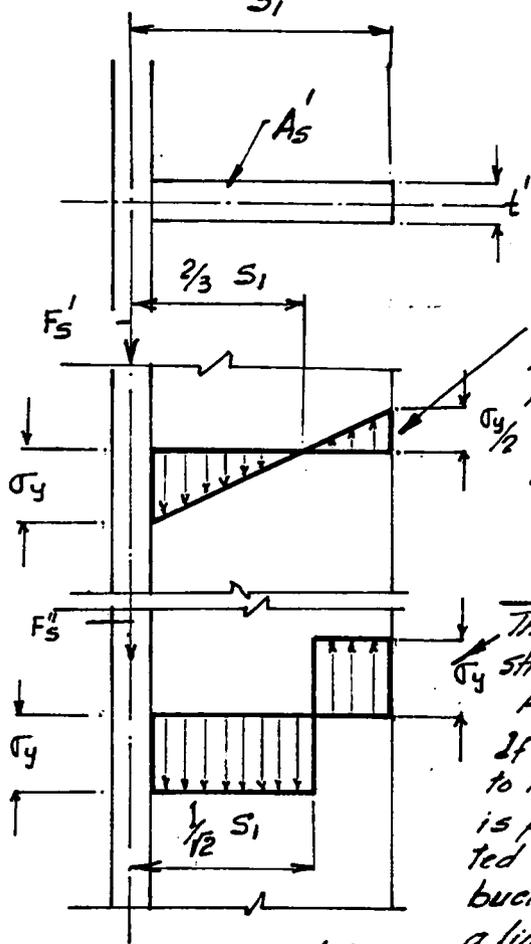
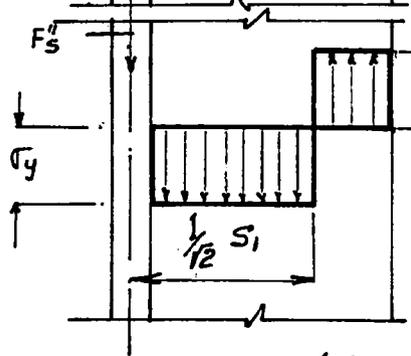


Fig. 6·2·10(a)
Two-sided stiffeners



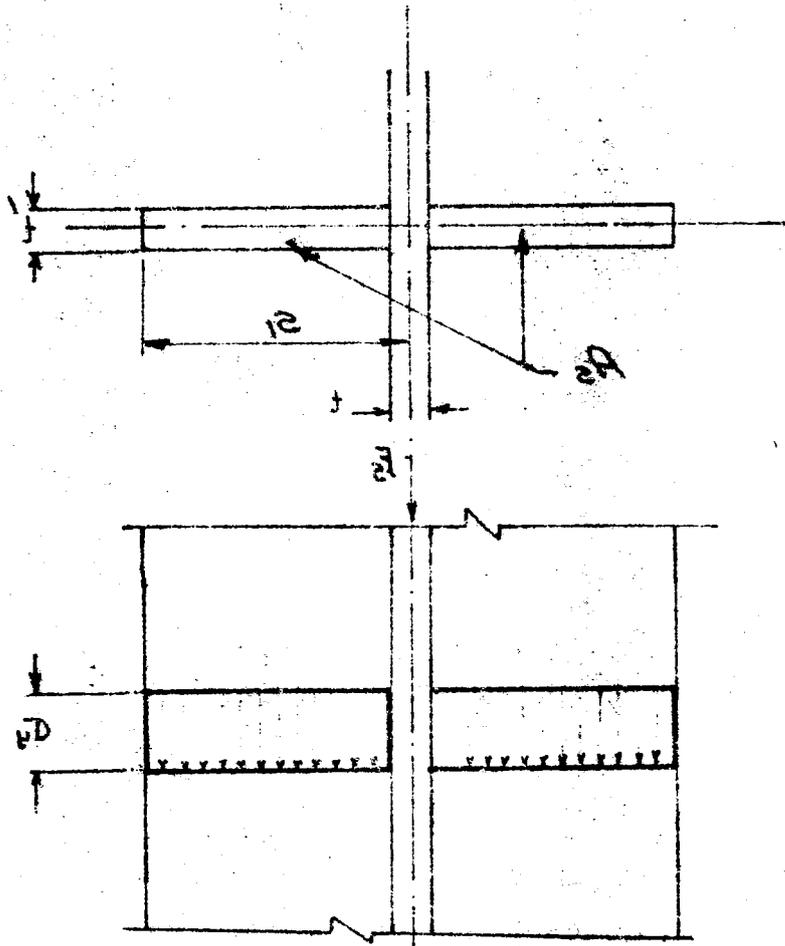
The axial load causes this stress distribution is $F_s' = 0.25 \sigma_y A_s'$.
The stiffener is loaded along one of its edge.



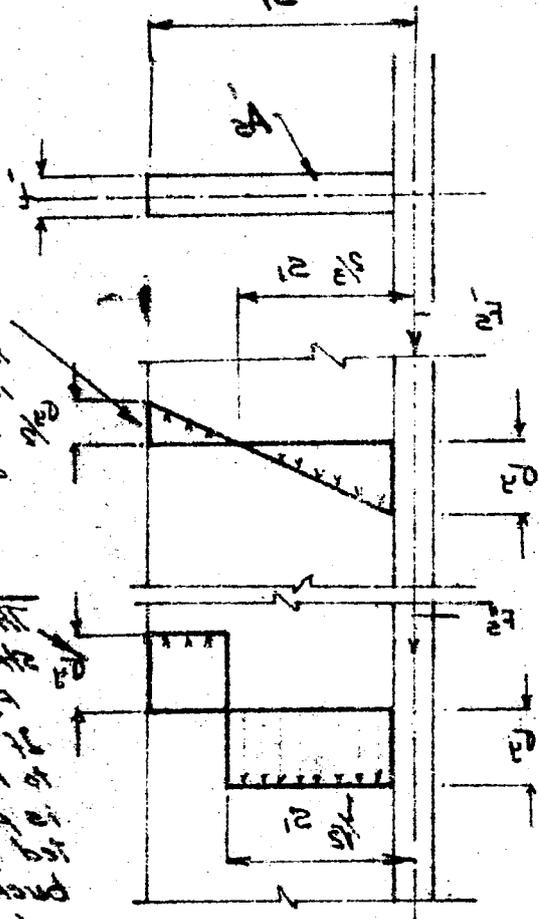
The axial load causes this stress distribution is $F_s'' = 0.414 \sigma_y A_s''$.
If the load F_s'' is increased to F_s''' and stiffener plate is proportioned to unrestricted yielding prior to plate buckling, the load will reach a limiting magnitude.

Fig. 6·2·10(b)

One-sided stiffeners



Two-sided stiffener
Fig. 2.10(a)



One-sided stiffener
Fig. 2.10(b)

The axial load causes this stress distribution is $f_s = 0.5P_2/t$. The stiffener is loaded along one of its edges.

The axial load causes this stress distribution is $f_s = 0.414P_2/t$. If the load P_2 is increased to P_2 and stiffener plate is proportional to increase in yielding over to plate buckling, the load will reach a limiting magnitude.

In the previous sections it was intended to describe Basler's design method of the plate girder, in shear. Now it will be appropriate to consider Rockey's method of dealing with the shear problem. Fig. 5.2.11 shows Rockey's assumption of modes of shear failure.

He stated that, if **there** are no imperfections in the web plate of a plate girder, then prior to buckling it does not impose any lateral loading upon the boundary members. But once the web has buckled, it has no capacity to carry any more compressive load across the diagonal ($x-y$) and therefore the web has to carry the additional shear loads by a diagonal tensile membrane action, which has been referred as "truss type action"⁽³²⁾. This membrane action may impose lateral forces upon the flanges and so the girder may fail due to the formation of the plastic hinges in the flanges. This type of failure depends on the stiffness of the flange plates and as the buckling stress increases, the membrane action decreases. If the flange plates are stiff enough, then the web and the stiffeners develop a full membrane action. When the web has developed the full membrane action, then it cannot carry any more shear force, and at that point the flanges and the stiffeners carry the additional shear force which together act as Vierendeel frame.

Basler⁽³⁰⁾ assumed that the web plate may fail due to the developments of the inclined plastic band (see fig 5.2.8) anchored against the vertical stiffeners. This assumption is based on the theory that the flanges of most of the plate girders are too flexible to resist the membrane action. Rockey said that, this type of collapse mode may "significantly underestimate and overestimate" the strength of the girder. According to Rockey, when a web plate in shear, buckles prior to yielding, then the failure is due to the development

of a diagonal tension band which is fully yielded together with the development of the plastic hinges in the flange to form a mechanism (see fig.5.2.11 and 12). Rockey has proved by experiments that the width of the diagonal band, s (see fig 5.2.8 c) and the position of the plastic hinges depends on the $I/a^3.t$ ratio, Where I = flexural rigidity of the compression flange about an axis through its centroid and perpendicular to web plate.

" a = spacing between the transverse stiffeners.

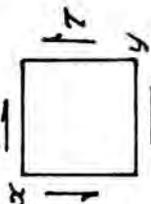
" t = thickness of the web plate.

Rockey's theory of shear web design: As in welded plate girders, flanges have low torsional rigidity⁽³²⁾, it has been assumed that the shear web is simply supported on all edges. Rockey's⁽³³⁾ previous experiments show that the web develops a tension band (see fig.5.2.12). The angle of inclination of the tension band, (ϕ) is equal to the inclination of the geometrical diagonal and the tension band is symmetric with respect to the geometric diagonal. The width of this tension band is assumed to be such that the intercept of its boundary with the flange coincides with the position of the plastic hinges in the flanges.

Rockey and Basler both agree that the angle of inclination of the tension field will be close to 45° . In a vertically web stiffened plate girder (see fig.5.2.13), it may be seen that there are two stress regions, i) two triangular wedges in which the critical shear stress subjected to act, ii) a yielded diagonal strip. According to Rockey, the tension stress, σ_t acts uniformly over the diagonal band and yield occurs, when σ_t reaches a value of σ_{ty} .

Stage I : Pure Shear behaviour

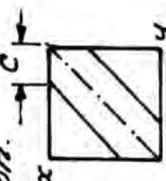
For $T \geq T_{cr}$ the web operates as a pure shear field.



No lateral loading imposed on boundary frame

Stage II : Post buckled behaviour

For $T > T_{cr}$ the web has to carry any shear load in excess of T_{cr} by a truss action.



As a result of this truss action the web imposes lateral & axial loading on the flanges & stiffener

Stage III : Collapse modes

Local instability of outstanding flanges or overall lateral instability of girder

Plastic hinges develop in flanges together with the web yielding over area shown to form a mechanism.



Position of hinge varies with flexibility of flanges

Flange rigidity is high such that the lateral loading imposed by



The tension field is sufficient to produce a collapse mechanism and web yields over entire area.

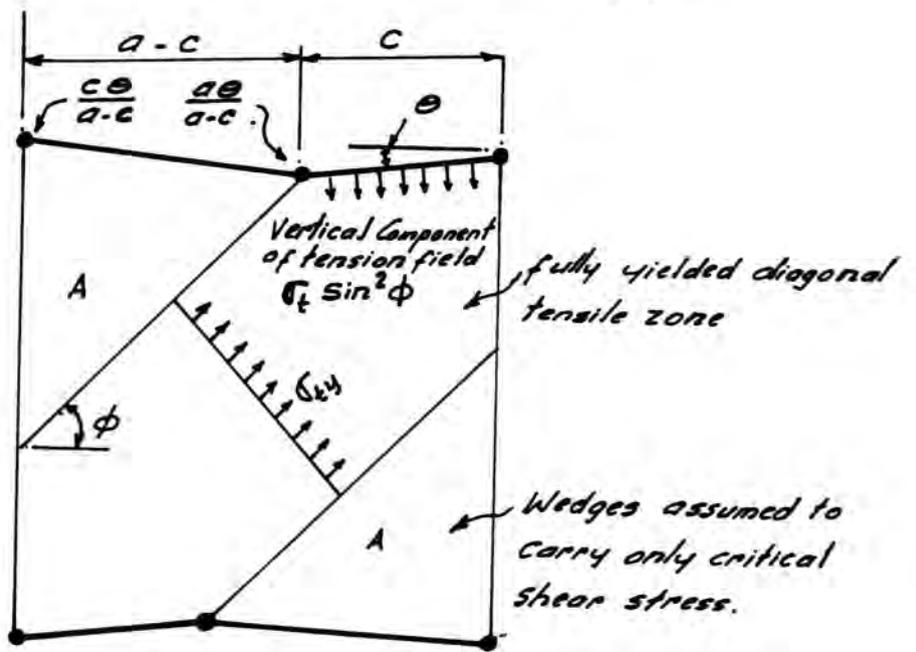
No additional load can be carried by web. All additional load carried by framework comprising flanges and stiffeners.

Girder then fails on the application of further load when either :

1. Creation of an overall frame failure mechanism of the boundary frame operating as a Vierendeel frame.
2. Web tears under tensile loading initiated at a stress concentration.
3. Attachments fail in the case of a bolted, riveted or welded girder or weld fractures in the case of welded plate girders.

Rockey's assumption of possible modes of failure.

Fig. 5.2.11.



Collapse Mechanism Proposed by Rockey
 Fig. 5.2.12.

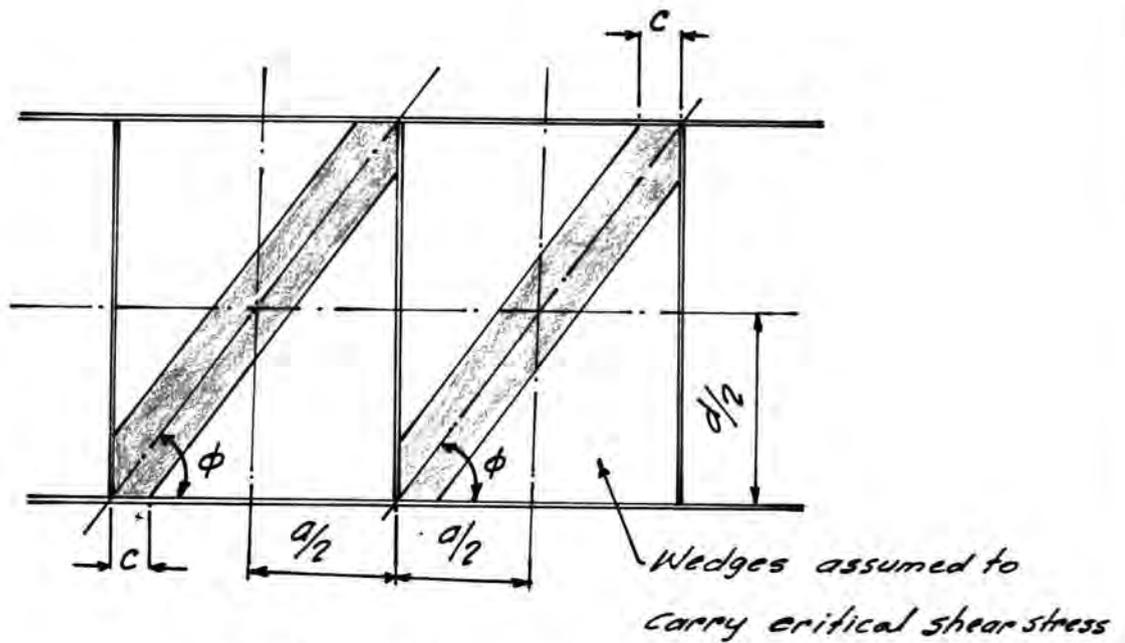


Fig. 5.2.13.

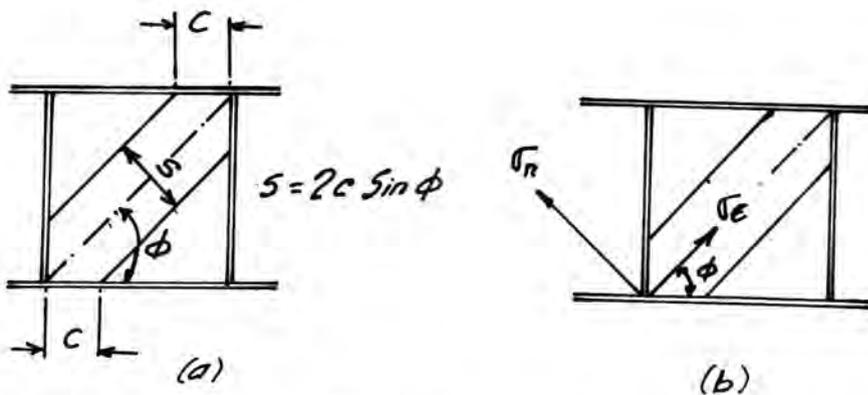


Fig. 5.2.14

Under the stress condition as shown in fig 5.2.14, the stress in the diagonal web strip is given by

$$\left. \begin{aligned} \sigma_E &= \tau_{cr} \sin 2\phi + \sigma_{ty} \\ \sigma_n &= \tau_{cr} \sin 2\phi \\ \& \quad \tau &= \tau_{cr} \cos 2\phi \end{aligned} \right\} \text{----- 42}$$

Now Rockey has used the Huber Von Mises plasticity condition, that is, $\sigma_c = \sigma_{yw}$ where, σ_c = maximum compressive stress as per Mises & σ_{yw} = tensile yield stress in web.

$$\text{but, } \sigma_c = \sqrt{\sigma_E^2 + \sigma_n^2 - \sigma_E \sigma_n + 3\tau^2} \text{----- 43}$$

substituting equations 42 into 43,

$$\sigma_{ty} = -\frac{3}{2} \tau_{cr} \sin 2\phi + \sqrt{\sigma_{yw}^2 + \tau_{cr}^2 \left[\left(\frac{3}{2} \sin 2\phi \right)^2 - 3 \right]} \text{----- 44}$$

The vertical component V_f due to the diagonal stress σ_{ty} may be written as

$$V_f = 2ct \sin^2 \phi \left(-\frac{3}{2} \tau_{cr} \sin^2 \phi + \sqrt{\sigma_{yw}^2 + \tau_{cr}^2 \left\{ \left(\frac{3}{2} \sin 2\phi \right)^2 - 3 \right\}} \right) \text{----- 45}$$

The critical shear force to cause the web plate to buckle,

$$V_{cr} = \tau_{cr} dt$$

$$\text{and } \tau_{yw} = \frac{\sigma_{yw}}{\sqrt{3}}$$

$$\text{Now, } V_u = V_{cr} + V_f$$

$$\therefore V_u = \frac{\tau_u}{\tau_{yw}} = \frac{\tau_{cr}}{\tau_{yw}} + 2\sqrt{3} \left(\frac{c}{d} \right) \sin^2 \phi \left[-\frac{\sqrt{3}}{2} \sin 2\phi \left(\frac{\tau_{cr}}{\tau_{yw}} \right) + \sqrt{1 + \left(\frac{\tau_{cr}}{\tau_{yw}} \right)^2 \left\{ \frac{3}{4} \sin^2(2\phi) - 1 \right\}} \right] \text{---46}$$

Values of C may be theoretically found by using Rockey's collapse mechanism as shown in fig. 5.2.12.

His collapse mechanism assumes that the hinge coincides with the edge of diagonal strip and the loading consists of the vertical component of the diagonal tensile membrane stress σ_{ty} . Then the solution of this mechanism reduces to the solution of the equation,

$$(c/a)^3 - (c/a)^2 + \frac{4Z_{pf}\sigma_{yf}}{a^2t\sin^2\phi(\sigma_{ty})} = 0 \quad \text{----- 47}$$

Where Z_{pf} = plastic modulus of the flange plate.

Rockey proposes that when the web buckling stress is less than half the shear yield stress, a depth of web plate

$Z = 30t(1 - \frac{2\tau_{cr}}{\tau_{yw}})$ be assumed to act with the flange assembly.

Rockey provides the following limiting conditions to satisfy the equation 46:

a) for very thin web and rigid flanges

$\tau_{cr} \approx 0$ then,

$$\frac{\tau_w}{\tau_{yw}} = 2\sqrt{3} \frac{c}{a} \sin^2\phi$$

but for rigid flanges $\frac{c}{a} = 0.5$ and for square web panel $\phi = 45^\circ$

then $\tau_w = \frac{\sqrt{3}}{2} \tau_{yw}$ or $\frac{\tau_{yw}}{2}$

b) for very thick web $\tau_{cr} \approx \tau_{yw}$

$\therefore \tau_w = \tau_{yw}$

c) for very flexible flanges

$\frac{c}{a} \approx 0$ if the flanges have zero stiffness and cannot withstand any lateral force,

then $\tau_w = \tau_{cr}$

If $\sqrt{3}\tau_{cr}$ exceeds the limit of proportional stress, then the effective modulus is less than modulus of elasticity (E) and this reduces the critical stress.

Basler has recommended that τ_{cr} should be replaced by τ'_{cr} when $\tau_{cr} > \frac{0.8\sigma_y}{\sqrt{3}}$

and Rockey stated that

$$\frac{\tau'_{cr}}{\tau_{yw}} = 1 - \frac{0.16\tau_{yw}}{\tau_{cr}} \quad \text{----- 48}$$

In fig. 5.2.9 dependence of the ultimate shear force on a/d and d/t have been plotted for both Rockey's and Basler's equations.

Ostapenko and others⁽²⁹⁾ derived the following formulas for the case of pure shear, on the basis of the beam action theory: For a plate girder, with the area and thickness of the tension and compression flanges equal (as in case of this project) and for $\lambda \leq 0.58$ (strain-hardening range),

$$V_u = V_p \left\{ [1 + 4.3(0.58 - \lambda)^{1.56}] + \sqrt{3} \frac{A_f T}{\sigma A_w} \left(\frac{\sigma_{yf}}{\sigma_{yw}} \right) \right\} \quad \text{----- 49}$$

for $0.58 < \lambda \leq \sqrt{2}$ (elastic-plastic range)

$$V_u = V_p \left\{ [1 - 0.615(\lambda - 0.58)^{1.18}] + \left[\frac{0.6\lambda - 0.348}{\sqrt{\lambda^2 + 1.6}} \right] + \sqrt{3} \frac{A_f T}{\sigma A_w} \left(\frac{\sigma_{yf}}{\sigma_{yw}} \right) \right\} \quad \text{----- 50}$$

for $\lambda > \sqrt{2}$ (elastic range)

$$V_u = V_p \left\{ \left[\frac{1}{\lambda^2} \right] + \left[\frac{0.9 - (0.787/\lambda^2)}{\sqrt{\lambda^2 + 1.6}} \right] + \sqrt{3} \frac{A_f T}{\sigma A_w} \left(\frac{\sigma_{yf}}{\sigma_{yw}} \right) \right\} \quad \text{----- 51}$$

Where V_p = plastic shear of the web = $\frac{1}{\sqrt{3}} \sigma_{yw} A_w$

$$\text{and } \lambda = \frac{d}{t} \sqrt{\frac{12(1-\nu^2)}{\sqrt{3} \pi^2 E}} \frac{\sigma_{yw}}{\kappa} \quad \text{----- 52}$$

κ = plate buckling coefficient. Assuming the web is fixed

at the flanges and simply supported at the transverse stiffeners (agrees with Rockey's statement⁽³²⁾),

$$K = \frac{5.34}{L^2} + \frac{6.55}{L} - 13.71 + 14.10L \quad \text{----- } 53$$

where $L < 1.0$

$$\text{or, } K = 8.98 + \frac{6.18}{L^2} - \frac{2.88}{L^3} \quad \text{----- } 54$$

where $L \geq 1.0$

It has been established by Rockey⁽³³⁾ that the position of the plastic flange hinge depends on the flange's relative stiffness. The partial tension field (see fig.5.2.13 and 14) renders the ultimate strength of a plate girder under pure shear as:-

$$V_u = \frac{ad^2t\sigma_{yw}}{4(a-c)^2 + d^2} \quad \text{----- } 55$$

according to Herzog⁽³⁴⁾, this equation is valid for

$$0.75 < L < 1.6 \text{ only.}$$

If $L < 0.75$ but > 1.6 then this equation to be multiplied by the factors

$$F_1 = \frac{3}{4} \frac{d}{t} > 1$$

$$F_2 = L - 0.6 > 1$$

$$\left. \begin{array}{l} \text{thus } V'_u = F_1 V_u \\ \text{or } V'_u = F_2 V_u \end{array} \right\} \text{----- } 56$$

Equation 55 and 56 are valid only when $\frac{d}{t} > 140$. For webs with an intermediate slenderness⁽³⁴⁾,

$$V_u = V_p - (V_p - V'_u) \left(\frac{d/t - 70}{70} \right) \quad \text{----- } 57$$

Fig.5.2.15 shows the pattern of a collapse mechanism which occurs in a longitudinally stiffened web of a plate girder. Considering Panel 1, it is stated⁽³²⁾ that the

panel will impose lateral loading on the flange and a yield zone will develop as indicated with hinges forming in the flanges, the position of the hinge, C_1 in the flange varying with the rigidity of the flange and the buckling stress in the panel. Panel 2 which is the adjacent panel, will act as a very stiff flange, at the position of the longitudinal stiffener. The position of the hinge is assumed by Rockey, at $0.5a$. Therefore the shear load in panel 1,

$$V_1 = [\tau_{cr1} d t + t \sin^2 \phi_1 \sigma_{ey1} (c_1 + 0.5a)] \text{ ----- } 58$$

and

$$V_2 = [\tau_{cr2} d t + (0.5a + c_2) \sin^2 \phi_2 \sigma_{ey2}] \text{ ----- } 59$$

But total shear load on the whole web,

$$V = V_1 + V_2, \text{ (i.e., equation 58 + 59) ----- } 60$$

Longbottom and Heyman⁽⁸⁾ derived the following design formulas, after experimenting with numbers of miniature welded plate girders with d/t ratio of 85, 241, 299 and 401, compared with B.S.153 normal limits BS, 240, 300 and 400:-

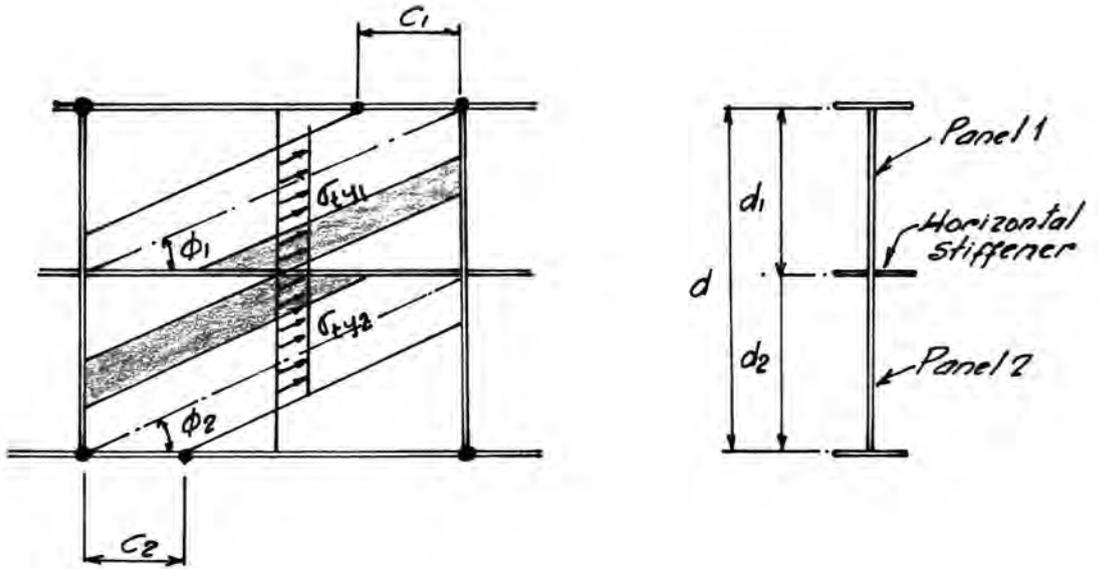
- i) the applied loads should be multiplied by a suitable load factor and the bending moment, and the shear force should be calculated at the plastic hinges.
- ii) the average shear stress, $\tau_0 = \frac{V}{dt}$ and should not exceed $\frac{1}{\sqrt{3}}$ times the yield stress, σ_{yw} (of the web)
- iii) the permissible plastic bending stress, σ_p in the web should be

$$\sigma_p = \sqrt{\sigma_{yw}^2 - 3\tau_0^2} \text{ ----- } 61$$

- iv) flange should be designed as $M = M_f + M_w$ ----- 62

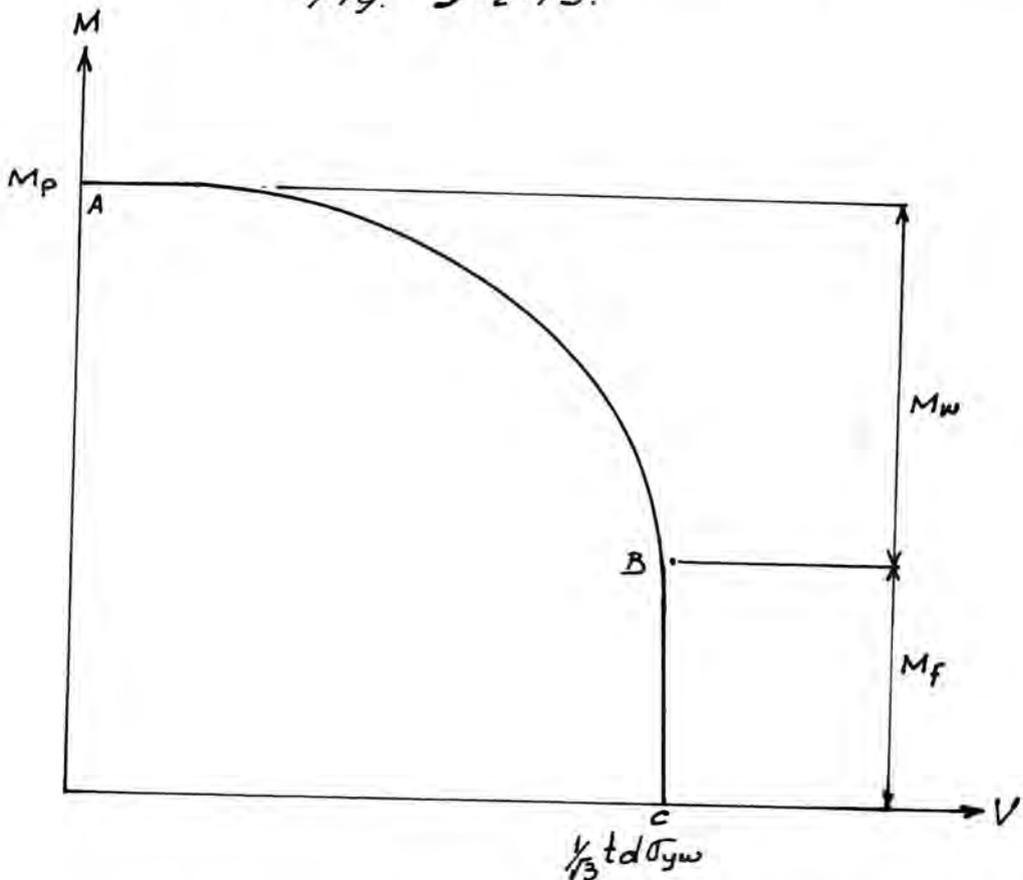
where $M_f = BT(d+T)\sigma_{yw}$

and $M_w = \frac{1}{4} t d^2 \sigma_p$.



Proposed Collapse Mechanism for a longitudinally stiffened web subjected to shear - by Roakey.

Fig. 5.2.15.



Moment & Shear force for fully Plasticity (schematic)

Fig. 5.2.16

Referring to fig 5.2.16, the vertical line Bc at $V = \frac{1}{\sqrt{3}} t d \sigma_{yn}$ implies that the web fails in shear and the girder can carry any amount of moment up to M_f , which is the moment of resistance of the flange above. The curve represents a girder with d/t ratio of 85. B.S.153 allows a maximum permissible shear stress of 6 tonf/in², but a mild steel (B.S.4360 grade 43A), having a yield stress of 15.25 ton f/in² will yield in shear at $\frac{15.25}{\sqrt{3}} = 8.80$ tonf/in². So, the load factor for a simply supported girder should not be greater than $\frac{8.80}{6} = 1.47$. Similarly, the load factor in bending (to 9.5 tonf/in² as stated in B.S.153) is about 1.85 for a simply supported girder. Longbottom and Heyman suggested that, a uniform load factor of 1.75 should be appropriate. Horne suggested that, if the depth of the plate girder is moderately larger than the flange thickness then

$$M = d \left(A_f + \frac{A_w}{6} \right) \sigma_b \quad \text{----- 63}$$

and $V = A_w \tau_a \quad \text{----- 64}$

If failure occurred with combined bending and shear deformation, then the mean web shear stress $N \tau_a$, where N is the load factor, and the equation of the moment of resistance should be

$$NM = d \left\{ A_f \sigma_{yn} + \frac{A_w}{4} \sqrt{\sigma_{yn}^2 - 3N^2 \tau_a^2} \right\} \quad \text{----- 65}$$

Substituting the value of V in equation 63 and denoting

$$r' = \frac{A_f}{A_w} \quad \text{ratio}$$

$$N = \frac{3 \sigma_{yn} \left\{ 8r'(1+6r')\sigma_b + \sqrt{[4(1+6r')^2\sigma_b^2 + 27(1-16(r')^2)\tau_a^2]} \right\}}{4(1+6r')^2\sigma_b^2 + 27\tau_a^2} \quad \text{----- 66}$$

When A_f is larger than A_w then the section may fail in shear without bending. In that case $N = \frac{1}{3} \left(\frac{\sigma_{yn}}{\tau_a} \right)$. Horne stated that the correct value of load factor is lower of the two above estimates. It has been assumed that

the design of the web plate is governed by the allowable shear stress. If some limitation of shear stress is made by τ_a' , then the equation 64 should be re-written as

$$V = \frac{2}{3} \left(\frac{1 + 6r'}{1 + 4r'} \right) A_w \tau_a' \text{ ----- 67}$$

At a load factor N , the mean shear stress in the web becomes

$\tau_a = \frac{2}{3} \left(\frac{1 + 6r'}{1 + 4r'} \right) N \tau_a'$ and that quantity should be substituted for $N \tau_a$ in the equation 65, also substituting the value for M from equation 63 to equation 65, the expression for the load factor becomes,

$$N = \frac{3(1 + 4r') \sigma_{yw} \{ 4r'(1 + 4r') \sigma_b + \sqrt{[(1 + 4r')^2 \sigma_b^2 + 9(1 - 16(r')^2)(\tau_a')^2]} \}}{4(1 + 6r')^2 \sigma_b^2 + 27(\tau_a')^2} \text{ ----- 68}$$

The values of N calculated from the equations 66 and 68 have been shown in fig. 5.2.17. The values for σ_b , τ_a and σ_y have been taken from B.S.153 part 3B, table 3. Horne stated that when $r' = \frac{3\tau_a - 2\tau_a'}{12(\tau_a' - \tau_a)}$, then the

minimum load factor for value is obtained.

In fig 5.2.18, Horne suggested that, this chart may be used to find the minimum practical load factor obtained under combined loading, because the maximum shear stress may not be so limited that, it will control the design of normal section. He also stated that the load factor of 1.85 for pure bending and 1.47 for pure shear failure which has been suggested by Longbottom and Heyman is not justifiable, because the difference is too much. Horne's suggestion is that safety factors higher than 1.5 are nowhere required.

A.I.S.C. proposed that when $\epsilon_y = 0.0011$, then the factors against ultimate load and yielding are $N_u = 1.65$ and $N_y = 1.46$.

Basler suggested that the $N_u = 1.83$ and $N_y = 1.73$. If Basler's suggested factors are considered then the equation 33 becomes:-

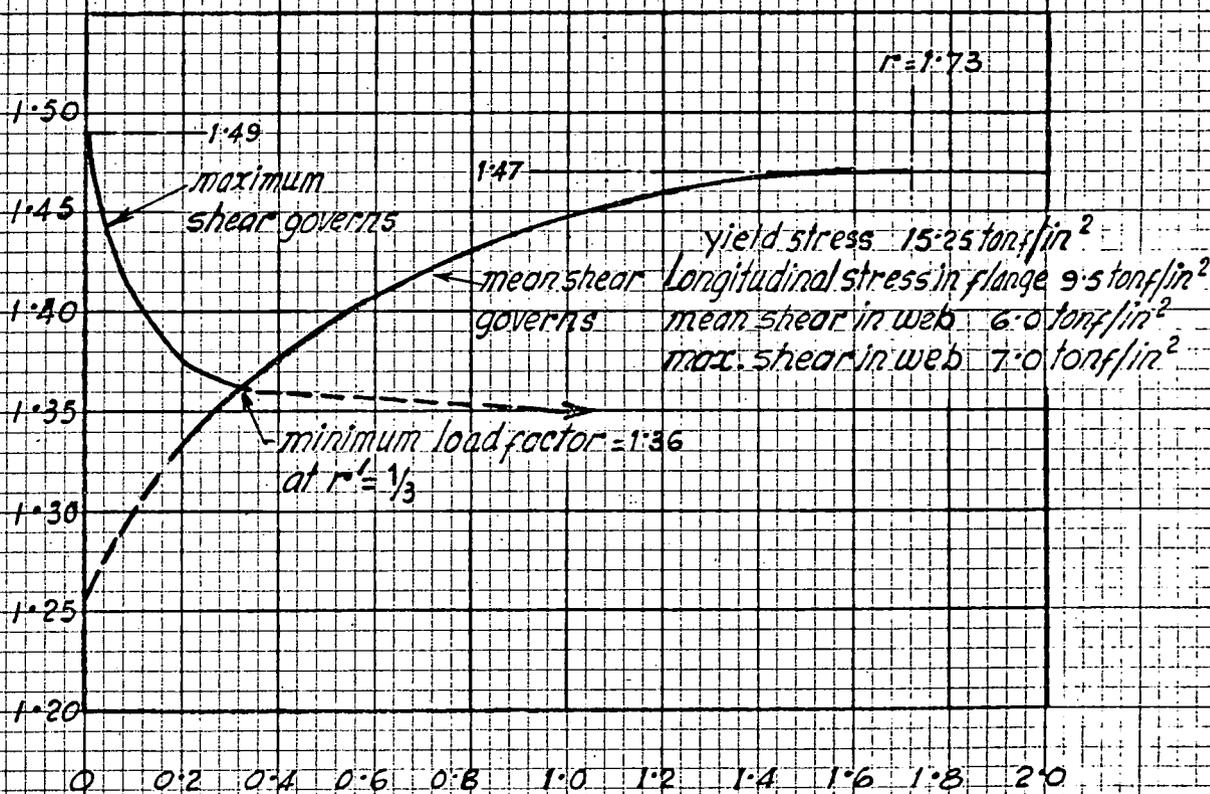
$$\tau_{al} = \frac{\sigma_y}{N_y \sqrt{3}} \left(\frac{\tau_{cr}}{\tau_y} + \frac{1 + \frac{\tau_{cr}}{\tau_y}}{\frac{2N_u}{\sqrt{3}N_y} \cdot \sqrt{1+\alpha^2}} \right) \text{ for } \frac{\tau_{cr}}{\tau_y} \leq 1$$

$$\tau_{al} = \frac{\sigma_y}{N_y \sqrt{3}} \cdot \frac{\tau_{cr}}{\tau_y} \text{ for } 1 < \frac{\tau_{cr}}{\tau_y}$$

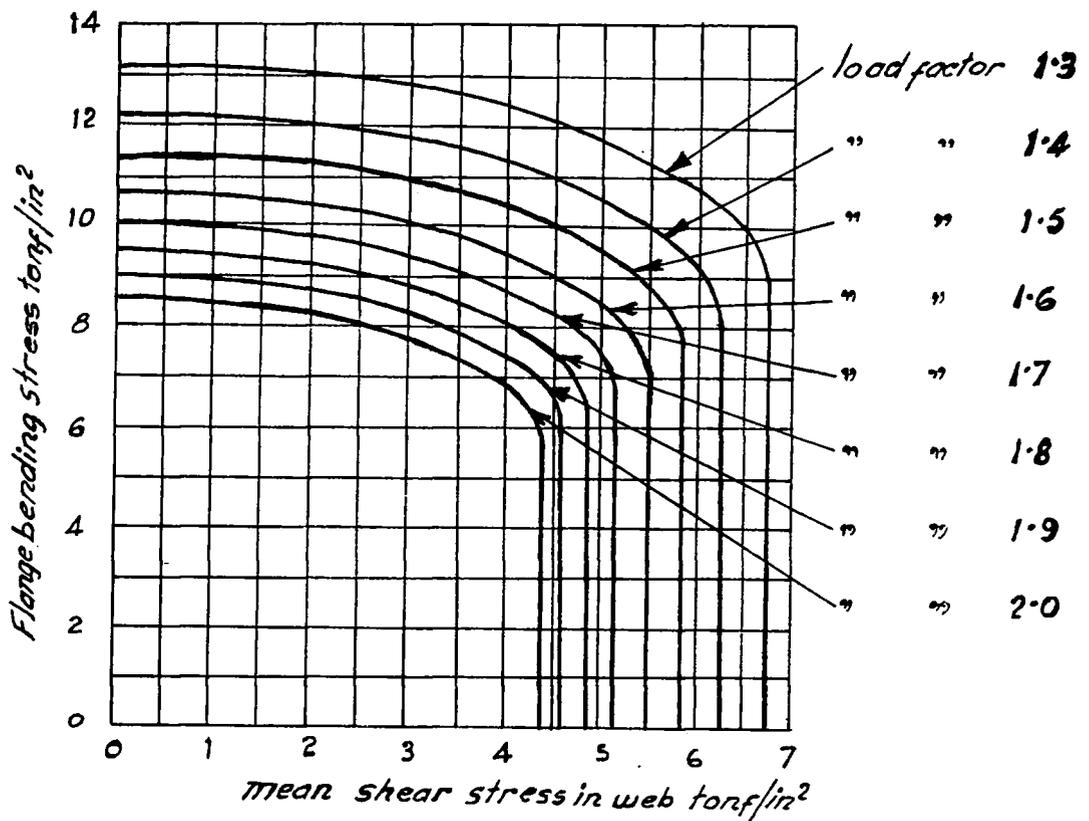
with $\frac{\tau_{cr}}{\tau_y} = \frac{\pi^2 \sqrt{3} E}{12(1-\nu^2)\sigma_y} \cdot \frac{K}{B^2} \text{ for } \frac{\tau_{cr}}{\tau_y} \leq 0.8$

$$\frac{\tau_{cr}}{\tau_y} = \sqrt{\frac{0.8\pi^2 \sqrt{3} E}{12(1-\nu^2)\sigma_y} \cdot \frac{K}{B^2}} \text{ for } \frac{\tau_{cr}}{\tau_y} > 0.8$$

} ----- 69



Load Factors for general yield in symmetrical Plate Girder - by Horns
Fig. 5.2.17



Load factors under combined shear force & bending — by Horne.
moment

Fig. 5.2.18

C. Main Girder under combined bending and shear:

Most plate girders are subjected to be under combined bending and shear. In some cases, a plate girder may be subjected to bending moments only, but not just the shear force. Basler⁽³¹⁾ stated that in a slender web of a plate girder the stress rearrangement is due to the web deflections, and in previous sections of this chapter it has already been stated. In stocky web of a plate girder, bending moment cannot be carried by the web, because of high concurrent shear is transferred to the flange through yielding.

Basler proposed the three following equations for moments of a symmetrical girder cross-section:-

$$M_f = \sigma_y d A_f \quad \text{----- 70}$$

$$M_y = \sigma_y d (A_f + \frac{1}{6} A_w) \quad \text{---- 71}$$

$$M_p = \sigma_y d (A_f + \frac{1}{4} A_w) \quad \text{----- 72}$$

Where M_f = flange moment which is carried by the flanges alone, when the stresses over the flange are equal to the yield stress.

M_y = yield moment which initiates yielding at the centroid of the compression flanges.

and M_p = plastic moment which is the resisting moment of a fully yielded cross-section.

After a loading condition is fixed, the shear force and the bending moment at any cross-section of a plate girder, denote the load intensity. Thus, the moment/shear ratio is independent of the load and characterises the loading condition.

Basler⁽³¹⁾ expressed the following equations in terms of stress:-

$$M = M_f + (M_p - M_f) [1 - (V/V_u)^2] \quad \text{----- 73}$$

$$\frac{M}{Z} \cdot \frac{Z}{M_y} = \frac{M_f}{M_y} + \frac{M_p - M_f}{M_y} [1 - (V/V_u \cdot \frac{A_w}{V_u})^2] \quad \text{---- 74}$$

$$\sigma = \sigma_y \cdot \frac{1 + \frac{1}{2} \frac{A_w}{A_f} [1 - (\tau/\tau_u)^2]}{1 + \frac{1}{6} \frac{A_w}{A_f}} \quad \text{---- 75}$$

$$\sigma = \frac{\sigma_y}{N} \cdot \frac{1 + \frac{1}{2} \frac{A_w}{A_f} [1 - (\tau/\tau_u)^2]}{1 + \frac{1}{6} \frac{A_w}{A_f}} \quad \text{---- 76}$$

The equation 76 has been derived from

M/Z = flange stress, σ , due to bending

M_y/Z = yield stress, σ_y

V/A_w = average shear stress in the web

V_u/A_w = ultimate shear stress in the web

and the ratio of M_f/M_y and $\frac{M_p - M_f}{M_y}$ which are expressed in equation 70, 71, and 72.

Considering $N = 1.65$ as per A. I. S. C.

$$\sigma < (27 - 12 \times \tau/\tau_u) \quad \text{----- 77}$$

Considering $N = 1.83$ as per AASHO*

$$\sigma < (24.5 - 11 \times \tau/\tau_u) \quad \text{----- 78}$$

(*American Association of State Highway Officials).

Basler stated that the presence of shear has both a detrimental and a beneficial aspect. The beneficial aspect is due to the fact that shear forces always imply a moment gradient and thus, only a short girder portion is affected by the maximum moment. The adverse aspect is that a web which is exhausted by shear cannot simultaneously take its allotted bending moment and the flanges will have to compensate for it, resulting in a higher flange stress than computed by the sectional modulus concept.

The overturning moment causing lateral buckling is made of a contribution by the compression flange and another by the web. A rearrangement of stresses between the web and the flange, does not change the overall or resulting overturning moment. Let the ultimate bending moment be M_u , which is due to the flange instability, then the equation $\frac{M}{M_u} = 1$. Now, this equation is independent of shear, but applies also to lateral buckling.

Basler's equation for lateral buckling are as follows:-

$$\frac{\sigma_{cr}}{\sigma_y} = 1 - \frac{(\lambda')^2}{4C_1} \quad \text{----- 79}$$

$$\text{when } 0 < \lambda' < \sqrt{2C_1}$$

$$\& \frac{\sigma_{cr}}{\sigma_y} = \frac{C_1}{(\lambda')^2} \quad \text{----- 80}$$

$$\text{when } \lambda' > \sqrt{2C_1}$$

$$\text{where } \lambda' = \frac{1}{r} \sqrt{\frac{E_y}{\pi^2}} = 1 \sqrt{\frac{E_y}{\pi^2} \cdot \frac{A_f + \frac{1}{6} A_w}{I_f}}$$

$$\text{and } C_1 = \text{a factor} = 1.75 - 1.05K + 0.3K^2$$

where K = ratio of the smaller end moments of a longitudinal girder segment **free** from interspan loads to the larger end moment.

Rockey⁽³²⁾ stated that the web plates are usually subjected to a combination of bending and shear. The following factors are to be considered when determining the failure load of a web plate loaded in shear and bending:-

- a) the reduction in the buckling stress of the web due to the presence of the bending or direct stress and this buckling stress τ_{cr} , can be calculated from

$$\left(\frac{\sigma_{mb}}{\tau_{crb}}\right)^2 + \left(\frac{\tau_m}{\tau_{cr}}\right) = 1 \quad \text{----- 81}$$

where σ_{mb} = critical bending stress when acting together with shear stress.

τ_{crb} = critical bending stress when the plate is subjected to pure bending.

τ_m = critical shear stress when acting combined with bending stress

τ_{cr} = critical bending stress when the plate is subjected to pure shear.

In case of all edges simply supported:-

$$\tau_{crb} = 23.9 \left\{ \frac{\pi^2 E}{12(1-\nu^2)} \right\} \left\{ \frac{t}{d} \right\}^2 \quad \text{----- 82}$$

$$\tau_{cr} = \left(5.35 + \frac{4d^2}{a^2} \right) \left(\frac{\pi^2 E}{12(1-\nu^2)} \right) \left(\frac{t}{d} \right)^2 \quad \text{When } a \gg d \quad \text{--- 83}$$

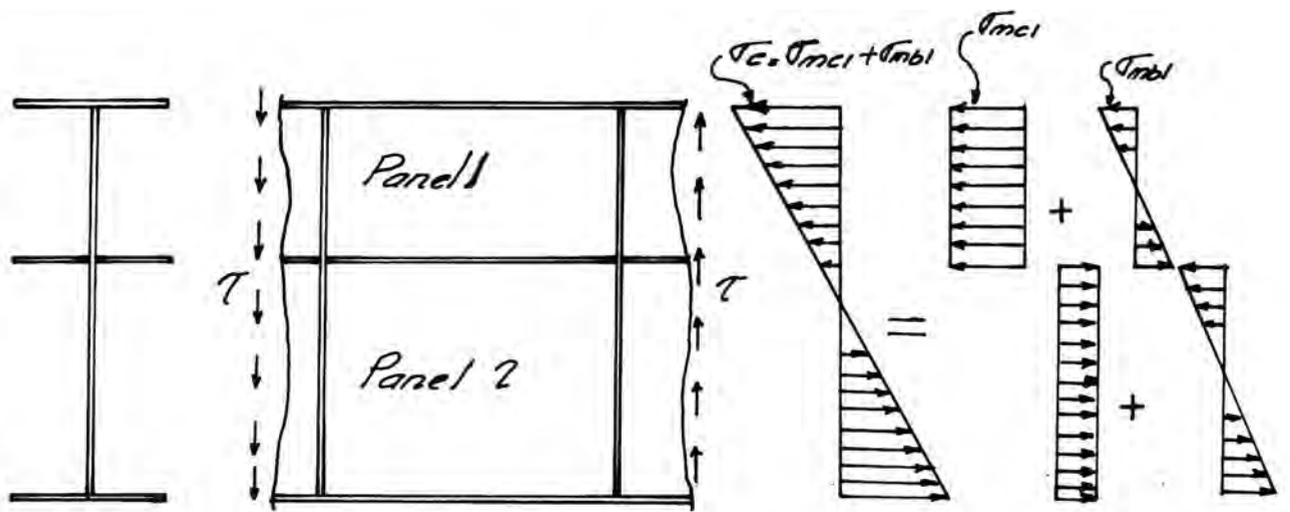
$$\tau_{cr} = \left(5.35 \frac{d^2}{a^2} + 4 \right) \left\{ \frac{\pi^2 E}{12(1-\nu^2)} \right\} \left\{ \frac{t}{d} \right\}^2 \quad \text{When } a \leq d \quad \text{--- 84}$$

- b) the reduction in the magnitude of the plastic modulus Z_p of the flanges due to the presence of the axial compressive and tensile stresses. For flanges having a simple rectangular cross-section the reduced modulus is:-

$$Z_p = Z_p [1 - (\frac{\sigma}{\sigma_y})^2] \quad \text{----- 85}$$

where σ = axial stress in the flange

and σ_y = yield stress of the flange plate.



actual direct stress distribution = Equivalent direct stress distribution

Stress distribution in panels of a plate girder subjected to shear & bending - Rockey.

Fig. 5.2.19

c) When a panel is loaded in direct compression, the central area of the panel buckles, becoming unable to carry any further direct stress and any additional direct load has to be carried by the web material adjacent to the flanges and stiffeners. As the stress distribution in a yielded panel which is subjected to both shear and bending, is very complicated, Rockey stated that, after the panel is buckled, the flanges alone carry the additional shear loads by the development of a diagonal membrane stress σ_{ty} . So, when a web plate is loaded by direct bending stresses and also by shear stresses, the value of the diagonal stress, σ_{ty} comes to

$$\sigma_{ty} = \frac{1}{2} \left[(3\tau_m \sin 2\phi + \sigma_m \sin^2\phi - 2\sigma_m \cos^2\phi) + \sqrt{(3\tau_m \sin 2\phi + \sigma_m \sin^2\phi - 2\sigma_m \cos^2\phi)^2 - 4[\sigma_m^2 + 3\tau_m^2 - \sigma_{yw}^2]} \right] \text{ --- 86}$$

Fig. 5.2.19 shows a typical example of a panel of a plate girder reinforced by both transverse and longitudinal stiffeners and subjected to bending and shear. In panel 1, there is a direct compressive stress of σ_{mc} and a pure bending stress at the flange to web junction of τ_{mb} . Rockey suggested that the critical stresses σ_{mc} , τ_{mb} , and τ_m may be calculated, within reasonable accuracy by the equation,

$$\left(\frac{\sigma_{mc}}{\sigma_{cre}} \right) + \left(\frac{\tau_{mb}}{\tau_{crb}} \right)^2 + \left(\frac{\tau_m}{\tau_{cr}} \right) = 1 \text{ --- 87}$$

Where σ_{cre} = the critical uniform direct axial stress to cause buckling

τ_{crb} = the compressive edge stress causing buckling in the panel when loaded in pure bending.

τ_{cr} = the uniform shear stress to cause buckling

Now,

$$\sigma_{cre} = 4 \left[\frac{\pi^2 E}{12(1-\nu^2)} \right] \left[\frac{t}{d} \right]^2 \text{ when all edges are simply supported. --- 88}$$

$$\sigma_{crc} = 5.41 \left[\frac{\pi^2 E}{12(1-\nu^2)} \right] \left[\frac{t}{d} \right]^2$$

When longitudinal edge is clamped and the others are simply supported ----- 89

$$\sigma_{crb} = 41.7 \left[\frac{\pi^2 E}{12(1-\nu^2)} \right] \left[\frac{t}{d} \right]^2$$

When the compressive longitudinal edge is clamped, the others simply supported --- 90

$$\sigma_{cr} = \left[7.07 + \frac{3.91}{(a/d)^{3/2}} \right] \left[\frac{\pi^2 E}{12(1-\nu^2)} \right] \left[\frac{t}{d} \right]^2$$

When one longitudinal edge is clamped, the others simply supported. ----- 99

Therefore, once the critical stresses, σ_{mc} , σ_{mb} & τ_m for the individual panels have been determined, the stress distribution at buckling will be known and the collapse load for each of the panels may be obtained by using the equations:-

$$\left(\frac{c}{a} \right)^3 - \left(\frac{c}{a} \right)^2 + \left(\frac{4Zp \sigma_y}{a^2 t \sin^2 \phi (\sigma_y)} \right) = 0. \quad -$$

and

$$\sigma_y = \frac{1}{2} \left[(3\tau_m \sin 2\phi + \sigma_m \sin^2 \phi - 2\sigma_m \cos^2 \phi) + \sqrt{(3\tau_m \sin 2\phi + \sigma_m \sin^2 \phi - 2\sigma_m \cos^2 \phi)^2 - 4(\sigma_m^2 + 3\tau_m^2 - \sigma_y^2)} \right] -$$

The solution of these two equations is quite difficult and to use them in practical design will be a very complicated work. Ostpenko and others⁽²⁹⁾ suggested that the "beam action" shear for the combined loads is

$$V_m = \tau_m A_w$$

where V_m = beam action shear under combined loads.

τ_m = shear buckling stress under combined loads.

A_w = area of the web

$$\text{Now, } \frac{\tau_m}{\sigma_{cr}} + \frac{1+R}{2} \left(\frac{\sigma_c}{\sigma_{crb}} \right) + \frac{1-R}{2} \left(\frac{\sigma_c}{\sigma_{crb}} \right)^2 = 1. \quad \text{----- 91}$$

$$\text{Where } \sigma_c = \left(\frac{\mu d A_w \gamma_c}{I} \right) \tau_m \quad \text{----- 91a}$$

$$\mu = \frac{M}{Vd}$$

- y_c = distance from the centroidal axis to the compression edge of the web
- σ_c = bending buckling stress at the extreme compression fibre of the web
- R = ratio of the maximum tensile stress or minimum compressive stress, to the maximum compressive stress.

$$\tau_{crb} = K_b \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{d}\right)^2 \leq \sigma_{yw} \quad \text{----- 91b}$$

$$K_b = 13.54 - 15.64R + 13.32R^2 + 3.38R^3 \quad \text{----- 91c}$$

$\therefore \sigma_c$ is directly related to τ_m

$$\text{Then, } \tau_m = \tau_{cr} \frac{\sqrt{F(3-R)^2 + 16 - (1+R\sqrt{F})}}{2\sqrt{2 + (1-R)F}} \quad \text{----- 92}$$

$$F = \left(\frac{\mu \cdot d y_c A_w}{I} \cdot \frac{\tau_{cr}}{\tau_{crb}} \right)^2 \quad \text{----- 92a}$$

Ostapenko's proposed formula for ultimate shear is, adding the results of the following equations:-

1. $V_m = \tau_m \cdot A_w$
2. $V_{cr} = V_{\sigma}$
3. $V_{fc} = \left(1 + \frac{\sigma_c}{\sigma_y}\right)$

The strength of a panel is controlled by the failure of the compression flange and at this point, moment is dominant. Ostapenko developed the following formula from the numerical computer output:-

$$V'_{fc} = \frac{(A_f + 30t^2)(\sigma_{cf} - \sigma_c) - \mu V_{fc}}{B \left(\frac{V_p}{V_{\sigma}}\right) \left(\frac{180}{\sigma_t}\right) \sqrt{\frac{33\sigma}{\sigma_{yw} y_c}} + \mu} \geq 0 \quad \text{----- 93}$$

Where $B = 0.338\lambda - 0.196$ for $0.58 \leq \lambda < \sqrt{2}$

$B = 0.235\lambda - 0.05$ for $\lambda \geq \sqrt{2}$

V'_{fc} = Incomplete tension field action shear under combined loads.

V_{fc} = frame action shear under combined loads

σ_{cf} = buckling stress of the compression flange column

V_p = plastic shear of the web

V_t = tension field action shear under pure shear

The ultimate shear is,

$$V_{th} = V_m + V'_{fc} + V_{fc} \quad \text{----- 94}$$

and the moment, $M_{th} = U.A. V_{th}$.

This moment should be checked so that it should not exceed M_u .

Herzog⁽³⁴⁾ stated that in plate girders with very thin webs, especially those welded automatically, crippling can become a problem. So he suggested that the web crippling load is

$$P_{cr} = 1430 t^2 \left[1.2 + 1.25 \frac{I_f \cdot d}{I_w \cdot t} \left(1 + \frac{p}{d} \right)^2 \left(0.85 + \frac{a}{100d} \right) \right] \sqrt[3]{1 - \left(\frac{\sigma_b}{\sigma_y} \right)^2} \quad \text{----- 96}$$

Where p = length of the patch load

" σ_b = maximum bending stress in compression flange.

A.A.S.H.O. recommends the following points for load factor design:-

for flexural members, assume that compact sections are capable of forming plastic hinges, which when the plastic moment is attained, rotate under increasing loads with little or no change in moment. Braced sections that do not qualify as compact are presumed incapable of developing moments in excess of that at first yield of the sections.

For fully braced, compact, symmetrical sections:-

1. Width-thickness ratio of flange projection should not exceed, $\frac{B_{2T}}{t} = \frac{50.6}{\sigma_y}$ ----- 97

2. Depth-thickness ratio of the web should not exceed, $\frac{d}{t} = \frac{421}{\sqrt{\sigma_y}}$ ----- 98

3. The compression flange should be supported laterally at intervals not exceeding the distances:-

$$L = \frac{222\sigma_y}{\sqrt{\sigma_y}} \quad \text{When } M_2 \geq 0.7M_1 \quad \text{--- 99}$$

$$L = \frac{379\sigma_y}{\sqrt{\sigma_y}} \quad \text{When } M_2 < 0.7M_1 \quad \text{--- 99a}$$

$$L = \frac{20000 A_f}{\sigma_y \cdot d} \quad \text{--- 99b}$$

M_1 = larger of the bending moments at two adjacent braced points

M_2 = Smaller of the bending moments at those braced points.

4. axial compression should not exceed,

$$\sigma = 0.15A\sigma_y \quad \text{----- 100}$$

where A = area of the cross-section

5. Shear should not exceed,

$$V = 0.55d \cdot t \sigma_y \quad \text{----- 101}$$

The moment capacity for fully braced sections may be calculated from,

$$M_u = \sigma_y Z_p$$

Members with axial loads in excess of $0.15A\sigma_y$ should be designed as beam columns.

Transverse stiffeners are required for the web members not satisfying the following equations:-

a) $V = 0.55 dt \cdot \sigma_y$

b) $V = \frac{3.5 E t^3}{D}$

where D = unsupported distance between flange components.

c) $V_p = 0.58 D \cdot t \cdot \sigma_y$

The web depth-thickness ratio with transverse stiffeners

should not exceed, $\frac{D}{T} = \frac{1155}{\sqrt{\sigma_y}} \dots\dots 102$

If a girder panel is subjected to combined shear and bending, with shear exceeding $0.6V_u$ where V_u is the ultimate shear force, then the moment capacity should not exceed,

$$M = M_u \left(1.375 - \frac{0.625V}{V_u} \right) \dots\dots 103$$

where the shear capacity

$$V_u = V_p \left[c + \frac{0.87(1-c)}{\sqrt{1 + (d/D)^2}} \right] \dots\dots 104$$

& $c = 569 \frac{t}{D} \sqrt{\frac{1 + (D/d)^2}{\sigma_y}} - 0.3 \leq 1 \dots\dots 104a$

Spacing of transverse stiffeners may be determined from equation 104 in accordance with required shear capacity.

But the spacing should not exceed $1.5D$. At the ends of simply supported girders, the first stiffener space should not exceed

$$d = 458 \sqrt{\frac{Dt^3}{V}} \dots\dots 105$$

Where V = end shear

The area of the transverse stiffener should be at least

$$A'_s = Y \left[0.15 B D t (1-c) \frac{V}{V_u} - 18 t^2 \right] \dots\dots 106$$

Where $B = 2.4$ for single plate

Maximum width-thickness ratio permitted for a transverse

stiffener is $\frac{b'}{t} = \frac{82.3}{\sqrt{\sigma_y}} \dots\dots 107$

Where b' = projecting width of stiffener

Longitudinal stiffeners are required when the web depth-thickness ratio exceeds the value given in equation 102. It should be placed at $D/5$ distance from the inner surface of the compression flange. The radius of gyration should be at least

$$r = \frac{d\sqrt{t}}{727} \text{ ----- } 108$$

In the design of transverse stiffeners used with a longitudinal stiffener, the depth of the deepest sub-panel should replace D in the formulas. Also, the section modulus of each transverse stiffeners should be at least

$$Z_{st} = \frac{Z_{sl} \cdot D}{3d} \text{ ----- } 109$$

Where Z_{st} = Section modulus of transverse stiffener
 " Z_{sl} = Section modulus of horizontal stiffener

All these above formulas are used in appendix E to work out the properties of the main plate girders, which has already been designed elastically, (see appendix A part 11).

d. Deflection:

The proportioning of the main girders is such that deflection is not likely to be worthy of consideration. The deflection of the suspended span is about $1\frac{1}{2}$ " and a camber of $1\frac{3}{4}$ " for the whole span is quite adequate. A cross camber of 1 in 48 is enough to allow drainage of the road surface. This is achieved by placing the girder bearings at the required variation in level on the supporting piers and abutments.

One of the advantages of continuous girder design based on plastic theory is that, the stresses, which are induced by the settlement of supports are already taken into account. Baker⁽²⁴⁾ stated that the settlements of supports is irrelevant to plastic design, because, the relative settlement of supports has no effect on the full moments at failure. The girders will deflect elastically as it is loaded until hinges developed⁽³⁵⁾ and the slight change of geometry will tend to cause a reduction in the collapse load. But if the plastic range is small or the plastic strains are large, strain-hardening will take place before collapse and the girder will tend to increase in strength with increasing deflection. However Horne and Chin⁽²⁵⁾ stated that for a simply supported beam

$$\delta_p = \frac{L}{4} \cdot \theta H$$

where δ_p = plastic component of deflection

L = Span

and θH = hinge rotation

but $\delta_c = \delta_p + \delta_e$

where δ_c = central deflection

and δ_e = elastic deflection.

So, for the main girders this is not applicable, but for for the cross girders, the above assumption may be applied

to find the central deflection.

B.S.153 does not specify any limit in deflection, but in part 4, clause 21, takes care of the lateral deflection of the plate girder. B.S.449 clause 15 stated that "the maximum deflection due to loads other than the weight of the structural floors or roof shall not exceed $1/360$ of the span"

5.3. Cross Girder - Limit state analysis:

It may be noticed from previous chapters (of elastic method of analysis) that the cross girders are placed at 11.5 feet centres. The cross girders are bolted (with H.S.F.G.Bolts) to the main girders and therefore it is assumed that the cross girders are functioning as simply supported beams.

From simply supported beam theory it is known that, if the load is increased steadily from Zero, the beam at first behaves elastically. Eventually, at a certain value of the load, the central bending moment reaches the plastic moment and plastic hinge is then formed. No further increase of load is possible if equilibrium is to be maintained, for the bending moment cannot rise above the value of the plastic moment. But the plastic hinge may (by theory) undergo rotation through any angle while the bending moment and the load remain constant. The beam can thus continue to deflect at constant load due to this hinge action and so fails by plastic collapse. The beam remains elastic everywhere except at the centre and since the load is constant, the bending moments during collapse implies constancy of the curvatures. The increase in deflection during collapse is due to the rotation at the central plastic hinge.

Consistent with the limitation and restriction and strength provisions, plastic design is best suited to beams which are fixed or continuous. Simply supported beams offer no advantage. When collapse load has been reached, a sufficient number of plastic hinges must form to create a mechanism. Usually, three such hinges are required, one at each (fixed) end and the other at the position of maximum span moment at collapse. At all these hinges the value of the plastic moment generally must be equal to give a perfect mechanism.

In appendix F an attempt has been made to see the difference in the values of actual plastic modulus value and the required plastic modulus value, using a typical floor beam section with deck plate, which has already been elastically analysed.

6 ORTHOTROPIC DECK SYSTEM: LIMIT STATE ANALYSIS

6.1 Design Considerations:

The deck system used in this project, consists of longitudinal ribs and the transverse cross girders, (@ 11.5 feet centres), both utilizing the deck plate as their top flange. This particular system may be treated as an independent member of the bridge and the horizontal shear connections between the main girders and the deck is not substantial. As this system of deck is an independent member of the bridge, no longitudinal force can be transferred from the main girders into the deck⁽¹⁸⁾. The longitudinal ribs act as continuous beams and transmit the local effect of external loads, into the cross girders, which carry their load to the main girder. The cross girders deflect proportionally to these loads and provide elastic supports for the longitudinal ribs. Due to the common flange (the deck plate), and the continuity of this common flange, the ribs cannot act independently to each other; so the ribs without any load should also deflect and become stressed as shown in fig.6.1 , section a - a.

It appears to be obvious that the cross-girders framed into single-web girders are simply supported, and have been treated separately in chapter 5.3. A.I.S.C.⁽¹⁸⁾ stated that the ribs and the cross-girders carrying locally applied loads are to act as purely flexural structural members. Experiments have been carried out and it has been established that if the load on the deck has been increased beyond the usual wheel load, or the ratio of the deflection to the

span of a rib is relatively large, then stresses begin to appear in the loaded ribs in addition to the purely flexural stresses, and an additional tension due to a membrane action of the deck plate occurs. Increasements in loads and the corresponding deflections, cause the redistribution of the stresses in the system and the membrane stresses replace the flexural stresses which are predominant under working loads. Therefore, with large deflections, the deck plate behaves differently from that predicted by the usual flexural theory which disregards the effect of the deformations of a system on its stresses and its strength has been found many times greater than predicted by the ordinary flexural theory⁽¹⁸⁾. This structural behaviour is due to the combination of the membrane action of the plate and the plastic strength reserve of the highly statically indeterminate system of the deck. However, a more precise understanding of this extremely complicated structural mechanism of a steel deck plate under large load is still lacking and no such experiment has yet been carried out to clarify this sort of structural behaviour.

Tests have been carried out by Hooke & Rawlings⁽³⁷⁾ to find out the deflection profile of clamped rectangular plates, and each plate was loaded well into the plastic range. The plates were of mild steel with width/thickness ratios covered the range from 50 to 160 and had aspect ratios ($B = b/s$) of $1. \frac{1}{2}$, $\frac{2}{3}$ and $\frac{1}{3}$. These tests results are relevant to this project, if it is assumed that the wheel load will be placed on the deck as shown in fig. 6.1 (e) Hooke & Rawlings experimental values are given by,

$$\frac{2b}{t} = 71.6 \left(\frac{\delta_c}{t} \right) + 31.25$$

Where b = the width of the strip of the plate

" t = thickness of the plate

" δ_c = central deflection .

Comparison with the predictions of the theory based on the assumption that yield first occurs when plastic sections form at the centres of the long sides showed that the theoretical values of $\frac{\delta c}{t}$ were in every case lower than those given by the equation. The experimental values did not lead to a straight-line relationship and for lower values of $\frac{2b}{t}$ tended to the origin and not 31.25. Also the theory predicted that plates having the same $\frac{2b}{t}$ would not yield at the same value of $\frac{\delta c}{t}$, but the value increased as β decreased .

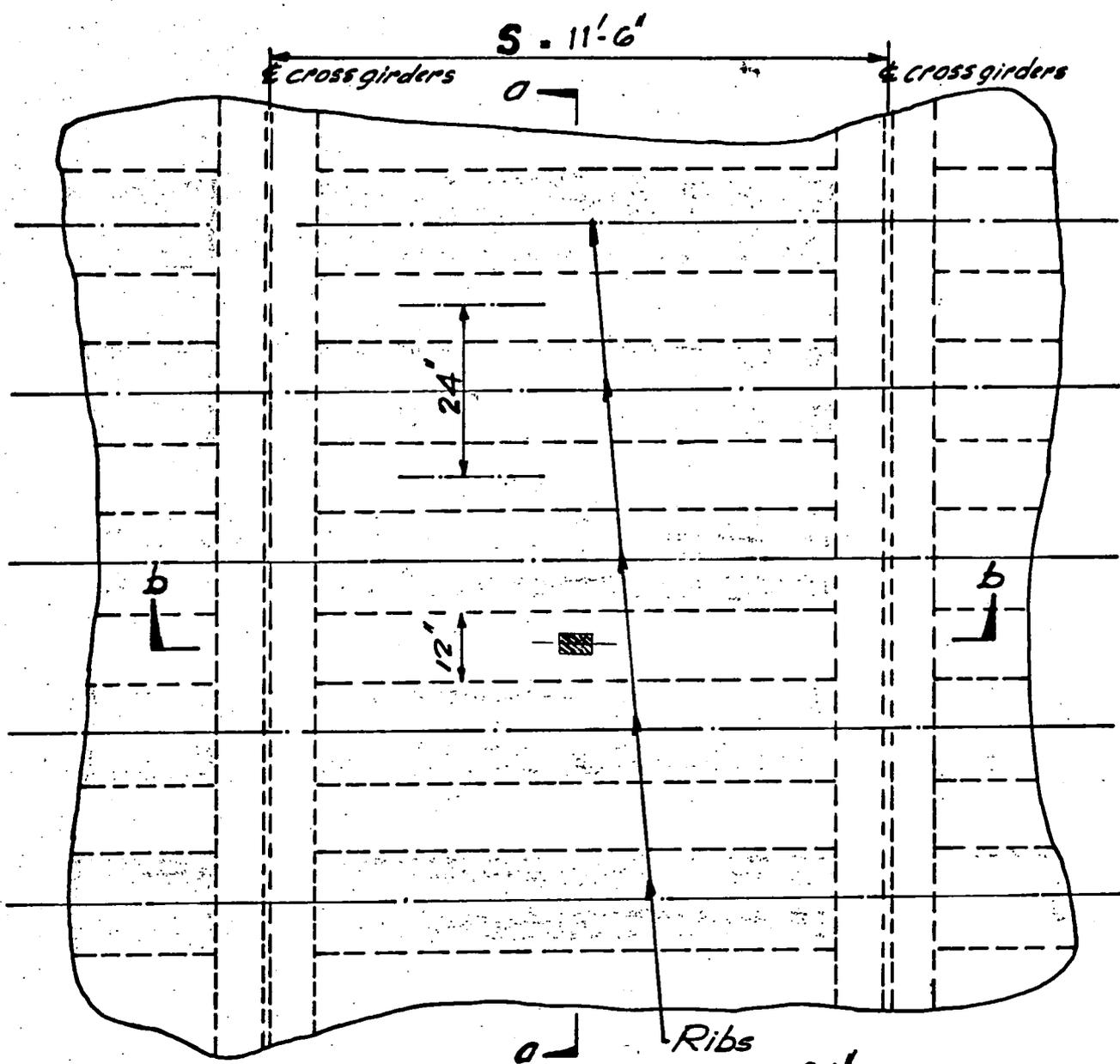
Experiments have been carried out⁽¹⁸⁾ on deck plate panels with closed ribs at University of Stuttgart where the dimensions of the test panel with trapezoidal ribs were for a half-scale model of an actual bridge deck panel. The purpose of these experiments was to determine the effect of the torsional rigidity, which is more pronounced with longer rib spans. The rib depth-to span ratio was chosen as $\frac{1}{44}$ and the material was mild steel. The characteristic values for a concentrated wheel load (W) were computed with the flexural theory and the values were as follows:-

allowable stress @ W = 1.45 tonf. , 8.84 tonf/in²
 yield stress in rib @ W = 2.56 tonf
 ultimate strength @ W = 3.8 ton f

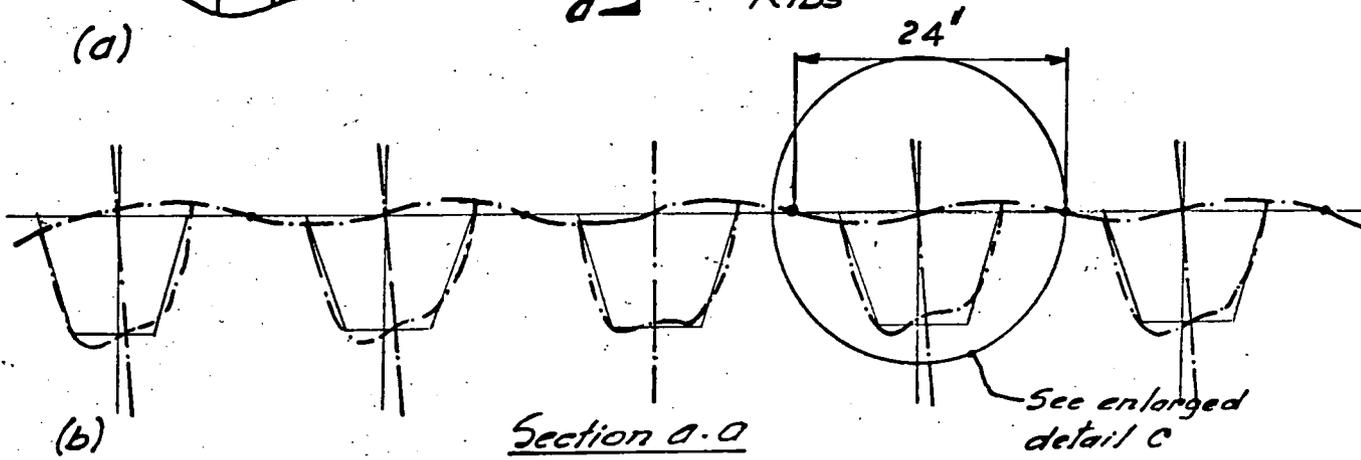
The deflection line of the loaded rib was almost a straight line. A measurable membrane condition occurred well within the elastic limit, before reaching the plastic limit at any point and with the deflection under load only $\frac{1}{500}$ of the rib span. At further load increments (W = 5 tonnes, which is beyond the theoretical load), an almost linear relationship between the load and the deflection was observed. At this loading a deflexion of $\frac{1}{3000}$ of the rib span was measured. The conclusion of these tests is that, the actual

ultimate capacity of the longitudinal ribs is much greater than the computed values. The actual safety against reaching the yield point stress is higher than the computed values. Unlike in a beam, reaching plasticity at one location of the deck plate does not mean a characteristic point at which the structural behaviour is changed in any significant way.

In Appendix G, calculations are provided to examine the inelastic behaviour of the orthotropic deck.



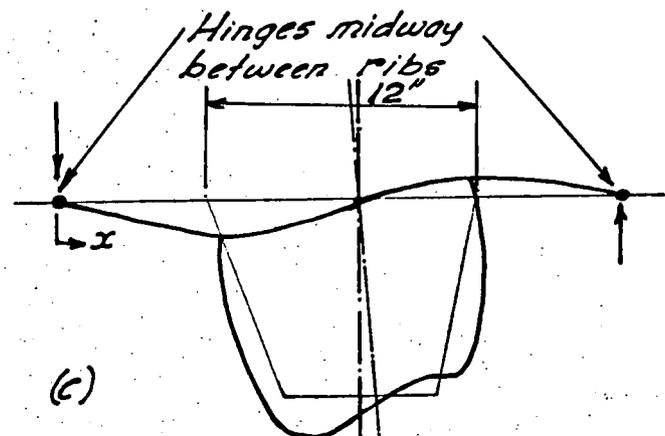
(a)



(b)

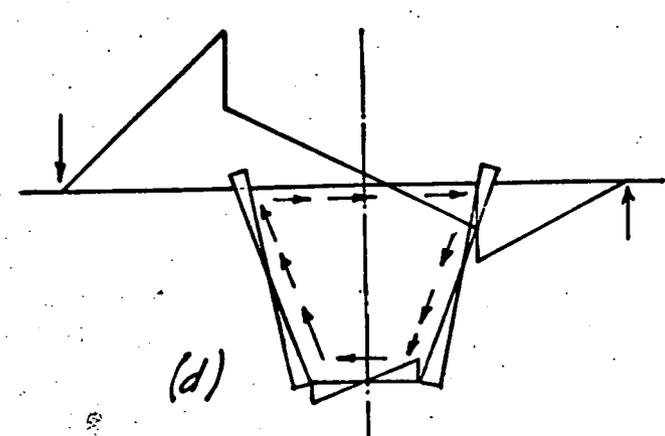
Section a-a

See enlarged detail c



(c)

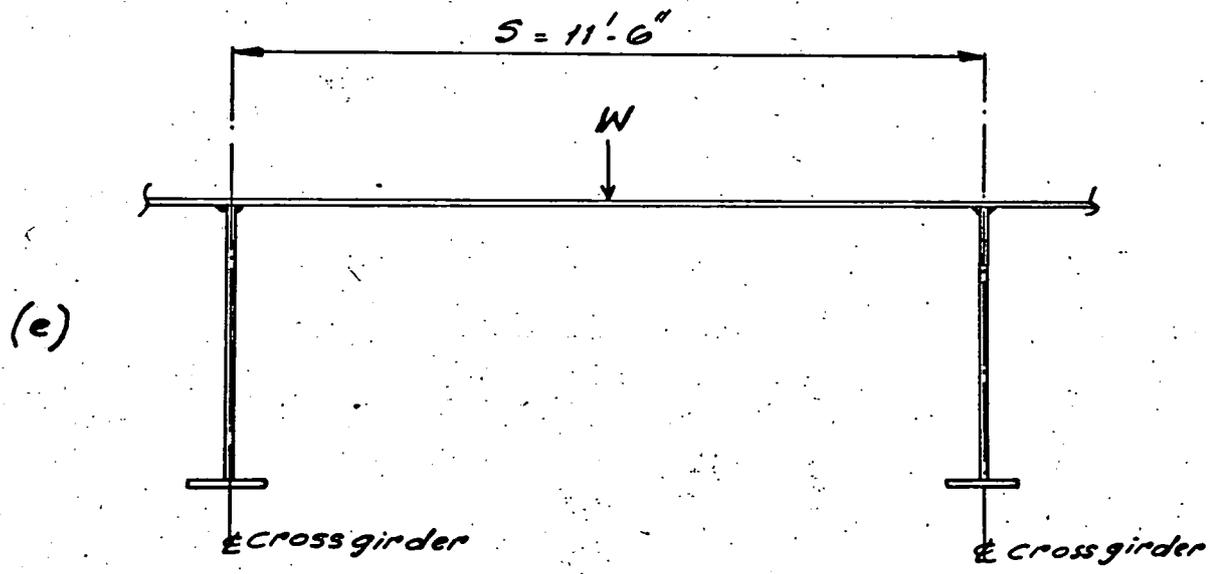
actual rib rotation
 Rotation of a rib due to shear transfer in x direction.



(d)

Moment & shear forces in a rib.

Fig. 6.1



Section b-b
Fig. 6.1

6.2 Ultimate load test on the deck:

It has been established⁽¹⁸⁾ that, under increasing loads, the elastic behaviour of the plate evidenced by the lack of deformations after removal of the loads, extends considerably beyond the limits predicted by the flexural theory. Membrane stresses are developed as the loads and deflections increases, which causes horizontal reactions at the supports and compressive stresses in the portions of the plate adjoining the loaded area, and at higher loading, plastic hinges are formed over the supports and the flexural stresses are replaced by the membrane stresses. Due to the restraining effect of the unloaded portions of the plate, the deflections are limited, even at the critical locations of the plate where plastic hinges are formed. At the final stage of loading, the strain-hardening phase has been observed and thus the full tensile strength of the material has been developed before the rupture at the ultimate load.

In figure 6.2 results of a full scale loading test has been shown⁽¹⁸⁾. This test was carried out on a mild steel deck plate. From the load-deflection diagram, it may be noted that, the behaviour of the test specimen was fully elastic up to the load of 32.5 tonnes; between 32.5 tonnes to 75 tonnes the load-deflection curve is flatter, which means, it is in plastic range, finally the deflection increments under higher loads (over 75 tonnes) are smaller, which is due to the strain-hardening effects. The ultimate strength of the plate reached at the load $W/2 = 276/2$ tonnes., so, if the design load is $W = 10$ tonnes, the safety against rupture is $\frac{276}{10} = 27.6$ times. It has been concluded⁽¹⁸⁾ that, the stresses in the plate under the working loads cannot be used to judge the ultimate static strength of the plate.

A.I.S.C.'s formula for ultimate capacity of the plate is based on the behaviour of a flat bar. It has stated that a flat steel bar fixed at both ends and uniformly loaded with W per unit length, acts as a cable, at failure, with the load carried by axial stresses only, sustained by the reactions at supports. The total elongation at failure was found to be very similar to the tensile test of a specimen of equal length.

For a bar of span, S , and cross-sectional area A , ultimate load, W_u is given by:-

$$W_u = \frac{4.9 f_u A}{S_1} \sqrt{E_u}$$

Where f_u = ultimate tensile stress of the material

$$E_u = \text{ultimate elongation} = \left(\frac{1}{24}\right) \left(\frac{W S_1}{f_u A}\right)^2$$

The value of E_u is 14.0% for a mild steel bar.

The ultimate loading capacity of a plate with a uniformly distributed loading system extending only over a portion of the width of the plate will be much greater than that of a bar loaded over its full width; therefore an empirical correction coefficient $R = 1.25$, has been proposed. (18).

Thus the above equation for the ultimate load, W_u may be written as:

$$W_u = 1.25 \left(\frac{4.9 f_u t}{S_1} \right) \sqrt{E_u}$$

Where t = plate thickness

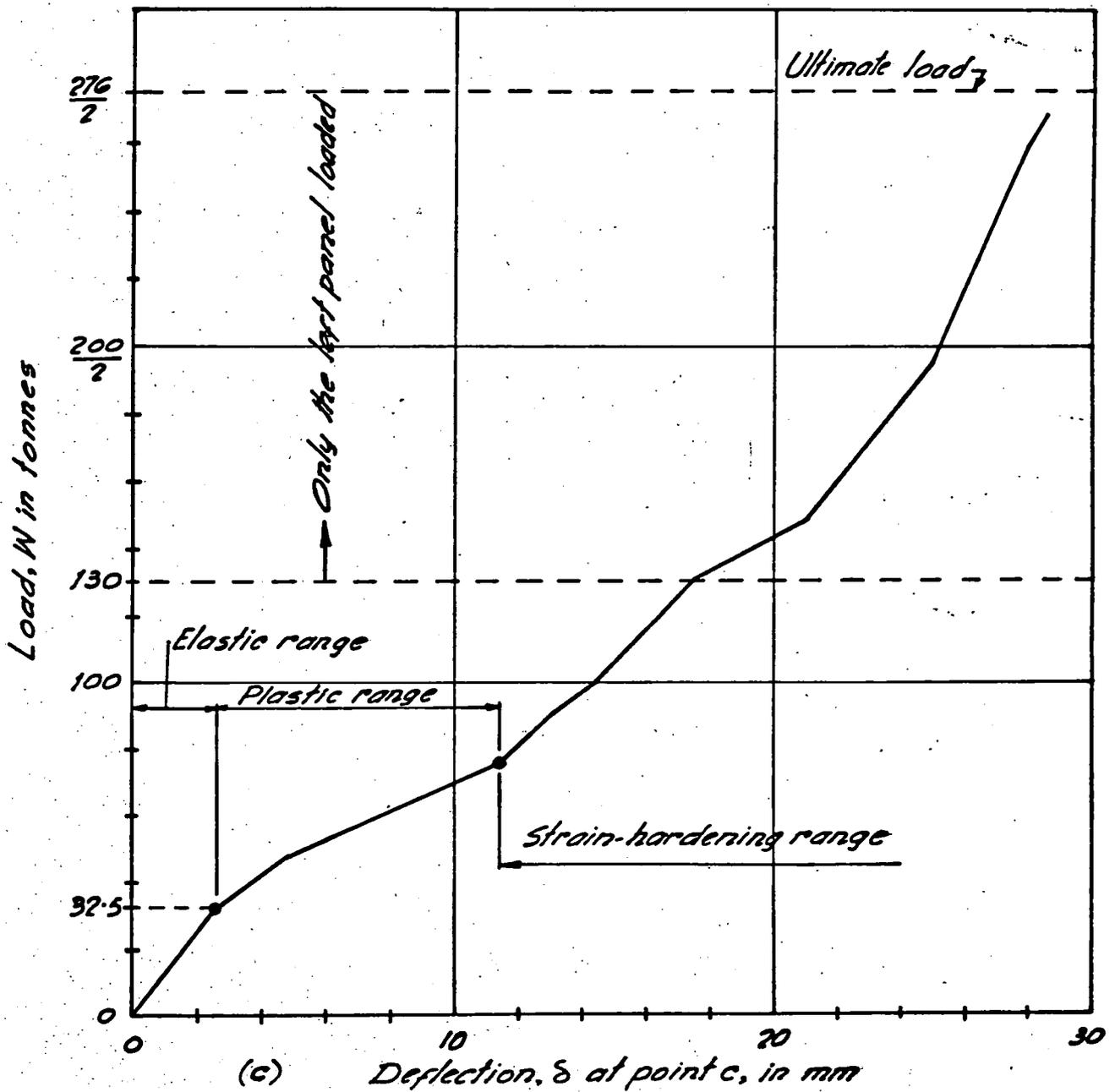
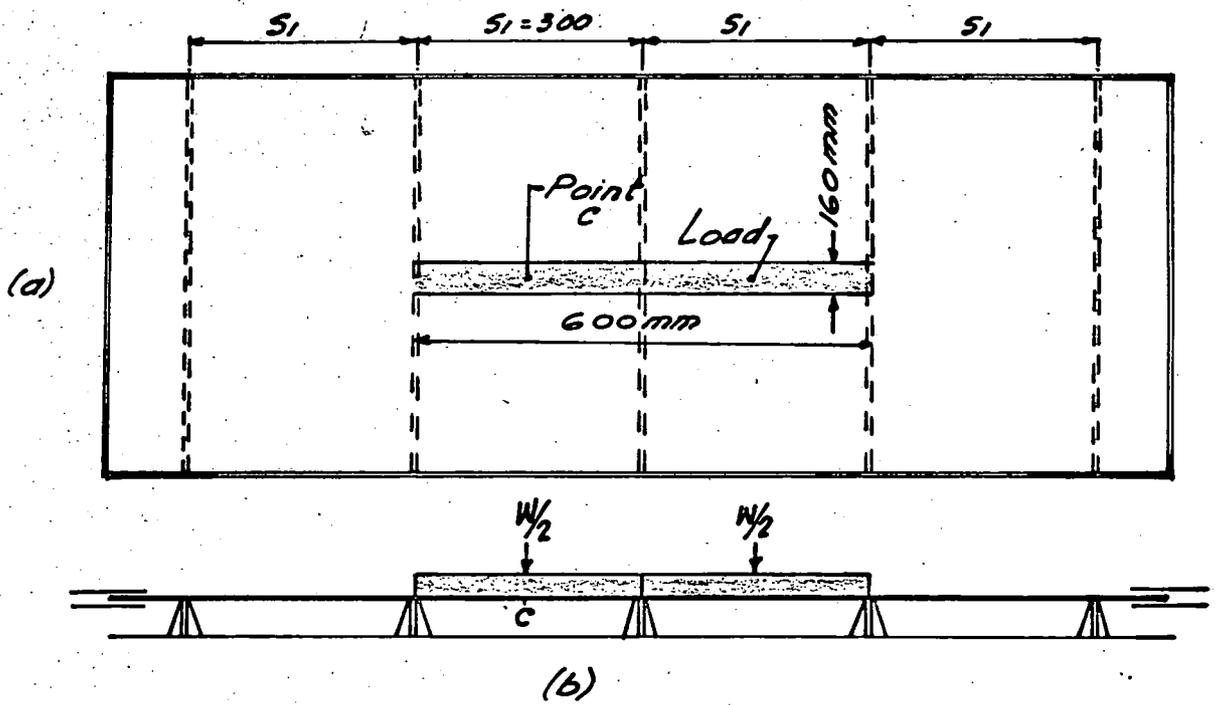
" W_u = ultimate load, lb_f per sq. in.

" f_u = ultimate tensile stress, lb_f per sq.in.

" S_1 = longitudinal rib spacing, in.

" E_u = ultimate elongation of the material, corresponding to the tensile stress, f_u .

However,, A.I.S.C. suggested that the above formula is correct enough, but further experimental varification of this formula is desirable.



Load-deflection diagram (18)

Fig. 6.2

7.0 Discussion

This thesis has illustrated the evolution and application of the elastic and limit state theories to a plate girder bridge. Design of plate girder bridges elastically, has been well established in the current edition of B.S. 153 and stress computations are no longer a problem to an engineer, once the loading conceptions are clear and the final bending moment and shear force diagrams are obtained. However, the maintenance of elastic stress conditions now appear to be less important than an understanding of the collapse behaviour of the structure. In limit state theory, it has been noticed that, for every structure, there exists a unique number of independent mechanisms of collapse and every other mechanism may be formed by combining these fundamental modes. By application of the Kinematic theorem, the actual collapse mode can be indentified as the lowest value of the collapse load consistent with the mechanics of the structure and every likely mode should be investigated. In this thesis, only a few probable modes have been investigated and the resulting mode has been confirmed as the true mode by computing the statically admissible bending moment in which the fully plastic moment of resistance is nowhere exceeded.

An extensive study has been made to understand the work done by Baster, Rockey, Ostapenko and others, on the ultimate load method of design for plate girders. In the following paragraphs, a concluding re-statement is made.

Ostapenko and others stated that the ultimate strength of a plate girder under pure bending and having the large portion of the web in compression may be assumed to be controlled by the failure of the compression flange column. Baster stated that the strength prediction of a plate girder cross section, subjected to pure bending is not so difficult, because, only three possible types of compression flange buckling have to be considered; but some difficulties in specifying admissible compressive flange stresses may appear. A slender web burdens the flanges with the stresses which the web cannot resist. This leads to an increase of the compressive flange stresses above the nominally calculated values.

Rockey stated that, if there are no imperfections in the web plate of a plate girder, then prior to buckling it does not impose any lateral loading upon the boundary members. But once the web has buckled, it has no capacity to carry any more compressive load across the diagonal and therefore the web has to carry the additional shear loads by a diagonal tensile membrane action. This membrane action may impose lateral forces upon the flanges and so the girder may fail due to the formation of the plastic hinges in the flanges with consequent local buckling. If the flange plates are stiff enough, then the web and the stiffeners develop a full membrane action. When the web has developed the full membrane action, then it cannot carry any more shear force and at that point the flanges and the stiffeners carry the additional shear force which together act as a Vierendeel frame. Baster proposed that the web plate may fail due to the developments of the inclined plastic band anchored against the vertical stiffeners. This is based on the theory that the flanges of most plate girders are too flexible to resist the membrane action. Rockey stated that this type of collapse mode may "significantly underestimate and over-estimate" the strength of the girder. According to Rockey, when a web plate in shear, buckles prior to yielding, then the failure is due to the development of a diagonal tension band which is fully yielded together with the development of the plastic hinges in the flange to form a mechanism. Rockey has proved by experiments that the width of the diagonal band and the position of the plastic hinges depends on the ratio

I/a^3t where I is the second moment of area of the compression flange about an axis through its centroid and perpendicular to web plate, a is the spacing between the transverse stiffeners and t is the thickness of the web plate.

Rockey has shown by experiments that the web develops a tension band. The angle of inclination of the tension band is equal to the inclination of the geometrical diagonal of a stiffened panel. The width of this tension band is such that, the intercept of its boundary with the flange coincides with the position of the plastic hinges in the flanges. Baster agreed with Rockey's statement and both agreed that the angle of inclination of this tension field will be close to 45° , unless web behaviour is modified by the stiffener geometry.

Baster and Rockey both stated that the web plates are usually subjected to a combination of bending and shear. Baster stated that the presence of shear has both a detrimental and a beneficial aspect. The beneficial aspect is due to the fact that shear force always imply a moment gradient and thus, only a short portion is affected by the maximum moment. The adverse aspect is that, a web which is "exhausted" by shear cannot simultaneously take its allotted bending moment and the flanges will have to compensate for it, resulting in a higher flange stress than computed by the sectional modulus concept. According to Rockey, when a panel is loaded in direct compression, the central area of the panel buckles, becoming unable to participate any further in resisting direct stress and any additional direct load has to be carried by the web material adjacent to the flanges and stiffeners. As the stress distribution in a yielded panel which is subjected to both shear and bending is very complicated, Rockey stated that, after the panel is buckled, the flanges alone carry the additional loads. Thus Baster and Rockey virtually agreed on these assumptions.

After experimenting with numbers of miniature welded plate girders with d/t ratio 85, 241, 299 and 401 (compared with B.S. 153 normal limits 85, 240 300 and 400), Longbottom & Heyman suggested that the load factor in bending should be about 1.85, and 1.47 in pure shear for a simply supported girder. But Horne stated that these values are not justifiable, because the difference is too great. Horne's suggestion is that, the load factors higher than 1.5 are nowhere required. Baster's suggestion is 1.83 and 1.73 respectively. B.S. 449 recommends a load factor of 1.75 for dead plus superimposed loading and an increase in working stresses of 25 per cent, if such increase is solely due to the effect of wind. This implies a reduction in collapse load factor to 1.4. Such a reduction is practically impossible, because, full wind and superload cannot act at the same time. For certain multi-storey buildings, braced against wind and where rigorous load patterns are warned, the load factor of 1.4 may be used on dead plus superimposed plus wind loads. A.I.S.C. proposed that these values should be 1.65 and 1.46 respectively. As the A.I.S.C. code is well established in the U.S. (and in offshore structure design in the U.K.), it may be advantageous to use a load factor of 1.65 on dead plus superimposed loading and carry out a second analysis at a load factor of 1.46 on dead plus superimposed plus wind loads. Finally, the more critical of these two cases will give the actual design.

From these conclusions of recent research, it appears to be certain that a new design code must change many of the present principles of the design of plate girders. These changes will, also, depend on the acceptance of new philosophies of design. Whereas previous procedures were aimed at the prevention of the occurrence in any component of too high stresses, the new approach must take account of the new knowledge of the mechanisms of failure,

and of such things as the post-buckling behaviour of web plates, with the accompanying changes in flange and stiffener function. Such changes, when applied to the design code, must result in modifications to the detailed design of the components of the girder, such as the provision of greater local bending stiffness in flanges and vertical web stiffeners. Some of these changes will result in more economical design, while some will not have this effect.

Returning to the detailed consideration of the girder bridge, which is the subject of this thesis, plastic design of the cross girders offers no advantage, because of the simple supported condition. Consistent with the changes implied in the application of ultimate load analysis, plastic design gives advantages of economy to beams which are fixed at their ends or are continuous. When the collapse load has been reached, a sufficient number of plastic hinges must form to create a mechanism. Usually, three such hinges are required, one at each (fixed) end and the position of maximum span moment at collapse.

The deck system used in this project is an orthotropic deck and to analyse this type of deck plastically, much more research should be carried out. Although, in this thesis, a study has been made to examine the possible modes of failure, it is not enough to arrive at any suggestive conclusion to analyse the whole deck system, plastically.

The calculated weight of the plate girder is 26 tons (approx.) when elastic design method is applied. On the other hand, if it has been designed using more recently acquired knowledge and using ultimate load techniques based on this, the weight could be reduced to 21 tons (approx.). Therefore, a saving of about 19 per cent may be achieved, if the bridge main girders had been designed plastically. No savings can be made on the cross girders. From the practical point of view, the cross section of the main girders are too shallow to fix the cross girders, especially the 38" deep pivot girder. If the depth of the girders are increased from 61 $\frac{1}{4}$ " to 108" (say), the saving will not be significant. It is quite clear from the evidence within the scope of this thesis that economies can be made in the design of plate girders through the greater understanding achieved by recent research, although no particular guidance is yet available from a published code. It is understood that in the next edition of B.S. 153, there will be a major change and a limit state approach will be adopted rather than the previous dominance of the elastic analysis.

8. Apprehension of the appendices.

8.1 Appendix A - Part I Loading conditions.

a. Dead Loads; dead loads have been considered for

- i. the deck and troughings
- ii. the cross girders
- iii. the asphalt surfacing
- iv. the walkway
- v. snow.

Snow load has been taken into consideration, as stated in B. S. 153 part 3A.

b. Live Loads;

i. Type H.A. loading system has been considered as recommended in B.S. 153, part 3A, together with the knife edge loading (for equivalent U.D.L. and knife edge load, see table 1 and fig. 1, abstracted from B.S. 153, part 3A).

ii. Type H.B. loading system has been considered separately as recommended by B.S. 153, part 3A.

The main girders have been checked for the stability when the bridge should be in cantilever condition (see appendix A paragraph 3.3). Maximum moments and shear forces have been computed for the girders at cantilever condition (see sketches sk. 1 and 2). In paragraph 4, the maximum moments and shear forces have been computed for the main girders at working position, by applying the dead loads and the live loads, in various ways, to confirm the worst possible combinations of the loads (see sketches sk. 3 to 10). Then the moments and the shear forces, due to HA and HB loading systems have been examined. However, it is noticed that, HA loading system is more critical than the HB loading system. In paragraph 4.2, effect of wind

load on the main girders has been considered.

8.2 Appendix A - Part II; Stress Computations.

Various stress computations have been provided in this section of the appendix and most of the formulas are obtained from B.S. 153 part 3B and 4. This section deals with the stresses for the main girders only.

In paragraph 5.3, a section has been selected to resist the maximum moments at the worst possible loading conditions. The sectional properties are recalculated and then stresses are computed to examine whether this section is capable of resisting the maximum moments and shear forces. Finally, the computed stresses are compared with the permissible stresses, as recommended in B.S. 153 part 3B and 4.

In Paragraph 5.4 a heavier section has been selected. This section of the main girder is at the centre pivot, which is situated at 114.5 ft. from the nose bearing. The reason of the heavier section is, due to the higher bending moment value at this point.

The sections of the main girder at various distances have been checked in paragraph 5.5, because heavier sections may not be required over certain lengths of the girder and so, a significant quantity of steel may be saved.

In paragraph 5.8, the requirement of web reinforcements has been computed as per the clauses stated in B.S. 153 part 3B.

8.3 Appendix B; Analysis of Orthotropic Deck.

Maximum moments have been computed by applying dead loads and live loads (HA and HB type loading systems), in various ways to obtain the worst possible loading conditions. The basic theory of the orthotropic deck system has been adopted from A.I.S.C. Manual (18). The stresses have been computed and again, the value of computed stresses have been compared with the permissible stress values as recommended by B.S. 153 parts 3B and 4.

8.4 Appendix C; Floor beams, connections etc.

This section of appendix provides the connection details of the bridge members. In general, high strength friction grip bolts are used at the site connections. This section also provides the stress computations for the following;

- i. deck over Kentledge box
- ii. floor beam at tail jacks
- iii. Kentledge box detail
- iv. slewing gear support steelwork
- v. ring girder
- vi. roller tracks

8.4 Appendix D; This appendix provides the computations of the plastic moments of the main girders. Various load conditions are considered and finally, the plastic moments are summarised in page D20.

8.5 Appendix E; In this part, attempts have been made to obtain lighter sections of the main girders by applying limit state theory. Many researchers have derived a number of formulas for the design of plate girders behaving inelastically. A few of those formulas have been used

to be satisfied about the final section chosen.

8.6 Appendix F; An attempt has been made to obtain a lighter section for the cross-girders; but it became obvious that, limit state theory is not appropriate to a simply supported beam condition.

8.7 Appendix G; Inelastic buckling of the orthotropic steel deck has been analysed in this part of the appendix.

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Appendix A

Part I

1. Loading Conditions: advised by "The Mersey Docks & Harbour Board" dated 9th April 1973.
 - a. Highway loading Type H.A. to B.S. 153 Part 3A, with appropriate E.U.D.L (equivalent uniformly distributed load), K.E.L (knife edge load) and wheel loads as shown in table 1 (see next page) and with impact allowance of 40% on nose and tail transverse girders.
 - b. Highway loading Type H.B. to B.S. 153 Part 3A, using 20 units of loading with an impact allowance of 40% on nose and tail transverse girders.
 - c. Footway loading to B.S. 153 Part 3A, to resist crowd conditions.
 - d. Handrails and standards, to resist crowd conditions.
 - e. Snow load is applicable to all cases @ 3 lbf/ft².
 - f. Steady wind pressure @ 20 lbf/ft² maximum.
 - g. Operating wind pressure @ 20 lbf/ft² for opening or closing the bridge in 2½ minutes.
 - h. Temperature variation, forces and movements resulting from a temperature variation within the range -7°C to +40°C to be catered for.
2. The resistances to be overcome in turning the swing bridge are:
 - a. the inertia of the span, i.e., the mass to be revolved through a required angle in a specified time.
 - b. the frictional resistance due to the dead load of the bridge resting on the centre pivot and tail rollers.
 - c. the friction due to side wind pressure:

A2

- i. on vertical surface of pivot
- ii. axle & rolling friction on trailing wheel due to the over-turning effect of a side wind.
- d. the unbalanced side wind pressure.
- e. the unbalanced load from snow etc.
- f. the friction of gearing, shaft bearing etc.

In the following chapters these items will be dealt with; but first of all, it is intended to find the maximum bending moments & shear forces under the worse loading conditions, as per B.S. 153 Part 3A.

3. Design Loads

3.1. Dead loads:

assumed	a. Deck & troughing	38.0	lb/ft ²
	b. Cross girders	10.5	"
	c. asphalt surfacing	42.0	"
	d. walkway	30.0	"
	e. Snow	3.0	"

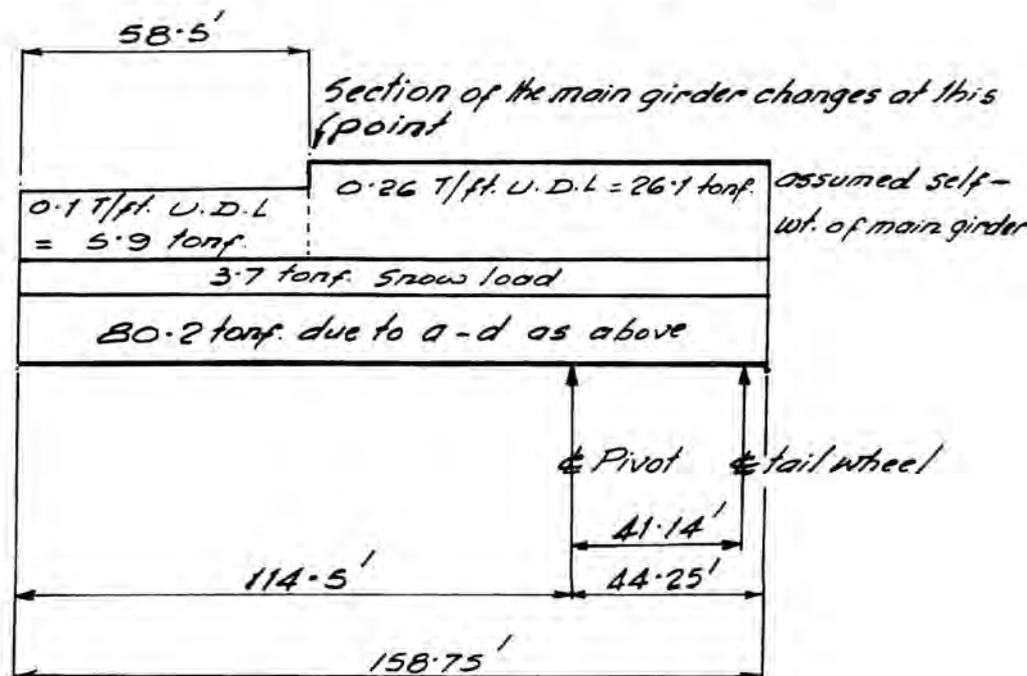


Fig. A1. Main Girder @ Cantilever Condition

3.2. Main Girder - Cantilever Condition :

$$\text{Dead load due to a \& b over the full length of the bridge} \left\{ \begin{array}{l} \text{d/o length} \quad \text{d/o width} \\ \frac{158.75 \times 25}{2 \text{ N}^\circ \text{ Main Girders}} \times \frac{48.5 \text{ lbf/ft}^2}{2240 \text{ (T)}} = 43.0 \text{ tonf.} \end{array} \right.$$

$$\text{Dead load due to c over the full length of the bridge} \left\{ \begin{array}{l} \text{Width of asphalt carriageway} \\ \frac{158.75 \times 20.0}{2 \text{ N}^\circ \text{ main girders}} \times \frac{42.0 \text{ lbf/ft}^2}{T} = 29.8 \text{ tonf.} \end{array} \right.$$

$$\text{Dead load due to d over the full length of the bridge} \left\{ \begin{array}{l} \text{Width of the walkway} \\ \text{@ each side of the bridge} \\ 158.75 \times 3.5 \times \frac{30 \text{ lbf/ft}^2}{T} = 7.4 \text{ tonf.} \end{array} \right.$$

$$\text{Dead load due to e over the full length of the bridge} \left\{ \begin{array}{l} \frac{158.75 \times 35.0}{2 \text{ N}^\circ \text{ main girder}} \times \frac{3.0 \text{ lbf/ft}^2}{T} = 3.7 \text{ tonf.} \end{array} \right.$$

$$\Sigma \text{ dead load due to a-e} = 83.9 \text{ tonf.}$$

3.3. Stability :

K = Kentledge load.

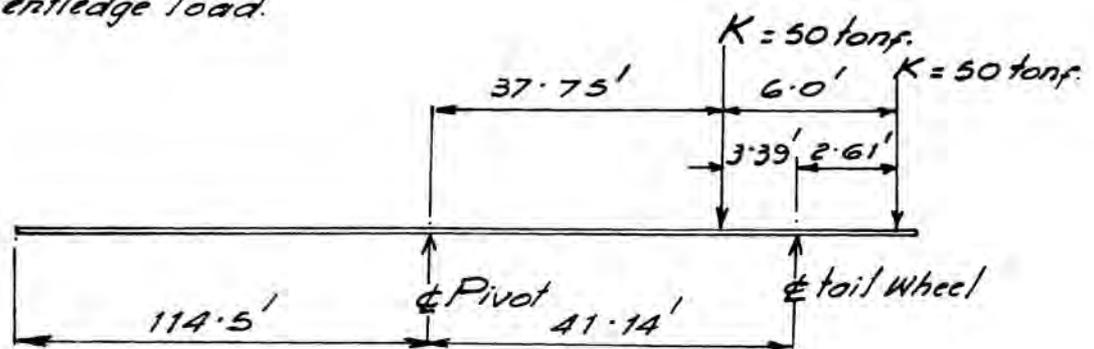


Fig. A2. Kentledge load Position

taking moments about pivot :

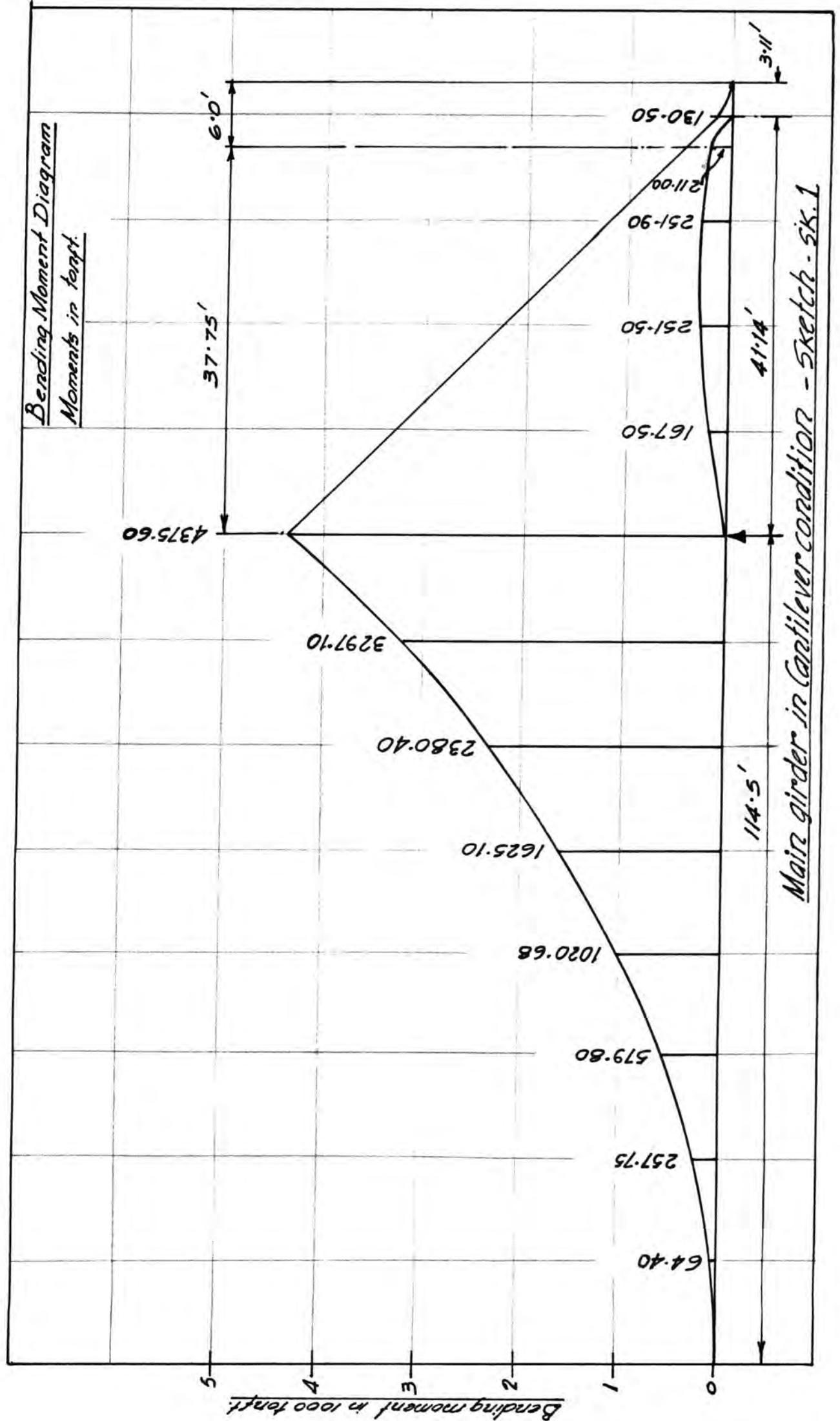
$$\text{Over-turning moments} \quad 5.9 \times 85.25 = 503 \text{ tonft.}$$

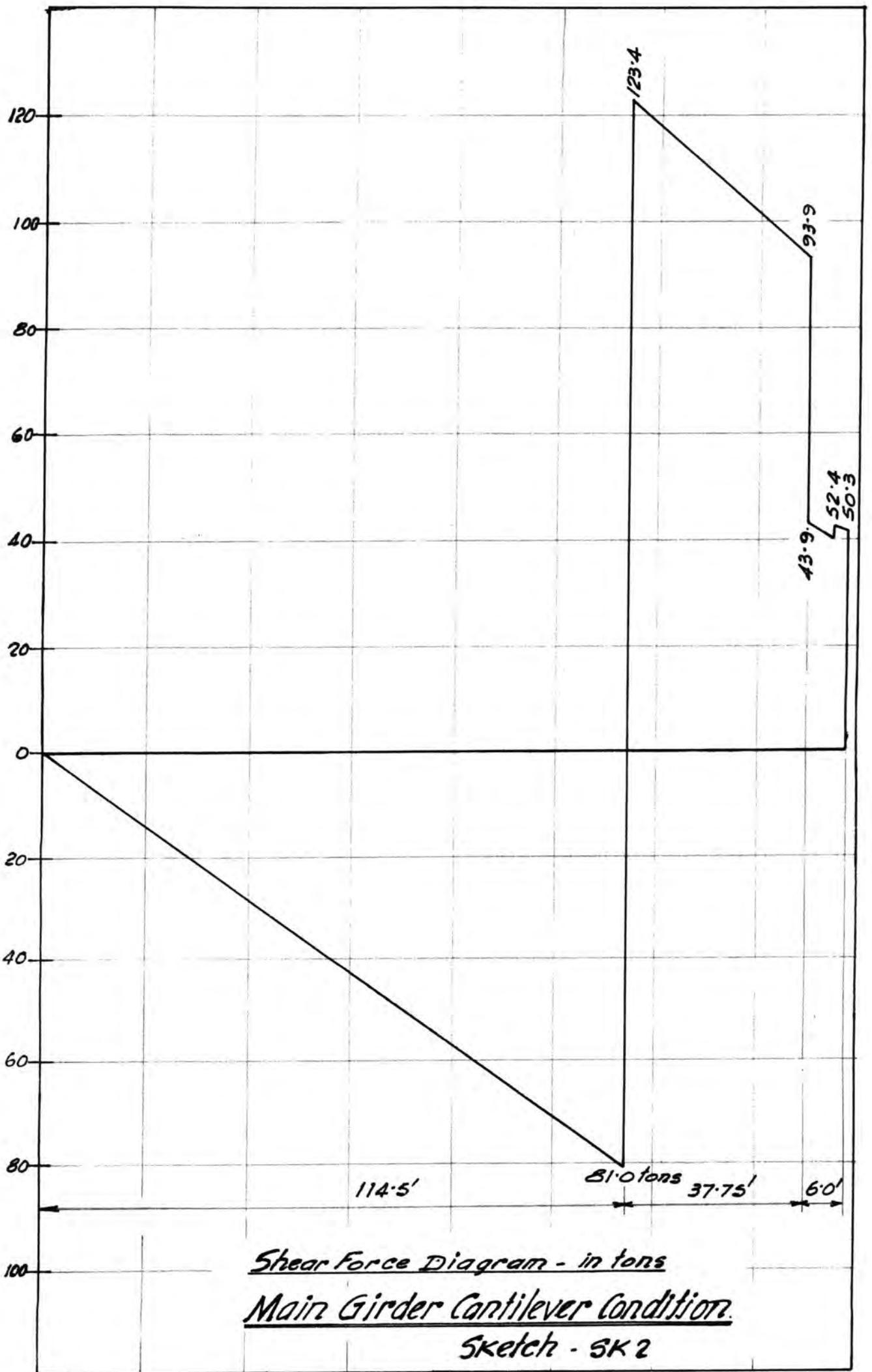
$$\frac{26.1 \times 50^2}{100.25 \times 2} = 408.2 \text{ ''}$$

$$\frac{83.9 \times 114.5^2}{158.75 \times 2} = 3464.4 \text{ ''}$$

$$\Sigma \text{ moments} = \underline{4375.6 \text{ tonft.}}$$

Bending Moment Diagram
Moments in tonft.





4. Main Girder - Working Position

4.1 Loading Cases:

- A. Dead loads
- B. Snow load
- C. Highway loading type H.A
- D. Walkway Super loading
- E. Highway loading type H.B.

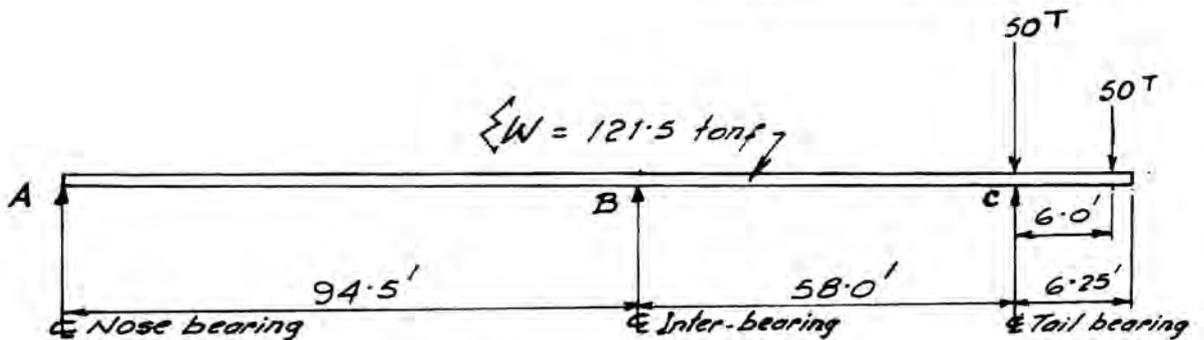
Case A - Dead loads

as assumed before,

dead load due to deck, asphalt etc. = 80.2 tonf.

dead load due to self weight of main girder @ 0.26 tonf/ft. length } = 41.3 "

$$\Sigma \text{dead load} = 121.5 \text{ tonf.}$$



applying moment distribution:

<u>Member</u>	<u>Stiffness factor</u>	<u>distribution factor</u>
BA	$\frac{3}{4} \times \frac{1}{94.5} = 0.0079$	0.38
BC	$\frac{3}{4} \times \frac{1}{58.0} = 0.0129$	0.62

fixing moments:

$$M_{BA} = \frac{121.5 \times 94.5^2}{158.75 \times 8} = 854.4 \text{ tonft.}$$

$$M_{BC} = \frac{121.5 \times 58.0^2}{158.75 \times 8} = 321.8 \text{ tonft.}$$

$$\text{Cantilever moment} = 50 \times 6 + \frac{121.5 \times 6.25^2}{158.75 \times 2} = 315 \text{ tonft.}$$

AB

Distribution table :

	A	B		C
		0.38	0.62	
Fixing moments	+854.4	-321.8	+315	-315
Distributed		+157.5		
	-262.2	-427.9		
Final moments	+592.2	-592.2	+315	-315

Reactions :

$$\text{at A : } \frac{121.5 \times 94.5}{158.75 \times 2} - \frac{592.2}{94.5} = 29.9 \text{ tonf.}$$

$$\text{at B : } \frac{121.5 \times 94.5}{158.75 \times 2} = 36.2 \text{ tonf.}$$

$$+ \frac{121.5 \times 58.0}{158.75 \times 2} = 22.2 \text{ "}$$

$$+ \frac{592.2}{94.5} = 6.3 \text{ "}$$

$$+ \frac{592.2 - 315}{58.0} = 4.8 \text{ "}$$

$$\Sigma = 69.5 \text{ tonf.}$$

$$\text{at C : } \frac{121.5 \times 58.0}{158.75 \times 2} = 22.2 \text{ tonf.}$$

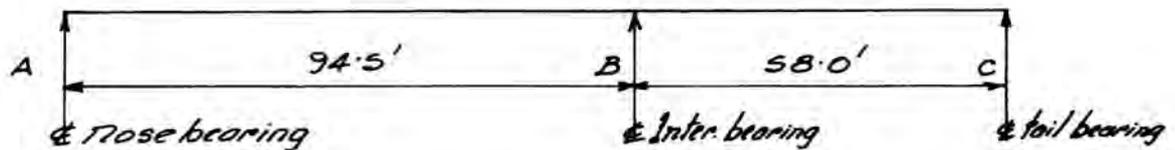
$$+ \frac{121.5 \times 6.25}{158.75} = 4.8 \text{ "}$$

$$- \frac{(592.2 - 315)}{58} = -4.8 \text{ "}$$

$$+ (50 + 50) = 100.0 \text{ "}$$

$$\Sigma = 122.2 \text{ tonf.}$$

Case B - Snow load = 3.7 tonf.



Distribution factors:

$$DA = 0.38 \quad \& \quad DC = 0.62$$

fixing moments:

$$M_{BA} = \frac{3.7 \times 94.5^2}{152.5 \times 8} = 27.1 \text{ tonft}$$

$$M_{BC} = \frac{3.7 \times 58^2}{152.5 \times 8} = 10.2 \text{ m}$$

	0.38	0.62
	+27.1	-10.2
	-6.4	-10.5

final moments +20.7 -20.7

Reactions:

$$\text{At A : } 1.15 - \frac{20.7}{94.5} = 0.9 \text{ tonf.}$$

$$\text{At B : } 1.14 + 0.7 + \frac{20.7}{94.5} + \frac{20.7}{58.0} = 2.4 \text{ tonf.}$$

$$\text{At C : } 0.7 - \frac{20.7}{58.0} = 0.4 \text{ tonf.}$$

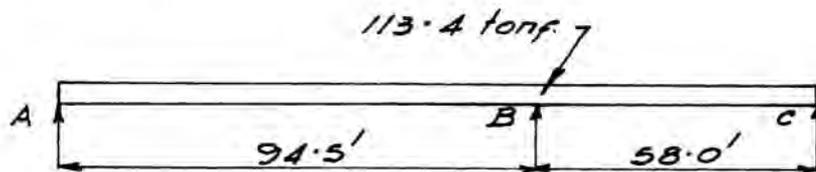
free bending moment:

$$\text{Span AB : } \frac{3.7}{152.5} \times \frac{94.5^2}{8} = 27.1 \text{ tonft.}$$

$$\text{Span BC : } \frac{3.7}{152.5} \times \frac{58.0^2}{8} = 8.3 \text{ tonft.}$$

Case C - Highway loading type H.A.

B.S. 153 Part 3A, Clause 4A, c recommends that, the H.A. type loading shall be taken to occupy one carriageway lane and to be uniformly distributed over the full width of the lane, and two lanes shall always be considered as occupied by full H.A. type loading, while other lanes shall be considered as occupied by $\frac{1}{3}$ the full lane loading.

1st Condition

assume H.A. loading over whole length A-C, then loaded length = 152.5 ft. \therefore H.A. load = 1665 lb/lin.ft., from table 1 B.S. 153, Part 3A (see page 12).

Hence H.A. load per girder = $\frac{1665 \times 152.5}{7} = 113.4 \text{ tonf.}$

fixing moments :

$$M_{BA} = \frac{113.4 \times 94.5^2}{152.5 \times 8} = 830.1 \text{ tonft.}$$

$$M_{BC} = \frac{113.4 \times 58.0^2}{152.5 \times 8} = 312.7 \text{ tonft.}$$

Distribution factors : $DF_{BA} = 0.38$ & $DF_{BC} = 0.62$

0.38	0.62
+830.1	-312.7
-196.6	-320.8
+633.5	-633.5

final moments

Reactions :

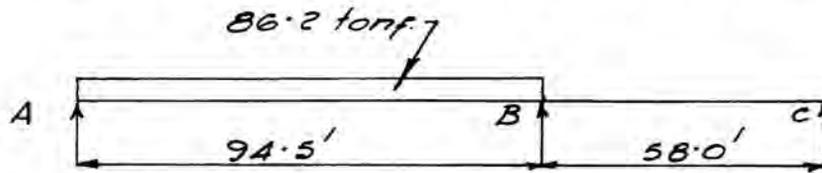
$$\text{at A} : 35.2 - \frac{633.5}{94.5} = 28.5 \text{ tonf.}$$

$$\text{at B} : 56.6 + \frac{633.5}{94.5} + \frac{633.5}{58.0} = 74.2 \text{ "}$$

$$\text{at C} : 21.6 - \frac{633.5}{58.0} = 10.7 \text{ "}$$

2nd Condition

assume H.A. loading over 94.5 ft. span, A-B :



Loaded length 94.5 ft. \therefore H.A. load = 2044 lbf/ft. from table 1 B.S. 153 Part 3A (see page 12).

$$\text{H.A. load per girder} = \frac{2044 \times 94.5}{7} = 86.2 \text{ tonf}$$

Fixing moments :

$$M_{BA} = \frac{86.2 \times 94.5}{8} = 1018.2 \text{ tonft}$$

Distribution factors : $DF_{BA} = 0.38$ & $DF_{BC} = 0.62$

	0.38	0.62
	+1018.2	
	-386.9	-631.3
final moments	+631.3	-631.3

Reactions :

$$\text{at A} : 431 - \frac{631.3}{94.5} = 36.4 \text{ tonf}$$

$$\text{at B} : 431 + \frac{631.3}{94.5} + \frac{631.3}{58.0} = 60.7 \text{ "}$$

$$\text{at C} : - \frac{631.3^3}{58.0} = -10.9 \text{ "}$$

B.S. 153 : Part 3A : 1954

TABLE 1. HIGHWAY LOADING. TYPE HA
Equivalent U.D.L. to be used in conjunction with the knife edge load. (See Fig. 1).

Loaded length, ft	U.D.L. for beams per linear foot of lane	U.D.L. for longitudinal slabs per linear foot of lane	U.D.L. for transverse slabs and cross girders per linear foot of lane	Loaded length, ft	U.D.L. for beams per linear foot of lane	U.D.L. for longitudinal slabs per linear foot of lane	U.D.L. for transverse slabs and cross girders per linear foot of lane
3	24 200	24 200	22 700	12	4 870	3 250	2 600
4	17 000	17 000	11 800	13	4 540	2 950	2 400
5	12 250	12 250	7 700	14	4 210	2 700	2 300
6	9 660	8 850	5 800	15	3 880	2 500	2 200
7	8 230	6 550	4 600	16	3 550	2 400	2 200
8	7 250	5 200	3 900	17	3 220	2 300	2 200
9	6 440	4 520	3 400	18	2 880	2 250	2 200
10	5 800	4 000	3 100	19	2 540	2 200	2 200
11	5 200	3 600	2 800	20-75	2 200	2 200	2 200

Loaded length	Load								
ft	lb/lin ft								
80	2 160	175	1 530	270	1 210	430	1 005	1 200	660
85	2 120	180	1 500	275	1 200	440	1 000	1 300	630
90	2 080	185	1 480	280	1 190	450	990	1 400	600
95	2 040	190	1 460	285	1 175	460	985	1 500	580
100	2 000	195	1 440	290	1 165	470	975	1 600	560
105	1 960	200	1 420	295	1 155	480	970	1 700	540
110	1 920	205	1 400	300	1 145	490	965	1 800	520
115	1 890	210	1 380	310	1 130	500	960	1 900	510
120	1 860	215	1 360	320	1 115	550	935	2 000	490
125	1 830	220	1 350	330	1 105	600	910	2 100	480
130	1 800	225	1 335	340	1 090	650	885	2 200	470
135	1 770	230	1 320	350	1 080	700	860	2 300	460
140	1 740	235	1 305	360	1 070	750	835	2 400	450
145	1 710	240	1 290	370	1 060	800	810	2 500	440
150	1 680	245	1 280	380	1 050	850	790	2 600	430
155	1 650	250	1 265	390	1 040	900	770	2 700	420
160	1 620	255	1 250	400	1 030	950	750	2 800	410
165	1 590	260	1 240	410	1 020	1 000	730	2 900	410
170	1 560	265	1 225	420	1 015	1 100	690	3 000	400

94.5 ft. by interpolation load = 2044 lb/lin ft.

152.5 ft. by interpolation load = 1665 lb/lin ft.

NOTE TO TABLE 1 AND FIG. 1

Normal loading (Type HA) approximately represents the effect of three vehicles, each 22 tons in weight, closely spaced, in each of two carriageway lanes, followed by 10-ton and 5-ton vehicles. For short span members, to allow for possible local concentration of loads, the effects of two 9-ton wheels 3 ft apart have been considered (i.e. approximately two 11¼-ton wheels with 25 per cent overstress).

In general, normal loading is sufficient to cover 30 units of abnormal loading (Type HB) for loaded lengths above 100 ft and for slabs (but see Clause A5); and at least 20 units of abnormal loading for beams having spans less than 100 ft carrying decks with a weight similar to that of an ordinary reinforced concrete slab. Where a bridge is definitely required to carry abnormal loads in excess of 20 units a check should be made.

A special case is a narrow bridge or one in which the carriageway is cantilevered beyond the beams, where high stresses can occur under abnormal loading.

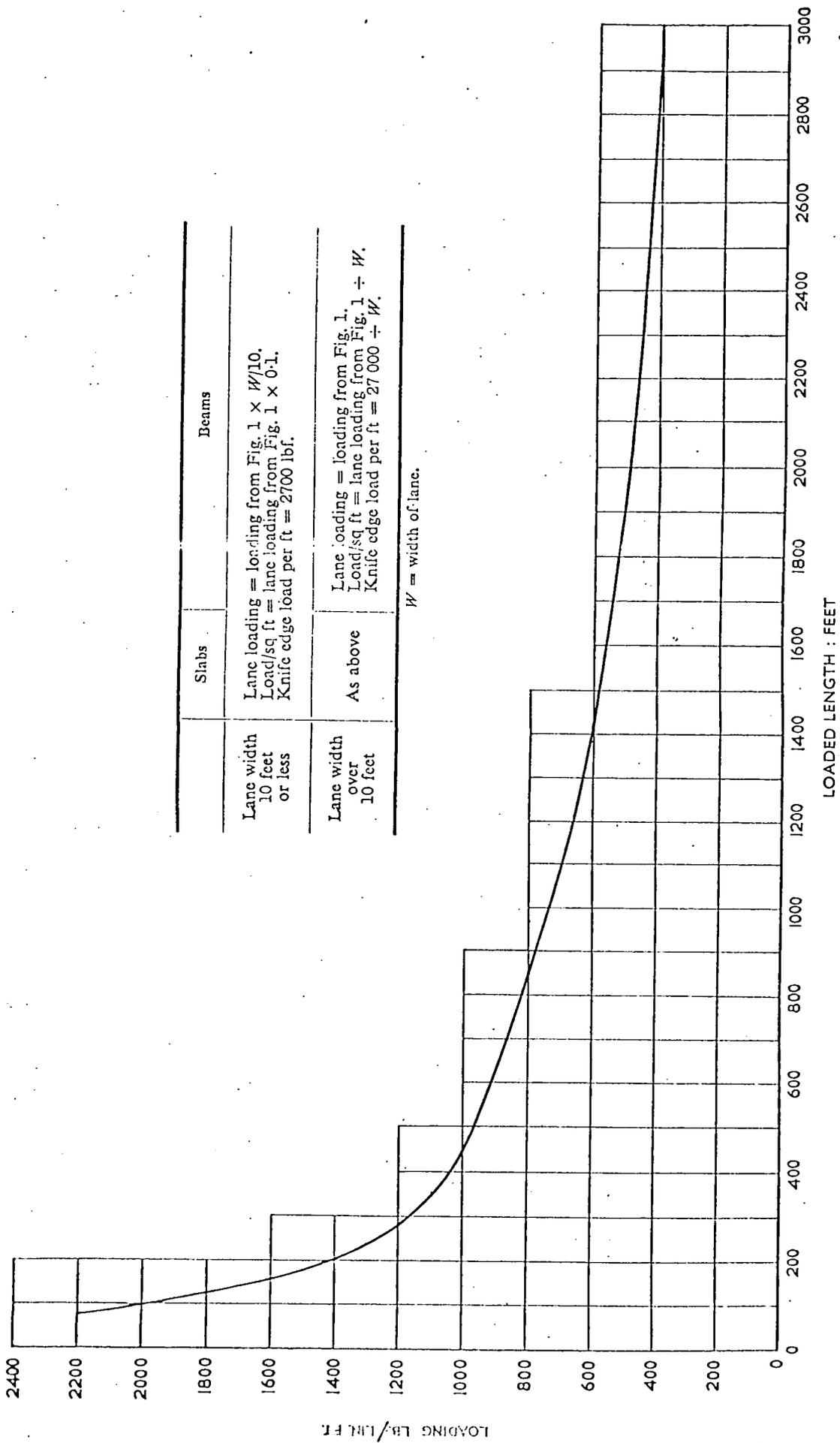
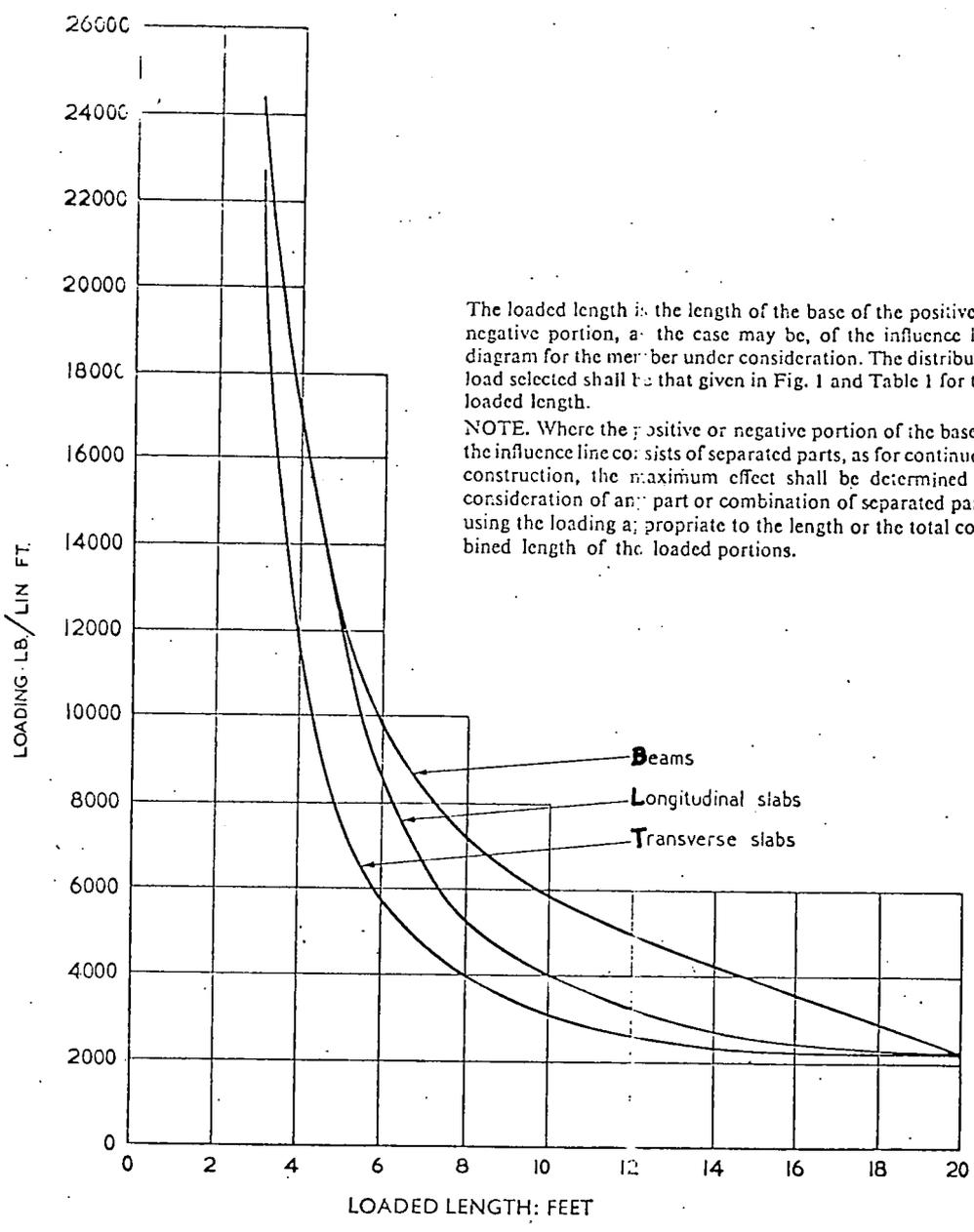


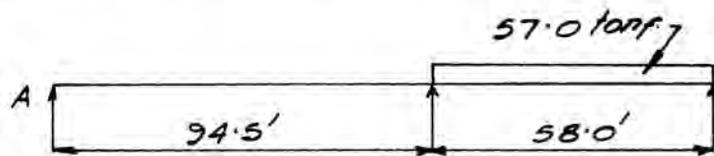
Fig. 1. Loading curve for type HA highway loading



3rd Condition

H.A loading over 58.0 ft. span, B-C

Loaded length 58.0 ft. \therefore H.A loading = 2200 lb/ft. from table 1, B.S. 153 Part 3A (See page 12).



$$\text{H.A. load per girder} = \frac{2200 \times 58}{7} = 57.0 \text{ tonf.}$$

fixing moment :

$$M_{BC} = \frac{57 \times 58}{8} = 413.3 \text{ tonft.}$$

Distribution factors: $Df_{BA} = 0.38$ & $Df_{BC} = 0.62$

0.38	0.62
	- 413.3
+ 157.1	+ 256.2
+ 157.1	- 157.1

final moments

Reactions :

$$\text{at A : } - \frac{157.1}{94.5} = - 1.7 \text{ tonf.}$$

$$\text{at B : } 28.5 + \frac{157.1}{94.5} + \frac{157.1}{58.0} = + 32.9 \text{ ''}$$

$$\text{at C : } 28.5 - \frac{157.1}{58.0} = + 25.8 \text{ ''}$$

Case C - HA loading (Cont.)Knife edge load - B.S. 153 Part 3A clause A3

In 1st, 2nd and 3rd conditions of loading systems, it has been shown that the uniformly distributed load varies with the loaded length, which represents the ordinary axle loads of the M.T. standard train, perfectly distributed.

Now, an invariable load of 2700lb. per foot of width will be applied at the section where it will, when combined with the U.D. load, be most effective. This is a knife edge load and it represents the excess in the M.T. standard train of the heavy axle over the other axles and this excess being undistributed. This load is applied for bending moment at mid-span, at mid-span point; for shear at the support, and for shear at any section.

In spans less than the axle spacing which is 10 ft, the concentration serves to counteract the over-dispersion of the distributed load. In slabs the K.E. load of 2700 lb/ft of width is taken as acting parallel to the supporting members, irrespective of the direction in which the slab spans. In longitudinal girders, stringers etc., this concentrated loading is taken as acting transversely to them, i.e. parallel with their supports. In transverse beams the concentrated loading is taken as acting in line with them, i.e. 2700 lb/ft run of beam.

However in all cases, irrespective of span length, one K.E. load of 2700 lb/ft run of width is taken as acting in conjunction with the U.D. load appropriate to the span or loaded length.

A 16

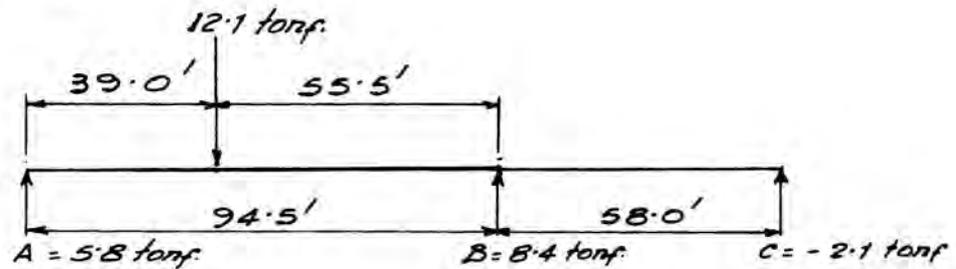
K.E load = 2700 lb/ft width of lane (see Fig 1, page 13 & 13a).

$$\therefore \text{K.E load per girder} = \frac{2700}{7} \times 10 = 12.1 \text{ tonf.}$$

This 12.1 tonf. load is now to be positioned to give maximum bending moment.

1. K.E. load on 94.5 ft span:

Position of K.E. load = $0.5875 \times 94.5 = 55.5$ ft. from the central support B.



fixing moment:

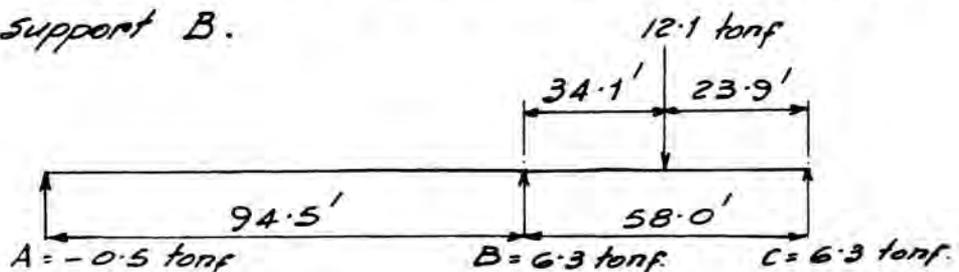
$$M_{BA} = \frac{12.1 \times 39.0 \times 55.5 (2 \times 39.0 + 55.5)}{2 \times 94.5^2} = 195.8 \text{ tonft.}$$

Distribution factors: $DF_{BA} = 0.38$ & $DF_{BC} = 0.62$

	0.38	0.62
	+195.8	
	-74.4	-121.4
final moments	+121.4	-121.4

2. K.E. load on 58.0 ft. span:

Position of K.E. load = $0.5875 \times 58.0 = 34.1$ ft. from central support B.



$$\text{fixing moment} = \frac{12.1 \times 34.1 \times 23.9 (2 \times 23.9 + 34.1)}{2 \times 58^2} = 120 \text{ tonft.}$$

Distribution factors: $DF_{BA} = 0.38$ & $DF_{BC} = 0.62$

final moments

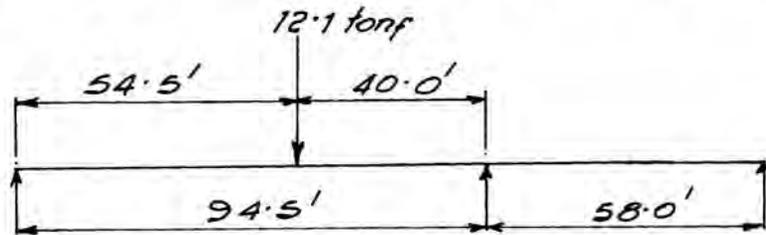
0.38	0.62
	- 120
+ 45.6	+ 74.4
+ 45.6	- 45.6

Reactions:

Maximum reaction will occur as K.E load passes over any support; i.e., at A, B or C max. reaction = 12.1 tonf.

3. K. E load on 94.5ft span for max. fixing moment at Support B :

Position of K.E. load = $0.577 \times 94.5 = 54.5$ ft. from support A.



fixing moment :

$$M_{BA} = \frac{12.1 \times 54.5 \times 40(2 \times 54.5 + 40)}{2 \times 94.5^2} = 220.1 \text{ tonft.}$$

Distribution factors: $DF_{BA} = 0.38$ & $DF_{BC} = 0.62$

final moment

0.38	0.62
+ 220.1	
- 83.6	- 136.5
+ 136.5	- 136.5

Thus, maximum moment at support B = 136.5 tonft.

Case D - Walkway super load.

In B.S. 153, Part 3A, it is stated that the standard footway loading is to be taken as 80 lbf./ft.^2 or loaded lengths upto and including 75 ft. Lengths above 75 ft., a load per sq. ft. equal to the standard U.D.L. for highways is to be taken from table 1 (see page 12), which should be multiplied by $\frac{80}{2200}$ with the provision that for crowd loading members exclusively supporting or forming the footway be designed for 100 lbf./ft.^2 . The value of 100 lbf./ft.^2 is taken as an inclusive load of pedestrian and light vehicular traffic, (eg. bi-cycle). There is no necessity to add an allowance for impact to this figure, because, the denser the crowd, the more nearly static becomes the load. It may be that, the sparser the pedestrians, the greater is the possibility for impact, but it will be appreciated that on balance, the value given is inclusive of all variations in concentrations and impact effects.

Three conditions will be considered to analyse the worst possible case.

Condition 1: Walkway loading over the full length of the bridge. Loaded length is then, 152.5 ft. & load = 1665 lbf./ft.
So, Walkway loading = $\frac{80}{2200} \times 1665 = 60.5 \text{ lbf./ft.}^2$

Thus, Walkway load = $\frac{60.5}{7} \times 3.5' \times 152.5' = 14.4 \text{ tonf. U.D.L.}$



fixing moments:

$$M_{BA} = \frac{14.4 \times 94.5^2}{152.5 \times 8} = 105.4 \text{ tonft.}$$

$$M_{BC} = \frac{14.4 \times 58^2}{152.5 \times 8} = 39.7 \text{ tonft.}$$

Distribution factors : $DF_{BA} = 0.38$ & $DF_{BC} = 0.62$

	0.38	0.62
	+ 105.4	- 39.7
	- 25.0	- 40.7
final moments	+ 80.4	- 80.4

Reactions :

$$\text{at A : } 4.45 - \frac{80.4}{94.5} = 3.6 \text{ tonf.}$$

$$\text{at B : } 7.2 + \frac{80.4}{94.5} + \frac{80.4}{58} = 9.4 \text{ "}$$

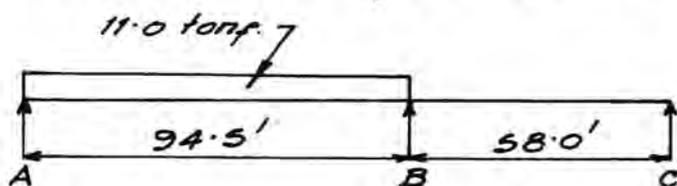
$$\text{at C : } 2.75 - \frac{80.4}{58} = 1.4 \text{ "}$$

Condition 2 : Walkway loading over 94.5 ft. span, A-B.

Loaded length 94.5 ft. \therefore load = 2044 lb./lin. ft. from table 1 B.5. 153 Part 3A.

$$\text{So, Walkway loading} = \frac{80}{2200} \times 2044 = 74.3 \text{ lb./ft.}^2$$

$$\text{Hence Walkway load} = \frac{74.3}{7} \times 94.5' \times 3.5' = 11.0 \text{ Tonf. U.D.L.}$$



fixing moments :

$$M_{BA} = \frac{11.0 \times 94.5}{8} = 130.0 \text{ tonft}$$

Distribution factors : $DF_{BA} = 0.38$ & $DF_{BC} = 0.62$.

	0.38	0.62
	+ 130	
	- 49.4	- 80.6
	+ 80.6	- 80.6

Reactions :

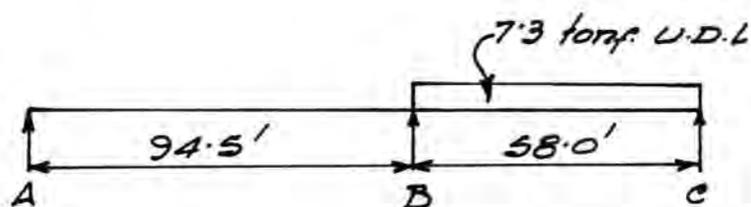
$$\text{at A : } 5.5 - \frac{80.6}{94.5} = 4.6 \text{ tonf.}$$

$$\text{at B : } 5.5 + \frac{80.6}{94.5} + \frac{80.6}{58} = 7.8 \text{ "}$$

$$\text{at C : } - \frac{80.6}{58} = -1.4 \text{ "}$$

Condition 3 : Walkway loading over 58.0 ft. span, B-C.
Loaded length 58.0 ft. \therefore load = 80 lb/ft² from table 1,
B.S. 153, Part 3A.

$$\text{So, Walkway loading} = \frac{80}{7} \times 3.5' \times 58.0' = 7.3 \text{ tonf. U.D.L.}$$



$$\text{fixing moment : } M_{Bc} = \frac{7.3 \times 58}{8} = 53 \text{ tonft.}$$

$$\text{Distribution factors : } DF_{BA} = 0.38 \text{ \& } DF_{Bc} = 0.62$$

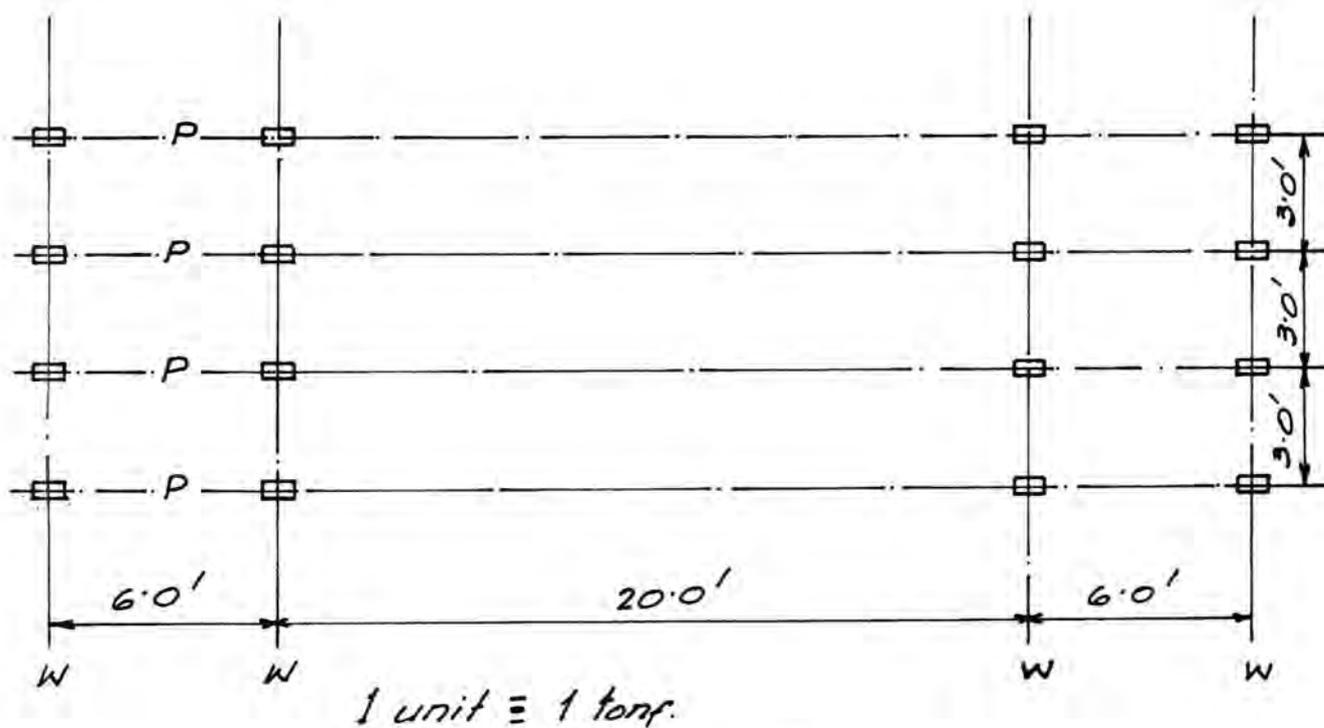
	0.38	0.62
		- 53
	+ 20	+ 33
final moments	+ 20	- 20

Reactions :

$$\text{At A : } -0.2 \text{ tonf.}$$

$$\text{At B : } +4.2 \text{ "}$$

$$\text{At C : } +3.3 \text{ "}$$

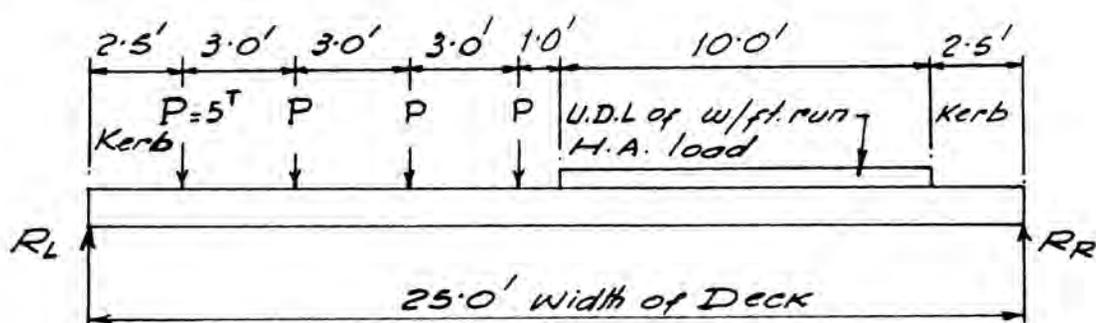
Case E : H.B. type loadingH.B. Unit loading

$P = 5$ tonf. taken, because B. S. 153 Part 3A recommends, "in general, normal loading is sufficient to cover 30 units of abnormal loading (type H.B) for loaded lengths above 100 ft. and for slabs and at least 20 units of abnormal loading for beams having spans less than 100 ft. carrying decks with a weight similar to that of an ordinary reinforced concrete slab."

Fig. A3.

Case E : H.B type loading.

In B.S. 153, Part 3A, it is stated that all parts of a bridge should be strong enough to carry H.A type loading and also should be strengthened, where necessary to carry type H.B loading system (see fig. A3 on page 21), as an alternative. Therefore it is necessary to do a check on the main girders with this type of loading system.



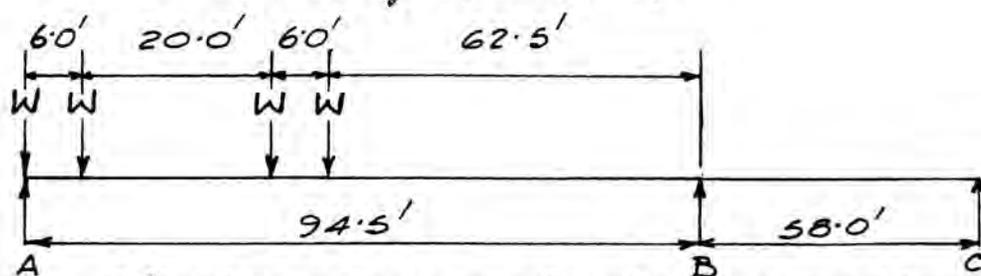
Distribution across Deck for Maximum Reaction on Main Girder - Fig. A4.

$$R_L = \frac{72}{25} P + \frac{75}{25} w = 2.88 P + 3 w$$

$$R_R = \frac{28}{25} P + \frac{175}{25} w = 1.12 P + 7 w$$

1. Determine the max. reaction R_A :

a. due to H.B loading system only -



Elevation of main girder at working condition

Distribution factors : $DF_{BA} = 0.38$ & $DF_{Bc} = 0.62$

Fixing moments :

$$M_A = \frac{W}{94.5^2} (88.5^2 \times 6 + 68.5^2 \times 26 + 62.5^2 \times 32) = 32.92 W$$

$$M_B = \frac{W}{94.5^2} (6^2 \times 88.5 + 26^2 \times 68.5 + 32^2 \times 62.5) = 12.71 W$$

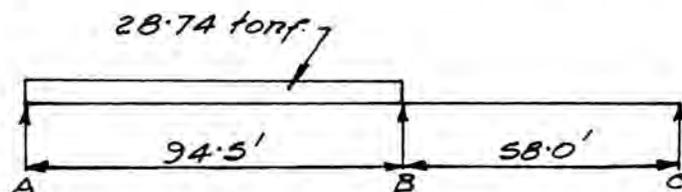
	0.38	0.62
	-32.92	+12.71
	+32.92	-4.88
		+16.46
		-6.25
		-10.21
Final moments	0	+18.09
		-18.09

$$\therefore R_A = \frac{314W - 18.09W}{94.5} = 3.13W$$

b. due to $\frac{1}{3}$ H.A. lane load only:

B.5.153, Part 3A, clause 4A(c) stated, where one carriage-way only is carried on a super-structure, all other lanes shall be considered as occupied by $\frac{1}{3}$ of the full lane loading.

From case c 2nd condition we get H.A. loading $\frac{1}{3}$ of 86.2 tonf = 28.74 tonf.



$$\text{fixing moment } M_{BA} = \frac{28.74 \times 94.5}{8} = 340 \text{ tonft.}$$

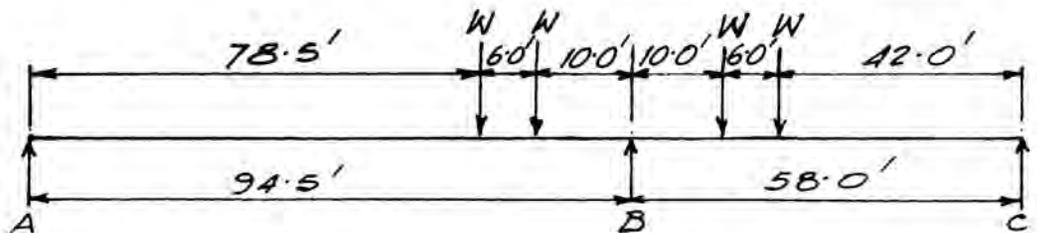
Distribution factors : $DF_{BA} = 0.38$ & $DF_{BC} = 0.62$

	0.38	0.62
	+340	
	-129	-211
final moments	+211	-211

$$\text{Reaction at A} = 14.37 - \frac{211}{94.5} = 12.2 \text{ tonf.}$$

C. due to Knife edge load over 94.5 ft. span:
 from case C K.E. loading system - final moment = 121.4
 tonft. \therefore Reaction at A = $\left(12.1 \div \frac{94.5}{39}\right) - \frac{121.4}{94.5} = 3.7$ tonf.
 Hence the maximum Reaction due to 20 units H.B. loading,
 $R_L = \left(\frac{2.88}{4} \times 20 \times 3.13\right) + \left(\frac{3}{10} \times 12.2\right) + \left(\frac{3}{10} \times 3.7\right) = 49.53$ tonf.
 This includes 25% impact allowance (B.S. 153 Part 3A, Clause
 A5 b).
 \therefore Maximum Reaction at Support A, including 40% impact
 is 55.92 tonf.

2. Determine maximum Reaction at support B:
 a. due to H.B. loading only:



Distribution factors: $Df_{BA} = 0.38$ & $Df_{BC} = 0.62$

Fixing moments:

Span AB:

$$M_A = -\frac{W}{94.5^2} (16^2 \times 78.5 + 10^2 \times 84.5) = -3.20W$$

$$M_B = +\frac{W}{94.5^2} (78.5^2 \times 16 + 10 \times 84.5^2) = +19.04W$$

Span BC:

$$M_B = -\frac{W}{94.5^2} (48^2 \times 10 + 42^2 \times 16) = -15.24W$$

$$M_C = +\frac{W}{58.0^2} (10^2 \times 48 + 16^2 \times 42) = +4.62W$$

	0.38		0.62	
	-3.20	+19.04	-15.24	+4.62
	+3.20	-1.44	-2.36	+4.62
		+1.60	-2.31	
		+0.27	+0.44	
final moments	0	+19.47	-19.47	0

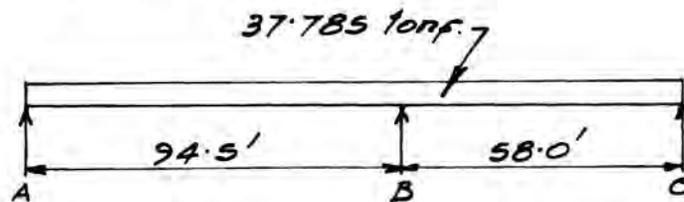
Reactions:

$$R_A = \frac{26W - 19.47W}{94.5} = 0.069W$$

$$R_C = \frac{26W - 19.47W}{58.0} = 0.113W$$

$$\therefore R_B = 3.818W.$$

b. due to $\frac{1}{3}$ H.A. lane loading only:
 from case c, HA type loading condition 1,
 $\frac{1}{3}$ of 113.4 tonf = 37.785 tonf.



Fixing moments:

$$M_{BA} = \frac{37.785 \times 94.5^2}{152.5 \times 8} = 276.6 \text{ tonft}$$

$$M_{BC} = \frac{37.785 \times 58.0^2}{152.5 \times 8} = 104.2 \text{ tonft}$$

	0.38	0.62
	+276.6	-104.2
	-65.5	-106.9
	+211.1	-211.1

Reactions :

$$\text{at } A = \left\{ \left(\frac{37.785}{152} \right) 94.5 \right\} \div 2 - \frac{211.1}{94.5} = 9.5 \text{ tonf.}$$

$$\text{at } B = \left[\left\{ \frac{37.785}{152} \times 58 \right\} \div 2 + 11.7 \right] + \frac{211.1}{94.5} + \frac{211.1}{58} = 24.73 \text{ tonf.}$$

C. due to K.E. loading system over span 94.5 ft. :

from case c - K.E. loading system, final moment = 121.4 tonft.

$$\therefore \text{Reaction at } B = \left(12.1 \div \frac{94.5}{55.5} \right) - \frac{121.4}{95.4} = 5.8 \text{ tonf.}$$

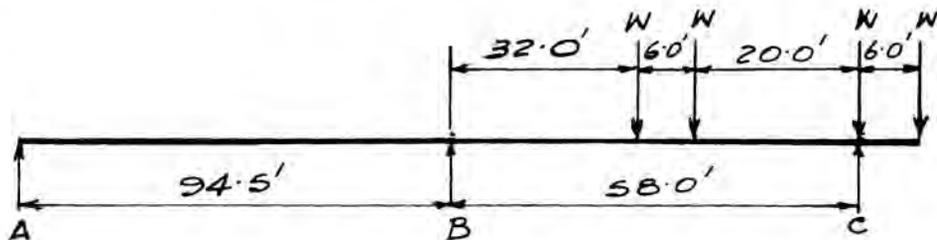
Hence the maximum reaction due to 20 unit H.B. loading

$$R_L = \frac{2.88}{4} \times 20 \times \frac{24.73}{4} + \frac{3}{10} \times 24.73 + \frac{3}{10} \times 5.8 = 64.14 \text{ tonf.}$$

this includes 25% impact allowance.

3. Determine maximum reaction on support C :

a. due to H.B. loading only :



fixing moments: span BC.

$$M_B = -\frac{W}{58^2} (26^2 \times 32 + 20^2 \times 38) = -10.95 W$$

$$M_C = +\frac{W}{58^2} (32^2 \times 26 + 38^2 \times 20) = +16.50 W$$

	0.38	0.62	
		-10.95	+16.50
+4.16	+6.79		-6
		-5.25	
+2	+3.25		
+6.16	-6.16	+6.0	-6.0

A 27

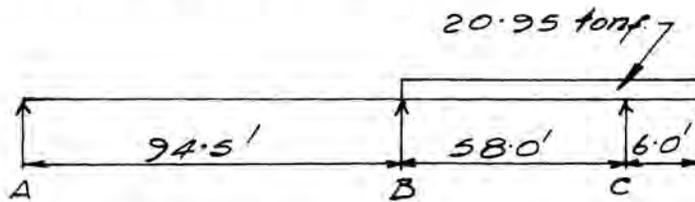
Reaction:

$$\text{At C} = \frac{192W - 6.16W}{58} = 3.21W$$

b. due to $\frac{1}{3}$ H.A. lane loading only:

from case C 3rd condition,

$$\frac{1}{3} \text{ of } \frac{2200}{7} \times 64 = 20.95 \text{ tonf.}$$



$$\text{Reaction on support C} = \frac{25.8}{3} + \frac{6}{64} \times 20.95 = 10.56 \text{ tonf.}$$

c. due to K.E. load over 58.0 ft. span:

from Case C - K.E. loading system, we got final moment = 45.6 tonf.

$$\therefore \text{Reaction on support C} = \left(12.1 \div \frac{58}{23.9}\right) - \frac{45.6}{58} = 4.2 \text{ tonf.}$$

Hence for maximum reaction due to 20 unit of H.B. loading

$$R_L = \frac{2.88}{4} \times 20 \times \frac{10.56}{3} + \frac{3}{10} \times 10.56 + \frac{3}{10} \times 4.2 = 50.59 \text{ tonf.}$$

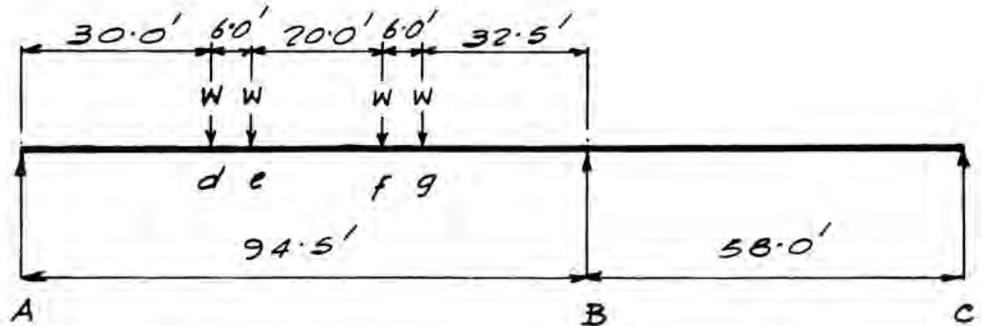
this includes 25% impact.

\therefore The maximum reaction on support C with 40% impact

is 56.67 tonf.

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4. Determine maximum bending moment at 94.5 ft. span due to H.B. loading System:



fixing moments:

$$M_{AB} = -\frac{W}{94.5^2} (64.5^2 \times 30 + 58.5^2 \times 36 + 38.5^2 \times 56 + 32.5^2 \times 62) = 44.4 W$$

$$M_{BA} = \frac{W}{94.5^2} (30^2 \times 64.5 + 36^2 \times 58.5 + 56^2 \times 38.5 + 62^2 \times 32.5) = 42.5 W$$

assuming 1 unit \equiv 1 tonf. $M_{AB} = 44.4$ tonft.

& $M_{BA} = 42.5$ tonft.

	0.38	0.62
-44.4	+42.5	
+44.4	-16.15	-26.35
	+22.20	
	-8.44	-13.76
0	+40.11	-40.11

Span A-B - assuming simply supported:

$$\text{reaction of A} = (64.5 + 58.5 + 38.5 + 32.5) \times \frac{\text{unit load}}{94.5} = 2.053 \text{ units}$$

or, $R_A = 2.053$ tonf \because 1 unit \equiv 1 tonf.

$$\therefore \text{Moment at d} = 30 \times 2.053 = 61.59 \text{ tonft}$$

$$\text{" " e} = 36 \times 2.053 - 6 \times 1 = 67.91 \text{ "}$$

$$\text{" " f} = 56 \times 2.053 - 26 \times 1 - 20 \times 1 = 68.97 \text{ "}$$

$$\text{" " g} = 62 \times 2.053 - 32 \times 1 - 26 \times 1 - 6 \times 1 = 63.28 \text{ "}$$

$$\begin{aligned}
 \text{Hence moment at } d &= 61.59 - \left(\frac{40.11 \times 30}{94.5} \right) = 48.86 \text{ tonft.} \\
 \text{" " e} &= 67.91 - \left(\frac{40.11 \times 36}{94.5} \right) = 52.63 \text{ " } \\
 \text{" " f} &= 68.97 - \left(\frac{40.11 \times 56}{94.5} \right) = 51.13 \text{ " } \\
 \text{" " g} &= 63.28 - \left(\frac{40.11 \times 62}{94.5} \right) = 36.96 \text{ " }
 \end{aligned}$$

Hence the maximum bending moment on main girder due to H.B. load is $52.63 \times 2.88 \times \frac{20}{4} = 760 \text{ tonft}$ - refer to sketch SK. 12.

Summary of Bending moments due to H.B. loading only:

max. B.M. due to $\frac{1}{3}$ H.A. lane load - from Case E 1 b, $R_A = 12.2 \text{ tonf}$.

$$\therefore \text{Max. B.M. at } d = 12.2 \times 36 - \frac{28.74}{94.5} \times \frac{36^2}{2} \times \frac{3}{10} = 75 \text{ tonft.}$$

Max. B.M. due to K.E. load over 94.5 ft. span = 30 tonft.

Max. B.M. due to H.B. loading system = 760 tonft

Hence Σ B.M. due to $\frac{1}{3}$ HA + K.E. load + H.B. load = 865 tonft.

Comparison of mid-span bending moments for Span AB:

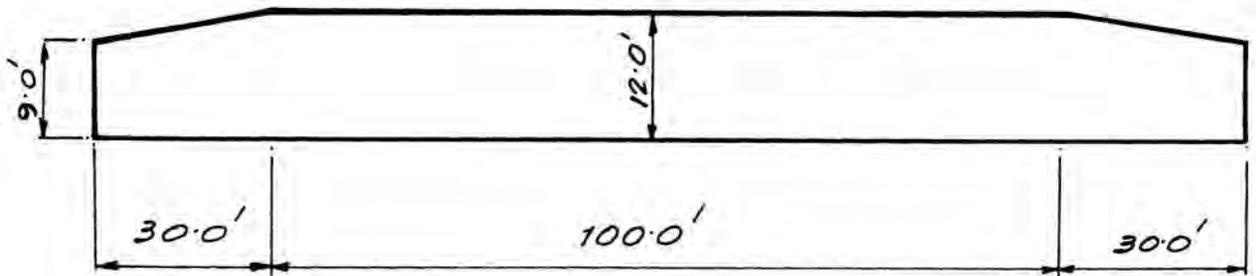
It is understandable that the dead load, walkway load, snow load etc. will be common to both type of loading system and so, these loads have been ignored.

Now, Σ B.M. due to full H.A. load + K.E. load = 952.1 tonft.

so, it is obvious that the B.M. due to the 20 unit of H.B. loading system is not critical.

4. Wind loadings

4.1. Exposed Wind Area :



Nett. exposed area of the main girder = $(160 \times 12 - 3 \times 30) = 1830 \text{ ft}^2$

According to B. S. 153, Part 3A, Clause 12 b(i) :-

area of windward girder = 1830 ft^2

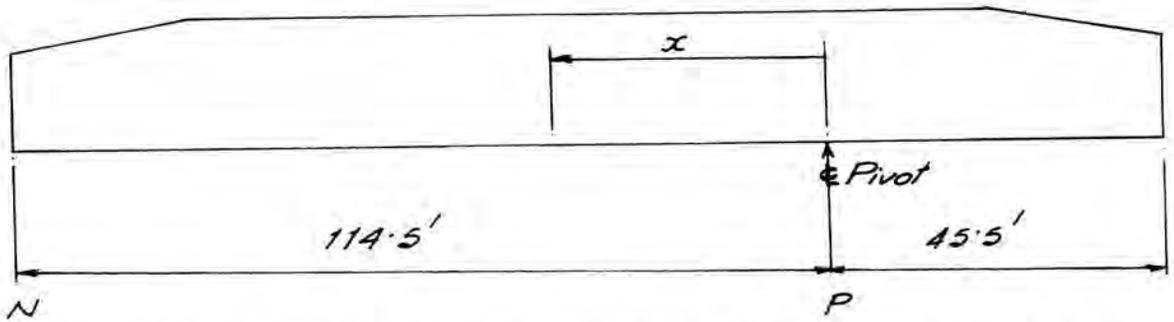
area of leeward girder = $\frac{r}{16} \times$ area of windward girder and this is valid only for Plate girders. r = ratio of distance, centre to centre between the windward and outermost leeward girder, to the depth of the windward girder.

Thus, $r = \frac{25}{12}$ where the distance between the windward and outermost leeward girder = 25 ft. and the depth of the windward girder = 12 ft.

Hence the area of leeward girder = $\frac{25}{12 \times 16} \times 1830 = 240 \text{ ft}^2$

Therefore, the total wind area = 2070 ft^2

4.2. Determine the line of action of Wind load :



Taking moments about Pivot bearing and considering wind load over length N to P only -

$$\text{Wind area of windward girder} = 114.5 \times 12 - \frac{1}{2} \times 3 \times 30 = 1329 \text{ ft}^2$$

$$\text{Wind area of leeward girder} = \frac{.25}{12 \times 8} \times 1329 = 173 \text{ "}$$

$$\underline{\sum \text{ area} = 1502 \text{ ft}^2}$$

$$\therefore \text{Wind force (@ } 20 \text{ lbf/ft}^2) = 13.41 \text{ tonft.}$$

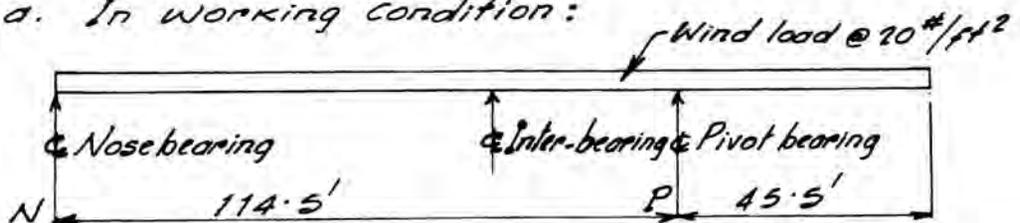
Line of action is given by, x :

$$114.5 \times 12 \times 57.25 - 30 \times 1.5 \times 104.5 = 1329 \cdot x$$

$$\text{or, } x = 55.65 \text{ ft. from Pivot bearing.}$$

Hence, horizontal wind force of 13.41 tonf acts through a lever arm of 55.65 ft. from the centre line of the Pivot bearing, which gives an applied moment of (13.41×55.65) , 746.3 tonft.

4.2 a. In working condition:



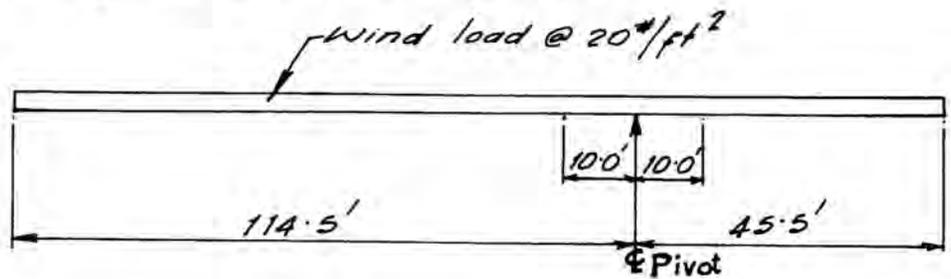
$$\text{Moments about nose bearing: Reaction at P} = \frac{2070 \times 20 \times 80}{114.5} = 12.91 \text{ tonf.}$$

$$\therefore \text{Reaction at N} = 18.48 - 12.91 = 5.57 \text{ "}$$

No lateral restraint has been assumed at the inter-bearing

support, when the bridge is in working condition.

4.2 b. Bridge in Cantilever Condition:



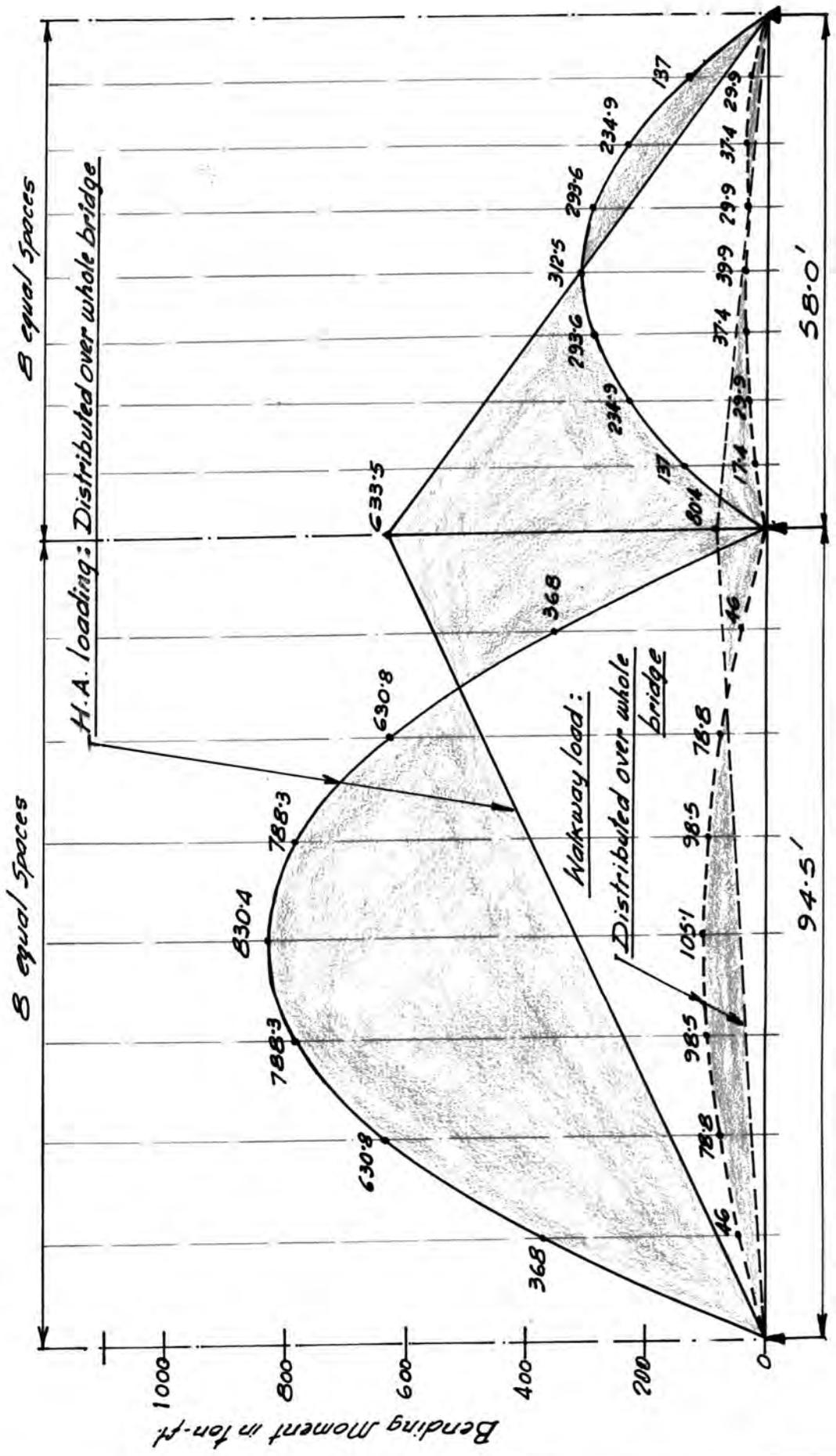
taking moments about Pivot:

$$2070 \times 20 \times 34.5 - 2P \times 10 = 0$$

or, $P = 31.88$ tonf for equilibrium.

Now, resolving, P horizontally, we get $R_p = 18.48$ tonf.

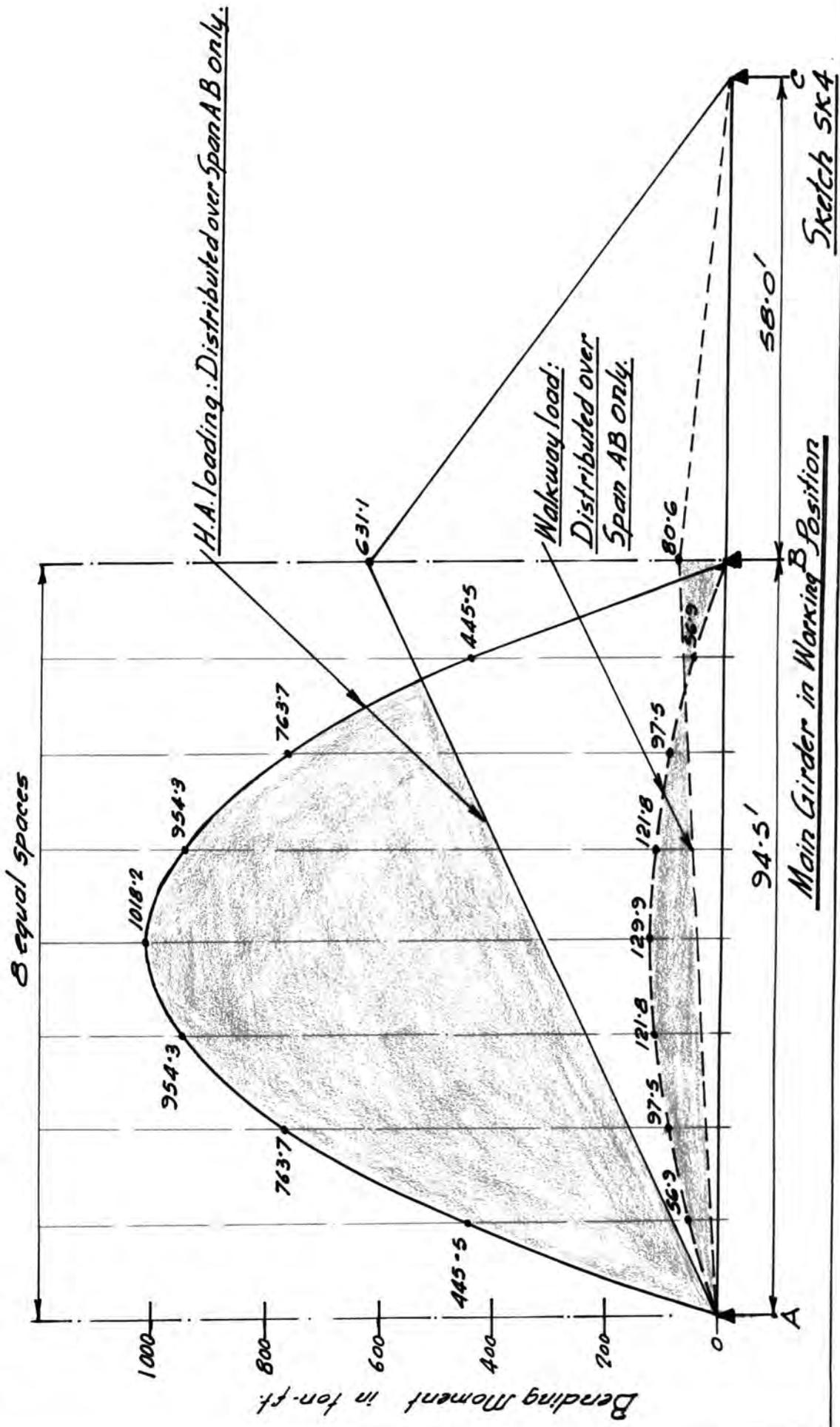
Bending moment diagram



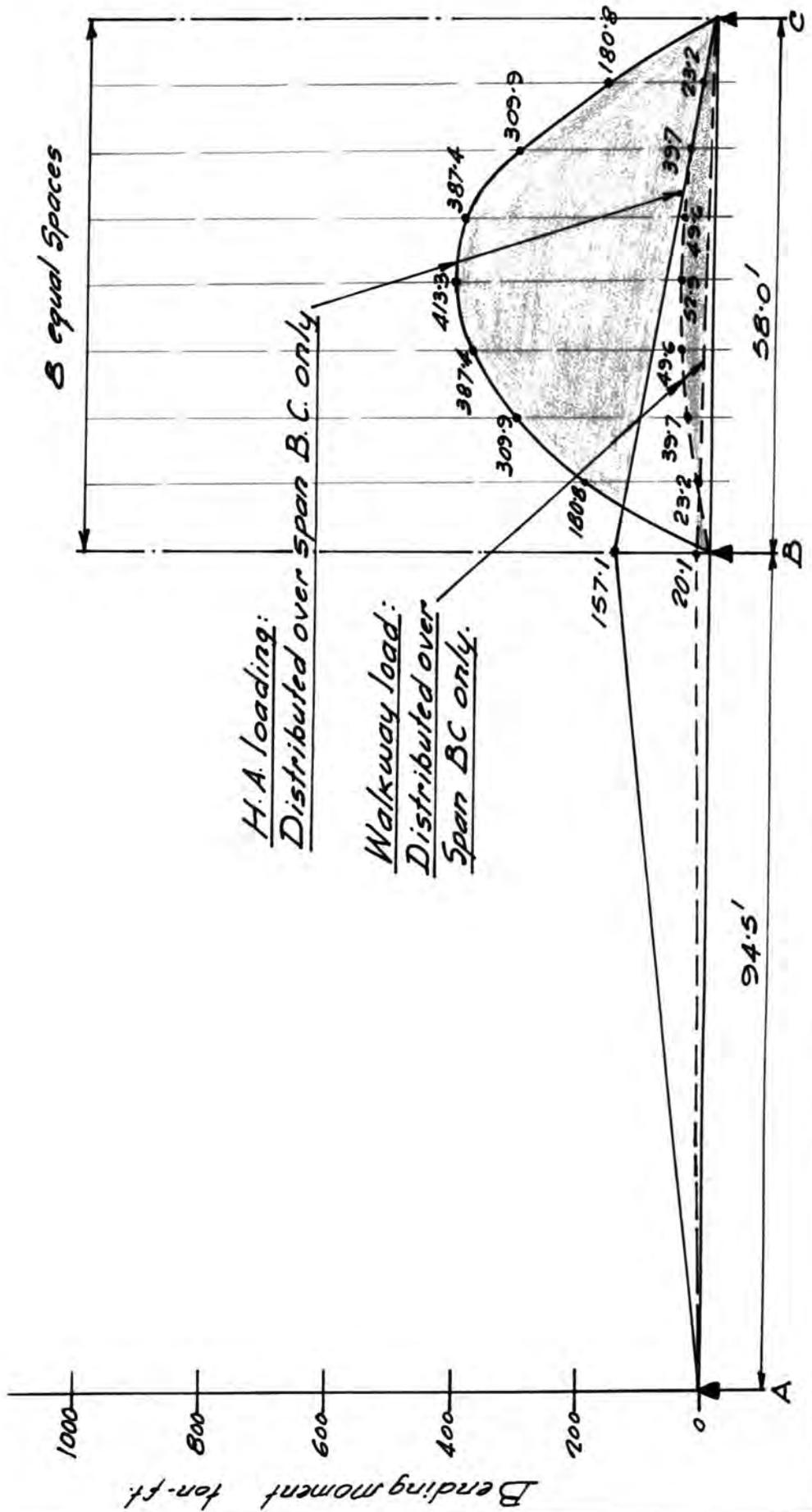
Main Girder in Working Position

Sketch Sx.3

Bending Moment diagram.



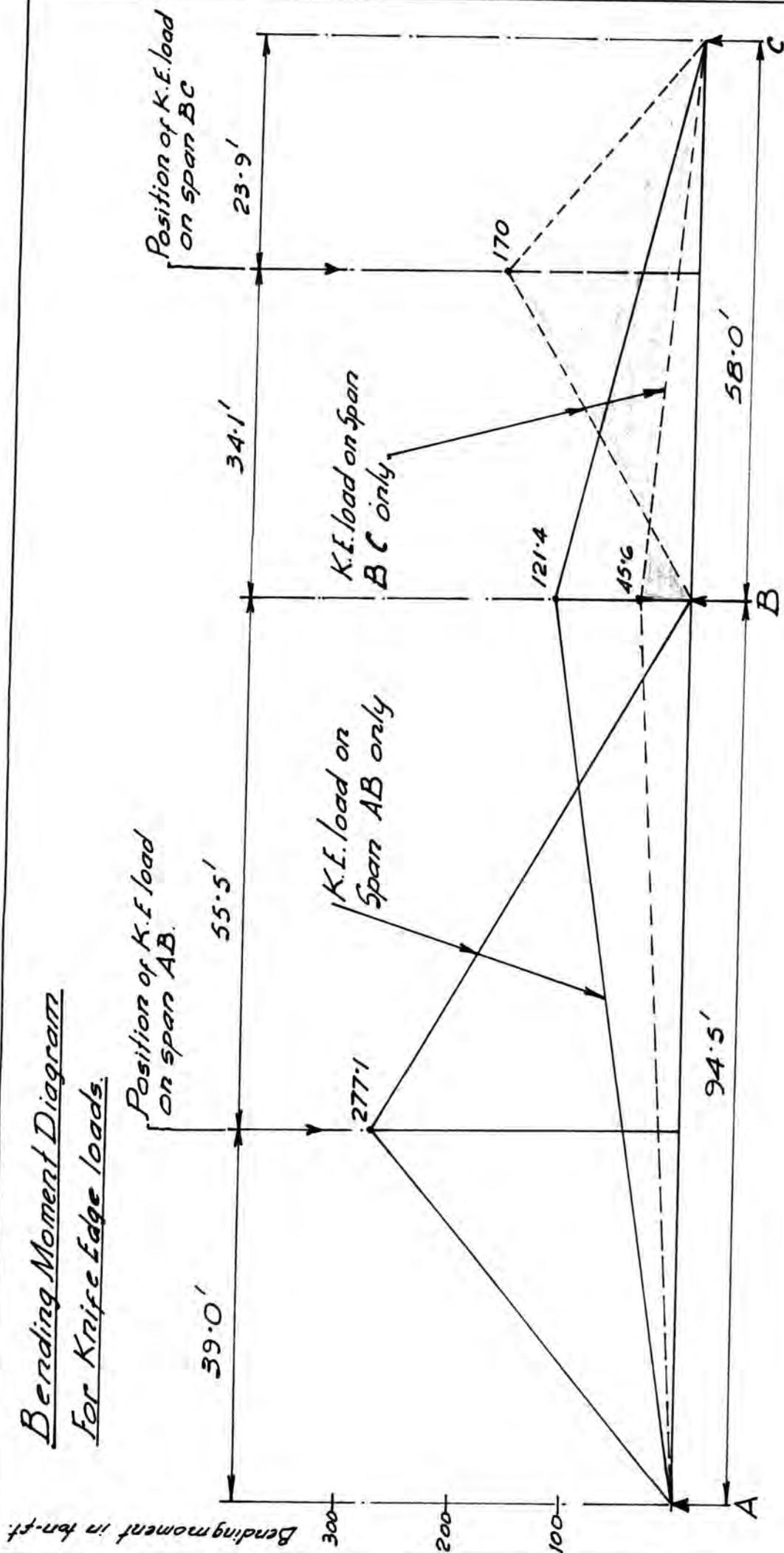
Bending Moment Diagram



Main Girder in Working Position

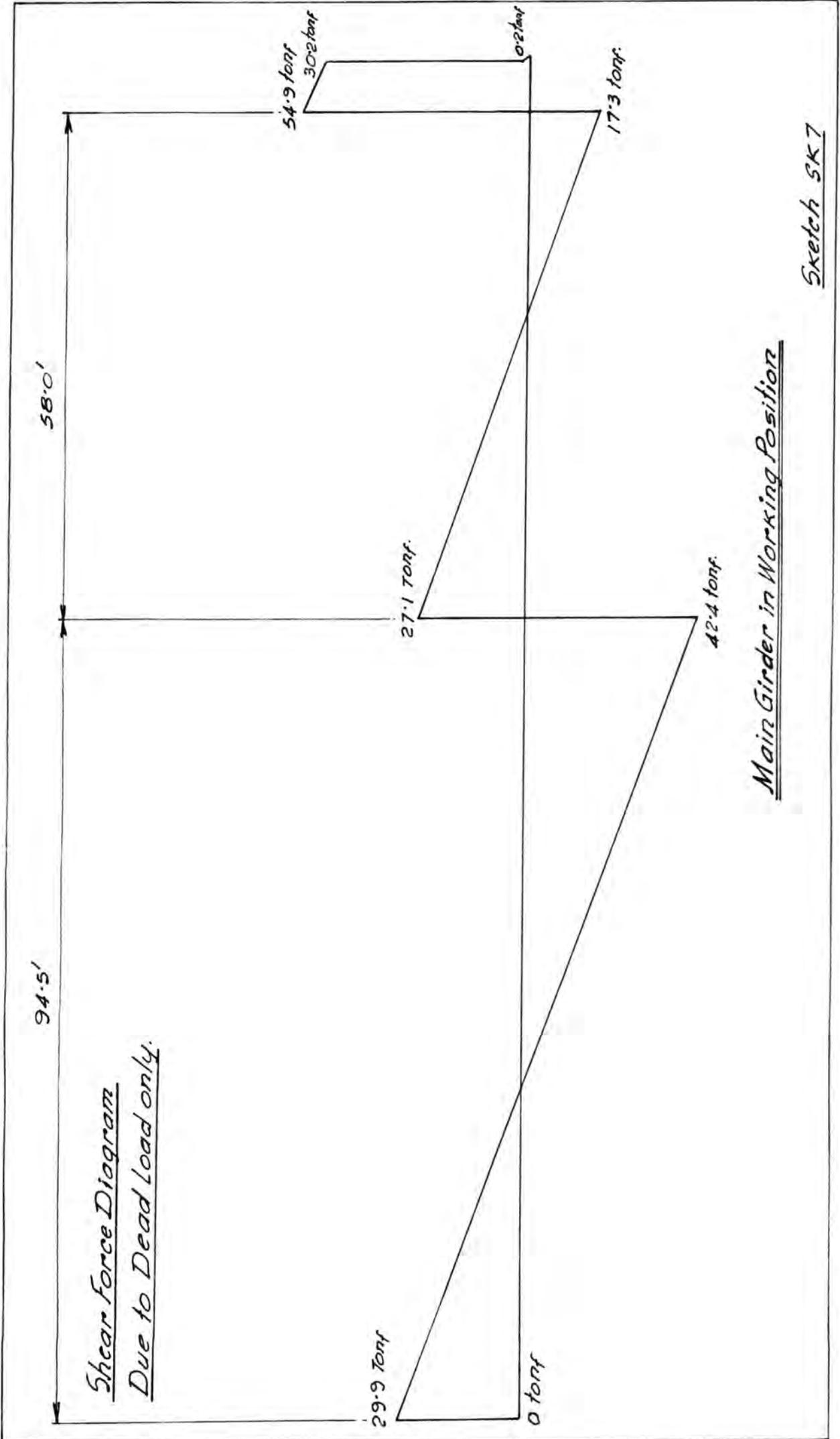
Sketch SK.5

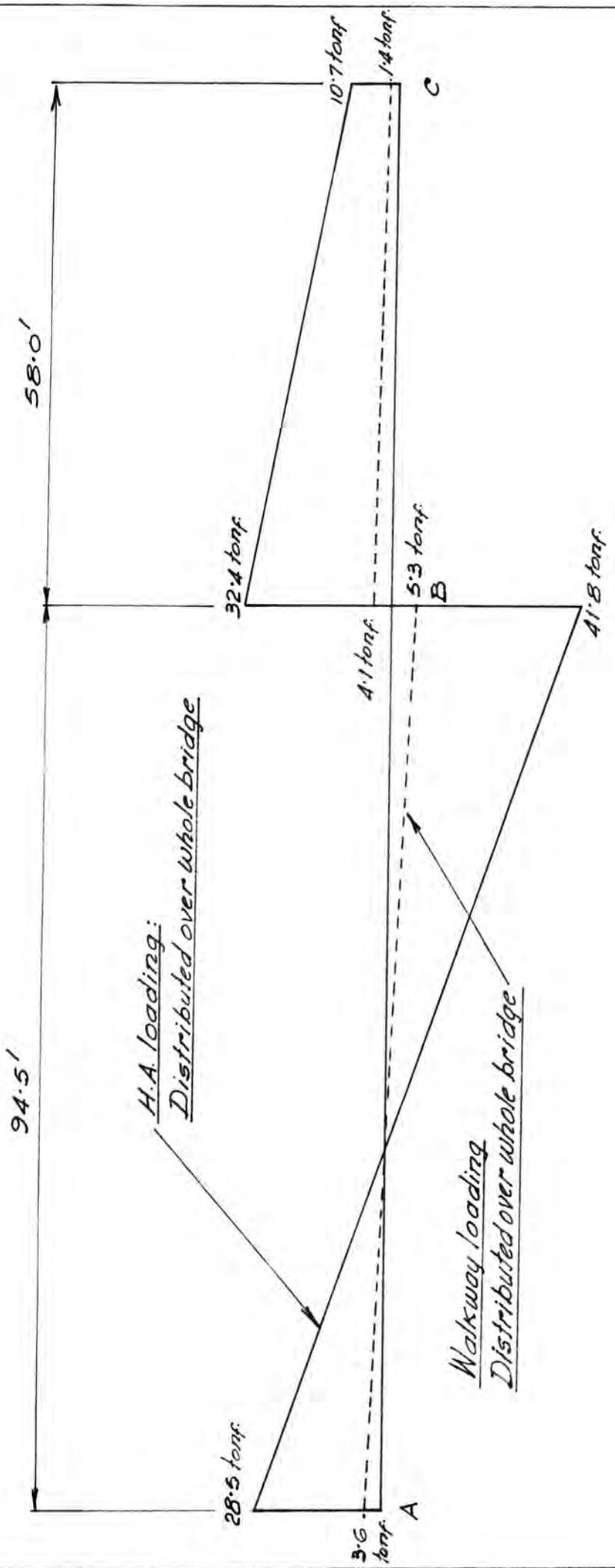
Bending Moment Diagram
For Knife Edge loads.



Main Girder in Working Position

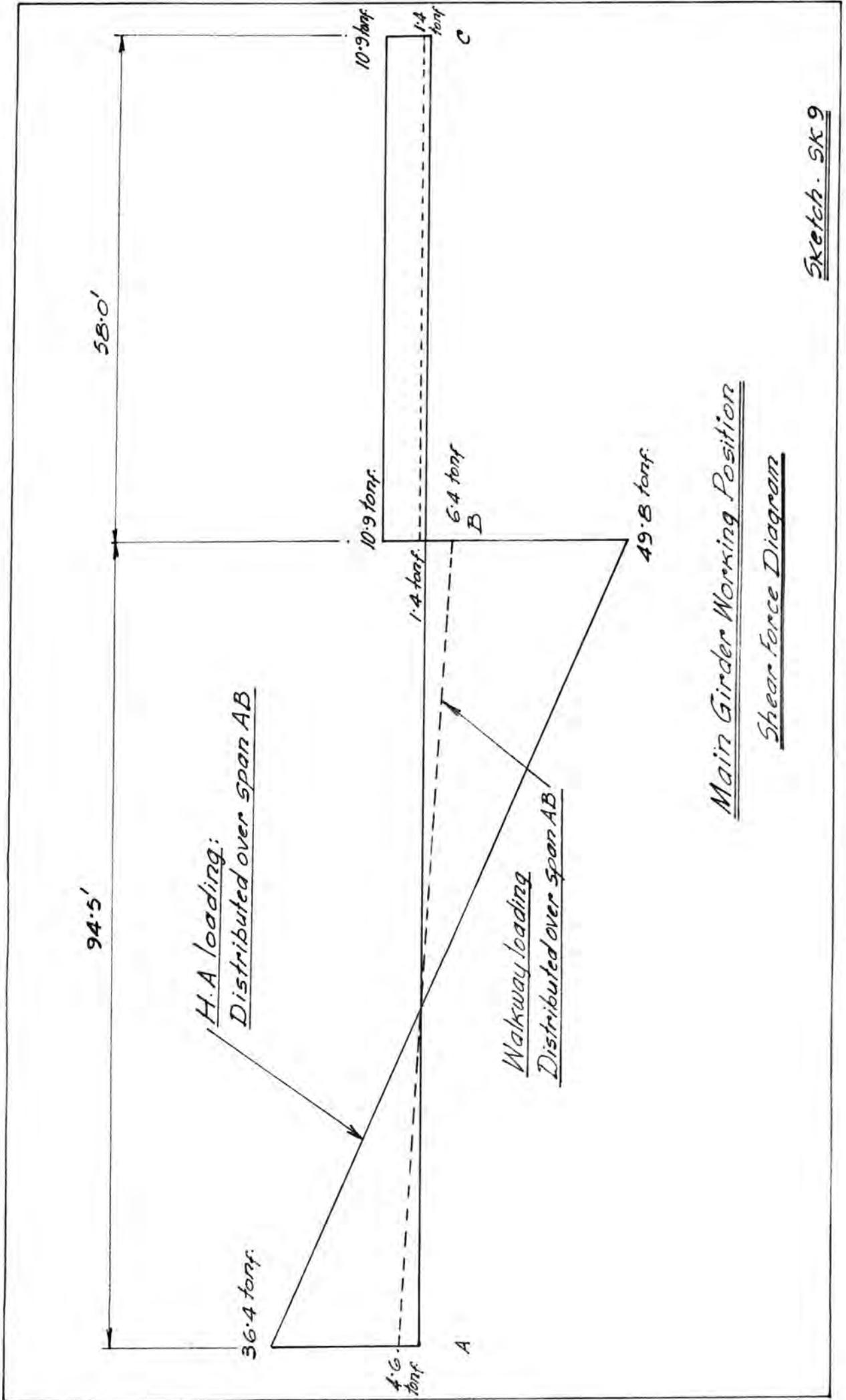
Sketch - SKG





Main Girder in Working Position
Shear Force Diagram

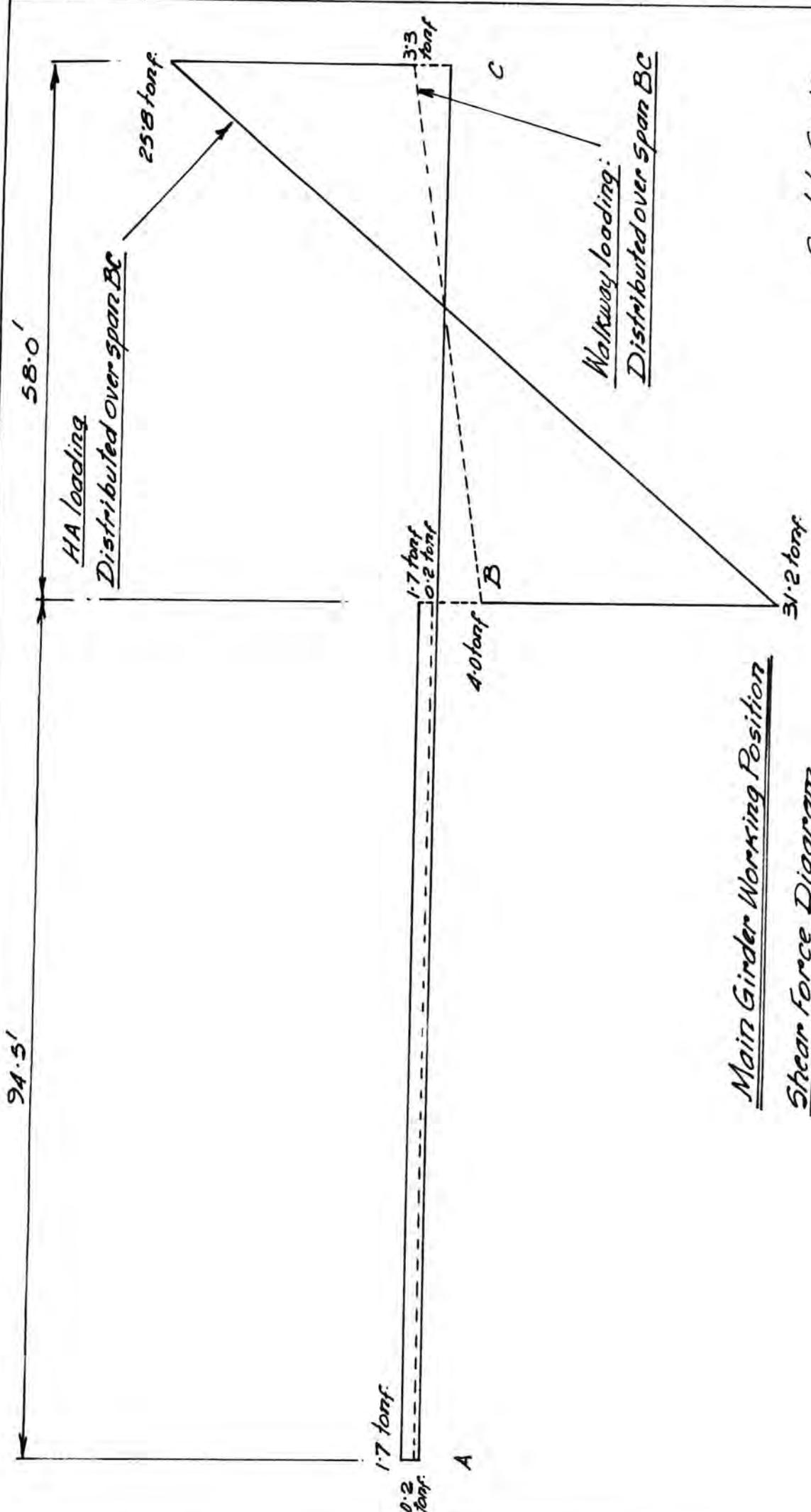
Sketch Sk. 8



Main Girder Working Position

Shear Force Diagram

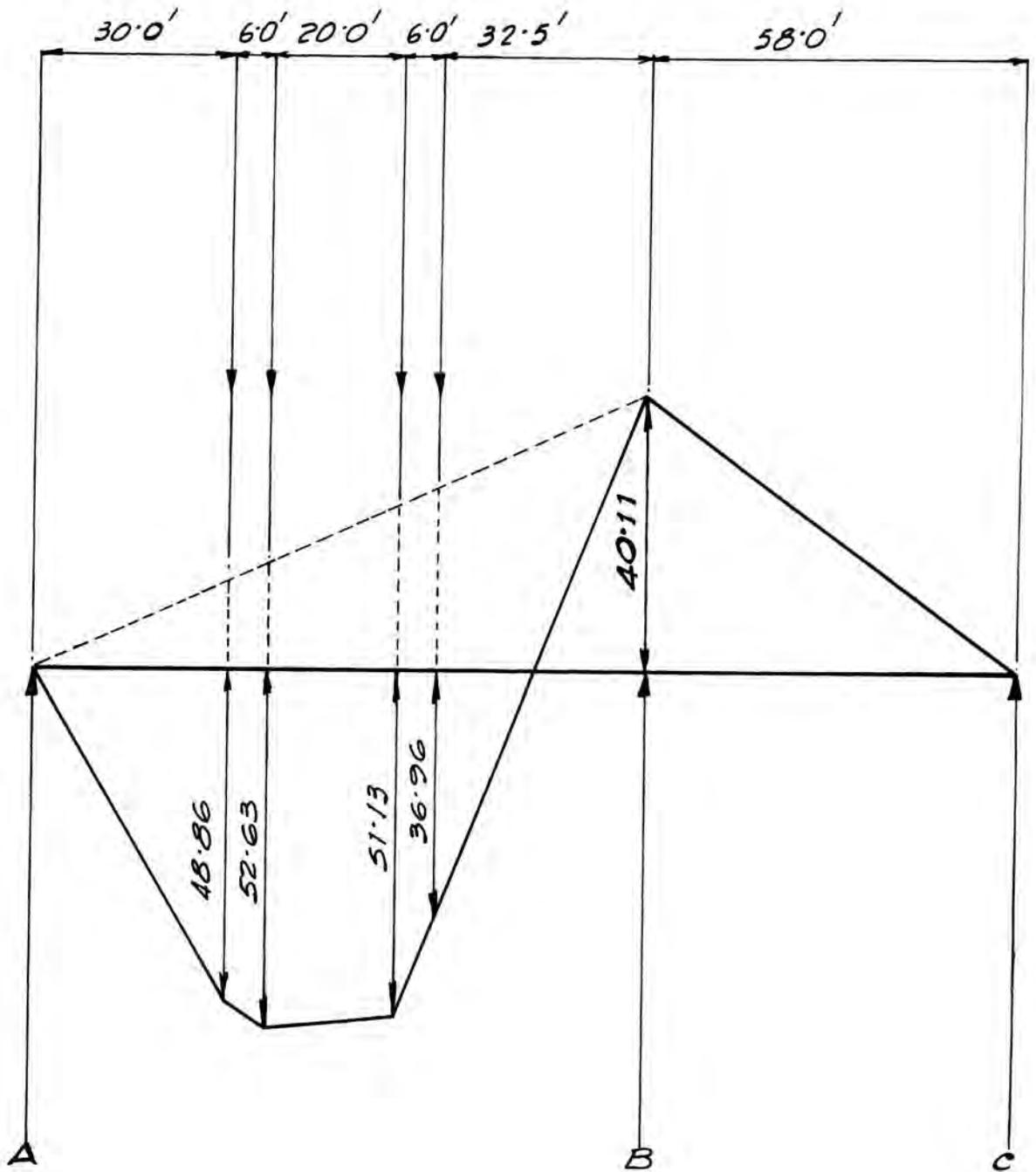
Sketch. SK 9



Main Girder Working Position
Shear Force Diagram

Sketch Sk. 10

A 41



B.M. Diagram at 94.5 ft. Span due to H.B. loading system

Sketch Sk. 12

Appendix APart II5. Stress Calculations

5.1. Main Girders - Working Condition:

From the bending moment diagrams, (sketches SK 1 to 11) it is noticeable that the maximum bending moment in span AB (ie., 94.5 ft. span), occurs at 39 ft from the support A.

Hence moment due to:

$$a. \text{ Dead load} = 29.9 \times 39.0 - \frac{121.5 \times 39^2}{158.75 \times 2} = 584.0 \text{ tonft.}$$

$$b. \text{ H.A. load on span AB only} = 36.4 \times 39.0 - \frac{86.2 \times 39^2}{94.5 \times 2} = 725.9 \text{ ''}$$

$$c. \text{ Walkway on span AB only} = 4.6 \times 39.0 - \frac{11.0 \times 39^2}{94.5 \times 2} = 90.9 \text{ ''}$$

$$d. \text{ Snow load} = 0.9 \times 39.0 - \frac{3.7 \times 39^2}{152.5 \times 2} = 16.6 \text{ ''}$$

$$e. \text{ Knife edge load} = 5.8 \times 39.0 = 226.2 \text{ ''}$$

$$\sum \text{moment} = 1643.6 \text{ tonft.}$$

Now, Shear force at support A due to:

$$a. \text{ Dead load} = 29.9 \text{ tonf.}$$

$$b. \text{ H.A. load on span AB} = 36.4 \text{ ''}$$

$$c. \text{ Walkway on span AB} = 4.6 \text{ ''}$$

$$d. \text{ Snow load} = 0.9 \text{ ''}$$

$$e. \text{ Knife edge load} = 12.1 \text{ ''}$$

$$\sum \text{Shear force} = 83.9 \text{ tonf.}$$

5.2. Properties of main girder at Maximum B.M. Point:
assume overall depth of the girder,

$d = 12.0 \text{ ft.} = 144''$ at max. B.M. point

Which is 39.0 ft. from the support A.

Try the section shown in Fig 5.2-1:

flange plates:

Top = $16'' \times \frac{3}{4}''$ thick

bottom = $16'' \times \frac{3}{4}''$ thick

Web plate = $142.5'' \times \frac{9}{16}''$ thick

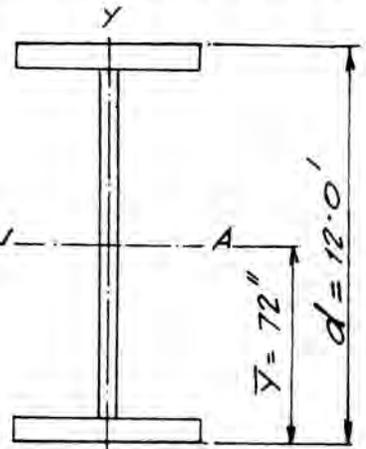


Fig A 5.2-1

Section	areas in in ²	I_{x-x} in ⁴	I_{y-y} in ⁴
flange plates	$2 \times 16 \times \frac{3}{4} = 24$	$\frac{16}{12} (144^3 - 142.5^3)$ $= 123125$	$\frac{3}{4} \times \frac{16^3}{12} \times 2$ $= 512$
Web plate	$142.5 \times \frac{9}{16} = 80.16$	$\frac{9}{16} \times \frac{142.5^3}{12}$ $= 135639$	Negligible

Hence $\Sigma \text{ areas} = 104.16 \text{ in}^2$ $\Sigma I_{x-x} = 258764 \text{ in}^4$ $\Sigma I_{y-y} = 512 \text{ in}^4$

Radius of gyration at y-y axis, $r_y = \sqrt{\frac{512}{104.16}} = 2.22 \text{ in.}$

Slenderness ratio, $\frac{l}{r} = \frac{201.97}{2.22} = 90.98$

Elastic modulus, $Z_{x-x} = \frac{I_{x-x}}{Y} = \frac{258764}{72} = 3594 \text{ in}^3$.

5.3. Restraints in stability:

No room is available to provide any lateral bracing of the compression flange, therefore, the cross members and stiffeners are forming U-frames which provide the lateral restraints.

Now, from chapter 3 and paragraph 3.7 and also from B.S. 153, Part 3B, clause 34 b, we get,

$l = 2.5 \sqrt[4]{EI \alpha \delta}$, but not less than a .

δ shall be computed on the assumption that the cross member is free to deflect vertically and the tangent to the deflection curve at the centre of its span remains

parallel to the neutral axis of the unrestrained cross member.

When δ is not greater than $\frac{a^3}{40EI}$, $l = a$.

In this project, the cross girder of symmetrical U-frames where cross girders and stiffeners are each of constant moment of inertia throughout their own length.

$$\delta = \frac{(d')^3}{3EI_1} + \frac{(d'')^2 \cdot b}{EI_2}$$

Where d' = distance of centroid of compression flange from top of the cross member.

d'' = distance of centroid of compression flange from the neutral axis of the cross member.

b = half the distance of centre to centre of the main girders.

I_1 = 2nd moment of area of a pair of stiffeners about the centre line of the web or of single stiffener about face of the web.

I_2 = 2nd moment of area of cross member in its plane of bending.

U-frames should have rigid connections and should be designed to resist, in addition to the effect of wind and other applied forces, the effect of a horizontal force F normal to compression flange of the girder at the level of its cross girder and having the value $2.5 \sqrt{EI \delta a}$.

Now, here δ is computed, assuming that the cross member is free to deflect vertically and the tangent to the deflection curve at the centre of its span remains parallel to the neutral axis of the unrestrained cross member.

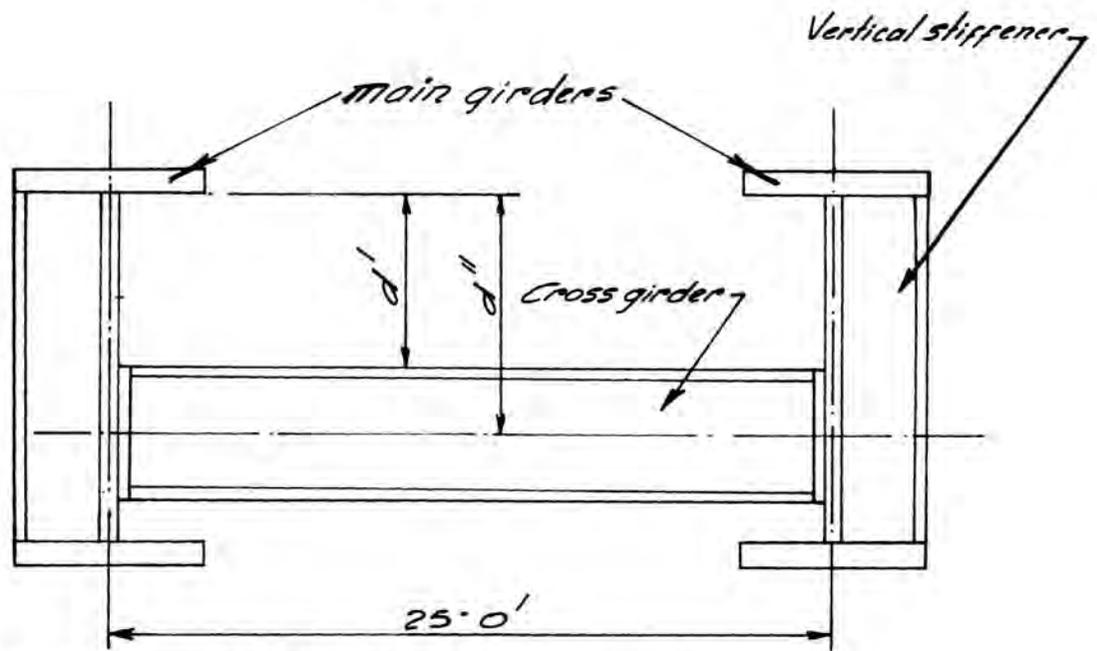


Fig. 5.3-1. Section thro' cross girder.

as mentioned above, $\delta = \frac{(d')^3}{3EI_1} + \frac{(d'')^2 b}{EI_2}$
 $d' = 92''$ & $d'' = 98.4''$; $b = \frac{1}{2} \cdot 25.0 \times 12 = 150''$

$I_1 = 2^{\text{nd}}$ moment of area of the vertical stiffener = 249.4 in^4

$I_2 = 2^{\text{nd}}$ moment of area of the cross girder = 8845 in^4

Note that the values of I_1 & I_2 have been worked out in later paragraphs of this appendix.

$$\text{Hence, } \delta = \frac{92^3}{3 \times 13000 \times 249.4} + \frac{98.4^2 \times 150}{13000 \times 8845} = 0.09''$$

B.S. 153 recommends that $l = a$ when, $\delta \leq \frac{a^3}{40EI}$

Where I is the maximum 2^{nd} moment of area of compression chord about y-y axis, which equals to 256 in^4 (ie, $\frac{3}{4} \times \frac{16^3}{12}$).

$$\text{check: } \frac{a^3}{40EI} = \frac{138^3}{40 \times E \times 256} = 0.0192''$$

$\therefore \delta > 0.0192 \quad \therefore l \neq a.$

So, effective length, $l = 2.5 \sqrt[4]{E \times 256 \times 138 \times 0.09} = 201.97''$

5.4. Check stresses of main girder at max. B.M. Point:

i) critical stress - according to B.S. 153, Part 3B, Clause 28 b i A, Where the tension & compression flange have equal 2nd moment of area, the critical stress,

$$C_s = \frac{170000}{\left(\frac{l}{r_y}\right)^2} \sqrt{\left\{1 + \frac{1}{20} \left(\frac{l}{r_y} \cdot \frac{T}{d}\right)^2\right\}} \text{ tonf/in}^2$$

Hence $C_s = 20.7 \text{ tonf/in}^2$

and allowable stress $p_{bc} = 6.4 \text{ tonf/in}^2$, from table B, B.S. 153 part 3B.

$$\therefore f_{bc} = \frac{M_{\max}}{Z} = \frac{1643.6}{3594} = 5.5 \text{ tonf/in}^2 < 6.4 \text{ tonf/in}^2$$

Therefore the section chosen is satisfactory.

ii) Shear stress: according to B.S. 153, Part 3B, Clause 29a, average shear stress, $p_g = 6 \left[1.3 - \frac{b/t}{250 \left\{ 1 + \frac{1}{2} \left(\frac{b}{a} \right)^2 \right\}} \right] \text{ tonf/in}^2$

Where, $a =$ the greater clear dimension of the web in a panel, not greater than $270t$.

$b =$ the lesser clear dimension of the web in a panel, not greater than $180t$

$t =$ thickness of the web plate

Refer to fig. 5.4-1 on page 47:

$$a = 138'' < 270t = 151.9''$$

$$b = 108 - 21.6 = 86.4'' < 180t = 101.25''$$

So, allowable shear stress,

$$p_g = 6 [1.3 - 0.51] = 4.74 \text{ tonf/in}^2$$

$$f_g = \frac{\text{max. shear at support}}{\text{Web area at support}} = \frac{83.9}{84 \times 0.5625} = 1.78 \text{ tonf/in}^2$$

Hence $p_g > f_g \therefore$ the section is satisfactory.

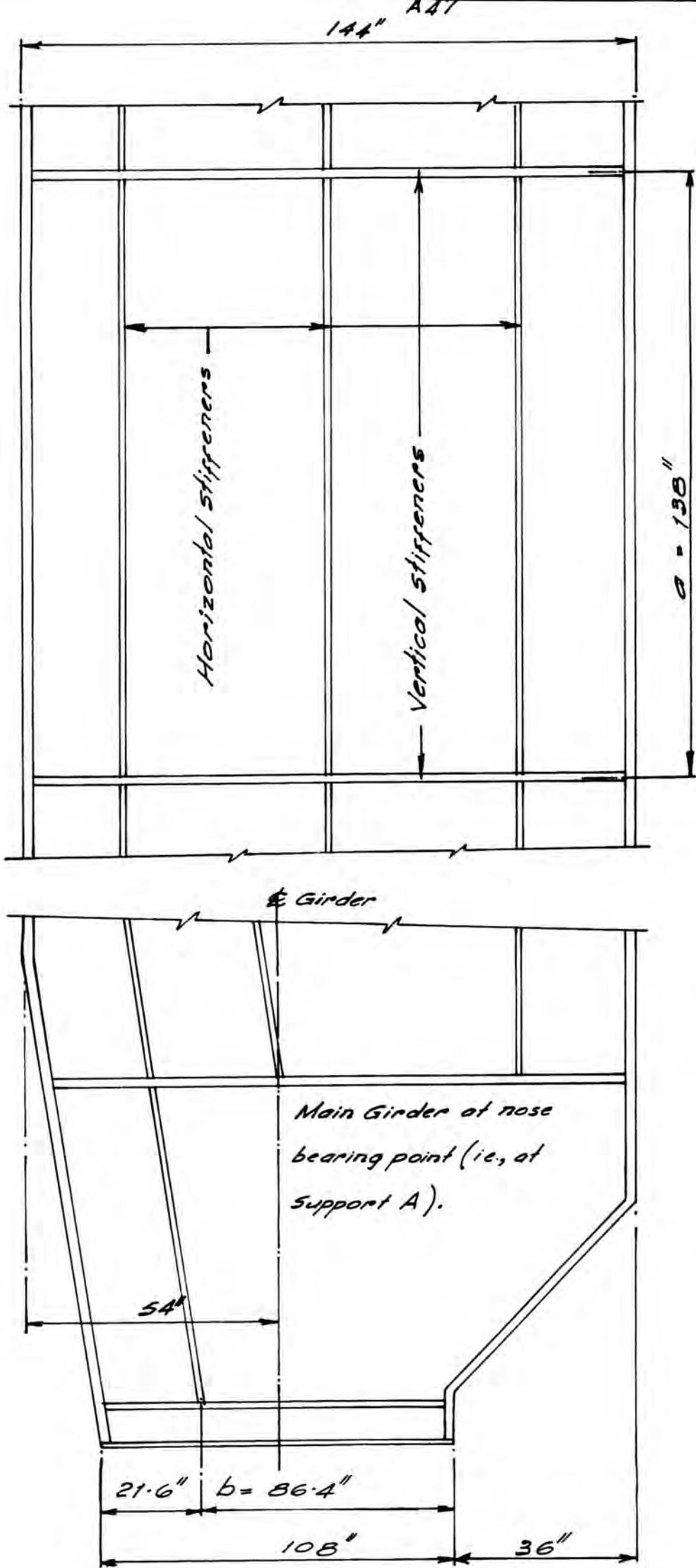


Fig. 5.4-1. Part Elevation of Main Girder

5.5. Section at centre Pivot - 114.5 ft. from nose bearing:

Main Girder at Cantilever Condition:

$$\text{Max. B. Moment} = 4375.6 \text{ tonft.}$$

$$\text{" shear force} = 123.4 \text{ tonf.}$$

Section:

Flange plates : $21'' \times 1\frac{1}{2}''$ thick

Web plate : $141'' \times \frac{9}{16}''$ thick

2nd moment of areas x-x axis:

$$\text{flange plates} : 2 \times 31.5 \times 71.25^2 = 319823 \text{ in}^4$$

$$\text{Web plate} : \frac{141^3 \times 0.5625}{12} = 131401 \text{ in}^4$$

$$\sum I_{x-x} = 451224 \text{ in}^4$$

Areas :

$$\text{flange plates} : 2 \times 21 \times 1\frac{1}{2} = 63.0 \text{ in}^2$$

$$\text{Web plate} : 141 \times \frac{9}{16} = 79.3 \text{ in}^2$$

$$\sum \text{areas} = 142.3 \text{ in}^2$$

2nd moment of area y-y axis:

$$\text{flange plates} : 2315.4 \text{ in}^4$$

Web plate : negligible

$$\sum I_{y-y} = 2315.4 \text{ in}^4$$

i) Working stresses:

$$r_y = \sqrt{\frac{I_{y-y}}{A}} = 4.034; \quad Z_{x-x} = \frac{I_{x-x}}{y} = 6267 \text{ in}^3$$

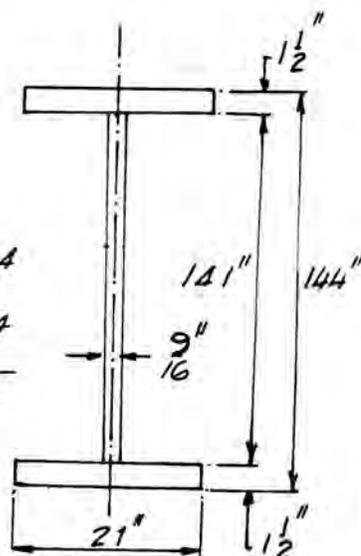
$$\frac{D}{T} = \frac{144}{1.5} = 96; \quad \frac{d}{t} = \frac{141}{0.5625} = 250.6$$

Bottom flange restrained at 11.5 ft. Centres by cross girders:

$$\therefore \frac{l}{r_y} = \frac{138}{4.034} = 34.2$$

$$\therefore p_{bc} = p_{bt} = 9.0 \text{ tonf/in}^2$$

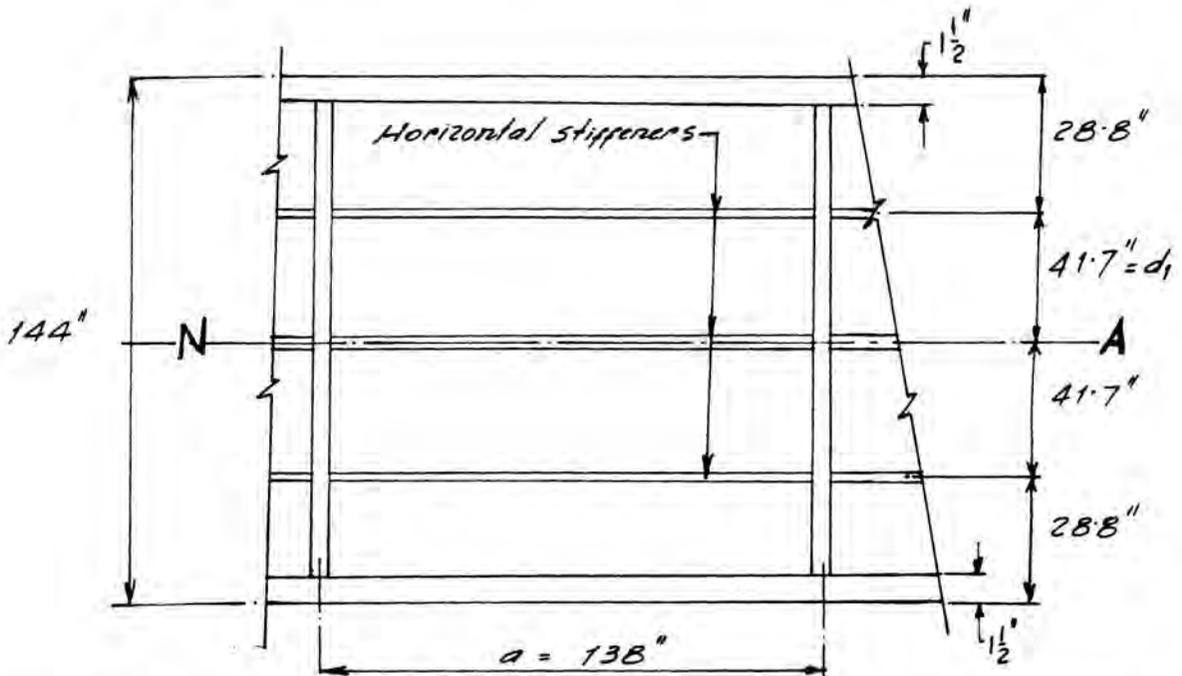
$$\text{But } f_{bc} = f_{bt} = \frac{M}{Z} = 8.34 \text{ tonf/in}^2 < p_{bc} \text{ or } p_{bt} \therefore \text{OKAY.}$$



ii) Shear Stresses:

Max. Shear force 123.4 tonf.

$\frac{d}{t} = 250.6 < 400$. \therefore Horizontal stiffeners are req'd. to limit the $\frac{d}{t}$ ratio (refer to clause 27, b(ii), B.S. 153 part 4).



for smaller panel $\frac{d_1}{t} = \frac{41.7}{0.5625} = 74.1 < 180$ see clause 27b(ii)

for greater panel $\frac{a}{t} = \frac{138}{0.5625} = 245.3 < 270$ See clause 27b(ii).

Hence satisfactory.

average web shear, $f_q = \frac{123.4}{A_w} = 1.6$ tonf/in²

allowable web shear $p_q = 6.0$ tonf/in² $> f_q$ \therefore satisfactory.

5.6. Change in section of Main Girder:

It is required to check the sections of the main girder at various distances. The heavier sections may not be required over certain lengths of the girders, and if not, which is most likely, then significant quantity steel may be saved.

i. Try the same section, which has been assumed in paragraph 5.2. at a distance of 75.0 ft. from the nose. But to use the same section, max. bending moment must be checked for cantilever condition at this point:

$$\text{Max. B.M.} = \frac{83.9 \times 75^2}{158.75 \times 2} + 5.9 \times 45.75 = 1756.3 \text{ tonf.}$$

A 50

Properties: $I_{x-x} = 257904 \text{ in}^4$; $Z_{x-x} = 3582 \text{ in}^3$ & $r_y = 2.22 \text{ in}$
 as bottom flange is restrained by cross girder at 138° c/c.,

$$\frac{l}{r_y} = 62.2 \quad \& \quad \frac{D}{T} = 192$$

$$\therefore C_s = \frac{170000}{(62.2)^2} \sqrt{1 + \frac{1}{20} \left(\frac{62.2 \times 0.75}{144} \right)^2} = 44.0 \text{ Tonf/in}^2$$

So, allowable critical stress, $p_{bc} = 8.8 \text{ tonf/in}^2$

$$\text{Working stress, } f_{bc} = \frac{M}{Z} = \frac{1756.3 \times 12}{3582} = 5.9 \text{ tonf/in}^2 < p_{bc}$$

Hence the section is satisfactory.

ii) Check the maximum positive moment at 23.9 ft. from the tail bearing; this may govern the top flange U-frame analysis & moments:

$$\text{dead load} = 122.2 \times 23.9 - 50(23.9 + 29.9) - \left(\frac{121.5 \times 30.15^2}{158.75 \times 2} \right) = -117.3 \text{ tonft.}$$

$$\text{H.A. load on span B-C only,} = 25.8 \times 23.9 - \left(\frac{57 \times 23.9^2}{58} \right) = 335.9 \text{ ''}$$

$$\text{Walkway load on span B-C only} = 3.3 \times 23.9 - \left(\frac{73 \times 23.9^2}{58} \right) = 43.0 \text{ ''}$$

$$\text{Snow load on span B-C only} = 0.4 \times 23.9 - \left(\frac{3.7 \times 23.9^2}{152.5} \right) = 2.7 \text{ ''}$$

$$\text{K.E load on span B-C only} = 6.3 \times 23.9 = 150.6 \text{ ''}$$

$$\underline{\Sigma \text{ moment} = 414.9 \text{ tonft.}}$$

iii) Max. moment on same type of section in cantilever condition,

15 ft. from pivot towards the tail bearing:

$$\text{Moments: } 5.9 \times 100.25 = 591.5 \text{ tonft}$$

$$\frac{83.9}{158.75} \times \frac{129.5^2}{2} = 4431.6 \text{ ''}$$

$$- 204.4 \times 15 = -3066 \text{ ''}$$

$$\underline{\Sigma \text{ moments} = 1957.1 \text{ tonft}}$$

Try a section 144" deep, with flange plates 18" x 1" and web plate 142" x $\frac{9}{16}$ " thick.

Properties of the section:

$$\Sigma A = 115.9 \text{ in}^2; \Sigma I_{x-x} = 318288 \text{ in}^4; \Sigma I_{y-y} = 972 \text{ in}^4;$$

AS1.

$$r_y = \sqrt{\frac{I_{y-y}}{A}} = 2.9 \text{ in}; Z_{x-x} = \frac{I_{x-x}}{y} = 4420.7 \text{ in}^3$$

$$\frac{d}{t} = 252.4; \frac{l}{r_y} = \frac{138}{2.9} = 47.6$$

Hence, $C_s = 75.2 \text{ tonf/in}^2 \therefore p_{bt} = p_{bc} = 9.0 \text{ tonf/in}^2$

Whereas, $f_{bt} = f_{bc} = \frac{M}{Z} = 5.3 \text{ tonf/in}^2 < p_{bt} \text{ or } p_{bc} \therefore \text{OKAY.}$

iv) Check span B-C for compression (in top) flange:

Max. moment = 414.9 tonft. occurs at 23.9 ft. from the tail bearing: Try the same section as above.

I_{y1} of Compression flange = 486 in⁴

$I_s = 2^{\text{nd}}$ moment of area of stiffener = 249.4 in⁴

$I_c = \text{" " " " of cross girder} = 8845 \text{ in}^4$

depth to top side of the deck, $d' = 92 \text{ in}$

" " Neutral axis of cross girder, $d'' = 98.4 \text{ in}$.

Check stability of compression flange as U-frame:

$$\frac{a_1^3}{40EI} = \frac{138^3}{40 \times 13000 \times 486} = 0.0101$$

$$\delta = \frac{(d')^3}{3EI_s} + \frac{(d'')^2 b}{EI_c} = 0.09''$$

Effective length, $l = 2.5 \sqrt{EI_{y1} a_1 \delta} = 237.1''$

Hence $\frac{l}{r_y} = 81.8 \therefore C_s = 25.6 \text{ tonf/in}^2$

So, $p_{bc} = 7.1 \text{ tonf/in}^2$

Hence $f_{bc} = \frac{M}{Z} = 1.1 \text{ tonf/in}^2 < p_{bc} \therefore \text{the section is satisfactory.}$

v) Check section 17 ft. from pivot towards nose bearing:

moment in cantilever condition:

$$\frac{83.9}{158.75 \times 2} \times 97.5^2 + \frac{26.1}{100.25} \times \frac{39.0^2}{2} + 5.9 \times 68.25 = 3113.7 \text{ tonft.}$$

try the same section as above: $f_{bc} = f_{bt} = \frac{M}{Z} = 8.45 \text{ tonf/in}^2$

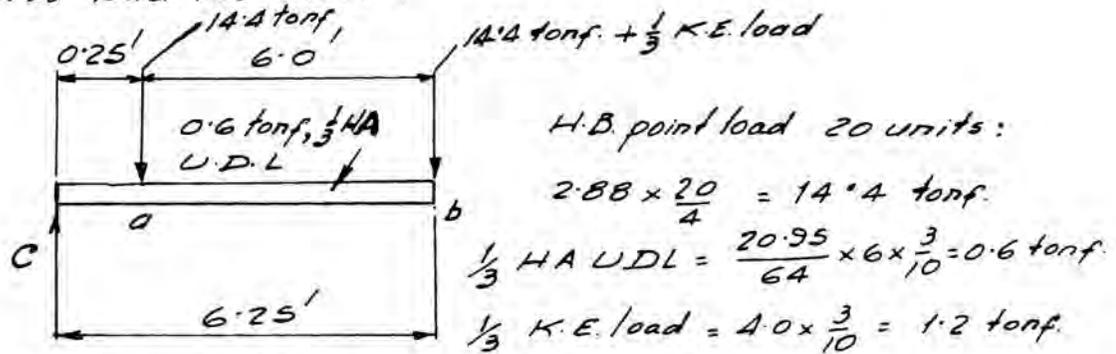
but $p_{bc} = p_{bt} = 9.0 \text{ tonf/in}^2 > f_{bc} \text{ or } f_{bt} \therefore \text{satisfactory.}$

5.7. Section at tail bearing:

Moments:

a. dead load moment 315 tonft.

b live load moments:



Hence moment at C:

$$\text{due to } \frac{1}{3} \text{ H.A. U.D.L.} = \frac{0.6 \times 6.25}{2} = 1.9 \text{ tonft.}$$

$$\left. \begin{array}{l} \text{due to } \frac{1}{3} \text{ K.E. load +} \\ \text{20 units H.B. load at b} \end{array} \right\} = (14.4 + 1.2) \times 6.25 = 97.5 \text{ ''}$$

$$\text{due to H.B. load at a} = 14.4 \times 0.25 = 3.6 \text{ ''}$$

$$\underline{\text{Moment} = 103.0 \text{ tonft.}}$$

\therefore total moment (due to live load + dead load) = 418.0 tonft.

Web shear force:

$$\text{due to dead load} = 54.9 \text{ tonf.}$$

$$\text{'' '' live ''} = 30.6 \text{ ''}$$

$$\underline{\text{Shear force} = 85.5 \text{ tonf.}}$$

Try a section 9' deep.

$$\text{Properties: } \Sigma A = 95.6 \text{ in}^2$$

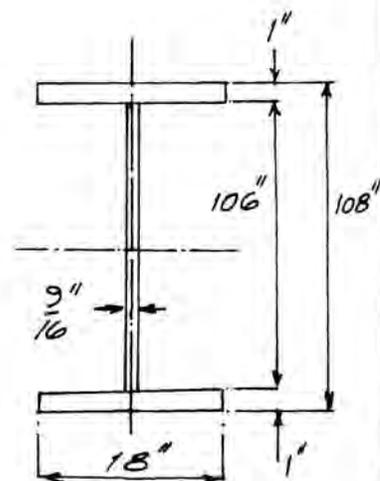
$$\Sigma I_{x-x} = 158870 \text{ in}^4$$

$$\Sigma I_{y-y} = 972 \text{ in}^4$$

$$r_y = \sqrt{\frac{I_{y-y}}{A}} = 3.19 \text{ in}$$

$$Z_{x-x} = \frac{I_{x-x}}{Y} = 2942 \text{ in}^3$$

$$\frac{D}{T} = 10.8; \quad \frac{d}{T} = 18.8 \neq \frac{1}{r_y} = 43.3.$$

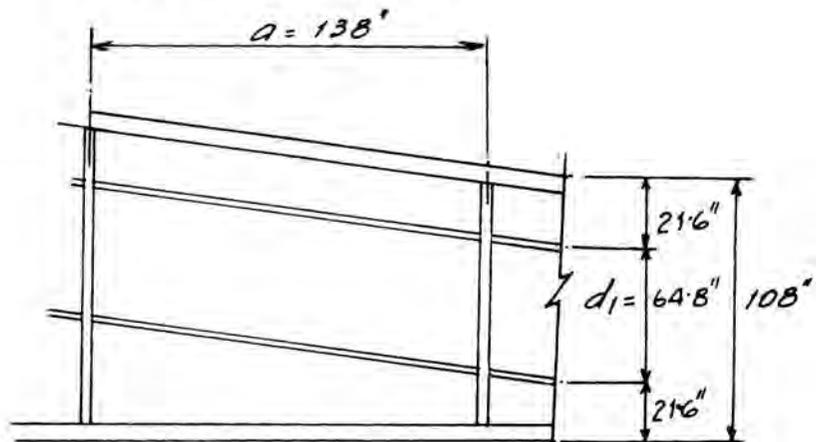


$$\text{Hence, } C_s = 91.0 \text{ tonf/in}^2 \therefore p_{bc} = p_{bt} = 9.0 \text{ tonf/in}^2$$

A 53

Hence $f_{bc} = f_{bt} = \frac{M}{Z} = 1.7 \text{ tonf/in}^2 < p_{bc} \text{ or } p_{bt}$
 therefore, satisfactory.

Web Shear: $V = 85.5 \text{ tonf}$



Position of horizontal stiffeners.

for smaller panel: $\frac{d_1}{t} = 115.2 < 180$

for greater panel: $\frac{a}{t} = 245.3 < 270$

average web shear, $f_q = \frac{V}{A_w} = 1.43 \text{ tonf/in}^2$

allowable average web shear $p_q = 5.28 \text{ tonf/in}^2 > f_q$

\therefore the section is satisfactory.

5.8. Check Combined Stress: as per clause 29 b(i) of B.S. 153 part 3B., $f_e = \sqrt{f_{bt}^2 + 3f_q^2}$ or, $\sqrt{f_{bc}^2 + 3f_q^2}$
 from para 5.5, we get, $f_{bc} = 8.34 \text{ tonf/in}^2$ which is at the top edge of the flange.

$\therefore f_{bc}$ at bottom edge of the flange (ie, at the edge of the web)
 $= \frac{8.34 \times 70.5}{72} = 8.17 \text{ tonf/in}^2$

and $f_q = 1.6 \text{ tonf/in}^2 \therefore f_e = 8.65 \text{ tonf/in}^2$ whereas
 $p_e = 14.0 \text{ tonf/in}^2$ see clause 29 b (iii) of B.S. 153 part 3B.
 Hence $f_e < p_e \therefore$ satisfactory.

5.9. Stiffeners

a. Horizontal Web stiffeners: B.5 153 part 4, clause 27 b (ii), indicates that, the horizontal stiffeners may be needed in addition to vertical stiffeners & they shall be on one or both sides of the web and shall be placed at a distance from the compression flange equal to $\frac{2}{5}$ th. of the distance from the compression flange to the neutral axis, when the thickness of the web is less than $\frac{d_2}{200}$ for grade 43 steel (mild steel) where d_2 is twice the clear distance from the compression flange plate to the neutral axis. This stiffener shall have a moment of inertia, I , not less than $45t^3$, where s is the actual distance between the stiffeners and t is the web thickness.

$$\therefore \text{Min. } I \text{ req'd} = 4 \times 138 \left(\frac{142.5}{400} \right) = 25 \text{ in}^4$$

Try $6'' \times \frac{1}{2}''$ flat.

$$\text{Moment of Inertia about the face of web} = \frac{0.5 \times 6^3}{3} = 36 \text{ in}^4$$

Hence satisfactory.

It also recommends that a second horizontal stiffener on one or both sides of the web shall be placed on the neutral axis of the girder, when the thickness of the web is less than $\frac{d_2}{250}$ for mild steel (grade 43) and this stiffener shall have a moment of inertia, I , not less than $d_2 t^3$.

$$\therefore \text{min. } I \text{ req'd.} = 142.5 \left(\frac{142.5}{400} \right)^3 = 6.4 \text{ in}^4$$

\therefore Use $6'' \times \frac{1}{2}''$ thick flat, which is adequate.

b. Intermediate Vertical stiffeners: B.5. 153 part 4, clause 27 b (i) recommends that intermediate stiffeners are to limit web buckling and shall be provided throughout the length of the girder at a distance not greater than $1\frac{1}{2}d_1$ when the thickness of the web is less than $\frac{d_1}{85}$ for mild steel, where d_1 = clear distance between the flange plates.

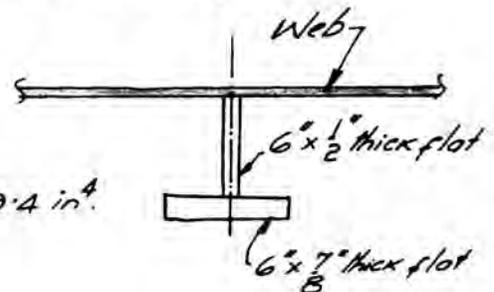
These stiffeners are to be designed with a minimum I value of $1.5 \times \frac{d_1^3 \times t^3}{S^2}$, where, S = min. permitted clear distance between stiffeners for thickness, t .

Hence, the min. I required, $= 1.5 \times \frac{142.5^3 \times (\frac{142.5}{400})^2}{(142.5 \times 1.5)^2} = 12.3 \text{ in}^4$

Try the section shown:

Moment of Inertia, I .

$$\frac{0.5 \times 6^3}{12} + 0.5 \times 6 \times 3^2 + 6 \times 0.875 \times 6.375^2 = 249.4 \text{ in}^4$$



Hence $I = 249.4 \text{ in}^4$

Assuming wind pressure @ 20 lb/ft^2 , wind pressure above deck level $= \frac{20 \times 11.5 \times 8}{7} = 0.82 \text{ tonf}$.

The restraint against torsion should be computed and the restraint element shall be designed to resist, in addition to the effects of wind or other applied lateral forces, the effect of a horizontal force, F acting normal to the compression flange of the girder at the level of its centroid where,

$$F = \frac{1.4 \times 10^{-3} l}{S \left(\frac{C_s}{f_{bc}} - 1.7 \right)}$$

from Clause 34b, B.S. 153, Part 4.

from paragraph 5.4 i)

$$C_s = 20.7 \text{ tonf/in}^2$$

$$l = 201.97'' \text{ from para } 5.3$$

$$S = 0.09'' \quad \text{''} \quad \text{''} \quad 5.3$$

$$f_{bc} = 5.5 \text{ tonf/in}^2 \text{ from para } 5.4$$

$$\therefore F = 1.49 \text{ tonf.} \quad \therefore \text{B.M.} = 1.49 \times 98.4'' = 146.6 \text{ ton in}$$

$$r = \sqrt{\frac{249.4}{8.25}} = 5.5'' \quad \therefore \frac{l}{r} = \frac{142.5}{5.5} = 26; \quad \frac{D}{T} = \frac{6.875}{0.875} = 8.$$

$$\therefore f_{bc} = 10.0 \text{ tonf/in}^2; \text{ but } f_{bc} = \frac{146.6 \times 6.875}{249.5} = 4.0 \text{ tonf/in}^2$$

$$\text{Now, } f_{bc} \text{ including wind,} = \left\{ \frac{146.6 + 40.4}{249.5} \right\} 6.875 = 5.2 \text{ tonf/in}^2$$

Thus, the section is satisfactory.

C. Load bearing stiffeners at Tail bearing:

i) loading :

dead load	122.2 tonf
H.A load on span BC	25.8 "
Walkway load on span BC	3.3 "
K.E. load	12.1 "
Snow load	0.4 "

$$\Sigma \text{load} = 63.8 \text{ tonf.}$$

ii) H.B loading :

dead load	122.2 tonf
Walkway on span B-C	3.3 "
Snow load	0.4 "
H.B. load + $\frac{1}{3}$ H.A load + $\frac{1}{3}$ K.E. load	50.59 "

$$\Sigma \text{load} = 176.49 \text{ tonf.}$$

Properties of the section :

$$\Sigma \text{area} = 37.5 \text{ in}^2$$

$$\Sigma I_{x-x} = 835.1 \text{ in}^4$$

$$r = \sqrt{\frac{I}{A}} = 4.72 "$$

depth (say) 114" in average

$$\therefore \frac{l}{r} = \frac{114 \times 0.7 (\text{factor})}{4.72} = 16.9 ; \frac{D}{T} = 16.4.$$

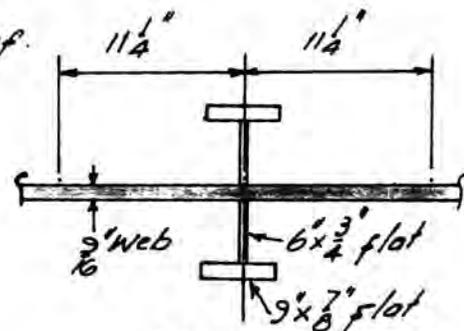
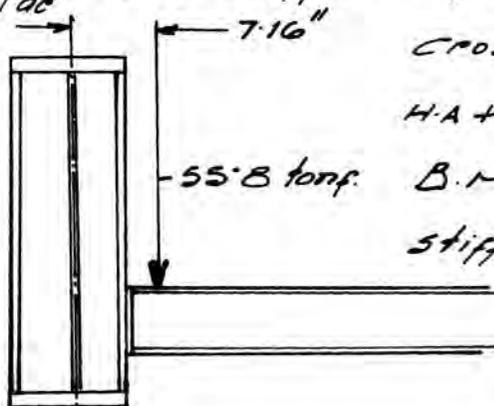


Fig. 5.9.c(ii)

$$\therefore p_{ac} = 8.5 \text{ tonf/in}^2 \quad \& \quad p_{bc} = 10.0 \text{ tonf/in}^2$$



Cross girder reaction, due to dead +
H.A + K.E. + Kentledge loads = 55.8 tonf

B.M. due to eccentricity on the
stiffener = $(55.8 \times 7.16) = 399.53 \text{ tonin.}$

Axial Compression force = 161.8 tonf.

$$f_{bc} = \frac{399.53 \times 7.16}{835.1} = 3.43 \text{ tonf/in}^2$$

$$f_{ac} = \frac{163.8}{37.5} = 4.37 \text{ tonf/in}^2$$

$$\text{Ratio: } \frac{f_{ac}}{p_{ac}} + \frac{f_{bc}}{p_{bc}} = \frac{4.37}{8.5} + \frac{3.43}{10} = 0.86 < 1.0 \therefore \text{okay.}$$

check bearing:

Bearing is taken on $6'' \times \frac{3}{4}''$ flats

$$\text{loads per flat} = \frac{163.8 \times 4.5}{21.7} = 34.0 \text{ tonf.}$$

$$\therefore f_b = \frac{34.0}{5.5 \times 0.75} = 8.2 \text{ tonf/in}^2$$

from B.S. 153 Part 3B, table 3, $p_b = 12.0 \text{ tonf/in}^2$

Hence, $f_b < p_b \therefore$ Satisfactory.

d. Load bearing stiffeners at intermediate bearing:
axial loads:

dead load 69.5 tonf

H.A " 74.2 "

Walkway " 9.4 "

K.E. " 12.1 "

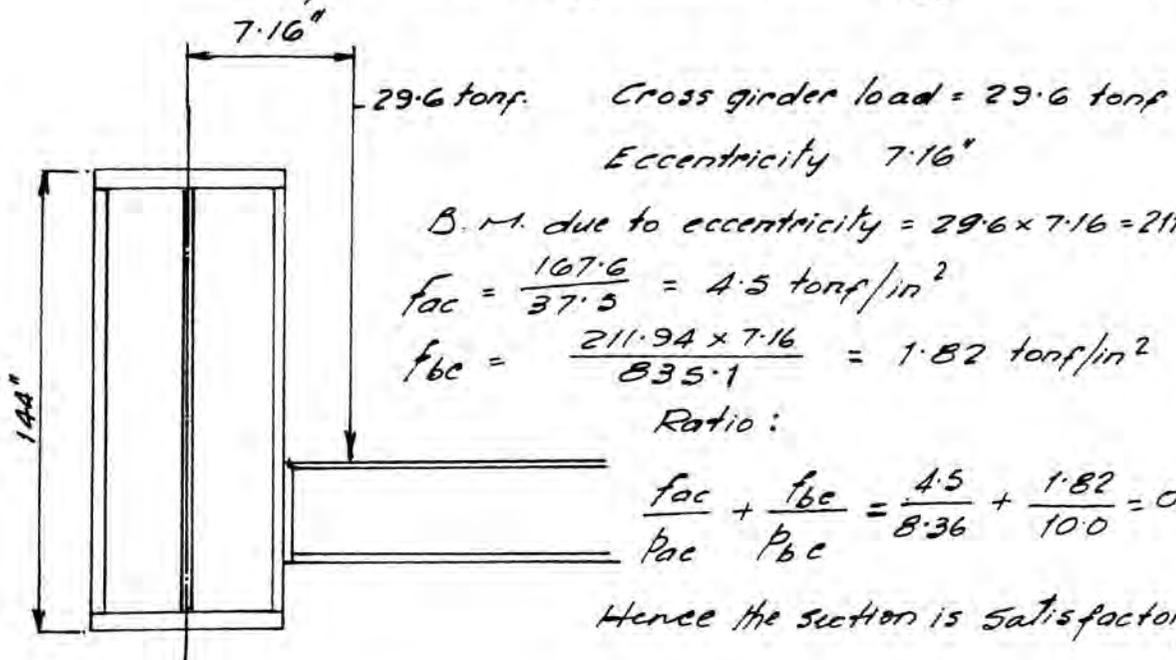
Stow " 2.4 "

$$\Sigma \text{load} = 107.6 \text{ tonf.}$$

Try the same section, as shown in Fig. 5.9. C(ii)

Properties: $\Sigma \text{area } 37.5 \text{ in}^2$; $\Sigma I_{x-x} = 835.1 \text{ in}^4$; $r = 4.72''$; $\frac{l}{r} = 16.9$

$$\frac{D}{T} = 16.4; p_{ac} = 8.5 \text{ tonf/in}^2 \text{ \& } p_{bc} = 10.0 \text{ tonf/in}^2$$



6.0. Welding: Flange to Web of Main Girder

The shear per linear inch between web plate and flange plates may be computed by the formula, $f = \frac{S \cdot A \cdot Y}{I}$ tonf.

Where, S = the max shear force in tonf.

A = Area of either flange plate in sq.in

Y = the vertical distance between the neutral axis of the girder and the centroid of either flange plate

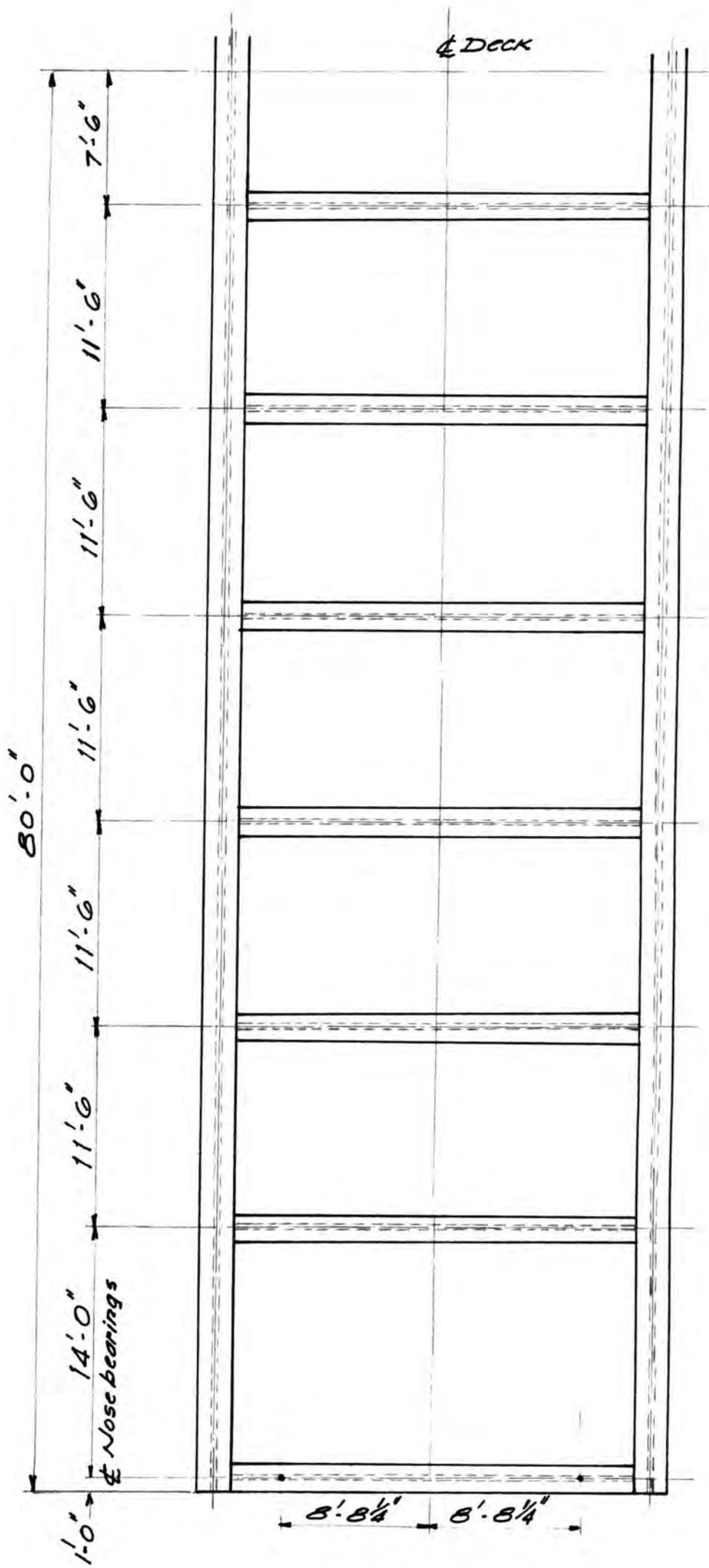
I = 2nd moment of area of the complete section of the girder in inches.

$$\text{Hence, } f = \frac{123.4 \text{ tonf} \times 31.5 \text{ in}^2 \times 71 \text{ in}}{451224 \text{ in}^4} = 0.61 \text{ tonf}$$

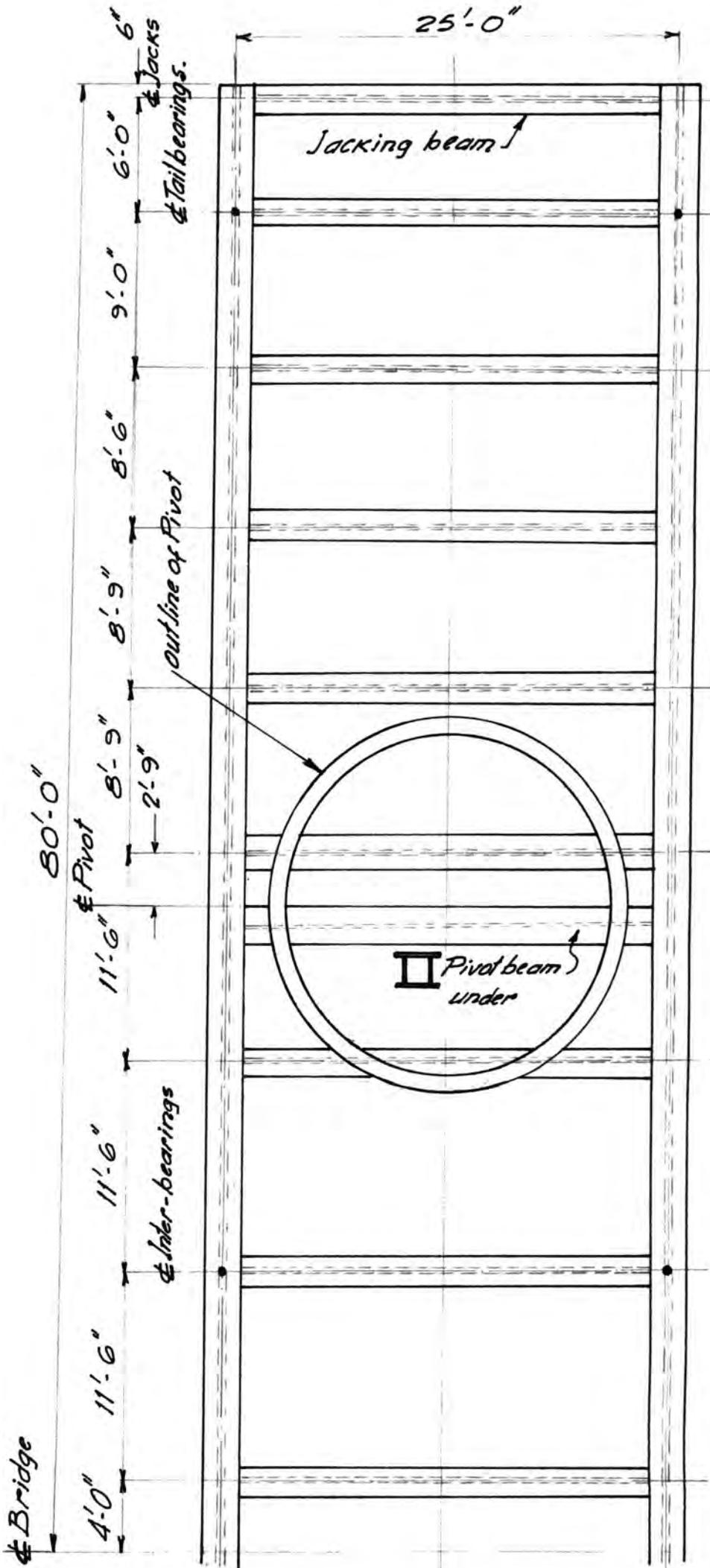
per linear inch. \therefore Use $\frac{1}{4}$ " fillet weld continuous on each side of web (strength of $\frac{1}{4}$ " \angle = 1.23 tonf/lin. in.).

Use $\frac{1}{4}$ " fillet weld (minimum) on all stiffeners.

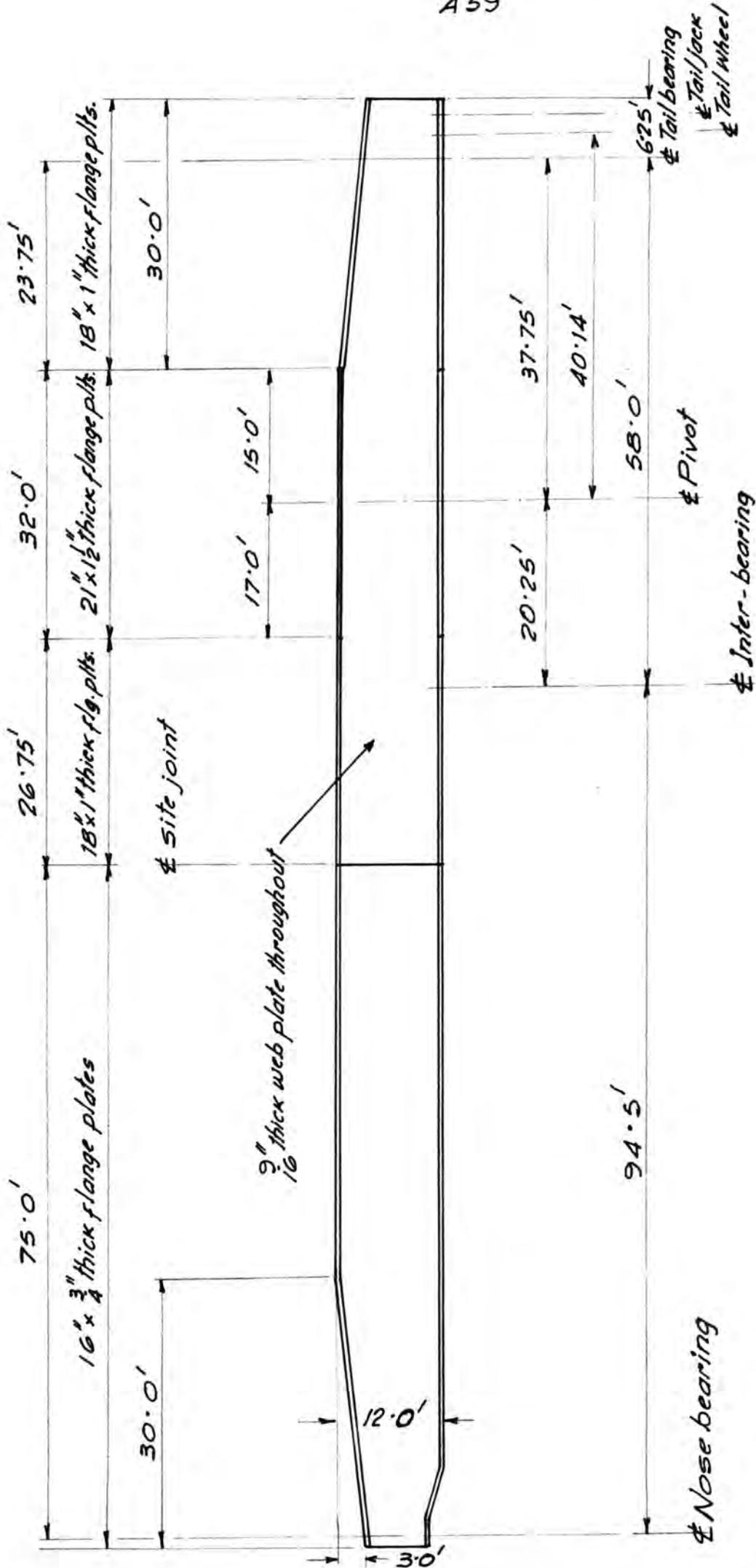




Plan Without Deck Plate

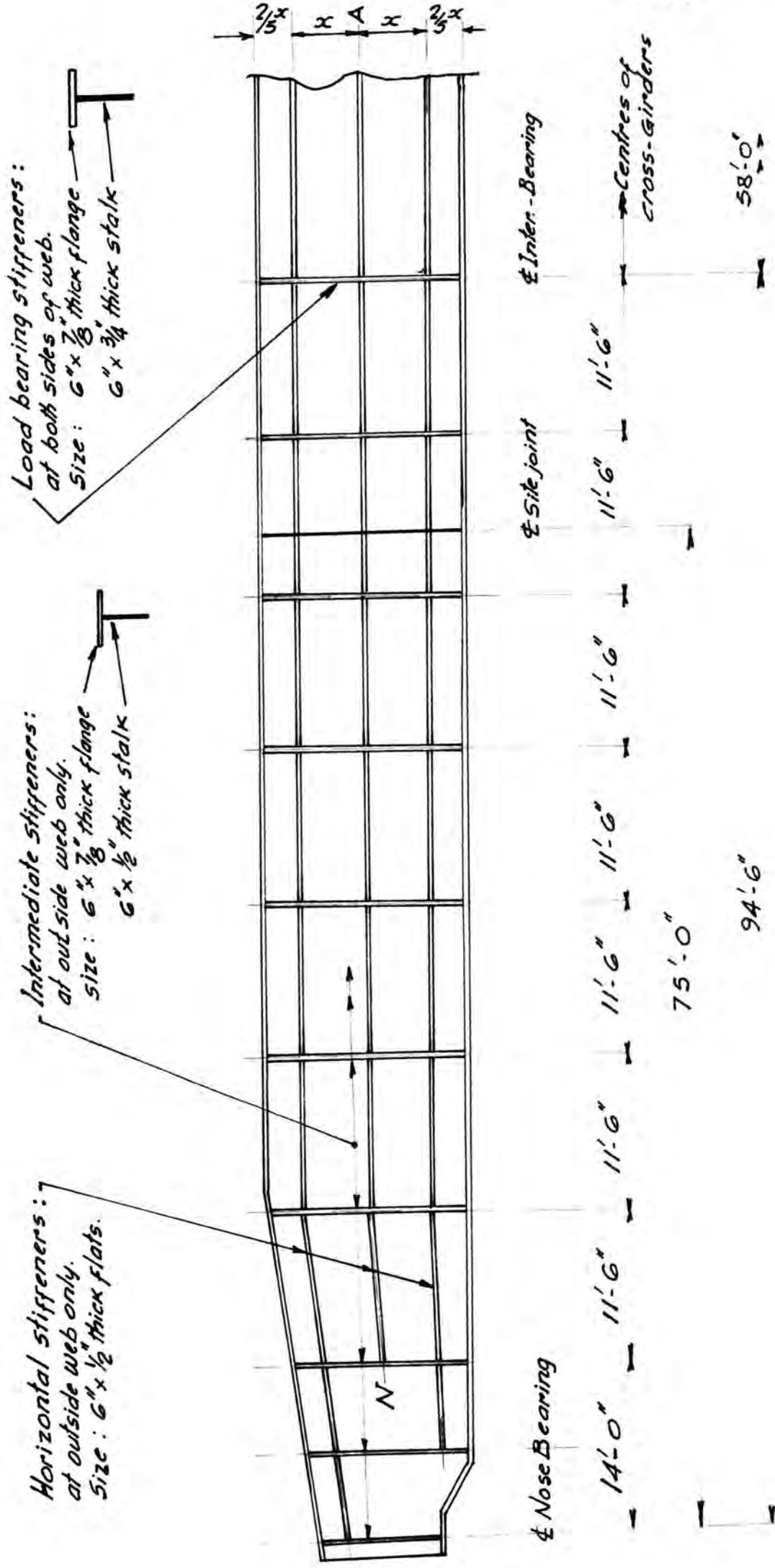


Plan Without Deck Plate

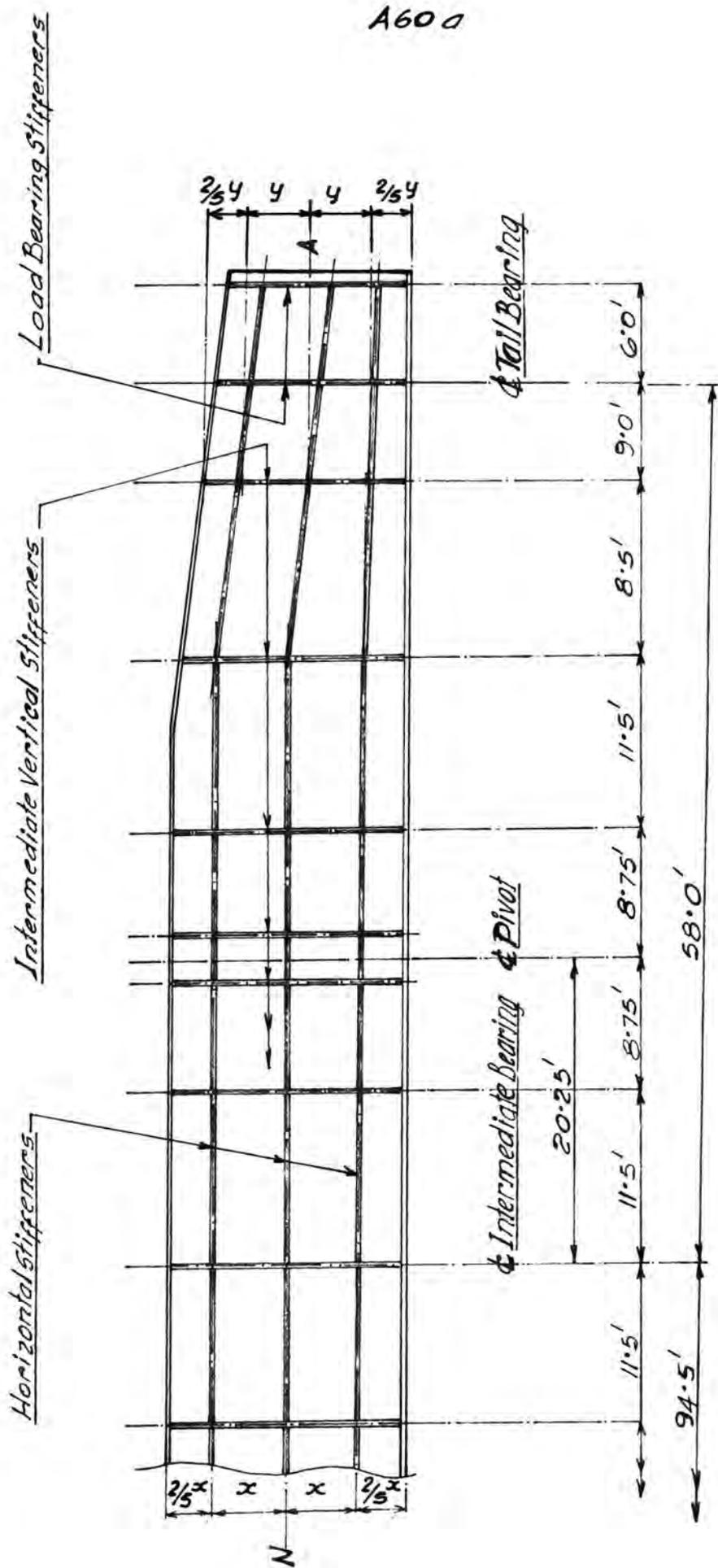


Elevation Of Main Girder

Showing section changes.



Elevation Of Main Plate Girder
Showing Position Of Stiffeners.



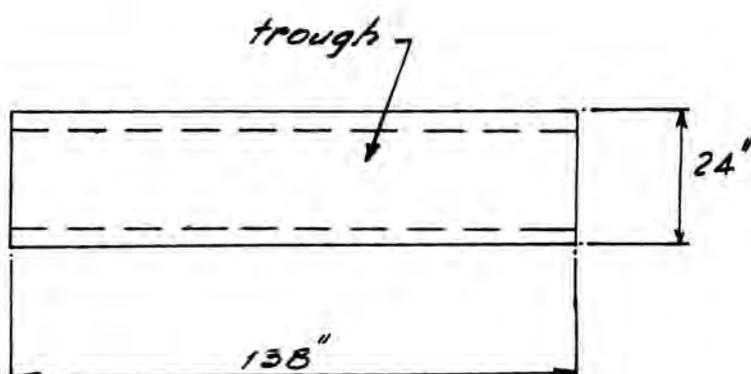
Elevation Of Main Plate Girder

Showing position of Stiffeners

Appendix B

1 Stress computations of Orthotropic Deck

1.1. Loadings: Consider a 24" width of deck, (i.e., troughs are at 24" centres) fixed at both ends to floor beams which are at 138" centres.



Dead load :

$$\text{deck } 24" \times 138" \times 16.75 \text{ lb/ft.} = 385.25 \text{ lb.}$$

$$\text{troughs } 138" \times 39.5 \text{ lb/ft.} = 454.25 \text{ lb.}$$

$$\text{Snow } 24" \times 138" \times 3 \text{ lb/ft}^2 = 69.00 \text{ "}$$

$$\text{asphalt } 24" \times 138" \times \frac{150}{3} \text{ lb/ft}^2 = 1150.00 \text{ "}$$

$$\Sigma \text{ dead load} = 2058.50 \text{ lb.}$$

$$\text{or, } \Sigma \text{ Dead load} = 0.92 \text{ tonf.}$$

Live load :

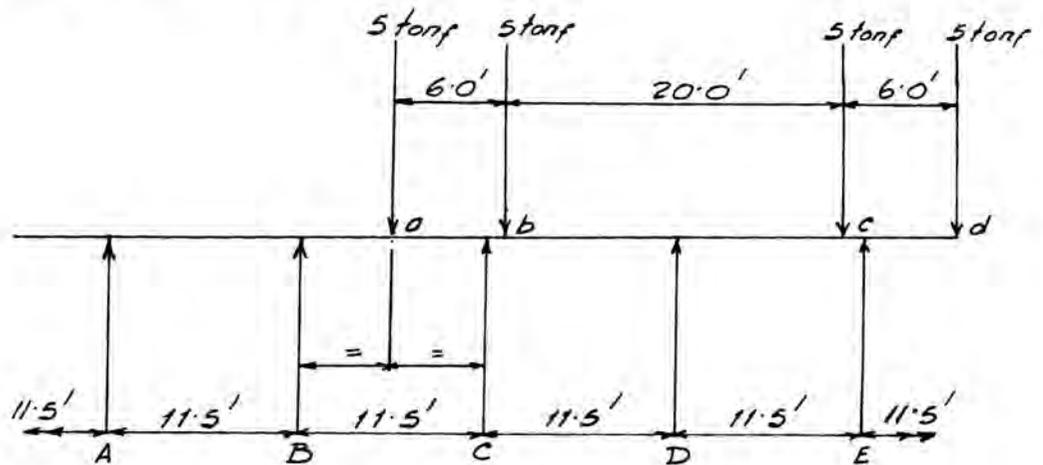
$$\text{H.A. loading @ } 10.0' \text{ lane} = \frac{3425}{10} = 342.5 \text{ lb/ft.}$$

$$\therefore \text{ on the } 24" \text{ wide strip of deck: } 24" \times 138" \times \frac{342.5}{T} = 3.52 \text{ tonf.}$$

$$\text{K.E. load} = \frac{24" \times 2700}{12 \times T} = 2.41 \text{ tonf as point load.}$$

$$\text{Hence } \Sigma \text{ dead + H.A loads} = 4.42 \text{ tonf.}$$

1.2. Position of Wheels for max. bending moment on deck due to H.B. loading:



Bending moment coefficients:

Using influence lines for four span beam, bending moment coefficient @ a = 0.173

Max. bending moment = $0.173 \times 138 \times 5 = 119.4$ tonin.
moments due to:

dead load and H.A load = $\frac{4.42 \times 138}{8} = 76.3$ tonin

K.E load as point load at mid-span = $\frac{2.41 \times 138}{4} = 83.2$ tonin

∴ B.M max. due to D.L, H.A & K.E = 159.5 tonin.

2. Stress Computation:

2.1. Effective width of the deck plate:

Table of effective width of plate: from Structural Steel Designer's Handbook, p.p. 4-47.

a_e/s_e	a_o/a_e	a_e/s_e	a_o/a_e	a_e/s_e	a_o/a_e	a_e/s_e	a_o/a_e
e_e/s_e	e_o/e_e	e_e/s_e	e_o/e_e	e_e/s_e	e_o/e_e	e_e/s_e	e_o/e_e
0	1.10	0.20	1.01	0.40	0.809	0.60	0.622
0.05	1.09	0.25	0.961	0.45	0.757	0.65	0.590
0.10	1.08	0.30	0.921	0.50	0.722	0.70	0.540
0.15	1.05	0.35	0.870	0.55	0.671	0.75	0.512

Here $S = 138''$

\therefore effective span, $S_e = 0.7 \times 138 = 96.6''$

For determination of effective width of the deck plate, as top flange, take the effective width, a_e at top of the rib equal to the actual width, a and the effective rib spacing, e_e equal to the actual spacing, e between ribs. Then,

$$\frac{a_e}{S_e} = \frac{12}{96.6} = 0.124$$

and $\frac{e_e}{S_e} = \frac{12}{96.6} = 0.124$

Then from the table on page B2, we get

$$\frac{a_0}{a_e} = 1.066 \quad \text{and} \quad \frac{e_0}{e_e} = 1.066$$

Hence the effective width of the top flange

$$= \frac{a_0}{a_e} \times a + \frac{e_0}{e_e} \times a = 1.066 \times 12 + 1.066 \times 12 = 25.6''$$

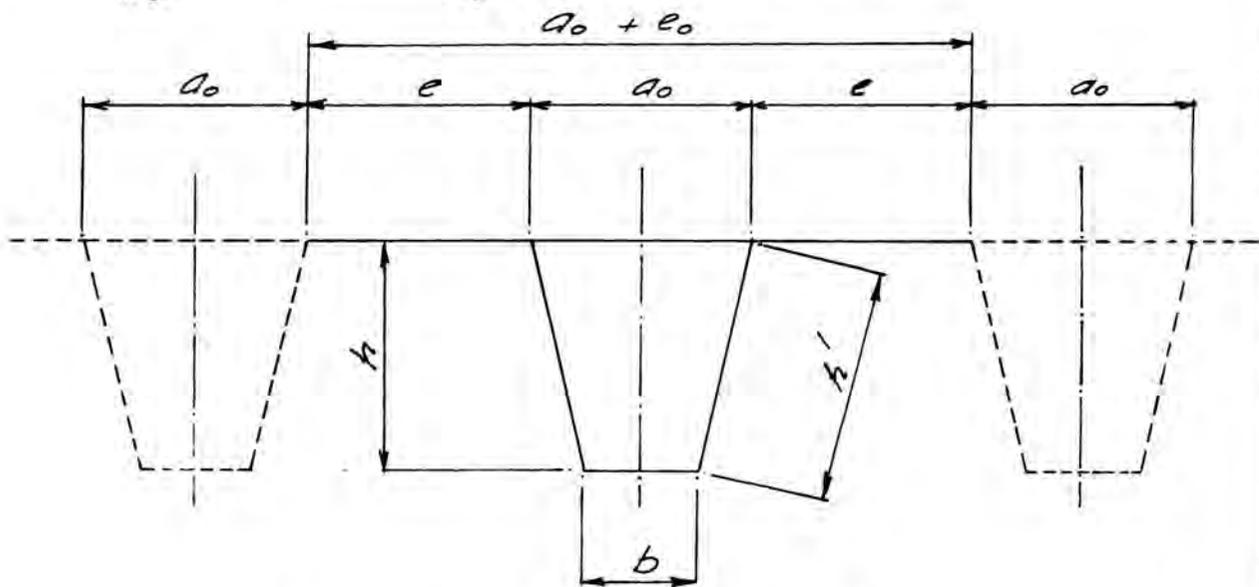


Fig. 2.1.1. Effective Width of Rib

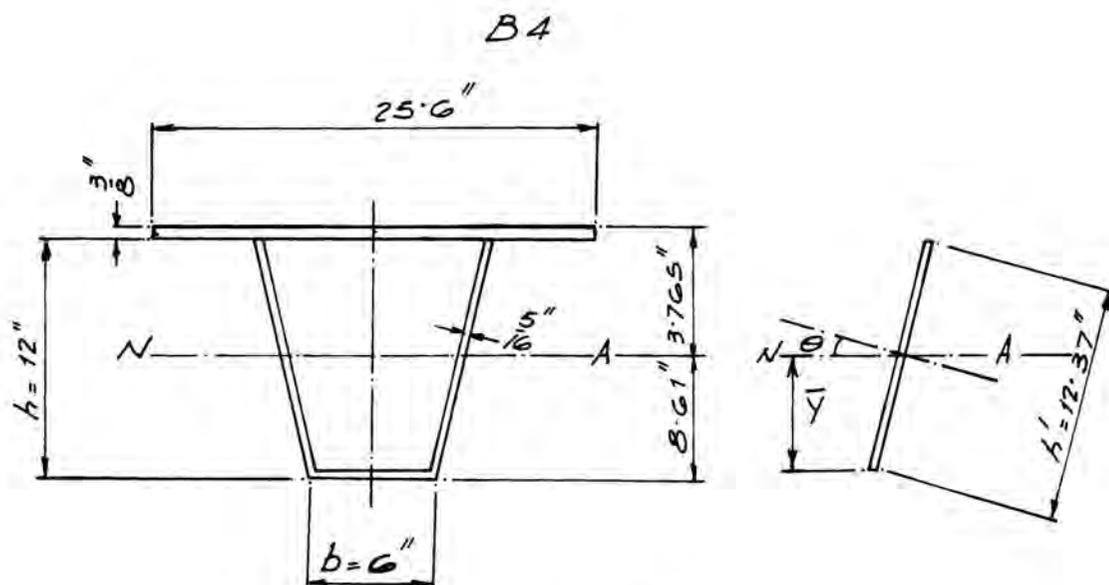


Fig. 2.1.2. Detail of Rib.

From Fig. 2.1.2, $b' = \frac{5}{16}$; $h' = 12.37$; $\theta = 14^\circ$.

Then, $\bar{y} = \frac{1}{2} (b' \sin \theta + h' \cos \theta) = 6$

and $I_{N-A} = \frac{1}{12} [b' h' \{ (b')^2 \sin^2 \theta + (h')^2 \cos^2 \theta \}] = 55.7 \text{ in}^4$

Note that the calculations do not take into account, the relief in the stresses, provided by the effect of adjacent trough section. The deck is designed as a transverse slab to carry one 10 ft. lane H.A. loading and 20 units H.B. loading.

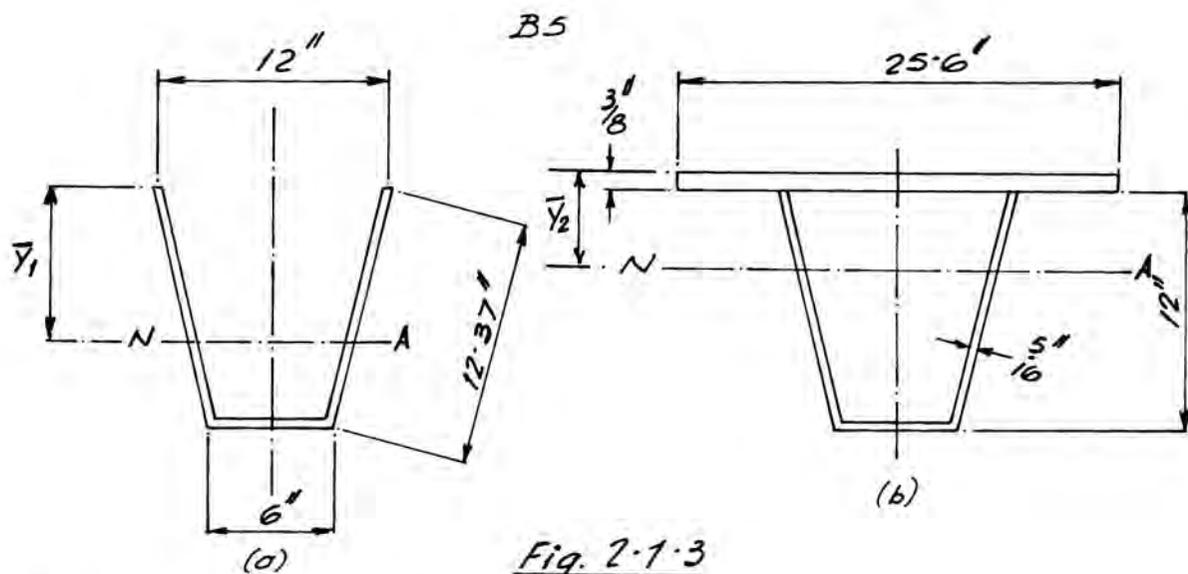


Fig. 2.1.3

Properties of Rib:

Rib without deck plate Fig. 2.1.3 (a)	Area A	Position of N-A: d	A × d	A × d ²	I ₀	I
2 No. Sides: 12.37" × $\frac{5}{16}$ "	7.73	6.00	46.40	278.00	79.00	357.00
Flange 5.38" × $\frac{5}{16}$ "	1.68	11.84	19.90	236.00		236.00
$\bar{y}_1 = \frac{66.3}{9.41}$ = 7.05						
	9.41		66.30			593.00

$$I_{N-A} = 593.00 - (7.05 \times 66.30) = 125.00 \text{ in}^4$$

Rib with deck plate Fig. 2.1.3 (b)	Area A	Position of N-A: d	A × d	A × d ²	I ₀	I
Rib only	9.41		66.30			593.00
Top flange deck plate 25.6" × $\frac{3}{8}$ "	9.60	0.1875	-1.80	0.34		
	19.01		64.50			593.00

$$I_{N-A} = 593.00 - (3.39 \times 64.50) = 374.00 \text{ in}^4$$

B6

Refer to Fig. 2.1.3 : distance from N-A to :

- a. top of deck plate, $\bar{Y}_2 = 0.375 + 3.39 = 3.765''$
- b. bottom of rib $= 12 - 3.39 = 8.61''$

Section moduli :

- i. top of deck plate, $Z_t = 99.3 \text{ in}^3$
- ii. bottom of rib, $Z_b = 43.4 \text{ in}^3$

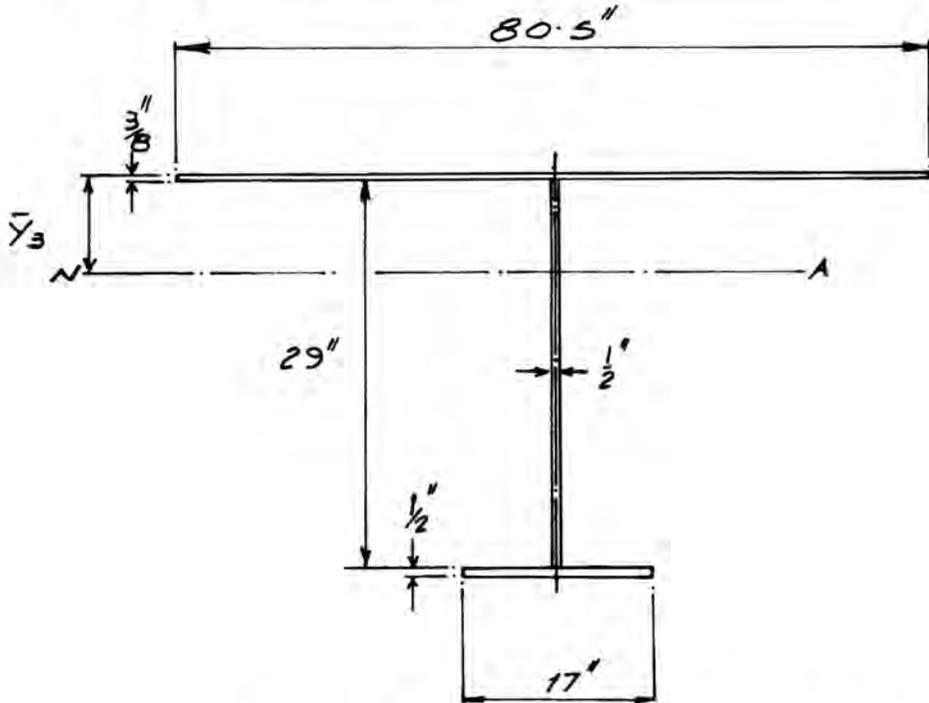


Fig. 2.1.4. Cross-Girder with deck plate

Fig. 2.1.4. Section	Area A	Position of N-A d	A x d	A x d ²	I ₀	I
Deck: 80.5" x 3/8"	30.19	14.69	443.50	6515.0		6515.00
Web: 29" x 1/2"	14.50				1016.00	1016.00
bottom flange: 17" x 1/2"	8.50	-14.75	-125.40	1850.0		1850.00
$\bar{Y}_3 = \frac{318.1}{53.19} = 5.98''$	53.19		318.1			9381.00

$$I_{N-A} = 9381.00 - (5.98 \times 318.1) = 7479.00 \text{ in}^4$$

B7

Refer to Fig. 2.1.4: distance from N-A to :-

- a. top of deck plate = $14.50 + 0.375 - 5.98 = 8.90''$
- b. bottom of rib = $14.50 + 0.50 + 5.98 = 20.98''$

Section moduli :

- i. top of deck plate, $Z_t = \frac{7479}{8.90} = 840.3 \text{ in}^3$
- ii. bottom of rib, $Z_b = \frac{7479}{20.98} = 356.5 \text{ in}^3$

Stresses :

From page B2, para' 1.2, We get max. B.M = 159.5 tonin.

$$\therefore f_{bt} = \frac{M}{Z_b} = \frac{159.5}{43.4} = 3.70 \text{ tonf/in}^2$$

$$f_{bc} = \frac{M}{Z_t} = \frac{159.5}{99.3} = 1.60 \text{ tonf/in}^2$$

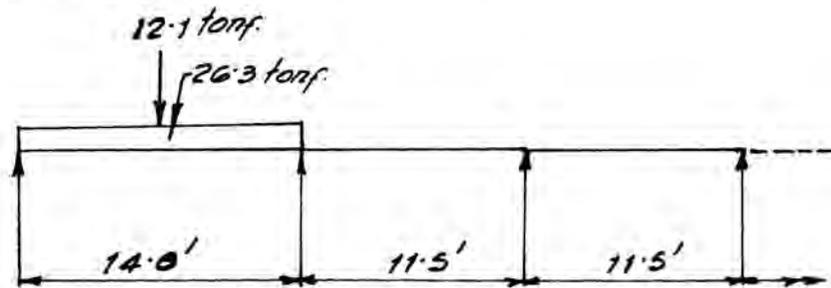
But max. moment due to dead load + 20 units H.B load = 135.3 tonin.

$$\therefore f_{bt} = \frac{135.3}{43.4} = 3.10 \text{ tonf/in}^2$$

$$\& f_{bc} = \frac{135.3}{99.3} = 1.40 \text{ tonf/in}^2$$

Hence the section is satisfactory.

3. Check end span :



loaded length = 14.0 ft.

\therefore H.A. lane load/ft = 4210 lbf.

$$\text{H.A. U.D.L} = \frac{4210 \times 14}{7} = 26.3 \text{ tonf.}$$

K.E. load/lane = 12.1 tonf.

$$\text{Dead load (load per rib)} = \frac{0.92 \times 14}{11.5} = 1.1 \text{ tonf}$$

$$\text{H.A. U.D.L} = \frac{26.3}{5} = 5.26 \text{ "}$$

$$\Sigma = 6.36 \text{ tonf}$$

$$\text{K.E. load} = 2.42 \text{ tonf}$$

free bending moments :

$$\text{due to dead + H.A. load} = \frac{6.36}{8} \times 14 \times 12 = 133.6 \text{ tonin}$$

$$\text{due to K.E. load} = \frac{2.42}{4} \times 14 \times 12 = 101.6 \text{ "}$$

$$\Sigma \text{moment} = 235.2 \text{ tonin.}$$

Stresses :

$$Z_b = 43.4 \text{ in}^3 \text{ \& } Z_t = 99.3 \text{ in}^3$$

$$\text{Hence } f_{bt} = \frac{M}{Z_b} = 5.4 \text{ tonf/in}^2$$

$$\text{and } f_{bc} = \frac{M}{Z_t} = 2.4 \text{ tonf/in}^2$$

3.1. Floor beams: Check stresses in typical floor beam :

Max. moment on floor beam :

$$\text{D/Load} = \frac{0.45}{8} \times 25^2 \times 12 = 442 \text{ tonin}$$

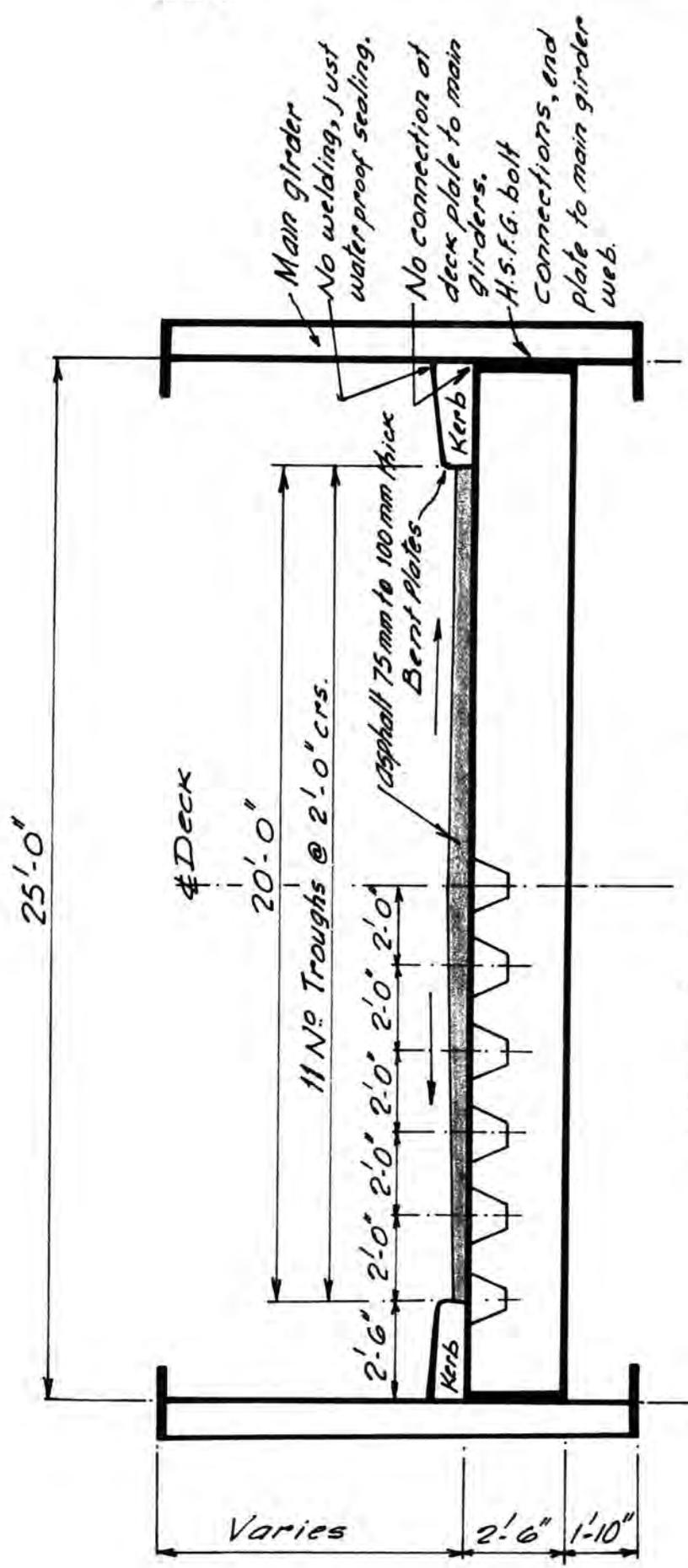
$$\text{H.A. + K.E load} = 2251 \text{ "}$$

$$\Sigma \text{moment} = 2673 \text{ tonin.}$$

From page B7, $Z_t = 840.3 \text{ in}^3$ & $Z_b = 356.5 \text{ in}^3$

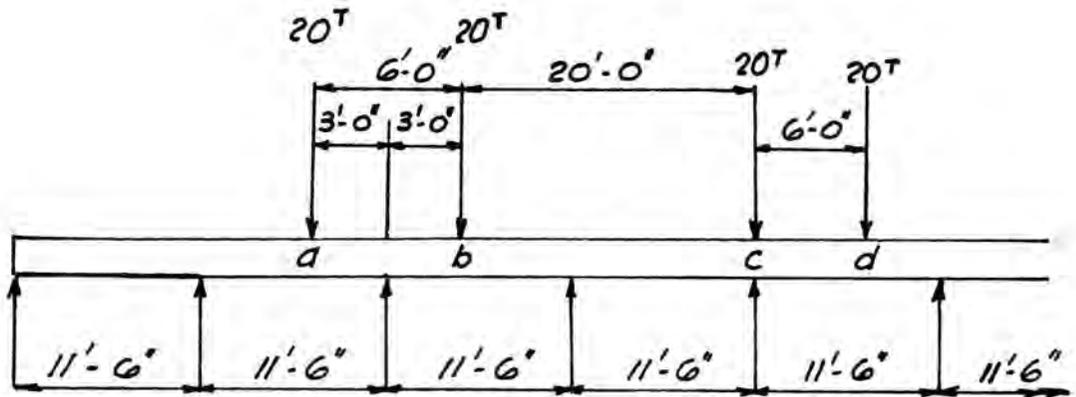
$$\therefore f_{bc} = \frac{2673}{840.3} = 3.2 \text{ tonf/in}^2 \text{ \& } f_{bt} = \frac{2673}{356.5} = 7.5 \text{ tonf/in}^2$$

\therefore the section is satisfactory.



Typical Cross-Section
through bridge.

4. Loading on floor beams



Coefficients taken from Influence line reactions & shear force for four span beam (see steel designer's manual - Crosby Lockwood pp. 193-196).

$$\text{Coefficient @ a} = +0.881$$

$$\text{@ b} = +0.881$$

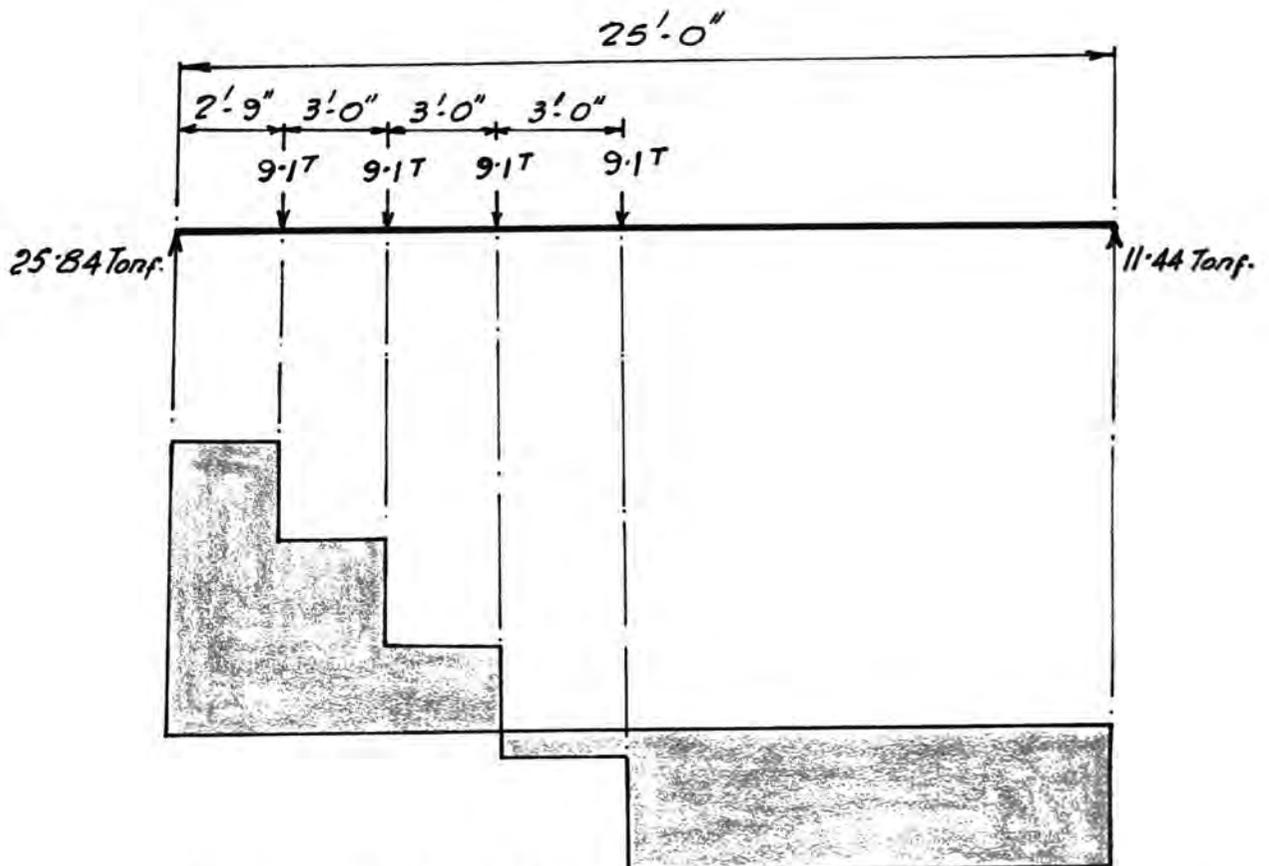
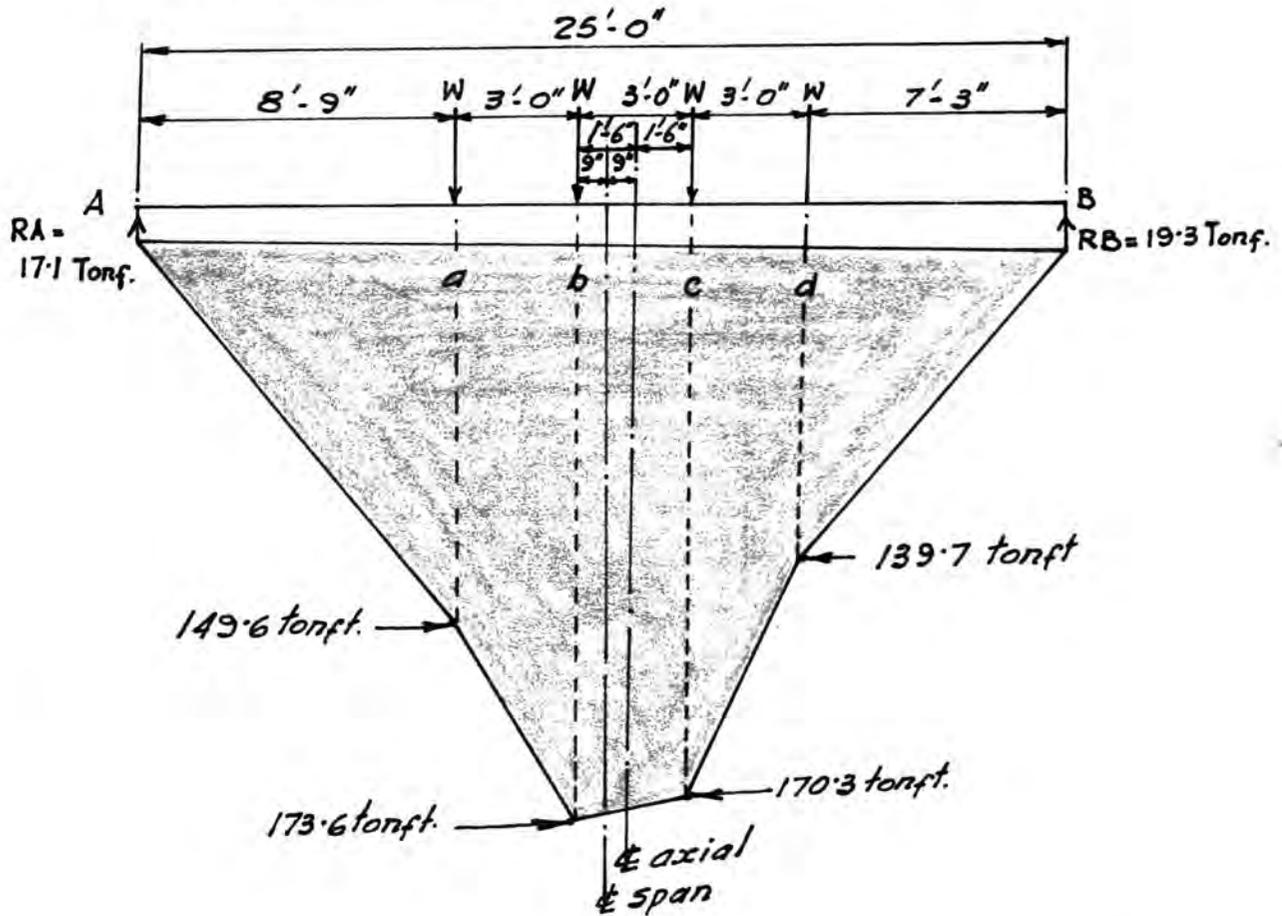
$$\text{@ c} = \text{---}$$

$$\text{@ d} = + \frac{0.161}{0.121} \times 0.04 = +0.053$$

Hence Σ coefficient, $R = 1.815$.

H.B. loading system: Position of wheels for maximum moments on floor beams (transverse).

$W = R \times 5 \text{ Tonf} = 9.1 \text{ tonf.}$

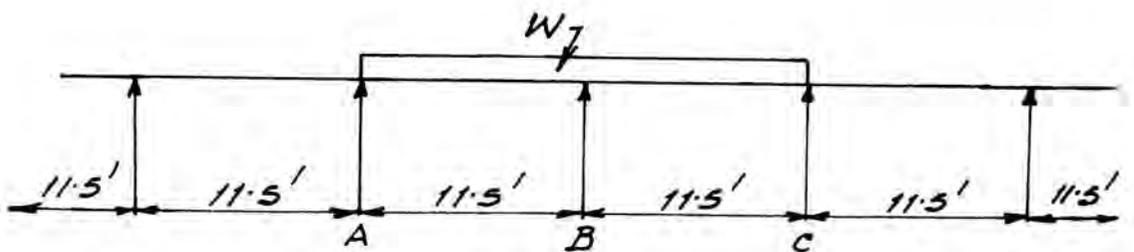
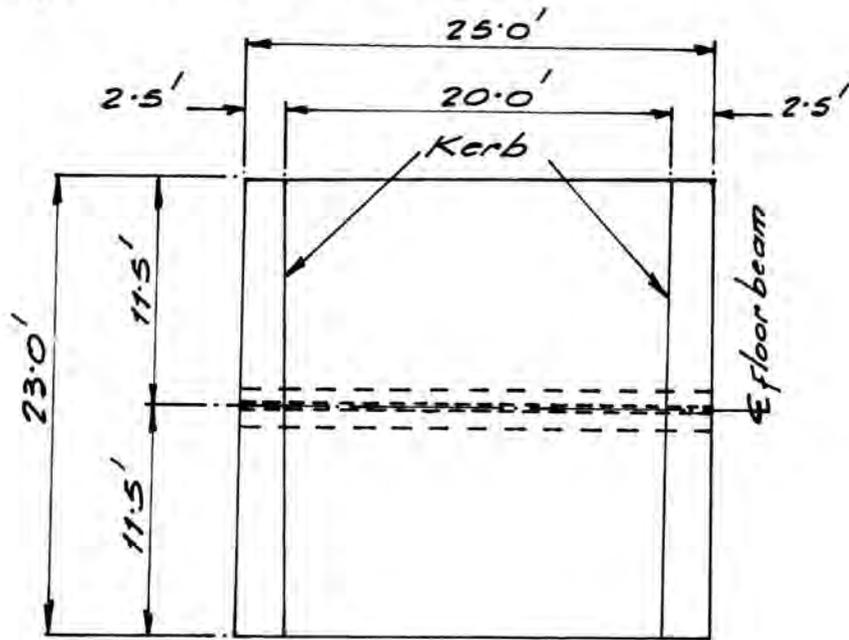


Maximum Shear at Connections - Position of Wheels

Hence max. shear force = 25.84 tonf.

B11

4.1. H.A. loading: Uniformly distributed lane loading; i.e., 10 ft. lane, load length 23.0 ft. = 2200 lbf/linear ft. of lane for cross girders.



$$U.D.L = \frac{2200 \times 23}{T} = 22.6 \text{ tonf over the span}$$

but due to the continuity of the span, a factor 1.143 to be taken in consideration to compute the load W .

$$\text{Hence } W = 1.143 \times 22.6 = 25.8 \text{ tonf.}$$

$$\therefore \text{H.A. U.D.L} = \frac{25.8}{20} = 1.3 \text{ tonf/ft.}$$

$$\& \text{ K.E} = \frac{2700}{T} = 1.2 \text{ tonf/ft.}$$

$$\therefore \text{Total load} = 2.5 \text{ tonf/ft.}$$

B 12

4.2. Floor beam with deck plate:

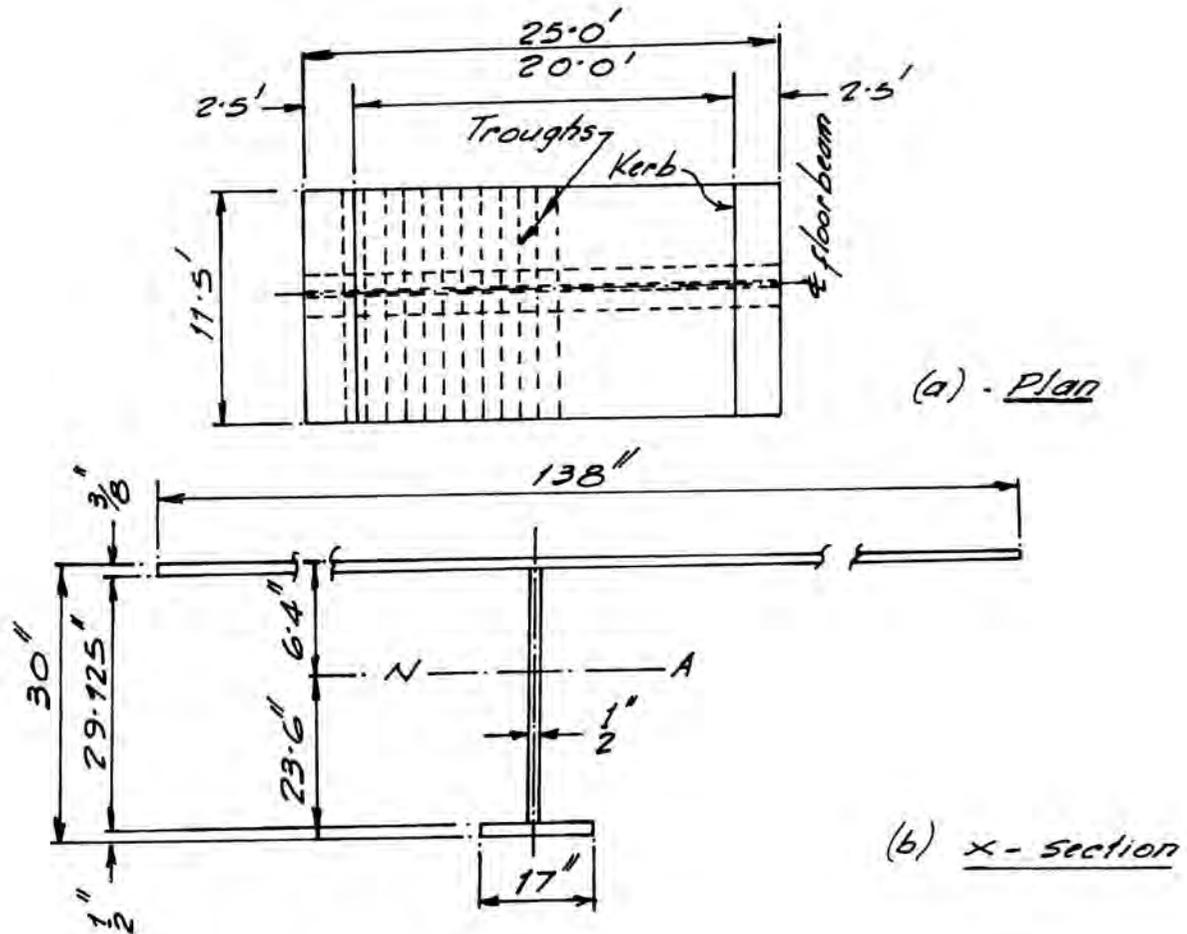


Fig. 4.2.1.

Properties:

areas:	top flange	$138 \times \frac{3}{8}$	=	51.75 in^2
	Web plate	$29.125 \times \frac{1}{2}$	=	14.56 in^2
	bottom flange	$17 \times \frac{1}{2}$	=	8.50 in^2
		<hr/>		
	Σarea		=	74.81 in^2

Moment about bottom flange:

$$74.81 \bar{y} = (8.5 \times 0.25) + (14.56 \times 15.06) + (51.75 \times 29.8125)$$

$$\text{or, } \bar{y} = 23.6''$$

2nd moment of area, I_{N-A} :

top flange	51.75×6.4^2	=	2120 in^4
Web plate	$0.5 \times \frac{(29.125)^3}{12} + 14.56 \times 8.54^2$	=	2091 in^4
bottom flange	8.5×23.35^2	=	4634 in^4
	<hr/>		
	ΣI_{N-A}	=	8845 in^4

B 13

Section moduli :

$$Z_{top} = 1382 \text{ in}^3 \text{ \& } Z_{bottom} = 375 \text{ in}^3$$

I_{y-y} :-

$$\text{top flange } \frac{0.375 (138)^3}{12} = 82127.3 \text{ in}^4$$

$$\text{bottom flange } \frac{0.5 (17)^3}{12} = 204.7 \text{ in}^4$$

Web plate ignored

$$\Sigma I_{y-y} = 82332 \text{ in}^4$$

$$r_y = \sqrt{\frac{I_{y-y}}{A}} = 33.3 \quad \therefore \frac{l}{r_y} = \frac{25 \times 12}{33.3} = 9.0$$

allowable bending stress :

$$C_s = \frac{170000}{9.0^2} \sqrt{1 + \frac{1}{20} \left(9.0 \times \frac{0.375}{29.125} \right)^2} > 136$$

$$\therefore p_{bc} = 10.0 \text{ tonf/in}^2$$

Loading on floor beam :

- Dead loads :
- i. Self weight $\approx 294.0 \text{ lb/ft}$
 - ii. asphalt = 482.0 "
 - iii. Snow = 34.5 "
 - iv. trough $\approx 200.0 \text{ "}$

$$\Sigma \text{dead load} = \frac{1010.5 \text{ lb/ft}}{7} = 0.45 \text{ tonf/ft.}$$

Moments :

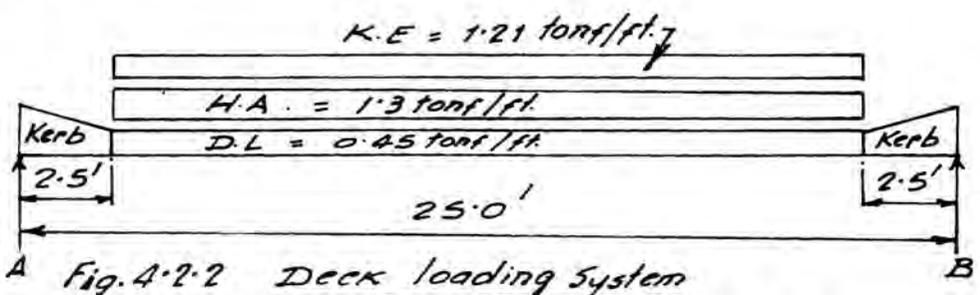


Fig. 4.2.2 Deck loading System

$$\Sigma \text{loads, } W = 2.96 \text{ tonf/ft.} \times 20.0' = 59.2 \text{ tonf.}$$

$$\text{Reaction at A} = \text{Reaction at B} = 29.6 \text{ tonf.}$$

B14

Assume, x_1 is the distance from A where shear force = 0

then, $x_1 = 2.5 + \frac{29.6 \times 20}{59.2} = 12.5 \text{ ft}$

\therefore Max. moment = $\frac{59.2}{20} \left(\frac{12.5^2 - 2.5^2}{2} \right) \times 12 = 2664 \text{ tonin.}$

Moment due to 20 unit H.B. loading system :

i. due to position of wheels only to get max. moment -

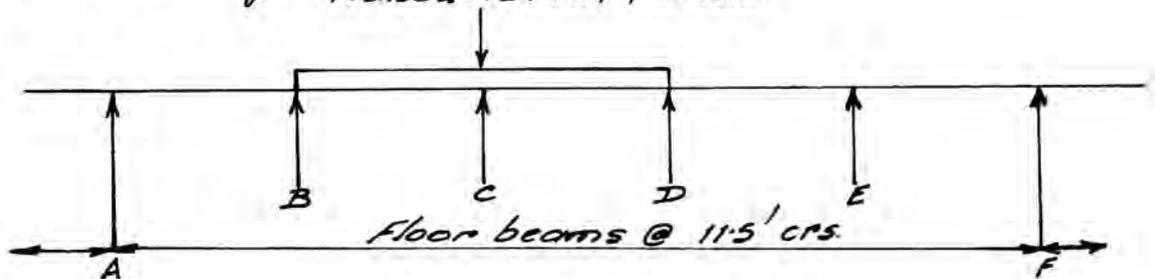
Max. moment = 2083 tonin

ii. due to dead load max. moment = $\frac{9}{20} \left(\frac{12.5^2 - 2.5^2}{2} \right) \times 12 = 405 \text{ tonin.}$

Hence Σ moment due to H.B. loading = 2488 tonin.

4.3 Check loadings for floor beams :

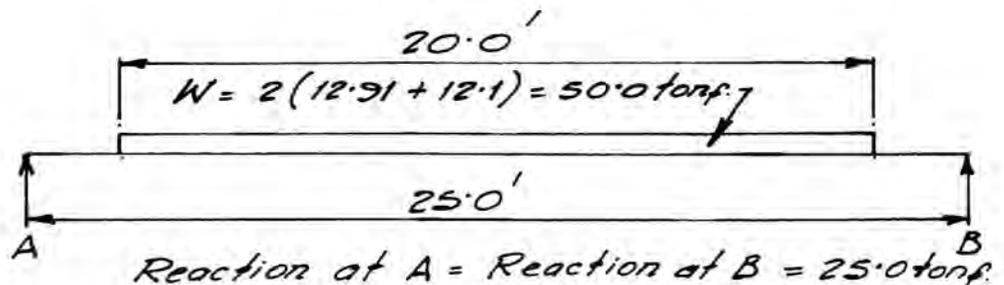
a. H.A. loading: K.E. load = 12.1 tonf per lane



Loaded length 23.0 ft. \therefore lane load/lin. ft. = 2200 lbf

\therefore Reaction at C is given by $\frac{2200}{7} (11.5 \times 1.143) = 12.91 \text{ tonf.}$

note 1.143 is the factor for the continuous span.

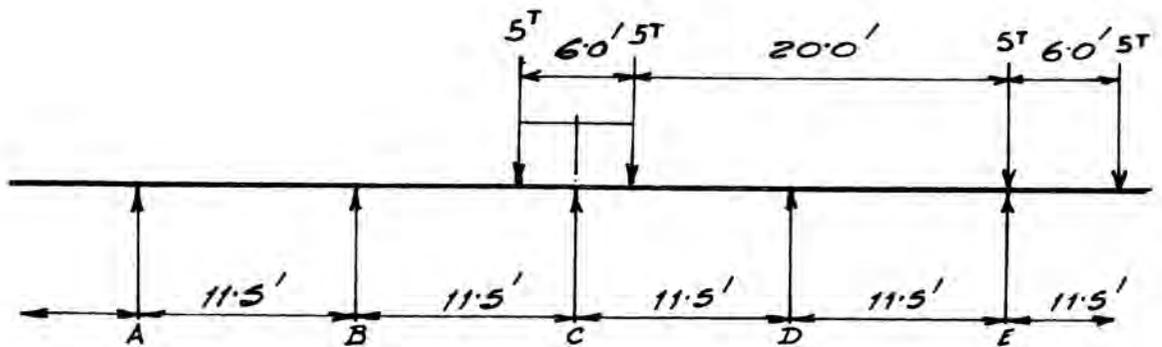


Reaction at A = Reaction at B = 25.0 tonf.

Mid-span bending moment = $25(12.5 - 5) \times 12 = 2251 \text{ tonin.}$

B 15

b. H.B loading:

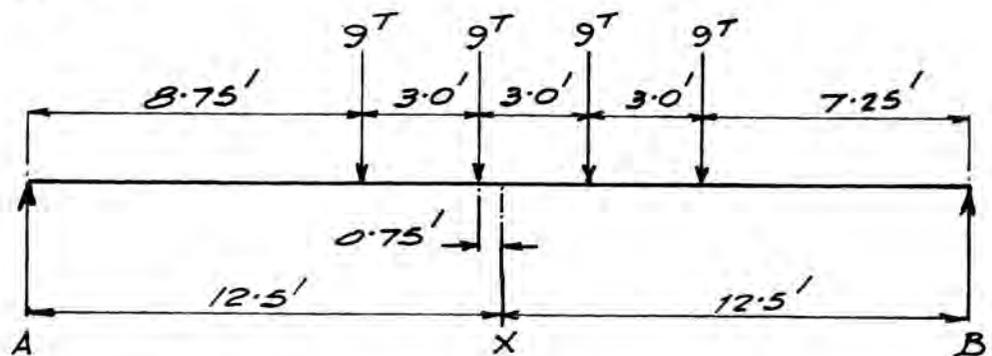


Position of H.B. load for Max. Reaction at C

Reaction at C = $5(0.8719 + 0.8719 + 0.0527) = 8.98 \text{ tonf.}$

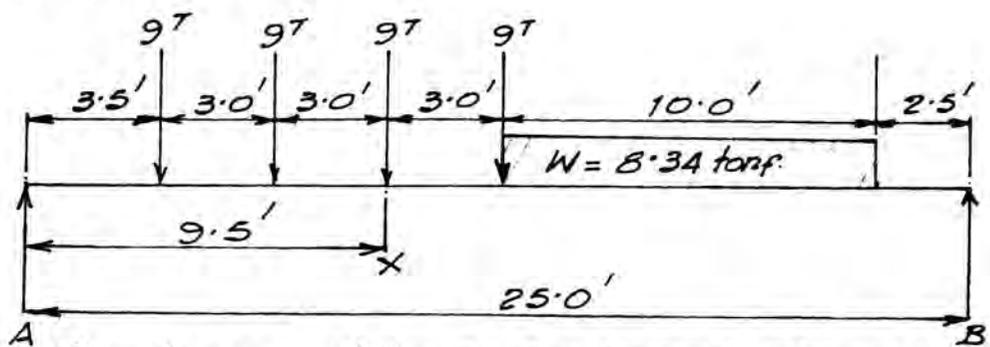
4.4. Check moments (transverse):

i. Central



Reaction at A = 16.92 tonf & Reaction at B = 19.08 tonf.
 Max. moment at X = $12(16.92 \times 11.75 - 9 \times 3) = 2062 \text{ ton in.}$

ii. In lane:

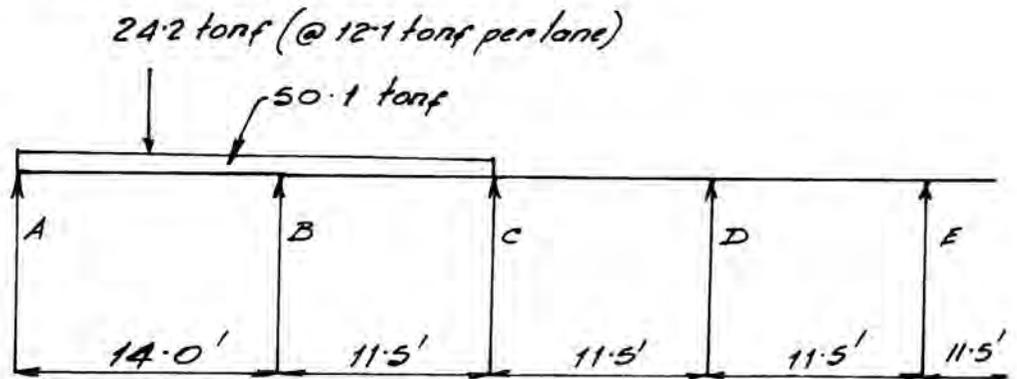


Reactions: at A = 26.98 tonf. & at B = 17.36 tonf.
 Max. moment at X = $12\{26.98 \times 9.5 - 9(6+3)\} = 2104 \text{ ton in.}$

B16

4.4. Floor beam local to end panel :

i. H. A. loading



loaded length 25.5 ft

load per ft. of lane = 2200 lbf/ft.

load on 2 lanes = $\frac{2200}{7} \times 2 \times 25.5 = 50.1$ tonf.

fixing moments

$$M_{BA} = \left(\frac{50.1 \times 14}{25.5} \right) \frac{14}{8} = + 48.1 \text{ tonft}$$

$$M_{BC} = - \left(\frac{50.1 \times 11.5}{25.5} \right) \times \frac{11.5}{12} = - 21.7 \text{ ''}$$

$$M_{CB} = + 21.7 \text{ ''}$$

Distribution factors:

DF_{BA} 0.38 ; DF_{BC} 0.62 ; DF_{CB} 0.5 ; DF_{CD} 0.5 , DF_{DC} 0.57 &

DF_{DE} 0.43.

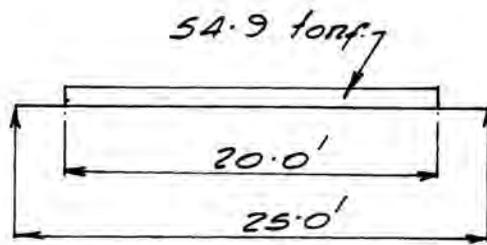
	0.38	0.62	0.5	0.5	0.57	0.43	
	+48.1	-21.7	+21.7				
	-10.0	-16.4	-10.9	-10.8			
		-5.4	-8.2		-5.4		
	+2.1	+3.3	+4.1	+4.1	+3.1	+2.3	
		+2.1	+1.7	+1.6	+2.1		
	-0.8	-1.3	-1.7	-1.6	-1.2	-0.9	
Final Moments	+39.4	-39.4	+6.7	-6.7	-1.4	+1.4	

Reaction at B:

$$\frac{50.1}{2} + \frac{39.4}{14} + \left(\frac{39.4 - 6.7}{11.5} \right) + 24.2 = 54.9 \text{ tonf.}$$

B17

ii. Bending on floor beam local to end panel :



Mid-span bending moment = $27.5(12.5-5) \times 12 = 2475$ ton in.

This includes H.A + KE loads.

Mid-span bending moment due to Dead load only

$$\left(\frac{0.45 \times 14}{11.5} \right) \times \left(\frac{25^2}{8} \right) 12 = 514 \text{ ton in.}$$

Hence Σ moment = 2989 ton in

Stresses :

From page B7, $Z_t = 840.3 \text{ in}^3$ & $Z_b = 356.5 \text{ in}^3$

$$\text{Hence } f_{bt} = \frac{2989}{840.3} = 3.6 \text{ tonf/in}^2$$

$$\text{and } f_{bc} = \frac{2989}{356.5} = 8.4 \text{ ''}$$

Thus the section is satisfactory.

4.5. Floor beam at rose bearing :

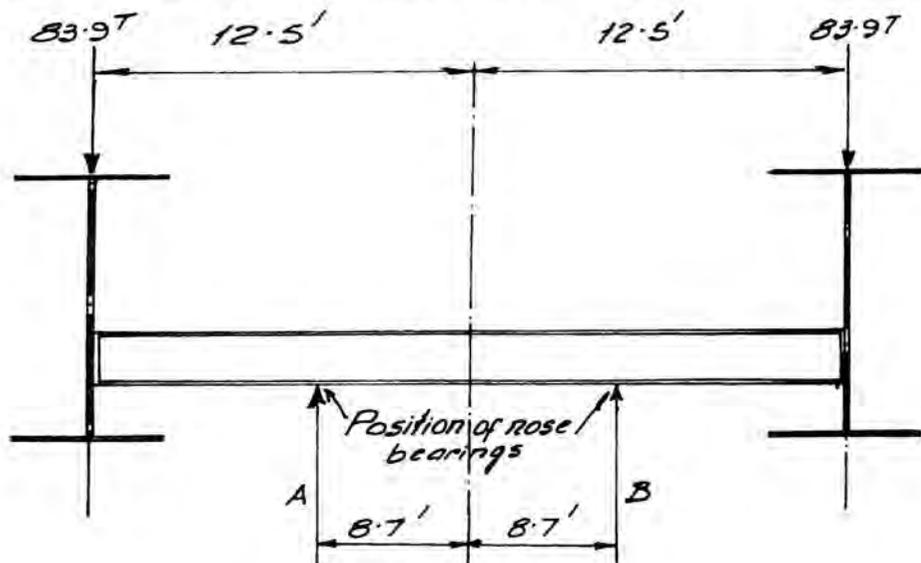


Fig. 4.5.1.

Section properties : refer to Fig 4.5.2.

Position of Neutral axis:

Member	area in ² , a	distance of Centroid y in	ay
Deck plate	$9 \times \frac{3}{8} = 34.9$	0.1875	6.5
Web "	$22 \frac{1}{8} \times \frac{3}{4} = 16.6$	11.44	189.9
bottom flange plate	$17 \times 1 \frac{1}{2} = 25.5$	23.25	592.9
Σ	77.0		789.3

$$\therefore \bar{y} = \frac{789.3}{77.0} = 10.3''$$

2nd moment of area - I_{N-A} :

Member	I	ay ²	I_{N-A}
Deck Plate		(34.9×9.7^2) = 3292.2	3292.2
Web Plate	$\frac{(22 \cdot 125^3 \times 0.75)}{12} = 676.9$	$16.6 \times 11.4^2 = 21.5$	698.4
Bottom flange Plate		25.5×12.95^2 = 4276.4	4276.4
Σ			8267.0

Position of y-y axis:

Member	area in ²	distance of Centroid x	ax
Deck Plate	$9 \times \frac{3}{8} = 34.9$	46.5	1622.9
Web "	$22 \frac{1}{8} \times \frac{3}{4} = 16.6$	12.0	199.2
Bottom flange Plate	$17 \times 1 \frac{1}{2} = 25.5$	12.0	306.0
Σ	77.0		2128.1

$$\therefore \bar{x} = \frac{2128.1}{77.0} = 27.6''$$

B20

2nd moment of area : I_{y-y}

Member	I	ax^2	I_{y-y}
Deck Plate	$\frac{1}{12}(93^3 \times 0.375) = 25136.2$	34.9×18.9^2 $= 12466.6$	37602.8
Web Plate		16.6×15.6^2 $= 4039.8$	4039.8
Bottom flange Plate	$\frac{1}{12}(73^3 \times 1.5) = 614.1$	25.5×15.6^2 $= 6205.7$	6819.8
Σ		25750.3	48462.4

$$r_y = \sqrt{\frac{48462.4}{77.0}} = 25.09''$$

Section moduli :

$$Z_{x-x} \text{ top of deck plate} = \frac{8267}{10.3} = 802.7 \text{ in}^3$$

$$Z_{x-x} \text{ bottom flange} = \frac{8267}{13.7} = 603.4 \text{ in}^3$$

$$\frac{l}{r_y} = \frac{8.7 \times 12}{25.09} = 4.16 \text{ restraint at mid-span of nose}$$

beam with stiffeners to under side of deck.

$$A = \frac{170000}{4.162} \sqrt{1 + \frac{1}{20} \left(\frac{4.16 \times 1.5}{24} \right)^2} = 9840$$

$$B = \frac{170000}{4.162} = 9823$$

Let I_T = 2nd moment of area, I_{y-y} of tension flange about its own axis.

I_c = 2nd moment of area, I_{y-y} of compression flange about its own axis.

$$M = \frac{I_c}{I_c + I_T} = \frac{614.1}{25750.3} = 0.024 \therefore K_2 = -1.0$$

$$\text{Now, } C_s = (A + K_2 B) \frac{y_c}{y_t} = (9840 - 1 \times 9823.4) \frac{13.7}{10.3} = 22.2 \text{ tonf/in}^2$$

$$\therefore p_{bc} = 6.6 \text{ tonf/in}^2 \ \& \ p_{bt} = 10.0 \text{ tonf/in}^2$$

$$\text{Hence } f_{bc} = \frac{3825.8}{603.4} = 6.34 \text{ tonf/in}^2 < p_{bc}$$

$$\& \ f_{bt} = \frac{3825.8}{802.7} = 4.77 \text{ tonf/in}^2 < p_{bt}$$

Therefore the section is adequate.

B21

Web Shear Stress:

$$f_q = \frac{83.9}{16.6} = 5.0 \text{ tonf/in}^2 < p_q = 6.0 \text{ tonf/in}^2 \therefore \text{okay}$$

Combined bending & shear stresses:

$$f_{bc} \text{ at edge of web plate} = \frac{6.34 \times 9.925}{10.3} = 6.11 \text{ tonf/in}^2$$

$$f_e = \sqrt{f_{bc}^2 + 3f_q^2} = \sqrt{6.11^2 + 5.0^2 \times 3} = 10.7 \text{ tonf/in}^2$$

Thus $f_e < p_e = 14.0 \text{ tonf/in}^2$ Hence satisfactory.

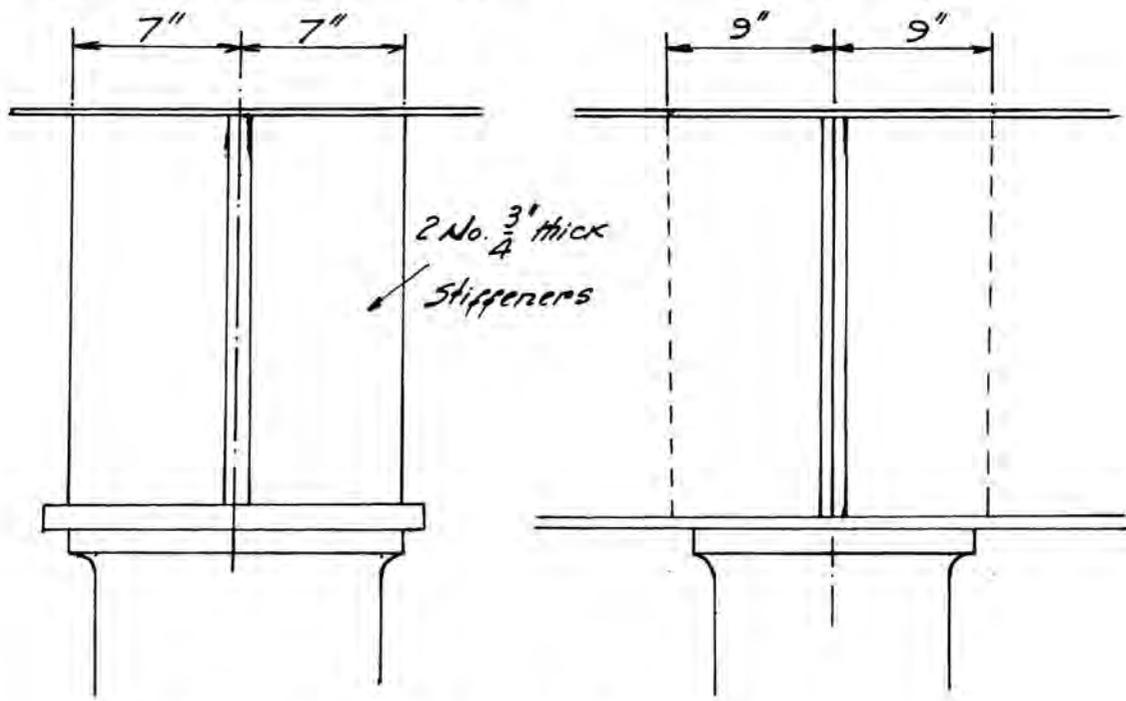


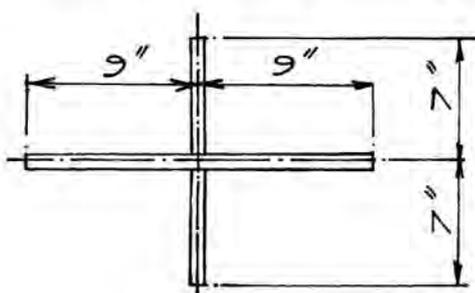
Fig. 4.5.3 Detail at Bearing

Stiffeners at bearing:

Min. required I value of stiffs. $\frac{D^3 T}{250} \cdot \frac{R}{W}$ From BS.153
Part 4 - clause
270 (iii)

$$= \frac{24^3 \times 1.125}{250} \cdot \frac{83.9}{167.8} = 31.1 \text{ in}^4$$

Provided I value = $\frac{14^3}{12} \cdot (0.75) = 228.7 \text{ in}^4$



Take 18" length of bearing acting with stiffs.:
areas: $(18.0 \times 0.75) + (14.0 \times 0.75) = 24 \text{ in}^2$

$$r_y = \sqrt{\frac{228.7}{24}} = 3.09 \text{ in}; \frac{l}{r_y} = \frac{22.5}{3.09} = 7.3$$

$$f_{ac} = \frac{83.9}{24} = 3.5 \text{ tonf/in}^2 > p_{ac} = 8.8 \text{ tonf/in}^2$$

Hence satisfactory.

Thus load on each stiff. = $\frac{83.9 \times 5.25}{24}$
= 18.4 tonf.

B22

Bearing stress on flange : $f_b = \frac{18.4}{6 \times 0.75} = 4.1 \text{ tonf/in}^2 < f_b = 12.0 \text{ tonf/in}^2$
 Therefore, use $\frac{3}{4}$ " thick stiffeners fitted with $\frac{1}{4}$ " L weld, Continuous.

Welding : Bottom flange to web : $F = \frac{5AY}{I} = 3.35 \text{ tonf/in.run}$

Capacity of 2 No. $\frac{3}{8}$ " L weld :

$$F = \frac{2 \times 0.375 \times 10.11}{8267} = 3.58 \text{ tonf/in.run}$$

Hence use $\frac{3}{8}$ " L weld at both sides of bottom flange to web.

Welding deck plate to web :

$$F = \frac{83.9 \times 34.9 \times 10.11}{8267} = 3.58 \text{ tonf/in.run.}$$

Hence $\frac{3}{8}$ " L weld at both sides of deck plate to web is adequate.

4.6. Connection of floor beam at nose bearing to main Girder :

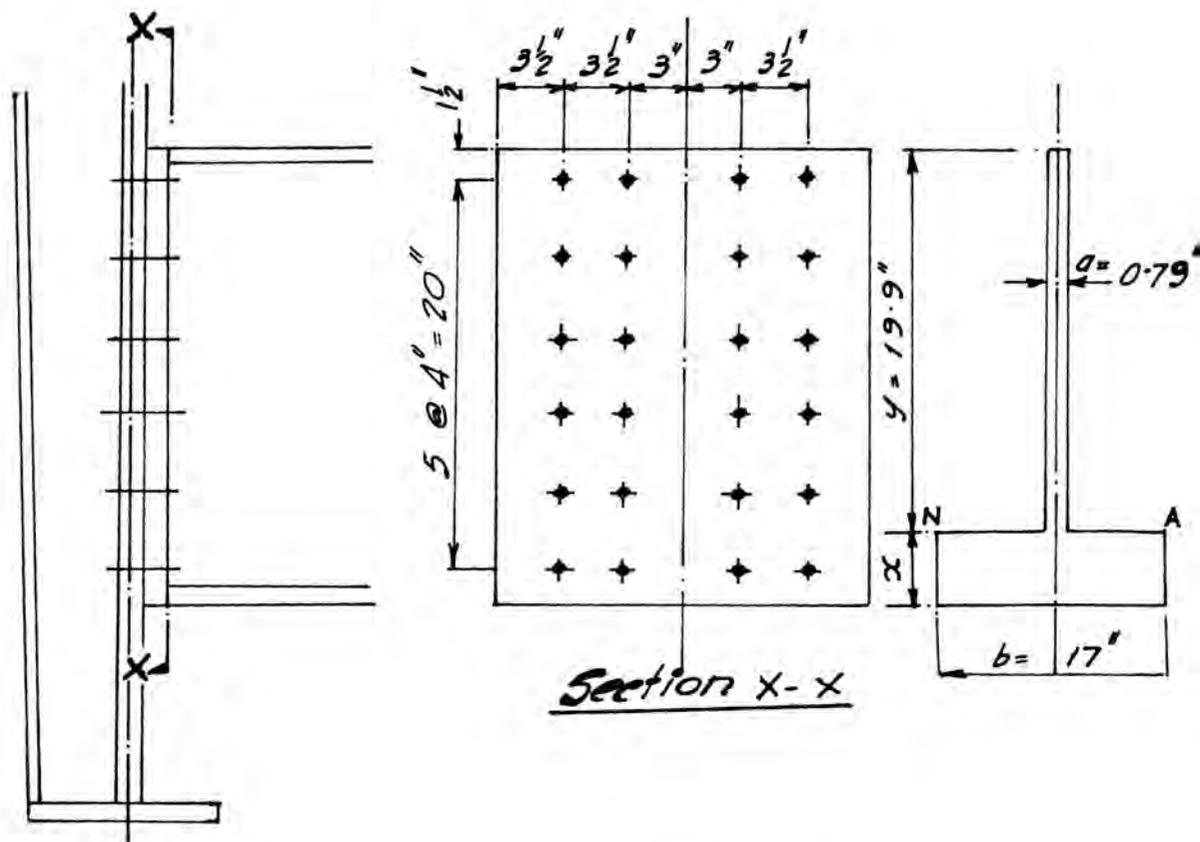


Fig. 4.6.1. Connection of floor beam to main girder.

Refer to Fig 4.6.1 :

area of 1" dia. H.S.F.G. bolts = 0.79 in^2

equivalent width of area in tension = $\frac{4 \times 0.79}{4} = 0.79'' = a$.

for equilibrium, $17 \times \frac{x^2}{2} = 0.79 \left(\frac{24}{2} - x \right)^2$

or, $x = 6.5$ or $4.13''$

distance of N.A, i.e., $y = 19.9''$

Ratio, $\frac{x}{y} = 0.206$ & $\sqrt{\frac{a}{b}} = 0.21$.

Hence satisfactory.

$$\begin{aligned} \downarrow N-A \left\{ \begin{aligned} &= \frac{0.79}{3} \cdot 19.9^3 = 2075.2 \text{ in}^4 \\ &= \frac{17 \times 4.13^3}{3} = 309.6 \text{ in}^4 \end{aligned} \right. \end{aligned}$$

Hence $\Sigma \downarrow N-A = 2465.8 \text{ in}^4$

Section moduli :

$$Z_c = \frac{2465.8}{4.13} = 601.4 \text{ in}^3$$

$$\& Z_t = \frac{2465.8}{19.9} = 123.9 \text{ in}^3$$

Stresses : Max. moment = 150.6 ton.in. & max. vertical load = 83.9 tonf.

Hence, Compressive stress, $f_c = \frac{150.6}{601.4} = 0.25 \text{ tonf./in}^2$

tensile stress, $f_t = \frac{150.6}{123.9} = 1.22 \text{ tonf./in}^2$

Tension of extreme bolt = area \times tensile stress

$$= \frac{6 \times 0.79}{4} \times \frac{1.22}{19.9} (19.9 - 3) = 1.2 \text{ tonf.}$$

actual shear on each bolt = $\frac{83.9}{20 \text{ No.}} = 4.195 \text{ tonf.}$

Slip factor = 0.45 ; load factor = 1.7

Shear capacity of each bolt = $\frac{0.45}{1.7} (21.09 - 1.7 \times 1.2) = 5.05 \text{ tonf.}$

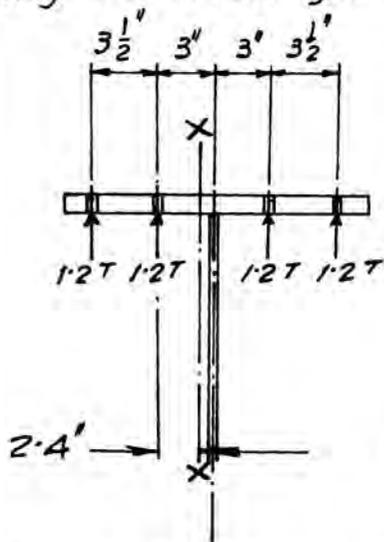
Hence 20 No. 1" dia. H.S.F.G. bolts are adequate.

Note:

The above method of computation has been obtained from

B. C. S. A. Publication No. 26 (1965).

4.7. End Plate : Connection of floor beam at rose bearing to main girder :



Moment due to cross bending

$$\text{on end plate} = 1.2(2.4 + 5.9) = 10 \text{ ton/in}$$

$$Z \text{ value of end plate} = \frac{1}{6} \times 1.25^2 = 1.04 \text{ in}^3$$

$$\therefore f_{bc} = \frac{M}{Z} = 9.6 \text{ tonf/in}^2 < p_{bc} = 10 \text{ tonf/in}^2$$

$$\text{Shear stress, } f_q = \frac{83.9 \times 0.5}{24 \times 1.25} = 1.4 \text{ tonf/in}^2$$

Which is less than $p_q = 6.0 \text{ tonf/in}^2$

Combined stresses: $f_e = \sqrt{9.6 + 3(1.4)^2} = 9.9 \text{ tonf/in}^2$, which is less than $p_e = 14.0 \text{ tonf/in}^2$.

Therefore $1\frac{1}{4}$ " thick end plate is adequate.

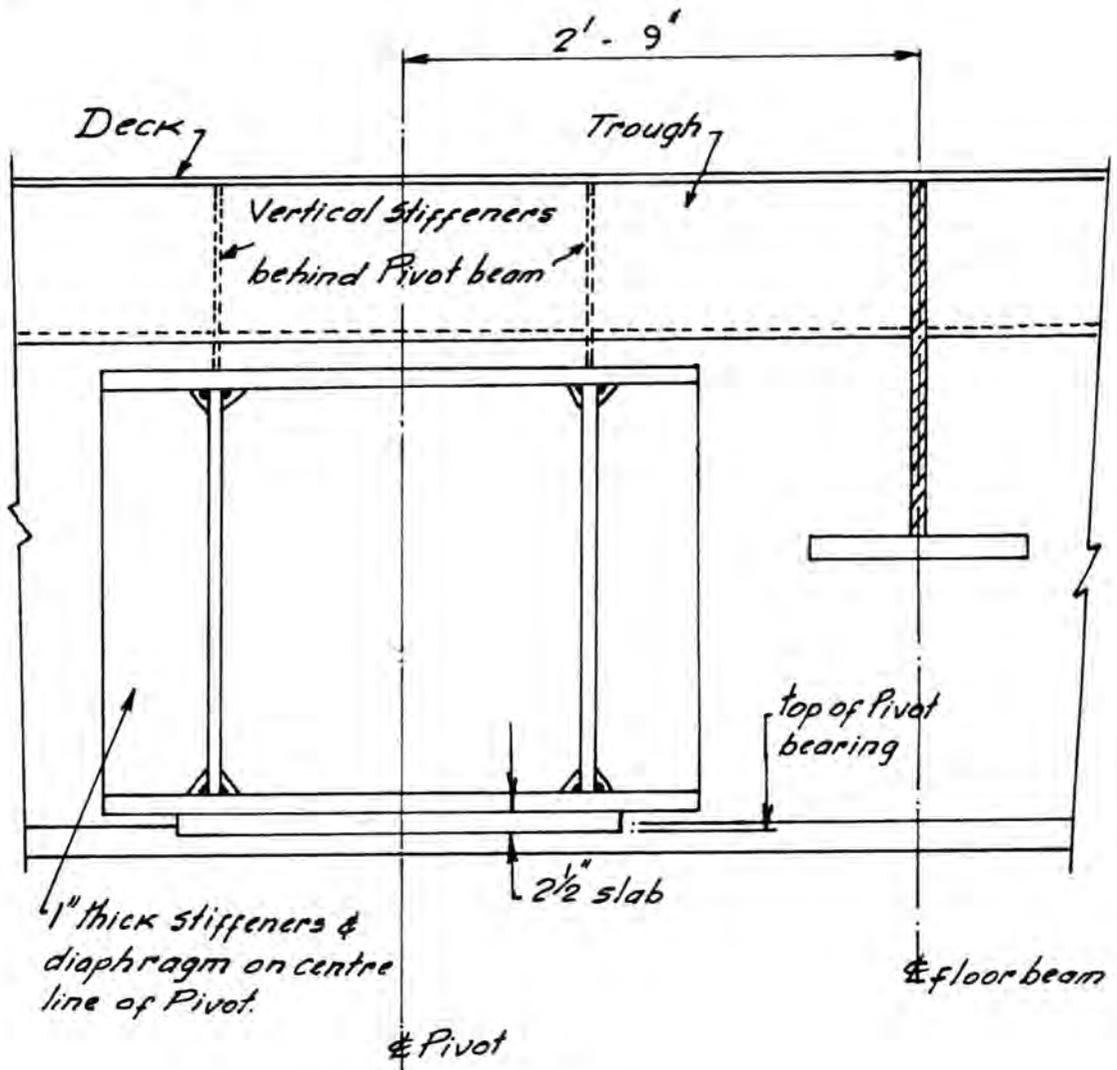
Check bearing stress:

$$\text{Compression on } 1'' \text{ strip of web plate} = 0.3 \times 17 = 5.1 \text{ tonf.}$$

$$\text{bearing stress, } f_b = \frac{5.1}{0.75 \times 1} = 6.8 \text{ tonf/in}^2 < p_b = 12.0 \text{ tonf/in}^2$$

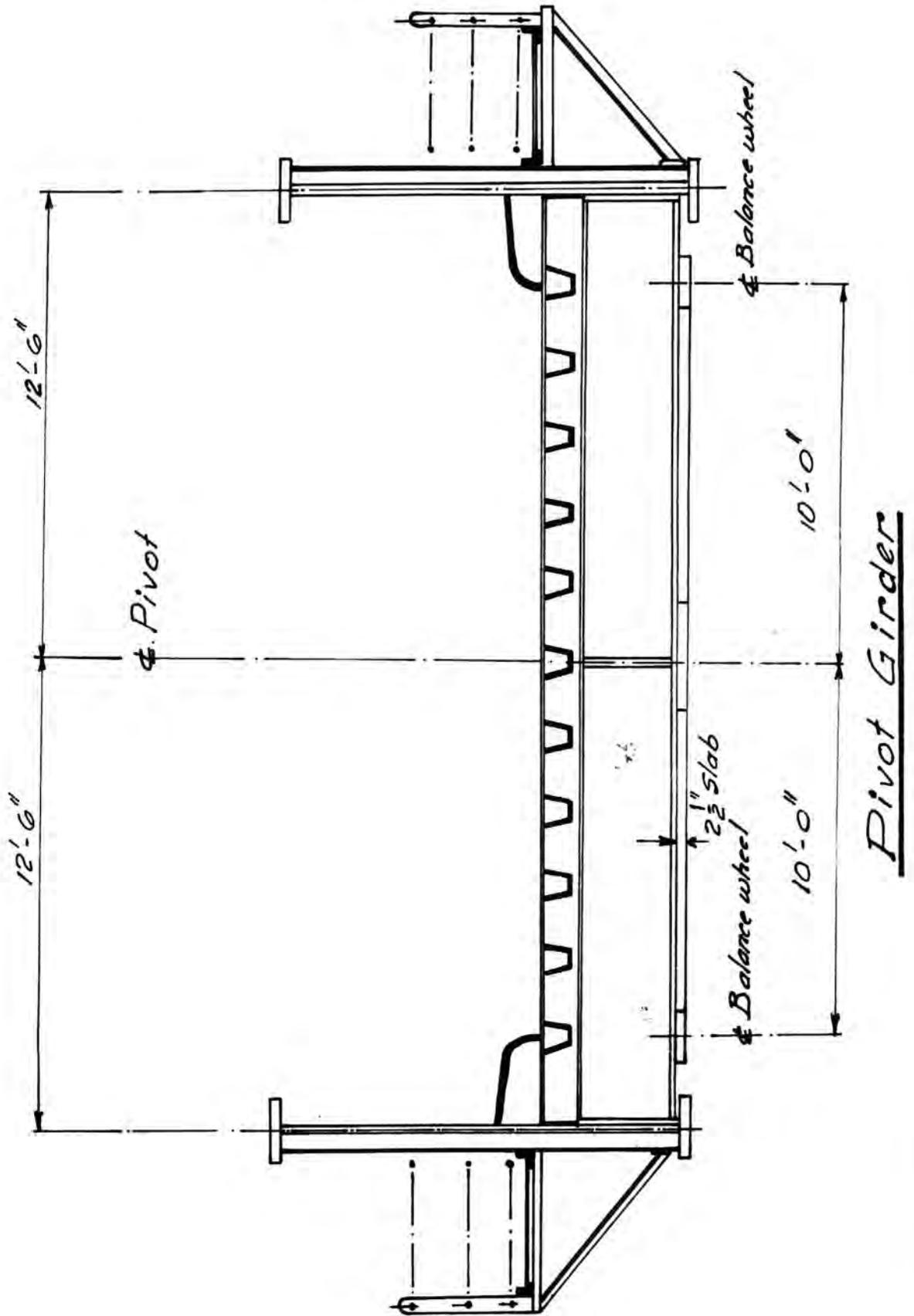
Hence satisfactory.

↓
floor beam web
plate thickness

5. Pivot GirderArrangement at Pivot

Note: Holes for Pivot are tapped holes in $2\frac{1}{2}"$ slab.
 Holes at balance wheels are similar detail
 tapped in an inch slab.

B 26



5.1. Pivot Girder Section:

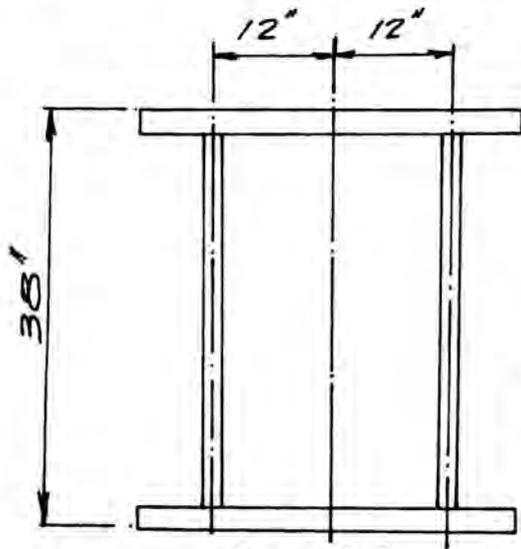


Fig. 5.1.1.

Pivot load = 448.5 tonf.

Moment on girder =

$$\frac{448.5}{4} \times 25.0 = 2803 \text{ tonft.}$$

Try a 36" deep section with
 2 No. 48" x 2" thick flange plates &
 2 No. 34" x $\frac{3}{4}$ " thick web plates.

Properties of the section:

$$I_{x-x} : \text{flange plates} - (48 \times 2) 18^2 \times 2 = 62208 \text{ in}^4$$

$$\text{Web plates} - \frac{2 \times 0.75 \times 34^3}{12} = 4914 \text{ in}^4$$

$$\Sigma I_{x-x} = 67122 \text{ in}^4$$

$$I_{y-y} : \text{flange plates} - 2 \times \frac{2}{12} \times 48^3 = 36864 \text{ in}^4$$

$$\text{Web} \quad \text{"} - (0.75 \times 34) \times 12^2 \times 2 = 7344 \text{ in}^4$$

$$\Sigma I_{y-y} = 44208 \text{ in}^4$$

$$\text{areas} : \text{flange plates} - 2 \times 48 \times 2 = 192 \text{ in}^2$$

$$\text{Web} \quad \text{"} - (34 \times 0.75) \times 2 = 51 \text{ in}^2$$

$$\Sigma \text{areas} = 243 \text{ in}^2$$

$$r_y = \sqrt{\frac{44208}{243}} = 13.5'' ; I_{x-x} = \frac{67122}{19} = 3533 \text{ in}^3$$

$$l/r_y = \frac{25.0 \times 12}{13.5} = 22.2 ; \frac{D}{T} = 19 \therefore p_{bc} = 10.0 \text{ tonf/in}^2 = p_{bt}$$

$$\text{Stresses} : f_{bc} = \frac{2803 \times 12}{3533} = 9.52 \text{ tonf/in}^2$$

$$\text{Shear stress, } f_q = \frac{224.3}{51} = 4.4 \text{ tonf/in}^2$$

$$f_{bc} \text{ at the edge of the web} = \frac{9.52 \times 17}{19} = 8.52 \text{ tonf/in}^2$$

$$\text{Combined stresses, } f_e = \sqrt{8.52^2 + 3 \times 4.4^2} = 11.5 \text{ tonf/in}^2 < p_e = 14.0 \text{ tonf/in}^2$$

Hence the section is satisfactory.

B28

Welds: flange to web.

$$F = \frac{5AY}{I} = \frac{224.3 \times 96 \times 18}{67122} = 5.8 \text{ tonf/in run.}$$

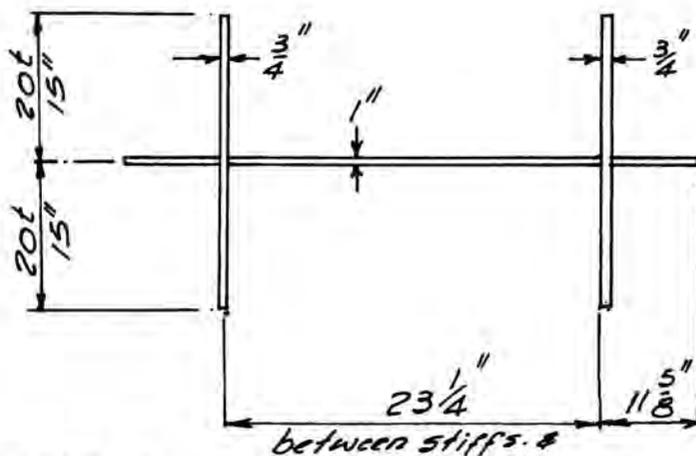
Shear per inch run per web = 2.9 tonf

Capacity of $2 / \frac{5}{16}$ Δ weld:

$$2 \times \frac{0.3125}{\sqrt{2}} \times 1.0 \times 7.0 = 3.1 \text{ tonf/in run}$$

Hence use $\frac{5}{16}$ Δ weld continuous on both sides of each web.

Bearing stiffeners:



areas :

$$23 \frac{1}{4} \times 1'' + 2 \times 11 \frac{5}{8} \times 1'' = 46.50 \text{ in}^2$$

$$30'' \times \frac{3}{4}'' \times 2 = 45.00 \text{ in}^2$$

$$\Sigma \text{ areas} = 91.50 \text{ in}^2$$

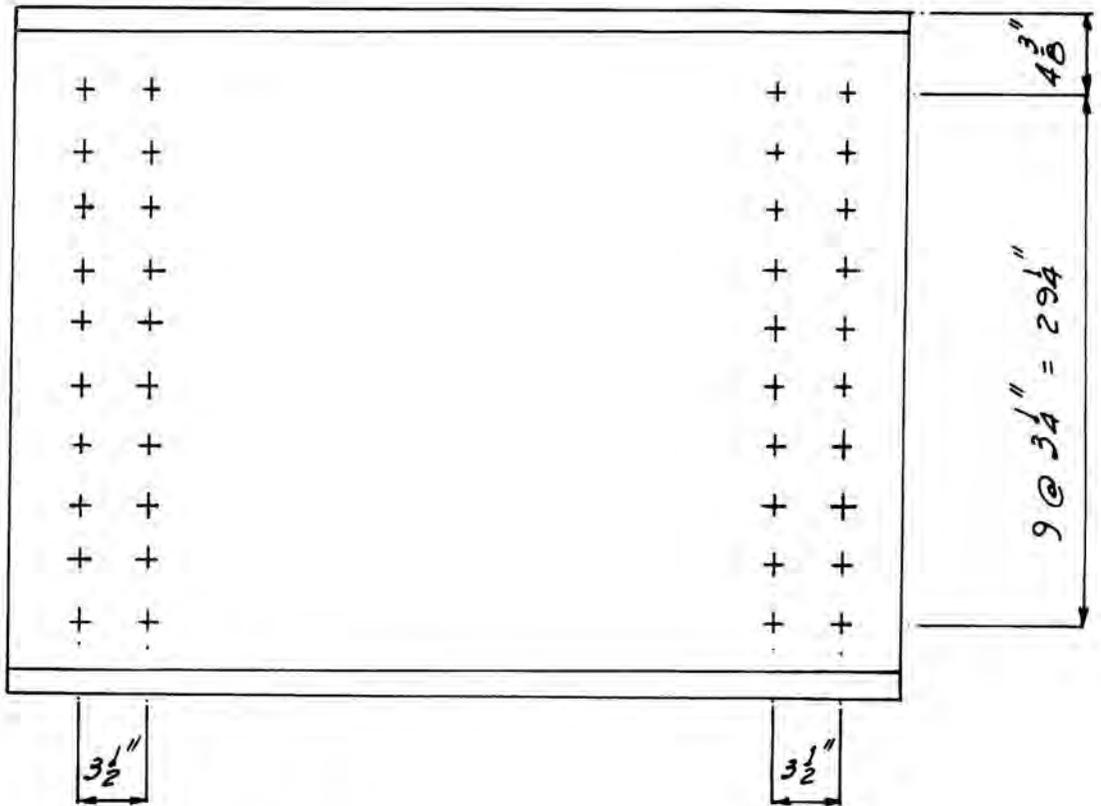
$$f_{ac} = \frac{\text{load}}{\text{area}} = \frac{448.5}{91.5} = 4.9 \text{ tonf/in}^2 < p_{ac} = 10.0 \text{ tonf/in}^2$$

Hence this arrangement is satisfactory.

Use $\frac{5}{16}$ Δ weld continuous to web only.

Appendix C

1. Connection to main girders:



Reaction 224.3 tonf.

assuming 40 No. bolts req'd. \therefore load per bolt = 5.61 tonf.

Capacity of 1" dia H.S.F.G. bolts (Proof load 23.0 tonf):

Slip factor = 0.45 ; load factor = 1.7

\therefore Capacity = $\frac{0.45}{1.7} (23.0 \times 1) = 6.08$ tonf, each bolt.

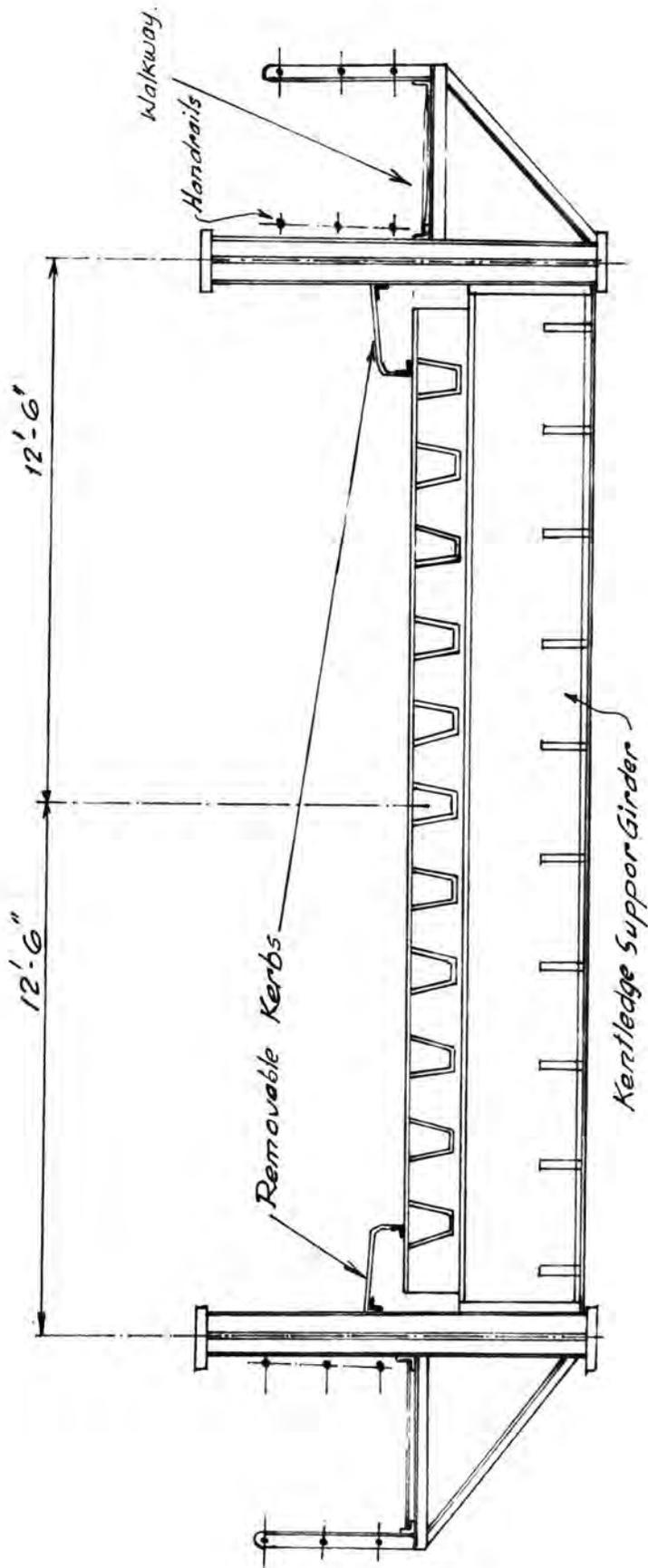
Hence 40 No. 1" dia H.S.F.G. bolts are adequate.

End Plate: Shear per web = $\frac{1}{2} \cdot 224.3 = 112.2$ tonf.

Try 1" thick plate: Shear stress, $f_q = \frac{112.2}{38" \times 1} = 3.0$ tonf/in²

Which is less than $f_q = 6.0$ tonf/in²

So, use 1" thick plate with $\frac{5}{16}$ " Δ Weld continuous.



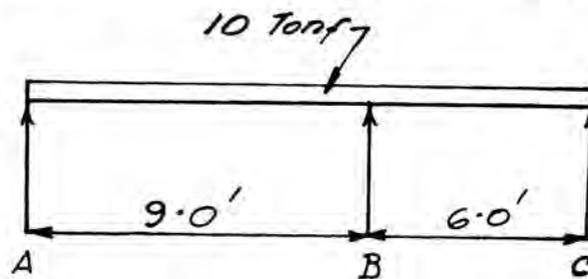
Section At Slip Bearing

C3

2. Deck over Kentledge box :

Loading over loose panel

i. H. A. load - loaded length 15.0 ft



Distribution factors: $DF_{BA} = 0.38$ & $DF_{BC} = 0.62$

Fixing moments: $M_{BA} = \frac{10}{15} \times \frac{9^2}{8} = 6.75 \text{ tonf}$

& $M_{BC} = \frac{10}{15} \times \frac{6^2}{8} = 3.00 \text{ "}$

	0.38	0.62
	+6.75	-3.00
	-1.43	-2.32
final moments	+5.32	-5.32

Reactions: at A $3.0 - \frac{5.32}{9} = 2.4 \text{ tonf}$

at B $5.0 + \frac{5.32}{9} + \frac{5.32}{6} = 6.5 \text{ tonf.}$

at C $2.0 - \frac{5.32}{6} = 1.1 \text{ tonf.}$

Since loaded length is 15 ft., load length per foot = 3880 lb/ft.

load on panel = $\frac{3880}{7} \times 15.0 = 26.0 \text{ tonf.}$

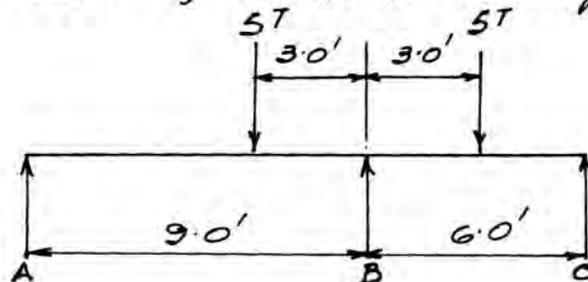
load on cross girder = $26 \times 0.65 = 16.9 \text{ tonf.}$

Hence load on cross girder due to HA 16.9 tonf.

due to KE 24.2 "

∴ load = 41.1 tonf.

ii. H. B. load local to girder at Tail bearing:



C4

Distribution factors: $DF_{BA} = 0.38$ & $DF_{BC} = 0.62$

fixing moments:

$$M_{BA} = \frac{5.0 \times 3.0 \times 6.0 (2 \times 6.0 + 3.0)}{2 \times 9.0^2} = 8.33 \text{ tonft.}$$

$$M_{BC} = \frac{3 \times 5.0 \times 6.0}{16} = 5.63 \text{ tonft.}$$

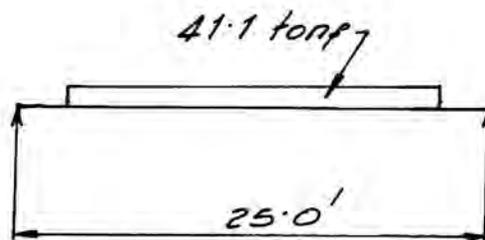
0.38	0.62
+8.33	-5.63
-1.03	-1.67
+7.30	-7.30

final moments.

Reaction at B $3.33 + 3.0 + \frac{7.30}{9.0} + \frac{7.30}{6.0} = 8.35 \text{ tonf.}$

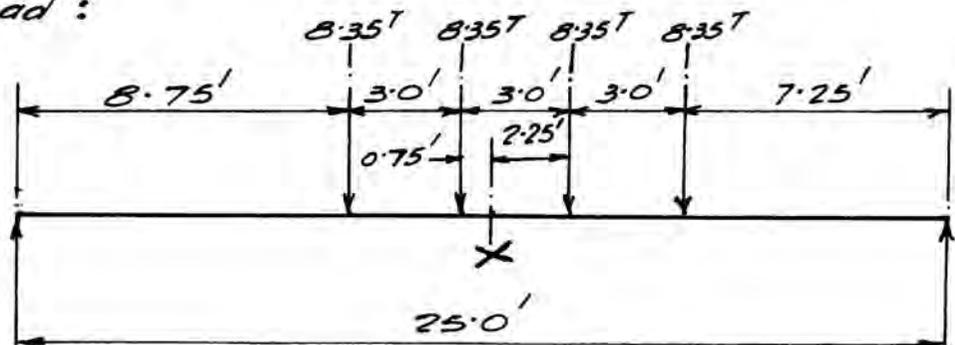
2.1. Floor beam at Tail Bearing:

H.A. load:



$$\text{Moment} = 20.55 (12.5 - 5.0) \times 12 = 1849.5 \text{ tonin.}$$

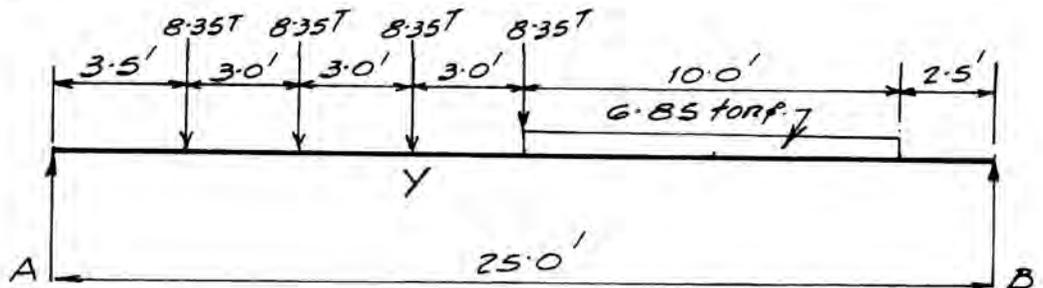
H.B. load:



$$\text{Moment at X} = 2062 \times \frac{8.35}{9} = 1913.1 \text{ tonin.}$$

C5

H.B + $\frac{1}{3}$ H.A load



$$\frac{1}{3}(H.A + KE) \text{ loads} = 6.85 \text{ tonf.}$$

Reactions : at A = at B = 24.8 tonf.

$$\text{moment at Y} = \{24.8 \times 9.5 - 8.35(6+3)\} 12 = 1925.4 \text{ tonin.}$$

$$\text{Kerfledge load on beam} = \frac{242}{15} \times 7.5 = 121 \text{ tonf.}$$

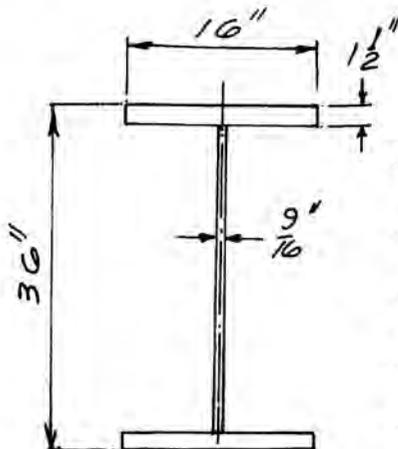
$$\text{Dead load due to deck etc.} = 0.45 \text{ tonf/ft.} \times \frac{25}{11.5} \times 7.5 = 7.3 \text{ tonf.}$$

moments :

$$\text{due to dead load} = \frac{128.3}{8} \times 25 \times 12 = 4810.8 \text{ tonin}$$

$$\text{due to H.A. load} = 1849.5 \text{ ,,}$$

$$\Sigma \text{moments} = 6650.3 \text{ tonin.}$$



Sectional Properties:

$$\Sigma \text{area} = 66.56 \text{ in}^2$$

$$\Sigma I_{x-x} = 15967.5 \text{ in}^4$$

$$\Sigma I_{y-y} = 1024 \text{ in}^4$$

$$r_y = \sqrt{\frac{1024}{66.56}} = 3.92 \text{ in}$$

$$Z_{x-x} = \frac{15967.5}{16 \times 1.5} = 887.1 \text{ in}^3$$

$$\frac{l}{r_y} = \frac{25 \times 12}{3.92} = 76.5; \quad \frac{D}{T} = \frac{36}{1.5} = 24$$

$$\therefore C_s = 37.7 \text{ tonf/in}^2 \quad \therefore p_{bc} = 8.4 \text{ tonf/in}^2$$

$$\text{Hence, } f_{bc} = \frac{M}{Z} = 7.5 \text{ tonf/in}^2 < p_{bc} = 8.4 \text{ tonf/in}^2$$

$$\text{Shear : Reaction} = 60.5 + 3.65 + 20.55 = 84.7 \text{ tonf.}$$

$$\therefore f_q = \frac{84.7}{18.56} = 4.56 \text{ tonf/in}^2 < p_q = 6.0 \text{ tonf/in}^2$$

Combined stresses : bending stress at the edge of web plate

$$f_{bc} = 8.4 \times \frac{16.5}{18} = 7.7 \text{ tonf/in}^2 \quad \therefore f_e = \sqrt{7.7^2 + 3(4.56)^2} = 11.0 \text{ tonf/in}^2$$

Which is less than $p_e = 14.0 \text{ tonf/in}^2$. Hence Satisfactory.

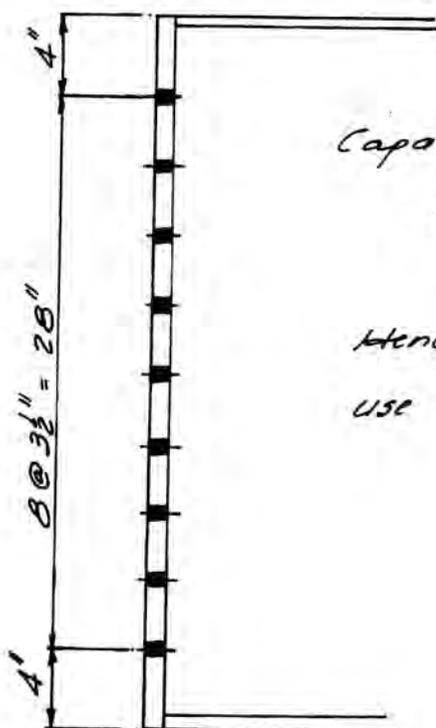
C6

Flange to web welding:

$$F = \frac{84.7 \times 24 \times 17.25}{15967.5} = 2.2 \text{ tonf/in run.}$$

therefore $\frac{5}{16}$ " weld continuous will be adequate.

Connection to main girder:



Reaction 84.7 tonf

$$\text{Shear per bolt if 18 No. used} = \frac{84.7}{18} = 4.71 \text{ tonf}$$

Capacity of 1" dia. H.S.F.G. bolt, (each)

$$\frac{0.45}{1.7} \times 23.0 = 6.1 \text{ tonf}$$

Hence, 18 No. bolts (H.S.F.G) are adequate.

use 1" thick end plate as before.

2.2. Floor beam at front of kentledge box:

$$\text{H.A. \& K.E. loads} = (1.21 + 1.3) \times 20.0 = 50.2 \text{ tonf.}$$

$$\text{Kentledge load} = \frac{242 \times 4.5}{15} = 72.6 \text{ "}$$

$$\text{Dead load} = 0.45 \times 250 = 11.3 \text{ "}$$

$$\text{Total load} = 134.1 \text{ tonf}$$

$$\text{moment} = 134.1 \times \frac{25}{8} \times 12 = 5030 \text{ tonin}$$

use the same section as stated in para. 2.1 in page C5.

2.3. Floor beam at tail jacks:

H.A. load on 6.0ft. length @ 9660 lbf per foot.

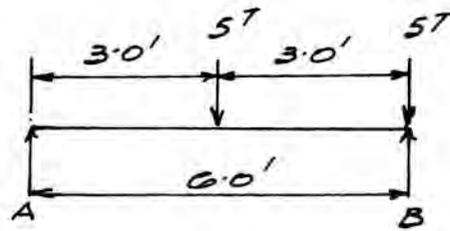
$$\therefore \text{load on the beam} = \frac{9660}{7} \times \frac{6}{2} = 12.9 \text{ tonf.}$$

$$\text{K.E. load} = 24.2 \text{ "}$$

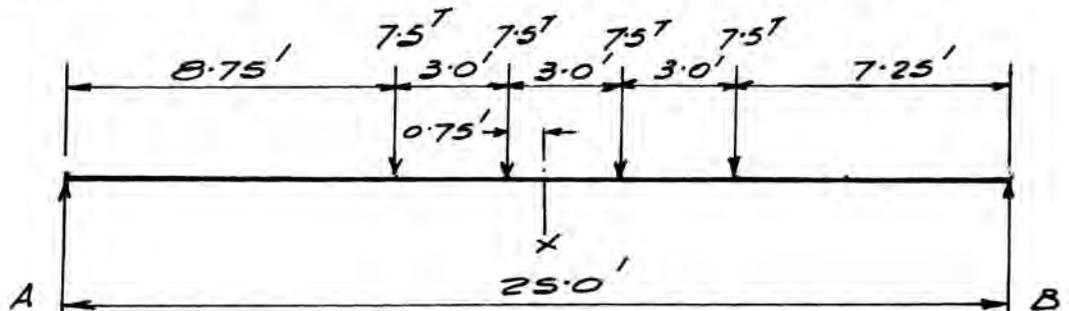
$$\text{Total load} = 37.1 \text{ tonf}$$

C7

H.B loading System:

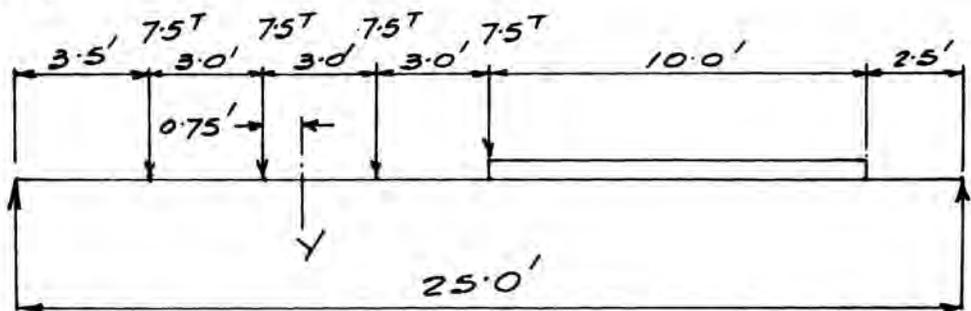


Reactions: at A = 2.5 tonf & at B = 7.5 tonf.



$$\text{moment at } x = \frac{2062}{9} \times 7.5 = 1718.3 \text{ tonin.}$$

H.B. + $\frac{1}{3}$ H.A. loads:



$$\text{moment at } Y = \{ 23.1 \times 9.5 - 7.5(6+3) \} 12 = 1823.4 \text{ tonin.}$$

$$\text{Kerledge load on beam} = \frac{242 \times 3}{15} = 48.4 \text{ tonf}$$

$$\text{Dead " " " " } = 0.45 \times \frac{3}{11.5} \times 25 = 3.0 \text{ "}$$

$$\underline{\text{Load} = 51.4 \text{ tonf.}}$$

$$\text{Moment} = \frac{51.4}{8} \times 25 \times 12 = 1927.5 \text{ tonin.}$$

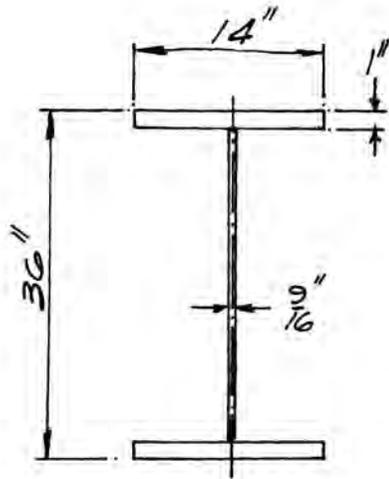
as calculated above, moment at Y, allows for 25% impact;

where as 40% impact must be allowed for tail floor beam;

$$\text{so, moment} = \frac{1823.4}{1.25} \times 1.4 = 2042.2 \text{ tonin.}$$

$$\text{Hence } \underline{\text{moments}} = (2042.2 + 1927.5) = 3969.7 \text{ tonin.}$$

CB



Sectional Properties:

$$Area, a = 46.56 \text{ in}^2$$

$$I_{x-x} = 10259.5 \text{ in}^4$$

$$I_{y-y} = 457.3 \text{ in}^4$$

$$r_y = \sqrt{\frac{I_{y-y}}{a}} = 3.13''$$

$$Z_{x-x} = 570 \text{ in}^3; \frac{I}{r_y} = 95.8; \frac{D}{r} = 36$$

$$C_s = 21.4 \text{ tonf/in}^2 \therefore p_{bc} = 6.5 \text{ tonf/in}^2 \text{ plus } 25\% \text{ for H.B. load}$$

Stresses:

$$\therefore p_{bc} = 8.13 \text{ tonf/in}^2$$

$$f_{bc} = \frac{M}{Z} = 6.96 \text{ tonf/in}^2 < p_{bc} = 8.13 \text{ tonf/in}^2$$

$$\text{Shear: Reaction} = 23.1 + \frac{51.4}{2} = 48.8 \text{ tonf}$$

$$\therefore \text{Shear stress, } f_q = \frac{48.8}{18.56} = 2.63 \text{ tonf/in}^2 < p_q = 6.0 \text{ tonf/in}^2$$

Welding, flange to web:

$$F = \frac{48.8 \times 14}{10259.5} \times 17.5 = 1.16 \text{ tonf/in. run.}$$

use $\frac{1}{4}$ " Δ weld continuous at both sides.

Stresses :

$$f_{bc} = \frac{16.4 \times 12}{26.7} = 7.4 \text{ tonf/in}^2 < p_{bc} = 10 \text{ tonf/in}^2$$

$$f_{bt} = \frac{16.4 \times 12}{122} = 1.6 \text{ tonf/in}^2 < p_{bt} = 10 \text{ tonf/in}^2$$

Shear stress in stiffeners, $f_q = \frac{7.32}{9} = 0.8 \text{ tonf/in}^2$

Check, plate spanning between stiffeners :

load on 10 ft. strip of plate = $0.65 \times 2.5 = 1.63 \text{ tonf}$.

moment = $\frac{wl}{10} = \frac{1.63 \times 2.5}{10} \times 12 = 5.0 \text{ tonin}$.

Z value of 1.0 ft. strip of plate = $\frac{0.75^2 \times 12}{6} = 1.125 \text{ in}^3$.

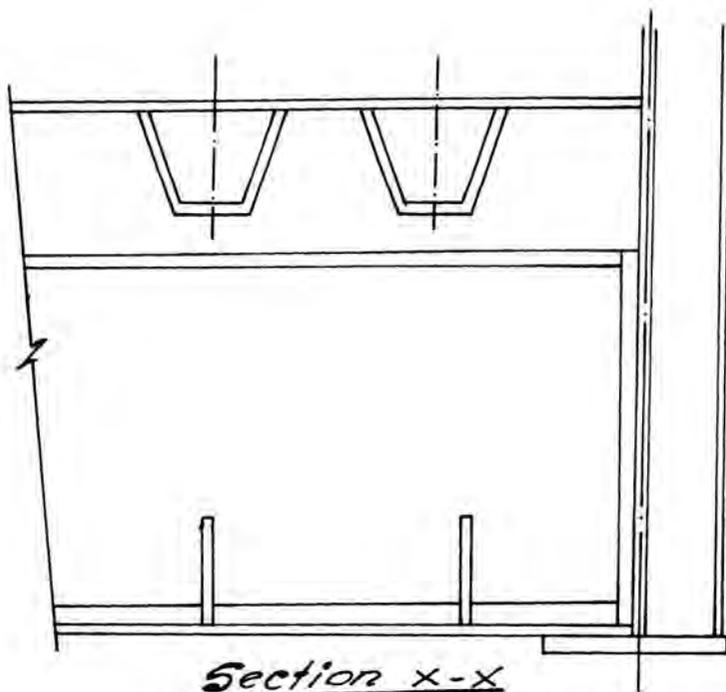
$\therefore f_{bc} = \frac{5.0}{1.125} = 4.4 \text{ tonf/in}^2$

Hence satisfactory.

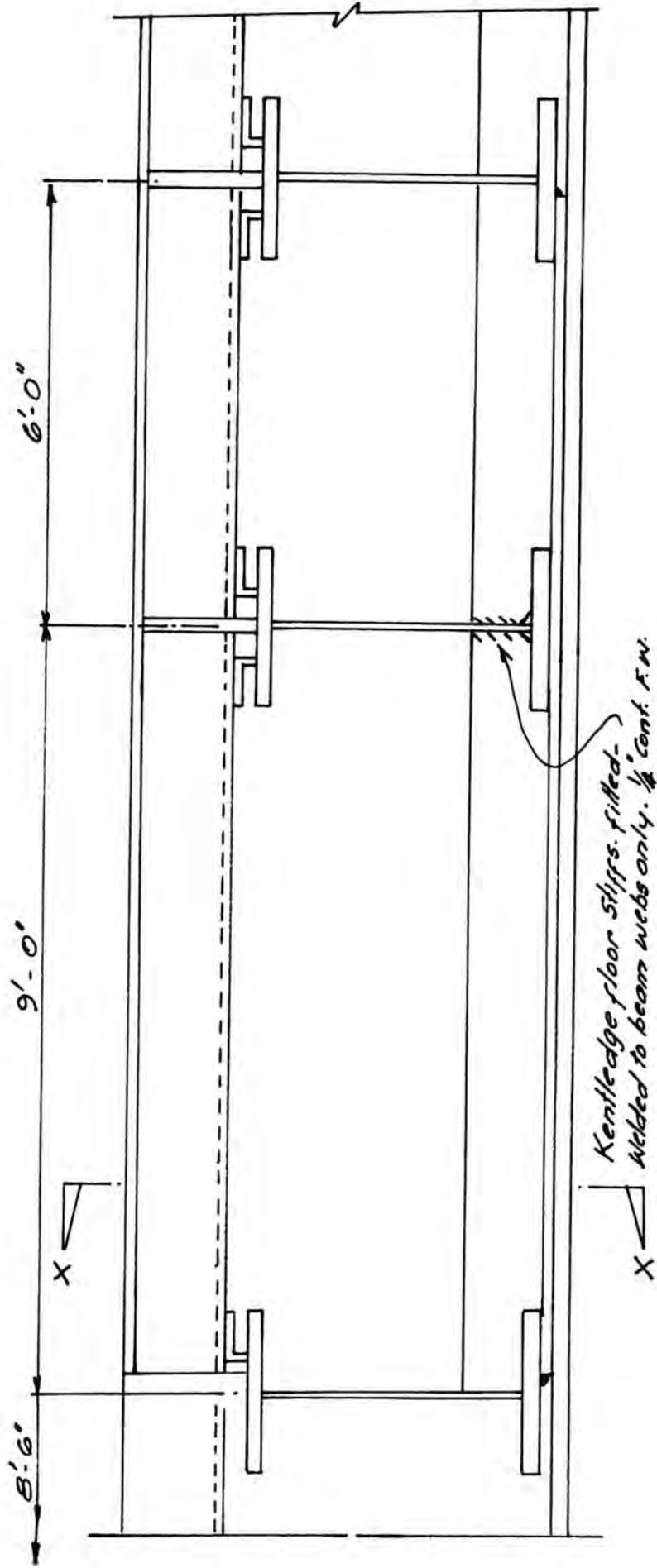
Welding stiffener to floor plate :

$$F = \frac{7.32 \times 22.5}{213.6} \times 1.375 = 1.06 \text{ tonf/in. max.}$$

Use $\frac{1}{4}$ " Δ weld continuous at both sides of stiffeners.

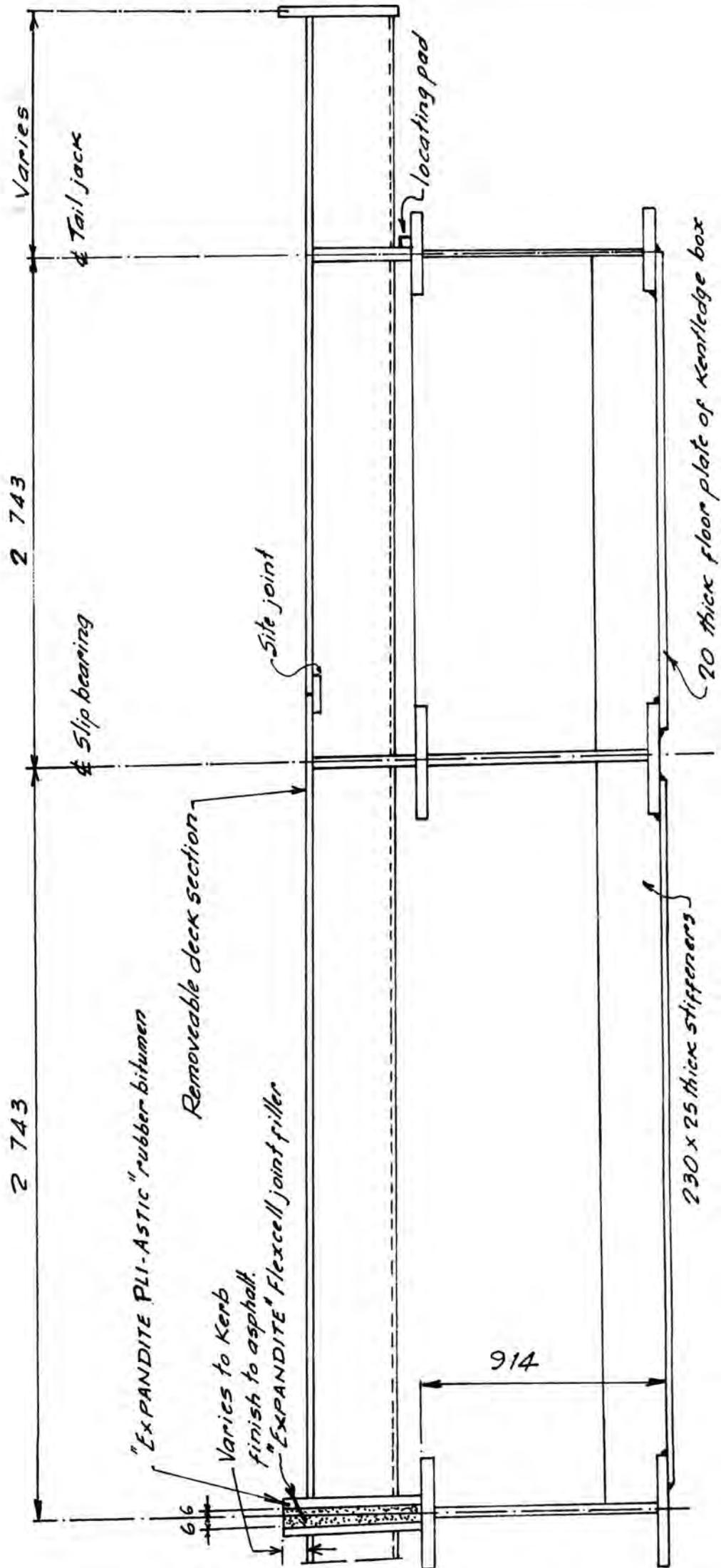


See Fig. 3.1 on page C12



Arrangement of Kentledge Box

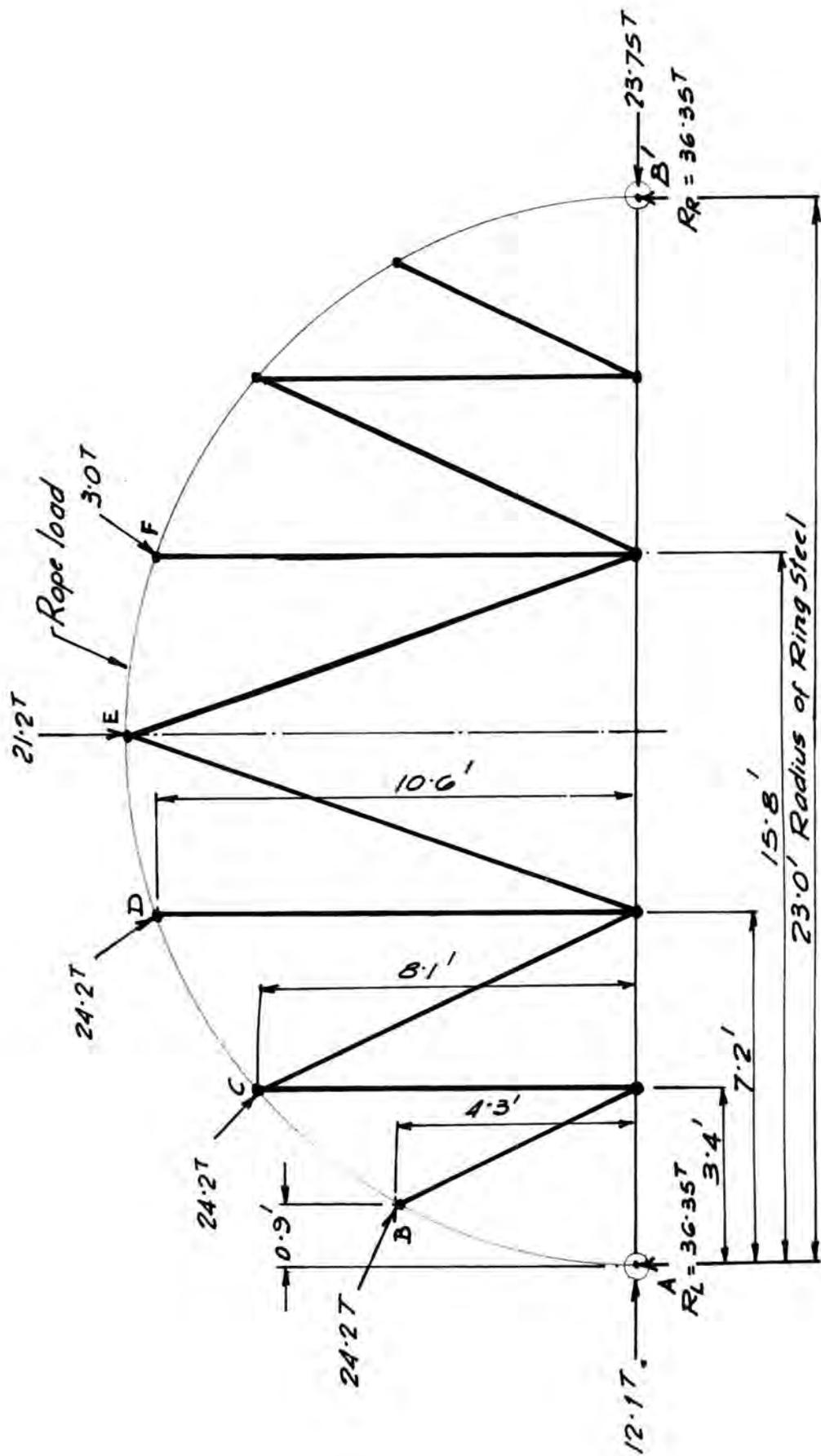
Fig 3.1.



Kerledge Box

Fig. 3.2

4. Slewing Gear Support Steel



Arrangement of Slewing Gear Support Steel

Fig. A.1.

Refer to Fig. 4.1 & Fig. 4.2 on pages C14 & 15
axial tension in rope, $S = 61.5$ tonf.

Radius, r , from pivot centre line to rope = 12 ft.

\therefore load acting normal to circumference of ring,

$$\frac{S}{r} = \frac{61.5}{12} = 5.13 \text{ tonf per foot run}$$

Circumference of 12 ft. radius rope = 75.4 ft.

\therefore U.D.L. on segment = $75.4 \times \frac{22.5^\circ}{360^\circ} \times 5.13 = 24.2$ tonf.

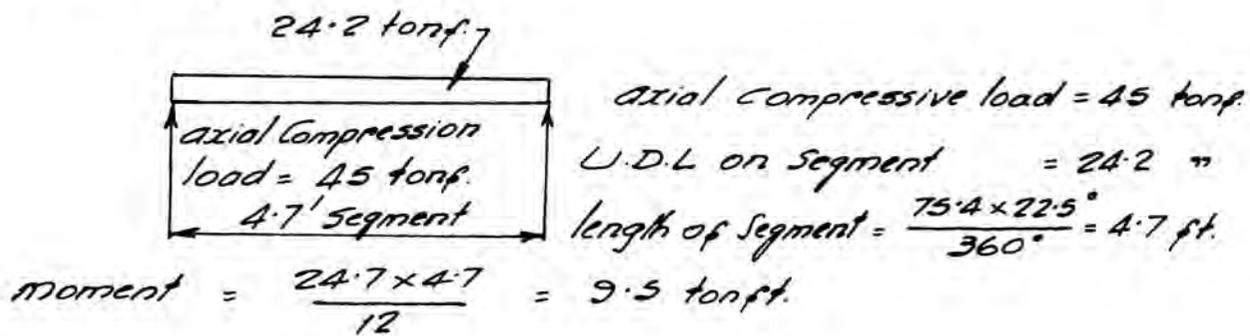
Referring to fig. 4.1, take moments about R_L to compute vertical reaction R_R :

Load Point	Vertical Component in tonf.	Horizontal Component in tonf.	Lever arm in feet	Vert./Horz. \times lever arm
A	-	-	-	
B	9.3	22.3	0.9 4.3	8.3 95.9
C	17.1	17.1	3.4 8.1	58.1 138.5
D	22.3	9.3	7.2 10.6	160.6 98.6
E	21.2		11.5	243.8
F	2.8	-1.2	15.8 10.6	44.2 -12.7
Total	72.7	47.5		835.3

$\therefore R_R = \frac{835.3}{23.0} = 36.35$ tonf & Similarly $R_L = 36.35$ tonf.

\therefore Horizontal reaction H_R Must be equal to H_L .

5. Ring Girder :



Try 2 No. $7'' \times 3\frac{1}{2}'' \times 18^\#$ R.S.C. I (ie, back to back) :

Properties: $\frac{l}{r} = \frac{4.7 \times 12 \times 0.85}{1.7} = 28.2$; $\frac{D}{T} = 13.2$

$p_{bc} = 10.0 \text{ tonf/in}^2$ & $p_{ac} = 8.6 \text{ tonf/in}^2$

Stresses : $f_{ac} = \frac{45}{10.29} = 4.4 \text{ tonf/in}^2$ & $f_{bc} = \frac{9.5 \times 12}{24.1} = 4.73 \text{ tonf/in}^2$

Ratio: $\frac{4.4}{8.6} + \frac{4.73}{10.0} = 0.98 < 1.0 \therefore$ satisfactory.

Note that the ring restrained at Inter-points.

Refer to Fig. 4.2 on page C15 : members m-e & n-e

loads $\left\{ \begin{array}{l} + 30.7 \text{ tonf.} \\ - 23.0 \text{ " } \end{array} \right\}$ reversible

member length = $\sqrt{11.5^2 + 4.3^2} = 12.3 \text{ ft.}$

Try 2 No. $6'' \times 3\frac{1}{2}'' \times \frac{1}{2}''$ I^S .

Properties: $\frac{l}{r} = \frac{12.3 \times 0.85 \times 12}{1.45} = 86.5 \therefore p_{ac} = 5.8 \text{ tonf/in}^2$

Stress, $f_{ac} = \frac{30.7}{8.99} = 3.4 \text{ tonf/in}^2 < p_{ac} = 5.8 \text{ tonf/in}^2$

Hence satisfactory.

members n-c & m-g :

axial load = $+15.1 \text{ tonf}$ & or -23.0 tonf .

length of member = 8.95 ft.

Try 2 No. $3'' \times 3'' \times \frac{3}{8}''$ I^S .

Properties: $\frac{l}{r} = \frac{8.95 \times 12 \times 0.85}{0.9} = 101.4 \therefore p_{ac} = 4.9 \text{ tonf/in}^2$

stress, $f_{ac} = \frac{15.1}{4.17} = 3.62 \text{ tonf/in}^2 < p_{ac} = 4.9 \text{ tonf/in}^2 \therefore$ okay.

axial tension, $f_{at} = \frac{23.0}{4.17} = 5.5 \text{ Tonf/in}^2$. Hence satisfactory.

members d-r & f-m:

axial load + 7.5 tonf. length = 10.6 ft.

Try 2 No. $3'' \times 3'' \times \frac{3}{8}'' \text{ JL}^S$

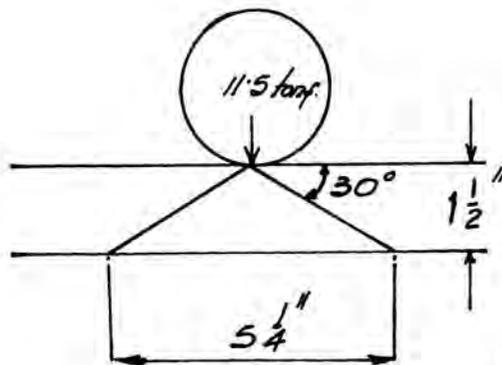
Properties: $\frac{I}{r} = \frac{10.6 \times 12 \times 0.85}{0.9} = 120 \therefore f_{ac} = 4.0 \text{ tonf/in}^2$

Stress, $f_{ac} = \frac{7.5}{4.17} = 1.8 \text{ tonf/in}^2 < f_{ac} \therefore \text{satisfactory}$

Hence, the remaining internal members are same as the member d-r. So, use 2 No. $3'' \times 3'' \times \frac{3}{8}'' \text{ JL}^S$ and fix gussets between the legs.

Calculations for the connections of the slewing gear support steelwork have been provided for detailing purposes, but it is felt that those minute details are irrelevant to include in this thesis.

6. Roller Track:



Let angle of dispersion = 30°

and the thickness of the plate = $1\frac{1}{2}''$

\therefore the effective length = $5\frac{1}{4}''$

assume the width of the plate is $5''$

\therefore area of the plate = $5 \times 5\frac{1}{4} = 26.25 \text{ in}^2$

$$Z = \frac{5 \times 1.5^2}{6} = 1.875 \text{ in}^3$$

assuming the system is simply supported

and a point load of 11.5 tonf is at the centre.

$$\therefore \text{Moment} = \frac{11.5 \times 5.25}{4} = 15.1 \text{ ton in.}$$

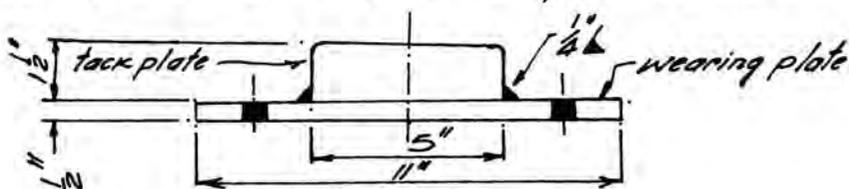
$$\therefore f_{bc} = \frac{M}{Z} = 8.0 \text{ tonf/in}^2 \text{ Hence satisfactory.}$$

Check thickness of the plate for bearing on concrete slab:

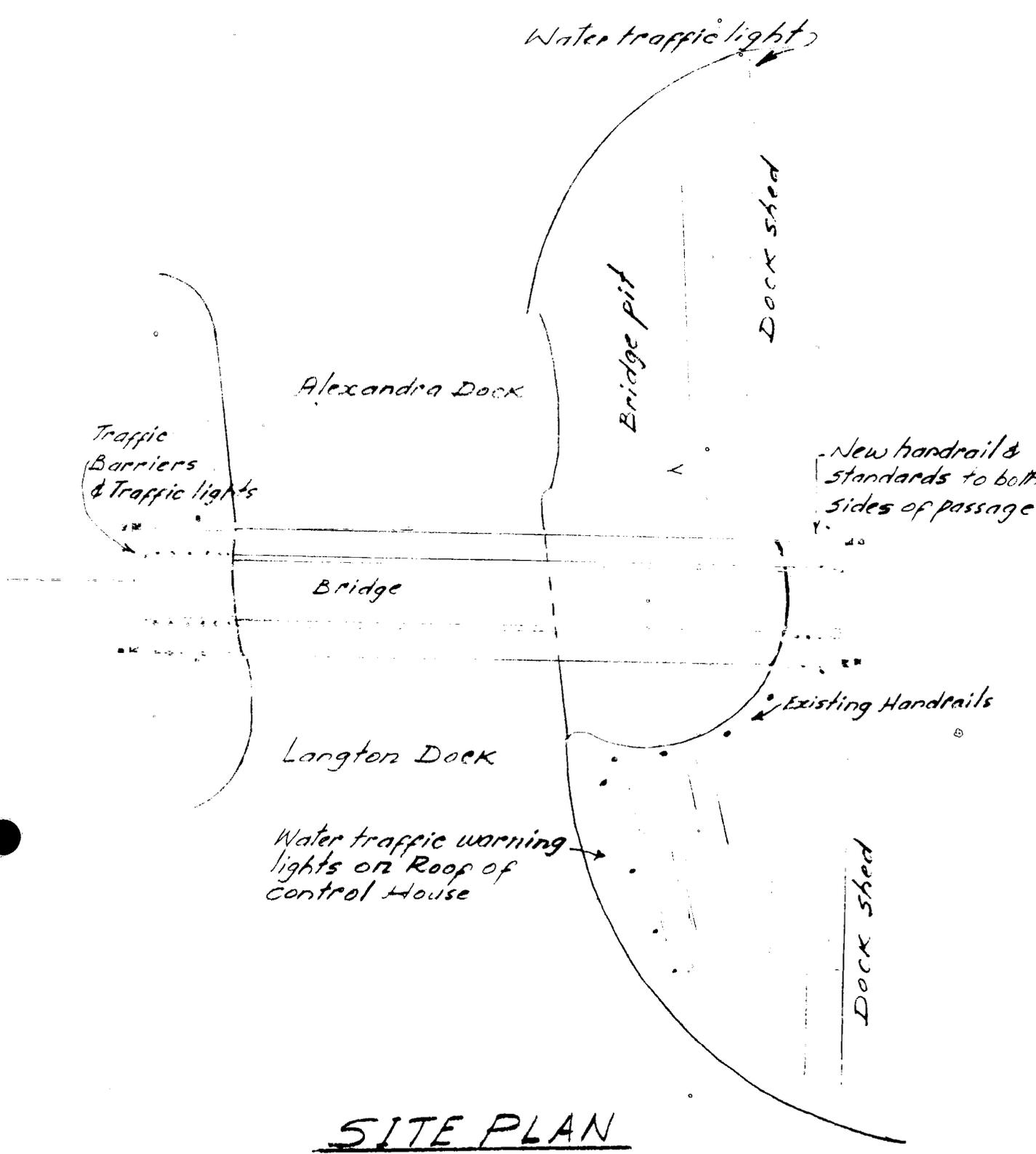
$$t = \sqrt{\frac{3W}{f_{bc}} \left(A^2 - \frac{B^2}{4} \right)}, \text{ where, } t = \text{required plate thickness,}$$

$$A = \frac{5.25}{2} = 2.625'', B = \frac{5}{2} = 2.5'' \text{ \& } W = \frac{11.5}{26.25} = 0.438 \text{ tonf/in}^2$$

Hence $t = 0.92''$. So, $1\frac{1}{2}''$ thick plate is more than adequate.



Detail of Track Plate

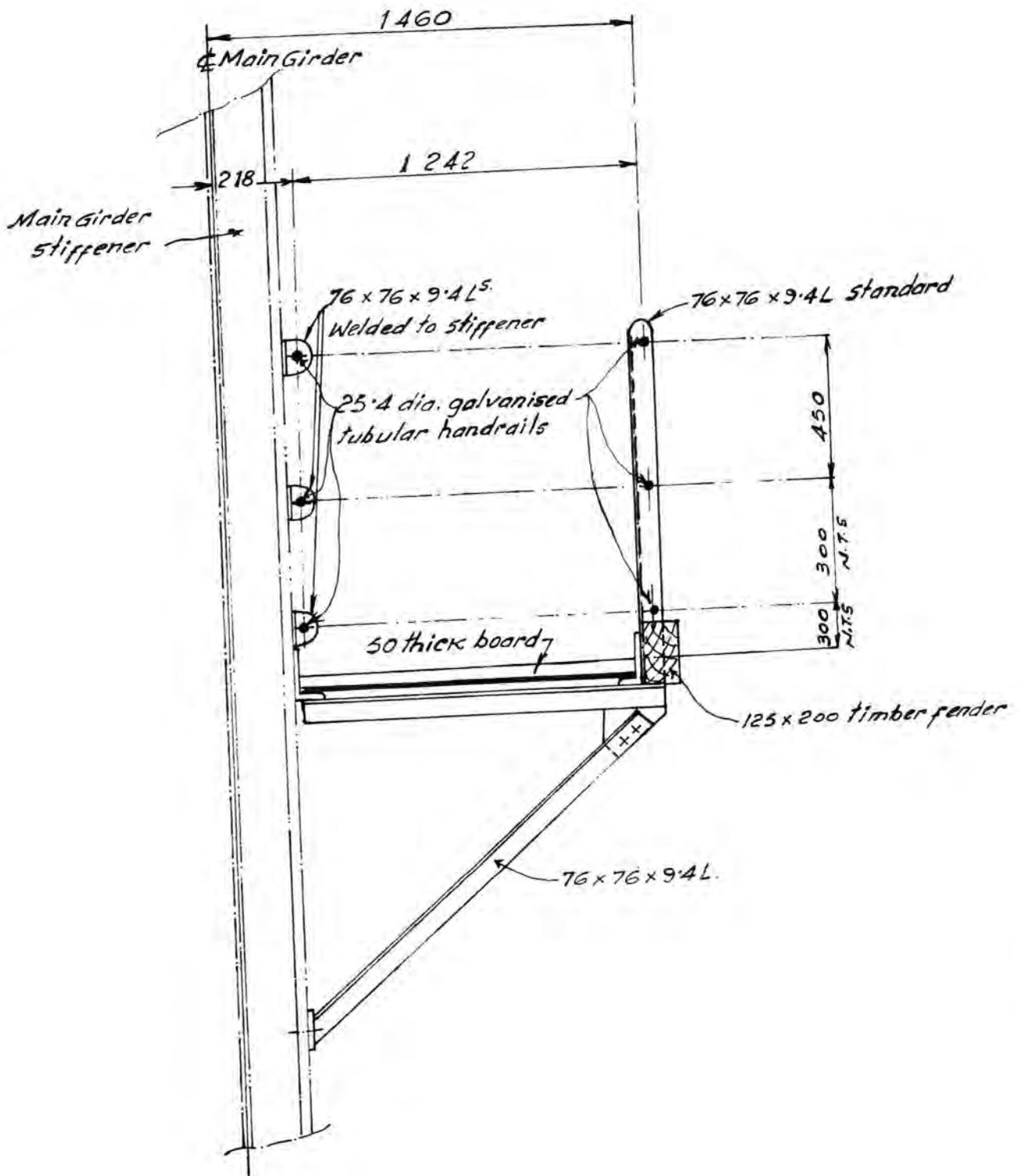


SITE PLAN

Minimum Clearance 7 010 (23'0")

6 096 (20'0") over Roadway

Section A-A



Detail Of Walkway

Appendix D

Limit state design of the Main Plate Girders.

A. Plastic Moments

i. Dead loads:

assume	a. deck + troughs	38.0	lb _f /ft ²
	b. cross girders	10.5	"
	c. asphalt surfacing	42.0	"
	d. walkway	30.0	"
	e. snow	3.0	"

for main girder, assume mass/ft. of individual girder = 0.2 ton/ft.

Breadth of roadway = 25.0 ft.

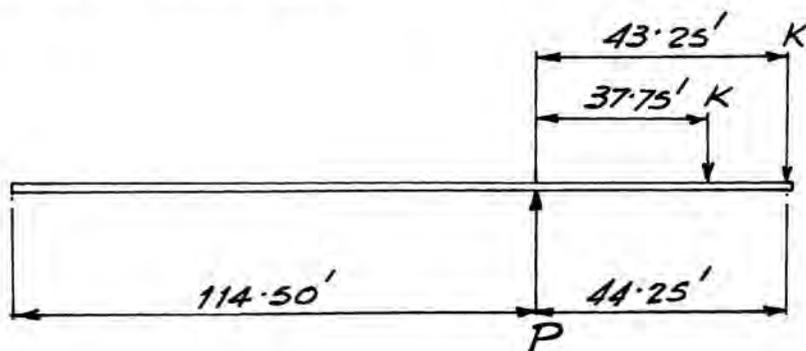
Combining items a, b & c above, dead load/ft. = 90.5 lb_f/ft.²

Width of walkway = 3.5 ft. ∴ Super load = 105.0 "

Hence total = 195.5 lb_f/ft.²

Say 200 lb_f/ft.²

Dead load will be evenly distributed between the two main girders, so, dead load per girder \approx 0.25 ton_f/ft. which includes self weight of the main girder.



for stability in cantilever condition: $1.1 \times O.T.M <$ restoring-moment - refer appendix A, Part 1.

or, Over turning moment (O.T.M) = $\frac{114.5^2}{2} \times 0.5 = 3278$ ton_fft.

D2

$$\text{Restoring moment} = \frac{44 \cdot 25^2}{2} \times 0.5 + K(37.75 + 43.25) = 489.4 \text{ tonft.} + 81K$$

where K represents the masses of the restoring kentledge blocks.

$$\text{Thus, } 1.1 \times 3278 < (489.4 + 81K)$$

$$\text{or, } K < 38.5 \text{ tonf.}$$

So, provide 2 no. 40 tonf. Kentledge blocks.

ii. H.A. loading :

for 94.5 ft. span loaded length = 94.5 ft.

$$\therefore \text{H.A.} = 2044 \text{ lbf/ft.}$$

$$\text{Thus, H.A. load per girder} = \frac{2044 \times 94.5}{2240} = 86.2 \text{ tonf.}$$

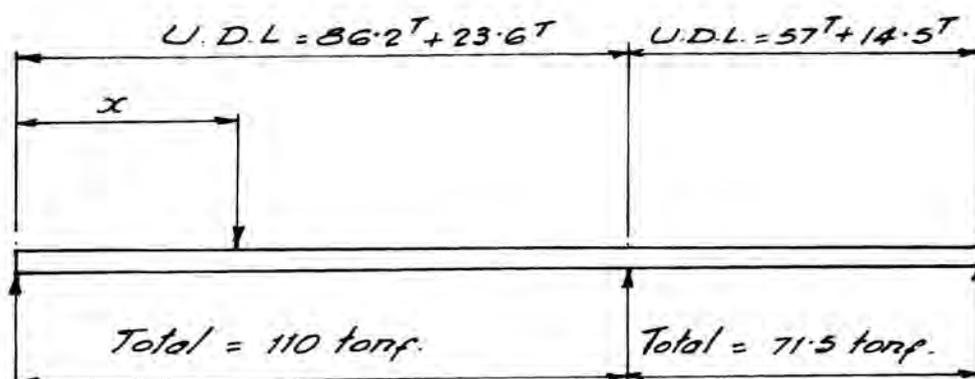
for 58.0 ft. span loaded length = 58.0 ft.

$$\therefore \text{H.A.} = 2200 \text{ lbf/ft.}$$

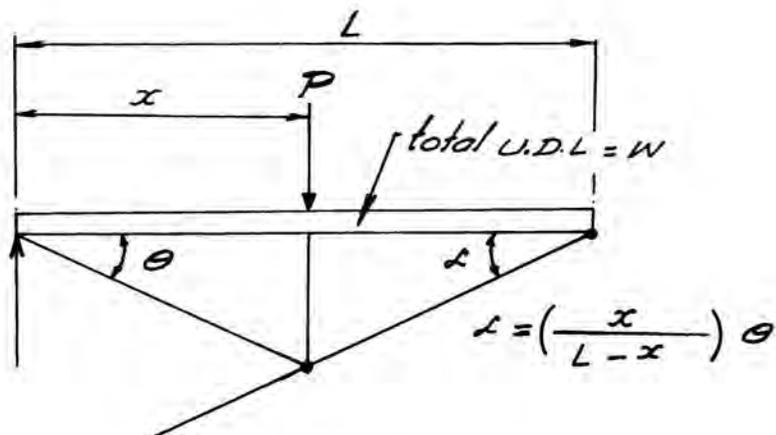
$$\text{Thus, H.A. load per girder} = \frac{2200 \times 58.0}{2240} = 57.0 \text{ tonf.}$$

$$\text{K.E. load per girder} = \frac{2700 \times 10}{2240} = 12.1 \text{ tonf.}$$

Hence the loading system is as shown below :



D3



$$\text{Total work done (internal)} = M_p \theta \left(\frac{L+x}{L-x} \right) \longrightarrow 1$$

$$\text{External work done} = P\theta x + \frac{W}{2} \theta x \longrightarrow 2$$

Equating equations 1 & 2

$$M_p = \left(P + \frac{W}{2} \right) \left(\frac{L-x}{L+x} \right) x \longrightarrow 3$$

for max. value of M_p , equate $\frac{dM_p}{dx}$ to 0

$$\frac{dM_p}{dx} = \left(P + \frac{W}{2} \right) \left\{ \frac{(L+x)(L-2x) - (Lx - x^2)}{(L+x)^2} \right\}$$

Equating $\frac{dM_p}{dx}$ to 0, we get

$$x = 0.4142L \text{ for max. value of } M_p.$$

$$\therefore M_p = P + \frac{W}{2} \left(\frac{0.5858}{1.4142} \right) 0.4142L$$

$$\text{or, } M_p = 0.17158L \left(P + \frac{W}{2} \right) \longrightarrow 4$$

Hence for 94.5 ft. span:

$$P = 12.1 \text{ tonf. \& } W = 86.2 \text{ tonf.}$$

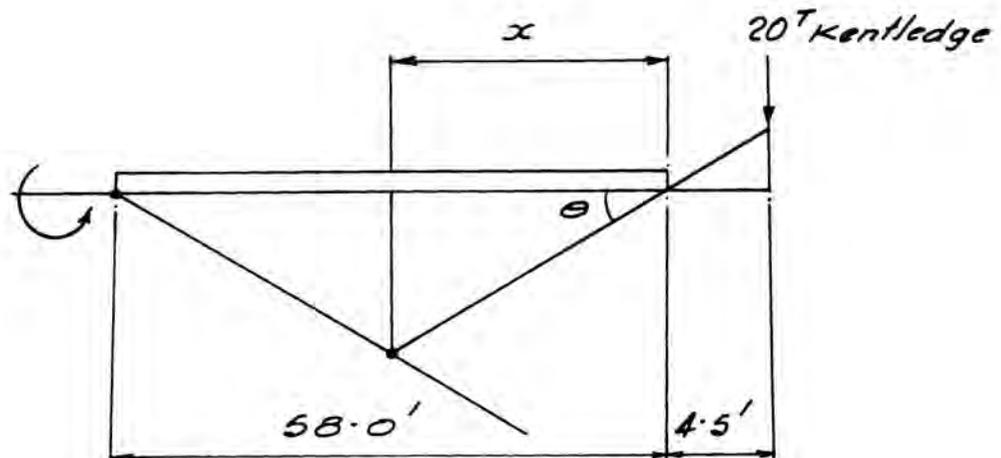
$$\therefore \underline{M_p = 1088 \text{ tonft.}}$$

For 58.0 ft. span:

$$P = 12.1 \text{ tonf. \& } W = 57 \text{ tonf.}$$

$$\therefore \underline{M_p = 476 \text{ tonft.}}$$

D4



$$\text{Internal work done} = M_p \theta \left(\frac{58 + x}{58 - x} \right) \rightarrow 5$$

$$\begin{aligned} \text{External work done} &= P \cdot x \cdot \theta + \frac{W}{2} \cdot x \cdot \theta - 20 \times 4.5 \theta \\ &= \left(P + \frac{W}{2} \right) x \cdot \theta - 90 \theta \rightarrow 6 \end{aligned}$$

From equation 5 & 6 we get

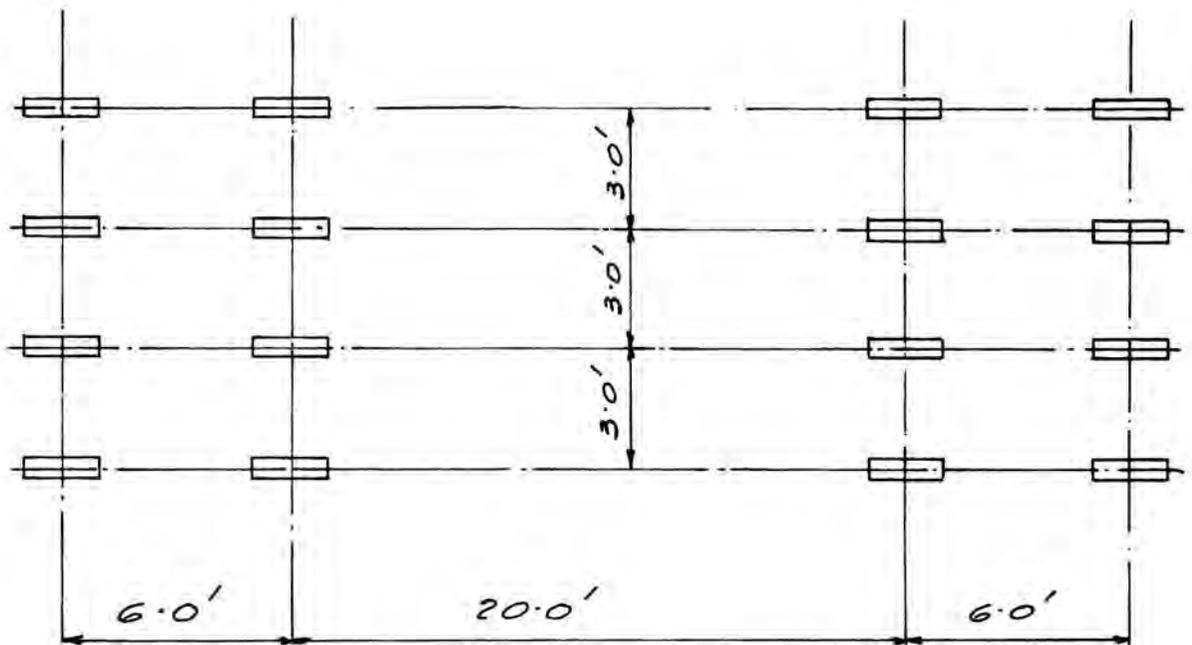
$$M_p = \frac{2865.3x - 5220 - 47.85x^2}{(58 + x)} \rightarrow 7$$

Equating $\frac{dM_p}{dx}$ to 0, {which will give the hinge position} for max. M_p .

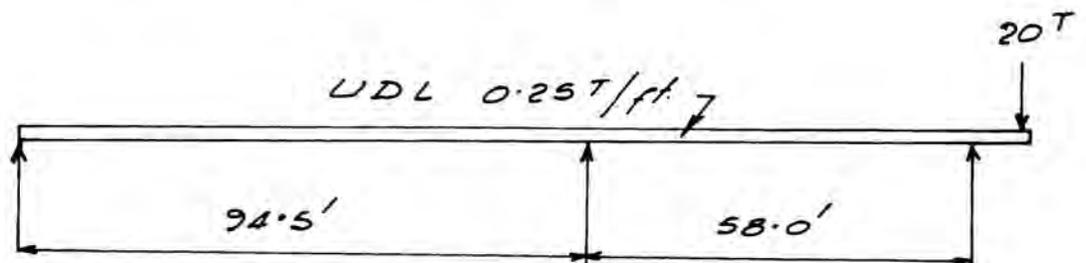
We get, $x = 25.34$ ft.

$$\text{Hence max. } M_p = \frac{36663.2}{83.34} = \underline{\underline{440 \text{ tonft.}}}$$

iii. H.B loading:



axle loads 20 tons each
Detail of load train system



Dead load system

Distribution of H.B loading between main girders :

When H.B loading is considered on a two lane bridge, adjacent lane to be loaded with $\frac{1}{3}$ H.A loading & this H.A loading value varies with the loaded length. Hence, load distribution is required for 94.5 ft. & 58.0 ft. spans.

For 94.5 ft. span :

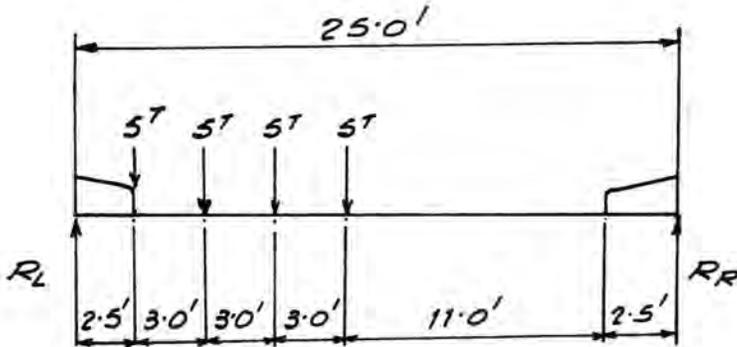
Full H.A. loading	is	2044	lb/ft.
$\therefore \frac{1}{3}$ H.A. loading	is	682	"
& $\frac{1}{3}$ K.E. loading	is	900	"

D6

and for a 10 ft. wide lane 4 tonf.

No distribution of the wheel load through the deck is assumed.

a. H.B. loading only:



$$R_R = \frac{1}{25} \{ 5 \times (2.5 + 5.5 + 8.5 + 11.5) \} = 5.6 \text{ tonf}$$

$$\therefore R_L = (20 - 5.6) = 14.4 \text{ tonf.}$$

b. $\frac{1}{3}$ H.A loading only:

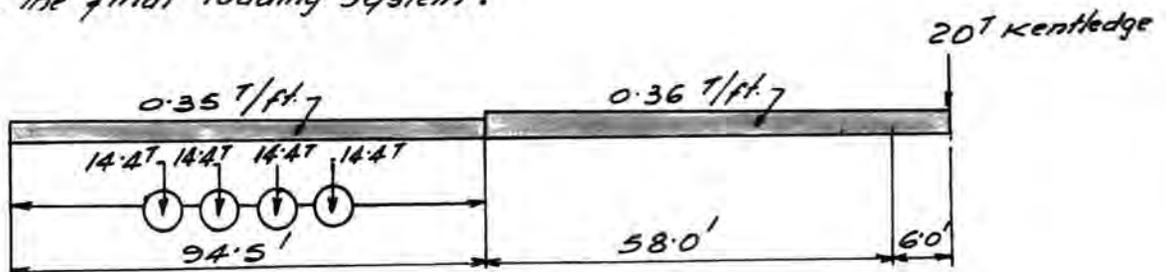


$$R_L = 0.1 \text{ tonf.}$$

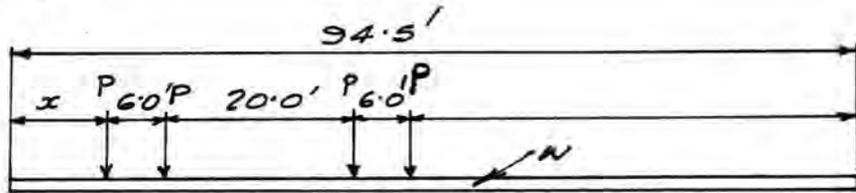
$$R_R = 0.2 \text{ tonf.}$$

Therefore, the girder adjacent to the loaded lane takes 14.4 tonf at every axle position. The effect of the $\frac{1}{3}$ H.A load in the opposite lane will be added to the dead load, for the analysis purpose, because both the loading system are similar.

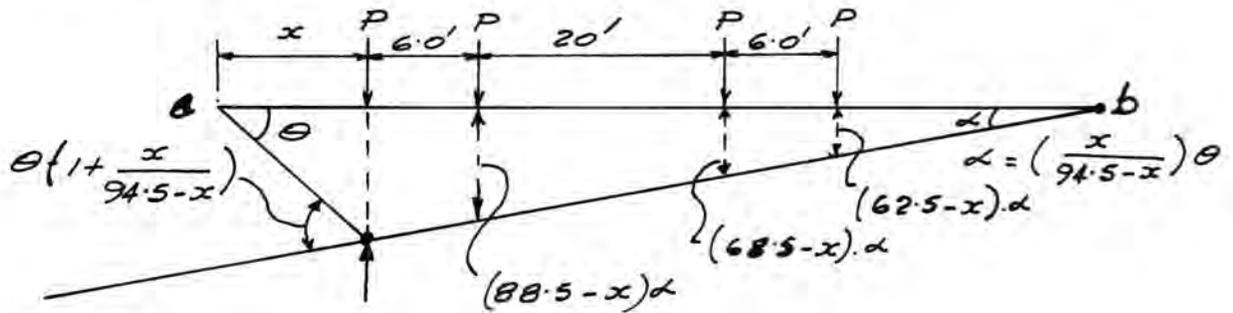
Now, the final loading system:



For 94.5 ft. span:



a. Plastic hinge at A:



assume rotation θ @ A

displacement @ hinge = $x\theta$

$$\text{Internal work done} = M_p \left(\frac{x}{94.5-x} \right) \theta + M_p \left(1 + \frac{x}{94.5-x} \right) \theta$$

$$= M_p \theta \left[1 + \frac{2x}{94.5-x} \right]$$

External work done: by U.D.L. = $W \cdot \frac{x\theta}{2}$

$$\text{by wheel load} = P \cdot x\theta + P \left(\frac{x}{94.5-x} \right) \theta \left[(88.5-x) + (68.5-x) + (62.5-x) \right]$$

Hence total external work done (ie., U.D.L + wheel load)

$$\frac{Wx\theta}{2} + Px\theta \left[1 + \frac{219.5-3x}{94.5-x} \right]$$

Equating work done

$$M_p \theta \left[1 + \frac{2x}{94.5-x} \right] = \frac{Wx\theta}{2} + Px\theta \left[1 + \frac{219.5-3x}{94.5-x} \right]$$

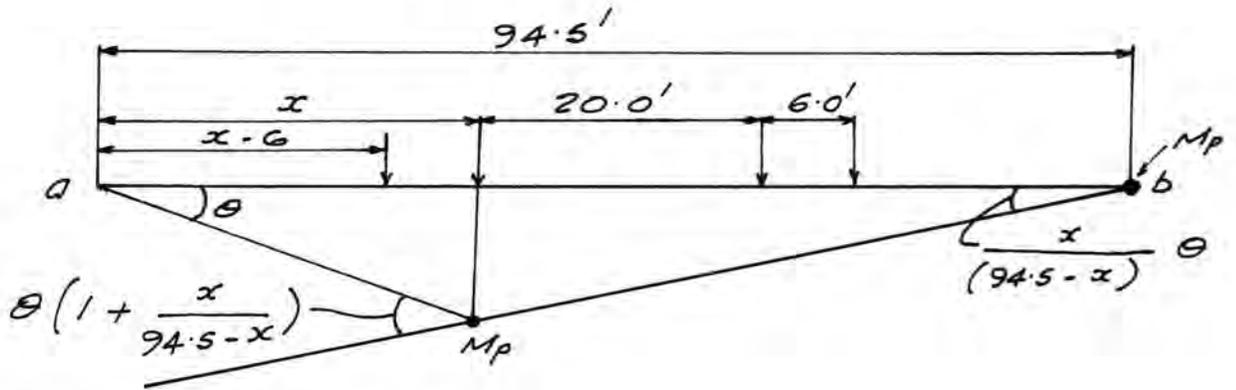
$$W = 0.35 \times 94.5 = 33.1 \text{ tonf.}$$

$$P = 14.4 \text{ tonf.}$$

$$\text{Hence } M_p = \frac{6085.6x - 74.15x^2}{94.5 + x}$$

Equating $\frac{dM_p}{dx}$ to zero, we get, $x = 34.67 \text{ ft.}$

b. Plastic hinge at B :



assume rotation θ at a

$$\text{Internal work done} = M_p \theta \left[1 + \frac{2x}{(94.5-x)} \right] \quad - 1$$

External work done :

$$i. \text{ by U.D.L} = W \times \frac{x\theta}{2}$$

$$ii \text{ by wheel load} = P \cdot x\theta + P \left(\frac{x}{94.5-x} \right) \theta \left[(74.5-x) + (68.5-x) \right] + P\theta(x-6)$$

$$\text{Hence total external work done (i + ii)} = \frac{W \cdot x\theta}{2} + P\theta \left[2x-6 + \frac{x(143-2x)}{94.5-x} \right]$$

$$W = 33.1 \text{ tonf} \quad \& \quad P = 14.4 \text{ tonf}$$

$$\therefore \text{external work done} = 14.4\theta \left(\frac{446.6x - 5.15x^2 - 567}{94.5 - x} \right) \quad - 2$$

Equating 1 & 2 we get,

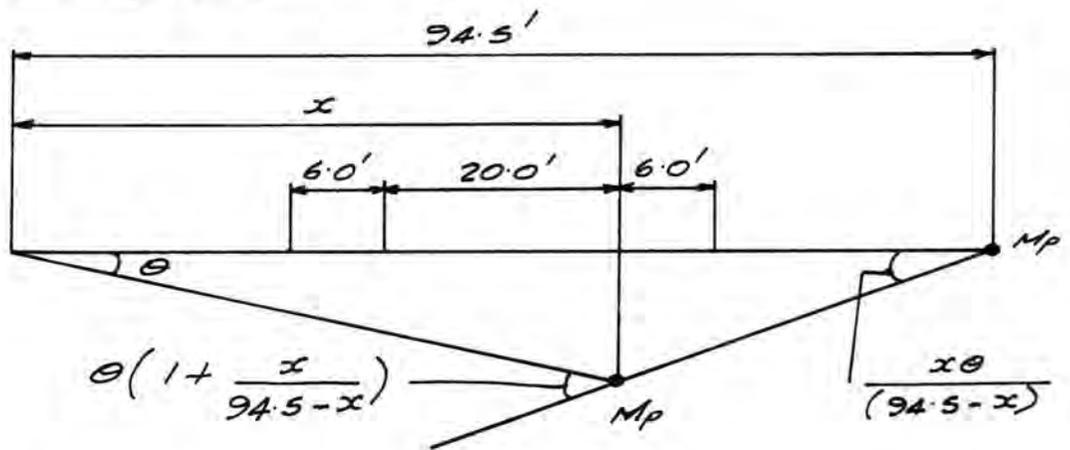
$$M_p = \frac{14.4 (446.6x - 5.15x^2 - 567)}{(94.5 - x)} \quad - 3$$

Now, M_p is maximum, when $\frac{dM_p}{dx} \rightarrow 0$

Hence from equation 3 we get

$$x = 36.78 \text{ ft.}$$

c. Plastic hinge at c



$$\text{Internal work done} = M_p \theta \left[1 + \frac{2x}{94.5 - x} \right] \quad - 1$$

External work done :

$$i. \text{ by U.D.L} = W \cdot \frac{x\theta}{2}$$

$$ii \text{ by Wheel load} = P(x-26)\theta + P(x-20)\theta + P \cdot x \cdot \theta + P \cdot \theta \cdot \frac{x(88.5-x)}{(94.5-x)}$$

$$= P\theta \left[\frac{418x - 4347 - 4x^2}{(94.5-x)} \right]$$

$$W = 33.1 \text{ tonf} \ \& \ P = 14.4 \text{ tonf}$$

$$\therefore \text{total work done (external)} = 14.4\theta \left[\frac{526.6x - 5.15x^2 - 4347}{(94.5-x)} \right] \quad - 2$$

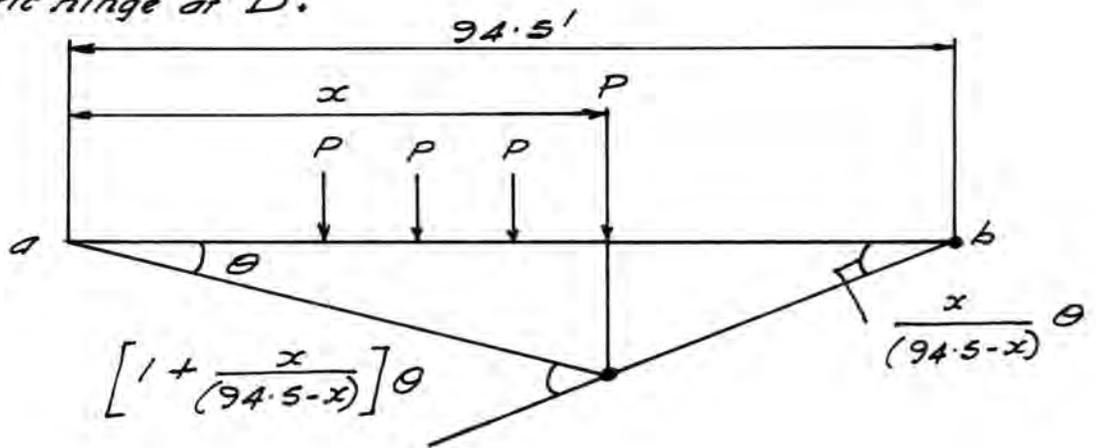
from equations 1 & 2 we get

$$M_p = \frac{14.4(526.6x - 5.15x^2 - 4347)}{(94.5-x)} \quad - 3$$

$$M_p \text{ is max. when } \frac{dM_p}{dx} \rightarrow 0$$

$$\therefore \text{from equation 3, } x = 44.917 \text{ ft.}$$

d. Plastic hinge at D:



assume rotation θ at a

$$\text{Internal work done} = M_p \theta \left[1 + \frac{x}{(94.5-x)} \right] \quad \text{--- 1}$$

External work done:

$$\text{i. by U.D.L.} = W \cdot \frac{x\theta}{2}$$

$$\text{ii. by wheel load} = P\theta(4x - 64)$$

$$P = 14.4 \text{ Tonf} \quad \& \quad W = 33.1 \text{ tonf.}$$

$$\therefore \text{total external work done} = (74.15x - 921.6)\theta \quad \text{--- 2}$$

from equations 1 & 2 we get

$$M_p = \frac{7928.78x - 74.15x^2 - 87091.2}{(94.5 - x)} \quad \text{--- 3.}$$

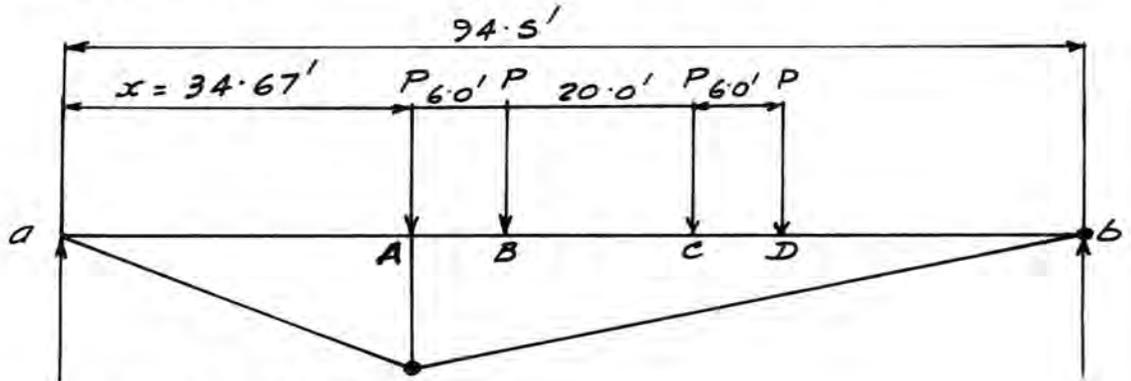
$$M_p \text{ is max when } \frac{dM_p}{dx} \rightarrow 0$$

$$\therefore \text{from equation 3 we get, } x = 47.66 \text{ ft.}$$

Summary: H.B. loading system on span 94.5 ft.

a. Plastic hinge at A

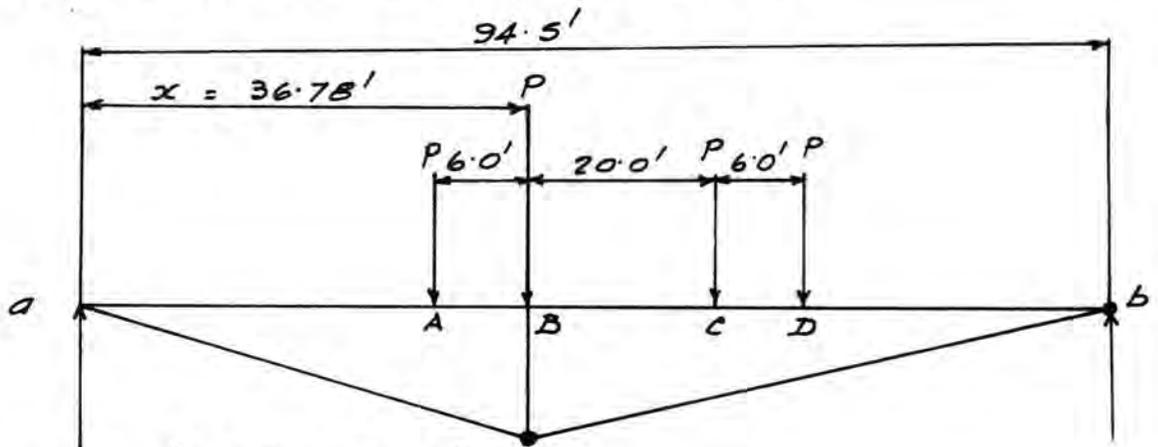
When $x = 34.67$ ft.



$$M_p = \frac{6085.6x - 74.15x^2}{94.5 + x}$$

or, $M_p = 943.4$ tonf ft.

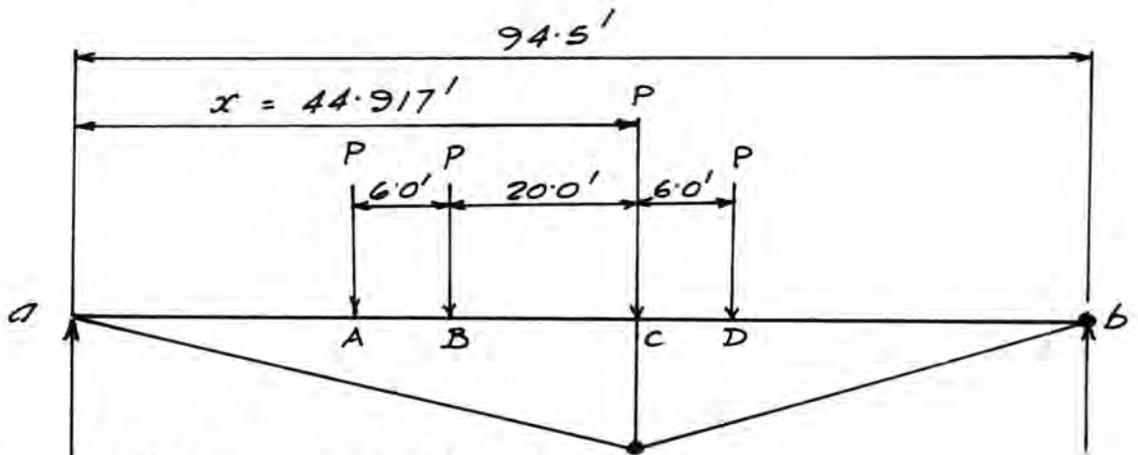
b. Plastic hinge at B - When $x = 36.78$ ft.



$$M_p = \frac{14.4(446.6x - 5.15x^2 - 576)}{(94.5 + x)}$$

or, $M_p = 975.4$ tonf ft.

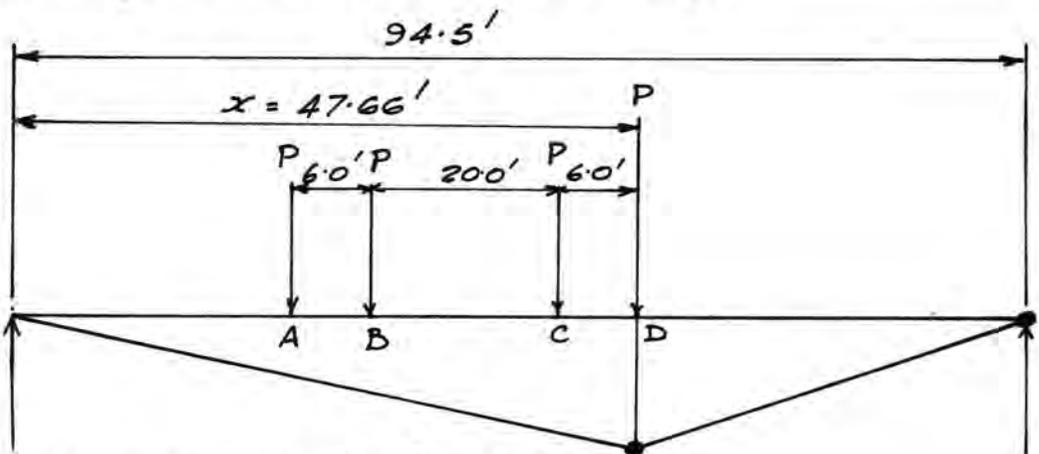
c. Plastic hinge at C - When $x = 44.917$ ft.



$$M_p = \frac{14.4(526.6x - 5.15x^2 - 4347)}{(94.5 + x)}$$

or, $M_p = 940.6$ tonf ft.

d. Plastic hinge at D - When $x = 47.66$ ft.

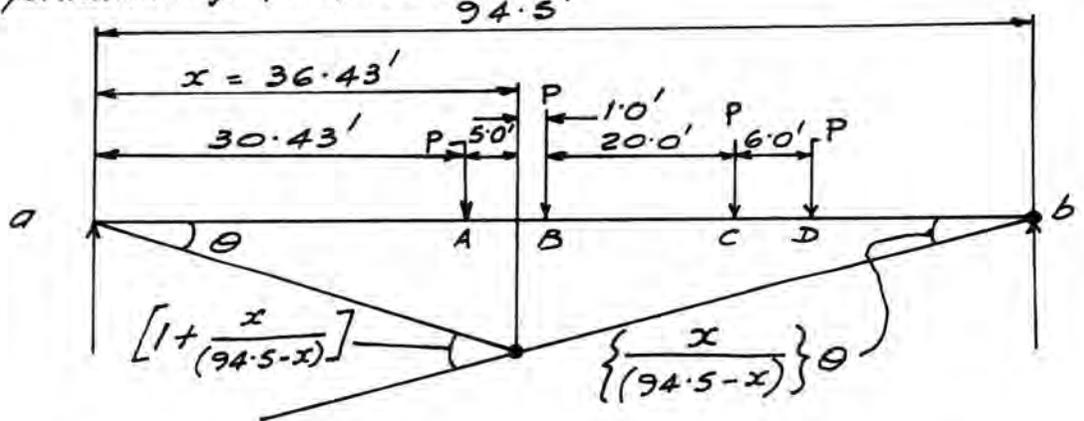


$$M_p = \frac{7928.78x - 74.15x^2 - 87091.2}{(94.5 + x)}$$

or, $M_p = 860.75$ tonf ft.

From sheet no. 11, it may be noted that the maximum value of $M_p = 975.4$ tonf ft. at mechanism B. But this might not be true. So an investigation is necessary to find the maximum value of M_p and the position of the plastic hinge, under wheel loads.

E. try formation hinge 1ft. left to axle B,



$$\text{Internal work done} = M_p \theta \left(\frac{94.5 + x}{94.5 - x} \right) \quad \text{--- 1.}$$

External work done:

$$i. \text{ by U.D.L} = W \frac{x\theta}{2}$$

$$ii. \text{ by wheel load} = P\theta(x-5) + \left(\frac{x}{94.5-x} \right) \cdot P\theta \left[(93.5-x) + (73.5-x) + (67.5-x) \right]$$

$$= \frac{P\theta}{(94.5-x)} (334x - 4x^2 - 472.5)$$

$$P = 14.4 \text{ tonf. } \& \ W = 33.55 \text{ tonf.}$$

Total external work done, i.e., (i + ii)

$$\frac{14.4\theta}{(94.5+x)} [442.61x - 5.15x^2 - 472.5] \quad \text{--- 2.}$$

from equation 1 & 2, we get

$$M_p \theta \left[\frac{94.5 + x}{94.5 - x} \right] = \frac{14.4\theta}{(94.5-x)} (442.61x - 5.15x^2 - 472.5)$$

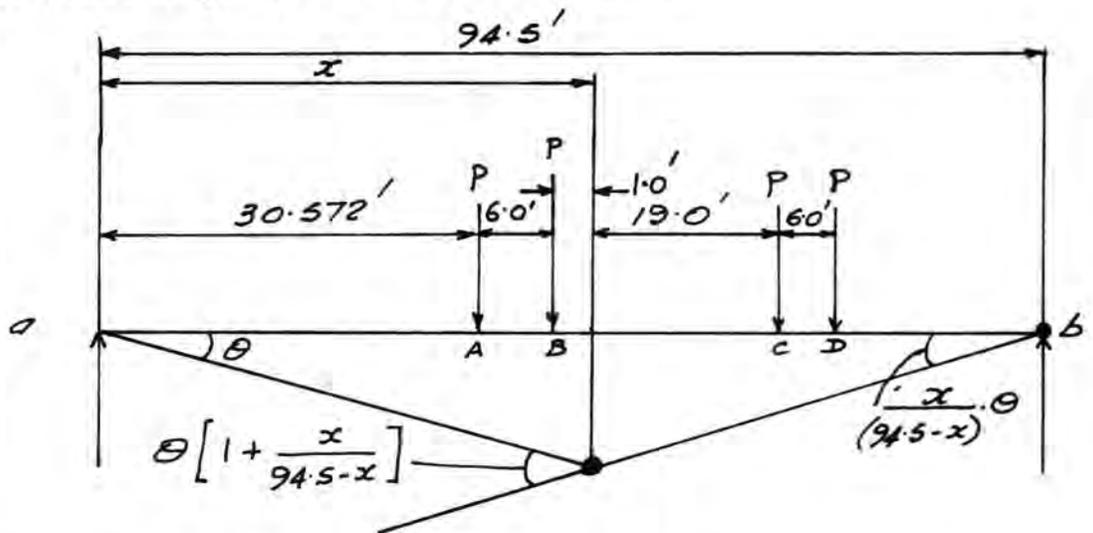
$$\text{or, } M_p = \frac{14.4}{(92.5+x)} (442.61x - 5.15x^2 - 472.5)$$

$$\text{For max. } M_p, \quad \frac{dM_p}{dx} \rightarrow 0$$

$$\text{so, } x = 36.43 \text{ ft.}$$

$$\text{Hence } \underline{M_p = 969.7 \text{ tonf ft.}}$$

f. try formation hinge 1 ft. to right of axle B.



$$\text{Internal work done} = M_p \theta \left(\frac{94.5 + x}{94.5 - x} \right) \quad \text{--- 1.}$$

External work done:

i. by U.D.L. = $W \cdot \frac{x \theta}{2}$

$$W = 33.1 \text{ tonf. } \therefore = 16.55 x \theta$$

ii. by wheel load =

$$\left[(x-7)\theta + (x-1)\theta + (75.5-x) \left(\frac{x}{94.5-x} \right) \theta + (69.5-x) \left(\frac{x}{94.5-x} \right) \theta \right] P$$

$$= \frac{P \theta}{(94.5-x)} [197x - 2x^2 - 756 + 145x - 2x^2]$$

$$= \frac{P \theta}{(94.5-x)} [342x - 4x^2 - 756]$$

$$P = 14.4 \text{ tonf.}$$

total external work done, i.e., i + ii

$$= \frac{14.4 \theta}{(94.5-x)} [450.16x - 5.086x^2 - 756] \quad \text{--- 2.}$$

from equation 1 & 2 we get

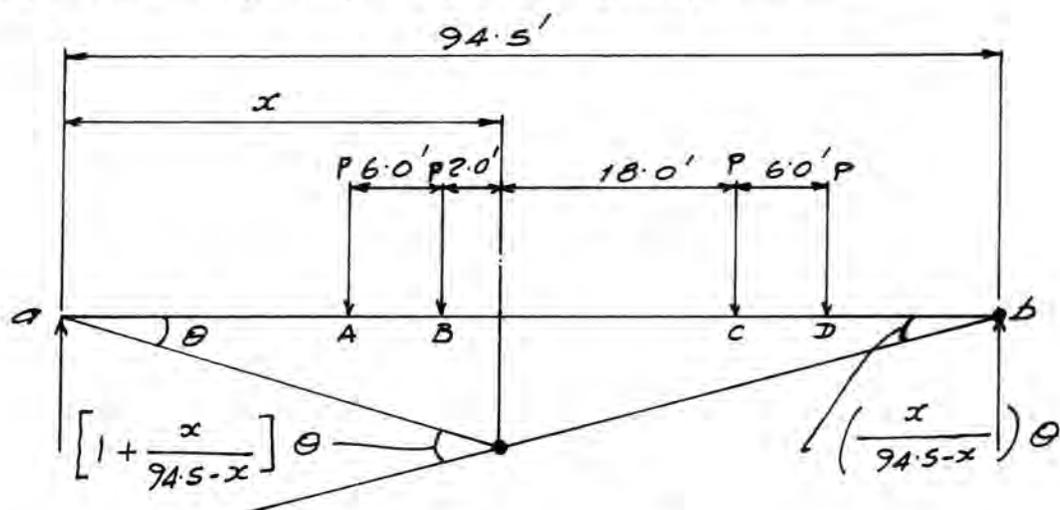
$$M_p = \frac{14.4 (450.16x - 5.086x^2 - 756)}{(94.5 + x)} \quad \text{--- 3.}$$

For max. M_p , $\frac{dM_p}{dx} \rightarrow 0$

So, $x = 37.572 \text{ ft.}$ — from equation 3.

$$\& \underline{M_p = 978.86 \text{ tonf ft.}}$$

g. try formation of hinge 2 ft to right of axle B :



$$\text{Internal work done} = M_p \theta \left(\frac{94.5 + x}{94.5 - x} \right) \quad \text{--- 1.}$$

External work done :

i. by U.D.L = $W \frac{x\theta}{2} = 16.55 x \theta \quad \therefore W = 33.1 \text{ tonf.}$

ii by wheel loads = $\left[\frac{346x - 4x^2 - 945}{(94.5 - x)} \right] 14.4 \theta \quad \therefore P = 14.4 \text{ tonf.}$

$$\therefore \text{total external work done} = \left[\frac{14.4 \theta (454.6x - 5.15x^2 - 945)}{(94.5 - x)} \right] \quad \text{--- 2.}$$

Equating work done (from equation 1 & 2), We get

$$M_p \theta \left[\frac{94.5 + x}{94.5 - x} \right] = \left[\frac{14.4 \theta (454.6x - 5.15x^2 - 945)}{(94.5 - x)} \right]$$

$$\text{or, } M_p = \left[\frac{14.4 (454.6x - 5.15x^2 - 945)}{(94.5 + x)} \right] \quad \text{--- 3.}$$

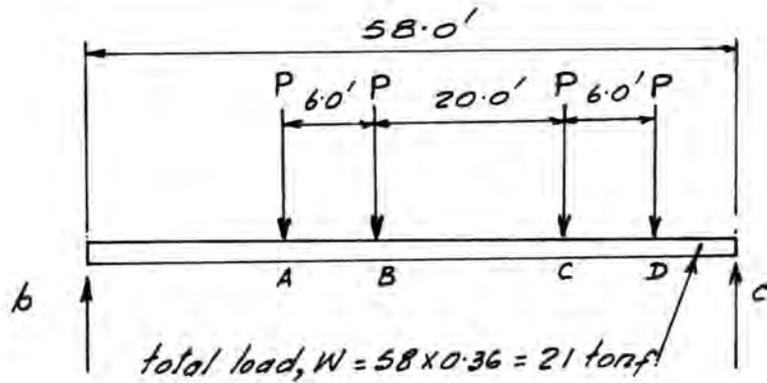
For max. M_p , $\frac{dM_p}{dx} \rightarrow 0$

So, equating 3, we get, $x = 37.62 \text{ ft.}$

$$\underline{\& M_p = 966.6 \text{ tonf ft.}}$$

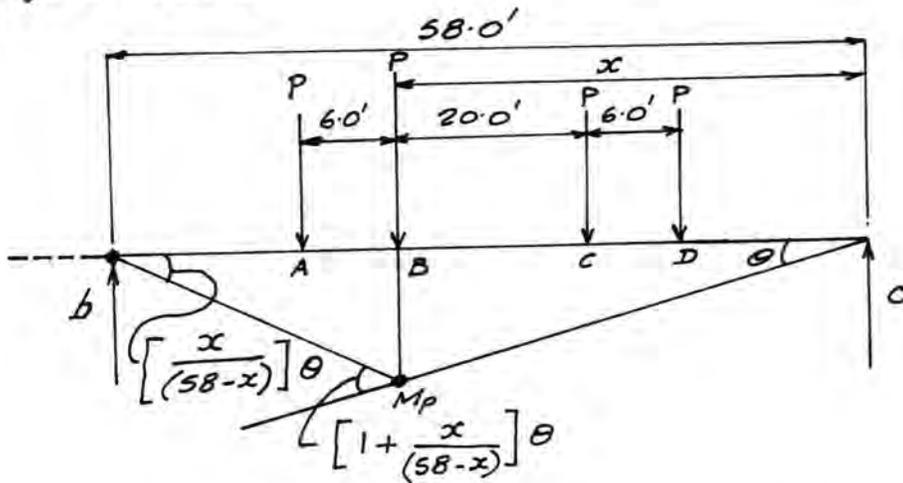
Therefore, it may be noted that the max. M_p value is obtained at the condition f. So, Maximum $M_p = 978.86 \text{ tonf ft.}$

H. B. loading system on span 58'-0": Effect of kentledge load is ignored.
From previous sheets, effect of axle load is 14.4 tonf. (= P.).



from previous calculations (i.e., at 94.5' span), it is obvious that the M_p will not be critical at A & D. Therefore, check on these points are not made.

a. Hinge at axle B.



$$\text{Internal work done} = M_p \theta \left[\frac{58+x}{58-x} \right] \quad - 1.$$

External work done:

$$i. \text{ by U.D.L.} = W \frac{x \theta}{2} = 10.5 x \theta.$$

$$ii \text{ by axle load} = P \theta \left[\frac{272x - 2668 - 4x^2}{(58-x)} \right]$$

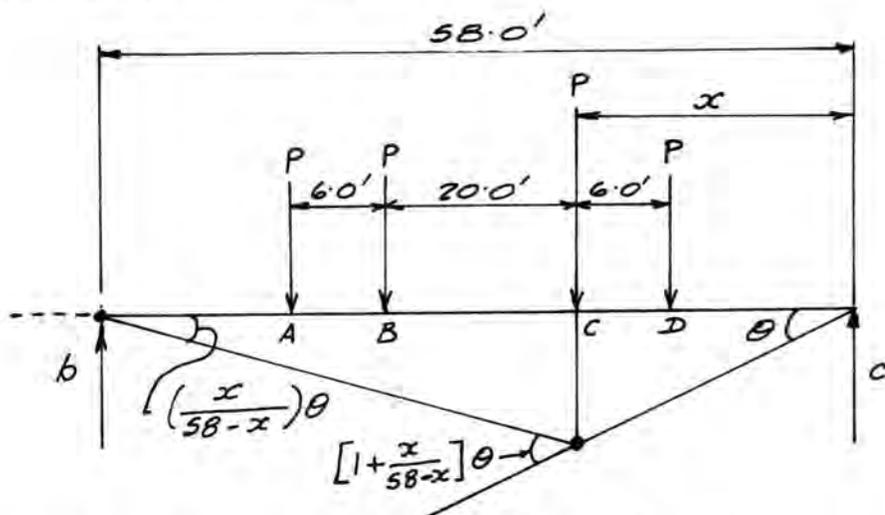
$$\text{Total external load} = \frac{14.4 \theta}{(58-x)} (314.3x - 2668 - 4.73x^2) \quad - 2$$

Equating work done (from equation 1 & 2)

$$M_p = 14.4 \left(\frac{314.3x - 2668 - 4.73x^2}{(58-x)} \right) \quad - 3$$

for max. M_p , $\frac{dM_p}{dx} \rightarrow 0 \therefore$ from equation 3, $x = 27.29 \text{ ft.}$ & $M_p = 402.95 \text{ tonf.ft.}$

b. Hinge at axle C.



$$\text{Internal work done} = M_p \theta \left(\frac{58+x}{58-x} \right) \quad \text{--- 1.}$$

External work done:

$$\text{i. by U.D.L} = W \frac{x\theta}{2} = 10.5x\theta$$

$$\text{ii by axle load} = P\theta \left(\frac{123x - 290 - 3x^2}{(58-x)} \right)$$

$$\text{total external work done} = 14.4\theta \left(\frac{165.3x - 290 - 3.73x^2}{(58-x)} \right) \quad \text{--- 2}$$

Equating work done (from equations 1 & 2),

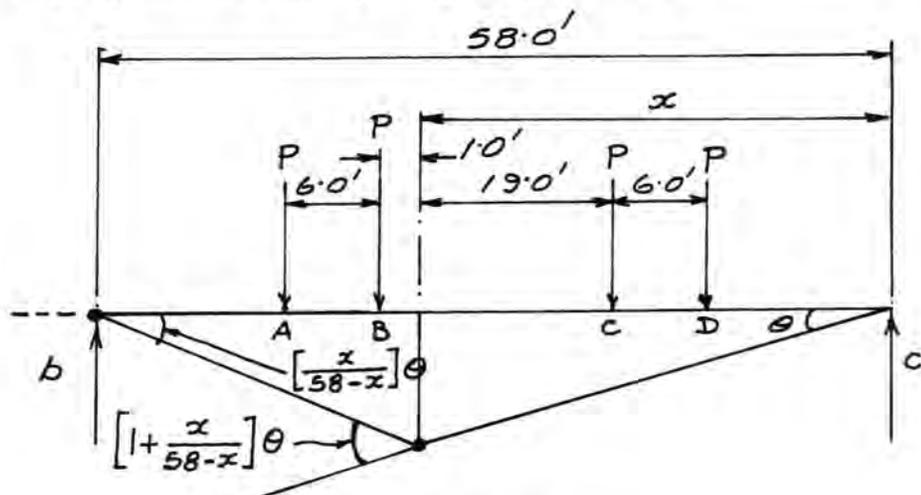
$$M_p = 14.4 \left[\frac{165.3x - 290 - 3.73x^2}{(58-x)} \right] \quad \text{--- 3}$$

for max. M_p , $\frac{dM_p}{dx} \rightarrow 0$

\therefore from equation 3, $x = 19.53 \text{ ft}$

$$\& M_p = \frac{14.4}{77.53} \times 1515.61 = \underline{\underline{281.5 \text{ tonf ft.}}}$$

c. try hinge formation 1ft right of axle B.



$$\text{Internal work done} = M_p \theta \left[\frac{58+x}{58-x} \right] \quad - 1$$

External work done:

$$i. \text{ by U.D.L} = W \frac{x\theta}{2} = 10.5 x \theta$$

$$ii. \text{ by axle load} = P\theta \left[\frac{268x - 2554 - 4x^2}{58-x} \right]$$

$$\text{total external work done} = 14.4\theta \left[\frac{310.3x - 2554 - 4.73x^2}{58-x} \right] \quad - 2$$

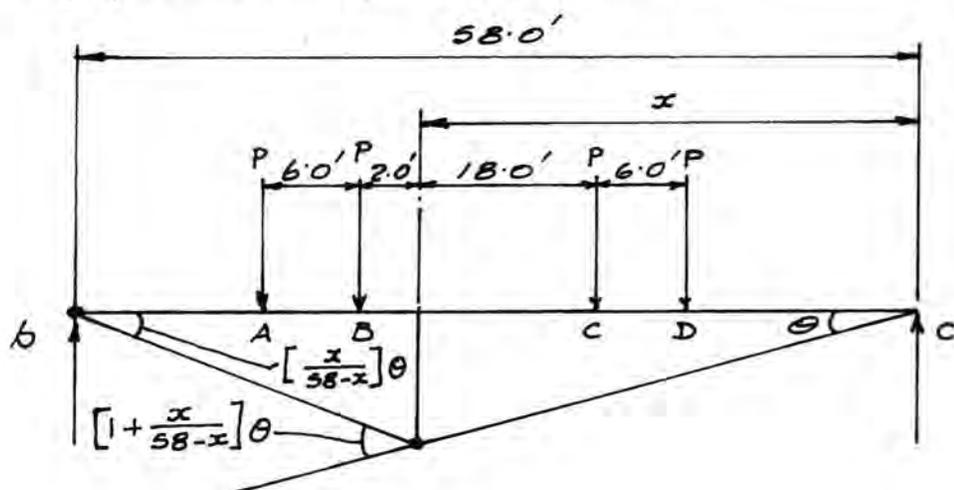
Equating Internal & external work done (equation 1 & 2)

$$M_p = 14.4 \left[\frac{310.3x - 2554 - 4.73x^2}{58-x} \right] \quad - 3.$$

for max. M_p , $\frac{dM_p}{dx} \rightarrow 0$

$$\therefore x = 29.8 \text{ ft.} \quad \& \quad \underline{M_p = 408.8 \text{ tonf ft.}}$$

d. try hinge formation 2 ft. right of axle B.



$$\text{Internal work done} = M_p \theta \left(\frac{58+x}{58-x} \right) \quad \text{--- 1.}$$

External work done :

$$i. \text{ by U.D.L} = W \frac{x\theta}{2} = 10.5 x \theta$$

$$ii. \text{ by axle load} = \frac{P\theta}{58-x} [264x - 2436 - 4x^2]$$

$$\text{total external work done} = \frac{14.4\theta}{58-x} [306.3x - 2436 - 4.73x^2] \quad \text{--- 2.}$$

Since,

Internal work done = External work done, by equating equations 1 & 2

$$\text{We get, } M_p = \frac{14.4}{58+x} [306.3x - 2436 - 4.73x^2] \quad \text{--- 3.}$$

$$\text{for max. } M_p, \frac{dM_p}{dx} \rightarrow 0$$

Thus from equation 3, we get,

$$\frac{dM_p}{dx} = 14.4 \left[\frac{(58+x)(306.3 - 9.46x) - (306.3x - 2436 - 4.73x^2)}{(58+x)^2} \right]$$

$$\text{or } 20201.4 - 548.68x - 4.73x^2 = 0 \quad \text{--- equating to 0.}$$

$$\text{Hence } x = 29.378 \text{ ft}$$

$$\& \quad \underline{M_p = 408.7 \text{ tonf ft.}}$$

Hence Maximum M_p Value is obtained 408.8 tonf ft. under the loading condition, as stated in case C (see page A/P30/18).

Summary of maximum plastic moments

1. For 94.5 ft. span:

- a. due to H.A. load, max. $M_p = 1088$ tonf ft.
 b. due to H.B. load, max. $M_p = 978.9$ tonf ft.

2 For 58.0 ft. span:

- a. due to H.A. load, max. $M_p = 476$ tonf ft.
 b. due to H.B. load, max. $M_p = 408.8$ tonf ft.

Hence it appears that the critical M_p occurs only due to H.A. loading system in both the spans.

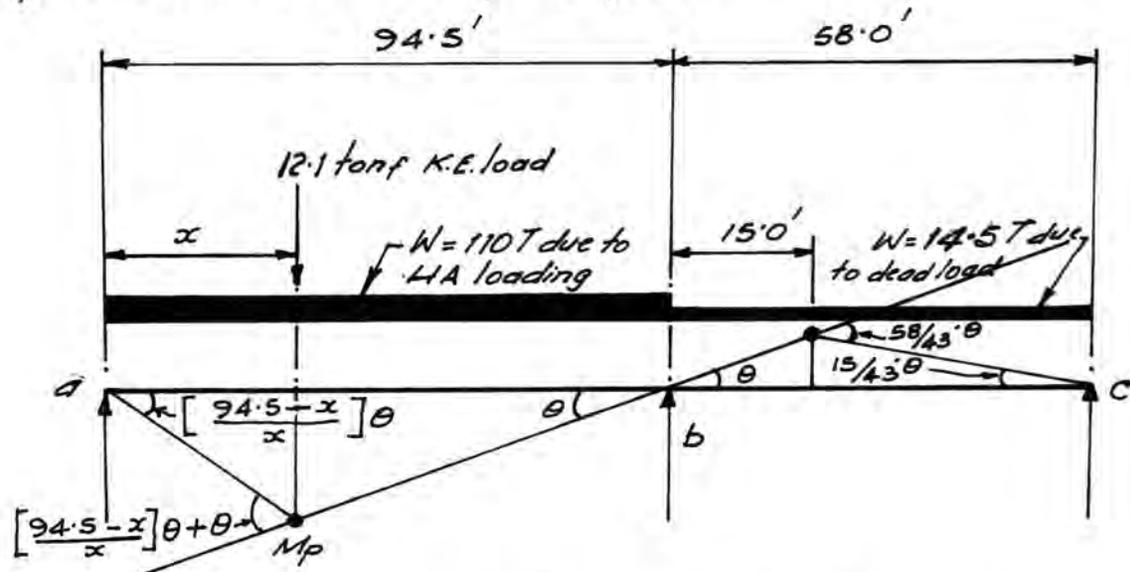
Using load factor 1.7, we get the following required moments:

$$\begin{aligned} \text{for } 94.5 \text{ ft. span, req'd. } M_p &= 1088 \times 1.7 = 1849.6 \text{ tonf ft.} \\ \text{" } 58.0 \text{ " " " " " } &= 476 \times 1.7 = 809.2 \text{ " } \end{aligned}$$

The change in sections of the main plate girders:

assume that the section of the main plate girder will be reduced at 15 ft into the 58.0 ft span. The M_p required at this portion must be less than 476 tonf ft.

Let us try the span 94.5 ft with H.A loading system and 58.0 ft span with dead load of the span.



$$\begin{aligned} \text{Internal work done} &= M_p \theta \left[1 + \frac{94.5-x}{x} \right] + \frac{58}{43} M_p \theta \\ &= M_p \theta / x \left[94.5 + 1.349x \right] \quad \text{--- 1} \end{aligned}$$

$$\begin{aligned} \text{External work done} &= \frac{110}{2} [(94.5-x) \theta] + 12.1 (94.5-x) \theta - 14.5 (15/2) \theta \\ &= 6232.2 \theta - 67.1 x \theta. \quad \text{--- 2} \end{aligned}$$

Equating work done, i.e., equations 1 & 2

$$M_p \theta / x [94.5 + 1.349x] = 6232.2 \theta - 67.1 x \theta.$$

$$\text{or, } M_p = \frac{6232.2x - 67.1x^2}{94.5 + 1.349x} \quad \text{--- 3}$$

for max. M_p , $\frac{dM_p}{dx} \rightarrow 0$

So, equating, equation 3 to 0, we get

$$x = 36.78 \text{ ft} \ \& \ M_p = 960.7 \text{ tonf ft.}$$

Since M_p at pivot is greater than 960.7 tonf ft., the failure mechanism is not possible, so the Z_p value may be changed at any convenient distance. This mechanism is only possible if M_p is inside the span ab & less than 960.7 tonf ft.

The section at the pivot, should be analyse elastically, due to the cantilever condition; since this condition is statically determinate, there is no advantage to analyse this section, plastically.

Appendix E

Stress Computations:

From page 20 of appendix D, it is noted that the maximum plastic moment will occur only due to H.A. loading system in both the spans, when the main girders are in working position.

Section chosen:

2 No. flanges (top & bottom) : $18'' \times \frac{7}{8}''$

Web plate : $60'' \times \frac{5}{8}''$

Using yield stress for flanges 15 tonf/in²

& " " " " Web 16 " " "

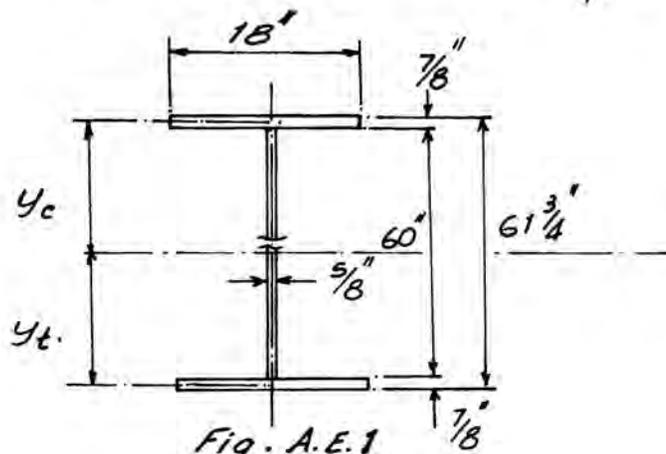


Fig. A.E.1

Plastic moment of resistance (P.M.R)

for flanges : $18 \times \frac{7}{8} \times 60 \cdot 875 \times \frac{15}{12} = 1198 \cdot 45$ tonf.ft.

" web : $\frac{0 \cdot 625 \times 60^2 \times 16}{4} \times \frac{16}{12} = 703 \cdot 13$ " "

total P.M.R = $1901 \cdot 58$ tonf.ft.

from page 20 of appendix D, we note that the max. Mp req'd. is 1849.6 tonf.ft. for 94.5 ft. span.

So, P.M.R. is greater than req'd Mp.

\therefore the section chosen appears to be adequate.

From appendix A, Part II para' 5.1, page A42 we have obtained that the max. shear force is 83.9 tonf at the support.

\therefore shear stress, $q = \frac{83 \cdot 9}{60 \times 0 \cdot 625} = 2 \cdot 24$ tonf/in².

Now reduced shear stress, $f_w = \sqrt{16^2 - (3 \times 2 \cdot 24)^2} = 14 \cdot 5$ tonf/in².

Thus $q < f_w$ \therefore may be adequate, although checks to be

made in the following pages.

$$\text{Required plastic modulus, } Z_{pr} = \frac{M_p \times f_y}{12} \text{ in}^3$$

$$\text{i.e., } Z_{pr} = \frac{1849.6 \times 12}{15} = 1479.7 \text{ in}^3$$

$$\text{Computed } Z_p = 2 \left\{ (18 \times 0.875 \times 30.437) + (30 \times 0.625 \times 15) \right\}$$

$$\text{or, } Z_p = 1521.4 \text{ in}^3$$

$$\text{Reduction in } Z_p = \frac{r t d^2}{4}, \text{ where,}$$

$$r = 1 - \sqrt{\left[1 - \frac{39^2}{f_y^2}\right]} = 0.04$$

$$\text{so, reduction} = \frac{0.04 \times 0.625 \times 60^2}{4} = 22.5 \text{ in}^3$$

$$\therefore \text{Nett Computed } Z_p = (1521.4 - 22.5) = 1498.9 \text{ in}^3$$

Thus, $Z_{pr} > Z_p \therefore \text{OKAY.}$

Therefore, the section chosen is satisfactory, but as mentioned before, load-factor design of flexural members assumes that the compact sections are capable of forming plastic hinges, which, when the plastic moment is attained, rotate under increasing loads with little or no change in moment; so, the following requirements are to be checked to insure the performance stated above. The following requirements are recommended by "Steel Designer's Handbook" (19).

1. Width-thickness ratio of flange projection should not

$$\text{exceed } \frac{B}{2t} = \frac{50.6}{\sqrt{f_y}} \approx 13$$

$$\text{Check } \frac{18}{2 \times \frac{7}{8}} = 10.285 < 13 \therefore \text{OK.}$$

2. Depth-thickness ratio of the web plate should not

$$\text{exceed } \frac{d}{t} = \frac{421}{\sqrt{f_y}} \approx 108.7$$

$$\text{check the ratio provided } \frac{60}{0.625} = 96 < 108.7 \therefore \text{OK.}$$

3. The compression flange should be supported laterally at intervals not exceeding the distance

$$L = \frac{2000 A_f}{f_y d} = \frac{2000 \times 18 \times 0.875}{15 \times 60} = 350''$$

We have provided stiffeners @ 138" centres \therefore O.K.

4. Axial compression force should not exceed

$$P = 0.15 \times \text{Area of the section} \times f_y$$

$$\text{or, } P = 0.15 \times \{(2 \times 18 \times 0.875) + 60 \times 0.625\} \times 15 = 155.25 \text{ tonf.}$$

This axial compression force value (155.25 tonf) will never occur under any loading condition.

5. Maximum shear force should not exceed

$$V = 0.55 d \cdot t \cdot f_y = 0.55 \times 60 \times 0.625 \times 16 = 309.3 \text{ tonf.}$$

The max. shear force we can expect is 83.9 tonf. \therefore O.K.

Hence this section appears to be quite adequate.

6. using "Steel Designer's Handbook" formula,

$$M_u = \sigma_y Z_p = 15.0 \times 1498.9 = 1873.6 \text{ tonft}$$

Which is greater than computed plastic moment (which the girder should resist) 1849.6 tonft.

Rockey⁽³²⁾ stated that the critical stresses which will cause buckling under their combined action, on web plates stiffened by both horizontal & transverse stiffeners, may be predicted "with reasonable accuracy", by the following equation:

$$\left\{ \left(\frac{\sigma_{mc}}{\sigma_{erc}} \right) + \left(\frac{\sigma_{mb}}{\sigma_{erb}} \right)^2 + \left(\frac{\tau_m}{\tau_{er}} \right) \right\} = 1$$

Where, σ_{mc} = direct compressive stress
 σ_{mb} = Compressive bending stress
 τ_m = shear stress

These are the critical stresses which may cause buckling under their combined action.

σ_{erc} = critical uniform direct axial stress

σ_{erb} = Compressive edge stress causing buckling in the panel when loaded in pure bending

τ_{er} = Uniform shear stress to cause buckling

assuming all edges are simply supported:

$$\begin{aligned} \sigma_{erc} &= 4 \left\{ \frac{\pi^2 E}{12(1-\nu^2)} \right\} \left\{ \frac{t}{d} \right\}^2 \\ &= 4 \left\{ \frac{\pi^2 \cdot 13000}{12(1-0.3^2)} \right\} \left\{ \frac{0.625}{60} \right\}^2 \end{aligned}$$

or, $\sigma_{erc} = 4.6 \text{ tonf/in}^2$

$$\sigma_{erb} = 23.9 \left\{ \frac{\pi^2 E}{12(1-\nu^2)} \right\} \left\{ \frac{t}{d} \right\}^2$$

or, $\sigma_{erb} = 27.7 \text{ tonf/in}^2$

When $s \geq d$ (we have, $s = 138''$ & $d = 60'' \therefore s > d$)

$$\tau_{er} = \left\{ 5.35 + \frac{4d^2}{s^2} \right\} \left\{ \frac{\pi^2 E}{12(1-\nu^2)} \right\} \left\{ \frac{t}{d} \right\}^2$$

or, $\tau_{er} = 7.1 \text{ tonf/in}^2$

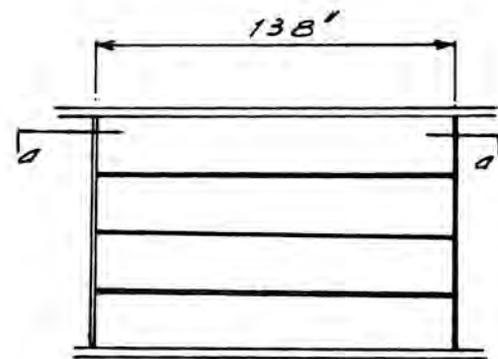
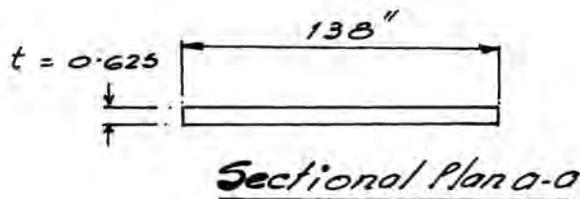


Fig. A-E.2

assuming that, the direct compressive stress is borne by the strip of that unstiffened portion of the web plate (as shown in sectional plan a-a of fig. A-E.2), We get

$$\text{area of the section} = 138 \times 0.625 = 86.25 \text{ in}^2$$

$$\text{max. load expected} = 83.9 \text{ tonf at the support}$$

$$\therefore \text{the direct compressive stress, } \sigma_{mc} = \frac{83.9}{86.25} = 0.972 \text{ tonf/in}^2$$

$$\text{Compressive bending stress} = \frac{P.M.R}{Z_p} = 15.22 \text{ tonf/in}^2$$

So, Substituting the values to equation stated in page 4, We get

$$\left\{ \left(\frac{0.972}{4.6} \right) + \left(\frac{15.22}{27.7} \right)^2 + \left(\frac{2.24}{7.1} \right) \right\} = 0.83$$

Which is less than 1 \therefore O.K.

Application of Ostapenko & Other's ⁽²⁹⁾ theory to find the Ultimate strength of the main plate girder:

Refer to Fig. A.E.1 & 2

Panel length, $s = 138''$

" depth, $d = 60''$

Comp. flange area = $18'' \times 0.875'' = 15.75 \text{ in}^2 = A_{fc}$

tension " " = $18'' \times 0.875'' = 15.75 \text{ in}^2 = A_{ft}$

Web plate " = $60'' \times 0.625'' = 37.50 \text{ in}^2 = A_w$

Unbraced length of the comp. flange, $L = 138''$

yield stress of comp. flange, $\sigma_{yf} = 15.0 \text{ tonf/in}^2$	} Obtained from B.C.S.A publication No 28 (1965).
" " " tension " , $\sigma_{yt} = 15.0 \text{ tonf/in}^2$	
" " " Web plate, $\sigma_w = 16.0 \text{ tonf/in}^2$	

Cross-sectional properties:

$I_{x-x} = 40434.867 \text{ in}^4$

$I_{web} = 11250 \text{ in}^4$

$A_{fc} = A_{ft} = 15.75 \text{ in}^2$

$A_w = 37.50 \text{ in}^2$

$y_c = y_t = 30.4375 \text{ in.}$

Non-dimensional parameters:

$\alpha = \frac{s}{d} = 2.3$; $\frac{d}{t} = 96$; $R = -\frac{y_t}{y_c} = -1$.

Loading for the combination of bending & shear: $\mu = \frac{M}{\sqrt{Vd}}$

$M = 1849.6 \text{ tonft} \rightarrow \text{from page 20}$

$V = 83.9 \text{ tonf} \quad \& \quad d = 60''$

$\therefore \mu = \frac{1849.6 \times 12}{83.9 \times 60} = 4.4 \text{ tonf.}$

Bending strength of the main girder:

Compression flange failure: Lateral buckling:

check: a) $\frac{B}{T} = 20.6$

b) $12 + \frac{L}{B} = \underline{19.7 < 20.6}$

When $\frac{B}{T} \leq 12 + \frac{L}{B}$, the buckling stress of the compression flange column, $\sigma_{cf} = \left(1 - \frac{\lambda_L^2}{4}\right) \sigma_{yc}$ for $0 < \lambda_L < \sqrt{2}$

Where $\lambda_L =$ lateral buckling parameter

* $\sigma_{yc} =$ yield stress of the compression flange. = 15 tonf/in²

$$\text{and } \lambda_L = L \sqrt{\frac{\sigma_{yc}}{E\pi^2} \cdot \frac{A_{fc} + \frac{1}{3} y_c \cdot t}{I_f}}$$

Where $I_f =$ 2nd moment of area of the compression flange

$$\text{or } \lambda_L = 138 \sqrt{\frac{15}{E\pi^2} \cdot \frac{15.75 + \frac{1}{3} \times 30.4375 \times 0.625}{\frac{1}{12} \times 0.875 \times (18)^3}} = 0.276$$

Thus, $\lambda_L < \sqrt{2}$

$$\text{Therefore } \sigma_{cf} = \left(1 - \frac{(0.276)^2}{4}\right) 15 = \underline{14.7 \text{ tonf/in}^2}$$

So, the yield stress of web, σ_w (which is 16 tonf/in²) $> \sigma_{cf}$
therefore use $\sigma_w = \sigma_{cf}$ and also check

$$\frac{y_c}{t} - 2.85 \sqrt{\frac{E}{\sigma_w}} = \frac{30.4375}{0.625} - 2.85 \sqrt{\frac{E}{14.7}} = -35.9$$

$$\text{Thus, use } \left(\frac{y_c}{t} - 2.85 \sqrt{\frac{E}{\sigma_w}}\right) = -35.9$$

Now the ultimate moment of the panel under pure bending,

$$M_u = \frac{I}{y_c} \cdot \sigma_{cf} \left\{ \frac{\sigma_w}{\sigma_{cf}} \left[\frac{I_w}{I} - 0.002 \frac{y_c t}{A_{fc}} \left(\frac{y_c}{t} - 2.85 \sqrt{\frac{E}{\sigma_w}} \right) \right] + \left(1 - \frac{I_w}{I}\right) \right\}$$

$$\text{or, } \underline{M_u = 59039 \text{ tonft.}}$$

The yielding of tension flange will be same as compression flange, because the sections are same.

Shear Strength of the main girder :

For $\alpha = 2.3 > 1.0$, plate buckling coefficient for pure shear, $k_v = 8.98 + \frac{6.18}{\alpha^2} - \frac{2.88}{\alpha^3} = 9.9$

Plastic shear of the web, $V_p = \frac{\sigma_w A_w}{\sqrt{3}} = 318.3 \text{ tonf.}$

Now the web buckling parameter,

$$\lambda = \frac{d}{t} \sqrt{\frac{12(1-\nu^2)}{\sqrt{3} \pi^2 E}} \frac{\sigma_w}{k_v} = 0.82$$

Hence $\lambda < \sqrt{2}$ but > 0.58 \therefore this is within elastic-plastic range. So, ultimate shear strength of the panel under pure shear, $V_u = V_p \left\{ [1 - 0.615(\lambda - 0.58)^{1.18}] + \frac{[0.6\lambda - 0.38]}{\sqrt{\lambda^2 + 1.6}} + \frac{\sqrt{3}}{2} \frac{A_{fcT} + A_{ftT}}{5 A_w} \left(\frac{\sigma_{yf}}{\sigma_{yw}} \right) \right\}$

Substituting all the values, we get,

$$\underline{V_u = 337.4 \text{ tonf.}}$$

Combined bending & Shear strength of the main girder:

The ratio of the max. tensile stress to the max. compressive stress, R , is negative when the stress is tensile and it has already been worked out that $R = -1$.

\therefore the plate buckling coefficient for pure bending,

$$K_b = 13.54 - 15.64(R) + 13.32(R)^2 + 3.38(R)^3$$

$$\text{or, } K_b = 14.6$$

Now, web buckling stress under pure buckling,

$$\sigma_{cp} = K_b \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{d} \right) = 18.6 \text{ tonf/in}^2$$

Now, the shear buckling stress for pure shear,

$$\tau_{cr} = \frac{\sigma_{yw}}{\sqrt{3}} [1 - 0.615(\lambda - 0.58)^{1.18}] = 2.7 \text{ tonf/in}^2$$

The shear buckling stress under combined load,

$$\tau_c = \tau_{cr} \frac{\sqrt{F(3-R)^2 + 16} - (1+R)\sqrt{F}}{2[2 + (1-R)F]}$$

Where F is a factor and $= \left(\frac{M d y_c A_w}{I} \cdot \frac{\tau_{cr}}{\sigma_{cp}} \right)^2$

Substituting all the computed values, we get, $F = 1.17$

Substituting the value of F in the equations, we get, $\tau_c = 1.6$

Now, the beam action shear under combined loads,

$$V_{rc} = \tau_c \times A_w \quad \text{that means } V_{rc} = 60.5 \text{ tonf.}$$

Ostapenko & others stated that, the tension field action contribution was found to vary very little (about 2%) due to the application of the bending stresses and so, the tension field action shear under the combined loads, V_{rc} equals the tension field action shear, under pure shear, V_r .

$$\text{i.e., } V_{rc} = V_r = 30.6 \text{ tonf/in}^2$$

$$\left\{ V_r = V_p \left[\frac{0.6\lambda - 0.38}{\sqrt{\lambda^2 + 1.6}} \right] = 30.6 \text{ tonf/in}^2 \right\}$$

The bending buckling stress at the extreme compression fibre of the web, $\tau_c = \left(\frac{M_d A_w y_c}{I} \right) \tau_c = 11.9 \text{ tonf/in}^2$

Frame action shear under combined loads, $V_{fc} = \left(1 - \frac{\tau_c}{\sigma_{yc}} \right) V_f$
 i.e., $V_{fc} = 0.37 \text{ tonf.}$

$$\left[\begin{array}{l} \text{Where } V_f = \text{frame action under pure shear} = V_p \left[\frac{\sqrt{3}}{2} \cdot \frac{A_{fc} T + A_{ft} T (\frac{\sigma_{yf}}{\sigma_{yw}})}{S A_w} \right] \\ \text{or, } V_f = 1.75 \text{ tonf.} \end{array} \right]$$

Now, the total shear strength of the panel is found as the sum of the individual contributions of all the shear, under combined load. i.e., $V_{th} = V_{rc} + V_{re} + V_{fc}$

$$\text{or } V_{th} = 60.5 + 30.6 + 0.37 = 91.47 \text{ tonf.}$$

Hence $V = 83.9 \text{ tonf} < V_{th} = 91.47 \text{ tonf.}$

Compression flange failure :

$$\lambda = 0.82 < \sqrt{2} \therefore \text{factor } B = 0.338\lambda - 0.196 = 0.081.$$

The incomplete tension field action shear under combined loads,

$$V'_{fc} = \frac{(A_{fc} + 30t^2)(\sigma_{ef} - \sigma_c) - \mu V_{fc}}{B \left(\frac{V_P}{V_c} \right) \left(\frac{180}{d/t} \right) \sqrt{\frac{33d}{\sigma_{yw} \sigma_c}} + \mu}$$

$$\text{or, } V'_{fc} = 8.8 \text{ tonf.}$$

$$\text{again, } V_{th} = 60.5 + 8.8 + 0.37 = 69.7 \text{ tonf}$$

Check maximum panel moment: according to Ostapenko & others, since in a panel under bending & shear, the moment at one end of the panel is greater than the design moment at mid-panel, this maximum moment may control the panel strength. It is sufficiently accurate to keep max. moment less than the moment which would produce yielding, according to the ordinary beam theory. So,

$$V_{th} = \frac{I \sigma_{yf}}{y_c \cdot d \cdot (\mu + \frac{1}{2} \alpha)} = \frac{40434 \times 15.0}{30.4375 \times 60 (4.4 + \frac{1}{2} \cdot 2.3)}$$

$$\text{or, } V_{th} = 59.84 \text{ tonf.}$$

Hence the ultimate strength of the selected section :

$$\text{from web failure } V_{th} = 91.47 \text{ tonf.}$$

$$\text{from Compression flange failure } V_{th} = 69.7 \text{ tonf.}$$

$$\text{from max. panel moment } V_{th} = 59.84 \text{ tonf.}$$

\therefore the max. panel moment failure is the mode of failure and the strength is 59.84 tonf. But, horizontal stiffeners are to be provided, as shown in elastic analysis, which will raise the I value of the section and thus will increase the V_{th} value.

Finally, the ultimate moment of the section,

$$M_{th} = \mu \cdot d \cdot V_{th} = 4.4 \times 60 \times 91.47 = 2012.3 \text{ tonft.}$$

Stiffeners : all formulas are obtained from A.A.S.H.O and these formulas have been already stated in previous chapters :

The transverse stiffeners are required for the web members not satisfying the following equations :

a. $V = 0.55 dt \sigma_y = 330 \text{ tonf}$ greater than expected shear force, 83.9 tonf.

b. $V = \frac{3.5 Et^3}{D} = 180 \text{ tonf}$ also greater than expected shear force, 83.9 tonf.

c. $V_p = 0.58 D t \sigma_y = 358 \text{ tonf}$.

The web depth-thickness ratio with transverse stiffeners should not exceed

$$\frac{D}{t} = \frac{1155}{\sqrt{\sigma_y}} = 288.75, \text{ but } \frac{D}{t} = 98.8.$$

The stiffener space should not exceed, $S = 458 \sqrt{\frac{Dt^3}{V}} = 245.5'$ but, we have taken $S = 138''$.

Maximum width-thickness ratio for the transverse stiffener

is $\frac{b'}{t} = \frac{82.3}{\sqrt{\sigma_y}}$

or, $t = \frac{b' \sqrt{\sigma_y}}{82.3} = 0.437''$

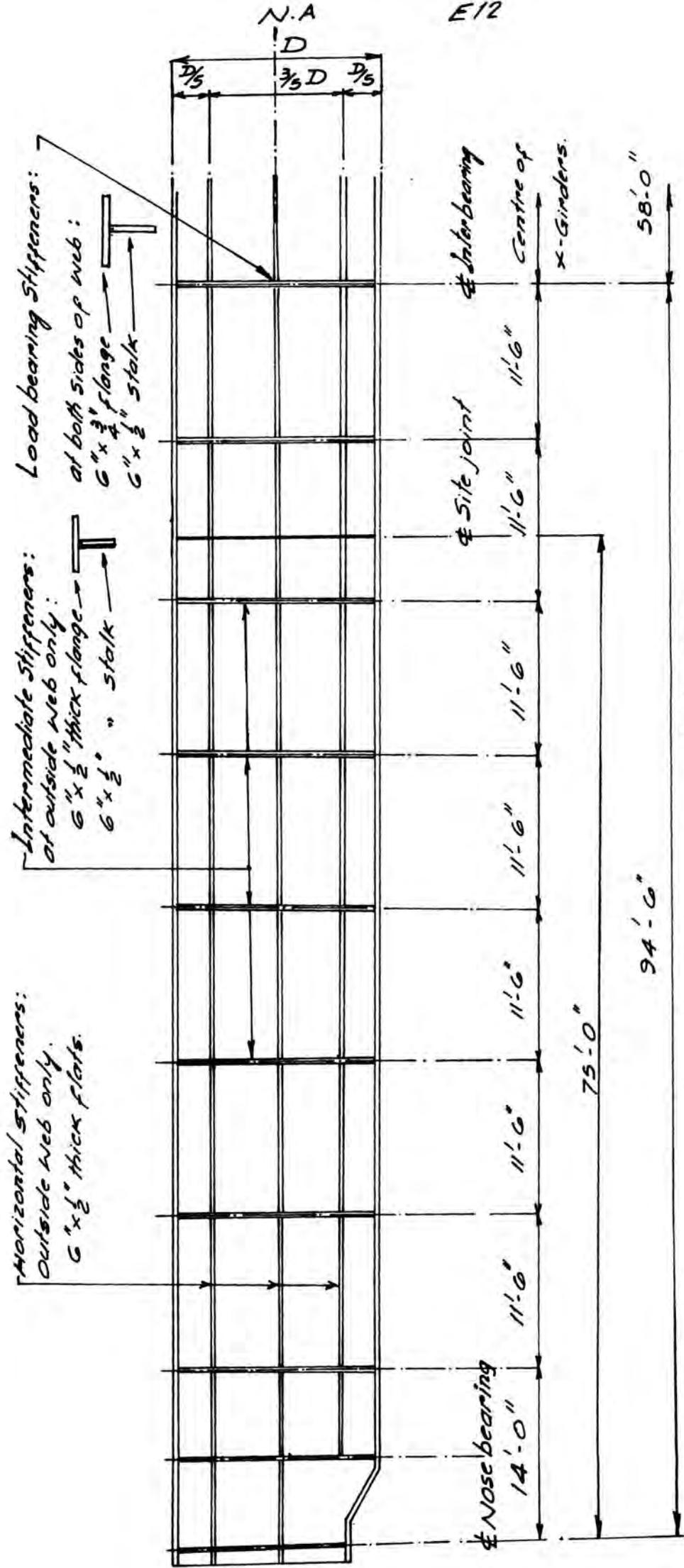
ie., the thickness (minimum) of the transverse stiffener should be 0.437''.

Longitudinal stiffener is required when the web depth-thickness ratio exceeds the equation : $\frac{D}{t} = \frac{1155}{\sqrt{\sigma_y}}$.

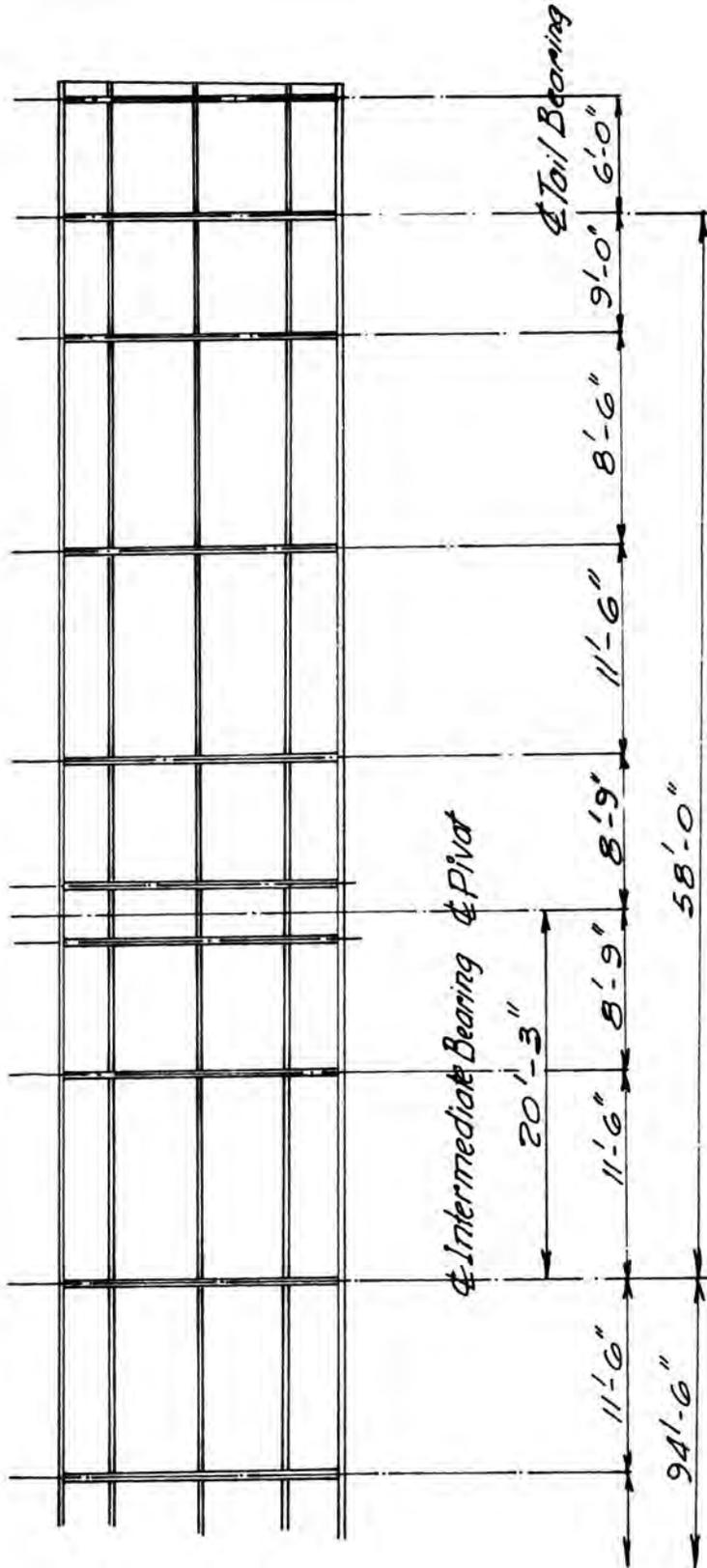
So, it is not necessary to provide any longitudinal stiffener. But due to the lower $V_{th} = 59.84 \text{ tonf}$ value (see page 32), a horizontal stiffener at the $\frac{D}{5}$ (ie., = 12.35'') distance from the inner surface of the compression flange, is desirable.

N.A
D

E12



Elevation of Main Plate Girder

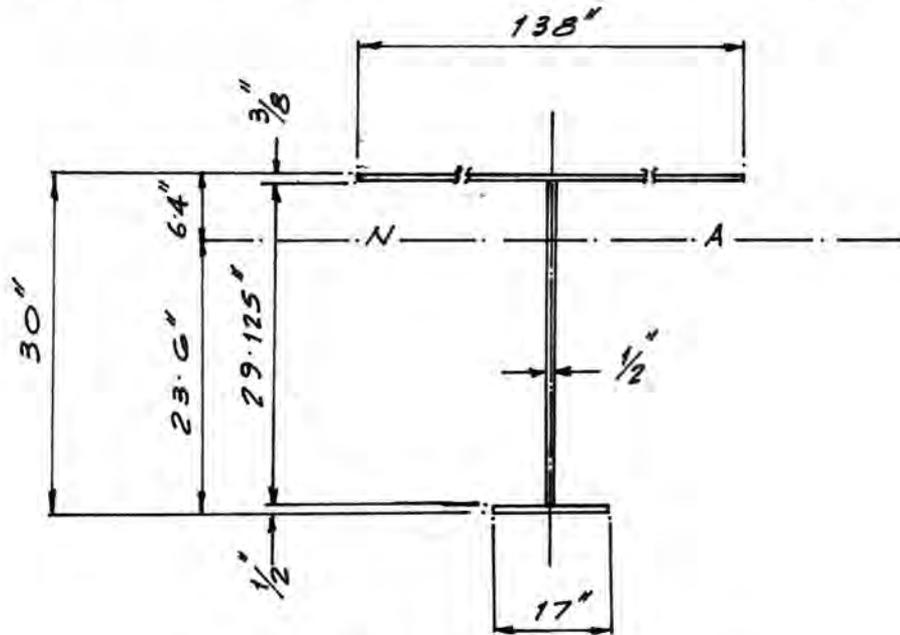


Elevation of Main Plate Girder

Appendix F

Typical floor beam with deck plate: Refer to chapter 5.3.

From appendix Part 2, the following properties are obtained, if the same section to be considered on limit state analysis:



Typical section of floor beam

$$\text{Area of the section} = 74.81 \text{ in}^2$$

$$\bar{y} = 23.6 \text{ from bottom flange}$$

$$\text{Loads} = 2.96 \text{ tonf/ft.}$$

Plastic modulus of the section:

$$Z_p = [138 \times \frac{3}{8} \times 6.2] + [29.125 \times \frac{1}{2} \times 14.56] + [17 \times \frac{1}{2} \times 23.35]$$

$$\text{or, } Z_p = 687.67 \text{ in}^3$$

$$\text{Full plastic moment, } M_p = \lambda \frac{\omega L^2}{12} = \frac{1.7 \times 2.96 \times 25^2}{12}$$

$$\text{or, } M_p = 262 \text{ tonf ft.}$$

$$\text{Required } Z_p = \frac{M_p}{f_y} = \frac{262 \times 12}{15.7/\text{in}^2} = 209.6 \text{ in}^3$$

Hence the actual plastic modulus value is far greater than required plastic modulus.

The effect of H.B. loading system has not been checked in this part, because by inspection, it is evident that, moments due to H.B. loading system are not critical.

Effects of shear force: It is assumed, as in elastic theory, that the shear is resisted by the web. Evidence exists elsewhere⁽³⁶⁾ to the effect that due to shear the yield stress f_y should be reduced from f_y to f'_y , i.e.,

$f'_y = \sqrt{f_y^2 - 3q^2}$, where q is the mean shear stress and must not exceed $f_y/\sqrt{3}$.

$$\text{Now, } q = \frac{\Sigma \text{load}}{\text{area of the section}} = \frac{2.96 \times 25}{74.81}$$

$$\text{Hence } q = 0.99 \text{ tonf/in}^2 < f_y/\sqrt{3}$$

$$\text{and reduced } f_y = \sqrt{15^2 - 3(0.99)^2} = 14.88 \text{ tonf/in}^2$$

Appendix G

Inelastic Buckling of Steel Deck

The following methods have been adopted from the A.I.S.C.⁽¹⁸⁾ handbook :

1. Local inelastic buckling of rib plate :

Let f_{cr} = critical stress

f_y = yield stress of the rib plate = 16 tonf/in²

f_p = proportional limit stress = 0.75 f_y = 12 tonf/in²

& f_i = ideal stress

$$\text{Ideal stress, } f_i = K \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t'}{h}\right)^2$$

Where K = a constant depends on the loading and edge conditions

$E = 13\,000$ tonf/in² & $\nu = 0.3$ for mild steel.

$t' = 12.37''$ taken from appendix B

$$\frac{f_{cr}}{f_y} = \frac{1}{1 + \frac{f_p}{f_y} \left(1 - \frac{f_p}{f_y}\right) \left(\frac{f_y}{f_i}\right)^2}$$

a. Sides of rib : The ribs are fixed at deck plate and it is assumed that the side of ribs are simply supported.

So, K is taken as 5.4

Therefore $f_i = 40.5$ tonf/in²

and $f_{cr} = 15.5$ tonf/in²

b. Bottom of rib : assuming both sides of the rib are simply supported, $K = 4.0$ should be taken. The buckling stress in the bottom of the rib will not be critical, unless the bottom width of the trapezoidal closed rib is greater than $0.86 t'$.

Check : $0.86 \times 12.37'' = 10.6''$ where as, we have assumed that the bottom width of the rib is 6''.

2. Overall Inelastic Buckling of the Deck :

$$\text{Ideal stress, } f_i = \frac{\pi^2 E}{\left(\frac{S}{r}\right)^2}$$

where, S = Spacing of floor beams = 138"

r = radius of gyration of a rib.

2nd moment of area about the neutral axis of the rib, with deck plate, $I_{N-A} = 374 \text{ in}^4$ taken from appendix B

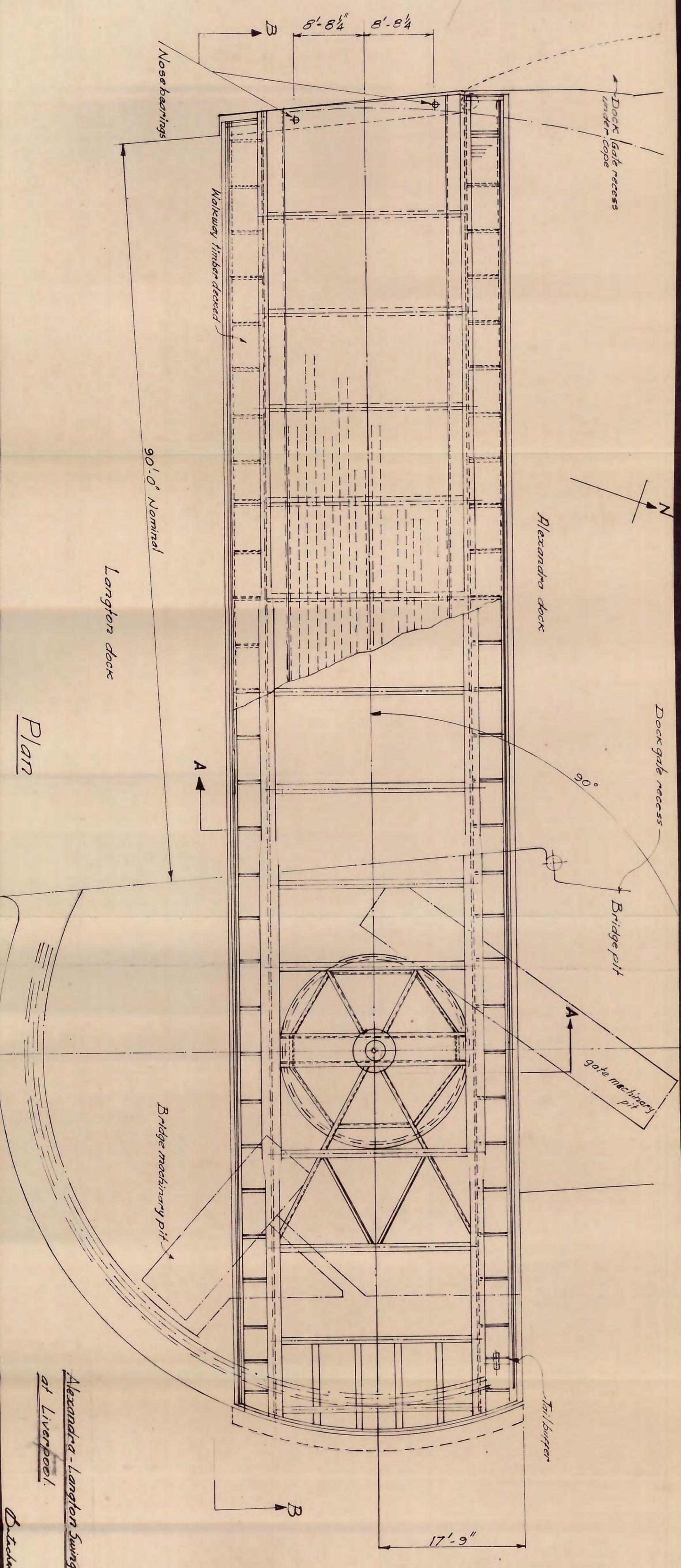
Area of rib with deck plate = 19.01 in² taken from appendix B

$$\therefore r = \sqrt{\frac{374}{19.01}} = 4.4$$

$$\therefore f_i = 130 \text{ tonf/in}^2$$

$$\frac{f_{cr}}{f_y} = 1 - \frac{f_p}{f_y} \left(1 - \frac{f_p}{f_y}\right) \frac{f_y}{f_i} = 0.98$$

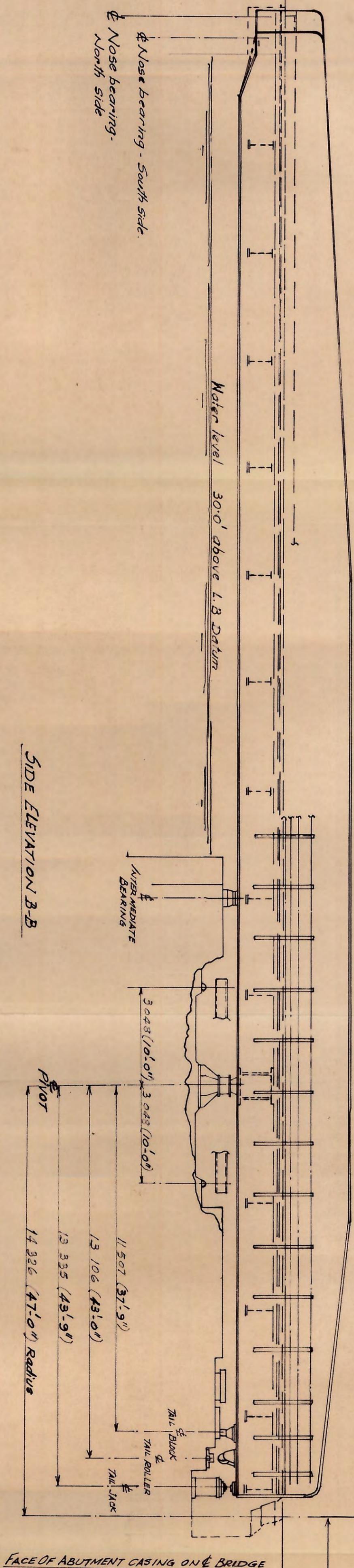
\therefore the critical stress, $f_{cr} = 15.7 \text{ tonf/in}^2$



Plan

Alexandra - Langton Swing Bridge
at Liverpool.

Butcher



SIDE ELEVATION B-B