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A STUDY OF THE NOISE TEMPERATURE AND BANDWIDTH
OF MICROWAVE PARAMETRIC AMPLIFIERS

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A Thesis Submitted in Candidature for the Degree of
Doctor of Philosophy in the Faculty of Science of the
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September 1968



Abstract

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Abstract

This study has arisen because of the requirement for a 4 GHz, 500 MHz bandwidth, parametric amplifier with a noise temperature of less than 20°K , for the proposed satellite communication system and is concerned entirely with nondegenerate parametric amplifiers using variable capacitance (varactor) diodes. Although all the measurements involved were at 4 GHz, much of the theory and most of the conclusions of the study can be applied to any part of the microwave frequency range (i.e. from 1 to at least 40 GHz).

A theoretical study of the limitations on bandwidth has been made and good agreement between the measured and theoretical bandwidth of a single diode amplifier is observed.

A novel form of liquid helium cooled broadband amplifier, using two single tuned, single diode amplifiers, separated by a quarterwave section of transmission line, has been realised. This type of amplifier has the advantage of requiring no external mechanical tuning readjustments on cooling to liquid helium temperatures. The $\text{gain}^{\frac{1}{2}}$. bandwidth product measured for this device is twice the $\text{gain}^{\frac{1}{2}}$. bandwidth of a single tuned amplifier, as predicted from the theory. A 4 GHz amplifier of this type producing a noise temperature of less than 20°K over a 250 MHz bandwidth at 30 dB gain or over a 500 MHz bandwidth at 15 dB gain, is described. Greater bandwidths should be available using the same broadbanding technique on two balanced diode amplifiers.

The magnitude of shot noise in parametric amplifiers has been determined by measurements on two single diode amplifiers at room temperature and the factors influencing amplifier noise at very low temperatures, have been investigated by a series of noise measurements on two liquid helium cooled devices.

Finally, some possible future trends in amplifier design are considered.

Acknowledgements

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List of Symbols

B	bandwidth
C	capacitance
C_j	varactor junction capacitance
C_t	varactor transition capacitance
C_1	varactor encapsulation capacitance
e	electronic charge
f	frequency
f_c	varactor cut-off frequency
g	amplifier gain
h	Planck's constant
j	complex operator
k	Boltzman's constant
k_e	electrical conductivity
k_{th}	thermal conductivity
K	parametric amplifier gain constant
K_a	heat transfer constant
l	length
L	inductance
L_s	varactor series stray inductance
n	varactor junction impurity profile constant
P	pump power
Q	resonant circuit quality factor
R	resistance
R_g	generator/load resistance
R_s	varactor total series resistance
T	temperature
v	velocity of light
V	voltage
Y	admittance
Z	impedance
Z_0	characteristic impedance of transmission line
γ	varactor capacitance variation factor
Γ	voltage reflection coefficient
λ	wavelength
τ	lifetime of carriers
ϕ	diode barrier potential
ω	angular frequency

Subscripts

- o refers to condition of resonance or zero volts bias as applicable.
- 1, 2, 3 refer to signal, idler and pump circuits respectively.

1. Introduction and General Theory

1.1 Introduction

The term parametric amplifier is used to describe a series of devices in which gain is obtained by a transfer of power from a periodic source via a non-linear reactance. This process, by its very nature, should be possible in any mechanical or electrical system, at any frequency, although the techniques by which it may be obtained will obviously differ greatly with frequency.

The amplification was possible by such a process was anticipated by Hartley in 1936 who described the possibility of oscillation in systems with non-linear reactance. A practical arrangement of a parametric amplifier was first proposed by Suhl in April 1957 using non-linear magnetic coupling in ferrite material at gyromagnetic resonance. Such a device, at a frequency of 4.5 GHz, was successfully built by Weiss (1957a) in that same year. Uhlir (1958a) anticipated the potential of pn junction diodes as non-linear elements and the first microwave parametric amplifiers using such varactor diodes, to be described in the literature were those of Heffner and Kotzebue (1958) and Hermann, Uenohara and Uhlir (1958). Uenohara (1963) described a liquid helium cooled 4 GHz parametric amplifier developed for a very low noise temperature ($< 20^{\circ}\text{K}$). Such devices, with comparable noise temperatures to a maser, but with a greater bandwidth potential, are beginning to replace the maser in many applications.

This study is concerned entirely with microwave amplifiers and has arisen because of the requirement for a wideband ($> 10\%$) low noise ($< 20^{\circ}\text{K}$ noise temperature) amplifier in the proposed satellite telecommunication system. The amplifiers described in this study were developed primarily to meet this requirement and therefore operate at 4 GHz, but most of the conclusions of the study apply to devices in any part of the microwave frequency range, (i.e. 1 to at least 40 GHz).

1.2 General Remarks

The amplification process can be illustrated in a general manner. If a signal f_s is mixed with a signal of higher frequency and power level, f_p , (fig. 1.1) in any system which stores energy non-linearly, then intermodulation products will be observed to an infinite order of harmonics. So $f_p - f_s$, $f_p + f_s$, $2f_p - f_s$, $2f_p + f_s$, $f_p - 2f_s$, $f_p + 2f_s$ etc. will be produced, (fig. 1.2), the magnitude of the higher order terms depending on the magnitudes of the coefficients in the algebraic expression which represents the non-linearity. If all frequencies other than f_s , f_p and $f_p - f_s$ are suppressed by applying short or open circuits, and currents are allowed to flow at $f_p - f_s$, (called the idler frequency, f_i), and at f_s and f_p , then further interaction between f_p and $f_p - f_s$

FIG.1.1. SIGNALS APPLIED TO NON-LINEAR REACTANCE

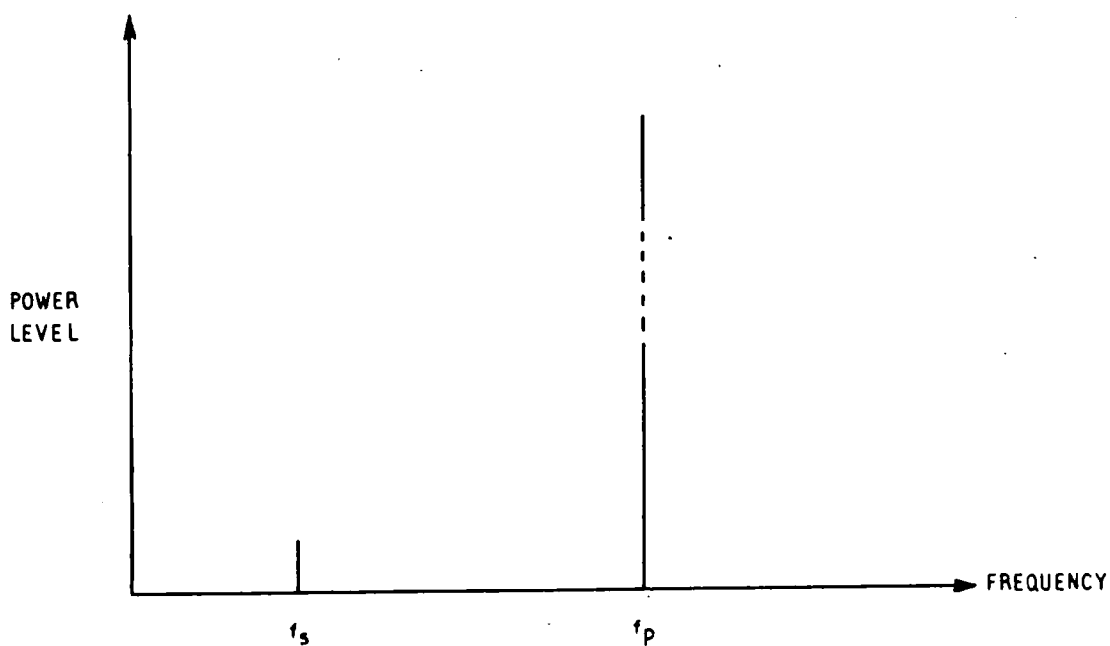
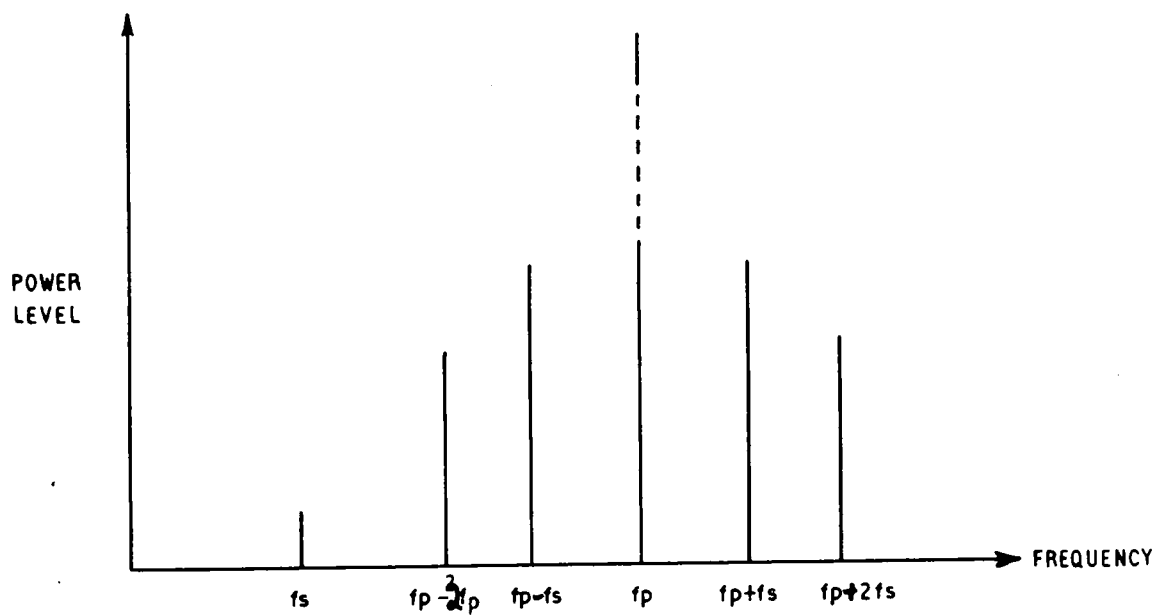


FIG. 1.2. SOME OF THE RESULTING INTERMODULATION PRODUCTS



will produce an output at f_s , the magnitude of which will depend on the power dissipated at the idler frequency and the magnitude of the input signal. This can be seen to be an amplification process, but it is necessary to construct an impedance matrix for the device, in terms of the currents flowing at the various frequencies, in order to establish the conditions for amplification.

The device that has just been described, in which currents flow at the signal frequency, a higher pump frequency, and the difference between the two, is known as a three frequency non-degenerate parametric amplifier and exhibits negative resistance at both signal and idler frequencies. If current was allowed to flow at the sum of the pump and signal frequencies, then it can be shown that this would contribute a positive resistance to the signal circuit; furthermore if equal currents flow at the sum and difference frequencies, their effect at the signal circuit would cancel out, to a first approximation, and the device would not produce gain.

A special case of the three frequency parametric amplifier is evident when the pump frequency is equal to twice the signal frequency and $f_p - f_s = f_i = f_s$, so that the idler circuit is indistinguishable from the signal circuit. This type of amplifier is referred to as "degenerate". As negative resistance occurs at both signal and idler frequencies about a centre frequency, $f_p/2$, the output of such an amplifier can be mixed with an oscillator at $f_p/2$, to give upper and lower sidebands, both containing signal information. Thus a degenerate device in such an arrangement will give 3 dB higher gain than a comparable non-degenerate device, and an improved noise figure. However, phase information from the pump source is directly transferred to the idler circuit, which means that, unless the device is used in a single-sideband configuration, it cannot be used for telecommunications without a phase-locked pumping system. As a single-sideband amplifier, the degenerate device is inferior to a nondegenerate device, from both noise and bandwidth points of view, and is thus not considered in this study which is directed towards the development of devices for telecommunications. However, whenever a band of noise has to be amplified, for example in radiometry or radioastronomy, the degenerate amplifier is often the best and simplest system to use. Although this study is concerned with non-degenerate devices, most of the conclusions can be directly applied to degenerate devices.

1.3 Manley-Rowe Equations

The power flow in non-dissipative non-linear elements is governed by the Manley-Rowe equations, which are themselves relatively simple, but their derivation as given by Manley and Rowe (1956) is complex. For any lossless non-linear element, with two impressed frequencies f_1 and f_2 , the power flow P into the reactance at any frequency, $mf_1 + nf_2$ is given by:

$$\sum_{m=0}^{\infty} \sum_{n=-\infty}^{\infty} \frac{mP_{m,n}}{mf_1 + nf_2} = 0$$

$$\sum_{n=0}^{\infty} \sum_{m=-\infty}^{\infty} \frac{nP_{m,n}}{mf_1 + nf_2} = 0$$

The simplicity of these equations and the fact that they are known to hold in many situations other than that of non-linear reactances, (for example they hold for gear boxes, induction motors, and masers), has encouraged others to find a more fundamental and general proof than that given by Manley and Rowe for non-linear reactances. A very simple demonstration of these equations can be obtained from quantum mechanics (Weiss 1957b and Brown 1965), but this has been objected to as a proof in so far that it would imply complete generality covering situations for which the Manley-Rowe equations are known not to hold (Schulz-DuBois and Seidel 1966). A rigorous and general proof is therefore still sought for these equations (Penfield P. 1966), but as far as non-linear reactances are concerned, their proof has been established by Manley and Rowe.

If the general relationships are reduced to the case of the three frequency non-degenerate parametric amplifier, when the frequencies present are f_s , f_p , and $f_p - f_s = f_1$, then:

$$\frac{P_1}{f_1} + \frac{P_p}{f_p} = 0 \quad \text{.....1.1}$$

$$\frac{-P_1}{f_1} + \frac{P_s}{f_s} = 0 \quad \text{.....1.2}$$

In equation 1.1, P_p , the power supplied at the pump frequency, is positive and hence P_1 must be negative representing flow out of the reactance. If P_1 is negative then P_s must also be negative and power must be delivered to the load at P_s . Negative resistance is thus evident at f_1 and at f_s with the possibility of oscillation. If additional impedances are included in the circuits at f_1 and f_s , then the device can give gain which will be controlled by the characteristics of these impedances. For this very reason that the gain and other characteristics of negative resistance parametric amplifiers are controlled by circuitry external to the non-linear reactance, these general power relationships are of little use in the final realisation of amplifiers. Circuitry external to an ideal non-linear reactance will include the resistive loss, stray inductance and stray capacitance that will be associated with a real non-linear element, and will modify or determine the gain, noise and bandwidth of an amplifier.

1.4 Noise Temperature Definition

The effective input noise temperature is a convenient measure of the noise performance of low noise amplifiers. This is defined as the temperature of the input termination to the amplifier which results in double the output noise power per unit bandwidth to that which would occur if the input termination were at a temperature of absolute zero. In other words, a noisy amplifier is represented as a noiseless amplifier, terminated in a resistance at T_e , the effective input noise temperature.

The effective input noise temperature is thus related to the noise figure, which may be defined as the total noise power per unit bandwidth available at the output of an amplifier, divided by the total noise power per unit bandwidth at the input from a termination at 290°K (IRE Standards on Electron Tubes 1957). That is;

$$F \text{ (noise figure)} = \frac{N_o}{gkT_oB} \quad \text{.....1.3}$$

where $T_o = 290^\circ\text{K}$. Thus with a termination at T_o the noise temperature referred to the input terminals is FT_o . Of this, T_o is due to the source, so the remainder, $(F - 1)T_o$, is what has been defined as the effective input noise temperature. Thus;

$$\begin{aligned} T_e &= T_o (F - 1) \\ \text{i.e.} \quad T_e &= T_o \left(\frac{N_o}{gkT_oB} - 1 \right) \quad \text{.....1.4} \end{aligned}$$

and N_o is the noise power delivered at the output from the termination at the input plus noise added from within the amplifier, i.e.;

$$\begin{aligned} N_o &= N_T + N_A \\ &= gkT_oB + N_A \\ T_e &= \frac{N_A}{gkB} \quad \text{.....1.5} \end{aligned}$$

where N_A is the noise power, generated within the amplifier, that is delivered to the output terminals.

2. Varactor Diode Amplifiers

2.1 General

Although a few other forms of non-linear reactive elements have been used in parametric amplifiers (e.g. Weiss 1957a and Bridges and Ashkin 1960), varactor diodes have shown most potential. To date the p-n junction varactor has been extensively used but may be superseded by other types, such as the Schottky barrier varactor (e.g. Foxell and Summers 1960) or the space-charge varactor (Howson, Owen and Wright).

2.2 Derivation of Diode Impedance Matrix

Whatever diode is used, all types exhibit voltage variable capacitance, which by definition is non-linear capacitance. That is, for linear capacitance $Q = CV$, but for a varactor $Q = C(v)V$, and so,

$$\frac{dQ}{dt} = \frac{d}{dt} (C(v)V(t))$$

If sinusoidal voltages are applied to the varactor and the condition is maintained at one of the voltages (the pump voltage) is much larger than the others, then $C(v)$ may be expressed as a time varying capacitance by the use of a Fourier series, and so;

$$\frac{dQ}{dt} = \frac{d}{dt} (C(t)V(t)) \quad \dots\dots\dots 2.1$$

Now it can be shown (Shockley 1949) from consideration of the charge distribution across linearly graded and abrupt p-n junctions that the capacitance variation with voltage can be expressed;

$$C(v) = \frac{C_0}{\left(1 - \frac{V}{\phi}\right)^{\frac{1}{n}}} \quad \dots\dots\dots 2.2$$

in which V is the applied voltage, ϕ is the diode contact potential and n is a constant depending on the junction impurity profile. This relationship assumes that the diode transition capacitance is much larger than the storage capacitance, a condition which precludes high forward currents. For a large sinusoidal voltage V_p (frequency ω) and bias V_b equation 2.2 becomes:

$$C(v, t) = \frac{C_0}{\left[1 - \frac{(V_b + V_p \cos \omega t)}{\phi}\right]^{\frac{1}{n}}} \quad \dots\dots\dots 2.3$$

which represents the conditions that exist in a three frequency parametric amplifier with a pump level much greater than signals at any other frequency. From the Fourier representation:

$$C(t) = C_0 (1 + a_1 \cos \omega t + a_2 \cos 2\omega t + a_3 \cos 3\omega t \dots)$$

and as

$$a_n = \frac{2}{\pi} \int_0^{\pi} C(t) \cos n\omega t \, d\omega t$$

the coefficients a_n can be calculated, e.g.:

$$a_1 = \frac{2}{\pi} \int_0^{\pi} \frac{\cos \omega t \, d\omega t}{\left[1 - \frac{(V_b + V_p \cos \omega t)}{\phi} \right]^{\frac{1}{n}}} \quad \dots\dots\dots 2.4$$

Equation 2.4, an elliptic integral, can be solved numerically for values of bias, contact potential, pump voltage and junction characteristic n ; which solution is plotted in figs. 2.1 and 2.2. Even for abrupt junction varactors the coefficient a_2 is an order of magnitude less than a_1 , for the same conditions of pumping, and since it is a factor in a_n^2 which determines the negative resistance at a given frequency (this is seen later), a_2^2 and higher order terms are usually neglected. Thus to a good approximation:

$$C_t = C_0 (1 + a_1 \cos \omega t)$$

or, in complex notation:

$$C_t = C_0 \left[1 + Y_1 (e^{j\omega t} + e^{-j\omega t}) \right] \quad \dots\dots\dots 2.5$$

where $Y_1 = \frac{a_1}{2}$

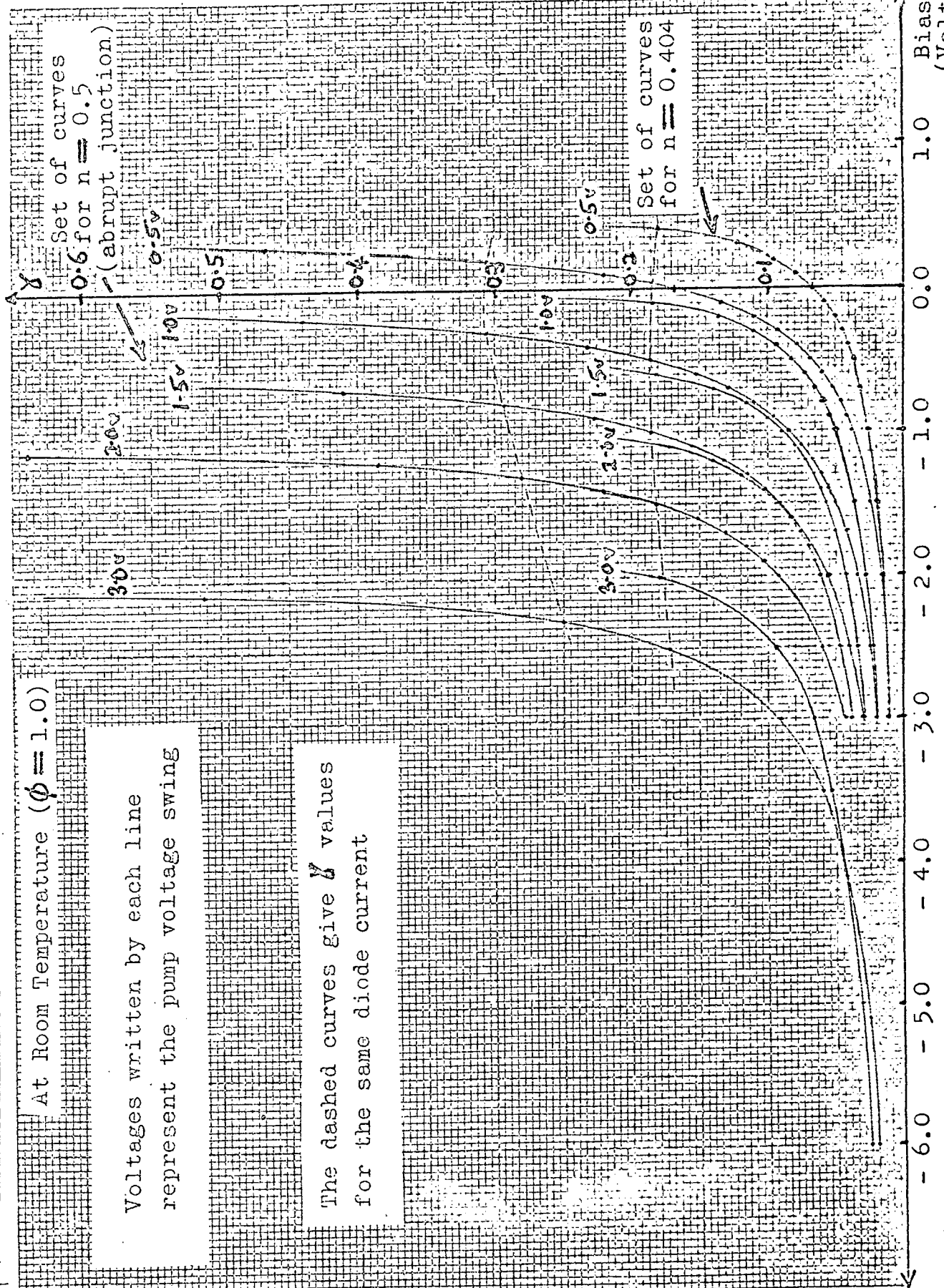
Y_1 is called the capacitance variation factor and represents the capacitance variation at the fundamental frequency for an applied sinusoidal voltage. (The subscript will not be used in further analysis).

An impedance matrix may now be constructed for the diode. If the only small signal currents and voltages present in the three frequency nondegenerate amplifier are:

$$V = V_1 e^{j\omega_1 t} + V_1^* e^{-j\omega_1 t} + V_2 e^{j\omega_2 t} + V_2^* e^{-j\omega_2 t}$$

$$I = I_1 e^{j\omega_1 t} + I_1^* e^{-j\omega_1 t} + I_2 e^{j\omega_2 t} + I_2^* e^{-j\omega_2 t}$$

(where ω_1 and ω_2 are the angular signal and idler frequencies respectively, so that, $\omega_1 + \omega_2 = \omega_3$, the angular pump frequency), then substituting these small signal voltages into equation 2.1 and using equation 2.5, gives two equations for current which can be expressed by the following matrix, if all voltages other than V_1 , V_2 and V_3 are set to zero. 6.

Fig. 2.1 χ versus Bias Voltage at various Pump Voltages

$$\begin{bmatrix} V_1 \\ V_2^* \end{bmatrix} = \begin{bmatrix} -\frac{1}{\omega_1 C_0} & -\frac{\gamma}{\omega_2 C_0} \\ \frac{\gamma}{\omega_1 C_0} & \frac{1}{\omega_2 C_0} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2^* \end{bmatrix}$$

It is impossible, because of integral resistive loss, to short circuit all voltages other than V_1 , V_2 and V_3 in a real diode, and it is therefore more realistic to construct a larger matrix involving harmonics to a high order, and to set unwanted currents to zero instead of voltages. This open circuit harmonic condition results in the following matrix;

$$\begin{bmatrix} V_1 \\ V_2^* \end{bmatrix} = \begin{bmatrix} -\frac{1}{\omega_1 C_0 (1 - \gamma^2)} & -\frac{\gamma}{\omega_2 C_0 (1 - \gamma^2)} \\ \frac{\gamma}{\omega_1 C_0 (1 - \gamma^2)} & \frac{1}{\omega_2 C_0 (1 - \gamma^2)} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2^* \end{bmatrix}$$

which is identical to the previous matrix if,

$$C_1 = C_0 (1 - \gamma^2)$$

It is seen from this so called small signal matrix, that when the idler circuit is terminated in R_2 , a negative resistance, $\frac{\gamma^2}{\omega_1 \omega_2 C_1 R_2}$, appears in the signal circuit. To prevent 3 dB power sharing between input and output and to improve stability, such a negative resistance is most successfully utilised in conjunction with a circulator. (A device in which power is guided from one port to the next with little loss and with little transfer of power to other ports.) All the amplifiers used in this study are of the reflection type for these reasons. Many attempts at operating parametric amplifiers without circulators have been reported, which have involved terminations at the upper-sideband frequency, quadrature pumping of phased diode arrays, or periodically loaded transmission lines, but these are all relatively complex and have so far given inferior results to reflection amplifiers (e.g. Bobisch and Sondhauss 1965 and Engelbrecht 1962). The operational bandwidth of present day circulators is in excess of that available from simple amplifier circuits, and they can therefore be used for this study. However, the theory relating to the bandwidth of reflection amplifiers as presented in this study, is for ideal circulators (power transferred from port to port without resistive or reactive loss, and

complete isolation between reverse ports) and may, especially for very wide bandwidths, have to be modified to account for circulator reactance. Resistive circulator losses are considered in the determination of amplifier noise temperature (sections 8 and 9).

2.3 Generalised Amplifier Equivalent Circuit

It is noted that the two products on the leading diagonal of the small signal impedance matrix for the diode, are the diode complex impedances at signal and idler frequencies. A circuit can therefore be drawn (fig. 2.3) with the diode elements at signal and idler frequencies outside a box $X_{12} X_{21}$ representing the transfer of impedance between the two circuits. A circulator is included in fig. 2.3 but the loosely coupled pump circuit is omitted. The transfer function $\frac{X_{12}X_{21}}{Z_2^*}$, will be real and negative if Z_2^* is real and positive. So that;

$$Z_a = - \frac{\gamma^2}{\omega_1 \omega_2 C_j^2 R_2} \quad \dots\dots\dots 2.6$$

This is the condition for resonance, when Z_a represents the signal circuit negative resistance. Now the power gain of a reflection amplifier is given by the square of the reflection coefficient;

$$\text{gain} = |\Gamma|^2 = \left| \frac{R_g - Z_{in}}{R_g + Z_{in}} \right|^2 \quad (\text{for } R_g = R_L) \quad \dots\dots\dots 2.7$$

in which, on resonance, Z_{in} is real and negative. The diode negative resistance must therefore be transformed to the value necessary to give the required gain from equation 2.7, which is the function of the transformer T_1 shown in fig. 2.3. If R'_g is the transformed value of input impedance, as seen by the diode, and the total signal circuit negative resistance (on resonance) is R , then for a given gain;

$$R'_g = -KR \quad \dots\dots\dots 2.8$$

in which K is given by;

$$K = \frac{(g+1)}{(g-1)} \pm \frac{2g^{\frac{1}{2}}}{(g-1)} \quad \dots\dots\dots 2.9$$

and g is the amplifier gain. At high gain (> 20 dB), $K \approx 1$.

2.4 Diode Parameters

The importance of the capacitance variation factor in determining the negative resistance (it is proportional to γ^2) has been seen, but unfortunately a real diode exhibits additional characteristics to non-linear capacitance.

FIG. 2.3 GENERAL AMPLIFIER EQUIVALENT CIRCUIT
WITH AN IDEAL LOSSLESS VARACTOR

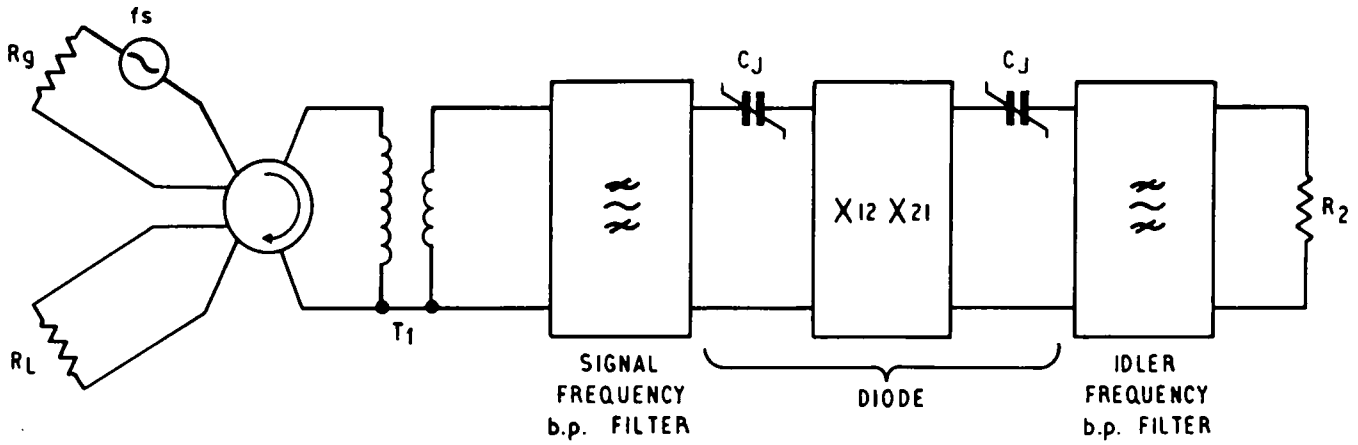
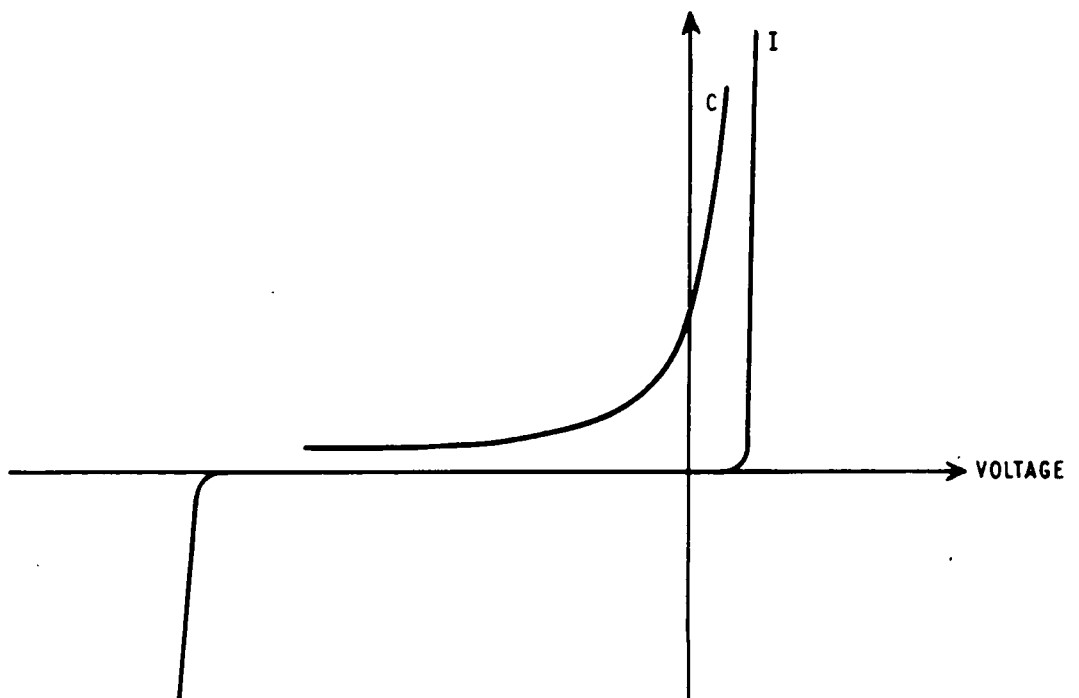


FIG. 2.4. FORM OF DIODE CURRENT VOLTAGE AND
CAPACITANCE-VOLTAGE CHARACTERISTICS



The rectifying property of p-n junction devices imposes a limit on the capacitance variation factor. The I/V characteristics are shown alongside the capacitance/voltage characteristics for a typical near-abrupt junction ($n = 0.4$) in fig. 2.4. Forward conduction will result in shot noise (discussed in section 8.2) and must be avoided, so the diode may be pumped as hard as the limit of forward conduction will allow, which defines a maximum γ value. Because most of the capacitance change occurs in or near forward conduction, there is little advantage in reverse biasing the diode. The dashed lines in figs. 2.1 and 2.2 show γ values for the limit of forward conduction and illustrate this.

The fact that there is a maximum value of γ for any diode, above which excessive D.C. current flows, leads to a method of tuning parametric amplifiers. If signal or idler circuits are off resonance and gain is obtained, then an excessively high value of γ and a high D.C. current will be evident, due to the impedance in these circuits. Moving towards resonance will increase amplifier gain, or after decreasing the pump drive, give the same gain for less diode current. Amplifiers may thus be tuned for minimum diode current at a given gain, when conditions closely approximating to signal and idler resonance will be attained.

A definition of γ based on a linear capacitance/voltage relationship can be used to indicate the maximum useful value i.e.:

$$\gamma = \frac{C_{j\max} - C_{j\min}}{2(C_{j\max} + C_{j\min})} \quad \dots\dots\dots 2.10$$

where $C_{j\max}$ = maximum available capacitance value (i.e. at $1 \mu A$ forward current)

and $C_{j\min}$ = capacitance value at -1 volt reverse bias. (There is little capacitance change in the reverse direction beyond 1 volt.)

As well as capacitance variation and rectifying properties, p-n junction varactors have additional reactance and resistive loss associated with them. An adequate equivalent circuit is given in fig. 2.5 which in most circumstances, can be replaced by fig. 2.6 (Foxell and Wilson 1965). In fig. 2.6:

$$R_s = \frac{R'_s}{\left(\frac{C_2}{C_1} + 1 \right)^2} \quad \dots\dots\dots 2.11$$

R'_s is resistive loss associated with the various diode regions, L_s is a stray inductance, largely from the leads between the diode structure and its encapsulation, and C_1 is the encapsulation capacitance. C_2 is a voltage independant capacitance across the diode structure. Since C_1 is a function of applied voltage, R_s must also be, but for low values of forward current

FIG. 2.5. VARACTOR EQUIVALENT CIRCUIT

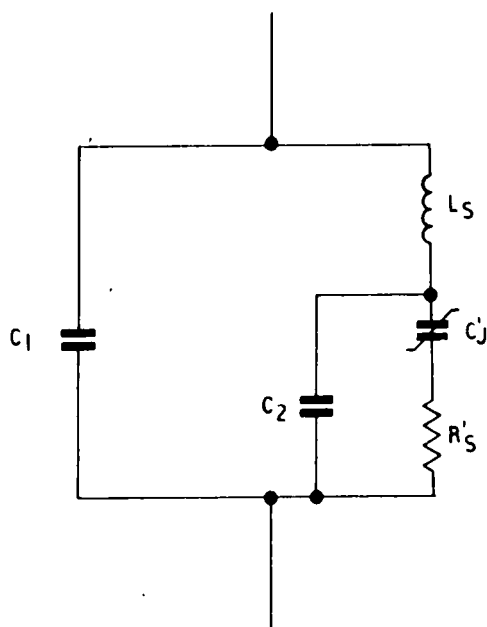
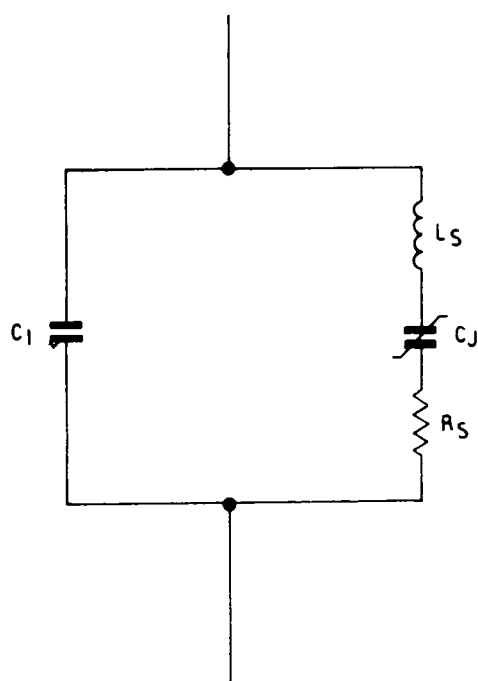


FIG. 2.6. MODIFIED VARACTOR EQUIVALENT CIRCUIT



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this variation may be neglected and C_j taken equal to C_j . Typical values of these parameters are as follows:-

L_s	0.1 to 0.8 nH
C_j	0.2 to 0.8 pF
C_1	0.05 to 0.3 pF
C_2	0.05 to 0.2 pF
R_s	1 to 4 Ω

Such an equivalent circuit regards the diode transition capacitance as the total junction capacitance, an assumption which is valid only for low forward currents.

It is usual to specify diode loss by the cut-off frequency:

$$f_c = \frac{1}{2\pi C_j R_s}$$

The capacitance variation factor, γ , as defined by equation 2.10 is often measured at the diode series resonance, when the encapsulation capacitance merely shunts the input signal. This form of measurement will give a higher γ value than can be realised in many applications, when L_s is comparatively small and C_1 can be regarded as a static stray across the junction. (See section 3.4.)

The general parametric amplifier equivalent circuit (fig. 2.3) must be modified to include diode loss and stray reactance, as shown in fig. 2.7.

2.5 Noise Temperature

2.5.1 General

An amplifier with an ideal lossless varactor and the idler circuit terminated in a resistive load, can be shown, from the matrix equations, to have an input noise temperature of:

$$T_e = T_2 \frac{\omega_1}{\omega_2} \quad \text{.....2.12}$$

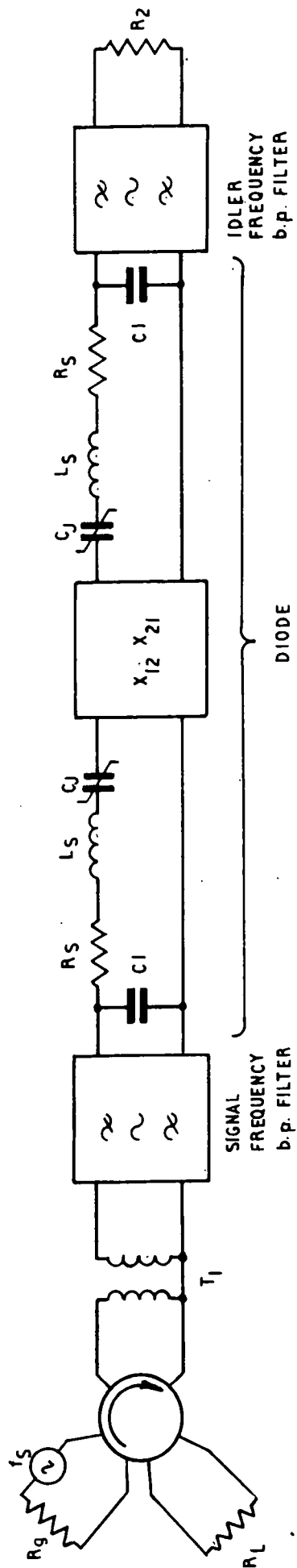
where T_2 is the temperature of the idler termination. It is therefore apparent that the noise temperature may be reduced by cooling the idler termination or by increasing the idler frequency.

If, however, loss is included with the varactor, it is seen that there is an optimum idler frequency, for minimum noise temperature at a given signal frequency, determined by the varactor loss. This can also be shown from the diode impedance matrix. If a voltage generator V_{g1} is present in the signal circuit then the current flowing in the output circuit is given by:

$$I_1 = \frac{V_{g1} |Z_2^*|}{|Z_1 Z_2^* - X_{12} X_{21}|}$$

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FIG. 2.7 GENERAL AMPLIFIER EQUIVALENT CIRCUIT INCLUDING DIODE LOSS AND REACTIVE STRAYS.



where Z_1 and Z_2 are the total impedances in the signal and idler circuits respectively. The output power may thus be written:

$$P_1 = \frac{(V_{g1})^2 |Z_2^*|^2}{|Z_1 Z_2^* - X_{12} X_{21}|^2}$$

but the amplifier gain is given by:

$$\text{gain} = \frac{4R_g |Z_2^*|^2}{|Z_1 Z_2^* - X_{12} X_{21}|^2} \quad (\text{for } R_g = R_L)$$

$$\text{so} \quad P_1 = (V_{g1})^2 \frac{\text{gain}}{4R_g} \quad \dots\dots\dots 2.13$$

Similarly the output power due to (V_{g2}) in the idler circuit is:

$$P_2 = \frac{(V_{g2}^*)^2 |X_{12}|^2 R_g}{|Z_1 Z_2^* - X_{12} X_{21}|^2}$$

$$\text{and so} \quad P_2 = \frac{(V_{g2}^*)^2 |X_{12}|^2}{4R_g |Z_2^*|^2} \cdot \text{gain} \quad \dots\dots\dots 2.14$$

If the varactor loss forms the idler termination then 2.14 becomes:

$$P_2 = \frac{(V_{g2}^*)^2}{4R_g} \left(\frac{\gamma f_c}{f_2} \right)^2 \cdot \text{gain} \quad \dots\dots\dots 2.15$$

where γ and f_c are the usual diode parameters (section 2.4). The input noise temperature due to sources $(\overline{v_{g1}^2})$ and $(\overline{v_{g2}^2})$ may be written:

$$T_e = \frac{1}{4kBR_g} \left[(\overline{v_{g1}^2}) + (\overline{v_{g2}^2}) \left(\frac{\gamma f_c}{f_2} \right)^2 \right] \dots\dots\dots 2.16$$

If the only signal circuit loss is R_g , then the excess noise temperature due to thermal sources may be written:

$$T_e = \frac{T_d R_s}{R_g} \left[1 + \left(\frac{\gamma f_c}{f_2} \right)^2 \right] \quad \dots\dots\dots 2.17$$

where T_d is the diode temperature. This expression applies for a given amplifier with a fixed value of generator/load impedance, which factor should therefore be as high as possible. (That is as high as is consistent with negligible diode current, see section 8.) Equation 2.8 defines the generator/load impedance for a given gain, viz:

$$\begin{aligned} R_g &= -KR \\ &= \frac{K(Yf_c)^2}{f_1 f_2} R_s \end{aligned}$$

and so 2.17 may be written:

$$T_e = \frac{T_d f_1}{K} \left[\frac{f_2}{(Yf_c)^2} + \frac{1}{f_2} \right] \quad \dots\dots\dots 2.18$$

A minimum noise temperature at a particular idler frequency, for a given signal frequency and diode Yf_c , is seen from this equation, (fig. 2.8). The higher Yf_c product, the lower the noise temperature at a given idler frequency, and the higher the idler frequency necessary for the minimum noise temperature. Thus the Yf_c product is a useful figure of merit for varactor diodes, in that it defines the noise temperature at a given signal frequency but it will be seen later that the product must be interpreted in relation to other parameters, when considered as a figure of merit for available bandwidth. It is apparent, from equation 2.18 that noise temperature increases linearly with signal frequency for a given Yf_c product and idler frequency, the latter being determined by the diode resonances for self resonant idler configurations, (see part 6 of this section). Fig. 2.9 shows a series of graphs of the noise temperature of room temperature devices versus signal frequency for various Yf_c products at two idler frequencies.

The noise temperature of an amplifier with a lossy diode (equations 2.17 and 2.18) shows a linear dependence on ambient temperature and noise temperatures approaching 1 or 2°K may thus be expected for liquid helium cooled devices. The reasons why these noise temperatures are not achieved are discussed in section 9.

2.5.2 The Noise Temperature of Broad-Band Amplifiers

Equations 2.13 and 2.14 can be used to calculate the noise temperature at any point within the band of a broad-band amplifier. These give; (neglecting signal circuit losses):

$$T_e = \frac{1}{4kBR_g} \left[(\overline{v_{g1}^2}) + (\overline{v_{g2}^2}) \left(\frac{(Yf_c)^2}{f_2} \right) \frac{(R_s)^2}{(R_s^2 + X_2^2)} \right]$$

Fig.2.8 Amplifier Noise Temperature versus Idler Frequency

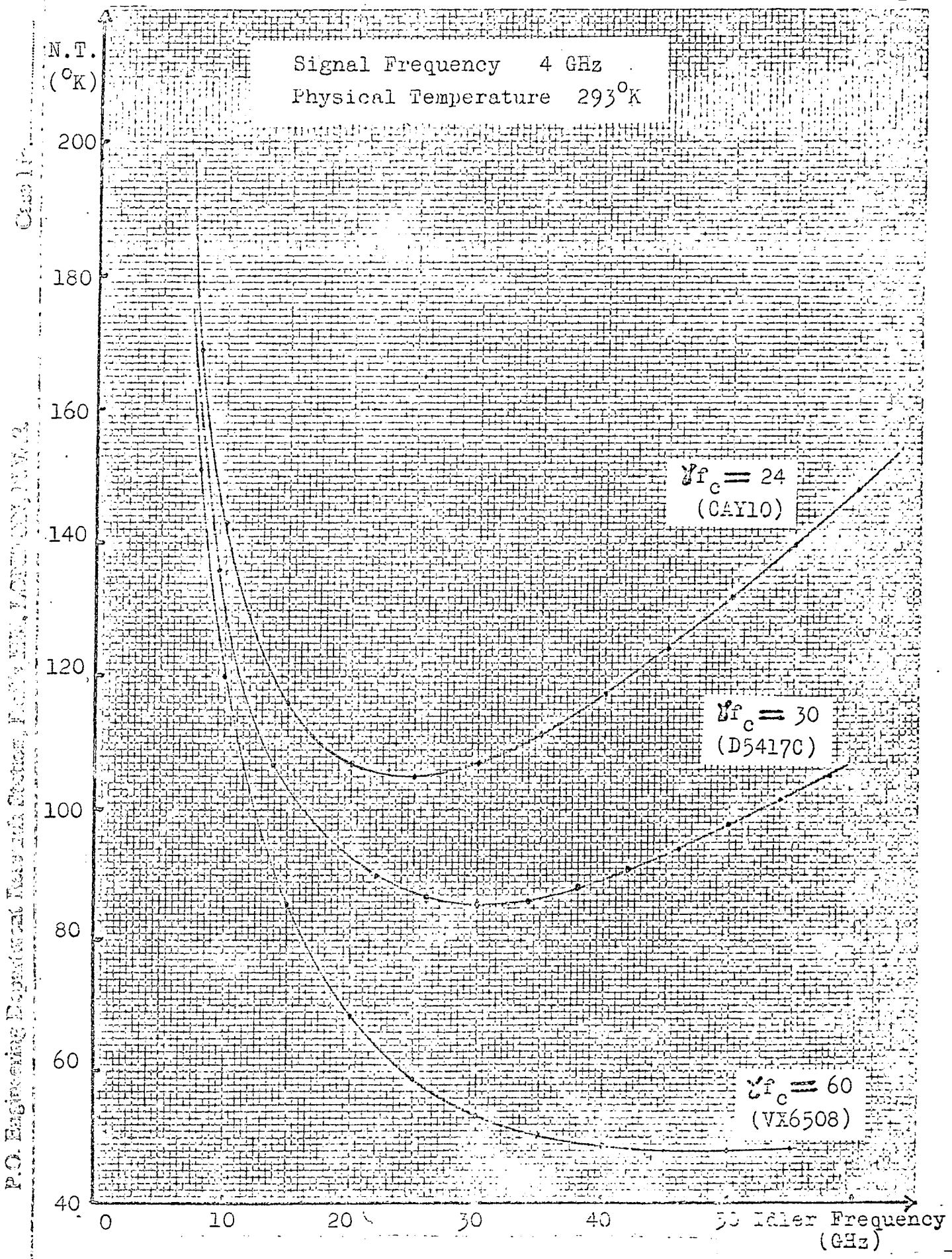
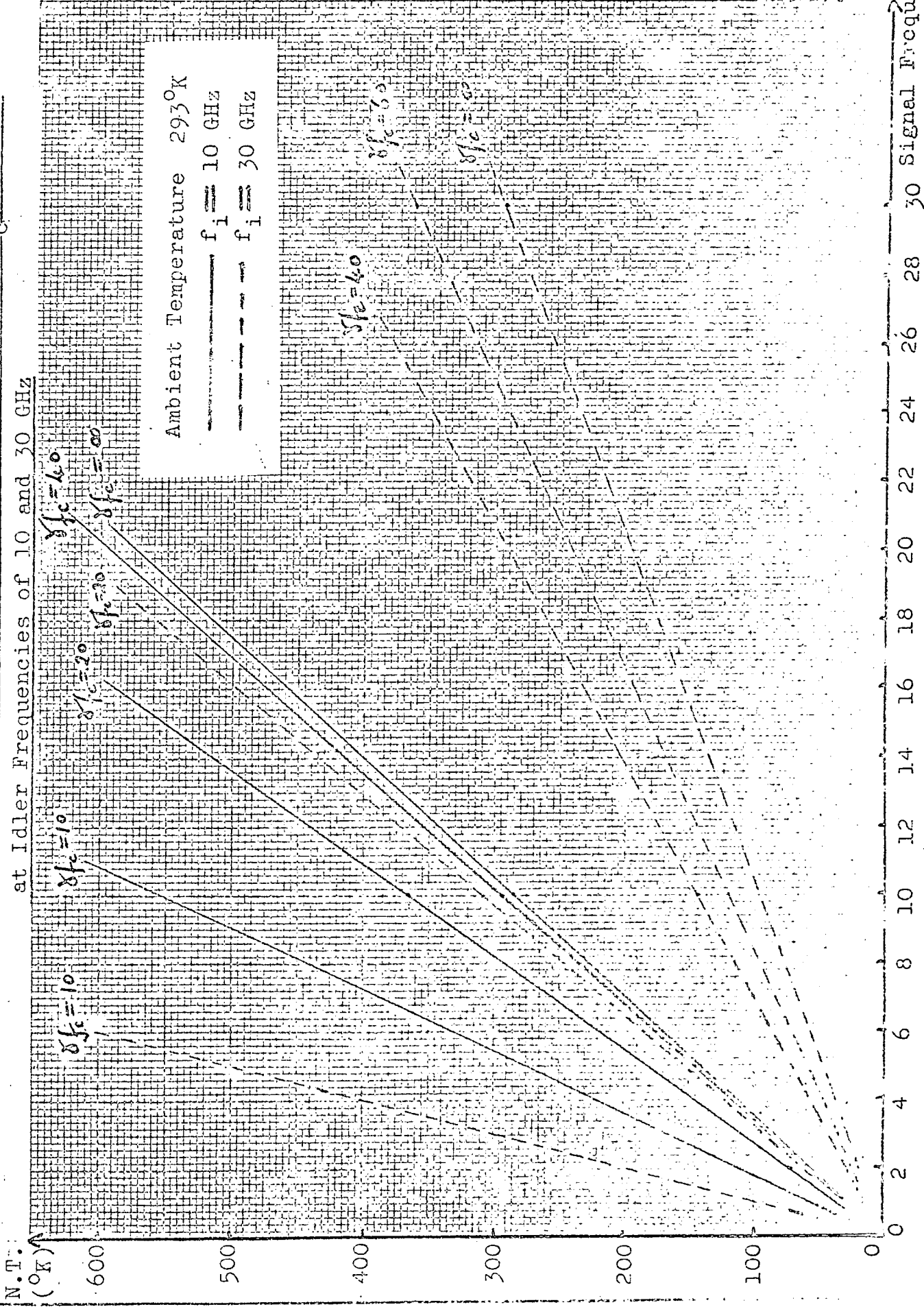


Fig.2.9 Noise Temperature versus Signal Frequency for various δf_c Products



where X_2 is the idler circuit reactance. It is therefore evident that circuit reactance in no way degrades noise temperature, so that a broad-band device, derived by multi-tuning a single tuned amplifier, should not show a higher noise temperature, at any frequency, than that at the centre frequency of the single tuned device. Circuit losses will probably be higher with a multi-tuned signal circuit, but at low temperatures this is relatively unimportant.

2.6 Possible Amplifier Types

2.6.1 General

The requirement that currents may flow at only three frequencies is the principal constraint in amplifier design. This is fulfilled by the schematic arrangement of fig. 2.7 and many earlier approaches to amplifier design involved the derivation of multi-element bandpass filter networks at signal and idler frequencies, in an attempt to realise such a scheme. It would appear however that not one successful amplifier design has been realised by approaching the problem in this way, for which there are many reasons apart from the fact that oversimplified diode circuits have often been used. Firstly, any elemental values of a filter network, derived from a Chebechev approximation to a desired frequency response, have (with present day techniques) to be realised with distributed circuits, which are not the same as lumped circuits. Secondly, any filter structure that is realised, has to have adjustable elements to permit tuning to a particular diode, or, since the diode parameters change with temperature, to permit retuning at a low temperature. (The latter requirement may make such an arrangement prohibitively complex for a liquid helium cooled device.) Finally, although an adjustable multi-element filter may be practicable for the signal circuit of room temperature amplifiers, the realisation of such a design in terms of distributed elements at the much higher idler frequency is usually impracticable.

Rather than starting with a complex multi-tuned design, it has thus been found more practical to start with the simplest of designs, that is, single tuned signal and idler circuits, assess its bandwidth capability (which may already be sufficient for many applications) and then proceed to broadband it by multi-tuning.

If single tuned circuits are to be used, it is apparent that the diode stray elements (section 2.4) would be neglected to the detriment of bandwidth. Thus it is usual to make use of one of the two basic resonances of the diode circuit. (fig. 2.6) as the idler circuit of the amplifier. (Getsinger and Matthaei 1964, DeJager 1964, Aitchison et al. 1967a and Edrich 1967.)

It is undesirable, in any form of amplifier, that the idler frequency should propagate, or that the upper sideband frequency should see a termination, in the signal or pump circuits. The signal circuit Q may not be sufficiently high, with single tuned circuits, to prevent such propagation, which could lead to a reduction in bandwidth and an increase in noise temperature. (An upper sideband termination reduces the signal circuit negative resistance at a given maximum permissible diode current and an external idler termination could well be at a much higher physical temperature than a cooled amplifier.) So even in the simpler forms of amplifier, involving single tuned circuits, low pass filters usually have to be used in the signal circuit, and bandpass filters in the pump circuit.

2.6.2 Amplifiers using the Diode Shunt Resonance

The diode encapsulation capacitance may be resonated with the stray inductance and junction capacitance, and used as a self-contained idler circuit, (Matthaei and Getsinger 1964). The circuit may be drawn as in fig. 2.10, or in expanded form as in fig. 2.11. An additional inductance L is usually required to resonate the diode to a given signal frequency, and it is noted that the idler circuit is terminated only in R_s .

The centre-frequency gain of such a device may be written:

$$\text{Power gain} = \left| \frac{R'_g + \frac{\gamma^2}{\omega_1 \omega_2 C_j R_s} - R_s}{R'_g - \frac{\gamma^2}{\omega_1 \omega_2 C_j R_s} + R_s} \right|^2 \quad \dots\dots 2.18$$

and the noise temperature, due to thermal sources, is given by equations 2.15 and 2.16. The frequency response and gain/bandwidth relationship for this type of amplifier are evolved in section 3.4, where it is noted that in some circumstances the encapsulation capacitance can be shunted with additional capacitance to advantage.

2.6.3 Amplifiers using the Diode Series Resonance

A short circuit can be placed across the diode at the resonant frequency of the junction capacitance and stray inductance, to form the idler circuit. At microwave frequencies it is difficult to present a wide-band short circuit across the diode, although a number of narrow band amplifiers have been constructed using half wavelength resonators. This type of amplifier will be inferior to the device using the shunt resonance, due to the nature of the half wavelength resonant line, unless additional idler circuit resistive loading is incorporated and this is difficult to realise without elaborate filters.

FIG. 2.10 CIRCUIT OF AMPLIFIER USING DIODE SHUNT RESONANCE

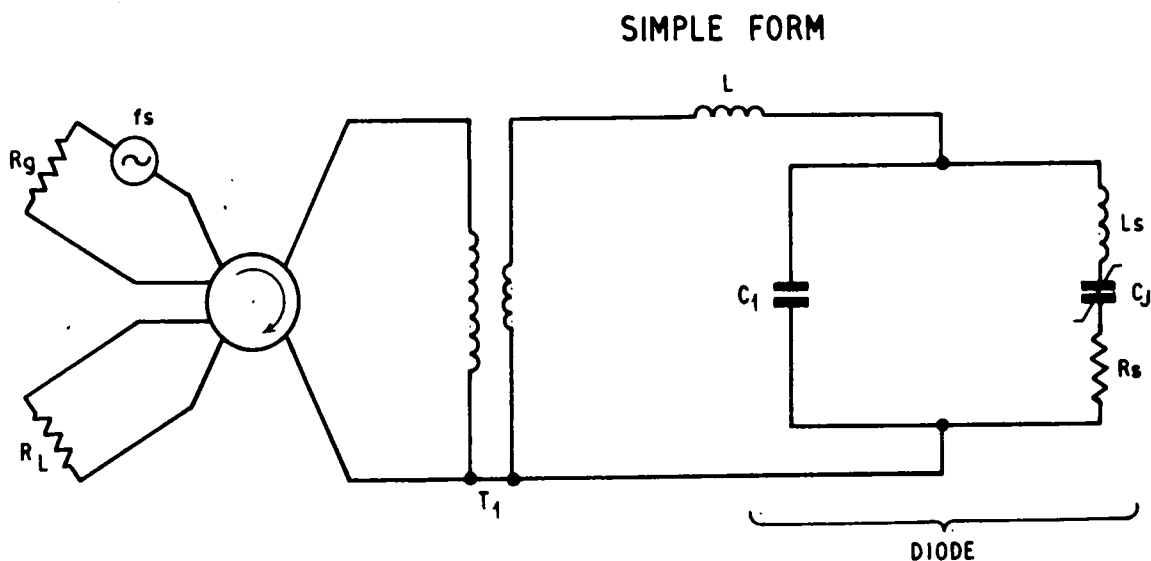
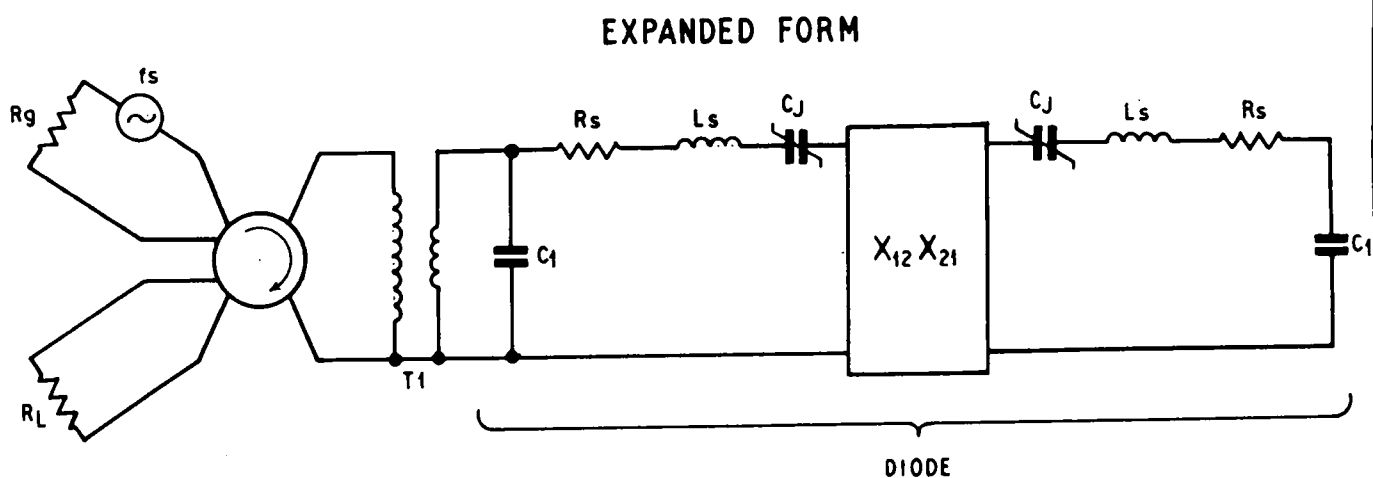


FIG. 2.11 CIRCUIT OF AMPLIFIER USING DIODE SHUNT RESONANCE



Simplicity of construction can be maintained by using two series resonant diodes in the balanced configuration (Pearson and Lunt 1964), a circuit of which is shown in fig. 2.12. This arrangement has an advantage over the single diode shunt resonant device, in that a voltage zero appears across the diodes at the idler frequency, if the two diodes are identical. This means that the tendency for the idler frequency to propagate in the signal and pump circuits is minimised, with the consequent simplification of design and greater likelihood of realising theoretical noise temperatures and bandwidths.

The same expressions for noise temperature (equations 2.15 and 2.16) apply to the balanced amplifier as the single diode shunt resonant device, since the idler circuit termination of the former is $2R_s$ and

$$(\bar{v}_{g2})^2 \text{ balanced} = 4(\bar{v}_{g2})^2 \text{ single diode}$$

A higher noise temperature than that of the single diode shunt resonant amplifier will, however, usually be evident, due to the lower idler frequency.

The bandwidth available from a balanced diode amplifier is discussed and compared with that available from other devices in section 3.

2.7 Stability

The dependence of amplifier gain on pump level (equation 2.4) allows an assessment of gain stability in terms of pump level stability. It can be shown (Richardson D. 1964, Carter J.W. 1966) that for N stages of amplification giving overall gain, g , the gain sensitivity factor, F , can be written:

$$F = \frac{N (g^{\frac{1}{N}} - 1)}{g^{\frac{1}{2}}} \quad \text{.....2.19}$$

$$F \text{ being defined as } \frac{\Delta g(\text{dB})}{\Delta P(\text{dB})} = \frac{\text{the change in amplifier gain (dB)}}{\text{the corresponding change in pump power (dB)}}$$

This expression is tabulated for a few values of gain in table 2.1.

Table 2.1

Gain (dB)	Stability Factor		
	1 stage	2 stages	4 stages
10	2.75	2.4	2.3
20	10	5.5	5
30	32	10	7.7

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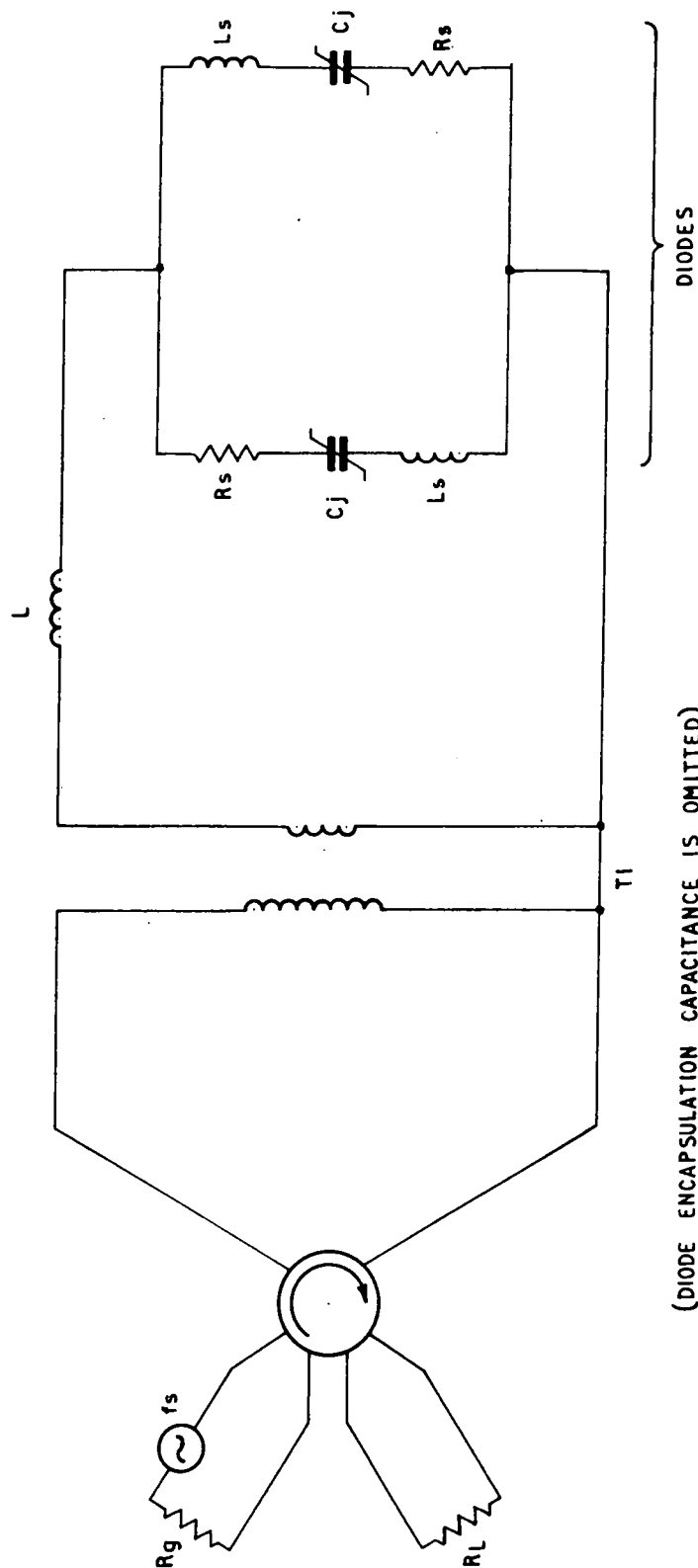


FIG.2.12. BALANCED DIODE AMPLIFIER

Allowing a 0.1 dB drift in pump power level it is seen that for a total gain of above 20 dB, at least two stages are required, to give less than 1 dB drift in amplifier gain. For below 30 dB gain there is little advantage in employing more than two stages.

Pump frequency stability is also an important factor, particularly in single tuned amplifiers. A change in pump frequency alters the band centre frequency and produces less gain at the new mid-band frequency. Richardson (1964) has shown that the change in phase, ϕ , due to a change in pump frequency, Δf_p , can be written:

$$\tan \phi = \frac{\left[\frac{2Q_1 \Delta f_p}{f_1 (R - 1)} \right]}{\left[\frac{(g + 1)}{(g - 1)} - \frac{(R - 1)}{(R_0 - 1)} \right]}$$

where f_1 and Q_1 are the signal frequency and circuit Q respectively and R and R_0 are the negative resistances in the signal circuit at the new and original pump frequencies, respectively. R thus depends on the circuit parameters as in equation 2.6 and also on the pump circuit attenuation/frequency characteristic. As gain is increased $\left(\frac{g + 1}{g - 1} \right)$ decreases to unity and since $\left(\frac{R - 1}{R_0 - 1} \right)$ may be slightly less than unity, output phase stability can be poor. Low gain stages (15 dB or less) are thus desirable.

Automatic klystron level and frequency stabilising apparatus may thus be necessary for an operational system in a receiving station.

3. Bandwidth Limitations

3.1 General

The fundamental limit on bandwidth for an ideal loss less varactor is determined by the efficiency of power transfer from a capacitance to a load. Such a limit is an academic one for microwave parametric amplifiers since loss and stray inductance are always present. Fano (1960) has considered the problem of matching an arbitrary impedance over a frequency band and produced solutions for some simple circuits. Following from this, a number of authors, for example Ku (1964) and Getsinger (1963), have considered the diode in a slightly simplified equivalent circuit and derived networks for achieving an optimum match over a given frequency band. This sort of exercise is interesting in that it indicates the fundamental bandwidth limitations when considered as a matching problem, but it has been pointed out (section 2.6) that such an approach is not very useful in the realisation of devices. However, if improved technology should allow the construction of entirely "lumped" microwave circuit elements with an integrated varactor, then this approach could be very useful.

At the present time, with distributed circuit elements, diodes that have to be individually mounted and marked variations in diode characteristics from diode to diode in the same production batch, the most successful approach to the realisation of wide bandwidths involves the construction of a simple amplifier with single tuned circuits, which may later be broad-banded with additional resonators.

3.2 Microwave Circuit Elements

The desirability of using "lumped" or "distributed" circuit elements at microwave frequencies must be considered. The Q factor of any resonant circuit may be written:

$$Q = 2\pi \frac{\text{Energy stored}}{\text{Energy dissipated/cycle}}$$

which becomes (Montgomery, Dicke and Purcell) at angular frequency, ω :

$$Q = \frac{\omega}{2R} \cdot \frac{dX}{d\omega}$$

The requirement of a low reactance slope for a broad bandwidth is evident and the reactance slopes of distributed and lumped circuits will now be considered.

Fig. 3.1 (line a) shows the reactance function of a half wavelength short circuit line as a function of frequency, superimposed with that for a series LC circuit (line b) with the same reactance slope at resonance. This condition can be written:

$$\left(\frac{dX}{d\omega}\right)_{\text{distributed}} = \left(\frac{dX}{d\omega}\right)_{\text{lumped}} \quad (\text{at resonance})$$

and

$$\left(\frac{dX}{d\omega}\right)_{\text{distributed}} = Z_0 \frac{l}{v} \sec^2 \left(\frac{\omega l}{v}\right)$$

$$\text{which at resonance} = Z_0 \frac{l}{v} = Z_0 \frac{n}{2f_0}$$

where l is the length of the short circuit line, v is the velocity of light, Z_0 is the characteristic impedance of the short circuit line, f_0 is the resonant frequency and where n is the number of half-wavelengths in the line. Since:

$$\left(\frac{dX}{d\omega}\right)_{\text{lumped}} = 2L \quad (\text{at resonance})$$

where L is the lumped circuit inductance, then:

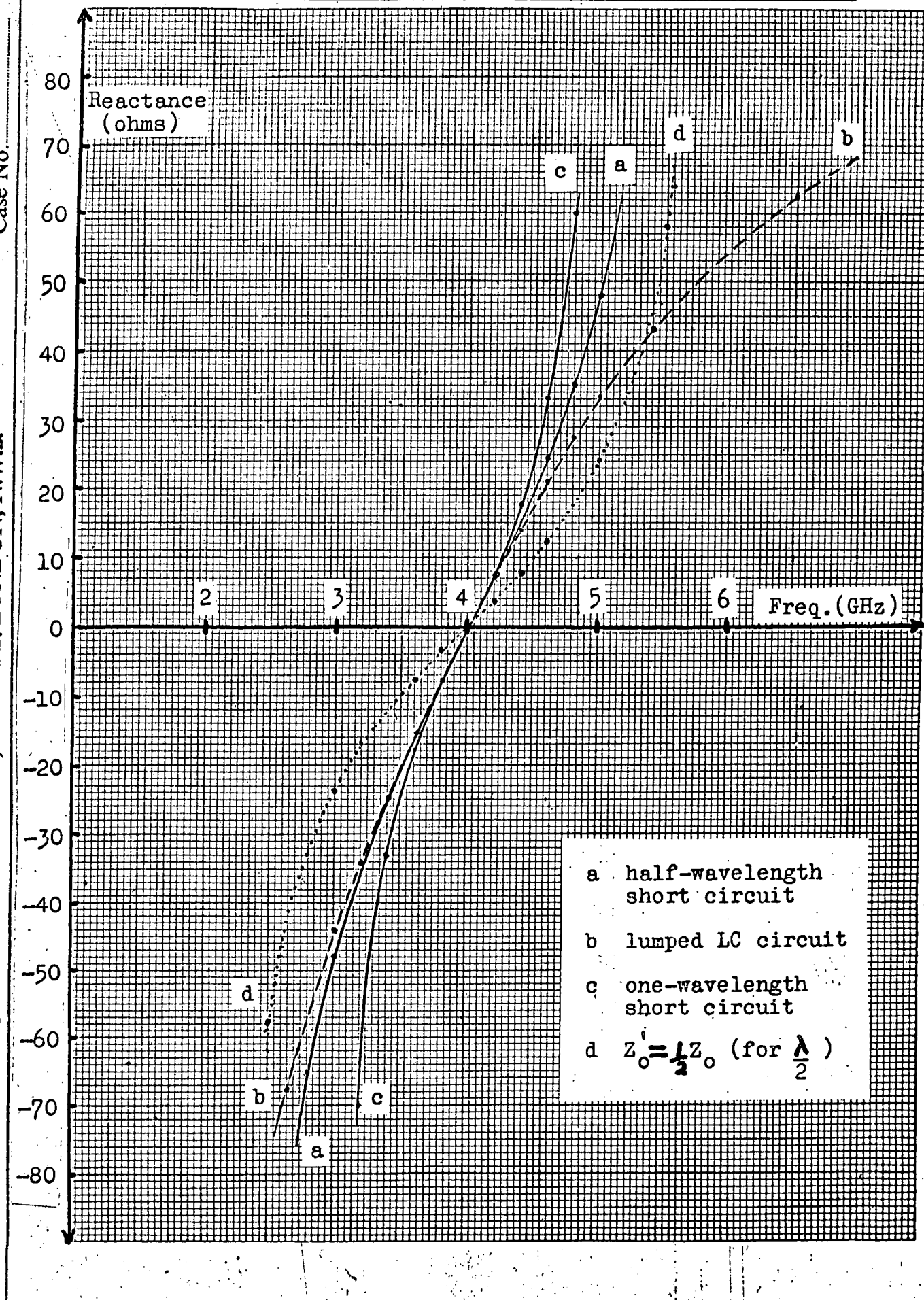
$$Z_0 = \frac{4Lf_0}{n} \quad \dots\dots\dots 3.1$$

The two circuits are thus equivalent at resonance if the above condition is maintained, but it is evident that at wide frequency deviations, the distributed circuit is more reactive than the equivalent LC circuit. Line c in Fig. 3.1 is the reactance function of an equivalent distributed circuit with $n = 2$ and demonstrates the undesirability of using high order harmonics. Since the reactance/frequency slope of a distributed circuit is directly proportional to Z_0 , it may be reduced at the same resonant frequency, by reducing Z_0 (e.g. line d in Fig. 3.1) but this is equivalent to changing the L/C ratio of the circuit, which invalidates the basis of comparison for the two circuits.

It may therefore be concluded that the available bandwidth from lossless distributed resonant circuits is always less than that from equivalent lossless lumped circuits. In other words, more energy is stored per cycle in a non-resonant distributed circuit than in the equivalent lumped circuit, because in the former, capacitance and inductance always occur together.

Fig. 3.1

Comparison of Reactance Functions of Distributed and Lumped Resonant Circuits



The case of mixed lumped and distributed circuits which often occurs in parametric amplifiers, will now be considered. If a capacitance C is resonant with the inductive line of length l at f_0 , then:

$$X = Z_0 \tan \frac{\omega l}{v} - \frac{1}{\omega C}$$

$$\begin{aligned} \text{and so } \frac{dX}{d\omega} &= Z_0 \cdot \frac{1}{v} \left[\sec^2 \left(\frac{\omega l}{v} \right) \right] + \frac{1}{\omega^2 C} \\ &= Z_0 \frac{1}{v} \left[1 + \tan^2 \left(\frac{\omega l}{v} \right) \right] + \frac{1}{\omega^2 C} \end{aligned}$$

$$\text{now if } l = \frac{\lambda_0}{4A} \text{ where } A > 1 \text{ (i.e. } l < \frac{\lambda_0}{4} \text{) then } l = \frac{v}{4Af_0}$$

$$\text{so } \frac{dX}{d\omega} = \frac{Z_0}{4Af_0} \left[1 + \tan^2 \left(\frac{2\pi}{4A} \right) \right] + \frac{1}{\omega^2 C} \quad \dots\dots\dots 3.2$$

Now the reactance slope of the equivalent lumped circuit = $\frac{2}{\omega^2 C}$ so:

$$\frac{\left(\frac{dX}{d\omega} \right) \text{ distributed}}{\left(\frac{dX}{d\omega} \right) \text{ lumped}} = \frac{\frac{Z_0}{4Af_0} \left[1 + \tan^2 \left(\frac{\pi}{2A} \right) \right] + \frac{1}{\omega^2 C}}{\frac{2}{\omega^2 C}}$$

but at resonance:

$$\frac{1}{\omega_0 C} = Z_0 \tan \frac{\pi}{2A}$$

so the ratio of the reactance slopes may be written:

$$\frac{\left(\frac{dX}{d\omega} \right) \text{ distributed}}{\left(\frac{dX}{d\omega} \right) \text{ lumped}} = \frac{Z_0}{2} \left[\omega_0 C + \frac{1}{\omega_0 C Z_0} \right] \left[\tan^{-1} \left(\frac{1}{Z_0 \omega_0 C} \right) \right] + \frac{1}{2} \quad \dots\dots\dots 3.3$$

As Z_0 increases $Z_0 \omega_0 C \left[\tan^{-1} \left(\frac{1}{Z_0 \omega_0 C} \right) \right]$ decreases to unity, so equation 3.3

decreases to unity as Z_0 increases. This is seen in Fig. 3.2, a plot of the ratio of distributed to lumped circuit reactance slopes (equation 3.3) against Z_0 for $C = 0.5 \text{ pF}$ and $f_0 = 4 \text{ GHz}$ and it is noted that for impedance values of

Fig. 3.2

Ratio of reactance slopes of a resonant circuit involving distributed inductance and an equivalent circuit with lumped inductance, versus the characteristic impedance (Z_0) of the line forming the distributed inductance

Ratio of
Reactance
Slopes

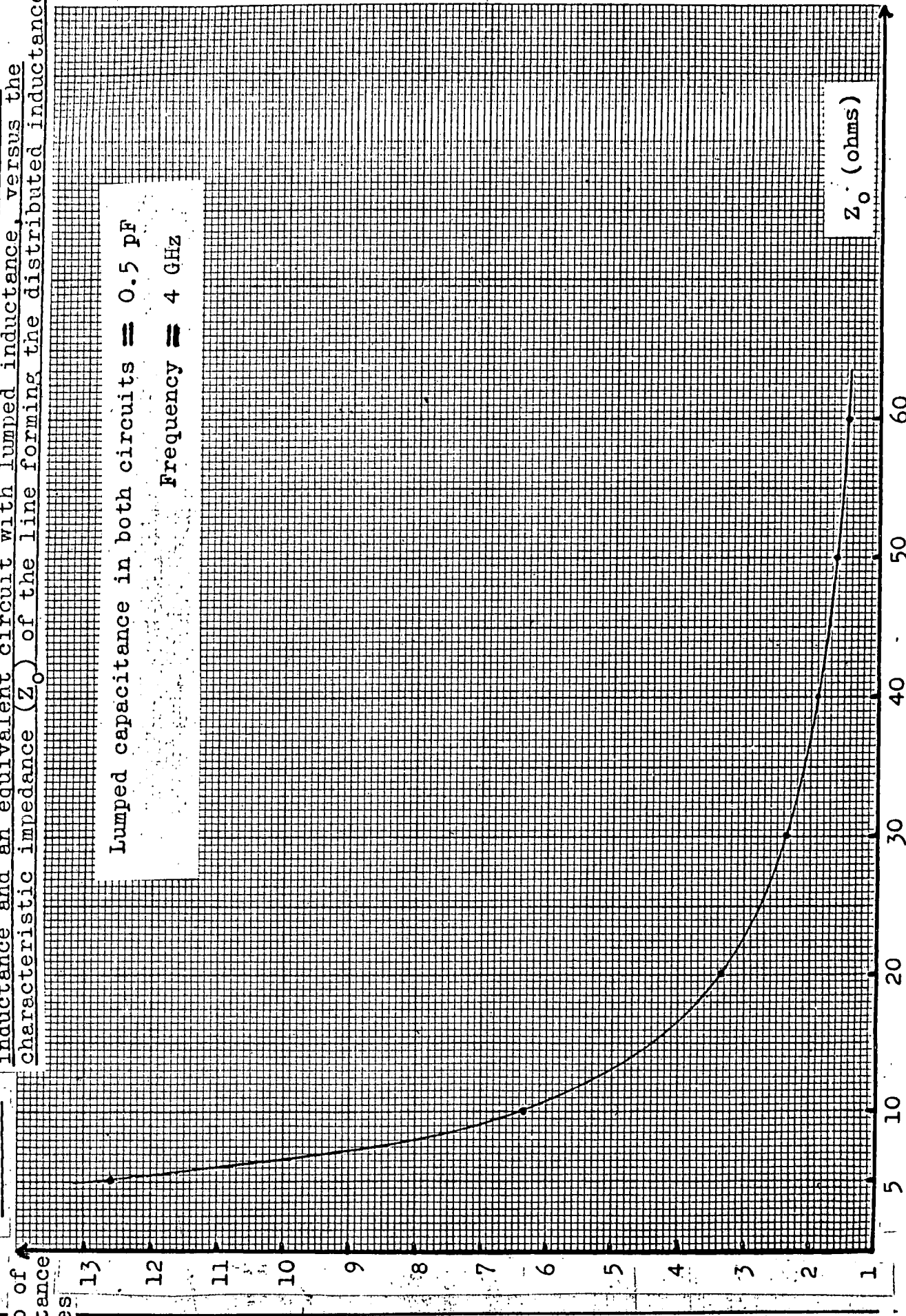
13
12
11
10
9
8
7
6
5
4
3
2
1

Lumped capacitance in both circuits = 0.5 pF

Frequency = 4 GHz

Z_0 (ohms)

5 10 20 30 40 50 60



60 to 10Ω (practicable values for most parametric amplifier circuits) the reactance slope of circuits containing a distributed inductive element is 1.5 to 6 times greater than the equivalent lumped circuit. The signal circuit Q of an amplifier will be increased by the same factor and thus amplifier bandwidth will be reduced according to the relationship discussed in section 3.5. An identical condition can be obtained for circuits with distributed capacitance.

It is thus concluded that individual non-resonant distributed elements are best avoided, but if used should be made with as high impedance lines as possible.

3.3 Determination of Frequency Response

The bandwidth of any circuit, no matter how complex, may of course be determined from the frequency response, if the driving point impedance function is known. For a given impedance function $Z(f)$, the gain at any frequency for a reflection amplifier is:

$$\text{gain} = |\Gamma|^2 = \left| \frac{R_g - Z(f)}{R_g + Z(f)} \right| \quad \dots\dots\dots 3.4$$

The accuracy of such a bandwidth calculation depends entirely on the validity of the amplifier equivalent circuit used, which in part depends on the validity of the small signal impedance matrix. The impedance function for the generalised equivalent circuit (Fig. 2.3) may be written:

$$Z_{in} = Z_1 - \frac{X_{12} X_{21}}{Z_2^*}$$

where Z_1 and Z_2 are the total impedances in the signal and idler circuits respectively. Substituting for $X_{12} X_{21}$ gives:

$$Z_{in} = Z_1 - \frac{\gamma^2}{\omega_1 \omega_2 C^2 Z_2^*} \quad \dots\dots\dots 3.5$$

The circuit of Fig. 2.11 applies for a single diode shunt resonant amplifier, with the following impedance function:

$$Z_{in} = R_s + jX_1 - \frac{\gamma^2}{\omega_1 \omega_2 C^2 (R_s + jX_2)} \quad \dots\dots\dots 3.6$$

This has been substituted in equation 3.4 using the parameters of a CAY10 diode (see section 5) and the resulting impedance function and gain/frequency responses are plotted in Figs. 3.3 and 3.4 using lumped inductance at 4 GHz.

Fig. 3.3

Impedance Function of a Single Diode Shunt Resonant Amplifier
using a CAY10 Diode

P.O. Engineering Department Research Station, Dollis Hill, LONDON, N.W.2.

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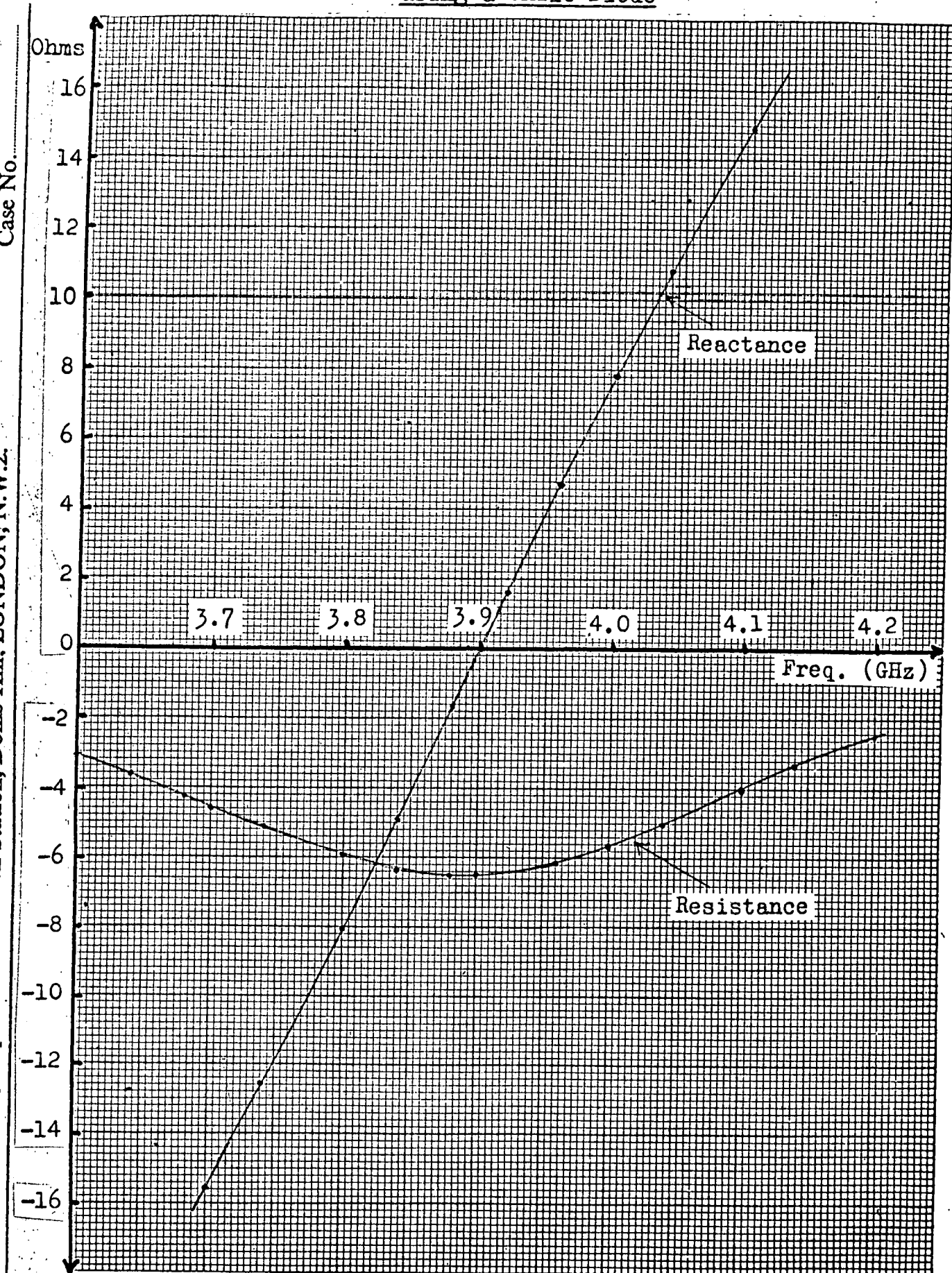
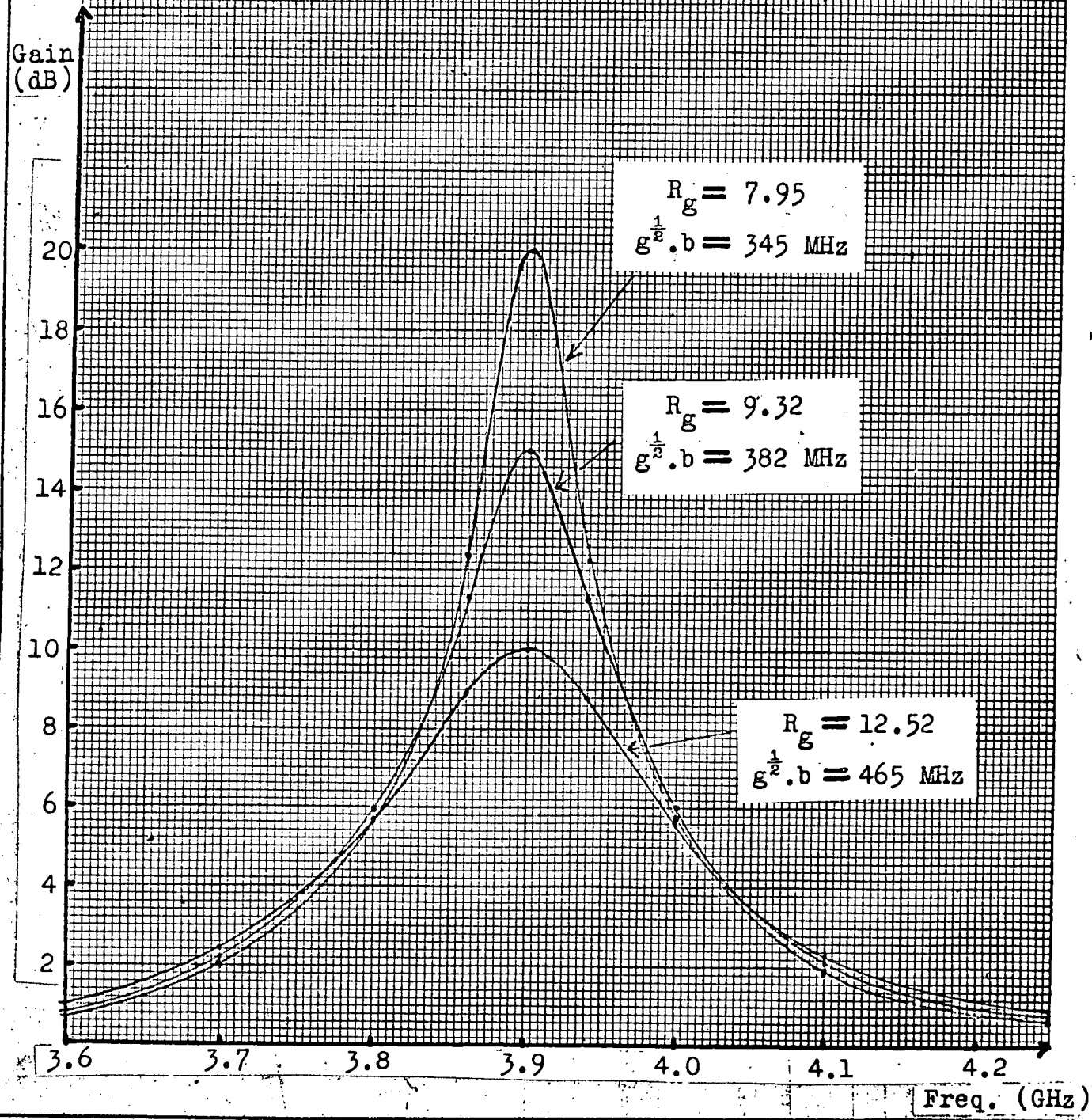


Fig. 3.4

Gain/Frequency Response of a Single Diode Shunt Resonant Amplifier using a CAY10 Diode

For various values of generator/load impedance (R_g)



The effects of the idler chokes on the signal circuit and the quarterwave idler stub on the idler circuit have been neglected in the equivalent circuit of Fig. 2.11, but the very good agreement between this analysis and bandwidth measurements (section 6.4) confirms the validity of this equivalent circuit. A slight negative resistance variation over the frequency band is seen for the shunt resonant amplifier, but the bandwidth is largely determined by the signal circuit reactance function. Broad-banding by multituning the signal circuit should therefore be possible.

3.4 Gain-Bandwidth Product

Approximations can be made in equation 3.4, for a particular amplifier structure, to derive an analytical expression for bandwidth in the form of a $\text{gain}^{\frac{1}{2n}}$ bandwidth product, dependent on the diode and circuit parameters (n = number of tuned circuits in the amplifier). The derivations which are often quoted for the expression describing the $\text{gain}^{\frac{1}{2}}$ bandwidth product of a single tuned amplifier (for example as in Blackwell and Kotzebue) involve many approximations which may sometimes be invalid for real amplifiers. An expression for the $\text{gain}^{\frac{1}{2}}$ bandwidth product will now be derived in which the significance of the approximations is carefully considered. The cruder approximations necessary to derive a $\text{gain}^{\frac{1}{2n}}$ bandwidth product for multituned circuits (Humphreys 1964) invalidate the use of the product as a constant for determining the relationship between gain and bandwidth for a given amplifier, but produce an expression that is useful in comparing the performance of various types of amplifier at a fixed gain.

The high Q approximation (Everitt and Anner) is usually valid for parametric amplifier circuits, so that the impedance of a series resonant circuit may be expressed:

$$Z = R + j2Q R_L \Delta\omega$$

where $\Delta\omega = \frac{\omega_{\text{res}} - \omega}{\omega}$ and Q_L is the loaded circuit Q .

So, for the signal and idler circuits of a single tuned parametric amplifier:

$$X_1 = j2Q_1 (R_g + R_1) \Delta\omega$$

and $X_2 = j2Q_2 R_2 \Delta\omega \frac{\omega_1}{\omega_2}$

since $\omega_{\text{res-1}} - \omega_1 = \omega_{\text{res-2}} - \omega_2$

Writing the total impedance function in the form $Z = R + jX$ gives, after rationalisation;

$$R = R_1 - \frac{\gamma^2}{\omega_1 \omega_2 C^2 R_2 \left[1 + 4Q_2^2 \Delta^2 \omega \left(\frac{\omega_1}{\omega_2} \right) \right]} \quad \dots\dots\dots 3.7$$

$$\text{and } X = R_g + R_1 2Q_1 \Delta \omega + \frac{\gamma^2 2Q_2 \Delta \omega \frac{\omega_1}{\omega_2}}{\omega_1 \omega_2 C^2 R_2 \left[1 + 4Q_2^2 \Delta^2 \omega \left(\frac{\omega_1}{\omega_2} \right)^2 \right]} \quad \dots\dots\dots 3.8$$

$$\text{so } R = R_1 - \frac{R_0}{(1 + M \Delta^2 \omega)} \quad \dots\dots\dots 3.9$$

$$\text{and } X = \Delta \omega P = \Delta \omega \left[N + \frac{R_0 S}{(1 + M \Delta^2 \omega)} \right] \quad \dots\dots\dots 3.10$$

where R_0 is the negative resistance at resonance and N , M , P and S represent the respective factors in equations 3.7 and 3.8. Now, from equation 3.4 the amplifier power gain may be written:

$$G = \left| \frac{(R_g^2 - R^2) - X^2}{(R_g + R)^2 + X^2} \right|^2 + \left| \frac{-2XR_g}{(R_g + R)^2 + X^2} \right|^2 \quad \dots\dots\dots 3.11$$

and the band-centre gain, G_0 , may be written:

$$G_0 = \left| \frac{R_g - R}{R_g + R} \right|^2 \quad \text{where } R = R_1 - R_0$$

So, at the 3 dB power points:

$$\frac{G_0}{2} = \frac{G_0^{\frac{1}{2}}}{\left[1 + \frac{X^2}{(R_g + R)^2} \right]} - \frac{\left[\frac{X^2}{(R_g + R)^2} \right]}{\left[1 + \frac{X^2}{(R_g + R)^2} \right]} + \left| \frac{-2XR_g}{(R_g + R)^2 + X^2} \right|^2 \quad \dots\dots\dots 3.12$$

At high gain $R_g = R$ and the term $\left(\frac{X}{R_g + R}\right)$ is large, so the first term in equation 3.12 is small compared with the second. Therefore:

$$\frac{\xi_o}{2} = \frac{1}{X^2} \left[\frac{2R_g}{\frac{(R_g + R)^2}{X^2} + 1} \right]^2 \quad \dots\dots\dots 3.13$$

to a good approximation.

or
$$\frac{\xi_o}{2} \Delta^2 \omega = \frac{4R_g^2}{P^2} \left[\frac{1}{\frac{(R_g + R)^2}{X^2} + 1} \right]^2$$

Now;
$$\frac{R_g + R}{X} = \frac{KR_o + R_1 - \frac{R_o}{(1 + M\Delta^2\omega)}}{\Delta\omega \left[N + \frac{R_o S}{(1 + M\Delta^2\omega)} \right]}$$
 from equations 2.8, 3.8, 3.9 and 3.10

At high gain K tends to unity and $\Delta\omega$ tends to zero so;

$$\frac{R_g + R}{X} \rightarrow \frac{R_1}{\left[(R_1 + R_o)2Q_1 + 2Q_2 \frac{\omega_1}{\omega_2} R_o \right]} \quad \text{at high gain,}$$

but $Q = \frac{\omega}{\Delta'\omega}$ where $\Delta'\omega = 3 \text{ dB bandwidth}$
 $= 2(\omega - \omega_{\text{res}})$
 $= 2\Delta\omega \cdot \omega$

So: $Q = \frac{1}{2\Delta\omega}$ (i.e. at the 3 dB power points, $2\Delta\omega Q$ is unity), and

$$\frac{R_g + R}{X} = \frac{R_1}{\left[R_1 + R_o + \frac{Q_2 \omega_1}{Q_1 \omega_2} R_o \right]} \quad \text{which is negligible if } R_o > 5R_1.$$

Thus at high gain $\left[\frac{1}{\frac{R_g + R}{K} + 1} \right]$ tends to unity if $R_o > 5R_1$, and so;

$$\frac{g_o}{2} \Delta^2 \omega = \frac{4R_g^2}{\left[(R_g + R_1) 2Q_1 + \left(\frac{2Q_2 R_o \frac{\omega_1}{\omega_2}}{1 + M\Delta^2 \omega} \right) \right]^2}$$

Now $R_o = \frac{R_g}{K} + R_1$ (from equation 2.8) and $1 + M\Delta^2 \omega \approx 1$ for $\Delta\omega < 0.1$ so;

$$\frac{1}{g_o^2} b = \frac{2\sqrt{2} \left(\frac{R_g}{R_g + R_1} \right)}{\left[Q_1 + Q_2 \frac{\omega_1}{\omega_2} \left(\frac{\frac{R_g + R_1}{K}}{R_g + R_1} \right) \right]} \dots\dots\dots 3.14$$

where b is the fractional bandwidth between the half-power points and equals $2\Delta\omega$. For greater than 20 dB gain $K \gg 1$ and 3.14 may be written:

$$\frac{1}{g_o^2} b = \frac{2\sqrt{2} \frac{R_g}{R_g + R_1}}{\left[Q_1 + Q_2 \frac{\omega_1}{\omega_2} \right]} \dots\dots\dots 3.15$$

Equation 3.15 applies only for the following conditions:

1. High Q circuits.
2. High gain, i.e. $K \gg 1$
3. R_o (band centre negative resistance) $> 5R_1$.
4. Narrow bandwidth $1 + M\Delta^2 \omega \approx 1$.

which are fulfilled for many amplifiers above 15 dB gain.

It is thus seen that $\text{gain}^{\frac{1}{2}} \cdot \text{bandwidth}$ is a constant for any single tuned amplifier at high gain. The validity of $\text{gain}^{\frac{1}{2}n} \cdot \text{bandwidth}$ products for multituned circuits is discussed in section 3.7.

3.5 Relationship between Bandwidth and the Diode Parameters for various types of Amplifier

3.5.1 Self-Resonant Idler Configurations

For a single diode amplifier using the shunt resonance:

$$R_1 = R_2 = R_s$$

$$Q_1 = \frac{1}{\omega_1 C (R_g + R_s)}$$

$$Q_2 = \frac{\omega_2 L_s}{R_s}$$

and at high gain $R_g = -R = R_o - R_1$

For a balanced diode amplifier using the series resonance:

$$R_1 = \frac{R_s}{2} \qquad R_2 = 2R_s$$

$$Q_1 = \frac{1}{\omega_1 2C \left(R_g + \frac{R_s}{2} \right)}$$

$$Q_2 = \frac{\omega_2 L_s}{R_s}$$

and at high gain $R_g = -R = R_o' - \frac{R_1}{2}$

where
$$R_o' = \frac{Y^2}{\omega_1 \omega_2 C^2 2R_s}$$

Equation 3.15 may thus be written, for both single and balanced diode amplifiers, as:

$$\frac{1}{\xi_o^2 b} = \frac{2\sqrt{2} \left(\frac{R_g}{R_g + R_1} \right)}{\left(Q_1 + \frac{\omega_1 L_s}{R_s} \right)} \qquad \dots\dots\dots 3.16$$

or, making the high gain approximation, $R_g = -R$, but not assuming that $R_o > 5R_1$, then:

$$g_{o.b}^{\frac{1}{2}} = \frac{2\sqrt{2} \left[1 - \frac{f_1 f_2}{(\gamma f_c)^2} \right]}{\left[\frac{f_2}{\gamma^2 f_c} + \frac{\omega_1 L_s}{R_s} \right]} \left[\frac{1}{\frac{(R_g + R)^2}{X^2} + 1} \right] \quad \dots\dots\dots 3.17$$

It is evident, from equation 3.16, that for both types of amplifier, as high values of negative resistance and generator/load impedance as possible, are required. At a given signal frequency these factors depend on the square of the diode γ value, which has been shown (section 2.4) to increase rapidly as the diode is biased towards forward conduction. Therefore, for greatest bandwidth, the highest possible R_g value is chosen consistent with the condition for negligible diode current, (see section 8).

The analysis so far has neglected the effect of diode encapsulation capacitance in the amplifier signal circuit. This can be regarded as a static stray across the junction capacitance, provided that the additional circuit inductance at the signal frequency is much greater than the diode series stray inductance. Such a static stray will reduce the diode γ value and the amplifier negative resistance. The reduced γ value can be calculated from equation 2.10 i.e.

$$\gamma' = \frac{C_{jmax} - C_{jmin}}{2(C_{jmax} + C_{jmin}) + 4C_1} \quad \dots\dots\dots 3.18$$

and the negative resistance becomes

$$R' = - \frac{\gamma \gamma'}{\omega_1 \omega_2 C_j^2 R_s + \omega_1 \omega_2 C_1 C_j R_s} \quad \dots\dots\dots 3.19$$

It is noted that the effect of the encapsulation capacitance can be neglected at the idler frequency, since it is incorporated in circuit in the shunt resonant amplifier and in the balanced diode amplifier has zero voltage across it.

Thus diodes with an encapsulation capacitance comparable with the junction capacitance have considerably reduced γ values and it may be regarded as impractical that C_1 should be greater than C_j . If $C_1 \leq C_j$ then:

$$\omega_{2(b.d.)} = \frac{1}{\sqrt{L_s C_j}} = (\sqrt{2}) \cdot \omega_{2(s.d.)} = \frac{1}{\sqrt{\frac{L_s C_j}{2}}}$$

It is thus seen from equations 3.16 and 3.17 that both balanced and single diode circuits have the same idler circuit bandwidth, $\left(Q_2 = \frac{\omega_1 L_s}{R_s}\right)$, but since $f_{2(b.d.)} \leq \sqrt{2} f_{2(s.d.)}$ or $Q_{1(b.d.)} \leq \sqrt{2} Q_{1(s.d.)}$, the available bandwidth from the balanced diode amplifier will always be greater than that from the single diode amplifier. For many diodes the ratio of the shunt to series resonant frequencies is 3 or 4, so that the signal circuit bandwidth of the balanced amplifier is 3 or 4 times that of the single diode amplifier. Diodes with comparatively large stray inductances so that little additional inductance is necessary to resonate at the signal frequency have $Q_1 \approx \frac{\omega_1 L_s}{R_s}$ and so if $C_1 \leq C_j$ then:

$$g_o^{\frac{1}{2}} \cdot b_{(b.d.)} \geq 1.33 g_o^{\frac{1}{2}} \cdot b_{(s.d.)} \quad \dots\dots\dots 3.20$$

Diodes with small stray inductances, so that $Q_1 \gg \frac{\omega_1 L_s}{R_s}$, give:

$$g_o^{\frac{1}{2}} \cdot b_{(b.d.)} \approx \frac{\omega_{shunt}}{\omega_{series}} \cdot g_o^{\frac{1}{2}} \cdot b_{(s.d.)} \quad \dots\dots\dots 3.21$$

Equation 3.17 has been evaluated for a 4 GHz amplifier using diodes with a range of values of stray inductance L_s and encapsulation capacitance C_1 , taking into account the effect of the encapsulation capacitance on the negative resistance, Figs. 3.5, 3.6, 3.7 and 3.8 show the results of this calculation and it is verified that, with many combinations of L_s and C_1 , the $gain^{\frac{1}{2}} \cdot bandwidth$ product of a balanced diode amplifier is 2 to 4 times that of a single diode amplifier. With large encapsulation capacitances, high inductances and high cut-off frequencies, the difference between the two devices is, however, minimised. (Note: the cut-off frequency is defined as $\frac{1}{2\pi C_j R_s}$ and does not include the encapsulation capacitance).

Fig. 3.5

Gain ^{$\frac{1}{2}$} . Bandwidth of a Balanced Diode Amplifier versus Diode Stray Inductance (L_s) for various values of Encapsulation Capacitance (C_1)

$$R_s = 2.5 \Omega$$

$g_{\frac{1}{2},b}$ values are for high gain
for values at 15 dB gain multiply by 1.4

No.
 $g_{\frac{1}{2},b}$
(MHz)

$$\begin{aligned} f_1 &= 4 \text{ GHz} \\ \gamma &= 0.15 \\ f_c &= 150 \text{ GHz} \\ C_j &= 0.45 \text{ pF} \end{aligned}$$

⊙ CAY10 Diode

$$C_1 = 0.01$$

$$C_1 = 0.05$$

$$C_1 = 0.1$$

$$C_1 = 0.3$$

$$C_1 = 0.5$$

L_s (nH)

Fig. 3.6

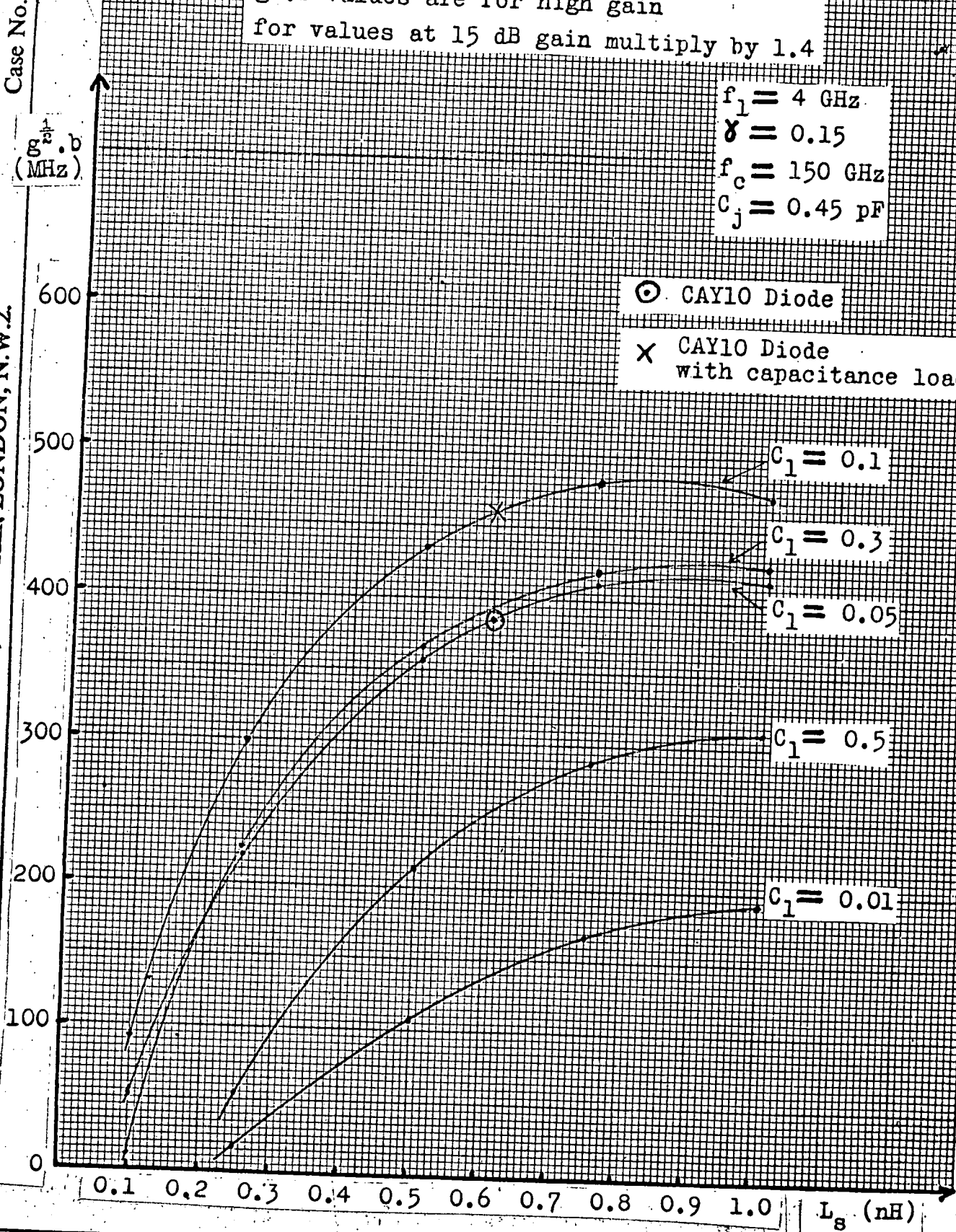
Gain^{1/2}. Bandwidth of a Single Diode Amplifier versus Diode Stray Inductance (L_s) for various values of Encapsulation Capacitance (C_1)

$$R_s = 2.5 \Omega$$

$g_{2.1}^{1/2} \cdot b$ values are for high gain
for values at 15 dB gain multiply by 1.4

$$\begin{aligned} f_1 &= 4 \text{ GHz} \\ \gamma &= 0.15 \\ f_c &= 150 \text{ GHz} \\ C_j &= 0.45 \text{ pF} \end{aligned}$$

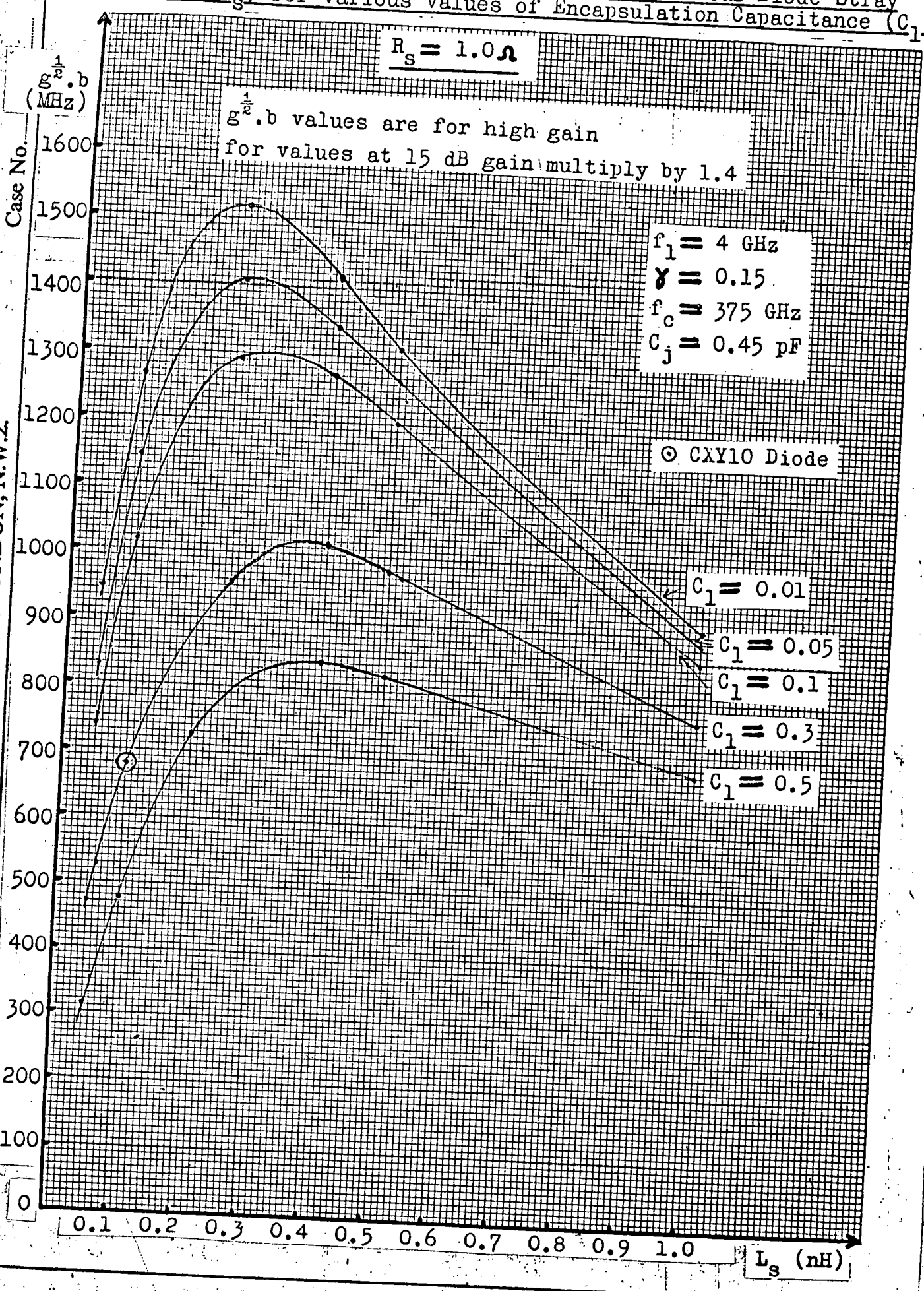
⊙ CAY10 Diode
× CAY10 Diode with capacitance loading



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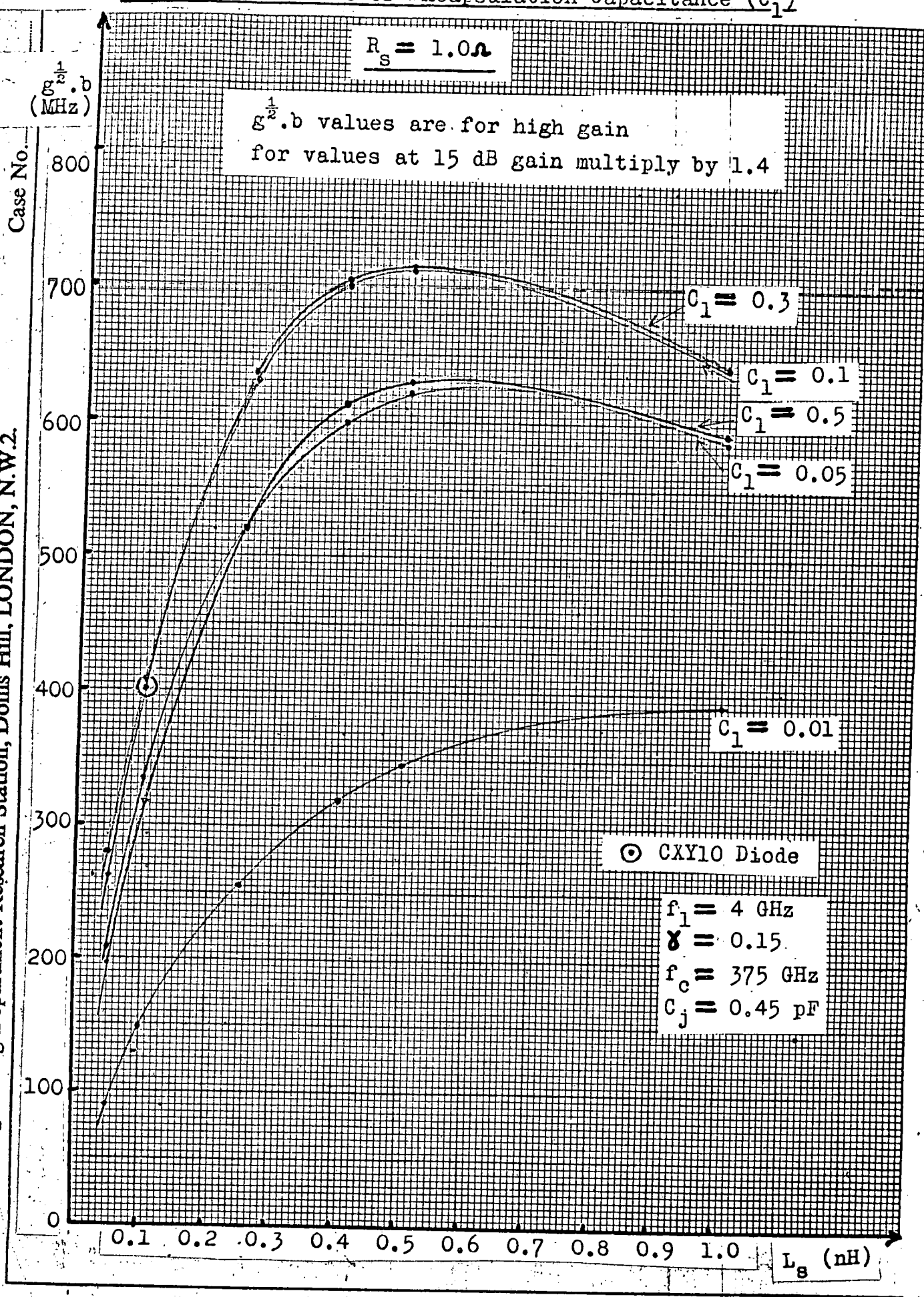
Fig. 3.7

Gain^{1/2}. Bandwidth of a Balanced Diode Amplifier versus Diode Stray Inductance (L_s) for various values of Encapsulation Capacitance (C_1)



Gain^{1/2}. Bandwidth of a Single Diode Amplifier versus Diode Stray Inductance (L_s) for various values of Encapsulation Capacitance (C_1)

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The parameters of a few currently available diodes are listed in table 5.1 and the performance of these diodes in the two types of amplifier is indicated on the graphs. Such present day diode structures permit either a small encapsulation capacitance and a relatively large stray inductance (e.g. Mullard CAY10) or, a relatively large encapsulation capacitance and a small series inductance (e.g. Mullard CXY10). Typically $C_s L_s = \text{constant} = 0.03 \text{ nH.pF}$.

The balanced diode amplifier requires the minimum possible encapsulation capacitance for greatest $\text{gain}^{\frac{1}{2}} \cdot \text{bandwidth}$, whereas there is an optimum value of encapsulation capacitance for the single diode shunt resonant amplifier, which is defined by L_s , and f_c , (see Figs. 3.6 and 3.8).

For a given cut-off frequency and diode γ value, there is an optimum value of series inductance for the balanced diode amplifier and an optimum combination of values of series inductance and encapsulation capacitance for the single diode amplifier. For a low cut-off frequency, the optimum value of series inductance is seen to be high (for a given γ value) and vice versa.

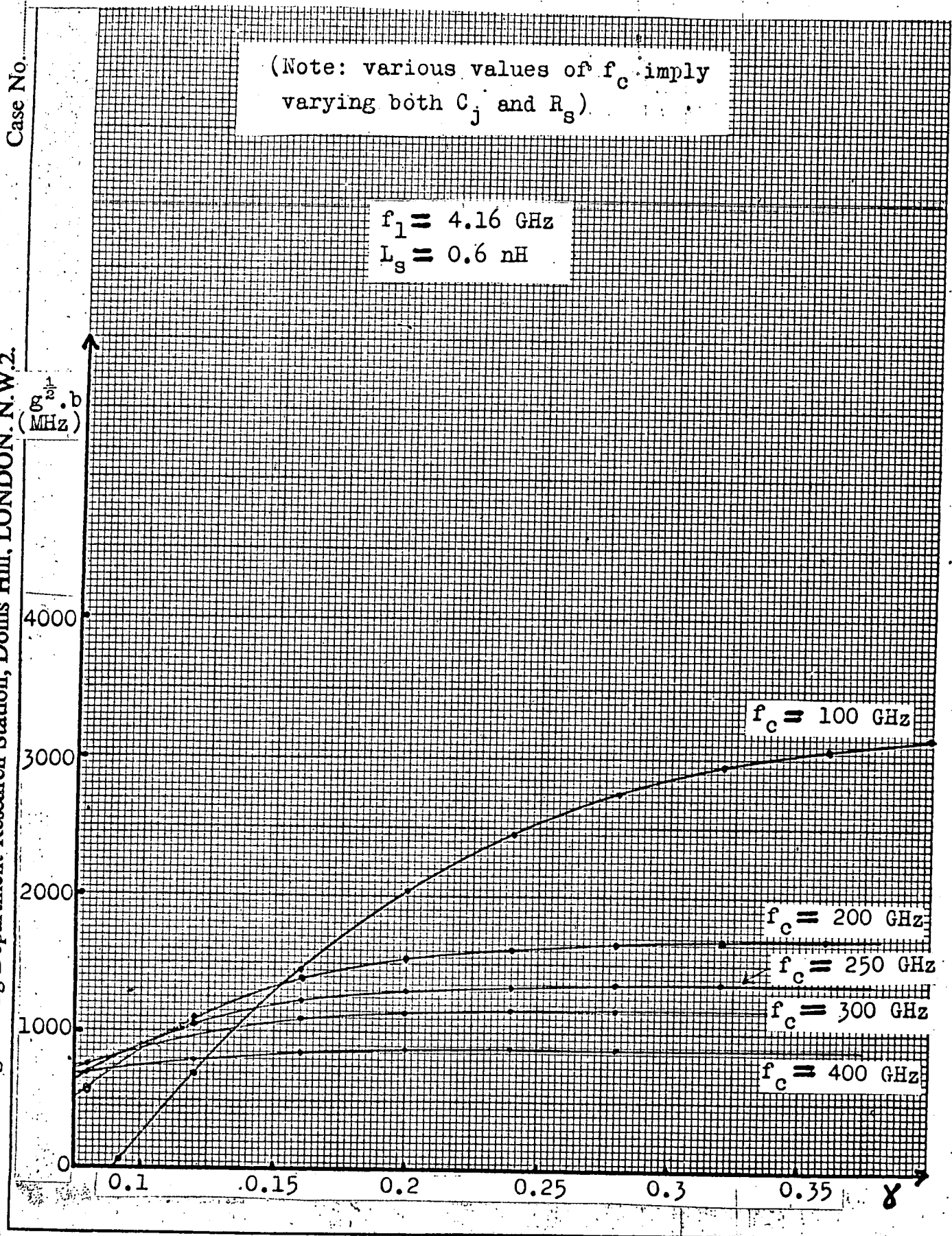
Similarly, with a given series inductance, there is an optimum value of cut-off frequency dependent on the diode γ value. This is seen for the balanced diode amplifier, from the graphs in Figs. 3.9-3.11, which were also obtained from equation 3.17. Diodes with very low series inductances give greater $\text{gain}^{\frac{1}{2}} \cdot \text{bandwidth}$ products for higher cut-off frequencies in all practical cases, but if L_s is large (e.g. Mullard CAY10, $L_s = 0.6 \text{ nH}$) then increasing f_c reduces $\text{gain}^{\frac{1}{2}} \cdot \text{bandwidth}$, due to increased circuit Q values. ^(Insertion) The graphs of Figs. 3.9 to 3.11 show that $\text{gain}^{\frac{1}{2}} \cdot \text{bandwidth}$ is a much more sensitive function of diode γ value than cut-off frequency, as high a γ value as possible being required.

It is noted that the $\text{gain}^{\frac{1}{2}} \cdot \text{bandwidth}$ product at high gain for a balanced diode amplifier, using the CAY10 varactor at 4 GHz, should be 1100 MHz (if no additional stray inductance is added; see section 3.7.3) and for a single diode shunt resonant amplifier should be 385 MHz (or 460 MHz with capacitance loading). The latter figure, for the shunt resonant amplifier without capacitance loading, shows reasonable agreement with the value of 345 MHz (at 20 dB gain) obtained from the gain/frequency response of the shunt resonant device with lumped circuit elements (section 3.2). This latter value, from the gain/frequency response, should be the more realistic figure, since no approximations or circuit simplifications have been made in its derivation, other than those connected with the small signal matrix theory (section 2). At low values of gain (<20 dB), $\text{gain}^{\frac{1}{2}} \cdot \text{bandwidth}$ is a function of gain so that, for example at 15 dB gain, 382 MHz should be obtained for the shunt resonant amplifier (see Fig. 3.4). The use of distributed inductance at the signal frequency, which will reduce these $\text{gain}^{\frac{1}{2}} \cdot \text{bandwidth}$ products, is considered in section 3.7.5.

Fig. 3.9

Initials.....
Date.....

Gain^{1/2}. Bandwidth of a Balanced Diode Amplifier versus Diode γ value
for various Diode Cut-Off Frequencies (f_c)



Gain ^{$\frac{1}{2}$} . Bandwidth of a Balanced Diode Amplifier versus Diode γ value for various Diode Cut-Off Frequencies (f_c)

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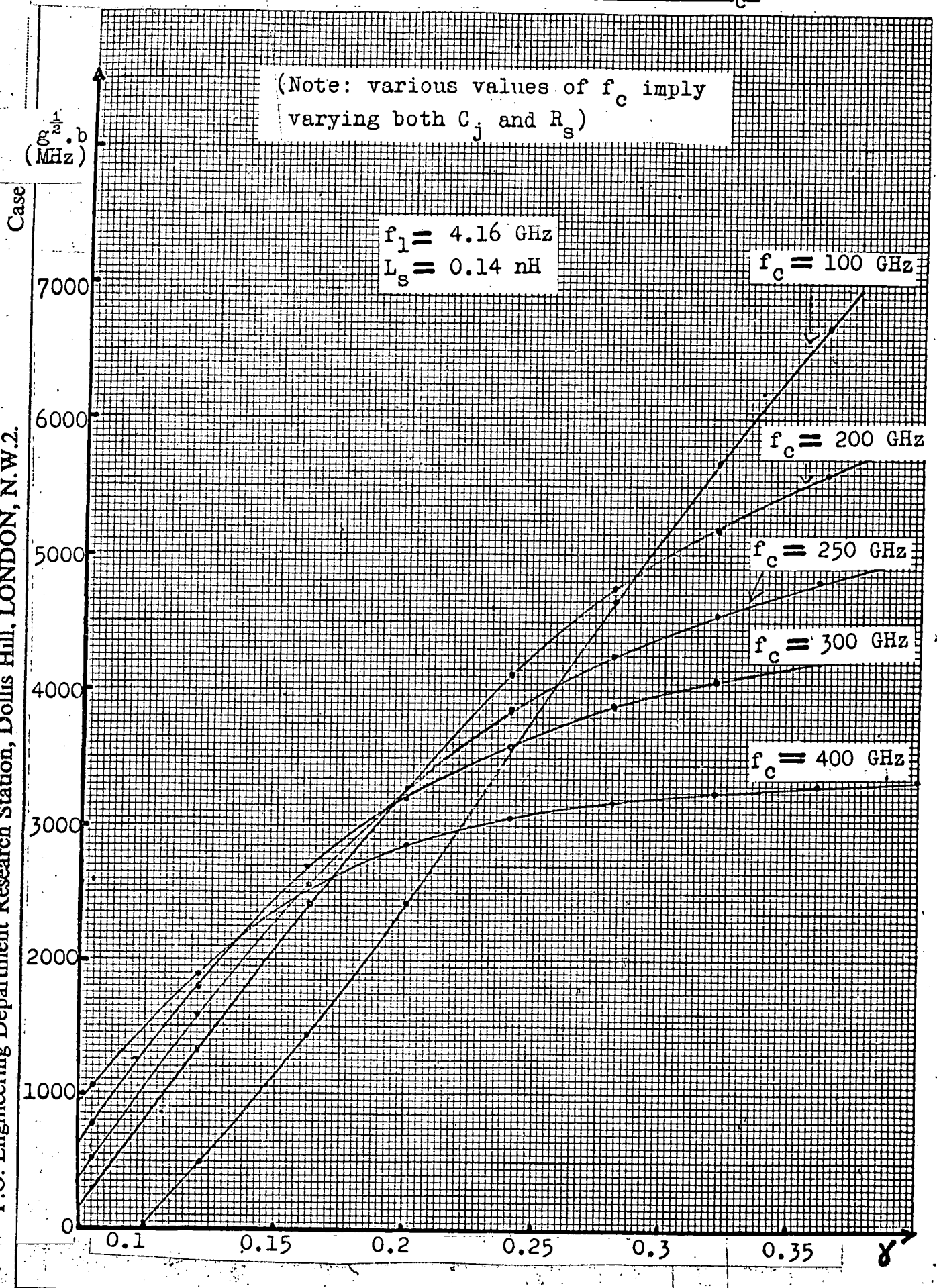
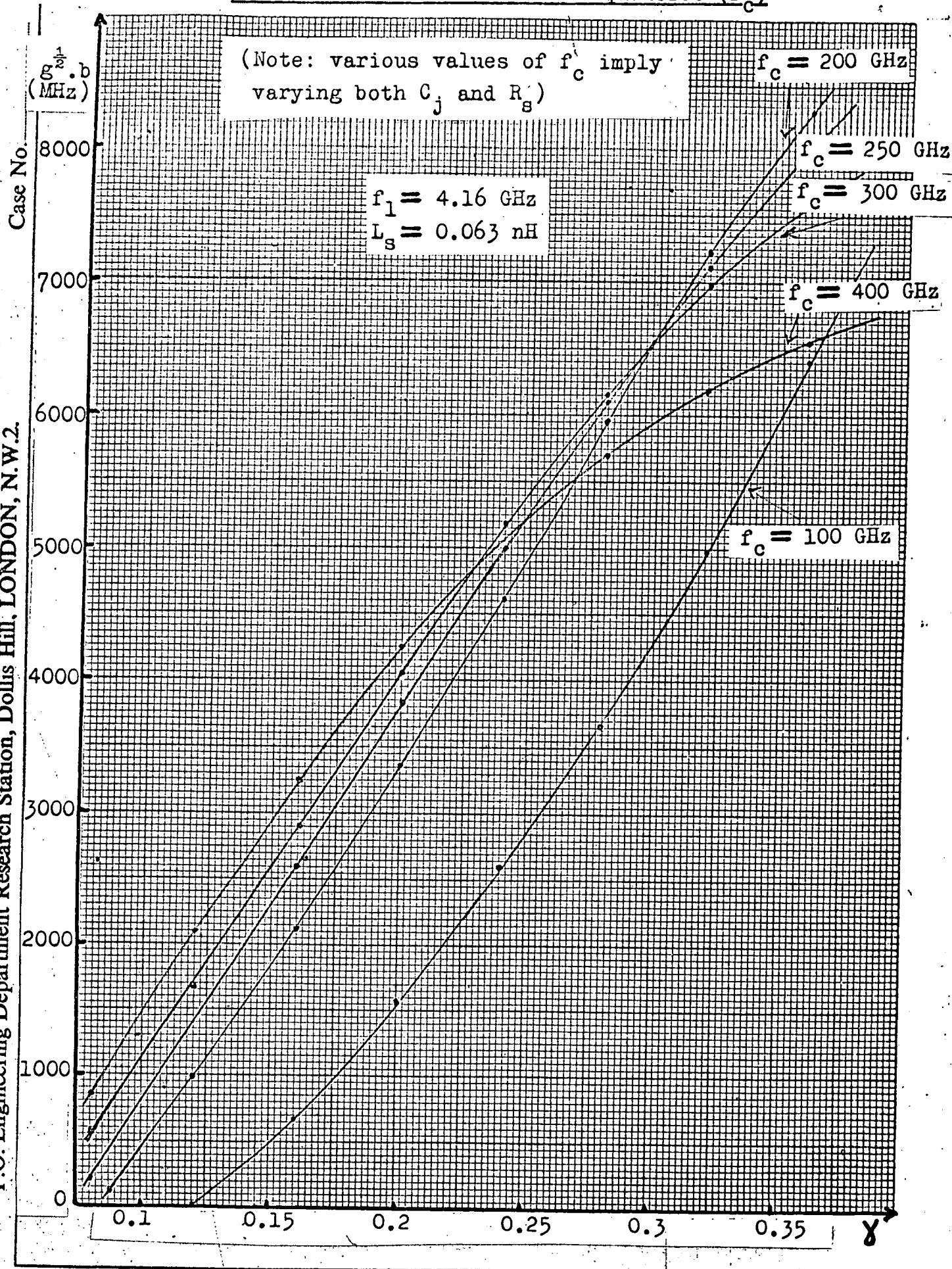


Fig. 3.10

Gain^{1/2}. Bandwidth of a Balanced Diode Amplifier versus Diode γ value
for various Diode Cut-Off Frequencies (f_c)

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• Fig. 3-11

Operation of amplifiers with self-resonant idler configurations at various signal frequencies

The signal circuit negative resistance may be written as $-\frac{\gamma^2}{\omega_1 \omega_2 C^2 R_2}$, for

either type of amplifier, where R_2 is the idler circuit resistance, which for the single diode amplifier is R_s and for the balanced diode amplifier is $2R_s$. Thus, for the single diode amplifier, the negative resistance can be written;

$$R_{(s.d.)} = \frac{(\gamma f_c)^2 R_s}{f_1 f_2}$$

and for the balanced diode amplifier;

$$R_{(b.d.)} = \frac{(\gamma f_c)^2 R_s}{2f_1 f_2}$$

Since the negative resistance is inversely proportional to the signal frequency for both types of amplifier, with given values of R_s , diode quality factor (γf_c) and idler frequency (specified by the relevant diode self-resonant frequency) there is a maximum signal frequency above which gain will not be obtained, defined by:

$$R > R_1$$

$$\text{that is, } \frac{(\gamma f_c)^2}{f_1 f_2} > 1 \text{ for both amplifier circuits,}$$

since R_1 equals R_s and $2R_s$ for the single and balanced diode amplifiers respectively. Hence:

$$f_1 < \frac{(\gamma f_c)^2}{f_2} \quad \dots\dots\dots 3.22$$

Since the idler frequency is defined by the relevant diode self-resonant frequency for each amplifier, an upper signal frequency limit is seen for a particular amplifier. However, it has been shown in section 2, that the noise temperature of an amplifier increases linearly with signal frequency for given values of γf_c and idler frequency, so that the upper operational limit on signal frequency may be determined by noise temperature requirements unless the

device is cooled. For a device, at ambient temperature T_d , to produce a noise figure of less than 3 dB, ($< 290^\circ\text{K}$ noise temperature), this limit may be expressed (from equation 2.17) as:

$$\frac{T_d}{290} \left[\frac{f_1 f_2}{(Y f_c)^2} + \frac{f_1}{f_2} \right] < 1$$

or

$$f_1 < \left[\frac{(Y f_c)^2 f_2}{(Y f_c)^2 + f_2^2} \right] \frac{290}{T_d} \dots\dots\dots 3.23$$

which dictates a lower signal frequency limit than equation 3.22, unless the device is cooled. Amplifier $\text{gain}^{\frac{1}{2}}$.bandwidth and noise temperature are thus interrelated.

It must be noted, however, that with many diodes, particularly in the balanced configuration, the condition $f_1 < f_2$ (onset of degeneracy) may determine the upper signal frequency limit. This means that for telecommunications, the single diode amplifier will operate at higher signal frequencies than the balanced diode amplifier, using the same diodes and Figs. 3.12 to 3.15, which represent an evaluation of equation 3.17 for $\text{gain}^{\frac{1}{2}}$.bandwidth over a range of signal frequencies, illustrate this fact. The dashed portions of these graphs indicate that the condition of degeneracy has been reached.

(The effect of encapsulation capacitance on the signal circuit, has been neglected in Figs. 3.12 ~~and~~ 3.15 for both types of amplifier, since as f_1 approaches $f_{\text{series resonance}}$ this capacitance merely shunts the amplifier signal circuit and could be resonated out at the signal frequency in the single diode amplifier. The $\text{gain}^{\frac{1}{2}}$.bandwidth of a balanced diode amplifier would, of course, be degraded by a high encapsulation capacitance, according to equations 3.18 and 3.19.)

3.5.2 Idler Circuit Resistive Loading

Circuit Q values may be reduced and bandwidth increased, within limits, by resistive idler loading. In a single diode circuit this is impractical because the idler load has to be separated, with filters, from the signal circuit. The problem then becomes that of broadband matching of the diode to a resistive load at the idler frequency, which, as previously discussed, is difficult with distributed circuits at high idler frequencies and with uncertain diode parameters.

Fig. 3.12 $\text{Gain}^{\frac{1}{2}}$ Bandwidth of a Single Diode Amplifier versus Signal Frequency

$R_s = 2.5 \Omega$ $C_1(\text{encapsulation}) = 0.05 \text{ pF}$
 Bandwidth expressed as a fraction of signal frequency
 Values of L_s are in nH

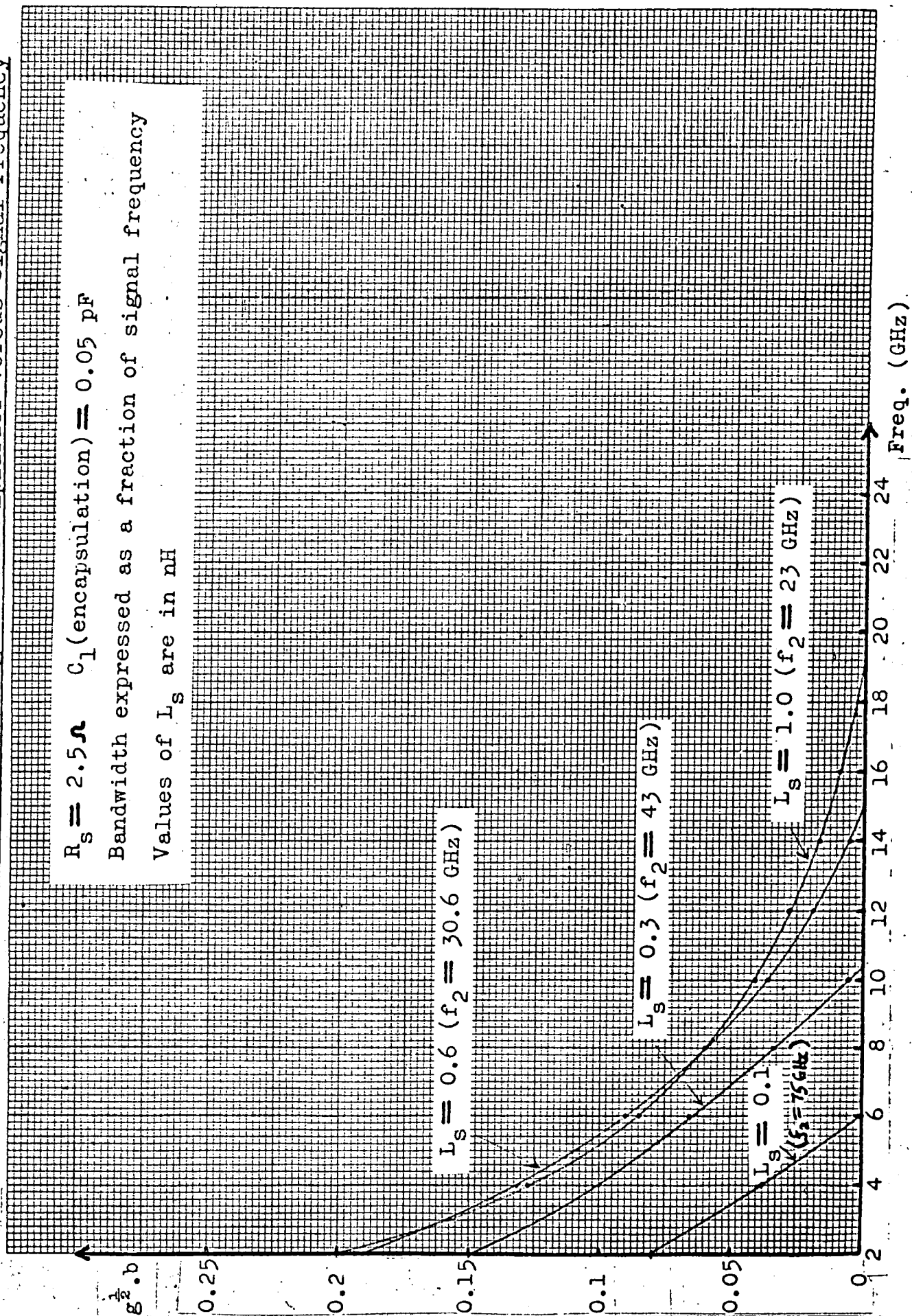


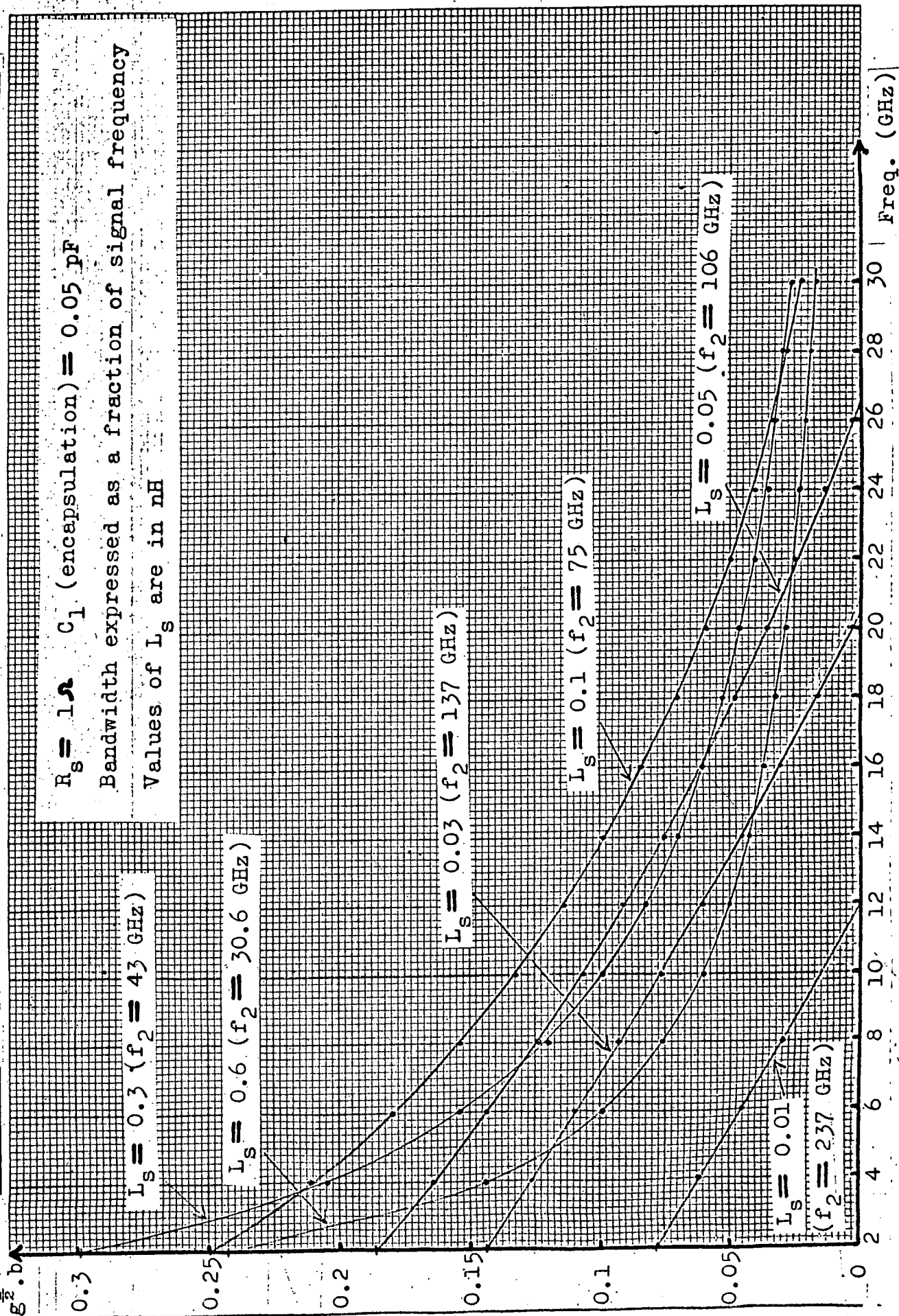
Fig. 3.13 Gain^{1/2}. Bandwidth of a Single Diode Amplifier versus Signal Frequency

Fig. 3.14 $\text{Gain}^{\frac{1}{2}}$. Bandwidth of a Balanced Diode Amplifier versus Signal Frequency

$$R_s = 2.5 \Omega \quad C_1(\text{encapsulation}) = 0.05 \text{ pF}$$

Bandwidth expressed as a fraction of signal frequency

Values of L_s are in nH

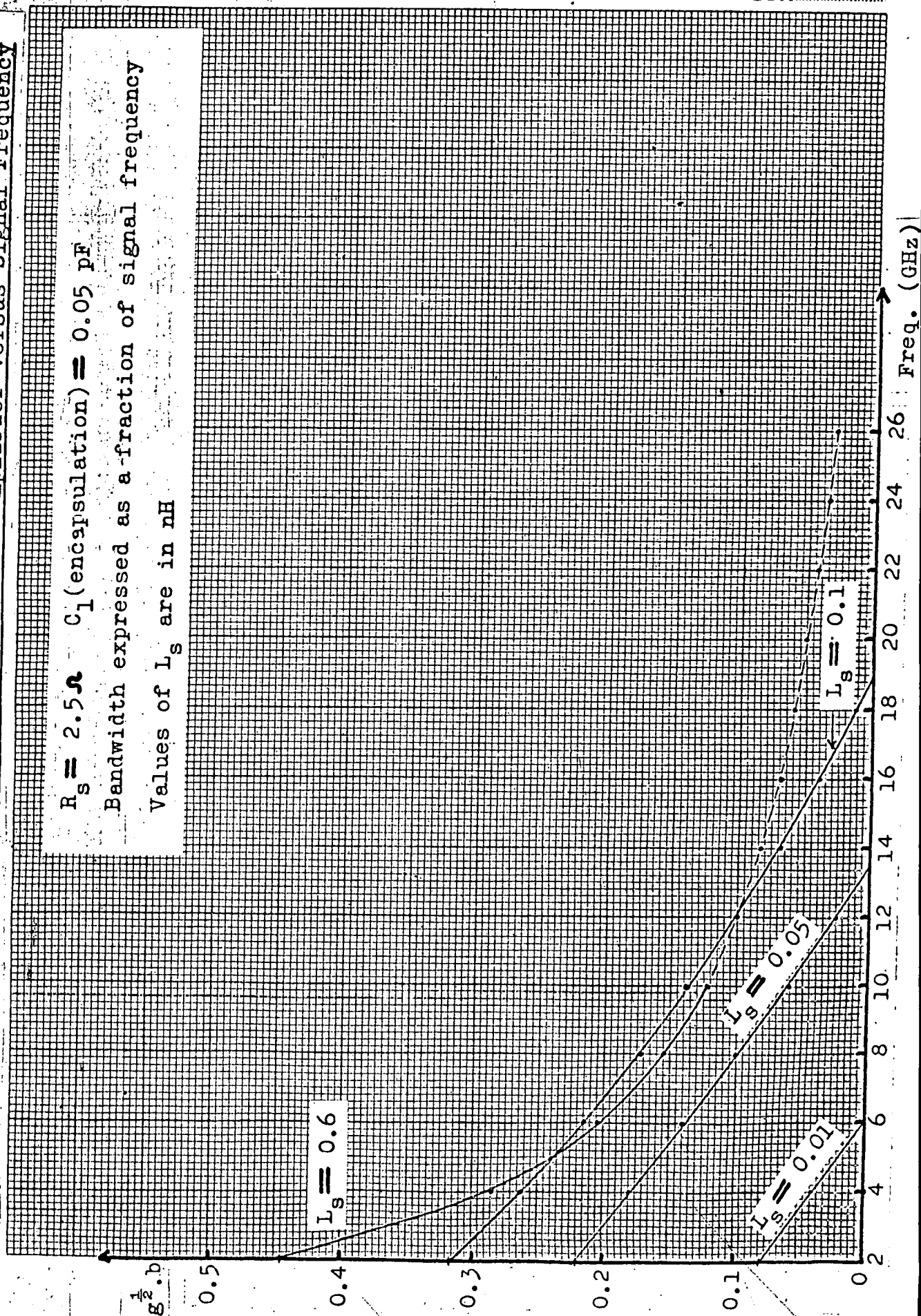
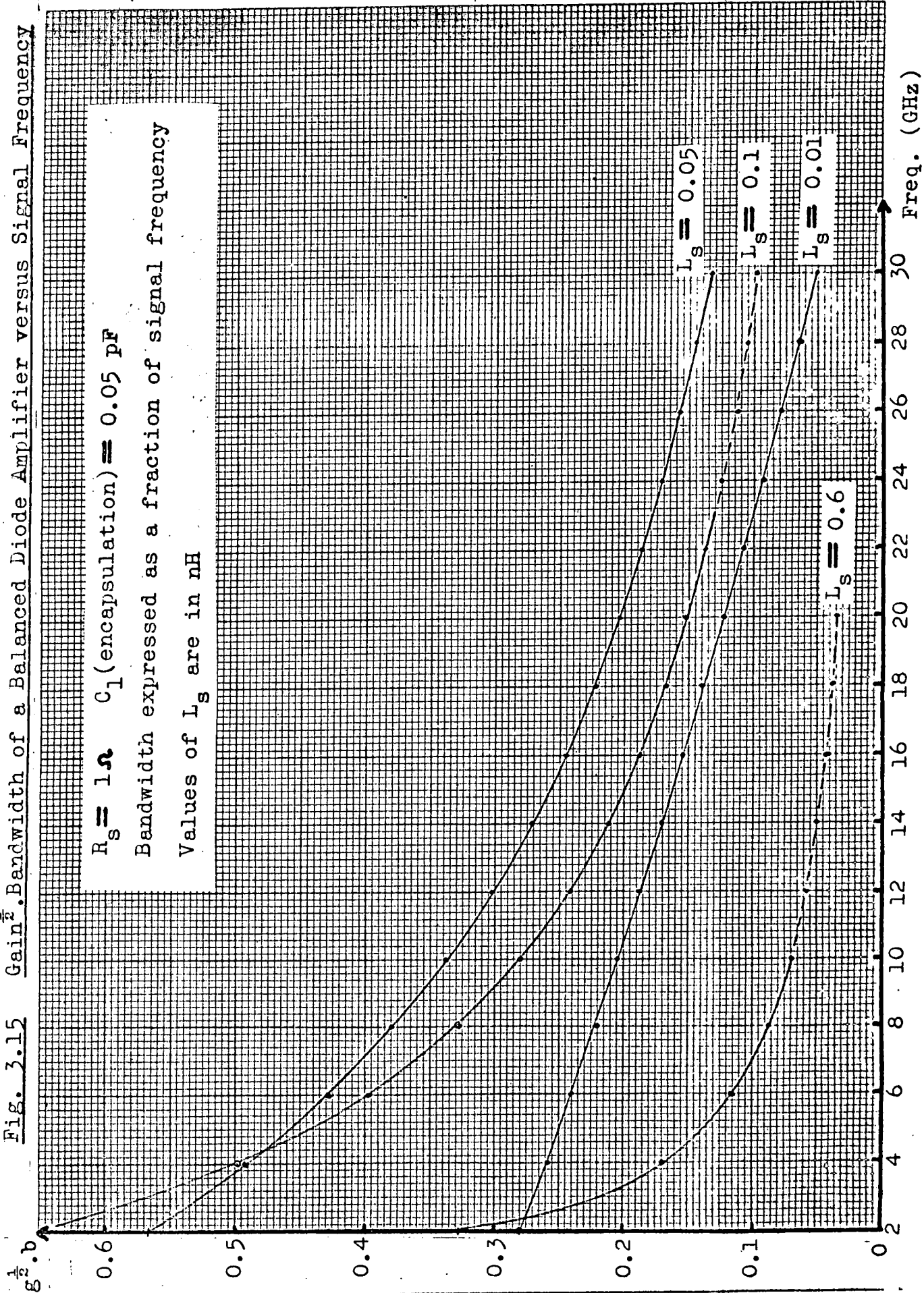


Fig. 3.15 $\text{Gain}^{\frac{1}{2}}$. Bandwidth of a Balanced Diode Amplifier versus Signal Frequency

$R_s = 1 \Omega$ $C_1(\text{encapsulation}) = 0.05 \text{ pF}$
 Bandwidth expressed as a fraction of signal frequency
 Values of L_s are in nH



A simple circuit could be used for a balanced diode, amplifier (Fig. 3.16) in which the resistance R_A is added to each diode. This resistance can, for the purpose of analysis, be regarded as integral with each diode, so that the cut-off frequency is effectively reduced. This is only of advantage when the diode stray inductance, L_s , and cut-off frequency are large, as for example for a diode with a cut-off frequency of 450 GHz ($R_s = 1\Omega$) and $L_s = 0.6$ nH, (see Fig. 3.5), when a small additional resistance might be added to advantage. Such a circuit could be realised by using a lossy material to support the diodes.

3.6 Detuning due to a change in Pump Power

The analysis of section 3.4 assumes that the diode circuits are at resonance. This condition can be realised if a device is tuned at a given gain for a minimum diode current, (see section 6.3) but if, as occurs in many practical circumstances, the gain of the device is changed by adjusting the pump power level, then resonant conditions are not maintained in the signal and idler circuits. If such an increase in pump power decreases the signal circuit resonant frequency by Δf_s , due to an increase in average junction capacitance, and decreases the idler resonant frequency by Δf_i , ($\Delta f_s \neq \Delta f_i$), then the new idler frequency, $f_p - (f_s - \Delta f_s)$, represents an increase and is $(\Delta f_s + \Delta f_i)$ away from the idler circuit resonant frequency. Equation 3.6 can be used to evaluate such detuned conditions, if the mean junction capacitance is known as a function of pump voltage. The diode capacitance variation factor γ can be defined by equation 2.9 and if the average junction capacitance is written:

$$C_{JA} = \frac{C_{jmax} + C_{jmin}}{2}$$

$$\text{then } C_{JA} = \frac{C_{jmin}}{(1 - 2\gamma)} \quad \dots\dots\dots 3.24$$

The minimum value of capacitance, C_{jmin} , can be taken as a constant for a wide range of pump voltage excursions, since most of the capacitance change occurs in the forward direction. Substitution of the values of γ and C_{JA} , determined at various pump levels for a CAY10 varactor, in equation 3.6 and subsequent evaluation of the frequency response from equation 3.4 gives a series of curves shown in Fig. 3.17. It is seen that the frequency of maximum gain is no longer the resonant frequency of the signal circuit, nor is it equal to the difference between the pump frequency and the idler circuit resonant frequency.

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FIG. 3.16 RESISTIVELY LOADED BALANCED IDLER CIRCUIT.

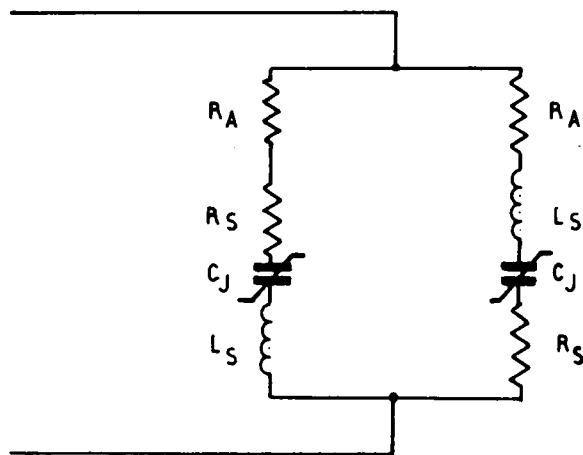
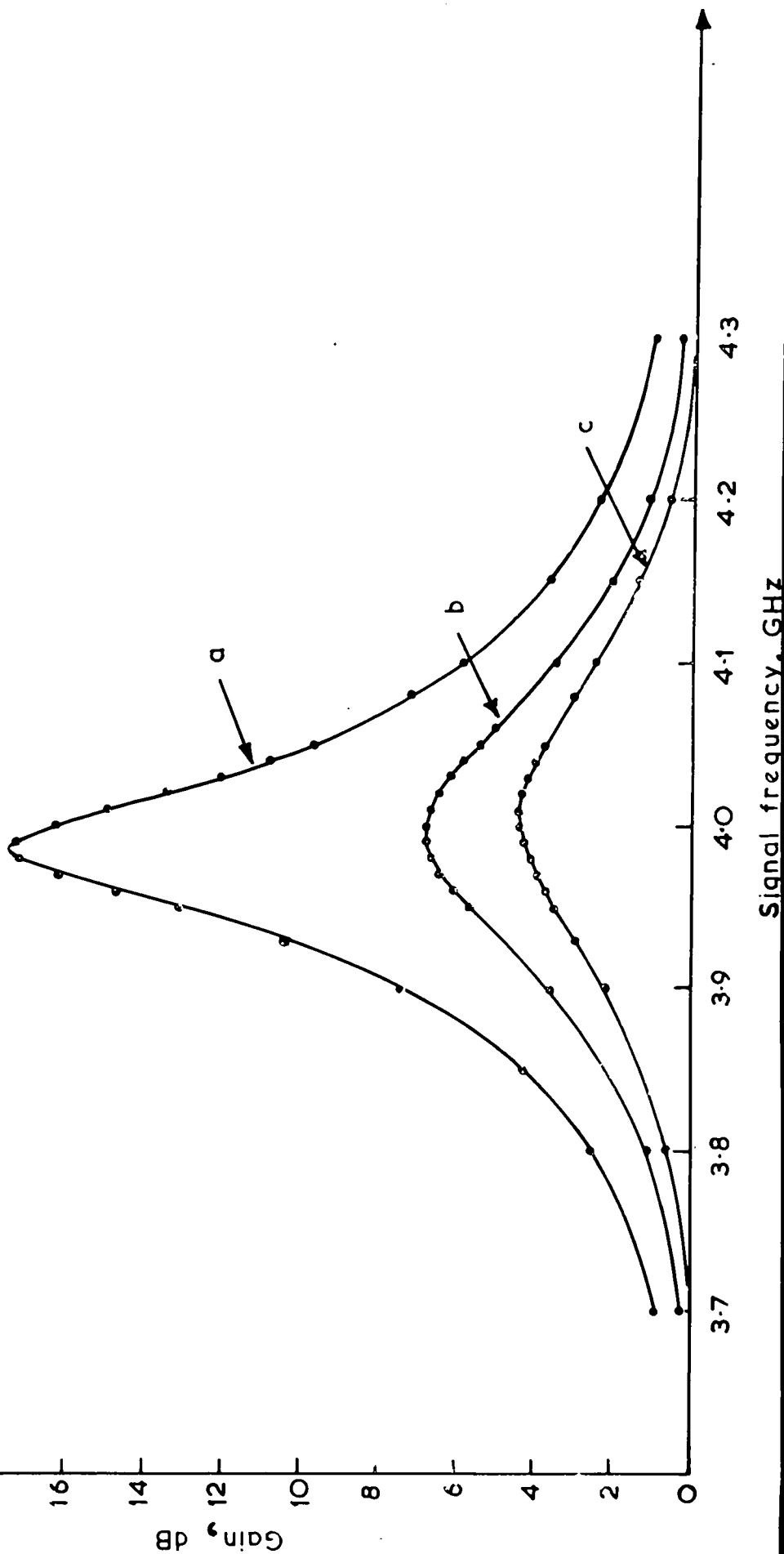


FIG. 3.17

▲ FIG. 3.17 SINGLE DIODE AMPLIFIER GAIN/FREQUENCY RESPONSE FOR A FIXED PUMP FREQUENCY AT VARIOUS PUMP LEVELS

curve	C_J (pF)	δ	f_{1res} (GHz)	f_{2res} (GHz)
a	0.47	0.157	3.9241	30.5462
b	0.46	0.120	3.9665	30.5961
c	0.45	0.105	4.0105	30.6204



3.7 Non-Ideal Amplifier Structures

3.7.1 General

It has been assumed that one of the diode self-resonances may be used as an idler circuit without the introduction of additional circuit elements that might degrade amplifier bandwidth. However, in a comparison of the bandwidth capabilities of the two amplifier types it is desirable to see how close to the ideal, operational devices may be. There are two principal ways in which the available bandwidth may differ from that predicted by the simple theory. Firstly, due to propagation of the idler frequency outside the diode and consequent inclusion of other reactive networks in the idler circuit, (enlarged idler circuit) and secondly due to stray reactance associated with the diode mount. The possibility of current at the upper sideband is also considered and the restrictions imposed by distributed circuits, (section 3.2) are evaluated for the two types of amplifier.

3.7.2 Enlarged Idler Circuit

The property of a perfectly symmetrical balanced diode amplifier in presenting an idler voltage zero across the pair of diodes, should ensure that idler current is confined to the diodes in this structure. However, slightly dissimilar diodes, dissimilarly mounted diodes, or dissimilarly pumped diodes, will encourage idler propagation into the diode vicinity. The single diode device has a finite idler voltage across the diode under all conditions and therefore will be more prone to the effects of idler propagation than a balanced amplifier.

Idler propagation in the signal or pump circuits must therefore be prevented by the use of suitable filters (see section 4.3), but care has to be taken that these circuits do not have steep reactance/frequency characteristics at the idler frequency. A convenient method of preventing idler propagation in the pump circuit is to use waveguide beyond cut-off to the idler frequency, but in either pump or signal circuits relatively simple techniques are available which should not significantly reduce idler bandwidth. (Low-pass coaxial filters in the signal circuit and a band-pass coupled cavity waveguide filter in the pump circuit are often used.)

The gain¹/₂ bandwidth of both single and balanced diode amplifiers should therefore not be significantly degraded by spurious idler propagation, but more care may have to be exercised in designing filters for the single diode device.

3.7.3 Diode Mount Strays

These are much more important for a balanced diode than a single diode device. It is relatively easy to mount a single diode and ensure that only a small capacitative stray is added to the idler circuit, which would be negligible (e.g. if $C_1 = 0.3$ pF) or advantageous, (e.g. if $C_1 = 0.05$ pF or less, see Figs. 3.6 and 3.8). However, it is impossible to mount two diodes in the balanced configuration without involving extra inductance or an additional resonant circuit. If the diodes are mounted as near together as possible, this will add stray inductance (possibly 0.05 to 0.2 nH), which will not be detrimental to amplifier performance if f_c and/or L_s are relatively low but will be, if these factors are high, (see Fig. 3.5 and 3.7). Another approach to the mounting of the two diodes (Aitchison et al 1967b) is to set them one half wavelength apart, on a transmission line, at the idler frequency. It is necessary to have a sufficiently low impedance line to ensure that the reactance slope of this does not significantly degrade idler bandwidth. This latter approach is probably more useful for diodes with a high cut-off frequency, when additional stray inductance would be most detrimental.

3.7.4 Upper Sideband Currents

If such currents are allowed to flow, $\text{gain}^{\frac{1}{2}} \cdot \text{bandwidth}$ is reduced. The low-pass filters included in the signal circuit and the band-pass filter included in the pump circuit of an amplifier, to prevent idler propagation, can be made to serve the additional purpose of upper sideband rejection.

It is convenient to couple pump power to a diode by a plunger one quarter wavelength away, and it must be considered whether or not this plunger can resonate at the upper sideband frequency for the signal and pump frequencies used. In any such amplifier structure, it is evident that the adjustment of this plunger will be critical, since an off-tune position would not only couple less pump power but may allow upper sideband propagation, which would be indicated by a high diode current for a relatively low amplifier gain, a condition that must be carefully avoided in tuning an amplifier. For this reason it may often be better to place the pump coupling plunger at 3 or 5 quarter-wavelengths from the diode, when upper sideband rejection may be better.

3.7.5 Restrictions imposed by Distributed Circuits

Use of a self-resonant idler configuration means that at most only one additional element need be used in the realisation of a single tuned parametric amplifier circuit, to form the additional inductance required to resonate the diodes to the signal frequency (e.g. L Fig. 2.10). (Unless the

stray inductance of the diode fulfils this function for a single diode amplifier in which circumstances the corresponding balanced diode amplifier would be degenerate.) Since the realisation of lumped elements at above 1 GHz is not yet within technological capabilities, such additional inductance has to be realised with distributed elements to the detriment of bandwidth, (see section 3.2). It is noted that for a given type of diode at a given signal frequency, operation in the balanced configuration involves half the additional inductance required for the shunt resonant configuration. This, in turn, means that restrictions imposed by distributed elements will be twice as severe for the single as the balanced diode amplifier.

The additional inductance, in the form of a coaxial line, is often mounted behind the diode (or diodes) in the signal circuit (e.g. Fig. 6.2), but it must be isolated from the idler circuit, particularly in the single diode amplifier. One way of achieving this is to make the line a quarter-wavelength long at the idler frequency, and to determine the line characteristic impedance by that value necessary to resonate the signal circuit. This impedance will usually be high ($\approx 50\Omega$) since the idler frequency may be 2 to 10 times the signal frequency, so the signal circuit bandwidth may not be significantly reduced (see Fig. 3.2). The quarter-wave line at the idler frequency which forms a parallel tuned circuit, should be made from as low an impedance line as possible for the least reactance slope, which requirement conflicts with that of a high impedance line for the signal circuit inductance. In many circumstances it may thus be better to include a low-pass filter or high frequency choke behind the diode to prevent idler and u.s.b. propagation, and then to add the necessary inductive line of as high an impedance as possible. As well as preventing restriction of the idler circuit bandwidth by the quarter-wave line, this arrangement allows a degree of signal circuit tuning by relaxing the requirement of maintaining a quarter-wave short circuit at the idler frequency. The highest impedance that can be used for the line forming the additional signal circuit inductance is usually defined by the diode cartridge size and the circulator coaxial line size.

The performance of a particular amplifier structure, involving distributed signal circuit inductance, may be assessed by increasing the signal circuit Q by the factor indicated in Fig. 3.2 for the particular value of Z_0 used, and by substituting this Q value in the $\text{gain}^{\frac{1}{2}} \cdot \text{bandwidth}$ equation, 3.15.

3.8 Broadbanding by Multituning

3.8.1 Passive Networks

This has been discussed in some detail by Humphreys (1964), who has evolved gain.bandwidth products in the form; $\text{gain}^{\frac{1}{2n}} \cdot \text{bandwidth} = \text{constant}$, where n is the number of resonant circuits in the circuit which is multituned, (i.e. the signal or idler circuit as the case may be). It has been shown that for single tuned circuits $\text{gain}^{\frac{1}{2}} \cdot \text{bandwidth}$ is a constant, provided that a few, practically realisable, conditions are fulfilled. However, the approximations, involved in the derivation of the higher order expression for multituned circuits, are poor for most practical devices, and in evaluating the theoretical performance of a device it is probably better to consider the driving point impedance function and to obtain the gain/frequency response under various circuit conditions. For a means of comparison between devices at a given gain, it is still useful, however, to specify a $\text{gain}^{\frac{1}{2n}} \cdot \text{bandwidth}$ product and for a double tuned signal circuit this may be written:

$$g^{\frac{1}{4}} \cdot b = \frac{k}{2} \left(\frac{1}{\sqrt{S}} + \sqrt{S} \right) \quad \dots\dots\dots 3.25$$

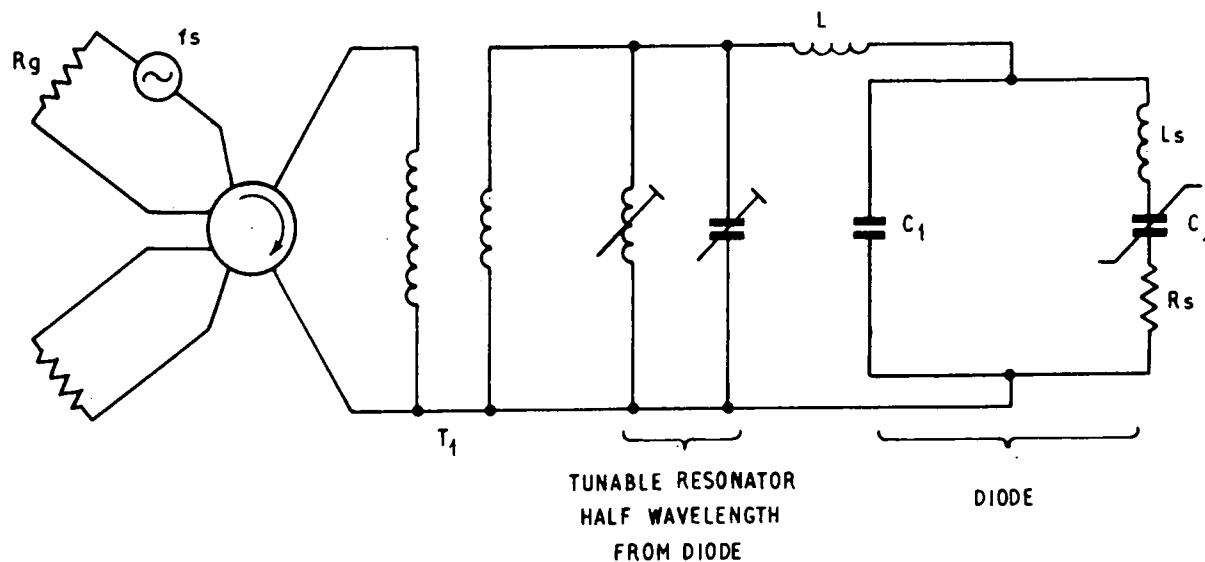
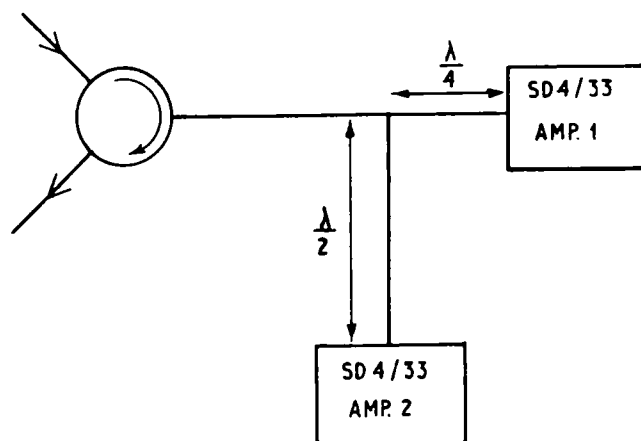
where k is the $\text{gain}^{\frac{1}{2}} \cdot \text{bandwidth}$ available with the basic single tuned amplifier, and $S = \frac{Q_1 f_2}{Q_2 f_1}$, which is often close to unity (from Humphreys 1964). Thus:

$$g^{\frac{1}{4}} \cdot b \text{ (for a double tuned device)} = g^{\frac{1}{2}} \cdot b \text{ (for a single tuned device)} = k.$$

The technique of broadbanding by signal circuit multituning involves the equalisation of the reactance slope of a single tuned device over a frequency band. For a single additional resonator the equation:

$$\left(\frac{dX}{df} \right)_{\text{resonator}} = \left(\frac{dX}{df} \right)_{\text{single tuned amplifier}} \quad \dots\dots\dots 3.26$$

completely defines the parameters of the required resonant circuit. Such a circuit is presented in anti-resonant form to the amplifier (i.e. placed in parallel to the signal line, the diode itself forming a series circuit) so that the reactance slopes of the two circuits are equal and opposite and compensation is achieved over a frequency band. Fig. 3.18 gives the equivalent circuit of such a device.

FIG. 3-18 EQUIVALENT CIRCUIT OF SD4/33 AMPLIFIER
WITH ONE ADDITIONAL RESONATORFIG. 3-19 SCHEMATIC ARRANGEMENT OF SIGNAL CIRCUIT
ACTIVELY COMPENSATED DEVICE

Further stages of signal circuit tuning may be added with diminishing advantage, since a point is reached when the idler circuit limits the overall amplifier bandwidth. It is therefore not usual to add more than two or possibly three stages of tuning in the signal circuit. The difficulties of constructing tunable resonant circuits at high idler frequencies discourage idler circuit multituning.

Any passive tuned circuit at microwave frequencies must usually be constructed with distributed elements and if an amplifier is to be cooled, the change in the diode junction capacitance will require that these elements be adjustable in order to restore matching conditions. Such a requirement can make coolable passively multituned amplifiers prohibitively complex.

3.8.2 Active Networks

The idea of using one or more additional pumped diodes to produce multituned signal or idler circuits, arose from consideration of quarter-wave coupled travelling wave devices (e.g. Bobisch and Sondhauss 1965). Such a circuit was built in February 1966 and found to give a greater bandwidth than an equivalent passively broadbanded device (see section 8.3) but required two pump sources and suffered from intermodulation between these. Because of these difficulties with intermodulation this approach was temporarily abandoned. Later, when the shortcomings of passive multituning at liquid helium temperature became apparent, the idea was taken up again (in April 1967).

It is evident that any passive distributed network, of the type required for multituning a single tuned amplifier, could be replaced, to advantage, by an unpumped diode resonated at the signal frequency with an additional inductance. Such a diode would need to be in parallel to the signal circuit, one quarter of a wavelength away from the pumped diode, when reactance compensation would be achieved over a frequency band. Furthermore, if the additional diode is itself pumped, then compensation would also be achieved for variation in negative resistance over the frequency band. With careful selection of diodes it should be possible to use the same pump source for both diodes and hence avoid the possibility of intermodulation.

Cooling such a device should only shift the band centre frequency and not involve circuit readjustments, so a simple coolable structure should be available.

The schematic arrangement of this device is shown in Fig. 3.19. If the input impedances of the two amplifier halves are $R_1 + jX_1$ and $R_2 + jX_2$, then the total input impedance is given by:

$$\frac{1}{Z_{\text{tot}}} = \frac{1}{R_1 + jX_1} + \frac{R_2 + jX_2}{Z_0^2}$$

where Z_0 is the impedance of the quarter-wave line separating the two devices. Rationalisation gives:

$$Z_{\text{tot}} = \frac{Z_0^2 R_1 + R_2 (R_1^2 + X_1^2)}{(Z_0^2 + 2R_1 R_2 - 2X_1 X_2)^2 + \frac{(R_1^2 + X_1^2)(R_2^2 + X_2^2)}{Z_0^2}} + j \frac{[Z_0^2 X_1 - X_2 (R_1^2 + X_1^2)]}{(Z_0^2 + 2R_1 R_2 - 2X_1 X_2)^2 + \frac{(R_1^2 + X_1^2)(R_2^2 + X_2^2)}{Z_0^2}} \quad \dots\dots\dots 3.27$$

and if $R_1 = R_2 = R$ and $X_1 = X_2 = X$ (identical diodes and pumping conditions) then:

$$Z_{\text{tot}} = R' + jX' = \frac{(Z_0^2 R Z_0^2 + R^2 + X^2)}{(Z_0^2 + R^2 - X^2)^2 + 4X^2 R^2} + j \frac{Z_0^2 X (Z_0^2 - X^2 - R^2)}{(Z_0^2 + R^2 - X^2)^2 + 4X^2 R^2} \quad \dots\dots\dots 3.28$$

The gain/frequency response of such an arrangement, using two single diode shunt resonant devices, has been calculated for various values of Z_0 , by using equation 3.28 in the gain equation 3.14. At a particular gain and generator impedance two optimum or near optimum tuning conditions are obtained for two values of Z_0 and the relevant impedance function and gain/frequency responses for 10 dB peak gain are shown in Figs. 3.20-3.23. Impedance functions and gain/frequency responses have also been evaluated with only one half of the amplifier pumped and these are presented in Fig. 3.24 and 3.25.

Fig. 3.20 Gain/Frequency Response of Two Diode Amplifier

$Z_o = 18\Omega$
Generator/load impedance (R_g) = 12Ω

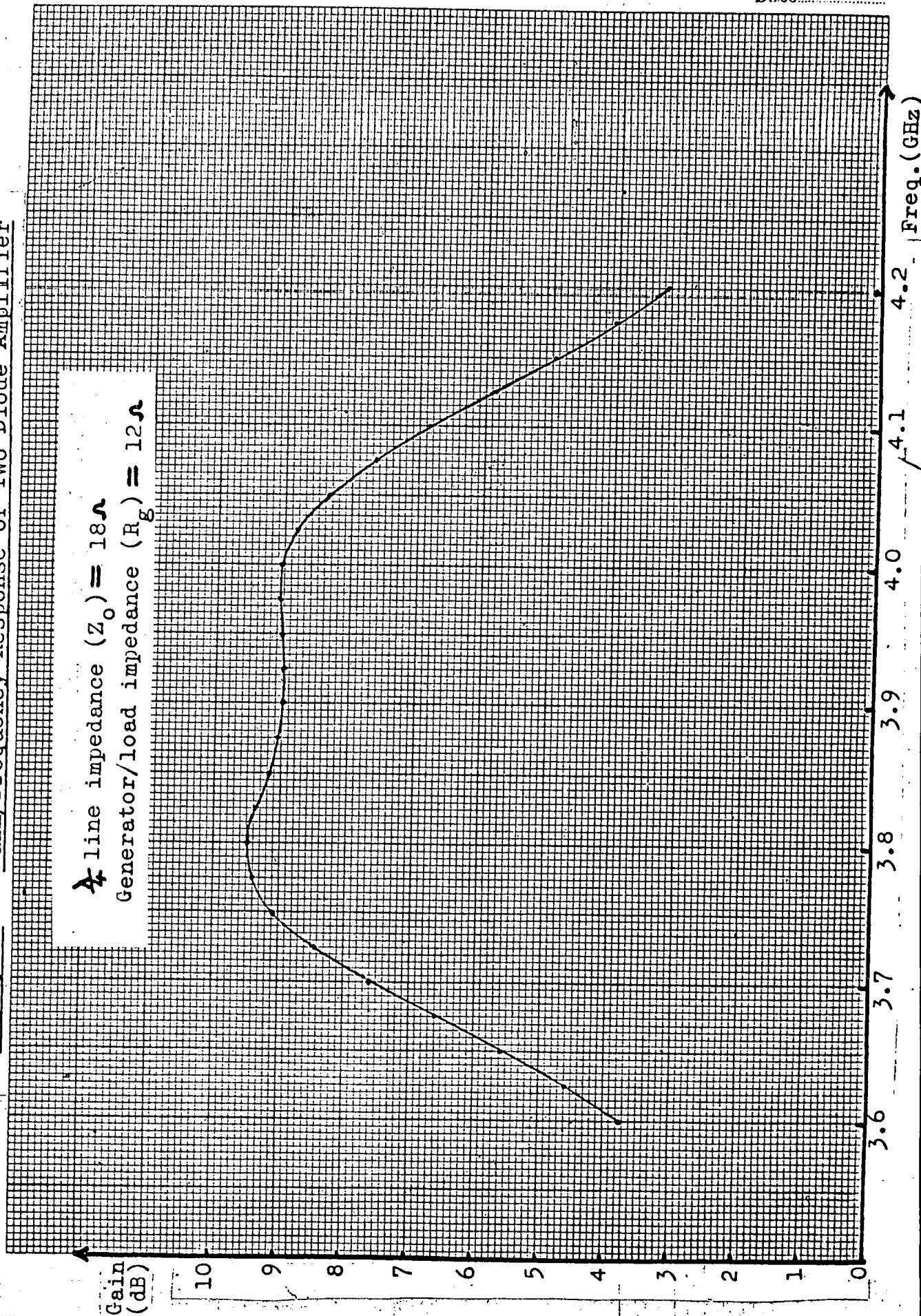


Fig. 3.21a

Two Diode Amplifier Impedance Function

Case No. _____

Initials _____
Date _____

Negative Resistance

Z_{line} impedance (Z_o) = 18Ω
negative resistance values have been
transformed from 12Ω (R_g) to 50Ω

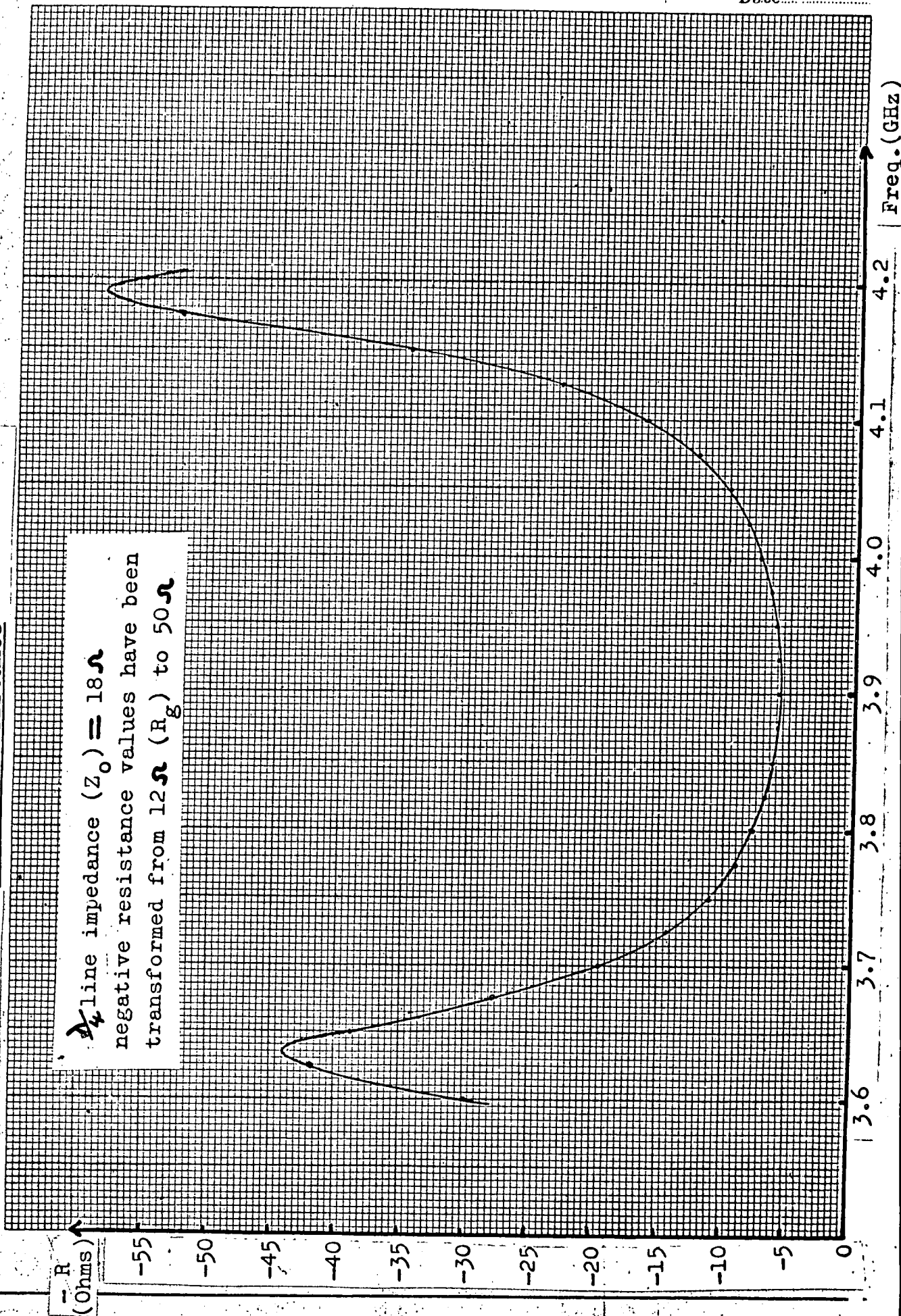


Fig. 3.21b Two Diode Amplifier Impedance Function

Reactance
(Ohms)

Reactance

$\frac{A}{4}$ line impedance (Z_o) = 18Ω
Reactance values have been
transformed from 12Ω (R_g) to 50Ω

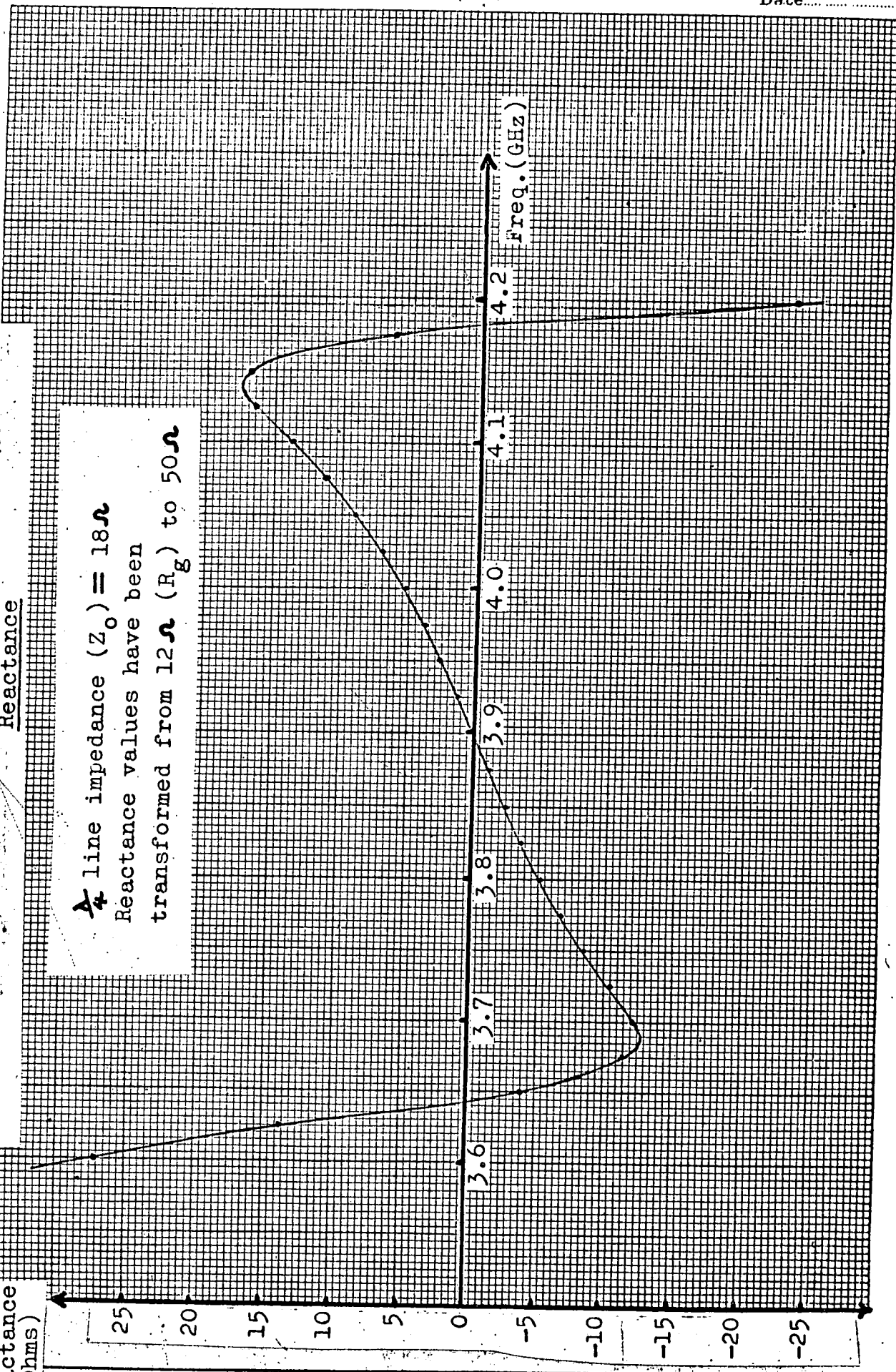


Fig. 3.22 Gain/Frequency Response of Two Diode Amplifier

$Z_0 = 8\Omega$
Generator/load impedance (R_g) = 12Ω

Gain
(dB)

10

9

8

7

6

5

4

3

2

1

0

3.6

3.7

3.8

3.9

4.0

4.1

4.2

Freq. (GHz)

Initials.....
Date.....

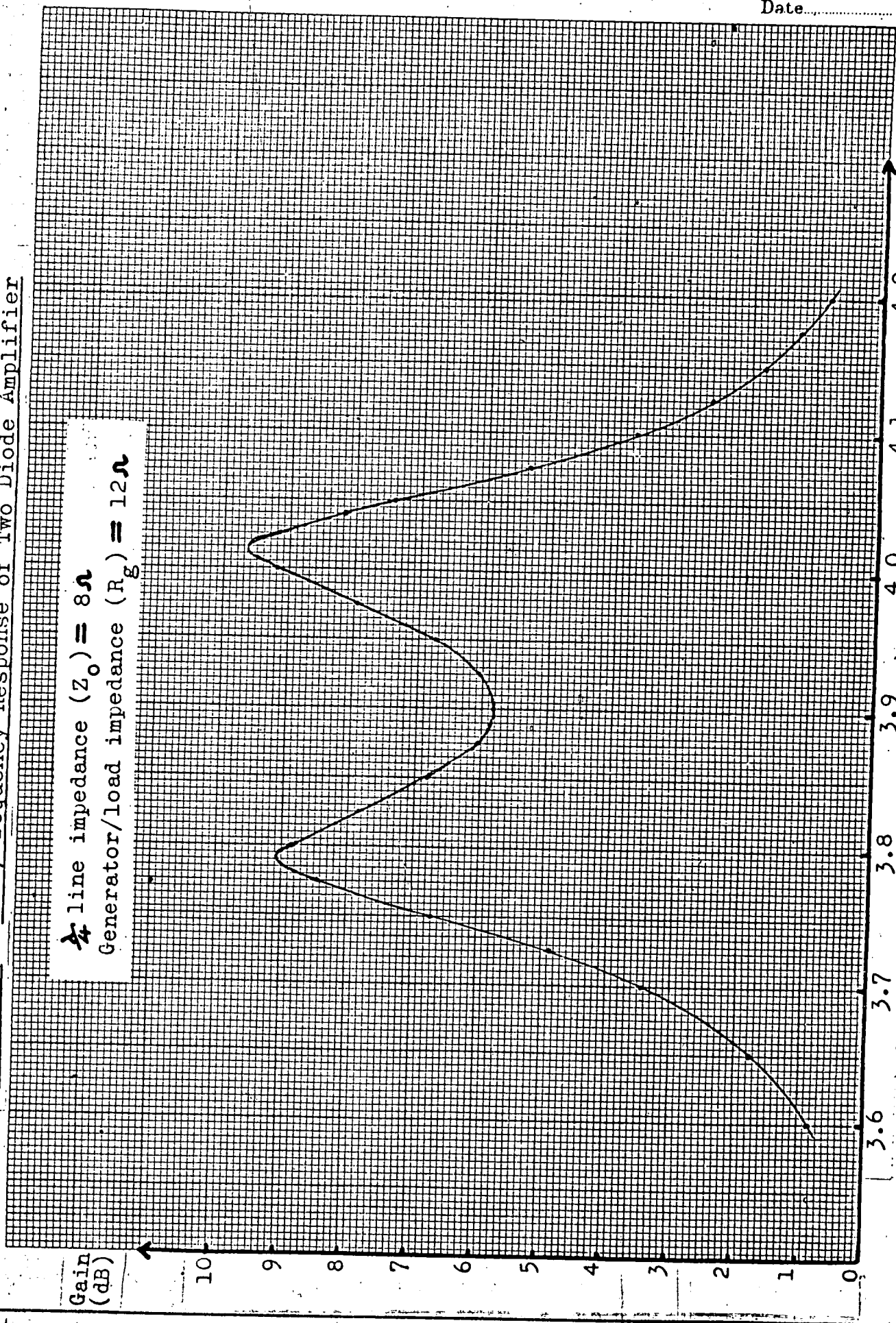


Fig. 3.23a Two Diode Amplifier Impedance Function

Negative Resistance

$\frac{1}{4}$ line impedance (Z_0) = 8Ω

Neg. res. values have been transformed from a 12Ω line (R_g) to a 50Ω line.

- R
(Ohms)

-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0

3.6

3.7

3.8

3.9

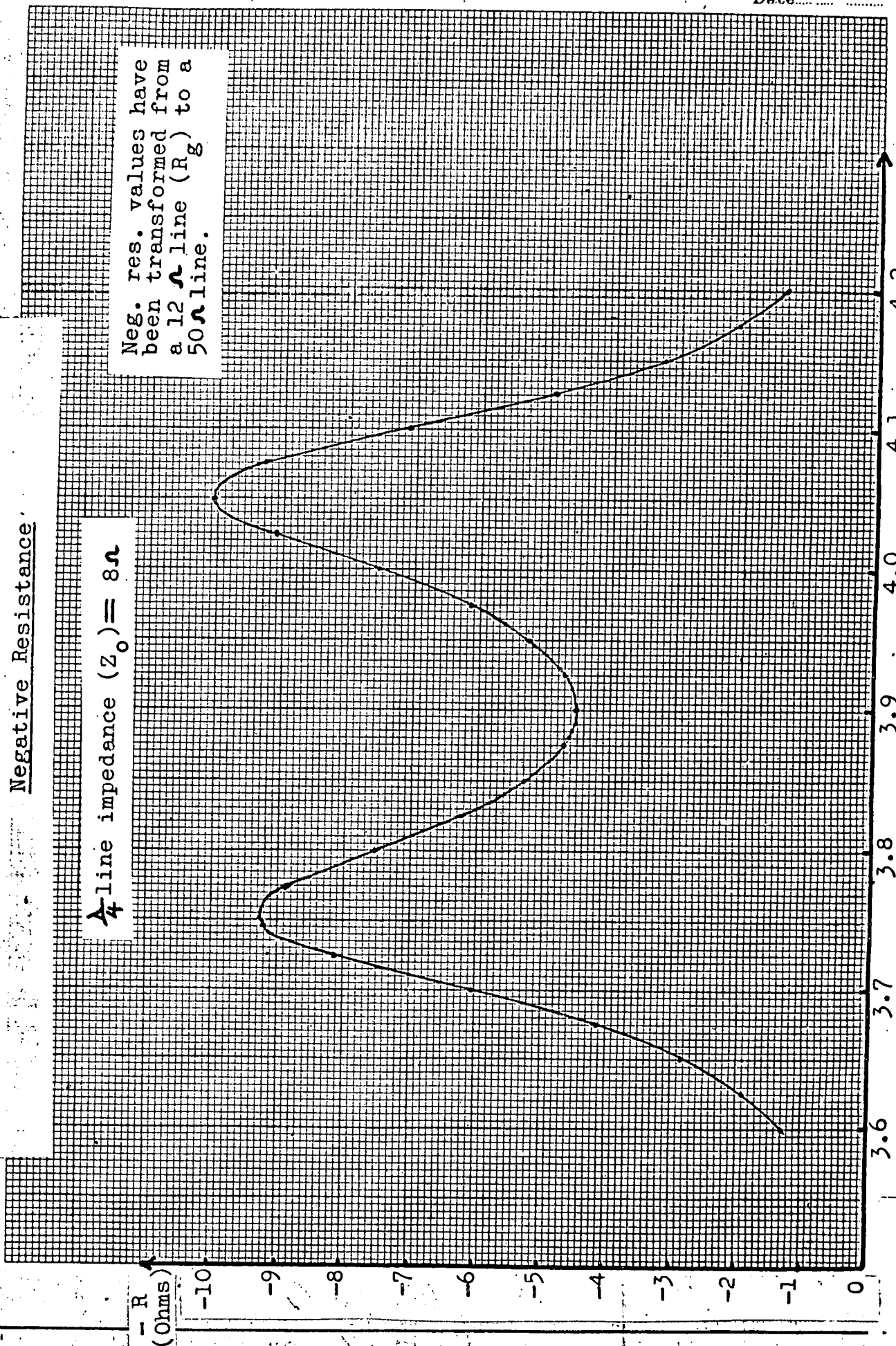
4.0

4.1

4.2

Freq. (GHz)

Initials.....
Date.....



Reactance

Reactance
(Ohms)

$\frac{A}{4}$ line impedance (Z_0) = 8Ω

Reactance values have been transformed from a 12Ω line (R_g) to a 50Ω line.

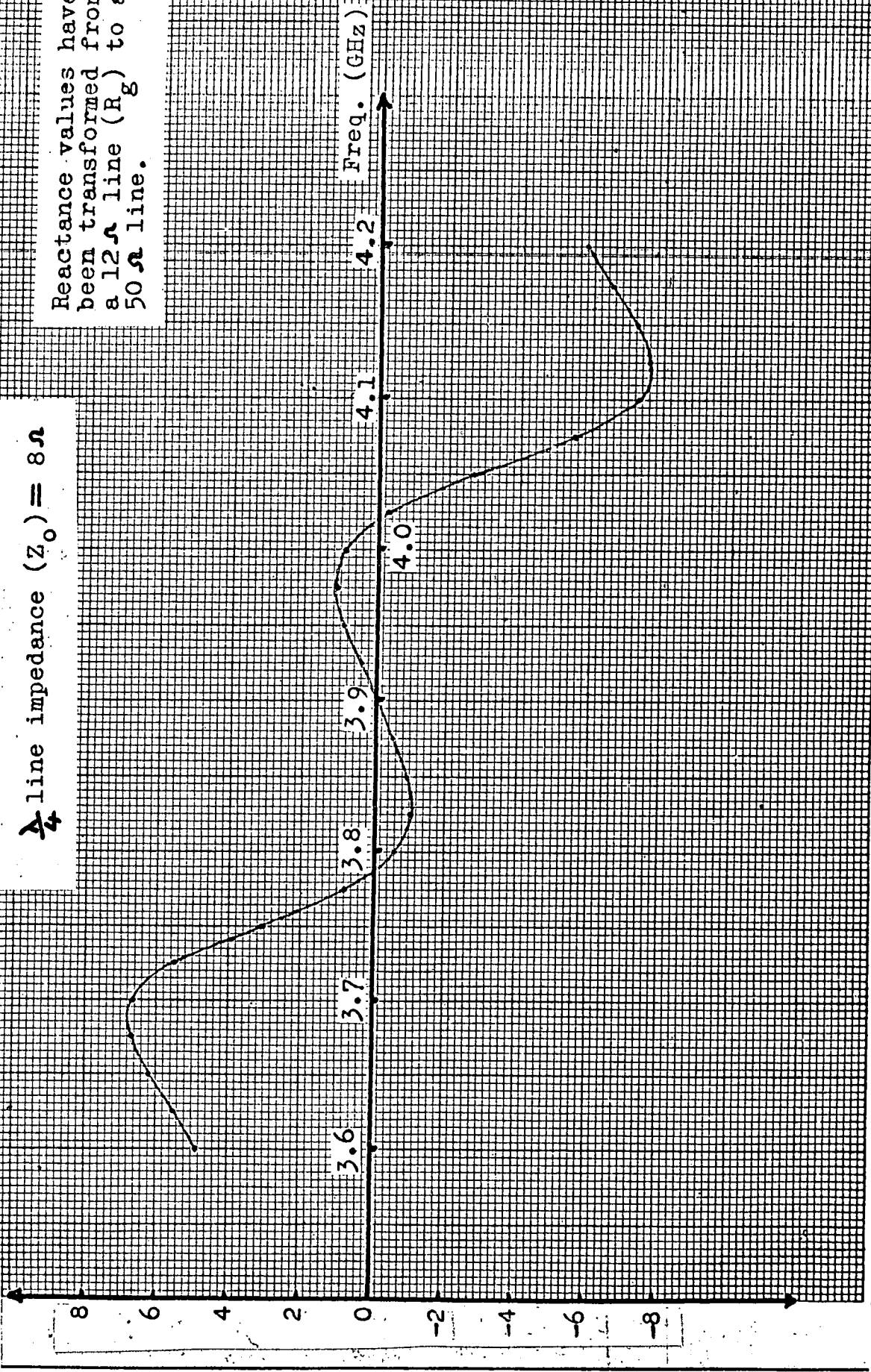


Fig. 3.24

Gain/Frequency of Two Diode Amplifier with One Diode Unpumped

$$\frac{\lambda}{4} \text{ line impedance } (Z_0) = 12\Omega$$

Maximally flat 400 MHz, 10 dB response produced with both diodes pumped.

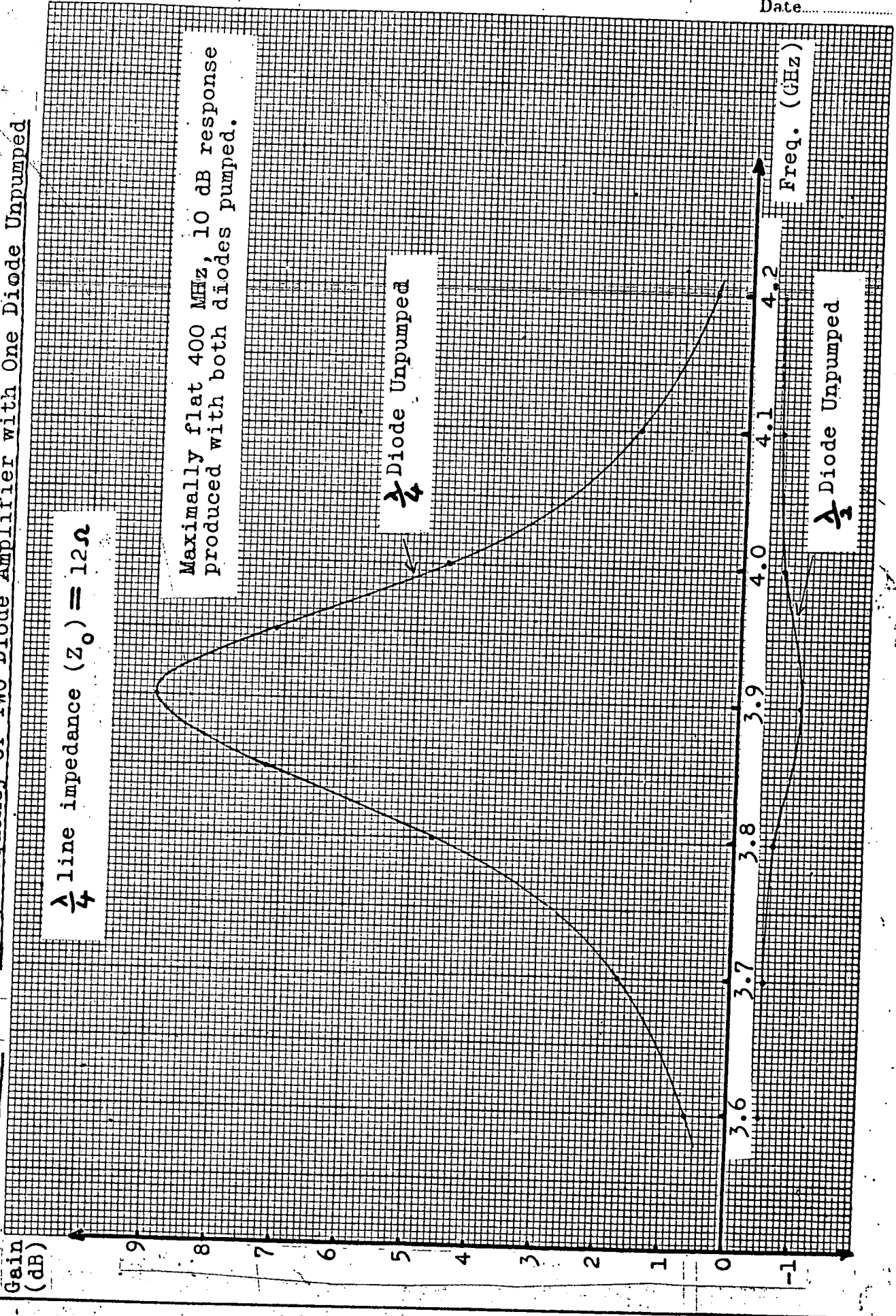
$\frac{\lambda}{4}$ Diode Unpumped

$\frac{\lambda}{2}$ Diode Unpumped

Gain (dB)

Freq. (GHz)

Initials
Date



Impedance Function of Two Diode Amplifier with $\frac{1}{4}$ Diode Unpumped

Case No.

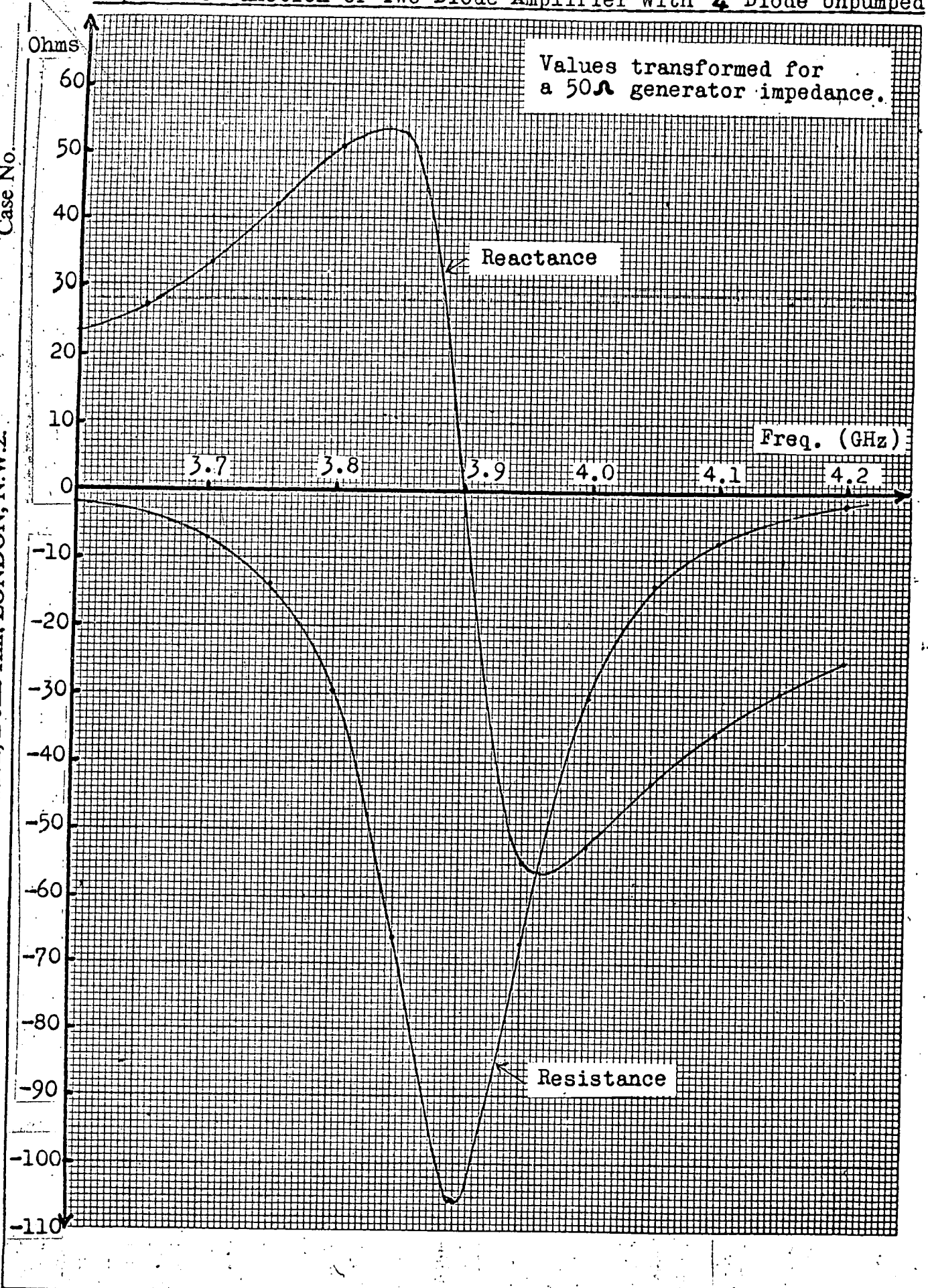
P.O. Engineering Department Research Station, Dollis Hill, LONDON, N.W.2.

Values transformed for
a 50Ω generator impedance.

Reactance

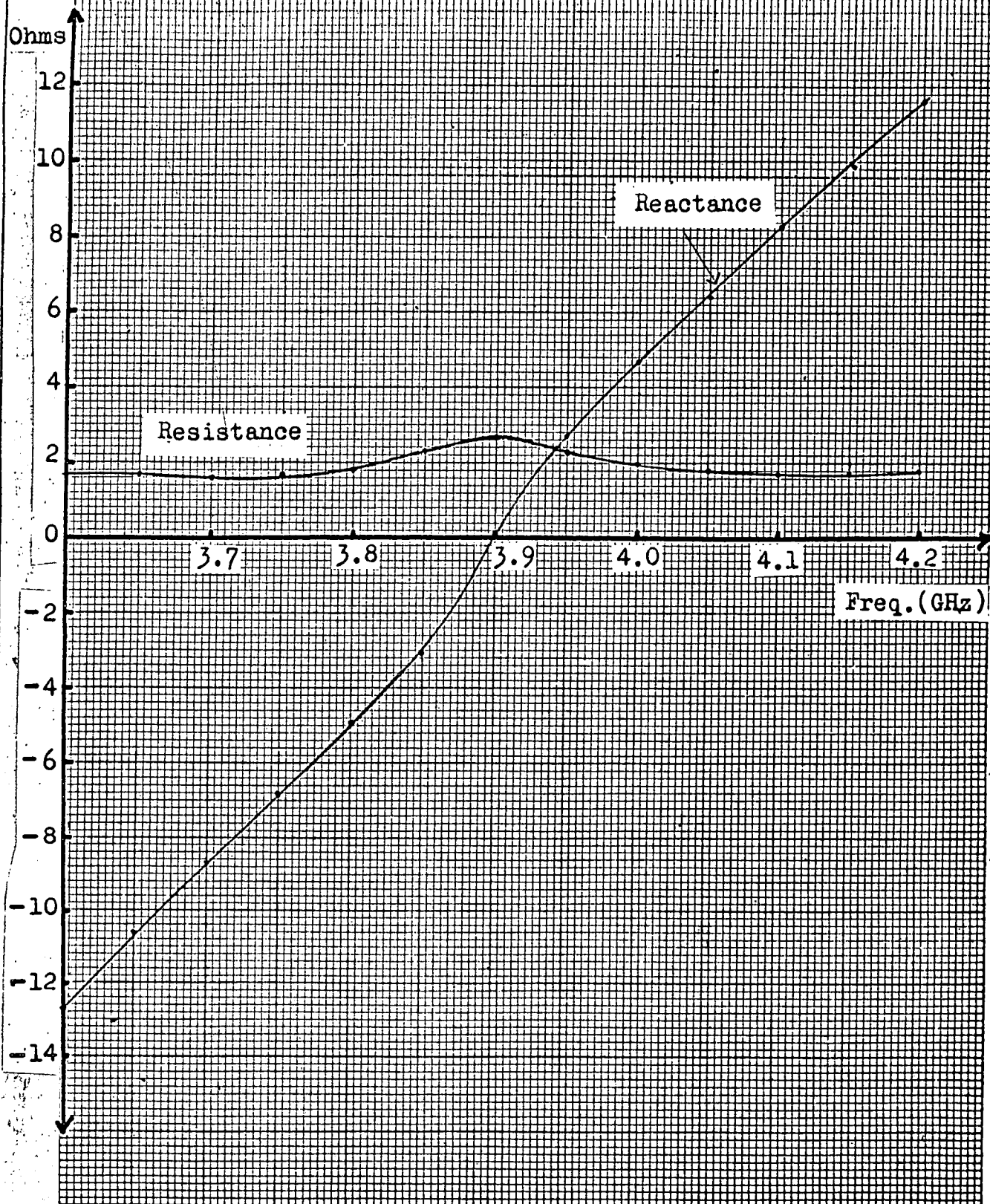
Freq. (GHz)

Resistance



Impedance Function of Two Diode Amplifier with $\frac{1}{2}$ Diode Unpumped

Values transformed for
a $50\ \Omega$ generator impedance



The conditions for optimum bandwidth can be obtained from the complete gain expression (i.e. equation 3.11 with $R' + jX'$ for $R + jX$), but the assumption, that the second term in equation 3.12 is much larger than the first, may not be valid. So if a gain^{1/4}.bandwidth product can be written it will be of the form:

$$g_o^{1/4} \cdot b = \frac{2 \cdot 2^{1/4}}{P} \left(|A|^2 + |B|^2 \right)^{1/4} \dots\dots\dots 3.29$$

where P is given by Equation 3.10, and:

A equals:

$$\frac{R_g^2 \left[\left(Z_o^2 + R^2 \right) - X^2 + 4X^2 R^2 \right]^2 - \left[Z_o^2 R \left(Z_o^2 + R^2 + X^2 \right) \right]^2 - \left[Z_o^2 X \left(Z_o^2 - R^2 - X^2 \right) \right]^2}{\left[Z_o^2 \left(Z_o^2 - X^2 - R^2 \right) \right]^2 + \left\{ \frac{R_g}{X} \left[\left(Z_o^2 + R^2 - X^2 \right)^2 + 4X^2 R^2 \right] + Z_o^2 \frac{R}{X} \left(Z_o^2 + R^2 + X^2 \right) \right\}^2}$$

and B is:

$$\frac{2R_g Z_o^2 X \left[Z_o^2 - X^2 - R^2 \right] \left[\left(Z_o^2 + R^2 - X^2 \right)^2 + 4X^2 R^2 \right]}{\left[Z_o^2 \left(Z_o^2 - X^2 - R^2 \right) \right]^2 + \left\{ \frac{R_g}{X} \left[\left(Z_o^2 + R^2 - X^2 \right)^2 + 4X^2 R^2 \right] + Z_o^2 \frac{R}{X} \left(Z_o^2 + R^2 + X^2 \right) \right\}^2}$$

Now, for a single diode amplifier:

$$g_o^{1/2} \cdot b = \frac{2\sqrt{2}R_g}{P} = k$$

$$\text{so } \frac{g_o^{1/4} \cdot b_{(t.d.)}}{g_o^{1/2} \cdot b_{(s.d.)}} = \frac{\left(A^2 + B^2 \right)^{1/4}}{2^{1/4} \cdot R_g} = k' \dots\dots\dots 3.30$$

$$\text{i.e. } g_o^{1/4} \cdot b_{(t.d.)} = k k'$$

Equation 3.30 has been evaluated for the combinations of values of R and X allowed from the single frequency response of the single diode shunt resonant amplifier (section 3.2) and for various values of Z_o . Fig. 3.26 shows $|k'|$ plotted against Z_o for 10 dB band-centre gain (similar curves can be obtained for other values of gain). However it is seen later that the required

Fig. 3.26

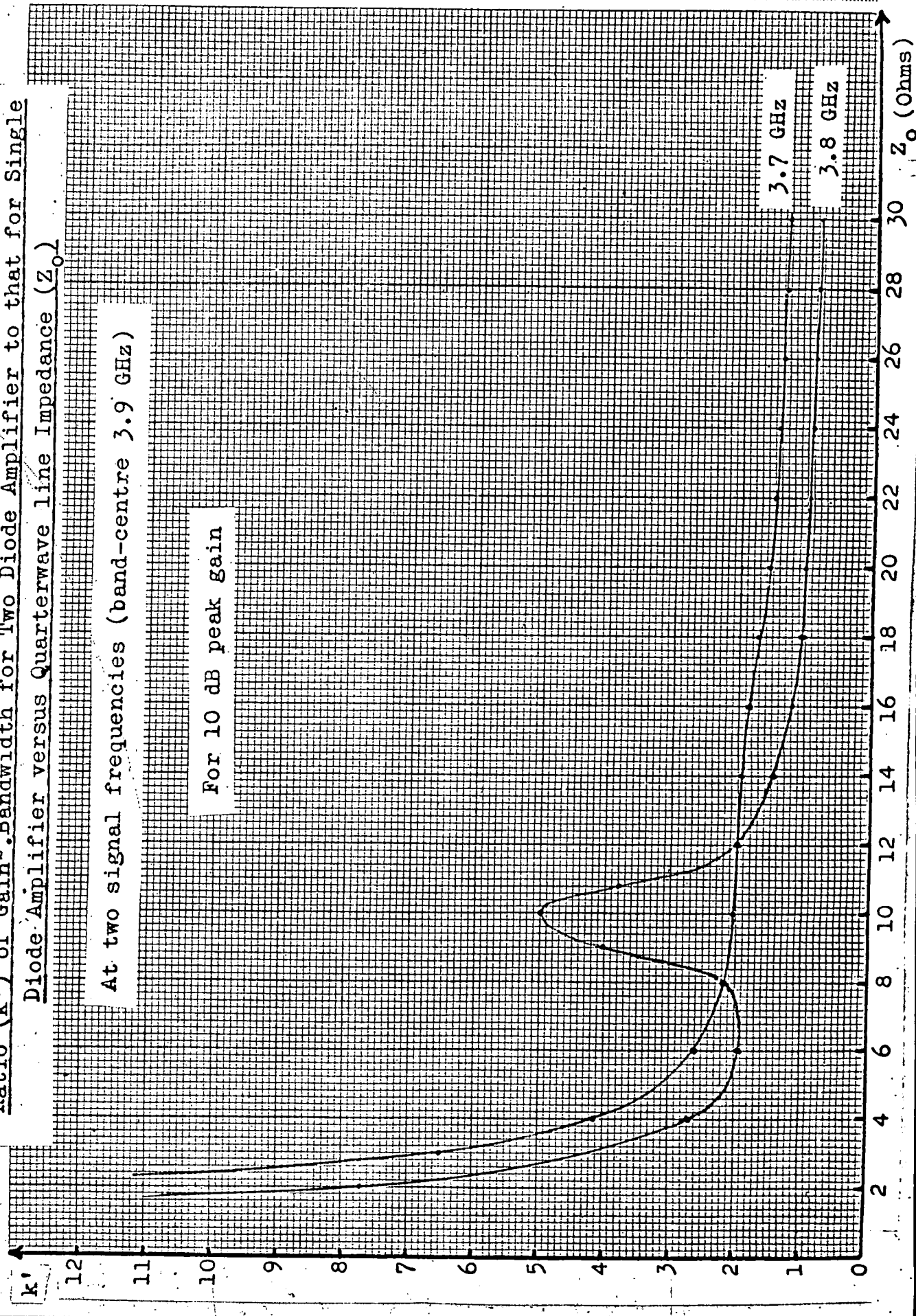
Ratio (k') of Gain ^{$\frac{1}{2}$} Bandwidth for Two Diode Amplifier to that for Single Diode Amplifier versus Quarterwave line Impedance (Z_0)

At two signal frequencies (band-centre 3.9 GHz)

For 10 dB peak gain

3.7 GHz

3.8 GHz



value of Z_0 is defined by the band-centre gain and in-band ripple. The generator/load impedance is determined by the required band centre gain and Z_0 , viz:

$$G_0 = \left\{ \frac{\left[\frac{R_0}{1 + \frac{R_0^2}{Z_0^2}} \right]^2 - R_g}{\left[\frac{R_0}{1 + \frac{R_0^2}{Z_0^2}} \right]^2 + R_g} \right\}^2 \quad \dots 3.31$$

where R_0 is the band-centre negative resistance of a single amplifier. Thus the generator/load impedance may be written:

$$R_g = \left[\frac{R_0}{1 + \frac{R_0^2}{Z_0^2}} \right] K \quad \dots 3.32$$

where K depends on the band-centre gain according to equation 2.9. R_g is plotted against band-centre gain for various values of Z_0 in Fig. 3.27.

The condition of maximum gain^{1/4} bandwidth from equation 3.29 is not sufficient to determine the optimum value of Z_0 . The amplifier gain/frequency slope must usually be zero, or less than some specified value, within the pass band. That is:

$$\frac{d(\text{gain})}{d\Delta f} \text{ or } \frac{dZ_{\text{tot}}}{d\Delta f} \rightarrow 0 \quad (\Delta f = f - f_0)$$

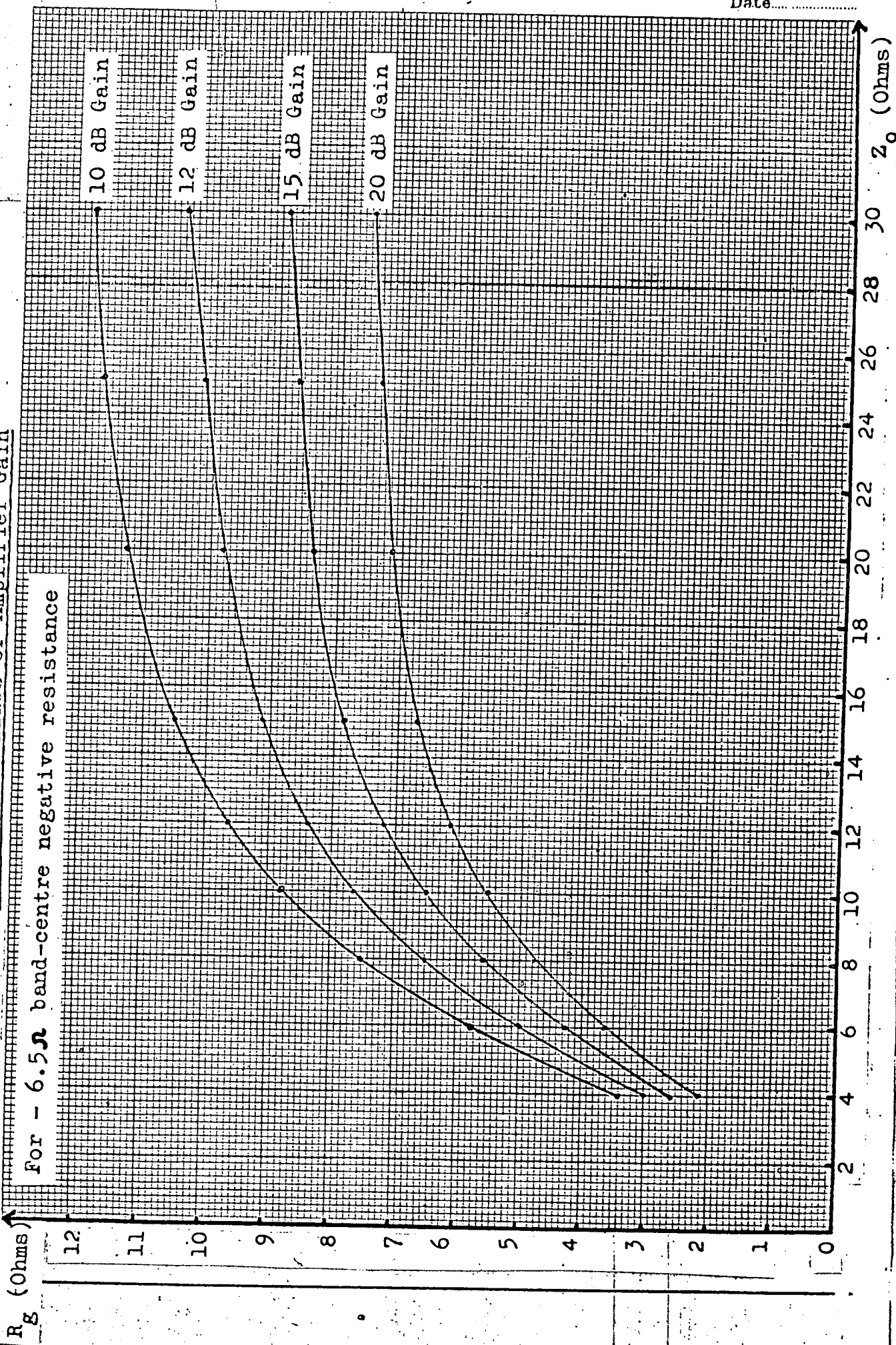
$$\text{Now: } \frac{d|Z_{\text{tot}}|^2}{d\Delta f} = 2X' \frac{dX'}{d\Delta f} + 2R' \frac{dR'}{d\Delta f}$$

and $V^2 \frac{dX'}{d\Delta f}$ equals;

Fig. 3.27

Generator Impedance (R_g) versus Quarterwave Line Impedance (Z_0)
at various Values of Amplifier Gain

Initials.....
Date.....



$$\left[\frac{dX}{d\Delta f} (Z_0^4 - Z_0^2 R^2 - 3Z_0^2 X^2) - 2Z_0^2 R X \frac{dR}{d\Delta f} \right] \left[Z_0^4 + X^4 + R^4 + 2X^2 R^2 - 2Z_0^2 X^2 + 2Z_0^2 R^2 \right]$$

$$- \left[Z_0^2 X (Z_0^2 - R^2 - X^2) \right] \left[\frac{dX}{d\Delta f} (4X^3 + 4R^2 X - 4Z_0^2 X) + \frac{dR}{d\Delta f} (4R^3 + 4RX^2 + 4Z_0^2 R) \right]$$

where $V = (Z_0^2 + R^2 - X^2)^2 + 4X^2 R^2$

also $V^2 \frac{dR}{d\Delta f}$ is;

$$\left[\frac{dR}{d\Delta f} (Z_0^4 + 3Z_0^2 R^2 + Z_0^2 X^2) + 2Z_0^2 R X \frac{dX}{d\Delta f} \right] \left[Z_0^4 + X^4 + R^4 + 2X^2 R^2 - 2Z_0^2 X^2 + 2Z_0^2 R^2 \right]$$

$$- \left[Z_0^2 R (Z_0^2 + R^2 + X^2) \right] \left[\frac{dX}{d\Delta f} (4X^3 + 4R^2 X - 4Z_0^2 X) + \frac{dR}{d\Delta f} (4R^3 + 4RX^2 + 4Z_0^2 R) \right]$$

Now; $\frac{dX}{d\Delta f} \gg \frac{dR}{d\Delta f}$ over a wide band (>10%) for many

single tuned amplifiers, so;

$$V^2 \frac{dX}{d\Delta f} = \frac{dX}{d\Delta f} \left\{ \left[Z_0^4 - Z_0^2 R^2 - 2Z_0^2 R X \right] \left[Z_0^4 + X^4 + R^4 + 2X^2 R^2 - 2Z_0^2 X^2 + 2Z_0^2 R^2 \right] \right.$$

$$\left. - \left[Z_0^2 X (Z_0^2 - R^2 - X^2) \right] \left[4X^3 + 4R^2 X - 4Z_0^2 X \right] \right\}$$

and;

$$V^2 \frac{dR}{d\Delta f} = \frac{dR}{d\Delta f} \left\{ 2Z_0^2 R X \left[Z_0^4 + X^4 + R^4 + 2X^2 R^2 - 2Z_0^2 X^2 + 2Z_0^2 R^2 \right] \right.$$

$$\left. - \left[Z_0^2 R (Z_0^2 + R^2 + X^2) \right] \left[4X^3 + 4R^2 X - 4Z_0^2 X \right] \right\}$$

Thus $\frac{d|Z_{tot}|^2}{d\Delta f}$ equals;

$$\begin{aligned} & \frac{2Z_0^2}{V^3} \cdot \frac{dX}{d\Delta f} \left\{ X(Z_0^2 - R^2 - X^2) (Z_0^4 - Z_0^2 R^2 - 2Z_0^2 R X) (Z_0^4 + X^4 + R^4 + 2X^2 R^2 - 2Z_0^2 X^2 \right. \\ & \qquad \qquad \qquad \left. + 2Z_0^2 R^2) \right. \\ & - \left[4X^3 + 4R^2 X - 4Z_0^2 X \right] \left[Z_0^2 R^2 (Z_0^2 + R^2 + X^2)^2 + Z_0^2 X^2 (Z_0^2 - R^2 - X^2)^2 \right] \\ & \left. + 2Z_0^2 R^2 X (Z_0^2 + R^2 + X^2) (Z_0^4 + X^4 + R^4 + 2X^2 R^2 - 2Z_0^2 X^2 + 2Z_0^2 R^2) \right\} \end{aligned}$$

If; $\frac{d|Z_{tot}|^2}{d\Delta f} = 0$, then, since $\frac{2Z_0^2}{V^3} \cdot \frac{dX}{d\Delta f} \neq 0$

$$\begin{aligned} & \left[Z_0^2 (4X^3 + 4R^2 X - 4Z_0^2 X) \right] \left[-X^2 (Z_0^2 - R^2 - X^2)^2 - R^2 (Z_0^2 + R^2 + X^2)^2 \right] \\ & + \left[Z_0^4 + R^4 + X^4 + 2X^2 R^2 - 2Z_0^2 X^2 + 2Z_0^2 R^2 \right] \left[X(Z_0^2 - R^2 - X^2) (Z_0^4 - Z_0^2 R^2 - 2Z_0^2 R X) \right. \\ & \qquad \qquad \qquad \left. + 2R X Z_0^2 (Z_0^2 + R^2 + X^2) \right] = 0 \end{aligned}$$

Thus, either;

$$Z_0^4 + R^4 + X^4 + 2X^2 R^2 - 2Z_0^2 X^2 + 2Z_0^2 R^2 = 0$$

$$\text{i.e. } (Z_0^2 - X^2 + R^2)^2 + 4X^2 R^2 = 0 \text{ which has}$$

no real roots, (but has a minimum value when $Z_0^2 = X^2 - R^2$), or;

$$Z_0^4 + Z_0^2 (3X^2 + 2R^2 - 2RX + 2R)$$

$$+ (-4X^4 - 7R^2 X^2 - 3R^4 + 2R^3 X + 2RX^3 + 2R^3 + 2RX^2) = 0 \quad \dots\dots 3.33$$

Equation 3.33 has been evaluated for the values of R and X allowed, at a particular frequency, for the single tuned shunt resonant amplifier response (fig. 3.4). Values of Z_0 obtained in this way are plotted against bandwidth in fig. 3.28. Any point on this curve gives the required value of Z_0 for $\frac{d(\text{gain})}{d\Delta f}$ to be zero at that particular frequency. As Z_0 is increased, the bandwidth is seen to increase as the frequency of maximum gain (when $\frac{d(\text{gain})}{d\Delta f} = 0$ if $g \neq 0$) moves from the band-centre. The maximum available bandwidth and the optimum value of Z_0 will thus be specified by the allowable ripple. The quarter wave line impedance, Z_0 , is related to the band-centre gain and generator/load impedance by equation 3.31. Thus if the values of peak gain at the frequencies implied from fig. 3.28 are plotted against Z_0 for a given band centre gain, the optimum values of Z_0 , for a particular in-band ripple, are evident. This has been done for 10, 12, 15 and 20 dB band centre gain and the results are shown in fig. 3.29. The dashed straight lines apply for the 12 dB curve only and show the required Z_0 values, for specified values of ripple, at their intersection with the curve. Similar lines can be drawn for the other curves and the optimum values of Z_0 , obtained in this way for a specified ripple at a given band centre gain, are summarised in table 3.1. Two possible values of Z_0 are apparent for a given gain and ripple but the larger of the two implies a much greater bandwidth (fig. 3.28). This is also demonstrated by the amplifier gain/frequency response curves (figs. 3.17 and 3.19) and by the $\text{gain}^{\frac{1}{4}}$. bandwidth curves (fig. 3.26).

Table 3.1

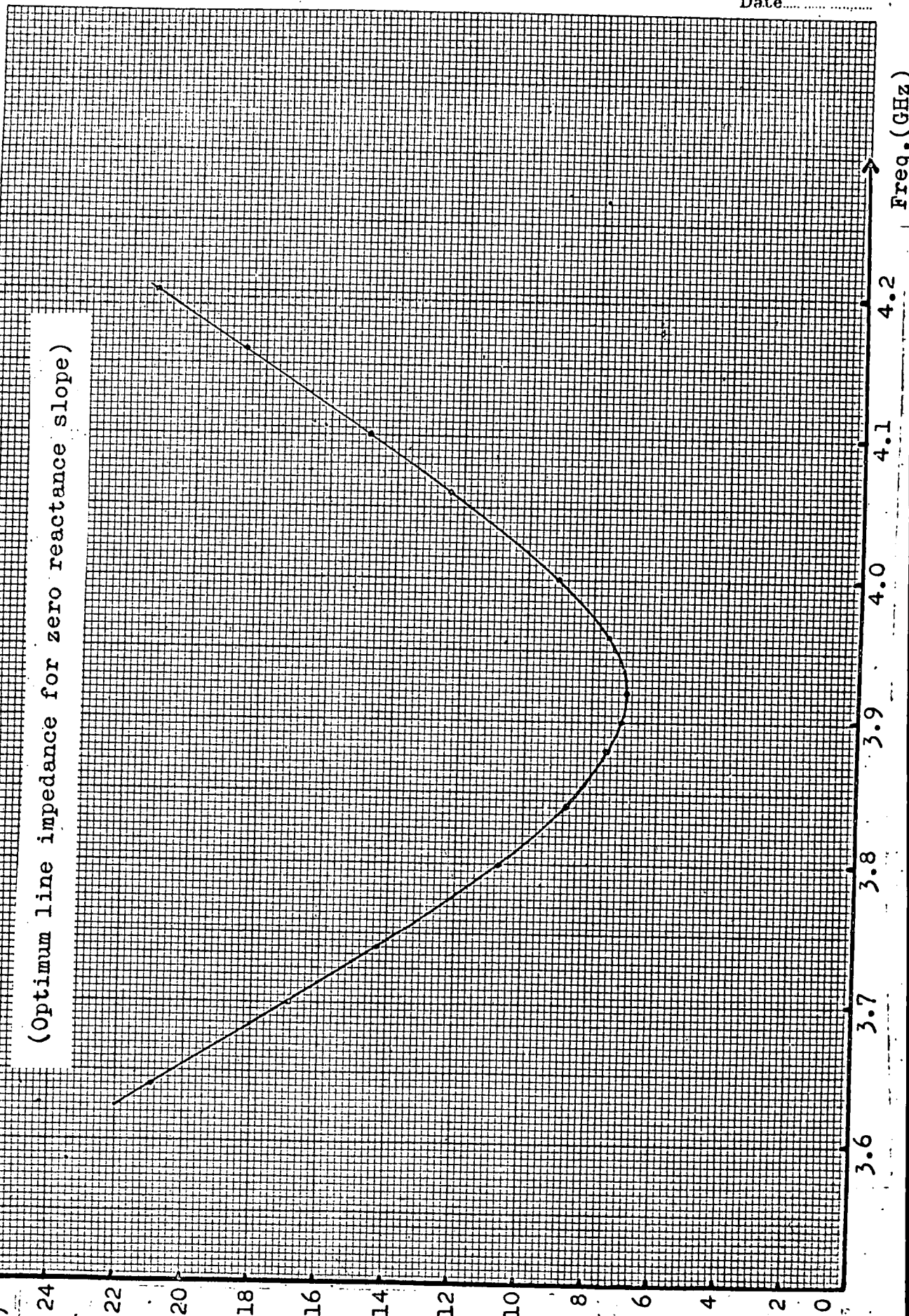
Values of Quarter Wave Line Impedance (Z_0) Ω
for a given Band-Centre Gain and In-Band Ripple

Ripple (dB)	Band-Centre Gain (dB)			
	10	12	15	20
3	8.55 13.4	8.3 12.0	7.95 10.5	7.7 9.05
1	7.7 14.15	7.65 12.6	7.55 10.9	7.38 9.25
0	7.0 14.65	7.0 12.9	7.0 11.15	7.0 9.35

Optimum Quarterwave Line Impedance (Z_0) for Two Diode Amplifier versus Signal Frequency

Optimum Z_0
(Ohms)

(Optimum line impedance for zero reactance slope)



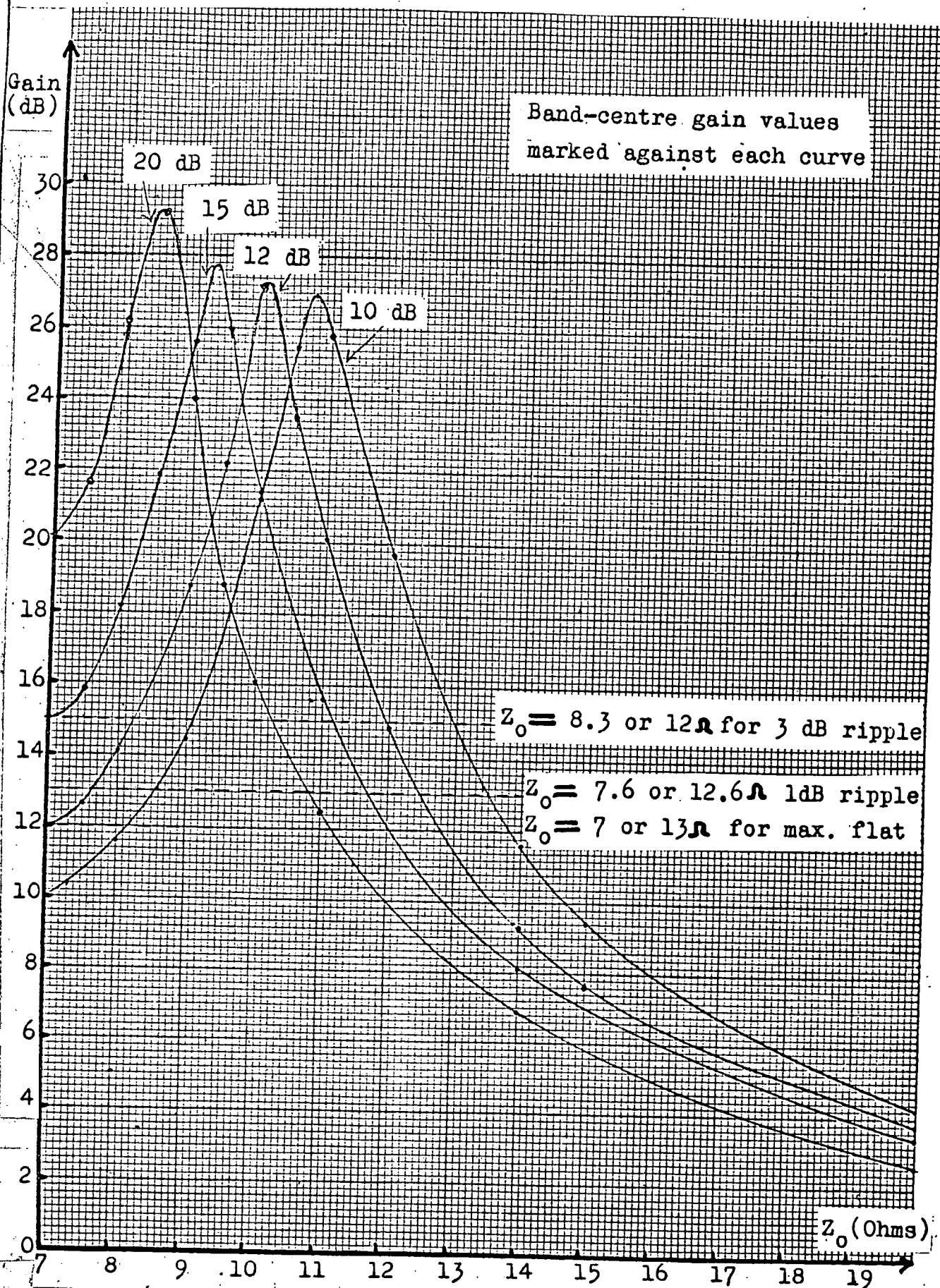
Initials.....
Date.....

Freq. (GHz)

Peak Gain versus Quarterwave Line Impedance (Z_0) for various values of Band-Centre Gain

P.O. Engineering Department Research Station, Dollis Hill, LONDON, N.W.2.

Case No.



When the $\text{gain}^{\frac{1}{4}}$. bandwidth product is evaluated for the allowed values of Z_0 (that is values of table 3.1 applied to fig. 3.26 for the frequencies implied from fig. 3.28), it is seen that, for the higher values of Z_0 , almost exactly twice the $\text{gain}^{\frac{1}{2}}$. bandwidth of the single tuned device is available in all cases, (or twice the $\text{gain}^{\frac{1}{4}}$. bandwidth of a passively double tuned device). It is noted that the peak in the $\text{gain}^{\frac{1}{4}}$. bandwidth versus Z_0 curve (fig. 3.26) does not occur with the value of Z_0 required for $\frac{d(\text{gain})}{df} = 0$, since this peak occurs when $Z_0 = X^2 - R^2$, which is not a root of equation 3.3.

An amplifier with active signal circuit compensation will thus provide twice the $\text{gain}^{\frac{1}{4}}$. bandwidth of a comparable passively compensated device, but the quarter wave line connecting the two single tuned amplifiers, must be of an impedance specified by the required band centre gain and in-band ripple.

3.9 Conclusions

There are many practical advantages in using one of the two diode self-resonances for the idler circuit of an amplifier, in the form of the balanced diode (two series resonant diodes) or the single diode shunt resonant amplifier. Of these two the balanced diode amplifier gives a greater $\text{gain}^{\frac{1}{2}}$. bandwidth product in all practical circumstances, which in many cases is greater by two or three times. However, the single diode amplifier is simpler to construct, has a lower input noise temperature and will operate at higher signal frequencies than the balanced diode amplifier, for the same diodes.

For wide bandwidths from a balanced diode amplifier the diodes used should possess high γ and f_c values with low L_s and C_1 values, but if the diodes have a significantly large series stray inductance, then a high cut-off frequency may not be to advantage. For a single diode amplifier there is an optimum value of encapsulation capacitance, for maximum bandwidth, defined by the diode cut-off frequency, γ value and series inductance.

In general, for high quality diodes (high γf_c compared with the signal and idler frequencies) the balanced diode structure is favoured when a wide bandwidth is required, but for narrow band applications and at high signal frequencies the single diode shunt resonant device is better.

Both balanced and single diode structures may be multituned to advantage, although more is gained, per stage of multituning, by broadbanding the shunt resonant device, since both structures have the same idler bandwidth. For a single diode amplifier one additional passive circuit, gives, to a good approximation, a $\text{gain}^{\frac{1}{4}}$. bandwidth equal to the $\text{gain}^{\frac{1}{2}}$. bandwidth product of the basic single tuned circuit, whereas one additional active circuit gives twice this value. Active compensation is thus a useful technique, particularly at low temperatures when the need for external tuning controls can be eliminated.

4. Measurement Techniques

4.1 Measuring bench for frequency response

Two systems were used; in the early measurements the output of the parametric amplifier was fed into a microwave mixer and detected at I.F., whereas in the later measurements a backward diode detector was available which, used in conjunction with an electronic chopper, allowed direct detection of the amplifier output. The two systems are summarised in figs. 4.1 and 4.2. Measurements of amplifier gain at any signal frequency could be read directly from the measuring attenuator A1, by operating switches S1 and S2, (which divert pump power from the amplifier and by-pass the amplifier), provided that the loss of the by-pass loop has been measured.

The signal frequency klystron could be changed for a swept frequency source and the output of the backward diode detector (without chopper) connected directly to a sensitive oscilloscope, which facilitated the display of a swept frequency response. When thus connected care has to be taken to avoid gain saturation which occurs at inputs of greater than about -45 dBm.

4.2 Noise measuring equipment

Two systems were used for noise measurements; a few early measurements on room temperature amplifiers were carried out with the use of a noise lamp, but the majority of measurements were taken on a specially developed noise-adding Y-radiometer*. The noise lamp circuit (fig. 4.3) was connected to input C of switch S2 (fig. 4.1) in place of the 4 GHz bench and either detecting systems was used with a microwave measuring attenuator inserted between the amplifier output and the detecting system. On/Off noise ratios for restored detector readings were read off directly from the measuring attenuator in the usual manner.

The major disadvantage of the noise lamp method is the uncertainty in the lamp calibration which leads to a possible systematic error of $\pm 10^\circ\text{K}$. A Y radiometer method, involving two loads at different temperatures, does not require calibration but suffers from the possibility of gain changes in the high gain amplifiers after the parametric amplifier whilst the hot/cold noise ratio is being measured. This has led to the development of a noise adding radiometer in which noise from a lamp is added to the input of the amplifier and the chopped noise output of the amplifier is compared with the chopped output from the noise lamp in a synchronous detector. In this way a null can be obtained when the two noise powers are equal. An increase in the amplifier load temperature now upsets the balance of the system which can be restored by introducing more noise from the lamp. If the same null position is observed at each reading then the

* See acknowledgements.

FIG. 4.1 MEASURING BENCH FOR FREQUENCY RESPONSE.

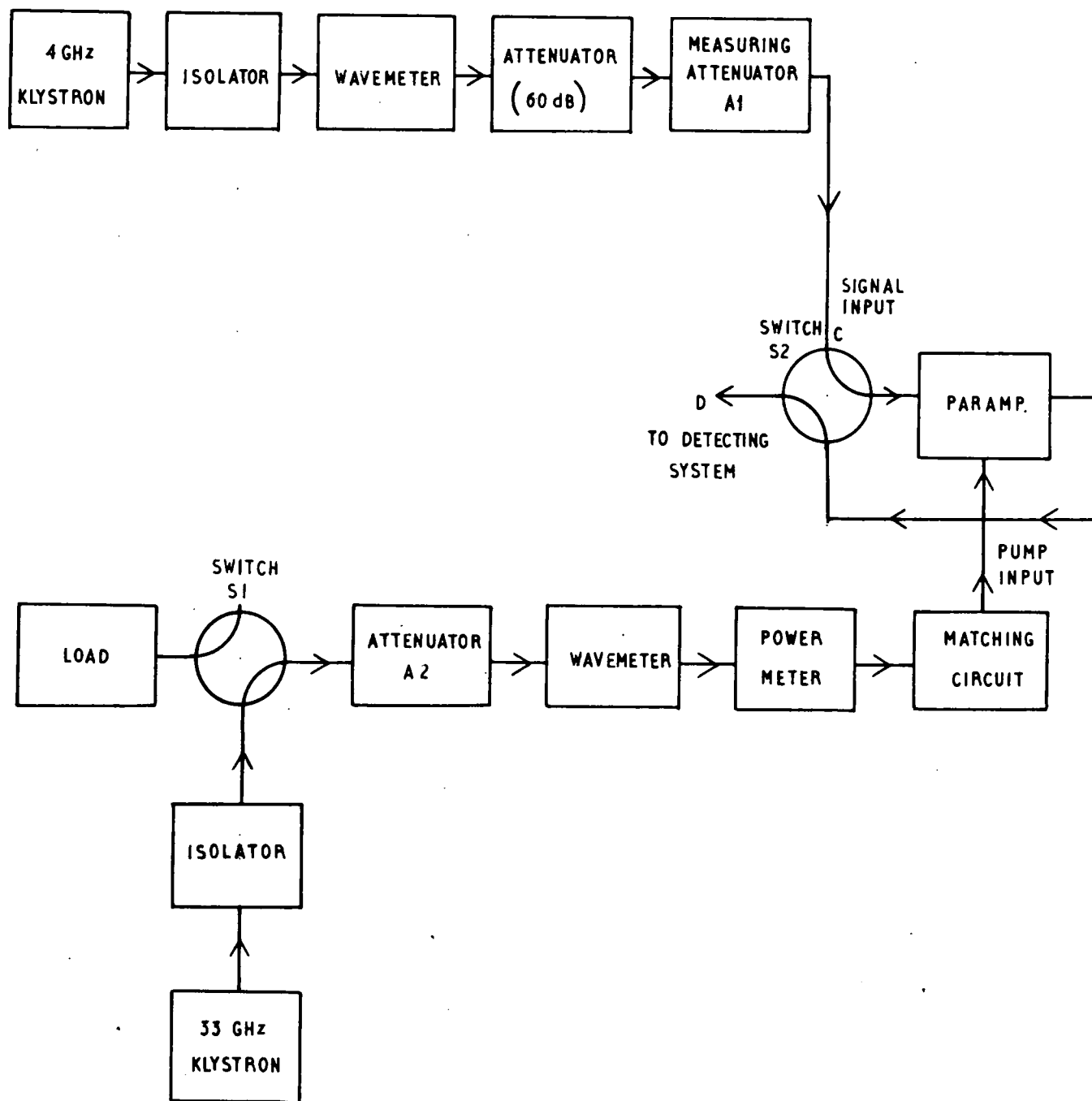
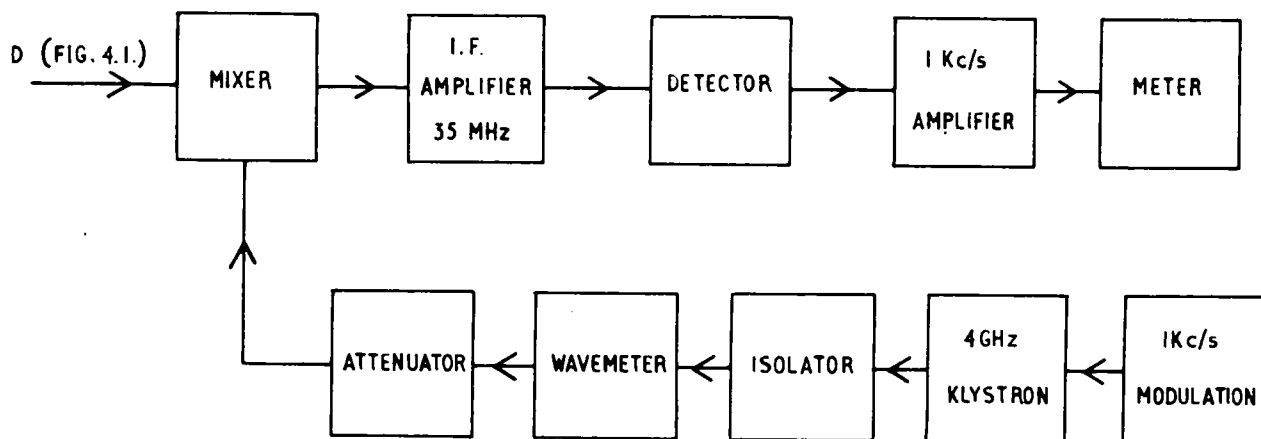


FIG.4.2 DETECTING SYSTEMS

SYSTEM 1



SYSTEM 2

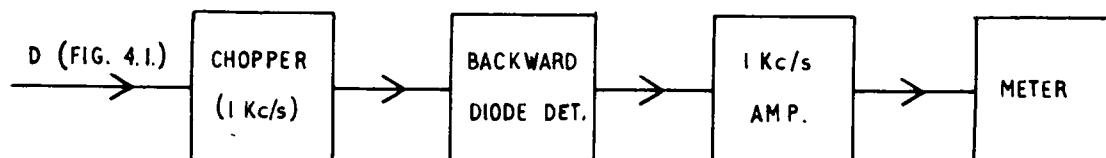
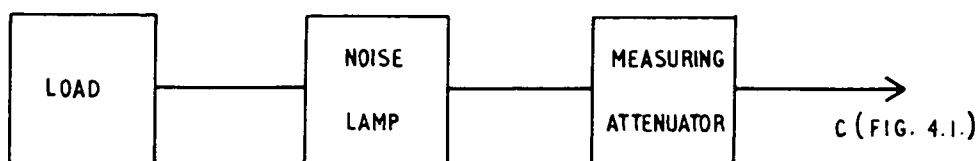


FIG.4.3 NOISE LAMP CIRCUIT



parametric amplifier gain (as well as the total system gain) has been constant. Thus the noise lamp is used as a stable noise reference whilst changing from cold to hot loads. A schematic diagram of the radiometer system is given in fig. 4.4.

The equation for amplifier noise temperature at the amplifier input terminals can be shown to be:

$$T_A = \frac{T_H - RT_c}{R - 1} + \frac{T_0}{g} \quad \dots\dots\dots(5.1)$$

where T_H , T_c are the hot and cold load temperatures

T_0 is the ambient temperature

R is the hot/cold load attenuation ratio

and g is the amplifier gain.

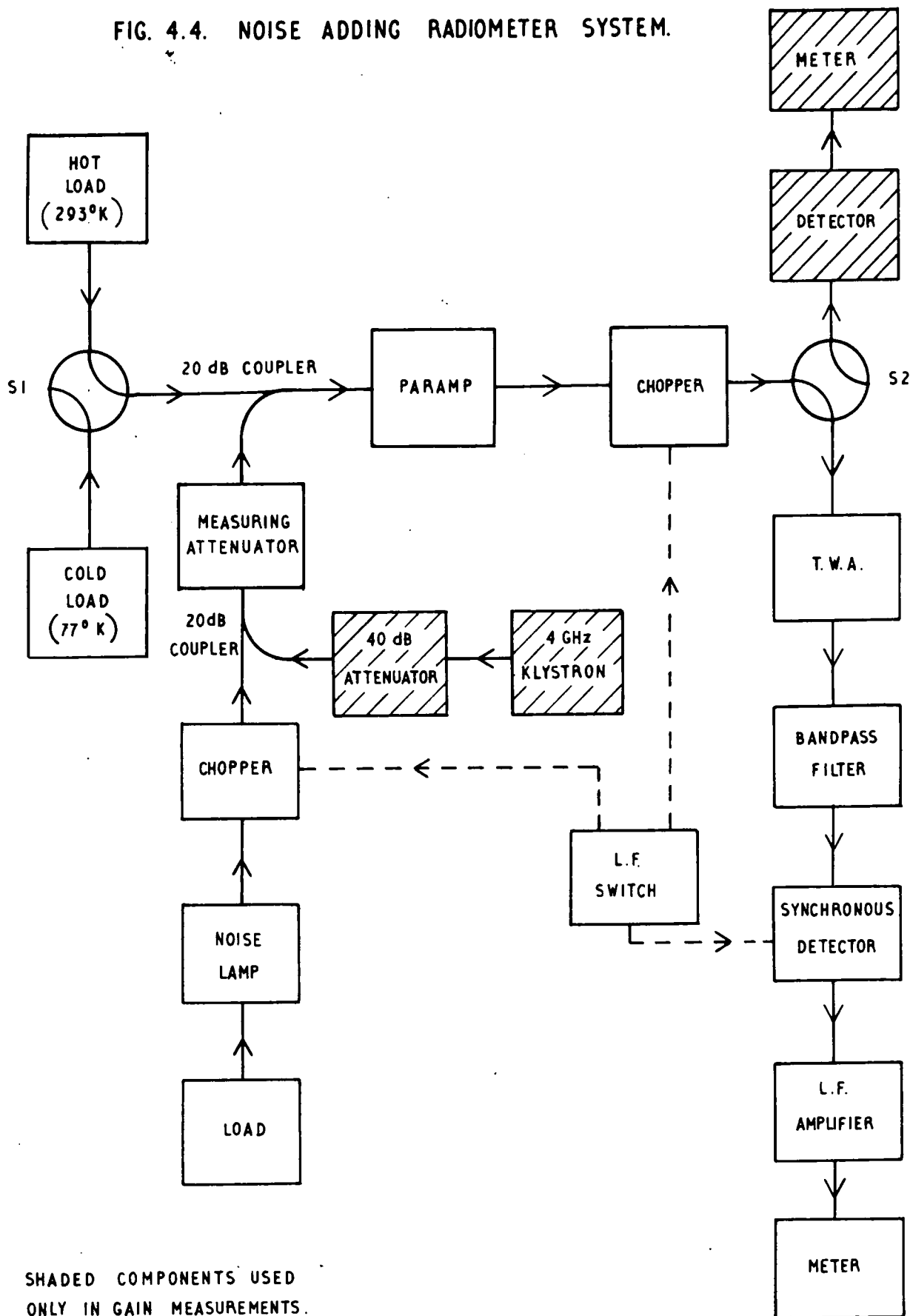
The major difficulty in this measurement is the maintenance of a stable parametric amplifier gain. For this reason each noise measurement was repeated at least five times, with a gain reading after each one. If the gain had drifted by more than 0.2 dB the result was discounted. The total errors, including those incurred during the measurement of input loss are listed in the table below.

Table 4.1

Maximum Errors in Noise Measurements for Temperatures <50°K

<u>Error</u>	<u>Type</u>	<u>Magnitude (°K)</u>									
1. Reading											
(i) Atten. calibration	Systematic	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
(ii) Balance stability	Random	1.3	1.25	1.2	1.15	1.1	1.05	1.0	0.95	0.8	
2. Gain measurement											
(i) Atten. calibration	Systematic	0.70	0.52	0.45	0.37	0.27	0.21	0.16	0.13	0.07	
(ii) Drift	Random	1.5	1.1	0.85	0.68	0.56	0.42	0.32	0.25	0.13	
3. Cold & hot load temp. uncertainty	Systematic	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	
4. Input loss measurement	Systematic	0.26	0.26	0.26	0.26	0.27	0.27	0.27	0.27	0.27	
Total random		2.8	2.35	2.05	1.83	1.66	1.57	1.32	1.20	0.93	
Total systematic		2.01	1.83	1.76	1.68	1.58	1.53	1.48	1.45	1.39	
Amplifier Gain (dB)		10	11	12	13	14	15	16	17	20	

FIG. 4.4. NOISE ADDING RADIOMETER SYSTEM.



Since most of the conclusions of this study are based on a comparison of noise readings, random errors are most important, but systematic errors must be normalised at a particular value of amplifier gain. For room temperature devices with noise temperatures between 200 and 500°K the total maximum systematic and random errors may be summarised as follows:

Table 4.2

Maximum Errors in Noise Measurements for Room Temperature Devices

Amplifier Gain (dB)	10	11	12	13	14	15	16	17	20
Random (°K)	8.5	7.9	7.45	7.1	6.76	6.4	6.1	5.9	5.13
Systematic (°K)	4.01	3.83	3.76	3.68	3.58	3.53	3.48	3.43	3.39

Loss before and after the amplifier must be taken into account. If losses of ratios L_1 , L_2 and L_3 at temperatures T_1 , T_2 and T_3 appear before the amplifier and L_4 at temperature T_4 appears after the amplifier, then:

$$T'_A g L_1 L_2 L_3 = g L_4 T_A + g L_4 \left[L_2 L_3 (1-L_1) T_1 + L_3 (1-L_2) T_2 + (1-L_3) T_3 \right] + (1-K_4) T_4 \quad \text{..4.2}$$

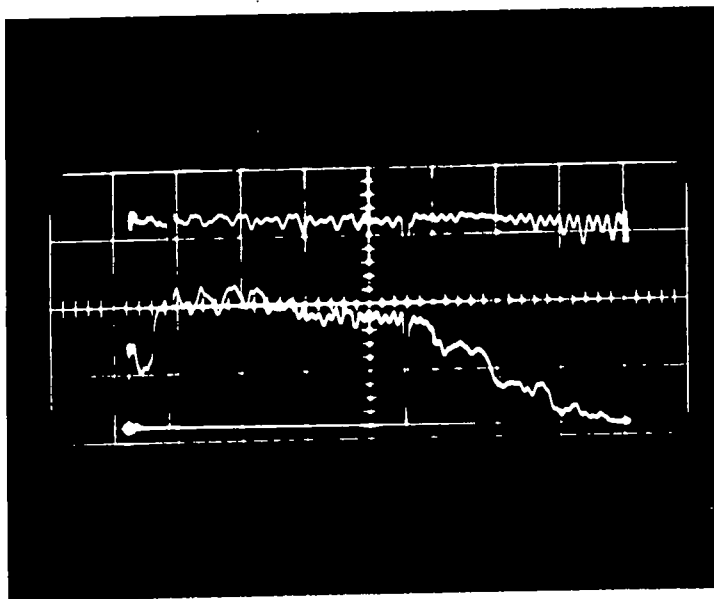
which gives the relationship between the system noise temperature T'_A , referred to the input of the cryogenic signal feed, and the amplifier noise temperature T_A referred to the input terminals of the amplifier. If the amplifier gain is measured between the input and output terminals of the cryogenic signal feed system then the radiometer equation (4.1) gives T'_A , after the subtraction of noise from input loss only (noise from output loss cancels in this definition). T_A , the amplifier noise temperature, may then be obtained from equation 4.2.

4.3 Circulators

Melabs four port coaxial circulators were used at room temperature, loss and VSWR measurements being made for each individual circulator. These are summarised in the tables below.

Marconi and Western Microwave Laboratories circulators were used at liquid helium temperatures. Marked differences in characteristics were noted between circulators in the same batch and with the same circulator over a period of time. The latter effect is attributed to the considerable mechanical stresses involved in the rapid cooling or warming up procedures which were sometimes necessary, and the effects of local fields and foreign material. Since these devices are of an open construction to allow liquid helium to flow freely through, care had to be taken whilst handling them at room temperature to exclude foreign material. Loss measurements were made immediately before or after each series of noise temperature measurements and these are summarised in the tables below. Fig. 4.5 shows a swept frequency response of the feed system with and without a WML circulator and hence also indicates the circulator losses.

Fig. 4.5 Frequency Response of Coolable Signal Feed.



Upper Trace: Feed alone

Lower Trace: Feed plus Circulator

Markers set at 3.6 and 4.3 GHz

Since the loss between individual ports of any of these circulators is, or should be, 0.1→0.3 dB, which is approaching the experimental error of a loss measurement by substitution, the transmission loss between two extreme ports was measured in most cases (i.e. between 1 and 4 or perhaps 2 and 4 or 1 and 3) and divided by the appropriate number of transitions of the ferrite material, to find the average loss between individual ports. All the 4-port circulators used were made up of two internally connected 3-port circulators, so that transferring from ports 1 to 2 involved one transition of the ferrite material and from port 2 to 3 involved two transitions.

Table 4.3

Room Temperature Circulator Measurements

Melabs 4 port Coaxial Circulators

Circulator Serial No.	Loss between ports 1→2 (dB)	Input VSWR (at 4 GHz)
14	0.10	>0.9
15	0.12	>0.9

Table 4.4

Liquid Helium Temperature Circulator Measurements

(i) Marconi Circulator SN203

Frequency (GHz)	Loss between ports 1→2 (dB)
3.8	0.72
4.0	0.39
4.1	0.79
4.2	0.36
4.3	0.49

4.4 (ii) WML Circulator SN11

Frequency (GHz)	Loss between ports		VSWR (including feed) load on 4
	1→3 + 3	2→1	
3.8	0.42	23	0.92
3.85	0.45	23	0.94
3.9	0.31	22	0.77
3.95	0.33	23	0.78
4.0	0.23	25	0.83
4.05	0.36	30	0.80
4.1	0.41	35	0.51
4.15	0.14	30	0.81
4.2	0.29	25	0.44

4.4 (iii) WML Circulator SN10

Frequency (GHz)	Loss between ports		VSWR (including feed) Load on 4
	1→3 + 3	2→1	
3.8	0.21	22	0.91
3.85	0.12	21	0.56
3.9	0.26	21	0.78
3.95	0.32	20	0.68
4.0	0.24	21	0.84
4.05	0.24	21	0.815
4.1	0.25	20	0.62
4.15	0.17	19	0.73
4.2	0.46	18	0.60

4.4 (iv) WML Circulator SN 5 Interpolated from Fig. 4.6

Frequency (GHz)	Loss between ports 2→4 + 3 (dB)
3.8	0.16
4.0	0.13
4.2	0.11

4.4 Liquid Helium Feed System

This is shown in figs. 4.6 and 4.7, with the appropriate loss components listed in table 4.5. VSWR measurements of the input and output signal feeds are given in table 4.6 and fig. 4.5 also gives a qualitative indication of the feed total VSWR. The signal and pump waveguides and the output coaxial line were of copper-plated stainless steel, to reduce heat loss. Although the heat exchange baffles were efficient with the feed in the vertical position, when tipped through 45° (as may occur on an aerial) they were found to encourage rapid helium boil-off, so a second feed was built having only two baffles near the top. These remaining baffles, close to the top, acted as a radiation shield at a temperature of 77°K . The type of tuning drive shown in fig. 4.6, involving gears, was used in only one type of amplifier (SD4/19 see section 6.4) and proved troublesome due to backlash, which led to gain_{in} stability.

Table 4.5

Loss Components of Liquid Helium Temperature Signal Feed

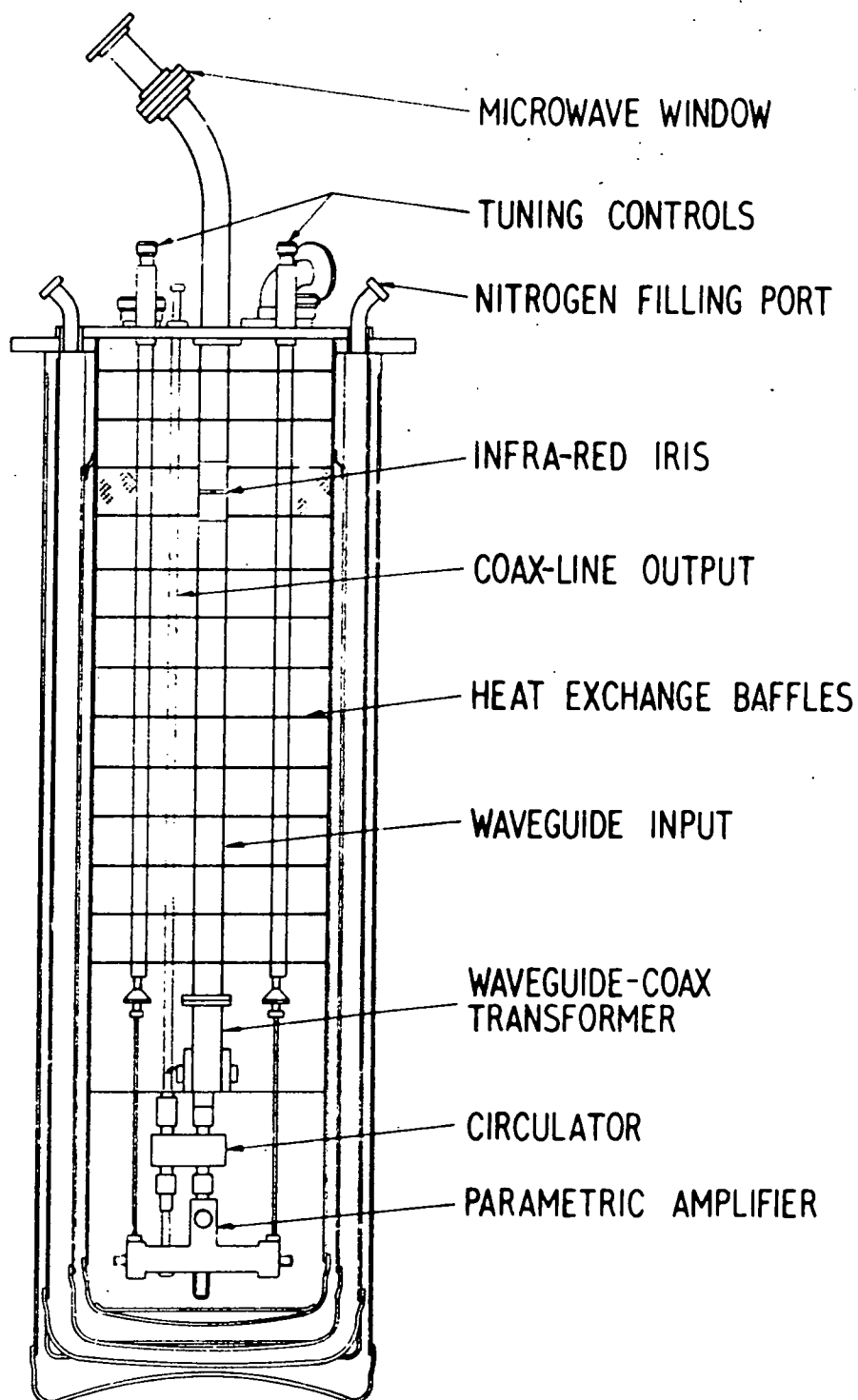
Component	Temp. (°K)	Loss (dB)
Microwave window	293	0.006
Bend	293	0.008
Waveguide	77	0.005
Infra-red iris	77	0.015
Waveguide	4	0.05
Waveguide/coax transducer	4	0.25
Output coax	4	1.0

Table 4.6

VSWR Measurements of Liquid Helium Temperature Signal Feed

Frequency (GHz)	Waveguide input feed (incl. transducer)	Coaxial output line
3.7	0.780	-
3.8	0.840	0.97
3.9	0.895	0.81
4.0	0.960	0.87
4.1	0.925	0.735
4.15	0.890	0.86
4.2	0.837	0.71
4.3	0.765	-

FIG. 4-6 LIQUID HELIUM COOLABLE AMPLIFIER FEED SYSTEM



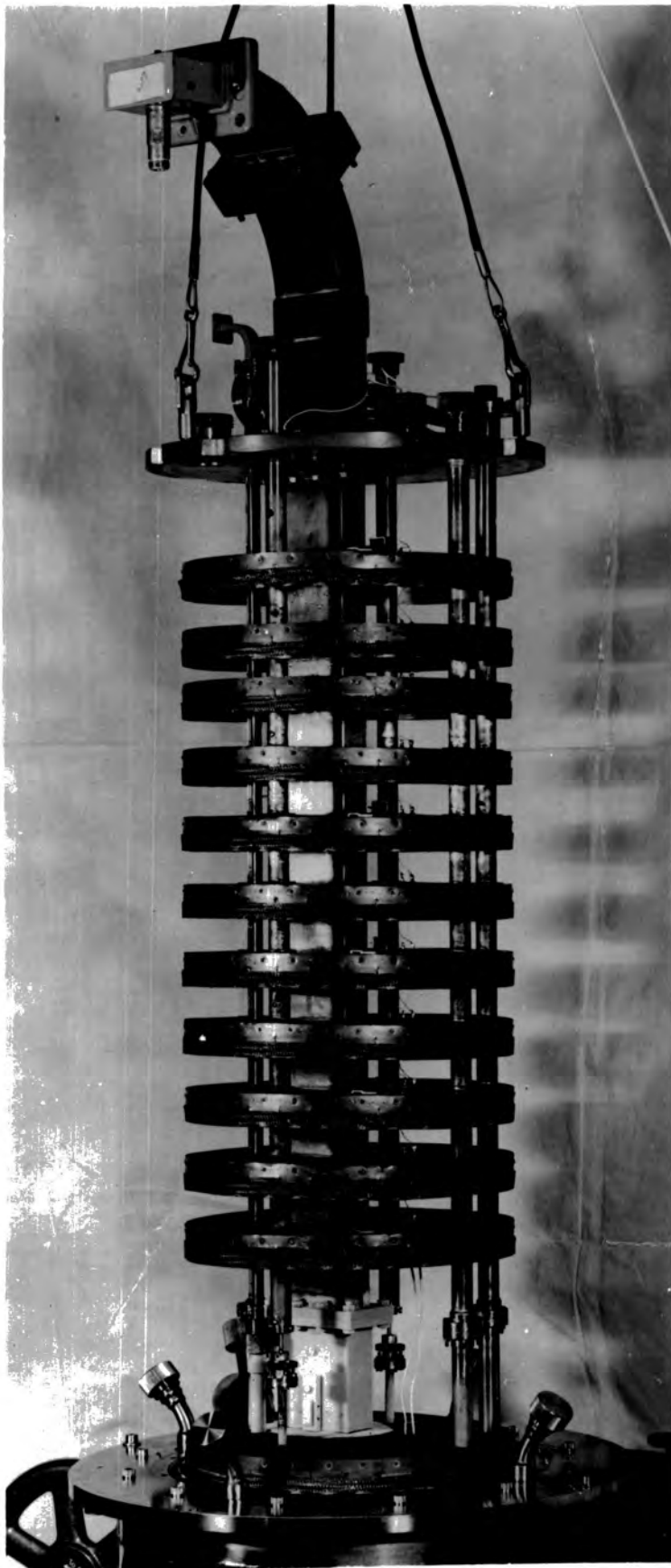


FIG. 47. LIQUID HELIUM COOLABLE SIGNAL FEED

It was decided to allow liquid helium to flow into the amplifier structure in order to simplify construction of the feed, but considerable difficulty was experienced with local heating in the diode when pumped. Bubbles of gaseous helium are produced which may rise up the pump waveguide, expanding as they rise, and producing large unstable pockets of gas at the top of the guide with an unstable meniscus. These instabilities modulate pump power to the diode, due to the differing dielectric constants between gaseous and liquid helium and produce amplifier gain fluctuations which may be 6 dB or more if the process is unchecked. Such instabilities may be reduced or eliminated by filling the pump waveguide with PTFE (taking care to allow for shrinkage) or by constructing the guide with suitable bends and holes so that the bubbles leave immediately after the diode. Complete stability was achieved with the latter method for an amplifier mounted as in fig. 4.8, but only a reduced amplitude of oscillation could be obtained with a solid dielectric in the pump waveguide, in the system shown in fig. 4.9 and the pump level was considerably attenuated (> 3 dB) due to dielectric loss. Where possible it is therefore thought better to divert helium bubbles from the pump waveguide at a position close to the diode.

With either system of bubble suppression care has to be taken not to introduce excessive reactance into the pump circuit, so that the bandwidth is not degraded. The ideal pump circuit should have zero attenuation between the idler and upper-sideband frequencies, with infinite attenuation at, and beyond these frequencies. With a practical cryogenic pump feed, even assuming that a flat bandpass coolable filter is available, reactance is still introduced at any discontinuity in the system and external tuning screws are usually required to match the circuit. This means that care has to be taken when tuning the amplifier for minimum pump power at liquid helium temperature, a process which becomes tedious and sometimes dubious with a highly tuned pump circuit.

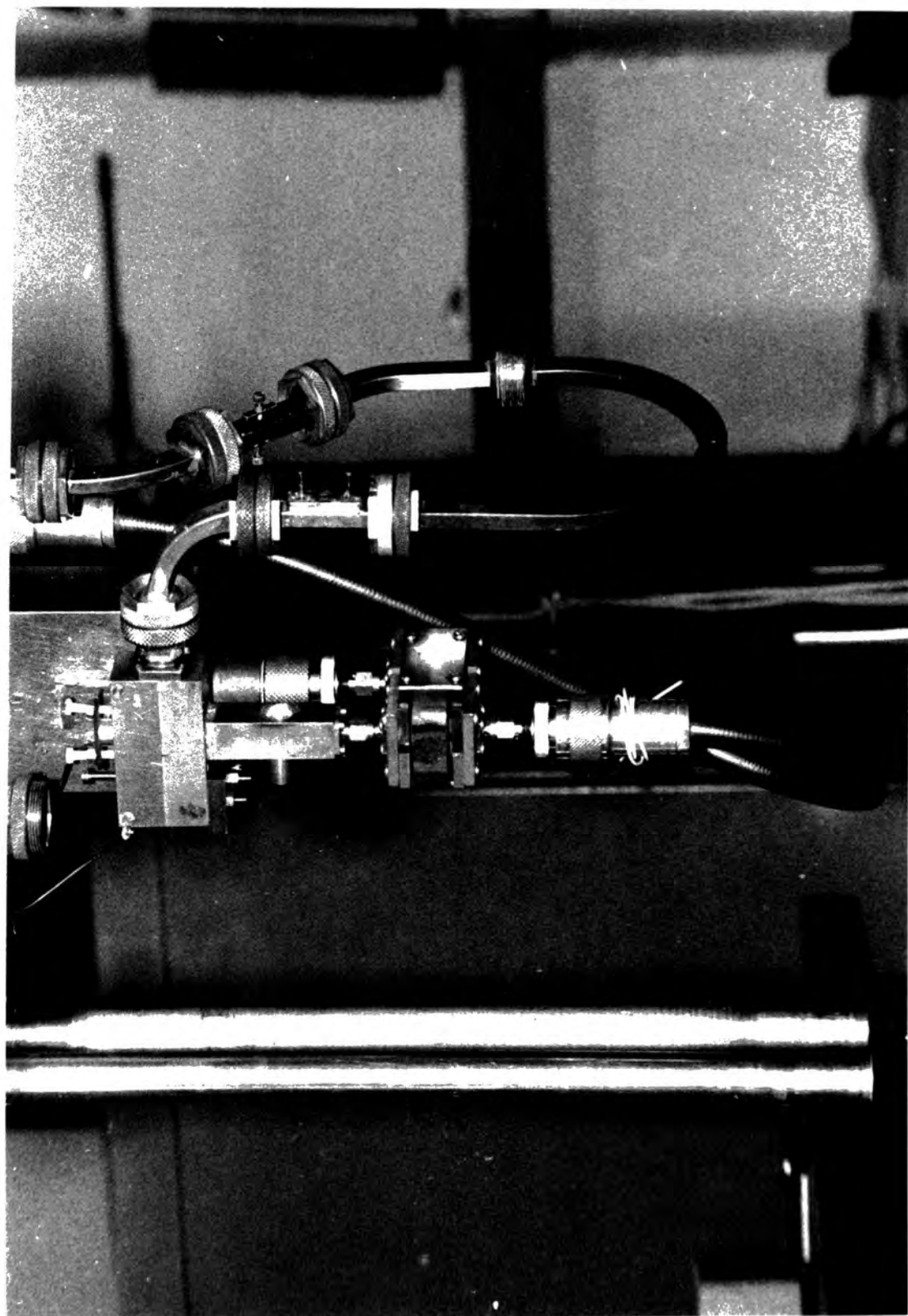
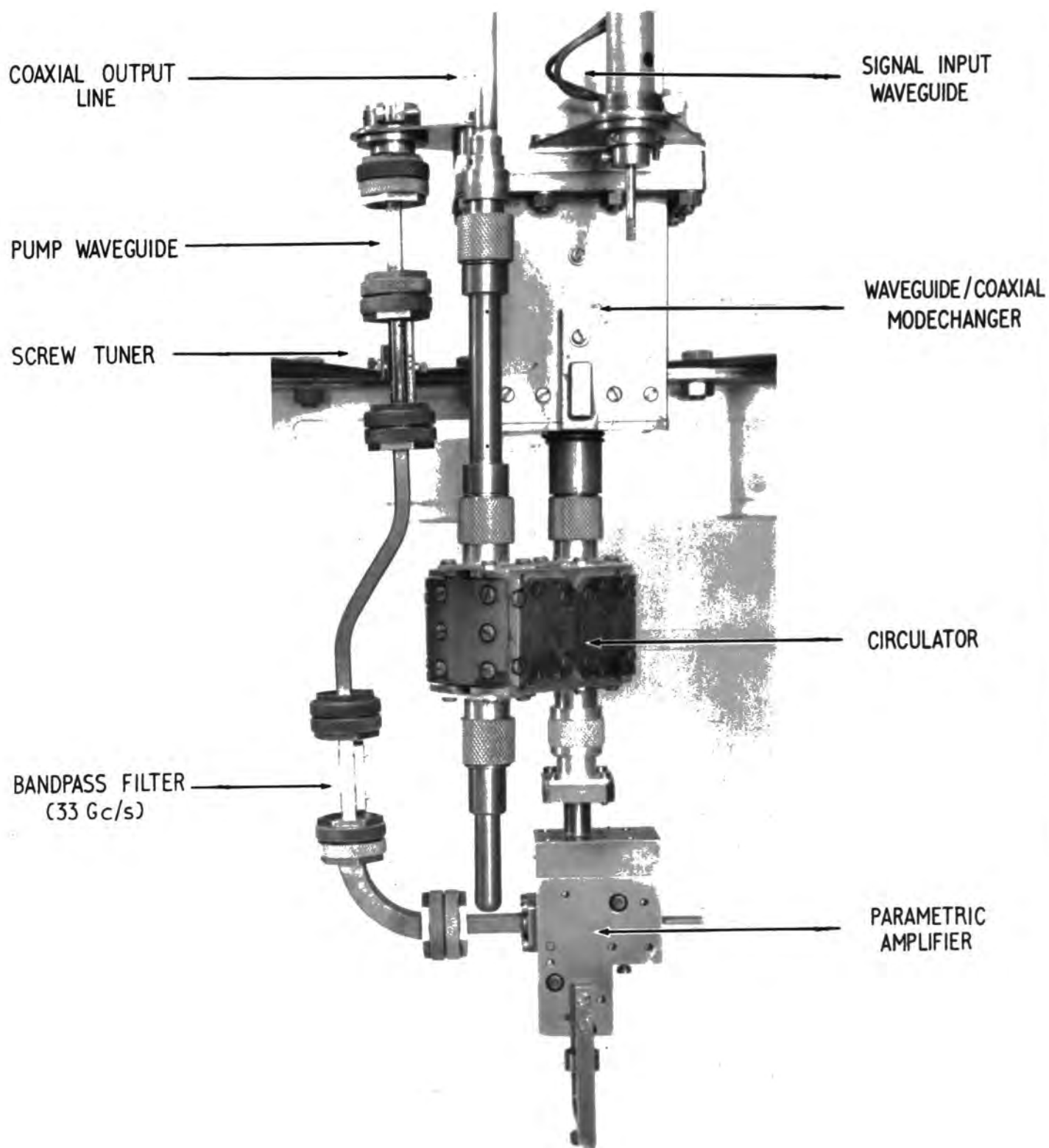


FIG.4-8

AMPLIFIER SD4/33A ON COOLABLE FEED

Fig. 4.9

4 Gc/s NON DEGENERATE PARAMETRIC AMPLIFIER
SD4/338 AND COOLABLE FEED



5. Diodes and Diode Mounts

5.1 General

Mullard CAY10 varactor diodes (also known as VX 3368) were used in most devices. These are degenerately doped gallium arsenide p-n junction varactors of a type known to function at liquid helium temperature, and at the outset of this study, exhibited the highest available Yf_c product at such temperatures, ($Yf_c = 24$). Sylvania diodes type D5417C (also gallium arsenide) have a higher room temperature Yf_c product (≈ 40) but, in their present form, show increased resistive loss at liquid helium temperature.

All silicon varactors so far produced suffer from carrier freeze-out at 40°K or above, which it seems cannot be avoided since further increase in the doping level produces varactors with impractically low breakdown voltages. However, at room temperature, the use of silicon varactors is usually favoured, since they exhibit a high capacitance variation factor ($\gamma = 0.2$ compared with 0.15 for GaAs). Other diode materials and/or other types of diode (e.g. Schottky barrier, space charge varactor) may ultimately prove superior either to gallium arsenide at cryogenic temperatures or to silicon at room temperature and some of these are discussed in section 10.

A few experimental gallium arsenide varactors (Mullard VX6508) with a higher cut-off frequency than the CAY10 (350 GHz compared to 150 GHz, due to lower series resistance and junction capacitance) and a lower series inductance (0.1 nH compared to 0.6 nH), became available during the course of this study and were tried in a number of amplifier structures without much success. These are of a smaller size than the CAY10 and have an encapsulation capacitance comparable with their junction capacitance, which, at a signal frequency of 4 GHz, reduces the effective γ value to less than 0.1. Similar diodes with an increased junction capacitance, or a pair mounted in one structure (see section 10) should give superior performance to the CAY10 varactor.

A further type of experimental varactor with a high Yf_c product (low R_s and C_j) was available, known as the "super" VX3368. These varactors have the same internal structure as a VX6508 but are mounted in the larger encapsulation of the CAY10, and thus have a smaller encapsulation capacitance but a much higher stray inductance than the VX6508. The high-cut-off frequency of these diodes allows an improvement in noise temperature over the CAY10, but means a reduction in available bandwidth, (see section 3).

The CAY10 varactor was thus chosen for the final design of a wideband liquid helium cooled amplifier. The room temperature parameters of this diode are listed in table 5.1, along with those for a few comparable currently available diodes. The γ values given here are the maximum values for negligible diode current. ($<1 \mu\text{A}$ at room temperature and $<0.1 \mu\text{A}$ at liquid helium temperature.) Curves of γ at various values of bias and pump voltage are shown in figs. 2.1 and 2.2 ($n = 0.404$ for the CAY10). The very slight advantage in operation at a small reverse bias is nullified by the requirement of additional pump power, which may lead to an increased amplifier noise temperature (see section 9). Diodes are therefore usually operated at zero bias (D.C. short circuited either directly or through a microammeter).

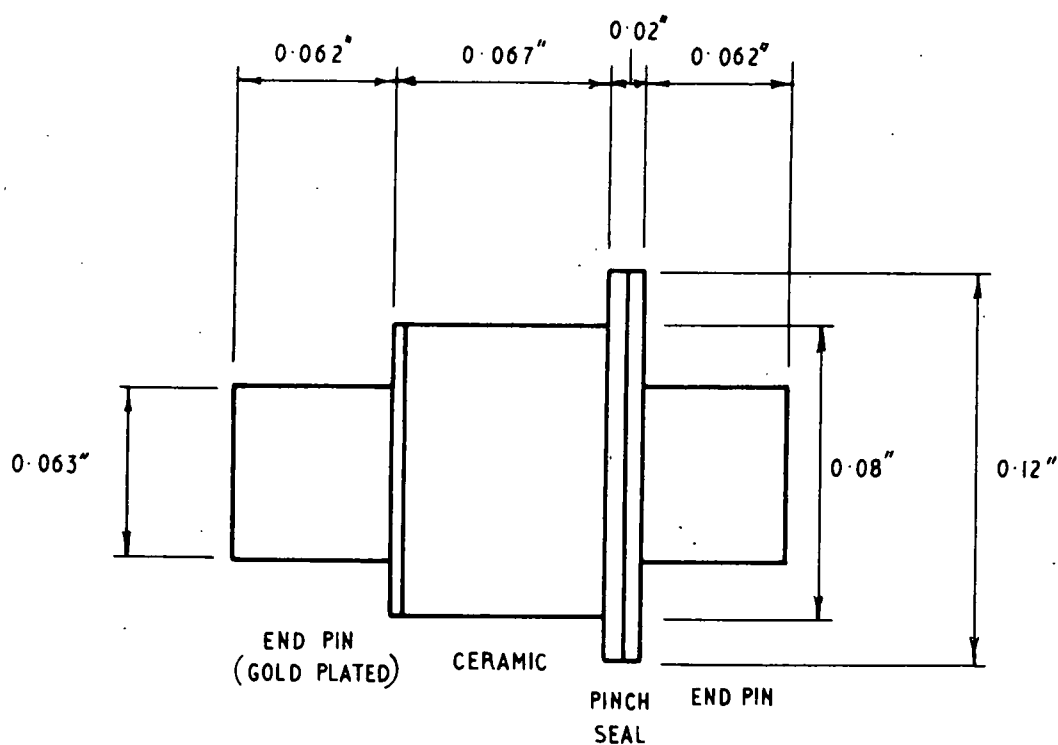
The physical construction of the CAY10 varactor is shown in figs. 5.1 and 5.2 (by courtesy of Mullard Ltd.) and the electrical equivalent circuit shown in fig. 2.6 is adequate. A breakdown of the components contributing to the series resistance, R_s , (as supplied by the manufacturers) is given in table 5.2. It is noted that the skin effect plays a part in determining this parameter.

At 4°K the capacitance variation coefficient of the CAY10 varactor is increased slightly over the room temperature value (from 0.15 to 0.175), the junction capacitance is decreased (from 0.45 to 0.42 pF) and the series resistance remains constant (within experimental limits, i.e. $\pm 1\Omega$). (Attested by measurements at Mullard Ltd. and by the performance of parametric amplifiers at 4°K , see sections 6 and 9.)

Table 5.1

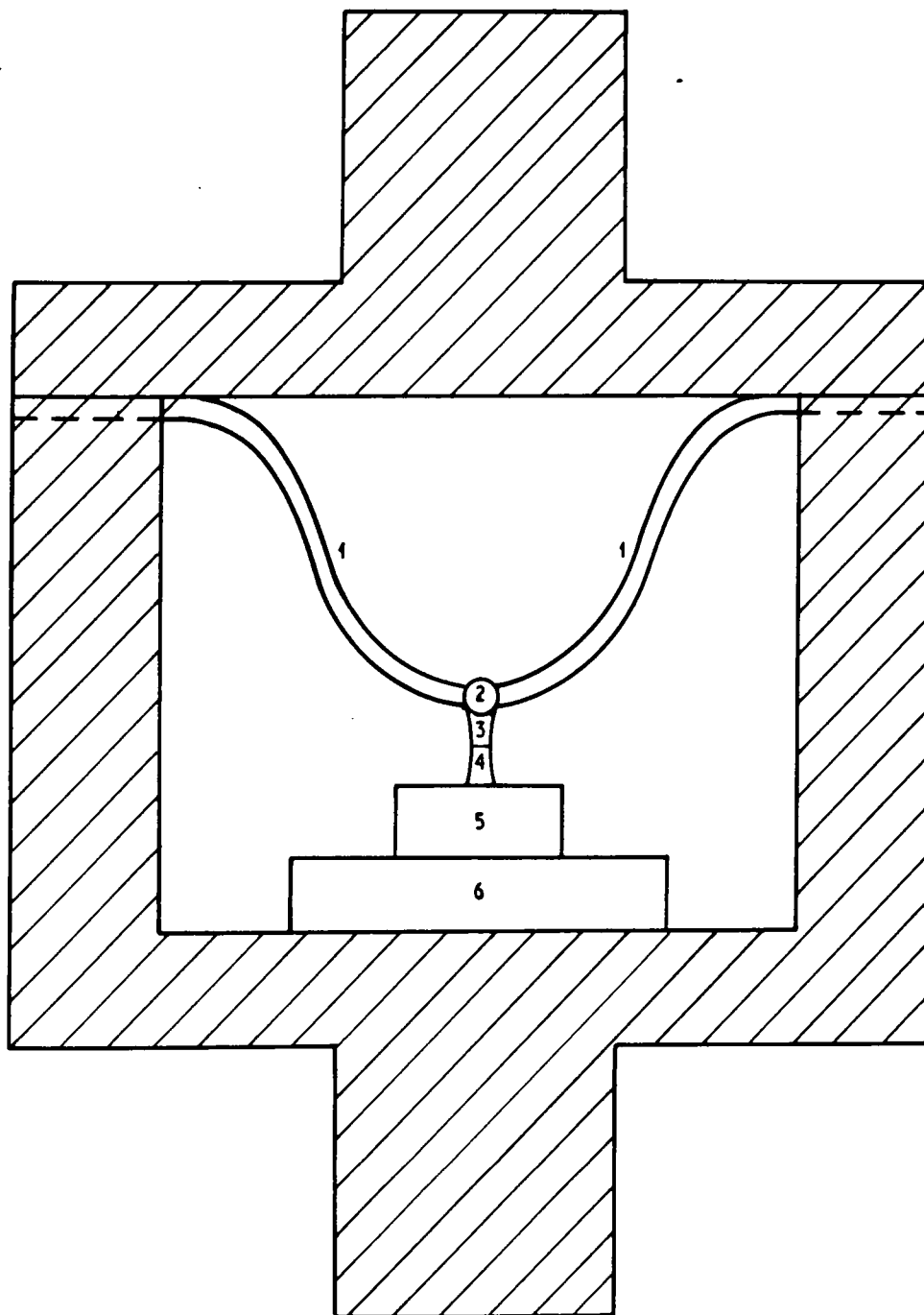
Diode	γ	f_c (Gc/s)	R_s (Ω)	C_j (pF)	L_s (nH)	C_1 pF
VX3368 (CAY10)	0.16	150	2.5	0.45	0.6	0.05
VX6508	0.127	400	2.0	0.2	0.120	0.2
"super" VX3368	0.16	400	1.0	0.40	0.6	0.05
D5417C (sylvania)	0.145	200	1.6	0.5	0.15	0.2

FIG. 5.1 CAYIO DIODE EXTERNAL DIMENSIONS



POST OFFICE RESEARCH STATION		LONDON, NW2		ASSEMBLY DRG	
DATE	7-9-68	APPD			
DD SERIAL NO	1915				
CASE NO	20836				

FIG. 5.2. INTERNAL CONSTRUCTION OF CAY10 DIODE.



1. Gold wires
2. Gold bead
3. p type GaAs $n_A = 10^{21} \rightarrow 10^{22} / \text{cm}^3$ (zinc) } mesa
4. n type GaAs $n_D = 10^{17} / \text{cm}^3$ } pillar
5. n type GaAs wafer $n_D = 10^{17} / \text{cm}^3$
6. Kovar base

Shaded area represents encapsulation

[illegible]

Table 5.2Components of the Diode Series Resistance at Room Temperature

Region (see fig. 5.2)	Resistance (Ω)	
	D.C.	Microwave
contacts & wires	0.5	0.75
1	0.083	0.083
4	0.826	0.826
5	0.59	0.85
total	2.0	2.5

5.2 Diode Mounts

The mount of any varactor diode will contribute stray inductance and capacitance to the equivalent circuit, and this must be taken into account in amplifier design (see section 6). The two mounting pins of the CAY10 type cartridge are usually slightly eccentric, which means that a closely engineered mount cannot be used, and since the coefficient of expansion of these devices will be less than that of an amplifier structure (brass), the mount must be sprung to accommodate relative contraction. These requirements have led to the design of sprung mounts with a close fit on one diode pin ($\pm .001$ ins.) and a pressure maintained loose fit on the other pin (e.g. see fig. 6). Such an arrangement has been found to be both mechanically and electrically stable.

Stray capacitance can be minimised by mounting the diode between a stub of the same diameter as the diode ceramic at one end and a stub of the diameter of the diode pinch seal at the other end. Stray inductance can be tuned out in a single diode shunt resonant amplifier, but will be introduced into the idler circuit of a balanced diode amplifier. Methods of reducing stray inductance in the mounting of balanced diodes are discussed in section 3.5.3.

5.3 Diode Measurements

A quality factor (Yf_c) measurement at liquid helium temperature has been carried out by other workers for the type of diode used (Davies R. 1967, Chakraborty D. and Coackley R. 1967.) and, considering the time necessary for such a measurement, was not repeated.

The D.C. characteristics at room and liquid helium temperatures are plotted in fig. 5.3. These were also measured with the diode situated in the SD4/33 amplifier at various pump levels, and a negative resistance effect was observed, which is most satisfactorily explained as an avalanche condition arising from the storage of minority carriers, (Hefni I. 1960). Fig. 5.4 shows the D.C.

Fig. 5.3 D.C. Characteristics of CAYLO Varactor

at Room and Liquid Helium Temperatures

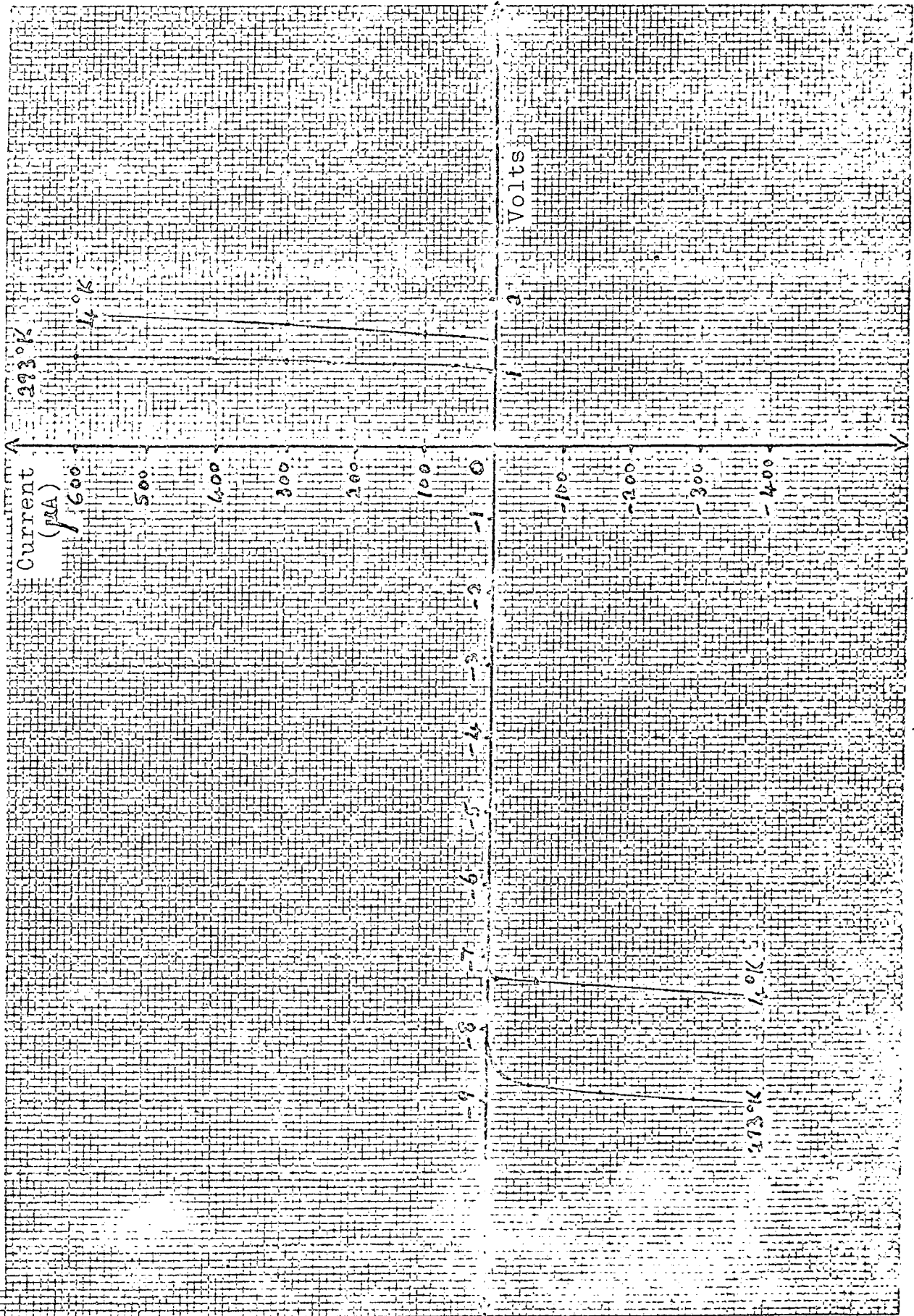
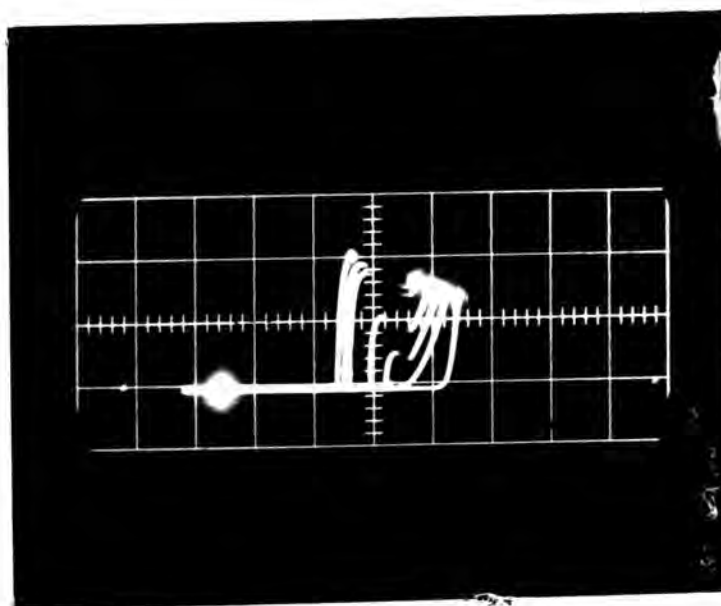
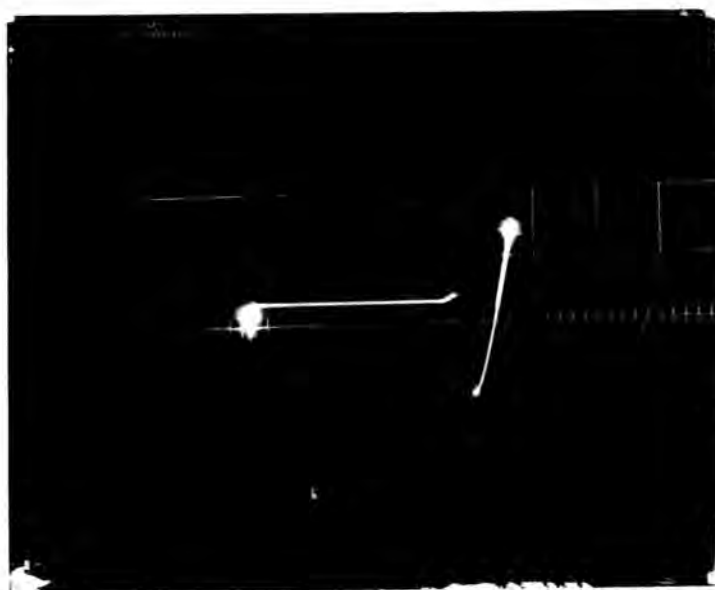


Fig. 5.4 Diode Dynamic Characteristics at the Pump Frequency.



Trace (from right to left)	Pump Power (mW)
1	0
2	15
3	25
4	35
5	50
6	55
7	60

Fig. 5.5 Showing Reverse Current Effect



Pump Power
150 mW

X plate sensitivity 0.5 v/cm

voltage/current characteristics of a CAY10 diode subject to radiation at the pump frequency (34 GHz). Separate traces are produced because of self bias and the negative resistance effect is at reverse bias. A reverse current effect was seen at much higher pump levels, (fig. 5.5). Both effects show a definite threshold with pump power, which was seen to vary from diode to diode and to occur with slightly lower pump levels at cryogenic temperatures, (77°K and 4°K).

If the minority carrier lifetime is comparable with the reciprocal pump frequency then, at a particular biasing condition, an equilibrium will be reached with a fraction of the stored carriers returning to their region of origin. This fraction will be high at zero bias, but will decrease as the junction is reverse biased until the pump voltage excursion is less than the bias, when injection ceases.

Forward current at reverse bias will be seen on a curve tracer when the pump swing is large enough for injection, but this is partially suppressed by the screening effect of the stored carriers. Thus the diode current/voltage curves should move further into reverse bias on increasing pump power and display a "softer" onset of forward conduction. However, at a certain pump level the rate of return of stored carriers will be sufficient to cause impact ionisation and consequent reverse current. This produces the observed negative resistance effect, which, at higher injection levels may be in the reverse current region due to the dominance of avalanching.

6. Single Tuned Amplifiers using the Diode Shunt Resonance

6.1 General

It was decided to build an amplifier using the diode shunt resonance for the idler circuit, as described in section 2.6, in order to study the factors governing amplifier noise at liquid helium temperature, and as a stage towards the development of a wide-band device.

The fact that the device would ultimately be used at 4°K, necessitated homogeneous construction and particular care in the design of the diode mount (section 5.2). Amplifier structures were made from brass, being easier to machine than copper and having acceptable thermal and electrical conductivities.

Since it was desirable to use a cooled circulator directly attached to the amplifier at liquid helium temperatures, the amplifier signal circuit had to be of a coaxial construction, waveguide circulators at 4 GHz being too large.

The facility for measurement of D.C. diode current is required for any device, particularly at room temperature, since this allows precise tuning (see section 2.4) and ensures the absence of shot noise (i.e. at very low diode currents, see section 8.2). Bias can always be applied by the same circuit as used for measuring diode current, but since most of the diode capacitance change occurs in or near the forward direction the application of bias does not improve amplifier performance, and diodes were therefore not externally biased.

Uncertainty about the diode encapsulation capacitance (CAY10 see section 5) lead to the development of two devices, designated SD4/19 (single diode, 4 GHz signal frequency, 19 GHz pump frequency) and SD4/33.

6.2 Realisation

6.2.1 Amplifier 1 (SD4/19)

Information from the diode manufacturers indicated that the shunt resonance might occur at about 20 GHz. A circuit was constructed as shown in fig. 6.1 which is of the form suggested by Matthaei and Getsinger (1964) with an additional signal circuit element, half a wavelength from the diode, to allow flexibility of tuning, but no gain was obtained. Pump power was coupled through a resonant iris opposite the diode from a waveguide through the plane of the diagram (fig. 6.1) and a choke C was included to prevent idler or pump signals propagating in the signal circuit. The failure of this structure to produce gain was thought to be due to the difficulty in establishing an open circuit at the idler frequency, at a small finite distance from the short circuit block B. The circuit was therefore modified to that of fig. 6.2 with two stubs at the idler frequency, the idea being that the stubs would be shunt resonant at the idler frequency and present the correct inductance to resonate

FIG. 6.1 AMPLIFIER SD4/19 EARLY VERSION.
CROSS SECTION

RES. 21964.
SH.15

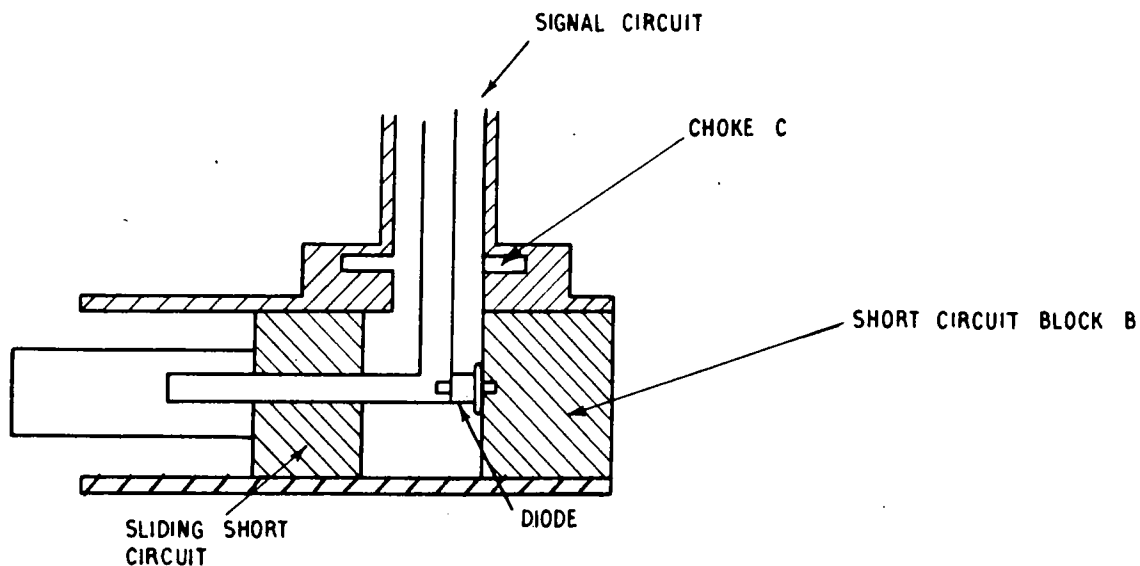
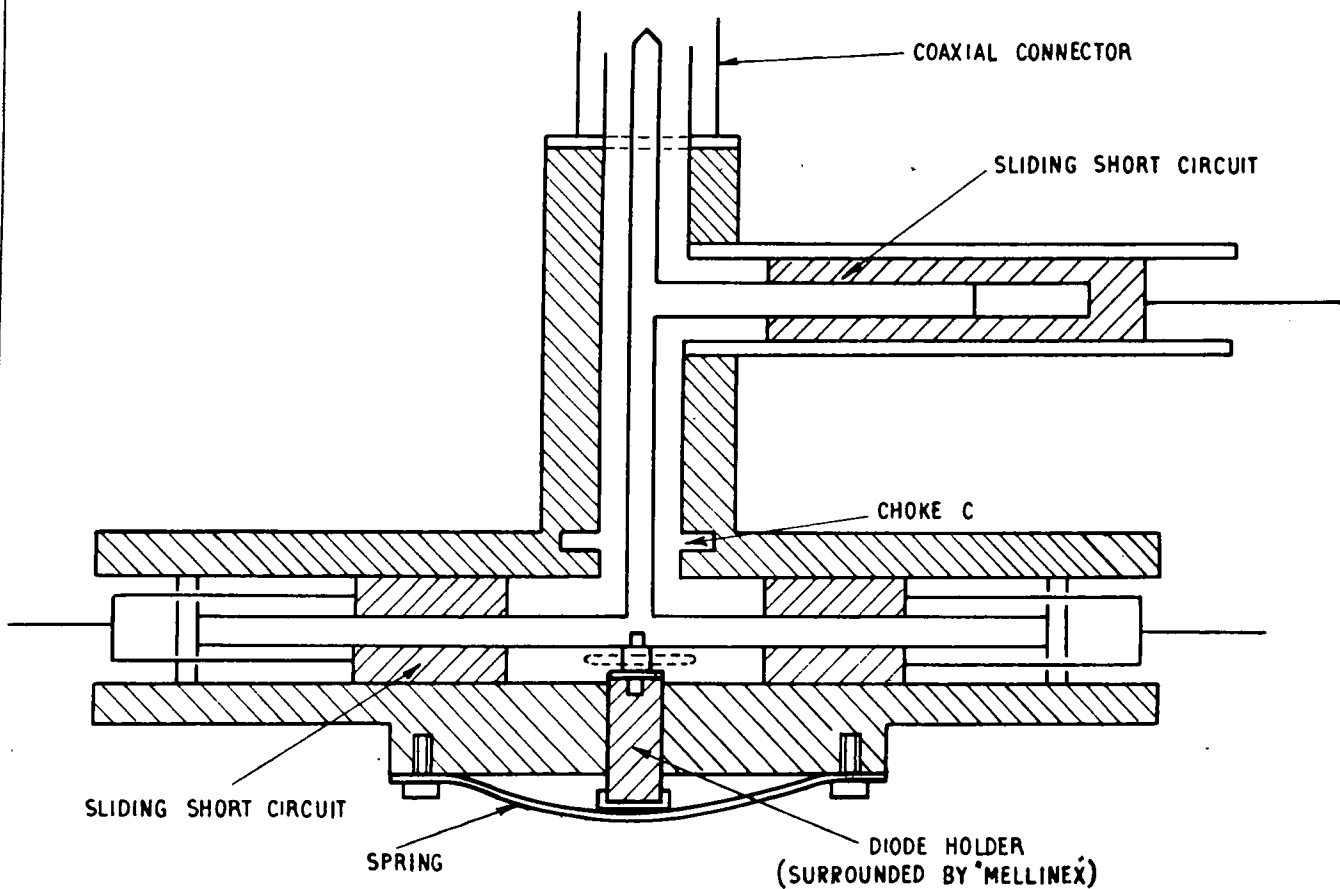


FIG 6.2 AMPLIFIER SD4/19 FINAL VERSION
CROSS SECTION



PUMP IS FED THROUGH A RESONANT IRIS (DOTTED) FROM WAVEGUIDE
THROUGH PLANE OF ~~PAPER~~ PAPER.

the diode at the signal frequency. The pump coupling iris was cut in the plane I-I' opposite the diode and an idler reject filter was included in the pump waveguide. Diode current was measured between the insulated diode holder and the amplifier block. Gain was obtained at a pump frequency of 19 GHz for two symmetrical settings of the tuning stubs, as might be expected, but the optimum inner rod diameter, found by experiment, was not that predicted by analysis of the simple circuit. The diode manufacturers then reported that the value of diode encapsulation capacitance should give a shunt resonance of 30 GHz, rather than 20 GHz, so the simple tuning conditions for this amplifier cannot apply. The idler circuit must therefore be shunted with external capacitance from the diode mount, and since a double tuned response was observed, (fig. 6.12) the stubs must be shunt resonant at the idler frequency and interchangeable with the external resonator at the signal frequency. Additional inductance at the signal frequency is provided by the short post supporting the diode, and double tuning is effected by the additional resonator. The equivalent circuit is thus thought to be as in fig. 6.3. Adjustment of the additional resonator (a quarterwave short circuit line) was seen to tune one peak of the response over a limited frequency band, but this could be restored by adjustment of the stubs connected to the diode. However, such a process was seen to alter the gain-bandwidth product of the amplifier and the diode D.C. current. It is evident that since these interchangeable resonators have different characteristic impedances, the transformed value of generator impedance seen by the diode can be adjusted which is verified by this tuning process.

Although this amplifier does not give the greatest available bandwidth from this varactor, and is unnecessarily complex, it was decided to take some noise measurements at liquid helium temperature whilst a broadband amplifier was being developed. The device is shown mounted on a signal feed for use at liquid helium temperature in figs. 6.4 and 6.5. The results of the noise temperature measurements on this device at room and liquid helium temperatures are discussed in sections 8 and 9, and the bandwidth performance is discussed in part 4 of this section.

6.2.2 Amplifier 2 (SD4/33)

The knowledge that the diode shunt resonance was at 30 GHz and not 20 GHz, encouraged the development of an amplifier pumped at 33-34 GHz. The final design, an electrical equivalent circuit and a photograph of the device appear in figs. 6.6, 2.10 and 6.7. The signal circuit transformer, T_1 , presents the correct impedance to the diode which, for greatest bandwidth and least noise temperature, should be the highest possible consistent with

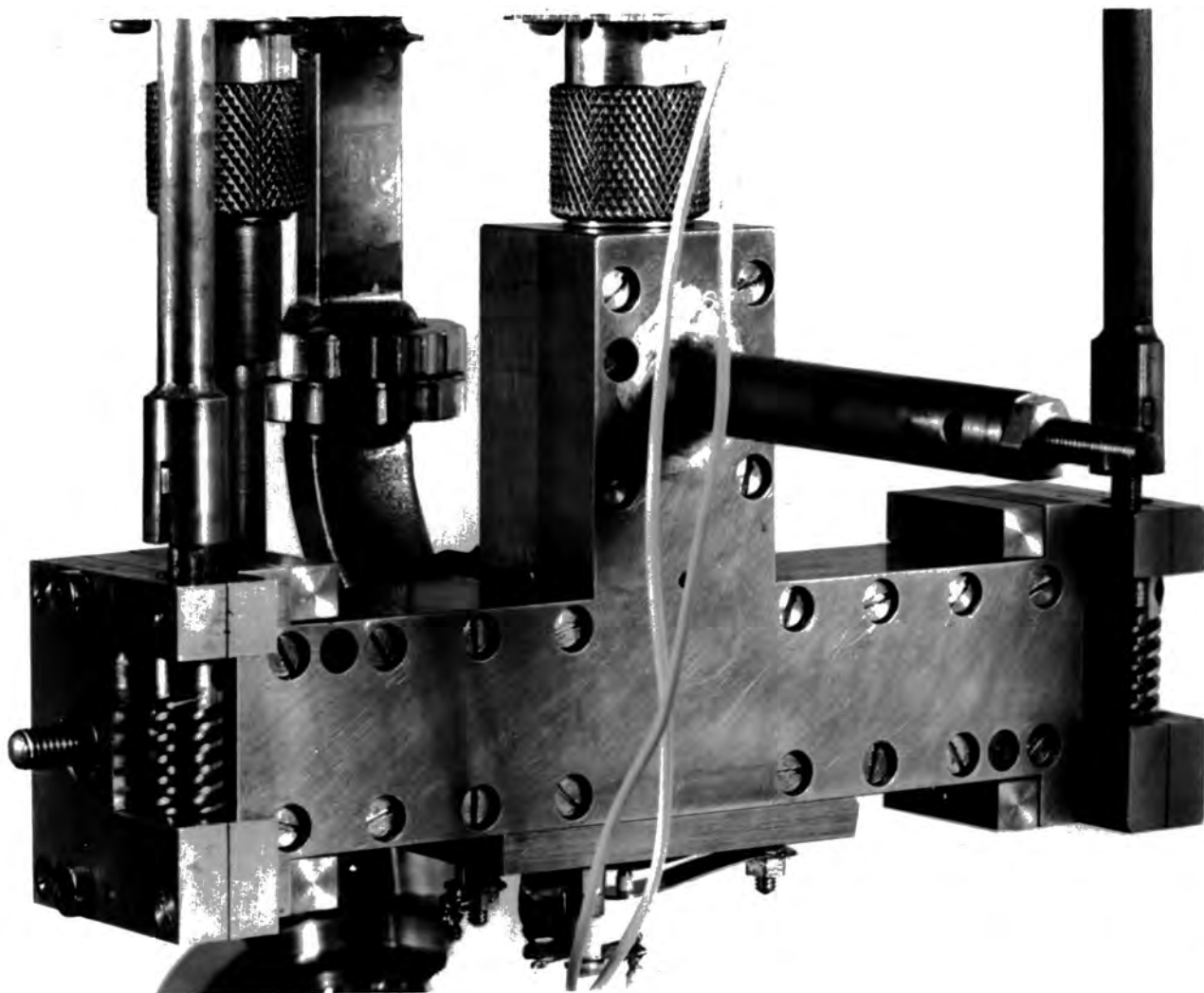


FIG. 64. AMPLIFIER TYPE SD4/19

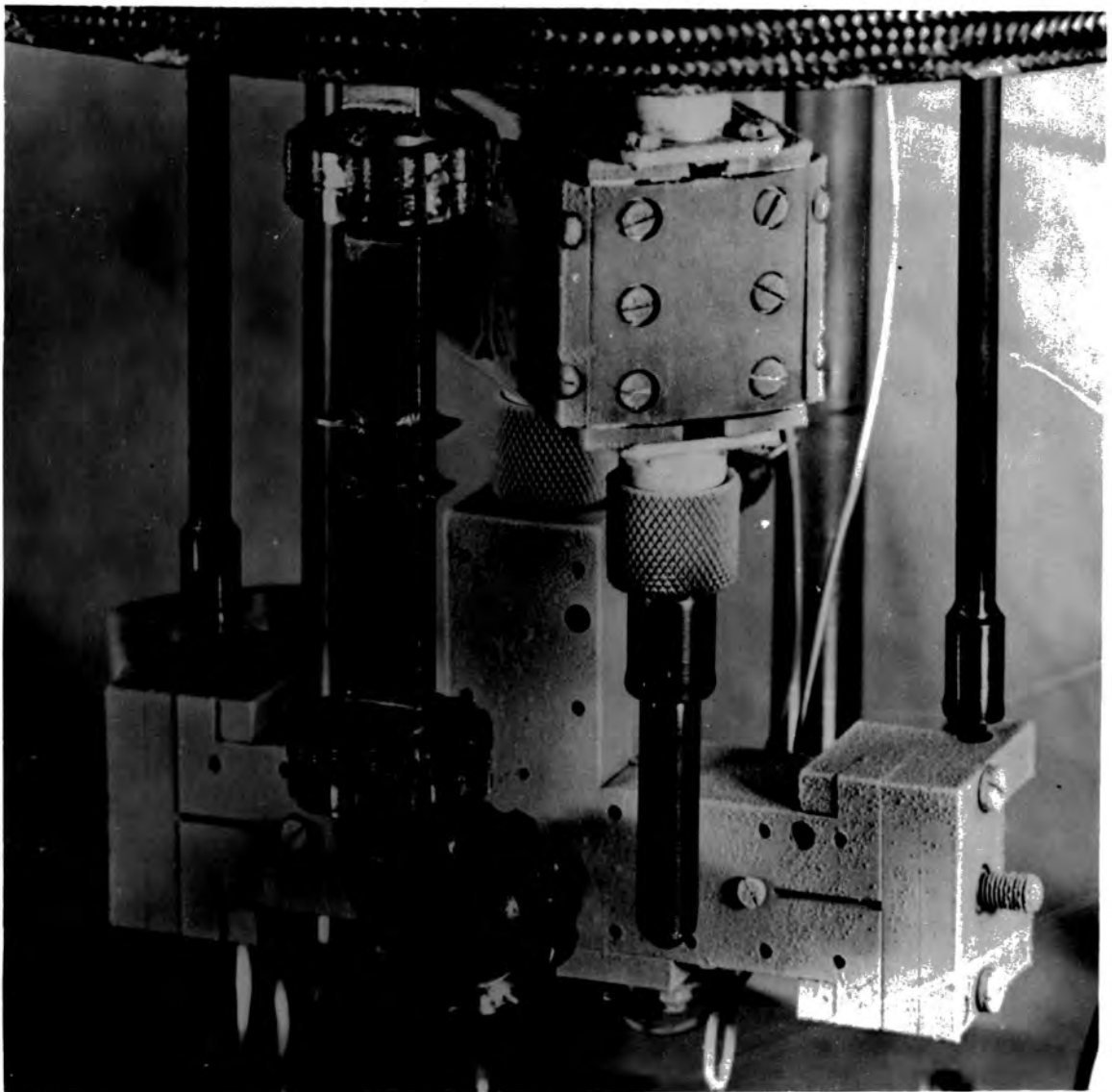
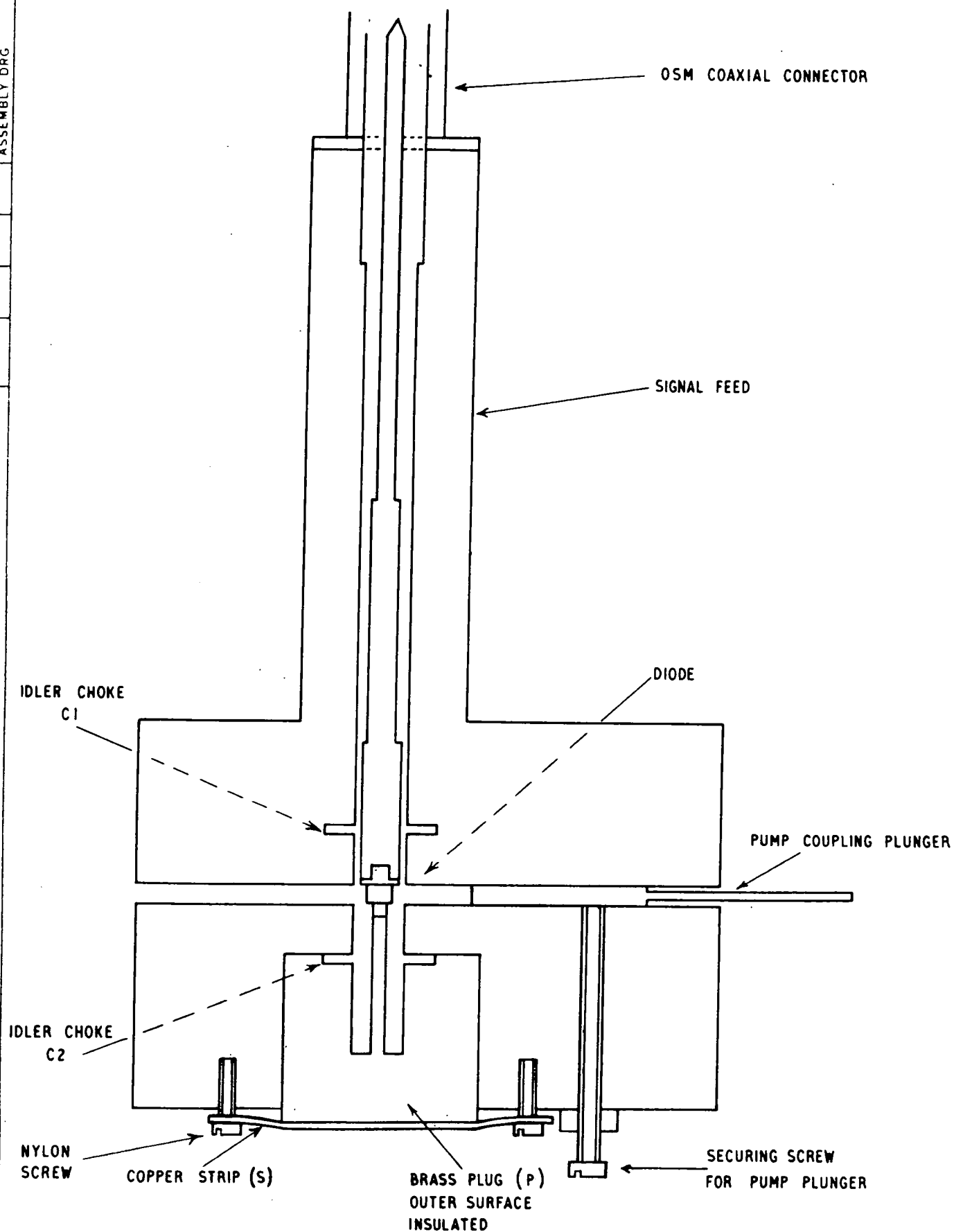


FIG. 65.

AMPLIFIER TYPE SD4/19
MOUNTED ON LIQUID HELIUM COOLABLE
SIGNAL FEED.

FIG. 6.6 CROSS SECTION THROUGH SINGLE DIODE AMPLIFIER SD4/33



4 GHz SINGLE TUNED PARAMETRIC AMPLIFIER
TYPE SD4/33

SCALE
INCHES

0

1

2

3

4

5



FIG. 6.7

negligible diode current, (see sections 2.3 and 8). For investigations of shot noise (section 8) it was desirable to have a means of changing this transformer ratio, so several interchangeable inner conductors were made, each of which, used with or without a PTFE dielectric, gave two possible values of transformer ratio. In this way the impedance seen by the diode at the signal frequency could be made either 8.0, 11.0, 12.3 or 13.4 Ω .

To prevent propagation of idler and upper sideband frequencies, the radial choke, C1, (fig. 6.6) was included in the signal circuit coaxial line. A low pass filter was tried in this circuit (shown in fig. 6.8) but was later removed, since it did not improve the amplifier noise performance at liquid helium temperature, and was detrimental to bandwidth.

The diode was supported by the back coaxial plug, P₁, under pressure from the spring S, (fig. 6.6) and leaving a gap C2 to form a choke at the idler frequency. This arrangement has proved to be both mechanically and electrically stable. Diode current was measured between the insulated stub S, and the main amplifier block, with a D.C. short circuit somewhere in the signal output line, usually at a bar and post coaxial/waveguide transducer. An earlier version of this device (fig. 4.10) involved a much larger insulated segment on the amplifier block, and suffered from pick-up in the diode D.C. circuit. This resulted in the destruction of a number of diodes, due to transient pulses induced in this circuit at the switching on or off of some ancillary laboratory apparatus (e.g. klystron power supply or cooling fan). Such pulses were observed on an oscilloscope to be of sufficient magnitude to damage a diode. Although the final arrangement did not demonstrate this ability to destroy diodes, care was normally taken to open the diode D.C. circuit before connecting or disconnecting any ancillary electrical equipment.

This experience is relevant to the installation of working devices in receiving stations. If diode current is measured, then the D.C. circuit must be well screened and, if possible, all ancillary electrical equipment well suppressed. Continued pulsing with transients not sufficient to completely destroy the diode, may have the long term effect of degrading the reverse characteristic due to the creation of local hot spots or avalanching spots, which in turn may degrade the amplifier noise temperature (see section 5.2). No long term operational information is yet available, but this factor is probably important in the determination of diode lifetime.

Pump power was coupled directly to the diode with a short circuit plunger a quarter wavelength away. The pump waveguide included a bandpass filter to reject idler and upper sideband frequencies; this is shown in fig. 6.9 and

LOW PASS COAXIAL FILTER CUT - OFF 6 GHz
(INNER SECTION)



SCALE
INCHES



FIG. 6-8

BAND PASS PUMP LINE FILTER
(CENTRE FREQUENCY 33 GHz)



SCALE
INCHES



FIG. 6-9

its frequency response in fig. 6.10. As a further measure to prevent idler propagation in the pump circuit, the section of pump waveguide integral with the amplifier was designed to cut-off at 31.5 GHz.

The exact length of the stub behind the diode, which resonates with the diode at the signal frequency, is determined by the idler frequency, since it is desirable that it should be a multiple of quarter-wavelengths long at this frequency, even though an idler reject choke is included. This stub length can therefore only be determined by experiment, since the diode does not approximate to a simple coaxial structure at the idler frequency. The diameter of the inner of this coaxial line was made equal to the diode end pin diameter, and the length was chosen to resonate the signal circuit to about 3.8 GHz, and also fulfil the idler circuit requirement. The amplifier could then be tuned over the signal frequency band (3.8 → 4.2 GHz) by slipping brass sleeves of varying thickness and fixed length over the inner of this coaxial stub, (fig. 6.5). This means that with a given sleeve and diode, there is only one correct operating condition for the amplifier, indicated by the tuning condition of minimum diode current, (see part 3 of this section). It must be remembered however, that the sleeve thickness or stub characteristic impedance will affect the available bandwidth from the amplifier according to the relationship discussed in section 3.2. The minimum diameter of the inner of this stub is 0.064 in. (the diode end pin diameter), which without a change in the coaxial line outer dimensions from the OSM miniature connector (required for a WML circulator), means that the maximum stub characteristic impedance is 47Ω . Such an impedance at 4 GHz means that 0.72 times the bandwidth from equivalent lumped circuit elements is available. The maximum sleeve diameter needed to cover the frequency band is 0.09 in, which results in 0.51 times the bandwidth from lumped elements.

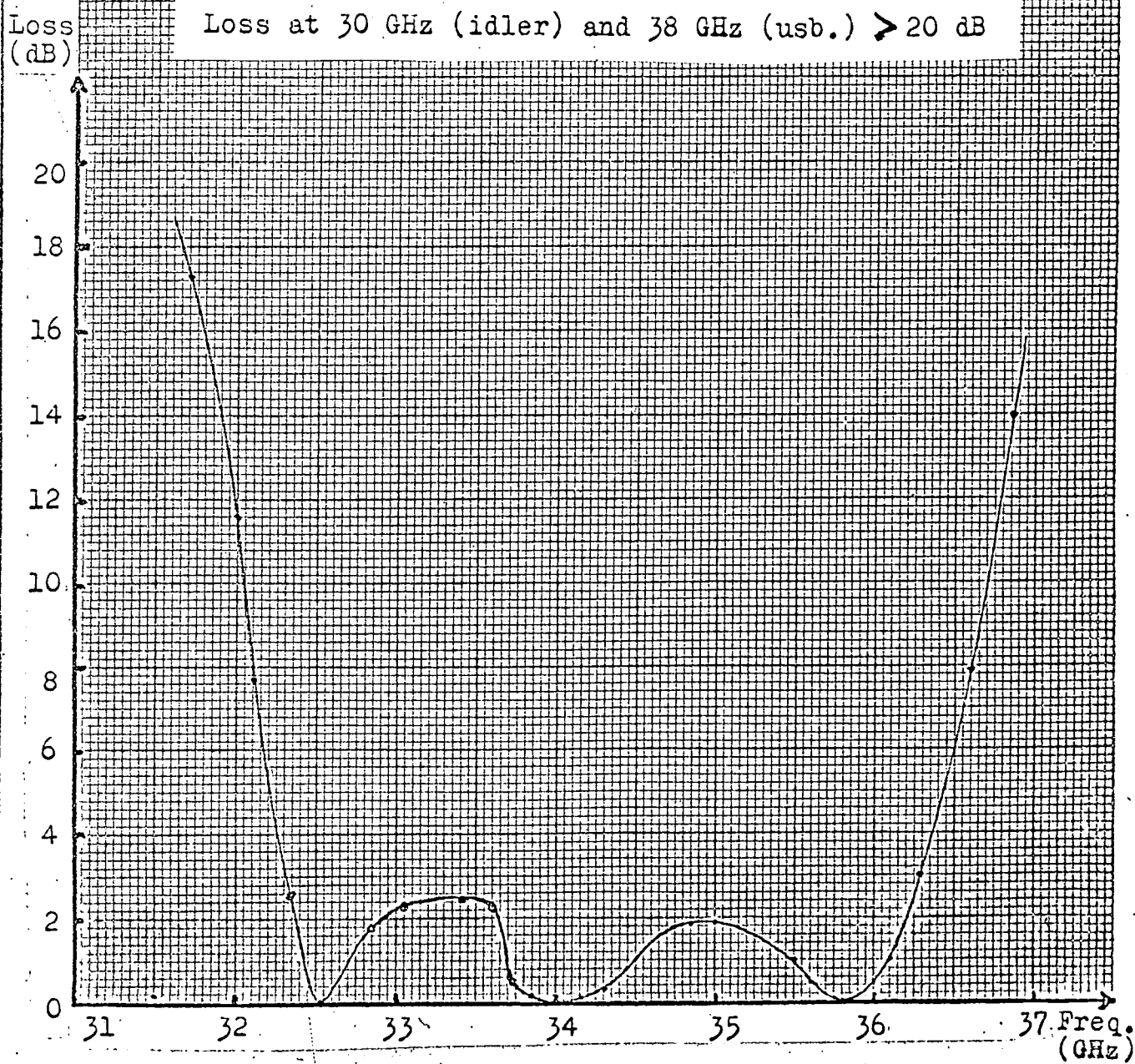
The earlier version of this amplifier (fig. 4.10, referred to as SD4/33B) was constructed with a variable tuning stub behind the diode, and although this stub may not be a multiple of a quarter wavelength at the idler frequency, a limited range of tuning frequencies could be accommodated, presumable because the coaxial idler reject choke was fulfilling its function. This variable tuning arrangement was, however, mechanically unstable and was finally abandoned in favour of the fixed stub device.

The fixed stub device (SD4/33A) is shown mounted on a liquid helium coolable feed system in fig. 4.9, attached to a Western Microwave circulator, and the SD4/33B device is shown on a similar feed in fig. 4.10 with a Marconi circulator.

Case No.

P.O. Engineering Department Research Station, Dollis Hill, LONDON, N.W.2.

Fig. 6.10
Frequency Response of Pump Line Bandpass Filter
Centred on 34 GHz



6.3 Tuning Procedure

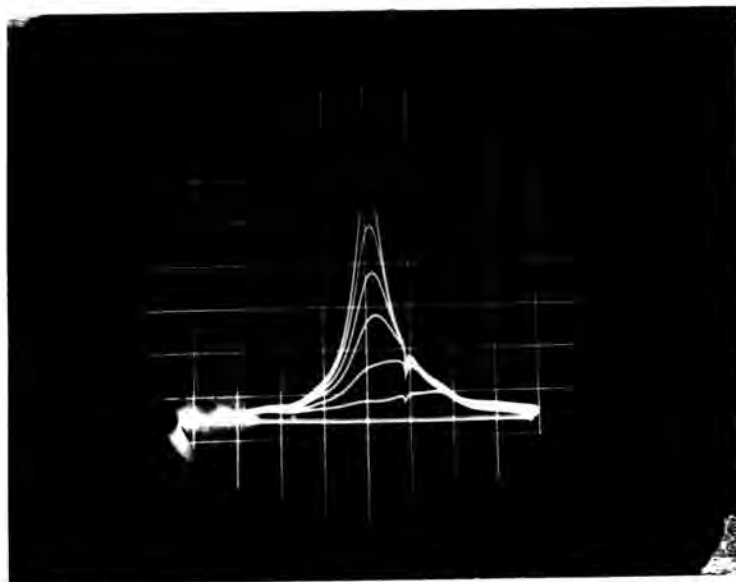
At room temperature an amplifier can be conveniently tuned for minimum diode current, which is obtained when the signal and idler circuits are at resonance. The amplifier is fed from a swept frequency signal source and with a given signal circuit resonant frequency (i.e. a given sleeve diameter in amplifier SD4/33A or a fixed signal circuit resonator position in SD4/19) and a pump input level of about 100 mW, the pump frequency is adjusted until a response is obtained, when the pump level is reduced to give the required gain. The pump frequency for minimum diode current at a fixed gain can then be found. Finally the pump circuit quarterwave plunger is adjusted so that the minimum pump power is required, which ensures that it is not, by chance, resonating the pump cavity at the upper sideband frequency. If the frequency response is not at the desired band centre frequency, the signal circuit resonant conditions are changed and the process is repeated until the desired centre frequency is achieved.

At liquid helium temperature the diode current is so small that the minimizing technique cannot be used. It is thus usual to pretune an amplifier at room temperature for minimum diode current, which must be done at about 200 MHz below the required signal frequency to take into account the decrease in diode junction capacitance on cooling. The final tuning when cool may be carried out for minimum pump power, but this is usually difficult since a coolable pump circuit will usually be long and highly reactive. At 4°K it is thus better to tune for a minimum noise temperature, when least pump heating will be observed (see section 9).

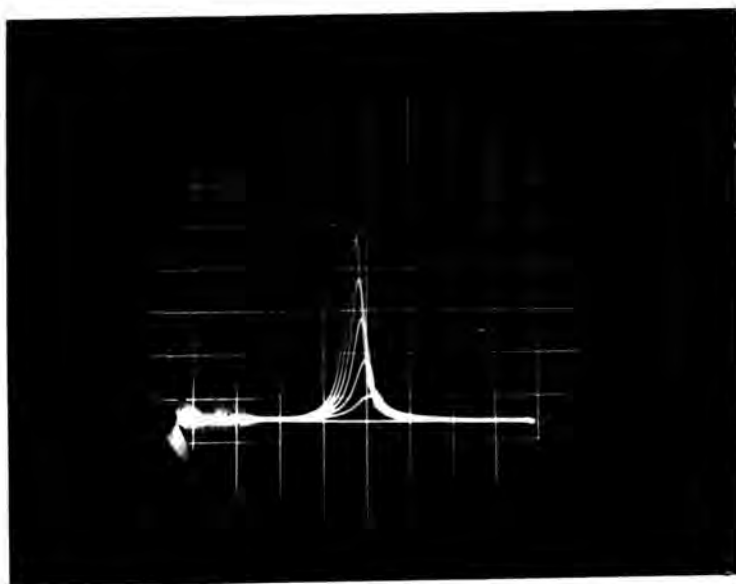
6.4 Bandwidth Measurements

Measurements of the frequency response of amplifier SD4/19 at room temperature and SD4/33 at liquid helium temperature, obtained using the circuit shown in fig. 4.1, are illustrated in figs. 6.11 to 6.14. Figs. 6.11 and 6.12 are for various tuning positions of the SD4/19 device, resulting in different gain bandwidth products and diode currents. It is shown in section 8 that a few microamps of diode current is permissible in room temperature amplifiers without serious degradation in noise temperature, which means that the maximum available bandwidth from the SD4/19 amplifier is 60 MHz at 15 dB gain. Greater bandwidths, which are produced at prohibitively high diode currents, (fig. 6.12) correspond to the significantly higher diode γ values that are obtained by pumping into forward conduction. At currents of 1 mA or more, such bandwidths are enhanced by lower circuit Q values due to increased diode series resistance, (see equation 2.11 section 2.4).

Fig. 6.13 Frequency Response of SD4/33 Amplifier at Room Temperature
showing Effect of Increasing Pump Power. (for $R_g = 10\Omega$)



Trace (from top)	Gain (dB)	Centre frequency (GHz)	Band- width (MHz)
1	16.0	3.914	65
2	15.5	3.917	68
3	14.5	3.920	75
4	12.5	3.931	92

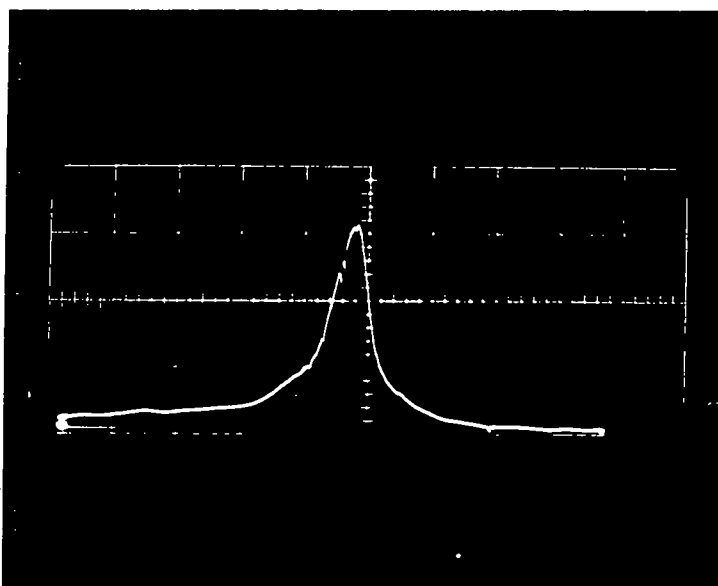


5	27.0	3.892	19
6	26.0	3.896	21
7	24.6	3.900	25
8	21.0	3.903	39
9	17.6	3.908	57
10	16.0	3.917	65

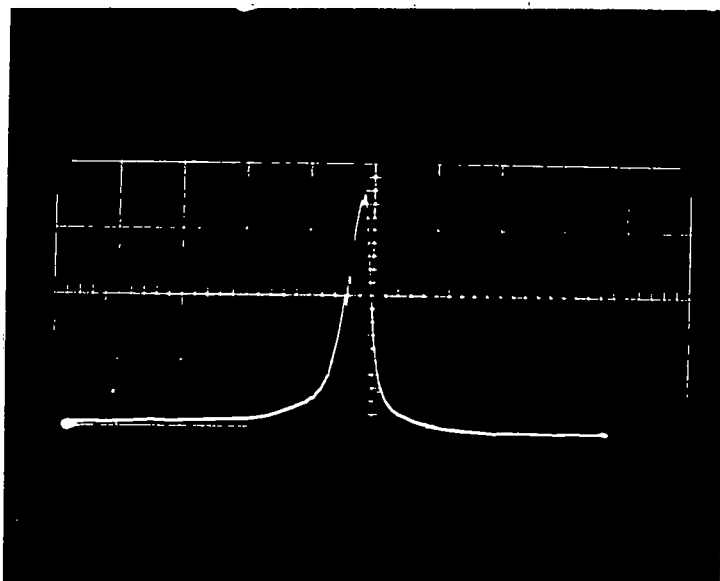
Calibration 50 MHz/Division

Fig.6.14

Gain/Frequency Response of SD4/33 Amplifier
at Liquid Helium Temperature.

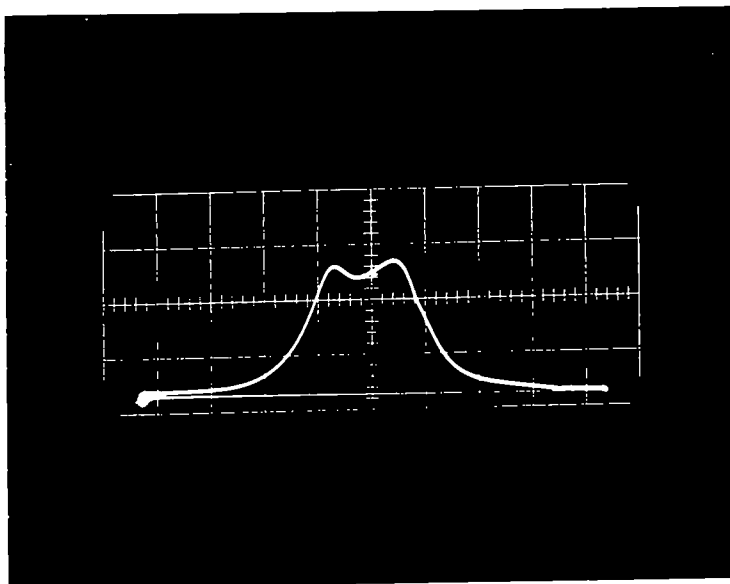


102 MHz Bandwidth
10 dB Peak Gain

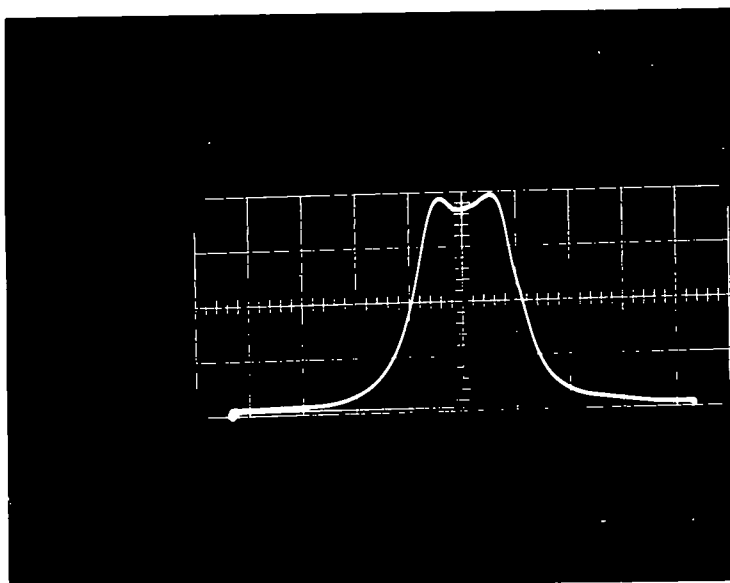


60 MHz Bandwidth
15 dB Peak Gain

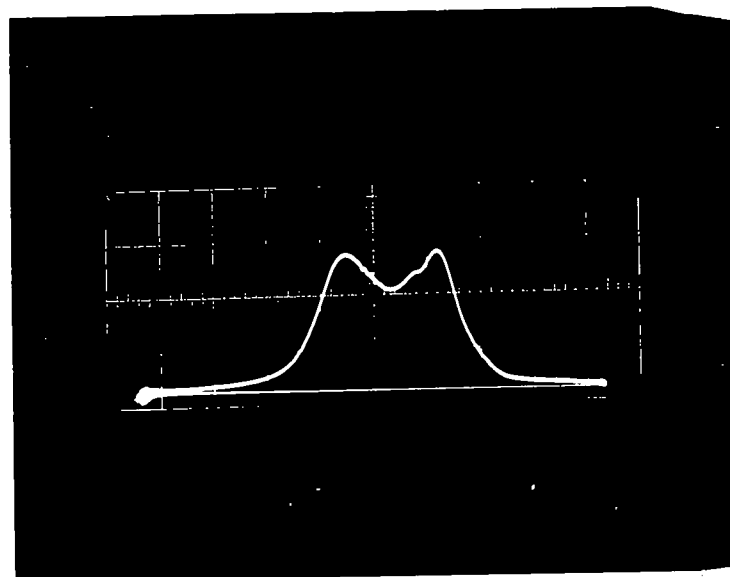
Fig. 6.11 Gain/Frequency Response of SD4/19 Amplifier
at various Pump Levels.



15 dB Peak gain
 50 MHz Bandwidth
 0.4 μ A Diode current

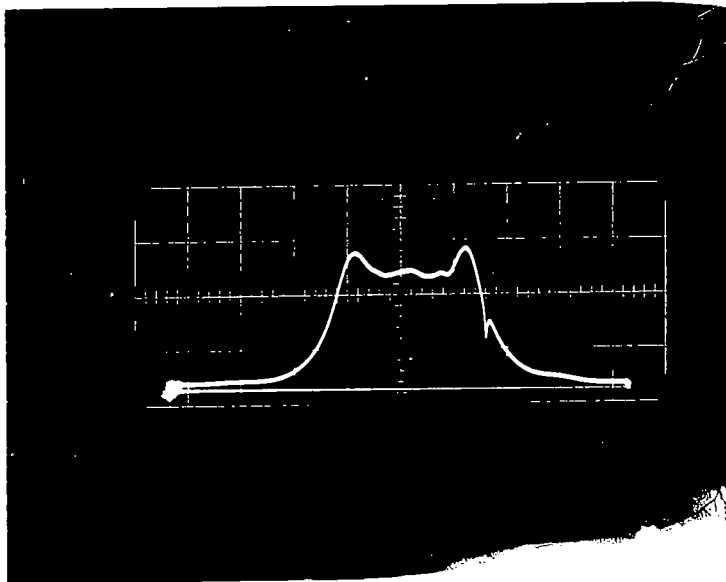


17 dB Peak gain
 45 MHz Bandwidth
 1.5 μ A Diode current

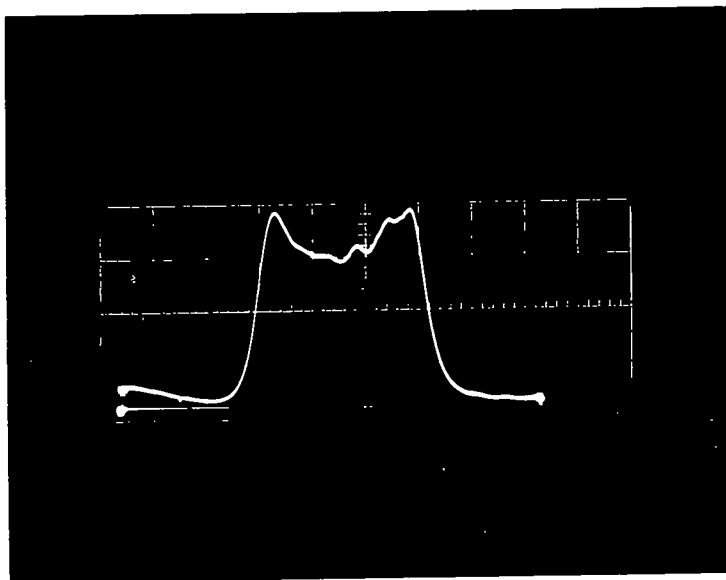


15 dB Peak gain
 60 MHz Bandwidth
 3.5 μ A Diode current

Fig. 6.12 Gain/Frequency Response of SD4/19 Amplifier
at various Pump Levels.



15 dB Peak gain
80 MHz Bandwidth
20 μ A Diode current



12 dB Peak gain
310 MHz Bandwidth
2 mA Diode current

Figs. 6.13 and 6.14 show a series of responses of amplifier type SD4/33 for various pump levels, at room and liquid helium temperatures respectively. With a generator/load impedance of 8Ω the acceptable limit of diode current for this device occurs at 15 dB gain, when a $\text{gain}^{\frac{1}{2}} \cdot \text{bandwidth}$ product of 310 MHz (varying slightly from diode to diode) is realised, which compares favourably with the value of 280 MHz predicted from section 3.5. (This theoretical figure has been reduced by 0.72 to account for distributed signal circuit inductance.) With such circuit conditions some 1 to 4 μA diode current is measured at room temperature, or $< 0.01 \mu\text{A}$ (due to an increase diode γ value) at liquid helium temperature.

Both SD4/19 and SD4/33 amplifiers demonstrate the theoretical prediction that the bandwidth at a given gain can be increased by increasing the generator/load impedance. Such an increase requires a higher diode γ value, which may only be attained when the diode is driven into forward conduction. Table 6.1 gives typical diode currents and $\text{gain}^{\frac{1}{2}} \cdot \text{bandwidth}$ products for a range of R_g values in this amplifier. The increasing bandwidth shown in figs. 6.11 and 6.12, for the SD4/19 device, is also for increasing R_g (approximately 10 to 20Ω) which in this device is infinitely variable but difficult to evaluate with accuracy.

Table 6.1
Typical $\text{Gain}^{\frac{1}{2}} \cdot \text{Bandwidth}$ Products and Diode Currents
for various Generator/Load Impedances
(amplifier SD4/33)

$R_g (\Omega)$	Current at 15 dB gain (μA)	$\text{gain}^{\frac{1}{2}} \cdot b (\text{MHz})$
8	3	310
11	12	360
12.3	20	400
13.4	30	450

The variation of centre frequency with amplifier gain, on increasing the pump power, and the maintenance of a constant $\text{gain}^{\frac{1}{2}} \cdot \text{bandwidth}$, are both clearly seen in fig. 6.13 to be as predicted in sections 3.6 (see fig. 3.4) and 3.4 respectively. A slight reduction in $\text{gain}^{\frac{1}{2}} \cdot \text{bandwidth}$ was observed on cooling to 4.2°K (300 compared with 350 MHz).

The photographs of the SD4/33 amplifier response at liquid helium temperature (fig. 6.14) were of sufficiently long exposure (0.1 sec) to indicate that any instability due to bubbling (section 4.4) was completely absent. (Similar exposures of some broad-band responses (fig. 7.12) illustrate the effect.)

7. Broad-band Structures

7.1 General

An amplifier producing 30 dB gain over a 500 MHz band centred at 4 GHz, with a noise temperature of 20°K or less, is required for the proposed satellite communication system. Such a bandwidth may be achieved with two 15 dB cascaded stages or two stagger tuned 250 MHz, 30 dB stages. However, an individual stage producing more than 15 dB is undesirable from the gain stability point of view (section 2.7), although an elaborate pump frequency/level stabilisation system may allow operation at a higher gain. Operation of three 10 dB stages in cascade would probably eliminate the need for any form of pump stabilisation circuitry, but the requirement of three amplifiers and associated pump supplies, power units etc. may be undesirable.

It has been seen that to achieve the required noise temperature with present day varactors, at least the first stage of amplification has to be cooled to less than 20°K and for stages of less than 20 dB gain the second stage must also be cooled to at least 77°K. It was therefore decided to construct two broad-band 15 dB gain, liquid helium cooled stages in cascade.

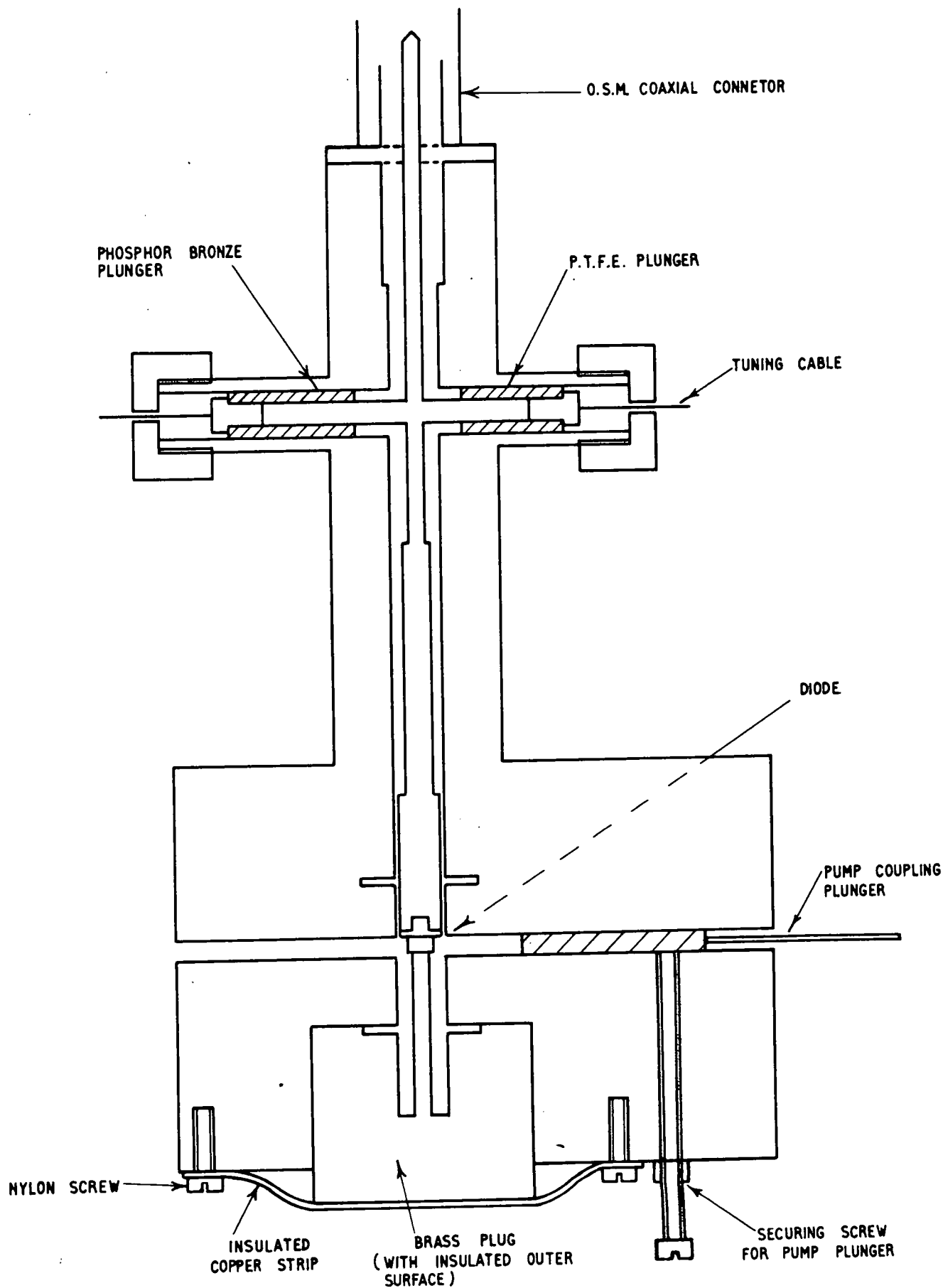
7.2 Additional Passive Signal Circuit Networks

7.2.1 Realisation

Passive reactance compensation of a 4 GHz signal circuit is most easily achieved with quarterwave coaxial resonators. One stage of additional tuning may be realised with a tunable, parallel fed, quarterwave short circuit, one half wavelength away from the diode, when the required series/parallel arrangement of fig. 3.15 is realised. The characteristic impedance of the quarterwave resonator must be chosen so that equation 3.16 is fulfilled, which means that for a given resonator at a required band centre frequency, there is only one correct tuning position and reactance slope, for a defined response shape. A more flexible arrangement (Aitchison et al 1967), producing a range of reactance slopes at a fixed band centre frequency, involves the use of a $\frac{1}{8}$ wavelength tunable short circuit and a $\frac{1}{8}$ wavelength tunable open circuit, one on either side of the signal circuit coaxial line. (Fig. 7.1). The short circuit may be formed with phosphor bronze or brass and the open circuit with a dielectric (e.g. PTFE). When both stubs are symmetrically placed about the coaxial signal line, the circuit is identical to a quarterwave resonator, but movement of the stubs away from such a position allows a range of resonant frequencies, or reactance slopes for the same resonant frequency, to be accommodated. The admittance of the pair of stubs may be written:

$$Y = Y_0 \tan \frac{2\pi l_1}{\lambda} - Y_0 \cot \frac{2\pi l_2}{\lambda}$$

FIG. 7.1. CROSS SECTION THROUGH SINGLE DIODE AMPLIFIER SD4/33
WITH ONE ADDITIONAL RESONATOR



AMENDMENT PARTICULARS		POST OFFICE RESEARCH STATION LONDON, N W 2		ASSEMBLY DRG	
ISSUE					
ORIGIN					
CKD.					
APPD.					
DATE	7-8-68				
D.O. SERIAL NO	C105				
CASE NO	20836				

where Y_0 is the characteristic admittance of the line forming the stubs, and l_1 and l_2 are the lengths of the open and short circuits respectively.

Additional quarterwave (or two $\frac{1}{8}$ wave) resonators can be added at quarter wavelength intervals to form additional stages of reactance compensation. An alternate series/parallel arrangement is maintained since the second of a pair of such resonators is transformed by the first to a series circuit.

A double tuned device (one additional resonator) based on the SD4/33 amplifier is illustrated in fig. 7.2 with part of the signal circuit removed to show the internal structure. A circuit, also based on the SD4/33 amplifier, with 2 additional resonators is shown in fig. 7.3.

In order to use a passively multituned device at liquid helium temperature the additional resonators must be externally adjustable. Previous experience with gears and long mechanical drives on the SD4/19 amplifier (fig. 6.5) was discouraging. A more successful method was developed involving the use of Bowden cables attached directly to the sliding open and short circuits and is shown in fig. 7.4. Cap C is fastened down with a vacuum seal whilst evacuating the dewar prior to filling with liquid helium. After filling, the cap can be removed for the necessary adjustments of the tuned circuits, the cables from which are individually locked with screws. The amplifier shown in fig. 7.2 was designed for use with this arrangement.

7.2.2 Measurements

The double tuned response of the SD4/33 device, at room temperature, with one additional resonator is shown in fig. 7.5. A bandwidth of 130 MHz at 15 dB gain, for less than $5 \mu\text{A}$ diode current, is produced which represents a $\text{gain}^{\frac{1}{4}}$. bandwidth product of 310, compared with the theoretical figure of 280. A response for the device with two additional resonators is illustrated in fig. 7.6, and that for a device with three additional resonators in fig. 7.7. These latter two responses involve considerable diode current ($> 100 \mu\text{A}$) and therefore represent gain^{2n} . bandwidth products in excess of theoretical values for zero current.

7.3 Additional Active Signal Circuit Networks

7.3.1 Realisation

The relatively poor bandwidth capability of the passively compensated device and complexity of tuning at liquid helium temperature (particularly if 2 or 3 devices are required) led to the development of the two diode, actively compensated device, described in section 3.8.2. An early device was made up of two individually constructed SD4/33 amplifiers and operated from two pump supplies. Excessive intermodulation between the two

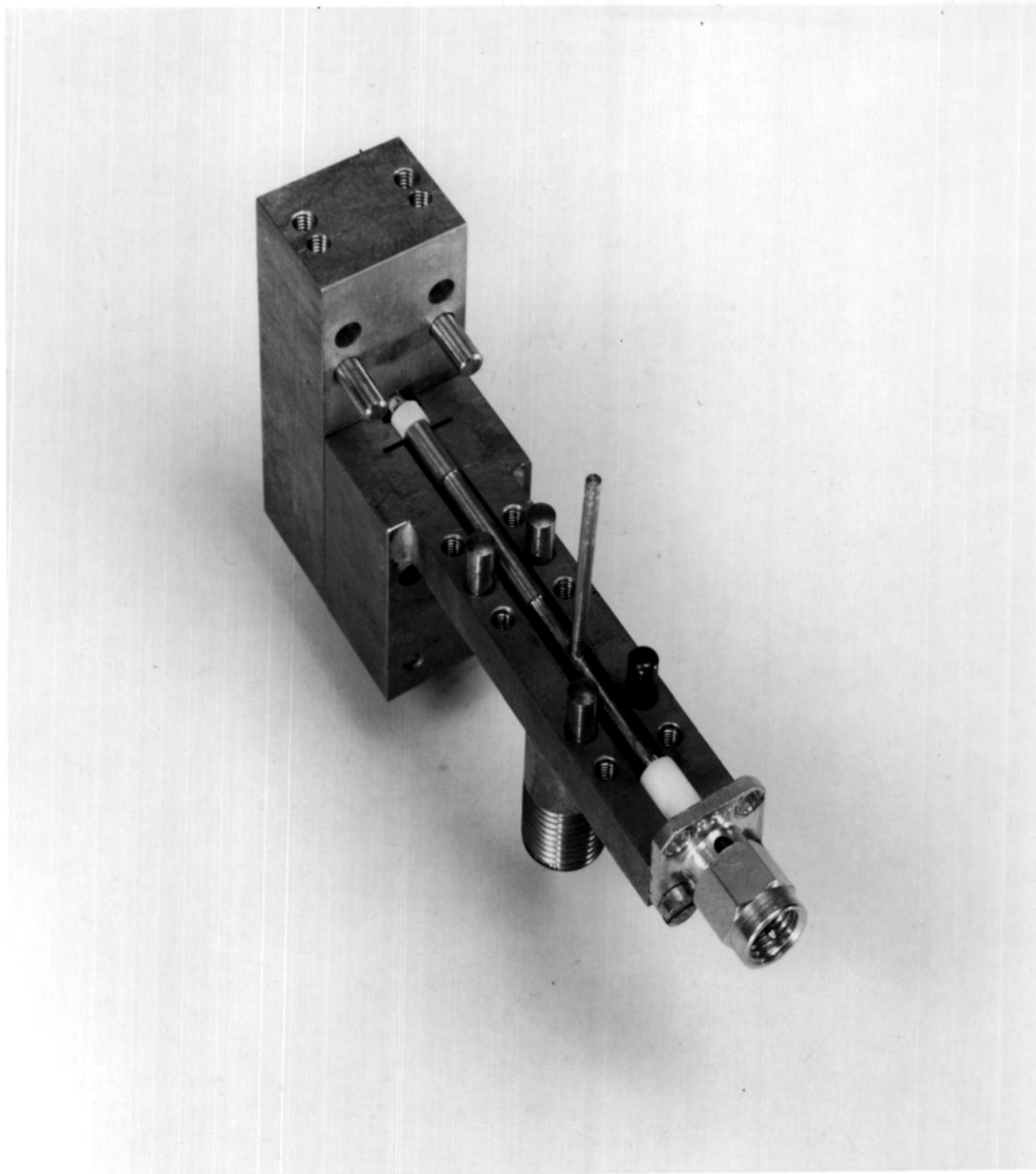


FIG. 7-2 SD4/33 AMPLIFIER WITH ONE ADDITIONAL RESONATOR
 SHOWING INTERNAL CONSTRUCTION

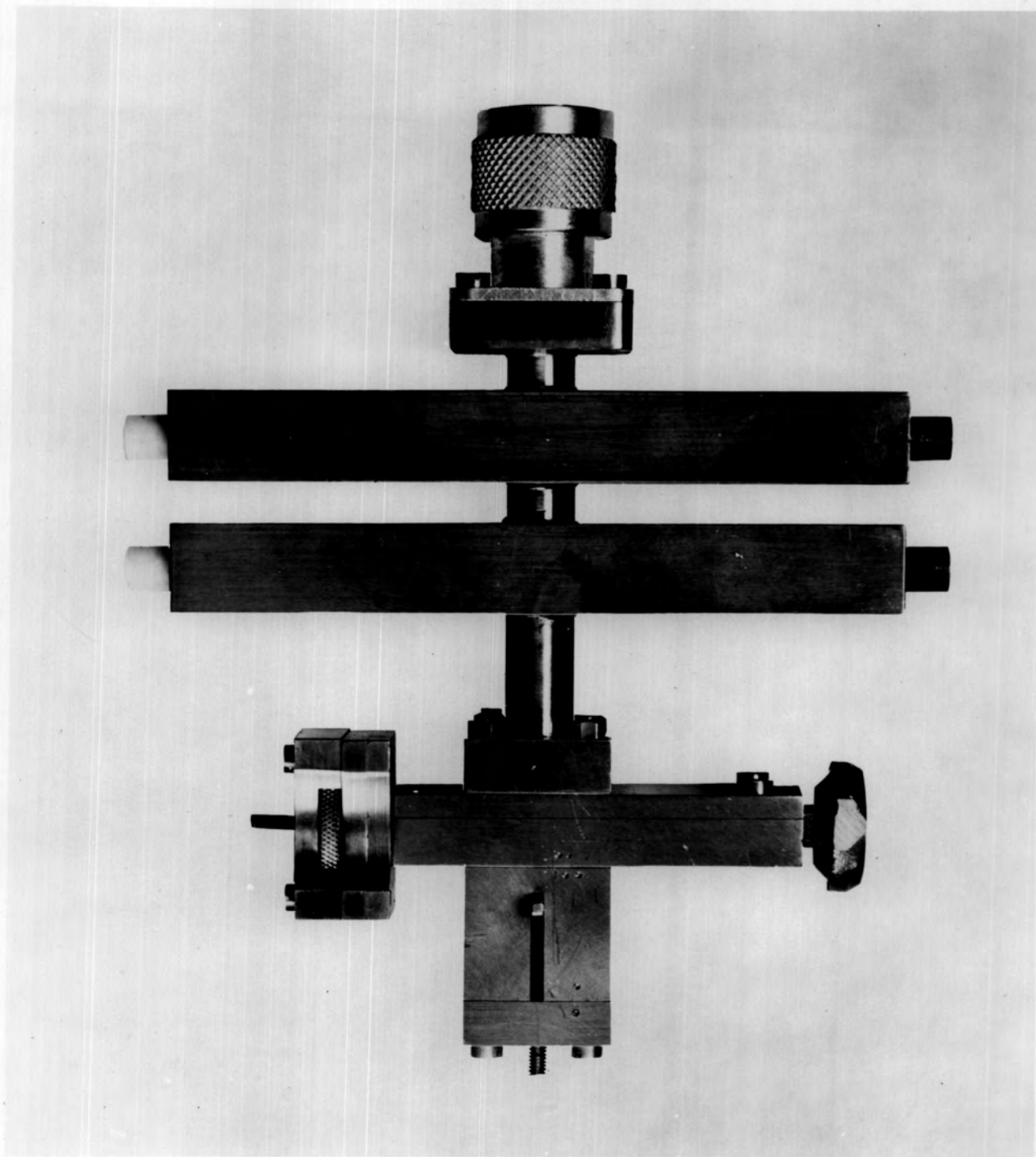
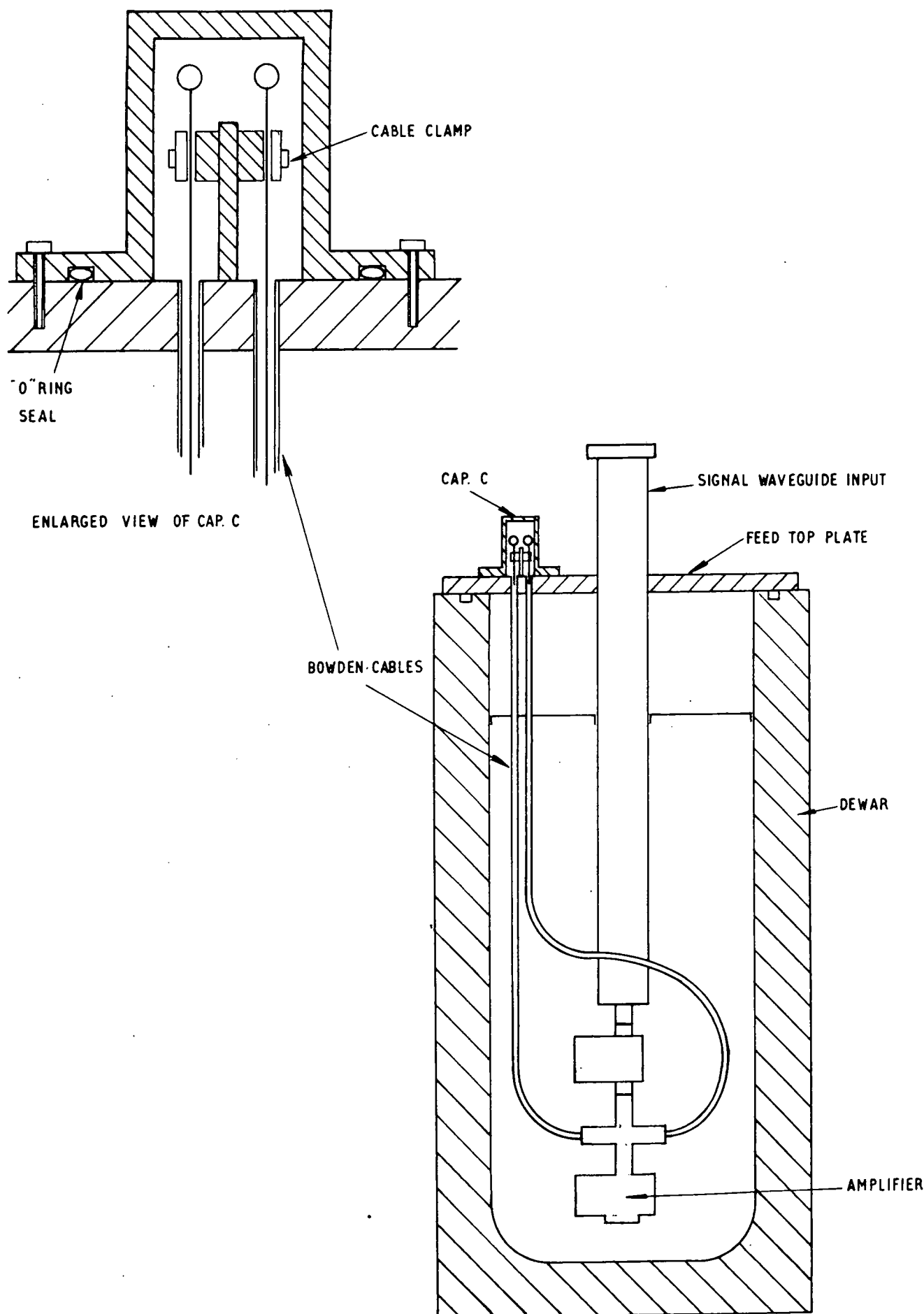


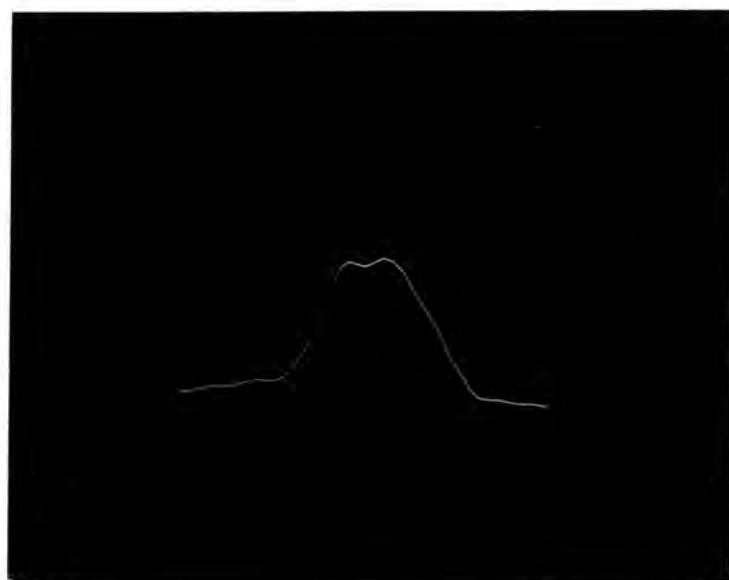
FIG. 7.3. AMPLIFIER TYPE SD4/33
WITH TWO ADDITIONAL RESONATORS.

FIG. 7-4 REMOTE TUNING SYSTEM FOR LIQUID HELIUM TEMPERATURE. SH. 19

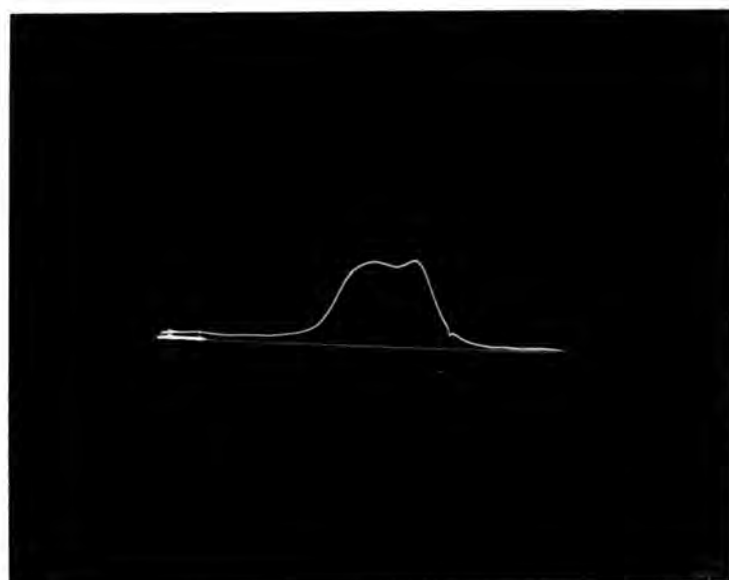


AMENDMENT PARTICULARS						DATE		INITIALS		NO. 20	
ISSUE											
DRN	1-1984										
CKD	Rd										
APPO	PAYABLE										
DATE	7-8-68										
DO SERIAL NO C 793											
CASE NO 20836											
										POST OFFICE RESEARCH STATION LONDON, NW 2	
										ASSEMBLY DRG	

Fig. 7.5 Gain/Frequency Response of SD4/33 Amplifier at Room Temperature with One Additional Resonator.

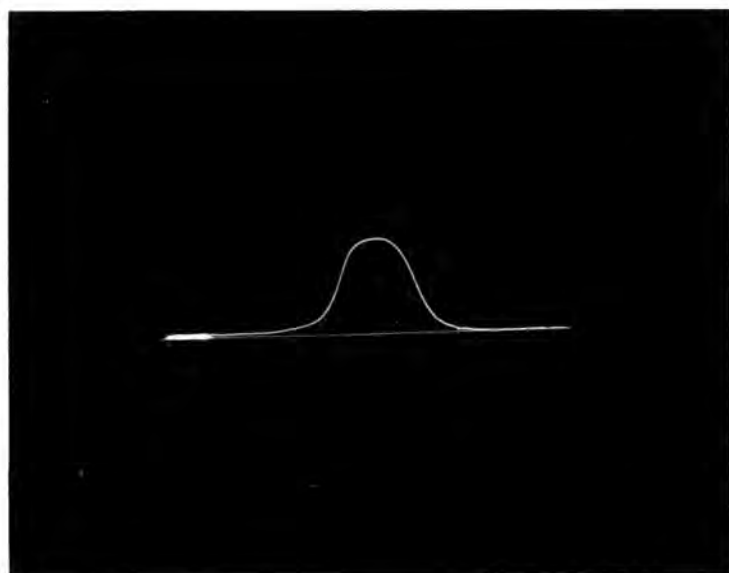


10 dB Gain
160 MHz Bandwidth



Calibration
50 MHz/Div.

15 dB Gain
120 MHz Bandwidth



20 dB Gain
90 MHz Bandwidth

Fig.7.6 Gain/Frequency Response of SD4/33 Amplifier
with Two Additional Signal Circuit Resonators

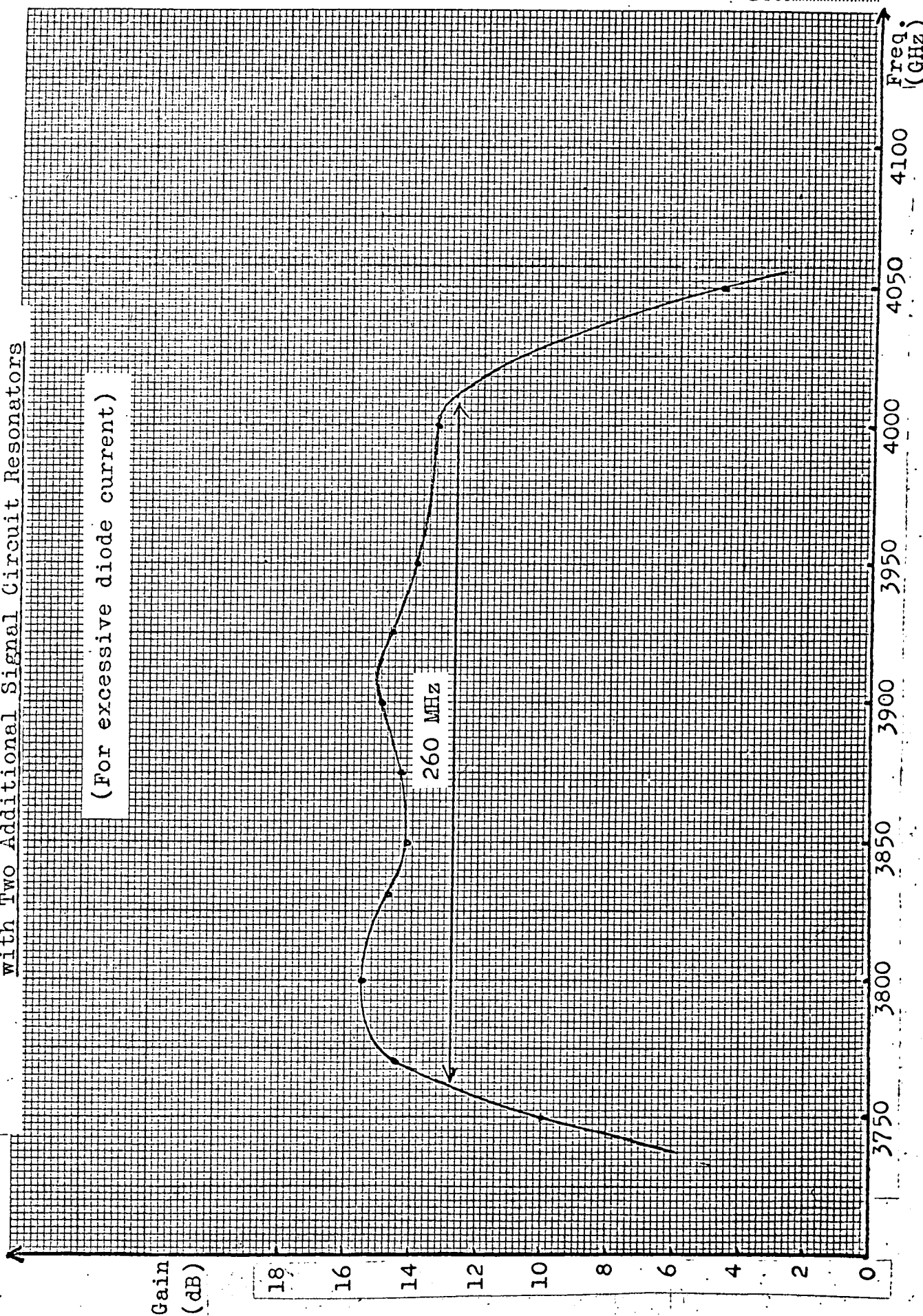
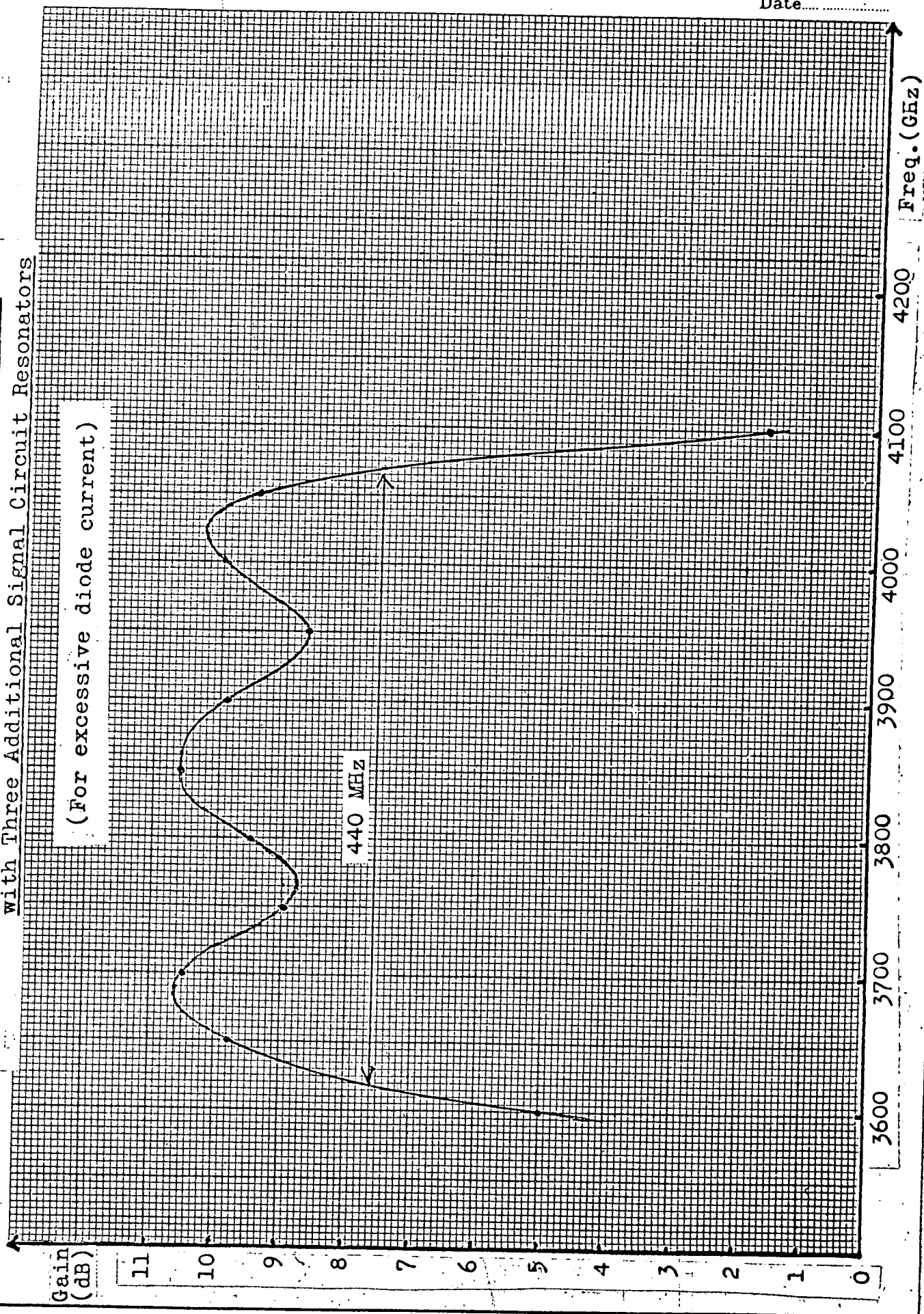


Fig. 7.7 Gain/Frequency Response of SD4/33 Amplifier
with Three Additional Signal Circuit Resonators



pump supplies was observed, so a composite structure of two identical amplifiers was made (fig. 7.8) which, with selected diodes, could be operated from one pump source. The circuit of the device is as drawn in fig. 3.16, and the construction is illustrated in fig. 7.9.

7.3.2 Measurements

The frequency responses shown in figs. 7.10 and 7.11 are realised at room temperature for the device shown in fig. 7.8 with negligible diode current. A $\text{gain}^{\frac{1}{2}}$. bandwidth product of 600-700 is thus realised, (250-280 MHz at 15 dB gain) twice that for a single tuned amplifier, which agrees with the theory of section 3.8.2. Considerable flexibility of tuning is available with this device, through dissimilar pumping and dissimilar tuning of the diodes, but the widest maximally flat bandwidth is realised (with optimum R_g and quarterwave line impedance Z_0), for identical diode conditions. One amplifier pumped and the other unpumped or vice versa gave exactly the gain/frequency responses predicted in figs. 3.24. and 3.26. The quarterwave line impedance (Z_0) was 10Ω and the R_g value was 10Ω , which, being slightly different from the optimum values predicted in the theory (12Ω and 8.6Ω respectively) meant that slightly different pumping conditions were required for each diode for a maximally flat response. Identical pumping for the two diodes produced a response with 4 dB ripple. Some slight improvement in $\text{gain}^{\frac{1}{2}}$. bandwidth may therefore be available if the values of R_g and Z_0 are corrected.

At liquid helium 250 MHz, 3 dB bandwidth, was achieved for two stages in cascade (30 dB gain) but instability from liquid helium bubbles was evident with the particular pump circuit used. Fig. 7.13 shows the response of a single stage at liquid helium temperature and illustrates this gain instability. (The ragged appearance of these responses is partially due to the feed and circulator responses as in fig. 4.6 and partially due to noise on a low frequency amplifier used in this particular measurement.) The noise temperature was checked with a single measurement at liquid helium temperature and found to be the same as that for a single diode device ($\approx 16-20^\circ\text{K}$), within experimental limits, but this measurement was made extremely difficult by the gain instability due to helium bubbles. A reorganisation of the pump circuit is expected to cure such instability as it did with the single diode device.

Fig. 7.14 shows one of these devices mounted on a liquid helium feed system and figs. 7.15 and 7.16 show two devices in cascade in the final realisation of the broad-band liquid helium cooled device. The two pairs of pump lines run out of the dewar, through attenuators (one in each line), until each pair is combined with a 3 dB coupler, which is fed from the pump klystron. This

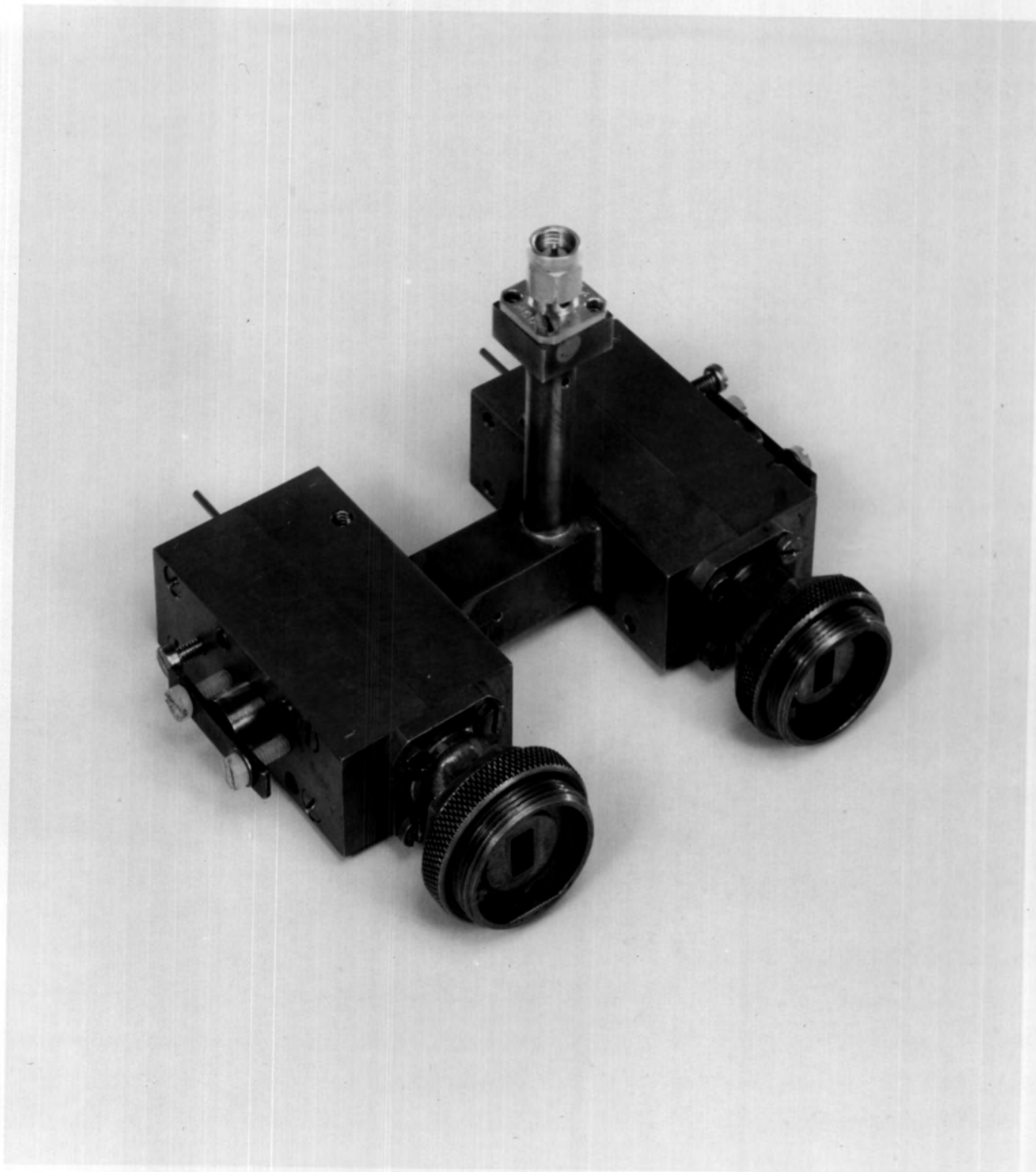
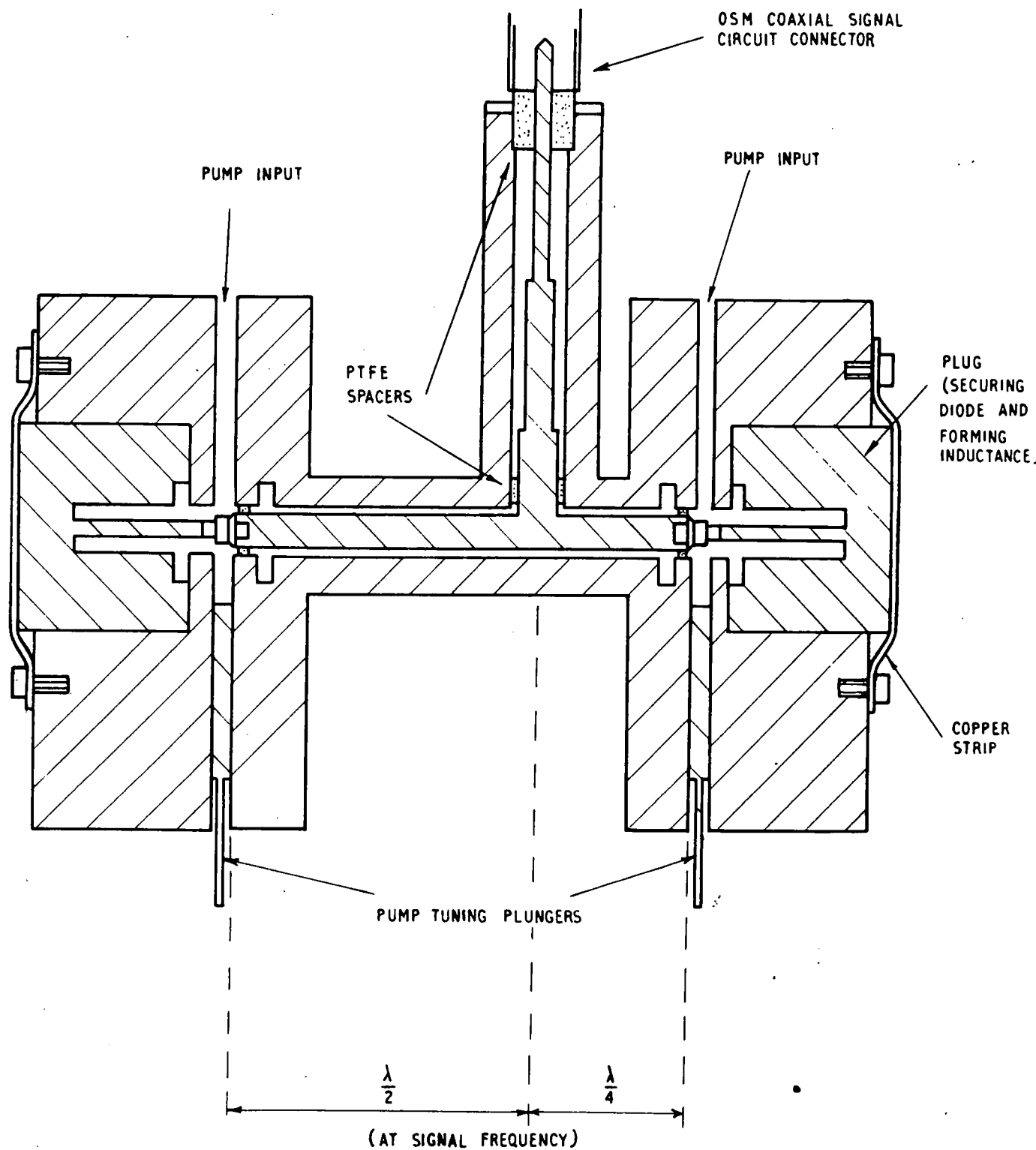
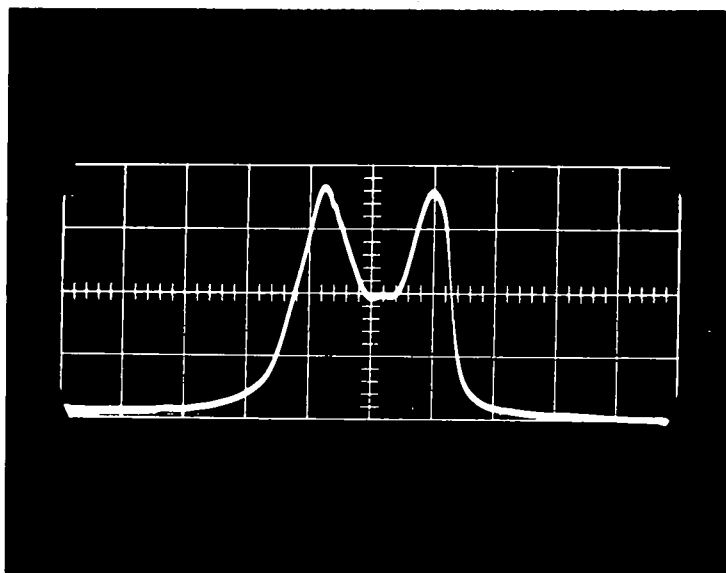


FIG. 7-8 FINAL TWO DIODE AMPLIFIER (T04/33) USING
SIGNAL CIRCUIT ACTIVE COMPENSATION

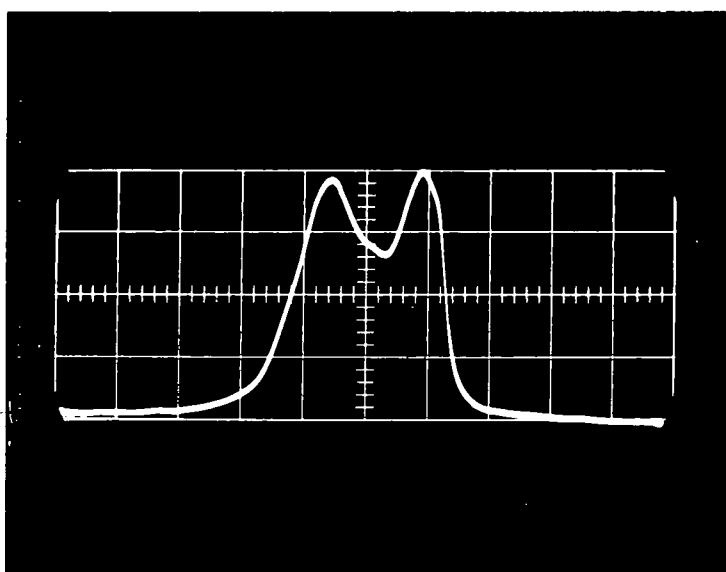
FIG.7.9 TWO DIODE ACTIVELY COMPENSATED AMPLIFIER
CROSS SECTION

DRN	Revised	ISSUE		AMENDMENT PARTICULARS	DATE	INITIALS	R1.4.5	R6.3.1.
CKD	WJ							
APPD	WJ							
DATE	7-8-68							
D.O. SERIAL NO	G195							
CASE NO	20836							
				POST OFFICE RESEARCH STATION	LONDON, NW2			
				ASSEMBLY DRG				

Fig. 7.10 Gain/Frequency Response of broad-Band Amplifier
with Signal Circuit Active Compensation.
At Room Temperature.

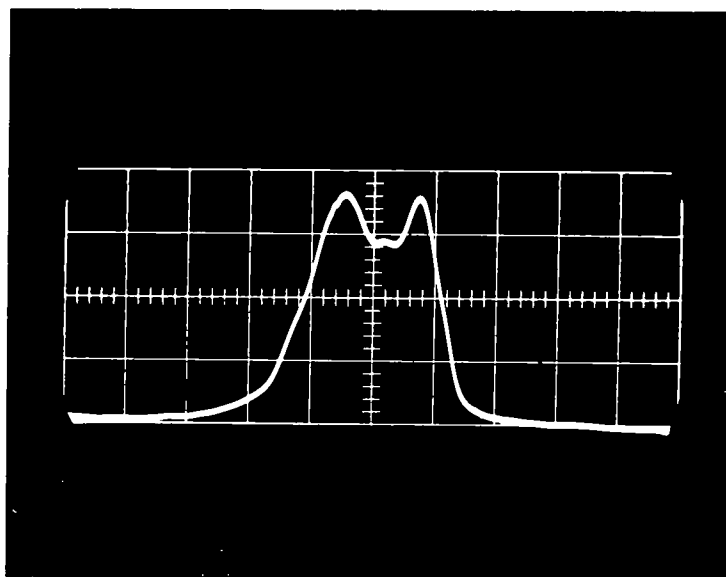


280 MHz Bandwidth
15 dB Peak Gain
3 dB Ripple

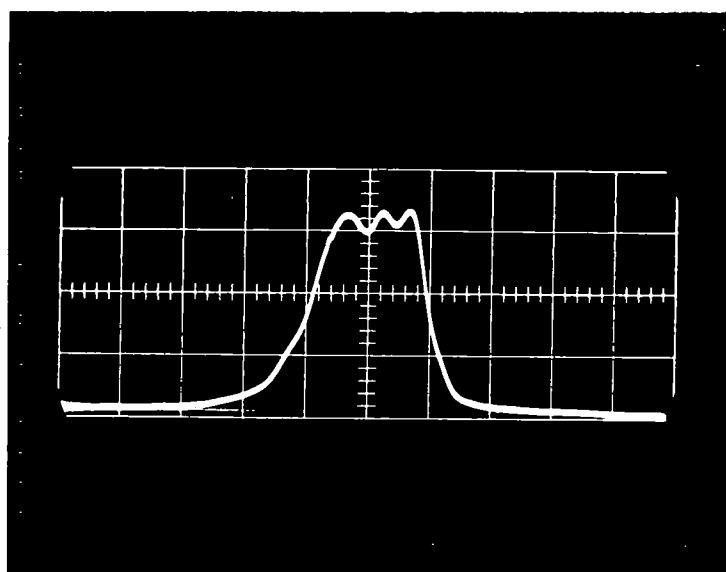


250 MHz Bandwidth
15 dB Peak Gain
1.5 dB Ripple

Fig. 7.11 Gain/Frequency Response of Broad-Band Amplifier
with Signal Circuit Active Compensation.
At Room Temperature.

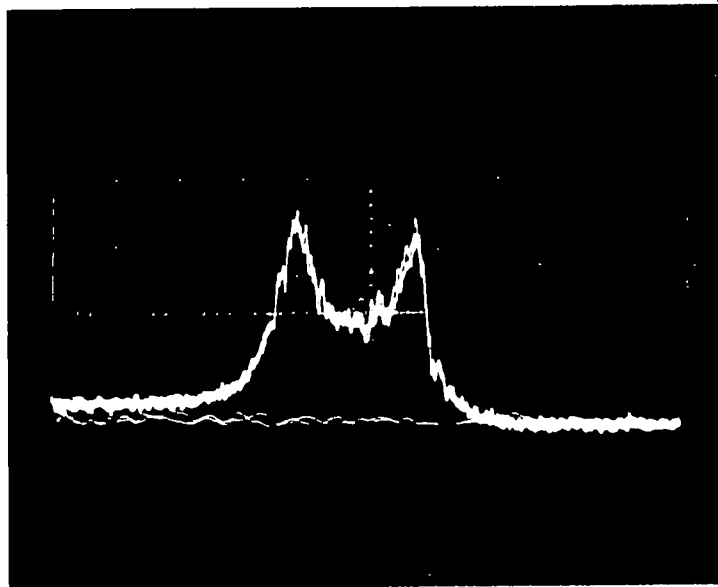


240 MHz Bandwidth
15 dB Peak Gain
1 dB Ripple

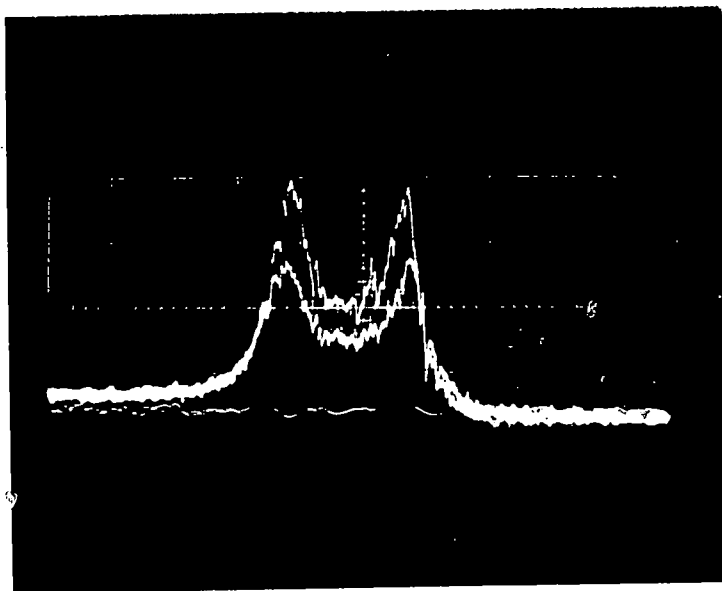


220 MHz Bandwidth
15 dB Peak Gain
0.1 dB Ripple

Fig. 7.12 Gain/Frequency Response of Broad-Band Amplifier
with Signal Circuit Active Compensation.
At Liquid Helium Temperature.



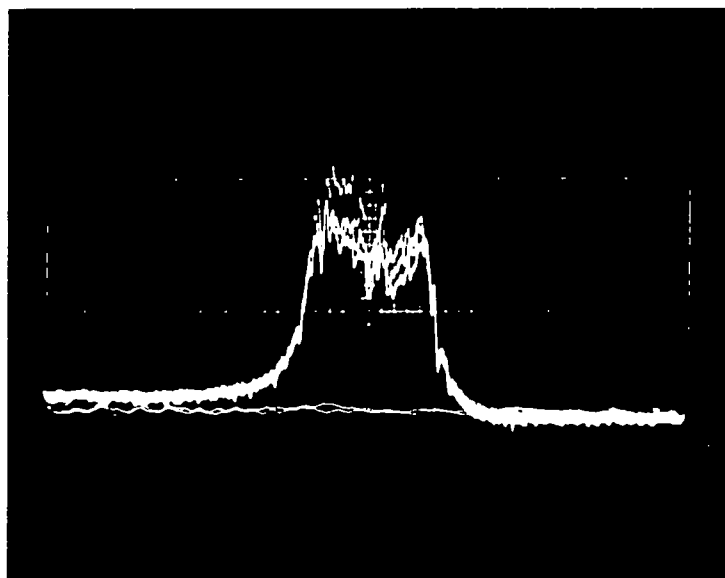
250 MHz Bandwidth
15 dB Peak Gain
3 dB Ripple



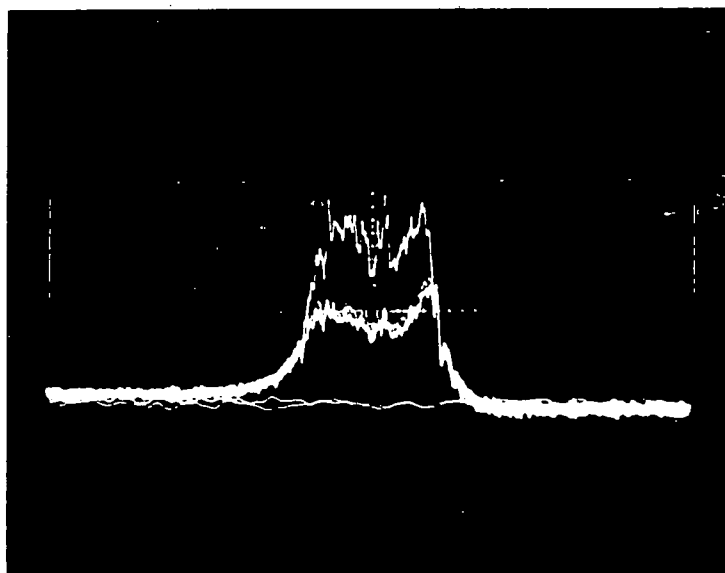
As above.
(showing pump level
instability due to
helium bubbles)

Fig. 7.13 Gain/Frequency Response of Broad-Band Amplifier
with Signal Circuit Active Compensation,
At Liquid Helium Temperature.

(Showing gain instability due to helium bubbles.)



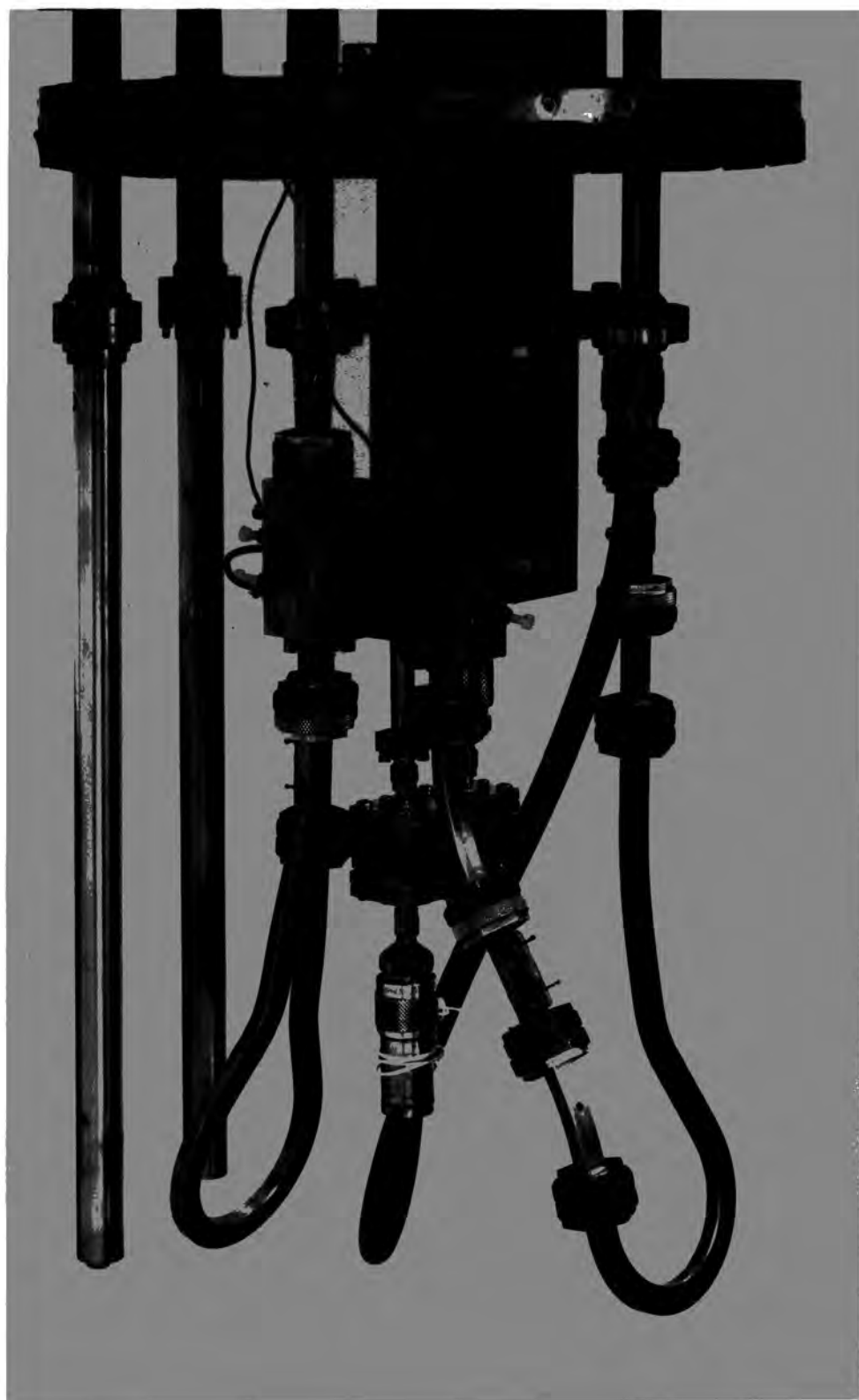
200 MHz Bandwidth
15 dB Peak Gain
(mean value)
1 dB Ripple



As above.

FIG. 7-14

TWO DIODE AMPLIFIER ON COOLABLE FEED



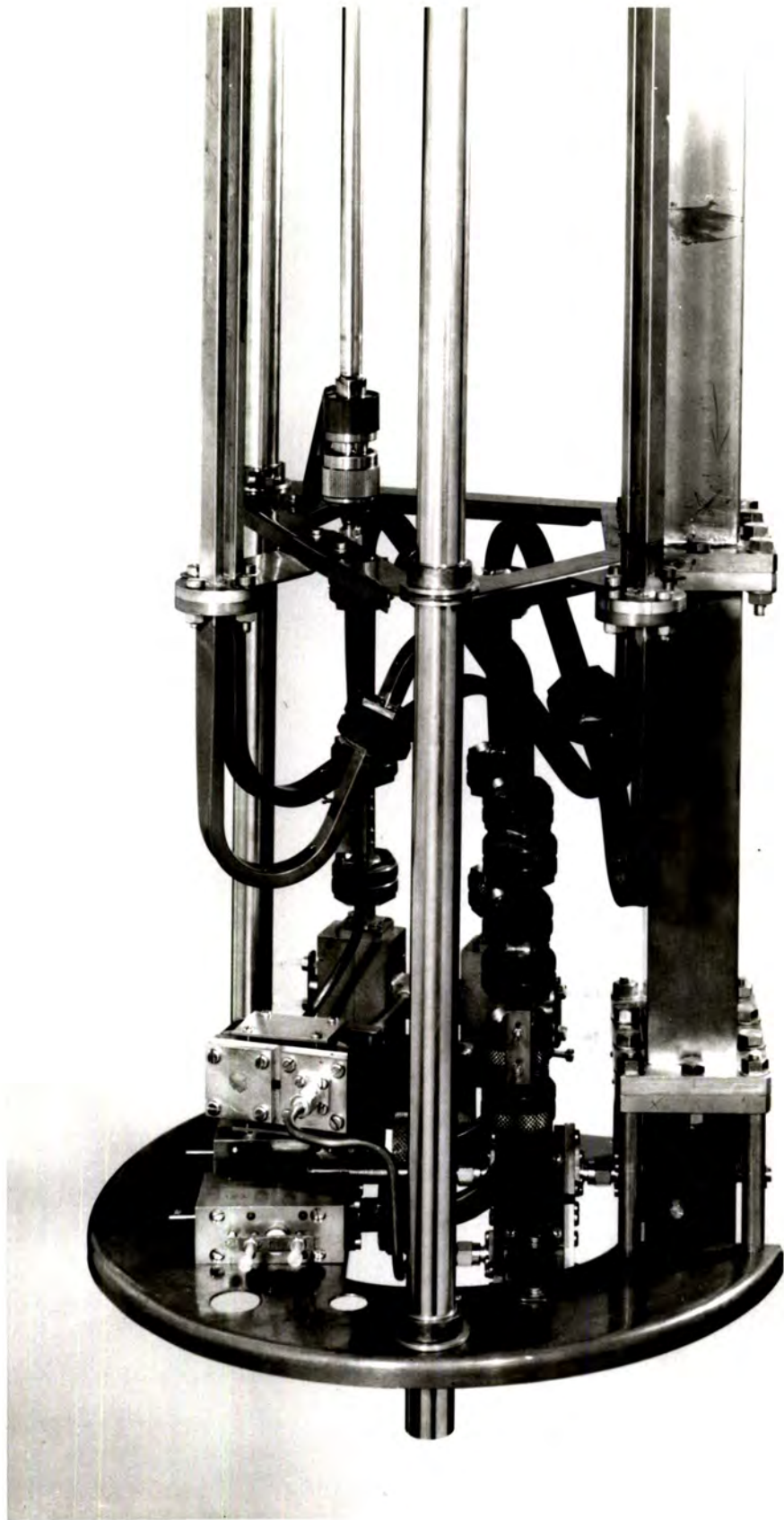
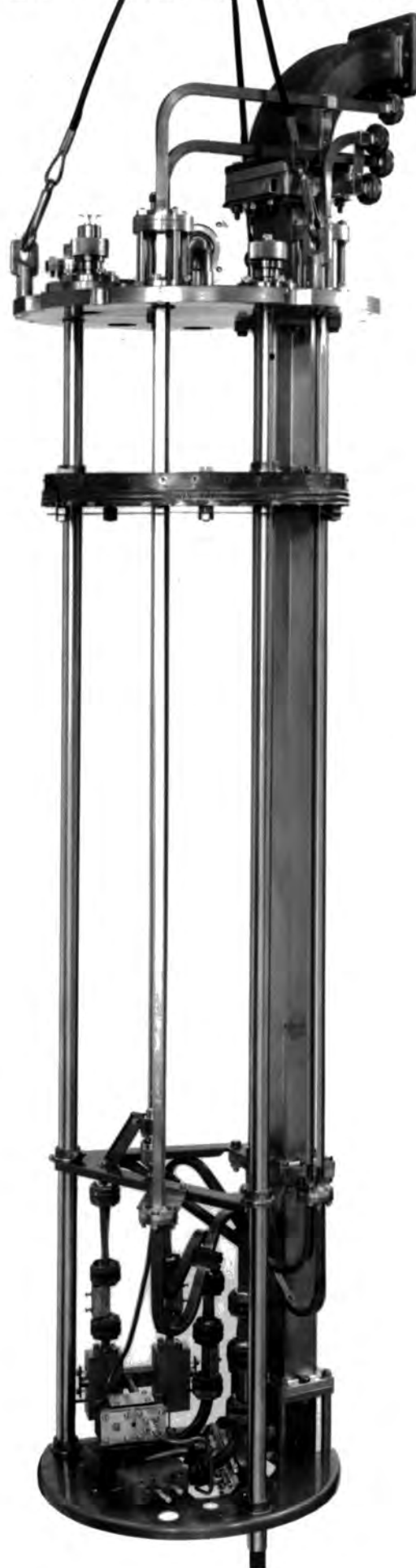


FIG.7-15

BROAD-BAND LIQUID HELIUM COOLED AMPLIFIER
USING TWO TQ4/33 STAGES IN CASCADE

FIG.7-16 BROAD-BAND LIQUID HELIUM COOLED AMPLIFIER SHOWING COOLABLE FEED



arrangement allows the necessary tuning of the individual diodes for a maximally flat response at liquid helium temperature: Each device was pretuned at room temperature, to some 200 MHz below the band centre frequency, by inserting the appropriate sleeve for each signal circuit inductance. On cooling, the band centre frequency difference was taken up by the change in diode capacitance, so that only the pump frequencies of the two stages and the pump line attenuators need be adjusted.

It was possible to stagger tune, rather than cascade, the two amplifier stages in the final design, to produce 500 MHz bandwidth at 15 dB gain as shown in fig. 7.17.

7.4 Additional Active Idler Circuit Networks

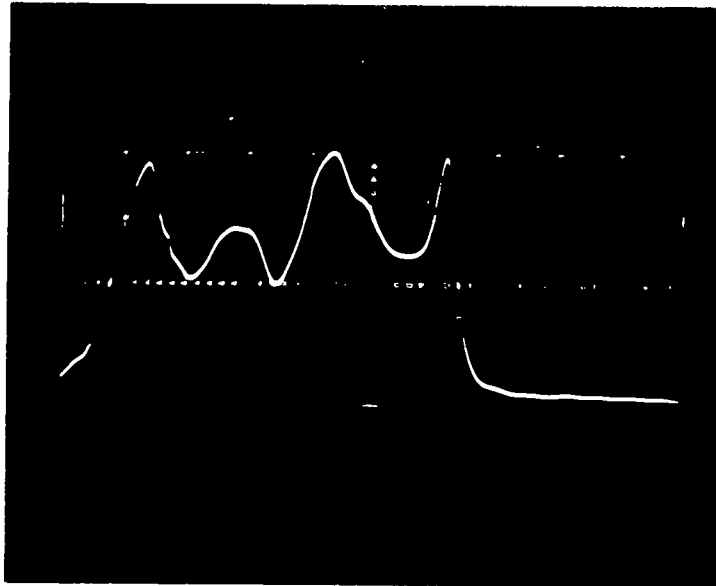
A device employing active compensation at the idler frequency was developed simultaneously as an alternative to the signal circuit actively compensated device and is illustrated in figs. 7.18 and 7.19. The circuit is basically two SD4/33 amplifiers combined in the idler circuit, with an additional quarterwave section in one diode circuit. Both diode circuits were made to appear identical (series resonant) at the signal frequency, from the signal circuit T junction, by choosing the necessary lengths for each signal circuit inductive plug (fig. 7.18). However, it is noted that the signal phase is slightly different for each diode with such an arrangement, but the idler phase is equal and opposite, so that pumping the amplifier with the same source will result in a reduction of gain ^{$\frac{1}{4}$} . bandwidth. The diameter of the half-wave section (at idler frequency) could be reduced, to increase the circuit inductance and hence equalise the signal phase relationship between the two diodes, but the resulting mismatch may be equally detrimental. Idler circuit active compensation is thus not as easy to realise as signal circuit active compensation.

The development of this device was not pursued further than the earlier stages, when success was achieved with the actively compensated signal circuit device, but a gain ^{$\frac{1}{4}$} . bandwidth product of about 500 MHz was obtained, (300 MHz bandwidth, 10 dB gain, with a 4 dB ripple). The frequency response indicated that idler resonances were occurring additional to those expected from the simple two diode circuit. At 30 GHz such resonances are difficult to eliminate from the two diode coaxial structure.

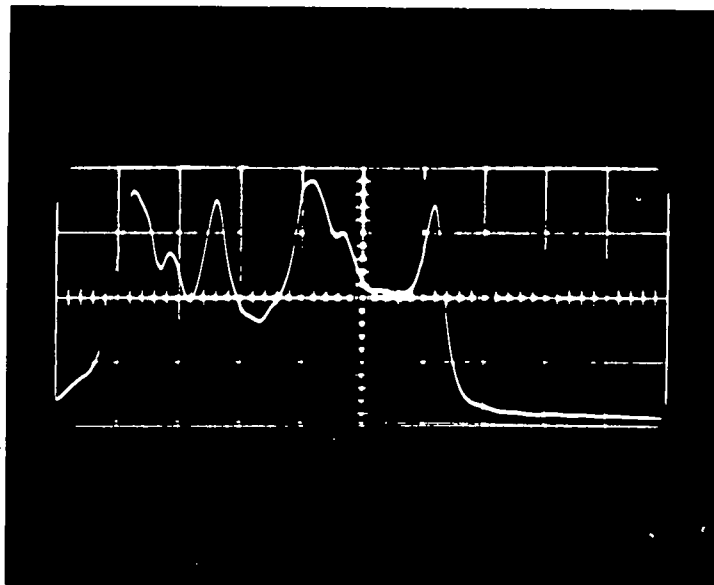
7.5 Conclusions

It is obvious that the simplest way to achieve a specified bandwidth is with single tuned amplifiers (e.g. with high quality diodes in a balanced diode circuit), if varactors of sufficiently high quality are available. Failing this single tuned amplifier circuits may be broad-banded with either passive or active circuits.

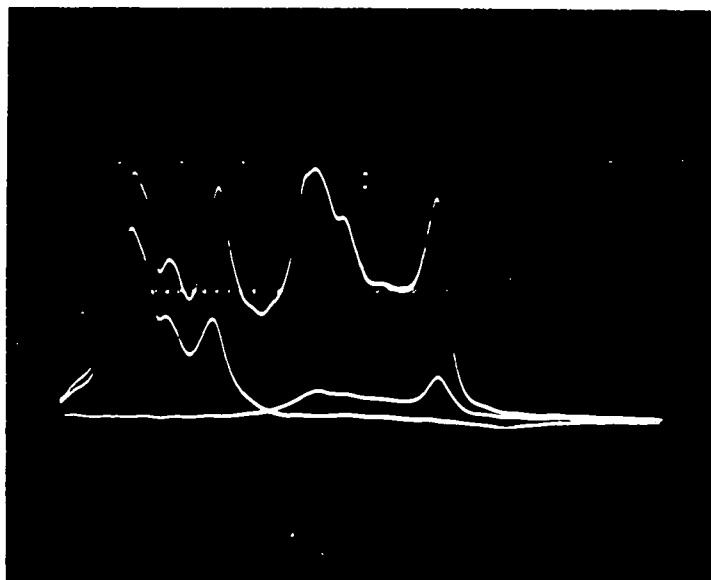
Fig. 7.17 Gain/Frequency Response of Two Stagger-Tuned Amplifiers
with Signal Circuit Active Compensation.
At Room Temperature.



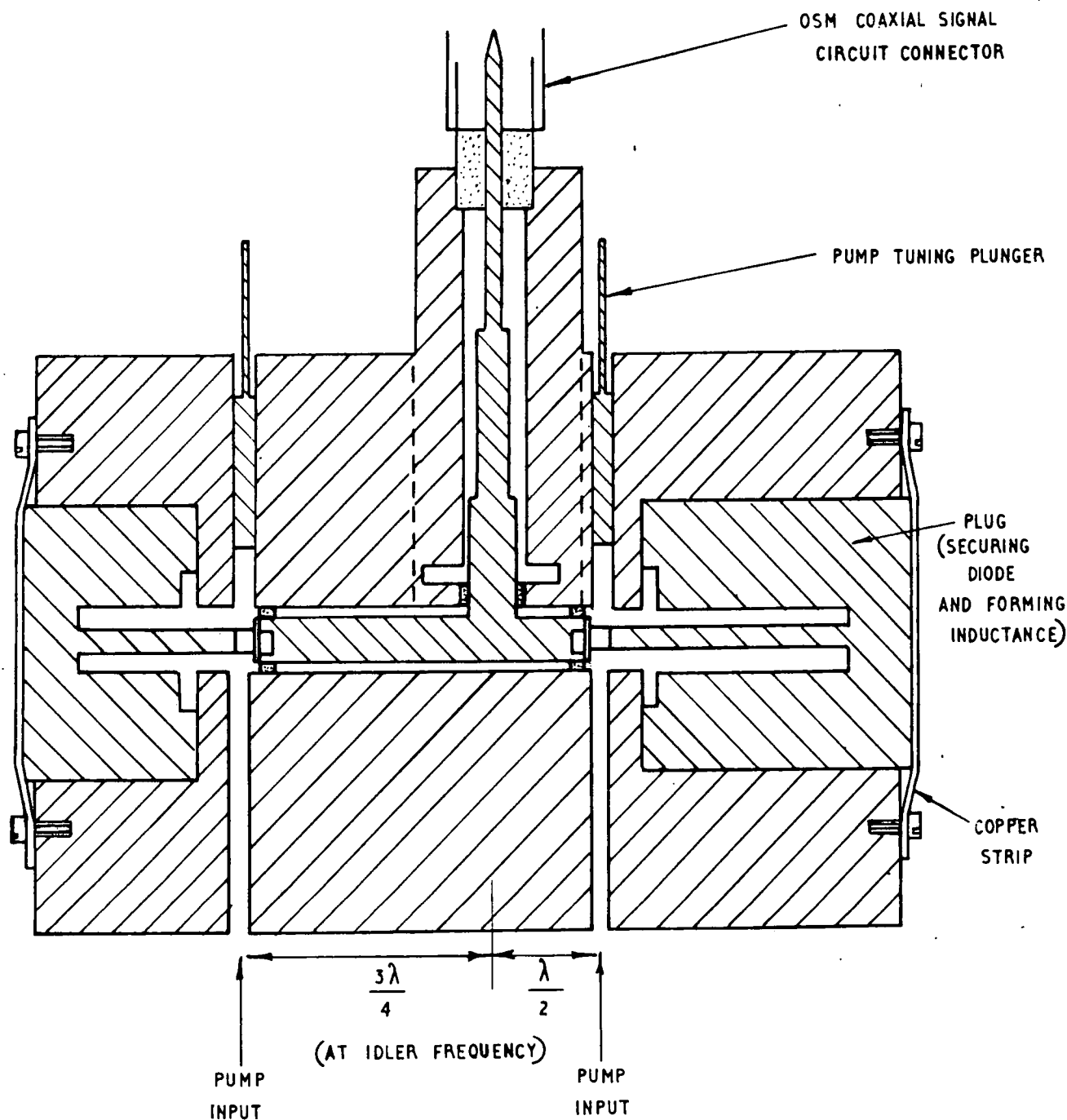
502 MHz Bandwidth
14 dB Peak Gain



495 MHz Bandwidth
14 dB Peak Gain



As above,
showing responses
of individual
amplifiers.

FIG. 7.18 TWO DIODE AMPLIFIER USING IDLER CIRCUIT ACTIVE COMPENSATION
CROSS SECTION

SIGNAL CIRCUIT SECTION (PART DOTTED) IS CONVENIENTLY ROTATED THROUGH
90° TO PLANE OF FIGURE.

POST OFFICE RESEARCH STATION LONDON, N.W.2		ASSEMBLY DRG.	
AMENDMENT PARTICULARS		DATE	INITIALS
ISSUE			
DRN	D. Fountain		
CKD.			
APPD			
DATE	7-8-68		
D.O. SERIAL NO.			
CASE NO.			

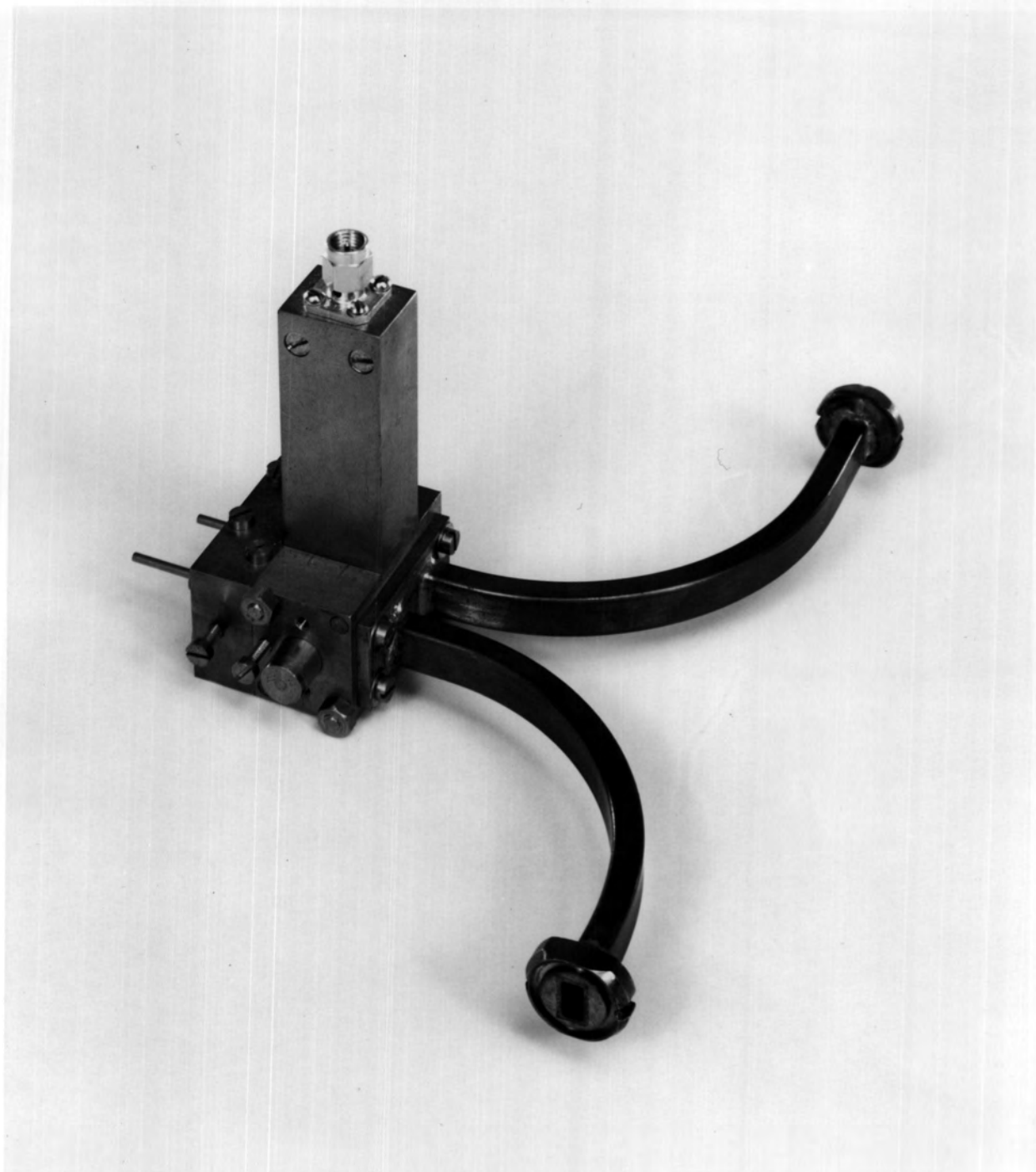


FIG. 7-19

TWO DIODE AMPLIFIER USING
VOLTAGE CIRCUIT ACTIVE COMPENSATION

Any additional passive circuits must be externally adjustable at low temperatures, which adds considerable complexity. Furthermore the available bandwidth with such circuits may not be sufficient for many applications. (To a good approximation the bandwidth of an amplifier with one additional resonator is $\text{gain}^{\frac{1}{2}}$ times the bandwidth of a single tuned amplifier at the same gain.)

Broad-banding with active circuits is more convenient at low temperatures and yields considerably more bandwidth (twice the $\text{gain}^{\frac{1}{2}}$. bandwidth of a single diode amplifier). Using a basic single diode structure with a $\text{gain}^{\frac{1}{2}}$. bandwidth of 350 MHz, a broad-band liquid helium structure of two cascaded amplifiers with active signal circuit compensation, has been developed, which gives 250 MHz 3 dB bandwidth, at 30 dB gain, for a 16°K noise temperature. This structure may alternatively be operated with 500 MHz bandwidth at 15 dB gain. Cooling from room to liquid helium temperature requires no mechanical readjustment of any part of the amplifier, since all the amplifier circuits involve the same capacitance change. It is evident that much greater bandwidths would be available if two single or balanced diode amplifiers, having greater $\text{gain}^{\frac{1}{2}}$. bandwidth products than 350 MHz, were used.

Idler circuit active compensation also appears to be practical, although slightly more difficult to realise.

8. The Noise Temperature of Room Temperature Devices

8.1 General

The two most important sources of noise at room temperature arise from thermal and shot effects. These have been investigated with amplifiers SD4/19 and SD4/33, for which the excess noise temperature (equation 2.14) due to noise sources $(\overline{v_{g1}^2})$ and $(\overline{v_{g2}^2})$ at the signal and idler frequencies should be:

$$T_e = \frac{1}{4kBR_g} \left[(\overline{v_{g1}^2}) + (\overline{v_{g2}^2}) \left(\frac{Yf_c}{f_2} \right)^2 \right]$$

8.2 Thermal Noise

The validity of equations 2.15 and 2.16 has been demonstrated by many workers (e.g. Uenohara 1960). Equation 2.16 is plotted in fig. 2.8 in which the curve for $Yf_c = 24$ applies for the CAY10 varactor. The theoretical noise temperatures of the two amplifiers type SD4/19 and SD4/33 can be read directly from this graph and are listed in the table below along with the measured noise temperatures at zero diode current, after the subtraction of noise from external input line loss. An attempt was made to measure the integral amplifier signal circuit loss by the reflection method, without a diode in the amplifier, and this varied between 0.2 and 0.5 dB depending on the R_g value. There is however a large possible error in this measurement, since the open circuit was found to radiate. The amplifier integral signal circuit loss probably accounts for most of the discrepancy between the theoretical and measured noise temperatures, but any remaining difference may be explained by uncertainty in the diode Yf_c product, uncertainty in the amplifier R_g value, or noise at an unwanted frequency, (see part 4 of this section).

Table 8.1

Amplifier	Noise Temperature °K	
	Theoretical	Measured $\pm 8^\circ\text{K}$
SD4/33		
$R_g = 8\Omega$	146	170
$R_g = 11\Omega$	106	140
$R_g = 13.4\Omega$	87	130
SD4/19	153	175

8.3 Shot Noise

A few varying figures for shot noise in parametric amplifiers have been quoted in the literature, (Uenohara and Elward 1964, Garbrecht and Metzger 1963) and a study has been made by Josenhans (1964) for silicon and gallium arsenide varactors. Josenhans found agreement with simple theory for gallium arsenide diodes, but disagreement by a factor of two or more for silicon diodes. A detailed study was therefore made.

8.3.1 Theory

The theory of shot noise in p-n junction diodes proposed by Van der Ziel (1958) and Uhler (1958b) can be used to derive a varactor equivalent circuit for shot noise components as in fig. 8.1. In this C_1 is the encapsulation capacitance, C_t is the transition capacitance, which is usually considered as the total junction capacitance, and R_s , the series resistance, is a function of bias. G_1 represents the low current junction conductance $\frac{eI_1}{kT}$, and the current generator $(\overline{i_1^2}) = 2eI_1B$ represents the noise contribution of the diffusion current, I_1 . At high frequencies the junction admittance is modified by Y_2 , due to minority carriers returning to their region of origin, and which shows full thermal noise $(\overline{i_2^2}) = 4kTG_2B$, where G_2 is the real part of Y_2 (Van der Ziel 1958, Uhler 1958b). The imaginary part of Y_2 can be expressed in terms of the storage capacitance C_s , which must be included in the total junction capacitance at high forward currents. Now;

$$C_s = G_1 \tau_p \quad (\text{if } n_A \gg n_D)$$

where τ_p is the lifetime of holes in the n region (Greiner).

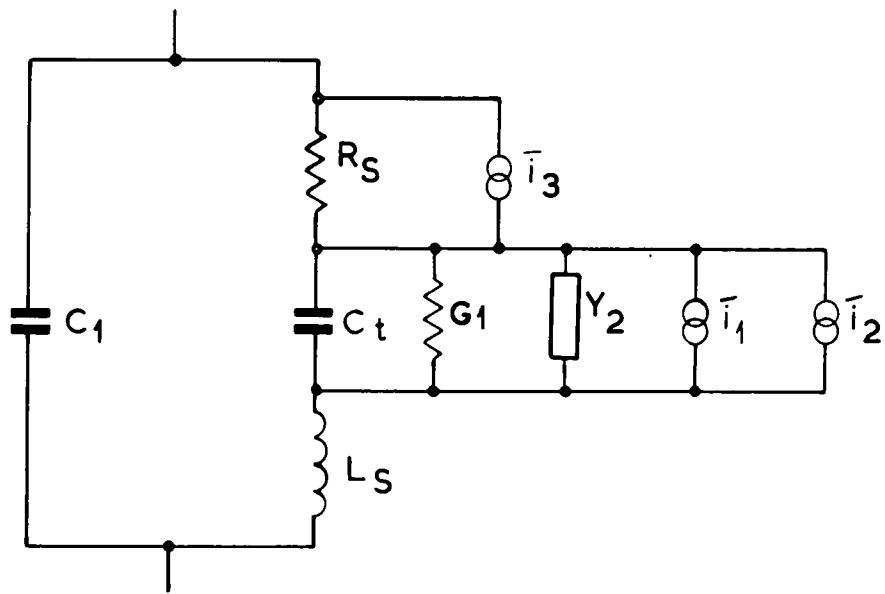
The total mean squared shot noise current may therefore be written:

$$(\overline{i_t^2}) = 2e(I_1 + I_0)B + 4kTG_2B + 2eI_0B \quad \dots\dots\dots 8.1$$

which includes the effect of holes generated in the n region and diffusing across the junction. In most varactors I_0 is less than 10^{-8} amp, being almost independent of applied voltage and therefore not of interest in the study of forward current shot noise. Thus:

$$(\overline{i_t^2}) = 2eI_1B + 4kTG_2B \quad \dots\dots\dots 8.2$$

FIG. 8.1



or, in terms of voltage generators in series with the diode elements;

$$(\overline{v_t^2}) = (2eI_1 B + 4kTG_2 B) \left[\frac{1}{(G_1 + G_2)^2 + \omega^2(C_t + C_s)^2} \right] \dots\dots\dots 8.3$$

at angular frequency ω . Since D.C. current also flows through the bulk resistance a small noise term $(\overline{i_j^2})$, should also be included. Substituting for $(\overline{v_{g1}^2})$ and $(\overline{v_{g2}^2})$ in equation 2.14 and including $(\overline{i_j^2})$ gives;

$$T_{e(\text{shot})} = \frac{1}{4kBR_g} \left[\frac{(2eI_1 B - 4kTG_2 B)}{(G_1 + G_2)^2 + \omega_1^2(C_t + C_s)^2} + 2eI_1 BR_g^2 \right] \\ \frac{1}{4kBR_g} \left[\frac{(2eI_1 B + 4kTG_2 B)}{(G_1 + G_2)^2 + \omega_2^2(C_t + C_s)^2} + 2eI_1 BR_g^2 \right] \left(\frac{Yf_c}{f_2} \right)^2 \dots\dots\dots 8.4$$

Now the factor $2eI_1 BR_g^2$ is small and G_1 is negligible for the range of currents considered. The storage capacitance C_s is also negligible for this range of currents if $\tau_p \leq 10^{-10}$ secs, so to a good approximation:

$$T_{e(\text{shot})} = \frac{2eI_1 + 4kTG_2}{4kR_g} \left[\frac{1}{(G_2^2 + \omega_1^2 C_j^2)} + \frac{1}{(G_2^2 + \omega_2^2 C_j^2)} \right] \left(\frac{Yf_c}{f_2} \right)^2 \dots\dots\dots 8.5$$

where $C_j = C_t$. For negligible noise from stored carriers this becomes:

$$T_{e(\text{shot})} = \frac{eI_1}{2kR_g} \left[\frac{1}{\omega_1 C_j} + \frac{1}{\omega_2 C_j} \right] \left(\frac{Yf_c}{f_2} \right)^2 \dots\dots\dots 8.6$$

An identical expression can be evolved for the balanced diode amplifier since;

$$(\overline{v_{g2}^2})_{\text{balanced}} = 4(\overline{v_{g2}^2})_{\text{single diode}} \quad \text{and}$$

$$(\overline{v_{g1}^2})_{\text{balanced}} = (\overline{v_{g1}^2})_{\text{single diode}}.$$

8.3.2 Noise Measurements

These were carried out on the SD4/33 type amplifier, since the R_g value of SD4/19 is difficult to assess. An increase in the pump level to the device gives increased gain and diode current against which measurements of noise temperature were made, and by changing the signal circuit transformer ratio, these were obtained for various values of generator/load impedance.

Using six individual diodes, about thirty series of measurements were made, from which five representative graphs are shown in fig. 8.2. The maximum error for these points is $\pm 8^\circ\text{K}$, most of which arises from uncertainty in the amplifier gain during measurement, and applies only for the points at lower gain. A measurement of gain was made before and after each noise reading and a change of greater than 0.2 dB meant that the noise reading was discounted. The effect of input line and circulator losses has been subtracted for these measurements, but loss integral with the amplifier structure remains, which could lead to a maximum systematic error in shot noise/diode current slope of +8%.

The diodes were self-biased and did not reach reverse breakdown, so that reverse current shot noise was avoided.

A number of interrelated factors must be considered in the interpretation of these graphs, namely, the effect of varying resonance conditions in the signal and idler circuits and the increase in Y , C_j , and R_s with forward current. The evaluation of the amplifier frequency response for various values of peak gain (or pump power level), at a fixed pump frequency, as was carried out in section 3.6 can be used to determine these factors. It was seen that an increase in the pump power to the amplifier, over that for the condition of minimum diode current (i.e. resonance), caused the frequency of maximum gain no longer to be the resonant frequency of the signal circuit, nor to be the difference between the pump frequency and the idler circuit resonant frequency (see fig. 3.14). Hence the variation of centre frequency gain with pump power does not directly indicate the variation of junction capacitance.

The fact must also be considered that an increase in the pump power to the amplifier, over that for the condition of minimum diode current, will introduce reactance into both signal and idler circuits, unless the pump frequency is adjusted. Reactance in these circuits does not directly degrade the amplifier noise temperature, but gives lower amplifier gain for a given noise temperature, or conversely, higher diode current for a given gain and will also mean that a higher value of negative resistance, and therefore Y , is required for a given gain.

The effect of Y and junction capacitance variations on amplifier noise temperature for a given generator/load impedance can thus be calculated from;

$$T_{e(\text{total})} = T_{e(\text{thermal})} + T_{e(\text{shot})}$$

$$\text{i.e.} \quad T_{e(\text{total})} = T_0 \frac{R_s}{R_g} \left[1 + \left(\frac{Y f_c}{f_2} \right)^2 \right] + \frac{e I_1}{2k R_g} \left[\frac{1}{\omega_1^2 C_j^2} + \frac{1}{\omega_2^2 C_j^2} \left(\frac{Y f_c}{f_2} \right)^2 \right] \dots\dots 8.7$$

remembering that the cut-off frequency, $f_c = \frac{1}{2\pi C_j R_s}$, depends on the variation of R_s and C_j with forward current. As forward current is increased C_j will increase, resulting in slightly less total shot noise than with a constant C_j , and Y will increase, resulting in slightly more thermal and shot noise (transferred from the idler circuit) than if Y was constant. Equation 8.7 has been evaluated for a change in amplifier gain from 10 to 15 dB and yields a negligible noise temperature change for all values of R_g . This is because the effects of increasing Y and junction capacitance are roughly equal and opposite, the effect of increasing R being itself negligible ($< 3^\circ\text{K}$). Table 8.2 summarizes this result, the first column being for constant C_j and the second for constant Y .

Table 8.2
Change in Noise Temperature due to variation in Y and C_j
(for an increase in gain from 10 to 15 dB)

$R_g (\Omega)$	$\Delta\text{NT } (^\circ\text{K})$ due to Y increase	$\Delta\text{NT } (^\circ\text{K})$ due to C_j increase
8	15.8	-20
10	16.3	-21
12.3	17.7	-22
13.4	16.8	-23

8.3.3 Diode Dynamic Characteristics at the Pump Frequency

These were described in section 5.3 and indicate the activity of stored carriers in these diodes at the pump frequency. It might be expected that even below the threshold of avalanching, significant stored carrier currents could be flowing, to contribute to the total shot noise.

8.3.4 Comparison of Measurements with Theory

The graphs of noise temperature versus diode current for various values of generator/load impedance plotted in fig. 8.2 along with the expression for noise temperature without high frequency terms (equation 8.6) show good agreement for low diode current and pump levels, and so in this region, $T_e = \frac{45.9}{R_g} ^\circ\text{K}/\mu\text{A}$. The greater slope which is evident for higher currents,

FIG. 8.2a

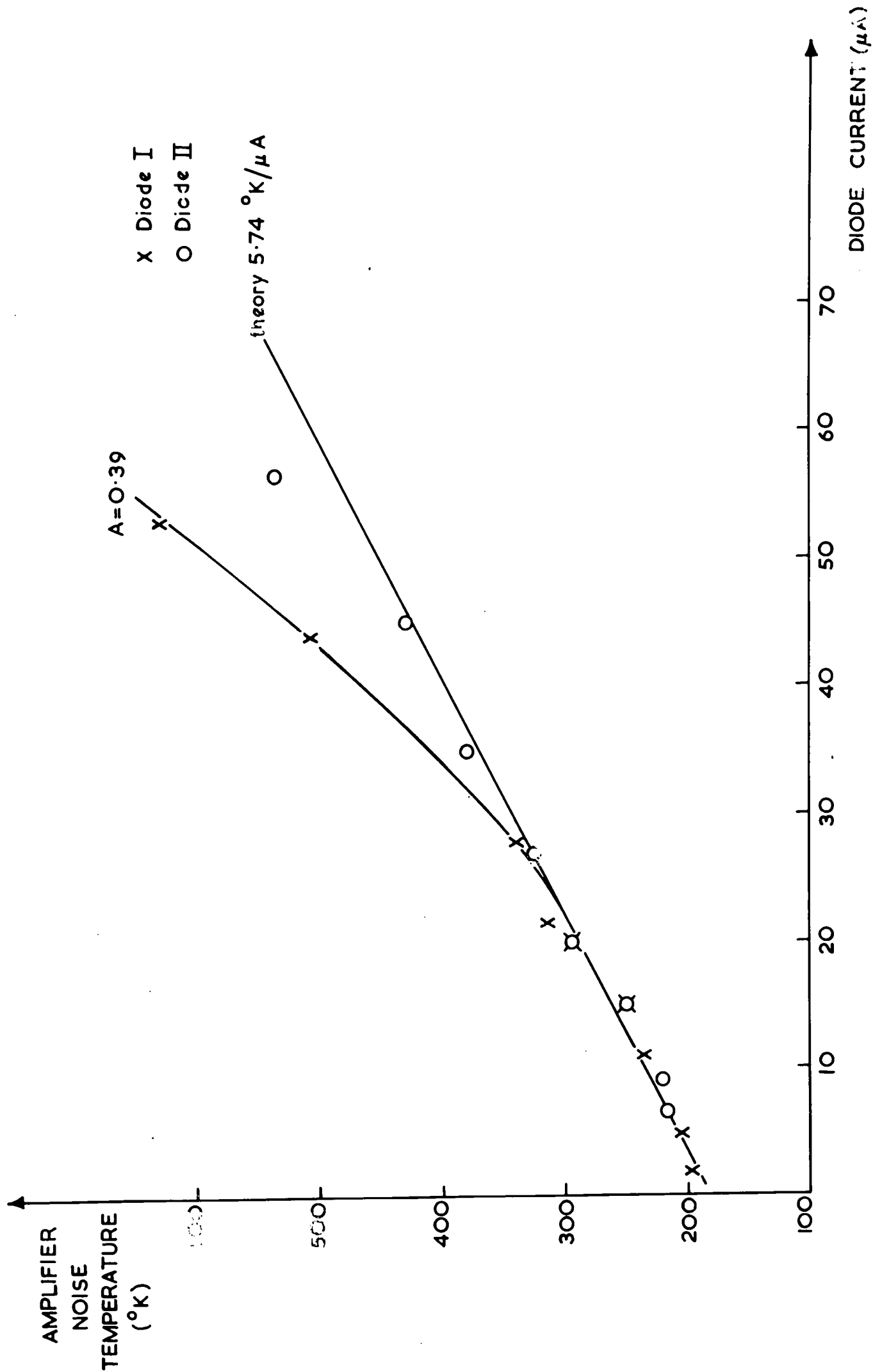


FIG. 8.2b

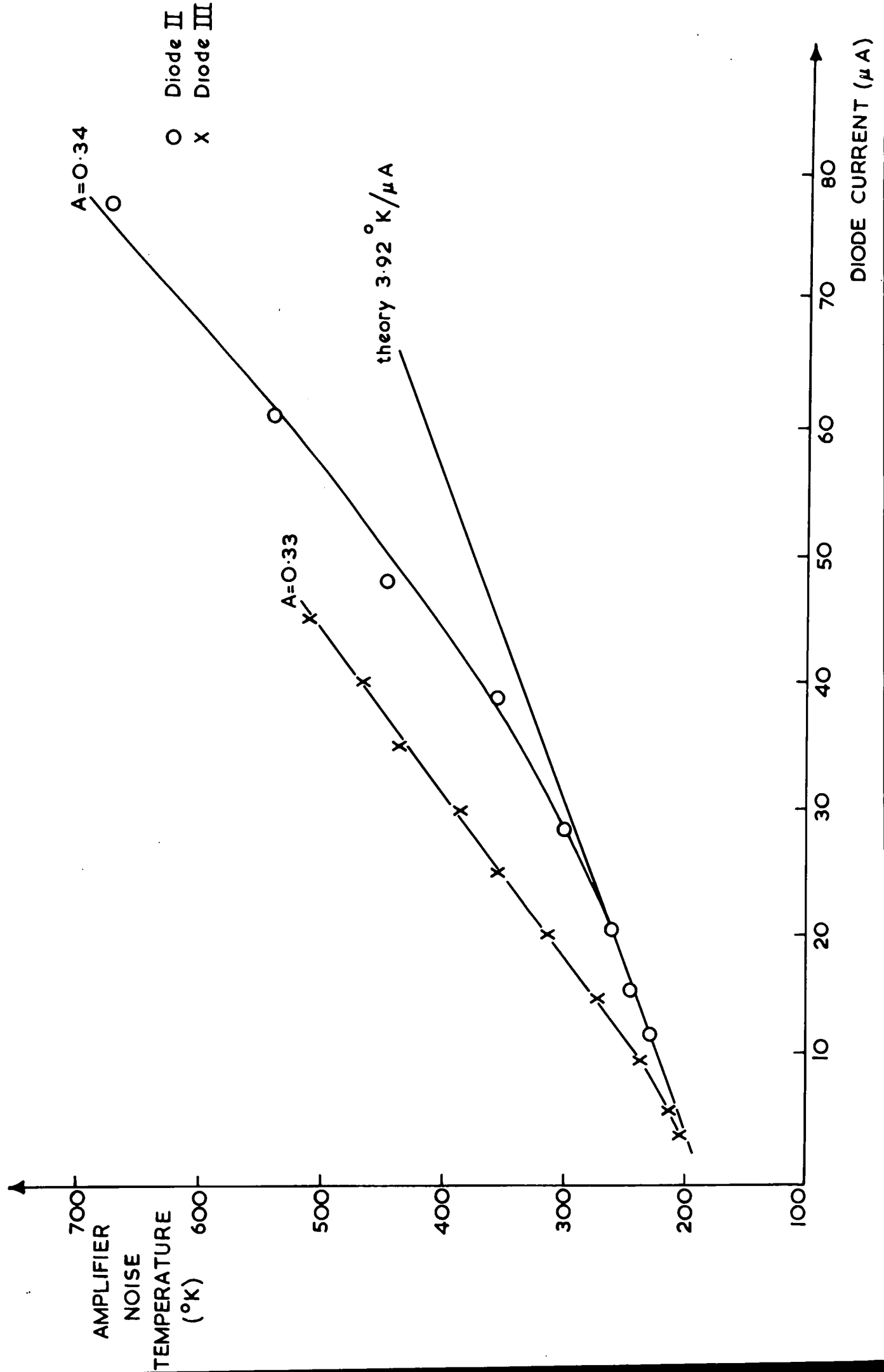
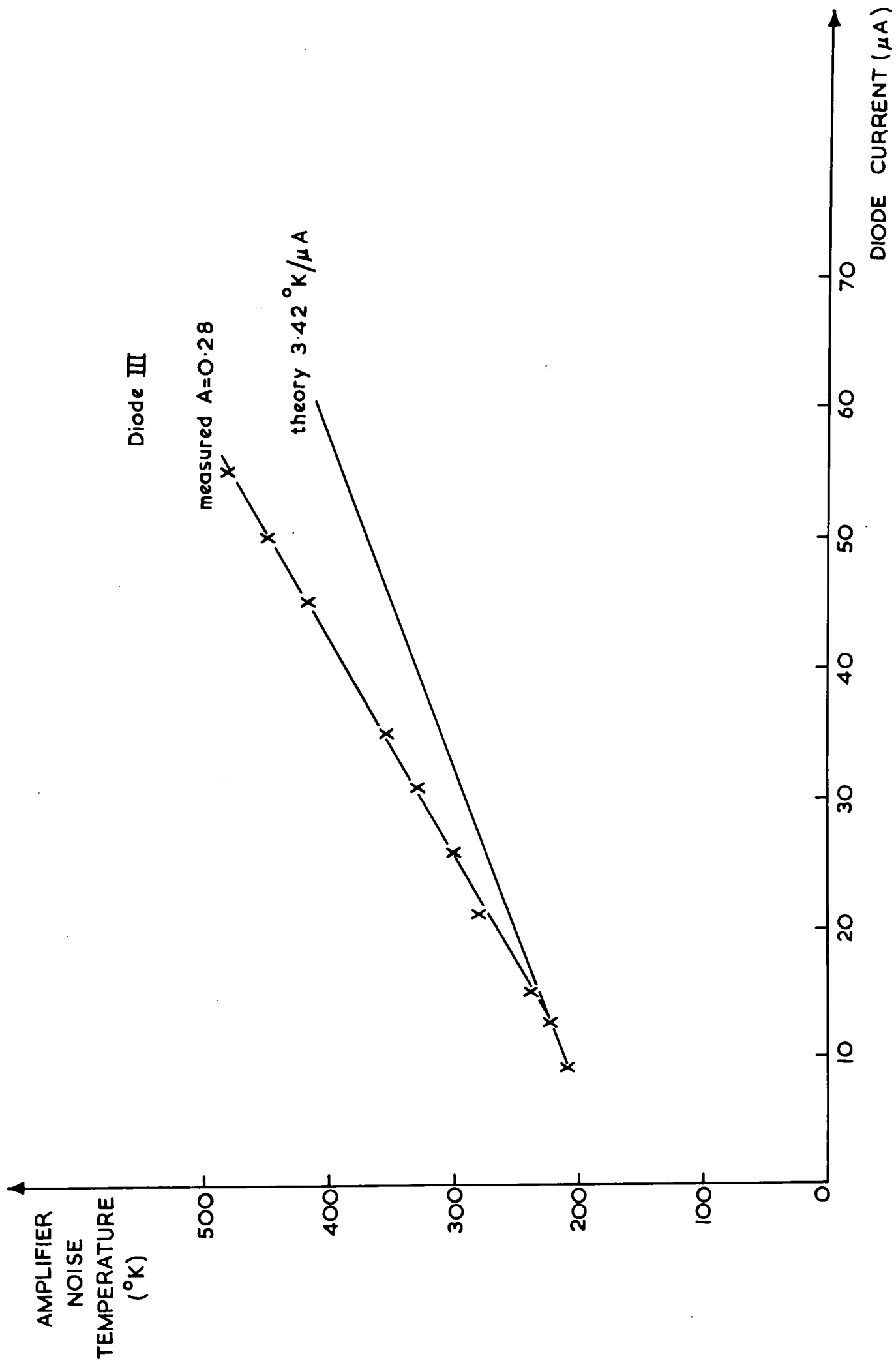


FIG. 8.2c



implies the presence of another noise source, and the diode dynamic characteristics under pumped conditions indicate the probability of significant minority carrier currents. Diodes which showed the anomalous negative resistance effect at low currents and pumping levels, departed from the simple noise theory at lower levels and vice versa. Thus some curves, particularly those for low values of generator impedance and therefore relatively low pump levels showed good agreement with the simple theory (e.g. fig. 8.2a diode II) and others, usually for high values of generator impedance, disagreed for the whole range of measurements (e.g. fig. 8.2b diode III and fig. 8.2c), but most diodes in most situations showed a significant change in noise temperature/diode current slope within the range of measurement (e.g. fig. 8.2a diode I and 8.2b diode II). The use of equation 8.5 to explain these conditions therefore seems to be justified.

The current from stored minority carriers may be estimated from the graphs. If this is equal to AI_1 and the total measured current is I_m then:

$$I_m = I_1 - AI_1$$

but the total current contributing to shot noise, I_n , is given by:

$$I_n = I_1 + AI_1$$

provided that forward and reverse current pulses are uncorrelated (Uhlir 1958). So:

$$I_n = I_m \left(1 + \frac{2A}{1-A} \right) \dots\dots\dots 8.8$$

If the noise temperature graphs in fig. 8.2 have straight lines drawn through those points which do not agree with simple theory, these are found to correspond to values of A between 0.25 and 0.4. From the equivalence $2eI_nB = 4kTG_2B$ it can be shown that values of conductance (G_2) required to agree with these points, lie in the range $33I_m$ to $47I_m$.

The possibility of pump heating of the diode spreading resistance must also be considered, but for a room temperature device this should not be significant, (see section 9). Two other phenomena are apparent which might add significant noise to parametric amplifiers on increasing the pump level. Firstly, an analogous process to shot noise due to the presence of a strong A.C. signal (in this case the pump signal) with the weak signal to be amplified (Bull and

Bozic 1967) and secondly, the possibility of non-sinusoidal pumping, which effectively reduces the junction admittance for a given diode current and hence increases noise power output. The assumption that a sinusoidal pump voltage is applied may be valid in the initial tuning conditions referred to in section 8.2.2, when the pump circuit is also brought to resonance, but may be upset on increasing the pump level.

From the good agreement with simple theory for low generator/load impedances, it would appear that the latter two effects are not significant.

It therefore seems probable that diodes with properties favouring minority carrier storage (e.g. a long minority carrier lifetime, a steep impurity gradient across the junction, and a small junction area), show high values of shot noise. Josenhans (1964) has reported high values of shot noise for silicon diodes, which may be explained by the long minority carrier lifetime (10^{-7} secs) compared with the reciprocal pump frequency. Although τ_p is less in gallium arsenide ($\approx 10^{-10}$ secs), for diodes with a small junction area and steep impurity gradient (as in a CAY10), it would appear to be long enough to allow significant storage at 34 GHz.

It is also noted that diodes with "soft" reverse characteristics will, as well as directly adding noise current (I_0 in equation 8.1 is then significant), encourage impact ionization and avalanching on the reverse cycle.

8.4 Noise at Related Frequencies

Noise can be transferred from a frequency related to the three frequencies of the device, if a termination exists for that frequency. It is assumed in the simple analysis, for open circuit harmonics, that all harmonics and intermodulation products see a complete open circuit. Any termination at one of these frequencies adds a further term to the noise temperature expression (equation 2.14), analogous to the term for idler circuit noise. An upper sideband termination is most detrimental (Pearson and Hughes 1966), but this is accompanied by reduced amplifier gain and apparently reduced diode efficiency. Finite terminations at other frequencies have much lower transfer functions and even if comparable to the idler circuit termination (R_s), would introduce a noise term at least an order of magnitude less than noise transferred from the idler termination. Care is taken in the design of amplifiers to eliminate any possible upper sideband termination (see section 3.5) but it is possible that a small proportion of the discrepancy between the theoretical and measured noise temperatures of amplifiers SD4/19 and SD4/33 arises in this way. Inclusion of upper sideband terms in the diode impedance matrix (section 2.2) indicates that a termination of 10Ω at the upper sideband

frequency would lead to an increase in noise temperature of only 7.5°K for the SD4/33 device with $R_g = 8\Omega$, but a termination approaching R_g (2.5Ω) would lead to 35°K extra noise, when the amplifier gain and apparent varactor efficiency would be much reduced.

8.5 Conclusions

The simple expressions for thermal noise are obeyed within experimental limits, and give room temperature noise temperatures for SD4/19 and SD4/33 amplifiers as approximately 160 and 150°K respectively.

Shot noise in parametric amplifiers using gallium arsenide varactors in self resonant idler configurations, at low forward currents and low input/output impedances is given by:

$$T_e = \frac{e}{2kR_g} \left[\frac{1}{\omega_1^2 C_j^2} + \frac{1}{\omega_2^2 C_j^2} \left(\frac{\gamma f_c}{f_2} \right)^2 \right] ^{\circ}\text{K}/\mu\text{A}$$

which is usually $2 \rightarrow 10^{\circ}\text{K}/\mu\text{A}$ depending on the circuit and diode parameters. For higher impedances (and hence higher pump input levels), and for higher currents, some $8 \rightarrow 15^{\circ}\text{K}/\mu\text{A}$ is measured which can be accounted for by the inclusion of an additional noise term to represent a significant stored minority carrier current.

Noise at related frequencies should not be significant if care is taken to ensure the absence of an upper sideband termination.

9. The Noise Temperature of Cooled Devices

9.1 Measurements

Noise temperature measurements were made for a number of diodes in amplifier SD4/33 at liquid air, liquid nitrogen and liquid helium temperatures, and for amplifier SD4/19, at liquid helium temperature.

The apparatus described in section 4 was used for all the measurements on cooled amplifiers and the results of the measurements at 4°K are shown, for various values of amplifier gain, in Figs. 9.1 and 9.2. These points represent the noise temperature at the input terminals of each amplifier, noise from resistive loss in the feed and circulator (tables 4.4 and 4.5) having been subtracted. Noise measurements at liquid air temperature were carried out on amplifier SD4/33, with liquid air in the outer wall of a helium dewar and gaseous helium surrounding the amplifier. A thermocouple on the body of the amplifier indicated the amplifier temperature. Liquid nitrogen temperature measurements on amplifier SD4/33 were, on the other hand, obtained by dipping the lower end of the amplifier block (Fig. 6.6) directly into liquid nitrogen contained in the small dewar. A room temperature circulator was used for this latter measurement.

Great care was exercised in taking noise measurements; amplifier gain was checked after each noise reading to ensure that no change had occurred and each point on the graphs represents the average of at least five noise measurements. The vertical lines on Figs. 9.1 and 9.2 represent the maximum random error at each point, not from the spread of readings, but from the error analysis of the noise measuring equipment. The spread of readings at each point was within the range of the vertical lines at all points.

A comparison of the results of the noise measurements at the various temperatures is presented in tables 9.1a and 9.1b, for the two amplifiers. It is seen that the noise temperatures at 4°K do not show a direct temperature proportionality. Whereas the liquid nitrogen temperature noise measurements, within experimental limits, are very nearly in proportion to the physical temperature as predicted by equation 2.16.

Fig. 9.1 Input Noise Temperature versus Gain for Amplifier SD4/19

3 sets of points are for different
signal circuit transformer ratios
(and therefore different R_g values)

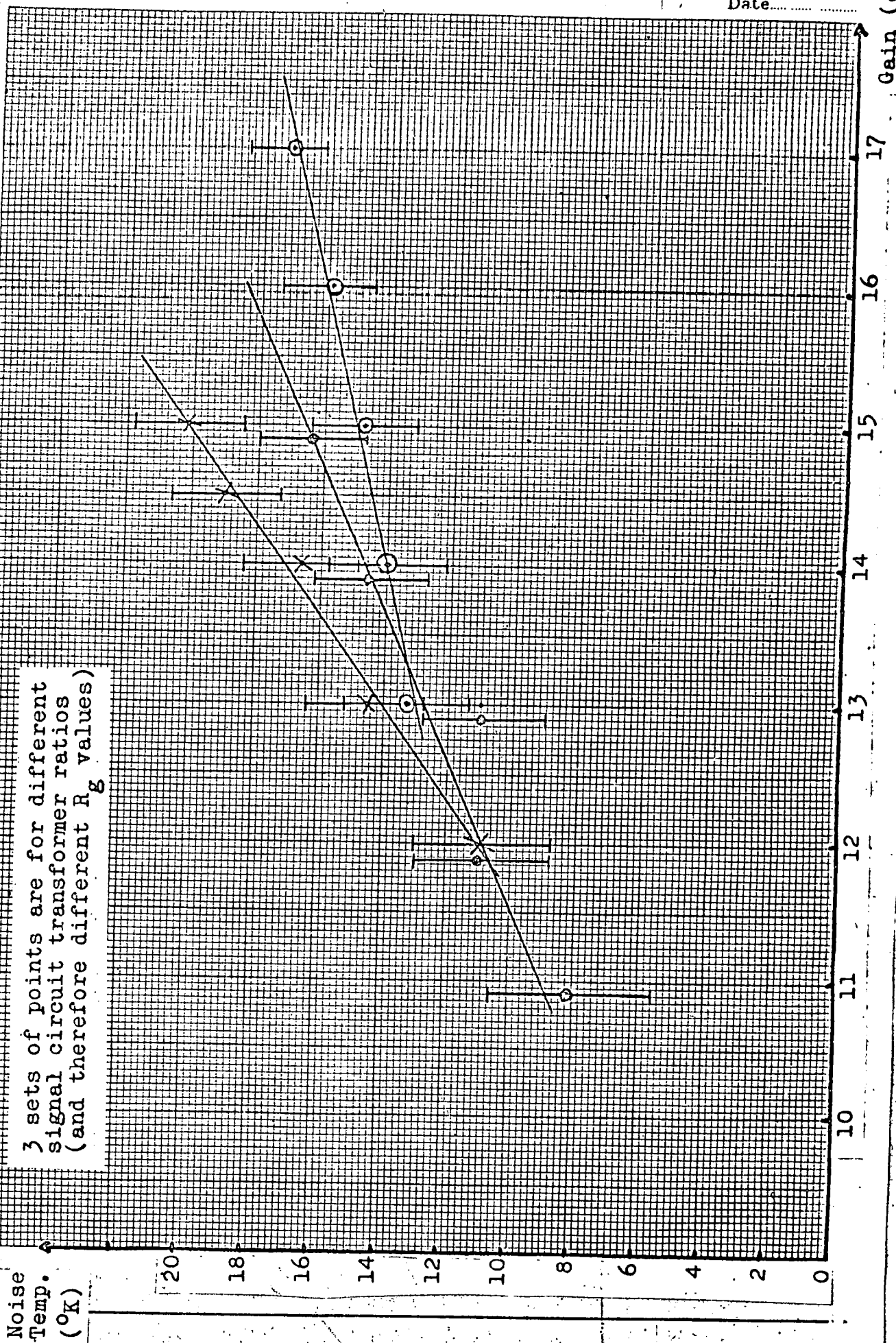


Fig. 9.2 Input Noise Temperature versus Gain for Amplifier SD4/33

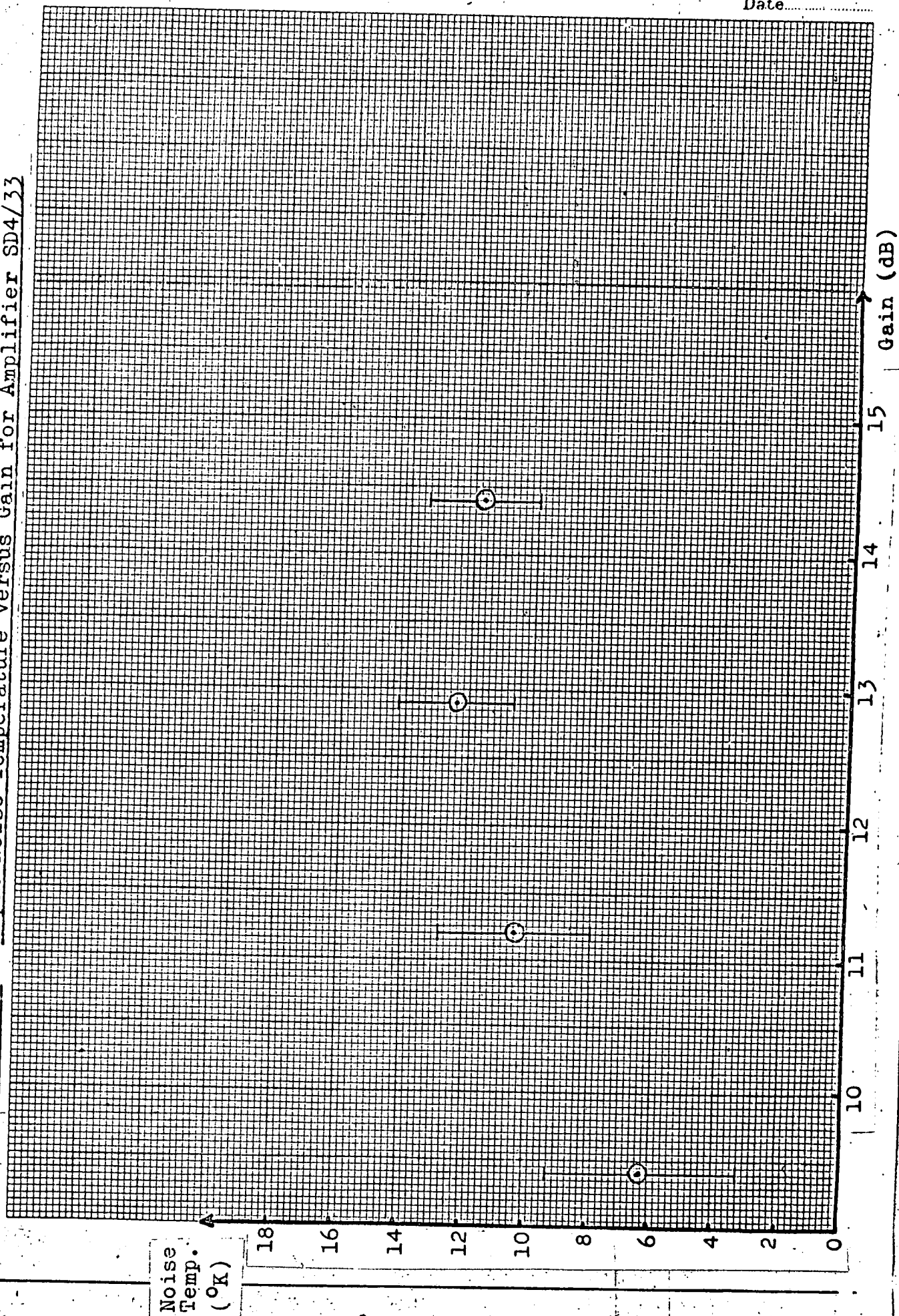


Table 9.1

Amplifier Noise Temperature at Various Ambient Temperatures

(a) SD4/19 (At 15 dB Gain)

Physical Temp. °K	Noise Temp. °K	Max. Possible error °K
290	180	± 10
4	15	± 3

(b) SD4/33 (At 15 dB Gain)

Physical Temp. °K	Noise Temp. °K	Max. Possible error °K
290	160	± 7
110	65	± 5
77	40	± 5
4	11	± 3

Furthermore it is noted from Figs. 9.1 and 9.2 that the noise temperature of liquid helium cooled devices increases with amplifier gain (or pump power level) yet the diode D.C. current which was monitored throughout each measurement did not exceed $0.01 \mu\text{A}$. The results for amplifier SD4/19 (Fig. 9.1) are for three values of generator/load impedance (R_g) and show that the noise temperature/diode current slope depends on this parameter. Since the pump power required for a given gain is proportional to R_g , this dependence of the noise temperature/diode current slope on R_g is further evidence that amplifier noise temperature depends on pump power. Higher noise temperatures than those shown in Figs. 9.1 and 9.2 were observed for both amplifiers when the signal and idler circuits were not on resonance (slightly detuned pump frequency).

At 15 dB amplifier gain the excess noise over that predicted from equation 2.16 is seen from Figs. 9.1 and 9.2 to be 9°K for amplifier SD4/33 and 11°K for amplifier SD4/19, (when each is correctly tuned) resulting in input noise temperatures of 11 and 15°K respectively. This means that, with each amplifier mounted on a cryogenic feed, a noise temperature of some 17°K (SD4/33) or 21°K (SD4/19) is measured at the feed input flange (see Fig. 4.6), for 15 dB gain. A few degrees of excess noise in a satellite receiving system can be critical and can mean considerable expense in terms of increased satellite transmitting power or receiving aerial efficiency. The extra 10°K of noise over the figure predicted from simple theory is therefore of practical as well as academic importance. For these reasons the possible sources of noise at liquid helium temperature are now examined in some detail.

9.2 Possible Sources of Noise at Liquid Helium Temperature

9.2.1 Thermal

Equation 2.16 predicts a linear decrease in thermal noise with physical temperature provided that all other factors in the equation remain constant. However, R_s , γ , f_c and f_2 all change slightly with temperature (section 5) and the theory must be modified accordingly. The fact that a gain¹.bandwidth product is obtained at liquid helium temperature comparable to that at room temperature, confirms that the change in these factors is slight or self-compensating. Allowing for a 1 Ω increase in R_s at liquid helium temperature, the noise temperatures predicted by equation 2.16 for amplifiers type SD4/33 and SD4/19 are 2.5 and 4.6 $^{\circ}$ K respectively. (A lower figure is expected for the SD4/33 device because of the higher idler frequency).

9.2.2 Shot Noise

This should not be evident for a correctly designed amplifier. If the diode D.C. current is less than 0.1 μ A it has been seen (section 8) that less than 0.4 to 0.8 $^{\circ}$ K of noise should be added, dependent on the amplifier circuit parameters. The possibility that the observation of zero D.C. current at 4 $^{\circ}$ K is due to the complete storage and return of diffused carriers (see section 8) must be considered but measurements of the diode dynamic characteristics at these temperatures do not indicate greater storage than at room temperature. However, storage at low currents will be negligible at room temperature, but may be significant at 4 $^{\circ}$ K. Furthermore, the increased diode contact potential and slightly shortened reverse characteristics (Fig. 5.3) will increase the likelihood of reverse current due to impact ionisation from stored carriers. The pumped dynamic characteristics of one particular diode measured at 4 $^{\circ}$ K revealed a peak forward current of 0.8 μ A on the curve tracer, yet zero total D.C. current was indicated on a microammeter. This means that the total current contributing to shot noise may be 1.6 μ A or more which would add at least 6 to 12 $^{\circ}$ K to the amplifier noise temperature. However, the reverse characteristic of this diode was seen to be slightly "soft" (breakdown 6 volts at room temperature) and the effect could not be repeated with other diodes at the pump levels required for the operation of either type of amplifier.

Shot noise should therefore not be important for a correctly designed amplifier at 4 $^{\circ}$ K but the possibility of an anomalously low D.C. current measurement due to minority carrier storage cannot be ruled out.

9.2.3 Quantum Noise

The parametric process depends on energy transfer from the pump frequency, and therefore a noise limitation is imposed due to the quantized nature of the pump input. If a pump frequency photon is considered to be absorbed with the simultaneous emission of signal and idler frequency photons then it can be shown (Louisell, Yariv and Siegman 1961) that the signal output will have an additional noise power of hfB , which is the same as the limiting noise power of a maser and corresponds to an excess input noise temperature of about 0.2°K at 4 GHz.

9.2.4 Capacitance Fluctuation Noise

The varactor junction capacitance will have a statistical fluctuation caused by the releasing and trapping of electrons by impurity centres (Giacoletto 1961). This may be written as a noise current generator in parallel with the junction capacitance, of magnitude:

$$\overline{(i^2)} = 2e^2 \frac{n_D AF}{\tau_D} \cdot \frac{(1 - \omega)}{\omega} \left[- \frac{e(V_b + \phi)}{kT} \right]^{-1} B \quad \dots 9.1$$

where n_D = number of donor atoms/mtr²

A = area of junction

F = width of junction

ω = equilibrium probability of ionisation of donors

V_b = bias voltage

ϕ = diode barrier potential

B = bandwidth

τ_D = ionised donor lifetime

Substituting the parameters for the CAY10 diode at 4°K gives

$$\overline{(i^2)} = \frac{2(1 - \omega)}{\omega \tau_D} \cdot 7.21 \cdot 10^{-31} B \quad (\tau \text{ in } \mu\text{sec.}) \quad \dots 9.2$$

and the equilibrium probability of ionisation of $n_D = 10^{17}$ n type gallium arsenide at 4.2°K is 0.6 (inferred from conductivity measurements) so this noise contribution is negligible for any realistic value of τ_D .

9.2.5 Pump Heating Noise

Dissipation of pump power in the diode resistive regions can raise the temperature of these regions and lead to additional thermal noise. Garbrecht (1966) the only person as yet to have published detailed noise measurements on liquid helium cooled parametric amplifiers, has endeavoured to explain the excess noise that he also measured in terms of this process.

However, two shortcomings of Garbrecht's analysis of pump heating are evident. Firstly the thermal resistance of the diode used has been measured by a method which assumes a temperature proportionality of resistance. That is, the diode is pulsed with a relatively high forward current (a few mA) so that heating occurs, the degree of which can be interpreted in the pulse off-time by an ~~electrical~~ resistance measurement, provided that the diode resistance/temperature calibration is known. However, the bulk of the diode ~~resistance~~^{electrical} is in the n region and the contact; the p region, which has a very high thermal resistance due to impurity scattering, (see Holland, 1966) is of negligible electrical resistance ($\approx 0.1\Omega$) and will not be detected in any such measurement of the total diode thermal resistance. Secondly, Garbrecht has assumed that the change in thermal conductivity of gallium arsenide is negligible over a temperature range around 4.2°K , but it has been shown (Holland 1966) that this factor is proportional to T^3 and cannot be taken as constant. It will be seen that the assumption of a temperature independent thermal conductivity allows an analysis of pump heating to give almost exact agreement with noise measurements, but that modification of the theory to take into account the temperature variation of thermal conductivity shows pump heating to be negligible.

Analysis

The thermal resistances (defined as the inverse of thermal conductance) of the various diode regions (as shown in Fig. 5.2) are listed in table 9.2 for room and liquid helium temperatures. The figures at 4°K have been obtained by applying results of low temperature thermal conductivity measurements (Holland 1966 and Rosenberg 1955) to the diode geometry. The high p region thermal resistance is noted, which is due to impurity scattering. (The error in this factor allows for the possible error in extending Holland's data for lower doping levels (up to $n_A = 10^{18}$) to give a figure ~~of 1×10^{-4}~~ for $n_A = 10^{21}$ gallium arsenide).

Table 9.2

Thermal Resistances of the Diode Regions

Region (see Fig. 5.2)	Thermal Resistance $^\circ\text{K}/\text{watt}$	
	at 290°K	at 4°K
p type GaAs	700	$10^5 \rightarrow 10^7$
n type GaAs mesa pillar	700	88 ± 10
n type GaAs wafer	500	40 ± 5
gold bead	70	55 ± 20
gold wires	1440	1100 ± 400
Kovar base	50	25 ± 10

The components of the diode total series resistance have been listed in table 5.2. It is noted that the skin effect plays a part in determining this factor but is neglected in the following analysis.

The diode model may be simplified to that of Fig. 9.4, in which a uniform cross sectional area has been taken for the various regions and the lengths of the regions adjusted accordingly. (This simplification is possible because most of the diode thermal and electrical resistance occurs in the mesa pillar, the gold contact and the gold wires).

Pump heating will cause a temperature rise over ambient for each region according to the power dissipated, which, provided that the change in electrical resistance with temperature is negligible for each region, will produce a noise temperature of:

$$T_e = \frac{1}{R_g} \left[\sum_{n=d}^{n=a} \frac{R_n}{l_n} \int_0^{l_n} T_n(x) dx \right] \left[1 + \left(\frac{Yf_c}{f_2} \right)^2 \right] \quad \dots 9.3$$

(for uniform dissipation throughout each region). Subscripts a, b, c and d refer to the n type GaAs, p type GaAs, contact and wires respectively, which form the four diode regions and l and R are the lengths and ~~electrical~~ resistances of each particular region.

The temperature profile T(x) in any region can be evaluated from the equation of heat conduction for the region. This may be written (Jacob):

$$\frac{d^2 T}{dx^2} + \frac{i^2}{A^2 k_{th} k_e} = 0 \quad \dots 9.4$$

which defines the temperature T at any point x in a uniform region of cross sectional area A. The factors k_{th} and k_e are the thermal and electrical conductivities respectively and i is the electrical current flowing through the region. If k_{th} and k_e are independent of temperature then integration gives:

$$T = -\frac{Kx^2}{2} + Cx + D \quad \dots 9.5$$

where $K = \frac{i^2}{A^2 k_{th} k_e}$ and C and D are arbitrary constants. However, for gallium arsenide at 4°K:

$$k_{th} = \frac{k_{th(4.2)} T^\alpha}{(4.2)^\alpha} \quad \dots 9.6$$

where $\alpha \approx 3$ (Holland 1966).

Fig. 9.3 Diode Regions Influencing Pump Heating

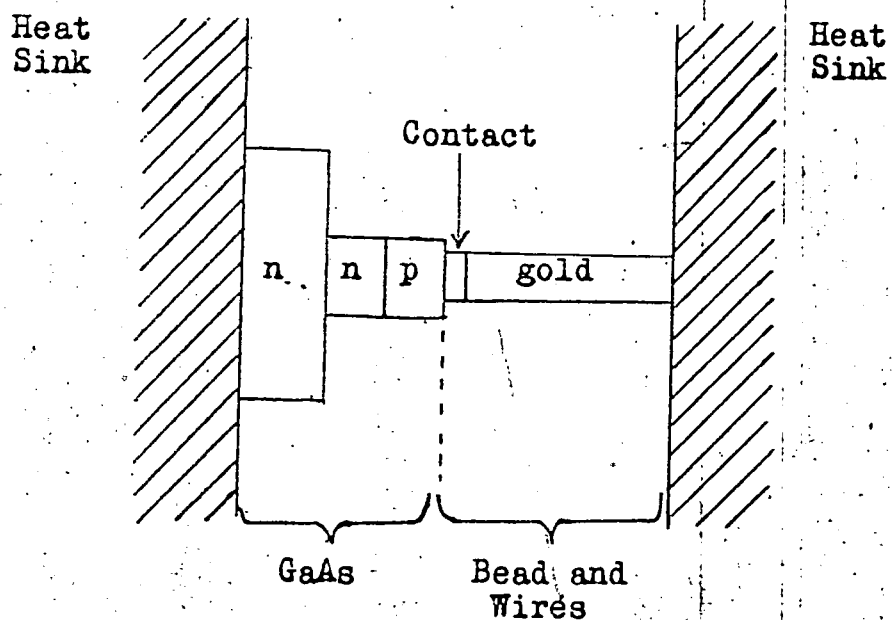
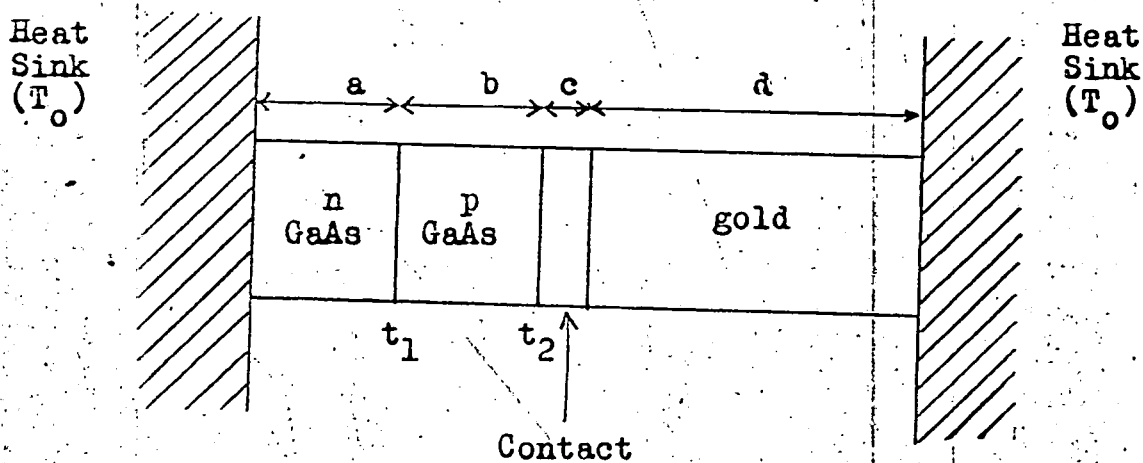


Fig. 9.4 Uniform Model of Diode Regions



Putting $\frac{dT}{dx} = p(x)$ and integrating gives:

$$T^2 = C(x + D)^2 - \frac{K}{C} \quad \dots 9.7$$

where $K = \frac{i^2(4.2)^3}{A^2 k_e k_{th}(4.2)}$ and C and D are arbitrary constants. Thus equation 9.7

can be applied to region a and b (GaAs) and equation 9.5 can be applied for region d (gold), the thermal conductivity of which is almost independent of temperature (Rosenburg 1955). The contact region is difficult to analyse, but to a first approximation it may be considered as extremely thin and of uniform temperature. That is, a fraction of the total pump power can be considered as dissipated in an infinitely thin region which has a total thermal resistance between it and ambient of:

$$\frac{R_{thd}(R_{tha} + R_{thb})}{R_{tha} + R_{thb} + R_{thd}} \quad ^\circ K/W \quad \dots 9.8$$

The following equations, obtained from the boundary conditions for the various regions may thus be written:

$$T_0^2 = C_a D_a^2 - \frac{K_a}{C_a} \quad \dots 9.9$$

$$t_1^2 = C_a(l_a + D_a)^2 - \frac{K_a}{C_a} \quad \dots 9.10$$

$$t_1^2 = C_b D_b^2 - \frac{K_b}{C_b} \quad \dots 9.11$$

$$t_2^2 = C_b(l_b + D_b)^2 - \frac{K_b}{C_b} \quad \dots 9.12$$

$$t_2 = D_d \quad \dots 9.13$$

$$T_0 = -\frac{K_d l_d^2}{2} + l_d C_d + D_d \quad \dots 9.14$$

where T_0 is the heat sink temperature and t_1 and t_2 are the temperatures at the boundaries between regions a and b and regions b and d respectively (t_2 is also the contact temperature). Now the heat continuity equation holds at each boundary, i.e.:

$$k_a \left(\frac{dT_a}{dx} \right)_{x_a = l_a} = k_b \left(\frac{dT_b}{dx} \right)_{x_b = 0}$$

$$k_b \left(\frac{dT_b}{dx} \right)_{x_b = l_b} = k_d \left(\frac{dT_d}{dx} \right)_{x_d = 0} \quad (\text{neglecting}$$

dissipation in the contact). So:

$$k_a (l_a + D_a) C_a = k_b D_b C_b \quad \dots 9.15$$

and
$$\left(\frac{t_2}{4.2} \right)^3 \frac{(l_b + D_b) C_b k_b}{t_2} = C_d k_d \quad \dots 9.16$$

The constants C and D for each region and the temperatures t_1 and t_2 can thus be evaluated from these equations if K_a and l_a etc. are known. Now:

$$K_a = \frac{i^2 (4.2)^3}{A^2 k_{ea} k_{tha} (4.2)}$$

but

$$k_{ea} = \frac{l_a}{AR_a} \text{ and } P = i_p^2 R_s \text{ where } P \text{ is the}$$

pump power supplied to the diode and i_p is the peak current in the diode at the pump frequency. So:

$$K_a = \frac{P (4.2)^3 R_a}{R_s k_{tha} (4.2) A l_a} \quad \dots 9.17$$

Similar expressions may be written for K_b and K_d .

Now, since:

$$|v_p| = \frac{|i_p|}{\omega C_j} \text{ where } v_p \text{ and } i_p \text{ are the peak}$$

pump voltage and current excursions respectively. Thus:

$$P = v_p^2 \omega^2 C_j^2 R_s \quad \dots 9.18$$

The pump power required for a given diode γ value can now be calculated via equation 2.4, and thus, in turn, the pump power required for a given gain can be calculated from equations 2.6 and 2.7 (for a given value of R_g). Fig. 9.5 is a plot of the required γ values and pump power levels for values of gain between 8 and 20 dB for the SD4/33 amplifier. Values of these factors, required at a given gain, for the SD4/19 amplifier are obtained by dividing those for the SD4/33 amplifier by 3.3 (the square of the pump frequency ratio for the two devices).

When the parameters for the CAY10 diode (tables 5.2 and 9.2) are substituted in equation 9.17 to give values of K for each region, (which allows t_1 and t_2 to be evaluated from equations 9.9 to 9.16), it is seen that t_1 and t_2 are little above 4.2°K for either amplifier if T_0 is 4.2°K , ($t_1 \approx t_2 \approx 4.33^\circ\text{K}$ for the diode parameters given in the tables). However, a temperature maximum does exist in the diode p region of 11°K but this is insufficient to contribute significant excess noise. Even if the extreme possible values of p and n region thermal resistance are taken, the maximum temperature in the p region is about 30°K and excess noise from pump heating is still not significant.

The possibility of heating in the contact must now be considered. The highest possible total thermal resistance between the contact and the heat sink (equation 9.8) is 1000°K/W . Thus the maximum temperature rise above ambient in an infinitely thin contact of resistance 0.5Ω is $T_c = P \cdot \frac{0.5}{R_g} \cdot 1000^\circ\text{K}$, where P is the pump power dissipated in the diode, which for the SD4/33 amplifier is 13.3 mW at 15 dB gain (fig. 9.5), so $T_c = 2.7^\circ\text{K}$. Thus, if t_2 is 7°K and t_1 is 4.33°K then the maximum temperature in region b is 12°K which is again negligible as far as adding excess noise is concerned. If t_2 were at a much higher temperature (due perhaps to a high contact thermal resistance) then the p region is a sufficiently good insulator to maintain the n region at little above 4.2°K . For example, if t_2 is 50°K then t_1 will be 4.34°K and the maximum temperature is at the contact, so that; for 15 dB amplifier gain:

$$\frac{R_c}{l_c} \int_0^{l_c} T_c(x) dx = 50R_c = 25$$

$$\frac{R_b}{l_b} \int_0^{l_b} T_b(x) dx = 26R_b = 2.6$$

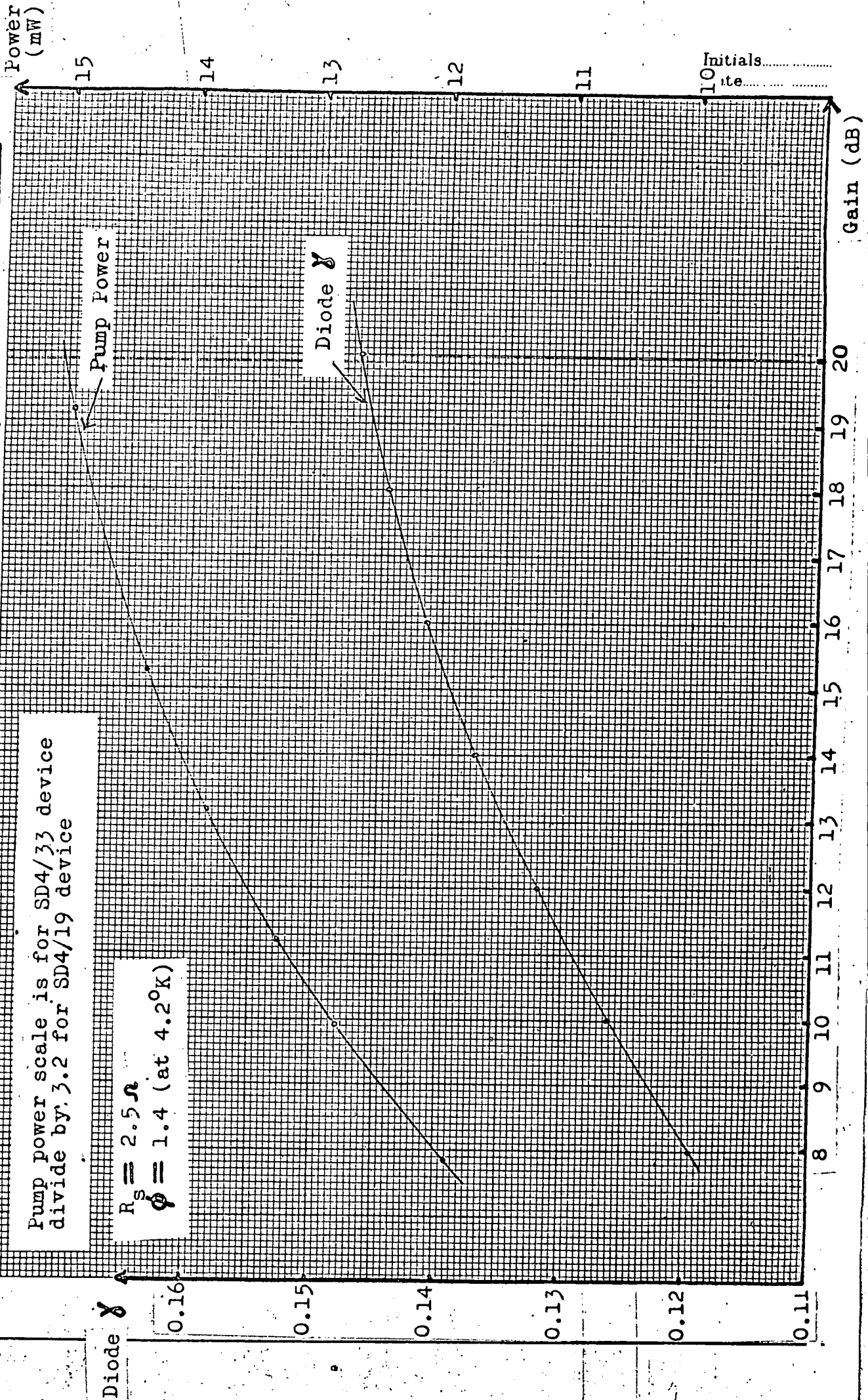
Fig. 9.5

Pump Power Level and Diode γ Value Required for a Given Amplifier Gain

Pump power scale is for SD4/33 device
divide by 3.2 for SD4/19 device

$$R_s = 2.5 \Omega$$

$$\phi = 1.4 \text{ (at } 4.2^\circ\text{K)}$$



Initials
ite

Gain (dB)

and T_e from equation 9.3 is 4.43°K (heating in regions a and d being negligible). So, even if the contact is maintained at an improbably high temperature, the heating in it and other regions is insufficient to explain the observed excess noise. A high temperature could conceivably occur in the contact if it was of significant thickness and of sufficiently low, temperature independent, thermal conductivity. The problem would then become very similar to considering the p region thermal conductivity as independent of temperature.

It is noted that if the thermal conductivity of gallium arsenide was taken as independent of temperature then much more heating would occur. This can be illustrated by considering a single section of p type GaAs of length l , with both ends at T_0 . Now $T = T_0$ when $x = 0$ and $x = l$, so from equation 9.6:

$$T = -\frac{Kx^2}{2} + \frac{Klx}{2} + T_0 \quad \dots 9.19$$

gives the temperature profile in the section. So at $x = \frac{l}{2}$ (maximum temperature due to symmetry):

$$T_{(\max)} = T_0 + \frac{Kl^2}{4} \quad \dots 9.20$$

But, if k_{th} is proportional to T^3 then equation 9.7 applies and the square of temperature profile becomes:

$$T^2 = \frac{2T_0^2 - 2(T_0^4 + Kl^2)^{\frac{1}{2}}}{l^2} \cdot (x - \frac{l}{2})^2 - \frac{Kl^2}{2T_0^2 - 2(T_0^4 + Kl^2)^{\frac{1}{2}}} \quad \dots 9.21$$

and $T_{(\max)}$ at $\frac{l}{2}$ is given by:

$$T_{(\max)}^2 = \frac{-Kl^2}{2T_0^2 - 2(T_0^4 + Kl^2)^{\frac{1}{2}}} \quad \dots 9.22$$

Substituting the values of K and l for the diode p region in equations 9.20 and 9.22 gives $T_{(\max)}$ equal to 171°K and 11°K respectively (for $T_0 = 4.2^\circ\text{K}$). The heating in the p region implied by a maximum temperature of 171°K would be sufficient to give an almost exact fit between measured and theoretical noise temperatures. However, as has already been stated, equation 9.14 cannot be applied to gallium arsenide (on the evidence of Holland) and the much lower temperature obtained from equations 9.7 and 9.22 must be accepted. Nevertheless, the possibility that the contact region has a large temperature independent thermal resistance, cannot be ruled out, and might permit significant heating.

Pump heating in a diode should therefore not produce significant excess noise unless the gold contact on the diode p region has a very high, temperature independent, thermal resistance. If the latter condition does not apply, it is difficult to explain more than a fraction of a degree of excess noise, even considering the possible extremes of the diode thermal parameters as being applicable. It is noted however, that the pump power required for a given gain increases approximately as the square of the pump frequency (equation 9.18) so that the significance of pump heating will be increased for amplifiers at higher signal frequencies (and therefore higher pump frequencies) unless more efficient varactors are produced.

9.2.6 Noise from the Pump Source

Rapid amplitude and frequency fluctuations in the pump supply will be directly transferred to the signal output of the amplifier. Small frequency fluctuations should not be troublesome if they are distributed about a mean, since they will merely change the amplifier centre frequency, which is of no consequence when the noise temperature is measured in a finite bandwidth. The effect of amplitude fluctuations can be obtained directly from the expression relating diode γ value to pump voltage (equation 2.4). If this integral is simplified, using a binomial expansion, then the mean square γ fluctuation due to a fluctuation in pump voltage, $(\overline{v_p^2})$, can be written:

$$\overline{(\gamma^2)} = \frac{(\overline{v_p^2})n^2}{4\phi^2} \quad \text{at zero bias}$$

where ϕ is the diode barrier potential and n is the junction impurity profile parameter. Now if the mean square noise power per unit bandwidth of the pump source is a factor 10^N down on the pump power level (i.e. 10N db down) then:

$$(\overline{v_p^2}) = V_p^2 \cdot 10^{-2N} B$$

and the excess input noise temperature may be written:

$$T_e = \frac{1}{4kBR_g} (\overline{\gamma^2}) \left(\frac{f_c}{f_2} \right)^2 \quad (\text{from equation 2.16})$$

$$T_e = \frac{1}{4kR_g} 10^{-2N} \left(\frac{\gamma f_c}{f_2} \right)^2 \quad \dots 9.23$$

This excess noise is negligible if N is greater than 11, that is if the mean square noise power associated with the pump source is greater than 110 dB down on the pump power level. Figures for klystron noise are available (Mueller 1954) and indicate that a noise power per unit bandwidth of - 150 dB or less, relative to the power output is to be expected. Noise from a klystron pump source should therefore be negligible for parametric amplifiers. It is noted however, that were a solid state source of microwave power used (e.g. a varactor multiplier or a Gunn diode) then care may have to be taken to ensure that the condition of $N > 11$ is fulfilled.

9.2.7 Noise at Related Frequencies

It has already been pointed out (section 8.4) that a termination at the upper-sideband frequency can be detrimental to amplifier noise temperature. However, if care is taken to ensure the absence of such a termination, so that the theoretical room temperature noise temperature is achieved, then no excess noise should be added at 4°K. Even if a finite termination does exist for the upper-sideband, which might add a few degrees to the room temperature noise temperature, this will not add significant noise at 4°K provided that the termination is at 4°K. If however, a room temperature upper-sideband termination, or an additional idler termination at room temperature, can be seen by the diode of a cooled amplifier, then either of these terminations could add significant noise to a cooled amplifier. Room temperature terminations will be evident if the idler or upper-sideband frequencies are allowed to propagate in the pump or signal circuits; filters must therefore be included in these circuits to prevent such propagation.

Adequate filters (see section 6.2) were present in the signal and pump circuits of both amplifiers for all the noise measurements described in this study. Noise from room temperature idler or upper-sideband terminations should therefore not be important.

9.2.8 Reactance

Reactance in the signal and idler circuits does not in itself degrade amplifier noise temperature (see section 2), but will mean that higher pump levels are required for a given amplifier gain.

The effect of detuning in the amplifier signal and idler circuits, on increasing the pump input to an amplifier was seen, in the investigation of shot noise (section 8), to be negligible at room temperature. At 4°K however, the effect may be exaggerated because tuning of the amplifier for zero diode current is usually impossible (since the diode current is very low due to the increased contact potential). The situation is further complicated by the fact

that an increase in band-centre frequency will occur on cooling, so that a device cannot be tuned at room temperature and expected to work at 4°K with the same pump frequency. A cooled amplifier is thus usually tuned for least pump power, which is a much less sensitive process than tuning for least diode current and may lead to a false tuning position because of reactance in the pump feed. Such a false tuning position will require an unnecessarily high pump power for a given gain, which in turn will give an increased noise temperature by whatever process is operative in producing the pump power dependent excess noise that has been observed.

A noise temperature minimum with pump frequency was observed for both the amplifiers used in the noise measurements and can thus be explained as representing the correct amplifier tuning position, with signal and idler circuits very near to resonance.

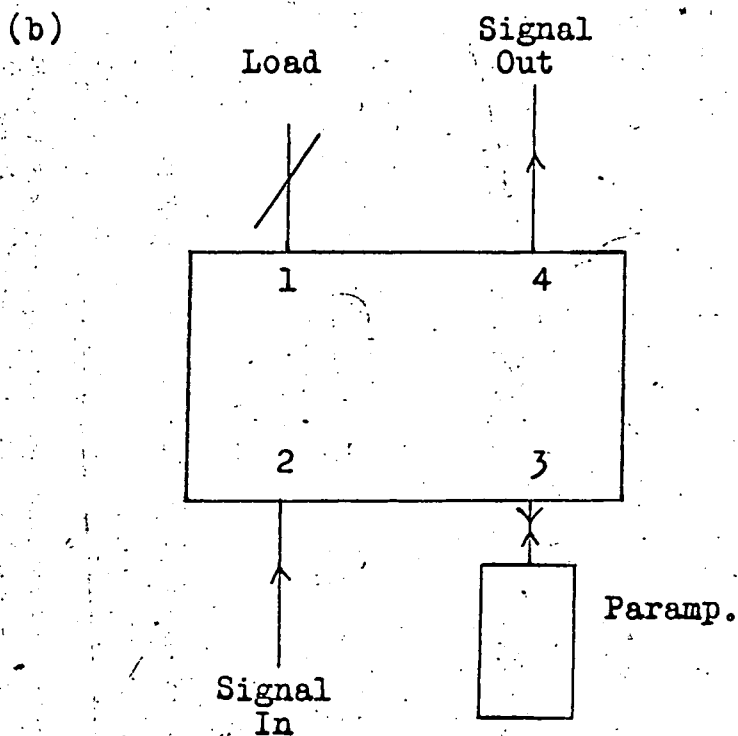
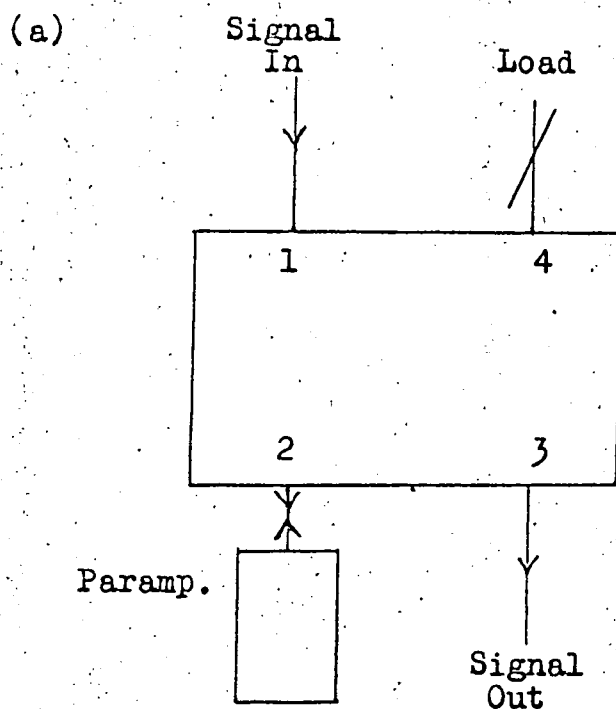
9.2.9 Non-Ideal Circulators

Apart from possessing resistive loss, which was measured after each noise measurement, the circulators used at 4°K are far from ideal (see section 4.3). Noise may also be transferred from the output to the input of a circulator, due to poor isolation. If the amplifier is on port 2 of a 4 port circulator (fig. 9.6a) then noise may be reflected back to the cooled load on port 4, if the output VSWR is poor. If this cooled load is also poor, noise will be fed into the input circuit and a small proportion may be reflected back into the amplifier. Poor direct isolation between ports 3 and 2 may also result in reflected noise from the amplifier output or noise from the output termination reaching the amplifier input terminal. Measurement of the isolation between ports 3 and 2 does not distinguish between the two routes 3 → 4 → 1 → 2 and 3 → 2, so these can be dealt with together. If the measured isolation between ports 3 and 2 is I_{3-2} and the voltage reflection coefficient at the output is Γ , then the noise power at the output terminals due to reflection and poor isolation is $N_p \cdot \frac{|\Gamma|^2}{I_{3-2}}$, where N_p is the amplifier total noise output. The amplifier input noise temperature may therefore be written, from equations 1.5 and 2.16 as:

$$T_e = \frac{T_d R_s}{R_g} \left[1 + \left(\frac{\gamma f_c}{f_2} \right)^2 \right] \left[1 + \frac{G |\Gamma|^2}{I_{3-2}} \right] + \frac{T_{out}}{I_{3-2}} \quad \dots 9.24$$

where G is the amplifier gain. The term $\frac{T_{out}}{I_{3-2}}$ represents noise transferred directly from the output termination at temperature T_{out} .

Fig. 9.6 Circulator Configurations for use at 4°K



Isolation between ports 3 and 2 is usually greater than 20 dB even with circulators for use at 4°K, since two transitions of the ferrite material are involved (see measurements in section 4, isolation between ports 2 and 1 should be approximately one half that between 3 and 2) and for the feed system used, the output VSWR was better than 0.6 at any frequency from 3.8 to 4.2 GHz. The noise temperature is therefore:

$$T_e = \frac{T_d R_s}{R_g} \left[1 + \left(\frac{Y f_c}{f_2} \right)^2 \right] \left[1 + G \cdot 6 \cdot 10^{-4} \right] + 2.9 \quad \dots 9.25$$

for the worst possible case ($I_{3-2} = 20$ dB) with $T_{out} = 290^\circ K$. Excess noise by reflection from the output should therefore be negligible and the $2.9^\circ K$ direct from the output termination, due to poor isolation, can be eliminated by including a second cooled stage of amplification or a second circulator between port 3 of the first circulator and the output line. If, however, a different circulator configuration is used and the amplifier is placed on port 3 (fig. 9.6b) with a load on port 1, then isolation between ports 4 to 3 may be less than 20 dB, since only one transition of the ferrite material is involved. The advantage of the other configuration (Fig. 9.6a) is thus evident.

The isolation between the two relevant circulator ports was 22 ± 2 dB for all the noise measurements on amplifier SD4/33 and greater than 30 dB for those on SD4/19. Thus $1.5^\circ K$ (with a possible error of $+ 1.4$ or $-0.4^\circ K$) should be added, independent of amplifier gain, for amplifier SD4/33 and less than $0.3^\circ K$ for SD4/19. An additional cooled circulator was added to both amplifiers and less change in noise temperature than the experimental error of the noise measurement, was observed in each case. An additional noise of $1.5^\circ K$ is thus possible, but not confirmed in the case of the SD4/33 device.

It is concluded that reflected noise from the amplifier output and noise from the output termination, transferred in each case to the amplifier input due to poor circulator isolation, was barely significant in the circumstances of the noise measurements.

9.2.10 Excess Noise due to Fluctuations in Characteristics when Signals are Applied

This form of noise has been detected in resistors, diodes, and transistors by Bull and Bozic (1966). Two types can be distinguished; a white noise spectrum, analagous to shot noise but produced with A.C. signals and a noise of non-uniform spectrum increasing as the measurement frequency

approaches an applied signal frequency. The enhanced shot noise due to an A.C. signal may be written as:

$$\overline{(i^2)} = \text{const. } I_p B \quad \dots 9.26$$

where I_p is the current at the applied signal frequency. The second kind of noise can be represented by:

$$\overline{(v^2)} = \text{const. } \frac{V_p^2}{\Delta f} \quad \dots 9.27$$

where V_p is the voltage at the applied signal frequency and Δf is the separation of the frequency under consideration from the applied signal frequency.

A parametric amplifier has a strong pump frequency relatively close to the weak signal to be amplified, so this form of noise may be evident, particularly at close pump and signal frequencies. It is noted that greater excess noise, over that expected from simple theory, is detected for the SD4/19 amplifier than the SD4/33 amplifier, which is contrary to the expectation of any other known explanation of the excess noise (e.g. pump heating or shot noise from carrier storage) and might be a point in favour of an explanation in terms of this form of noise. However, measurements on amplifiers at many other pump/signal frequency ratios must be performed, preferably using the same diode or diodes in each amplifier, before significance could be attached to this observation.

Bull and Bozic do not give values of the constants for equations 9.26 and 9.27, so an estimate of the magnitude of this noise cannot be made.

9.3 Summary

An excess noise of some 8 to 10°K dependent (at least in part) on pump power level or amplifier gain, is observed in parametric amplifiers at 4°K. An attempt has been made to describe all the possible relevant noise sources, but a satisfactory explanation of the excess noise is still sought. The amount of noise expected from each noise source is summarised in the table following.

Table 9.3
Summary of Possible Noise Sources

Source	Theoretical Magnitude °K (at amp. input)		Comments
	SD4/33	SD4/19	
Thermal	2.5	4.6	-
Shot	<0.1	<0.1	May be underestimated due to anomalous D.C. current measurement from carrier storage.
Quantum	0.2	0.2	-
Capacitance	<0.01	<0.01	-
Fluctuation			
Pump Heating	<1.0	<1.0	A high, temperature independent contact thermal conductivity could lead to more noise.
Pump Source	<0.01	<0.01	-
Related	<0.01	<0.01	-
Frequencies			
Non-Ideal	1.5	0.3	Possible error of +1.4 or -0.4 for SD4/33
Circulator			
Excess Noise	-	-	Magnitude uncertain
due to			
A.C. Signals			
Total	4.2 → 5.3	5.1 → 6.2	
Measured	11	15	

Of these possible noise sources, it is seen that uncertainty exists in the analysis of the type of noise described by Bull and Bozic as excess noise due to fluctuations in characteristics of diodes (or resistors and transistors) when signals are applied, and an explanation may be possible in terms of this form of noise. In addition, the possibility that carrier storage, enhanced by the increased diode barrier potential, gives anomalously low D.C. current readings at 4°K, cannot be ruled out.

Explanation of most of the excess noise in terms of pump heating seems, however, to be excluded by a detailed analysis of this phenomenon in the various diode regions; unless the gold contact on the diode p region has a sufficiently high, temperature independent, thermal resistance. Pump heating, may, however, be more significant at higher pump and signal frequencies (e.g. for amplifiers at millimetric frequencies and above) if similar varactors are used.

9.4 Conclusions

Parametric amplifiers can be realised at a signal frequency of 4 GHz with input noise temperatures of 11°K ($\pm 2^{\circ}\text{K}$) at liquid helium temperature. Including a cooled feed and circulator a noise temperature, at the input to the feed, of 15°K has been measured for 15 dB amplifier gain. These figures indicate an excess noise of some 9°K , a satisfactory explanation of which is still sought.

10. Future Trends

10.1 General

Cryogenically cooled microwave parametric amplifiers are bulky, complex devices and for this reason are likely to be superseded by a different amplifier or be subject to improvement, at any given signal frequency, to allow operation at room temperature. Solid state travelling wave amplifiers (not parametric amplifiers e.g. Sumi 1966), low noise transistor amplifiers, or solid state bulk effect devices may ultimately supersede parametric amplifiers in many applications, more particularly so if improvements of the type discussed in parts 2-4 of this section are not realised.

The possibility of travelling wave parametric amplification has been investigated for some time (earliest reference is Engelbrecht 1959), but such devices have not progressed beyond the experimental stage, exhibiting all the disadvantages of present day single or balanced varactor diode amplifiers with other disadvantages (inferior noise performance, bulky, complex) and few advantages.

10.2 Integrated Circuits

A fully integrated parametric amplifier including its own pump supply and circulator may ultimately be achieved, with the obvious advantage of convenience and the additional advantage of increased operational bandwidth. The development of miniature "lumped" microwave circuit elements to replace currently used distributed elements, which are bulky and detrimental to bandwidth, is an important factor in the realisation of such devices. Also, as signal frequencies are increased, the impracticality of mounting a tiny individual varactor becomes increasingly evident, and indeed, at 4 GHz, a significant bandwidth reduction may be introduced by diode mount strays. Bandwidth may be further increased and size reduced by eliminating detachable connectors between a circulator and an amplifier, and the size of an overall amplifier system could be considerably reduced by integrating with a solid state source, such as the Gunn diode. Integrated parametric amplifier circuit modules are therefore a logical extension of the art.

A step towards integration might be achieved using existing varactors, with high self-resonant frequencies compared with the signal frequency. For example, two of the Mullard CXY10 (VX6508)^{diodes}, which have a large case capacitance and low series inductance, could be successfully incorporated in one encapsulation to form a balanced diode structure for use at 2-20 GHz. The proposed circuit is shown in fig. 10.1, the whole of which is inside one encapsulation. L_1 is an inductance that will be introduced from the interconnecting leads between the

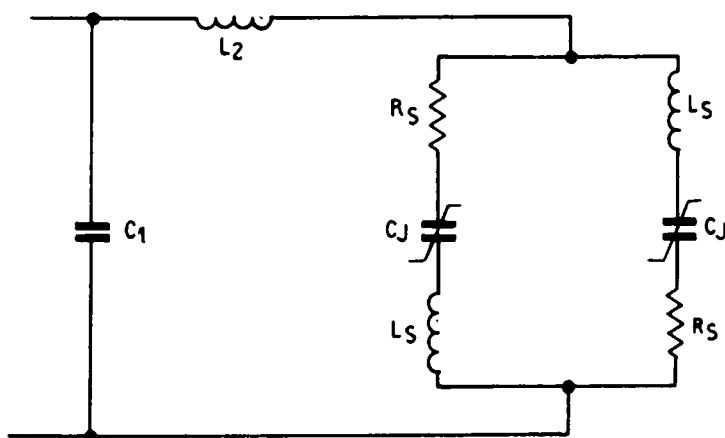
FIG 10.1 TWO CXY10 DIODES MOUNTED IN ONE CASE.
EQUIVALENT CIRCUIT

FIG 10.2 POSSIBLE MECHANICAL STRUCTURE

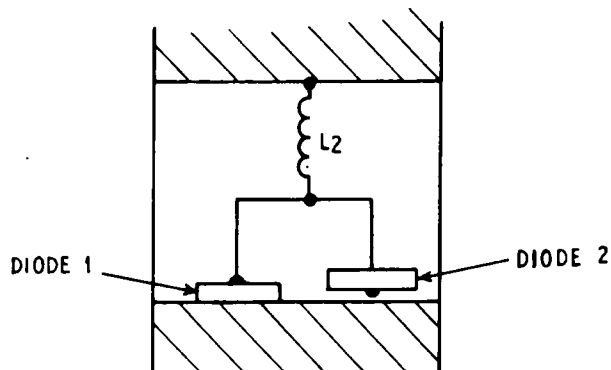
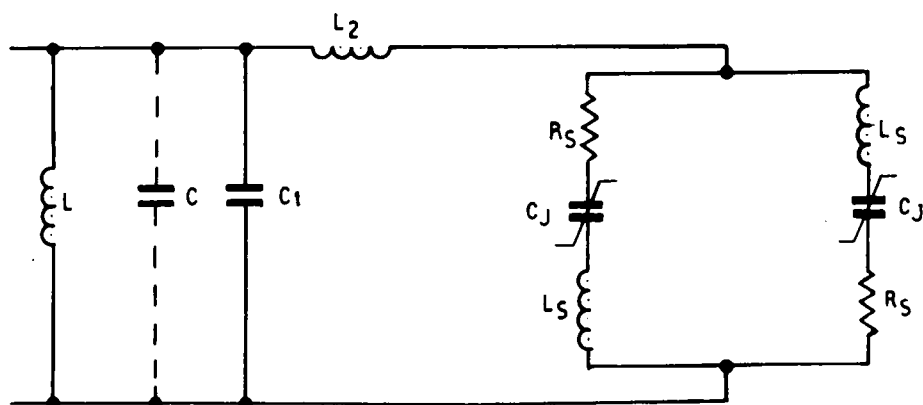


FIG 10.3 EQUIVALENT CIRCUIT WITH ONE ADDITIONAL RESONATOR



two diodes and should be as low as possible. L_2 is added between the junction of the two diodes and the case to resonate the whole structure at the required signal frequency. The mechanical structure of such a circuit might take the form of fig. 10.2, which could be enclosed in a comparatively large encapsulation. Such an arrangement has numerous advantages. Firstly, the capacitance variation factor γ (and hence amplifier gain ^{$\frac{1}{2}$} . bandwidth) is increased by a significant amount over that for a single encapsulated diode. This is because the inductance L_2 , which resonates the structure to the signal frequency, isolates the encapsulation capacitance from the diodes at the signal frequency. γ and gain ^{$\frac{1}{2}$} . bandwidth may be increased by as much as one half in this way. Secondly, L_1 might be less than that for a single diode in a case, and hence the idler circuit Q would be reduced. Finally, no additional tuning elements are required for the device to work as a parametric amplifier, but the addition of a single tuning element, as shown in fig. 10.3, would produce an amplifier with a double tuned input circuit, capable of giving a bandwidth in excess of 10% at 15 dB gain for a 4 GHz signal frequency. Only parallel inductance would be required to tune out the encapsulation capacitance at a given signal frequency, but in order to obtain the optimum L/C ratio for broad-band matching, additional capacitance may also have to be added.

10.3 Reliability

This is dependent almost entirely on the reliability of present day pump sources, with the additional factor of cryostat reliability for cooled devices. Klystrons, particularly at high frequencies, with encumbent power supplies are expensive, complicated, unstable, relatively unreliable and of short lifetime. The development of a reliable solid state source of microwave power is therefore an important goal related directly to parametric amplifier development.

10.4 Improved Varactors

Uncooled amplifiers are relatively simple and have found many applications to date. The integration of such devices with solid state sources should lead to many more applications. However, the need for ultra-low noise temperatures, as in satellite communications or radio astronomy, cannot be met at present without cooling devices to 20°K or less. Improved varactor quality factors (γf_c) would remove the need for cooling in many applications, and also allow operation at higher frequencies. For example fig. 2.9 shows that a γf_c of 60 GHz would produce a noise temperature of 47°K for an uncooled 4 GHz amplifier, or would permit operation at 30 GHz with a noise temperature of 360°K (uncooled) or 92°K cooled to 77°K. Such a quality factor may be realised

with new forms of diodes using new materials or techniques giving increased γ and/or cut-off frequency, for example hyper-abrupt junction diodes using new diffusion techniques (Sukegawa, Fujikawa and Nishizawa 1963 and Antell 1968) metal-semiconductor diodes, (so-called Schottky barrier diodes, Oxley and Summers 1966, Foxell and Summers 1966) or space charge diodes (Howson, Owen and Wright 1965). The use of integrated unencapsulated diodes would in itself, realise higher γ values, due to the reduction of static parasitic capacitance.

Although varactor quality may improve, the demand for low noise amplifiers at yet higher frequencies will no doubt occur, so that the techniques of cooling to low temperatures and the factors discussed in this study which limit noise temperature at these temperatures, would still be relevant.

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