A practical development of multi-attribute decision making using fuzzy set theory

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A PRACTICAL DEVELOPMENT OF MULTI-ATTRIBUTE DECISION MAKING USING FUZZY SET THEORY

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ABSTRACT

The foundations of multi-attribute utility theory are reviewed and compared with the author's practical experience and other psychological studies of decision-making. The case is presented for a new approach to decision-making, moving away from the strictly numerical techniques. Instead of concentrating on the normative or descriptive aspects of decision-making, the meta-problem of decision-making is studied, thereby giving the decision-maker more control over the decision-making process and ensuring a more truly participative approach to design and decision-making. The problem of uncertainty is also tackled by considering it from both the stochastic and fuzzy standpoints. A revised approach to the assessment of uncertainty and its incorporation in the decision-making process is advocated. The theoretical framework behind these ideas is expressed using fuzzy set theory. Previous attempts to apply fuzzy set theory to multi-attribute decision-making are reviewed and criticised for their failure to tackle the basic assumptions of multi-attribute utility theory. A practical methodology for using verbal descriptions is derived, and illustrated with a worked example. A practical description of how to apply the method is included, and the results of some applications are presented.

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I am especially grateful to Pat Croft and Margaret Compton for typing the manuscript so painstakingly, and being more than just secretaries. And a personal note of thanks to my husband, George.

QUOTATIONS

"One of the chief difficulties lies in properly describing the assumptions which have to be made about the motives of the individual."

Von Neumann & Morgenstern
pg.8, "The Theory of Games
and Economic Behaviour", 1947

"I'm not fascinated, but I am very, very interested."

Xander Jones (age 8)
upon being shown a Jacob's
Ladder

"If we have a correct theory but merely prate about it, pigeonhole it and do not put it into practice, then that theory, however good, is of no significance."

Mao Tsetung
1937

CHAPTER 1
INTRODUCTION

By now it is a platitude to state again the importance of decision analysis in our modern complex world. Multi-attribute decision-making is only one part of the field, but has received fair attention and there have been a selection of ways in which this problem has been tackled. One of the reasons for such a variety of approaches is that multi-attribute problems are so common and occur in every field of endeavour, so that the various investigators have brought different beliefs and practices to choose a solution.

The main thrust of the study of multi-attribute decision-making has come from the disciplines of psychology and management science. The psychologists seek to understand the underlying mechanism of the largely intuitive process of choosing between alternatives. Management scientists are interested because 'good' and 'bad' decisions may have serious commercial consequences, so they require a reliable method of making decisions so as to enjoy a successful outcome.

However, despite a flurry of interest in the sixties, multi-attribute decision-making is not so fashionable with psychologists now, with the early models having proved unsatisfactory. From the management science point of view, little seems to have changed lately. The recent books on decision analysis still religiously quote the work of von Neumann and Morgenstern, which was published in 1947 ("the most unread best seller in the social sciences other than the Kinsey report", Edwards, 1971), and each book seems to be largely a reiteration of all those previous.

To explain this apparent stagnation of the topic, we may look at the history of the subject and the present cultural environment. Decision analysis took off with opertional research, which arose around the time of World War II. The Armed Forces wanted to know how best to deploy their resources so as to defeat the enemy. The best minds were involved in the study and once the War was over, operational research remained with its emphasis on measurement and modelling. The goal of the original operational research teams was relatively value-free, although emotive beyond its context, i.e. destroy enemy forces. This encouraged a scientific, objective attitude, with wholly appropriate tools and results. In the post-war period, the cultural environment encouraged this 'rational' attitude and so it persisted and is obvious in the textbooks of the fifties. The development of decision analysis reflected this attitude and emphasised the decision technique and the skills of the analyst, ignoring the decision-maker.

However, the cultural environment changes again, moving away from strict objectivity in the fields which involve human reasoning, and some parts of decision analysis are being tugged in this direction. Instead of concentrating on the application of the decision technique, decision analysis, and multi-attribute decision-making in particular, are looking more closely at the decision-maker himself.

This is the attitude which this thesis adopts. Rather than concentrating on mathematical rigour in decision analysis, I shall take a looser approach to the problem of multi-attribute decision-making, involving the user in a participative design role, with a corresponding de-emphasis of the analyst and his traditional techniques. The burden of decision-making is returned to the decision-maker, but with tools and a structure to assist him.

Summary of Chapters

Chapter 2 provides an introduction to multi-attribute decision-making, as it is normally pursued. General human problem-solving is described briefly and multi-attribute decision-making is placed within context. The problems of assessing both objective and subjective criteria are examined and previous attempts to do this are reviewed and their drawbacks considered.

Chapters 3 and 4 move on to consider the two techniques which are almost always used, i.e estimation of the probability of events and the assessment of the utility of the consequences of events. The many faces of uncertainty are considered only briefly but its origins in lack of knowledge and lack of insight are presented. The axioms of utility are discussed, together with their practical difficulties, and a reassessment of the role and treatment of probability in decision-making is proposed.

Chapter 5 introduces the notion of fuzzy sets and discusses their appropriateness to handling this subject, beset as it is with vagueness, subjective opinions and lack of information. The choice and definition of adjectives, the use of pairs of statements and the advantages and disadvantages of truth functional modification and its inverse are considered.

Chapter 6 presents a review of other attempts to apply fuzzy set theory to multi-attribute decision-making and concludes that they have failed to attack the root of the problem.

In Chapter 7 we have an attempt to do just that. Rejecting many of the approximations of traditional multi-attribute utility theory in a fundamental postulate and introducing a more human-oriented, fuzzy attitude, a new approach to multi-attribute decision-making is presented. This is illustrated by examples, and its philosophical and axiomatic foundations are discussed.

Chapter 8 presents a recipe for the application of such an approach, within a participative framework and considers how this will affect the role of the analyst with his consequent loss of control. Some attempts to apply this fuzzy approach are described in Chapter 9.

The thesis closes with a summary of its broad conclusions and some suggestions for further development of the ideas and the direction such work is likely to take in the future.

CHAPTER 2
HUMAN PROBLEM SOLVING AND MULTI-ATTRIBUTE DECISION-MAKING

Before proceeding to discuss multi-attribute decision-making, it is appropriate now to set it within the context of problem-solving in general. Human problem-solving behaviour has been described, notably, by Newell and Simon (1972). They studied the behaviour of individuals who were presented with problems in chess, cryptarithmetic and theorem proving. They formulated a general model of such behaviour, depicted in Figure 2.1.

The sort of problems Newell and Simon's subjects faced had some features in common. These were, firstly, that the problem-solver was the only person involved in the task and its outcome; no one else could be affected. Secondly, if the first solution method failed, the subject could continue to apply trial methods until one succeeded or he gave up. The environment imposed few constraints upon the problem-solver - he was not restricted by time, money or the effects of his actions upon other people.

However, in real life, the environment does impose constraints upon problem-solving behaviour. The environment may not be able to permit the expenditure of large amounts of time and effort upon the repeated implementation of trial solutions. The problem must be solved at the first attempt, and the luxury of failure is denied. The problem-solver must focus his attention upon selecting the method which will solve the problem at the first attempt. Not able to sequentially apply the suggested methods, the problem-solver will be required to forecast from his own experience what the outcome of the various methods is likely to be. He is faced with a decision to make under uncertainty.

In order to find out which problem-solving method to use, the decision-maker is faced with another problem - the meta-problem - which is how to select the best method. To do so he may seek some sort of decision technique to assist the problem-solving activity. This leads to a meta-meta-problem of how to select the best technique. This state of affairs is depicted in Figure 2.2. To solve the external problem in the environment, the problem-solver's output must be a selected, best method. To solve the problem of selecting a method, a meta-problem-solver could be

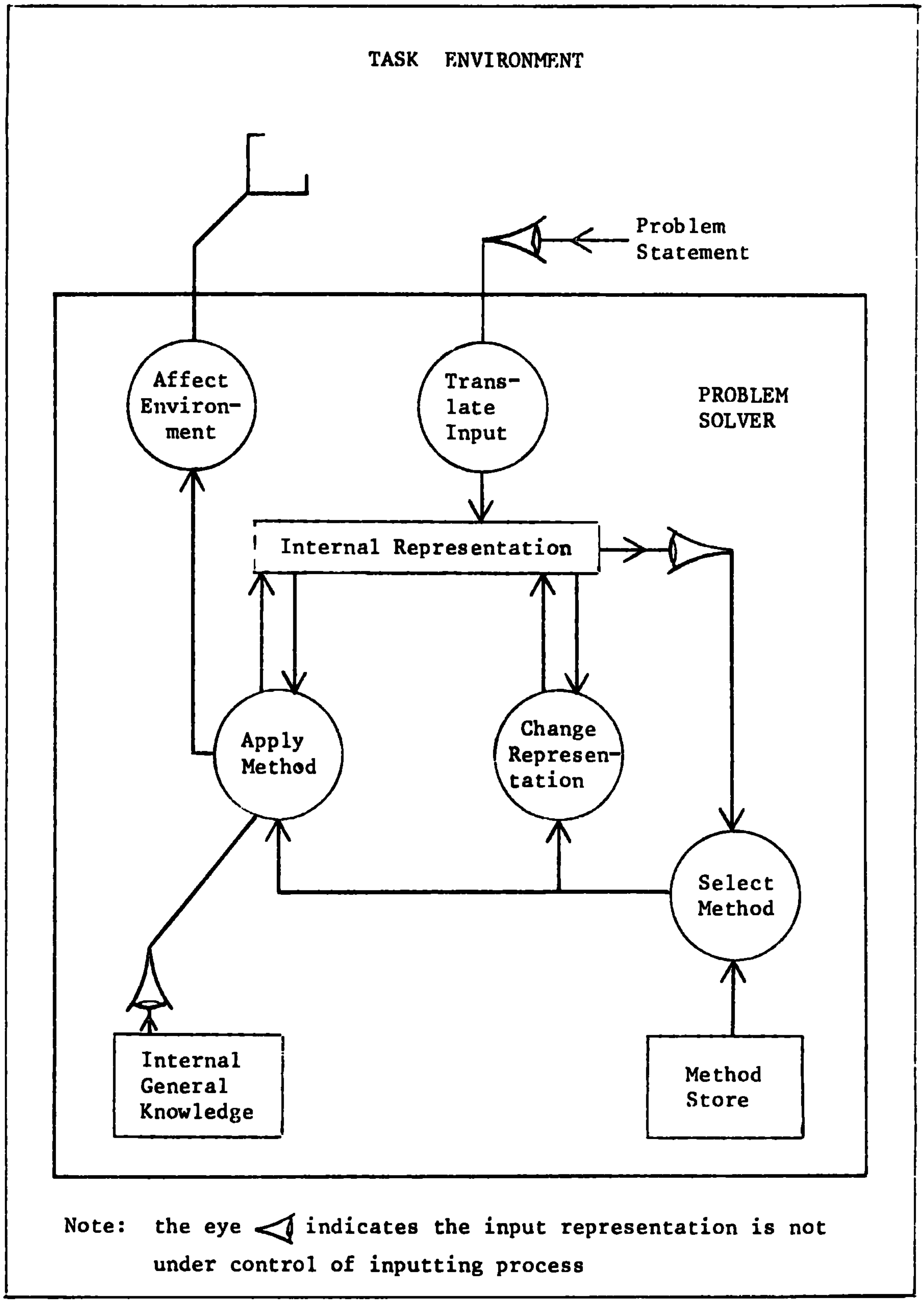


Figure 2.1. Newell & Simon's Problem Solver

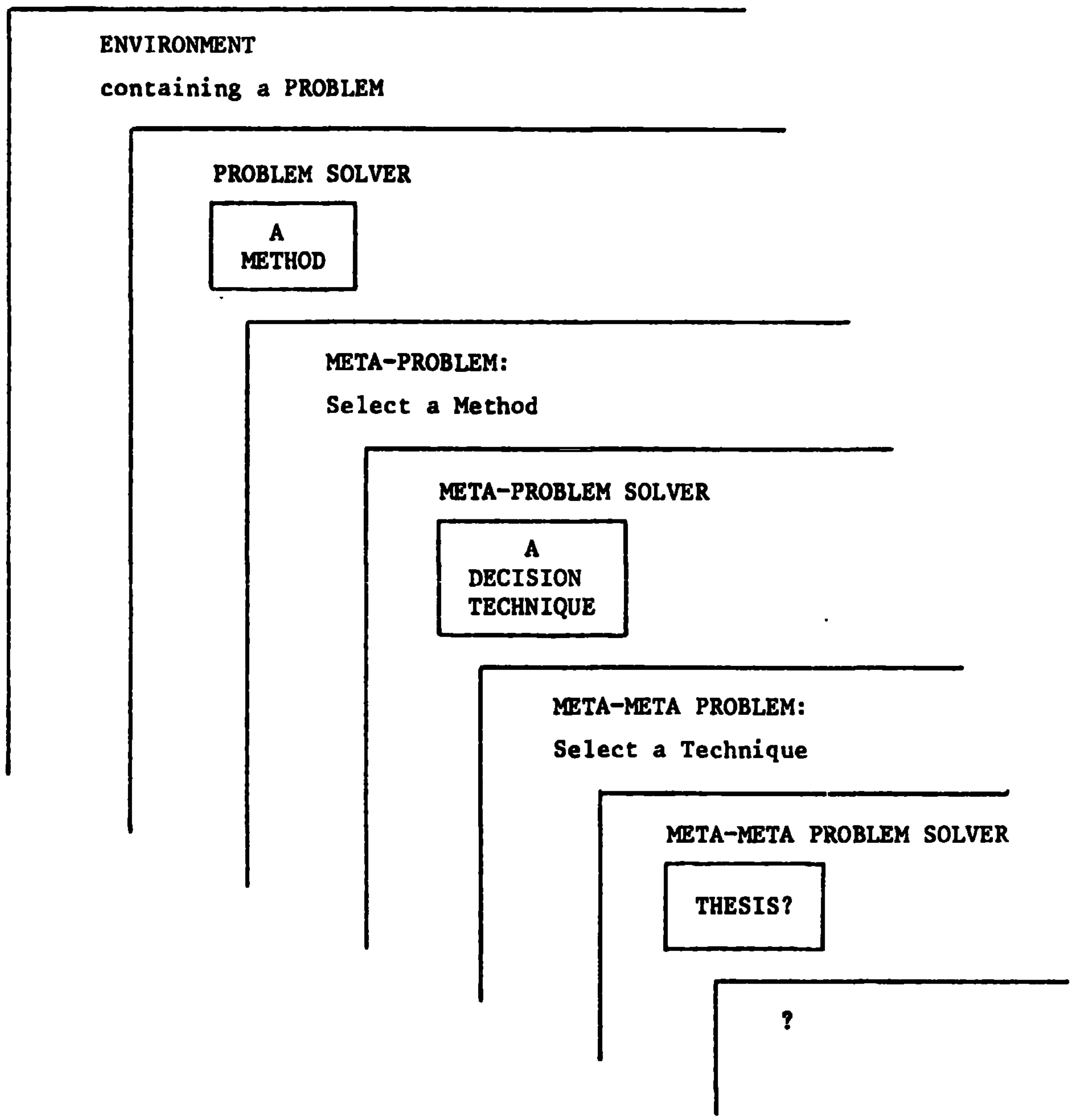


Figure 2.2. The meta-problem

a decision-technique, with a selected method as its output. But what sort of meta-meta-problem-solver is required to select the best techniques?

In practice, the meta-meta-problem is given little consideration. The decision-maker chooses the meta-problem-solver which is most conveniently to hand or best presented by its devotees and advocates. In some ways, this thesis is just such a meta-meta-problem-solver. By means of argument and persuasion, it will try to compare and discuss some meta-problem-solving techniques, with the hope that the output will be a "best technique".

In many cases the decision technique will be determined, more or less, by the type of meta-problem to be solved. Dilemmas in scheduling and transportation have their own methods of solution, but the meta-problem-solving method which we shall be concerned with is multi-attribute decision-making. This technique is used when the external problem in the environment requires the attainment of more than one objective, and each problem-solving method only attains some of the objectives, to a greater or lesser degree. The interests of many groups of people may be involved and there may be only limited opportunity to apply the selected method.

Within the context of human problem-solving activity, multi-attribute decision-making must be seen as one method of producing an output from the circle on Newell and Simon's diagram (Figure 2.1) labelled "Select Method". The multi-attribute decision-making problem only exists when the problem which is to be solved is within an environment which imposes constraints upon the problem-solver, so that repeated attempts at solving the problem are forbidden and only a small number, perhaps no more than one, method may be applied.

MULTI-ATTRIBUTE DECISION-MAKING AND THE CYCLE OF ACTIVITY

As part of the general problem-solving activity, multi-attribute decision-making will also involve steps which require the building up of an internal, mental representation of the external problem, acquiring further internal general knowledge and inventing or discovering more methods of solving the problem (See Figure 2.1). As the decision-maker examines the meta-problem of the selection of the method, he will build up his knowledge of the problem in a learning process. Thus, the decision-making problem is at two levels. Firstly, there is the level of learning about the problem, and this proceeds via the second level which is the examination of the meta-problem of selecting a problem-solving method. The multi-attribute decision process will be cyclical, since the process of learning and perception affects the selection of a method of solving the problem. If we attempt to delineate the activities involved in solving a multi-attribute problem, they will be joined cyclically, because of the feedback to earlier stages.

Many such attempts to classify the stages of multi-attribute decision-making have been made and a few are contained in Appendix 1 to this chapter. The differences between classifications of the stages of activity are due to the level of generalisation intended and are biased towards each author's preferred approach. Any attempt to divide up the decision-making process into a series of stages will be open to dispute, because of the cyclical nature of the activity and the overlapping stages. However, the value of such classifications is in their usefulness as a tool to help the decision-maker to think more clearly about the decision-making process, since before the final selection, all stages should be complete. A suggested classification is depicted in Figure 2.3. The cyclical and reiterative nature of the process is emphasised by the feedback loops in the diagram.

Using this diagram as a framework, we shall consider each stage of the activity in turn. Although the stages will be considered in the order in which they occur in the diagram, this does not necessarily mean that this is the only order in which to perform each action.

Problem Recognition

The first stage, problem recognition, is a preliminary to every decision problem. It is a stage which is sometimes hastily done so that the definition of the problem to be solved is only an impression of what the problem is, and some ideas for solving it. For example, an overburdened library may decide that it needs a computerised issue system to solve its problems of congestion and paperwork, and may proceed to evaluate

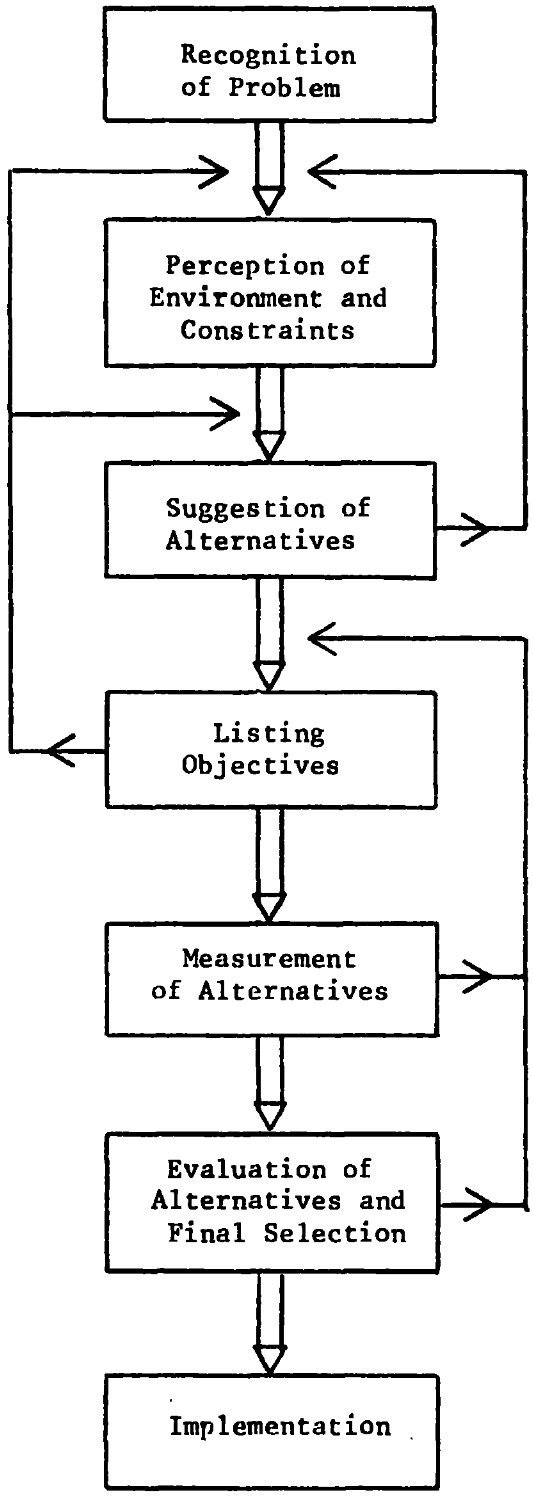


Figure 2.3. Cycle of multi-attribute decision making

different (expensive) issue systems without ever questioning if their problem could not be solved by the elimination of repetitive and unproductive work elsewhere. This stage needs to be treated with as open a mind as possible. Something like the 'system approach' is very useful at this stage for questioning the assumptions which underlie the problem statement, obtaining an overall understanding of the environment and clarifying the real difficulties. If this stage is not given proper attention, and the terms of reference in which the problem is couched are too restrictive, it may turn out later that interesting and useful solutions are unnecessarily discarded.

Perception of Environment

The second stage refers to the environment of the problem. People will be affected by the outcome. In some cases, this will be the decision-maker himself alone, but in the decisions of corporations and governments the number of people involved will run into millions, whole nations. The target population can be divided into groups which will be affected in different ways, although certain individuals may be members of more than one group. The interests of the groups may be conflicting, and it is more of a moral decision whether the outcome should favour the underprivileged, the vociferous, the loyal or the party members. Pareto-optimality is one strategy for allocating benefits and a state of affairs is Pareto-optimal when "no individual can move to a more preferred position except by causing another to move to a less preferred position" (Seldon and Pennance, 1976). This is just one social welfare function among many possible, such as Kaldor's Compensation Principle or the Pigovian social welfare function, which is also known as dollar democracy.

Such a statement as Pareto-optimality provides one set of constraints on the solutions of the problem. Those which can be shown to violate Pareto-optimality would be immediately discarded. But the environment can provide other constraints such as the amount of money available, no foreign equipment to be bought, staff must not be reduced or no change in public image. These constraints can come in many forms and usually impose limits on the resources available or the changes which can be made. They are often stated as negative imperatives. Constraints are often mixed up with the objectives of the organisation and indeed they are very similar. As a means of distinguishing between them, Eilon (1972) suggests that constraints are the Don'ts and Goals are the Dos. I would suggest that constraints are imposed by the environment and goals or objectives are determined from within the organisation. Thus, if one problem is to buy the most suitable car, and the ideal consists of high reliability combined in the same model with low cost, such a car does not exist and this is a constraint which cannot be violated within the present environment. The constraint of avoiding staff reductions could be violated, if an alternative could be presented which was outstanding in every other way. So this constraint is really an objective of the organisation.

These two stages are linked since they are both part of the learning process involving the environment and the problem which it contains. The next stages will be concerned with the analysis of the problem and the search for the best solution.

Suggestions of alternatives

The third stage can take many forms. For a system change, such as the library example above, the solutions would be possible alternative systems. For the prospective car buyer the alternatives are the cars available on the market. The businessman may find that the solutions to his problems are policy changes or strategies which he may adopt, affecting his future plans and behaviour. The alternatives may be tangible items, such as the motorcars, or plans for the future. There will be some uncertainty associated with any alternative, but some will be more uncertain than others (particularly if the car is secondhand).

This is the most creative stage of the problem-solving process. It requires the imagination of the decision-maker to conjure up new and pertinent approaches to the problem. The environment's constraints may mean that traditional methods of solving the problem may not be appropriate, and so new methods will be needed. There have been suggestions of ways of improving this process, because the resulting decision will only be as good as the proposed alternatives, no matter how worthy the decision-making technique. Again, the systems approach is useful here in generating new perspectives upon the problem, but the decision-maker himself will be the source of these ideas. As the analysis of the problem proceeds, new ideas may occur to the decision-maker, and so this is one stage to and from which looping will forever occur.

Listing objectives

The fourth stage of problem-solving is in determining the objectives of the organisation. In this discussion, organisation is taken as meaning any decision-making entity, whether large corporation or single individual. The organisation will have objectives at many levels, some acting as banner statements of general policy, objectives within departments, the objectives of individuals and many more. For example, in the recent film, Superman stated his objectives to be "To fight for truth, justice and the American way". These would be his banner objectives, but he has lesser objectives as well, such as to avoid lumps of deadly Kryptonite and to court Lois Lane. He achieves these objectives by fighting the bad guys and upholding law and order, and in the execution of these self-imposed duties, he fulfills many lesser, more immediate objectives as well.

The statement of the higher order objectives will often be made as part of the definition of the problem, but this must be reduced to more day-to-day objectives. One way of doing this is to weight the higher order objectives, and in turn resolve each of these to lower order objectives which are weighted at this lower level. This tree ordering of the objectives is continued until the objectives are stated in terms of directly measurable quantities, such as CPU store or queuing time (Land, 1975). This means that there is a means of directly measuring every objective and so the separate alternatives can be assessed. However, this does have the effect of forcing objectives which refer to intangible quantities, such as prestige or trust, to be measured using objective, tangible quantities. This problem has been recognised, and the assessment of both subjective and objective aspects has been incorporated in a single assessment technique (Mumford, Land & Hawgood, 1978).

We must also distinguish carefully between attributes and objectives. In some multi-criteria techniques the terms are used interchangeably, but we shall distinguish between them here. An objective is an aim or intention of the organisation, whereas an attribute is a characteristic of the alternative solutions to the problem. A solution will fulfill the objectives, to greater or lesser extent, and it will possess the attributes similarly. The assessment of how a solution will fulfill the objectives may be done by expressing the objectives in terms of the attributes and weighting them as seen fit, or by measuring or estimating the fulfilment of the objectives directly. This distinction is not often made, and in many cases is not necessary. However, we must be aware of the fact that objectives are often subjective or qualitative in nature, and the introduction of so-called objective measurements may be misleading or artificial.

The Measurement of Alternatives

In the discussion of the previous stage, we touched on ways that this can be done, i.e. by reducing the higher level objectives to directly quantifiable attributes. When tangible and intangible aspects are being assessed together, it is sometimes suggested that, as far as possible, the measurement of intangibles should be reduced to quantitative aspects. For example, some libnrarians wanted to measure the friendliness of the atmosphere of the library and they suggested using 'Litres of Green Paint on the Walls' as an indicator of the presence of an official attitude. However, this approach can run into difficulties, because no single objective attribute can be used to capture the essence of the subjective quality it is attempting to measure. If more than one objective measure is used for each objective, then difficulties can arise from two additional sources, apart from its inability to capture the essence. Firstly, we may find that the same index is being used as part of the measure of two or more objectives. This leads to that index being counted twice, and hence being given greater weight than it should have. The second problem is that when two or more measures are being used to assess one objective, then there must be some means of combining these separate measurements to give a single measurement. This can be done either by the weighting of lesser objectives methods, mentioned earlier, or by an unvoiced, intuitive method, as has been used. In the second case it might be simpler to dispense with the objective measures and just rely on the subjective assessment at source.

On a slightly different tack, the substitution of objective measures for subjective objectives can lead to 'goal displacement'. The fulfilment of the measurement becomes the objective. For example, an employment agency wishes to improve its performance as a body set up with the purpose of finding people suitable employment. As a possible measure of success "Number of people interviewed each week" is adopted, since the more people who are being interviewed, the more people who will find jobs. However, in order to increase the number of interviews, staff attend to their clients hastily and superficially, so the final outcome may be fewer people placed in satisfactory jobs, but more people presented for interview, over and over again.

It had been hoped that the eventual advances of science would yield objective measurements for subjective aspects too.

> It is more difficult to construct measures of effectiveness where qualitative objectives are involved than where only quantitative objectives are concerned. This fact should not discourage efforts to transform qualitative objectives into quantitative ones. The history of science has repeatedly demonstrated that a property that appears in one era to require qualitative treatement is converted into quantitative terms in another era. At one time such qualitative properties as "red", "hard", "intelligent" and "communicative" were thought to be inherently qualitative. Today we know better. There is no logical or methodological reason (though there may be a practical one) why such concepts as "good will", "morale" and "responsibility" cannot be reduced to quantitative terms. (Churchman, Ackoff & Arnoff, 1957)

While holding out hope for a quantitative, scientific future, such statements offer little consolation to the analyst faced with intransigently qualitative criteria now, and aware of the dangers of overenthusiastic quantisation.

In the twenty years since that passage was written, we are no nearer objective, operational measurements of "good will", "morale" and "responsibility". This lack of progress may be attributed to the inherent and undeniable subjectivity of such concepts. The nearest that we have come is the unsatisfactory method of awarding points out of ten, which is neither reproducible nor independent of the observer.

It seems that we must accept that there are some aspects of life and decision-making which cannot be measured precisely and objectively. To assess any alternative, these aspects must be measured in some way. Awarding marks out of ten is one method which may be adopted, allowing the subjective and objective aspects to be treated similarly. Once they have been included in the calculations, the 'subjective numbers' are assumed to be accurate, within the error limits which it may be possible to include. However, these 'subjective numbers' when compared to objective measurements are meaningless and to treat them on a par is to accord them greater respectability than they deserve. The answers which emerge may appear to be acurate to several places of decimals, but unless such answers can be shown to be stable over a range of values on the subjective attributes, within all the error limits, say, then they cannot be trusted.

Objective and subjective aspects are fundamentally different. Until this is proved to be wrong, we must consider what is the best way in which to handle them jointly. This difference means that applying objective techniques to the estimation of subjective aspects can lead to error and misunderstanding, e.g. the substitution of inappropriate objective aspects, multiple use of objective aspects and goal displacement. See Appendix 2 for an illustration.

Subjective aspects are not precise and the precision of numerical measurement is out of place. Objective and subjective aspects are both important, but decision-makers require some method of treating them equally, and with the minimum amount of distortion.

I would argue that the substitution of objective measurements for subjective criteria is a dangerous practice. The objective measurements, by their very nature, can never express the essence of the subjective objective and can lead to the over-emphasis of unimportant attributes and displacement from the true objective.

Evaluation of Alternatives and Final Selection

I have discussed the process of measurement along the various attributes or objectives of the organisation, without saying anything on how these measurements are to be combined. In many cases, the mesurements will be along different scales and hence impossible to combine without some sort of rationalisation. Cost-benefit analysis does this by converting all the units to £ and expressing the overall value of each alternative as a financial benefit or cost. Other methods use "utility" measurements which convert the quantities of each attribute into "utiles", which measure the subjective value placed upon commodities. The subjective value of any commodity is supposed to be expressible in utiles, which can be freely converted to any other commodity.

Once the attributes of the proposed solutions have been measured, they require some sort of combination so as to give an index of merit, upon which to base the selection of the 'best' alternative. The objectives or attributes (we shall refer to them both as 'criteria' where there is no need to distinguish between them) assume different importances in the eyes of the decision-maker and so the relative weights of the criteria must have some representation. This is often done by assigning a numerical weight to each criterion. We now have a list of criteria, each of which has a numerical weight, and a set of solutions, each of which is separately scored for each of the attributes.

There have been many ways suggestged of combining these figures. See Table 2.1. The simplest method is to take the product of weights and scores and add them up. This method begs assumptions on the interactions between the attributes, but is often used. Another method, which begs similar assumptions, is to multiply the products, obtained as before. These will be considered in Chapter 4. This method is not so commonly used, presumably because it can cause wider variation and one must be careful not to include weights or scores which equal zero. Dujmovic (1977) and Easton (1973) both suggest higher order averages. Some of these have been investigated and are depicted in Figures 2.4-2.5 and Tables 2.1-2.5.

There are three decision rules (with variations) common in the literature which evaluate multi-criteria alternatives in non-additive ways:

(1) conjunctive - each criterion has a standard fixed, and the alternative must pass on a number of these standards. The standards must be set at a minimal level or few alternatives will pass.

(2) disjunctive - standards are set of each criterion, but they are set at a maximal level, and an alternative need only pass on one.

(3) lexicographic - see Dawes (1964). The attributes are ranked in order of importance, and the intra-attribute values are placed on an ordinal scale. All the alternatives are compared on the top-ranking criterion, and the highest scoring is selected. If there is a tie, they are compared on the next criterion and the process is continued until all but one of the alternatives have been eliminated.

MacCrimmon (1973) also suggests Dominance and Eliminating by Aspect. An alternative is dominant over another if it is better than or equal to the

Figure 2.4 Comparison of 6 Aggregation Methods for 10 Separate Interest Groups

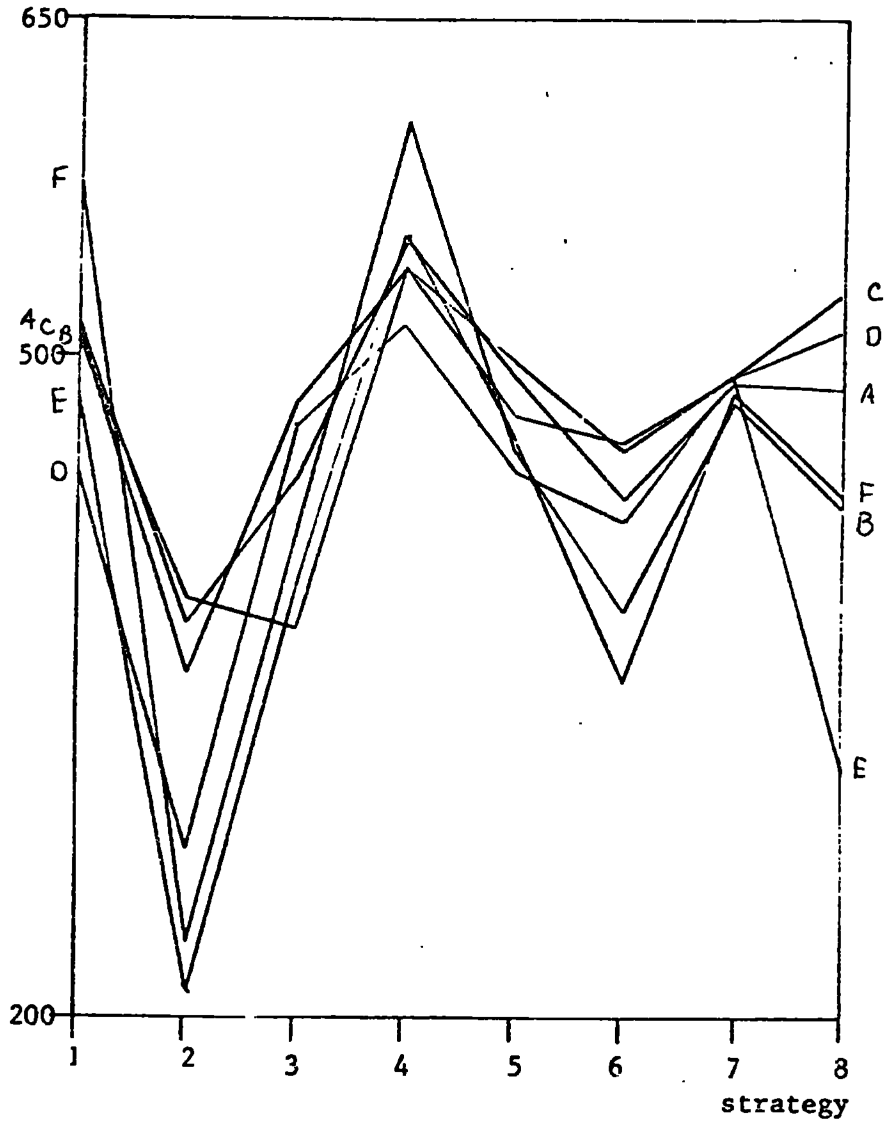


Figure 2.4(a) Group 1

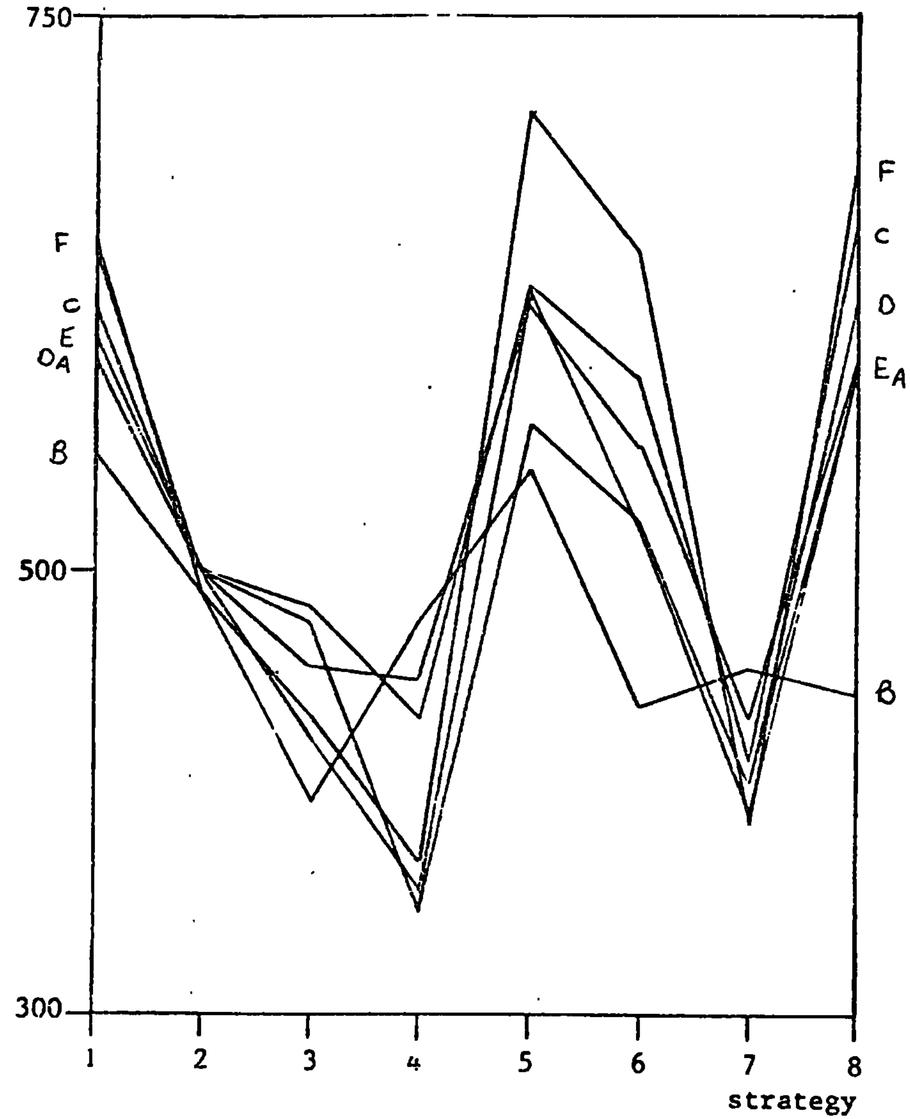


Figure 2.4(b) Group 2

See footnote on p.22.

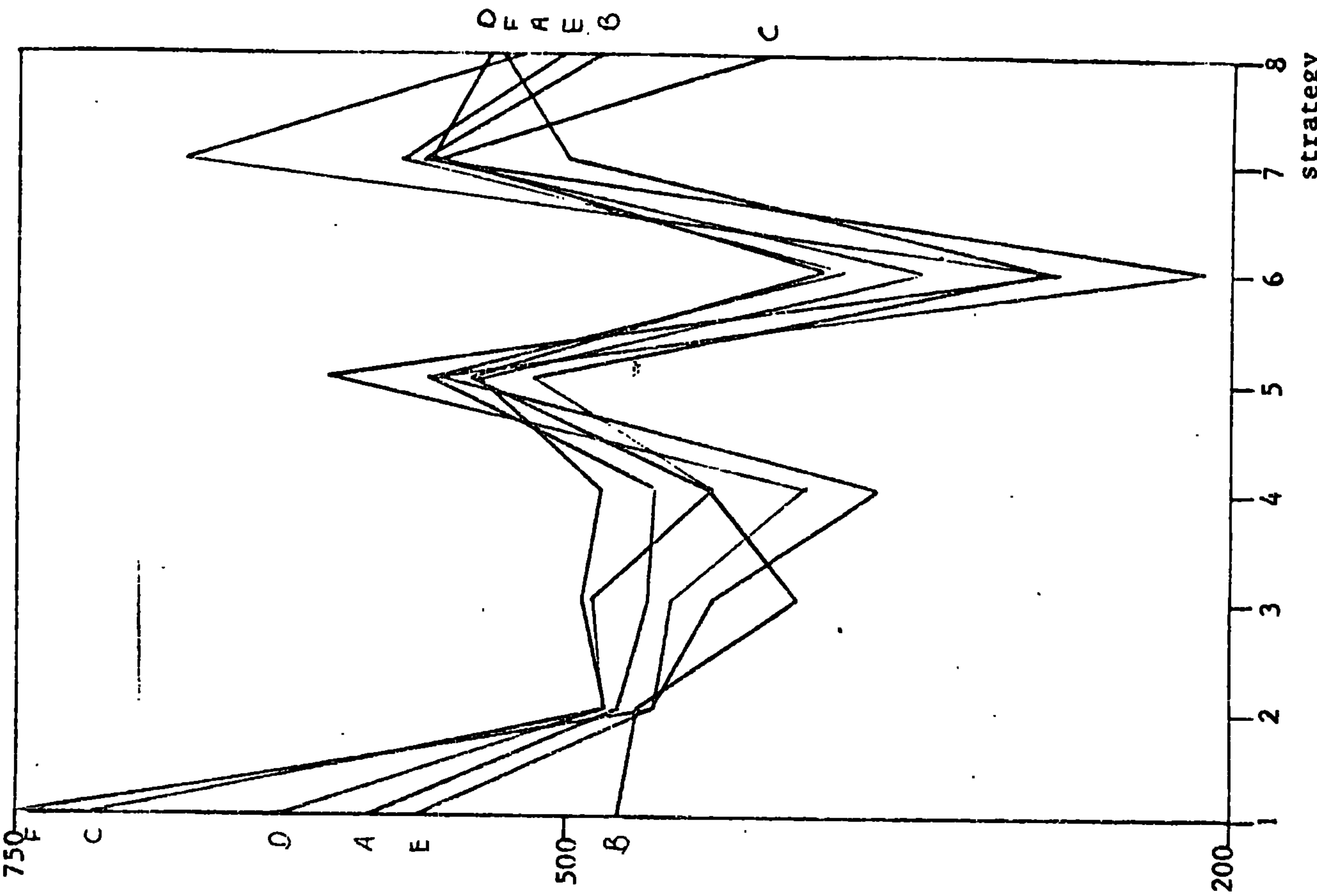


Figure 2.4(d) Group 4

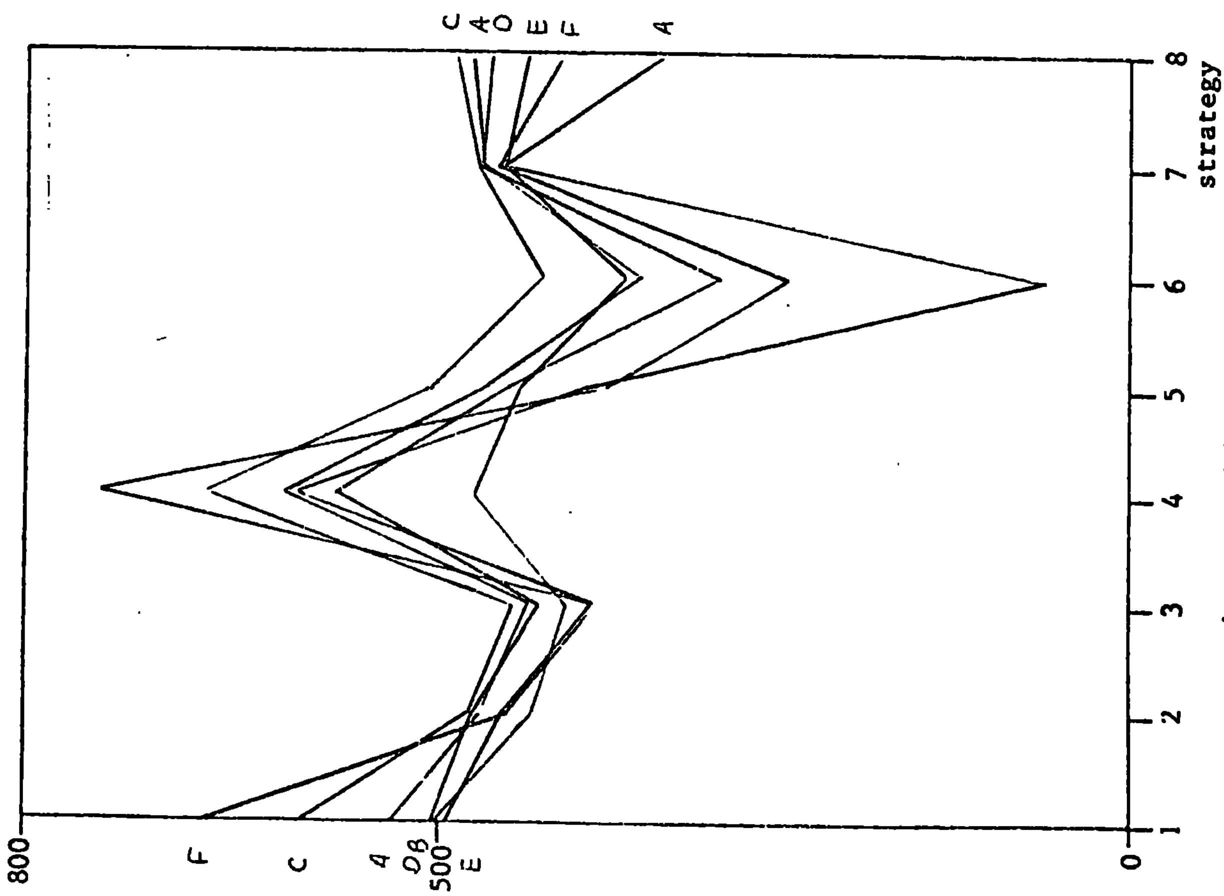


Figure 2.4(c) Group 3

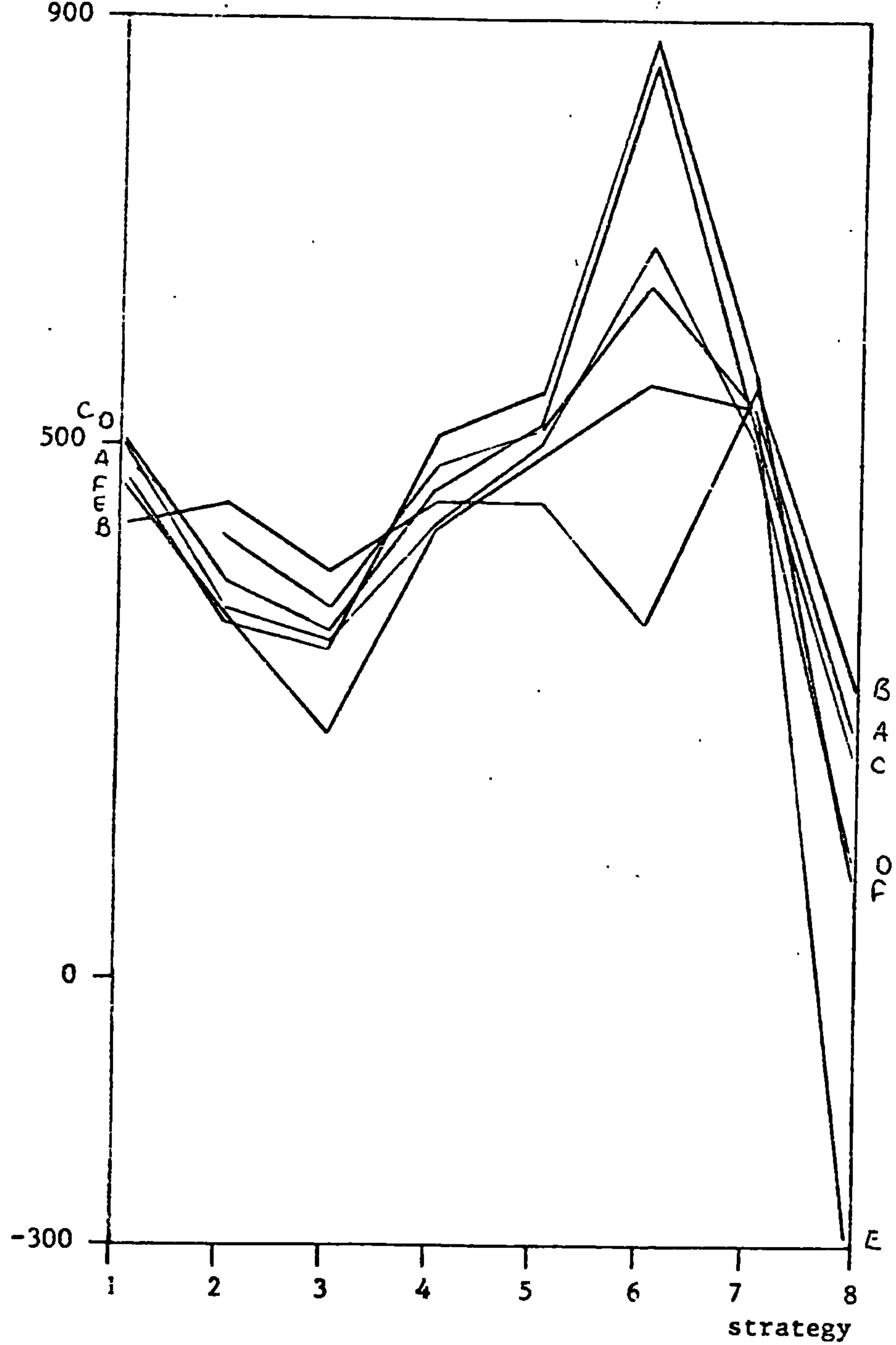


Figure 2.4(e) Group 5

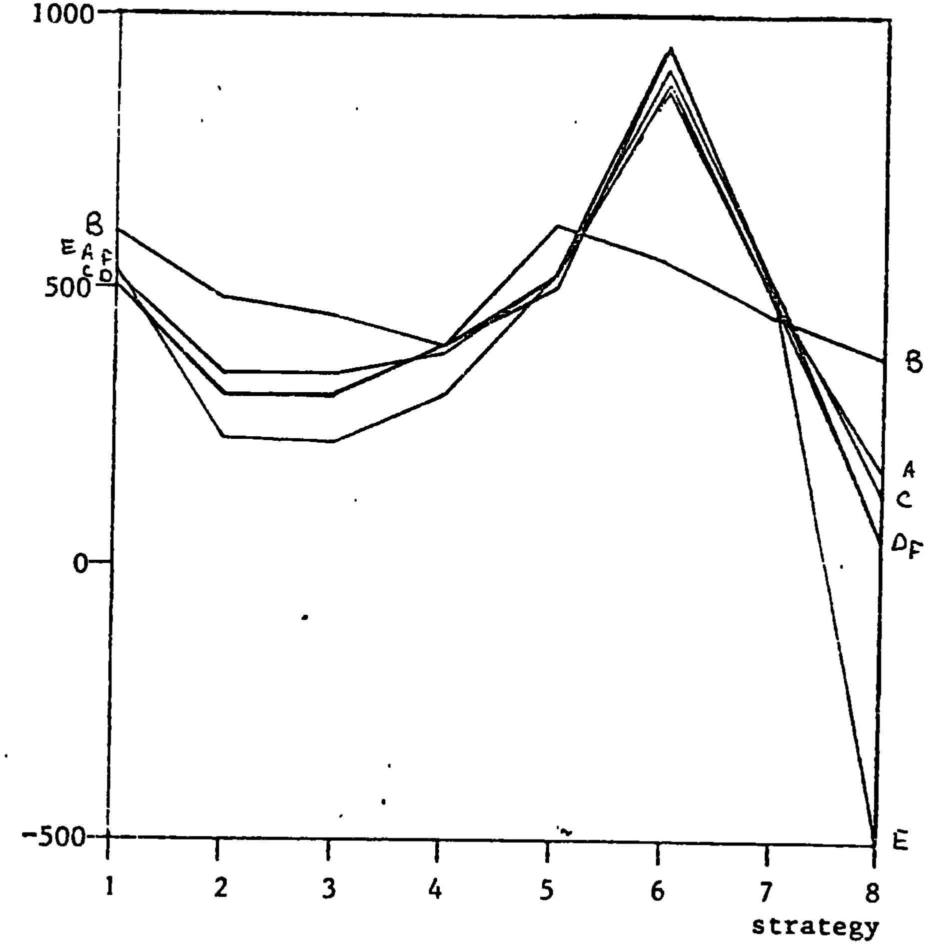


Figure 2.4(f) Group 6

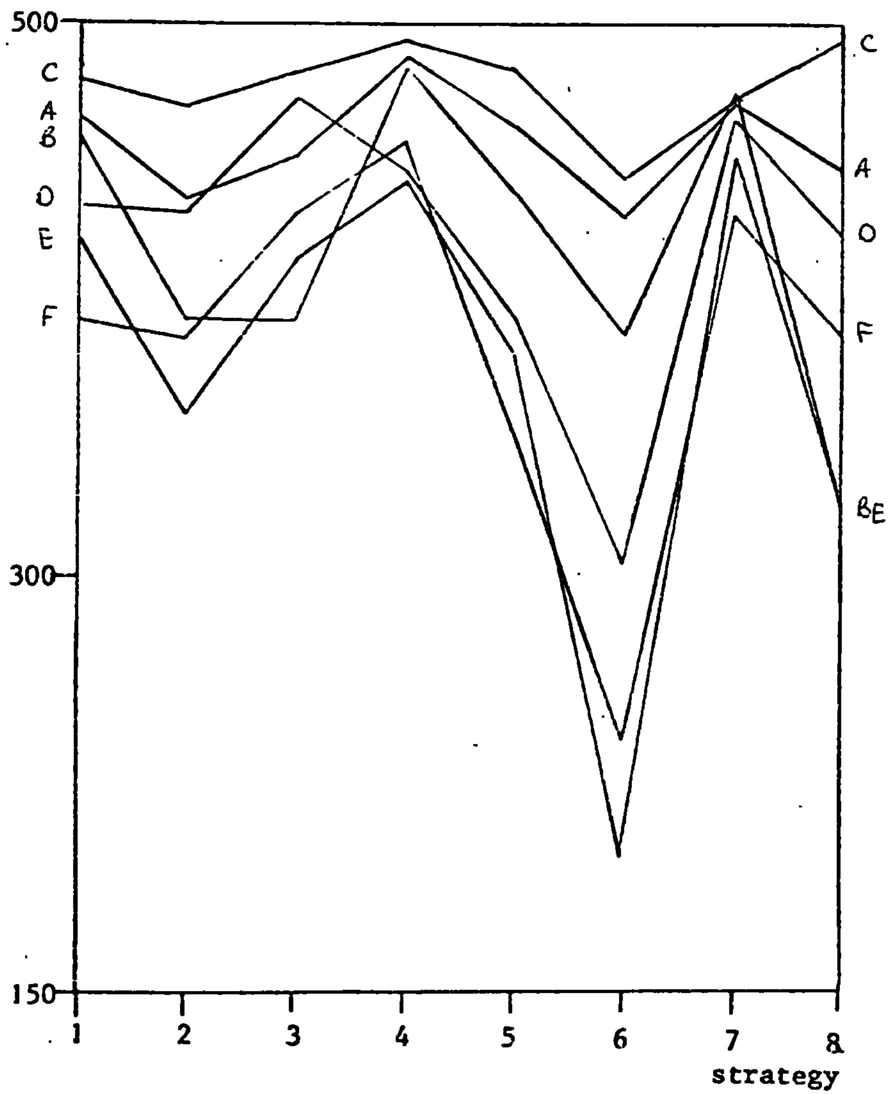


Figure 2.4(g) Group 7

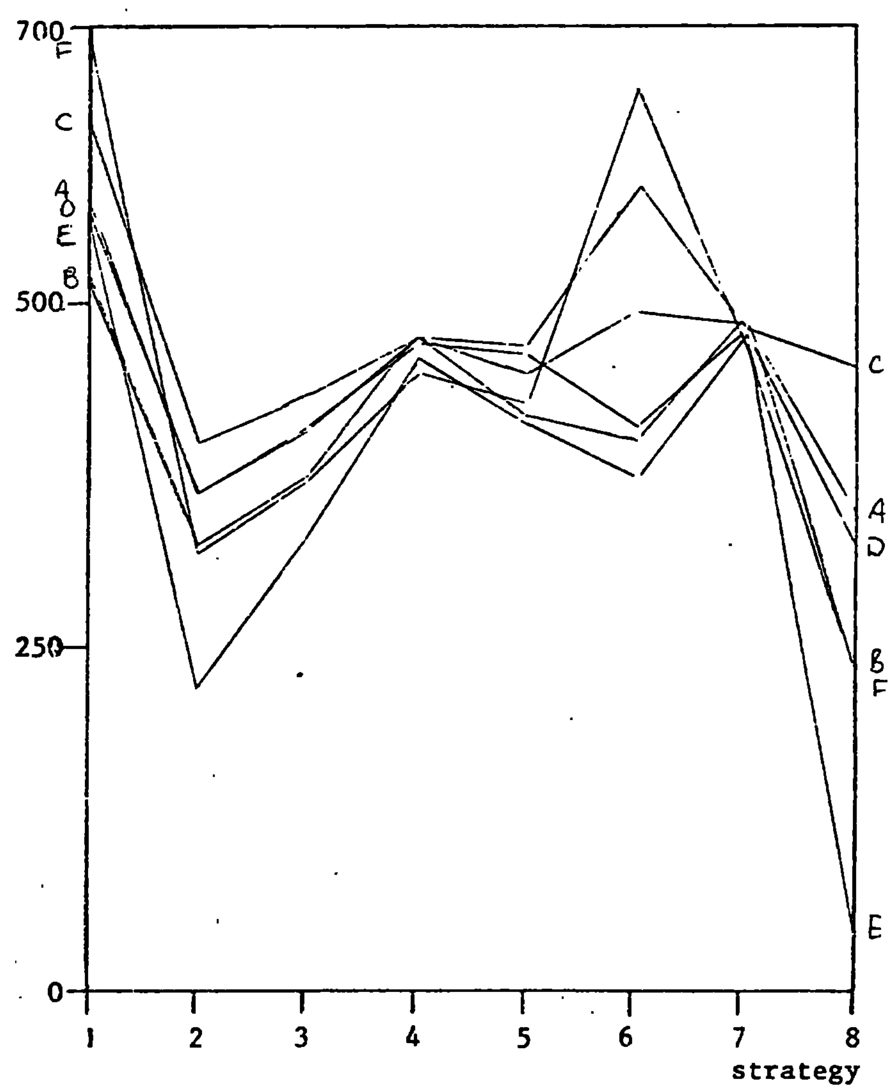


Figure 2.4(h) Group 8

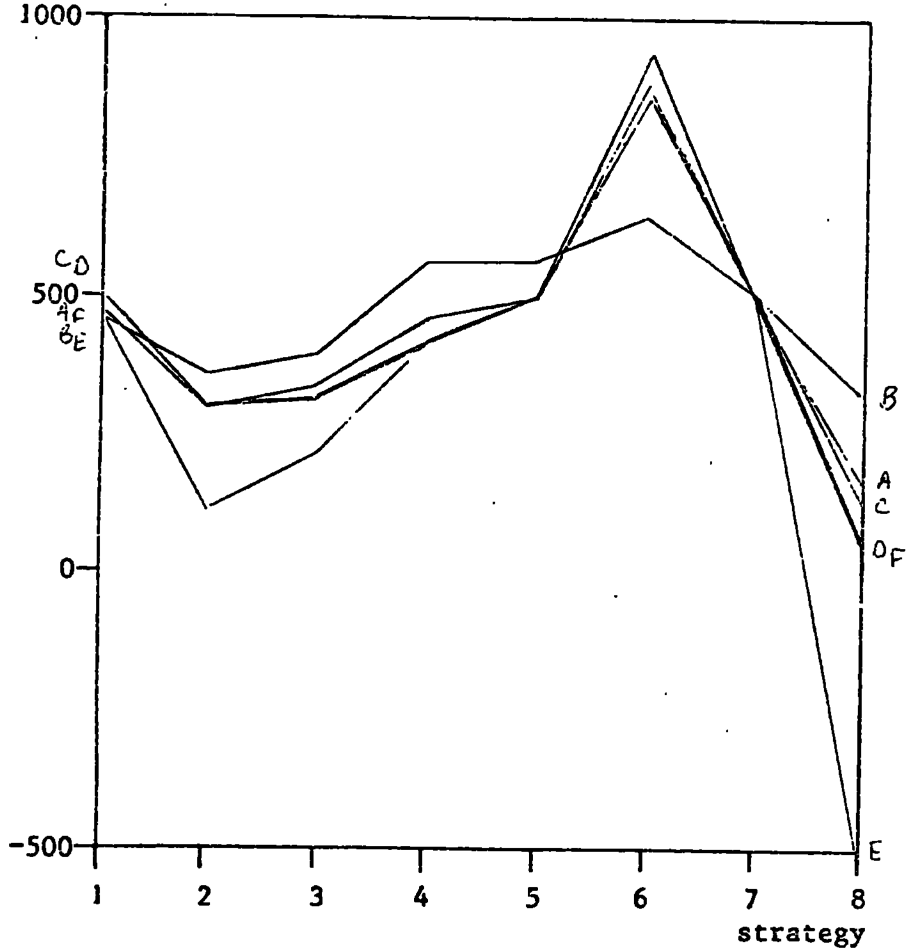


Figure 2.4(i) Group 9

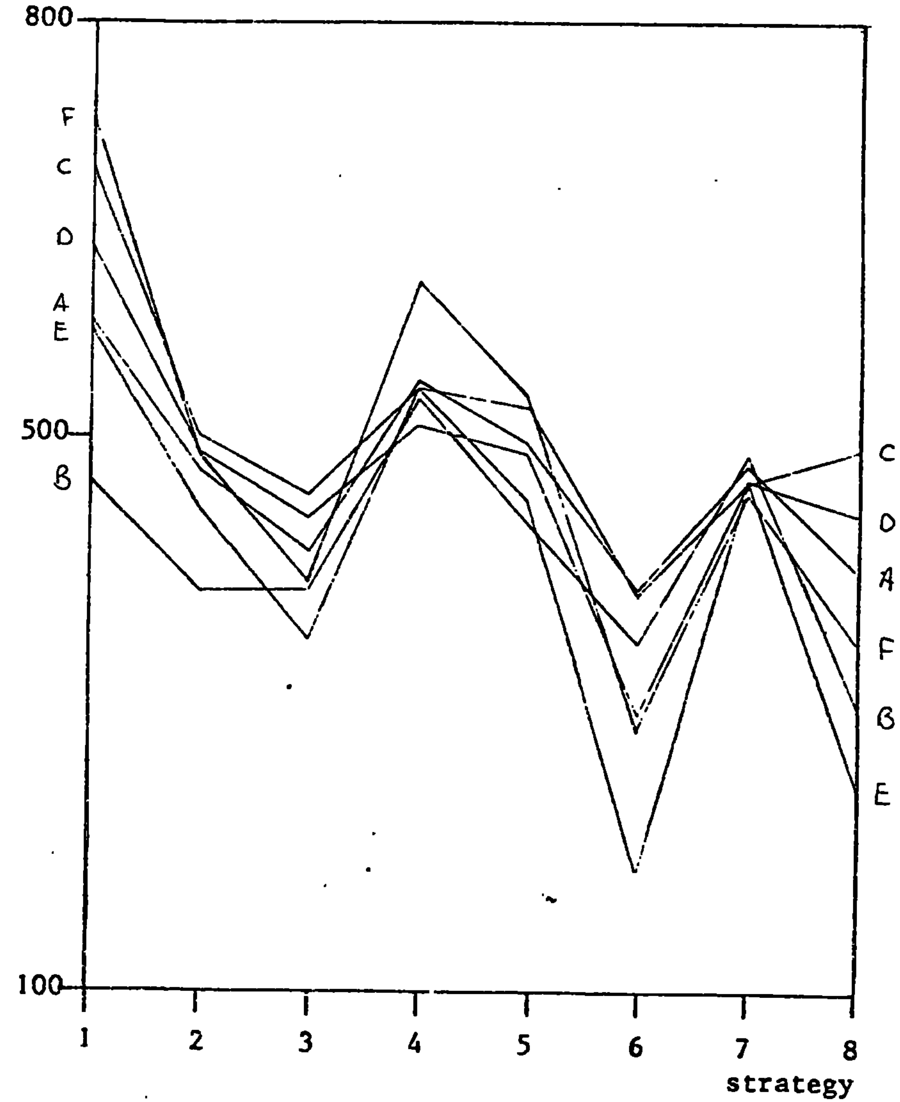
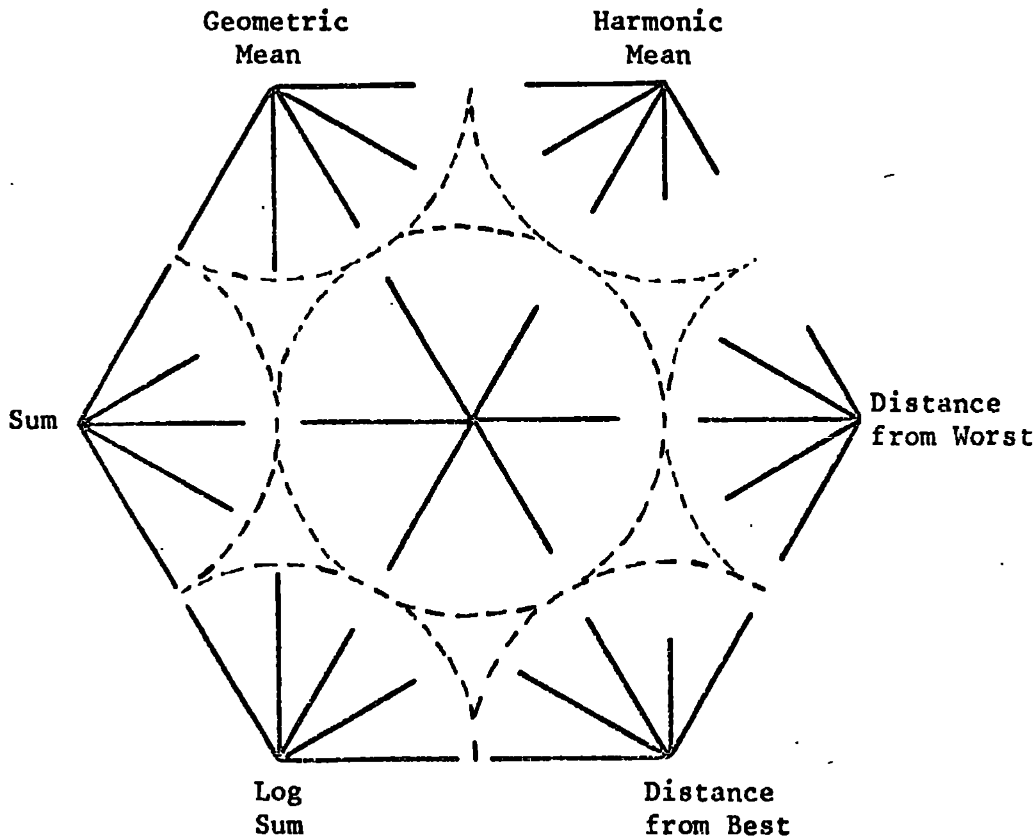


Figure 2.4(j) Group 10

Figure 2.5 Comparison of Correlation Coefficients for Each Rule-Pair



The length of a line directed outwards from any corner indicates the rank correlation between the rule represented by that corner and the rule towards which the line is directed. The circles indicate the maximum correlation. The central rosette displays the average of the correlations of each rule with the other five.

Each diagram in Figure 2.4 represents a separate group. The weights which a particular group has assigned to the objectives are combined with the scores of each of the 8 decision stategies or alternatives to produce a figure of merit for each strategy. Using the six combination rules of Table 2.1 produces six differing figures of merit for each strategy, for each group. These are plotted in Figure 2.4, to display the similarities and differences between the rules.

m objectives $\sum_{i=1}^{m} w_i = 100$ $\qquad -10 \leqslant S_i \leqslant +10$

	Name	Rule	Normalised Version
A	Sum	$F = \sum_{i=1}^{m} w_i S_i$	$F^1 = 500 + \frac{1}{2} F$
B	Geometric Mean *	$F = \sqrt[n]{\prod_{i=1}^{n} w_i S_i'}$ $S_i' = S_i + 11$	$F^1 = \dfrac{50F}{\sqrt[n]{\prod_{i=1}^{n} w_i}} - 50$
C	Distance from worst	$F = \sqrt{\sum_{i=1}^{m} w_i^2 S_i'^2}$ $S_i' = S_i + 10$	$F^1 = \dfrac{50F}{\sqrt{\sum_{i=1}^{m} w_i^2}}$
D	Distance from best	$F = K - \sqrt{\sum_{i=1}^{m} w_i^2 S_i'^2}$ $S_i' = 10 - S_i$	$F^1 = 1000 - \dfrac{50}{\sqrt{\sum w_i^2}} \sqrt{\sum_{i=1}^{m} w_i^2 S_i'^2}$
E	Log sum	$F = \log\left(\prod_{i=1}^{m} S_i'^{\,w_i}\right)$ $S_i' = S_i + 11$	$F^1 = \dfrac{5F}{\log\left(\frac{21}{11}\right)} + 500\left(1 - \dfrac{\log 11}{\log\left(21/11\right)}\right)$
F	Harmonic Mean **	$F = \sqrt{\sum_{i=1}^{m} w_i^2 S_i^2}$	$F^1 = \dfrac{50F}{\sqrt{\sum w_i^2}} + 500$

For each rule, when all $S_i = 0$, $F^1 = 500$, and when all $S_i = 10$, $F^1 = 1000$

* When a $w_i = 0$, this is not included in the calculations and the n^{th} root is taken, when n is the number of non-zero weights. This maintains F^1 at 500 when the S_i approach 0.

** Squaring negative scores loses the information on the scores being less than zero. To allow for this, the squared negative scores are subtracted.

The letters in the leftmost column are used to distinguish between the rules in Figure 2.4.

Table 2.1
Rules Investigated

WEIGHTS

Group	1	2	3	4	5	6	7	8	9	10
Objective										
1	12	4	0	6	15	0	4	16	0	14
2	1	0	1	0	46	79	9	28	76	10
3	0	39	7	0	0	13	19	0	0	0
4	0	0	0	22	8	0	0	0	0	0
5	38	0	0	0	0	0	13	22	16	2
6	0	0	26	0	4	0	26	0	0	10
7	23	18	36	48	0	2	0	34	0	36
8	12	16	30	11	12	0	10	0	7	12
9	0	23	0	13	0	4	0	0	0	0
10	14	0	0	0	15	2	19	0	1	16

SCORES

Strategy	1	2	3	4	5	6	7	8
Objective								
1	0	-1	-5	0	-4	-6	0	-7
2	0	-4	-6	-4	0	9	0	-9
3	3	0	0	-5	0	0	-3	5
4	-4	-2	0	-7	0	-5	9	0
5	-3	-7	0	0	-2	0	0	1
6	-4	-2	0	0	-6	-9	0	-4
7	6	0	0	0	0	-5	-1	1
8	-2	0	-4	9	3	4	0	0
9	0	0	0	-2	9	7	0	0
10	2	3	0	0	5	0	0	0

Table 2.2

Weights and Scores

Figures of Merit

Each group of weights is combined with the scores for each of the eight strategies, using each of the six combination rules in Table 1. The resulting 480 figures of merit are listed in Tables .3a to .3j below, together with the rank orderings.

Table 2.3a Group 1

Rule \ Strategy	1	2	3	4	5	6	7	8
Sum	514	380	444	553	490	435	488	484
	2	8	6	1	3	7	4	5
Geom Mean	508	391	377	538	497	457	491	310
	2	6	7	1	3	5	4	8
Dist Worst	512	357	479	537	472	460	490	528
	3	8	5	1	6	7	4	2
Dist Best	447	278	469	515	447	425	489	510
	5.5	8	4	1	5.5	7	3	2
Log Sum	483	216	398	554	462	352	483	434
	2.5	8	6	1	4	7	2.5	5
Harm Mean	580	235	422	608	456	383	477	427
	2	8	6	1	4	7	3	5

Table 2.3b Group 2

Rule \ Strategy	1	2	3	4	5	6	7	8
Sum	597	498	458	451	619	555	432	592
	2	5	6	7	1	4	8	3
Geom Mean	555	490	395	477	544	438	456	443
	1	3	8	4	2	7	5	6
Dist Worst	623	500	483	432	629	586	413	659
	3	5	6	7	1	3	8	1
Dist Best	598	500	475	346	565	521	403	627
	2	5	6	8	3	4	7	1
Log Sum	608	497	425	355	622	517	390	594
	2	5	6	8	1	43	7	3
Harm Mean	652	470	434	368	706	642	385	688
	3	5	6	8	1	4	7	2

Table 2.3c Group 3

Rule \ Strategy	1	2	3	4	5	6	7	8
Sum	536	472	438	616	467	357	471	479
	2	4	7	1	6	8	5	3
Geom Mean	503	433	409	477	443	365	456	342
	1	3	6	2	4	7	3	8
Dist Worst	601	479	448	669	505	427	476	491
	2	5	7	1	3	8	6	4
Dist Best	506	475	431	577	451	300	474	469
	2	3	7	1	6	8	4	5
Log Sum	497	456	392	604	397	65	456	440
	2	3.5	7	1	6	8	3.5	5
Harm Mean	667	452	389	747	382	251	461	415
	2	4	6	1	7	8	3	5

Table 2.3d Group 4

Rule \ Strategy	1	2	3	4	5	6	7	8
Sum	589	475	463	459	563	374	575	503
	1	5	6	7	3	8	2	4
Geom Mean	477	468	395	436	544	383	558	407
	3	4	7	5	2	8	1	6
Dist Worst	714	485	492	485	539	339	562	335
	1	6.5	5	6.5	3	8	2	4
Dist Best	631	483	487	434	516	282	500	531
	1	6	5	7	3	8	4	2
Log Sum	568	461	433	359	560	211	566	485
	1	5	6	7	3	8	2	4
Harm Mean	745	460	452	392	607	279	672	521
	1	5	6	7	3	8	2	4

Table 2.3e Group 5

Rule \ Strategy	1	2	3	4	5	6	7	8
Sum	479	411	346	480	513	648	536	232
	5	6	7	4	3	1	2	8
Geom Mean	420	439	378	446	446	332	558	274
	5	4	6	2.5	2.5	7	1	8
Dist Worst	500	369	324	458	520	857	515	207
	4	6	7	5	2	1	3	8
Dist Best	495	345	315	424	500	690	505	114
	4	6	7	5	3	1	2	8
Log Sum	459	337	227	421	489	560	537	-238
	4	6	7	5	3	1	2	8
Harm Mean	472	330	306	508	550	882	568	95
	5	6	7	4	3	1	2	8

Table 2.3f Group 6

Rule \ Strategy	1	2	3	4	5	6	7	8
Sum	527	345	342	384	523	864	479	178
	2	6	7	5	3	1	4	8
Geom Mean	601	477	452	400	618	556	456	379
	2	4	6	7	1	3	5	8
Dist Worst	505	308	308	397	502	940	497	135
	2	6.5	6.5	5	3	1	4	8
Dist Best	504	305	305	396	501	902	495	63
	2	6.5	6.5	5	3	1	4	8
Log Sum	533	228	224	310	524	871	466	-502
	2	6	7	5	3	1	4	8
Harm Mean	526	303	303	393	523	943	475	58
	2	6.5	6.5	2	3	1	4	8

Table 2.3g Group 7

Strategy / Rule	1	2	3	4	5	6	7	8
Sum	466 3	437 7	452 5	488 1	463 4	431 8	471 2	447 6
Geom Mean	457 3	393 5.5	393 5.5	484 1	439 4	388 7	475 2	323 8
Dist Worst	479 5	470 7	481 4	494 1.5	483 3	444 8	473 6	494 1.5
Dist Best	434 4	432 5	473 1	448 3	395 7	305 8	466 2	423 6
Log Sum	422 3	359 6	415 4	443 2	381 5	198 8	453 1	323 7
Harm Mean	393 4	387 5.5	432 2.5	458 1	352 7	241 8	432 2.5	387 5.5

Table 2.3h Group 8

Stragegy / Rule	1	2	3	4	5	6	7	8
Sum	569 1	359 7	404 6	472 4	446 5	493 1	483 3	346 8
Geom Mean	516 1	322 7	372 6	473 3	417 4	401 5	487 2	241 8
Dist Worst	629 1	397 8	430 7	473 3	467 5	583 2	479 4	434 6
Dist Best	563 1	359 7	408 6	469 3	459 4	409 5	478 2	324 8
Log Sum	560 1	218 7	327 6	456 3	410 4	372 5	475 2	43 8
Harm Mean	686 1	315 7	367 6	446 4	425 5	654 2	467 3	237 8

Table 2.3i Group 9

Rule \ Stragegy	1	2	3	4	5	6	7	8
Sum	470	293	334	455	497	856	500	166
	4	7	6	5	3	1	2	8
Geom Mean	454	355	389	557	560	640	500	317
	5	7	6	3	2	1	4	8
Dist Worst	494	297	311	412	498	934	500	131
	4	7	6	5	3	1	2	8
Dist Best	492	294	307	406	496	883	500	68
	4	7	6	5	3	1	2	8
Log Sum	451	110	210	414	491	868	500	-491
	4	7	6	5	3	1	2	8
Harm Mean	468	292	304	411	484	938	500	61
	4	7	6	5	3	1	2	8

Table 2.3j Group 10

Rule \ Strategy	1	2	3	4	5	6	7	8
Sum	589	480	421	544	498	392	482	405
	1	5	6	2	3	8	4	7
Geom Mean	472	393	393	532	439	352	492	308
	3	5.5	5.5	1	4	7	2	8
Dist Worst	700	504	462	539	524	388	470	492
	1	4	7	2	3	8	6	5
Dist Best	642	493	446	512	490	301	469	446
	1	3	6.5	2	4	8	5	6.5
Log Sum	583	453	357	540	456	189	473	249
	1	5	6	2	4	8	3	7
Harm Mean	733	490	398	616	533	289	461	353
	1	4	6	2	3	8	5	7

The correlation coefficients are calculated between the rank orderings of the strategies. Each group has the coefficients calculated separately for all pairs of rules.

Group Rule	Pair	1	2	3	4	5	6	7	8	9	10
Sum	GM	.81	.45	.55	.81	.47	.78	.91	.86	.88	.85
	DW	.76	.93	.86	.96	.95	.99	.49	.50	1.00	.88
	DB	.61	.88	.93	.88	.98	.99	.60	.60	1.00	.90
	LS	.96	.98	.92	1.00	.98	1.00	.93	.93	1.00	.98
	HM	.98	.95	.88	1.00	1.00	.99	.77	1.00	1.00	.98
GM	DW	.31	.24	.64	.73	.37	.77	.18	.76	.88	.56
	DB	.14	.24	.74	.50	.41	.77	.52	1.00	.88	.71
	LS	.74	.43	.77	.81	.41	.78	.91	1.00	.88	.90
	HM	.75	.31	.74	.81	.47	.77	.68	.86	.88	.77
DW	DB	.90	.95	.79	.90	.98	1.00	.20	.76	1.00	.96
	LS	.99	.95	.98	1.00	.98	.99	.84	.86	1.00	.93
	HM	.81	.95	.67	.96	.95	1.00	.40	.90	1.00	.93
DB	LS	.73	.90	.99	.88	1.00	.99	.79	1.00	1.00	.90
	HM	.67	.93	.67	.88	.98	1.00	.92	.86	1.00	.97
LS	HM	.99	.95	.98	1.00	.98	.99	.84	.86	1.00	.93

Table 2.4
Rank Correlation Coefficients

For each Rule Pair, the mean and standard deviation of the correlation coefficients are calculated from the ten values in Table 2.4.

Mean Stan.Dev.	HM	LS	DB	DW	GM
Sum	.955 .075	.968 .312	.837 .167	.832 .191	.737 .176
GM	.704 .179	.763 .197	.591 .277	.544 .250	
DW	.857 .189	.821 .288	.844 .241		
DB	.888 .124	.918 .097			
LS	.952 .058				

Table 2.5
Mean and Standard Deviation of
Correlation Coefficients

other on all attributes, and better than one at least once. In practice, such a clearcut decision seldom arises. Elimination by Aspects is similar to lexicography because it eliminates alternatives by comparison attribute by attribute. The attributes are not arranged in order of descending importance, but in order of descending discrimination power. All alternatives are compared on a particular criterion with a set standard. Those which fail to comply are eliminated. The remainder are tested on the second attribute, and some more eliminated, until eventually only one is left. Because of its similarity with lexicography, this technique willl not be considered separately.

Einhorn (1970) considers these three models and tests them against liner models on how judges select applicants for graduate school. Each applicant is considered on three criteria and the judges were asked to rank order 20 hypothetical applicants according to their acceptability on the graduate program. These results yield the judges' regression weights on each criterion, and given a further 20 hypothetical applicants it was possible to predict according to the different models how a particular judge would rank this second group of 20 applicants. He found the three judges used different models, one conjunctive, one disjunctive and one using a complex combination of models.

Wiggins and Hoffman (1968) explore combination of models and take into account second orders of the decision criteria, as well as cross terms, e.g. x_i^2 and $x_i x_j$. They tested 29 judges who were deciding whether patients were neurotic or psychotic according to their MMPI scores (Minnesota Multiphasic Personality Inventory). The models used were:

1) Linear $\sum_{i=1}^{11} b_i x_i$

   b_i represents least-square regression weights.

2) Quadratic $\sum b_i x_i + \sum b_i x_i^2 + \sum_{\substack{j=1 \\ i \neq j}}^{11} \sum_{i=1}^{11} b_{ij} x_i x_j$

3) Sign model

   "A sign model consists of a linear combination of 70 clinical signs where a sign is any scale score or combination of scores however simple or complex which can be specified precisely." (Goldberg, 1965)

The sign model contained cross terms (called configural) and rationally chosen variables based on, e.g., sums and differences of the 11 scales, as found in the clinical literature. They found

13 sign model
3 quadratic
12 linear
1 sign and linear equally well

They found 16 judges took account of configural terms.

Smith and Greenlaw (1967) use an essentially lexicographic process to simulate the psychological decision processes in selection of personnel. Each job applicant sits a battery of tests, providing 19 test scores, as well as some personal information, e.g. age, previous experience. Psychologists were asked to 'verbalise' how they selected an applicant as being suitable or otherwise for the job. They wrote a computer program with about 300 IF statements to simulate the process, and print out diagnostic remarks along the way. They found a surprising degree of similarity between the classification and comments produced by psychologists and the computer on assessment of test cases.

Tversky (1969) describes the lexicographic semiorder which is appropriate when the "relevant dimension is noisy as a consequence of imperfect discrimination on or unreliability of available information". For example, if choosing a job applicant from a pair for a job where intelligence was most important, it would be correct to pick the brighter of the pair. However, if their IQ scores were sufficiently close together, e.g. 3 points separate them, as to make little difference, the decision-maker would consider them as equal and decide on the next criterion, e.g. experience.

The process of aggregation is crucial to the decision-making process, since it can affect the eventual choice of a solution to an unexpected degree. It is important to be aware of the idiosyncracies of the various aggregation techniques, and when they are applicable. I feel that the truth is that people do not always use one rule or another, but combinations of some. A certain rule may be obeyed locally in some part of the decision space, and a completely different rule at some other part. Without being sure of what one is doing, it could lead to error to assume that what seems to be a locally accurate rule will remain accurate over the entire space, globally.

Throughout this discussion, I have avoided any mention of probability. This will be dealt with in Chapter 3. We shall take it that the scores and weights are single-valued, with no estimate of error or probability. The aggregation methods discussed are valid in each case.

Implementation

Once the final selection has been made, the next stage is implementation. This stage has received very little attention in the literature and this could be for a variety of reasons. Once the decision has been made, the analyst or consultant feels that his job is done and so is gone before the job is finally finished. Or the implementation process is not so successful as people would like to think. Or there have been so few applications of decision making techniques that successful applications are thinner on the ground than we would imagine. Implementation can be a traumatic and irksome business. If a change is imposed upon an organisation without prior consultation of the people involved, then the implementation of the project will almost certainly be fraught with difficulty. To evaluate such a change requires the assessment of the reaction of crucial interest groups, and the decision-maker may find that his estimates of their reaction can be sadly out. This is one reason for consulting them at the planning stage, apart from gaining their moral support and interest.

For successful implementation of a system change, consultation is very important, if not vital. These remarks only apply when the decision-making entity is a corporation of some kind. For an individual, the implementation of such decisions is not often reported. To be sure, I would guess that few people use such techniques to make important personal decisions such as the choice of a spouse, with perhaps the notable exception of Charles Darwin. Having decided he needed a wife, he surveyed his eligible cousins and selected that one who most accurately answered his needs. Apparently his choice was a good one. (Litchfield, 1915)

TYPES OF PROBLEM

The types of problem which have been studied according to the pattern of multi-criterial techniques may be divided into two classes. These are the classes of

1) repeated decisions
2) one-off decisions

Examples of the first class are the admission of students to college and the classification of patients as maladjusted or not. Examples of the second class are decisions such as to install a new organisational system, or allocate a large amount of money on one particular project.

The class of repeated decisions may seem to contradict the ideas in the first section, but this is not so. Although the decision-maker makes the same sort of decision repeatedly, each individual case represents a separate problem, which can only be solved once. Because the decisions in the first class are repeated, they may be subject to statistical analysis. With the example of the psychotic patients, their mental health can be measured by the Minnesota Multiphasic Personality Inventory, which consists of 16 scales. The judges sort patients into 'maladjusted' or 'adjusted' according to these scores. If linear regression is applied to the patients' test scores and the judges' decisions, a model can be obtained which ought to predict the answers the judges would give. The predictive ability of these models is, in general, not very high, but we shall discuss possible reasons for this later on. With one-off decisions, no statistical analysis can be performed. With the first class, the decision-maker does have the opportunity to learn if his decisions were good and to modify his future behaviour accordingly. With the second class, the only learning that can be done is prior to the decision, and if the decision is bad, then tough.

The first class of decision, the repeated decision, has received a great deal of attention in the psychology literature, because it enables the psychologists to test their theories on the aggregation of information by human judges. Because of their repetitive nature, these decisions are learnt according to a set of rules, which are passed on. Thus, the stock broker has a set of rules for governing his behaviour in the market, and these are passed on to his apprentices. This sort of decision behaviour is governed by a series of pragmatic rules and so to try to model the decisions using a linear regression model would seem to be the wrong approach. Wiggins and Hoffman (1968) used other decision rules apart from the linear model and they found that the berhaviour of some judges was better described by quadratic or sign models. If it is the case that the repeated decisions are made according to a set of rules, which may be revealed by statistical analysis, then we may suppose that the value judgements from one-off decisions are also rule-based, and do not depend upon mathematical aggregation, which is the modelling technique commonly used. Since such decisions are only implemented once, the fault in the aggregation method can only be observed when it is too late to amend the procedure.

REVIEW OF TECHNIQUES

We shall move on to consider some multi-attribute techniques and discuss them in the light of the previous part of this section. Chief amongst these is cost-benefit analysis, which illustrates many of the earlier points.

> Cost benefit analysis is thus a particular kind of economic appraisal. It assumes that the considerations in an appraisal can be reduced to economic values, which will represent the way people themselves value different goods. This may sometimes include 'shadow prices' for goods in which there is no market, such as Norman churches and fine landscapes. It further logically assumes that all these can be rendered in money terms, and so made co-measurable. It then follows that these values can be aggregated to find a single best solution, in terms of benefits versus costs. Money valuation and aggregation are crucial features of cost benefit analysis.

The above succinct description of cost benefit analysis comes from the Leitch report (1978, ch.4) on the assessment of trunk road schemes. The characteristic feature of cost benefit analysis is indeed its attempt to reduce the effects of a policy decision to monetary values, in order to form some figure of merit which will assist in the choice between altgernative policies.

According to Turvey (1969) the purpose of cost benefit analysis is two-fold:

> In one it consists simply of the work necessary to present a decision taker with the information which he requires in order to

take a decision. In the other sense it goes further and includes the task of taking the decision.

It is in this duplicity of roles, I feel, that many of the criticisms of cost benefit analysis are founded.

There seem to be three ways generally of dealing with somewhat subjective benefits and costs. The first is to infer their value indirectly from people's behaviour. For example, during the enquiry into a third London airport, the value of a noise-free home was deduced by seeing how house prices in quiet suburbs compared with those of similar properties near a large airport. The second method is to go and ask people directly what they think by carrying out social surveys. The third method is to ignore their measurement completely and list such extra benefits and costs alongside the economic considerations.

Cost benefit analysis tends to emphasise the easily quantifiable aspects of any proposed scheme. Williams (1972) blames this on the "scientific sub-culture within our society", so that "quantifiable things tend to take precedence over non-quantifiable things, and hence undue weight tends to be given to the insignificant things that CBA is able to measure with precision, while the crucial unmeasurables get neglected." He regretfully concedes this as a likelihood, but cannot proffer any resolution of the dilemma between things quantifiable and non-quantifiable.

In his book, Easton (1973) suggests many aggregation rules, or amalgamation rules as he prefers to call them. Many of these have been investigated in the earlier section on aggregation. He also advocates rules such as dominance and simple binary choice, which do not have very wide discriminatory powers.

The Hawgood, Land and Mumford (1978) technique, called BASYC (Benefit Assessment for SYstem Change) pays careful attention to all the stages mentioned in the earlier discussion. The aggregation method it employs is the addition of the products of weights and scores. However, it incorporates an additional feature not normally treated explicitly, that of uncertainty. The decision-making group is asked possible outcomes of the alternative strategies, assuming both favourable and unfavourable states of nature. They are asked to estimate the outcomes which would lie roughly at the upper and lower quartiles of the distribution of outcomes. These forecasts are referred to as 'optimistic' and 'pessimistic', although in most cases they are probably not the quartile values. The optimistic and pessimistic levels are treated separately throughout the analysis, so that each alternative has two figures of merit. The decision-maker now has extra information on the stability of the proposals to unknown future events, and can use his own attitude towards risk to trade off between high potential benefit and lack of stability. This additional feature is well understood and treated in practice. My own real objection to this technique is in its lack of flexibility when it comes to the aggregation stage.

The BASYC technique is also particularly careful in its treatement of separate interest groups. Each group is allowed to set its own goals and to weight them accordingly. Their benefits are calculated separately and there is no attempt to impose a weighting upon the groups.It is up to the decision-maker himself, as this is essentially a political decision.

Another markedly similar technique is that of Morris Hill (1968). He distinguishes between constraints and objectives as follows:

> Constraints are a particular type of requisite. The achievement of specified levels of particular objectives may serve as constraints on the acceptability of the alternative plans, irrespective of the weight of these objectives in the total array of objectives. Thus, the maintenance of air pollution below specified levels may serve as a constraint on the choice of alternative transportation plan even though the reduction of air pollution, expressed as an open-ended objective, may not be highly valued by the community.

He has an interesting interpretation of weights:

> The weights applied to the incidence of objectives can be interpreted as the community's desired distribution of benefits relating to particular objectives.

He also realises the importance of uncertainty, but fails to offer any concrete suggestions for treating it, other than "supplementary comparisons where outcomes are sensitive to a particular contingency". Each goal is given a value and every group is given a weight, either for each goal individually or for all the goals together. Costs and benefit are treated separately, allowing some element of risk assessment, since costs and benefits may not cancel out in the manner that such techniques suppose. Rather unexpectedly, he sums the costs and benefits for each goal, across the groups. Compare this with the BASYC technique, which produces a much more sensible sum across goals for each group. In general, this is a fairly sound technique, but doesn't seem to go as far as it should. The aims behind it are to be applauded.

Bell (1970) used linear programming to select Research and Development projects. His technique seems to have narrow usefulness and it took four years to develop his first model. Geoffrion et al used a method which involved eliciting weights and trade-offs from people on the various aspects of an academic department's administration. He used an iterative process which he claims is easy to use with computer assistance. The mathematics with this method, in contrast to the others mentioned so far, is non-trivial.

Moore and Baker (1969) classify R&D selection techniques into 4 groups:

1) Scoring Models compute an overall project score based on various ratings assigned to each project for each relevant decision criterion and are designed to operate with the subjective input data which exists at the research stages of the project life.

2) Economic Models base project rankings on such economiccriteria as rate of return and present worth.

3) Constrained Optimisation methods which seek to optimise some objective economic function subject to specified resource constraints.

3) Risk Analysis based on a simulation analysis of input data in distribution form and which provide output distribution of such factors as rate of return, market share, etc. They proceed to compare the four models, warning against the excessive data required by some.

In another paper, Pessemier and Baker (1971) identify three methods, comparative, scoring and benefit contribution methods. They explain the three methods thus:

1) Dollar Metric Each judge is given a pair of projects and asked to say which is preferred, and how much the price would have to change before he would reverse his choice.

2) Successive Rating Give 100 points to the most preferred project and an appropriate number to all the others.

3) Successive Comparisons Each project is compared against selected subsets of alternative projects.

They find that in practice the three methods produce similar results, but prefer the Dollar Metric method. Note that this is a decision-making case, in which the separate attributes of the problem are never stated. Perhaps this is because the members of the decision-making group are already familiar with the attributes involved, and there is no point in stating them over again. This also means that there is no aggregation method explicitly involved. Direct preferences between alternatives are used.

Dean and Nishry (1965) use two models: the scoring model which is appropriate at the early stages, and the profitability model which uses more detailed information. The scoring model is no different to the many other models we have seen. A single estimate of the weights and scores are provided, and the sum of their products provides a rating value of each project. To determine the weights of the attributes, or 'factors' as they call them, the following procedure is used:

> Board members were asked to rank order factors. The rank orders were converted into numerical values, assuming equal intervals between adjacent ranks. These values were averaged across the review board members, assuming approximately equal degrees of knowledge.

Without some justification, these assumptions would seem questionable. The profitability model is based on purely economic aspects, and since it involves no consideration of multiple criteria need not concern us.

A very neat and thoughtful technique is that of Goodwin (1972). He emphasises the need to take care over the assignation of the weights because of the mixture of objective and subjective aspects, lack of knowledge and haphazard thinking. He tried a number of aggregation methods, but in the end used a simple additive method. To convert the assessment of subjective attributes into numbers, he suggested a scale, as shown in Figure 2.6. To take account of error or uncertainty in the weights and scores, he used a computer program which allowed the weights to vary by +/- 10% of their original values and the scores by +/-.5. In this way, he obtained a range of values for the figure of merit of each alternative. He considers some of the cases when overlap of the ranges of different alternatives, and considers how to choose the optimal alternative. I would suggest that this last stage should be left to the decision-maker himself, knowing his own faith in the figures he provided.

The Litchfield, Hansen and Beck (1972) model is very thorough and tries to take account of uncertainty by rethinking the classical approach to multi-attribute decision-making. The classical model consists of a set of alternatives and a set of possible states of nature. Each of the alternatives has a payoff value associated with each state of nature. The probabilities of the various states of nature are assumed to be known. They replace the states of nature with a set of objectives or goals which the decision-maker wishes to achieve. The payoff matrix is sustituted by a set of probabilities of each objective being realised by each alternative. The probability of occurrence is replaced by a measure of the relative worth of each objective. The information required by this technique is quite considerable:

1) Each decision-maker produces a probability curve for the level of attainment of each attribute for each alternative.

2) A 5-point utility curve is obtained for each attribute.

3) The attributes are weighted by taking successive pairs after ranking and assigning figures to give relative value.

The analysis produces graphs of probability against utility for each alternative. These probability distributions are not scaled to include the relative importance of the attributes. This is done using a Monte Carlo method to estimate total utility. Where a consensus of utility functions is

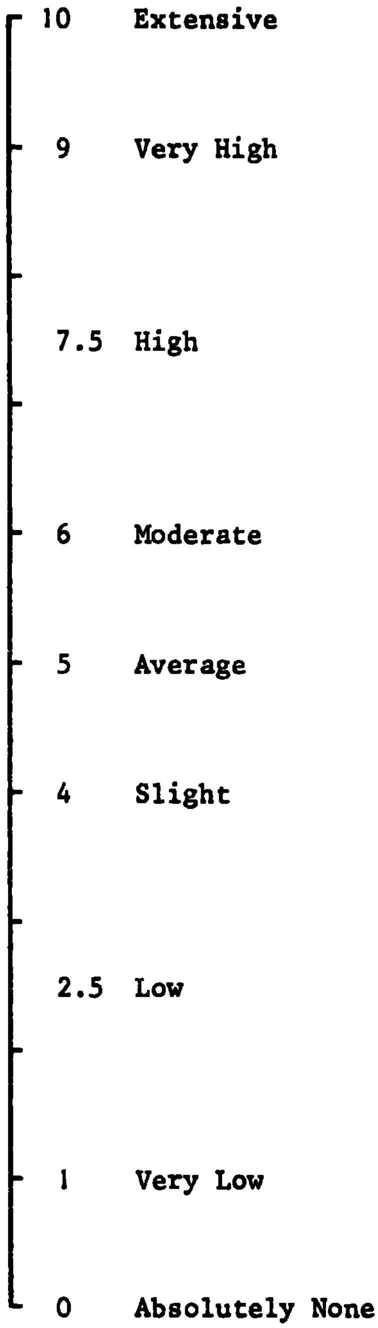


Figure 2. 6 Goodwin's Scale for Converting the Assessment of Subjective Attributes to Numbers

impossible within the decision-making group, a number of runs can be done using the 'best', 'worst' and expected utility functions. The output from this method is a set of cumulative probability estimates on the utility of each alternative. To rank order these graphs the authors demonstrate how the graphs may be characterised by a mean and variance, and may be plotted on a mean-variance plane, to allow the decision-makers to make their own choice. For asymmetric distributions they suggest a transformation method to make the distributions symmetrical and allow them to be placed on the plane.

This method seems admirable in its intentions, but does require a large amount of information. Also, I would question its transparency to the decision-makers; do they really understand what becomes of the data they supply, and how sensitive is the final answer to the information they supply? It could happen that by the time the decision-makers have supplied all the information asked of them, they could have already made the decision. To structure thinking is an important part of the use of any technique, and this method achieves that in good measure.

This section has consisted of a review of some of the decision-making techniques to have appeared in the literature over the past few years. For more papers on the subject, see the excellent review articles by MacCrimmon (1971), Dujmovič (1977) and Baker and Freedland (1975). We shall return to some of the points made in this review later in the work.

SUMMARY AND CONCLUSIONS

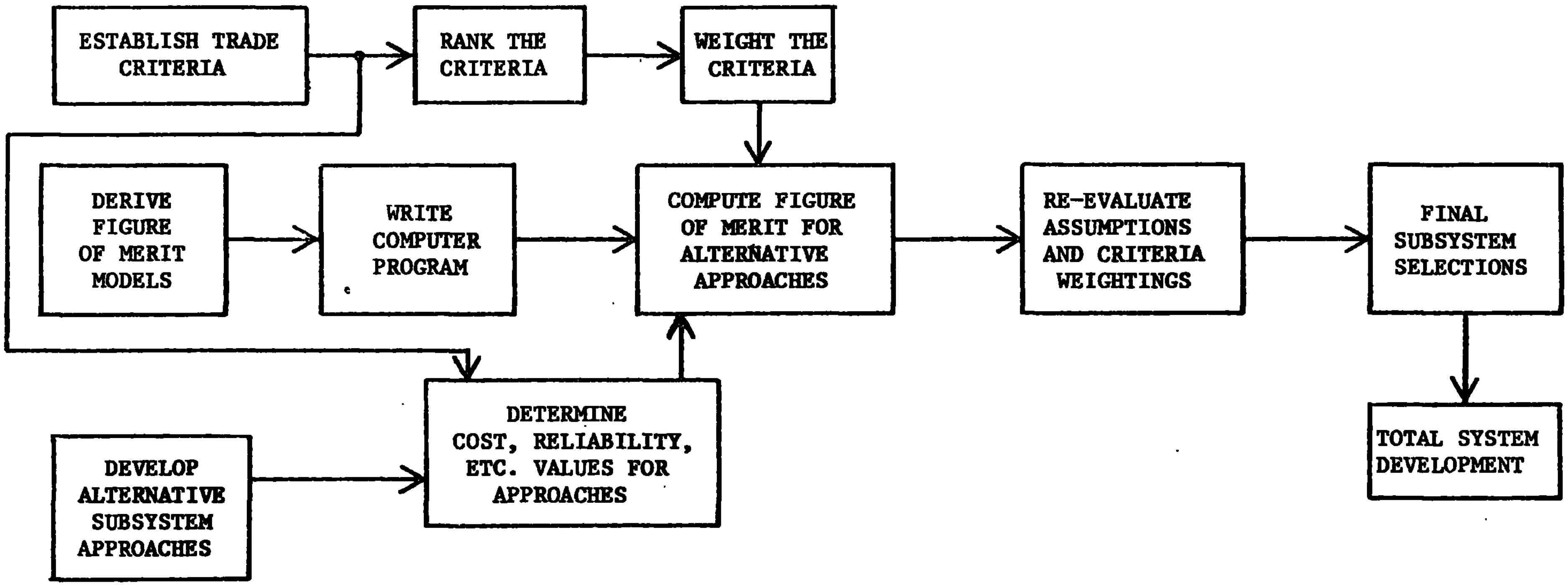
This chapter opened by considering the role of multi-attribute decision-making within general human problem-solving. It was pointed out that the need for multi-attribute decision-making arises from the imposition of constraints upon the problem-solver by the environment.

The cycle of activity in multi-attribute decision-making was considered, and how this cycle accommodated the learning process. Some stages of the cycle were discussed at length, i.e. Measurement of the Alternatives and Evaluation of the Alternatives and Final Selection. From this it is argued that present methods of assessing subjective criteria are inadequate and misleading, because of the spurious emphasis on objectivity. The process of aggregating measurements is often arbitrary and may bear no relation to the decision-maker's intutive aggregation method. Indeed, such intuitive methods may not be similar to the mathematical aggregation techniques, but may be rule-based.

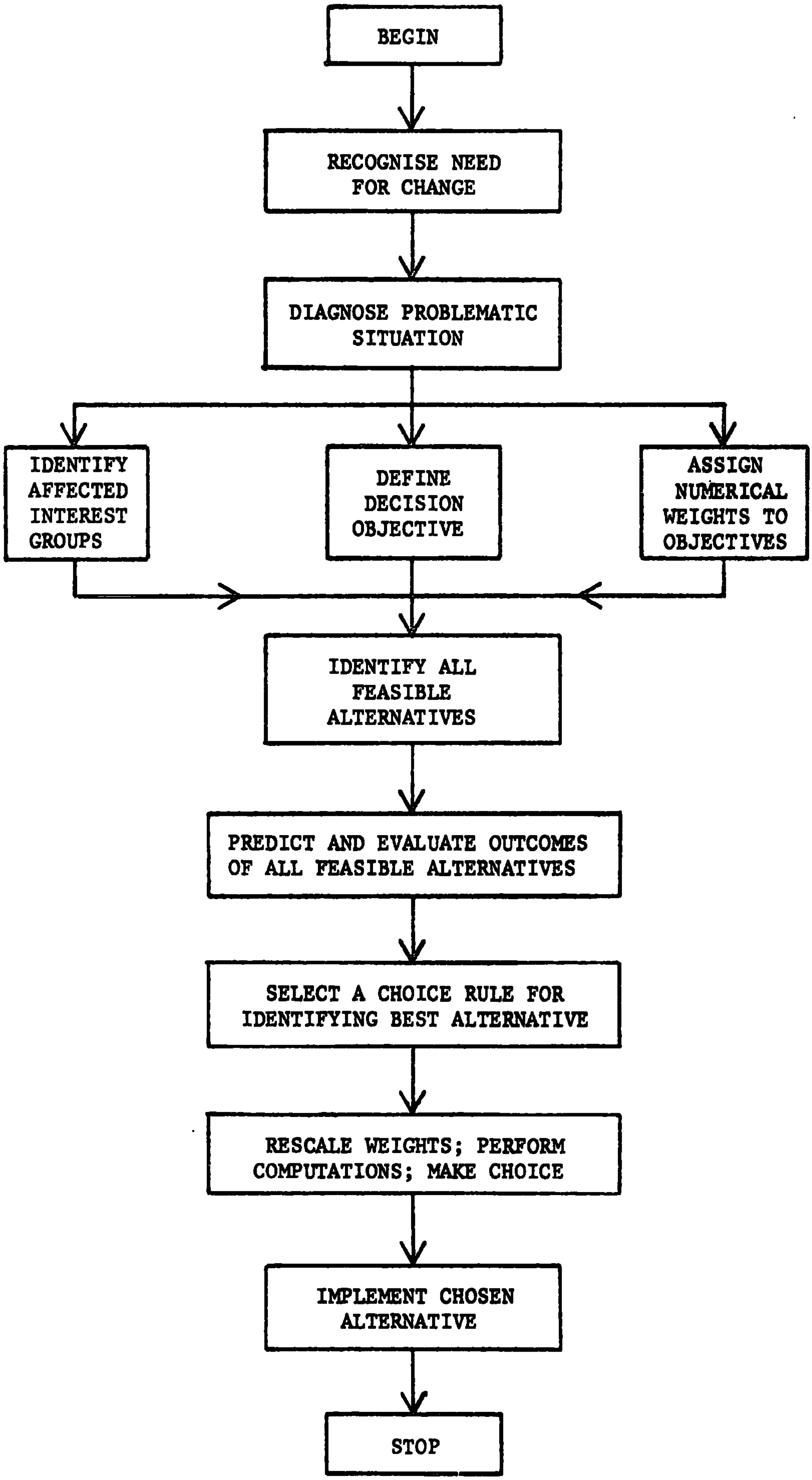
The chapter closed with a review of techniques and their solutions of the multi-attribute decision problem.

APPENDIX 1

SOME CLASSIFICATIONS OF THE MULTI-ATTRIBUTE DECISION-MAKING PROCESS



<u>Goodwin : Method of System Evaluation</u>



Easton (1973) : Basic Optimising Model

Moore & Thomas (1975)

The decision-maker has a set of objectives whose attainment depends upon the decision that he takes.

The decision-maker must systematically and creatively search for a range of possible options from which a set of alternative courses of action (or strategies) can be determined for consideration in a particular context.

The decision problems exist in an uncertain environment, and the decision-maker may have the option of collecting further information.

A measure is needed of the value or payoff of each possible outcome in terms of the decision-maker's objectives.

Johnson & Huber (1977)

Utility Assessment Process

1) Identify the perspective from which utility is to be assessed, i.e. "utility to whom", which individual or organisational unit.

2) Determine the scope of the problem and identify the objectives, purposes or uses of the objects or events whose utilities are to be assessed.

3) Identify the set of alternatives to be evaluated.

4) Determine the relevant attributes or factors on which each of the alternatives are to be assessed.

5) Develop operational measures for each attribute or factor.

6) Choose an appropriate technique for assessing the utility of each attrribute or factor, i.e. for converting the physical measure into a utility or value measure.

7) Assess the utility or value of each alternative on each attribute or factor.

8) Choose an assessment model.

9) Evaluate each alternative using this model.

10) Select the "best" alternative.

APPENDIX 2

The following is a true account of an attempt to place a numerical, quantitative measurement on the library objective "Equalise Book Provision". It was written after a discussion of how best to quantify this goal, before the author realised the paradox involved in such attempts. It is intended as an illustration of the problems involved in such attempts.

Equalise Book Provision

This is a very laudable goal for any Area Librarian, but it is very difficult to know when the goal has been achieved. That goal implies that each ratepayer served by the local library should have equal access to books, or as near equal as possible. Thus people living near Duffield library should receive as good a selection of books as those living near Derby. Clearly, this is an ideal, because not every library can offer the specialised services available in some libraries, but on the level of general fiction and non-fiction reading the user of the small library ought to be as well supplied with a supply of titles to suit his taste as anywhere else.

This leads us to one of the first requirements of this goal, that the stock should match the catchment area population, i.e. that each library should have x books per capita in its catchment area. This leads us to difficulties in the measuring of the library's catchment area, which may be constantly changing. So we can try to match books to the actual number of readers registered with the library.

This leads to the second requirement, that existing stock be taken account of. Obviously, a library with an ageing, dated stock is not providing for its users as well as a library with fresh, new stock available, all else being equal. It would be foolish to embark upon a policy of completely restoring all libraries in order to obtain equality of provision, ignoring the resources already at hand.

Some indication of the books' popularity is necessary, so this leads us to try 'issues per reader' as a possible measure. By bringing issues per 1000 readers to the same level at each library, we know that readers in both places are being provided with an equitable supply of books.

A problem with this measure is how well one can forecast its likely value in the future. To predict how annual issues and readership will be influenced by changes in book purchase policy are separate tasks fraught with error, and compounding them to the same measure will produce a large error. This is where optimistic and pessimistic estimates will show their merit.

One point which must be stressed is that the numbers obtained as issues per 1000 readers are no guide whatsoever to the allocation of funds. At present Long Eaton records 270,000 issues annually and has 12,000 readers, giving approximately 22.5 issues per reader. Alfreton has 8,218 readers and 240,000 issues, approximately 30 per reader. To jump straight in and decide to allocate money accordingly, say £3,000 to Alfreton and £2,250 to Long Eaton, or inversely £3,000 to Long Eaton and 2,250 to Alfreton, is absurd. It is only the similar size of these libraries which tempts us to this conclusion. Had we done the same calculation for Derby and Duffield, and found 20 issues/reader at Derby and 30 issues/reader at Duffield, no one would have suggested giving Derby £2,000 and Duffield £3,000. Issues per reader itself bears no linear relationship to money spent, and it would be reckless to allocate money assuming such a relationship.

Of course, this was not the only measure considered, many others suggested themselves. Some sort of ratio comparing some value at either library was essential, aiming for the ratio to tend to unity, i.e. that the number obtained at each library is the same. Because of special features at either library, e.g. records, photocopying, books alone should not be considered, and for the sake of simplicity, reference books ignored.

Some sort of formula was also sought, but this could not be clearly defined. As already mentioned, measures relating, for example, to recently acquired books, shelf stock, issues, etc. to the catchment area population were considered. Catchment area population is an unsatisfactory concept for two reasons:

1) catchment area itself is hard to define, particularly at the boundaries, where library usage overlaps and nothing is known

of how it may change as a result of a library policy change, and

2) the population of the area is a transient thing, constantly changing and attending work, school and shops in another library's catchment area, and so preferring to use it. For these reasons 'readership' as chosen instead as more tangible and easier to measure.

The search for a fair measure for this goal is made more difficult by the dual aims of such a measure to reflect the catchment area as well as the existing stock. That one proposed goes some way towards doing both, and has the added advantage of being easy to measure and calculate. However, it stands to be corrected.

CHAPTER 3
PROBABILITY AND UNCERTAINTY

INTRODUCTION

In this chapter, we shall consider some ways in which decision-making is affected by uncertainty. The decision-maker is often faced with both a lack of available information, and with only probabilistic knowledge of the future. We consider how lack of knowledge of the future and probability have been treated, and examine the psychological studies of this process. From this, we shall argue for a different treatment of uncertainty, taking account of decision-maker's personal attitudes.

TYPES OF UNCERTAINTY

The taking of any decision is ridden with uncertainty. The uncertainty may arise from many different sources but there are two main types of uncertainty which we shall examine separately. The classification involved follows a theme which will emerge in the treatment of this thesis.

The first type of uncertainty arises from the decision-makers's internal environment. The evalution of any of the alternatives presented to him requires the decision-maker to place value judgements upon the different attributes of the alternative. In many cases he will be asked to rank order the attributes (inter-attribute), but he will also be asked to give an estimate of the utility associated with the various levels of attainment of a single attribute (intra-attribute). This is an introspective process and requires the decision-maker to examine his motives and experience very carefully. When faced with a decision which he has never had to make before, this process of introspection will involve learning both more about the decision to be made and of his own priorities and objectives. Because this type of uncertainty is derived from lack of knowledge in its fullest sense, rather than lack of information, this uncertainty has nothing to do with randomness or probability but is a mental phenomenon.

Some of the internal uncertainty or imprecision is language-based and may be described as fuzzy. The language which we use to describe normal life is not scientifically precise, simply because it need not be. Scientific precision is based upon numbers, and language is not adequate for these purposes. Many people find the precision of numbers incomprehensible and prefer to reason in the imprecise fashion of humans. This topic will be discussed at greater length in Chapter 5.

The uncertainty of the second type is part of the external environment and due to our incomplete knowledge of it. If a decision-maker is required to see into the future, he might be correct part of the time but the art of prophecy is not perfect. Thus there will be uncertainty because of the imperfect information available. A skilled decision-maker or manager may acquire some feel for what the future may hold but some uncertainty will always remain. It is this second type of probabilistic or stochastic uncertainty that I wish to examine in this section.

PROBABILISTIC UNCERTAINTY

There seems to be three possible ways of treating future uncertainty. The first way is to assume that there is no uncertainty and that there is only one relevant state of nature in the future. The second way is to assume that there are many possible future states of nature but that we have no knowledge of which is likely to hold, and so they must all be assumed equally likely. The third way is to consider many possible states of nature but to have some information on their relative likelihood, via probability estimates. We shall consider each of these methods.

In the examples of decision-making techniques which were discussed at the end of the last section, we saw how some took account of uncertainty due to the unforeseeable future. We shall ignore for the time being uncertainty due to value judgements. The BASYC method (Mumford et al (1978)) asked the decision-making group to estimate 'optimistic' and 'pessimistic' forecasts for the attributes of the alternatives. The Litchfield, Hansen and Beck model (1976) assumes that the probabilities of the various states of nature are known. Some methods state the need to recognise lack of certainty but do not seem to know how best to handle it and seem to assume that only one state of nature will prevail. There may be some circumstances where this may be a reasonable assumption, i.e. that there is no uncertainty.

The No-Uncertainty Case

The assumption of no uncertainty might be reasonable for the selection of some R&D projects, for example, where the department is fairly insulated from the outside world and any changes likely to affect their work are judged too unlikely to be considered. The case where the state of nature does not affect the outcome also occurs when the multi-attribute decision-making problem is to select some item which is tangible and present. For example, in the case of selecting a pocket calculator, all the attributes of the alternative models can be easily measured and tested. They do not lie in the future but in the here and now. However, if one of the attributes had been, say, reliability which can only be measured as time passes, then some uncertainty will enter. But since pocket calculators are a cheap commodity and have a reputation for becoming obsolete rather than wearing out, then this attribute may not be worth considering and the problem is governed by only one state of nature.

The Many States of Nature but No Knowledge Case

Let us move on to consider the case of many states of nature but there is no knowledge of their relative likelihood. In this case they are treated as equiprobable. Consider Table 3.1. There are n possible alternatives or strategies for the future and m possible states of nature.

States of Nature	E_1	E_2	E_3	E_m
Strategies				
S_1	V_{11}	V_{12}	V_{13}	V_{1m}
S_2	V_{21}	V_{22}	V_{23}	V_{2m}
S_3	V_{31}			
.				
.				
S_n	V_{n1}			V_{nm}

Table 3.1 - Payoff under State of Nature - Strategy Pairs

E_j denotes a state of nature and S_i a strategy. The expected value of payoff of the state of nature - strategy pair is denoted by V_{ij}. If there is no knowledge about the likelihoods of the various states of nature, then we may assume them to be equally likely. A number of different criteria for selecting a strategy have been suggested. These are listed in Table 3.2.

It is easy to imagine the case might arise where each decision criterion

Decision Criteria for the second case when there are many possible states of nature and no knowledge about which will occur. See Fishburn (1964), Milnor (1954), Pappis (1976).

m possible states, j = 1, . . ,m
n possible alternatives, i = 1, . . ,n
V_{ij} denotes expected payoff of alternative i under state-of-nature j

1) Laplace criterion. Choose that alternative which maximises

$$\sum_{j=1}^{m} V_{ij}$$

2) Maximim criterion. Choose that alternative which maximises

$$\min_{j} \{V_{ij}\}$$

3) Maximax criterion. Choose that alternative which maximises

$$\max_{j} \{V_{ij}\}$$

4) Hurwicz criterion. This a combination of the two previous methods which are pessimistic and optimistic respectively. A constant $0 \leq \alpha \leq 1$ is chosen to take account of the decision-maker's in-between attitude. When $\alpha = 1$ the decision-maker is completely optimistic and when $\alpha = 0$ he is completely pessimistic. The criterion is chosen to maximise

$$\alpha \times \max_{j} \{V_{ij}\} + (1-\alpha) \times \min_{j} \{V_{ij}\}$$

5) Minimax regret criterion. Choose the criterion to minimise

$$\max_{j} \left\{ \max_{i} \{V_{ij}\} - V_{ij} \right\}$$

Table 3.2- Decision Criteria

meant that a different alternative would be selected as the best possible, see Table 3.3 and Figure 3.1. Again the choice of the decision criterion to use is up to the decision-maker. It is his attitude towards risk which is important.

The Many States of Nature and Some Knowledge Case

We shall consider now the case where there is some information about the relative probabilities of the states of nature. If we assume that the choice of strategy has no effect upon the states of nature, we may draw up a list of probabilities of them, P_j. The decision criterion now used to compute the expected value of each of the S-E pairs and to select the maximum. The table above would be replaced by one with entries reading $V_{ij} \times P_j$, see Table 3.4. To select the best strategy from those possible, we could take that one which offers the maximum expected payoff, which would be, in this case, S_3 which combines a high payoff with a probable occurence.

These criteria are quite arbitrary. Their only distinction is that they can be expressed neatly in mathematics. We could adopt any decision criterion we choose, and apply it, regardless of its 'mathematicalness'. For example, "If the most probable outcome is expected to occur with greater than 50% probability and one of the strategies is expected to have a payoff in the top quarter of the range, then we'll try it. Otherwise choose that alternative which is least likely to give us a bad deal".

This is a reasonably practical decision criterion but only the first part could be expressed mathematically. The second sentence contains expressions with vague definitions which may be understood by the decision-maker's colleagues but not by a computer, for example.

THE PSYCHOLOGY OF PROBABILITY ESTIMATION

The psychology of decision-making and probability estimation has received a great deal of attention and many of its conclusions are relevant to this problem.

	E_1	E_2	E_3	$\sum_{j=1}^{m} V_{ij}$	$\min_j V_{ij}$	$\max_j V_{ij}$	Max Regret	Hurcwicz
S_1	2	8	8	18*	2	8	3	
S_2	5	5	5	15	5*	5	5	.4
S_3	1	1	10	12	1	10*	7	.5
S_3	3	6	8	17	3	8	2*	.5
$\max_i V_{ij}$	5	8	10					

* denotes the chosen alternative

<u>Table 3.3</u> <u>Comparison of Decision Criteria with No Knowledge of the Future</u>

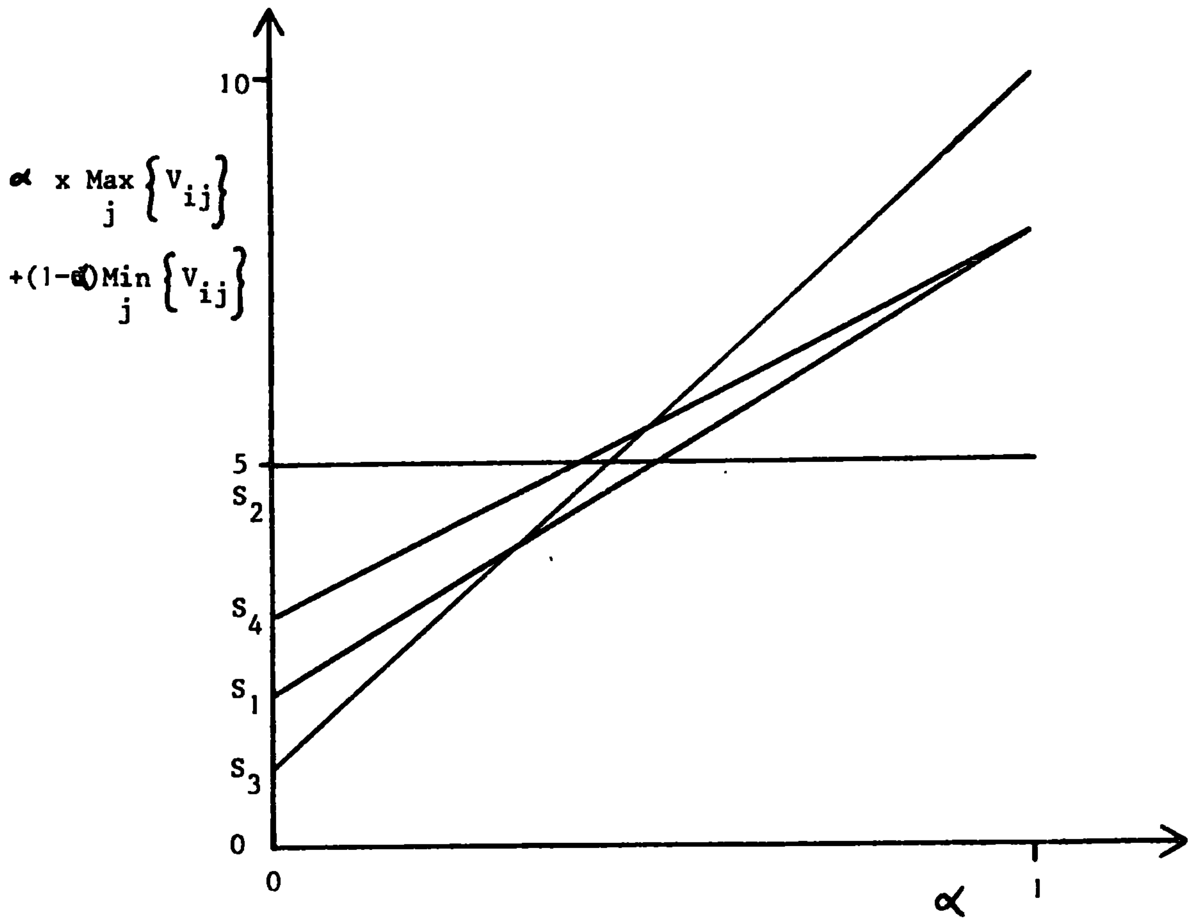


Figure 3.1 The Hurwicz Criterion

V_{ij}	E_1	E_2	E_3	$\mathcal{E}(S_i)$
S_1	2	8	8	6.00 *
S_2	5	5	5	5.00
S_3	1	1	10	4.00
S_4	3	6	8	5.67
P_j	.3	.1	.6	$\mathcal{E}_p(S_i)$
P_jV_{ij}				
S_1	.6	.8	4.8	6.2
S_2	1.5	.5	3.0	5.0
S_3	.3	.1	6.0	6.4
S_4	.9	.6	4.8	6.3

* denotes chosen alternative

$$\mathcal{E}_p(S_i) = \frac{1}{m} \sum_{j=1}^{m} V_{ij} \qquad \mathcal{E}(S_i) = \sum_{j=1}^{m} P_j V_{ij}$$

Table 3.4 Comparison of the No-Knowledge and Knowledge Criteria

The first attempts at quantitative decision-making sought to combine objective probability estimates with objective values of the alternatives. These values were stated in terms of some objective payoff, say money, and the probabilities were estimated as accurately as possible. The objective payoff was replaced with something to take account if the subjective worth of the alternatives in the manner of utility theory. Eventually the objective probability was also changed to the subjective probability of the decision-maker leading to the so-called Subjective Expected Utility models.

There are two mathematical techniques available with which to study how people estimate probability and how their utility compares with the external stimulus. These are Bayes theorem and the calculation of the mathematical expectation . We shall discuss these techniques and the conclusions which can be drawn.

Bayes Theorem

Bayes theorem may be written as follows:

$$P(E_j \mid D) = \frac{P(E_j).\ P(D \mid E_j)}{\sum_{j=1}^{m} P(E_j).\ P(D \mid E_j)}$$

where E_j denotes the possible state of nature, $P(E_j)$ denotes the prior probability and $P(E_j \mid D)$ denotes the posterior probability after receiving the information D. $P(D \mid E_j)$ is the probability of D occurring if the state of nature is E_j.

Consider again the payoff table of Tables 3.3 and 3.4 We may start off with no information on the relative probabilities of the future states of nature and assume that they are all equally likely. If we know that a piece of information D will give us a better indication of which state of nature will be true, then it may be worth our while to obtain this extra information. The acquisition of this extra information might involve some expense and Bayes theorem allows the calculation of how much extra outlay is reasonable so as to reduce uncertainty.

Suppose the payoff table is as before (see Table 3.5) but we know that the conditional probabilities of D occuring, given each of the E_j. Using Bayes theorem we may calculate revised probabilities for each E_j in the light of

V_{ij}	E_1	E_2	E_3	$\mathcal{E}(S_i)$	$\mathcal{E}(S_i\|D)$	$\mathcal{E}(S_i\|D)$
S_1	2	8	8	6.00*	6.61	5.53
S_2	5	5	5	5.00	5.00	5.00
S_3	1	1	1	4.00	7.23*	1.53
S_4	3	6	8	5.67	6.61	4.88
$P(E_j)$	.33	.33	.33			
$P(D\dagger E_j)$	.3	.1	.9			
$P(\neg D\|E_j)$	.7	.9	.1			
$P(E_j\|D)$	$\frac{.3}{1.3}$	$\frac{.1}{1.3}$	$\frac{.9}{1.3}$			
$P(E_m\|\neg D)$	$\frac{.7}{1.7}$	$\frac{.9}{1.7}$	$\frac{.1}{1.7}$			

$$P(D) = \sum_{j=1}^{m} P(D|E_i).P(E_j) = .33 \times 1.3 = .429$$

$$P(\neg D) = .561$$

$$\text{Expected payoff} = P(D).\mathcal{E}^*(S_i|D) + P(\neg D).\mathcal{E}^*(S_i|D)$$
$$= .429 \times 7.23 + .561 \times 5.53$$
$$= 6.204$$

<u>Table 3.5</u> <u>Bayesian Decision-Making</u>

the extra information, the posterior probabilities. The expected payoff from each strategy may be calculated, given D or ¬D. These are $E(S_i|D)$ and $E(S_i|\neg D)$. We may now choose which strategy should be adopted to give the maximum payoff - S_3 when D is true and S_1 when D is not true. The probabilities of D and ¬ D are calculated simply and combined with the maximum payoffs in either case to give the overall expected payoff. In this case we see the expected payoff would be 6.204, compared to 6.0 in the no-information case. Thus it would be foolish to pay more than 0.204 to find out if D or ¬D is true.

The psychological studies of behaviour under Bayesian conditions involve tasks which require the decision-maker to revise his probability estimates upon the receipt of new information. Phillips and Edwards (1966) asked people to consider bags containing 100 poker chips, red or blue chips predominated in the bag, and the subjects were shown 20 chips from the bag one at a time with replacement. After each new chip was shown, the subjects revised their previous intuitive estimates of whether a predominantly red or blue bad had been chosen. Phillips and Edwards found that -

> Revision was consistently smaller for subjects in this experiment than the amount predicted by Bayes theorem.

This tendency to extract less certainty from the information than the theoretical amount available has been called conservatism. Phillips and Edwards conclude that -

> The failure of Subjects to extract from the data all the certainty that is theoretically available is consistent and orderly and may reflect a general limitation on human ability to process information.

The results obtained by Phillips and Edwards have been duplicated by other workers (see the review by Slovic and Lichtenstein, 1971). The most popular explanation for this conservative behaviour is that the subjects have great difficulty in aggregating various pieces of information to produce a single response. Man's limitations as an information processor prevents him from making full use of the information available.

Mathematical Expectation

When a decision-maker is faced with a risky decision, he may use the mathematically expected profit of the alternatives to choose which to implement. This requires him to know the probabilities of the various outcomes and their payoffs. A risky decision is one which may produce profit or incur loss.

		S_1	S_2
State of Nature		S_1	S_2
Payoff	A	-10	+5
	¬A	0	0
Probability $P(S_i)$		0.3	0.7

The decision-maker must decide whether or not to implement the risky strategy A. The mathematically expected profit of A is -

$$(P_A) = 0.3 \times -10 + 0.7 \times +5 = -0.3 + 0.35$$
$$= +0.05$$
$$(P_{\neg A}) = 0.0$$

The method advises him to implement A. However, the decision-maker may feel that he is not willing to take the chance of incurring a heavy loss (-10) for such a small return (+5). He might prefer not to implement A and play safe.

When processing Bayesian information, the decision-maker is playing a game against Nature which he may assume will perversely take the course to do him the most harm. Hence, if the decision-maker does not want to incur loss, he will want to be sure when guessing Nature's future behaviour.

The behaviour of the decision-maker described above is known as risk-averse; he is wary of incurring loss. People may be risk-averse to a greater or lesser degree and some are almost risk-neutral, i.e. their behaviour is close to that predicted by mathematical expectation. A much rarer individual is the risk-seeker, who seems to enjoy the gamble for its own sake and choose high profit-low probability gambles.

Attitude towards risk may be studied mathematically by presenting people with a series of 50-50 gambles of, for example, winning £x or £0 and asking the gambler to state the amount of money which he would accept in exchange for this gamble. We may then plot the "certainty equivalent" against the mathematical expectation (here, x/2). See Figure 3.2. Such a graph is only true for probabilities of 50-50 which are unlikely to occur in Nature. It would be wrong to assume, for example, that the certainty equivalent of £5 with probability 0.8 is the same as that for £400 with probability 0.01. Such graphs can demonstrate behaviour within a fixed range but can hardly be used as a serious prescriptive tool.

For such graphs it is tempting to seek a mathematical relationship (e.g. Lindley, 1975).

Using such relationships the decision criterion could be built into the decision-making technique. However, a decision-maker's attitude is not constant and any changes would have to be checked and the technique corrected to prevent it from becoming misleading. One reason for a change in attitude could be a change in the decision-maker's financial position. As his starting position grows weaker he may become more risk-averse and, similarly, may become risk-neutral or risk-seeking as his position strengthens.

To summarise, mathematical expectation by itself is not an adequate guide to choosing between strategies, since it ignores the decision-maker's attitude towards risk. This attitude may be explored by comparing mathematically expected quantities with their certainty equivalents. However, for every combination of probabilities in the gamble a different certainty equivalent is required. Not only is the extent to which a decision-maker deviates from risk-neutrality dependent upon the probabilities presented him, but also upon his starting position.

These effects can become important in real life as in a hierarchical organisation, where the lower echelons are often responsible for preparing reports upon which the higher members base their decisions. The alternatives which are presented will require some assessment of the probabilities associated with them. Now it can be that the attitude towards risk of the compiler of the report will influence his estimate of the probabilities, tending to increase his estimates for the safer, more likely alternatives, whilst decreasing his estimates for the more risky or maverick options. This is hard to avoid and can only be stated as another instance of the subjective nature of much of the art of estimating probability.

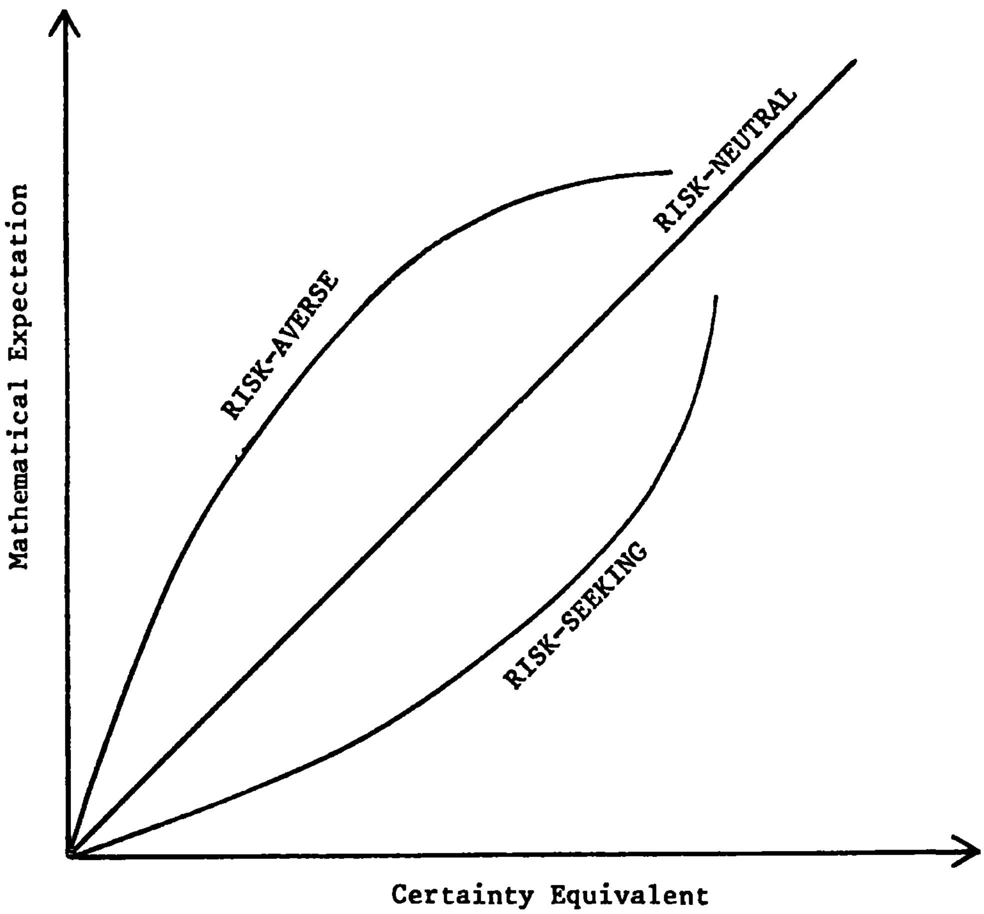


Figure 3.2 Behaviour towards Risk

The Connection between Utility and Probability

For the Subjective Expected Utility model to hold the determination of probability and utility must be independent. The SEU model proposes that in a gamble the decision-maker maximises -

$$S(P_W)\ U(£_W) + S(P_L)\ U(£_L)$$

where $S(P_W)$ is the subjective probability of winning
$S(P_L)$ is the subjective probability of losing
$U(£_W)$ is the utility of the amount to win
$U(£_L)$ is the utility of the amount to lose

If the estimation of utility and probability were in any way related then the SEU model would be unusable. In fact such a relationship has been established.

Irwin (1953) asked people to state whether or not they expected to draw a marked card from a pack. He found that:

> A significantly greater number of "yes" responses occurred when a marked card was desirable than when it was undesirable.

Edwards (1955) found that subjective probability functions obtained from bets on which subjects could only win or break even indicated that subjective probability exceeded objective probability at all points between 0 and 1. But functions obtained from bets on which subjects could only lose or break even indicated that subjective probability equalled objective probability.

Slovic and Lichtenstein (1968) used a rather unusual model of risky decision-making; namely:

$$A(G) = \mu + w_1 P_W + w_2 £_W + w_3 P_L + w_4 £_L$$

where A(G) is the attractiveness of the gamble. This model combines the variables additively instead of multiplicatively. They found "enormous" differences in the weights, both within and between subjects which suggests that the responses of many subjects were overwhelmingly determined by one or two of the risk dimensions and were remarkably unresponsive to large changes in the values of the less important factors. Slovic and Lichtenstein also asked the subjects to write a paragraph describing how they went about evaluating the attractiveness of the gambles. They found that -

> A large number clearly indicated that they believed certain dimensions to be far more important than others in determining their responses. For the most part, the regression weights derived from their responses reflected their stated strategies.

When evaluating the attractiveness of gambles, people seem to use strategies which reflect their own beliefs on which are the important dimensions and their limited information processing capacity forces them to concentrate on some dimensions and ignore others. Thus, people's behaviour is not adequately modelled by the SEU models but they can state their own decision-making strategies.

PROBABILITY ESTIMATION AS AN ART

Up to now I have been arguing that the estimation of probability is more of an art than a science. The estimates of probability are required to be stated in the form of numbers. This is inconsistent with its highly subjective nature. The subjectivity of probability may be illustrated by -

1) lack of consistency amongst decision-makers being asked to estimate the probability of the same event,

2) tendency to be influenced by the attitude of the assessor towards risk and utility of the outcome,

3) conservatism of people in their processing of information and subsequent estimation of probability.

If it is the case that probability is a subjective aspect of any alternative, then the treatment which it has received in the last has been on the wrong track. The payoffs associated with the alternatives can involve subjective valuations and to multiply these by further subjective, error-prone figures to calculate expected payoff would appear to be unsound. The decision-maker's choice of the alternative will depend upon his attitude towards risk and the process of multiplying the subjective valuation of an alternative by its subjective probability assumes that the valuation of the alternative is entirely independent of its likelihood.

Using the two premises that -

1) probability is a subjective aspect and

2) the value of an alternative is not independent of its probability,

the only sensible solution is to bring probability down to the level of an attribute of the multi-attribute decision-making problem, discarding models such as the SEU model. To be sure, it is a very special attribute and can not be treated in the same way as the other attributes, but we shall look at this difficulty again later. (See Chapter 8)

If we accord probability the status of a subjective attribute, then we must consider carefuly how it is to be measured. In everyday terms probability is discussed using verbal statement of relative likelihood. For example -

> "I don't think it will rain tomorrow."
> "It's more likely that it will rain tomorrow than that it will snow."

Fine (1973) argues the case of the various means of stating probability and the theories behind them. His conclusion is that the only real sort of probability is comparative probability and I would agree with this position. In many books (see the Review by Slovic, Fischhoff and Lichtenstein, 1977, pp.18-20), which attempt to train the decision-maker in the art of estimating probability, they use comparative probability as a means of getting across the relative likelihood of certain events. For instance, that an elephant will walk down the road or that a penny will come down heads when tossed. Through everyday experience we become accustomed to the relative likelihood of certain events and can assess which are the more likely without ever taking recourse to numbers. No one ever states in ordinary conversation that -

> "It will rain tomorrow with probability .5."

Such a statement could be made, if repeated measurements could be taken over the year to give an annual average, or over a long span of time assessing the probability of it raining on tomorrow's date. But the trouble with such 'objective' measurements is that they do not take account of the weather conditions which are prevailing at this time. It is a feature of human reasoning that we take account of facts and intuitions in a strange and subtle way. The conclusions that are reached by this process are not those which the numerical methods predict, but that need not necessarily means that they are wrong. Rather than being scornfully dismissed as unscientific, the process of human reasoning itself should be examined more closely for the insight which it can give.

If we adopt the position that probability is a subjective attribute of a decision-making alternative, then this will have implications upon the way in which it may be treated. As examples of other subjective attributes, we may cite Norman churches, peaceful surroundings, country walks and prestige. These aspects cannot be measured in meaningful units and it is certainly difficult to measure them in units of money. However, they may be described verbally and their quality is felt. Probability can be treated in this way too. We can describe our feelings about probability in words, as in the two sentences on the weather earlier. Recall too the decision criterion mentioned earlier which involved verbal statements. We may attempt to measure probability verbally since it has many of the features of other subjective attributes. But because of decision-makers' special attitudes towards risk it will have to be accorded a different status in the decision criteria. Verbal measurements of probability have always been ignored in the past for reasons such as "words are only useful to convey meaning provided that the writer and the reader (or speaker and listener) agree on the meaning to be ascribed to the words", (Moore and Thomas (1975).)

Moore and Thomas took a list of ten expressions commonly used to convey uncertainty and asked 250 executives at the London Business School to rank order the phrases in decreasing order of uncertainty. The found that each phrase was placed within a wide range of ranks. For example, "Expected" was ranked anywhere between 1st and 6th most uncertain! Moore and Thomas conclude that -

> there is a clear need for the scale to be a numerical one in order to eliminate doubts as to the relative orderings of different uncertainty statements.

We may challenge Moore and Thomas' conclusion on four points

a) The executives they used were "on middle senior management programmes at the School and elsewhere". These people could hardly be expected to form a coherent group, sharing a common understanding of the meaning of words. Within a group, the formal, verbal transfer of information is effective, and so one would expect that within a decision-making group, accustomed to discussing uncertainty, the ranking of phrases would show less deviation than Moore and Thomas found.

b) The expressions which Moore and Thomas used were obtained from an article and not from the conversation of the group members. One would also expect that not all of these words would be commonly used by all the executives they tested. In practice, only a few graduations of probability are perceived and so only a small vocabulary is needed. When asking people to rank order expressions which may not be part of their usual vocabulary, one would expect to find inconsistency. This is because the reader may not have any fixed meaning ascribed to the words but only a vague notion, rather than sharing a meaning with the writer.

c) Thirdly, the executives were asked to order the phrases in "decreasing order of uncertainty". We have already seen in this chapter how uncertainty may be derived from lack of knowledge as well as stochastic probability but no such distinction is made here although Moore and Thomas move directly to the assessment of probability.

d) Finally, the purpose of adopting a numerical scale would seem to be to ensure understanding of the "relative orderings of different uncertainty statements". If ordering is all the information which is required, then a numerical scale provides more information than is necessary, since an ordinal scale would be enough. Such a scale can be provided by verbal statements. We may also recall that the use of words may be carefully structured, using a simple grammar and thereby eliminating doubts as to relative orderings.

The possibility of using verbal measurement of probability seems to have been overlooked. In a culture which regards scientific precision as desirable, the use of numbers is regarded as respectable and the use of words as an unnecessary defeat.

SOURCES OF UNCERTAINTY AND METHODS OF HANDLING IT

To complete the picture on uncertainty, let us look at where uncertainty may arise. These are three main sources which may be identified with various parts of the decision-making process.

The first source is in the process of measuring. Where the alternatives are tangible the measuring process may not be complete or exact. This type of uncertainty is due to lack of information and should be reducible.

The second source lies in the problem of forecasting. The choice of a strategy is like a game with Nature where Nature is an unpredictable opponent. One cannot know which strategy she will adopt. This type of uncertainty can never be removed entirely although the decision-maker may have some idea of the relative likelihood of the possible states of nature. Some strategies involve forecasting to estimate their likely benefits and so the measurement of the alternatives can incorporate uncertainty from this source as well.

The third source is in value judgements. This may be best described as due to lack of self-knowledge. The decision-maker may be unsure of the value which he places upon the different aspects of the alternatives, both when comparing aspects and when comparing levels of the same aspect. But if through practice the decision-maker does get to know himself better, there will remain a hard core of imprecision which is due to the nature of the human reasoning process and the language which we use.

Looking back to the decision techniques presented in an earlier section, we may see which types of uncertainty they tackled. For example, the BASYC technique (Mumford et al, 1978) which used optimistic and pessimistic estimates of the measurement of attributes, effectively took both forecasting error and measurement error into account. The within-attribute error may appear in the optimistic and pessimistic estimates but the inter-attribute error is not so immediate. The weights which are assigned to the attributes are a value judgement but it is stressed in the BASYC technique that the results which are obtained must be tested for sensitivity to fluctuations in the weighting values. For example, a heavy weight when combined with a high score may contribute a substantial amount to the total value of that alternative. There are processes for testing for this possibility but so far they remain rather ad hoc.

The method of Goodwin (1972) tests in particular for sensitivity due to slight changes in the weights, so he does test for error in the value judgements. He also changes the scores or measurements slightly, thereby testing for error of the other two types.

The Dean and Nishry (1965) method makes no explicit treatment of error or uncertainty at all and indeed would seem to incorporate some procedures which could increase the overall error.

The Litchfield, Hansen and Beck (1976) model requires a great deal of subjective information. This information could be very uncertain and I wonder how important they think this is. Their use of a Monte Carlo method would give some indication of the possible spread of results and some indication of their sensitivity.

In this section, I have examined some of the aspects of uncertainty. This is by no means an exhaustive treatment, and many of the views expressed can be disputed. The treatment of probability or uncertainty is highly problematic but a suggestion has been made, reducing the status of probability to a subjective attribute, although retaining the possibility of according it special treatment in the decision algorithm. This idea will be developed further in later chapters.

SUMMARY AND CONCLUSIONS

The nature of uncertainty as it is manifested in decision-making was examined in this chapter. The sources of uncertainty were listed but in this chapter only stochastic uncertainty was treated.

Methods of dealing with this type of uncertainty were demonstrated under three different sets of assumptions of knowledge about the future. It was shown that the criteria for selecting a strategy, under the assumption of no knowledge of the future and many states of nature, could each lead to a different answer so that the choice of strategy depended upon the criterion adopted.

Where there is some knowledge about the relative likelihood of the many future states of nature this is usually stated as a numerical probability. This was examined more thoroughly under the paradigms of Bayes' theorem and expected value. The decision-maker's attitude towards risk was seen to be important and how this pessimism showed itself as conservatism in the Bayesian processing of information. Furthermore, the evidence for an interaction between the assessment of probability and utility was cited and how this is also usually attributed to the subjects' limited information processing capacity.

From this it was argued that expected value models, whether using objective or subjective values of the probability and financial value were misleading and that the probability of an alternative producing the desired result should be considered as a special attribute of that alternative. The usual reasons for using numerical probability as a measurement of uncertainty rather than verbal statements were examined and it was argued that the assessment of uncertainty could be done with careful use of words.

The chapter closed with a brief review of those methods presented in Chapter 2 considering how uncertainty affected their answers.

CHAPTER 4
UTILITY

MORAL AS OPPOSED TO MONETARY WORTH

The concept of "utility" arose from the study of gambles - how to decide which of several available cash gambles was the most advantageous. The method of deciding between them was to invoke the principle of mathematical expectation. This follows from the law of large numbers, that in the long run, over repeated trials, the gambler's overall gain or loss may be estimated. Mathematicians in the eighteenth century saw it as paradoxical that seemingly prudent individuals would reject the gambles which this law recommended (Savage, 1954, p.92).

Daniel Bernoulli (1700-1782) seems to have been the first to point out that this principle was only a rule of thumb, and quotes the so-called St. Petersburg paradox as an illustration. The argument proceeds roughly as follows:

Suppose upon paying an entrance fee, a person had the opportunity to participate in a gamble which could provide an infinite amount of wealth. The gamble is this: a coin is tossed and when a head appears, the game stops. If the game ends on the nth toss, the gambler wins £2^n. Thus, the mathematical expectation is

$$\frac{1}{2} \cdot 2 + \frac{1}{2^2} \cdot 2^2 + \ldots + \frac{1}{2^n} \cdot 2^n + \ldots = 1 + 1 + 1 + \ldots$$

$$= \infty$$

Hence, no matter how large the entrance fee, even if it were one's entire existing wealth, the gambler should still accept the bet, despite the 50% chance of the game ending on the first toss and the player being left with only £1.

Hence the value of a gamble is not its expected monetary worth, but depends upon the gambler himself, and the "moral worth" of the outcomes to him. Bernoulli postulated that people seek to maximise not the mathematically expected amount, but the expected moral value of the gamble.

The notion of utility in gambling problems was considered by Ramsey in the 1920's, but interest in the subject was revived by von Neumann and Morgenstern's axiomatic treatment of utility in 1947. Since then, other systems of axions leading to the utility concept have been proposed, by Herstein and Milnor (1953), Hausner (1954), Savage (1954), Luce and Raiffa (1957), Pratt, Raiffa and Schaifer (1965) and Fishburn (1970), amongst others. We shall consider axioms from these systems in a later section.

THE PURPOSE OF UTILITY THEORY

With the axiomistion of the concept of utility, it became possible to assign a numerical value to the "moral expectation" of a gamble and thereby predict a grambler's choice between gambles, assuming he wishes to maximise expected utility. Fishburn (1970) states what he calls the "fundamental theorem of utility".

> This has to do with axioms for preferences which guarantee, in a formal mathematical sense, the ability to assign a number (utility) to each alternative so that, for any two alternatives, one is preferred to the other if and only if the utility of the first is greater than the utility of the second.

The purpose of the theory would seem to be the prediction of preferences, purely on whether one number is greater than another. However, because of the power of numbers and their association with the physical sciences and measurements, many have been inclined to believe that it is possible to measure the "moral value" of an alternative, i.e. to make an "objective measurement" of the subjective feelings of the decision-maker.

Most theories of utility appeal to the existence of the "rational" person, i.e. one who obeys the axioms.

> I am about to build up a highly idealised theory of the behaviour of a "rational" person with respect to decisions. In doing so I

> will, of course, have to ask you to agree with me that such and such maxims of behaviour are "rational". (Savage, 1954, p70)

The implication becomes that if a decision-maker does not obey the laws of rational behaviour (which have been laid down to achieve the goal of the numerical measurement of utility) then that decision-maker is irrational. The definition of a rational decision-maker is, therefore, one who takes his decision in a manner which is consistent with the numerical measurement of utility. This is becoming painfully reminiscent of the attitude of the 18th Century mathematicians who believed in the principle of mathematical expectation as being the criterion for rationality.

We can see a dilemma or paradox emerging in utility theory. If the purpose of a utility theory is to predict preferences then that theory is purely descriptive of the decision-maker's behaviour. If it must lay down definitions of rational behaviour in order to do so, then it becomes a normative theory. This choice between normative and descriptive theories has plagued decision theorists for decades.

In our imperfect, irrational way, decision-maker's may choose to obey some of the axioms which the theory requires, thereby setting standards of consistency and rationality. This is similar to logic where rules of consistency are set down and voluntarily obeyed. A decision-maker may accept an axiom, such as transitivity of choice, as a standard of rational behaviour and so long as that axiom never declares that he should make a decision which he could not accept, then that axiom if fair. The purpose of a normative theory is to prescribe courses of action for the decison-maker, on the basis of his beliefs, values and standards of consistent behaviour.

Most decision aids are inclined to be normative in the sense that they study beliefs and values, test for standards of consistent behaviour and prescribe courses of action. A purely descriptive theory would still leave the burden of decision-making with the decision-maker, since it only aims to describe the beliefs and values of the decision-maker and the manner in which he incorporates them into a theory.

Fishburn (1968) suggests three purposes of the normative theory of utility. These may be summarised:

1) to serve as a normative guide, indicating when the decision-maker's preferences appear to violate a "rational" preference assumption;

2) to help a decision-maker to determine his preferences amongst complex alternatives, characterised by multi-dimensionality and uncertainty;

3) to enable the decision-maker's "preferences to be transformed into a numerical utility structure to be used in an optimisation algorithm".

These purposes seem fair and reasonable and in the next section we shall look at the axioms which normative theories require, and discuss whether they are indeed suitable normative guides for decision-makers to adopt.

CLASSIFICATION OF THE AXIOMS

In this section, we shall examine some of the axioms which are presented as bases for theories of utility, both with and without risk, and in the single and milti-dimensional cases.

As a broad classification ot the types of axiom presented, we suggest the following:

1) Simple Ordering
2) Archimedean
3) Monotonicity
4) Combining
5) Multi-attribute

I shall discuss the axioms under these headings but will explain notation and the concept of a mixture set beforehand.

Notation

The separate authors tend to adopt their own system of notation, but, within the chapter, we shall use von Neumann and Morgenstern's notation and Keeney and Raiffa's multi-attribute notation.

Thus, $u, v, w, \ldots$	are items to be compared
$\alpha, \beta, \gamma, \ldots$	are probabilities associated with the items
$\alpha u + (1-\alpha)v$	denotes a gamble in which the gambler may expect to win u with probability α and v with probability $(1-\alpha)$.
$u \succcurlyeq_p v$	means that u is preferred or indifferent to v
$u =_p v$	means that the decision-maker is indifferent
X_i	is attribute i
x_i	is the level of attribute i
$\bar{x}_i$	is the set of levels of the attributes, complementary to x_i

The Mixture Set

The utility axioms apply to preferences upon a commodity which must obey certain rules. When von Neumann and Morgenstern laid down their axioms, many of these defined the mixture set itself. The distinction between the mixture set and the axioms of preference has been made since, but the best description of the mixture set belongs to von Neumann and Morgenstern, although they do not use the term.

> We shall therefore assume that the aim of all participants in the economic system, consumers as well as entrepreneurs, is money, or equivalently a single monetary commodity. This is supposed to be unrestrictedly divisible and substitutable, freely transferable and identical, even in the quantitative sense, with whatever "satisfaction" or "utility" is desired by each participant.

We list Herstein and Milnor's definition of the mixture set below, but see also Hausner (1954)

> A set $\mathcal{S}$ is said to be a mixture set if for any $u, v \in \mathcal{S}$ and for any α we can associate another element, which we write as

$$\alpha u + (1-\alpha)v,$$

which is again in $\mathcal{S}$, and where:

1) $1u + (1-1)v =_p u$

2) $\alpha u + (1-\alpha)v =_p (1-\alpha)v + \alpha u$

3) $\beta \left[\alpha u + (1-\alpha)v \right] + (1-\beta)v = (\beta\alpha)u + (1-\beta\alpha)v$

The notation has been changed to agree with earlier remarks.

The third condition in this list sometimes appears as a separate axiom, known as Substitutability. This states that a lottery may have as a prize another lottery ticket and that the probabilities are the objective probabilities calculated from the lotteries.

Simple Ordering

A relation $\succcurlyeq_p$ is a simple ordering among items u,v,w,... if and only if for every u,v,w:

Either $u \succcurlyeq_p v$ or $v \succcurlyeq_p u$

If $u \succcurlyeq_p v$, and $v \succcurlyeq_p w$, then $u \succcurlyeq_p w$.

The first requirement above defines complete ordering. Each system of axioms requires a complete ordering amongst alternative together with transitivity of preferences. The relation $\succcurlyeq_p$ is also reflexive since:

$x \succcurlyeq_p x$

The ordering axioms require that every alternative can be compared with every other. Thus, a utility can not be derived for any two alternatives which cannot be compared. The notion of transitivity of preferences is more controversial. For any system of assigning utility values, the transitivity axiom gives it meaning, since the numerical utilities will always predict a strict ordering of the alternatives, and this must match the preference ordering of the decision-maker.

Archimedean

The Archimedean axiom is as follows:

If $u \succ_p v \succ_p w$, there exists an α such that

$$\alpha u + (1-\alpha)\, w =_p v$$

This axiom is included in all the axiomatic systems quoted herein, with the highly deliberate exception of Hausner. The purpose of the Archimedean postulate is to bring the measurement of utility onto the real number line. It is this postulate or axiom which gives utility its handle onto numerical measurement, so for this reason, it is one of the most important axioms which is commonly used.

Monotonicity

This axiom is similar to the previous in that it allows that preference between a pair of items to be predicted as a result of preferences between gambles, but this time involving different probabilities:

If $u \succ_p v$, then

$$\alpha u + (1-\alpha)v \succ_p \beta u + (1-\beta)v$$

if and only if $\alpha > \beta$

Combining

These axioms allow preferences amongst prizes to be deduced from the decision-maker's preferences amongst gambles, and vice versa. The axioms are:

If $\alpha u + (1-\alpha)v \succ_p \alpha u + (1-\alpha)w$ for $0 < \alpha < 1$,

$$\Leftrightarrow v \succ_p w$$

Multi-Attribute

We present the conditions necessary for a decision-maker's utility function to be either additive of multiplicative. The conditions for probability-less utility or 'value' are presented with those for utility, assessed using probability.

1. Additive probability-less multi-attribute value theory

Given attributes $X_1,\ldots,X_n$, $n \geqslant 3$, an additive probability-less utility function

$$v(x_1, x_2,\ldots,x_n) = \sum_{i=1}^{n} v_i(x_i)$$

exists if and only if the attributes are mutually preferentially independent

$$\text{i.e. if } \left[(y', z') \succcurlyeq_p (y'', z')\right] \implies$$

$$\left[(y', z) \succcurlyeq_p (y'',z)\right] \quad \text{for all } z,y',y''.$$

where y and z represent complementary subsets of the attributes

This may also be stated as, for any pair of attributes (X_i, X_j) to be preferentially independent of the other n-2 attributes, preferences between (X_i, X_j) pairs, given that the levels of the other n-2 attributes are held fixed, do not depend on the level at which those attributes are fixed. Preferential independence implies that the trade-offs between attributes X_i and X_j do not depend on $X_1,\ldots,X_k,\ldots,X_n$, $k = i,j$.

See Keeney and Raiffa (1976, p. 111) for this result and references to other workers.

2. Additive multi-attribute utility function with probability

The n-attribute additive utility function

$$u(x) = \sum_{i=1}^{n} k_i \, u_i(x_i) = \sum_{i=1}^{m} u(x_i, \bar{x}_i^{\circ})$$

is appropriate if and only if the additive independence condition holds among attributes, $X_1, X_2,\ldots, X_n$ where:

1) u is normalised by $u(x_1^{\circ}, x_2^{\circ},\ldots, x_n^{\circ}) = 0$ and

$$u(x_1^{*}, x_2^{*},\ldots, x_n^{*}) = 1$$

2) u_i is a conditional utility function of X_i normalised by

$$u_i(x_i^{o}) = 0 \text{ and } u_i(x_i^{*}) = 1$$

3) $k_i = u(x_i^{*}, \bar{x}_i^{o})$, $i = 1,2,\ldots,n$

See Keeney and Raiffa (1976, p. 295) and Fishburn (1964, 1968, 1970). See Figure 4.1.

OR

An individual's utility function is additive if and only if his preference between any two lotteries

$$L_1 = \tfrac{1}{2}(x_i , \bar{x}_i') + \tfrac{1}{2} (x_i^{a} , x_i^{a})$$

and

$$L_2 = \tfrac{1}{2} (x_i, x_i'') + \tfrac{1}{2} (x_i^{b} , x_i^{b})$$

is the same for all x_i, for any $\bar{x}_i'$, $\bar{x}_i''$, x_i^{a}, $\bar{x}_i^{a}$, x_i^{b}, $\bar{x}_i^{b}$

See Figure 4.2

$$\text{If } L_1 >_p L_2 \Longleftrightarrow L_1' >_p L_2'$$

for additive utility.

See Pollak (1967).

<u>3. Multiplicative, multi-attribute utility function with probability</u>

If the additive, independence condition holds and

$$\sum_{i=1}^{n} k_i = 1$$

then the additive utility function is appropriate. If

$$\sum_{i=1}^{n} k_i \neq 1$$

then the utility function is multiplicative:

$$1 + ku(x) = \prod_{i=1}^{n} 1 + kk_iu_i(x_i)$$

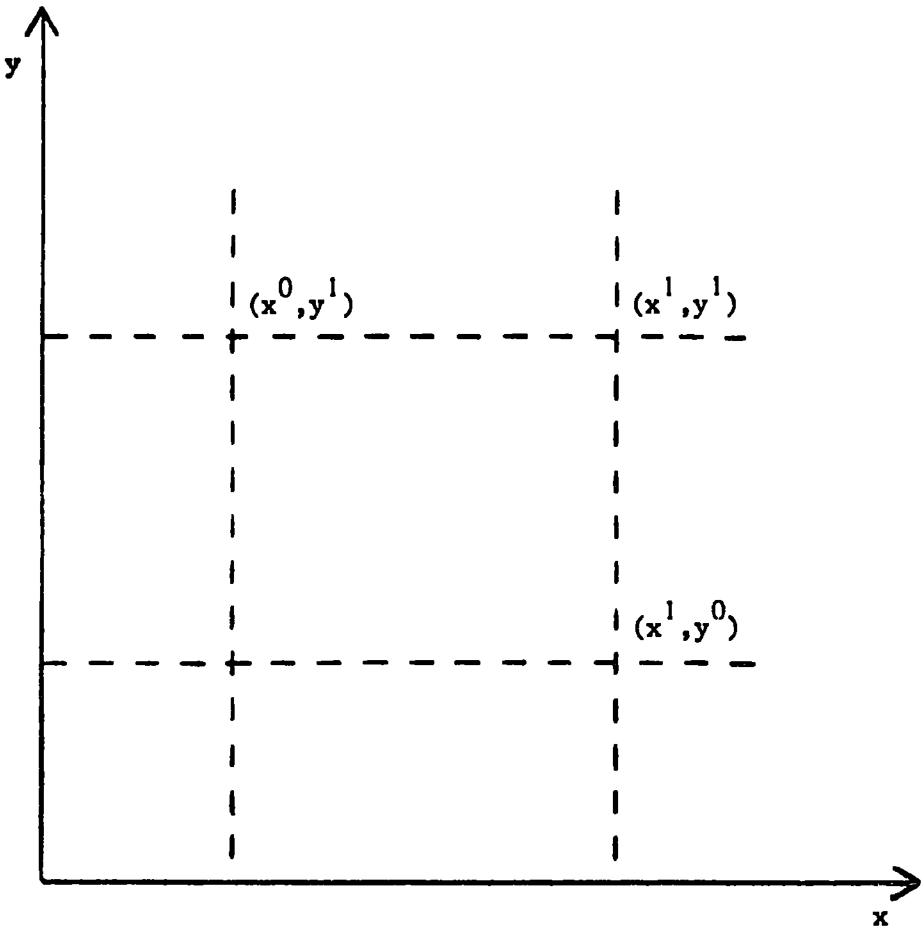
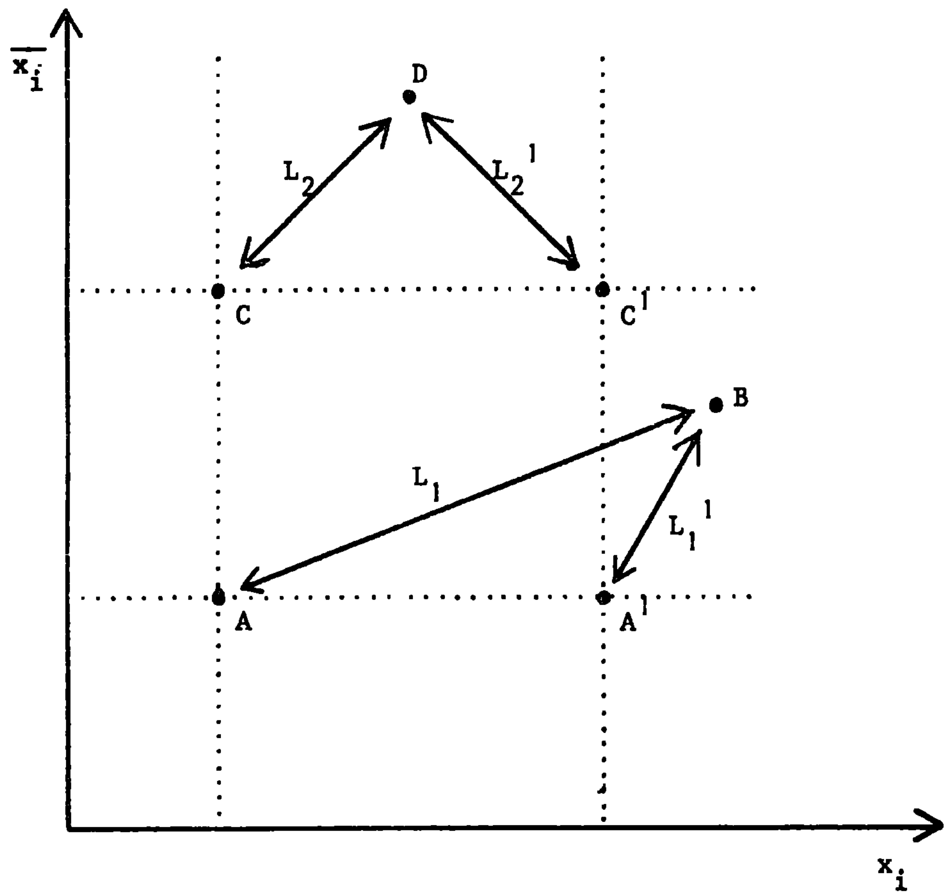


Figure 4.1 Additive Multi-Attribute Utility

$u(x^1,y^1)=u(x^0,y^1)+u(x^1,y^0)$



L_1 is ½A + ½B, L_2 is ½C + ½D. Note that A and C have the same amount of attribute X_i. Whatever preference we have between L_1 and L_2 we must also have, if the level x_i in A and C is changed, e.g. both are moved to A' and C'.

<u>Figure 4.2</u> <u>Lotteries and Additive Utility</u>

k may be determined by evaluating at x^*

$$1 + k = \prod_{i=1}^{n} (1 + kk_i)$$

The necessary and sufficient conditions for the additive value function are necessary for the multiplicative value function.

If attributes $X_1, X_2, \ldots X_n$ are mutually utility independent, then

$$\begin{aligned} u(x) = & \sum_{i=1}^{n} k_i u_i(x_i) \\ & + k \sum_{\substack{i=1 \\ j>i}}^{n} k_i k_j u_i(x_i)\, u_j(x_j) \\ & + k^2 \sum_{\substack{i=1 \\ j>i \\ l>j}}^{n} k_i k_j k_l u_i(x_i)\, u_j(x_j)\, u_l(x_l) \\ & + \ldots \\ & + k^{n-1} k_1 k_2 \ldots k_n u_1(x_1)\, u_2(x_2) \ldots u_n(x_n) \end{aligned}$$

where

1) u is normalised by $u(x_1^{\circ}, x_2^{\circ}, \ldots x_n^{\circ}) = 0$ and

$$u(x_1^*, x_2^*, \ldots x_n^*) = 1$$

2) $u_i(x_i)$ is a conditional utility function on X_i normalised by

$$u_i(x_i^{\circ}) = 0 \text{ and } u_i(x_i^*) = 1, \; i = 1,2,..n$$

3) $k_i = u(x_i^*, \bar{x}_i^{\circ})$

4) k is a scaling constant that is a solution to

$$1 + k = \prod_{i=1}^{n} (1 + kk_i)$$

See Keeney and Raiffa (1976, p.289)

EXAMINATION OF THE AXIOMS

We shall now examine the axioms of utility as listed in the previous section, remembering all the while that a theory of utility, however convenient its use may be, is only as good as its axioms.

Ordering

The two main assumptions here concern the comparability of alternatives and the transitivity of preference.

It may be the case that a decision-maker feels unable to compare two alternatives which are presented to him because they bear no similarity to one another. An example might be choosing between a new car or a lifetime's supply of cornflakes, but as a decision-maker's inability to state a preference between the two is a different matter and may be due to indifference or lack of discriminatory power.

Transitivity of preference is more interesting and is one axiom of choice which has been experimentally tested. Papandreou (1953) found that transitivity of choice occurred amongst commodity bundles, i.e. using probability-less, multi-attribute utilities. Edwards (1953) found that intransitivity of choice was marked in paired comparisons of bets, i.e. single attribute utilities with probability. In a later paper, Edwards (1961) is sceptical about the transitivity of choices. The notion of stochastic transitivity has since been developed, based on assumptions of the probability of choosing A over B. He says:

> As a basis for psychological theorising algebraic transitivity is dead and stochastic transitivity, strong and weak, has yet to be exposed to the adverse climate of hostile experiments ... the question for experimenters to answer is not whether any form of transitivity holds but rather under what circumstances do various assumptions about transitivity hold and under what circumstances they do not.

Consider Table 4.1.

Does Transitivity of Choice hold?
Experimental Results:

	Single Attribute	Multi-Attribute
Probability-less	Yes (More is better than less)	Yes (Papandreou)
With Probability	No (Edwards)	? (Probably No)

Table 4.1

We must also consider the question of transitivity of indifference. Many utility theories assume that the binary relation of indifference will define equivalence classes on the set of possible alternatives,

$x \not>_p x$ reflexive

$x \not>_p y \quad y \not>_p x$ symmetric

$x \not>_p y \, , \, y \not>_p z \Rightarrow x \not>_p z$ transitive

However, the condition of transitivity of indifference may be violated:

$x =_p y, \; y =_p z$ but $x \not>_p z$

Some experiments (e.g. May, 1954) tested transitivity of choice, but did not permit respondents to record indifference between alternatives.

Let us conclude this discussion on transitivity of choice by saying that transitivity is generally obeyed for riskless alternatives, and may also be adopted as a normative rule. Transitivity of choice does not hold so well for risky alternatives, and it is doubtful whether is should be adopted in this case as a normative rule because of the gambler's preference or aversion for certain probabilities, (Edwards, 1953).

Tversky (1969) shows that transitivity of choice between gambles may be violated when people choose according to lexicographic semi-ordering decision rules. Transitivity of indifference is only correct over short ranges. This may be because of "indifference thresholds", i.e. two alternatives are preferentially indifferent, but once the preference difference between them becomes greater than the threshold value, then there is a distinct preference between them.

The Archimedean Postulate

The purpose of the Archimedean postulate is to give a numerical assessment of the utilities. It requires that the utilities are defined with lotteries, so the Archimedean postulate does not work for probability-less utilities. Given the Archimedean postulate, it is possible to define a utility which is unique up to a positive linear transformation.

The Archimedean postulate:

If $u >_p v >_p w$, there exists an α such that

$$\alpha u + (1-\alpha)w =_p v$$

is difficult to test in practice. It seems plausible that if α is near 1 then $\alpha u + (1-\alpha)w$ is preferred and as α is near 0, so v is preferred over $\alpha u + (1-\alpha)w$. As α is smoothly varied, so there must be point of inversion where the two alternatives are indifferent.

This Archimedean postulate is often used as a direct means of assessing the businessman's attitude towards risk (Hammond, 1967; Swalm, 1966; Pratt, Raiffa and Schlaiffer, 1964; Miller and Starr, 1969).

There are two variations on the theme. The first is to fix α at 0.5, say, choose values for u and w, and ask the decision-maker to provide v, i.e. the amount which he would exchange for the chance to play the bet when he had an equal possibility of winning u and w. The other method is to fix u,v and w, and ask the decision-maker for the value of α which would make him indifferent. It would be interesting to compare results obtained by both methods, but I am not aware of any serious attempt to do so.

Hausner (1954) omits the Archimedean postulate so as to generalise the von Neumann and Morgenstern theory of utility. In commenting on Hausner's work, Thrall (1954) says that the Archimedean postulate "limits utility spaces to one dimension, i.e. to real numbers". As a counter example of the Archimedean postulate, he suggests that if u = "be given 2 common pins", v = "be given 1 common pin" and w = "be hanged at sundown", then we would expect

$$u \succ_p v \succ_p w,$$

but no α exists such that

$$\alpha u + (1-\alpha)w =_p v.$$

Thrall does say that in much of economics, the Archimedean postulate is likely to hold and non-Archimedean, multi-dimensional utilities may be useful in game theory.

My basic objections to the Archimedean postulate are as follows:

1) It tends to place a high accuracy on the values of which the decision-maker may produce. This may lead to inconsistency later on.

2) It assumes that the subjective probability is equal to objective probability.

3) Its use as a normative rule of behaviour is questionable.

Monotonicity

This axiom is similar to the previous, but considers the effects of probability, rather than preferences between alternatives.

If $u \succ_p v$,

$$\alpha u + (1-\alpha)v \succ_p \beta u + (1-\beta)v \quad \text{if and only} \quad \alpha \succ \beta$$

This means that with two lotteries, one should select the lottery which renders the more preferred alternative more probable.

In suggesting this axiom, Luce and Raiffa also put forward some examples in which it may be invalid. For instance, a mountaineer almost certainly prefers the alternative "life" to "death", but while climbing mountains, he is preferring the lottery of life and death to the safer bet of life, i.e. staying at home. But, the thrill of climbing is more than the lottery of life and death and it is probably this extra dimension of excitement which makes this axiom invalid in such cases.

As a normative criterion, this axiom would be fair for one-dimensional alternatives, as in gambles for money. As the dimensionality of the problem grows, the psychological interaction between the probabilities and alternatives makes this axiom more strained and less useful as a normative criterion.

Combing and the Sure-Thing Principle

These two axioms will be treated together since one is the converse of the other.

$$\alpha u + (1-\alpha)v \succsim_p \alpha u + (1-\alpha)w \quad \overset{\Longrightarrow}{\Longleftarrow} \quad v \succsim_p w$$

The leftwards pointing arrow is an axiom of Hausner and Herstein and Milnor, although they use indifference. Hausner has the rightwards pointing arrow as an axiom as well.

Herstein and Milnor state that the combining axiom means that:

> If an individual is indifferent as to a choice between a and a', then he is also indifferent to a choice of A and A', where A represents a 50-50 change of getting a or b and A' a 50-50 change of getting a' or b, for any prospect b.

but they extend the axiom to cover any α , $0 \leq \alpha \leq 1$

Fishburn (1970, p. 108) cites this axiom, with leftward arrow as a normative criterion. Consider the following payoff matrix:

	α	$1-\alpha$
Option A	u	v
Option B	u	w

If v is preferred to w, then the gambler should prefer A to B, whatever the probability, α .

Whatever, the direction of the arrow, this axiom would seem to be a good guide towards consistent behaviour, i.e. as Savage's theory of personal probability and is known by him as the Sure-Thing Principle.

As a foundation of behavioural decision theory, Savage's sure-thing principle has had its ups and downs. In 1953, Allais published a famous counter example, and in 1961, Ellsberg suggested another. In the same year, Edwards defines and comments on the sure-thing principle as follows:

> The sure thing principle ... asserts that if a course of action A is at least as good as course of action B in all possible future states of the world, and is definitely better in one or more, then B should never be preferred to A; it is about the only universally accepted and universally empirically confirmed principle in decision theory.

However, in 1974, Slovic and Tversky challenge this claim. They point out that individuals are quick to revise their preferences and conform to transitivity once their violation of it is demonstrated to them (MacCrimmon, 1968; Tversky, 1969), and wondered if the same were true of the sure-thing principle. Subjects were asked to choose amongst the gambles as shown in Table 4.2. The sure thing principle says to choose either gambles 1 and 3 or 2 and 4, but Allais and Ellsberg recommend 1 and 4. Having made their choices, subjects were presented with either the Allais or Savage argument against their choice. It was found that 17 of 29 chose according to Allais on their first choice, and after hearing the counter-arguments, 19 of 29 chose according to Allais on the second choice.

At the end of their paper, Slovic and Tversky present a hypothetical dialogue between Savage and Allais, which could suggest that they too are unsure which position to adopt. Savage's axiom does require a very cool look at the probabilities involved, and is a normative procedure. Allais tends to appeal to caution, making more of the subjective importance of risk. The arguments in favour of choice in both the Allais and Savage ways are presented in an appendix to this chapter.

Since the arguments for and against the sure-thing principle are both compelling and accepted by ordinary decision-makers, then there seems no reasonable criterion for choosing between them. If Savage's axiom is to be used as part of a theory of utility, then any decision-maker who wishes to use that theory should either intuitively obey Savage's axiom or accept it as a normative criterion. Any theory of utility, embodying this axiom will give inconsistent results when applied to a decision-maker who does not wish to choose according to the sure-thing principle. There is a case for separate theories of utility, one to be applied to sure-thing principle believers, and another for those who do not believe.

Imagine the following two decision situations - each involving a pair of gambles:

	Probability of winning	Amount to win
Situation X		
Gamble 1	100%	£1,000,000
Gamble 2	10%	£5,000,000
	89%	£1,000,000
	1%	£ 0
Situation Y		
Gamble 3	11%	£1,000,000
	89%	£ 0
Gamble 4	10%	£5,000,000
	90%	£ 0

Table 4.2 Choice Between Gambles

Multi-Attribute Utility

The aim of multi-attribute utility theory is to enable the decision-maker to express his utility of the multi-attribute alternatives as a simple combination of the utilities of the separate attributes. Thus, the overall utility may be obtained conveniently by addition or multiplication of the separate utility functions. The conditions necessary to achieve these goals have already been listed. We may make the following comments.

Firstly, the criteria of independence are not normative. The decision-maker may or may not adhere to the standards of behaviour required for a conveniently modelled utility function. In descriptions of practical application, the decision-makers do usually obey these criteria, but some methods have been suggested to handle less convenient utility models.

In order to decide whether a decision-maker does obey these criteria, the analyst must have sensitive testing techniques (Keeney and Raiffa, 1976). However, as the complexity of the problem grows and more and more attributes are involved, these testing techniques become more and more time-consuming and boring. Since the testing procedures provide little in the way of training or self-insight for the decision-maker, one wonders how valuable the testing procedures are, and consequently the value of the decisions based upon them.

This long process of preliminary testing keeps the control of the decision analysis in the hands of the analyst, and not the decision-maker. Any decision aid must "interface" with the decision-maker. If the decision aid is simple to use, the decision-maker may not even be aware of the interface. As the complexity of the technique increases, the interface between decision-maker and technique widens, and help may be needed. An analyst can advise and explain. As the technique grows ever more complex, and only the mathematically able can comprehend, so the analyst, who understands, acquires more and more control over the decision-making process and may exert pressure on the decision-maker to adopt his perspective. This becomes apparent from the hypothetical dialogues between Analyst and Assessor in Keeney and Raiffa's book (1976).

The preferential indifference assumption requires that constant trade-offs be maintained throughout the range of variation of x_i. Thus, when considering a system change, for example, the importance of the attributes must not vary as the level of performance moves from the intolerable to the ideal. This is unlikely in practice, as for example with job satisfaction. The BASYC technique (Mumford et al, 1978) is prone to errors of this kind because of its assumption that the midpoint of the utility range coincides with the present position.

WEIGHTS AND SCALING FACTORS

Let us look closely at the scaling factors involved, the k_i. Each k_i is defined so that:

$$u(x_i^*, x_i^0) = k_i$$

Hence each k_i represents the amount of satisfaction which is obtained from that attribute at its highest level, when the levels of all the other preference variables are held at their lowest level. If the attributes are mutually preferentially independent, then these weights will remain constant, whatever the level of fulfilment of the other attributes, including the status quo levels. Hence, if the weights accorded to each attribute are determined with respect to the status quo, these weights are only valid so long as all the attributes are mutually preferentially independent.

The failure of preferential independence is manifested as non-constant weights, as in an asymmetry of weights about the status quo. As an illustration, a group of workers were asked to rank order a number of objectives. Some of the objectives referred to their own working life, and others were of a more altruistic nature, dealing with the interests of other user groups and the purpose of their organisation. To our surprise, their own job satisfaction was ranked well down the list, with the more altruistic objectives ranked higher. When asked why this was so, they replied that the improvement of their own conditions was relatively unimportant, because they were reasonably happy already.

In a second case, a group of managers were having to face impending cuts in their budget and wanted to decide beforehand how these cuts should be distributed over the service which they provided. They were asked to list the objectives of the organisation, but to state them as if improving them from the present level, i.e. "Improve the level of provision". This assumed that the weights which were assigned for the "Improve" objectives would be the same as for "Maintain" and "Do not decrease" objectives on the same performance variable. The managers realised that what they were being asked to do was not pertinent to the problem, and so the project was abandoned.

There is an observable asymmetry about the status quo in the allocation of weights. A group may perceive the importance of improving on the level of a performance variable as quite different from the importance of maintaining or not decreasing that level. This is particularly true when highly subjective or emotive issues are involved. This means that in such cases the assumption of preferential independence is unreasonable, and so additive utility cannot be used.

NATURAL OPERATIONS

We have looked, in some detail, at the foundations of utility theory and the axioms it supposes. Some axioms have a greater intuitive appeal than others, and some may be considered as normative criteria of rational behaviour. A generalised theory of utility should be based on the fewest and most transparent axioms, but to do this we may need to relinquish the ideal of numerically measurable utilities and lower our sights towards a descriptive theory of utility, which is in danger of becoming tautological.

> I prefer x to y because u(x) is greater than u(y) because
> I prefer x to y ....

This encourages the attitude that "the only 'natural' datum in this domain is the relation "greater", i.e. the concept of preference" (von Neumann and Morgenstern, 1947 p.3).

One wonders if we are in a position to account for the failure of utility theory to be taken up as a convincing tool (see Bell, Keeney and Raiffa, 1977, p. 431-432). It requires standards of consistency and rationality which are difficult even for decision analysts themselves to attain. However, since multi-attribute decision-making is a difficult and complex task, some method of assisting that process is required. An offshoot or development of the classical utility theory may be required.

We have argued that in systems which strongly involve the human factor, preferential independence among attributes becomes less and less strong, and may be observed in the statements of relative importance which decision-makers make. If we look at the multi-attribute problems involving highly subjective attributes, we are likely to find the failure of preferential independence and hence of additive value theory.

Of what use, then, is a multi-attribute value theory, in these circumstances? The conditions for preferential independence are time-consuming to test, and are likely not to hold. Fishburn (1964) says:

> In virtually all decision situations in which independence is used it must be taken as an assumption. The first reason for this is that it is simply too great a task to attempt complete verification of the independence hypothesis - the effort might be better spent on other things. The second and perhaps more important reason for stating independence as an assumption is because it often yields an approximation (to a very complicated state of affairs). Although in many instances it may be a good approximation, it is nevertheless an approximation and should be recognised as such. It may also be an approximation which must be made in order to treat a problem analytically or "objectively".

The value of multi-attribute value theory would seem to be in its ability to structure the decision-making process. But the mathematical model is only an approximation to the true subjective valuation, which must be the final criterion for deciding which alternative to choose. The mathematical method can assist the subjective assessment but can never wholly replace it.

We have argued that the theoretical weaknesses of multi-attribute utility theory render it an approximation only, but as such is should have some uses as a decision aid. However, we shall now argue that the practical use of multi-attribute utility theory introduces errors and distortions which make it disadvantageous and inaccurate.

DISADVANTAGES OF MULTI-ATTRIBUTE UTILITY THEORY

It may be argued that the root of many of these difficulties lies in the use of numbers to describe what are fundamentally subjective opinions and feelings. These ideas are vague and imprecise but assume a quite spurious impression of accuracy when stated numerically.

For example, we may say, colloquially, that one objective is twice as important as another; we do not mean exactly that if one has a weight of 10 points the other should have a weight of 20, although this is the interpretation the numerical methods use. We are merely talking about a big difference in the relative importance of the objectives. To translate these subjective comments into precise numbers can create suspicion, depending on the numeracy of the users.

The best example of this "cult of numbers" is cost benefit analysis, where the costs and benefits of a proposed system change are compared directly by reducing everything to the lowest common denominator of financial value. The complex mathematics involved leaves most of the affected parties baffled, bewildered and alienated (Leitch, 1977). While it is reasonable and possible to forecast deaths, injuries and increased noise levels, etc., which will be caused by the construction of a new road or airport, to place monetary values on these emotive aspects is repugnant and unreal to many people. Cost benefit analysis, despite some success, has been inadequate for coping with the subjectively valued issues which affect people's lives.

This emphasis on numbers has another identifiable effect - upon the type of objectives which are selected as relevant to the problem. To compare the alternatives they must be measured according to scales derived from the objectives. In practice, this means that the only objectives which can be handled satisfactorily are those which are quantifiable, such as speed of service, CPU store, frequency of breakdown and cost. Those attributes of any system change which impinge directly upon the qualify of life of the people who are affected are often relegated to a "second division" simply because they cannot be easily measured and not because of lesser importance. In fact, their very subjectivity may indicate a deeper relevance. So, objectives involving good working relationships, job satisfaction, a pleasant environment and prestige, etc., are used to resolve the finer points of decisions made according to the quantifiable objectives, when in effect ignoring such qualities has led to the premature downfall of apparently impeccable system designs.

The precision which numerical estimates demands can cause distraction and time-wasting in the decision-making process. Subjective quantities are inherently imprecise and so it becomes pointless to try to place an accurate measurment upon them. The estimates are imprecise but this is often forgotten later when the subjective estimats are treated together with genuine objective measurements to produce an answer quoted to several decimal places. The Archimedean postulate in utility estimation is a prime culprit in this respect, since it requires very high discriminatory powers on the part of the decision-maker.

We have already examined the theory of multi-attribute utility from a practical perspective and found it lacking as a model of human behaviour. The above discussion is intended to demonstrate that multi-attribute utility theory is inadequate in practice, even as an approximation to human behaviour. The reason for this is the emphasis which it places on the precise numerical measurement of human opinions. This causes several undesirable effects, such as the financial measurement of emotive quantities, a selection effect on the types of objectives relevant to the problem and a dangerous and time-wasting over-precisiation of the subjective estimates.

SUMMARY AND CONCLUSIONS

In this chapter, we have reviewed the ideas of utility theory and discussed them in the light of normative and descriptive theories. The theoretical and practical difficulties of multi-attribute utility theory were considered.

At this stage we may conclude that any value theory should adopt only those axioms which are accepted as reasonable, normative standards of behaviour. These are:

1) that prefernces can be made
2) that preference is transitive, but not indifference.

Axioms involving lotteries are only appropriate in cases where the probabilities are objectively known, which is rarely, if ever, the case in real life. Multi-attribute utility theory acts as an approximation and has value as a means of structuring the decision-making process. However, it has disadvantages due to its emphasis on numbers. The two standards mentioned above are not enough to derive a unique, numerical utility, which may not be a bad thing.

A better theory of utility or value should incorporate the structuring ideas of multi-attribute utility theory, but avoid its time-wasting and distorting side-effects. An attempt to do so is presented in Chapters 7 and 8.

Imagine the following two decision situations - each involving a pair of gambles:

	Probability of winning	Amount to win
Situation X		
Gamble 1	100%	£1,000,000
Gamble 2	10%	£5,000,000
	89%	£1,000,000
	1%	£ 0
Situation Y		
Gamble 3	11%	£1,000,000
	89%	£ 0
Gamble 4	10%	£5,000,000
	90%	£ 0

Table 4.2 Choice Between Gambles

Allais' argument

I would choose Gamble 1 over Gamble 2 in Situation X and Gamble 4 over Gamble 3 in Situation Y. In Situation X, I have a choice between £1,000,000 for certain and a gamble where I might end up with nothing. Why gamble? The small probability of missing the chance of a lifetime seems very unattractive to me. In Situation Y, there is a good chance that I will end up with nothing no matter what I do. The chance of getting £5,000,000 is almost as good as getting £1,000,000, so I might as well go for the £5,000,000 and choose Gamble 4 over Gamble 3.

Savage's argument

One way in which Gambles 1,2,3 and 4 could be played is by means of a lottery. Suppose we had 100 numbered tickets in a bowl where 1 ticket would be selected at random to determine the outcome. The four gambles can thus be represented as in the table below. The payoffs are the amounts that would be won if a ticket whose number appears at the top of the column is drawn.

	Ticket Number		
	1	2-11	12-100
Situation X			
Gamble 1	£1,000,000	£1,000,000	£1,000,000
Gamble 2	£ 0	£5,000,000	£1,000,000
Situation Y			
Gamble 3	£1,000,000	£1,000,000	£ 0
Gamble 4	£ 0	£5,000,000	£ 0

Now, if one of the tickets numbered from 12 through 100 is drawn, it will not matter, in either situation, which gamble I choose. I therefore focus on the possibility that one of the tickets numbered 1-11 will be drawn, in which case Situations X and Y are exactly parallel. My decision in both situations depends on whether I would rather have an outright gift of £1,000,000 or gamble to win £5,000,000.

(a) If I prefer the gift of £1,000,000, I should choose Gamble 1 over Gamble 2 and Gamble 3 over Gamble 4.

(b) If I prefer the gamble for £5,000,000, I should choose Gamble 2 over Gamble 1 and Gamble 4 over Gamble 3.

No other pairs of choices are logical. I imagine that I would choose Gamble 1 over Gamble 3.

CHAPTER 5
FUZZY SET THEORY

In the previous chapters, we have argued against the use of numbers in the decision-making process. A new branch of mathematics is being developed which looks more closely at the human process of reasoning. This is fuzzy set theory and fuzzy logic. See the bibliography by Gaines & Kohout (1977) for references. In this chapter we shall examine the notion of fuzzy sets, its aims, usefulness, weaknesses and relevance to this problem.

EXPLANATION

To explain what is meant by fuzzy set theory is almost impossible without saying something of its purpose as well. However, we shall try.

With ordinary set theory, we name a set and decide which items are elements and which are not. Thus, we may define the set of men and can choose easily which people are members of this set and which are not. We may define subsets of the set of men, such as those men who are parents. However, we can define the set of parents separately, taking as its members parents of both sexes. The intersection of the set of parents with the set of men tells us which men are parents. The union of both these sets gives the set of parents and men.

This is straightforward enough, because it is usually easy to decide whether a given individual is male or is a parent. However, if the sets in question had been the sets of tall men or the set of kind parents, then we would have found it less easy to be completely sure about whether a given individual was a member of either set or not. The adjectives 'tall' and 'kind', although commonly and naturally used in natural language, do not lend themselves easily to precisiation, which is necessary if we need to make 'yes-no' replies.

Fuzzy set theory attempts to ease this dilemma by allowing items to be members of sets with something less than the firmness of the 'yes-no' decision. Ordinary set theory allows items to be members with a grade of membership of either 0 or 1. They are either members of the set or they are not. Fuzzy set theory allows items to be elements of the set with a grade of membership which can lie anywhere in the range 0 to 1. These grades of membership are defined subjectively, reflecting the individual's own opinion of the truth of the statement that "Item A is a member of the set AA".

As with ordinary sets, fuzzy sets can be subject to the laws of union, intersection, complementation, idempotence, etc., in such a way that ordinary sets appear as a special case of fuzzy sets. For further details see, e.g. Zadeh (1973,1977).

PURPOSE

In the above section, it was mentioned that the grades of membership were defined subjectively. This is a very important aspect of fuzzy set theory, and has been the source of its greatest strength and loudest controversy. In a seminal work on the subject, Zadeh (1973) has this to say about the purpose of fuzzy set theory:

> Essentially, our contention is that the conventional quantitative techniques of system analysis are intrinsically unsuited for dealing with humanistic systems or, for that matter, any system whose complexity is comparable to that of humanistic systems.... An alternative approach ... is based on the premise that the key elements in human thinking are not numbers, but labels of fuzzy sets, that is, classes of objects in which the transition from membership to non-membership is gradual rather than abrupt.

He seems to take the view here that human reasoning is too complex to be handled by conventional mathematics. This may be true. Certainly, conventional mathematics is not a good language in which to embody human reasoning, but I would guess that the reason is not just the complexity of the subject, but its nature as a language-based activity. If one does try to describe human reasoning using conventional mathematics, then the problem does become complex. Perhaps it is only complex because mathematics makes it so. For complex, non-human systems, fuzzy set theory does provide a new insight.

THE PROBLEM OF VAGUENESS AND FUZZY SET THEORY

One of the inherent difficulties with human reasoning is its vagueness, which has long been recognised by philosophers as a problem (Black, 1937; Russell, 1923). With statements involving vague propositions, it is very difficult to decide whether they are true or false. One of the more recent studies on this subject is that of Haack (1974). She distinguishes between uncertainty about the applicability of a predicate and imprecision thus:

(1) The qualifications for being F are imprecise.

(2) The qualifications for being F are precise, but there is difficulty in determining whether certain subjects satisfy them.

She proceeds to specify some of the ways in which the qualifications for being F may be imprecise:

(a) The qualifications are complex (in the form of an open conjunction, or conjunction of disjunctions) and it is indeterminate how many of the qualifications must be satisfied, and how the qualifications are to be weighted.

(b) The qualifications are complex, and in certain cases conflicting.

(c) The qualifications are simple (in the form of a single condition, or of a straightforward conjunction of all of whose conjuncts must be satisfied), but in certain cases it is indeterminate whether the condition, or one of the conditions, is satisfied. To avoid confusion with uncertainties of type (2), it is necessary to add that the indeterminacy about whether the qualifications are satisfied should not be due to any lack of information about the object in question.

Haack is concerned with the implications of vague sentences upon the use of classical logic. She argues that vague sentences do not require the step of replacing classical, bivalent logic with a logic of some other type. Her argument proceeds roughly as follows. For every vague predicate, there is a scientific predicate which may be substituted for it. Instead of stating that an apple is red, we may state the precise wavelengths of the light it reflects. However, in making this statement, we give ourselves the means of testing whether the statement is true or not. In most cases, the instruments will not be availble to test the validity of the statements, but in principle they are testable, and hence may be shown to be true or false. This replaces the uncertainty which had been of type (1) with another type of uncertainty, type (2), while preserving the bivalence of the logic. Her position is presented much more clearly than I have space to permit, and is beautifully argued.

While accepting the arguments she presents, I question the usefulness of such a procedure. Haack states that

> I admit that a legitimate aim of the construction of a formal calculus is to formalise arguments which occur in ordinary non-mathematical discourse. I only suggest that it may be necessary, and desirable, for the logician to tidy up - or, ..., to 'regiment' - this discourse.

Fuzzy set theory provides a logic which can cope with imprecision without requiring the formalisation of ordinary discourse, while sacrificing the bivalence of classic logic. It is possible for fuzzy logic to formalise the arguments which occur in ordinary, non-mathematical discourse, but perhaps without causing the regimentation Haack has in mind. Both arguments and logics are valid - it is a matter of choosing the logic to suit the purpose. I have already argued that the regimentation of human reasoning as part of decision-making should be avoided (Chapter 4).

The purpose of fuzzy set theory is to handle complex or humanistic systems. It is essentially practical, and must therefore represent a deviation from the classical bivalent logic. In 1904, Duhem stated that

> The laws of physics can acquire this minuteness of detail only by sacrificing some of the fixed and absolute certainty of common-sense laws. There is a sort of balance between

> precision and uncertainty: one cannot be increased except to the detriment of the other.

Compare this with a remark by Zadeh (1973):

> The essence of this principle (of incompatibility) is that as the complexity of the system increases, our ability to make precise and yet significant statements about its behaviour diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics.

The imprecision with which fuzzy set theory deals must be considered. Haack divided uncertainty into two types, due to imprecise qualifications, or precise qualifications which are difficult to determine if they are satisfied. Earlier (see Chapter 3) I distinguished between uncertainty due to the internal environment of the decision-maker and that due to the external environment and our incomplete knowledge of it. These two classifications of uncertainty are roughly similar, except that Haack's type (2) uncertainty should also include uncertainty due to the future, which she does not mention, although it seems consistent with her earlier remarks. Fuzzy set theory is meant to cope with the first type of uncertainty, due to imprecision, rather than lack of knowledge.

Fuzzy set theory has been applied to human reasoning, which is an imprecise process. The concepts with which humans reason are fuzzy sets in many cases, as Zadeh suggests. Vague statements may be only approximately true, but they can be manipulated to yield premises which are of value although their truth may still be only approximate. One of the results of fuzzy set theory application has been to show how imprecise human statements can be used to regulate machinery. These vague statements were translated onto numerical scales, using fuzzy set theory, and then built into a controller and successfully used. (See references in Chapter 6.) In this way fuzzy set theory can be used to convert pragmatic rules stated in language into a machine-readable form. The rules may be learned or acquired through experience by a human operator, and would otherwise be very difficult to implement in a precise fashion.

Critics of fuzzy set theory regard it as unnecessary that a special method should be devised to cope with this type of uncertainty, since probability theory could be used. The stochastic type of uncertainty is covered by probability and some attempts have been made to extend it to cover lack of certainty about mental phenomena, notably by Savage (1954). One could say that fuzzy set theory is preferable because it allows the use of words, but Savage requires people to use numbers in their estimation of probability. However, fuzzy sets eventually require conversion to numbers too, which may involve arbitrary decisions, although this problem may be avoided in the case of fuzzy logic. Fuzzy sets have been applied in this way to fuzzy controllers (see Chapter 6), requiring conversion of the sets of numbers, but the fuzzy set conversions may be tuned to give a better performance if they do not seem to work well at first. This tuning procedure is most obviously applicable to complex systems, but should be useful in human applications as well. The answers a fuzzy model predicts may not match up to those the decision-maker expects, and the reason may be either inconsistency on the behalf of the decision-maker or misrepresentation by the model of his feelings.

One of the defendants of fuzzy set theory as opposed to probability theory is Goguen (1969) who makes the distinction clearly as follows: "We are not concerned with the likelihood that a man is short, after many trials; we are concerned with the shortness of one observation".

It is still not certain that the problems which fuzzy set theory attempts to solve could not have been tackled by a modification of probability theory, but perhaps the most valuable contribution of fuzzy set theory to such problems is the novelty of its approach. Proponents of probability too can become embroiled in the precisiation of a vague and subjective phenomenon, and the fuzzy approach challenges this much at least.

In this section I have tried to show that imprecision is inescapable. If we want a practical logic, classical bivalent logic will not suffice, and so fuzzy logic may be adopted instead. Fuzzy logic was intended by its progenitor, Zadeh, to challenge the precisiation of human and complex systems, which he felt was reaching a threshold of relevance as the precisiation increased. He proposes that human reasoning is a vague process, using the labels of fuzzy sets as the material of reason. This attitude has been challenged by probability theory and others (e.g. Watanabe, 1978) who point to the eventual conversion from words to numbers, with its associated arbitrariness, as a flaw in the approach. However, I propose that these objections may indeed be valid, but because of the pragmatic spirit of fuzzy logic, they are not insurmountable.

FUZZY SET THEORY AND LANGUAGE

Zadeh set up fuzzy set theory to deal with complex systems and human reasoning. We shall consider the application of fuzzy set theory to the study of language, since we require a method which uses language, but in a way which is both precise and natural, as far as these divergent aims may be reconciled.

Any language is constructed of words and a grammar which defines the rules which the words must obey in relation to one another. In English, at least, the words which we use make up some well defined classes, such as nouns, verbs, adjectives, adverbs, prepositions, etc. According to Zadeh "the key elements in human thinking are ... labels of fuzzy sets". In this way a chair is the label of a whole range of objects ranging from the leather-bound, winged armchair to the modern steel and plastic kitchen chair to the most rudimentary of seats. 'Chair' is therefore a fuzzy concept, and is a subset of the set of 'Furniture', which in turn is a subset of 'Household articles'. There is a sort of infinite regression, each set is a subset of another, back to who knows what fundamental quantity. Because of this regression, in practical use, a universe of discourse is set up, which limits the extent of the regression.

Verbs are a different case, and I have not seen them treated in any discussion of fuzzy set theory, except in how they define relations between other fuzzy sets. For example, we may discuss the truth of a statement such as 'Pete lives near Palo Alto' without saying anything about the universe of discourse of which 'lives' is a subset. The statement may be subject to a truth-value, but the verb itself is not. We cannot modify the meaning of the sentence by operating on the verb, but only on the other parts of the sentence. Thus, we may say 'Pete lives very near Palo Alto' or 'Pete does not live near Palo Alto'. Since verbs are only modified directly by the word 'not' we could argue that verbs are non-fuzzy sets. However, this is only a suggestion.

Adjectives and adverbs are better examples of fuzzy sets. 'Tall' and 'quickly' are subjectively defined upon the universe of discourse of 'meters' and 'meters/sec' respectively, say. One person's notion of tallness may differ from another's, and along the range from 1.5m to 2.5m the truth value of a person of that height being described as tall will increase from 0 to 1 roughly. The meaning of adjectives and adverbs may be modified using hedges, such as 'very', 'fairly', 'somewhat', 'terribly', and many more. For a fairly comprehensive list of hedges in contemporary American usage, see Lakoff (1973). These modifiers or hedges are supposed to operate upon the original adjectives or adverbs in some way. The set of 'very tall men' will be a fuzzy subset of the set of 'tall men'. It is supposed that by operating upon the grades of membership of the set 'tall men', one would arrive at a set which is at least a good approximation to the grades of membership of the set of 'very tall men'.

Another type of word is prepositions, e.g. close to, beside, on top of. These prepositions are subjectively defined too. There is a range of distance along which one object may be more or less close to another. Many prepositions are imprecise. Even such which are apparently well defined as 'beneath' can be shown to be vague and circumstantial. If an elephant had a small bird perched on its back, would the elephant be described as being beneath the bird? The relative sizes of the two objects affect our description of their relation to one another beyond the usual definition of relative position. As the size of the upper item changes from much smaller to much larger than the lower, so the appropriate preposition would range from 'on top of' to 'beneath' as a means of describing the positioning of the two animals.

The next class of word which I wish to discuss is hedges and connectives. Because this is an important and relevant topic, I shall devote a new section to them.

Hedges

Hedges are usually taken as operators upon the fuzzy sets to which they refer. The type of operation depends upon the type of hedge and the use to which it is put, as well as depending to some extent upon the group using it. This is because a hedge can have different meanings depending upon who is using it. This all demonstrates that to lay down rigorous rules for the application of hedges is dangerous, because their meaning is so very imprecise.

Zadeh (1972) divides hedges into two categories:

<u>Type I</u> Hedges in this category can be represented as operators on a fuzzy set. Typical hedges in this category are: very, more or less, much, slightly, highly.

Type II Hedges in this category require a description of how they act on the components of the operand. Typical hedges in this category are: essentially, technically, actually, strictly, in a sense, practically, virtually, regular, etc.

The sort of operation which can be used to produce Type I hedges depends upon the context. Zadeh suggests the use of:

1) direct operation on the grades of membeship
2) support fuzzification
3) grade fuzzification

To this list we may also add

4) shift operations

The simplest method is probably the first, and usually involves a power operation on the grades of membership,

$$\text{e.g.} \quad \mu_{\text{very } x}(u) = \mu^2_x(u)$$

$$\mu_{\text{fairly } x}(u) = \mu^{1/2}_x(u) \quad , \quad u \in U$$

Since μ is in the range 0 to 1,

$$\text{very } x \subseteq x$$

$$x \subseteq \text{fairly } x$$

Before looking at the other Type I hedges, let us compare 'very' as a Type I hedge with 'essentially', the Type II hedge. In general, Type II hedges involve a fuzzy algorithmic definition, using Type I hedges. Zadeh's example is that 'decent' is a weighted combination of the components 'kind, 'honest', 'polite' and 'attractive'. The hedge 'essentially' has the effect of increasing the weights of the most important attributes and decreasing the weights of the least important. If a person has a high grade of membership in the hevily weighted components, their grade of membership within 'essentially decent' will be higher than within 'decent'. This means that 'essentially decent' is not a subset of 'decent' in the way that 'very tall' is a subset of 'tall'.

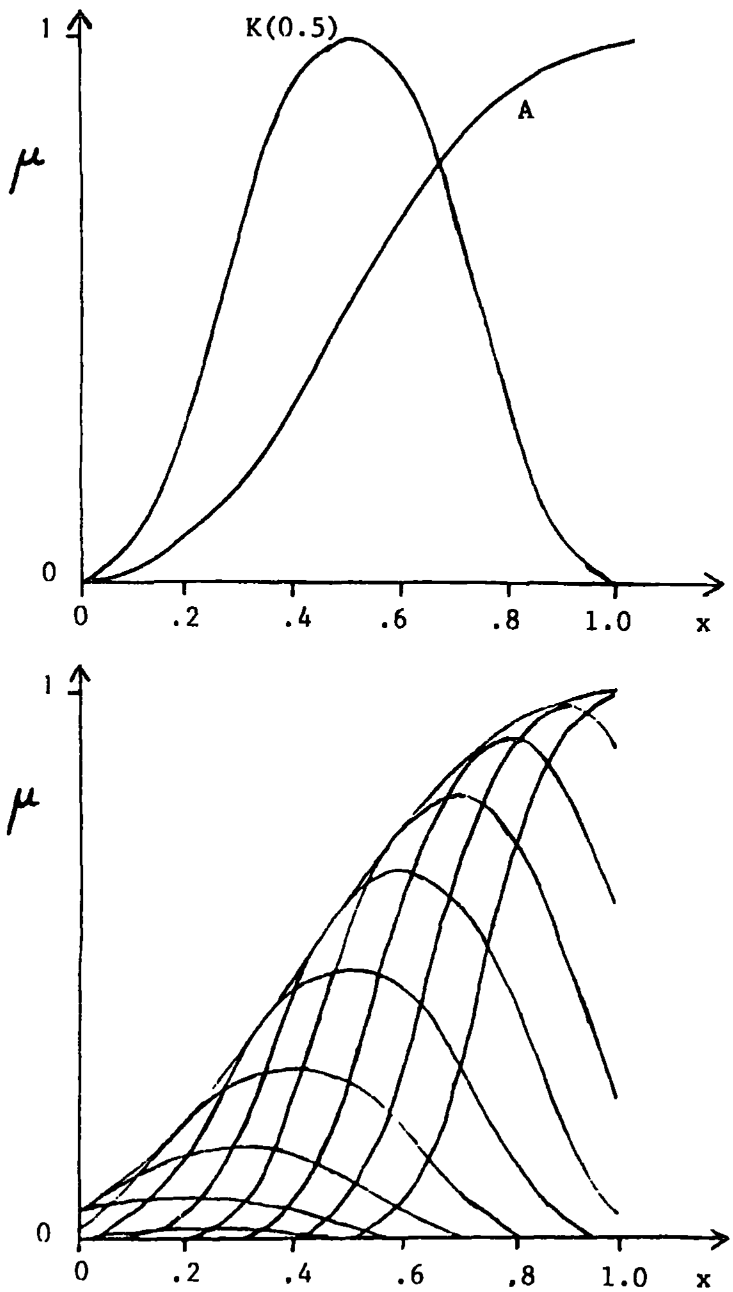
This sort of Type II procedure does not seem to have been widely used, possibly because it involves highly quantitative combination of subjective values, requiring accuracy of the grades of membership and weights of the components of 'decent'. The non-fuzzy operations on fuzzy quantities is undesirable, and a fuzzy algorithmic approach seems more promising.

To explain the meaning of support and grade fuzzification, see Figure 5.1. Zadeh does not suggest any hedges which might be examples of grade fuzzification hedges, but he suggests 'more or less' as an example of a support fuzzification hedge. This type of hedge is seldom seen in the literature, possibly because it too involves many arbitrary parameters. As can been seen from Figure 5.2, the choice of kernel set has to be made to match the hedge, otherwise unexpected effects are obtained.

Hersh and Caramazza (1976) carried out some psychological studies of the use of hedges, using a fuzzy approach. They performed a number of carefully controlled experiments on undergraduate students to test their use of the words 'large' and 'small', together with the hedges 'very' and 'not'. The stimuli used were slides depicting a black square on a white background. There were 12 different sizes, ranging from 4 to 48 inches along one side. The subjects were presented with answer sheets with a list of phrases in a random order. They had to decide whether a given phrase applied to the square that they had seen. The proportions of yes and no answers provided the grades of membership of the resulting fuzzy sets. Their main results were that the hedge 'very' "served to simply translate the function along the abscissa", i.e. was a shift operator. They found that the slope of the function did not increase after operation by 'very', as Zadeh's power model should predict. They present graphs comparing the experimental results of 'very large (y)' with 'large (y+2)' and the agreement is most convincing. The root mean square errors overall were 0.074, 0.214 and 0.061 for the best fitting power, exponential and shift functions, respectively. MacVicar-Wheelan (1978) also found that shift operators were a better model of hedges than Zadeh's power operators.

In one experiment, designed to test the consistency of the individual as opposed to averaging the performance of a group, Hersh and Caramazza found that one subject gave results which were different from those obtained from all the others, although this person still gave logical results. Whereas the other subjects perceived the operation of 'very' on the concept 'small' as defining successively smaller subsets, this subject interpreted the

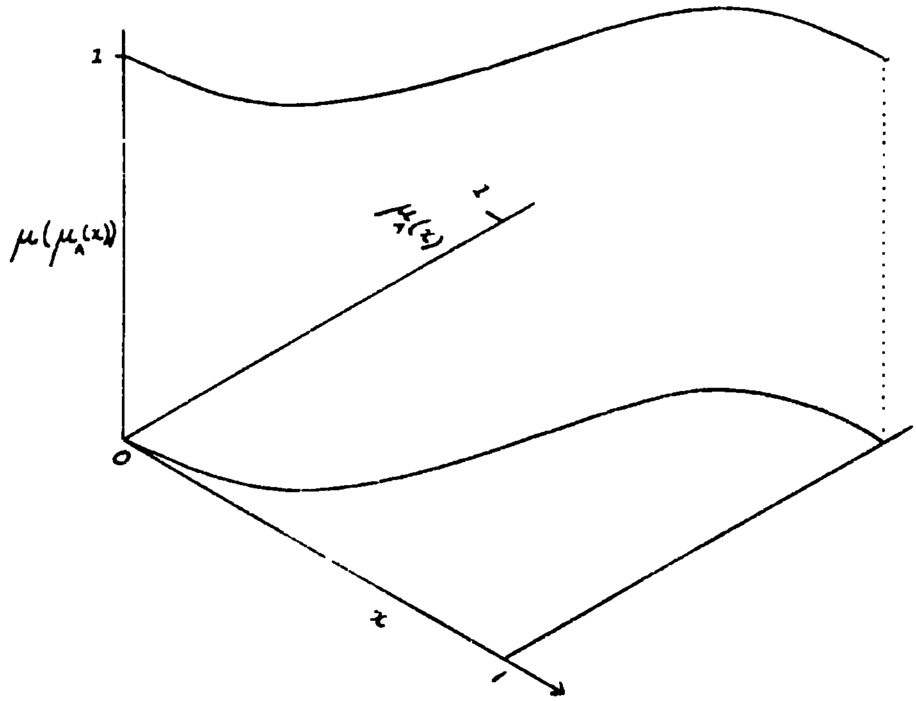
Figure 5.1(a) Support Fuzzification Hedges

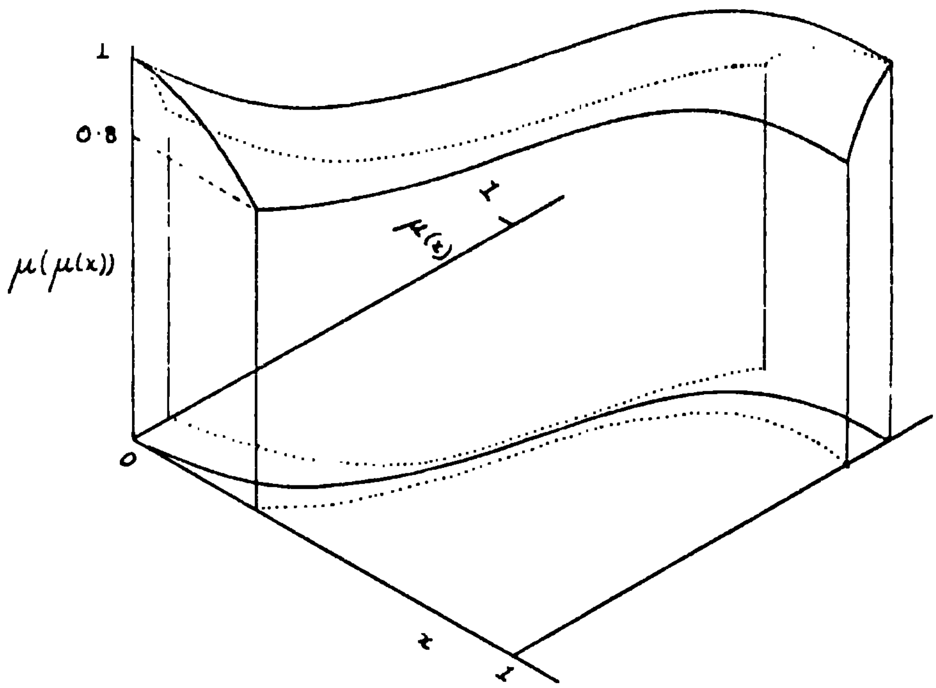


The set A is to be support fuzzified. The kernel set K(0.5) is shown. Each kernel set has the same shape and is normal. Each kernel set (K(x) is multiplied by $\mu_A(x)$. This produces a series of curves, and their envelope is SF(A,K).

$$SF(A;K) = \int_X \mu_A(x)K(x)$$

Figure 5.1(b) Grade Fuzzification Hedges

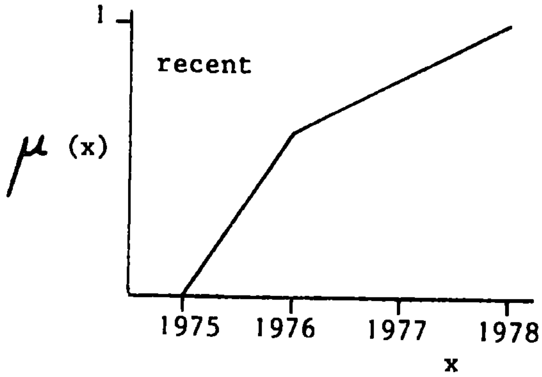




The set A is to be grade fuzzified. Ordinarily, the grade of membership is crisp, i.e. non-fuzzy. However, the grade of membership may be fuzzified, using kernel sets such as K(.6) = 1/.6 + .8/.5 + .8/.7. This is depicted above in three dimensions.

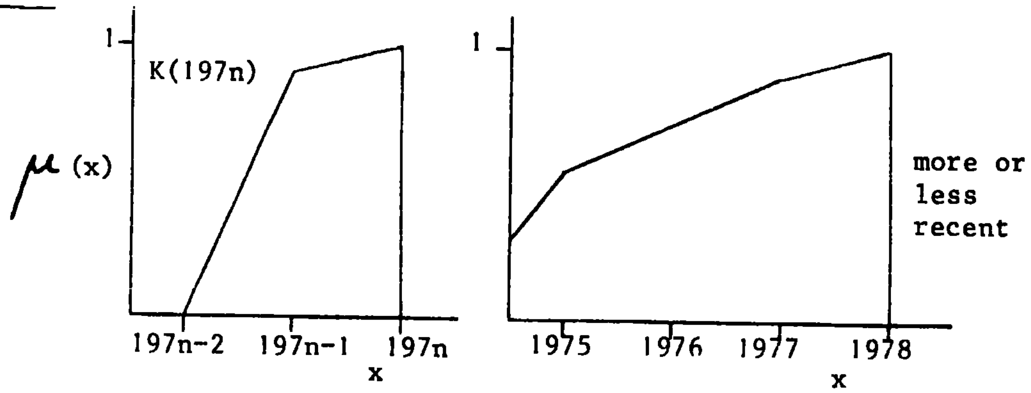
$$GF(A;K) = \int_x \tilde{\mu}_A(x)/2 \text{ where } \tilde{\mu}_A(x) = K(\mu_A(x))$$

Figure 5.2

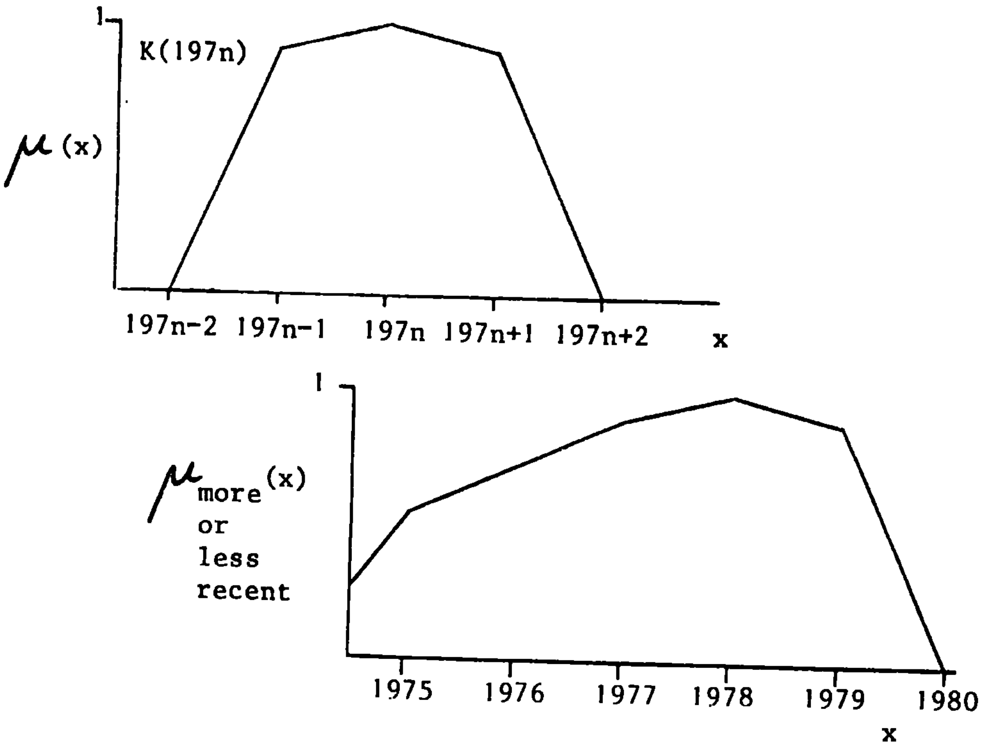


More or less recent = $\mu(1975)K(1975) + \ldots + \mu(1978)K(1978)$

Example 1



Example 2



phrases as fuzzy, overlapping categories. A square that is judged to have a maximum grade of membership in the set 'very very small' is considered to be a marginal (f=0.56) member of the set 'small'. The concept of entailment was not functioning for that subject.

Hersh and Caramazza also discuss the context dependent nature of the operator 'very'. They state:

> It would have been nicer if Zadeh's hypothesis of 'very' operating as a power function of the grade of membership was applicable. Although such a finding would have supported the generality of the operation of 'very', the form of the operation is an empirical question. 'Very' defined as a translation operation is obviously class dependent: it can certainly be generalised to other relative adjectives (e.g. good, short, hot). How such an intensifier would operate on the class of absolute adjectives (e.g. he is very British, this is very red) remains to be determined. Intuitively, it appears that the meaning of 'very' in 'very large' is qualitatively different from 'very' in 'very British'. The former implies an extreme of a continuum; the latter implies a greater emphasis on characteristic features.

This is a very interesting problem. Lakoff (1973) has considered it also and objects to the fact that the Zadeh definition of 'very' as a squaring operation would mean that the sets 'very tall', 'very very tall' and 'very very very tall', etc. all hit the value 1 at the same place as 'tall'. Lakoff suggests a shift of 'tall' followed by a concentration or power operation to produce the effect which he desires.

Lakoff also discusses the effects of 'very' when combined with other words. He considers cases such as 'very similar' and 'sort of similar'. Because things are similar with respect to many different attributes and to different degrees, then the assessment of similarity is very subjective. He states that there are two possible ways of deciding if a pair of ideas are 'very similar'. The first way is that the values assigned to the various criteria are closer, and in the second case the number of criteria is greater. 'Sort of' has the opposite effect when applied to 'similar'. He goes on to argue that when applied to 'strictly speaking', 'very' decreases the number of criteria which are to be taken into account. He says that the behaviour of 'very' may:

> be viewed as changing the weights assigned to various criteria at the upper end of the spectrum.

This is the sort of treatment which Zadeh proposed for the Type II hedge 'essentially'. Hersh and Caramazza had the same problems in mind when they pointed out the emphasis of characteristics in the phrase 'very British'. To describe something with an absolute adjective, such as 'British', 'pregnant', 'dead', etc. means that we can decide with a yes-no precision in most cases whether that adjective is appropriate or not. If someone is described as 'very pregnant', this takes into account other characteristics, such as girth and attitude. Lakoff says:

> There may not be a strict division between primary and secondary criteria; rather there may be a continuum of weighted criteria, with different hedges picking out different cut-off points in different situations.

If we consider the application of the hedge 'very' to common adjectives, there seem to be three classes of adjective in this context. These are:

1) absolute e.g. British, pregnant, dead
2) objective e.g. tall, quick, hot
2) subjective e.g. good, kind, happy

The first case has already been discussed a little. These are adjectives which may be applied if the subject fits the conditions laid down by some definition. The wording of the definition may change or be subject to debate, as with both 'dead' and 'British'. Since these conditions are precise, there should be no meaning associated with the phrase 'very dead', yet it does have a meaning in common usage. This is because there are certain secondary characteristics displayed by any individual which satisfies the definition, and when the word 'very' is applied, these secondary characteristics receive attention. The association of these secondary characteristics is only possible when the word is in common usage, and the characteristics are firmly recognised. If we look at an adjective which has a scientifically precise definition, such as 'nuclear', it has no secondary characteristics. We may talk about 'nuclear particles', 'nuclear forces', 'nuclear physics', etc. Place the word 'very' before any of these phrases and they immediately become meaningless. The only groups who could ascribe meaning to such statements would be perhaps enthusiastic undergraduates, or people working in the field, for whom secondary characteristics exist. Thus, 'very' can be applied to absolute adjectives, but only when they have acquired secondary characteristics which are modified or weighted by the word 'very'.

The second class of adjectives is those I have labelled objective, such as 'tall', 'quick' and 'hot'. The reason for this is that these adjectives may be replaced by scientific, measurable statements. 'This man is tall' could become 'This man is 6ft. high'. This is the sort of replacement which Haack advocated in her defence of classical logic. If we were to define a fuzzy set on a universe of discourse to describe 'tall', the obvious choice would be the range of heights from, say, 0ft. to 9ft. 'Tall' is measurable with one attribute only. The range of heights which is taken as the universe of discourse is divided up into fuzzy regions which represent 'tall', 'short', 'average', 'very tall', etc. Since these adjectives are measurable, there are not usually secondary attributes associated with them. A 'big' man is both 'tall' and 'fat', but tallness on its own is not usually associated with other characteristics, I would guess. It is true that all the studies of the psychological meaning of fuzzy adjectives, such as 'tall', 'large', etc. have tended to use adjectives from this class. Whether this is a coincidence since such adjectives are the only ones which provide a reasonable horizontal axis, or in recognition of the anomalous results which would have been obtained from other non-objective adjectives, I do not know. Zadeh, too, usually concentrates on objective adjectives, whose universe of discourse is known and measurable. For such adjectives, the application of the hedge 'very' is much simpler, since there are no secondary characteristics to complicate matters. The choice of a translation operator or a power operator seems to be a matter of choice. Given the undeniable imprecision of the topic, the practical difference is almost negligible.

The third class of adjectives is the subjective adjectives. These are adjectives such as 'good', 'kind', and 'happy'. Such adjectives are simple to apply, but upon inspection, extremely complex. To be 'good' requires the fulfilment of such conditions as 'kind', 'patient', 'loyal', 'honest' and many more. The choice of attributes is unique to each individual, together with the weighting allocated to each. The emphasis which 'very' causes means that some attributes are weighted more than others, but which attributes are affected, and in which way, is impossible to tell. It might be possible to ask a person to describe what features cause a person to be 'good' and what must be the additional features or change in weighting for a person to be 'very good'. If such a test were possible, it would be of doubtful value in a practical sense. That definition of 'good' would only be true for that one individual, and perhaps for quite a short length of time. It would seem that 'very' requires a fuzzy-algorithmic definition when it is applied to subjective adjectives.

Zadeh (1972) defined two types of hedges, those which act as operators on a fuzzy set and those which require a description of how they act upon the components of the operand. It would seem from the above discussion that 'very' is a hedge of the first type when it is applied to objective adjectives, and a hedge of the second type when it is applied to absolute or subjective adjectives.

In Haack's defence of classical logic, she argued for the replacement of vague statements with those which had a scientifically establishable precision. We can see that this would be possible, when dealing with statements involving absolute or objective adjectives. 'Fred is British' is not a vague statement. 'This tea is hot' may be replaced with a statement of the temperature of the tea. Whether I consider that this temperature justifies calling the tea hot is debatable, but the statement may be brought back within the realm of classical logic. 'John is happy' cannot be replaced with a scientifically measurable statement, and cannot be resolved to a yes-no answer, to everybody's satisfaction. It will not be absolutely true or false that 'John is happy' and opinion would disagree. The truth of such remarks is not absolutely determinable, i.e. true or false in every case, and independent of the observer. However, it should be possible for one person to decide on balance whether a group of such statements were each true or false. These statements could then be handled using classical logic. We must then ask if the answers which were deduced using bivalent logic were true. In other words, would classical logic produce false conclusions from 'true' premises? If the answer is 'No', then classical logic remains adequate. This argument brings us back to the Sorites paradox - the definition of a heap is vague, and when the logic constrains statements to yes-no truth values, then false or unacceptable conclusions may be obtained from true premises.

We may argue then for a deviant logic for two reasons, firstly that classical logic is impracticable when it means replacing statements with scientific statements which are not immediately verifiable. And secondly, that subject-referenced statements are not replaceable by scientific, object-referenced statements, and are therefore beyond classical logic. To constrain such statements to belong to the absolute class of adjectives can cause classical logic to produce false or unacceptable conclusions.

Hedges must be treated with great care. For absolute and subjective adjectives, 'very' can introduce changes in the attributes which are considered. For objective adjectives, the situation is probably simpler, although the assumption of ever decreasing subsets on further application of 'very' may not be true for everyone. There seems to be little to choose in practice between power operators for 'very' and shift operators. Although the discussion referred to adjectives and 'very', the conclusions would seem to be true for adverbs also and other common classes of hedge.

There is another important hedge whose special status requires careful study. This is the word 'not'. In classical logic, this is taken as the negation of its subject, but the common usage of this word is not so rigid as this. Negation is the central use of 'not' but in use its effect is not always as strong as the negation. Haack is discussing the ordinary language usage of 'and', 'not', 'if', etc. when she says

> The sense of the sentential connectives of classical propositional calculus fails to coincide exactly with that of their usual ordinary-language readings.

For example, in a recent report from the Stock Exchange, their opinions of the prospects of industry for the future were 'not unpromising'. The law of the excluded middle would state that this is equivalent to 'promising', but that is not what these cautious gentlemen meant.

Hersh and Caramazza (1976) examined the use of 'not' in their psychological studies. They plot graphs comparing the following pairs of phrases:

Small	Not Small
Large	Not Large
Very Small	Not Very Small
Very Large	Not Very Large
Very Very Small	Not Very Very Small
Very Very Large	Not Very Very Large

They conclude that

> The fact that the graphs indicate a reasonably good fit, and the fact that the average root mean square error over all positive-negative pairs was less than 0.07 supports the fuzzy set notion of negation as being the complement of the positive set.

There are two definitions of 'not' which appear in the literature, sometimes. The more common is the complement definition, and the other is the reverse definition:

complement: $$\mu_{Not\ A}(x) = 1 - \mu_A(x)$$

reverse: $$\mu_{Not\ A}(x) = \mu_A(1 - x)$$

Zadeh prefers the complement definition, but the reverse definition is more in the spirit of the shift operators, although it can only be applied when the universe of discourse is limited to the range $[0,1]$.

The law of the excluded middle implies that 'not true' is identical with 'false'. We may see how 'not true' and 'false' can have different meanings in the fuzzy logic. The definition of 'not', like many of the other definitions of fuzzy set theory, is probably best left open to the circumstances of its intended use.

Connectives

Connectives have been omitted from the discussion so far, but they are very important in the linguistic description of things. The most commonly cited connectives are 'and' and 'or'. They are usually represented in fuzzy set calculations by the operators 'min' and 'max' respectively, although Zadeh (1977) proposed alternative formulations, called the interactive 'and' and 'or'. These are supposed to be less harsh than the min and max versions, and involve multiplication of the grades of membership instead of strict comparison.

$$\mu_{a \text{ and } b}(x) = \min\{\mu_a(x), \mu_b(x)\}$$
$$\mu_{a \text{ and}^* b}(x) = \mu_a(x) \times \mu_b(x)$$

$$\mu_{a \text{ or } b}(x) = \max\{\mu_a(x), \mu_b(x)\}$$
$$\mu_{a \text{ or}^* b}(x) = \mu_a(x) + \mu_b(x) - \mu_a(x) \times \mu_b(x)$$

* denotes the interactive version.

Connectives do not seem to have received much attention in the literature, which is a pity because of their undeniable importance. Zadeh sets up a simple grammar to show how the connectives may be combined with the terminal adjectives and the hedges to produce sensible statements. These English statements may then be given a fuzzy set representation.

One objection to the min operator is that the fuzzy sets it produces are not always normal, i.e. the grade of membership never reaches the level unity. This seems counter-intuitive, because when an item is described in some manner, there must be some level where the description is true and should attain the level of unity. This problem is overcome by normalising the fuzzy sets after the min operation. Although this gives fuzzy sets more close to intuition, these sets are not so useful for calculations. The min and max operations cause a loss of information from the original sets, and normalisation increases this information loss still further.

The connective 'and' has some uses in fuzzy logic that would be excluded under classical logic. The law of the excluded middle states that it is impossible to state both P and not P whereas such a statement can be given a fuzzy meaning. Lakoff considers this problem for both connectives. He says that

> Incidentally, I consider it a virtue of this system that '$\neg P \vee P$' is not a tautology. Suppose 'P' is 'This wall is red'. Suppose the wall is pretty red, say, to degree 0.6. Then 'This wall is red or not red' will be true to degree 0.6 according to the given semantics. This seems to me to be within the range of plausibility. Certainly one would not want to say that the sentence was true in such a situation. Similarly, '$P \wedge \neg P$' is not a contradiction in the above system. And similarly, the

sentence 'This wall is red and not red' in the situation given where the wall is red to some extent seems to me not to be false, but rather to have a degree of truth.

It seems to be the case that the connectives which are used in natural language are not the same as the logical connective. This was noted by Haack (1974, p.119). Just as a fuzzy set represents something of our understanding of a word, so the logical connectives represent a core use of the linguistic connectives. Each operation upon the original fuzzy set produces another error. What is remarkable then is that the fuzzy sets of complex statements bear any resemblance to the meaning which the speaker wishes to express.

The interactive versions of the connectives represent one extreme, and the non-interactive are the other. The real meaning of the connective will depend on the context, and will probably lie somewhere between the two. An attempt to fuzzify the connectives has been made by Baldwin (1978). Throughout this work, the min and max versions have been used.

Truth Functional Modification

The statements which we have been considering so far have involved truth. Instead of adopting the bivalent truth values of classical logic, we have adopted the range of values between 0 and 1 to describe the imprecision associated with the subjective evaluation of truth. This is a practice not confined to fuzzy logic alone, but adopted too by multiple valued logic. The advance which fuzzy logic makes is in the use of verbal rather than numerical descriptions of truth. The truth values are linguistic, such as 'true', 'not true', 'very true', 'more or less true', etc. If we make a statement such as

John is very tall (1)

we would like to know how equivalent this is to

John is tall is very true (2)

A linguistic truth value has been placed on the statement 'John is tall'. A method has been proposed to convert from statements of Type (2) to statements such as (1). This is commonly known as Truth Functional Modification.

Zadeh (1977,1976b,1975) defines truth functional modification, or truth qualification as follows:

If $X \text{ is } F \text{ is } \tau$

then $X \text{ is } F^{+}$

where $\mu_{F^{+}}(u) = \mu_{\tau}(\mu_{F}(u))$

See Figure 5.3 for an explanation.

Semantic Entailment

This is a technique which has been employed in argument for a long time, but which has been given a new perspective by fuzzy set theory. It is generally assumed that statements such as

John is very tall

could not be made without it also being true that

John is tall.

It is said that the first statement semantically entails the second. We can see that the power law definitions of 'very' would be consistent with this remark, because 'very tall' is a subset of 'tall'. This is known as strong semantic entailment.

Weak semantic entailment occurs when statements such as

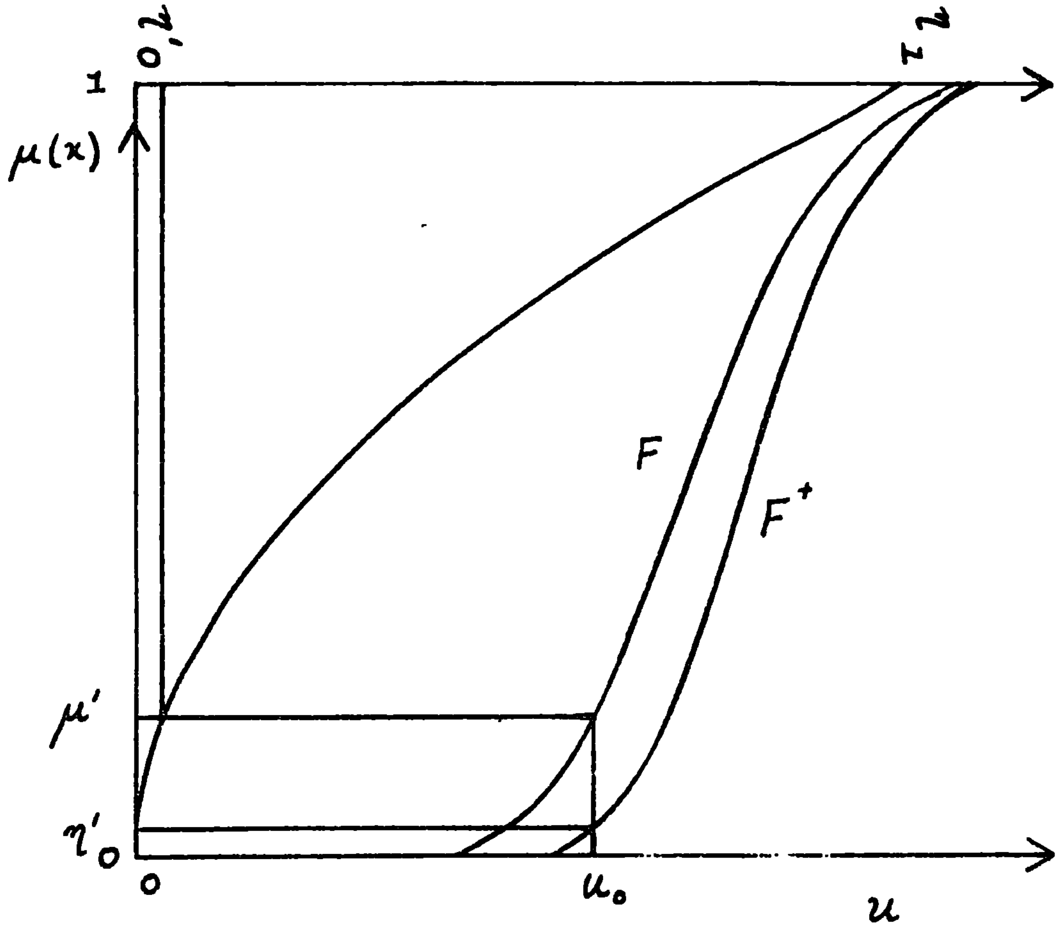
This box is not large

is said to semantically entail

This box is small.

In this case, the semantic entailment depends upon the definitions of the words 'large' and 'small'.

Semantic entailment might seem a fairly intuitively reasonable concept, but it is open to dispute (Gaines, 1976). One experimental study which showed that it does not always hold for every individual was that of Hersh and Carramazza. One of their subjects gave answers which deviated from the others'. For this person, semantic entailment did not hold. "The fact that a square is judged to be 'very, very large' does not entail that it is also 'large'."



$$\mu_{F^+}(u) = \mu_\tau(\mu_F(u))$$

From diag. $\mu_F(u_0) = \mu'$

$$\mu_\tau(\mu') = \eta'$$

$$\therefore \mu_{F^+}(u_0) = \eta'$$

Note that 'very true' is drawn with the horizontal axis running from 1 to 0.

Figure 5.3 Truth Functional Modification

Truth Functional Modification, Semantic Entailment and Hedges

Let us return to the concept of hedges and see how they will be affected by truth functional modification and semantic entailment. We mentioned earlier that Zadeh proposed a simple grammar to describe how statements involving hedges may be generated, without demonstrating it. The grammar he proposes is in Figure 5.4. With such a grammar, we can form phrases such as:

young
not young
young and not very young
young and not old
.....

But, we can also form less meaningful phrases:

young and very young
very young and not young
young and not young
very young and very very young

Phrases like these seem unnatural, because they involve semantic entailment, in the usual sense of defining successively smaller subsets of the original phrase. The statement

John is very young and young

provides no more information from a fuzzy set point of view than

John is very young.

The fuzzy set would be

$$F \subseteq F \wedge G$$

where

$$F = \{ \mu_{very\ young}(x), x \}$$
$$G = \{ \mu_{young}(x), x \} \quad , \quad x \in [0, 100]$$

We would need to exclude from the grammar such statements as

young and very young

but retain

young and not very young.

But do we include

```
T ———> A

T ———> A or T
.....................
A ———> B

A ———> A and B
.....................
B ———> C

B ———> not C
.....................
C ———> (T)

C ———> D

C ———> E
.....................
D ———> very D

E ———> very E

D ———> old

E ———> very E
```

<u>Figure 5.4</u> <u>Zadeh's Grammar</u>

S → B

S → B or S

B → C

B → D

B → C and D

B → D and C

D → F

D → not F

C → E

C → not E

E → Y

E → (S)

F → O

F → (S)

Y → very Y

Y → young

O → very O

O → old

E → Y

E → (S)

E → Y but not Y

F → O

F → (S)

F → O but not O

Figure 5.5 Extended Version of Zadeh's Grammar very young but not young

young but not young ?

The last two statements can be given fuzzy set representations but their meaning is debatable. Lakoff accepts 'red and not red' as a plausible statement. The phrase 'very young and not young' is more interesting. It denies the semantic entailment implicit in 'very young', but still has an intuitive feel about it, probably due to the introduction of secondary characteristics. Zadeh's grammar may be modified so that

1) we can say 'young but not young' or 'old but not old',
2) avoid 'young and very young',
3) allow 'very young and not young' and 'young and not young'.

See Figure 5.5.

Let us consider again the power of semantic entailment. If we say

John is very young

we may also state that

John is young is true.

In fuzzy set terms, 'Young' $\supset$ 'Very Young'

Now, it is also true that 'Fairly Young' $\supset$ 'Young'
where $\mu_{\text{fairly young}}(x) = \mu^{\frac{1}{2}}_{\text{young}}(x)$

But can we reasonably deduce that

John is fairly young

from the statement

John is young ?

Is it reasonable to deduce 'John is fairly young' from the statement 'John is very young'? It me, this does not seem correct, and there are two possible reasons. Either by 'fairly young' I mean something other than ' young', e.g.

$$\mu_{\text{fairly young}}(x) = \mu_{\text{young}}(x) \wedge 1 - \mu^2_{\text{young}}(x)$$

in which case

fairly young $\not\supset$ young

Or, it could be that one can only semantically entail from very^{n}x onto x, but not from x onto lower powers of x.

To represent this diagrammatically, given the statement

John is very, very young

we may say that every statement along the continuum from

very very young $\longrightarrow$ young

is also true.

See Figure 5.6. Note that 'fairly young' is to 'young' as 'young' is to 'very young'.

In "A Theory of Approximate Reasoning" (1977), Zadeh extends the notion of semantic entailment to incorporate something of his ideas of Truth Valuation. He says that there are two types of semantic entailment,strong and approximate, as mentioned earlier. He states that

very(X is F) $\longleftrightarrow$ X is very F   equ.(5.1)

very(X is F is τ) $\longleftrightarrow$ X is F is very τ equ.(5.2)

X is F is τ $\longleftrightarrow$ X is very F is $^2\tau$

where

$$^2\tau = \int_0^1 \mu_\tau(\eta) / \eta^2$$

$$\mu_{^2\tau}(\eta) = \mu_\tau(\sqrt{\eta})$$

Now, substituting τ for 'true' in equ.(5.2) gives

X is F is very true $\longleftrightarrow$ very(X is F is true)

Now, assuming

X is F is true $\longleftrightarrow$ X is F

we obtain

X is F is very true $\longleftrightarrow$ very(X is F)

and from equ.(5.1) this gives

X is F is very true $\longleftrightarrow$ X is very F

Truth functional modification also demonstrates this deduction but is only valid for a particular definition of 'true', which may demonstrate that semantic entailment is not so powerful as one would imagine.

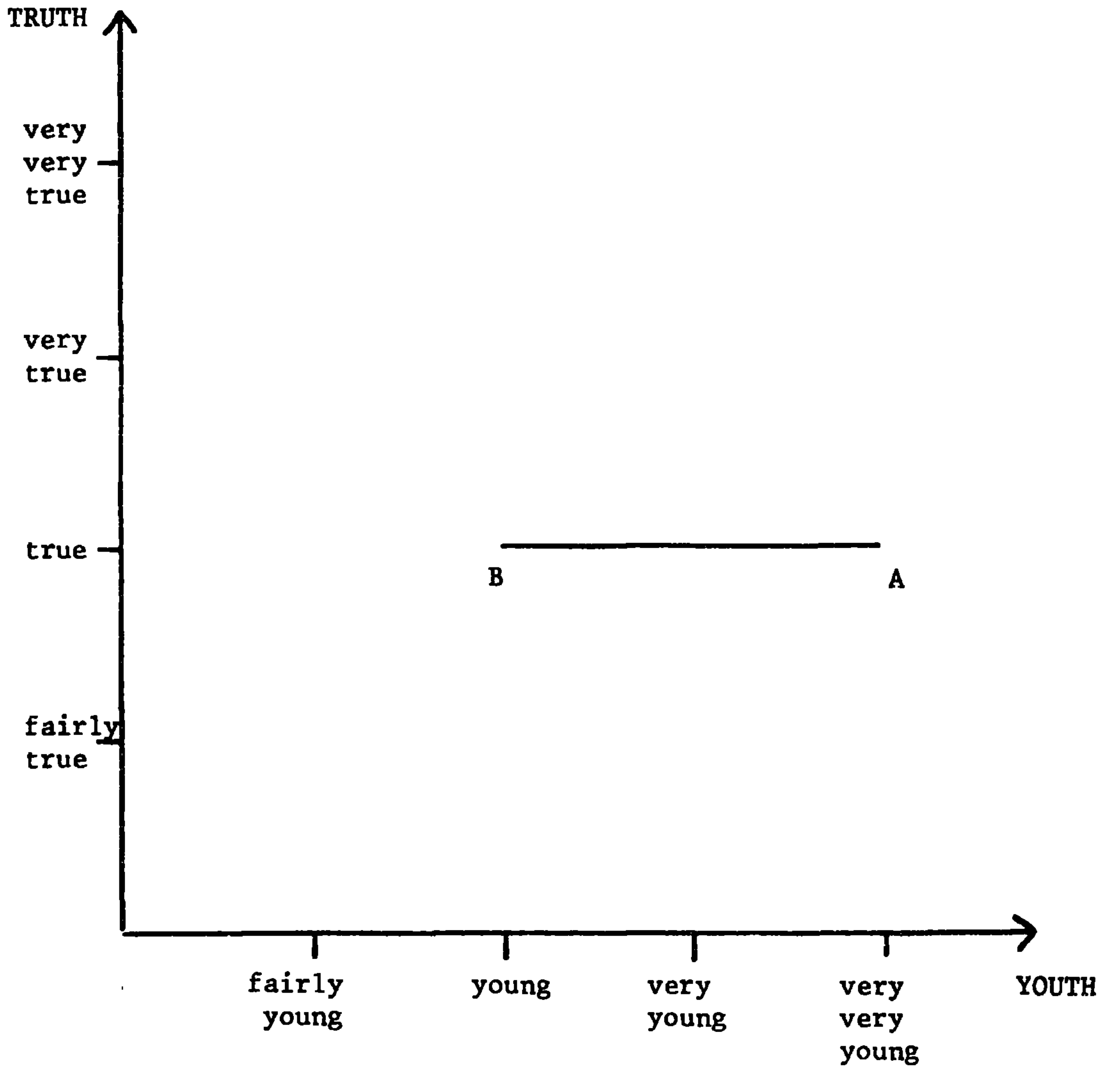


Figure 5.6 Semantic Entailment

Given point A, this entails the range up to point B.

When F and F^+ in Figure 5.3 are the same, we find a value of τ where τ must be 'true'. The only solution for 'true' which holds for all cases of F and F^+ is

$$\mu_\tau(\eta) = \eta$$

This is the unitary definition of 'true' and is the only definition of 'true' for which both semantic entailment and truth functional modification hold.

Using this definition of 'true', we may start with the statement

(John is very very young) is true

and deduce

(John is very young) is very true

(John is young) is very very true

(John is fairly young) is very very very true.

See Figure 5.7 for the graphic presentation.

The process of truth functional modification produces more and more restrictive truth values on the statement

John is m young

where m is some modifier, as m becomes less and less restrictive. Not only does truth functional modification allow extrapolation backwards to less restrictive values of John's age, it also permits extrapolation forwards, which is not allowed under semantic entailment.

Semantic entailment represents the most modest deductions which can be made about the truth of one statement, given that another statement is true. It only allows statements about whether a proposition is true or undetermined. Zadeh extends semantic entailment, allowing the use of modified truth values, other than 'true' and 'unrestricted'. However, this sort of semantic entailment does not seem so strong. If one approaches the same statements from a truth functional modification point of view;, only one definition of 'true' will permit Zadeh's deductions, which would otherwise appear to be valid.

Truth functional modification allows the calculation of F^+, given F and τ :

$$X \text{ is } F \text{ is } \tau \longrightarrow X \text{ is } F^+$$

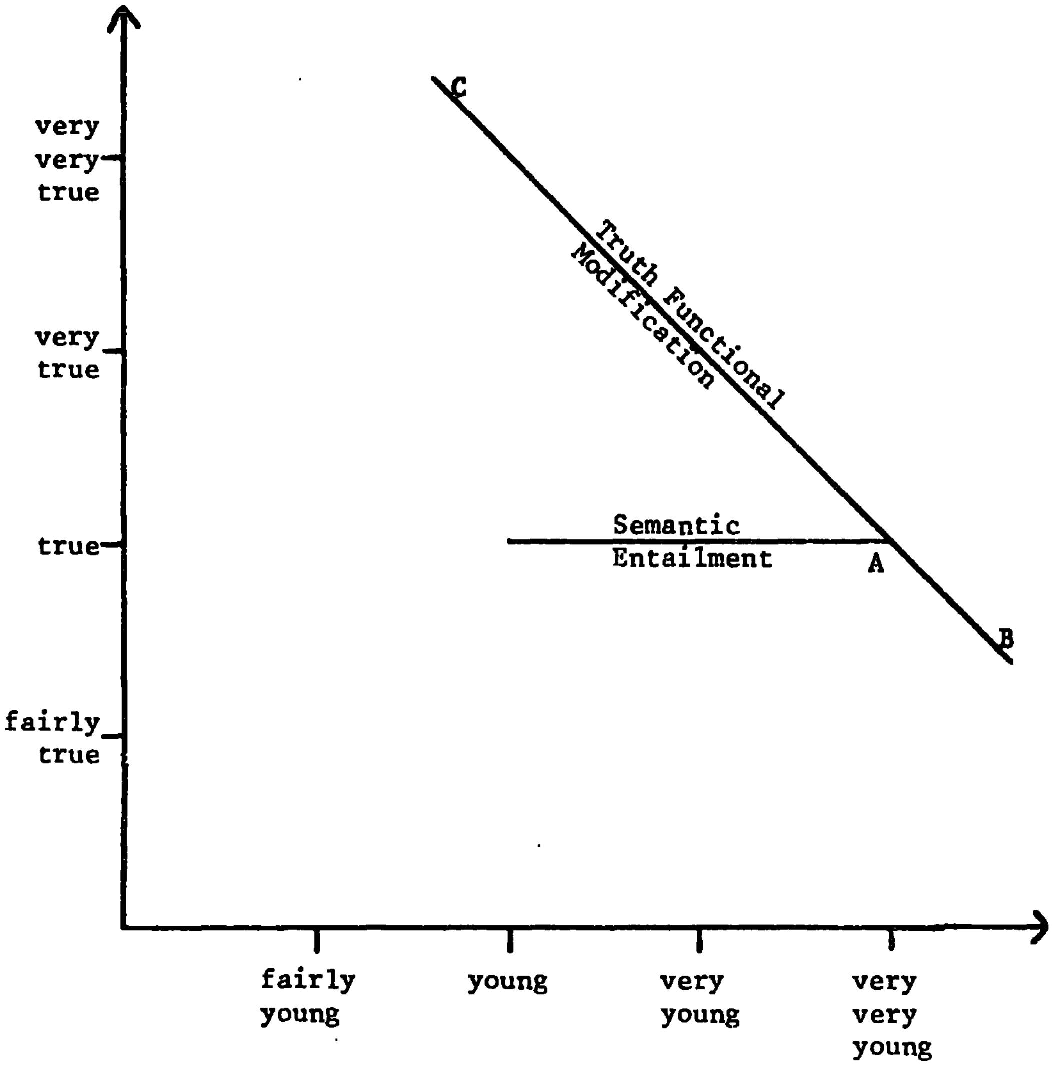


Figure 5.7 Semantic Entailment and Truth Functional Modification

Given point A, truth functional modification implies all points along BC extended.

The inverse process is to calculate τ , given F and F^+:

$$\tau = v(X \text{ is } F \mid X \text{ is } F^+)$$

where v denotes 'truth value'.

Baldwin (1978) suggests:

$$\mu_\tau(\eta) = \bigvee_{\substack{u \in U \\ \mu_F(u)=\eta}} \left[\mu_{F^+}(u)\right]; \qquad F, F^+ \subset U, \quad \eta \in [0,1]$$

This formula works well in most cases, yielding the following results:

v(X is F | X is very F) = very true
v(X is very F | X is F) = fairly true
v(X is very F | X is fairly F) = fairly fairly true

These results follow naturally from the definitions of 'very' and 'fairly' as power operators.

This formula relies upon there being an overlap of the fuzzy sets F and F^+. Had the 'very' operator involved a shift so that successive operations upon convex sets eventually produce non-overlapping sets, then

v(John is very very F | John is F) = impossible

where $\mu_{\text{impossible}}(\eta) = 0 \quad , \forall \eta \in [0,1]$

The formula tends to give anomalous results in some cases when the leftwards premise provides little restriction on truth values. For example,

v(X is uknknown | X is F^+) = absolutely true

where $\mu_{\text{unknown}}(x) = 1 \quad , \quad \forall\ x \in X$

and $\mu_{\text{absolutely true}}(\eta) = \begin{cases} 1 \text{ if } y = 1 \\ 0 \text{ otherwise} \end{cases}$

The opposite result is more reasonable:

$$v(X \text{ is } F \mid X \text{ is unknown}) = \text{unrestricted}$$

$$\text{where } \mu_{\text{unrestricted}}(\eta) = 1, \forall \eta \in [0,1]$$

However, inverse truth functional modification in this form is generally useful, so long as the premises are informative and do not involve shift operators. It is also subject to the same restrictions on applicability as truth functional modification.

Pairs of Statements

So far, we have been considering the validity of deductions which may be made given one statement. However, when we have more than one statement about a particular linguistic variable, different conclusions may be drawn. We shall consider the one adjective case at present, and discuss the two adjective case later.

<u>One adjective case</u>

When we obtain pairs of statements which describe the same item, there may be some inconsistency between them, in the truth functional modification sense. For example,

John is fairly young is true
John is young is very true

People may not perceive the truth of statements in a way which is consistent with truth functional modification and semantic equivalence. We shall consider the possible reasons for such inconsistency, and how it may be handled in practice.

The first and most obvious reason for lack of consistency is that the respondent has not properly understood what is required of him. Explaining the problem as carefully as possible to the respondents and trying to understand their difficulties is the only fair way to avoid this problem. One must be careful to avoid instructing the respondent so precisely that the answers they give are what they think the questioner wants, and not what the respondent actually thinks.

Assuming the respondent understands what he is supposed to do and produces statements which he feels to be correct, inconsistency may still appear. There are three possible reasons for this which I shall call doubt, ambivalence and uncertainty.

Doubt is an internal, mental phenomenon. One may not be clear about one's own exact contextual meaning of 'good' or 'true' when applied to the particular example in hand. To describe the library service as 'good' and one's mother's cooking as 'good' implies different things in either case. In both cases the library and one's mother produce a degree of contentment as a result of their actions and one's own consumption of the products. So, with an arbitrary, well-used word, one is trying to use it to be fairly precise about degrees of satisfaction.

We could suggest defining a new vocabulary, making up a new word to use, but the same prolems of reference exist as before.

The concept of what is good is only vaguely defined in one's mind at any time. When asked "What is good?", one can answer by either describing it as something which evokes a certain set of internal emotions or by pointing to examples of things which are good. By asking "Is X good?" and replying "Yes", the notion of what is good may have been extended from its previous meaning. The concept of good is capable of being stretched as a result of asking questions about whether something is good, when it might have never before been thought of in that light.

I would argue that when people give answers which might appear to be inconsistent, one reason could be that their concepts are being stretched. This inconsistency might disappear with practice as the vocabulary being used "firms up".

This is what I call doubt. The respondent is unsure of himself and trying to become accustomed to a more careful definition of his concepts.

The second reason, ambivalence, is related to the previous one of doubt. But this time, instead of the concepts behind the vocabulary being vague, the actual concept which is to be described is "spread out" because of the respondent's ambivalence towards it. To describe A as good may not be true, because of certain aspects of A which are not good. They may be only fairly good, or even very good. A single truth value may not seem descriptive enough, and rather than producing a composite truth value to describe his overall feelings, the respondent may be happier to produce apparently inconsistent statements. If we plot a respondent's replies, we find something like Figure 5.8. This depicts the spread of feeling about a concept which is only imprecisely or fuzzily formulated in the mind.

The third reason, uncertainty, is different from the previous two and recognises that the respondent may lack information on the subject about which he is being asked questions. In most cases, this cannot be avoided. The respondent may be asked to make forecasts or he may lack information about a present system. This means that he would not be able to make accurate statements which place tight restrictions on truth values.We could test for the three types of inconsistency as follows:

doubt is caused by concept stretching and lack of practice. It ought to disappear with practice, but if the vocabulary appears to be inadequate, it may be altered to use words which match the respondent's feelings better.

ambivalence is caused by mixed feelings about the questions and is indicated by answers which are inconsistent with truth functional modification. This is to be expected when asking questions about multiple aspect problems. However, if ambivalence appears when asking questions about what are meant to be single aspects, the presence of ambivalence could indicate either the wrong vocabulary or that two or more aspects are being considered by the respondent.

uncertainty When uncertainty occurs, the respondents usually say so. The information available is inadequate and so the answers they give are so unrestictive as to be meaningless. Before proceeding with the analysis, try to fill in any knowledge gaps. This may be a good way of distinguishing between spurious information hunts and those which are really necessary.

Two Adjective Case

When two or more adjectives are used to describe a linguistic variable, we may test for consistency between them.

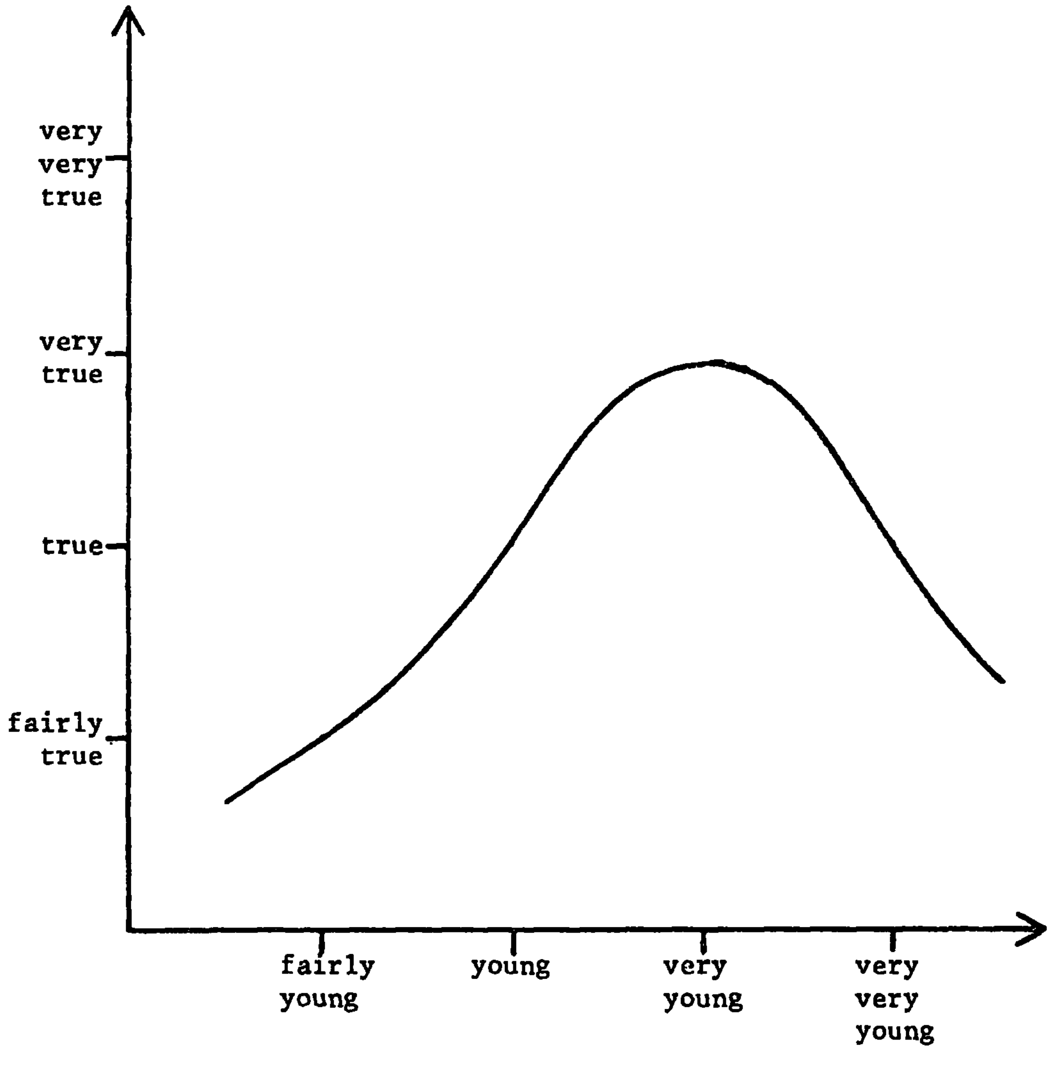


Figure 5.8 Ambivalence

The respondent's replies may appear inconsistent, according to semantic entailment and truth functional modification.

<u>Example 1</u>

u is reas is not true
u is dear is very true

$\Longrightarrow$ u is not reas
u is very dear

$\Longrightarrow$ not reas $\equiv$ very dear
$\Longrightarrow$ not not reas $\equiv$ not very dear
$\Longrightarrow$ reas $\equiv$ not very dear

For this simple example, the result is intuitively correct. The assumptions made were that the two statements provide the same information content on u.

With compound truth statements, involving connectives such as 'and' and 'or', we may not be able to make this assumption.

<u>Example 2</u>

u is nons is not true
u is sci is true <u>but</u> not very true

where nons stands for nonscientific
sci stands for scientific NB nons $\neq$ not sci
'<u>but</u>' is used in the same way as '<u>and</u>'

$\Longrightarrow$ u is not nons
u is sci <u>but</u> not very sci

$\Longrightarrow$ not nons $\equiv$ sci <u>but</u> not very sci
$\Longrightarrow$ nons $\equiv$ not sci <u>or</u> very sci

This result is counterintuitive, and arises because of the assumption of equal information in the two statements. It would appear, by inspection of the statements, that the item in question could be better described by the word 'sci' rather than 'nons', so that the phrase 'not nons' places only a fairly loose constraint on the value of u.

Let us deduce as follows:

u is not nons
u is sci <u>but</u> not very sci

$\Longrightarrow$ u is not nons <u>and</u> sci <u>but</u> not very sci or from Example 1

u is not reas and very dear

The connective 'and' is the min, $\wedge$, operator.

So, from Example 1, with the assumption on both statements having the same meaning,

u is not reas and very dear

$\equiv$ u is not reas

$\equiv$ u is very dear

However, under the 'and' operation, if the statement 'not nons' is more unrestricted than the other, we obtain

{not nons} and {sci but not very sci}

$\equiv$ sci but not very sci

So, we may say of the fuzzy sets,

$$\mu_{\text{not nons}}(x) \geqslant \mu_{\text{sci but not very sci}}(x)$$

$$\mu_{\text{not nons}}(x) \geqslant \mu_{\text{sci}}(x) \wedge \mu_{\text{not very sci}}(x)$$

$$\mu_{\text{nons}}(x) \leqslant \mu_{\text{not sci}}(x) \vee \mu_{\text{very sci}}(x)$$

or 'nons' $\subseteq$ 'not sci' $\cup$ 'very sci'

Compare with the previous result:

'nons' $\equiv$ 'not sci' $\cup$ 'very sci'

The equality sign only holds when both statements impose equal truth value restrictions on u. See Figure 5.9.

To place a greater restriction on the meaning of nons requires a further statement. This could be another description of a different item, or it could be a definition of one adjective in terms of the other. For example, we could take as a definition:

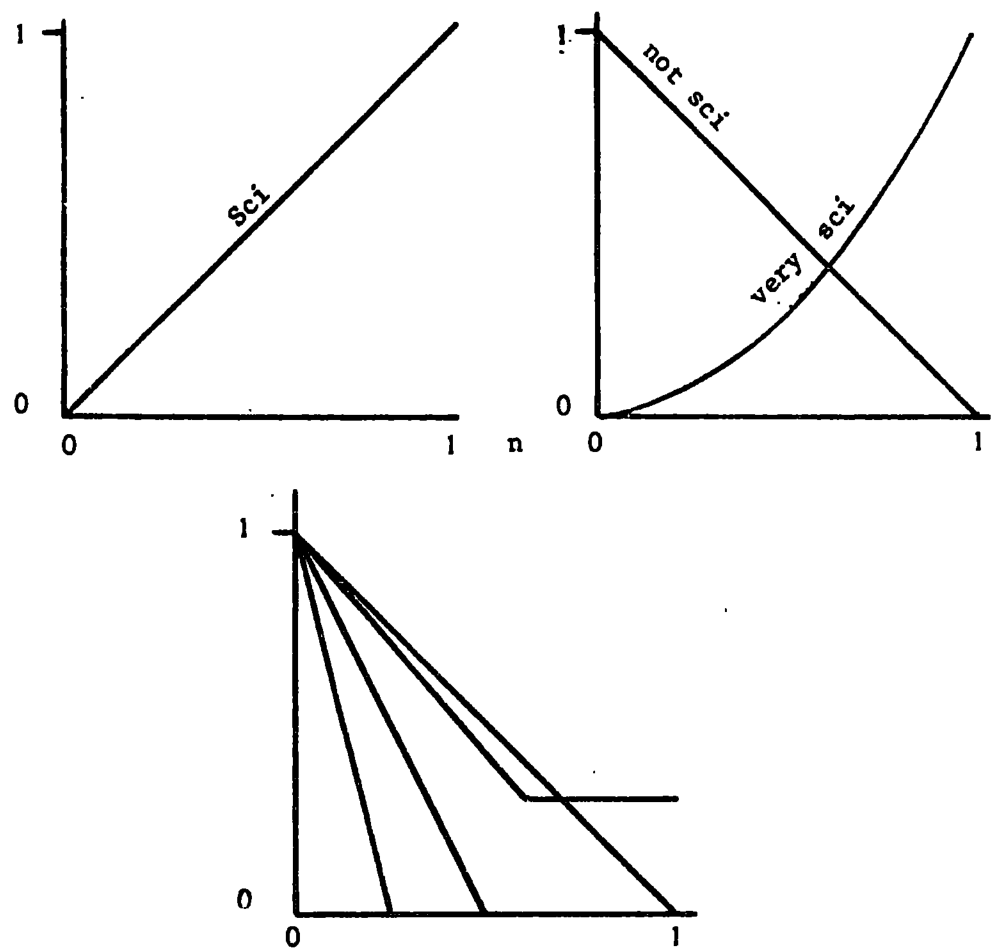


Figure 5.9 Possible Meanings for 'Nons'

Some are more intuitive than others

<u>Example 3</u>

sci $\triangleq$ not nons <u>and</u> not very not nons

and see if this is consistent with

nons $\subseteq$ not sci <u>or</u> very sci.

From definition

not sci or very sci

$$= \text{not } \{ \text{not nons} \cap \text{not very not nons} \}$$

$$\cup \quad \text{very } \{ \text{not nons} \cap \text{not very not nons} \}$$

$$= \{ \text{nons} \cup \text{very not nons} \}$$

$$\cup \{ \text{very not nons} \cap \text{very not very not nons} \}$$

$$= \{ (\text{nons} \cup \text{very not nons}) \cup \text{very not nons} \}$$

$$\cap \{ (\text{nons} \cup \text{very not nons}) \cup \text{very not very not nons} \}$$

by the Distributive Law

$$= \{ (\text{nons} \cup \text{very not nons}) \} \cap$$

$$\{ (\text{nons} \cup \text{very not nons})$$

$$\cup \quad \text{very not very not nons} \}$$

$$= \{ \text{nons} \cup \text{very not nons} \}$$ by the Absorption Law

which is consistent with

nons $\subseteq$ not sci <u>or</u> very sci.

This sort of logical argument isn't one that could be easily demonstrated to naive users.

Alternatively, suppose we take a description of another alternative. This alternative is less scientific than the previous one and is described by:

<u>Example 4</u> v is fairly nons

v is not sci

Let us say that 'fairly nons' is the more informative (truer) or accurate description, so that

$$\text{fairly nons} \subseteq \text{not sci} \qquad \text{equ.(5.3)}$$

As a result of the first alternative we could say

$$\text{nons} \subseteq \text{not sci or very sci}$$

and we can see that equ.(5.3) is consistent with this statement, given the usual definitions of 'fairly', etc.

<u>Example 5</u> Let us also consider the case

$$\text{not sci} \subseteq \text{fairly nons}$$

$$\Rightarrow \text{very not sci} \subseteq \text{nons}$$

and we recall that

$$\text{nons} \subseteq \text{not sci or very sci}$$

and since

$$\text{very not sci} \subseteq \text{not sci or very sci},$$

this constraint on 'nons' is consistent also.

Hence we could reasonably say that:

$$\text{fairly nons} \equiv \text{not sci}$$

$$\text{nons} \equiv \text{very not sci}$$

And we now have a definition of 'nons' in terms of 'sci'.

This work suggests that when two or more adjectives are used to describe the same linguistic variables, then we may test for consistency within the meanings of the adjectives. This is done by taking two or more of the respondent's statements and determining, from the respondent, which is the better description. This will place constraints on the meaning of one adjective with respect to another, and this may be tested in other pairs of statements for other alternatives. Or, the respondent may define one adjective in terms of another and again we may test for consistency.

Applicability of Truth Functional Modification

Let us consider Figure 5.8. The internal distribution of truth values does not obey that of either semantic entailment or truth functional modification. The reason why this may be so has already been suggested in the discussion on ambivalence. When the respondent is discussing a multiple-aspect problem, his ambivalence towards the many aspects will produce a spread in the truth values. It would be anover-simplification, therefore, to apply truth functional modification or semantic entailment to statements on multiple aspect problems.

Hence, it might be reasonable to state

(John is tall) is very true
(John is very tall) is true

but not

(John is good) is very true
(John is very good) is true

The personal quality of being good is a function of many traits, e.g. generosity, kindness, honesty, etc. It is not reasonable to deduce truth values for 'John is very good' given that 'John is good' is true, because the truth of 'John is good' is determined by John's character along the many other dimensions. But, when we are discussing simple quantities, such as height or age, which could be measured objectively, truth functional modification may apply. Note that this argument is very similar to that applied earlier in the chapter.

In order to handle multiple-aspect adjectives, we require a greater information input. This may be achieved by using two or more adjectives to assess a particular item, and testing for consistency between them in the manner outlined in the chapter. This method relies upon the existence of some abiguity in the respondent's statements to provide different amounts of information in the statements.

Therefore, we may conclude that truth functional modification can only be applied to statements about variabes which could be measured objectively, but where some sort of subjective aggregation of many aspects is involved, be they measurable or not, then truth functional modification in its simplest form does not apply, and pairs of statements are necessary to provide a workable method.

SUMMARY AND CONCLUSIONS

This section opened with a brief explanation of the ideas of fuzzy set theory. The purpose of fuzzy set theory was discussed and its relation to the problems of vagueness. Haack's defence of classical logic was reviewed and its impracticability demonstrated. The importance and necessity of vagueness and uncertainty, together with its inevitability, were taken as justification of a deviant logic.

Since we are interested in applying fuzzy set theory to verbal statements, the role of fuzzy set theory in their representation was considered, and the use of hedges and connectives. Three classes of adjectives were proposed to explain the anomalous behaviour and meaning of adjectival phrases.

The tools for handling such statements were discussed next, i.e. semantic entailment and truth functional modification. It was argued that with subjective or multiple-aspect adjectives, truth functional modification did not strictly hold. However, such adjectives will be encountered in any practical problem, so it was suggested that pairs of statements could be used to test for consistency and obtain more information on the quantity or quality involved.

CHAPTER 6
ATTEMPTS TO APPLY FUZZY THEORY TO MULTI-ATTRIBUTE DECISION-MAKING

In the past some workers have recognised the obvious usefulness of fuzzy set theory in the field of decision-making. Indeed, one of Zadeh's seminal works on the subject (1973) was entitled "Outline of a New Approach to the Analysis of Complex Systems and Decision Processes". There are many types of decision-making activity of which multi-attribute decision-making is only one. An important class is the class of decisions which must be repeated with time, as in the control of machinery or plant. The theory of such decisions has been considered by Bellman and Zadeh (1970) and others. Practical applications have also emerged, such as the control of traffic lights (Pappis and Mamdani, 1977), a hot water plant (Kickert and Lemke, 1976) and the celebtrated steam engine (Mamdani and Assilian, 1975; Mamdani, 1974). See Mamdani and Gaines (1976) for further examples.

The control of these machines has been fuzzified by adapting the pragmatic rules of the human controller and expressing them as rules upon fuzzy sets, giving a rule-based control process. With a few variables which may be observed, the human controller can derive a practical set of rules to adjust the input of the process so as to maintain as near a steady state of the output as possible.

This type of control process has many similarities to the approach proposed herein. The emphasis on rules supplied by the human link is the most obvious similarity. The main difference is that the decision in a control plant is repeated many times and the rules may be tuned to give the optimum response. With a once-made decision, there is no opportunity for improvement if it does not go well the first time. But the idea of setting up a rule-based procedure, taking measurements and passing them through the rules to arrive at a decision is strikingly similar. Real differences are hard to pin down but I can suggest four.

The first is that control decisions are repeated in time, and the output of one control decision will act as the input to the next. With multi-attribute decisions of the type considered herein, usually one decision only is made and is final and may be irreversible. This leads to the second difference. Because control decisions are repeated with time, the rules may be tuned as a result of experiment. Trial decisions may be made using multi-attribute techniques and may be inspected and compared with the intuitive decisions, but the rules may only be tuned through introspection. This in turn reflects the third difference, i.e. that although the rules in both cases are human-derived, the control rules apply to machines and the multi-attribute rules apply to the mind. This brings out the fourth difference, which is one of purpose. The purpose of control is to produce better control through better decisions. With multi-attribute, one-off decisions, the purpose is to produce better decisions through self-learning.

It is difficult to point to these applications of fuzzy set theory to decision processes and point to some definite difference between the two. The reason is probably because they are both applications of fuzzy set theory and it has left its mark of a similarity of approach upon them. Rather than being remarkable for their similarity, it would have been remarkable had they been different.

In this review of techniques, I shall confine myelf to those techniques which lay claim to model the goal- or objective-seeking behaviour of humans and organisations, although noting the relevance of fuzzy controllers to much of what is to follow.

MULTI-ATTRIBUTE DECISION-MAKING AND FUZZY SET THEORY

The traditional approach to this type of decision-making (see Chapter 2) involves setting the various goals and assigning to each a numerical weight, reflecting their relative importance to the decision-maker. If there are m goals, this list of weights may be viewed as a 1 x m matrix. Each alternative is then measured according to its ability to satisfy separately each of the m goals. These measurements or scores are taken along normalised scales, so that each alternative is described by m scores. If there are n alternatives, the table of scores constitutes a m x n matrix.

The next step is to multiply the scores by the corresponding weight for that goal and then add up the products of weight x score to obtain an overall rating for each alternative. Different permutations of this basic procedure have been suggested - novel methods of weighting, assorted scoring scales and a multitude of ways of aggregating the products. However, they have little to offer over the simplest method.

Expressed in matrix notation, this is:

Let there be m goals and n alternatives

Let W = matrix of weights, $1 \times m$

S = matrix of scores, $m \times n$

R = matrix of final ratings, $1 \times n$

Then, $W \cdot S = R$

The best alternative is taken to be the one with the highest final rating, and the decision is made. Under the matrix formalism, the desired alternative is that which has the highest entry in the matrix R. We shall use this technique as the starting point for this discussion.

In "Outline of a New Approach to the Analysis of Complex Systems and Decision Processes" (1973), Zadeh describes the compositional rule of inference and gives examples with fuzzy sets and relational matrices, similar in form to W, S and R. The matrices are combined according to the max-min rule, which is exactly similar to matrix multiplication, but with the 'x' operation replaced by 'min' and the '+' operation replaced by 'max'.

It is a simple step, then, to proceed from the matrices W and S with matrix multiplication and to call them fuzzy sets to be combined by the max-min rule. The idea of selecting the best alternative, i.e. that with the maximum overall rating, also tends to suggest fuzzy set theory, since the 'max' operator is so well-established therein.

In his Ph.D. thesis (1976), Pappis does just this. He considers n alternatives described by m attributes, and sets up an $m \times n$ matrix "R = r_{ij} , where r_{ij} is a number in the interval 0,1 indicating the extent to which alternative v_j is ascribed attribute u_i." With a fuzzy subset A of U, where U is the set of attributes,

$$A = \left\{ (u_i, a_i) \ / \ i = 1,\ldots,m \right\}$$

and a_i characterises the importance assigned to attribute u_i, he forms the fuzzy set B, such that

$$B = A \circ R$$

$$\text{i.e. } b_j = \max_i(a_i \wedge r_{ij})$$

He proceeds "Under this interpretation, alternative v_k will be chosen, corresponding to b_k such that

$$b_k = \max_j(\max_i(a_i \wedge r_{ij})) \quad "$$

He extends the idea a little further by introducing a set S of l objectives and the l x m matrix $P = p_{hi}$ defined on S x U, i.e. the objective - attributes space, so that p_{hi} describes how well attribute U_i fulfils objective S_h. This second relational matrix does not seem to advance multi-attribute decision-making very far and I fear that in practice he may have difficulty distinguishing clearly between how well an attribute satisfies an objective and how well an alternative is ascribed a certain attribute. Given an existing, well-known theory, fuzzy sets have been applied in an attempt to rejuvenate it without giving any serious consideration to the validity of use a 'min' rule in comparing weights and scores. The same problems would remain were this method put into practice, e.g. assigning accurate, realistic numerical weights and scores, but the language in which it is expressed is a little more elegant. To obtain a satisfactory model of decision-making, it is better to start off with "fuzzy thinking" and build your model from a few basic premises, rather than tacking fuzzy set theory onto an existing method.

One of the first attempts to do this was in 1970 by Bellman and Zadeh. They set up $X = \{x\}$ to be a "set of alternatives", and identify a goal G to be a fuzzy set on X. The use of the word 'alternatives' is perhaps misleading here, because they seem to be referring to what one would refer to as the complete decision space, and not just a subset of points within that space. Constraints are similarly defined, and a decision is the fuzzy set resulting from the intersection of G and C

$$D = G \cap C$$

or, for n goals and m constraints,

$$D = G_1 \cap G_2 \cap \dots \cap G_n \cap C_1 \cap C_2 \cap \dots \cap C_m$$

In order to obtain a single x from the set X, the x with the maximum grade of membership in D is selected.

the decision, not only that with the smallest degree of attainment.

They propose two alternative methods:

$$\mu_D(x) = \min_i \frac{\mu_i}{W_i}(x)$$

and

$$\mu_D(x) = \min_i \mu_i(x)^{W_i}$$

Since all the weights are in the interval $[0,1]$ some of the less important goals have values of μ_i / W_i greater than 1, but this "unaesthetic" effect is not present in the second method.

Nurminen and Paasio's techniques are to select the alternative x_{opt} such that

1) $$\mu_D(x_{opt}) = \max \min \frac{\mu_i(x)}{W_i}$$

2) $$\mu_D(x_{opt}) = \max \min \mu_i(x)^{W_i}$$

This is the maximin decision rule modified to include weights. Nurminen and Paasio do not provide any criteria for choosing between the two methods, and say that it may not be relevant to use the same weight parameters in both methods! With these rules an alternative is chosen according to its worst performance on any goal. There is no means of trading off between goals, simply that less important goals may be made to play a smaller role in the decision-making process.

Another piece of work which is derived from the Bellman and Zadeh paper, although borrowing from it much more heavily than Nurminen and Paasio, is that by Yager and Basson (1975). They too define goals and constraints as fuzzy sets on the space of alternatives, and a decision as the intersection of goals and constraints. They propose a weighting scheme the same as Nurminen and Paasio's exponential use of weights, with the same maximin rule. They also discuss the problems of constraints over different spaces and of constraints which are conditional upon each other, taking their definition from Bellman and Zadeh. The examples which they use to illustrate the discusssion seem to indicate a lack of appreciation of the meaning of fuzzy sets and the purpose of decision-making. Their "conditioned" sets could be expressed more easily using a fuzzy relation, and when discussing constraints over different spaces, the example fails to make the point. They comment on the subjectivity inherent in Bayesian Theory and Utility Theory, "two of the most useful tools in decision-making", and regret that "the subjectivity in the assignment of membership is one of the most important drawbacks of the fuzzy set approach to decision-making". (See my comments in Chapters 3 and 4.)

In more recent papers (1977,1978), Yager pursues his approach to decision-making a little further. He uses Saaty's technique of determining weights of goals. Pairs of goals are compared and a number representing the strength of one over the other is written down. A matrix is formed such that if a_{ij} of the matrix is

$$a_{ji} = \frac{1}{a_{ij}}$$

The maximum eigenvalue of the matrix is obtained and the elements of the corresponding eigenvector give the weights of the goals. Using the same definition of goals, and these weights as exponents, the intersection of the goals gives the decision.

In this paper, Yager mentions Zadeh's interactive and non-interactive 'and', with the interactive 'and' requiring that the product of goals and constraints should be taken. He fails to mention that in his example the decision would have been changed had the interactive been used rather than the non-interactive. He does comment on the two forms, that they "may represent in the oriental sense the Ying and Yang ... and all other rules lie between them". He leaves the choice of method to the decision-maker.

For a more mathematical treatment of this type of interpretation one may refer to Negoita and Ralescu (1974). They consider

1) the fuzzy decision

$$D(x) = max \{ G_i(x), C_j(x) \}$$

This emphasis of the similarity between goals and constraints is very neat, since in practice it is often difficult to distinguish between them. When does a goal become a constraint? Does the goal "Achieve a higher profit than last year" act as the constraint "Profit must not be lower than last year"? In this interpretation goals are defined as a fuzzy subset of the set of alternatives. In defining the grades of membership of G_i

$$G_i = \{ \mu_{G_i}(x) \,/\, x \} \qquad G_i \subset X$$

we have to score each alternative according to its ability to fulfil a particular objective, and the list of scores is given the name 'goal'.

In the case of equal weights, Bellman and Zadeh's intersection rule of goals and constraints is really the 'maximin' decision rule, i.e. select the alternative which has the highest minimum score on any attribute. See Chapter 3.

Since all the goals and constraints may not be of equal importance, some method of weighting the goals or constraints is needed. Bellman and Zadeh propose a convex combination:

$$\mu_D(x) = \sum_{i=1}^{n} \alpha_i(x)\,\mu_{G_i}(x) + \sum_{j=1}^{m} \beta_j(x)\,\mu_{C_j}(x)$$

where

$$\sum_{i=1}^{n} \alpha_i(x) + \sum_{j=1}^{m} \beta_j(x) = 1$$

We see that in order to tackle the problem of weighting goals and constraints, Bellman and Zadeh have had to fall back on the traditional weighted-sum rule.

Nurminen and Paasio (1976) start off with the same interpretations of the meaning of goals and decision and say they cannot agree with Bellman and Zadeh's use of the convex combination in the case of unequal weights because

1) the fuzzy measure is not additive,
2) in the case of equal weights the two methods are in contradiction,
3) in the convex combination all goals have an effect on

2) the product fuzzy decision

$$D_{pr}(x) = \prod_{i=1}^{n} G_i(x) \cdot \prod_{j=1}^{m} C_j(x)$$

3) the convex fuzzy decision

$$D_{co}(x) = \sum_{i=1}^{n} \alpha_i G_i(x) + \sum_{j=1}^{m} \beta_j C_j(x)$$

and point out that

$$D_{co}(x) \geqslant D(x) \geqslant D_{pr}(x)$$

contradicting Yager's interpretation of D(x) and $D_{pr}(x)$ as Ying and Yang.

The papers which have been mentioned so far have been based on the traditional weighted-sum method of decision-making. Fuzzy sets have been added on, with greater or lesser success, but apart from the occasional insight, little really has been gained.

A rather more original approach comes from Jain (1976a,1976b,1977). He considers a system with n states, governed by some parameter setting. There are m available alternatives from which one must be selected and each alternative-state pair has a utility associated with it. For a given state of the system, the problem is to select the one giving the highest utility in combination with that state. However, if either the knowledge about the system state is fuzzy, or the utilities associated with alternatives are fuzzy, or both, the problem is not so simple. The decision maker is faced with the problem of ranking the fuzzy sets which describe the utility of each alternative. He says (1976b)

> One may be tempted to make a decision either on the basis of the maximum utility associated with alternatives or on the basis of the utilities having the maximum grade of membership in the sets. However, both of these may lead to the selection of the improper alternative as the optimal alternative.... The balanced approach for the selection of the optimal alternative should consider both the maximum utility associated with various alternatives and the grade of membership of the utilities.

The problem is reduced to that of ranking fuzzy sets. Note that Jain has not attempted to obtain the utilities of the states, but assumes them as given. Hence, the emphasis is not on multi-attribute decision-making, but on one aspect, the ranking of fuzzy utilities. Jain's method is based on the notion of the 'maximising set' and the method is explained in Figure 6.1, using the example he uses in his paper, "Decision Making in the Presence of Fuzzy Variables" (1976b).

A more recent development of multi-attribute decision-making is that of Baldwin and Guild (1978b). They make good use of fuzzy set theory and set up a new approach to decision-making. They state that in this approach

> The rationale for making a choice is expressed in the form of an argument in fuzzy logic, consisting of a number of statements which connect propositions. This formalises the way that "goodness" in a choice is characterised. Any information available about the propositions is codified as truth value restrictions and then used to evaluate a fuzzy impression of "goodness".

A set of n independent statements are made, to correspond with n criteria of the decision. These statements are of the form

"IF (the alternative is an Effective choice)
THEN (the alternative satisfies Criterion 1)"

which is written as

$$E \supset C_i \quad , \quad i \in \{1, 2, \ldots, n\}$$

To account for different weightings of the criteria, truth functional modification is applied:

$$E \supset C_i \text{ is } \tau_i \quad \text{where } \tau_i \text{ is a truth value.}$$

In the example which they provide, the propositions are such as

(E (Space required is small)) is <u>true</u>
(E (Response time is short)) is <u>f true</u>
(E (Coordination is easy)) is <u>v true</u>

For each alternative, the criteria are measured and, using Inverse Truth Functional Modification, a fuzzy truth value is obtained for (Space required

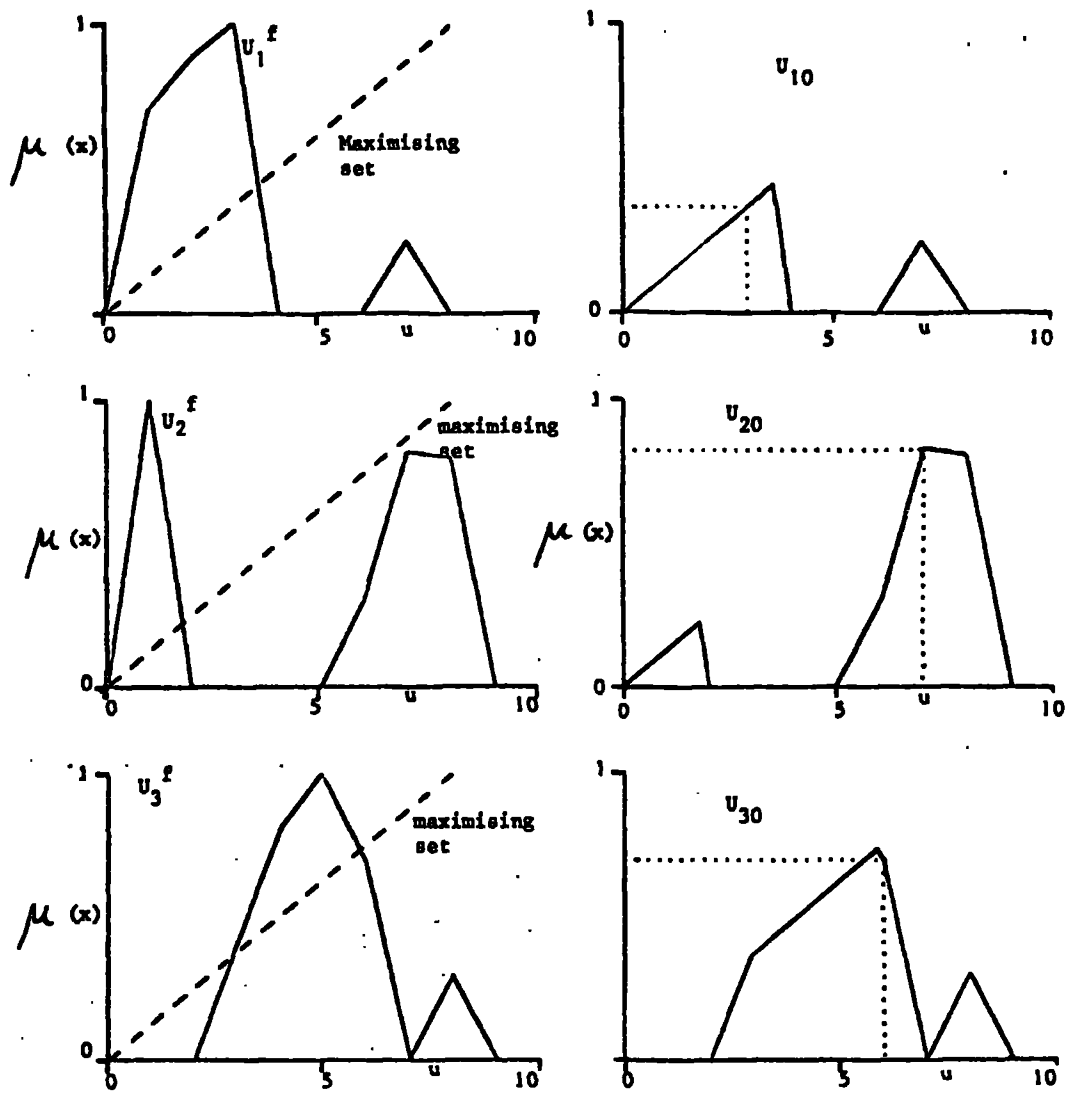
The fuzzy sets are:

$U_1^f = \{(.88,2),(1.0,3),(0.7,1),(0.3,7)\}$

$U_2^f = \{(.82,7),(0.8,8),(1.0,1),(0.3,6)\}$

$U_3^f = \{(0.4,3),(0.8,4),(1.0,5),(0.7,6),(0.3,8)\}$

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 0\}$



Select the maximising set $U_{im} = \{(fu_{im}(U_x), U_x)\}$

where $fu_{im}(U_x) = (U_x/U_{max})^n$ (n = 1 here)

$U_{max} = 8$

$U_{i0} = U_i^f \wedge U_{im}$

Select the maximum grade of membership of each U_i. This gives the grade of membership of each alternative in A_o, the fuzzy set of optimal alternatives.

$$A_o = \{(.375, a_1), (.82, a_2), (.7, a_3)\}$$

Hence, implement a_2 with the highest grade of membership in A_o.

Figure 6.1
Jain's method for ranking fuzzy sets

is small/Space required for Option A). Max-min composition of this value with the relation for Implication(true) results in v(E/Space required for Option A). By sequentially rejecting the alternatives with the most false truth value restrictions, one alternative is left. Where the difference between the truth value restrictions of alternatives is small, another method of choosing is used, although this relies quite heavily on the accuracy of the fuzzy set memberships.

There are two main objections which I suggest, although the method proposed is quite ingenious. Firstly, for the modus tollens argument to work satisfactorily, there must be no interaction between the criteria. This prevents any trade-offs between criteria and produces a fairly unrealistic model of human reasoning. For the criteria to be independent, the same conditions would have to be fulfilled as with numerical multi-attribute utility. There may be some cases when this might be so, but it will not in general be so. The second objection is that truth functional modification and inverse truth functional modification have been applied to non-objective criteria. I have discussed objections to this practice elsewhere (Chapter 5). There is also the problem of accuracy. The method relies upon accurate estimates of the importance of the criteria, the measurements for each option and the standards required for the criterion to satisfy the effectiveness criterion of "goodness".

Thus, although this method represents an advance in the approach to decision-making, it does not solve any of the problems. The criteria must still be objectively measurable and they must also be independent in their contribution to the overall effectiveness of the option.

Baas and Kwakernaak (1977) claim that "Fuzzy sets theory may be used to solve multiple-attribute decision problems under unertainty". They make two main assumptions, and compute a rank ordering of the alternatives according to a method of their own devising. While we do not wish to comment upon the mathematics of the ranking method, their two assumptions may be challenged. The assumptions are

> that all alternatives in the choice set can be characterised by a number of aspects, and that information is available to assign weights to these aspects and to construct a rating scheme for the various aspects of each alternative.

and

> The uncertainty that is assumed to be inherent in the assessment of the ratings and weights is accounted for by considering each of these variables as fuzzy quantities characterised by appropriate membership functions.

The uncertainty in decision-making comes from two sources - lack of knowledge about the future and the internal uncertainty of the decision-maker. These two sources of uncertainty may be labelled probabilistic and imprecise. Whereas most decision techniques concentrate upon a probabilistic treatment of uncertainty, ignoring imprecision, Baas and Kwakernaak have moved to the other extreme, favouring imprecision over probability. To claim that multiple-attribute problems can be solved under uncertainty, both these aspects must be accommodated.

Their assumption of constant (fuzzy) weights has been challenged in Chapter 4, and the criticisms mentioned therein apply to most of the other methods discussed in this chapter.

In more recent papers, Zadeh (1977) has adopted a different stance on the defining of preference relations over many alternatives. He has emphasised the use of linguistic statements of preference, such as

> The preference of I_1 for a_5 over a_3 is strong.
> The preference of I_1 for a_5 over a_4 is much stronger than the preference of I_1 for a_5 over a_3.
> If the preference of I_1 for a_5 is strong, then the preference of I_2 for a_5 over a_3 is very strong.

where I_i is an individual.

His paper, "Linguistic Characterisation of Preference Relations as a Basis for Choice in Social Systems" (1976a) describes the various forms of the statements of preference, but does not explore the idea much further, concentrating instead on the linguistic rules.

In a separate paper (1976b), he develops the idea of multi-criterial decision-making:

> when more than one criterion of performance is involved the trade-offs between the criteria are usually poorly defined. In such cases, the linguistic characterisations of trade-offs or preference relations provide a more realistic conceptual framework for decision analysis than the conventional methods employing binary-valued preference relations.

He gives a small example to explain the idea. With only two decision variables, it is easy to obtain the set of points which are preferred to each point. If the constraint set is C, then a point in C is undominated if its intersection with the set of dominating points is that point only i.e. is the singleton. This will be the Pareto-optimal set. Had the preference relation been fuzzy, the Pareto-optimal set would also be fuzzy. "In general, the extent to which the size of the Pareto-optimal set is reduced in this fashion depends on the linguistic information provided by the trade-offs."

Zadeh points out the difficulties of this sort of approach:

> In the first place, the preference relation ρ which results from translation of linguistic propositions ... is a fuzzy set of type 2 (i.e. has a fuzzy-set-valued membership function), which makes it more difficult to find the intersection of $D(u_0)$ (sic) with the constraint set as well as to compute the grade of membership of u^0 in the fuzzy set of Pareto-optimal solutions. Secondly, the preference relation represented by ρ may not be transitive, ... in which case it may be necessary to construct the transitive closure of ρ. And finally, it may not be a simple matter to apply linguistic approximation to $\mu\ p(u^0)$

where $\mu\ p(u^0)$ is the membership of the point u^0 in the Pareto-optimal set and u^0 is a point in the decision space.

CONCLUSIONS

In this section, we have considered some approaches to the problem of multi-attribute decision-making which have been inspired by fuzzy set theory, in some part. Some of the approaches failed to solve any of the problems and only used fuzzy set theory in an ad hoc manner, failing to take advantage of its power. More serious attempts have used fuzzy set theory in an imaginative and consistent manner, but still failed to overcome the fundamental difficulties of multi-attribute decision-making, i.e. non-constant, interactive weights, subjective attributes and the importance of uncertainty.

More modern methods have accepted that multi-attribute decision-making is a difficult problem, and that it cannot be solved facilely. Zadeh's most recent work on the subject, as well as the idea propounded herein, are adopting a different approach, looking at the preference itself, rather than the atomic criteria. Although fuzzy set theory may provide a better tool for the subject, the problem of multi-attribute decision-making will not be easy to solve. The pendulum swings from strictly numerical approaches to strictly fuzzy approaches, without much improvement, and is now swinging towards a more loose, fuzzy approach.

CHAPTER 7
A NEW APPROACH TO MULTI-ATTRIBUTE DECISION-MAKING

In this chapter we shall summarise the arguments for a new approach to multi-attribute decision-making, explain how we intend to use fuzzy set theory, explain and discuss the new approach and illustrate it with examples.

WHY IS A NEW APPROACH NECESSARY?

Previous approaches to multi-attribute decision-making have had some success but the general lack of application displays a mistrust of existing methods and the need for change. Existing methods, relying upon multi-attribute utility theory, place an emphasis upon measurement and precision, which has a number of identifiable effects.

The first effect is the removal of control over the decision-making process from the people involved, transferring it to the consultant or analyst, who may or may not be a member of the organisation. If a highly mathematical technique is being used, non-technical groups are automatically excluded and the analyst may have control over the inputs to the technique. For participative decision-making to work effectively, the decision technique should be simple to use and understand so that the participants can retain control.

The second effect of the numerical techniques is an emphasis on the objective aspects, which can be measured. Subjective aspects are nonetheless important too in determining the success or failure of an implemented decision. If the attributes are required to be measurable this can exclude the subjective attributes from the central decision-making process relegating them to a secondary role.

The third effect is concerned with the assessment of utilities for separate attributes. Ignoring for a moment the justification for dealing with single attributes, the assessment of utility often involves preferences between lotteries and the fulfilment of other inappropriate axioms of rationality.

When utilities are being assessed the conditions of the utility assessment may be unlike the problem in hand to such an extent that there may be actual distortion involved. This may be the case, particularly when Archimedean probability is compared to conditions of genuine uncertainty.

Fourthly, where uncertainty is involved, most methods tend to treat it using subjective probability and use methods involving mathematical expectation, for example. This fails to distinguish between the sources of uncertainty whether subjective or stochastic and may not follow the true method of assessing uncertainty which the decision-maker may employ.

Fifthly, where the measurement of an attribute is imprecise, for whatever reason, most methods tend to ignore or lose the imprecision of the measurement. When an index of merit is calculated for each alternative, the imprecision in this figure must be determined through a sensitivity analysis. The effect is sometimes to give a spurious impression of accuracy to the final answers, ignoring the imprecision of inputs.

Apart from relying upon a numerical approach, multi-attribute utility theory is difficult to apply because of the strict conditions which must be met to allow the convenient weighted sum methods to be used. The violation of the conditions (e.g. mutual preference indifference) is met in pracrtice, so the multi-attribute theory can only be used as a guide or approximation to decision-making. However, when this is compounded with the distorting effects mentioned earlier, real difficulties can arise.

However, applying a multi-attribute utility approach does have some advantages which arise mainly from the rigourous structuring of the decision-making process which is necessary for any numerical decision-making method to be applied at all.

An ideal decision-making aid should aim to contain the advantages of multi-attribute utility theory but avoid the disadvantages which numerical methods involve.

THE NEW TOOLS

In this new approach to decision-making we propose to use fuzzy set theory and fuzzy logic. The reason for this choice is that we believe the use of fuzzy sets can avoid some of the problems connected with precise measurement already discussed and can encourage a more pragmatic and realistic approach to the complicated problems of balancing many attributes.

We wish to use fuzzy set theory in two ways. The first is in the measurement of attributes. Where these are subjective, appropriate adjectives or adverbs may be used. Where the attribute is objectively measurable, e.g. price, size, etc., numbers may be used if required, otherwise adjectives may be substituted to take account of the "graininess" in the human perception of such variables, (see also Lakoff, 1973). In order to structure the use of such adjectives, we suggest the use of two or three basic adjectives, together with a simple grammar. This is to ensure that members of the same group attach roughly the similar meanings to words and enable the terms to be used in an orderly manner.

The second use of fuzzy set theory is in the assessment of overall utility. This too is described verbally where desired. The utility surface is described using heuristics or rules. Given any alternative, we may then determine its utility.

THE MODEL

We propose that the utility of a multi-attribute alternative may be described as a fuzzy set on a one-dimensional utility space. An alternative may be represented as a fuzzy region of a multi-dimensional decision space, where the dimensions are the relevant performance variables. The decision space maps onto the utility space under a relation which describes the decision-maker's algorithm for decision-making. The fuzzy set utilities may be compared for preference and indifference.

Each performance variable, D_i, is described using a set,

$$D_i = \left\{ d_i, \right\} \qquad d_i \in D_i$$

The decision space $\mathcal{D}$ is the Cartesian product of the D_i

$$\mathcal{D} = D_1 \times D_2 \times D_3 \times . . . \times D_n$$

where n is the number of performance variables. A point in the decision space may be described as an n-tuple $(d_1,d_2,d_3,...,d_n)$. If a point can be measured along every performance variable, it can be represented as a point in the space $\mathcal{D}$. Generally values of the performance variables are imprecisely measured so that an alternative A is a fuzzy region of $\mathcal{D}$, and may be presented as the Cartesian product of fuzzy subsets on the D_i.

$$A = \mathcal{F}_A(D_1) \times \mathcal{F}_A(D_2) \times ... \times \mathcal{F}_A(D_n)$$

where $\mathcal{F}_A(D_1)$ is a fuzzy measurement of alternative A along D_i. Every point in $\mathcal{D}$ can be given a grade of membership in alternative A.

The total or aggregate utility of apoint in the performance space $\mathcal{D}$ can be represented as a single scalar value or as a fuzzy set of values. Under the fuzzy representation the grades of membership at a particular point express the compatibility between that point and each utility value. The utility dimension may also be a set of adjectives (a universe of discourse), and may be chosen by the decision-m,aker himself. The mapping from the multi-dimensional space onto the utility surface, U, is called Φ , and is a fuzzy set of $\mathcal{D} \times$ U, the Cartesian product of $\mathcal{D}$ and U.

The relation Φ is obtained from the heuristic information provided by the decision-maker. It may be encoded in a table of values of utility for some of the n-tuples in $\mathcal{D}$. The missing points may be obtained by interpolation and checked by the decision-maker.

Once the Φ has been obtained it may be used to predict a utility for the given alternatives. Each alternative, A, is a fuzzy subset of $\mathcal{D}$, and its corresponding utility U_A, may be calculated. If the alternatives are represented by Type I fuzzy sets, then the max-min relation may be used:

$$U_A = A \circ \Phi$$

or

$$\mu_{U_A}(u) = \max_{d^{(n)} \in \mathcal{D}} \left\{ \min \left\{ \mu_\Phi (d^{(n)}, u), \mu_A (d^{(n)}) \right\} \right\}$$

This approach to determining a utility function omits the usual intermediary step of assessing the partial utility of each performance variable separately. The reasons for the undesirability of this practice have already been discussed. By trying to assess the 'utility' of separate dimensions, we lose information on the aggregation of these dimensions and so must impose some method, e.g. weighted addition. Thse subjective aggregation methods may be very complex and, at best, partial utilities can only supply a cross section through the overall utility surface. We suppose that for many cases,partial utilities cannot be correctly and operationally aggregated by general mathematical functions because the aggregation methods actually used are unique for each individiual and too complex to be satisfactorily described by an arbitrary comination rule. See Efstathiou and Rajkovic (1979).

EXAMPLE

In the following example, we shall consider the problem of buying a domestic toaster. Two methods of handling the fuzzy set calculations will be presented, although much of the preliminary analysis is common to both. The first method uses ordinary fuzzy sets with a max-min relation mapping them onto utility spae. The second uses Type-2 fuzzy sets and a crude interpolation. The relative merits of the two methods will be discussed at the end. See Efstathiou and Rajkovic (1980).

Preliminary Analysis

The problem is to choose a toaster, for own use, from a list of possible alternatives as found in Which? The attributes which we shall consider are:

1) price
2) depth of slot - should be able to take a slice from a standard loaf
3) evenness - not burnt in the middle and white round the edges
4) consistency - same result between batches at the same setting

The ideal toaster would be cheap, toast evenly and consistently and have a deep slot.

Each attribute is described using an appropriate vocabulary:

Price	-	dear, cheap
Depth	-	deep, not deep enough
Evenness	-	good, medium, bad
Consistency	-	good, medium, bad

We may write down statements to describe our opinions on the attributes:

1) Price is not very important. Since the item is intended for own use, it is worthwhile to pay for something to suit one's needs.

2) Consistency is less important than evenness, but they must both be at least medium.

3) The depth of the slot is important. There is no point in producing evenly done toast if 30mm of the bread stick out at the top.

The vocabularies are chosen to be as close as possible to the language which one would normally use to describe the attributes. Although some could have been measured along objective scales, such as mm or £, the way in which we perceive these attributes is not so finely graduated. The difference between 120 and 121mm is imperceptible in everyday usage, but the difference between 'deep' and 'not deep enough' is realistic and pertinent to the choice.

Other attributes which could have been considered are safety and durability, but since there seems to be nothing to choose amongst these alternatives on these attributes, they will not be considered. Appearance too could be a deciding factor, but will not be considered at this stage.

To help us draw up a complete table defining utility, we will look in more detail at the interaction between evenness and consistency.

price	depth	evenness	consistency	utility
cheap	deep	good	good	vv high
"	"	good	med.	high
"	"	med.	good	med. and low
"	"	med.	med.	low

The vocabulary to describe utility is:

$$U = (\text{high, medium, low})$$

together with the hedges 'fairly' and 'very', abbreviated to 'f' and 'v' respectively, where

$$\mu_{\text{fairly } a}(x) = \mu_a^{1/2}(x)$$

$$\mu_{\text{very } a}(x) = \mu_a^{2}(x)$$

The connective 'and', denoted by &, is used. On the fuzzy set calculations,

$$\mu_{med \,\&\, low}(x) = \frac{\min\{\mu_{med}(x), \mu_{low}(x)\}}{\max_x\{\min\{\mu_{med}(x), \mu_{low}(x)\}\}}$$

i.e. the sets are normalised after the 'min' operation.

A complete table may be drawn up to map from attribute space to utility space. Each of the sixteen n-tuples is assigned a utility value, consistent with the statements given earlier. See Table 7.1.

Alternatives

From the seventeen alternatives available, we select five for further investigation. The other twelve are removed because either the slot was too small, or the toaster performed poorly on either consistency or evenness or both.

The alternatives are presented in Table 7.2. The information on the alternatives has been presented using objective measurements, i.e. £ and mm. This is to show the common basis of the two methods, which require different types of fuzzy set.

Method 1

We shall define the fuzzy subsets 'dear' and 'cheap' on the £ scale and 'deep' and 'not deep enough' on the mm scale, as shown in Figure 7.1. Using these definitions, the alternatives are described in Table 7.3.

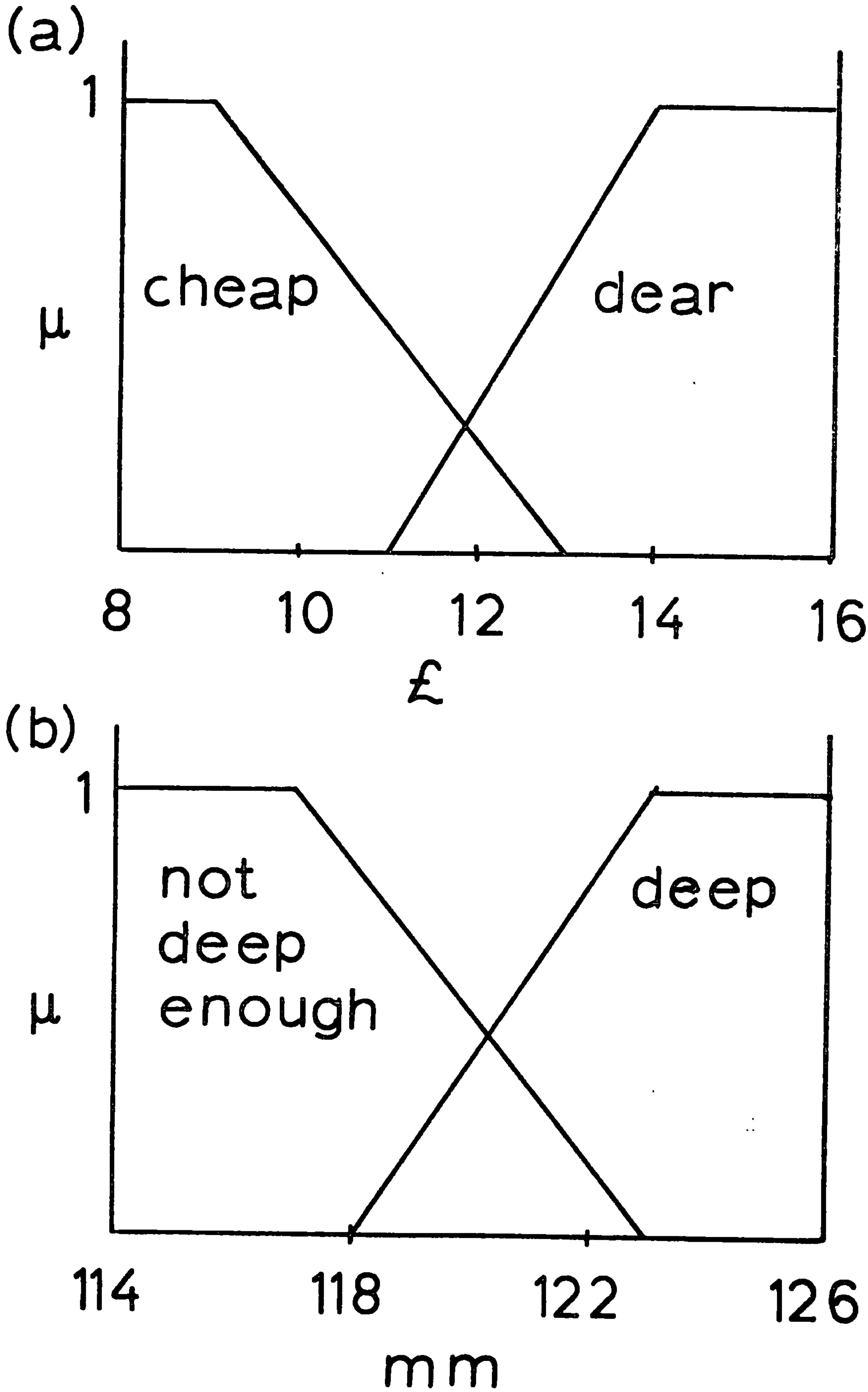


Figure 7.1 Definition of fuzzy set vocabularies on £ and mm scales

Price	Depth	Evenness	Consistency	Utility
cheap	deep	good	good	v v high
		good	med	v high
		med	good	high
		med	med	med & high
cheap	not deep enough	good	good	med
		good	med	med & f low
		med	good	low
		med	med	v low
dear	deep	good	good	v high
		good	med	high
		med	good	f high
		med	med	med
dear	not deep enough	good	good	med & low
		good	med	med & low
		med	good	v low
		med	med	v v low

Table 7.1
Utility of n-tuples

Alternative	Price £	Depth mm	Evenness	Consistency
1	14	125	good	good
2	9	121	med	good
3	12	112	good	med
4	13	120	med	good
5	12	120	med	med

Table 7.2
Assessment of the alternatives

Alternatives	
1	dear, deep, good, good
2	cheap, .6/deep, .3/nde, med, good
3	.33/dear, .25/cheap, nde, good, med
4	.67/dear, .4/deep, .5/nde, med, good
5	.33/dear, .25/cheap, .4/deep, .5/nde, med, good

Table 7.3
Type-1 Fuzzy Set descriptions of alternatives

Alternative	Price	Depth	Evenness	Consistency
1	very true/dear	true/deep	true/good	true/good
2	very true/cheap	true/deep	true/med	true/good
3	fairly true/dear	true/nde	true/good	true/med
4	true/dear	true/deep	true/med	true/good
5	fairly true/dear	true/deep	true/med	true/med

Table 7.4
Type-2 Fuzzy Set descriptions of alternatives

Using a max-min relation, the utilities would be:

U(Alt 1) = 1/ v high
U(Alt 2) = .6/ high + .3/ low
U(Alt 3) = .33/ med & low + .25/ med & f low
U(Alt 4) = .4/ f high + .5/ v low
U(Alt 5) = .33/med + .25/ med & low + .33/ v v low + .25/ v low

where + denotes the union.

With 'high', 'med' and 'low' defined on the interval $[0,1]$ as in Figure 7.2, we may represent these utilities as in Figure 7.3.

Ranking these sets presents a problem. Methods have been proposed to rank fuzzy sets, but are not so satisfactory with bi-modal sets, or sets with plateau maxima (Baldwin & Guild, 1978a).

Another point with pre-determined ranking methods is in the decision-maker's attitude towards risk. These fuzzy sets indicate that in many cases there may be some 'disutility' involved with some of the alternatives, and it should be up to the decision-maker himself to state how he wishes to trade-off utility and disutility. We may also remember that because of the inherent fuzziness, to try to rank the sets too accurately would be a mistake. Some sort of visual presentation, or a semi-ordering is as much as we can reasonably produce, at a first attempt.

Method 2
In this case we use Type-2 fuzzy sets (Mizumoto & Tanaka, 1976) to describe the alternatives. A Type-2 fuzzy set is one for which the grades of membership of the elements are in turn fuzzy subsets of another set. Instead of saying

The grade of membership of a in F is 0.9

one would say

The grade of membership of a in F is very high

where 'very high' is defined on the range $[0,1]$. We effectively replace the number in the range $[0,1]$ with a verbal truth table. The alternatives can

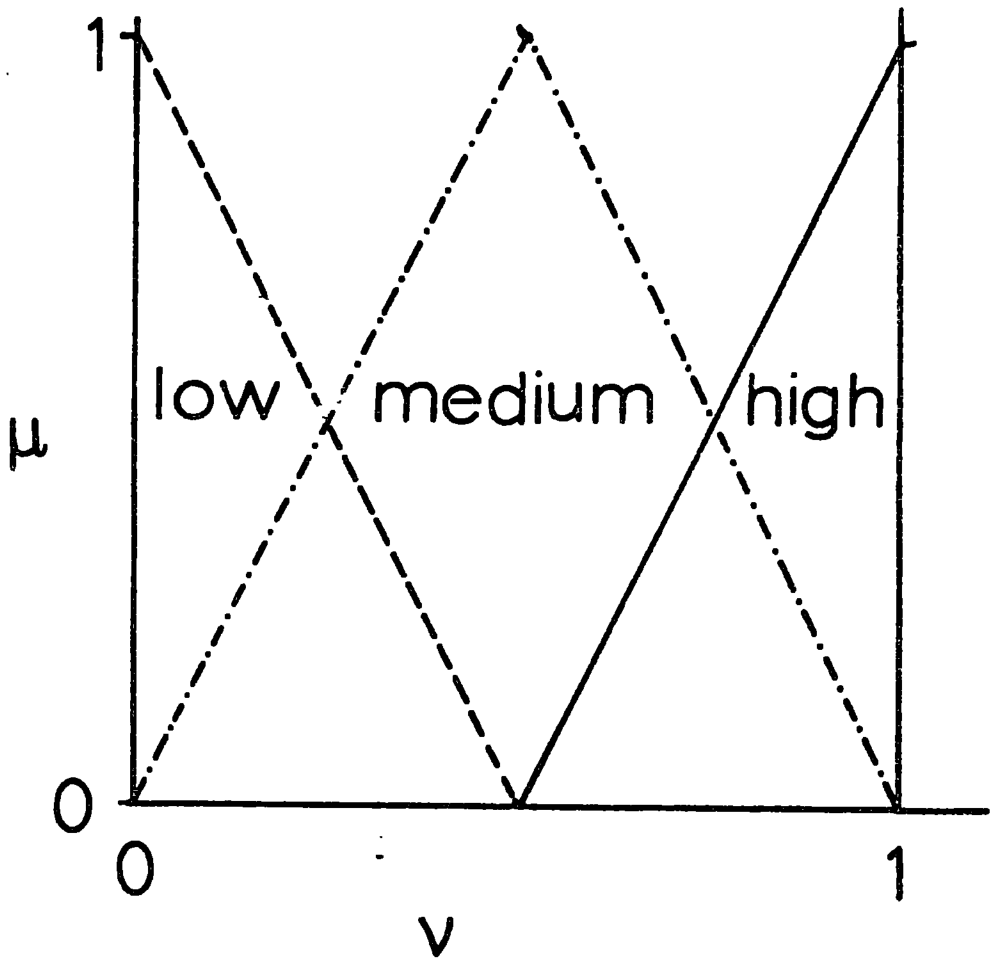


Figure 7.2 Definition of utility vocabulary on [0,1]

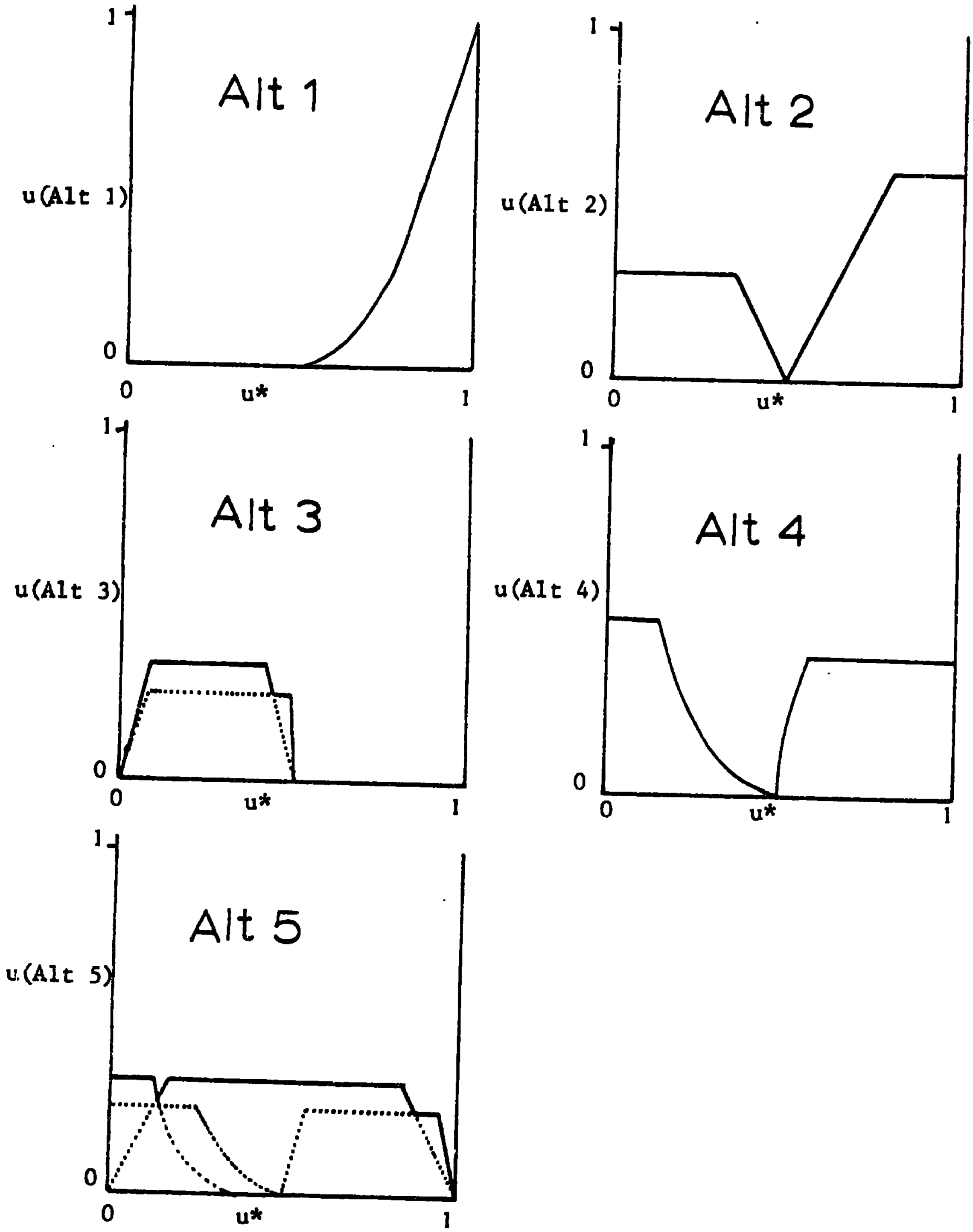


Figure 7.3 Utilities obtained using method I

now be presented as in Table 7.4.

Note that for Alternative 1 the grade of membership of 'dear' is 'very true'. Table 1 only provides values of 'true' for 'cheap' and 'dear'.

true/cheap	v v high
true/dear	v high
very true/dear	?

Some means of extrapolating from the points given to those we wish to use is necessary. Truth functional modification (Zadeh, 1977) by itself may not be adequate because the relationship between price and utility may not be quite linear, as truth functional modification supposes. The only reasonable solution is to follow the spirit of the statements in the preliminary analysis and adopt an ad hoc answer. This is probably as good as one can expect. It would seem fair to substitute 'high' for the question mark in the table above. Similarly, for Alternative 2, the answer would be 'v high'.

For Alternatives 3 and 5 the truth values are 'fairly true' for 'dear'. In this case, this must lie between 'true/cheap' and 'true/dear', so for Alternative 3 we could say

true/dear	med & low
true/cheap	med & f low
fairly true/dear	med & low & not(med & f low)

Hence we obtain utilities as follows:

U(Alt 1) = high
U(Alt 2) = v high
U(Alt 3) = med & f low and not(med & low)
U(Alt 4) = f high
U(Alt 5) = med and not(med & high)

This method offers the advantage of expressing the answer directly as English statements, but using Figure 7.2 we may also produce the fuzzy sets to compare with Method 1, see Figure 7.4.

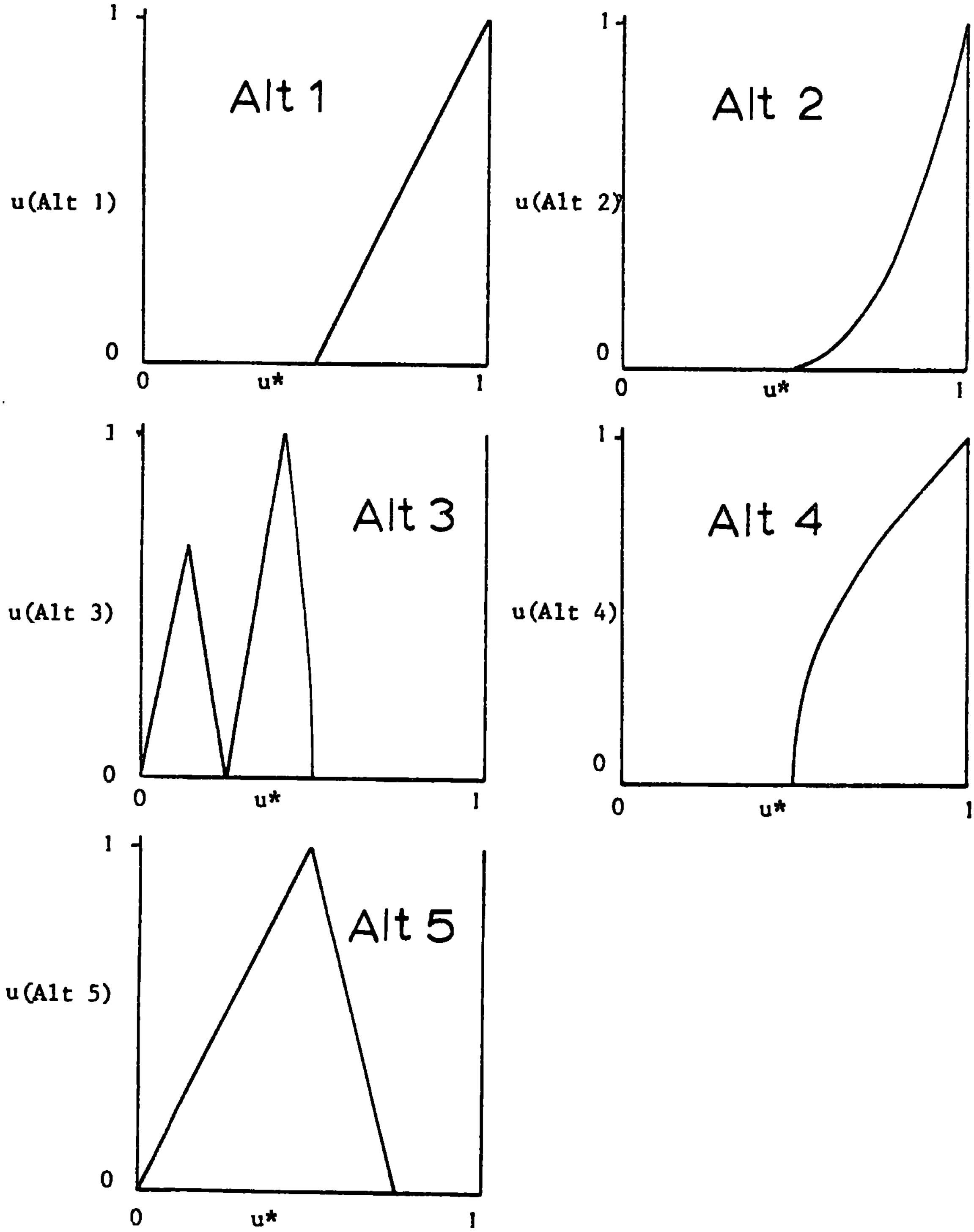


Figure 7.4 Utilities obtained using method 2

Discussion of Methods

Although the selection of a toaster may seem a trivially simple problem, it illustrates this technique quite well, as well as showing up the problems of any multi-attribute decision technique, which are:

1) handling the attribute interactions
2) ranking the outcomes

The attribute interactions may be expressed using a fuzzy relation as in Table 7.1. Any alternative is a more or less fuzzy region of the space described by the table.

We have seen two ways of obtaining the utility of any region of that table. The first method tends to give pessimistic answers. If an alternative has grade of membership 0.33 at one n-tuple, 0.5 at an adjacent and 0 everywhere else, one would suppose that, at some point in between, the alternative would have grade of membership 1.0. Unfortunately, the simple max-min relation does not take this into account, and we obtain the flat-topped, multi-modal sets, which miss the point. The max-min relation fails to interpolate between the n-tuples. However, this method is computationally simple and where the vocabularies are reasonably discrete, it can work well.

The second method might seem more appealing to the non-numerate. No numerical grades of membership are involved in the measurement of alternatives, just statements of belief. This method, though, has the disadvantage of being computationally awkward and would be more difficult to incorporate into a computer program. Because of the interactions between the attributes, truth functional modification cannot be properly applied and some sort of ad hoc method substituted.

THE PHILOSOPHY OF THE MODEL

How can we know if such a model is a 'true' representation of a decision-maker's feelings? The answer is probably that we can never know. It is a fact that many dimensions are involved in making decisions, and that people use heuristic statements to explain their preferences. Just as an orrery can be adjusted and improved until it produces a reasonably accurate representation of the solar system, so a fuzzy model of an individual's decision-making process may be adjusted until it appears to predict his preferences reasonably accurately. But the real solar system does not function with rods and cogs, and so the real decision-making process may not function with a multi-dimensional space and fuzzy relations.

However, a fuzzy model does seem to have some advantages over previous models - the inputs and outputs of decision-makers under real-life conditions are more like the inputs and outputs of a fuzzy model than of a multi-dimensional utility model. When making a decision, a human being receives vague information and he states heuristics to explain how this information is being processed. Because humans are poor information processors, heuristics such as this are necessary to assimilate and use as much information as possible. A fuzzy utility model can use vague information and heuristics, whereas a multi-attribute utility model requires precise information and can only process this information under limited classes of models. In this way, a fuzzy utility model seems to be more like the human decision-maker.

But should we use a fuzzy model which seems to incorporate what might be considered the human weaknesses when making decisions? If humans are poor information processors, a model which uses the maximum amount of information should make a 'better' decision than a limited, fallible human could. A decision aid which only reproduces the simplest decision-making heuristics would not be useful, since it does not extend the capacity of the human decision-makers. To be useful the decision aid must help the decision-maker to explore the decision space in greater detail than he could unaided, and to examine his own motives and inconsistencies. Whereas a multi-attribute utility model requires plenty of information, it cannot effectively use heuristic information, particularly on aggregation, and does not encourage constructive introspection. Once the flaws in his decision-making procedures have been realised, a decision-maker may adjust so as to be, for example, less risk averse or more altruistic. Such a model may combine descriptive decision-making with the deliberate incorporation of normative standards. This seems to be the best way to resolve the conflict between descriptive and normative decision-making.

ASSUMPTIONS ABOUT UTILITY

In the derivation of a measurable utility, the necessary axioms reflected this aim. We ae seeking a fuzzy utility, and the assumptions will reflect this too, no doubt.

Assumption 1
For any two multi-attribute alternatives, the decision-maker can state that he prefers one to the other or is indifferent between them.

Assumption 2
Preference of choice is transitive; indifference of choice is not.

Assumption 3
A decision-maker can provide a verbal measurement of the utility of two alternatives, and can provide heuristics with which to explore the decision space.

THE MODEL AND THE ASSUMPTIONS

The first axiom requires that for the fuzzy sets which represent the utility of two alternatives, we may declare them to be equal or that one is greater than the other. Ranking fuzzy sets has been a problem Jain, 1976; Baldwin & Guild, 1978a; Baas & Kwakernaak, 1977) and no single satisfactory method seems to have emerged. Ideally, the ranking algorithm should be derived from empirical observations of the behaviour of the subject, but for such a model the many variables involved would make the construction of an ordering relation time-consuming and could add considerably to the time required to perform the analysis. Hence, we must build in some form of ordering relation.

The simplest ordering relation which could be used might be of the form:

$$A \succcurlyeq_p B \text{ iff } \mathrm{rep}(A) \succcurlyeq_p \mathrm{rep}(B)$$

where A,B are fuzzy sets and rep(A) is the representative element of A, i.e. the element with the maximum grade of membership in A. Such an ordering relation applies when the sets are simple and an obvious 'shift' relation exists, so that 'high' $>_p$ 'medium', as in Figure 7.2. For a comparison of 'very high' and 'high', this type of ordering relation would not be able to distinguish between them if the usual power operator definition of 'very' is used. However, if a shift operator definition is used:

$$\mu_{\text{very } A}(x) = \mu_A(x - c)$$

where c is some context-dependent constant, and $x >_p x - c$ if the sets have 'plateau' maxima, i.e. more than one element attains the maximum grade of membership, then the representative element is not uniquely defined. Such sets would be obtained from 'not' or 'or' operations on the original sets. We could take as the representative element the average of all the elements which reach the maximum grade of membership, and use the ordering definition above.

However, we could also introduce a threshold below which the sets cannot be distinguished:

$$\text{If } \left| \text{rep}(A) - \text{rep}(B) \right| < \epsilon \text{, then } A =_p B$$

where is some constant, which might be chosen as a fraction of c, e.g.

$$\epsilon = \tfrac{1}{2}c$$

This is only a suggestion and the choice of c and ϵ is entirely context-dependent.

Within this problem, the class of operations which are performed on the fuzzy sets is quite limited, 'and', 'or', 'not' and a few modifiers, say 'very' and perhaps 'fairly'. This means that the fuzzy sets which are obtained from these operations will be simple, so long as the original definitions are simple, as is usually the case. Hence, the sort of ordering relation which has been proposed would be adequate for most problems.

We refer briefly to the ranking method of Baas and Kwakernaak (1977). They defined the 'preferability' of one alternative over another, based on fuzzy arithmetic, but it does not seem to be any more powerful as a means of distinguishing between alternatives than the simple ordering relation proposed herein.

We have already discussed the problem of knowing whether the model accurately reflects the decision-maker's own ideas, and the same is true of the ordering relation. There will be a problem of knowing whether the ordering is as sensitive or more sensitive to 'utility' differences than the decision-maker. The threshold value may not be true and can lead to error. The table below illustrates the type of error which is likely to occur.

		Model's State of Affairs		
		$A >_p B$	$A =_p B$	$A <_p B$
Real	$A >_p B$	✓	b	X
State of	$A =_p B$	a	✓	a
Affairs	$A <_p B$	X	b	✓

If the ordering relation predicts that $A >_p B$, when the real state of affairs is that $B >_p A$, then a serious error occurs, denoted by a cross in the Table above. If all the information is correct and consistent, then the ordering relation must be at fault.

As the ordering relation is made tighter and tighter, so we would expect to find more Type a errors occurring, i.e. a preference is predicted between alternatives when they are judged to be indifferent. As the relation is loosened, Type b errors will become more common; alternatives will be predicted to be indifferent when a preference ordering exists. These errors are more difficult to cure than those of the previous paragraph, since to fix the tightness of the relation at a particular level assumes that the human judge has the same threshold of judgement whatever the level of utility. The tightness of the ordering relation may also be a function of the level of utility of the alternatives being compared. As models become more and more refined, so faults in the ordering relation are likely to become important. However, we would suppose that in most practical cases these problems would not arise because of the high level of accuracy which is required on the decision-maker's part.

The second axiom imposes a vagueness upon the ordering relation because of the non-transitivity of indifference. A useful ordering relation must predict transitivity of preference, but as the chain of indifference lengthens, so we would expect the alternatives at either end to eventually cross the threshold of indifference.

It would seem that the weakness of a fuzzy model of utilities in the ordering relation. With precise numerical measurements of utility, this problem does not arise. The introduction of imprecision, whether via a sensitivity analysis of numerical estimates or the introduction of fuzzy set theory, makes the ranking of alternatives more difficult.

In practice, we may avoid this problem by:

1) imposing constraints on the measurement of alternatives so as to ensure that they can always be definitelydistinguished;

2) keeping the problems so simple that the ordering relation is clear;

3) leaving the ordering relation to the decision-maker.

The first method might be to divide each performance variable into a number of classes, thereby dividing the decision space into cells. Each alternative can be designated as fitting into a particular cell, and definite preferences exist between cells. This method is undesirable because of the constraints it imposes on the decision-maker, requiring a precision of a different sort. The second method is also undesirable for similar reasons, since it only permits simple problems to be used, rendering the decision aid less useful.

The third method would seem to be the easiest in times of confusion. Where the utilities of two alternatives are almost equal, they can be presented to the decision-maker for him to discriminate between them. However, if these utilities have been calculated using fuzzy sets, their translation back into words may cause another element of arbitrariness to enter the analysis. An alternative method of presenting the fuzzy utilities is by using graphs, to convey the balance between good and bad and the spread of values. If the decision-maker's method of choosing between two alternatives depends on the possibility of untoward effects, then he must choose between alternatives in his own way, taking account of this risk. With this method, there would be no need for the decision aid to do the ranking automatically, since that could be left to the decision-maker himself.

To obtain these utilities, we must consider the range of values over which these utilities may vary. With Archimedean utilities, two quantities could be chosen to define 0 and 10 utiles, say, and all other quantities measured using this scale. We must also fix our scale, but ask the decision-maker himself to provide the values. So, we describe two points in the decision space and attach values of utility to them. For convenience, these are taken as the present position and the ideal position, but the decision-maker must apply his own description, e.g. 'poor' and 'very, very good' respectively. If heuristic information is provided, the meanings attached to the words 'poor' and 'good' should remain consistent. The utility of any alternative may then be calculated. See Efstathiou, Hawgood and Rajkovic (1979).

RULE BASED DECISION-MAKING AND COMPUTATIONAL EFFICIENCY

As the number of performance variables increases, so the number of n-tuples will increase very rapidly. If the process of assessing utility is automated, we must consider how we may improve computational efficiency. The decision-making process which we have discussed so far may be represented by Figure 7.5(a). The process might seem more efficient if we used instead the paradigm of Figure 7.5(b). The other methods require the assessment of Φ from these rules, so we might consider the possibility of calculating the utility of each alternative by operating upon the heuristics directly. Unfortunately, such an approach does not provide a solution because of the need to express interactions and the lack of tools to handle such statements.

Consider a simple example. Suppose a decision-maker wishes to purchase an armchair and the only relevant performance variables are Price and Comfort. The vocabularies could be:

Price = cheap, reas, dear
Comfort = good, medium, poor
and Utility = low, medium, high

Simple heuristics might be:

If the price is cheap then the utility is medium (7.1)
If the price is dear then the utility is low (7.2)
If the price is reas then the utility is high (7.3)
If the comfort is good then the utility is high
If the comfort is low then the utility is low

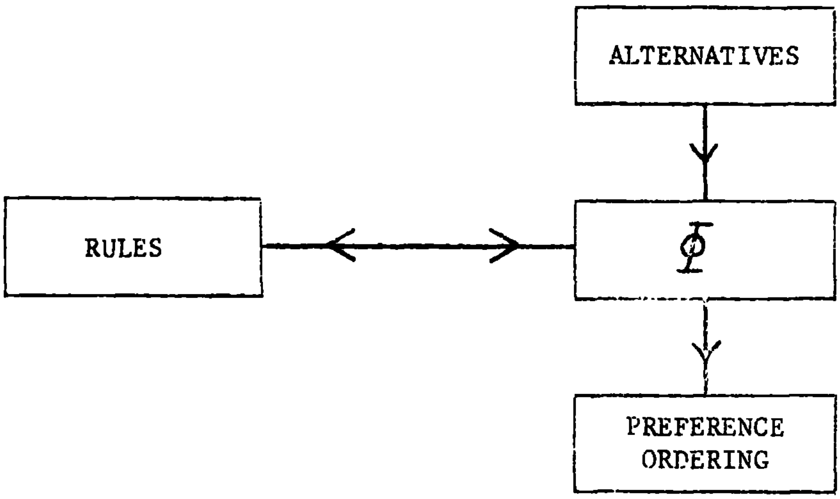


Figure 7.5(a)

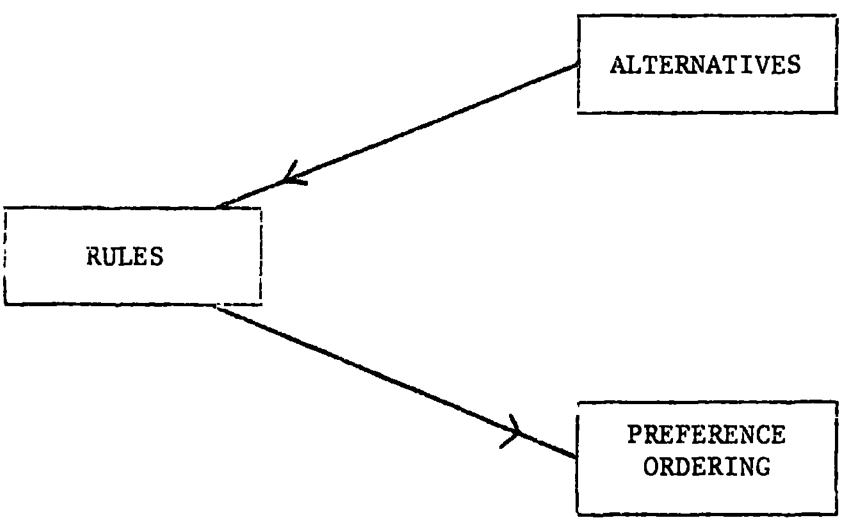


Figure 7.5(b)
Rule-based decision-making and Φ

These heuristics provide no information on the interaction between price and comfort of the armchair, but we can see where inconsistencies arise. For example, is a dear armchair with good comfort of high or low utility? Unless a simple interpolation rule is used, assuming no interaction, the decision-maker will have to be queried at those points of inconsistency. Thus, we have returned to the identification of the utility of each n-tuple. The statement of utility at each of these points is in fact a more detailed rule than the simple, preliminary heuristics, and it is necessary to contain information on the interaction of performance variables.

The other reason for being uncomfortable in the use of such rules is the lack of adequate tools to handle them. We may have the following situation:

The price is very cheap.
<u>If the price is cheap then the utility is medium</u>
The utility is ?

Zadeh (1975) suggests two possible ways of solving such syllogisms:

x is P
<u>If x is Q then y is S</u>
x is $P \circ (\overline{Q'} \oplus \overline{S})$ Lukasiewicz
or y is $P \circ (Q \times S + \overline{Q'})$ Max min

where $\overline{Q'}$ is the cylindrical extension of Q'
$\oplus$ is the bounded sum $\mu_1 \oplus \mu_2 = \min(1, \mu_1 + \mu_2)$

Given the three rules above relating price and utility, equs.(7.1) to (7.3), the definitions of price and utility in Figures 7.6(a) and (b), and the two composition methods above, together with a third, we obtain the fuzzy sets of Figures 7.7(a,b,c). The third composition method has been chosen from many presented in Baldwin and Pilsworth (1978) as that which most accurately produces the answers from such syllogisms. It may be seen that the max min method provides very little information and would not act as an effective way of using rules to arrive at decisions. However, the Lukasiewicz and Divisive rules are more useful.

Another method would be to employ truth functional modification in some way. For example,

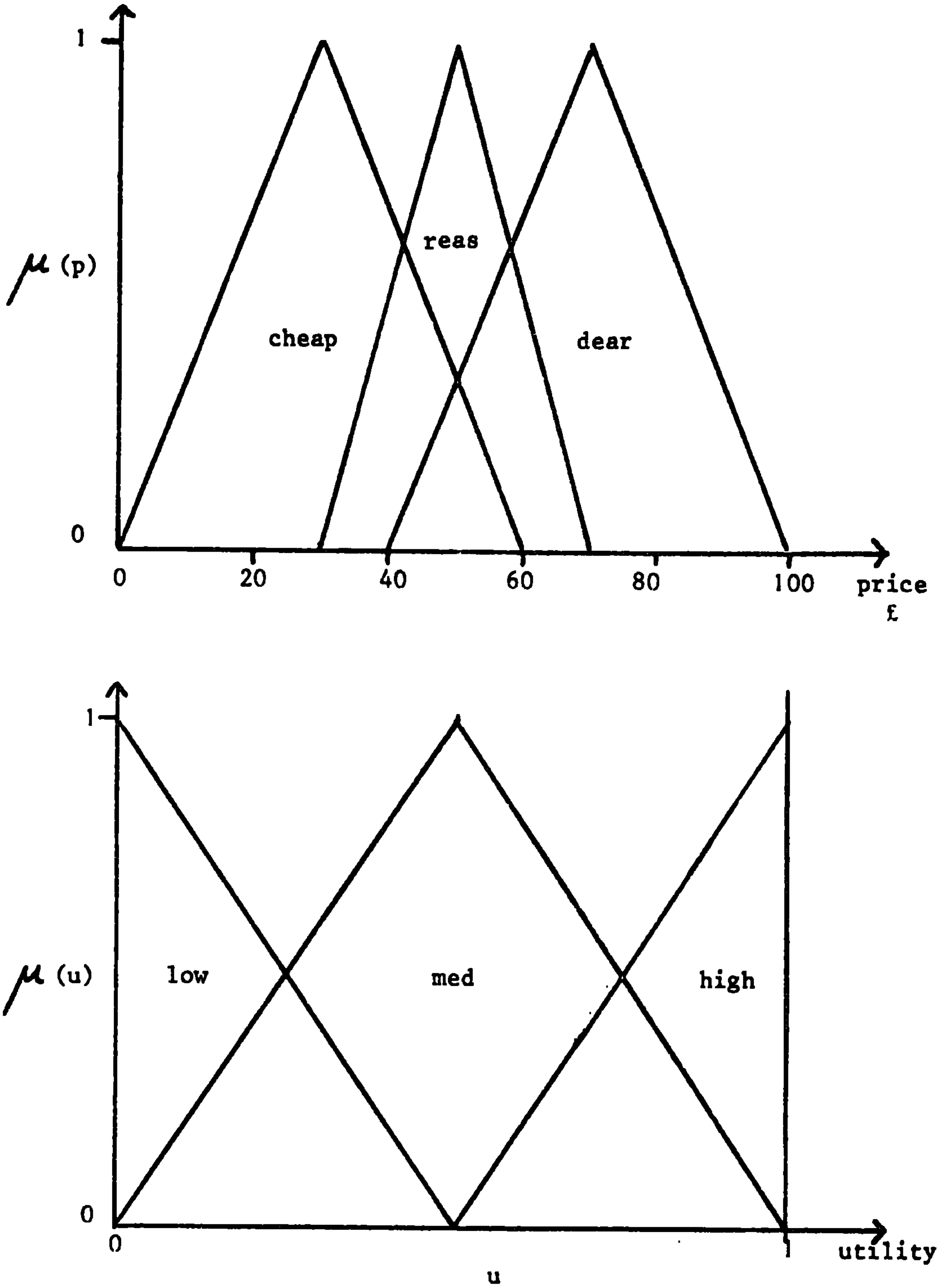
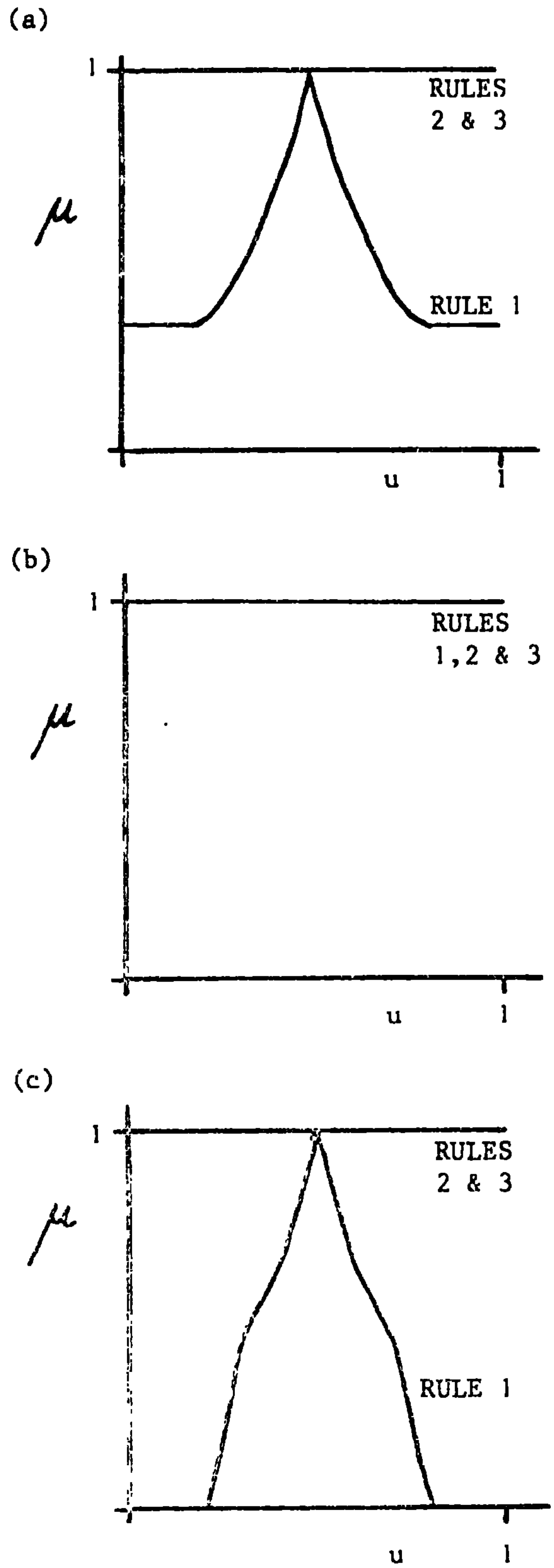


Figure 7.6 Definitions on £ and utility scales



Lukasiewicz

$P \circ (\bar{Q}' \oplus \bar{S})$

Q = cheap, reas, dear

S = med, high, low

P = very cheap

Max Min

$P \circ (Q \times S + \bar{Q}')$

Division Rule

$P \circ R$

where $R_{ij} = 1 \wedge \dfrac{\mu_S(u_i)}{\mu_Q(p_j)}$

Figure 7.7 A comparison of deductions from 3 decision rules x is P

<u>If x is Q then y is S</u>

v(x is Q x is P) = τ — Inverse Truth Functional Modification

v(y is S) = τ

y is S_τ — Truth Functional Modification

The min of the results from all the rules can then provide an estimate of utility. The three rules relating price to utility have been treated in this way, for the statements

The price is cheap

and

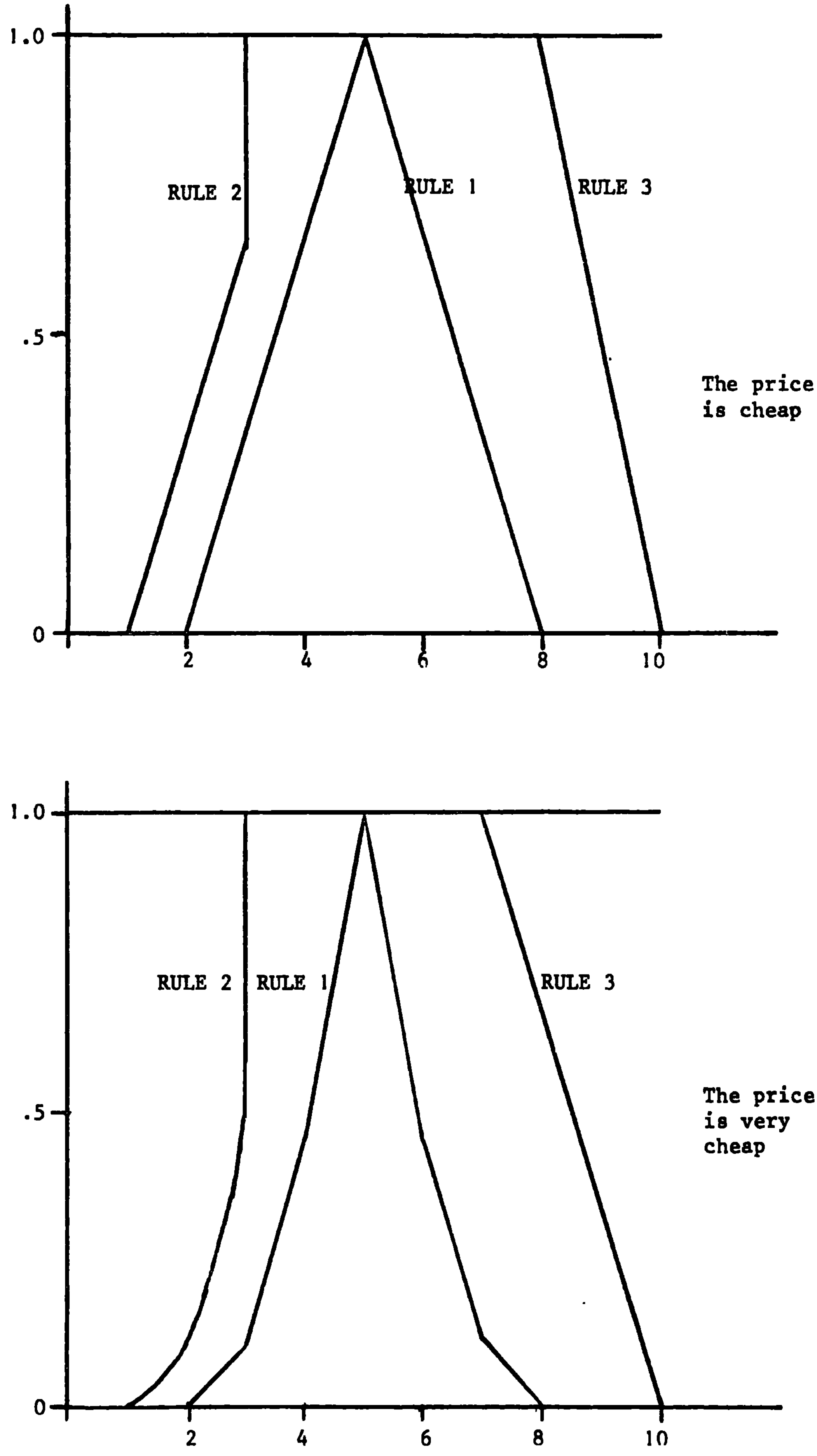
The price is very cheap.

See Figure 7.8 for the results.

It would seem that in cases where the utilities of the performance variables are non-interacting, we could use these rules to obtain a value for utility, given the measurement of an alternative. This is similar to the method used by Baldwin & Guild (1978b) except that they use only one rule and one adjective for each performance variable, assuming utility varies evenly with the level of the performance variable.

The above method provides a satisfactory output from a number of rules for one performance variable. When many performance variables are involved, the min rule can amalgamate the many performance variables, but again only if the variables do not interact utility-wise, and we assume that the utility of a particular n-tuple is the lowest utility of the combining values (conjunctive).

It would appear that to improve the speed of computation, we would need to ignore the interactions between attributes, defeating the object of the process. However, in such applications, speed of computation is not so important as in others, e.g. controllers. The decision-maker must be encouraged to learn and this will not be adversely affected so long as the computation of utility takes of the order of one or two minutes. A program has been written by a group at the "Jozef Stefan" Institut (Rajkovic & Bohanic, unpublished) and they find that with 5 or 6 performance variables and 300 n-tuples, the calculation of utility for a particular alternative takes about 1 or 2 minutes. With 3 to 5



<u>Figure 7.8</u> <u>Inverse truth functional modification operating on decision rules</u>

performance variables and 30 n-tuples, the response time is only a few seconds.

Their program uses the max-min relation to calculate the utility of each alternative, which takes account of every n-tuple in the space. Computation time increases very rapidly, therefore, with the number of n-tuples. They are hoping to decrease computation time during the interactive process of setting up $\underline{\tilde{\Phi}}$ by only taking account of neighbouring n-tuples in the computer's calculation of utilities against which the decision-maker may compare his own ideas. The evaluation of real alternatives is non-interactive, in the learning sense, and should be more accurate. This is done using the complete $\underline{\tilde{\Phi}}$, because response time is not important and accuracy is.

We would conclude by saying that within a practical context, speed of computation is not important because the decision-maker must be encouraged to learn. The time lapse between issuing the instruction and receiving the result of the calculation of utility should not be much longer than about a minute in the worst case, and preferably less. Also, since any method of improving speed of calculation would require sacrificing some of the generality of the paradigm, we would consider it unwise.

SUMMARY AND CONCLUSIONS

The chapter opened with a review of the fundamental disadvantages of traditional multi-attribute decision-making techniques, and explained how fuzzy set theory may be useful in overcoming them. A new approach to multi-attribute decision-making was explained and illustrated with an example. The three assumptions about utility which are required for such a model were presented and discussed. Rule-based decision-making and computational efficiency were considered, and the computer program of Rajkovic and Bohanic cited.

I conclude that it is possible to revise the approach to multi-attribute decision-making to produce a technique based on fuzzy set theory which is more personal in its approach and less prone to arbitrary distortion.

CHAPTER 8
A PRACTICAL TECHNIQUE FOR MAKING COMPLEX DECISIONS

The purpose of this chapter is to provide a practical recipe, which may be used by a decision-maker who is about to make a complex multi-attribute decision, such as the selection of a portfolio or design. This recipe emphasises throughout the importance of user participation in the design of complex systems and the recognition of the subjective value of uncertainty, objective-setting, etc. This recipe is also simple, and does not require the assistance of a special computer program. Some of the ideas developed in earlier chapters have had to be omitted, therefore. The chapter will finish with an expanded discussion on the topic of participation and the role of the analyst.

HOW TO PERFORM THE ANALYSIS

The design team

Many complex decisions have identifiable effects on the life of many people. These effects may be near at hand and immediate or less important and without major impact. For example, the construction of a new motorway will have a profound effect upon the lives of those living nearby, both during and after building, whilst the weekend driver may experience a fairly marginal improvement to his life.

According to the way in which they are likely to be affected and their objectives connected with the decision, the people may be roughly classified into various interest groups. Some groups may exercise considerable control over the success or failure of the decision, so their co-operation is important because

1) their goodwill may be necessary for a successful implementation, and

2) their knowledge and experience of the practical system will provide valuable information.

Thus, the design team should contain one member of each group. These members may be appointed through either

> selection by the high-level decision-makers in charge of the task, or
>
> election by other members of the group.

The method of appointment depends on the nature of the group, e.g. whether it is loosely organised or well bound together and easily identifiable, and the style of management adopted. Whichever method is used, the other members of the group who are not in the desugn team should be kept well informed of the progress of the analysis, and their opinions and comments fed back to the team.

The level of consultation with the interest groups will depend on how easily they can be contacted by the design team. If a group consists of a large section of the general public, then that group cannot be kept as well-informed as the group of people in the office next door. When dealing with interest groups which are easily identified and conveniently located, e.g. office staff, a <u>consensus</u> approach may be adopted. This attempts to involve all the group members continuously throughout the design process. With more diffuse groups, <u>representative</u> participation will be required, when representatives of the groups involved meet together to put the case for their groups. This will probably lead to the setting up of a team of representatives from the present group who are consulted and informed regularly by the design team.

Another important member of the team is the consultant. His role is to assist in the process of analysis and change. He is not there to control the process, as an expert who knows best, but as a guide, encouraging and assisting the people themselves to take the decisions.

The decision-making process which proceeds under the control of such a team will be PARTICIPATIVE. The users and affected groups will have some say in the decision, and their knowledge is part of the information required to take the decision. The consultant's status is diminished, therefore, since control over the process is no longer entirely in his hands. He will act as a bridge between groups, an organiser and a guide along the path. See Mumford et al (1978).

Analysing the problem

Within the participative framework, communication is very important. Members of the design team represent different groups and so they must be able to report back to their groups. Also, within the team, everyone must understand what the problem is, so that they can communicate effectively. Thus, the analysis of the problem is very important.

The process of analysing the problem follows several steps which may be distinguished. However, these steps are not followed in a strict sequence, because ideas from later steps often relate back to the earlier steps, which may then require revision. The process finishes when the final step has been completed, and there appear to be no more changes to be made to the earlier steps.

The steps involved in problem analysis are roughly as follows:

Problem Recognition
Perception of Environment
Listing Objectives

Let us consider each of these steps in turn.

Problem Recognition

Before the design team was set up, the existence of a problem had been recognised. The team must now acquaint themselves with the problem. This will require an overall understanding of the environment and how the problem has arisen. The flow of information and goods within the system may have to be explained, so as to initiate new-comers, and fill gaps in the knowledge of others. It is important not to become too engrossed in measuring precisely the operations of the system because the purpose of this stage is to understand and recognise the problem. To assist this, the team may undertake such tasks as:

1) listing the basic operations of the organisation

2) mapping the flow of information and goods

3) considering when the roots of the problem lie, e.g. when variances (i.e. deviations from ideal behaviour) in the existing system occur.

Perception of the environment

The purpose of this stage is to understand how the environment determines the kinds of solutions of the problem which may be offered. These solutions are governed by the constraints which the environment imposes and by the solutions' effects upon people.

Constraints The constraints upon the solutions come from many sources, which may be inside or outside the organisation. Internal constraints may be limits on the amount of time spent planning and implementing the solution and might be considered as objectives. External constraints may be legal or government policy restrictions or the influence of public opinion. As a generalisation, internal constraints are set by the organisation and are within its control and may be more flexible, whereas external constraints are set outside the organisation and are beyond its control. This is not a hard and fast classification, and it would be wasting effort to decide whether a particular constraint arose from within or beyond the organisation. This classification of inside or outside the organisation, within or beyond control is intended to assist the design team in understanding and generating ideas about the environment in which the problem is set.

People The second aspect of understanding the environment of the problem is in its effects upon people. Many different people are involved in the day-to-day operation of an organisation, and may be affected by the decisions which are made. The employees, shareholders and directors of a commercial organisation will have an interest, as will the people who buy its product or services. Public organisations must also consider politicians, and the public at large may also be affected. Within these broad groups are smaller subgroups. These subgroups may be distinguished from one another by two things, the impact of the decision upon them and their objectives (see below) with respect to the organisation. Thus, two groups with the same objectives and who suffer the same impact need not be considered separately and may be treated together as one group.

Particular individuals may find that they belong to more than one group; for example, an employee might also use the service his organisation provides. His objectives may be, in this case, a mixture of the objectives of both groups.

The design team, because it is participative and requires information, should have contacts with as many of these groups as possible, and certainly those who are most likely to be affected by a decision. Where possible, a member of these high impact groups should be part of the actual team. However, there are limits to the workable size of a design team, and some team members may feel qualified to play the role of a member of more than one group, because of the reasons outlined above. However, role-playing of group members should only be undertaken when the number of groups is large, and the group concerned is a "low impact" group. Where possible, the role-players' judgments should be presented to members of the group for ratification and comment.

To summarise, the stage entitled "Perception of the Environment" requires the decision-making team to examine constraints which the environment places upon them, and to consider the groups of people who will be affected by the decision and how to obtain their involvement and co-operation.

Listing Objectives

In the previous section, different groups were distinguished by the different impacts they suffered and by their different sets of objectives. We will consider these objectives next.

The purpose of listing objectives is to clarify the needs of the groups. When we come to the step of evaluating solutions to the problem, we will need some means of assessing the benefits of its implementation. Estimating its effect upon these objectives gives us the means of comparing alternatives.

How are these lists of objectives generated? Once the previous two steps have been considered, many objectives may have come to mind. Studying the problem will show up faults in the system, and groups may see the correction or improvement of these faults as some of their objectives. Looking at the interest groups and their attitudes towards how things should be changed will suggest more objectives.

At this stage it will become apparent that there are different levels within the organisation's objectives, so that they form themselves into a hierarchy. For example, the objective

"Provide opportunity for cultural development" may be broken down into objectives at a lower level such as

"Preserve historic sites and areas of natural beauty"

"Provide adequate public libraries, museums and cultural activities"

"Protect meaningful local tradition and encourage civic pride"

Similarly, it may be clear that a group of objectives may be grouped together under a single, broader objective.

Generally though, it will be found that the hierarchy is developed from the top downwards. Starting off with a broad objective, such as "Improve the quality of life", which is so vague to be unworkable, we can list the objectives which contribute towards this, and decompose each of these objectives in turn.

The outcome of this step should be a list or a hierarchy of objectives for each group. There are no restrictions upon the type of objectives which are stated, simply that they are relevant to that group. The objectives may be quantitative or qualitative in nature, e.g.

"Reduce queuing time by half"

or

"Preserve attitude of trust between managers and secretaries"

Some groups may tend to state their objectives as vague, higher-level aims, and some groups' objectives will be very much concerned with changes in specific day-to-day activities.

The tree-structuring of objectives is not essential, some groups may prefer a simple list. The choice between tree or list will depend upon the complexity of the problem and the feelings of the group concerned.

How do we know when the list of objectives is complete? At this stage in the analysis that is difficult to answer. We have already seen how past steps can influence the present step and how it, in turn, feeds back new ideas to previous steps. It may be that a set of objectives which appears to be complete may turn out later to have some serious omissions. It is easy, at this stage, to "err on the side of caution" and include more objectives than are necessary. To avoid this fault, ask if the decision is likely to be altered if this objective were not included in this list. In other words, how significant is this objective in comparison with the most important of the other objectives? If it seems completely insignificant and unlikely to have an effect on the final decision, then it may be safely excluded. As a rough guide, lists of objectives should contain no more than about a dozen, and each branch of a tree should branch in turn to no more than about five or seven sub-objectives.

Comment

So far, the analysis of the problem has concentrated on studying the issues in some depth. We may emphasise two points:

1) the iterative nature of the analysis. Each stage leads on from the previous stage so smoothly that the beginning of one stage becomes blurred into the end of the previous stage. Each stage may feed back or feed forward into other stages. To avoid spending too much time on this process, some "indicators of adequacy" have been built in.

2) the learning process involved. Since the decision-making team are acquiring information for their own use and not for passing on to outside analysts, they must become engaged in a process of learning and understanding. The mixture of groups is intended to encourage a pooling of knowledge and insight, and the members of the decision-making team should learn from one another.

Uncertainty

At this beginning of this paper, we identified three components of the complex decision. Two of these, many objectives and many groups, have already been introduced during the phase labelled "Analysis of the Problem". We must now consider how to cope with uncertainty.

When a decision-maker has to forecast future events, he is bound to meet uncertainty. He may have a greater or lesser amount of confidence in which state of nature will prevail. His attitude towards implementing a particular act will often depend upon its riskiness, i.e. the amount of money he stands to lose or the disruption which may occur. Willingness to take a risk will depend as well upon the decision-maker's present position.

Not all uncertainty is due to an inability to predict the future. Some uncertainty also arises from a lack of knowledge about the present position. In the case of objectively mesurable quantities, this uncertainty could be eliminated by measurement, but if the quantity is subjective, then there is bound to be imprecision. Just as a decision-maker may be more or less confident in his judgements on probability, so he may be uncertain too in his judgment on the quality of subjective attributes.

Discussions on subjective attributes are possible using words, whereas attempting numerical measurements may be time-wasting and misleading. It is well-understood to say that "John is good", but it soon becomes fruitless to argue what measurement John's behaviour deserves on a 10 point scale, whether he should be given 7, 8 or 9 points, and to how many significant figures the statement is accurate.

When such numerical assessments of subjective qualities are obtained, it is an easy step into the trap of according a spurious, scientific objectivity to them, and attempting to apply to them the common arithmetic operations. However, such numbers are only a crude representation of subjective ideas, and it would be misleading to mix them with genuinely objective measurements.

To avoid this confusion, we may take the unusual step of recommending two ways of representing decision-makers' feelings about quantities. Where the quantities are objective, they may be measured numerically, and where we are considering subjective "quantities", they may be assessed verbally. The natural method of expressing uncertainty about the future, where objectively measurable, is by using probability theory, and where uncertainty is only intuitive or internal to the decision-maker, it may be expressed using words.

For example, we may draw up a probability distribution to show the probabilities of different numbers of people being out of work in six months or a year's time. However, if the prediction was for 10 years from now, the probability distribution would be much harder to draw, because of the unknown effects of many possible events in the meantime. Hence, the uncertainty about the probability distribution will grow, until it may become difficult or impossible to convey information this way. Probability is no longer objectively measurable, but relies upon a succession of subjective impressions of the likelihood of events. It might be more informative, and a better admission of the uncertainty involved, to say instead:

> "It is very likely that unemployment will double in the next ten years"

whereas it might be quite reasonable to say:

> "There is a 50% chance that unemployment will increase by 5% in the next 6 months"

Uncertainty is a very special attribute of any design strategy. We have discussed the problems involved with treating it objectively, and how the decision-maker's attitude is very important. We shall propose a special means of handling it later, but at this stage uncertainty must be recognised as inevitable. Further investigations and analyses may reduce it, but it may not always be worthwhile to spend money to reduce uncertainty. These problems will be considered later.

Assess the Status Quo

This is the next stage in the decision analysis, after the Analysis of the Problem. Again, this stage consists of a number of substages, which will be discussed separately. These are:

Define vocabularies
Measure the status quo
State the ideal solution(s)

Define vocabularies

This stage is necessary because we have removed the assumption that everything must be expressed in numbers. The language of numbers had been always used, but, having recognised its inadequacy for many purposes, we must devise some more suitable vocabularies.

The purpose of numbers was to assist in the process of measurement - measuring the importance of objectives, measuring progress towards them and measuring the resulting "utility". These new vocabularies must fulfil similar purposes and should be chosen with this in mind.

For example, if the problem is to choose which camera to buy, the decision-maker may be concerned with such objectives as:

1) the camera should be small and light
2) the camera should use the cheaper types of film
3) the camera's reliability should be high

Each of these three objectives refers to one or more "performance variables" of the camera. The performance variables are attributes of the desired solution. The objectives place a desired value upon the level of the performance variable, and each alternative solution will fulfil the objective, i.e. measure against the performance variable more or less well.

In this example the performance variables might be:

1a size
1b weight
2 price of film type required
3 reliability

In some cases, e.g. 3, the objective may actually mention the attribute or performance variable directly, but in other cases, e.g. 1, a measurement or desired level may be stated, and so the underlying performance variable may be deduced.

Now,a vocabulary must be chosen which will assess performance along the attribute. For "size" the vocabulary might be:

V1a (tiny, small, medium, large)

or, for weight, the vocabulary could be:

V1b (light, comfortable, heavy)

Since the size and weight of a camera are closely connected, as the objective suggests, the chosen vocabulary for Objective 1 might be:

V1 (good, fair, inadequate)

The performance variable would now be something like "handiness", to embrace the connected concepts of "smallness" and "lightness". It is true that a large camera tends to be heavier and small cameras are light, so we may refer to the camera's overall handiness.

The size and weight of a camera could have also been assessed using numerical values of the performance variables. Size may be measured using all three dimensions of the camera, length, width and depth, and the weight could have been measured in grams. These methods are equally valid, particularly when assessing amounts of money, and are particularly useful when communicating between groups, where the meaning of terms such as "small" may be different. However, we are concerned primarily with the decision-makers' perception of the alternatives, and they may not perceive a great distinction between 250g and 270g. Again, over-precisiation can distract from the learning process. The vocabulary should reflect those variations along the performance variables which are perceptible and sufficient to cause a change in the desirability of that item.

It will often be the case that the list of adjectives set up as the vocabulary will not be sufficiently fine to express all the perceived distinctions. For example, two items may both be described as "small" yet one is obviously smaller than the other, although not small enough to be described as tiny. We can extend the basic vocabulary, and structure its use, by introducing additional words such as "very", "not", "and", "or", "but", etc. Other words may be substituted for those suggested above, but I would not recommend using a greater number because their use tends to become disorganised and confusion may arise within the group as to the meaning of these "modifying" words.

So, we may describe one alternative as "small" and another as "very small" or "very small but not tiny", or, if we wish to express more uncertainty about its measurement, "small or very small" where "or" denotes that it could be either "small" or "very small" but we are uncertain which.

The purpose of this stage has been twofold. Firstly to show how to select a vocabulary to match each objective, by considering which performance variable is involved, and secondly to demonstrate how the use of words may be organised, by setting up vocabularies and using a few extra modifying words, so as to express feelings carefully and precisely in convincing natural English yet avoiding the use of numbers, with its associated pitfalls.

Measure the Status Quo

The purpose of this stage is to consolidate much of the study done during the Analysis of the Problem. The existing situation was carefully studied and objectives set up. Each objective has a vocabulary associated with it, and we may now use these vocabularies to assess the status quo, by estimating verbally how well it fulfils each objective. This stage will clarify the status quo and may feed back to earlier stages of the Problem Analysis. It will also test the suitability of the vocabularies and how easy they are to use.

In some decisions, no status quo may exist, e.g. a decision-maker may wish to buy a car, camera or calculator, or select a school for a child, but he does not own one of these items, or has never sent a child to school before. In this case, the stage of measuring the status quo is inapplicable, and the decision-maker should proceed to the next stage.

State the Ideal Solution

It will have become apparent, during the analysis, that the ideal solution of the problem will remain exactly that - ideal and unrealisable. This may be for two main reasons. Firstly, what one group perceives as ideal may be quite unacceptable to another. The groups have different objectives and standards of performance, so that if the ideal solution was to give everyone their ideal solution it could never be done. The second reason for unrealisable ideals is that the ideal solution is not available. For example, a car which is safe, reliable, has a low petrol consumption and costs less than £2,000 is not available on the market, and may be impossible to produce. The choice will have to be made amongst alternatives which satisfy the ideal conditions to a greater or less degree. The statement of the ideal, together with the measurement of the status quo, provides standards for comparing alternatives.

All that this stage requires is the conception of an ideal alternative and the standards of the performance variables it should attain. Having listed the objectives, this stage should be straightforward and ensure that the vocabularies are adequate and the objectives thorough.

What about the Analyst?

During an earlier section, we mentioned that the presence of an analyst was necessary as a helper and guide. Although his function has not been specified in the last sections, it should be clear that his presence would be helpful. Each group may have different objectives, vocabularies, measurements of the status quo and ideals. The analyst's job is to assist and encourage the groups in their task, ensure their work follows the guidelines herein and to reassure when the requests seem unusual or too difficult or easy. His role is to answer questions on how to apply this technique, rather than to control the acquisition and use of information.

The earlier stage of the Analysis of the Problem would be performed as a team of different group members. During this stage, the groups may perceive common areas of agreement, and may choose to proceed with the analysis as a team, although agreeing to differ on the importance of some objectives or on their nature. This will have extra advantages later, but at present it will encourage cohesion between the groups, towards a common purpose.

Should the groups decide to set objectives and assess the status quo apart from other groups, the analyst must ensure that communication between the groups is maintained. This may be done by meetings to compare progress and share ideas on alternative solutions, discussions on objectives and attributes, and any new ideas on the problem or environment, including the role of the constraints.

Preference Rules and the Assessment of Alternatives

Before considering the mechanics of this next stage, let us review the results and working material which is available. Each group has determined what its ideal solution is, and has established vocabularies for assessing performance towards each objective. To describe any alternative solution fully, its measurement along each of these objectives must be supplied.

The next steps which we shall consider are:

Rank order the importance of objectives
Consider hypothetical alternatives
Devise and describe possible alternatives
Negotiate between groups

This last stage is probably the most demanding of all, because the information can no longer be obtained from the environment and is completely dependent upon the judgments and introspections of the decision-makers themselves.

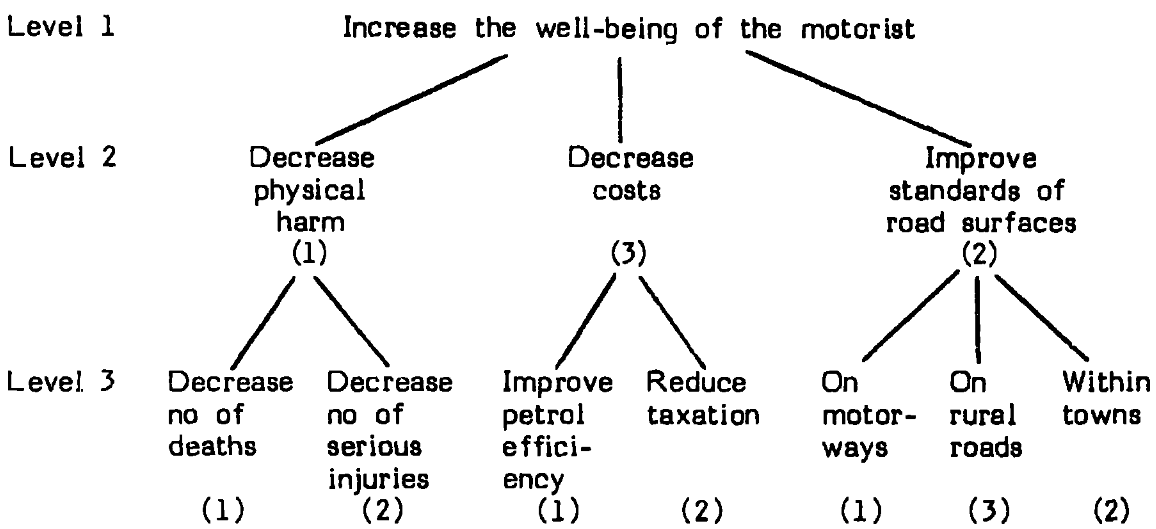
Rank order the importance of objectives

When considering the status quo, the decision-making group will realise that to improve upon it, it is more important that some objectives be satisfied than others. The group should attempt to rank-order the objectives in decreasing order of importance. They should consider the present position, as it has already been explained, and sort out the priorities. We do not require the ordering of the priorities, given some future time when achievements may have been made elsewhere, but the present, existing priorities of improvement.

If the objectives have been arranged in a list, the rank-ordering process is straightforward, and operates upon this list. However, if the objectives have been structured as a tree, the rank-ordering process is slightly different. Consider Figure 8.1. The highest-order objective is at Level 1, and several lower-order objectives are at Level 2. Each of these has some sub-objectives at Level 3. To rank-order these objectives, consider, firstly, only those objectives at Level 2, and order them, as if they were a simple list. When that is done, consider a group of objectives at Level 3 which branches from one Level 2 objective, and rank-order these. Now, take another group of Level 3 objectives, all proceeding from Level 2, and order these. Proceed in this way through and down the Levels until all the objectives have been considered.

It is important to realise that the hierarchical arrangement of the objectives is to assist the decision-maker in structuring his ideas, and for no other reason. The level at which an objective occurs within the hierarchy does not necessarily determine its importance. For example, in Figure 1 "Decrease number of deaths" may be seen as a more important objective than "Improve standards of road surfaces". This is not apparent from the rank ordering, as displayed, but should emerge during the verbal assessment of importance.

Now, it is not only more or less important to improve upon the standards of achievement of the performance variables, but it is also important that these standards do not deteriorate below their present levels to a less satisfactory position. For example, a contented staff may consider the improvement of their job satisfaction as not very important, but may feel



The numbers in brackets refer to the rank order within that branch

Figure 8.1
A Hierarchy of Objectives

that it is very important that their job satisfaction does not fall below its existing level. To obtain a more complete picture of the group's attitudes, they should rank-order the importance of "NOT DECREASING" the present levels, as well as the importance of "INCREASING" those levels.

We should now have two lists of rank-ordered objectives. Rank-ordering on its own provides no information on the separation or distance between the importance of the objectives. Some may be very closely spaced in importance and others well apart. This may be represented again using verbal assessments. A vocabulary such as:

(crucial, important, desirable)

could be used, together with the modifying words "and", "or", "but", "very", "not" listed earlier.

Two effects may be observed in practice if the group has to be polled rather than consulted directly. The first is that there is a tendency for people to exaggerate the importance of objectives, stating most of them as "crucial" or "very, very important". The second, which may be a result of the first, is that the distribution of responses may be bi-modal.

The first effect, exaggeration as opposed to genuine strength of conviction, may be checked carrying out a further study. The first study, a pilot survey showing these results, should now be presented as a histogram of responses, and further group members polled to see if they agree with that finding. It will usually be the case that a few individuals are prone to exaggeration, and this may show up in their other responses.

If the distribution of responses is bi-modal, and the effect is not the result of exaggeration, then the group may have to be split into two subgroups. For example, schoolchildren were asked how important they thought it was that their local public library should provide study facilities. Not unexpectedly, the result emerged that some thought it very important and some were unconcerned. The group had split into two subgroups, one of which liked studying and one which did not. It may be that in identifying the interest groups, the design team unwittingly failed to distinguish between two or more separate groups. The group's perception of the objectives will reveal this, since this is what distinguishes them. See Figure 8.2.

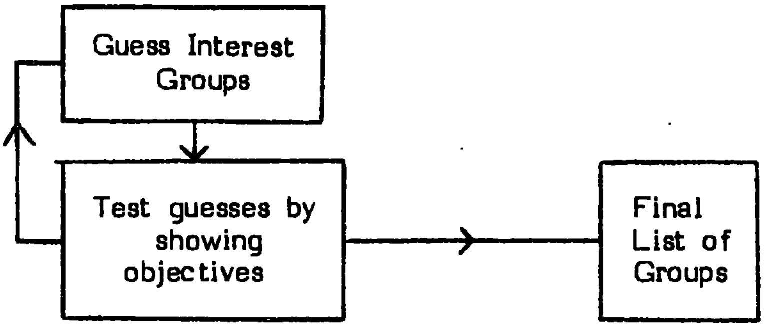


Figure 8.2
Identification of Interest Groups

To ensure consistency between the lists, the importance of an objective which has been rated on the Improve list as "very important" should be compared with the importance of the "very important" objectives on the Maintain or Not Decrease list. This exercise may prove taxing and need not be attempted for more than one or two cases.

The process will help to structure the design team's thinking and will be useful in carrying out the next phase.

Consider hypothetical alternatives

By this stage, the design team will have built up a considerable store of knowledge and should be ready to consider some hypothetical alternatives. Given the measurements of the status quo and the ideal alternative, the rank-ordered lists of the objectives and some practice, the process of determining the quality of each alternative should be fairly straightforward.

The generation of the hypothetical alternatives may be done in two ways. The first way is to think up a design strategy or select an item and measure it along each of the performance variables. The second way is to vary the level of performance of one or two attributes at a time to produce an alternative which we may not know how to implement or whether it exists.

For each alternative, the design team should compare it with the status quo, the ideal and any other alternative and state the quality of that alternative under favourable circumstances or its "preferability" as compared to the others.

It is this part of the process which is different from most other decision aids. We have emphasised throughout the importance of acquiring information and educating the design team or decision-maker to understand the issues involved. The problem of multi-objective decision-making is in handling the information required to make a reasonable decision. Man's information processing capacty is limited, and the burden may be removed from him by converting much of the information into numbers and using mathematical operations to process it. We restore the information-processing burden to the decision-maker because this is the only way to overcome the problems which mathematical methods introduce. However, to assist the decision-maker in handling the information, the process has been organised so that the burden is eased and the decision-maker has become more educated and better informed.

The outcome of this stage should be a "table" of alternatives, each of which is measured along each performance variable and has an assessment of its quality under optimistic, favourable conditions. This stage is highly iterative because the decision-maker may wish to revise earlier trial estimates and consider how the rank-ordering of the objectives changes as progress is made.

This "table" may be considered as a list of rules for determining the quality of any alternative (as we shall see later) but there is a more fundamental and insightful type of rule which may emerge during the consideration of hypothetical alternatives. These are known as heuristics. Heuristics are usually rules-of-thumb or inexact problem-solving methods, and the word "heuristic" is often applied to these rules which express an insight into human decision-making. Examples of heuristics are:

> If the price is very dear then regardless of the other performance variables, that alternative will never be better than good.
>
> If the reliability is poor, then that car is unacceptable.
>
> I value accuracy over appearance in every case, i.e. if the accuracy of two items is the same, then appearance can resolve the preference difference between them.

When it comes to considering the hypothetical alternatives, the table of rules or the heuristics may be built up by a number of methods. These are outlined below:

1) by the inclusion of a new rule not previously considered.

2) by the removal, replacement or correction of an existing rule, which no longer seems correct.

3) by the addition of an extra performance variable,

whose performance is important, although hitherto overlooked.

4) by the grouping together of objectives within a hierarchy.

This stage is complete when the table has been filled to such an extent that the preference ordering of test alternatives is correctly obtained by interpolating between existing rules, and the decision-maker agrees that all alternatives which the rules predict are equally preferred agree with the decision-maker's own perceptions. Once the rules have been determined, the decision-makers are able to assess real alternatives, and are aware of their genuine needs. We may now proceed to the next stage.

Devise and describe possible alternatives

This stage differs from the previous stage in that the alternatives are no longer hypothetical, but that real, feasible alternatives must be devised. It is probable that some ideas for real alternatives have already emerged from the previous stage. If so, these should be carefully considered and a detailed plan of their implementation drawn up, explaining how the estimates of the measurements are obtained and the chain of reasoning involved. If ideas on possible alternatives are lacking, then the list of objectives may prove stimulating.

There is no set of instructions which can be given here for devising alternatives. The process demands imagination and creativity and an ability to see beyond the trivia to the underlying scheme of things. This was encouraged in the earlier phases, in the hope of producing ideas to develop at this stage.

The alternatives may be measured along each performance variable and rank-ordered preference-wise either intuitively by the trained decision-makers or, in the case of more complex problems, with the assistance of a computer and fuzzy logic.

By this stage, the decision-making group should have a clear idea of the sort of alternative it requires, but the participative decision-making process is not yet complete.

Negotiate between groups

When many groups are involved in the decision-making process, their perceptions of the ideal alternative may differ to a large degree, so that there would appear to be no solution which can be adopted to please all. The participative approach tries to prevent such intransigent stalemates from occurring by

1) encouraging communication between different groups and between members of the same group.

2) encouraging the exchange of information and ideas throughout the decision-making process.

3) encouraging groups to recognise and understand the objectives of other groups whom they may not have previously understood.

It is hoped that as the decision-making process proceeds, the similarities between groups will emerge, rather than their differences. Personal rivalries sometimes exist between groups, so that an alternative which would otherwise be deemed acceptable is rejected because another rival group accepts it too. If such attitudes can be brought into the open by such a process of analysis, then we may achieve some progress towards co-operation and mutual benefit. However, if these attitudes are supported by stubbornness and obduracy, then no further progress can be made. The participative approach to decision-making is founded upon a belief that co-operation is possible and to everyone's ultimate benefit. If any group does not accept this belief, then the participative approach fails and decision by confrontation and conflict will result.

The final step of the participative decision-making approach advocated herein is to consolidate communication between groups and agree upon a selected alternative. If communication between the groups has been by consensus throughout the process, and the design team has remained intact, then this step will not be necesary. Each group should now have a set of objectives and a "table" of rules which describes its attitude towards several alternatives.

During this step, groups may be able to exchange ideas on alternatives which other groups had not thought of. Eventually, an alternative should emerge which is acceptable to all the groups. The details of the alternative may require re-negotiation, but its broader effects and fulfilment of particular objectives should be identifiable.

In addition to the decision-making rules to describe their attitude towards alternatives, groups may have higher level rules which describe how their attitudes towards an alternative may depend upon other groups' opinions. For example, a less than perfect alternative may become preferable over an apparently better alternative if it is seen to benefit a less fortunate group. This "altruism" between groups is an important aid towards negotiation, as opposed to the attitudes of conflict discussed earlier.

Thus, the final part of the analysis may require the groups to examine their attitude towards one another.

The output of this stage should be a design plan or strategy which suits all groups and fulfils the various objectives as well as possible. Equally, the output should now be an "educated" group of decision-makers who understand the organisation, its environment, the objectives and motivation of other groups, as well as the factors which exert a deciding influence upon their own behaviour.

How to handle uncertainty

At the end of the previous section, the decision-making team should have a list of possible alternatives and an optimistic estimate of the quality of that alternative. Negotiation with other groups may have reduced this list to one alternative only, but if this is not the case, then we must introduce uncertainty as an extra decisive attribute of each alternative.

The decision-making team is required now to estimate the likelihood of each alternative achieving the desired result. This may be contingent upon a number of factors, such as good staff reactions, no sudden, unexpected market changes during implementation, etc. It is helpful to write down the favourable conditions necessary to achieve the desired result. The estimation of likelihood may be, as desired, numerical or verbal, in the manner outlined in the section "Define Vocabularies".

In that section, we examined the nature of uncertainty and saw how it arose from two sources. One is due to the decision-maker's own feelings and may be labelled fuzzy. The other is due to an inevitable lack of knowledge about the future and may be labelled stochastic. It is the second type of uncertainty which we shall consider now.

The stochastic uncertainty may also arise from two sources which lie roughly within or beyond the control of the organisation. Those variables which are within the organisation's control are more predictable, in the sense that the organisation has better knowledge of them. The behaviour of those variables beyond its control may be completely unknown and may even be treated as random.

To reduce uncertainty, the organisation will need to acquire more information. The participartive design process is one way of doing this, because it supplies more information about and from the people affected by the decision, and should make their behaviour more predictable, although it will never be completely certain. Another way of acquiring information, particularly about those external variables beyond the organisation's control, is to employ external advisers to assess the future possibilities. Since these advisers must be paid, it is important to know how much money should be invested to reduce uncertainty in this way.

In order to decide whether or not to employ extra outside help, the decision-making team must consider their attitude towards risk, with respect to the available alternatives. Do they prefer the less good alternative which will not be affected by external factors, or do they want to try the very promising alternative which will only succeed if everything goes well? If they decide upon the stable alternative which is very likely to achieve fairly good results, then it would not be necessary to employ a consultant. If they want to consider the risky alternative the question is now:

> "This alternative will achieve very good results only if certain conditions prevail. We are uncertain, but think these conditions may exist. An external consultant can reduce our uncertainty by supplying extra information. His fee is x $,000. Is the extra benefit which we are likely to gain if we implement this alternative worth spending x $,000, given that the consultant too may recommend the stable alternative?"

See figure 8.3.

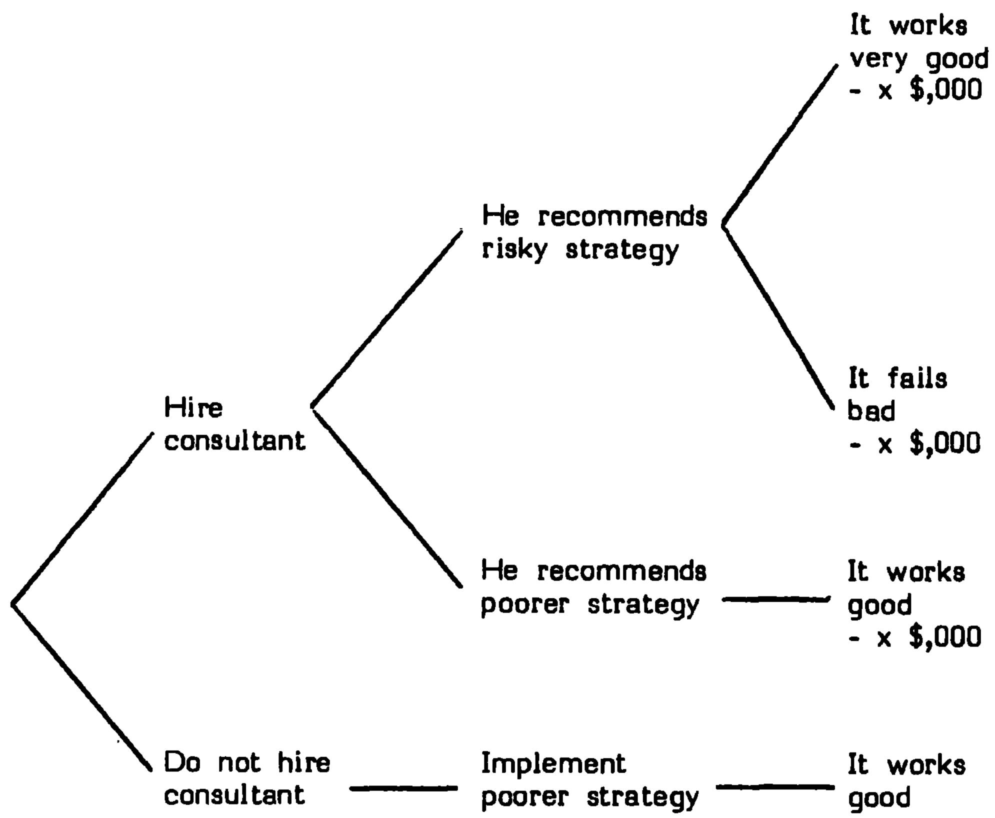


Figure 8.3
Is the Consultant worth it?

The problem of whether or not to hire the consultant has reduced to a lottery. Each decision-making team member should try to make the binary choice of hire or not hire, given the consultant's fee and reputation. Each team member will have his own estimate of the likelihood of the various events, and opinions of the value of the alternative strategies. A simple ballot amongst the team members may be necessary to decide which course of action to pursue. Should this consultant be rejected, the procedure could be repeated for another consultant, with, perhaps, a lower fee and a less good reputation.

This sort of problem is usually treated so as to maximise the mathematically expected value. That requires placing a numerical estimate of value upon each of the possible outcomes and numerical estimates of the probability of each event. However, the values are subjective and the probabilities uncertain, so the mathematical expectation cannot be used. (This is probably true in most cases, if it were admitted.)

Let us now assume that uncertainty has now been reduced as far as is reasonable. The decision-makers must now consider the available uncertain alternatives and consider how to trade-off between the possibility of a very good result and the certainty of a not very good result. This sort of behaviour is, again, likely to be rule-based, except that two attributes only are being considered, but very important ones.

This phase is likely to be very difficult if there are many alternatives, but should be fairly clear if a small number of carefully studied alternatives are being considered. It is hoped that the previous stages will have reduced much of the uncertainty which derives from lack of knowledge and unnecessarily unpredictable aspects. But the decision-makers' own attitudes towards risk are under the microscope now.

Implementation

The purpose of initiating any decision-making process is not to educate the decision-makers but to make a decision! Once this decision has been made, it must be implemented, and the responsibilities of the decision-makers must carry over to this phase too. There are two main reasons for this.

The first is that during the implementation, revision of the decision-makers' specification may be necessary. Swift changes may have to be made, and so the groups or their representatives may need to be consulted rapidly. The groups' tables of rules may be useful here, but the implementors ought not to consult them without the help of group representatives, since they may make errors of understanding otherwise. Any change in plans should be communicated back to the group, together with the reasons.

The second reason, connected with the first, is to ensure the co-operation during implementation of the groups involved. Their role in the decision-making process should have been apparent and so should the reasons for accepting and implementing this alternative. The choice of a particular alternative may have depended upon the favourable reaction of interest groups, and for the alternative to be a success, the reactions should be as expected. The decision-makers have a responsibility to assist during implementation in order to justify their choice.

Implementation is a neglected area of the decision-making process, and by maintaining the participation of the interest groups during this phase, we would hope for more success.

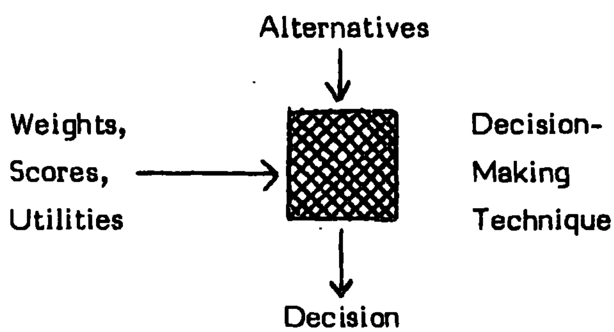
PARTICIPATION AND THE ROLE OF THE ANALYST

In this section we shall consider how some of the problems facing the 'participative analyst' may be resolved by adopting such an approach to decision-making as has been suggested herein.

But firstly let us define some terms. By 'participation' we refer to the involvement of users from many levels in the design and implementation of a system. Their opinions and knowledge, together with the impact of the system on their day-to-day or working life, must be taken into account. It is believed that the extra effort required for such a design process is worthwhile for a number of reasons, some of which have already been mentioned (See also Mumford, 1978).

The 'analyst' in the participative design process is the person who directs the process and who instructs the design group in the use of the tools available to them. The role of the analyst in participative design is very different from that which he usually assumes. Since the knowledge and skills of the users are being used, the analyst's own ideas on how the organisation 'is' or (worse still) 'ought to be' run must be deliberately avoided. The analyst's own value system has no place in the process. This implies a downgrading of the analyst from the 'expert' to the 'guide'.

If design is to become truly participative, and be seen to be truly participative, then the control of the design process must be with the users involved and not with the analyst. So long as control is left with the analyst, there will be fears that participation is nothing more than manipulation with a smiling face. To avoid these accusations, we must look at ways in which the control over the design or decision-making process is retained by the analyst. To do this, we must consider not just the meta-problem (Select an alternative) but also the meta-meta-problem (Select a technique). See Chapter 2. Participative problem-solving allows the users to suggest alternative methods of solving the problem, but may only offer a prescribed method of selecting between the alternatives, i.e. solving the meta-problem. We should look at the process of selecting the technique to choose between alternatives. The process might appear to the users as follows:



The decision-making technique is a black box which allows certain 'participative' inputs in the form of weights and scores, but offers no control over the machinery within. Hence, at this level, the participative design process is essentially normative.

The techniques which we advocate are intended to give the participative decision-maker more say about the machinery within the black box. He should be able to state his own decision-making rules, and set his own standards of behaviour. The decision-makers should be given control over the meta-problem solver too, i.e. selecting their own decision-technique.

One can always take the argument one step further back and point out that the decision-maker is still unaware of the method whereby a computer can accept these rules and predict decision-making behaviour, and that this is a stage that only a small number of highly skilled decision-makers could aspire to understand. This is true, but we would hope that the decision-maker would never need to. The computer is not intended to replace the decision-maker, but to act as a decision aid, in its truest meaning. The computer should enable the decision-maker to discover his own decision rules, and train him in their use, so that as the decision-maker learns and becomes competent, so the computer is replaced.

Thus, the analyst must not bring to the participative design process a prescribed meta-problem-solving technique, but only the meta-meta-problem-solving technique, since it is, by definition, the participative design team who must provide the ideas to solve the problem. The analyst should find himself in the background, but still with an important role to play. No longer should he be the controller, but a guide and educator.

SUMMARY AND CONCLUSIONS

The purpose of this chapter has been to show how complex decisions can be made in an ordered manner. The information involved may be handled simply and naturally, without unnecessary recourse to numbers. The participation of the interest groups involved is encouraged so as to reduce the uncertainty involved in predicting their reaction to change, and to use information and experience which would not have otherwise been available to the decision-maker or design team. This philosophy has two other effects: the reduction in status and control of the consultant to assistant and guide rather than ultimate decision-maker, and the education of the decision-making team and groups in the operation of the organisation and values, objectives and priorities of the groups. Rather than the analyst providing norms for decision-making, the interest groups are required to consider more deeply their own decision-making behaviour.

CHAPTER 9
PRACTICAL EVALUATION

Some of the ideas which have been presented in earlier chapters have been subject to practical trials. These experiments will be described separately, together with the conclusions which may be drawn.

THE ICI SECRETARIES

The secretaries at Fulshaw Hall, ICI's Central Management Services Department, operate a word processing system. They were invited by their top management to design the system, since it would be their responsibility to operate it, and to decide what the role of the secretary should be in this service. Enid Mumford from the Manchester Business School was called in to help them with their enquiries. They took a detailed look at the responsibilities of secretaries and how they distributed their time between word processing machines and typewriters. They already had two word processors but felt that the work was not organised round the machines as efficiently as it could be, and neither the secretaries nor their clients, the managers, were satisfied with the arrangement.

The design objectives were to increase the secretaries' job satisfaction and efficiency while at the same time effectively meeting the needs of the managers who were their clients.

The secretaries had already visited other ICI Departments which had word processing systems to see how these were organised. They had prepared a questionnaire so as to standardise the information which they obtained from all the departments. By this stage, they were fairly sure what they wanted in their own system, and what they wanted to avoid. I was kindly invited to work with the group for one day, and we proceeded to detail more carefully the secretaries' needs.

For the purposes of goal setting, the secretaries' needs were divided into Job Satisfaction needs and Efficiency needs. A careful distinction was made between the importance of maintaining the goals' performance objectives at their present levels and the importance of improving above the present levels. This idea was easily explained to the secretaries, and they were very quick to understand and draw up the two lists. (The information obtained is presented in an Appendix to this chapter.) The next exercise was for the secretaries to guess what the objectives of the managers were. These were written down and later, when the managers were called in, they were told that this had been done. They were asked to write down what their objectives were, and at the end of the afternoon, the two lists were compared. It was found that the two lists were essentially the same, although the managers expressed their objectives in more general terms, whilst the secretaries were more concerned with day-to-day matters, with which they were directly concerned. This had the purpose of identifying areas of potential conflict, and to everyone's satisfaction, there were none.

From this exercise, the conclusions were firstly, that without the restriction of the objectives being measurable, they tended to be very subjective and concerned with such highly important and subjective aspects of work as 'trust'. The second main conclusion was that the asymmetry of weights was obvious and readily accepted and estimated. The rank ordering was expressed verbally, which gave more information than the usual numerical rank ordering, because the secretaries were able to cross-check between the list of efficiency and job satisfaction lists and between the improve and maintain options.

I was not able to participate further in this problem, but in the final report of the analysis, the list of objectives which we obtained was used. The next stage was the setting up of alternatives and comparing them. A decision was reached and the analysis is now complete.

UNDERGRADUATES AND THE LIBRARY

As part of their degree course, some undergraduates were to undertake a systems analysis of the University Library, a system with which they were already familiar. For three hour-long sessions, I worked with a group of about eight.

We began by listing the unit activities which they do in the library - every activity which consumes their time. See Table 9.1.

The next step was to define the objectives of the library, so as to improve the lot of undergraduates. These goals were chosen, again avoiding the use of numerical measurements. See Table 9.2.

We proceeded to examine each goal separately, and defined a vocabulary with which to describe each goal. The present level of each performance variable was written down and the desired future level. At this point it was clear that the students realised the importance of precision in the setting of the vocabularies, and the definition of the words which they used in terms of one another. There was a fair degree of consensus within the group on the meaning of words, and where disagreement arose, they settled the issue by discussion. See Table 9.3 for this stage.

Our next exercise was to rank order the goals in terms of their importance. This rank ordering was again done verbally. We distinguished between the improving and maintaining options once more, see Table 9.4. In order to test the validity of truth functional modification, the students were presented with a page (see the Appendix) which asked them to be more precise about their present feeling about the overall library service. This exercise showed that some people upheld truth functional modification, but some did not. See Table 9.5.

The final stage in the exercise was to study briefly the available alternatives and to try to assess their impact on the goals which had been laid down. This stage was hurried and not investigated as fully as it ought. However, a few systems were measured verbally, and it soon became apparent which areas of their present knowledge required further investigation. See Table 9.6. For example, on the problem of security, it was not known what proportion of the present losses was due to deliberate theft, and what proportion was due to accidental loss. To accurately assess the impact of a secure system on overall losses, it is necessary to know what the existing reasons for loss are, and this information is not provided at present.

Despite the short time which could be devoted to this study, a few interesting conclusions emerged. It was clear that we were able to think

Finding books - in catalogue
- on shelves

Filling in forms - Short loan
- Reserve short loan
- Borrowing
- Recall

Using Abstracts

Queuing

Sitting down to work

Retrieving pinched books

Climbing stairs

Table 9.1
Activities in Library

1) Immediate finding books for essay writing
by subject or
by author or
by title

2) Improve security

3) Signing out books less tedious

4) Good opening hours

5) Better recall, i.e. quicker and more certain

6) More books

Table 9.2
Goals of Library

Measuring the Goals

4) **Opening hours**

Vocab: 9 a.m. - 1 p.m.   1-5 p.m.   5-7 p.m.   7-10 p.m.

	present level	desired level
Term	all	all
Week before term starts	9-5	all
Out of term	9-5	9-5

2) **Security**

Vocab: insecure $\equiv$ not secure

	present level	desired level
Accidental loss	very secure	
Deliberate dishonesty	very insecure	

3) **Signing out books**

Vocab: tedious $\triangleq$ time-consuming and boring and involving writing

Present level	Desired
tedious but not very tedious	not tedious at all and avoid writing, fun ($\triangleq$ not boring and not much restriction on time-consuming)

1) Find a book in catalogue by subject — fairly easy
   title — very difficult
   author — easy

   Find a book on shelves in library (assuming it is in the correct position) — easy

   difficult $\triangleq$ not easy

5) Recall

2 conflicting viewpoints - the person who recalls a book, recaller, and the person from whom the book is being recalled, recallee.

Only one person in the group had ever recalled a book, and so a measure could not really be made.

6) More books

2 ways of extending collection by greater variety
or by more duplicate copies

	present level	ideally
variety	good	very good
duplication	bad	very good

Table 9.3

Importance of improving goals

2	Security	Extremely important
1	Finding books	Very important
6	More books (duplicated)	Very important
3	Less tedious signing out	Important
4	Opening hours improved	Not important
5	Better recall	Not important

Importance of NOT decreasing, i.e. maintaining at present level

6 More books
4 Opening hours
3 Signing out
1 Finding books
2 Security
5 Better recall

Table 9.4

Subject No.	Statements	Truth Value	Subject's Remarks
1	good fairly good	fairly true true	
2	fairly good	fairly true	
3	fairly good fairly bad	true	almost fairly good
4	good fairly good	true	
5	fairly good	fairly true	
6	good fairly good	true	
7	good but not quite very good	fairly true personally very true	
8	good fairly good	true fairly true	

Table 9.5
Truth Functional Modification Tested

PLESSEY

Telepen system - books are barcoded.

Information can be stored online or on cassette for batch input.

System only registers withdrawn books, not the whole catalogue.

No security against theft.

Library needs restricted exit passage - e.g. turnstile.

Batch system represents no improvement - need online system with manual back-up, not using NUMAC.

Changes to Goals

(1) No change.

(2) Accidental loss - no change.

Deliberate dishonesty - improved to secure.

(3) Not tedious at all.

(4) No change.

(5) Slightly more certain.

No change in speed since rate determining factor is beyond library's control.

(6) Makes worse.

TELEPEN

As Plessey, but enquiries may be made directly to the system.

Changes to Goals

(1) Slightly better.

(2) Insecure.

(3) Not tedious at all.

(4) No change.

(5) Slightly more certain.

(6) Worse.

Need to know more about costs and the proportions of books lost from the library by phantom borrowing and straight theft.

Table 9.6
Measurement of Alternatives

clearly about the library's activities, and were able to identify areas where we lacked information, for example on the problem of losses and on the efficiency of the recall service, which only one member of the groups had had occasion to use. They did describe the existence of interactions between the importances of various attributes, although it was not possible to write them all down at the time. For example, such statements were made as "I don't care about the security so long as the books are on the shelves when I want them". They took care over the definition of the vocabularies and realied that precision was necessary. They made their own efforts to be consistent, without my prompting and although they knew nothing of the fuzzy set representation behind what they said. They knew, for example, that 'good' was not necessarily equal to 'not bad', and became aware of the complexities of meaning. Due to the lack of time available, it was not possible to perform a complete analysis of the problem, evaluating the alternatives according to the statements which were made, and to properly obtain the interactions which existed. But, we have demonstrated, I think, that the method is feasible, this far, and that it seems to be enjoyable and sufficiently searching to be useful.

THE COMPUTER UNIT AND DEPARTMENT OF COMPUTING

A further project has been carried out by the author to investigate the value of a fuzzy approach. At Durham University, the Department of Computing is housed in the same building as the computer service, known as the Computer Unit. They are linked formally as well as physically because both entities are under the direction of Dr. John Hawgood. Space is at a premium and, with expanding degree courses in the Department and heavy demand on the Unit's services, some changes will have to be made. This is the problem which I investigated. Any changes which could be made would have to be far-reaching, and require careful negotiation with the bodies concerned over a long period of time. For this reason my investigation was fairly low-key, as a first cycle, so as to avoid antagonising any of the parties involved before negotiations could be complete. Also, my purpose was to test the fuzzy approach, and this could be done without involving many people, prematurely to the main negotiations.

Analysis of the Problem

Apart from the Department and Unit, two other entities exist which may be affected by the problem, i.e. the University Library and the Department of Applied Physics and Electronics. If the link between the Computer Unit and Department of Computing were weakened, there could be a consequent strengthening of links between the university-wide services, i.e. Library and Computer Unit, and the academic departments of Computing and Applied Physics and Electronics, who already run a joint degree course. Individuals from the Department and Unit were interviewed separately.

The environment was analysed through these discussions, looking at the interactions which already exist between the four entities, and the staff involved.

An important constraint upon the problem and any possible solutions was the lack of buildings. As already mentioned, the Department and Unit share the same two-storey building. The Department occupies the top floor, the Unit the first floor, and the Unit and Department both have rooms on the ground floor. Offices have been acquired on Chemistry/Geology roof, which causes an undesirable physical separation of the staff, whilst offices in the main building are subject to further subdivision.

The Department of Applied Physics and Electronics is also physically divided, with some people housed in the Physics Department. The main Aplied Physics building is being extended, with offices being built on the roof, but further extension is probable. It is hoped that the whole Applied Physics Department may be brought under one roof. The Applied Physics and Electronics Department is also willing to accommodate Computing within the building, but only after all its own staff have been united.

I shall describe separately the investigations performed within each entity, since for reasons mentioned earlier, it would have been premature to embark upon full, joint discussions.

The Computer Unit

A programming adviser in the Computer Unit very kindly agreed to answer my questions, and altogether we spent about 6 hours investigating the problem.

After an initial discussion of the problem, we proceeded to draw up a list of objectives. A preliminary tree was drafted and then modified to produce the tree shown in Table 9.7. There were obvious links between the objectives, and the respondent drew the diagram (Figure 9.1) to illustrate them. The objectives were rank ordered and their importance measured verbally. See Table 9.8. Using a verbal scale, he assessed the present performance of the Unit towards fulfilling these objectives.

We devised two possible alternative strategies for the future, and described these in sufficient detail for them to be assessed using the verbal scales. Briefly, A1 involved establishing a complete information service, with the Computer Unit housed in the Library, as it is now. Some Unit staff would have to be housed on Chemistry/Geology roof because of the constraints on space. The cataloguing and query services would be expanded and possibly computerised. The second strategy, A2, involved no change with respect to the Library, but weakened the link between the Department and the Unit. In this case, the Unit took over the whole building, bringing all its staff together, and instigating other simultaneous policy changes. The preference ordering of the three possibilities
(A2, A1, present) was obtained by evaluating, for each alternative, the fulfilment of the subobjectives, and then combining these to produce an overall assessment. The verbal assessments are present in Table 9.9.

During this analysis, a number of points emerged. Firstly, in producing the assessments of future possibilities, the respondent found it easier to give comparisons, e.g. "a bit better", "a lot, lot better", rather than stating "well" and "very well". Both sets of responses have been recorded. Secondly, where uncertainty of prediction occurred, it ws easy to state the conditions under which one or the other circumstance would prevail. For example:

> "If the right living space is provided, and if the work is organised better, then Objective 5 would be fulfilled very well. Otherwise, poor working relationships and poor communication so that you don't know what you're doing could lead to disaster."

This seems to me a more constructive way of approaching the evaluation of such qualities than using probability estimates of either set of circumstances occurring.

Objectives of Computer Unit

Broad Objective:

To provide for all the needs of an expanding community of computer users as seen now and as can be predicted. To maximise the amount of information relating to computing reaching all sectors of academic pursuits with concern to providing the right information to the various branches of academia.

Sub-Objectives:

1. Educate computer users.
2. Inform the educated users.
3. Extend the network of computers.
4. Liaise with other departments, providing information.
5. Maintain and improve job satisfaction.

Goals:

1a. Review courses to naive users regularly so that they don't stagnate.
1b. Take account of feedback from users in assessment of courses.
1c. Branch into undergraduate and M.Sc. teaching and relate teaching to specialist areas.

2a. Inform users of changes in general software provision, e.g. editors, file systems.
2b. Inform specific user groups of changes which are pertinent to them. (Involves keeping records of what particular people do.)
2c. Inform users of new facilities and new software which may replace existing services.
2d. Maintain standard of day-to-day programming advice.
2e. Improve standard and amount of documentation.
2f. Improve cataloguing of documentation with respect to general topic books held in library.

3a. Provide easy access to computer databases.
3b. Provide easy access to computers at any university in country and elsewhere.

4a. Advertise services within other departments.
4b. Advise on problem solving at a level where Unit could recommend purchase of departmental systems.

5a. Maintain friendly, relaxed working relationships.
5b. Improve in-house communication.
5c. Improve delegation of responsibility and duties to avoid areas of neglect.
5d. Improve feedback from management on assessment of own performance.
5e. Improve feedback from users on individual performance.

Table 9.7

Rank Ordered Goals

1	5c	Crucial
2	1a	Very Very Important
3	1c 1b 4a 5a	Very Important
4	2a 2b 2e 2d	Important but not Very Important
5	5b 4b	Important
6	5b 5e	Very Very Desirable
7	3a 2e	Very Desirable
8	3b	Desirable
9	2f	Sort of Desirable

Table 9.8

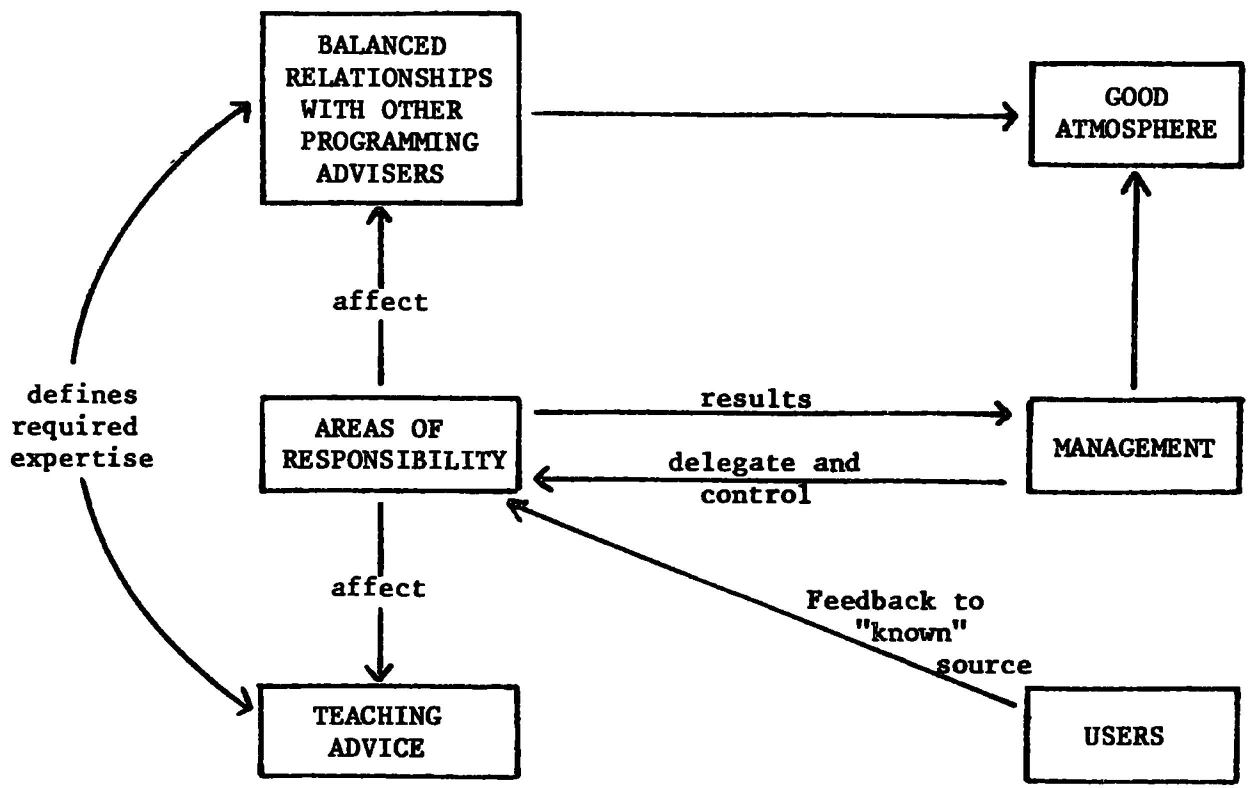
Verbal Assessment of Present Position and Two Alternatives

Goal		Alternative 1	Alternative 2
1a	Well	Well (Same)	Well (Same)
1b	Sort of Well	V Well (A bit better)	Sort of Well (Same)
1c	V V Badly	Well (Better)	V V Badly (Same)
2a	Badly	Badly (Same)	Badly (Same)
2b	V Badly	Well (A lot lot better)	V Badly (Same)
2c	Not Well but Not Badly	Reasonably Well (A Small Amount)	Not Well but Not Badly (Same)
2d	V Well	V Well (Same)	V V Well (Better)
2e	Well	V Well (Really improved)	Well (Same)
2f	V V Badly	V V Well (Tremendous improvement)	V V Badly (Same)
3a	V V Badly	V Badly (Improved)	V V Badly (Same)
3b	V V V Badly	V V V Badly to Well *	V V V Badly (Same)
4a	Not Well but Not Badly	Well but Not V Well (Improved)	Not Well but Not Badly (Same)
4b	Not Sure	Not Sure (Same)	Not Sure (Same)
5a	Well	Bad to Well (Better or Worse)	V Well (Improved)
5b	Not Well but Not Badly	V Badly or Well	Well (Improved)
5c	Badly	Not Sure	Well (Improved)
5d	V V V Badly	Not Sure	Well (Improved)
5e	V Badly	Well (Improved)	V Badly (Same)

* The uncertainty in this response depends on the improvement of delegation to ensure the appropriate effort to achieve good results. The other uncertain responses also depend upon the achievement of other objectives, or upon the degree of planning in the design of the change.

Table 9.9

Figure 9.1 The Links that exist between Aspects of the Computer Unit



The respondent's comments at the end of the analysis were illuminating too. He considers himself a wholistic judge of such alternatives, but said he might have ended up breaking things down in such a way if he had had to make such a decision on his own. The reiterative process was useful as one thought about the problem.

On the question of setting objectives and numerical versus verbal measurement, he said that it is easy to make measurements of objectives, but that linear programming techniques of combining this information are not necessarily good guides to making decisions. Such techniques also encounter the problem of the vagueness of the constraints, and the difficulty of evaluating what will happen in the future. A program to process such information would be interesting just to see what emerges, rather than as a decision-maker, but a program to handle verbal information would be equally interesting. In using the verbal scale, he said that he tended to reduce the statements to a sort of numeric code, and in this way it was only another variation on the 0(unimportant) - 10(very important) type of scale. He did think that the ways in which verbal and numerical scales were used are different. With a numerical scale, each alternative could be placed on the scale by considering its merits alone. A verbal scale required checking back to remember whereabouts on the scale previous alternatives had been placed.

This last is an interesting point. Numerical scales may be used to consider each alternative in isolation because the 0-10 concept remains fixed in one's mind in some sort of 'absolute' sense. The verbal scale is less fixed, and requires constant re-iteration as the alternatives are evaluated. So long as the number of alternatives does not grow too large, verbal assessment could ensure a useful built-in reiteration mechanism.

The Department of Computing

A lecturer in the Department kindly assisted in this investigation.

Again, we began the analysis by discussing the problem and drawing up a tree of objectives. This list of objectives involves a large amount of interaction between different objectives, in the sense that improving any single objective will also cause improvement in many of the others.

For example, the Department is expanding, with a new lecturer just

Objectives of Department of Computing

Main Objective:

Improve morale within Department.

Sub-Objectives:

1 Improve undergraduate teaching.

2 Improve status of Department from eyes of other departments.

Goals:

1a Offer more disciplines within Department by having more specialists on staff to provide more varied courses.

1b Recognise computing as a laboratory subject and supply machines and appropriate staff.

1c Remove restrictions on undergraduate computing time and avoid interfering with Unit's services.

1d Improve organisation and communication.

2a Improve interface and communication with Applied Physics on Joint Honours course.

2b Distinguish clearly between Unit and Department.

Table 9.10

appointed, and the first group of 3rd year students about to join the joint Honours course in Computing and Electronics. The informal system of in-house communication which was previously effective is becoming less so now, as outside Departments become involved. A more effective communications system would be desirable, but this must be achieved, as far as possible, without disrupting the existing relaxed atmosphere.

In this case, the overriding objective appeared to be to improve the morale of the Department, and the respondent was happy to evaluate alternatives on this attribute alone, without undergoing any further formal analysis. One part of the morale is the status of the Department, and already the Unit and Department are as one in the eyes of many members of other university departments. Thus, the alternatives which are likely to be suggested under the problem to be investigated here may be considered with respect to their effect on the link between Unit and Department.

As a first approximation, two alternatives and the present situation may be rated as follows:

A1	Sever completely the link between Unit and Department, making the Department equal in every respect with all other Departments.	Very good
A2	The present position	Poor
A3	The Department and Unit completely integrated with teaching done jointly by Departmental and Unit staff.	Very Poor

This may seem an over-simplification and I apologise if I do the respondent an injustice. However, these statements of alternatives are simple, and require further development, with respect to how undergraduate teaching may be improved. For example, the other main sore point with the Department is that it lacks a dedicated machine, properly maintained by own technicians. If any of the alternatives involved improvements to or deteriorations of the present arrangement, then this would be important in determining the success of any strategy. So far, this aspect does not appear to have been taken into account in this preliminary, first cycle evaluation of the alternatives.

There are few conclusions which can be drawn from this study, except that we observe how a preference ordering may be made using one attribute only. However, I must emphasise that this was not fixed, and that the respondent did express a strong willingness to re-consider the problem more fully as it developed in the future.

THE WORK AT "JOZEF STEFAN" INSTITUT

I have already mentioned (Chapter 7) the work of Rajkovič and Bohanic at the "Jozef Stefan" Institute, Ljubljana, Yugoslavia, in developing a computer program to help in the construction of $\underset{\sim}{\overset{\sim}{\phi}}$, and I shall now describe it more thoroughly.

Their program, DMP, supports a dialogue between the computer and decision-maker, so as to:

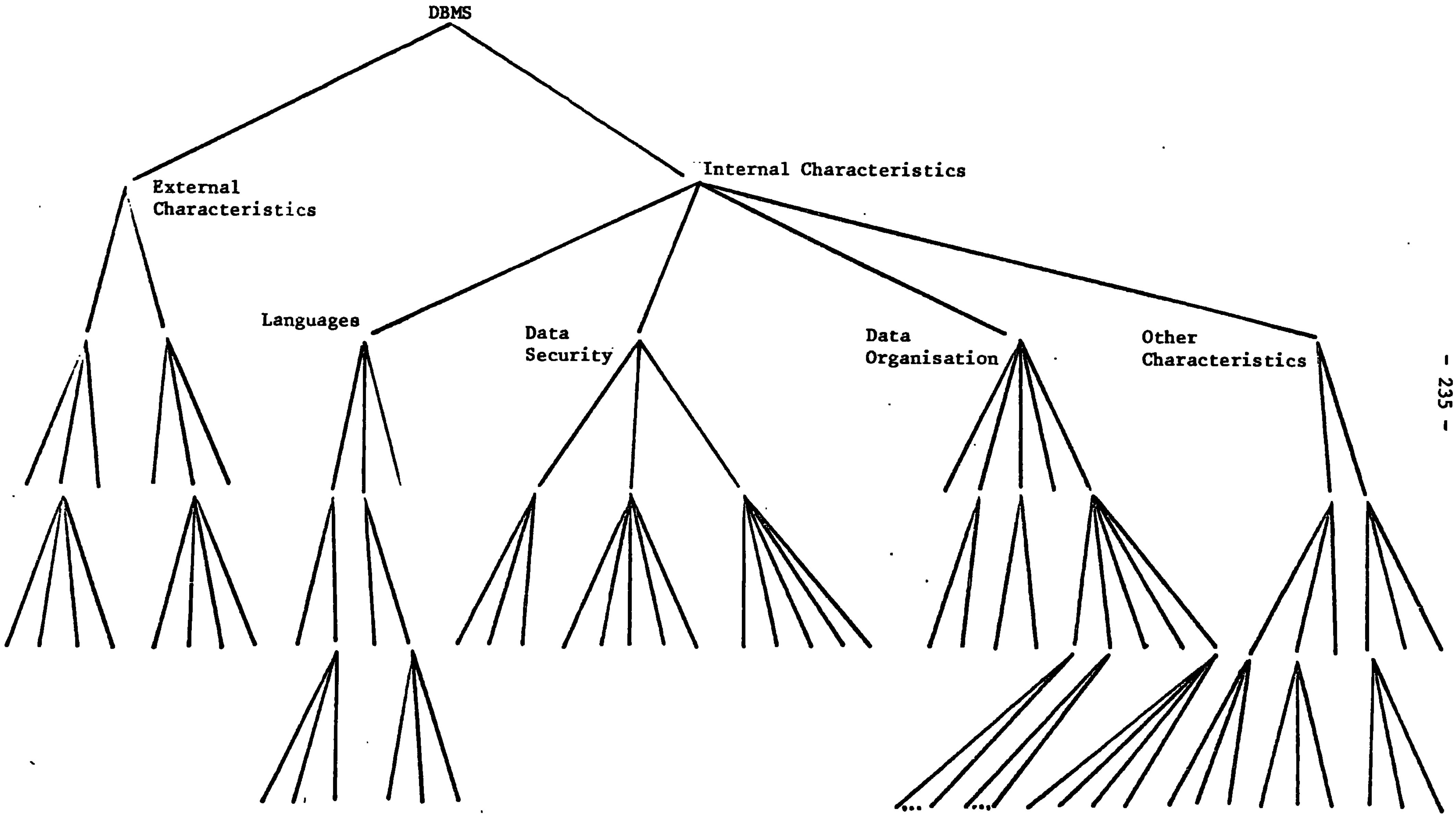
1) define a decision space,
2) obtain insight and 'utility' construction, $\underset{\sim}{\overset{\sim}{\phi}}$,
3) evaluate alternatives.

The emphasis of the program, though, is on the second point. Typically, they use 3 to 5 performance variables and about 30 n-tuples. Two examples are contained in this chapter's appendix.

They encourage the decision-maker to provide percentage weights for each performance variable, to express their relative importance. In practice they find this useful as a way of forcing people to think about all the performance variables at once, so as to distribute the percentage weights amongst them.

The group is carrying out a project to compare the results of a fuzzy approach with those of a more traditional approach, namely that of Dujmovič. The problem is on the selection of a database management system. A tree of the performance variables has been constructed, and one branch of this is being investigated under the two approaches. See Figure 9.2 for a skeleton of this tree. Note that there are never more than five branches from any node.

Figure 9.2 Decision Tree for Database Management System



They found that, using the fuzzy approach, it was not difficult to identify the partial Φ , represented by each branch of the tree. The process of aggregation and evaluation was simple and clear. They found that in some cases the full Φ tables were large, and so general, descriptive rules summarising the information would be useful. In comparison, Dujmovič's method requires the decision-maker to specify precisely a logical operator which aggregates the information. This task ws difficult to perform precisely enough, and also prevented backtracking through the analysis, to identify the sources of particular results.

The following table shows the results obtained by both methods for 3 database management systems.

	Fuzzy Method	Dujmovič Method (0 - 100)
DBMS1	very good	70.88
DBMS2	good	61.51
DBMS3	very good	68.47

During the analysis, the Dujmovic method forced them "to cheat" in binary situations, where the performance variables required Yes-No answers. The values of 0 and 1 were having too great an influence because of the continuous logic involved. Mgr. Rajkovič also comments that the first and third are equal under the fuzzy method, which is a more realistic conclusion than 68.47 vs 70.88. "Results precise on two decimal figures are stupid." To test this apparent equality is genuine, the fuzzy approach allowed backtracking through Φ , but the Dujmovič method loses all such information and so this cannot be done.

Their program is being used to compare the fuzzy and classical approaches in a very pertinent test of the methods. Of the examples in the appendix, one concerns a group of school teachers who were evaluating computer hardware for secondary schools, and in the other, an individual explored his decision space prior to purchasing a camera.

From this work we may conclude that fuzzy information may produce the same decision as information which has been precisely measured and aggregated. The information is presented in such a way as will allow identification of the sources of particular answers. Also, the dialogue is reported by users to be enjoyable and useful.

THE DERBYSHIRE COUNTY LIBRARY PROJECTS

For a period extending over about none months, I was involved in a project with some of the staff from Derbyshire County Library. Using the BASYC methodology, we investigated the problem of how a branch library, Long Eaton, could be made more attractive to users. After the departure of the Durham team, a further project was set up by the staff themselves to find the best layout of stock and public service areas at Long Eaton library. Some members of the original team were also involved in the second project.

The following comments are extracted from a report by K.H. Mantell, Assistant County Librarian, who was involved in both projects. This report was presented at a meeting of people involved in other library projects using the BASYC approach and the derived Verbal Assessment techniques.

> "BASYC, Benefit Assessment for System Change, is of course about effecting change, and if I had only two more words with which to describe it I would say that it was about Participation and about Measurement. Of both we had full experience in the original Durham project.
>
> "We fully took the point that if change is to be effective it should be planned with the members of the staff concerned or with their representatives. We thought that the prescription in original BASYC for measuring staff satisfaction was over-fussy and indeed impossible to apply without the aid of outside consultants. And we saw too, experienced too, some of the problems of Participation: how to secure genuine informed articulate representation: how to form an in-group without also forming an out-group. That would be an aspect we would have in mind in any post-Durham projects. So, Participation.
>
> "Now Measurement. On this the great claim of BASYC was that it enabled you to quantify benefits without necessarily giving them monetary values. You determined what your goals were; you ranked them; you weighted them, percentage wise, you scored the effect of what you were going to do, you multiplied your weights by your scores and you got your utility contributions. You had a total benefit. You also had a niggling doubt. Had you achieved a false precision?

. . . . .

"After our own project with Durham had ended we were very glad to hear of a completely alternative method of scoring, using verbal assessment rather than numbers. That was something we would want to try."

and on the second Long Eaton project:

"A list was compiled of service components with associated benefits, and the second meeting produced a ranking order of user goals and staff goals. And subsequent meetings tested by Verbal Assessment, using Worksheet B5, alternative strategies for the layout of Long Eaton library. The Worksheet was found to be helpful in ensuring systematic consideration of the effect of each strategy on all the important goals for each group, but in fact the team slipped into a quasi-numeric scoring - X, very Y, Y, very Y - rather than a purely verbal one.

"The merit of the whole process was seen to lie in the preliminary discussion of objectives and in the detailed subsequent discussion of alternative strategies which did generate ideas about the layout at Long Eaton which the local managers freely admit would not otherwise have been produced. So, perhaps here Participation was more important than Measurement."

The Worksheet B5, to which he refers, is a simple worksheet developed by Dr. John Hawgood to assist the design team in their assessment of alternatives with respect to many objectives.

Throughout the original Long Eaton project, Mr. Mantell did not express, to me anyhow, any doubts about the validity of the Weight, Score, Combine approach. However, he has said later that doubts did exist all along. The processes of assigning weights and scores meant that a figure was always required. He said that such a decision aid would only be used to reinforce a more intuitive decision. If the answers did not look right, then it would not make one change one's mind, but instead there was a tendency to work backwards adjusting the figures until the answer came out right.

The merit of the approach lay in the structure which it imposed upon the discussions - determining groups and objectives and carefully describing the alternatives. The team was forced to think and look more deeply. The reiteration of the process was very important, because ideas were continually developing.

OTHER LIBRARY PROJECTS

The idea of verbal assessment has been used within a full-scale investigation of the American Library Association Planning Process. Durham University undertook an examination of this Process, with librarians from Public Libraries at Sheffield and Islington, under the sponsorship of the British Library Research and Development Department. The results of this investigation are in an unpublished report written by Lt-Col. W.E.M. Morris, who acted as the analyst in both cases. The studies at Sheffield and Islington were carried out separately, in parallel. A joint meeting was held at the conclusion of the project. The analysis in both cases began by appointing a design team of staff members to carry out the analysis.

The Islington team produced a list of 16 Interest Groups, and identified 30 goals which could be relevant to them. They found it a difficult task to define the relative importance of the goals to the Interest Groups, involving much discussion and heart searching. These are the problems usually encountered in such role-playing activities. User surveys were carried out to reduce this uncertainty, and a number of verbal measures of importance were changed before the completion of the second cycle.

The design team devised three separate strategies which were measured using the previously mentioned Worksheet B5. The design team's task was complicated by having to combine the opinions of all sixteen Interest Groups, to yield an overall opinion on the strategy's projected performance. However, they were able to decide which groups fell into the following classes:

(1) Strategy A preferred over Strategy B, both being better than Strategy O.

(2) Little preference if any for either Strategy over Strategy 0.

(3) Strategy 0 preferred over both alternatives.

The second cycle was simplified by reducing the number of groups to 10, and using 24 goals instead of 30. A third strategy was devised, and again this was compared with the status quo, Strategy 0, using Worksheet B5.

The Sheffield team had a similar pattern of behaviour - many groups and goals first time around, dissatisfaction with role-playing and a survey undertaken. This team also benefited from a high degree of involvement by local Councillors and community representatives. The table of importances for interest groups and goals which they produced is in the appendix to this chapter.

From this study, we see some of the problems and successes of participation in decision-making. The recognition of uncertainty by the design team in their estimates of importance is very healthy. The fact that the analysis could be performed without the use of numerical measurements of importance or degree of fulfilment of the objectives is encouraging.

SUMMARY AND CONCLUSIONS

The investigations described in this chapter have demonstrated, I believe, the feasibility of such an approach. The traditional methods of multi-attribute decision analysis required the input of a large amount of information, yet the decision-makers themselves say that the output of such techniques would not be used to change their minds away from an intuitive decisions, but only to reinforce what they already know. The important part of decision-making is in the analysis of the problem, identifying the relevant features and the structured discussions within a participative process.

Specifically on the fuzzy approach, we have demonstrated the asymmetry of weights, the use of verbal measurements, the subjective aggregation of vague information and the learning process involved. There is a tendency for verbal assessments to be used as an ordinal measure only, but this seems to provide enough information. There is a demand for an automated technique, but its design must be influenced by the way in which people want to use it, and not by the particular favoured notions of the designer. The analysis seems to have been enjoyable for the participants, and the introspection required is useful.

APPENDIX

PRACTICAL RESULTS

These objectives were weighted as follows:

Scale

Extremely important
Very, very important
Very important
Quite important

Weighting of Objectives
Job Satisfaction

Working Relationships	1
Very important to maintain	
Important not to decrease	
Secretary's Competence	6
Very important to maintain	
Extremely important not to decrease. (without an acceptable level good working relationship cannot exist)	
Work without Supervision	3
Fairly important to maintain	
Very important not to decrease	
Promotion Opportunities	4
Important to improve	
Important not to decrease	

Job Satisfaction Objectives and Related Factors

Good Working Relationships	Require	Trust	EXTREMELY IMPORTANT NOT TO DECREASE
VERY IMPORTANT TO MAINTAIN		Taking over parts of manager's job	IMPORTANT TO INCREASE
IMPORTANT NOT TO DECREASE		Providing high quality service, administration 1 information 2 word processing 2 typing 4	
Ability to Work without Supervision		Maintenance of skills	Very important
FAIRLY IMPORTANT TO MAINTAIN		Responsibility	
VERY IMPORTANT NOT TO DECREASE		Trust	
Good Promotion Opportunities		IMPORTANT NOT TO DECREASE	
Necessary Skills	TECHNOLOGICAL SKILLS LESS IMPORTANT		MANAGERIAL SKILLS VERY IMPORTANT
Good Environment		IMPORTANT	

Committee A agreed that the Secretarial Group had the following job satisfaction and efficiency objectives.

Secretaries
Job Satisfaction Objectives

Maintain	Improve		Ranking 1 = Most importan
✓		Good working relationships	1
	✓	Varied and challenging work	2
	✓	Responsibility	2
✓		Work without supervision	3
	✓	Promotion opportunities	4
	✓	High level use of skill	5
	✓	Develop new skills	5
	✓	Help manager by taking over certain parts of his job	5
	✓	Secretary provides high quality service for manager e.g. information, word processing, typing, administration	5
✓		Manager can trust secretary's competence	6
✓		Work in pleasant environment	7

Secretaries
Efficiency Objectives

Maintain	Improve		Ranking
	✓	Access to word processing machines	1
	✓	More word processing machines	1
✓		No increase in centralized control (e.g. organics)	2
	✓	More knowledge of what w.p. machines can do in future	3
	✓	W.P. machine with capacity for: a) Registry b) Electronic mail c) Producing diagrams d) Mathematical calculations	3
	✓	Better accommodation	4
	✓	Better planning by authors	5
	✓	Better presentation by authors	5
	✓	Better knowledge of author of w.p. capability	5

Weighting of Objectives

Efficiency

Access to w.p. machines

EXTREMELY IMPORTANT TO IMPROVE
EXTREMELY IMPORTANT NOT TO DECREASE

No increase in centralized control

VERY, VERY IMPORTANT TO MAINTAIN
EXTREMELY IMPORTANT CONTROL DOES NOT INCREASE

Accommodation

VERY IMPORTANT TO IMPROVE
VERY IMPORTANT NOT TO DECREASE

*Possible conflict area

Knowledge of word processing capabilities in general

(understanding of these)
(can't be recognized without knowledge)

QUITE IMPORTANT TO IMPROVE
IMPORTANT NOT TO DECREASE

Knowledge of capability of wordplex and other machines

IMPORTANT TO IMPROVE
IMPORTANT NOT TO DECREASE

Author's planning and presentation of work and knowledge of w.p.

IMPORTANT TO IMPROVE
VERY IMPORTANT NOT TO DECREASE

* POSSIBLE CONFLICT AREA

Committee A's Perceptions of Managers' Objectives

Job Satisfaction

Now and Future	Maintain	Increase
To do job to best of his ability	✓	
Responsibility for making management decisions	✓	
Good working relationships with colleagues	✓	
Promotion opportunities	✓	
Varied and challenging work	✓	
Develop new skills and knowledge		✓
To persuade and influence other people	✓	
Get sense of achievement	✓	
To work in pleasant environment	✓	

Efficiency

Now	Maintain	Increase
Ease of correction with word processing	✓	
Less time required to check work		✓
Fast service		✓
Confidence in secretary's competence	✓	
Well presented reports		✓
Manager able to delegate job to secretary to give himself more time		✓
Manager can get information from secretary		✓
To have good secretarial service		✓

Future	Maintain	Increase
Faster appropriate information to and from other groups (by word processor, computer, or telephone)		✓
Verbal, not written communication to secretary. For typing or word processor and from secretary to manager		✓
? Visual tele conferencing		

Committee A's Perceptions of Top Management Objectives

As for managers plus:

Acquiring necessary resources from ICI

Running an efficient organization

Ensuring good relationships between Fulshaw Hall and rest of ICI

Providing job satisfaction for all staff

Making effective policies and setting standards

Ensuring two way communication between Fulshaw Hall staff and ICI top management

TOP MANAGEMENTS LIST OF THEIR OWN OBJECTIVES

LEADERSHIP WITHIN ICI RE. EXPLOITATION OF TECHNOLOGY

VALUES

- PROFITABILITY
- MAJOR CHANGE
- INTEGRATION
- SCOPE - ACCROSS BOUNDARIES/IMPACT
- INNOVATION
- CORPORATE
- LONG TERM/MISSION/ON GOING/DEVELOPMENT
- SUCCESS
- CAREER PROSPECTS FOR STAFF

FUN/COMMITMENT

FULSHAW HALL EXPLOITING TECHNOLOGY FOR ITS OWN USE

PRODUCTIVITY AND WORK ENRICHMENT

FACILITATE LEARNING OF DIFFERENT GROUPS

EFFICIENCY AND PRODUCTIVITY
WHAT AS WELL AS HOW

CREATE RESEARCH TYPE ENVIRONMENT

OTHER MANAGEMENT OBJECTIVES AS DEFINED BY MANAGERS

TO ENSURE THE ACCEPTABILITY OF TECHNOLOGY
HARDWARE/SOFTWARE/COMMUNICATIONS CAPABILITY

TO DEVELOP THE CONCEPT OF THE OFFICE OF THE FUTURE

TO ENSURE THAT TECHNOLOGY IS FLEXIBLE ENOUGH TO ACCEPT NEW DEVELOPMENTS

TO ENSURE THE WORD PROCESSING PRODUCTIVITY OF: SECRETARIES / MANAGERS

TO MATCH RESOURCES TO NEEDS

TO DEVELOP STRATEGIES TO TAKE THE COMPANY INTO THE FUTURE

TO PRODUCE AN <u>IMPROVED SECRETARIAL SERVICE</u>

DEFINITION OF 'IMPROVED SECRETARIAL SERVICE'

TO PROVIDE SCOPE FOR SECRETARIES TO BECOME PART OF THE MANAGEMENT RESOURCE

TO REDUCE THE AMOUNT OF TYPING - FEWER KEY DEPRESSIONS
- FREE PEOPLE TO IMPROVE THE SERVICE

TO HELP DEPARTMENTS TO MANAGE THEIR TIME

TO OPTIMISE THE USE OF THE BOSS, SECRETARY RESOURCE

TO ENSURE THE SECRETARY FEELS VALUED

TO ENABLE LESS TIME TO BE SPENT DEALING WITH PAPER AND TYPING; LESS REDRAFTING

THE DELEGATING OF TASKS TO THE SECRETARY

TO MAINTAIN FLEXIBILITY (WORK, ORGANIZATIONAL AND TECHNOLOGICAL)

TO INFLUENCE THE FUTURE

TO HAVE MORE INFORMATION ON ORGANIZATIONAL VALUES

TO UNDERSTAND EMPHASIS AND PRIORITIES

TO PROVIDE DEVELOPMENT OPPORTUNITIES AND THE TIME TO REALISE THESE.

Please indicate which of the following statements is most correct.

The library service is extremely good.
The library service is very good.
The library service is good.
The library service is fairly good.
The library service is fairly bad.
The library service is bad.
The library service is very bad.
The library service is extremely bad.

If you feel that your true opinion lies somewhere between a pair of these statements, try the next stage.

Choose the pair of statements you agree with most:

The library service is ______________

The library service is ______________

Just as we had used a range of 'goodness' to describe the library service, we will use a similar range to describe the truth of the statements you have chosen. For example, the range could be:

extremely true   very true   true   fairly true

Try to pick a 'truth value' to describe your feelings about the truth of your chosen statements.

____

____

DECISION MAKING PROCESS 01-JUL-79

Operating file: D:EXPER.DMP /OLD

UTILITY = COMPUTER HARDWARE SELECTION FOR SECONDARY SCHOOLS - EXPERTS

UTILITY KNOWLEDGE CONSTRUCTION

Group name: EXPERTS

File: D:EXPER.DMU/OLD

DECISION SPACE LIST

PS=(v good;good;acc;poor) 25%

CAL=(v good;good;acc;poor) 25%

CML=(good;acc;poor;none) 15%

ADMINIST=(good;acc;poor;none) 10%

MAINT=(none;min;acc;demand) 05%

LOCAT=(v conv;conv;l conv) 10%

PRICE=(low;med;high) 10%

UTILITY=(ex;v good;good;r good;acc;cond acc;not acc)

32 combinations entered.

LIST (ALL)

PS	CAL	CML	ADMINIST	MAINT	LOCAT	PRICE	UTILITY
v good	v good	good	good	none	v conv	low	ex
v good	v good	good	good	none	v conv	med	v good
v good	v good	good	good	none	v conv	high	good
v good	v good	good	good	none	conv	low	good
v good	v good	good	good	none	conv	med	

							good
v good	v good	good	good	none	l conv	high	acc
v good	v good	good	good	min	v conv	low	good
v good	v good	good	good	min	v conv	med	v good
v good	v good	good	good	demand	v conv	high	v good
v good	v good	good	acc	none	v conv	low	v good
v good	v good	good	acc	none	v conv	med	good
v good	v good	acc	poor	acc	conv	med	r good
v good	v good	poor	poor	none	conv	med	v good
v good	v good	poor	poor	acc	v conv	low	r good
v good	good	good	good	none	v conv	low	good
v good	good	good	good	min	v conv	low	good
v good	acc	acc	acc	demand	v conv	med	good
v good	acc	acc	acc	demand	v conv	high	good
good	v good	good	good	none	v conv	low	good
good	v good	good	good	min	v conv	low	good
good	good	good	good	none	v conv	low	good
good	good	good	good	min	v conv	low	

							good
good	good	good	acc	none	v conv	low	good
good	good	acc	acc	min	conv	med	r good
good	acc	good	good	acc	v conv	med	good
good	acc	acc	acc	acc	conv	med	acc
good	poor	poor	poor	acc	l conv	med	cond acc
acc	good	acc	acc	acc	conv	med	acc
acc	acc	acc	acc	acc	conv	med	acc
acc	poor	acc	poor	acc	conv	med	cond acc
acc	poor	none	none	none	conv	low	cond acc
poor	poor	none	none	min	v conv	low	not acc

Operating file: D:PHOTO.DMP /OLD

UTILITY = PHOTO CAMERA QUALITY

UTILITY KNOWLEDGE CONSTRUCTION

Group name: PHMARE

File: D:PHMAR.DMU/OLD

DECISION SPACE LIST

PRICE=(100;100 150;150 200;200 250;250) 15%

TYPE=(aut;s aut) 15%

FACIL=(many;q few;few;v few) 30%

PERF=(v good;f good;fair;r poor;poor) 30%

CONV=(very;fairly;conv;not very;not) 10%

UTILITY=(exc;good;accept;poor;n accept)

31 combinations entered.

LIST (ALL)

PRICE	TYPE	FACIL	PERF	CONV	UTILITY
100	aut	many	v good	very	exc
100	aut	many	v good	fairly	exc
100	aut	many	fair	very	good
100	aut	q few	v good	fairly	good
100	aut	q few	r poor	very	accept
100	aut	few	v good	fairly	

					accept
100	s aut	many	f good	very	exc
100	s aut	few	fair	fairly	poor
100	s aut	v few	f good	fairly	accept
100 150	aut	many	v good	very	exc
100 150	aut	many	f good	fairly	good
100 150	aut	q few	f good	fairly	good
100 150	aut	q few	fair	fairly	accept
100 150	aut	few	v good	fairly	good
100 150	aut	v few	f good	fairly	poor
100 150	s aut	many	f good	fairly	good
150 200	aut	many	v good	very	exc
150 200	aut	v few	f good	fairly	poor
150 200	aut	v few	poor	not	n accept
150 200	s aut	many	v good	very	exc
150 200	s aut	v few	v good	very	good
150 200	s aut	v few	f good	very	poor
200 250	aut	many	[illegible]	[illegible]	

					poor
200 250	aut	many	poor	not very	n accept
200 250	aut	few	r poor	not	n accept
250	aut	many	v good	very	exc
250	s aut	q few	r poor	fairly	n accept
250	s aut	q few	poor	conv	n accept
250	s aut	few	fair	very	n accept
250	s aut	few	r poor	very	n accept
250	s aut	v few	r poor	very	n accept

CHAPTER 10
CONCLUSIONS

In writing this thesis, my purpose has been to reconsider the multi-attribute decision-making problem. I have argued that the existing methods for tackling this problem are unsatisfactory when extended to problems with a strong human factor. The reasons for this are many, but boil down to the belief that the traditional mathematical methods may not be used to model human reasoning. These require that the decision-maker should obey some axioms of 'rationality' and should express his feelings about the value of alternatives numerically. In practice this is inadequate.

A suitable tool for dealing with the real problems of multi-attribute decision-making was sought, and fuzzy set theory and fuzzy reasoning seemed suitable. This tool was appraised and methods suggested for how it might be applied in practice. The previous attempts to apply fuzzy set theory to multi-attribute decision-making were considered, and deemed ad hoc.

A new approach to the problem was suggested, together with a practical recipe for its use. Worked examples and real trials have demonstrated that such an approach has potential as a personal or management tool.

Such an approach relies upon the concept of participative decision-making. Despite the extra effort involved, participative decision-making is gaining credence as a constructive and useful way to design and implement systems. The present climate of worker participation, educated users and shop-floor involvement in decision-making ensures that autocratic management practices are becoming unacceptable. At the same time, there is much to be gained in achieving successful implementation if the users of a system are involved in its design. The end-product of a decision, i.e. an implemented choice, has received less attention than it should, but it is hoped that participative decision-making will help to achieve greater success here.

In the participative decision-making process, the analyst takes a back seat, relinquishing much of the status and control which he once enjoyed. The decision aid should be free of his value system, even if this is only in the way in which information is aggregated. The choice of the method of processing information must be open to the decision-makers, so that truly participative decision-making will consider the meta-levels of the problems as well. The analyst must become an educator, in the classical sense.

The analyst's status is derived, in part, from the decision-making technique which he advocates. If this technique is complex and highly mathematical, say, then the analyst, who understands it, will retain control. Participative decision-making demands techniques which are easy to understand and apply. Because of the large amount of information processing involved in decision-making, the decision-maker needs a structure upon which to base his ideas. Multi-attribute utility theory did, in fact, provide such a structure, although the language involved was difficult for users to trust and understand. A technique based upon the use of verbal statements and a means of choosing the way to combine information seems best.

Fuzzy set theory and fuzzy logic provided the mathematical foundation upon which to build such a theory. In complex or human systems, there is an exchange between precision and significance, and it was this realisation which fostered the conception and development of fuzzy set theory, with its pragmatic attitude towards problem solving. Despite continuing objections, fuzzy set theory enjoys an expanding sphere of interest. Perhaps its chief contribution has been in making subjectivity respectable, and in removing the spurious need to express subjective opinions numerically. 'Rationality' had been defined as obedience to axioms whose only purpose had been to extend the realm of objectivity to include subjective phenomena as well.

FURTHER WORK

Looking towards the future, it would seem that fuzzy set theory might play a role in the development of the next generation of management tools. The work of people such as Machin (1977), Radford (1975), Howard (1975) and Boxer (1978) show new approaches based on the need for communication within an organisation, and an expression of the opinions of the people involved. We might speculate that this represents a movement towards the top of Maslow's (1954) hierarchy of human needs, i.e. to fulfil the need for growth and self-actualisation of the individual. Rather than obey prescribed decision-making behaviour, the decision-maker must play the meta-game and understand better his own motives. This is a demanding task, and does not permit the decision-maker to relinquish the chore of decision-making to an automated technique. Rather, the burden of the analysis is with the decision-maker himself, so that the decision-maker is a vital part of the decision-making process.

These ideas must determine the way in which multi-attribute decision-making tools will develop. If the process is to become computer-assisted (as those mentioned above already are), then there are some guidelines which it ought to follow. Most importantly, such a program is intended to educate the user by gaining, not knowledge, but insight into his own decision-making behaviour. His information-processing capacity may be expanded, because a greater amount of information is readily available. Such a program is intended to train and not necessarily to replace the decision-maker, although he should have the option to hand over responsibility to the program, once satisfied with the results.

This requires that the program to assist multi-attribute decision-making should be flexible. Such a program may be put to many uses, both personal and commercial, and it should be able to cope with all such cases. Traditional decision-making tools such as multi-attribute utility theory and mathematical expectation should be available to the decision-maker when he wishes to apply them over any range of the decision space. Such methods are examples of information aggregation processes, and if their axioms of rationality appeal to the decision-maker, then he should be able to use them as he wishes.

The program should also be flexible enough to accommodate the highly reiterative nature of such an enquiry. The decision-maker should be able to move freely to and from previous steps, adding and deleting attributes, vocabularies, alternatives and preference statements. The process of exploration should result in the insightful generation or adoption of heuristic rules of problem-solving, and a handy means of updating a list of heuristics should also be available.

Such a program should enable the investigator to look at both the problem and the meta-problem. The analysis of the problem should follow the lines outlined in Chapter 8, i.e. definition of objectives, recognition of uncertainty and assessment of status quo. The meta-problem stages involve the definition of an ideal solution, rank ordering the objectives and considering the alternatives. In these stages, the computer acts as an information handling device, and cannot contribute directly to stages such as analysis of variance, devising new alternatives and the statement of heuristics, which require imagination and insight. The division of labour between man and machine must be recognised, and we should avoid the pitfalls of giving the machine more of the task than necessary.

The process of finding the preference ordering of alternatives and the rules which are associated with choice are particularly the concern of such a program, however. It is assumed that the decision-maker can state the satisfaction he obtains from the ideal solution, as well as from the status quo and perhaps an intolerable solution. He can devise alternatives and describe their outcomes, and state a 'utility' value which they also represent. The decision-maker may explore his decision space in one of two ways - either he generates the description of an alternative himself and assesses its value to him, or the computer generates a description by changing a previous description slightly and suggests it for the decision-maker to evaluate or compare with the computer's assessment derived from previous information it already has available. This second method provides no description of the policy whereby a particular alternative as described may be achieved. Both methods may be checked by presenting all the aalternatives to which the decision-maker has given the same 'utility' value, and checking that they are indeed equally preferable.

The eventual output of such a program should be a table of preference statements, a list of heuristics, an educated decision-maker and a decision.

A simpler, less flexible version of such a learning program is being developed and tried out in practice at the Jozef Stefan Institut, but it is the author's intention to develop a more sophisticated version. This work has already begun.

The development of such a program must go hand-in-hand with applications of this approach. The program must suit the way in which people desire to use such a tool, and so a program written without testing the ideas behind it is less likely to be genuinely useful. It must be emphasised that practical trials are very important, and the next stage in the development of the ideas of this thesis must involve both rigorous practical tests and computer assistance.

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