Supernovae and the origin of cosmic rays

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SUPERNOVAE AND THE ORIGIN OF COSMIC RAYS

A thesis submitted to the University of Durham for the degree of Master of Science

by

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To my parents

ABSTRACT

This thesis contains a review of our present knowledge about the cosmic ray flux and about supernovae and their remnants. It contains an attempt to examine the widely held belief that cosmic rays are produced principally in supernova remnants by work on essentially four topics connected with the production of cosmic rays in supernova remnants:-

a) The likely surface density of supernova remnants in the Galaxy as a function of distance from the Galactic centre. It is shown that the general shape of the surface density curve consists of a large peak near the Galactic centre followed by a fairly rapid fall to a low value at the edge of the Galaxy. The actual values of surface density are very dependent on the distances of the observed supernova remnants, which in turn depend on the adopted surface brightness - diameter relationship.

b) The predicted slope of the electron spectrum between energies of 4×10^8 and 10^{10} eV, assuming it results from the addition of the energy spectra of all the supernova remnants in the Galaxy. The observed electron energy spectrum in this range is at present too uncertain to be able to say whether or not it disagrees with that predicted.

c) The time of production of cosmic rays in supernova remnants, if they are to traverse no more matter than that which is inferred from the ratio of abundances of the L to M groups in the cosmic rays. It is found that if the cosmic rays are produced inside the expanding remnant, then the matter traversal constraint means that they must be produced 2 years or so after the explosion.

d) The expected distribution of gamma rays in the Galaxy from supernova remnants of diameters less than 50 parsecs. It is shown that in all probability the gamma ray results give direct evidence of the presence of cosmic rays in supernova remnants but that supernova remnants alone cannot be responsible for the whole of the gamma ray flux, at least on present gamma ray production models.

PREFACE

The work presented in this thesis was carried out during the period October 1973 to September 1975, while the author was a research student under the supervision of Doctor J.L. Osborne, in the Department of Physics of the University of Durham.

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CHAPTER 1 INTRODUCTION

1.1 General remarks

If the bulk of the cosmic ray flux is to be Galactic in origin then it seems to have become generally accepted that cosmic rays are in some way associated with supernova explosions. The synchrotron radiation observed to come from known supernova remnants, such as the Crab Nebula, is a direct indication of the presence of relativistic electrons in such remnants, and it therefore seems likely that relativistic nuclei are also present. Since their discovery in 1968 pulsars have also been considered to be likely producers of cosmic rays, but since they themselves are almost certainly formed in supernova explosions they are really just an extra cause (indeed possibly the main cause) of particle acceleration in supernova remnants.

Normal stars like the sun are sources of cosmic rays of energy up to about 10^{11} eV but the mean flux of cosmic rays arriving directly from the sun in the relativistic energy range ($>10^{9}$ eV) does not exceed, in order of magnitude, 0.1 percent of the total cosmic ray flux. Ordinary nova explosions could possibly generate particles of energy up to about 10^{14}eV, whereas the cosmic ray energy spectrum extends up to at least 10^{20}eV.

Assuming that an explosion of the Galactic core gives out 3×10^{55} erg in cosmic rays (Burbidge and Hoyle, 1963), then an explosion of the Galactic core about once every 5×10^{7} years could give the cosmic ray energy density observed. If however relativistic particles are associated with normal stars, novae and exploding Galactic cores, then they must certainly be associated with supernova explosions.

Since the energy output and frequency of supernovae is sufficient to provide the observed cosmic ray energy density and since it is known that relativistic electrons exist in supernova remnants then it would seem that supernovae may well be the main contributors to the cosmic ray flux on a Galactic origin theory of cosmic rays.

The aim of this thesis is to look for direct evidence that cosmic ray nuclei come from supernovae.

1.2 Energy balance and requirements for cosmic ray sources

The radius of the Galaxy $R \simeq 15$kpc and the thickness of the Galaxy $d \simeq 600$ pc. The volume of a spherical region of this radius is:

$$V_h = \frac{4}{3} \pi R^3.$$

$$= 4 \times 10^{68} \text{ cm}^3$$

This volume is the volume which the cosmic rays fill if one assumes they are confined to a spherical halo around the Galaxy.

If cosmic rays are confined only to the disc of the Galaxy, then they fill a volume:

$$V_d = \pi R^2 d$$

$$= 1.25 \times 10^{67} \text{ cm.}^3$$

The mean energy density of cosmic rays in the Galaxy is usually taken to be the same as that near the earth, ie. $W_g = 10^{-12}$ erg cm^{-3}

Cosmic rays are believed to be trapped in the Galaxy for times of the order of 10^6 to 10^7 years if confined to the Galactic disc, and for approximately 3×10^8 years if confined to a Galactic halo.

One can therefore estimate the rate at which cosmic rays must be supplied to the Galaxy in order to maintain the observed energy density assuming disc or halo confinement.

a.) Disc confinement

The total energy of cosmic rays in the disc

$$= V_d \times W_g = 1.25 \times 10^{55} \text{ erg.}$$

The confinement time in the disc is $T_d = 10^6$ to 10^7 years, so that the rate at which energy must be supplied is:

$$U = \frac{V_d \times W_g}{T_d} = 4 \times 10^{40} \text{ to } 4 \times 10^{41} \text{ erg s}^{-1}$$

b.) Halo confinement

The total energy of cosmic rays in the halo

$= V_h \times W_g = 4 \times 10^{56}$ erg.

The confinement time in the halo is $T_h = 3 \times 10^8$ years, so that the rate at which particles must be supplied is:

$$U = \frac{V_h \times W_g}{T_h} = 4 \times 10^{40} \text{ erg s}^{-1}$$

The energy converted into cosmic rays in a supernova explosion is likely to be of the order of 10^{49} to 10^{50} erg. The actual figure depends on the assumed ratio of ions to electrons in supernova remnants. Only the total energy of electrons in supernova remnants is known from their synchrotron radiation, and is generally of the order of 10^{48} erg. It is generally assumed that the ratio of ions to electrons in a supernova remnant is 100, though this figure is simply an assumption based on the observed ratio in the cosmic ray flux and on general considerations and observations relating to the Galaxy as a whole. It should however be noted that in the most well known supernova remnant, namely the Crab Nebula, this ratio is believed to be 1.

The frequency of supernova explosions in the Galaxy is believed to be about 1 every 30 years (Tammann, 1970).

Therefore the rate at which energy in the form of relativistic particles may be supplied by supernovae is simply the cosmic ray energy output per supernova multiplied by the frequency of supernovae giving

$U_{sn} = 10^{40}$ to 10^{41} erg s^{-1}.

One can see that the energy output in relativistic particles by supernovae is sufficient to maintain the observed cosmic ray energy density on either a disc or halo confinement model.

For comparison one can do the same calculation for nova explosions. The total energy of cosmic rays generated in nova explosions is probably about 10^{45} to 10^{46} erg.
The frequency of nova explosions is very uncertain and lies in the range 50 to 200 per year. Hence the energy output in relativistic particles by novae is. $U_n = 10^{39}$ to 10^{41} erg s^{-1}.

Thus novae could just about supply the required energy density of cosmic rays. However since novae could not accelerate particles to as high an energy as supernovae and since supernovae can more easily satisfy the required energy density, their contribution of relativistic particles to the cosmic ray flux is probably insignificant compared to that of supernovae.

Discussions of the various possible Galactic sources of cosmic rays and the above type of arquments can be found in chapter IV of Ginzburg and Syrovatskii (1964).

1.3. Extragalactic Cosmic Rays

While most people concerned with the origin of cosmic rays would agree that supernova explosions are the main source of cosmic rays produced in the Galaxy, there is a school of thought which maintains that the bulk of the cosmic ray flux is extragalactic in origin. A detailed analysis of the arguments in favour of extragalactic cosmic rays can be found in Brecher and Burbidge (1972). The reason extragalactic sources have been sought is in an attempt to explain the fact that high energy protons of energies between 10^{11} and 10^{14} eV are extremely isotropic with $(I_{max} - I_{min}) / (I_{max} + I_{min}) \approx 10^{-4}$ to 10^{-3}, where I max and I min are the maximum and minimum secondary particle intensities in one cycle of observation (one sidereal day). Brecher and Burbidge (1972) claim that when the low anisotropy is considered in conjunction with the short storage or galactic traversal time of cosmic ray heavy nuclei derived from the (Li, Be, B) / (C, N, O) ratio (Shapiro, 1970) and the equally short proton lifetime in the Galaxy derived from the secondary positron / electron ratio (Fanselow et al., 1969), the production and storage of cosmic rays within the Galactic disc appears highly unlikely.

Brecher and Burbidge (1972) admit that the electron - positron component of cosmic rays with energy above 500 MeV cannot be universal because of the microwave background radiation. The maximum linear distance Rmax travelled by an electron from a distant source to the earth is given by:

$R_{max} = 1.57 \times 10^{-4} (\rho E)^{-1}$ parsecs where ρ is the density of background radiation and magnetic field in erg cm^{-3}, and E is the initial electron energy in GeV. Using this expression it can easily be shown that the bulk of the electrons in primary cosmic rays could not have reached us from external galaxies.

If however the nucleonic component of cosmic rays is to be universal, then the ratio of the cosmic ray energy density (10^{-12} erg cm^{-3}) to the rest energy density in visible condensed matter (3×10^{-31} g cm^{-3}) must be about 4×10^{-3}. This implies that each galaxy of mass 10^{11} Mo (Mo = 1 solar mass) must on average emit nearly 10^{63} erg of relativistic particles in a time of approximately Ho^{-1}, where Ho is the Hubble constant ($Ho^{-1} \simeq 10^{10}$ years). The evidence for the hypothesis that the cosmic rays can be universal comes from the properties of strong radio galaxies where the minimum energy caculations lead to the view that the total energy per galaxy exceeds 10^{61} erg (approximately equally divided between particles and magnetic flux with a nucleon -electron ratio assumed to be about 100), and that these cosmic rays have been generated on a short timescale (10^6 to 10^7 years).

Since galaxies appear to form clusters and superclusters, the cosmic ray energy density which we observe may simply be characteristic of our own cluster of galaxies and need not fill the whole of the universe. This implies a metagalactic containment volume for the local cosmic rays. It is possible to deduce the size of this containment volume as described for example in Lingenfelter (1973).

The most severe limit on the size of a possible metagalactic containment volume for the local cosmic rays comes from the upper limits on the background gamma ray intensity at energies of 100 MeV and above (Clark et al., 1970).
Since the cosmic ray nucleons interacting with intergalactic gas would produce neutral pi - mesons which decay to gamma rays in this energy range, the measured upper limit to the gamma ray

intensity places a bound on the product of the cosmic ray energy density, the intergalactic gas density and the linear dimension of the containment volume. Using the local cosmic ray density of about 10^{-12} erg cm^{-3} and assuming an intergalactic gas density of about 10^{-5} hydrogen atoms per cubic centimetre, then the measured upper limit on the gamma ray intensity requires that the dimensions of the containment volume be less than or equal to 30Mpc.This is of the same scale as the Virgo supercluster of galaxies, which Brecher and Burbidge (1972) suggest as a metagalctic containment volume for the bulk of the local cosmic rays.

The minimum metagalactic cosmic ray power density needed to fill such a volume, taking a maximum cosmic ray life equal to the Hubble time ($\simeq 10^{10}$ years) is: $P_{mg} \geq 3 \times 10^{-30}$ erg cm^{-3} s^{-1}.

This is three orders of magnitude less than the required galactic power density, where the confinement time in the Galaxy is expected to be about 10^{7} years.

The total cosmic ray power in this containment volume of about 3×10^{78} cm^{3} is : $U_{mg} \geq 10^{49}$ erg s^{-1}.

This is more than 10^{8} times greater than the required galactic power, and since there are less than 10^{4} galaxies in the supercluster it is obvious that galaxies such as our own are not typical sources, at least at the present time.

Brecher and Burbidge (1972) suggest that if all of the galaxies go through a Seyfert phase during which they emit about 10^{61} erg in nonthermal emission from relativistic electrons, and if their source power in cosmic ray nucleons is about 100 times that in electrons, then the total cosmic ray power in the supercluster could average better than the required 10^{49} erg s^{-1}.

Though one cannot rule out the possibility that the cosmic ray density in the Galaxy extends throughout the whole universe or a restricted part of it such as the local supercluster, the extragalactic cosmic ray origin theory relies on even more uncertain parameters than the Galactic theory. The intergalactic gas density and magnetic field are certainly not known as well as the Galactic gas density and magnetic field. There is little doubt that certain extragalactic objects must produce high energy particles (eg. radio galaxies, Seyfert galaxies, quasars) and no doubt some of these particles reach our Galaxy, but there is also little doubt that supernova explosions produce high energy particles which reach the earth. The argument is really about whether the main contribution to the cosmic ray flux at the earth comes from Galactic or extragalactic sources. In the absence of definite evidence for or against either origin theory it will be assumed in the rest of this thesis that cosmic rays in the main result from Galactic sources, in particular supernova remnants.

1.4 Recent evidence in favour of a Galactic origin of cosmic rays.

A recent paper by Dickel (1974) describes some work which suggests that supernova remnants provide the cosmic ray electrons in the Galaxy. By means of an analysis of recently improved parameters of the radio structure and evolution of supernova remnants he determined the total radio luminosity of the relativistic electrons from all the SNR's in the Galaxy. Although the results are uncertain to within a factor of 10, the radio luminosity of all the electrons which have escaped from the old supernova remnants appears to be equal to that of the general galactic background.

According to data analysed by Baldwin (1967), the integrated luminosity of the Galaxy at 400 MHz (assuming a radius 15 Kpc) is 5.8×10^{21} W Hz $^{-1}$. This estimate should not be uncertain by more than a factor of 2 or 3. Taking account of the way the luminosity of a typical supernova remnant would vary with time during the three main phases of its evolution, and assuming a rate of supernova explosions of 1 every 50 years, Dickel obtains a value of

$$L_{\nu} = 8.0 \times 10^{21} \text{ W Hz}^{-1}$$

as the integrated luminosity at 400 MHz from all of the relativistic electrons produced in supernova remnants. This value is uncertain to within a factor of 13.

Since supernova remnants appear to be able to supply the cosmic ray electrons in the Galaxy, it lends more credence to the idea that they can also supply cosmic ray nuclei.

CHAPTER 2 OBSERVED PROPERTIES OF SUPERNOVAE

2.1. General remarks on observed properties of supernovae

Nova and supernova explosions have been observed by astronomers for many centuries being recorded as very bright stars, often visible in daylight, which had suddenly appeared in the sky in places where only faint stars or no stars at all were previously visible. All such phenomena were called novae until 1933 when W. Baade and F. Zwicky proposed that there was a more powerful group of nova explosions, which they designated supernovae. This became apparent from studies of nova explosions in external galaxies, since novae in a particular galaxy are all essentially at the same distance from the earth. Supernova and nova explosions at the same distance can differ by ten magnitudes.

Zwicky classified supernovae into five types based on their light curves and spectra. About 36% of observed supernovae in spiral galaxies belong to type I, 54% to type II and 10% to the other three types. Type I supernovae are observed in all types of galaxies, whereas type II occur predominantly in spiral galaxies and never in elliptical galaxies. The spectra of type I supernovae are characterised by the absence of hydrogen, which has led to belief that they are the result of the explosion of old stars approximately 10^{10} years old. Such old stars would have a narrow mass range $1.16\ M_\odot < M < 2M_\odot$, and this could account for the uniformity of the type I light curve. Fig. (2.1) shows a comparison of typical light curves of types I, II and III supernovae.

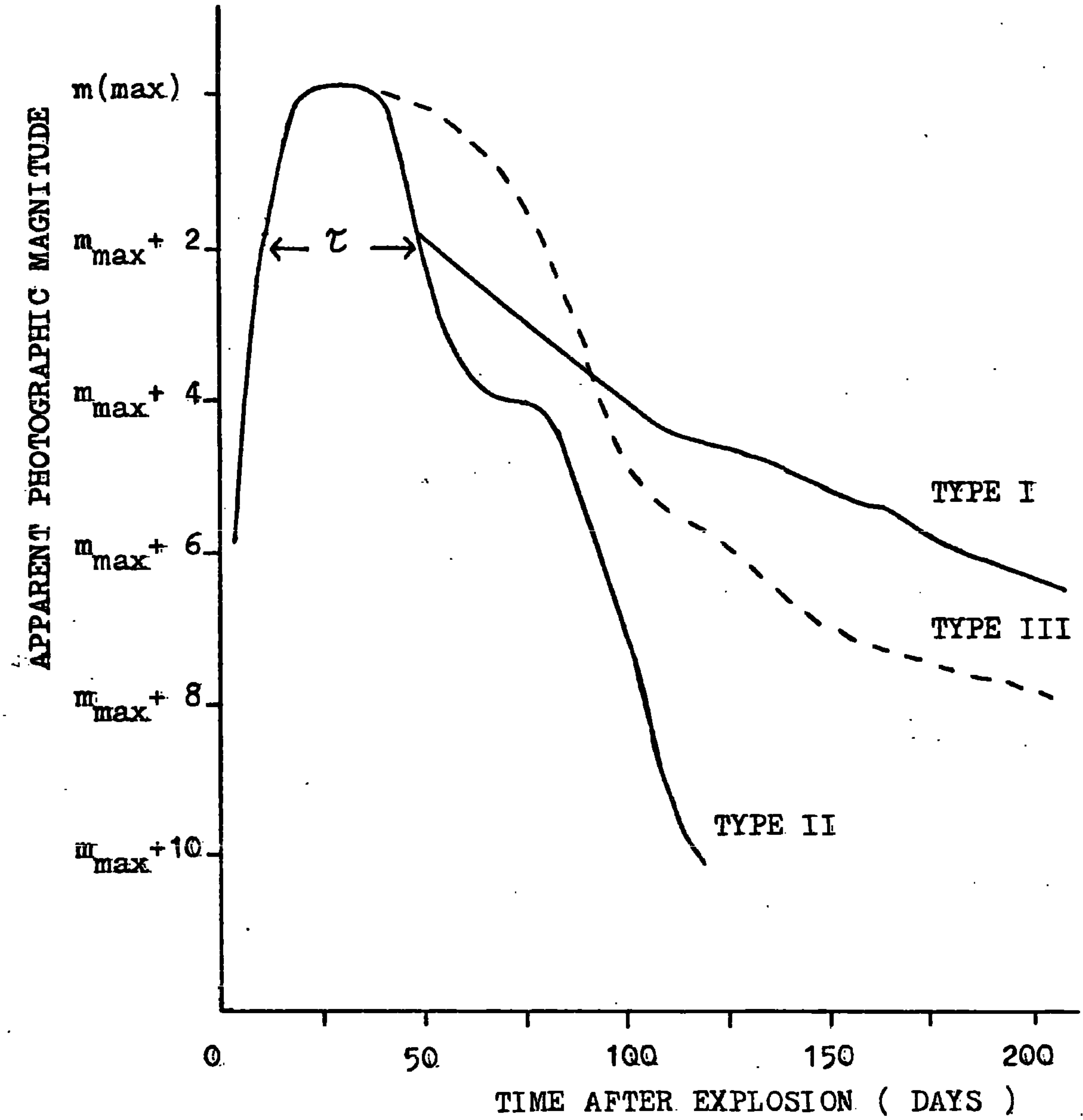


Fig. 2.1. TYPICAL SUPERNOVA LIGHT CURVES.

τ = 40 TO 50 DAYS TYPICALLY.

Supernova light curves are characterised by the very steep rise to maximum and the period of maximum brightness lasting typically 40 to 50 days. It can be seen that the types differ mainly in the length of time taken for the light intensity to die away, the type I supernova taking the longest time to decay in brightness.

Type II supernovae are believed to result from the collapse of comparitively young stars approximately 10^8 years old, having large masses, ie. greater than 2 M⊙ and possibly as much as 20 to 30 M⊙. The wide range of possible masses of type II supernovae could account for the wide variation in their light curves. Type III supernovae are simply very big type II explosions in which a mass equal to several tens of solar masses must be ejected. Fig (2.2) summarises in tabular form the typical parameters of novae, type I and type II supernovae The sources of information for this table and the light curves in fig. (2.1) are Weekes (1969) and Shklovsky (1968).

2.2 Notes on individual remnants

The most well known supernova remnant in the Galaxy is the Crab Nebula. The supernova explosion responsible for it was carefully observed and recorded by Chinese astronomers in 1054. Its high radio brightness and the fact that it lies in a convenient position in the sky for observation means that it is probably also the most well studied supernova remnant.
The discovery of a pulsar in the Crab Nebular in 1968 by Staelin and Reifenstein has made it of even more interest. The Crab Nebula lies at a distance from the earth of about 2 Kpc and its diameter is about 2 or 3 pc. Its mass may be anywhere between 1 and 10 M⊙.

	NOVAE	TYPE I SUPERNOVAE	TYPE II SUPERNOVAE
MASS ($M_\odot$) EJECTED	$10^{-6} \rightarrow 10^{-3}$	$0.1 \rightarrow 1$	$1 \rightarrow 10$
VELOCITY OF SHELL ($km\ s^{-1}$)	$300 \rightarrow 1000$	$10,000 \rightarrow 20,000$	$5000 \rightarrow 10,000$
KINETIC ENERGY OF SHELL (erg)	$9x10^{41} \rightarrow 10^{46}$	$10^{50} \rightarrow 4x10^{51}$	$2.5x10^{50} \rightarrow 10^{52}$
ABSOLUTE MAGNITUDE AT MAXIMUM	$-6 \rightarrow -9$	-19.0 ± 0.3	-17.7 ± 0.3
CHANGE IN MAGNITUDE	11	15	13 - 14
RATE PER YEAR IN GALAXY	$50 \rightarrow 200$	1/60	1/40
ASSOCIATED STELLAR TYPE	Population II, hot sub-dwarfs, binary pair of red star and U.V. dwarf.	Population II, age = 10^{10}yrs. $1.16M_\odot < M < 2M_\odot$	Population I, age = 10^8yrs. $M > 2M_\odot$

Fig. 2.2. TYPICAL NOVAE AND SUPERNOVAE PARAMETERS.

The expansion velocity of the remnant is 1700 Km. s^{-1} along the major axis and 1100 Km. s^{-1} along the minor axis. The expansion of the nebula is in fact accelerating at a rate of about 0.0014 cm. s^{-2}. Swept up interstellar matter accounts for only a few percent of the mass of the nebula.

The pulsar which exists at the centre of the Crab Nebula is the youngest on record and has the shortest period, which is in fact 0. 033 seconds. Optical, infrared, X - ray, and γ - ray emission emanates from the pulsar. The Crab nebula does not seem to be typical of either of supernova types I or II because of its low expansion velocity and its irregular shape, showing no signs of shell structure. It probably belongs to the 10% or so of strange supernovae forming types III, IV and V on the Zwicky classification.

A typical type I supernova is Tycho's supernova. The supernova outburst giving rise to the present remnant occurred in 1572 and was carefully observed by Tycho Brahe as well as others. It is believed to lie at a distance of about 3 Kpc from the earth. Its present diameter is about 6 pc, and it is expanding at a velocity of approximately 13000 Km. s^{-1}. Another type I supernova was Kepler's supernova, which was observed in 1604 and left a remnant which lies at a distance of 8 Kpc. Its diameter is about 7 pc, and its expansion velocity is approximately 18000 Km. s^{-1}.

The strongest radio source in the sky apart from the sun is also believed to be a supernova remnant. This is the remnant Cassiopeia A. This remnant lies at a distance of about 2.8 Kpc and its diameter is 3.2 pc. The supernova outburst which gave rise to this remnant was not observed optically, but a study of proper motions in the remnant indicate that it exploded in 1667 $\pm$ 8 A.D.

The reason it was not observed is probably because it exploded in a region of the Galaxy where strong optical obscuration occurred. The expansion velocity is about 10,000 Km. s^{-1}.

The Cygnus Filaments is a very old remnant probably of a type II supernova, according to Shklovsky (1968), which occurred about 70 000 years ago. Distinguishing between the remnants of types I and II supernovae when the remnants are very old is not very easy. In general the remnants containing a large amount of matter and of very old age are believed to be of type II. Only when one has records of the light curve of the observed supernova itself, can one be sure of the type. In general more energy is imparted to the shell (which is also of larger mass) in a type II supernova explosion, than in a type I. One would therefore expect remnants of type II supernovae to last longer. The present diameter of the Cygnus filaments is about 40 pc and it lies at a distance of about 800 pc. The expansion velocity at the outer boundary of the filaments is 115 Km. s^{-1}. This remnant is an example of a supernova remnant in its final stages of evolution.

IC 443 is another old remnant of a type II supernova, lying at a distance of about 1.5 Kpc. It is about 60,000 years old, has a diameter of 20 pc and a velocity of expansion of 65 Km. s^{-1}. The shell structure of this remnant is well defined at both optical and radio wavelengths.

The other supernova remnant worth mentioning is the Vela supernova remnant. This remnant lies only 500 pc away, is 30 pc in diameter and has an age of about 10^4 years. The notable thing about this remnant is that it contains a pulsar PSR 0833, which has a period of about 0.0892 seconds.

Only the Crab Nebula and Vela are known to be definitely associated with pulsars, though several other associations are suspected.

2.3 Evolution of a supernova remnant

A supernova explosion is essentially the result of a catastrophic process occurring in a star which results in the ejection of the outer layers of the star. The ejected shell of matter would possess a "frozen-in" magnetic field which would help to hold the shell together. This expanding shell acts like a snowplow sweeping up the interstellar medium into which it is progressing. J.H. Oort (1946) used these simple ideas and the law of conservation of momentum to predict a relation between radius and time for supernova remnants.

If one imagines the shell to be a perfectly spherical continuous object which is increasing in radius, then as it sweeps up the interstellar medium it will also increase in mass. The law of conservation of momentum thus gives

$$\left(M + \frac{4}{3} \pi R^3 \rho \right) v = M v_0 \qquad (2.1)$$

where : M = mass of shell at ejection, v_0 = initial velocity of shell, R = radius of shell, ρ = density of surrounding medium.

It has been assumed here that the shell is free from all forces.

$v = \frac{dR}{dt}$ so that equ. (2.1) becomes

$$M \frac{dR}{dt} + \frac{4}{3} \pi \rho R^3 \frac{dR}{dt} = M v_0$$

Integrating gives

$$\frac{1}{3} \pi R^4 \rho + M R = M v_0 t \qquad (2.2)$$

The motion of the shell is entirely determined by equations (2.1) and (2.2). In the case of a strongly decelerated shell, where $4 \pi R^3 \rho / 3 \gg M$, one gets from equations (2.1) and (2.2)

$$v = \frac{3 M v_0}{4 \pi \rho R^3}$$

$$R = \left(\frac{3 M v_0 t}{\pi \rho} \right)^{1/4}$$

$$R = 4 v t \qquad (2.3)$$

The above equations derived by Oort give a rough idea of the radius of a supernova remnant at a given time or the velocity at a given radius etc. A rather more exact treatment of the problem taking into account the shock wave associated with the explosion has been worked out by Shklovsky (1962) using the treatment of an adiabatic explosion in a gas of constant heat capacity given by Sedov (1959). The results obtained by this treatment of the problem will be described below.

The phenomenon of a supernova outburst in interstellar matter must be treated as a powerful explosion in a gas with constant specific heat.

The specific heat is constant because the ionisation energy of the gas is much smaller than the Kinetic energy of the ionising particles. Assuming the interstellar medium is homogeneous with a particle concentration n_1, mainly consisting of hydrogen atoms one can consider the explosion of a supernova as a sudden release of thermal energy at a point identical with the origin of the coordinate frame, occurring at time $t = 0$.

One then finds the following relationships using the Sedov treatment.

$$R_2 = \left(\frac{2.2\,E}{\rho_1}\right)^{1/5} t^{2/5}$$

$$= 10^{15} \left(\frac{E}{E_o\ n_1}\right)^{1/5} t^{2/5} \qquad \text{——— (2.4)}$$

$$T_2 = 1.44 \times 10^{21} \left(\frac{E}{n_1 E_o}\right)^{2/5} t^{-6/5} \qquad \text{——— (2.5)}$$

$$\rho_2 = 4\ \rho_1 \qquad \text{——— (2.6)}$$

where ρ_1 is $n_1 \times$ mass of a proton, R_2 is the radius of the shock front (cm), T_2 is the ion temperature behind the front ($^\circ$K), ρ_2 is the density behind the front (g cm $^{-3}$) and E is the energy of the explosion (erg). $E_o = 0.75 \times 10^{51}$ erg, which is taken as the standard amount of energy released in the explosion, being typical for a type II supernova with an ejected mass of 1 $M_\odot$. The specific heat ratio $C_p / C_v = 5/3$ for the interstellar gas.

Equation (2.6) represents the well known discontinuity of density behind the front of a strong shock wave. Equation (2.5) indicates that the explosion energy (under adiabatic conditions) is converted into thermal energy of all the particles present within a sphere of radius R_2. Equation (2.4) describes the rate of expansion of the hot gas inside the sphere of radius R_2. Formulae (2.4) and (2.6) have been verified experimentally in nuclear explosions in the atmosphere.

The gas is concentrated in a relatively thin shell, measuring approximately R_2 / 10 behind the front, whereas the gas density in the central region is insignificant. Differentiating equation (2.4) with respect to time gives the velocity of the wave front.

$$v = \frac{dR_2}{dt} = \frac{2}{5}\left(\frac{2.2E}{\rho_i}\right)^{1/5} t^{-3/5} \qquad \text{(2.7)}$$

Equation (2.7) describes the deceleration of the shock wave. From equations (2.4) and (2.7) one can derive the following simple relation $R_2 = \frac{5}{2} vt$ (2.8)

This can be compared with the formula derived from Oort's equations, namely equation (2.3). A comparison of the two equations shows that neglect of the adiabatic behaviour of the shock wave leads to an underestimate of the ages of the remnants by a factor of 1.6.

It is interesting to plot supernovae of known diameter and age on a graph showing the Oort equation and the Sedov equation relating diameter and age. This is done in fig. (2.3). It can be seen that Kepler's and Tycho's supernova remnants fit quite reasonably on the line defined by equation (2.4), ie. the Sedov equation. Cassiopeia A lies closer to the line defined by the Oort equation, namely equ. (2.2). The Crab remnant being a rather unusual supernova remnant does not lie particularly close to either line. Two other historically recorded supernovae can be correlated reasonably well with catalogued supernova remnants. PKS 1459 - 41 is the likely remnant of a supernova remnant which occurred in 1006, and was recorded by European, Arabic and Far Eastern observers. The present remnant has a radius of about 15 pc. The supernova remnant 3C58 is the probable remnant of a supernova observed in 1181. Its radius is 6.9 pc. 3C58 has also been plotted on fig (2.3) and lies not far away from the line corresponding to the Sedov equation. PKS 1459 - 41 lies off the graph, but would obviously be somewhat distant from this line. Interesting work and comments on the correlation of historically observed supernovae with catalogued supernova remnants can be found in Stephenson (1975).

Woltjer (1972) has distinguished four phases in the expansion of a supernova remnant:-

Phase I This is the free expansion phase when the mass of the ejected shell is very much greater than the mass of swept up matter. Everything will depend on the details of the explosive process.

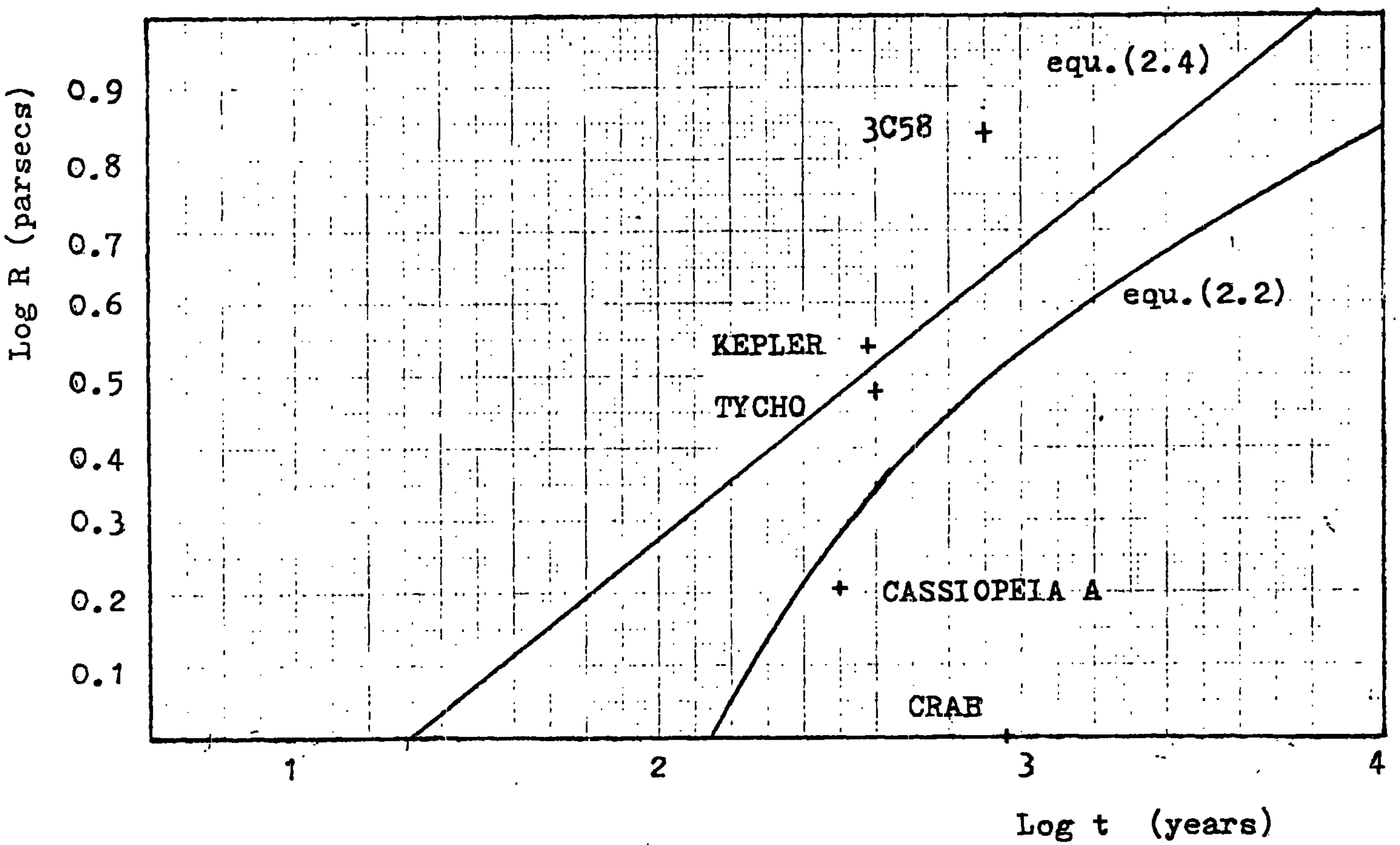


Fig 2.3. RADIUS VERSUS TIME FOR A TYPICAL SUPERNOVA REMNANT ACCORDING TO TWO EQUATIONS (SEE SECTION 2.3).

Phase II This is the adiabatic phase when the mass of the ejected shell is very much less than the mass of the swept up matter. In this phase radiative losses are negligible and energy is conserved. This is the phase in which the Sedov Equation (2.4) is valid. As the shell slows down, radiative losses become important. Above a temperature of 5×10^6 K the rate of cooling is proportional to $T^{1/2}$, being due to bremsstrahlung radiation, while the energy per unit mass is proportional to T. Consequently the cooling is unimportant as long as the shock velocity is high. As the expansion slows down the cooling time becomes smaller and ultimately the situation is one where the shock heated gas cools almost immediately. Cooling effects are of practical importance only at temperatures below 5×10^6 K. Woltjer (1972) shows that the velocity at which radiation loss begins to be of dominant importance is about 200 Km s^{-1} for typical parameters and is only very weakly dependent upon those parameters.

Phase III In this phase the radiative cooling time has become short and the matter behind the shock cools quickly. As a consequence, pressure forces are no longer important and the shell moves at constant radial momentum, that is $(4\pi/3)\, R^3 \rho_0 V$ = constant.

Phase IV This is the phase when the expansion velocity of the remnant becomes comparable to the random velocities of the interstellar medium (about 10 Km s^{-1}) and the remnant loses its identity.

Woljer gives a table showing the age t , radius R , velocity V , and mass M at the transition points between the four evolutionary phases for a supernova remnant having an initial expansion velocity of 10,000 Km s^{-1} and an ejected mass of 0.1 $M_\odot$, taking the number density of the interstellar medium to be 1 cm $^{-3}$ as usual. The table , from Woltjer (1972), is reproduced below.

TABLE 2.1 SCHEMATIC EVOLUTION OF A SUPERNOVA REMNANT.

PHASE	t (yr)	R (pc)	V (Km s^{-1})	M ($M_\odot$)
I - II	90	0.9	10,000	0.2
II - III	22,000	11	200	180
III - IV	750,000	30	10	3600

(For a supernova with E_o = 10 erg)

Most observed supernova remnants are in the adiabatic phase, ie. phase II .

2.4 Magnetic fields in supernova remnants

The theories presented in the previous section have not taken account of the presence of a magnetic field in the matter swept up by the shell, or the pressure of cosmic rays within the shell. It is known that an interstellar magnetic field of about 3×10^{-6} gauss exists and since the density behind a strong shock wave is increased by a factor of four, the swept up interstellar matter will have a magnetic field of about 1.2×10^{-5} gauss (since in a highly conducting medium the magnetic field is essentially frozen to the medium). Since the relativistic particles move along helical paths around the magnetic lines of force, any compression of the magnetic field will involve an increase in concentration of the relativistic particles. If the magnetic field is sufficiently entangled, the raised concentration of relativistic particles will be maintained for a longer time. The local intensification of the field will, by virtue of the conservation of the adiabatic invariant, entail an increase in energy of each relativistic particle and thus an increase in power of the synchrotron radiation.

It is possible that a noticeable part of the synchrotron radiation from a supernova remnant may be attributed to the compression of interstellar matter by the shock wave and the resulting intensification of the magnetic field. This could certainly be a very significant cause of synchrotron radiation in old supernova remnants such as IC 443 or the Cygnus Filaments. The theory is described in detail in Van der Laan (1962)

It is generally assumed that the magnetic field energy in the supernova remnant and the energy of the relativistic particles have the same order of magnitude. This corresponds to the minimum total energy of the system of the field and the particles for a given magnetic bremsstrahlung emission power.

A magnetic field with an energy density considerably less than the relativistic particle energy density would not be able to keep the relativistic particles in the limited volume of the remnant, and as a result of their leakage the system itself would come to a state close to a state of quasi - equilibrium of energy between the magnetic field and the relativistic particles. Therefore to a first approximation it is reasonable to write

$E_H \simeq E_{cr}$, where $E_H = (H^2 / 8\pi) \times$ (volume of remnant) is the total energy of the magnetic field. E_{cr} is the energy of the relativistic particles (ions and electrons) in the radio emitting nebula.

The data obtained from radio observations allows one to estimate only the number and energy of the electrons in the remnant, so that in order to measure the total energy of all the particles in the remnant one needs to establish the connection between this quantity and the energy of the electrons. It is generally assumed, in the absence of any better assumption, that the energy of all the cosmic rays in the supernova remnant is simply proportional to the energy of the relativistic electrons.

$$E_{cr} = K \quad E_e$$

where K is the coefficient of proportionality usually put equal to 100, as mentioned in Chapter 1.

The observed radio flux from a supernova remnant can (using the above assumptions) be used to make a direct determination of the magnetic field strength and the total energy of the cosmic rays and electrons in the remnant, if the remnant's radio spectrum, angular size and distance away are known. The equations needed to do this can be found in chapter 4. If one assumes no continuous accelerating source for the electrons in a supernova remnant, then the expansion of the remnant will be accompanied by a decrease in the relativistic particle energy and a decrease in the magnetic field strength. In this case the relativistic particle energy varies adiabatically in accordance with the law $E \propto V^{-1/3} \propto R^{-1}$, where V is the volume of the nebula and R is its radius. From the maintenance of the total magnetic flux in the nebula, which must occur since the coductivity of the medium is very high, it must be that $H \propto R^{-2}$ at least to a first approximation.

When relativistic particles are present inside the shell with total energy comparable to that of the kinetic energy of the shell, their pressure will tend to decrease the deceleration of the shell. During the adiabatic phase of the expansion of a supernova remnant, the kinetic energy of the shell is constant, while the relativistic particle energy decreases as R^{-1}, assuming escape and acceleration of new particles is negligible. Consequently the cosmic ray pressure is probably not very important during this phase, except perhaps at the beginning. During phase III of the expansion, the kinetic energy varies as V^{-1} or R^{-3}, and later in this phase the cosmic ray energy may become comparable to the kinetic energy of the shell. A solution for this case was given by Kahn and Woltjer (1967), who showed that after the cosmic rays became dynamically important one had $R \propto t^{1/3}$ and a cosmic ray energy equal to twice the kinetic energy of the shell. Under these circumstances

$$D = (E_{cr}\, D_{cr})^{1/6}\, n_H^{-1/6}\, t^{1/3} \qquad (2.9)$$

where D_{cr} is the diameter at which the total cosmic ray energy was E_{cr} (see Woltjer, 1970).

CHAPTER 3 DISTRIBUTION AND FREQUENCY OF SUPERNOVAE IN THE GALAXY

3.1 General remarks

A recent catalogue of Galactic nonthermal radio sources such as the one produced by Ilovaisky and Lequeux (1972) lists 112 objects which are believed to be the remnants of supernova explosions. Obviously not all the supernova remnants which exist in the Galaxy can be seen. Whether or not a supernova remnant (henceforth abbreviated to SNR) can be detected will depend upon its radio power, its distance from the earth, its diameter, and the sensitivity of the detecting equipment. It is possible to get a very rough idea of the probable number of SNRs in the Galaxy from the following simple argument. Ilovaisky and Lequeux's catalogue (henceforth abbreviated to IL 72) lists 28 SNRs within 3 Kpc of the sun. This gives a surface density of SNRs of $28/9\pi$ SNR Kpc^{-2}. The Galaxy has a radius of about 15 Kpc. Therefore if this surface density is maintained over the whole of the Galaxy, one would expect to find about $225\pi \times 28/9\pi = 700$ SNRs in the entire Galaxy. Clearly selection effects mean that only a small fraction of the SNRs known to exist in the Galaxy can be observed.

By correcting for the main selection effects it ought to be possible to use the distribution of observed SNRs in the Galaxy to predict the real distribution of SNRs in the Galaxy. Observations of SNRs in external galaxies (which are free of the selection effects operating in our Galaxy) indicate that for most galaxies the surface density of SNRs decreases approximately exponentially with distance from the Galactic centre.

The paper already referred to by Ilovaisky and Lequeux contains an attempt to correct for selection effects which will be briefly described below.

3.2. The correction for selection effects used by Ilovaisky and Lequeux

Using SNRs whose distance is reasonably well known it is possible to establish a relationship between surface brightness and linear diameter of the form

$$\Sigma_{1\ \mathrm{GHZ}} = 3.6 \times 10^{-15}\ D_{pc}^{\ -4.0 \pm 0.2}\ \mathrm{W\ m^{-2}\ Hz^{-1}\ sr^{-1}} \qquad (3.1)$$

where $\Sigma_{1\ \mathrm{GHZ}}$ = surface brightness of the SNR at a frequency of 1GHz

D_{pc} = diameter of the SNR in parsecs.

All of the distances to the SNRs in IL72 were obtained using this relation, except for the distances to Cassiopeia A and the Crab Nebula.

$$\Sigma_{1\ \mathrm{GHZ}} = \frac{1.19\ S_o}{\langle \phi \rangle^2} \times 10^{-19}\ \mathrm{W\ m^{-2}\ Hz^{-1}\ sr^{-1}}$$

by their definition of surface brightness, where

S_o = flux density of the radio emission in flux units (1f.u. = 10^{26} $\mathrm{W\,m^{-2}\ Hz^{-1}}$).

$\langle \phi \rangle$ = mean angular diameter in minutes of arc.

If d = distance of the SNR from the earth then

$$\langle \phi \rangle = \frac{D}{d} \text{ radians} = 3440 \frac{D}{d} \text{ mins.}$$

$$\frac{1.19 \; S_o \times 10^{-19}}{\langle \phi \rangle^2} = 3.6 \times 10^{-15} \; D^{-4.0}$$

The catalogue is believed to be complete only for those remnants whose flux density at 1GHz is greater than 10 f.u., and whose angular diameter is greater than 2 arc mins. Hence using this fact and expressing $\langle \phi \rangle$ in terms of D and d leads to

$$d_{kpc} < \frac{190}{D_{pc}} \qquad \text{———————— (3.2)}$$

Equation (3.2) thus sets an upper limit to the distance at which a SNR of given diameter can be observed, eg. a SNR of diameter 30pc can be detected up to a maximum distance of 6.3 Kpc from the earth. Equation (3.2) is based simply on the inverse square law for intensity of radiation and the detection limit of 10 f.u.

If one plots a graph of the number of sources having diameters smaller than a given diameter as a function of diameter, then one obtains approximately a straight line up to a diameter of about 30pc. This plot of $(n < D)$ versus D is called a luminosity function. In the range of diameters 10-30 pc and within 6.3 Kpc of the sun (where no selection effects should be operative) then a statistically correct approximation to the luminosity function is

$$n(<D) = \text{const. } D^{2.40 \pm 0.60} \qquad \text{———————— (3.3)}$$

the slope of the luminosity function decreases beyond D=30 pc probably due to another selection effect such as a limiting surface brightness effect. However for SNRs of diameters up to 30 pc one can use equations (3.3) and (3.2) to determine the real distribution of SNRs in the Galaxy in the following way.

At a given distance d one observes SNRs with diameters less than a maximum diameter given by equation (3.2). At this distance d the fraction of SNRs seen is $n(<D(d)) / n(<30)$ where the luminosity functions are given by equation (3.3).

One may thus statistically determine for each distance and direction in the Galaxy the total density of remnants up to D=30 pc by multiplying the observed number of objects by $n(<30) / n(<D(d))$.

The only assumption made here is that the shape of the luminosity fuction is the same throughout the Galaxy. Ilovaisky & Lequeux restrict their analysis to objects with distances smaller than 11 Kpc in order to avoid excessive corrections, and by the above method were able to plot out the surface density of SNRs as a function of distance R from the Galactic centre, assuming symmetry around the centre. The result is shown in fig (3.1).

3.3 Theoretical derivation of distribution of SNRs

Below is described an alternative way of deriving the radial distribution of SNRs, which also uses equation (3.2) but involves a theoretical equation for the expansion of a SNR.

The Sedov equation for the adiabatic expansion of a supernova remnant is given by

$$D_{pc} = 4.0 \times 10^{-11} \quad (Eo / n_H)^{1/5} \, t^{2/5} \qquad (3.4)$$

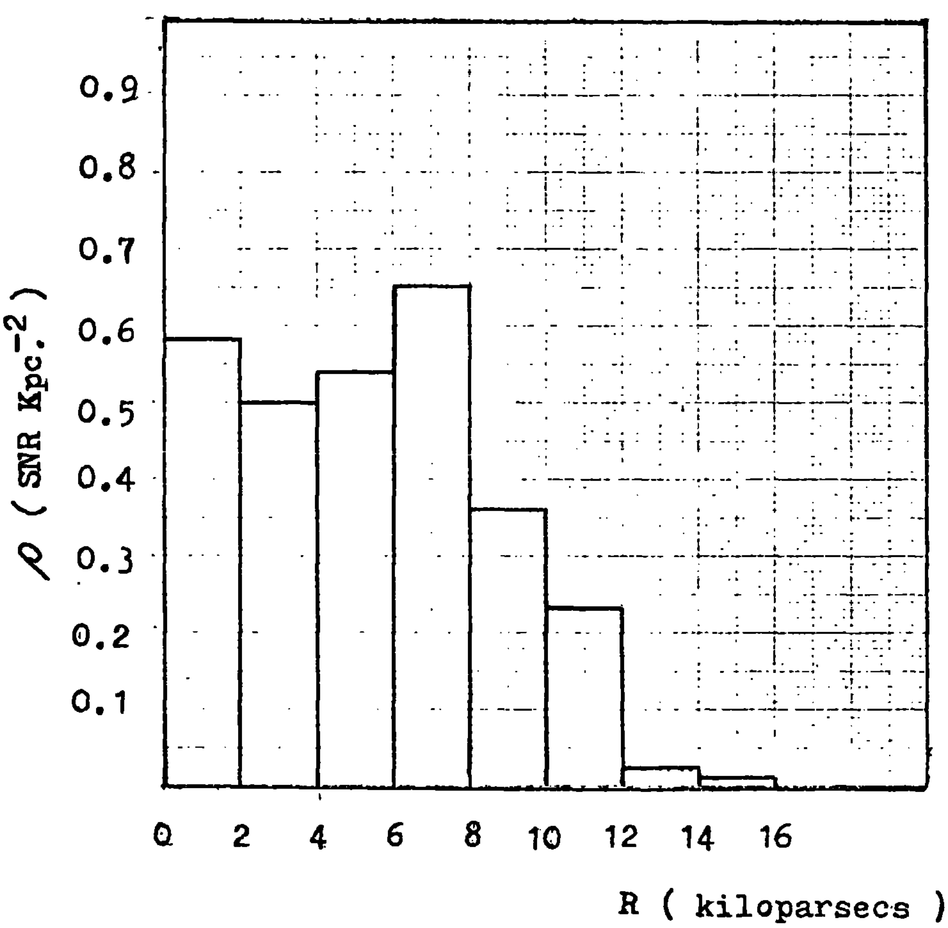


Fig. 3.1. SURFACE DENSITY OF SUPERNOVA REMNANTS IN THE GALAXY AS DERIVED IN IL72.

where t (years) = time after explosion, E_0(erg) = initial energy of the explosion, n_H(cm^{-3}) = hydrogen number density.

This equation should hold until $D \simeq$ 60pc, ie. equation (3.4) should describe the expansion of a SNR throughout its observable life.

Thus we can assume that the shell diameter D depends on time according to $D = At^{2/5}$ where A is a constant. The number of SNRs with diameters between D and D + dD is equal to the number of supernovae which occurred between times t and t + dt. Therefore the fraction of SNRs with diameters in the range D to D + dD is the fraction with ages t to t + dt, where $t = (D/A)^{5/2}$ and $dt = 5D^{3/2} / 2A^{5/2}$.

Thus for a uniform rate of birth of SNRs, the fraction of SNRs with diameters between D and D + dD is proportional to dt.

$$F(D)\, dD \propto dt$$

$$F(D)\, dD = \frac{D^{3/2}}{\int_0^{D_{max}} D^{3/2}\, dD}$$

Denoting the observed number of SNR by M and the real number by N, then by referring to fig (3.2) it can be seen that the observed number of SNRs between angle ϕ and $\phi + d\phi$ is

$$M(\phi)\, d\phi = \frac{N(R)\, dR}{2\pi R} R d\phi \int_0^{D=190/d} F(D)\, dD$$

$$M(\phi) d\phi = \frac{\frac{N(R) dR}{2\pi R} R d\phi \int_0^{D=190/d} D^{3/2}\, dD}{\int_0^{D_{max}} D^{3/2}\, dD}$$

$$\therefore \quad M(\phi) d\phi = \frac{N(R)\, dR\, d\phi}{2\pi} \left[\frac{190}{d\, D_{max}} \right]^{5/2}$$

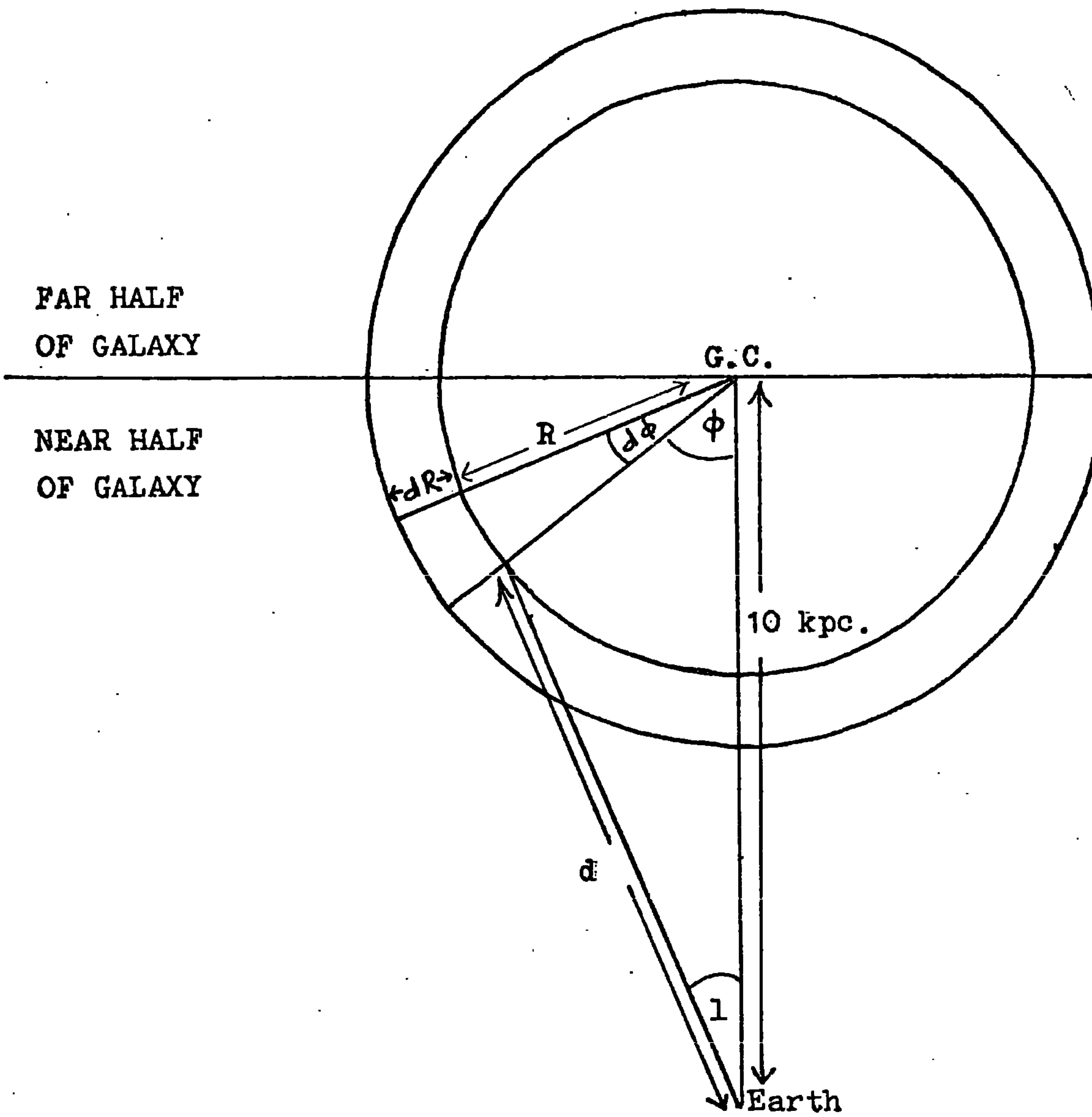


Fig. 3.2. DIAGRAM ILLUSTRATING RELATIONS BETWEEN PARAMETERS USED IN THE THEORY PRESENTED IN SECTION 3.3.

From fig. (3.2) we can see that $d = (100 + R^2 - 20R\cos\phi)^{1/2}$ using the cosine rule.

The total number of SNRs seen between R and R + dR for $d > 190/D_{max}$ is

$$M(R)dR = \frac{N(R)dR}{\pi}\int_0^{\pi}\left(\frac{190}{D_{max}}\right)^{5/2}\frac{d\phi}{(100+R^2-20R\cos\phi)^{5/4}}$$

For $d < 190/D_{max}$ the total number of SNRs observed is

$$M(R)dR = \frac{N(R)}{\pi}dR\int_0^{\phi m} d\phi$$

since one should see all the SNRs in the field of view when $d < 190/D_{max}$.

If $d < \frac{190}{D_{max}}$ then it follows that

$$\cos\phi > \frac{100 + R^2 - (190/D_{max})^2}{20R}$$

This defines $\phi_m = \cos^{-1}\left(\frac{100 + R^2 - (190/D_{max})^2}{20R}\right)$ ——— (3.5)

$d < \frac{190}{D_{max}}$ if $\phi < \phi_m$

Therefore the total number of SNR seen between R and R + dR is

$$M(R)dR = N(R)dR\,\frac{1}{\pi}\left[\phi_m + \left(\frac{190}{D_{max}}\right)^{5/2}\int_{\phi_m}^{\pi}\frac{d\phi}{(100 + R^2 - 20R\cos\phi)^{5/4}}\right] \qquad (3.6)$$

Therefore $M(R)dR = N(R)dR\ A(R)$

Where $$A(R) = \frac{1}{\pi}\left[\phi_m + \left(\frac{190}{D_{max}}\right)^{5/2}\int_{\phi_m}^{\pi}\frac{d\phi}{(100 + R^2 - 20R\cos\phi)^{5/4}}\right]$$

One can imagine the Galaxy divided into two halves by a line through the Galactic centre perpendicular to the line joining the earth to the Galactic centre.

The hemisphere containing the earth will be called the near half (subscript N) and the other hemisphere will be called the far half (subscript F). One can then write equation (3.6) as

$$M_N (R)\ dR + M_F (R)\ dR = N (R)\ dR \left[A_N (R) + A_F (R)\right]$$

This gives two equations for N(R) viz:

$$N (R)\ dR = \frac{M_N (R)\ dR}{A_N (R)} \qquad (3.7)$$

where $$A_N (R) = \frac{1}{180}\left[\phi_m + \left(\frac{190}{D_{max}}\right)^{5/2}\int_{\phi_m}^{90}\frac{d\phi}{(100 + R^2 - \cos\phi)^{5/4}}\right]$$

$$N (R)\ dR = \frac{M_F (R)\ dR}{A_F (R)} \qquad (3.8)$$

where $$A_F (R) = \frac{1}{180}\left[\left(\frac{190}{D_{max}}\right)^{5/2}\int_{90}^{180}\frac{d\phi}{(100 + R^2 - 20R\cos\phi)^{5/4}}\right]$$

3.4 Predicted distribution of SNRs in the Galaxy

Below is an attempt to use the theory developed in section 3.3 and the data contained in IL 72 to predict the real distribution of SNRs in the Galaxy.

Only the SNRs known to lie in the near half of the Galaxy will be considered, since statistics are very poor for SNRs in the far half of the Galaxy. Equation (3.7) relating the real number of SNRs between R and R + dR to the observed number in that distance interval will be used to predict the real distribution (R being distance from the Galactic centre). Since another selection effect seems to be operative for $D > 30$ pc as stated earlier, and since it would be interesting to compare the above results with those of IL 72, D_{max} was taken to be 30pc. Using this value graphs of ϕ_{min} and hence A_N (R) were plotted. The resulting curves are shown in figs. (3.3) and (3.4). For comparison the curve which was obtained for A_F (R) is also shown in graph (3.4).

IL 72 gives the positions and diameters of 112 observed SNRs. Their positions , as seen by an observer looking down on the plane of the Galaxy, are shown in fig. (3.5). Only SNRs of diameters less than 30pc are indicated in fig. (3.5).

Using this plot of observed supernovae it was possible to count the number of SNRs of diameter less than or equal to 30pc within intervals of 2Kpc from the Galactic centre. Taking a mean value of A_N (R) from fig. (3.4) over the distance interval it was thus possible to calculate $N(R) = M_N(R) / A_N(R)$ for each 2Kpc distance interval. This ought to be the actual number of SNRs in that distance interval.

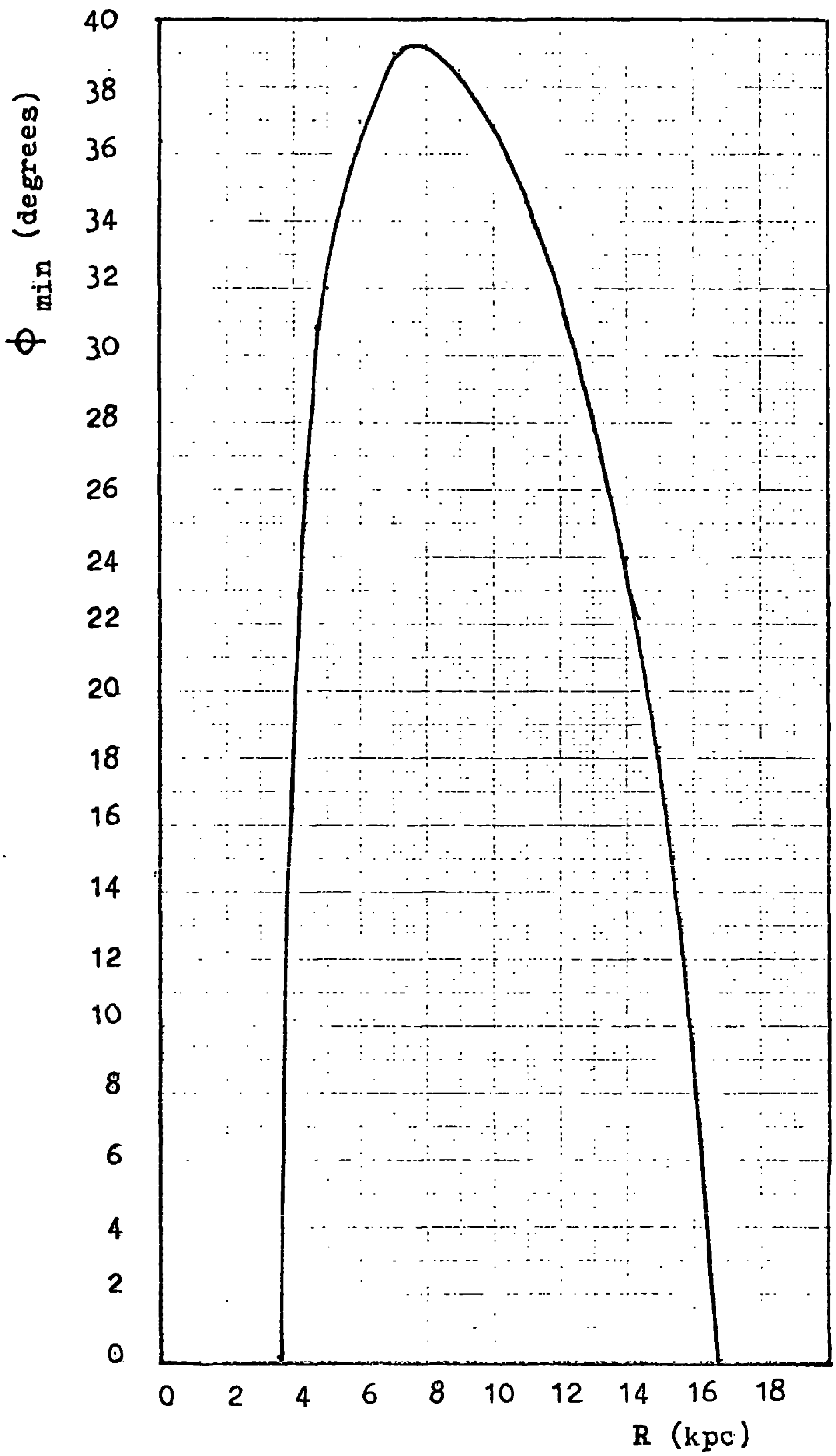


Fig. 3.3. RELATION BETWEEN ϕ_m AND R AS DEFINED BY EQUATION (3.5) IN SECTION 3.3, FOR D_{max} = 30 pc.

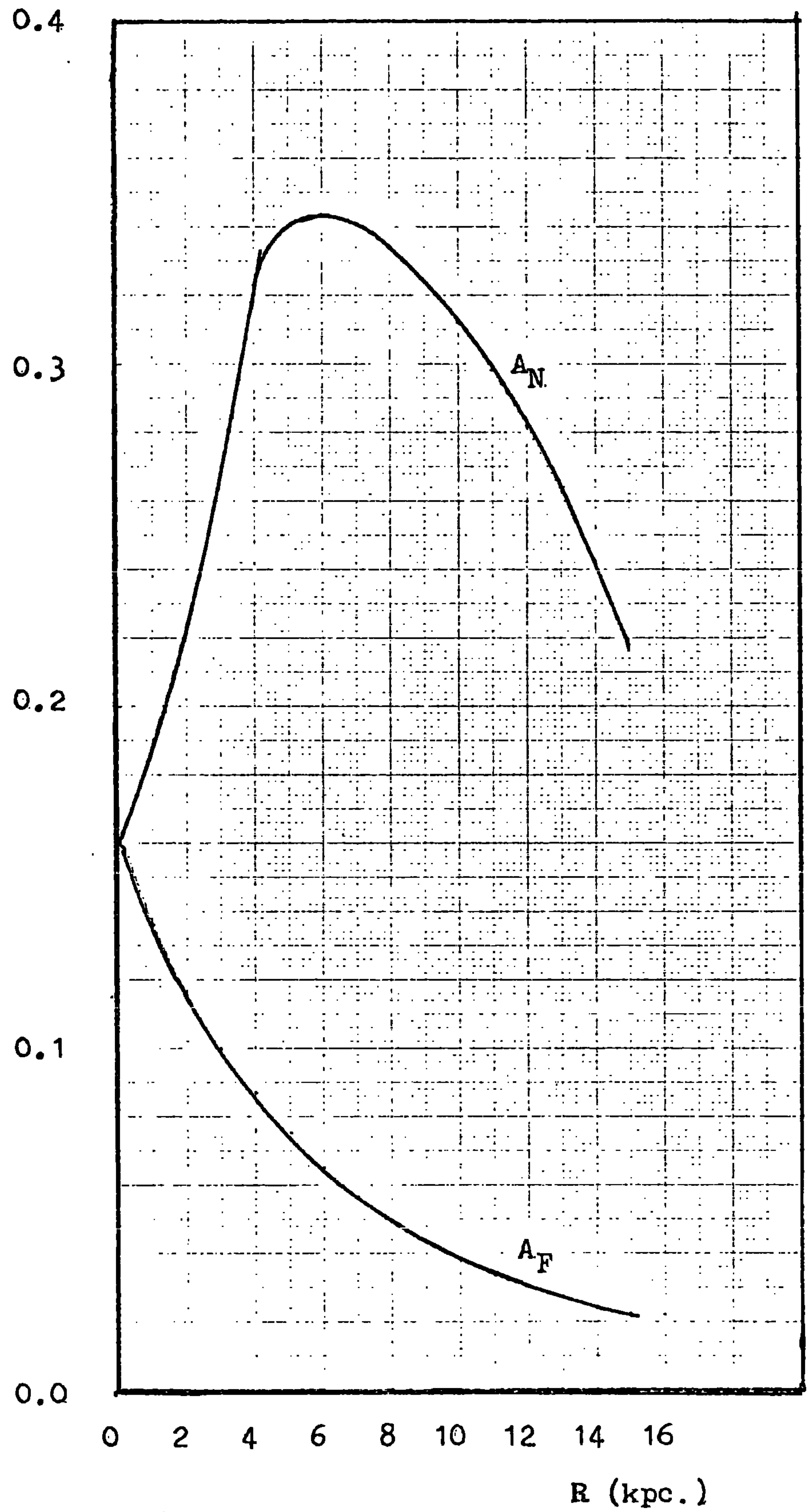


Fig. 3.4. RELATION BETWEEN A_N, A_F AND R AS DEFINED IN SECTION 3.3, FOR D_{max} = 30 pc.

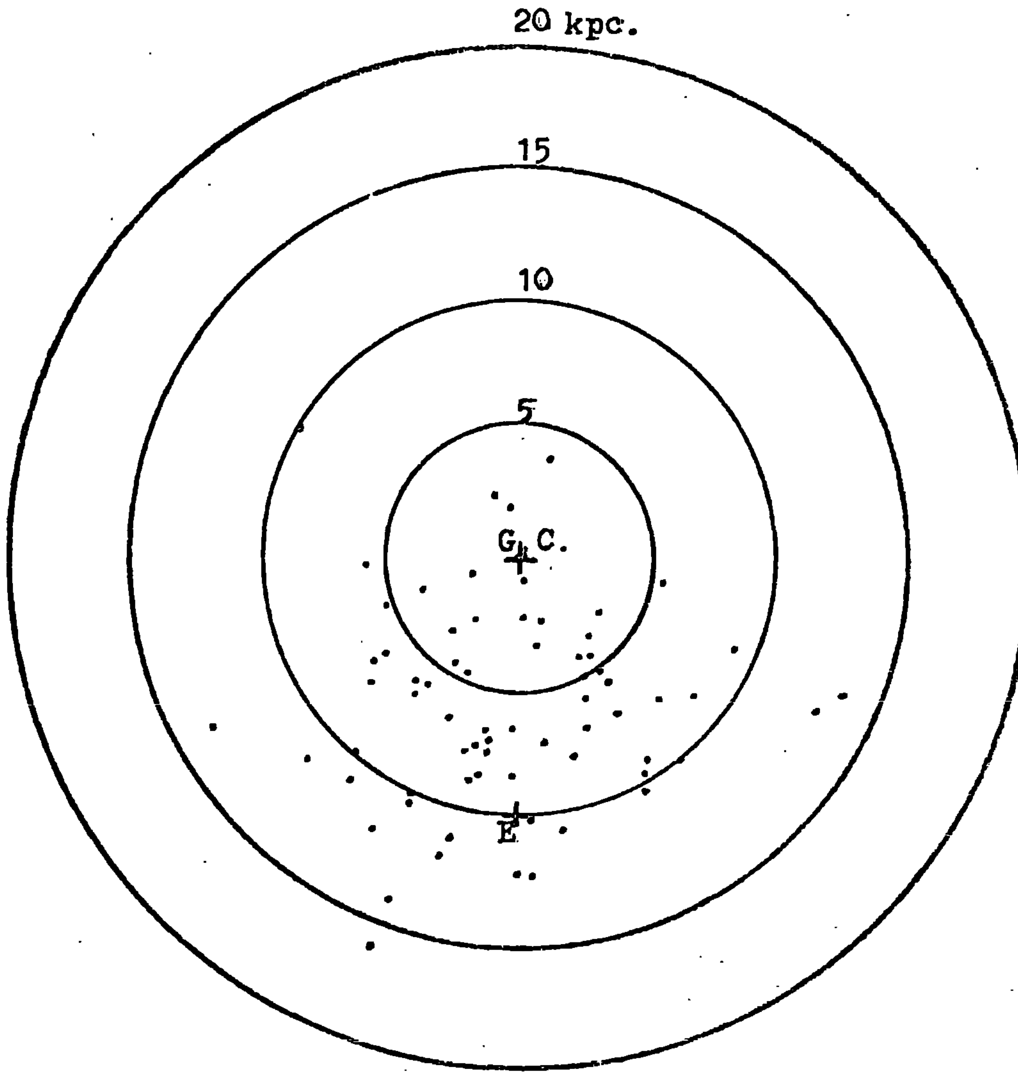


Fig. 3.5. SNRs IN IL72 WITH DIAMETER $D < 30$ pc, WITH $-10^{o} < b^{II} < 10^{o}$, FLUX DENSITY >10 f.u. AND ANGULAR DIAMETER $>2'$. THE DIAGRAM SHOWS THE DISTRIBUTION OF THESE OBSERVED SNRs AS SEEN BY AN OBSERVER LOOKING DOWN ON THE PLANE OF THE GALAXY.

The suface density of SNRs between distances R_1 and R_2 would be defined as

$$\rho(R) = \frac{N(R)}{(R_2^2 - R_1^2)}$$

Since $(R_2^2 - R_1^2) = (R_2 + R_1)(R_2 - R_1)$

and within annuli of thickness 2 Kpc, $(R_2 - R_1) = 2$, then

$$\rho(R) = \frac{N(R)}{2\pi(R_1 + R_2)}$$

Let R = mean radius of annulus

$$= (R_1 + R_2)/2$$

Therefore $\rho(R) = \frac{N(R)}{4\pi R}$

$$= \frac{M_N(R) / A_N(R)}{4\pi R} \qquad (3.8)$$

It was thus possible to predict the real suface density of SNRs as a function of distance from the Galactic centre. The resulting bar chart is shown in fig. (3.6).

The errors in $\rho(R)$ shown in fig. (3.6) were caculated by combining the statistical error in M_N and the uncertainty in the mean A_N over the distance interval. $\delta M_N = \sqrt{M_N}$, and δA_N = half the difference between the maximum and minimum values of A_N over the distance interval. The vertical lines on each bar of fig. (3.6) thus define one standard deviation in the height of each bar.

Taking into account the large errors in fig. (3.6) and the fact that errors are not shown in fig. (3.1) it can be seen that either the method used here or the method used in IL 72 give essentially the same distribution of SNRs in the Galaxy.

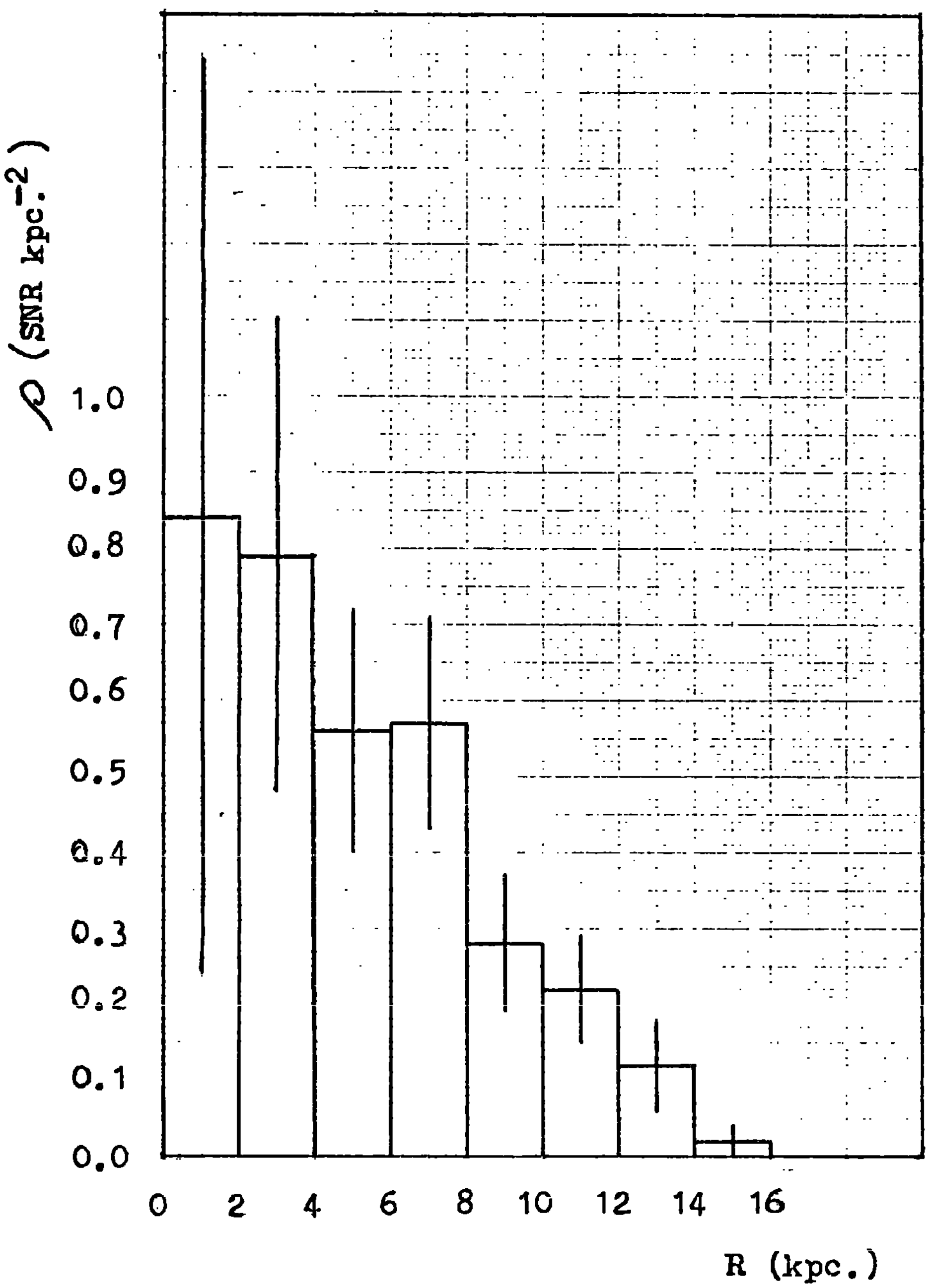


Fig. 3.6. PREDICTED SURFACE DENSITY OF GALACTIC SNRs ACCORDING TO THEORY DEVELOPED IN SECTION 3.3, FOR SNRs OF DIAMETER < 30 pc ONLY.

3.5 Effect of variations in the density of the interstellar medium on selection effects.

It is possible to think of other factors which perhaps ought to be taken into account when trying to correct for selection effects. Clearly the time for a supernova to reach a given diameter will depend on the density of the surrounding medium into which it is expanding. Since the average interstellar gas density decreases with distance from the Galactic plane in a manner somewhere between an exponential and a Gaussian, then supernovae appearing at large distances from the Galactic plane will give rise to remnants that will expand slightly faster and consequently fade in radio brightness faster than remnants near the plane. Hence at first sight there ought to be a selection effect against detection of SNRs at high latitudes. This effect would be mitigated to some extent by the fact that the radio background tends to decrease with latitude facilitating the detection of weak extended objects.

Table 3 in IL 72 gives the mean height $\langle |Z| \rangle$ above the Galactic plane for SNRs in various ranges of distance from the Galactic centre. The relevant part of the table is reproduced below.

Table (3.1)

	DISTANCE FROM GALACTIC CENTRE (Kpc)				
	0-4	4-6	6-8	8-10	10-16
NUMBER OF SNRs	11	11	15	13	12
$\langle \lvert Z \rvert \rangle$ (pc)	153	33	33	67	175

The density of hydrogen gas falls off with height above the Galactic plane according to

$$n(h) = \left[n_o \exp\left(-\frac{h}{h_A} \right) \right]_R \qquad \text{———} \quad (3.9)$$

h_A = scale height of gas
= $1.44\ h_{1/2}$

Using information on the Galactic half thickness of HI, such as given by Jackson and Kellman (1974), one can find the corresponding mean scale height above the Galactic plane of hydrogen gas in the above distance intervals. The result is as below.

TABLE (3.2)

	DISTANCE FROM GALACTIC CENTRE (Kpc)				
	0-4	4-6	6-8	8-10	10-16
h_A (pc)	232	302	357	416	751

Using an accepted distribution of mean gas density in the Galactic plane such as given by Puget and Stecker (1974) and Westerhout (1970), one has values for the neutral hydrogen density as a function of radial distance from the Galactic centre, ie. one has values of n_o. It is thus possible to calculate the average hydrogen density at heights corresponding to the mean height of SNRs over the distance intervals. This gives some idea of the mean density of the interstellar medium in the region of the Galaxy occupied by the SNRs in each distance interval.
Using the above information the following table was constructed.

TABLE (3.3)

R (Kpc)	h_A (pc)	<1Z1> (pc)	n_o cm^{-3}	n_H (<1Z1>) cm^{-3}	$n_H^{1/2}$
0-4	232	153	0.30	0.155	0.39
4-6	302	33	0.70	0.63	0.79
6-8	357	33	0.80	0.73	0.85
8-10	416	67	0.70	0.60	0.77
10-16	751	175	0.40	0.32	0.57

Since the diameter of a SNR depends on time according to

$$D \propto \frac{t^{2/5}}{n_H^{1/5}} \qquad \text{(see equation (3.4))}$$

then $t \propto D^{5/2} \quad n_H^{1/2}$

ie. the time for a SNR to reach a given diameter depends on the square root of the density of the surrounding medium. From the last column of table (3.3) it can be seen that $n_H^{1/2}$ does not vary by more than a factor of 2 over the distances under consideration, thus the time for a SNR to reach a given diameter would also not vary by more than a factor of 2, due to local variations in the neutral hydrogen density. Since the energy output of a supernova explosion is uncertain by at least a factor of ten, this effect is clearly not particularly significant in determining the diameter of a SNR at a given time, and is therefore not important in considering selection effects.

3.6 Frequency of supernova explosions

Using fig. (3.6) and equation (3.4) it is possible to work out the expected frequency of supernova explosions in the Galaxy. Using fig. (3.6) one can reconvert the surface densities back into numbers, and it is then found that there are 260 SNRs with diameters less than 30 pc.

The typical time for a SNR to reach a diameter of 30 pc can be got from equation (3.4)

$$t_{yr} = \left[\frac{D_{pc}}{4.0 \times 10^{-11}}\right]^{5/2} \left(\frac{n_H}{E_o}\right)^{1/2}$$

A mean value for the hydrogen number density can be got from table (3.3), $n_H = 0.49 \text{ cm}^{-3}$.

A typical value for the initial energy involved in the explosion is $E_o \simeq 10^{50}$ erg.

$$\therefore t = 3.41 \times 10^4 \text{ years}$$

If there are 260 remnants of diameter less than 30 pc, then this gives a rate for the entire Galaxy of

$$R = \frac{260}{3.41 \times 10^4} \text{ events per year}$$

ie. about 1 supernova every 130 years.

Because E_o is uncertain by about a factor of 10, the rate derived is uncertain by a factor of at least 3.

3.7 Effect of a change in the $\Sigma - D$ relation

Recently the relation between surface brightness and distance obtained in IL 72 has been called into question. This relationship (equation (3.1)) was used by them to derive all the distances of the supernovae in their catalogue except for the Crab Nebula and Cassiopeia A. Mathewson and Clarke (1973) obtained a different relationship using fourteen supernova remnants detected in the Magellanic Clouds. Their relationship was

$$\Sigma_{408} = 10^{-15} \; D_{pc}^{-3} \; W \; m^{-2} \; Hz^{-1} \; Sr^{-1} \qquad (3.10)$$

This relationship has the advantage that the distance of all the calibrators is reliably known, whereas for many of the Galactic calibrators uncertainties in their distances of up to 50% exist. However as noted by Mathewson and Clarke (1973) the Magellanic Cloud SNRs seem to be inherently brighter objects than Galactic SNRs of the same linear diameter.

Clark, Caswell and Green (1973) rederived the $\Sigma - D$ relationship using the class 1 and class 2 calibrators listed in IL 72 with surface brightness values in the range 3×10^{-21} to 3×10^{-19} $W \; m^{-2} \; Hz^{-1} \; Sr^{-1}$, this being the range over which they wished to apply the relation for distance determination This range excludes Cassiopeia A, the Crab Nebula and the Cygnus Loop. The data were well fitted by either of the expressions

$$\Sigma_{408} = 0.7 \; D_{pc}^{-3.0} \; 10^{-15} \; W \; m^{-2} \; Hz^{-1} \; Sr^{-1} \qquad (3.11)$$

or

$$\Sigma_{408} = D_{pc}^{-3.11} \; 10^{-15} \; W \; m^{-2} \; Hz^{-1} \; Sr^{-1} \qquad (3.12)$$

Equation (3.11) was arbitrarily adopted by Clark et. al. in their subsequent calculations . Note that this expression retains the Mathewson and Clarke exponent in equation (3.10). The difference in coefficient then indicates a mean initial energy for the Magellanic Cloud SNRs about 25% greater than for the Galactic SNRs.

Clark et al. (1973) list 27 new supernova remnants obtained by comparing 408 MHz observations from the Molonglo radio telescope and 5000 MHz observations of comparable resolution from the Parkes 64 metre radio telescope.

Several supernova remnants (in fact 15 more besides the 4 mentioned in a post script) in IL 72 have been subsequently found to be H II regions as pointed out by Caswell (1972), Dickel and Milne (1972), and Dickel et al. (1973).

In order to investigate the sensitivity of the derived supernova distribution to a change in the Σ-D relation the analysis of section 3.3 was repeated using equation (3.11) extrapolated to 1000 MHz. At the same time the H II regions were deleted and the new remnants of Clark et al. were added.

Once again only supernovae with flux densities greater than 10 f.u. and angular diameter greater than 2 arc minutes were used in the analysis. In fact only 6 of the 27 new supernova remnants listed by Clark et al. satisfied these criteria.

Assuming a mean spectral index of - 0.5, equation (3.11) can be extrapolated to 1000 MHz to give

$$\Sigma_{1000} = 4.5 \times 10^{-16} \; D_{pc}^{-3} \; W \, m^{-2} \; Hz^{-1} \; Sr^{-1} \qquad (3.13)$$

This equation was used to recalibrate the distances in IL 72.

Surface brightness in this equation has the usual definition

$\Sigma = 4 S_o / \pi \langle \phi \rangle^2$ where S_o is the flux density (at 1000 MH_z) and $\langle \phi \rangle$ is the mean angular diameter.

It should be noted that in IL 72 and in section 3.2, a different definition $\Sigma = S_o / \langle \phi \rangle^2$ is consistantly used. The distance of a remnant is then

$$d = 49.4 \left[S_o \langle \phi \rangle \right]^{-1/3} \text{ Kpc} \qquad \text{———(3.14)}$$

where S_o and $\langle \phi \rangle$ are expressed in flux units and minutes of arc respectively. Also the diameter of the remnant is given by:

$$D = 14.4 \langle \phi \rangle^{2/3} S_o^{-1/3} \text{ pc} \qquad \text{———(3.15)}$$

Equations (3.14) and (3.15) were therefore used to work out the distances and diameters of supernova remnants in IL 72.

The equivalent relation to equation (3.2) is now, using the new $\Sigma - D$ relation

$$d_{Kpc} < \frac{70.5}{D_{pc}^{1/2}} \qquad \text{———(3.16)}$$

This is the distance out to which one should see all supernova remnants of diameter D.

Previously it was found that a plot of number of SNRs less than diameter D versus D gave a line of constant slope on a log-log plot up to a diameter of 30 pc. It was suspected that other selection effects operate at diameters above 30pc and the analysis was done only for SNRs below this diameter.

This critical diameter will be different for a different $\Sigma - D$ relation, so it was necessary to draw $(n<D)$ versus D graphs to find the new critical diameter. Such graphs were drawn for SNRs of distances $d < 7.9$ Kpc which meant one should see all SNRs of $D<80$pc and for $d < 6.5$ Kpc where one should see all SNRs of $D < 118$ pc. In both graphs the luminosity function began to change slope at a diameter of about 50pc (see figs. (3.7a) and (3.7b) Hence this was taken as the critical diameter above which other selection effects operate.

The equations derived in section 3.3 had to be appropriately altered because of the replacement of equation (3.2) by equation (3.16), but otherwise the analysis was the same. The total number of SNRs seen between R and R + dR is now

$$M(R)dR = N(R)dR\,\frac{1}{\pi}\left[\phi_m + \left(\frac{4970}{D_{max}}\right)^{5/2}\int_{\phi_m}^{\pi}\frac{d\phi}{(100 + R^2 - 20R\cos\phi)^{5/2}}\right] \qquad (3.17)$$

so that

$$N(R)dR = \frac{M_N(R)\,dR}{A_N(R)}$$

where $$A_N(R) = \frac{1}{180}\left[\phi_m + \left(\frac{4970}{D_{max}}\right)^{5/2}\int_{\phi_m}^{90}\frac{d\phi}{(100 + R^2 - 20R\cos\phi)^{5/2}}\right] \qquad (3.18)$$

or

$$N(R)dR = \frac{M_F(R)\,dR}{A_F(R)}$$

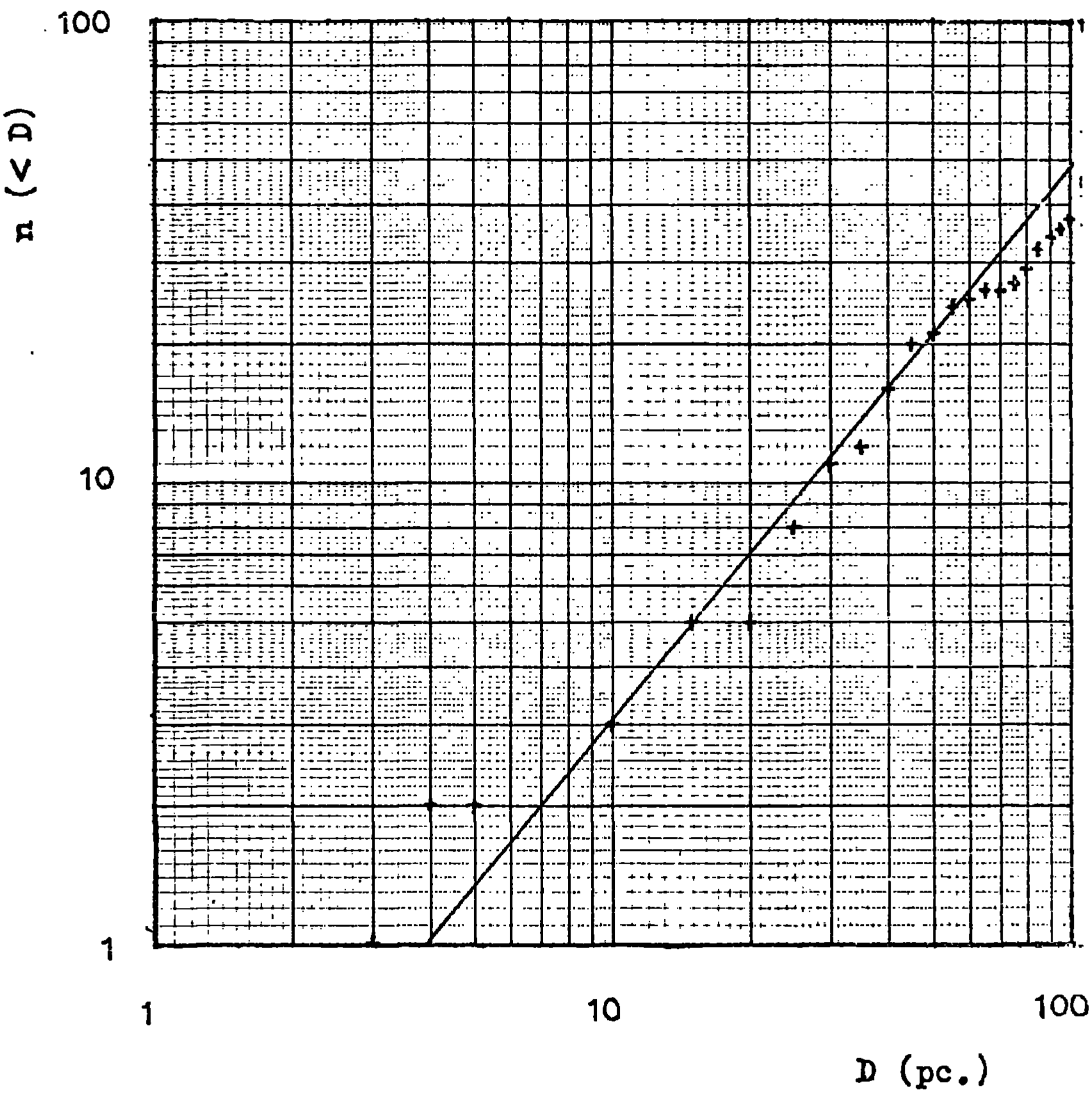


Fig. 3.7a. NUMBER OF SNRs LESS THAN A GIVEN DIAMETER D VERSUS D FOR OBSERVED SNRs WITHIN 6.5 kpc OF THE EARTH, AFTER RECALIBRATION OF THE DISTANCES IN IL72 (ALL SNRs OF D < 118 pc SHOULD BE SEEN).

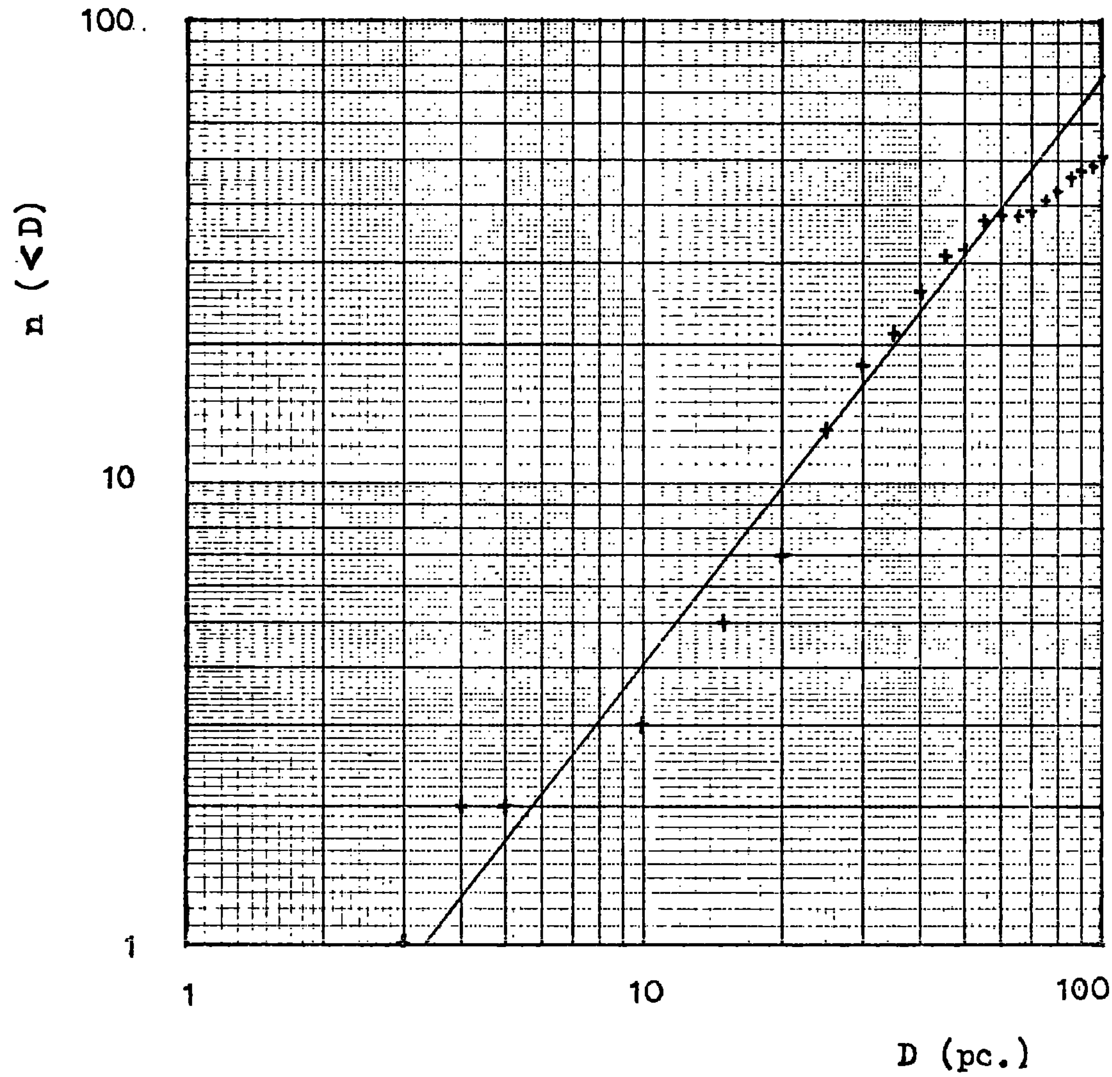


Fig. 3.7b. NUMBER OF SNRs LESS THAN A GIVEN DIAMETER D VERSUS D FOR OBSERVED SNRs WITHIN 7.9 kpc OF THE EARTH, AFTER RECALIBRATION OF THE DISTANCES IN IL72 (ALL SNRs OF D<80 pc SHOULD BE SEEN).

where $$A_F(R) = \frac{1}{180}\left[\left(\frac{4970}{D_{max}}\right)^{5/2}\int_{90}^{180}\frac{d\,\phi}{(100 + R^2 - 20R\cos\phi)^{5/2}}\right] \qquad (3.19)$$

$$\phi_m = \cos^{-1}\left(\frac{100 + R^2 - (4970/D_{max})}{20R}\right) \qquad (3.20)$$

Using the new equations and only observed supernova remnants in the near half of the Galaxy the same analysis as in section 3.4 was again carried out. This time of course D_{max} was taken to be 50 pc. Fig.(3.8) shows the distribution of the SNRs used. The corresponding graph of A_N versus R and A_F versus R is shown in fig.(3.9).

As before the number of SNRs of diameter $\leqslant$ 50 pc were counted within intervals of 2 Kpc from the Galactic centre using fig.(3.8). The surface density of SNRs in each interval was then calculated using equation (3.8). The resulting bar chart is shown in fig.(3.10). The errors on each bar represent one standard deviation as before.

The surface densities in fig.(3.10) are somewhat lower than in fig.(3.6). This is partly because several of the previously included SNRs have been excluded because they are H II regions, but mainly because the new Σ - D relation significantly increases the distance of all SNR. The shapes of the distributions in fig.(3.10) and fig.(3.6) are not however very different when one takes account of the errors. In conclusion it can be seen that the new Σ - D relation does not drastically alter the predicted real distribution of SNRs in the Galaxy.

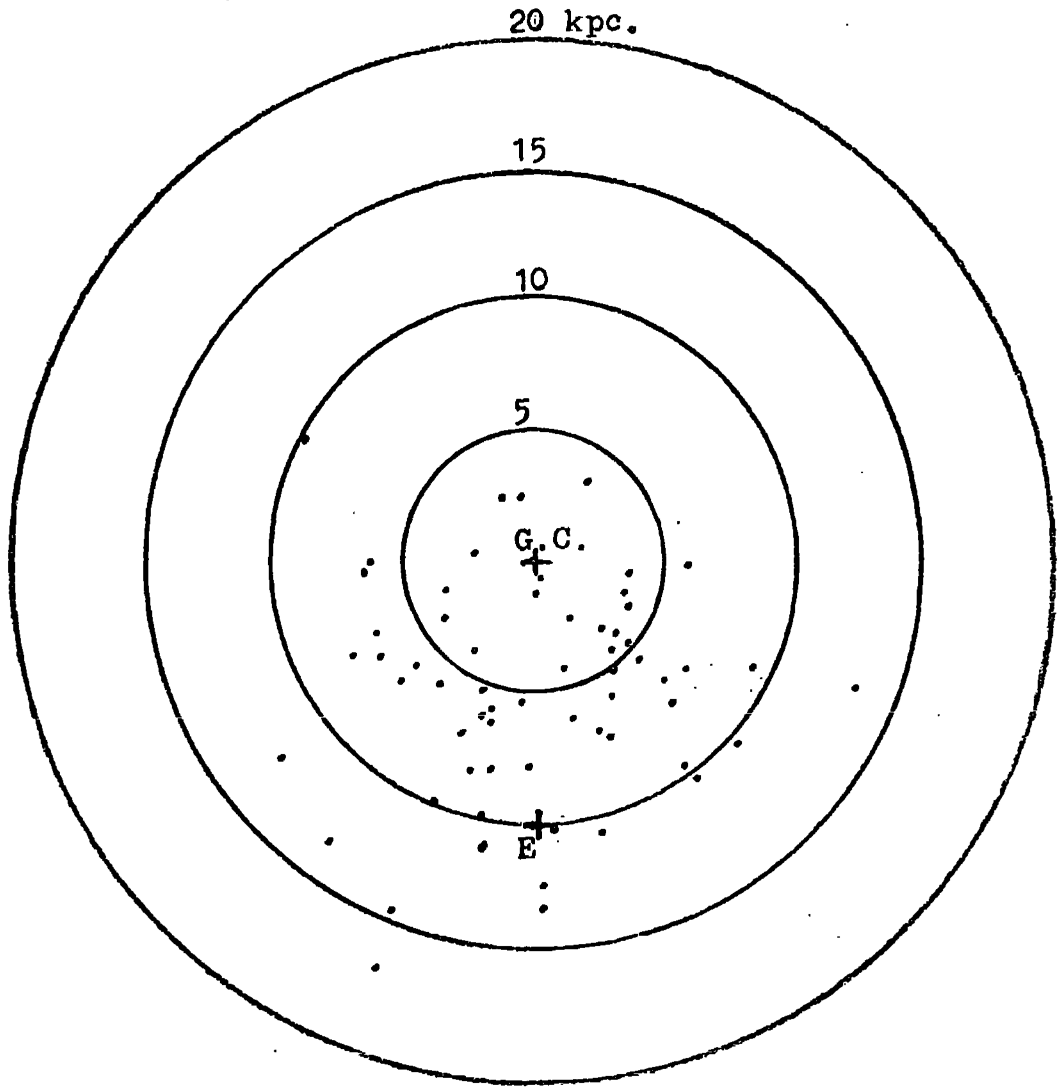
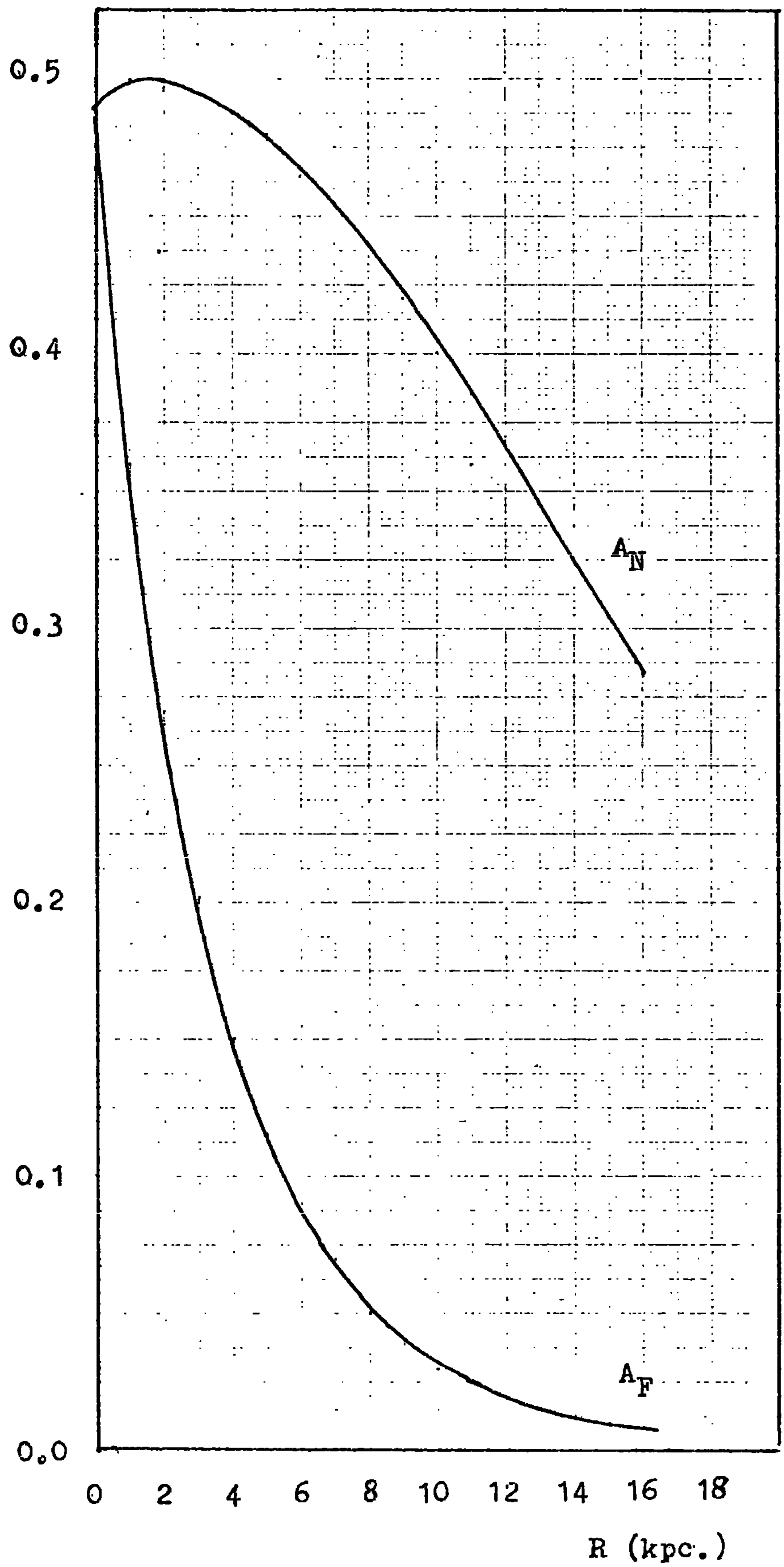


Fig. 3.8. SNRs IN IL72 WITH DIAMETER $D < 50$ pc, AFTER RECALIBRATION OF THE DISTANCES. AS BEFORE ONLY SNRs WITH FLUX DENSITY >10 f.u., ANGULAR DIAMETER $> 2'$, AND $-10^{\circ} < b^{II} < 10^{\circ}$ ARE SHOWN. THE DIAGRAM SHOWS THE DISTRIBUTION OF THESE OBSERVED SNRs AS SEEN BY AN OBSERVER LOOKING DOWN ON THE PLANE OF THE GALAXY.



<u>Fig. 3.9.</u> RELATION BETWEEN A_N, A_F, AND R AS DEFINED IN SECTION 3.7, FOR D_{max} = 50 pc.

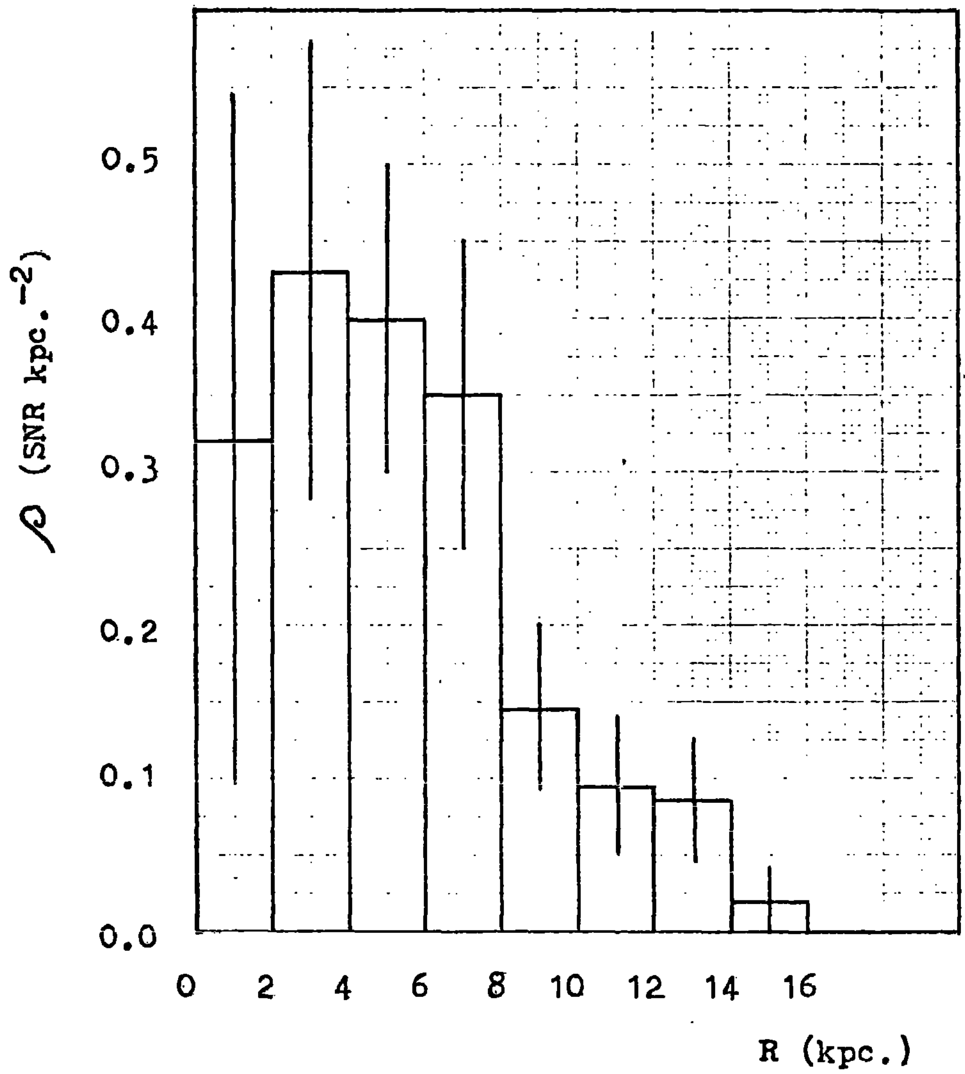


Fig. 3.10. PREDICTED SURFACE DENSITY OF GALACTIC SNRs ACCORDING TO THEORY DEVELOPED IN SECTION 3.7, USING REVISED DISTANCES; FOR SNRs OF DIAMETER $<$ 50pc ONLY.

It does however alter the frequency of supernova explosions derived in the manner described in section 3.6. A similar calculation gives 248 SNRs of diameter less than 50 pc in the Galaxy and a rate of 1 SNR every 490 years.

3.8. An alternative derivation of the radial distribution of SNRs.

A recent paper by Kodaira (1974) attempts to take account of the confusion of supernova remnants with radio noise, in predicting the real distribution of SNRs in the Galaxy from the observed distribution. The degree of the confusion is empirically determined as a function of the distance from the observer, independent of the direction.

The observed apparent surface density N(R,r) depends upon the distance to the Galactic centre R and the distance between the centre of the surface element in question and the solar system r.

Kodaira shows a plot of N(R,r) versus R for different values of r. N(R,r) is the number of observed SNRs with diameter smaller than 30 pc, for a surface element of a circle with diameter of 4 Kpc centred on (R,r). In principle the surface density should depend only on distance from the Galactic centre and not on distance from the solar system. Hence the fact that the curves for $r \geqslant 6$Kpc are systematically lower than those for $r \leqslant 4$ Kpc is attributed to loss of remnants due to confusion with noise.

Curves are drawn for r = 0, 2,4,6,8,10 Kpc and so Kodaira shifts the curves for 6, 8, 10 Kpc to coincide with the curves for 0, 2, 4 Kpc thus empirically increasing the observed surface density for r $\geqslant$ 6 Kpc. Using this method Kodaira is able to correct the observed distribution of SNRs assuming the fall off in number of sources with distance from the earth is due primarily to confusion with noise, the confusion being independent of direction.

Kodaira compares his surface density curve with IL 72 and a curve in Johnson and Macleod (1963) showing the surface density of the supernovae in spiral galaxies as a function of the normalised radial distance from the centre (which is 1 at the outer edge of the visible disks of the galaxies). This comparison is shown in fig.(3.11). Kodaira's curve shows a pronounced peak at R $\simeq$ 5 Kpc in contrast to the curve in IL 72 and is very similar to the surface density curve for supernovae in spiral galaxies by Johnson and Macleod (1963).

The method used by Kodaira is certainly of interest, but it seems that the statistical errors which are not shown on the radial distribution curve are too large to place great reliance on the results. Also it seems unrealistic to assume that confusion with noise depends only on distance. The background radio brightness is a function of direction and so one intuitively expects the confusion to depend on direction as well. In principle the detection probability of SNRs as a function of both distance and direction could be determined empirically, but the small number of observed remnants leads to unacceptably large statistical errors.

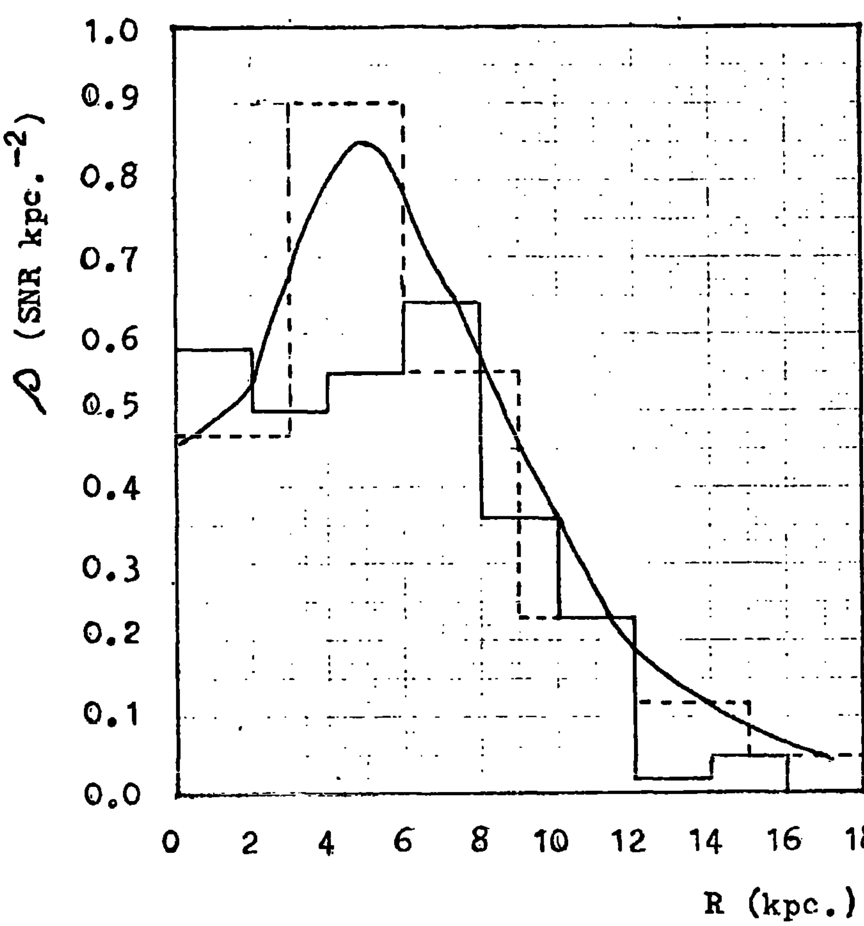


Fig. 3.11. COMPARISON OF SUPERNOVA REMNANT SURFACE DENSITY CURVES FROM KODAIRA (1974).

———(smooth curve) KODAIRA (1974).

———(bar chart) ILOVAISKY AND LEQUEUX (1972).

------JOHNSON AND MACLEOD (1963), FOR SNRs OBSERVED IN SPIRAL GALAXIES.

CHAPTER 4 COSMIC RAYS AND SUPERNOVAE

4.1 The cosmic ray energy spectrum

A recent discussion of the cosmic ray energy spectrum can be found in Wolfendale (1973). Fig.4.1 is taken from this book and shows the primary cosmic ray spectrum for protons and nuclei corrected for geomagnetic effects. The main methods of measurement in the various energy ranges are also included in this figure.

In the region below about 10^9eV/nucleon the interplanetary magnetic field reduces the primary intensity below its value far from the sun. The galactic spectrum is probably more nearly a linear extrapolation to lower energies of the spectrum above 10 GeV, at least for energies down to about 100 MeV / nucleon. Information on cosmic rays in this region comes mainly from satellite measurements and the use of geomagnetic effects.

Between 10^9 and 5×10^{11} eV / nucleon information is derived mainly from balloon detectors. In this region the slope of the differential energy spectrum is approximately constant for protons at about 2.7 ± 0.1, the heavier nuclei having a similar though less precisely measured slope.

In this region of the spectrum there is evidence from very recent work (eg. Juliusson et al. 1972) which suggests that the slopes of the different components of the cosmic ray flux are beginning to diverge. There appears to be a reduction of relative intensity of the secondary component with respect to that of primordial origin. This appears to imply that the higher energy particles have passed through a smaller thickness of matter. More about this significant result and about the cosmic ray composition will be discussed in later sections of this thesis.

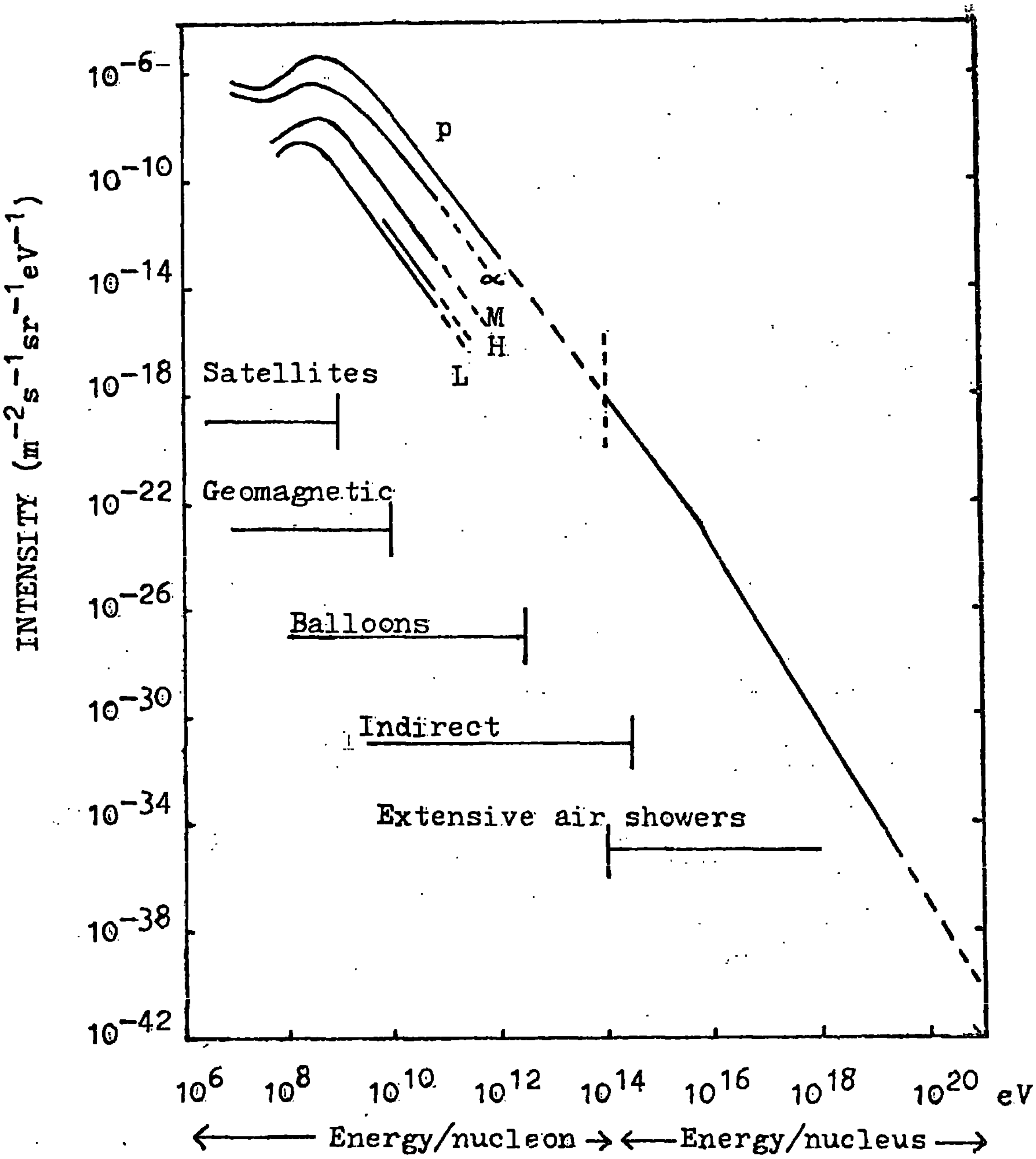


Fig. 4.1. THE PRIMARY ENERGY SPECTRUM OF NUCLEONIC COSMIC RAYS CORRECTED FOR GEOMAGNETIC EFFECTS, FROM WOLFENDALE (1973).

However the mass composition of cosmic rays is only known with certainty at energies below about 10^{10} eV.

At energies above 5×10^{11} eV / nucleon there is virtually no direct information about particles with $Z > 1$ although a few individual nuclei have been detected in balloon borne nuclear emulsions. Direct measurements on protons extend to about 3×10^{12} eV and above this indirect measurements take over. Up to about 10^{14} eV the cosmic ray primary spectrum is inferred from the sea level muon flux and above this energy it is inferred from extensive air shower data.

There is some evidence provided by the proton satellite measurements (see Grigorov et al. 1971) which suggests that the slope of the proton spectrum increases rather rapidly at 10^{12} eV. These measurements appear to indicate that heavier nuclei predominate above this energy. Because of this the proton intensity is shown as being uncertain in the range 10^{12} - 10^{14} eV.

A steepening of the cosmic ray energy spectrum occurs at about 3×10^{15} eV. The energy spectrum follows a power law of the form quoted below, where E is the energy per nucleus

$$J(E) = A_1 E^{-\gamma_1} \quad \text{for } 10^{11} < E < 3 \times 10^{15} \text{ eV}$$

where $A_1 = 3.1 \times 10^{18}$ m^{-2} s^{-1} Sr^{-1} eV^{-1}

$\gamma_1 \simeq 2.6$

$$J(E) = A_2 E^{-\gamma_2} \quad \text{for } 3 \times 10^{15} \text{ eV} < E < 10^{20} \text{ eV}$$

where $A_2 = 1.0 \times 10^{28}$ m^{-2} s^{-1} sr^{-1} eV^{-1}

$\gamma_2 \simeq 3.2$

The energy density of the primary radiation is given by:

$$\sigma(>E) = \frac{4\pi}{c}\int_{E}^{\infty} E\, J(E)\, dE$$

The total energy is about 0 .6 eV cm^{-3}, a value which is close to the energy carried by starlight at the Earth.

The spectrum of cosmic ray electrons between 20 and 800 GeV is now known reasonably well. The slope of the differental energy spectrum is again about 2.7. A recent summary of measurements of the electron spectrum has been made by Meyer (1971). The e^{+}/e^{-} ratio in the cosmic ray electron flux drops from a value less than unity at about 5 x 10^{8} eV to only some 5% above 10^{9} eV. This reduction implies that above 10^{9} eV, at least, the electrons are not due to secondary interactions produced in the interstellar matter. This conclusion arises because these interactions would produce π^{o} mesons from which γ-rays and in turn e^{+} e^{-} pairs would result. If the primary electrons are produced in supernova remnants as Dickel (1974) seems to indicate, then a study of the electron spectrum could throw some light on the nucleon spectrum, since the acceleration processes are presumably similar.

4.2 Addition of the energy spectra of known supernova remnants

About a hundred discrete objects known to lie in the Galaxy are observed to be emitters of synchrotron radiation. These objects are believed to be supernova remnants. Several recent catalogues of them have been formed listing the essential parameters of the remnants including the flux density and spectral index of the synchrotron radiation from each remnant.

The synchrotron radiation is due to the presence of relativistic electrons spiralling around the magnetic field in each remnant. Hence one has information on the spectral index of the electron differential energy spectrum in about one hundred SNRs. One has no direct information on the energy spectrum of relativistic ions in the SNRs, but it is not unreasonable to assume that it is similar to the electron energy spectrum in slope. Certainly the slopes of the primary cosmic ray electron and ion spectra at the Earth are approximately the same.

At first sight anyway it seems reasonable to use the distribution of spectral indices among the observed SNRs to predict the likely distribution of spectral slopes of SNRs in the Galaxy, and then to add up all the spectra and see if the resulting slope is the same as the observed slope in a particular energy range. This is what has been attempted in this section.

There does not appear to be any relationship between the surface brightness of a SNR and the spectral index α. Surface brightness is proportional to flux density over angular diameter squared. This means there is no relation between spectral index and diameter, ie. no relation between spectral index and age (Milne(1970), Downes (1971)). Since this is so one does not need to worry about selection effects when taking the observed SNRs as a representative sample of the spectral indices of SNRs in the Galaxy.

It was decided to use the SNR catalogue of Downes(1971) for the addition of the spectral indices, since this lists the flux density of the radiation at 400, 1400 and 5000 MHs aswell as the spectral index α over that frequency range for each remnant.

Initially a χ^2 - test was done to see if the distribution of spectral indices in Downes' catalogue was Gaussian or not. Some of the SNRs in Downes' catalogue are considered by him to be suspect and are indicated as such (eg. they may be extragalactic objects). Excluding SNRs in the Local Magellanic Clouds and the ones marked as suspect, he lists 67 Galactic SNRs whose spectral indices are known. If the suspect Galactic SNRs are included there are 96. The distribution was considered including and excluding the suspect SNRs. The results are given below.

a.) Excluding suspect SNRs

Mean spectral index of energy spectrum of electrons in the 67 SNRs is :-

$$\bar{\gamma} = 2\bar{\alpha} + 1 = 1.96$$

Standard deviation $\sigma = 0.38$

b.) Including suspect SNRs

Mean spectral index $\bar{\gamma} = 1.89$

Standard deviation $\sigma = 0.44$

A bar chart showing the numbers in each class interval is shown in fig.(4.2) for both cases. The normal frequency curve for each case is also plotted in this figure.

Provided one neglects the two SNRs having abnormally high spectral indices of 3.2 and 3.6 then a χ^2 test shows that the data is Gaussian. It thus seems reasonable to assume that the spectral indices of SNRs in the Galaxy are distributed normally about a mean value $\bar{\gamma} = 1.96$.

$$P(\gamma)d\gamma = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(\gamma-\bar{\gamma})^2}{2\sigma^2}} d\gamma \qquad (4.1)$$

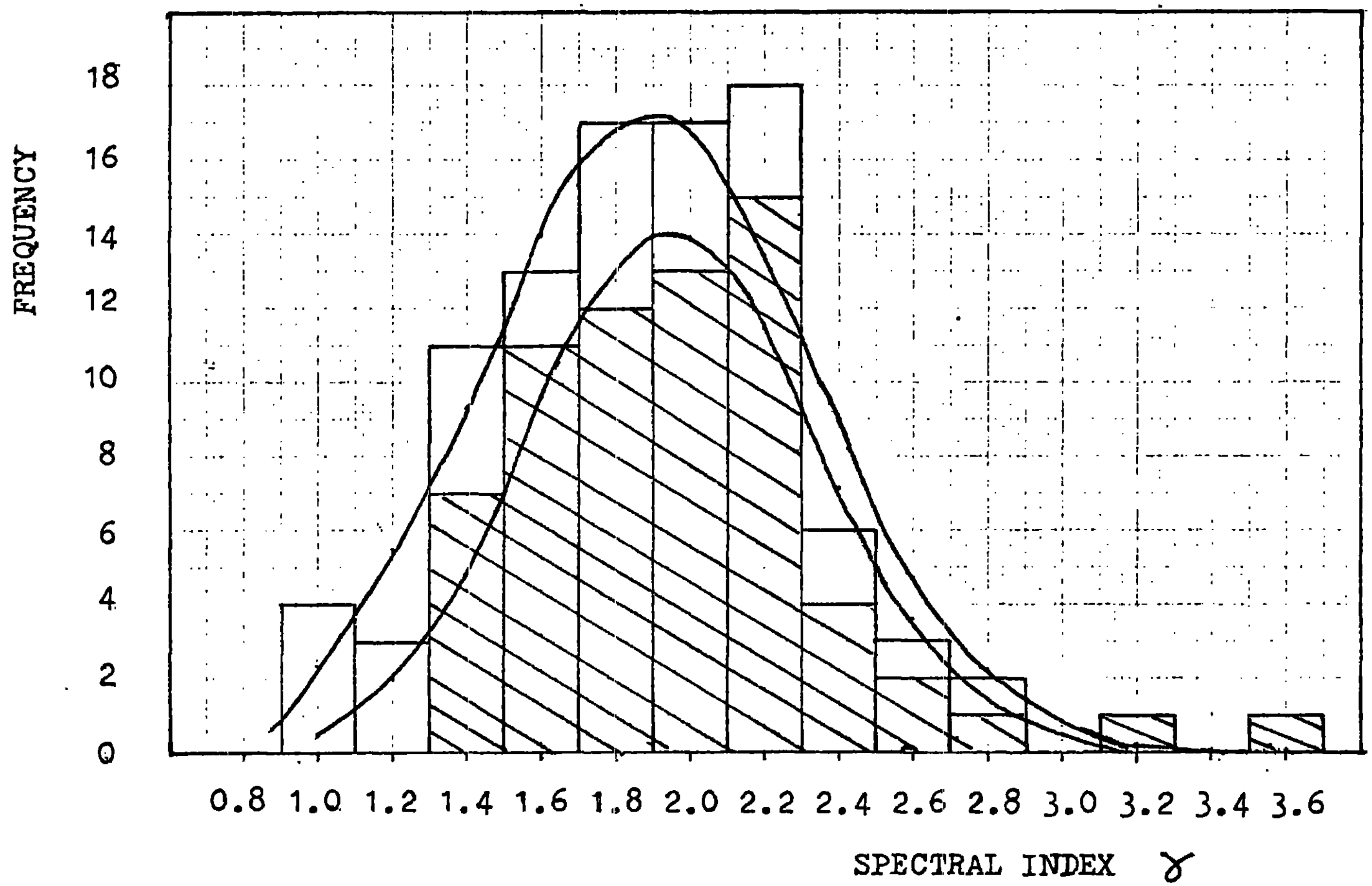


Fig. 4.2. DISTRIBUTION OF SPECTRAL INDICES FOR SNRs IN THE CATALOGUE OF DOWNES(1971).

☐ FOR ALL 96 SNRs (INCLUDING SUSPECT ONES), AND CORRESPONDING NORMAL CURVE.

▧ FOR 67 SNRs (EXCLUDING SUSPECT ONES), AND CORRESPONDING NORMAL CURVE.

gives the fraction of Galactic SNRs whose spectral indices lie in the range γ to $\gamma + d\gamma$

The electron energy spectrum in a SNR is of the form

$$n(E)dE = K_{\gamma} E^{-\gamma} dE \qquad \text{———} \quad (4.2)$$

This equation gives the number of electrons with energy between E and E + dE in the source.

The coefficient K_{γ} can be obtained from the flux density of the synchrotron radiation received from the source at a particular frequency, F_{ν}.

$$K_{\nu} = \frac{7.4 \times 10^{21} R^2}{a(\gamma) H} F_{\nu} \left(\frac{\nu}{6.26 \times 10^{18} H} \right)^{\frac{\gamma-1}{2}} \qquad \text{———} \quad (4.3)$$

where R = distance to source in centimetres

H = magnetic field in source in gauss

ν = frequency of radiation in cycles per second.

γ = $2\alpha + 1$

The magnetic field H is given by

$$H = \left[48 \text{ K A } (\gamma, \nu) \frac{F_{\nu}}{R \phi^3} \right]^{2/7} \qquad \text{———} \quad (4.4)$$

where ϕ = angular diameter of the source

K = assumed ratio of ions to electrons in the source (usually taken to be 100)

Equation (4.4) results from equating the magnetic energy to the particle energy in the source, which one would expect to be approximately true. $A(\gamma, \nu)$ and $a(\gamma)$ are constants defined at a particular spectral index and frequency, which come from the theory of synchrotron radiation.

$$A(\gamma,\nu) = \begin{cases} \dfrac{2.96 \times 10^{12}\,\nu^{1/2}}{(\gamma-2)\,a(\gamma)} \left[\dfrac{y_1(\gamma)\nu}{\nu_1}\right]^{\frac{\gamma-2}{2}} \left[1 - \left[\dfrac{y_2(\gamma)\nu_1}{y_1(\gamma)\nu_2}\right]^{\frac{\gamma-2}{2}}\right] & (\text{if } \gamma > 2) \\ 1.44 \times 10^{13}\,\nu^{1/2} \log_e \left[\dfrac{y_1(\gamma)\nu_2}{y_2(\gamma)\nu_1}\right] & (\text{if } \gamma = 2) \\ \dfrac{2.96 \times 10^{12}\,\nu^{1/2}}{(2-\gamma)\,a(\gamma)} \left[\dfrac{y_2(\gamma)\nu}{\nu_2}\right]^{\frac{\gamma-2}{2}} \left[1 - \left[\dfrac{y_2(\gamma)\nu_1}{y_1(\gamma)\nu_2}\right]^{\frac{2-\gamma}{2}}\right] & (\text{if } \gamma < 2) \end{cases} \qquad (4.5)$$

ν_1 and ν_2 are the frequency extremes of the observed radio band in which the spectral index $\alpha = (\gamma-1)/2$ has a constant value $(\nu_1 \leq \nu \leq \nu_2)$.

A graph showing how the constants $a(\gamma)$, $y_1(\gamma)$ and $y_2(\gamma)$ vary with γ is shown in fig.(4.3). The dependence of $A(\gamma,\nu)$ on γ using the above equations and fig.(4.3) is shown in fig.(4.4) for two different frequencies.

It was possible therefore to work out H for each of the SNRs in Downes' catalogue using the data given in the table and equation (4.4). Once the values of H were known it was then

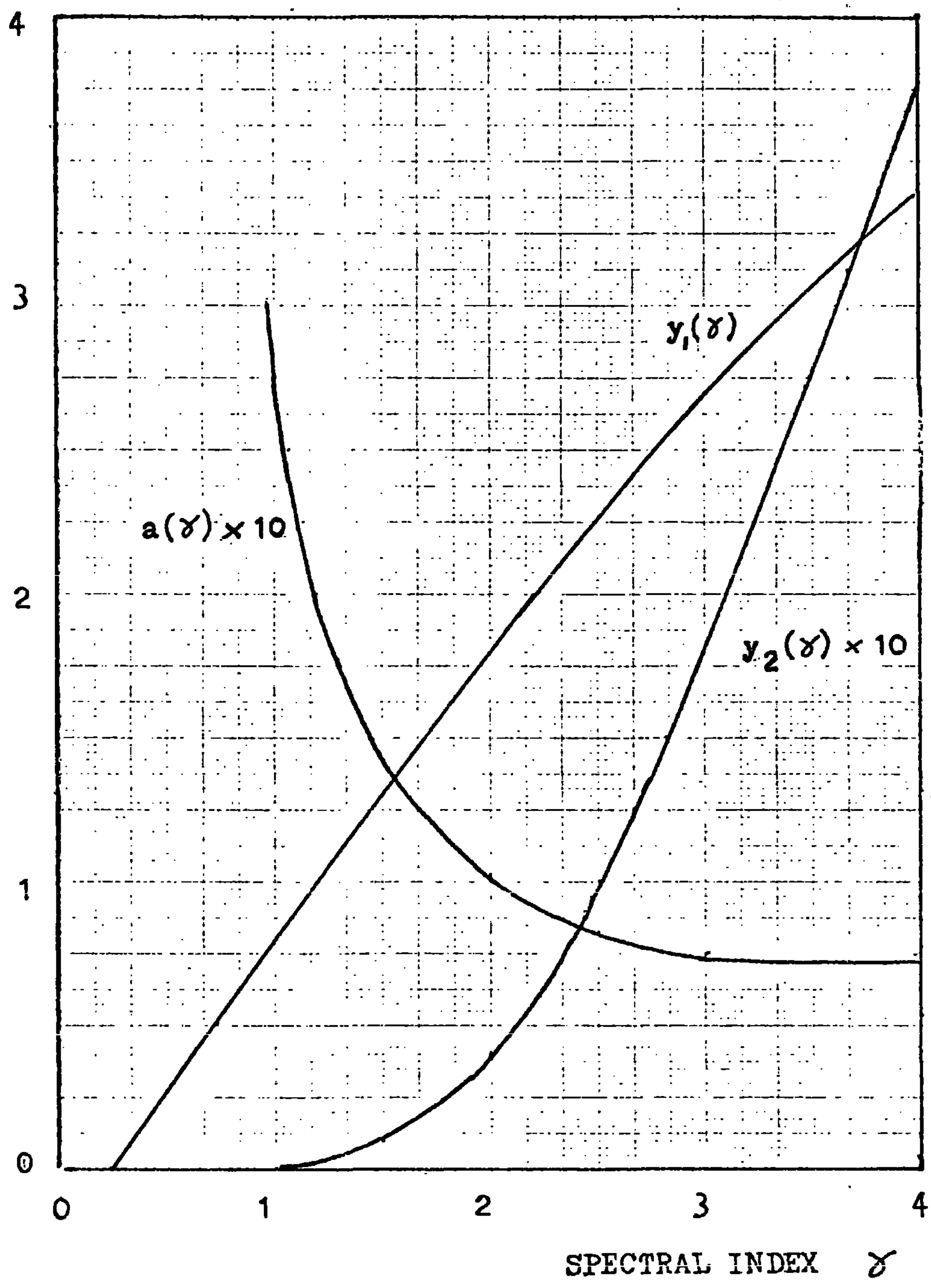


Fig. 4.3. DEPENDENCE OF SYNCHROTRON RADIATION PARAMETERS USED IN SECTION 4.2 ON SPECTRAL INDEX.

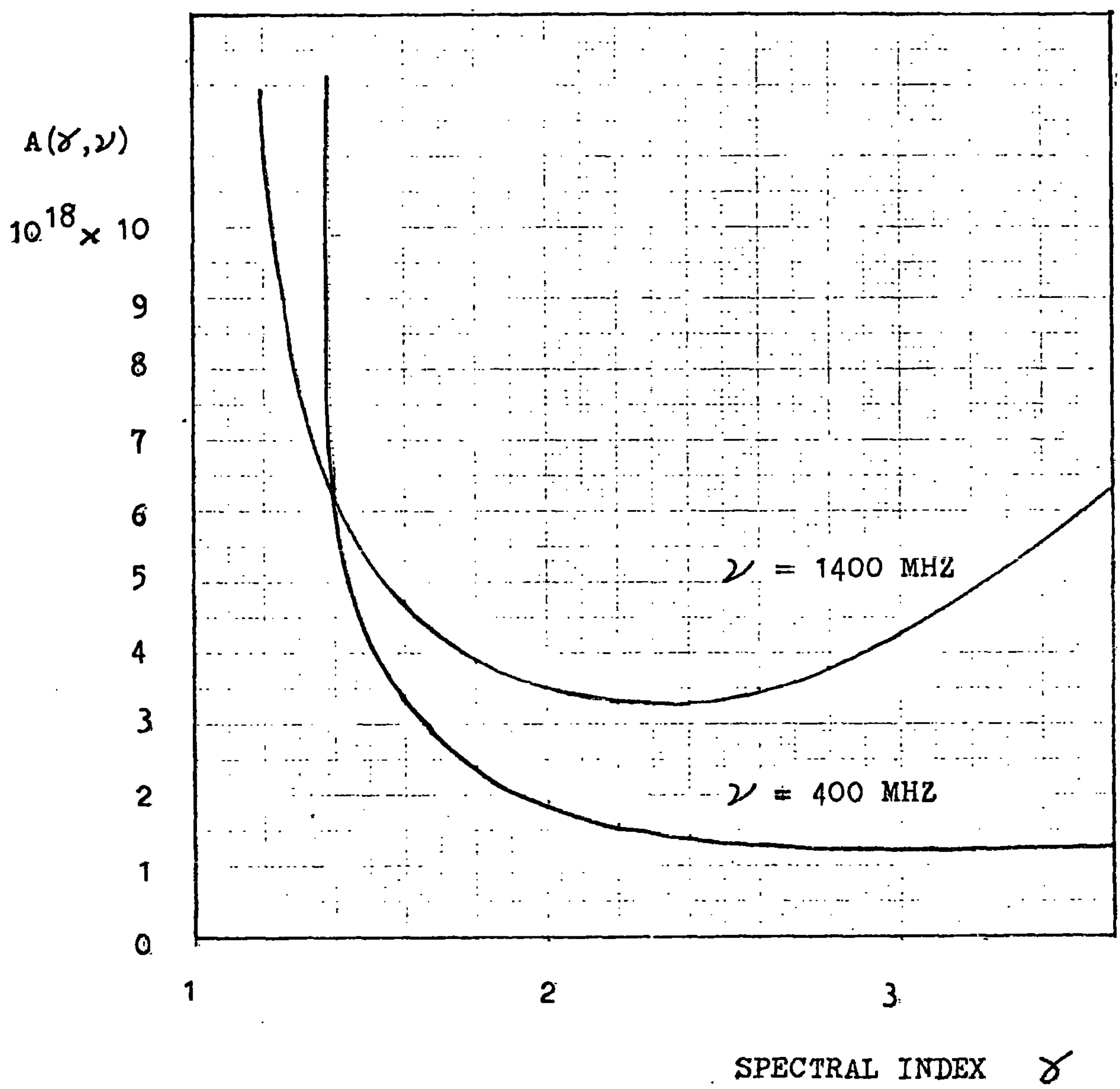


Fig. 4.4. DEPENDENCE OF SYNCHROTRON RADIATION CONSTANT $A(\gamma,\nu)$ ON SPECTRAL INDEX AT TWO FREQUENCIES.

possible to work out K_γ from equation (4.3) for each of the SNRs in the catalogue.

The K_γ values were grouped according to their corresponding γ - value and an arithmetic mean $\bar{K}_\gamma$ found for each group. This was done using values of K_γ worked out at 400 MHz and 1400 MHz. At 400 MHz only non- suspect SNRs in Downes' catalogue were used, and at 1400 MHz all SNRs in the catalogue except for the four in the Local Magellanic Clouds were used. A graph of log $\bar{K}_\gamma$ versus γ was drawn as shown in fig. (4.5). The points at 400 and 1400 MHz coincided quite well and a best straight line was drawn through them. The results indicate that to a good approximation log K_γ is linearly dependent on γ

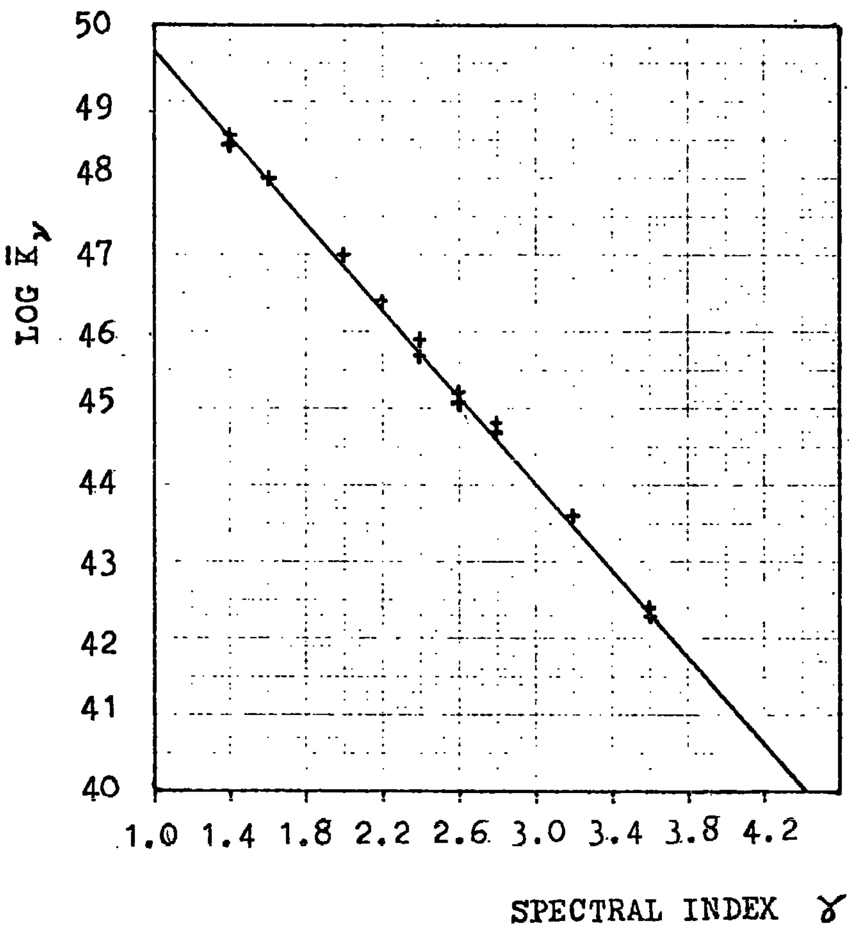
Therefore $K_\gamma = A e^{-B\gamma}$ ——————(4.6)

where A and B are constants.

Therefore $n(E)\, dE = Ae^{-B\gamma} E^{-\gamma} dE$ ——(4.7)

(substituting equation (4.6) in equation (4.2)).

Using the fact that the distribution of γ - values for the sample of SNRs observed in the Galaxy is Gaussian it was therefore possible to add up the spectra of all the various SNRs in the Galaxy by integrating equation (4.7) after weighting it with the normal distribution function, equation (4.1).



<u>Fig. 4.5.</u> DEPENDENCE OF $\bar{K}_\nu$ (AS DEFINED IN SECTION 4.2) ON SPECTRAL INDEX. THE POINTS PLOTTED WERE WORKED OUT FROM MEASUREMENTS OF FLUX DENSITY AT 400 AND 1400 MHZ.

Letting the total number of SNRs in the Galaxy be M then the total number of electrons , obtained from all the SNRs in the Galaxy, with energy between E and E + dE is given by

$$N(E)dE = \frac{MA}{\sigma\sqrt{2\pi}}\int_{\gamma_{min}}^{\gamma_{max}} \exp\left[\frac{-(\gamma-\bar{\gamma})^2}{2\sigma^2} - B\gamma\right] E^{-\gamma}\, dE\, d\gamma \qquad (4.8)$$

Adopting values for γ_{max}, γ_{min}, A, B, $\bar{\gamma}$, σ, and M, equation (4.8) could be integrated numerically. A and B were obtained from fig. (4.5) and $\bar{\gamma}$ and σ from the earlier results quoted in this chapter for the non - suspect SNR in Downes' catalogue. γ_{max} and γ_{min} were obtained by taking the maximum and minimum values of γ for non - suspect SNRs in Downes' catalogue. The total number of SNRs in the Galaxy was taken to be about 1000. The parameters used were therefore

$$M = 1000$$
$$A = 3.07 \times 10^{52}$$
$$B = 6.47$$
$$\gamma_{max} = 3.6$$
$$\gamma_{min} = 1.4$$
$$\bar{\gamma} = 1.96$$
$$\sigma = 0.38$$

The numerical integration was done for suitable values of E, fig.(4.6) shows a plot of log N(E) versus E.

The energy range over which the radio emission directly gives information on the electron energy spectrum is given by

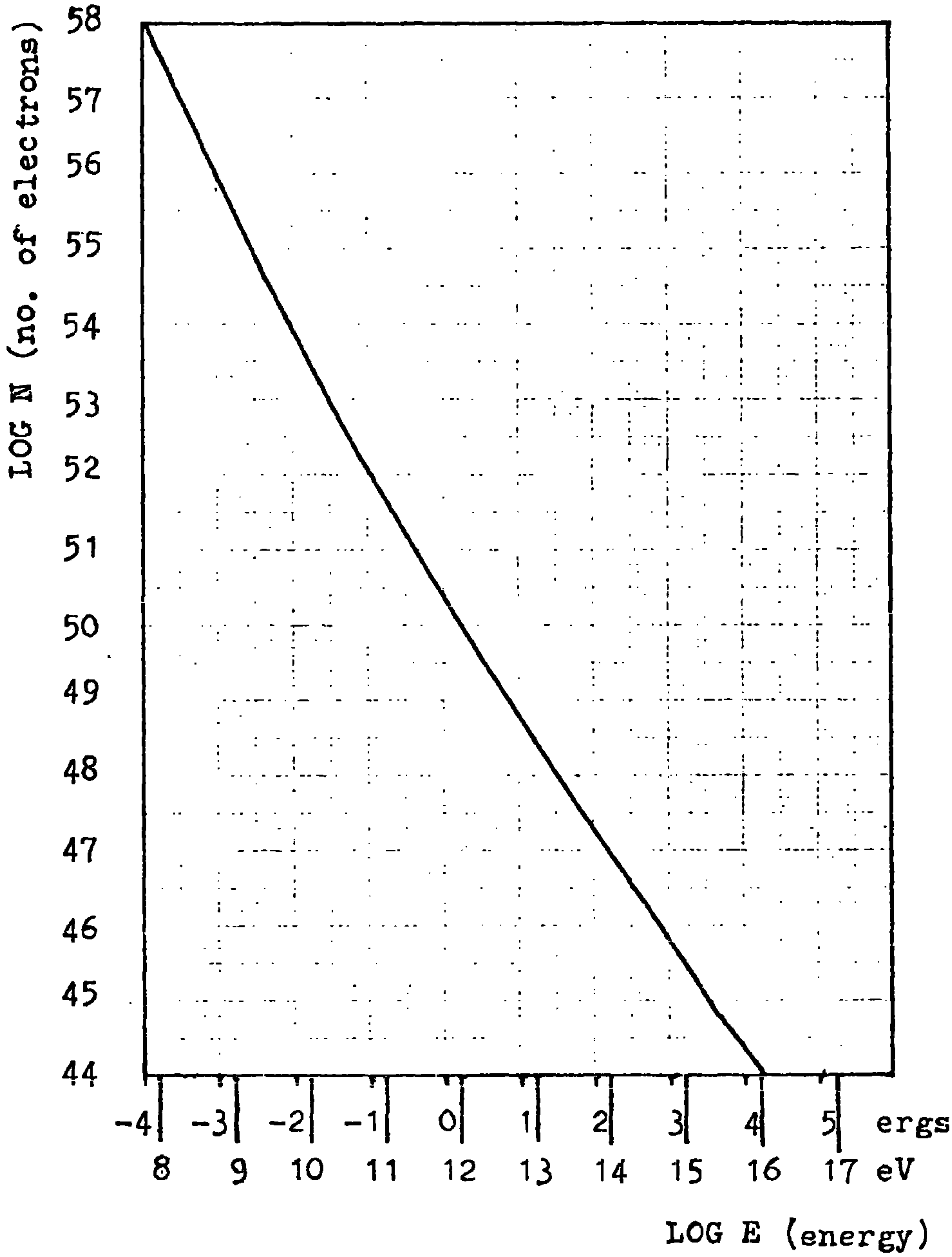


Fig. 4.6. ANALYTICAL ADDITION OF SUPERNOVA ENERGY SPECTRA ASSUMING A NORMAL DISTRIBUTION OF SPECTRAL INDICES.

$$2.5 \times 10^2 \left(\frac{\nu_1}{H\ y_1(\gamma)} \right)^{1/2} \leqslant \quad E_{ev} \leqslant 2.5 \times 10^2 \left(\frac{\nu_2}{H\ y_2(\gamma)} \right)^{1/2} \qquad (4.9)$$

In Downes' catalogue $\nu_1 = 400$ MHz and $\nu_2 = 5000$ MHz. An average value for H is 10^{-4} gauss.

Using these values then the energy range depends on γ as follows (energy in eV).

TABLE 4.1.

SPECTRAL SLOPE	ENERGY RANGE	
γ	E_{min} /10^8 eV	E_{max} /10^9 eV
1	5.6	83
1.5	4.4	17
2.0	3.7	10
2.5	3.4	5.1
3.0	3.1	4.2
4.0	2.7	2.2

The maximum and minimum values of γ used in the numerical integration were 3.6 and 1.4 respectively. The mean γ - value was about 2.0 and for this value direct evidence on the slope of the electron spectrum is available for energies between 4×10^8 and 10^{10} eV only. Hence it seems reasonable to assume that the analytical addition is only valid between 4×10^8 and 10^{10} eV. The slope of fig.(4.6) in this energy range is 2.1. The observed slope of the cosmic ray electron spectrum between 2 and 10 GeV seems to be somewhat steeper than this, more like 2.3 or 2.5 according to most experimental results (eg. Muller and Meyer (1973). The electron spectrum appears to have a constant slope of 2.66 $\pm$ 0.10 between 20 and 250 GeV. It begins to flatten out considerably below 2 GeV.

If one extrapolates the spectral indices to higher energies the predicted slope decreases, to about 2.6 to give an even greater discrepancy. If measurements became available of the spectral indices of SNRs at frequencies above the maximum of 5 GHz in Downes' catologue (1971), then if the cosmic ray spectrum is to be reproduced, the spectral index of the analytical addition should increase. If the discrepancy still remained, a confinement time in the Galaxy decreasing with energy would be needed. This could then be tied in with the observed energy dependence of the matter traversed by cosmic rays. However at the moment it would seem that the analytical addition does not give sufficient information on the slope of the spectrum at a high enough energy to enable one to make a definite statement as to whether or not the analytical addition agrees with the observed electron spectrum.

4.3 Composition of cosmic radiation

The chemical composition of the cosmic radiation is quite well known up to about 1 GeV/nucleon. It is interesting to compare the relative abundances of elements in cosmic rays with the relative abundances in the universe as a whole. Below is an extract from a table in Shklovsky (1968) which brings out the essential results. More up to date information on the cosmic ray composition can be found in Rasmussen (1975) and Reeves (1975), but the table below is particularly convenient and brings out the important points

TABLE 4.2.

GROUP OF NUCLEI	RANGE OF ATOMIC NOS IN GROUP	$\left[\frac{N}{N_H}\right]$ C.R.	$\left[\frac{N}{N_H}\right]$ UNIV.

P	1	680	6830
α	2	46	1040
L	3 - 5	1.0	10^{-5}
M	6 - 9	3.0	10
H	≥ 10	1.0	1
VH	≥ 20	0.28	0.05

Column 3 gives the relative concentration of the nuclei in cosmic radiation referred to group H. Column 4 gives the relative concentration of the nuclei in the universe referred to group H according to Cameron (1959).

The most striking differences between these abundances are that the cosmic radiation is relatively rich in nuclei of groups L, whereas these nuclei are actually rare in the universe, and also the relatively high abundance of nuclei of the groups M,H and VH compared to the protons and α - particles. The light group of elements in the cosmic rays, ie. the L - group is believed to be produced mainly by the fragmentation of heavier elements. The elements of the M - groups mainly carbon, nitrogen and oxygen undergo fragmentation during traversal of matter and form the elements of the L - group, Lithium, Beryllium and Boron. The ratio of Li, Be, B to C, N, O in the cosmic radiation is generally taken as an indication of the amount of matter traversed by the cosmic rays. The observed ratio indicates a traversal of matter of 3 to 5 gm cm^{-2} at least for cosmic rays up to about 1 GeV/nucleon in energy. The greater abundance of heavy elements in cosmic rays is generally taken to indicate the highly evolved nature of the sources or some sort of preferential acceleration process favouring elements of higher atomic number.

Reeves (1973) shows graphs of the abundances of universal matter as a function of atomic number and also of abundances in cosmic rays as a function of atomic number after correction for spallation in space. He refers to matter with the former abundance ratios as "POP I" matter, and that with the latter abundance ratio as "POP O" matter. The abundance curves of POP I and POP O matter are very similar. The relative abundance of elements in each curve varies by ten orders of magnitude but (after correction for interstellar spallation) the differences between the two curves never differ by more than one order of magnitude.

The recent analyses by Cassé and Goret (1973) and by Havnes (1973) have shown that the so called "source" overabundances of the Galactic cosmic rays exhibit a fair degree of correlation with the first ionisation potential (preferential acceleration ?) of the element under consideration, the overabundance being defined as the ratio of POP O to POP I matter. The general trend is for the overabundance to decrease as the ionisation potential increases.

Recent data on solar cosmic rays by Mogro - Campero and Simpson (1972) gives evidence that solar cosmic rays do not have the solar photospheric abundances. Hence it seems likely that preferential acceleration takes place in the sun and so could occur elsewhere in the universe.

The close but not exact similarity between POP I and POP O matter suggests that the cosmic rays are accelerated in matter which has undergone a fair amount of mixing with the interstellar matter. The most likely way in which this could happen is to assume that the cosmic rays are accelerated in supernova remnants. As a supernova remnant expands it mixes with the interstellar matter which is swept up and, by choosing the correct time during the expansion for acceleration to begin, one could get the correct composition for POP 0 matter. The time at which acceleration begins would have to be somewhere between the time of the explosion and the time at which the remnant merges into the interstellar medium, ie. between 0 and 10^6 years. On this model therefore the Galactic cosmic ray abundances will be made of two components: one component ejected from the supernova (which may not have the same composition for all supernovae) and one component of interstellar matter (ie. POP I matter).

Reeves considers hypothetical supernova mass compositions and concludes that if the acceleration of cosmic rays from a typical supernova takes place on the average after the ejecta has been mixed with a few solar masses, then the cosmic rays will contain abundances in good agreement with observation. The corresponding time delay before acceleration is about 100 years. As pointed out in Reeves' paper it is of interest to note that the intrinsic radio luminosity of very young supernovae of type II like the supernovae in M 101 and NGC 1058, which have exploded three or four years ago is much smaller than in the radio source Cassiopeia A, a supernova remnant which exploded over 200 years ago. The radio emission is produced by relativistic electrons and so the age difference between these sources may represent the time delay for the onset of the acceleration mechanisms; for example in the first few years the densities may be too high for any acceleration to take place.

Some recent measurements on the energy dependance of the cosmic ray charge composition obtained from balloon borne instruments have provided some interesting new facts. Results discussed in Balasubrahmanyan and Ormes (1973) show that the spectrum of the iron nuclei is flatter than that of the carbon and oxygen nuclei by 0.5 of a power. Below a few GeV previous experimental results were consistent with an energy independent composition of the cosmic ray flux. Recent results however have indicated that composition varies with energy above 1 GeV/ nucleon. Results described in the above paper are also cosistant with those of Smith et al. (1973) and Juliuson et al. (1972) in that they indicate that the spectra of secondary nuclei are steeper than those of primaries in the range 1 to 50 GeV. The decrease in the L/M ratio with increasing energy would seem to indicate that the matter traversed by the cosmic rays is energy dependent, since the L - group of nuclei is believed to be formed mainly by spallation of the M - group. A paper by Ramaty et al. (1973) shows a graph of matter traversed versus energy using this interpretation. The indicated energy dependence of the matter traversal is

$$X = 5 E^{-0.22 \pm 0.12} \qquad (4.10)$$

(between 1 and 50 GeV / nucleon) where X is the matter traversed in g cm^{-2} and E is the energy in GeV/nucleon.

This matter traversal dependence probably results either from an energy dependent confinement time in the sources or in the Galaxy.

The flatness of the iron spectrum compared to the energy spectrum of other charge groups in the range 3 to 50 GeV/nucleon, as shown also in Ramaty et al. (1973), seems to indicate either a preferential acceleration mechanism in the source or a different source for the Fe - component of the cosmic rays. Ramaty et al. (1973) favour this latter hypothesis and suggest the iron nuclei are accelerated in pulsars, since the surfaces of the neutron stars believed to form pulsars are likely to consist principally of iron. They then suggest a common origin for the other primary nuclei (which seem to have the same spectrum) such as in supernova envelopes or supernova remnants.

4.4 Traversal of matter by cosmic rays in supernova remnants

The amount of matter traversed by cosmic rays in a supernova remnants is given by

$$X = \int_{t_p}^{t_e} \rho(t)\, c\, dt \qquad \text{------------} (4.11)$$

where X is the amount of matter traversed in g cm^{-2}, t_p is the time at which the cosmic rays are produced in the remnant, t_e is the time at which cosmic rays escape from the remnant, c is the velocity of the cosmic rays (assumed to be that of light), and $\rho(t)$ is the function describing the variation in density of the supernova remnant with time.

The density of the remnant at time t will be approximately

$$\rho(t) = \frac{3M}{4\pi} R^{-3} \qquad \text{------------} (4.12)$$

(ie. mass M ejected divided by the volume of a sphere of radius R).

The simplest expression for R valid in the initial stages of the explosion at least is

$$R = v_o t$$

Therefore $$\rho(t) = \frac{3M}{4\pi v_o^3} t^{-3} \qquad (4.13)$$

(v_o is the initial velocity of the ejected shell).

Substituting equation (4.13) into equation (4.11) and performing the integration gives

$$X = \frac{3Mc}{8\pi v_o^3}\left[\frac{1}{t_p^2} - \frac{1}{t_e^2}\right] \qquad (4.14)$$

There are two limiting forms of equation (4.14). If one believes that the cosmic rays are formed in the outer envelope of a highly evolved star by an outward moving shock wave resulting from such a star undergoing a supernova explosion, as envisaged by Colgate and Johnson (1960), then the trapping time $t_e - t_p = 0$ and therefore equation (4.14) is equal to zero. The cosmic rays in this model are formed in the outer layers of the star and would not be expected to traverse much matter on escaping, in fact one would expect them to precede the ejected shell of matter.

On the other hand if one assumes the cosmic rays are accelerated in the actual supernova remnant itself and furthermore become trapped for long periods in the shell by the shell magnetic field then $t_e \gg t_p$. One can then obtain an expression for the time after the explosion at which the cosmic rays are produced from equation (4.14).

$$t_p = \left[\frac{3Mc}{8\pi v_o^3 X}\right]^{1/2} \qquad (4.15)$$

Assuming a value for the amount of matter traversed and putting in typical values for M and v_0, one can predict the time after the explosion at which cosmic rays must be formed using equation (4.15). The velocity at which the mass M is initially ejected is unlikely to vary by more than a factor of two from 7000 Km s^{-1} whether the supernova is type I or II, and would certainly be within a factor of 3 of this figure. Using equation (4.15) figure (4.7) was produced. This graph shows the dependence of matter traversed in the supernova remnant versus the time after the explosion at which the cosmic rays are produced, for various shell masses ejected. It has been assumed in this graph that v_0 = 7000 Km s^{-1} and that the cosmic rays are trapped for times much greater than the production time.

The ratio of light to medium nuclei in the cosmic ray flux indicates the traversal of no more than about 5 g cm^{-2} of material. What proportion of this matter is traversed in the interstellar medium and what proportion is traversed in the source itself is not really known. However whether one assumes practically all of it is traversed in the source or in the interstellar medium one can see from fig.(4.7) that the cosmic rays must be produced some time after the supernova explosion, indeed several years after if one assumes only about 1 g cm^{-2} of matter traversal in the source region. This type of argument can be found in Peters (1959).

Since the work of Reeves (1973) discussed earlier (and some gamma - ray work by Higdon (1975), to be discussed later) suggests that a fair proportion of the matter traversal by cosmic rays could occur in the source region, it is of interest to see if the observed energy dependence of the matter traversal given by equation (4.10) could be explained in this way.

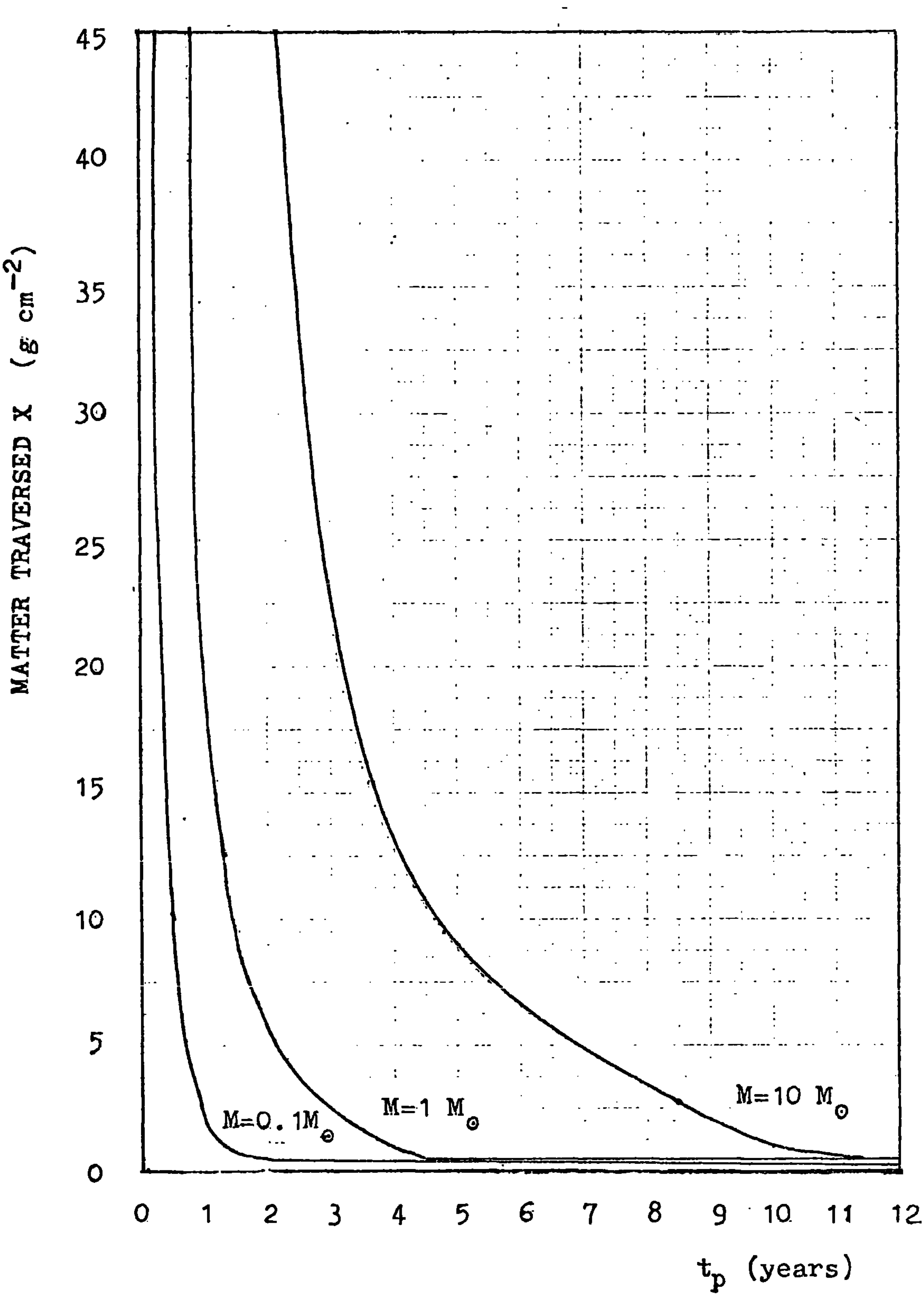


Fig. 4.7. AMOUNT OF MATTER TRAVERSED IN A SNR VERSUS TIME AT WHICH COSMIC RAYS ARE FORMED AFTER THE EXPLOSION, ASSUMING THEY ARE TRAPPED IN THE SNR FOR EFFECTIVELY INFINITE TIMES. THE RESULTS ARE SHOWN FOR VARIOUS MASSES M, ASSUMING THE SNR EXPANDS WITH A CONSTANT VELOCITY OF 7×10^8 cm s^{-1}.

At first sight an energy dependent trapping time of cosmic rays in supernova remnants would be expected. As the supernova remnant expands its magnetic field will decrease due to conservation of magnetic flux and so the maximum energy of particles which it can trap will decrease with time. The SNR cannot contain a particle whose Larmor radius is greater than the radius of the remnant itself, and this enables one to predict a very simple path length dependence on energy, as described below.

The maximum possible energy of an ion contained in a SNR is given by

$$E_{max} = 300 \frac{Z}{A} H R \quad \text{eV/nucleon} \qquad (4.16)$$

where Z is the atomic number of the ion, A is the atomic mass number of the ion, H is the magnetic field in the remnant in gauss and R is the radius of the remnant in centimetres.

Conservation of magnetic flux as the SNR expands means that at times t_1 and t_2

$$H_1 R_1^2 = H_2 R_2^2 \qquad (4.17)$$

Substituting for $H_1 R_1$ from equation (4.17) in equation (4.16) leads to

$$E_{2\,max} = E_{1\,max} \frac{R_1}{R_2} \qquad (4.18)$$

Therefore if $R_2 > R_1$, then $E_{2\,max.} < E_{1}\,max.$ Equation (4.14) gives after putting $R = v_o t$.

$$X = \frac{3\,M\,c}{8\pi v_o}\left[\frac{1}{R_1^{\,2}} - \frac{1}{R_2^{\,2}}\right] \qquad (4.19)$$

Substitution for R_2 from (4.18) gives

$$X = \frac{3\,M\,c}{8\pi v_o}\,\frac{1}{R_1^{\,2}}\left[1 - \left(\frac{E_{2\ max}}{E_{1\ max}}\right)^2\right] \qquad (4.20)$$

As $R_2 \rightarrow$ infinity, $E_{2\ max} \rightarrow 0$ and equation (4.20) becomes

$$X_o = \frac{3\,M\,c}{8\pi v_o}\,\frac{1}{R_1^{\,2}} \qquad (4.21)$$

where X_o is the amount of matter traversed by particles trapped for infinite times. One can thus substitute for R_1 in equation (4.20) to give

$$X = X_o\left[1 - \left(\frac{E_{2\ max}}{E_{1\ max}}\right)^2\right] \qquad (4.22)$$

$E_{1\ max}$ is defined by equation (4.16) where R_1 is given by equation (4.21).

Choosing M = 1 solar mass

$= 2 \times 10^{33}$ g

$v_o = 7 \times 10^{8}$ cm s^{-1}

$X_o = 5$ g cm^{-2}

gives $R_1 = 4.52 \times 10^{16}$ cm.

Taking $Z/A \simeq 1/2$ and using $H \simeq 10^{-3}$ gauss when $R \lesssim 1$ pc as observed in the Crab Nebula , conservation of magnetic flux (ie. equation (4.17)) gives $H_1 = 4.7$ gauss. Therefore equation (4.16) gives $E_{1\ max} = 3.2 \times 10^{10}$ GeV.

Substitution for $E_{1\ max}$ in equation (4.22) gives

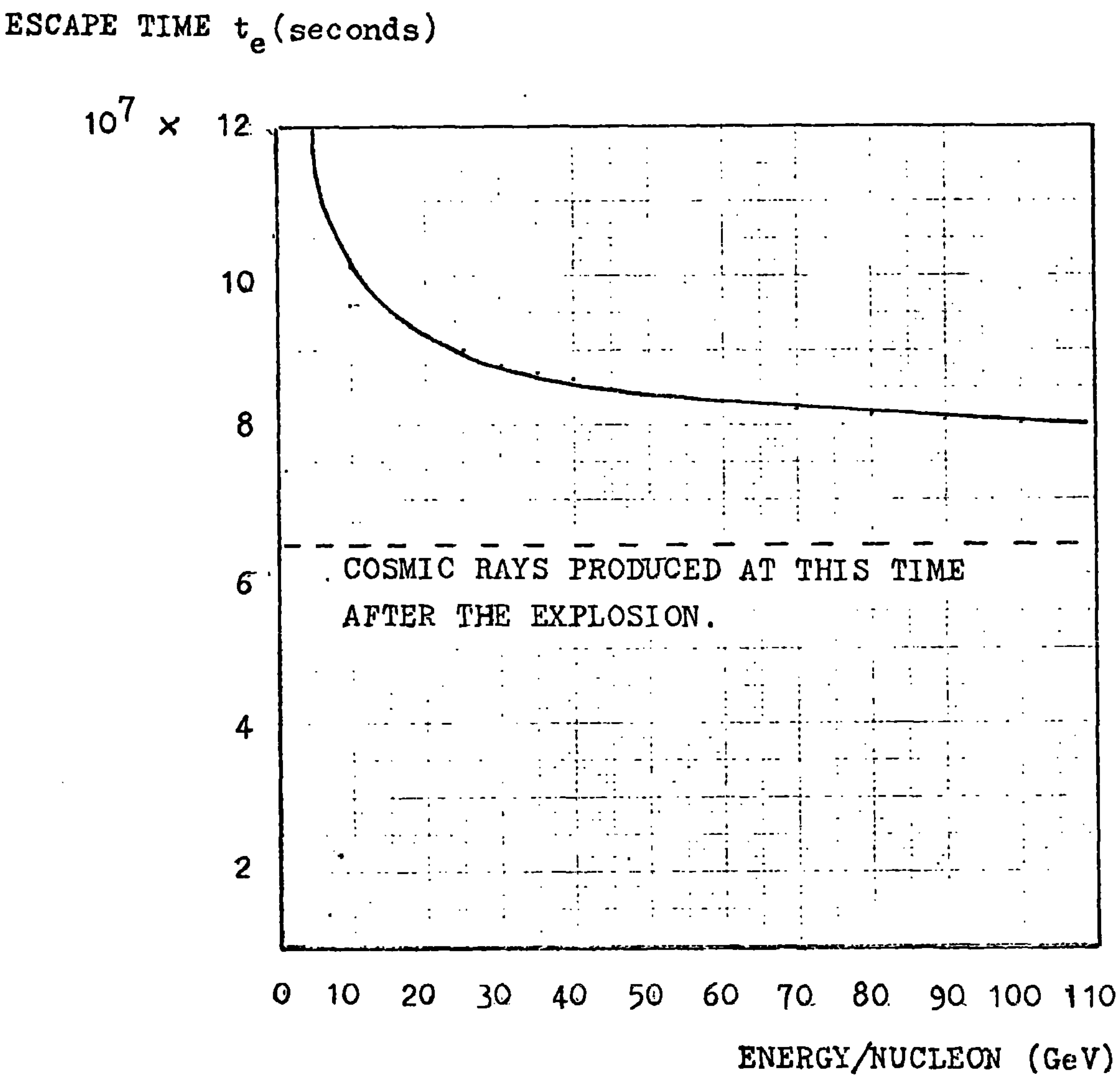
$$X = 5 \left[1 - \frac{E^2}{10^{21}} \right] \text{g cm}^{-2} \qquad (4.23)$$

Equation (4.23) thus gives the matter traversal dependence on energy (in GeV) assuming that a particle escapes from a SNR when its Larmor radius exceeds the size of the remnant. It can be seen immediately that equation (4.23) would predict a neglible dependence in the range 1 to 50 GeV in contrast to the steep dependence observed. In fact equation (4.23) would only predict a noticeable dependence at energies of a few times 10^{10} GeV.

It is of interest to see what dependence of escape time on energy is required to explain the observed X(E) dependence described by equation (4.10). Substituting equation (4.10) in equation (4.14) gives

$$\frac{1}{t_e^2} = \frac{1}{t_p^2} - \frac{2.5 \times 10^{-16}}{E^{0.22}} \qquad (4.24)$$

using the previously adopted supernova parameters. Defining t_p by equation (4.15) a graph of t_e versus energy was plotted and is shown in fig.(4.8). This graph shows that one can explain the dependence by postulating that all the cosmic rays are produced about 2 years after the explosion but all of those above 5×10^9 eV escape within 2 years of being produced while the particles of



<u>Fig. 4.8.</u> ESCAPE TIME OF COSMIC RAYS FROM SNR VERSUS ENERGY, IN ORDER TO SATISFY OBSERVED PATH LENGTH DEPENDENCE ON ENERGY.

energy below about 10^9 eV remain trapped for effectively infinite times. This appears to require a very energy sensitive trapping mechanism which seems somewhat unlikely.

An alternative way of explaining the observed dependence is to assume that the cosmic rays are trapped in the supernova remnant for very long times but that the time at which they begin to traverse the main shell matter is energy dependent. In effect this means letting the escape time, $t_e \rightarrow$ infinity, and varying the production time, t_p in equation(4.24).

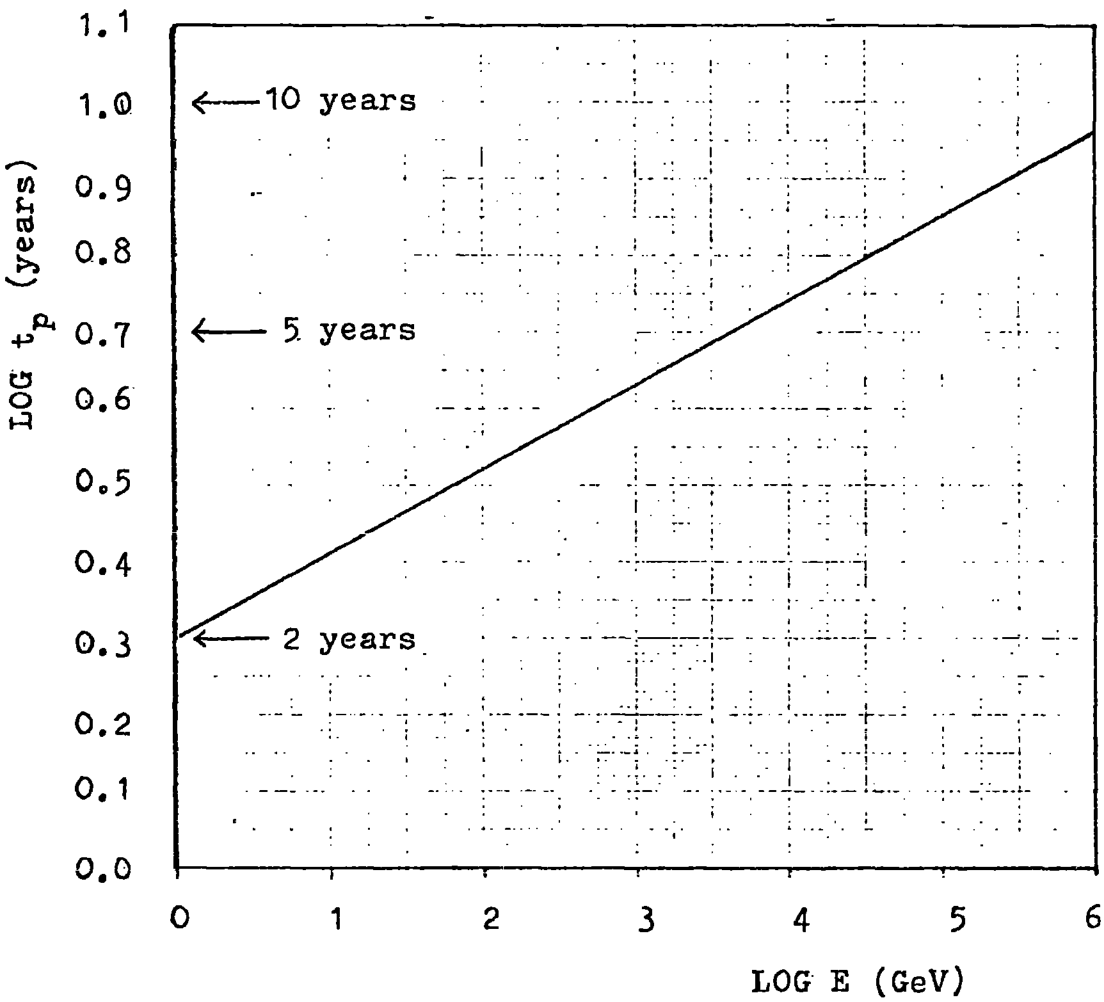
$$\text{Therefore } t_p = 6.3 \times 10^7 \; E^{0.11} \text{ seconds}$$
$$= 2 \; E^{0.11} \text{ years.} \qquad (4.25)$$

Fig.(4.9) shows how t_p would depend on energy to satisfy this relationship.

A possible physical model which could qualitatively account for this behaviour is to imagine that the cosmic rays are accelerated in a cavity essentially devoid of matter between the remnant star left after the supernova explosion and the ejected shell. If one assumes they have a certain probability of escape from the cavity, but that the longer they remain in it the higher the energy they attain, then since the remnant is expanding with time the higher energy particles (having been in the cavity longer) will traverse less matter on getting out. Note that t_p on this model is the time of escape from the cavity, since it is then that the matter traversal begins.

$$E \simeq 0.002 \; t_p^9 \quad \text{GeV} \qquad (4.26)$$

where t_p is in years.



<u>Fig. 4.9.</u> PRODUCTION TIME VERSUS ENERGY IN A TYPICAL SNR IN ORDER TO SATISFY THE OBSERVED PATH LENGTH DEPENDENCE ON ENERGY. IT HAS BEEN ASSUMED THAT A MASS OF ABOUT 1 $M_\odot$ IS EJECTED AT A VELOCITY OF 7×10^8 cm s^{-2}.

This equation implies that particles escaping from the cavity 2 years after the explosion have energy of about 10^9 eV and traverse about 5 g cm^{-2} in the shell, whereas those escaping about 12 years after the explosion have energy of about 10^{16} eV and traverse about 2 g cm^{-2} in the shell. This idea is qualitatively similar to ideas on pulsar acceleration mechanisms but little is known about the early stages of pulsar formation. The youngest pulsar observed is the one in the Crab Nebula which is about 920 years old. All current theories of pulsar acceleration mechanisms predict that the very high energy particles are produced in the early stages of pulsar evolution, when it is spinning rapidly , and particles of lower and lower energy are produced as the pulsar slows down. This means that the lower energy particles ought to traverse less matter than the higher energy particles, since they are produced at later times, which is contrary to observation. Since pulsars are observed to be slowing down then if the current theories of particle acceleration are correct observations would require there to be a speeding up phase of pulsar evolution lasting a few years in the very early stages of the supernova remnant. This only follows of course if one tries to explain the energy dependence of the matter traversal by processes in the supernova remnant itself rather than by energy dependent escape from the Galaxy.

In conclusion therefore it can be seen that if one does attempt to explain the observed energy dependence of the cosmic ray matter traversal by mechanisms within a supernova remnant, then an explanation effectively varying the production time of the cosmic rays in the remnant, rather than the escape time from the remnant, would offer the best hope of success. Theories and discussion of pulsar acceleration can be found in papers such as Kulsrud, Ostriker and Gunn(1972) or Pacini(1975). One of the earliest detailed theories of pulsars can be found in Gunn(1969).

CHAPTER 5 DIRECT EVIDENCE OF COSMIC RAY NUCLEI IN SUPERNOVA REMNANTS FROM GAMMA RAY DATA

5.1. General remarks on gamma ray results

As stated earlier in this thesis synchrotron radiation from SNRs only gives direct information on the presence of relativistic electrons in the remnants, the radiation from relativistic ions being negligible in comparison. However one would expect that collisions between cosmic ray protons and gas in a SNR would result in the production of gamma rays from the decay of neutral pions produced in the collisions. The possibility of detecting these gamma rays has become more of a reality in recent years due to satellite born experiments such as SAS - 2, and such experiments appear to be the means whereby one can observe cosmic ray nuclei in their sources directly.

A plot of gamma ray intensity versus Galactic longitude obtained from the SAS - 2 experiment is shown in fig.(5.1). This graph is taken from a figure in Fichtel et al.(1975), the figure having been redrawn on semi-log paper. The graph shows distinct peaks at the positions of the Crab Nebula , Vela X and the Cygnus Filaments, all well known SNRs. Hence the results seem to show enhancement of gamma rays at the positions of some known SNRs. However there is no noticeable peak at the position of Cassiopeia A, the youngest and most powerful radio emitter of the SNRs known to exist in the Galaxy. The large peak at the Galactic centre may be due to SNRs which we do not observe

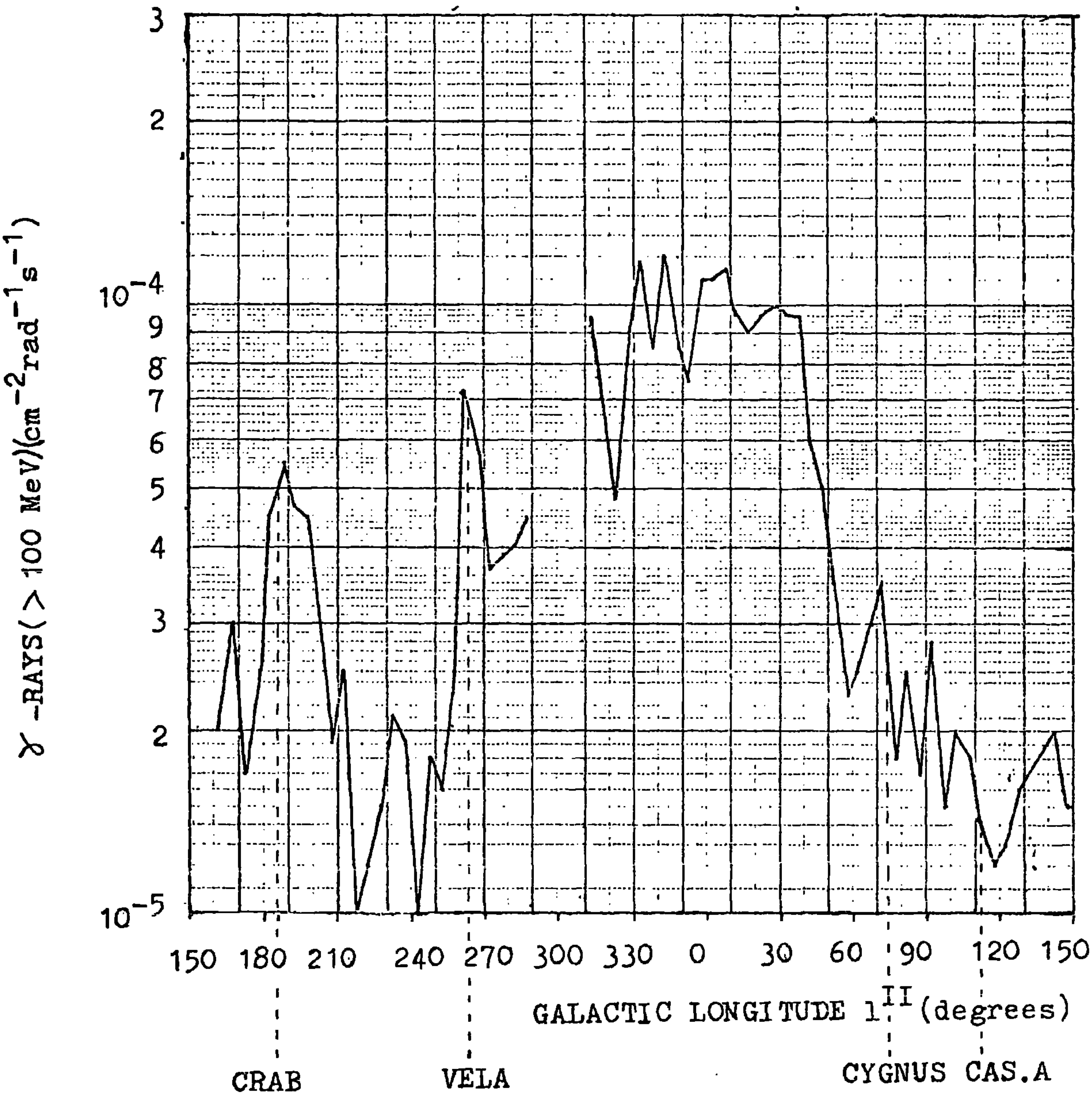


Fig. 5.1. DISTRIBUTION OF HIGH ENERGY (> 100 MeV) GAMMA RAYS ALONG THE GALACTIC PLANE FROM THE SAS-2 DATA (SUMMED FROM $b^{II} = -10^{\circ}$ to $b^{II} = +10^{\circ}$). THE GRAPH IS TAKEN FROM FICHTEL ET AL. (1975).

because of selection effects as discussed in chapter 3.
The Galactic distribution of gamma rays may be due predominantly to contributions from SNRs or it may be due mainly to cosmic ray interactions in the interstellar medium, with a few peaks superimposed on the distribution by SNRs. It is not yet clear just how significant the supernova contribution is.

5.2. Models of gamma ray production in supernova remnants.

According to De Freitas Pacheco(1973) it is very difficult to explain the gamma - ray Galactic background flux by the pion decay mechanism in SNRs, unless we drastically change our current ideas on the energetics of supernovae. In order to compute the gamma ray flux from a remnant one must know the variation of the relativistic particle energy with the shell radius. Pacheco believes that the SNR model proposed by Ostriker and Gunn (1971) provides the most favourable situation for gamma ray production by neutral pion decay. This model assumes that the energetic output of a pulsar is the main factor controlling the early stages of a supernova and the evolution of the remnant. Taking into account the relativistic particle pressure and the ram pressure of the interstellar medium (which decelerates the remnant), Pacheco obtains the equation of motion of the shell and is able to calculate numerically the total relativistic particle energy as a function of radius using typical SNR parameters.

Assuming the gamma rays result from π^{o} decays produced in proton - proton collisions, he calculates the total gamma ray luminosity for a typical SNR as a function of shell radius.

He then uses this result to obtain the equivalent line source strength in the Galactic plane caused by all the remnants, assuming one supernova event every 70 years and also assuming that particles are trapped in the remnant until it attains a radius of 35 parsecs.

The result obtained for the gamma ray line source strength in the Galactic plane is about 100 times smaller than the value reported by Clark et al.(1968). According to Browning et al.(1972) three gamma ray sources may be indentified with SNRs. These sources are the Cygnus Loop, Cassiopeia A and Tycho's remnant. Pacheco compares the computed gamma ray luminosity from the measured flux density given in Browning et al.(1972) with the theoretical gamma ray luminosity for a SNR of the appropriate radius on his model, for each of the three remnants. The theoretical luminosities based on the simple model are one to three orders of magnitude smaller than the computed luminosities. He claims that since his model maximises the gamma ray production by pion decay and is still not able to reproduce the observed luminosities then the gamma ray emission mechanism is not that of pion decay. On his model to obtain agreement with these observations one has to suppose that the total energy released by the pulsar is about 10^{54} erg, which is a rather high value (model assumed 6.6×10^{51} erg). If one assumes this high value then there is no longer agreement between the observed parameters of the remnants such as radius,expansion velocity and age.

The results for the SAS - 2 satellite (Fichtel et al. 1975) however strongly contradict the balloon flight measurements of Browning et al. They are able to give upper limits only to the γ ray flux ($>$ 100 MeV) but these are much lower than the positive values claimed by Browning et al. The SAS-2 upper limits (at the 95% confidence level) are given in the following table together with the predictions of Pacheco.

TABLE 5.1.

OBJECT	UPPER LIMIT TO MEASURED γ - RAY FLUX (> 100 MeV) FROM SAS-2 (photons cm^{-2} s^{-1})	COMPUTED LUMINOSITY (photons s^{-1})	THEORETICAL LUMINOSITY (photons s^{-1}) ACCORDING TO PACHECO(1973)
CYGNUS LOOP	1.4×10^{-6}	9.2×10^{37}	1.6×10^{37}
CASSIOPEIA A	1.1×10^{-6}	9.8×10^{38}	1.0×10^{39}
TYCHO	1.1×10^{-6}	4.3×10^{39}	5.0×10^{37}

It can be seen from table 5.1 that the predicted luminosities fall below the observational upper limits although for Cassiopeia A the computed and theoretical luminosities are very close indeed. The SAS-2 results give a positive flux for the Vela SNR (> 100 MeV) of $(5.0 \pm 1.2) \times 10^{-6}$ photons cm^{-2} s^{-1}. Assuming a distance of 460 pc. for the remnant, this gives a computed luminosity of 1.27×10^{38} photons s^{-1}. The predicted luminosity for the Vela SNR of radius 15 pc is 2×10^{37} photons s^{-1} using the luminosity - radius graph given by Pacheco.

Apart from this factor of six excess of observed flux over the prediction for Vela the Pacheco model is consistent with observations and it is not necessary to invoke a γ - ray emission mechanism in addition to pion decay. Using the Pacheco predictions and the distribution of SNRs in the Galaxy derived in chapter 3, the Galactic distribution of γ - rays from SNRs is calculated in section 5.3.

The implications of the measured flux from the Vela SNR have been considered in some detail by Higdon (1975). He claims that the observed gamma ray excess from Vela (from the SAS-2 results) provides the most direct evidence so far that supernovae do in fact produce sufficient energy in relativistic protons and heavier nuclei to be the principal source of cosmic rays. The gamma ray emission from the region around the Vela SNR has an intensity which is a factor of two greater than the surrounding background intensity and is seven standard deviations above that background. Timing analyses of the emission show that the Vela pulsar PSR 0833 contributes less than 15% of the gamma ray excess. The γ - rays could result from a "cloud" of cosmic rays surrounding the Vela supernova, as suggested by Pinkau (1970). The flux of gamma rays from the decay of pions produced by cosmic ray interactions with the interstellar gas can be written

$$F = \frac{\bar{n}_H \, W_{SN} \, q}{4 \pi d^2} \qquad (5.1)$$

where d is the distance of the cosmic ray cloud from the earth. $\bar{n}_H$ is the average hydrogen density seen by the cosmic rays, W_{SN} is the total energy of protons and heavier nuclei in the cosmic ray cloud produced by the supernova, and q is the yield of pion decay gamma rays per second per erg of cosmic ray protons and heavier nuclei.

Using his own estimates and others Higdon takes as a mean gamma ray yield for the Vela region a value of

$$q = 1.4 \times 10^{-13} \ \gamma \ (> 100 \text{ MeV}) \ \text{erg}^{-1} \ \text{s}^{-1}$$

The observed flux then requires that the product of interstellar gas density and the total cosmic ray energy released by the supernova be

$$n_H \ W_{SN} = 0.9 \times 10^{51} \ \text{erg cm}^{-3}.$$

Adopting a value of $n_H = 0.6 \text{ cm}^{-3}$ in the Vela region for which there is some observational evidence, the total energy in cosmic ray protons and heavier nuclei of energy greater than 500 MeV/ nucleon produced by the Vela supernova is $W_{SN} = 1.5 \times 10^{51}$ erg. Higdon takes into account the gamma ray yield from electron bremsstrahlung and concludes that its maximum contribution to the gamma ray excess is about 20%.

A total energy release in the Vela supernova of about 10^{51} erg in cosmic ray protons is an order of magnitude larger than that required from each supernova if all Galactic supernovae are cosmic ray sources (see chapter 1). The observed gamma ray excess from Vela, with an energy release of only 3×10^{49} to 10^{50} erg, would require an anomalously high mean gas density seen by the cosmic rays around Vela, of 9 to 30 cm^{-3}.

The apparent release of about 10^{51} erg of cosmic rays from the Vela supernova therefore suggests to Higdon two possible interpretations. One is that only a small fraction of all supernovae produce the bulk of the cosmic rays. Such sources must have a frequency in the Galaxy of one every 500 to 1500 years, thus making up only 3 to 10 percent of all Galactic supernovae. The alternative possibility is that the bulk of the matter traversed by the cosmic rays is in their sources. The mean amount of matter X, and the anisotropy can be related as follows (Lingenfelter, 1973).

$$X = \frac{h\rho}{\delta} \qquad \text{—————— (5.2)}$$

where h and ρ are the scale height and density of the gas in the Galactic disc.

For cosmic rays of energy greater than 10^{13} eV the lowest limit on the anisotropy is $\delta \leqslant 1.3 \times 10^{-3}$ (Gombosi et al.,1975). This limit on the anisotropy requires that 0.5 g cm^{-2} or only 10% of the total amount of matter be traversed in the interstellar medium. This limit on X means that the cosmic ray source power may be as much as 2×10^{41} to 6×10^{41} erg s^{-1} (as opposed to 2×10^{40} to 6×10^{40} erg s^{-1}, assuming all matter traversal is in the interstellar medium). To maintain this cosmic ray source power, supernovae which produce about 10^{51} erg should occur once every 50 to 150 years in the Galaxy. This is roughly consistent with the estimated rate of all supernovae in the Galaxy. It Is interesting that Higdon's latter interpretation ties in with some of the ideas, about the bulk of the matter being traversed by cosmic rays in the sources, described in chapter 4.

A point which Higdon overlooks in his calculation of the total energy released by the Vela supernova is that if the cosmic rays are trapped in the SNR for considerable periods before being released then they would undergo adiabatic energy losses as the SNR expands. The total particle energy varies as the inverse of the radius of the SNR. Assuming that the pre-supernova object has a radius of about 10^{13} cm, and that the supernova releases its cosmic rays into the interstellar medium when its radius is 30 pc, then if the cosmic ray energy at a particular radius is known, simple proportionality arguments give the energy of the cosmic rays at any other radius. Higdon estimates the cosmic ray energy in Vela at the present radius of 15 pc to be 1.5×10^{51} erg. If the cosmic rays are contained in the SNR until it reaches a radius of 30 pc then the energy of the cosmic rays on release is 7.5×10^{50} erg. This is still somewhat larger than the required energy release per supernova if all supernovae are to be cosmic ray sources. However it could be reduced to some extent by making the cosmic ray "release radius" somewhat larger. For example Pacheco (1973) estimates the maximum radius of a supernova, ie. the cosmic ray "release radius" by assuming a value for the energy output per supernova which would give the observed cosmic ray energy density, assuming a supernova frequency of 1 every 70 years. He gets the maximum radius to be 35 pc. However to reduce the cosmic ray energy of the Vela supernova to about 10^{50} erg, the "release radius" would have to be about 200 pc, which is rather unlikely. Therefore Higdon's general conclusions are not really altered by neglecting the adiabatic expansion of the remnant.

5.3 Galactic distribution of gamma rays from SNRs assuming Pacheco's model of gamma ray production.

A calculation was made of the Galactic distribution of gamma rays produced in supernova remnants in the manner envisaged by Pacheco (1973). This could then be compared with the gamma ray distribution obtained from SAS-2 and shown in fig.(5.1).

In order to do this it was necessary to know the distance and radius of each of the observed SNRs in our Galaxy, and also how the real surface density of SNRs depended on distance, in order to take account of SNRs which cannot be seen because of selection effects.

It was decided to adopt the new surface brightness - diameter relation defined by equation (3.13), and to recalculate the distances and diameters of all the Galactic SNRs in the catalogue of Ilovaisky and Lequeux (1972). Fig. (3.10) gives the real surface density of SNRs of diameter less than 50 pc, as a function of distance from the Galactic centre, according to the theory developed in section 3.7. According to equation (3.16) one should see all SNRs of diameter less than 50 pc, within 10 Kpc of the earth. Hence within the 10 Kpc circle the gamma ray flux from each observed SNR of diameter less than 50 pc was worked out using Pacheco's theory, and the contribution to the gamma ray flux from SNRs outside the circle was estimated using the surface density graph shown in fig.(3.10). In doing this one was essentially assuming that SNRs of diameters greater than 50 pc. do not produce any gamma rays. This is probably not quite true but one has to impose a gamma ray cut-off at some arbitrary diameter in order to do the analysis. There is some arbitrariness as to when one considers a SNR to have merged into the interstellar medium. Clearly when the cosmic rays have escaped from the SNR they will continue to produce gamma rays due to collisions in the interstellar medium, so eventually the gamma ray contribution from sufficiently large SNRs will be of the same order of magnitude as that from the interstellar medium. Pacheco assumes a cut - off diameter of 70 pc based simply on the observed cosmic ray energy density at the earth and on an assumed frequency of SNRs of 1 every 70 years. The cut-off diameter of 50 pc adopted here is based simply on the change in slope of the luminosity functions in figs (3.7 a) and (3.7 b) as described in chapter 3. It should be noted that the well known Cygnus SNR is not included, having a recalibrated diameter of 84.1 pc. If it were included it would contribute a line flux of 3.25×10^{-7} photons cm^{-2} rad^{-1} s^{-1}.

Using Pacheco's graph relating gamma ray luminosity (photons s^{-1}) to SNR radius the flux at the earth from observed SNRs within 10 Kpc of the earth could be worked out simply by dividing the luminosity by $4\pi d^2$ (where d = distance from the earth of the SNR). All the observed SNRs within 10 Kpc of the earth were then grouped into 5 degree Galactic longitude intervals to correspond to the way in which the SAS-2 data are presented. The fluxes from the SNRs in each interval were then added together to give a total for that interval. Dividing this total by 5 degrees expressed in radians gave a total gamma ray line flux from the observed SNRs for that longitude interval in units of photons cm^{-2} rad^{-1}. These are the same units as in fig.(5.1).

It was then necessary to take account of the contribution to the gamma ray flux from all the SNRs in the Galaxy outside of the 10 Kpc circle. This was done in the following way. Consider a surface element of the Galaxy of distance d from the earth and at an angle θ from the Galactic centre as shown in fig.(5.2). The element has dimensions dd and $dd\theta$ and a surface density $\rho(\theta,d)$. If F(r) is the fraction of SNRs having radii between r and r+dr, then following the same reasoning as in section 3.3

it follows from the Sedov equation that

$$F(r) = \frac{r^{3/2}}{\int_0^{r_{max}} r^{3/2}\, dr} \qquad (5.3)$$

Since Dmax = 50_{pc}, we must use r_{max} = 25_{pc} in equation (5.3)

The number of SNRs with radii between r and r+dr at distance d from the earth is equal to d x dθ x dd x $\rho(\theta,d)$x F(r). The total number of gamma ray photons per second emitted from a suface element of the Galaxy at distance d from the earth is therefore equal to

$$d \times d\theta \times dd \times \rho(\theta,d) \int_0^{25} F(r)\, L_\gamma(r)\, dr$$

where $L_\gamma(r)$ is the gamma ray luminosity for a SNR of radius r according to Pacheco.

It therefore follows that at Galactic longitude θ the total gamma ray line flux from SNRs more than 10 Kpc from the earth is given by

$$Q_\gamma(>10) = \int_0^{25} F(r)\, L_\gamma(r)\, dr \int_{10}^{y(\theta)} \frac{\rho(\theta,d)\, dd}{4\pi d} \qquad (5.4)$$

where y(θ) is the distance from the earth to the edge of the Galaxy in Kpc. at Galactic longitude θ. Note that the units of equation (5.4) are photons $Kpc^{-2}\ rad^{-1}\ s^{-1}$.

Equation (5.4) was evaluated numerically as a function of θ. The integration limit y(θ) was obtained for a particular θ from a scale drawing of the Galaxy, assuming a radius of 15 Kpc and a distance from the Earth to the Galactic centre of 10 Kpc. For a given θ and d the surface density $\rho(\theta,d)$ was read off from fig.(3.10) using the relation

$R = (100 + d^2 - 20d \cos \theta)^{1/2}$ which can be seen from fig.(5.2).

For each 5 degree longitude interval it was thus possible to add the gamma ray flux from the observed SNRs in the 10 Kpc circle to the gamma ray flux from the other inferred SNRs outside the circle and produce a total gamma ray line flux in photons cm^{-2} rad^{-1} s^{-1} for that longitude interval.

The resulting distribution of gamma rays in the Galaxy is shown in fig.(5.3). The dotted line is the contribution from SNRs further than 10 Kpc from the earth. As is obvious from fig. (5.2) these latter SNRs make no contribution to the flux between approximately 90 and 270 degrees (since $y(\theta)$ is less than 10 Kpc at these longitudes).

A comparison of fig (5.1) and fig.(5.3) immediately shows that SNRs cannot be the main contributors to the gamma - ray flux if the Pacheco model of production is correct. The contribution to the flux from the Galactic centre is two orders of magnitude lower than the results obtained from SAS-2.

According to fig (5.3) however Cassiopeia A should have been detected by SAS-2 giving a line flux of 3.43×10^{-5} photons cm^{-2} rad^{-1} s^{-1}. No peak appears at this position however in fig.(5.1). The peaks at the positions of the Crab and Vela SNRs are about an order of magnitude lower in fig.(5.3) than in fig.(5.1).

It should be remembered however that all the distances and radii used in the calculations were based on the new surface

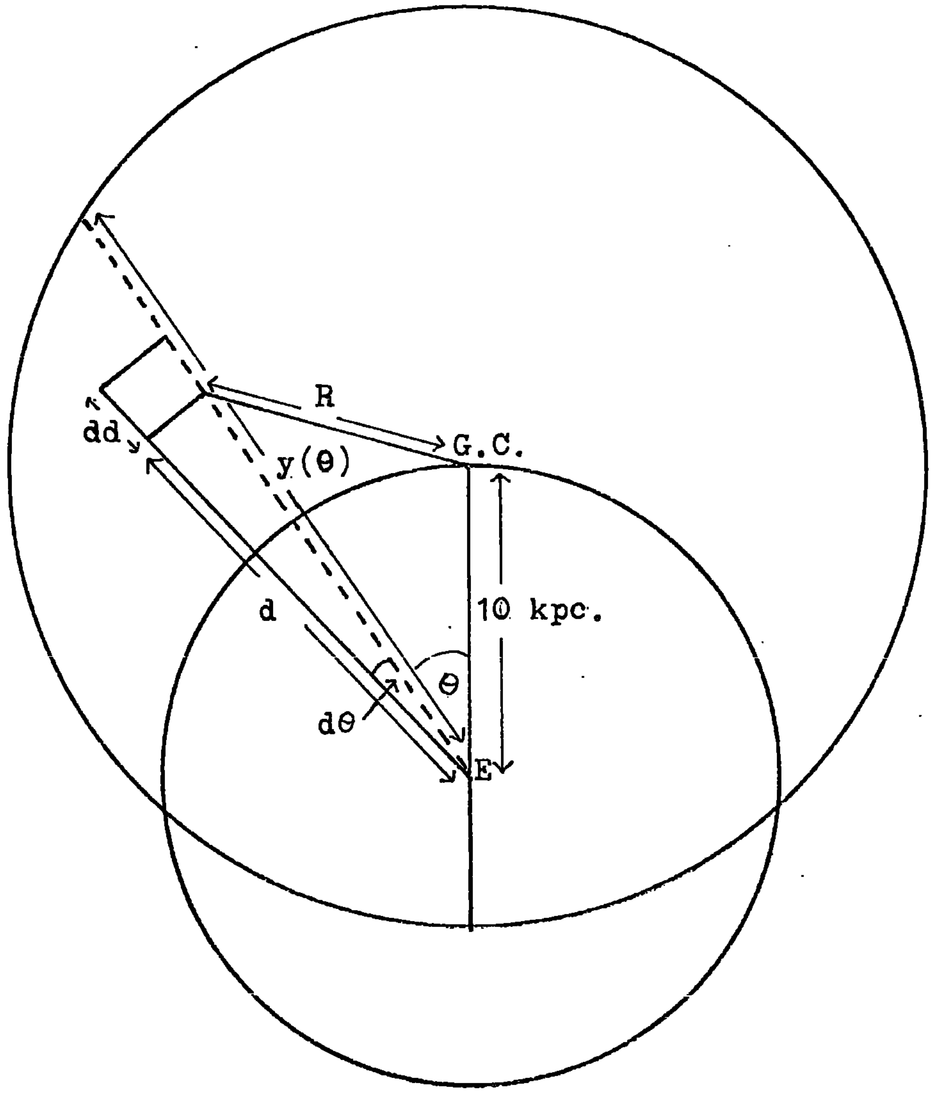


Fig. 5.2. DIAGRAM TO ACCOMPANY THEORY IN SECTION 5.3.

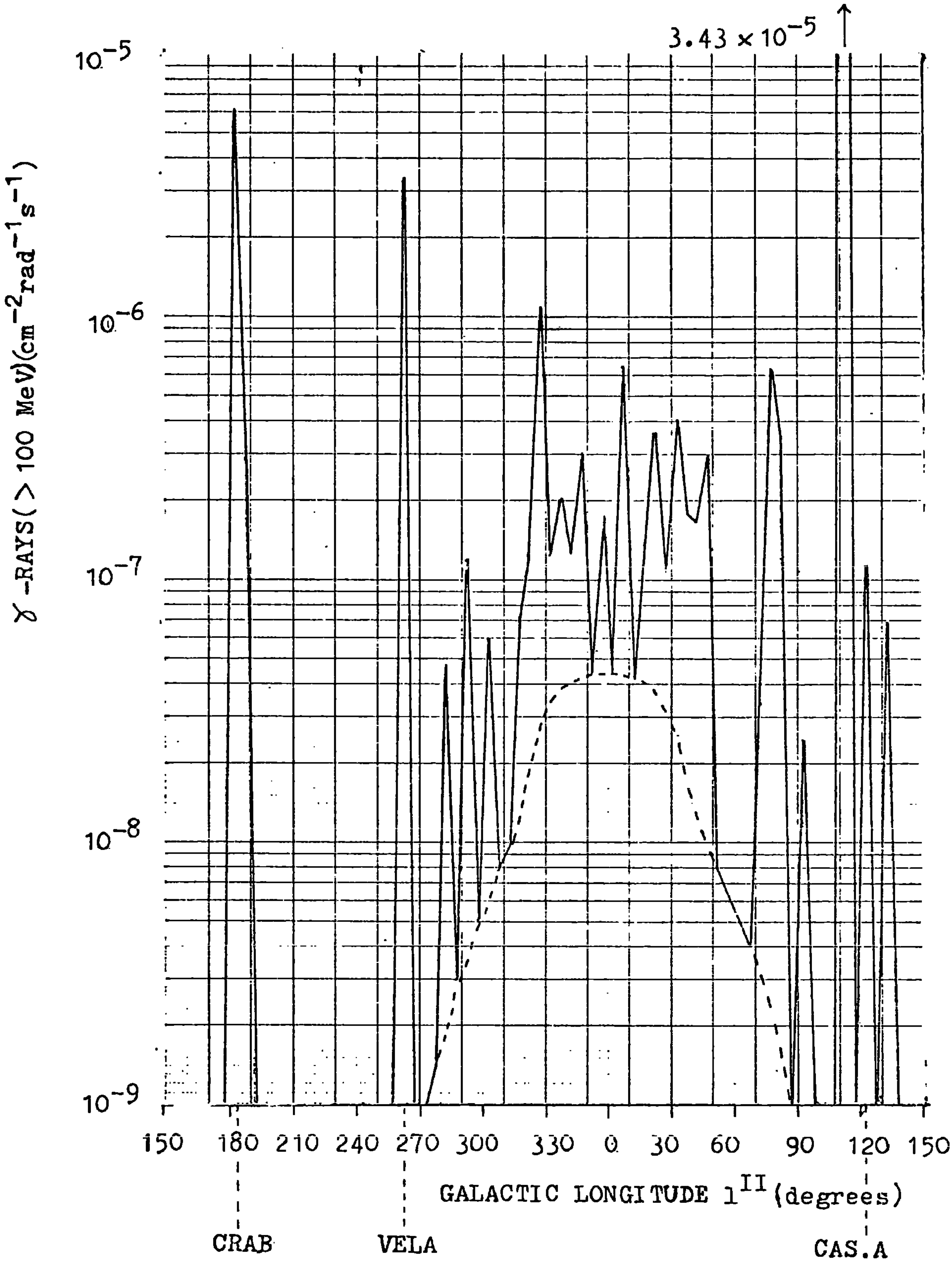


Fig. 5.3. DISTRIBUTION OF HIGH ENERGY (>100 MeV) GAMMA RAYS ALONG THE GALACTIC PLANE, ASSUMING THEY ARE PRODUCED IN SNRs ACCORDING TO THE PACHECO (1973) MODEL AND ASSUMING THE SURFACE DENSITY OF SNRs IN THE GALAXY IS AS SHOWN IN FIG. 3.10. (FOR SNRs OF DIAMETERS < 50 pc ONLY).

brightness - diameter relation . If one uses the distances and radii quoted in IL 72 for the Crab, Vela, Cassiopeia A and Cygnus SNRs then one gets gamma ray line fluxes in photons cm^{-2} rad^{-1} s^{-1} of respectively 7.6×10^{-5}, 7.6×10^{-6}, 7.7×10^{-6} and 1.5×10^{-6}. In this case Cassiopeia A would not be visible in the SAS-2 results, nor would Vela but the Crab would show up significantly.

In conclusion it can be seen that the peaks in the SAS-2 results at the positions of the Crab and Vela SNRs may well be directly attributable to those SNRs. The gamma ray luminosity is directly proportional to the local gas density in which the supernova occurs, and this can vary widely from one supernova position to another. For example the gas density can be as little as 0.1 cm^{-3} in intercloud regions and 10 cm^{-3} in gas clouds themselves, ie. a variation of 100.

This together with uncertainties in the distances and diameters of SNRs and in Pacheco's theory itself could account for the predicted flux from the Vela SNR being an order of magnitude lower than what is observed, for example. For these reasons it is also quite conceivable that Cassiopeia A would not show up as a gamma ray peak in the SAS-2 results.
However though certain prominent peaks in the Galactic gamma ray distribution may be directly attributable to SNRs it would seem that the general background level of gamma radiation, particularly the large flux from the Galactic centre, cannot be explained by "unseen" SNRs alone.

CHAPTER 6 GENERAL CONCLUSIONS AND DISCUSSION

The purpose of this chapter is to summarise the results and conclusions arrived at in the various chapters of this thesis and to try and fit them together to see what light they throw on the production of cosmic rays in supernova remnants.

The method used to derive the dependance of the surface density of SNRs on distance from the Galactic centre (chapter 3) did not produce a result radically different from that of earlier workers such as Ilovaisky and Lequeux (1972), once the statistical errors were taken into account. However recalibration of the observed SNR distances did reduce the actual surface density values by as much as a factor of two in some distance intervals as can be seen by comparing fig. (3.10) with fig. (3.6) or fig. (3.1). The sharp peak in the surface density curve for spiral galaxies derived by Johnson and Macleod (1963) (shown in fig. (3.11)), between 3 and 6 kpc. from the galactic centre, is not quite so prominent in the curve for our galaxy. Fig.(3.10) does show a peak at about this position and the shape of the curve is very similar, though the magnitudes are down by a factor of two.

The supernova frequency derived from the surface density graphs was 1 every 130 years using the surface brightness - diameter relation in IL72 and 1 every 490 years using the corresponding relation in Mathewson and Clarke (1973). Both these frequencies are uncertain by at least a factor of 3.

The commonly quoted frequency of supernova explosions is 1 every 30 years. If the frequency of 1 every 490 years is correct, then supernovae could not maintain the cosmic ray energy density unless their energy output in cosmic rays is higher than is thought by at least a factor of 10 (chapter 1). However if the energy output in cosmic rays of the Vela SNR, suggested by its gamma ray emission to be about 10^{51} ergs, is typical of SNRs in general, then supernovae could still maintain the observed energy density of cosmic rays even with a frequency as low as 1 every 500 years.

The analytical addition of the SNR electron energy spectra shown in fig.(4.6) gave a slope of 2.1, between energies of 4×10^8 and 10^{10} eV., which was the energy range over which the radio emission directly gave information on the SNR electron spectra. This addition was done because at first sight, it seemed to offer a possible way of deciding whether energy dependent confinement time of cosmic rays operated in the Galaxy,which would lead to energy dependent matter traversal, as is observed.

Since there appears to be no correlation between supernova remnant spectral indices and remnant ages, then it seems reasonable to assume that the slope of the energy spectrum of electrons in a remnant remains the same until all the particles are released into the Galaxy. By adding up the energy spectra of all the Galactic SNRs, then this gives a mean value for the spectral slope of electrons being injected into the Galaxy. If this were shown to be significantly flatter than the observed slope then it would imply an energy dependent confinement time for cosmic ray electrons in the Galaxy. This would then lend some support for an energy dependent confinement time for cosmic ray nuclei in the Galaxy, which would be reflected in an observed dependence of matter traversal on energy. However the energy range over which the analytic addition had to be done was one in which the slope of the observed electron spectrum is very uncertain. It could be anything from 2.1 to 2.4, between 2×10^9 and 10^{10} eV. Hence all that one can say is that the analytic addition is not inconsistent with an energy dependent confinement time of cosmic rays in the Galaxy. Since it is certain that the cosmic ray electrons are of Galactic origin, then improved measurements of the electron spectrum in this energy range could throw some light on this problem, if it is accepted that they originate in SNRs.

By considering typical parameters of supernova remnants such as expansion velocity and mass of shell, it was shown that if cosmic rays are produced behind the ejected shell matter, then they must be produced at least 2 years after the explosion (for an ejected mass of 1 $M_\odot$), if they are to traverse no more than 5 g cm^{-2} of matter, as suggested by the ratio of L to M groups of nuclei in cosmic rays. It should be noted however that on the Colgate shock wave production model of cosmic rays in supernovae the cosmic rays would be formed at the outer edge of the exploding star and would precede the bulk of the shell matter. One would then expect the matter traversal to occur predominantly in the interstellar medium. Later developments of Colgate's first paper on this topic (1960) are described in Colgate (1975). The considerations about matter traversal apply only to models in which the cosmic rays are accelerated in the ejected shell matter, such as pulsar acceleration models or statistical acceleration models. The considerations apply of course only to complex nuclei, one obviously cannot tell how much matter a cosmic ray proton has passed through, since it cannot fragment.

If one attempts to explain the observed dependence of matter traversal on energy by postulating energy dependent trapping times in the remnant itself, then it was shown that this was very difficult to achieve by simply varying the escape time of particles from the remnant according to their energy. It was shown that one was more likely to succeed by varying the production time of cosmic rays of different energies in the remnant. However on present theories of pulsar acceleration in SNRs, the production of higher energy particles later in time would seem to require the pulsar to speed up in its early stages of evolution during the first few years. For this to occur it would be necessary to postulate pulsar formation in a system where angular momentum transfer was possible, from some other body. It should be noted however that in producing fig.(4.9) no accout has been taken of adiabatic energy losses during expansion of the remnant. If one assumes that the particles are stored in the SNR until it reaches a radius of 30 pc. and that their energy is inversely proportional to the radius of the SNR, then particles of energy 10^9 eV, produced 2 years after the explosion, would be released with an energy of 5×10^5 eV, and particles of energy 10^{11} eV, produced 3 years after the explosion, would be released with an energy of 7×10^7 eV. However a paper by Wentzel (1973) shows that the cosmic ray deceleration by adiabatic expansion can be reduced from a factor of about 10^3

predicted by the simple ideas described above, to a factor of 2 on a somewhat extreme theoretical model attempting to minimise the coupling between cosmic rays and the interstellar gas, which normally occurs in the presence of an ambient magnetic field, due to the growth of resonant hydromagnetic waves which scatter the cosmic rays.

The SAS-2 gamma ray results have perhaps given the first good direct evidence that SNRs do contain cosmic ray protons at least, in the same way that synchrotron radiation gave the first direct evidence of the presence of cosmic ray electrons in SNRs. It was shown however that using Pacheco's model of gamma ray production in SNRs then the whole of the gamma ray flux cannot be attributed to SNRs, even allowing for ones which are not seen due to selection effects. Some of the peaks in the distribution are almost certainly due to gamma rays from SNRs such as the Crab and Vela.

In conclusion , perhaps there is now more evidence for the presence of relativistic nuclei and electrons in supernova remnants, though their mode of acceleration is certainly still unclear. Though it is now over sixty years since Hess in 1912 first established that the cosmic radiation was coming from outside the Earth, from measurements of ionisation in an ascending balloon, it is still not possible to locate their main site of production precisely. It seems very likely that supernova remnants are one site of cosmic ray production, but what fraction of the flux received at the earth can be attributed to them still remains uncertain.

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