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MODELLING GLACIAL EROSIONAL LANDFORM DEVELOPMENT

VOLUME 2

BY

R. C. A. HINDMARSH

A thesis presented for the degree of
Doctor of Philosophy in the
University of Durham

Graduate Society

January 1985



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1985/HIN

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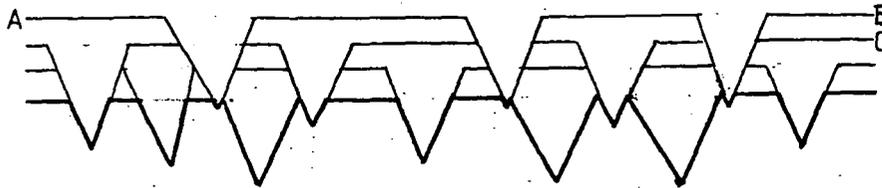
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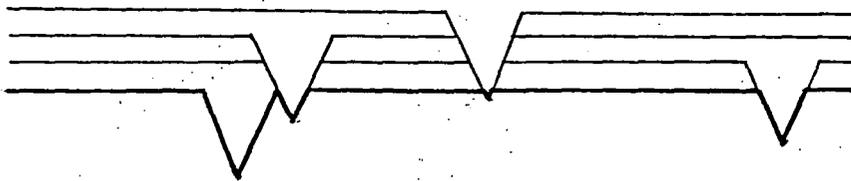
Overlay 1 Scaling lines for uniaxial flow.

FIGURE 21 DEVELOPMENT OF GLACIALLY ERODED SURFACES BY SCRATCHING AND POLISHING



(a) 3 scratches/unit area/unit time

KEY	
AB	= UNIT WIDTH
BC	= POLISHING EFFECTED IN UNIT TIME
	UNIT SCRATCH



(b) 1 scratch/unit area/unit time

FIGURE 2.2 DERIVATION OF THE ABRASION EQUATION

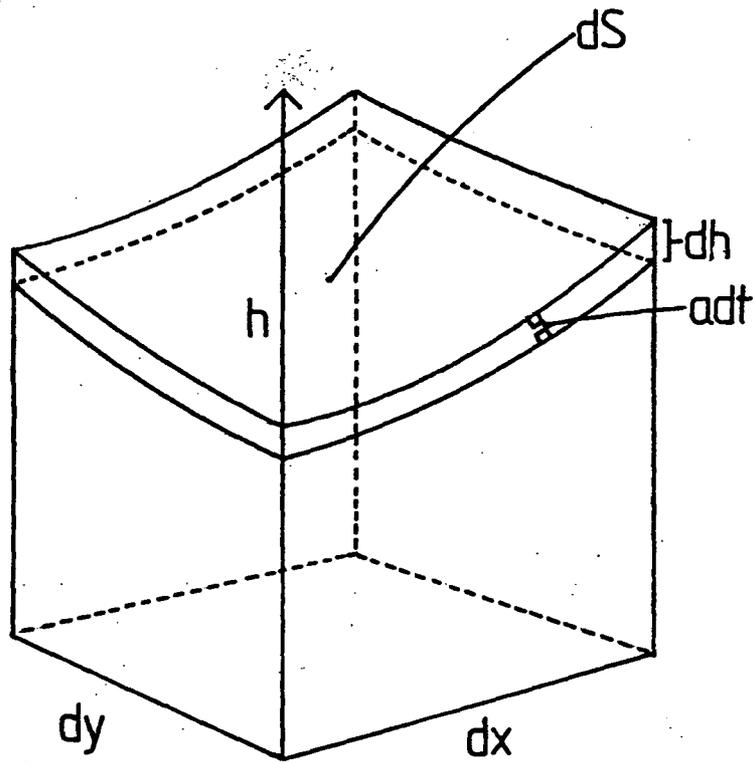


Figure 2.3

Contour plot of surface persistent under constant abrasion: $a = 1, \ell = 2, m_x = m_y = 0.$

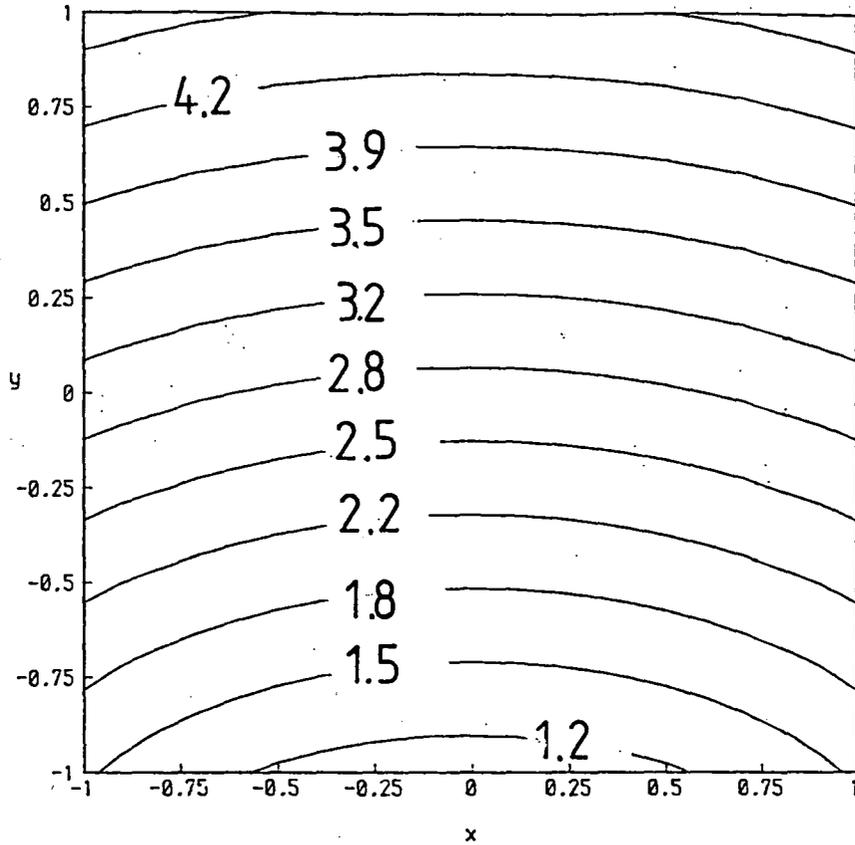


Figure 2.4

Contour plot of surface persistent under constant abrasion: $a = 1, \ell = 2, m_x = 0.0, m_y = -0.5.$

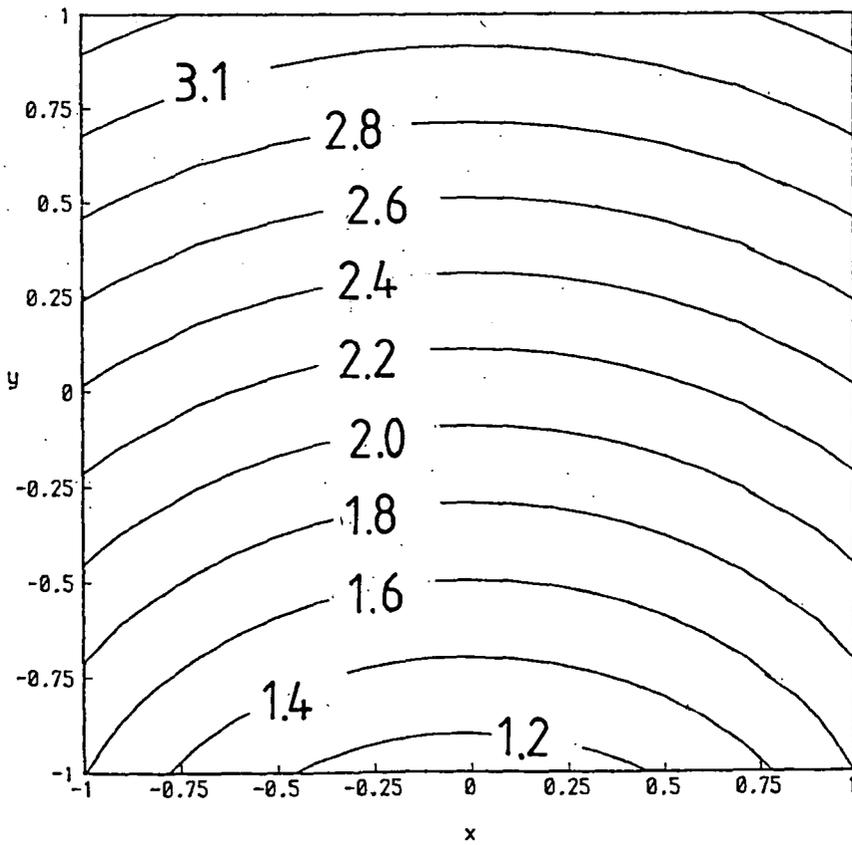


Figure 2.5

Contour plot of surface persistent under constant abrasion: $a = 1, \ell = 2, m_x = m_y = -0.5.$

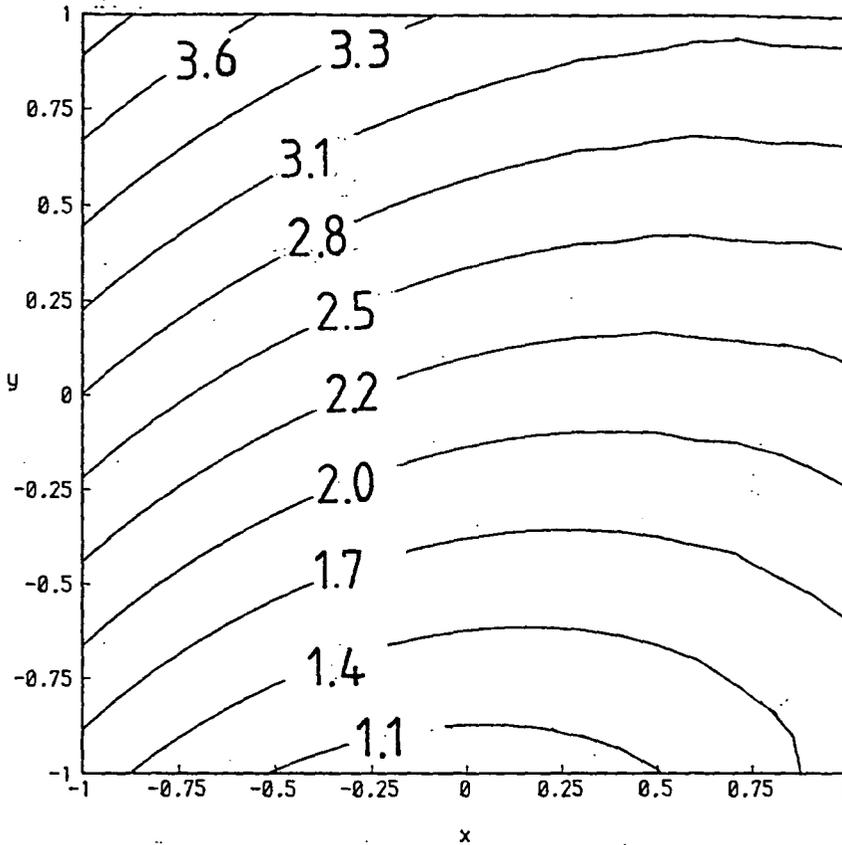


Figure 2.6

Contour plot of surface persistent under constant abrasion: $a = 1, \ell = 2, m_x = m_y = -0.9.$

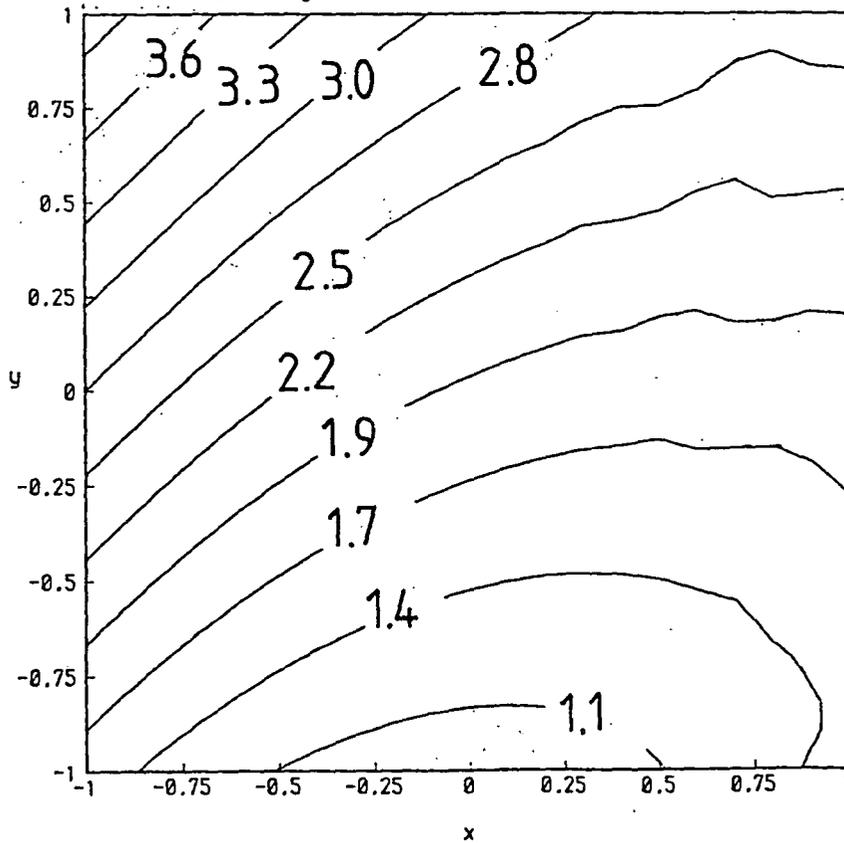


Figure 2.7

Dependence of abrasion on position for persistence of a sine-wave under lowering.

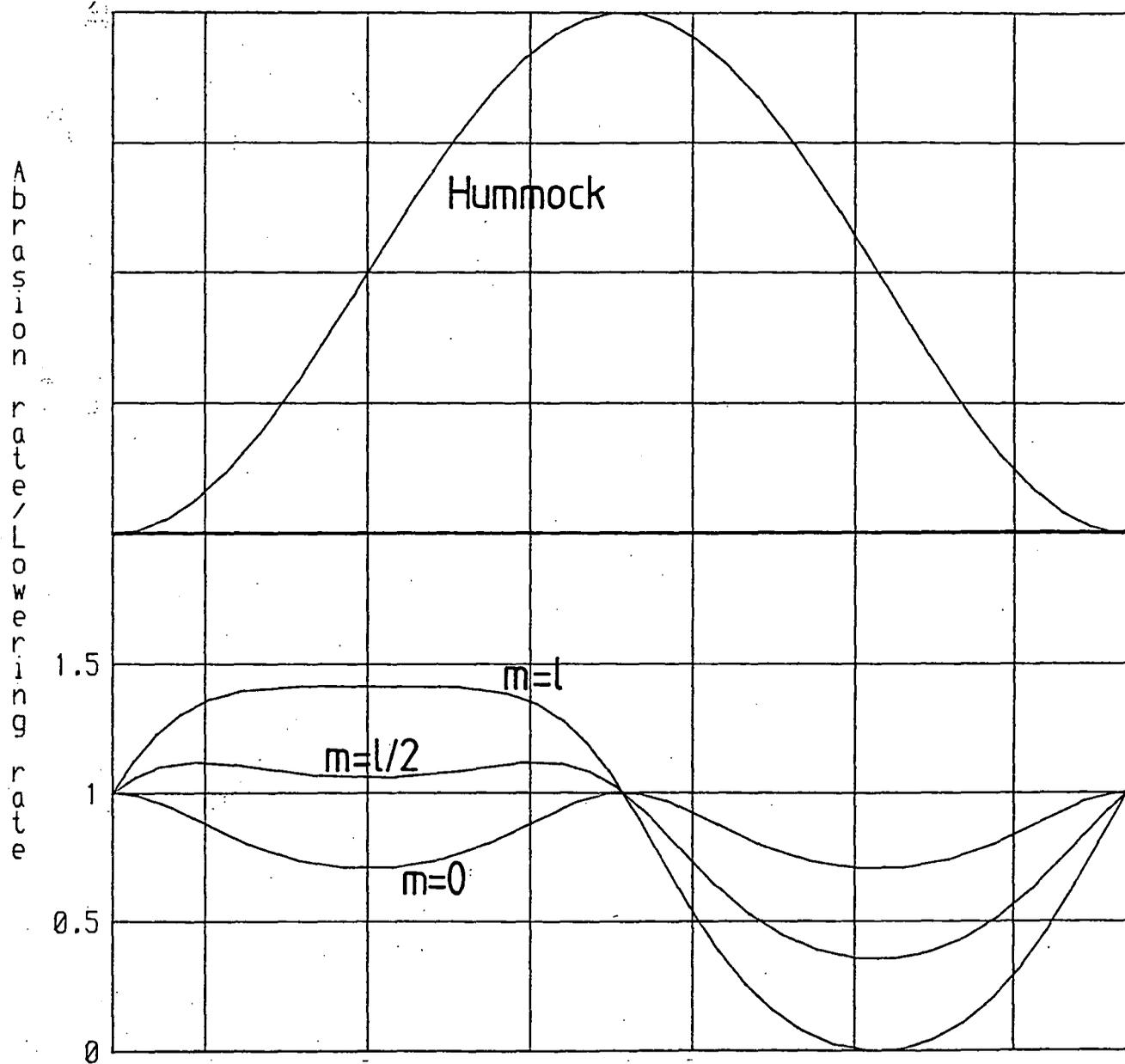


Figure 2.8

Evolution of a slope and plateau under constant abrasion using the method of characteristics.

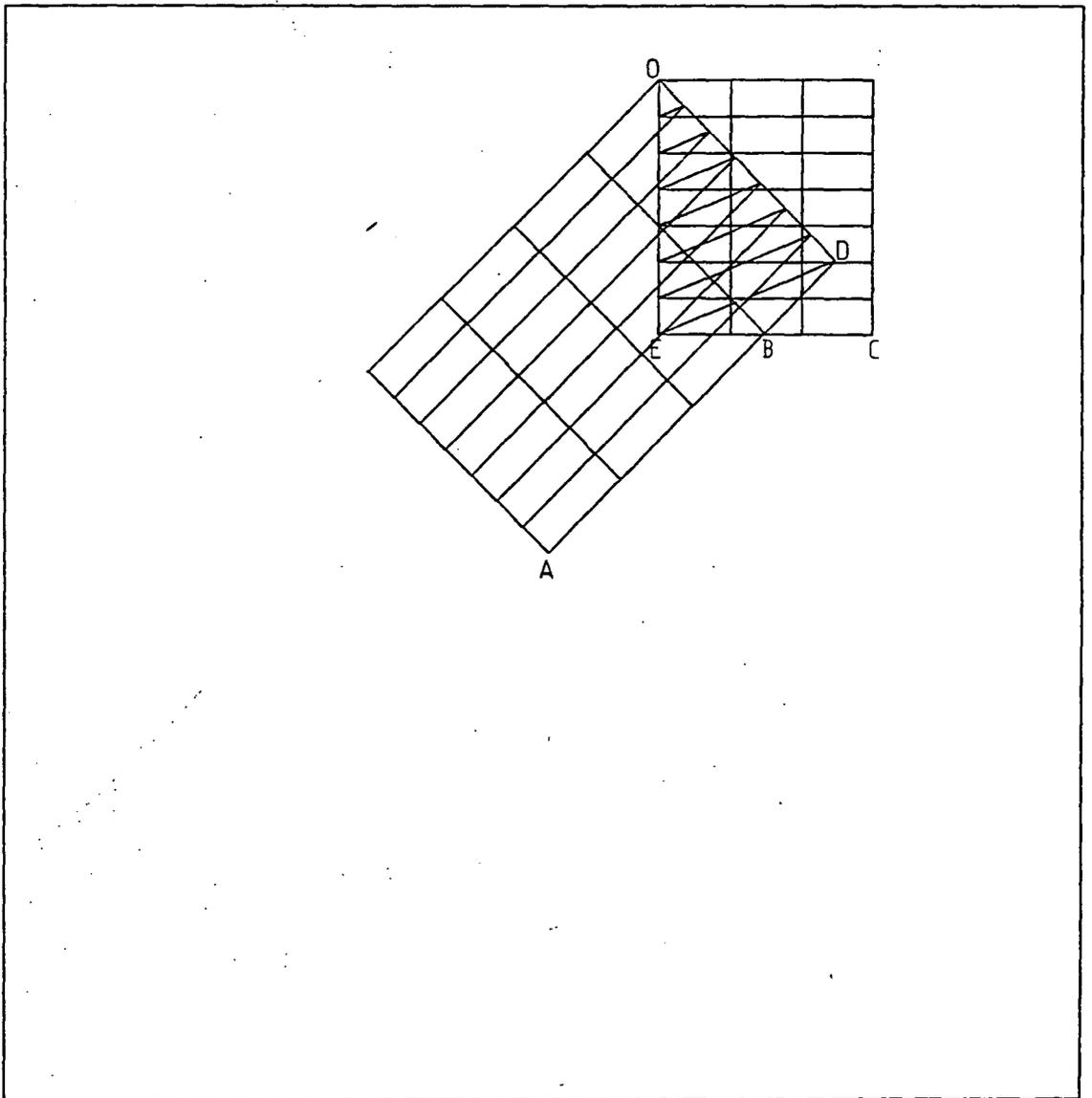


Figure 2.9

Evolution of a bell-shape under constant abrasion using the method of characteristics.

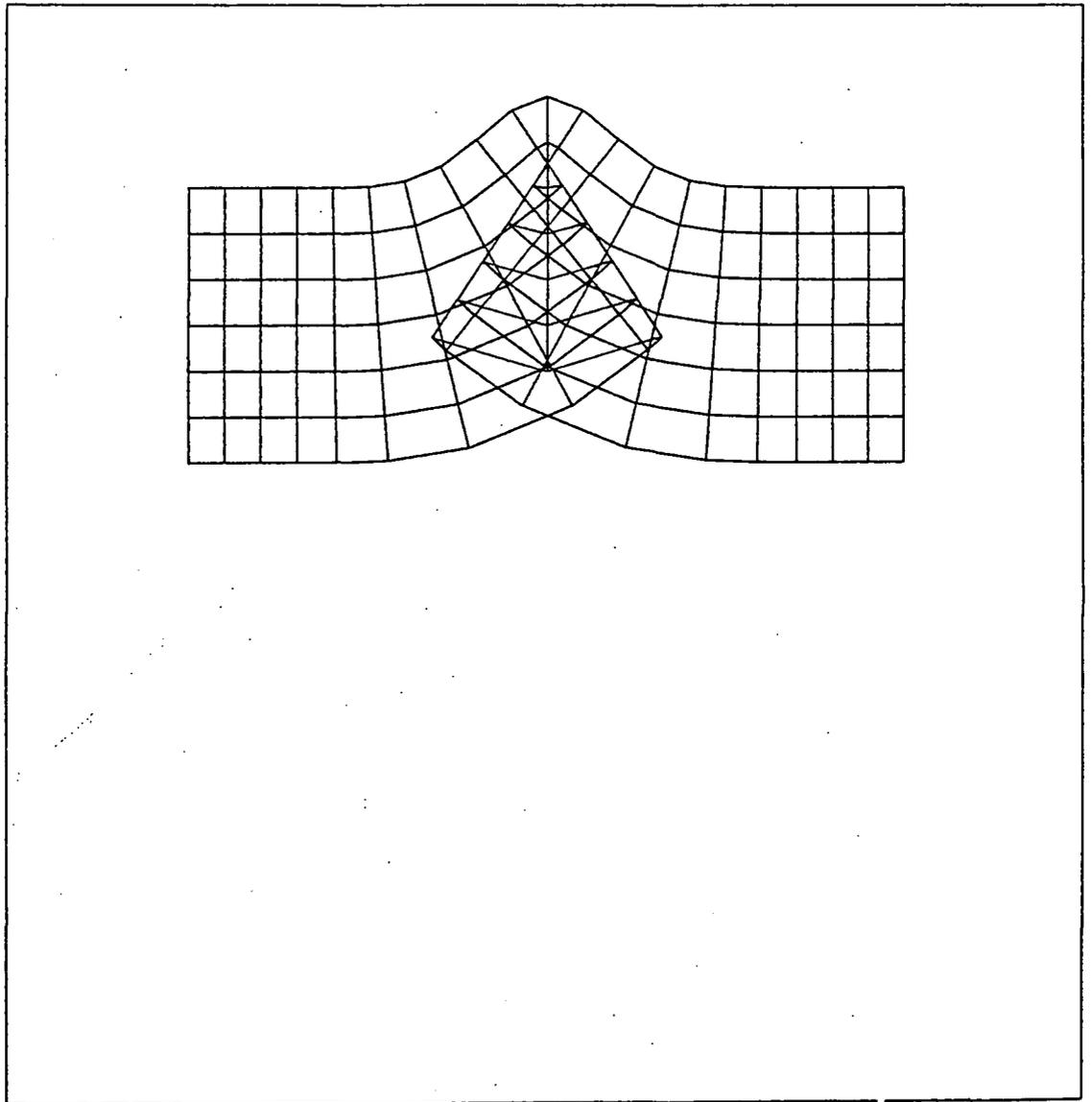


Figure 2.10

Evolution of a V-shaped valley under constant abrasion using a direct finite-difference formulation of the abrasion equation.

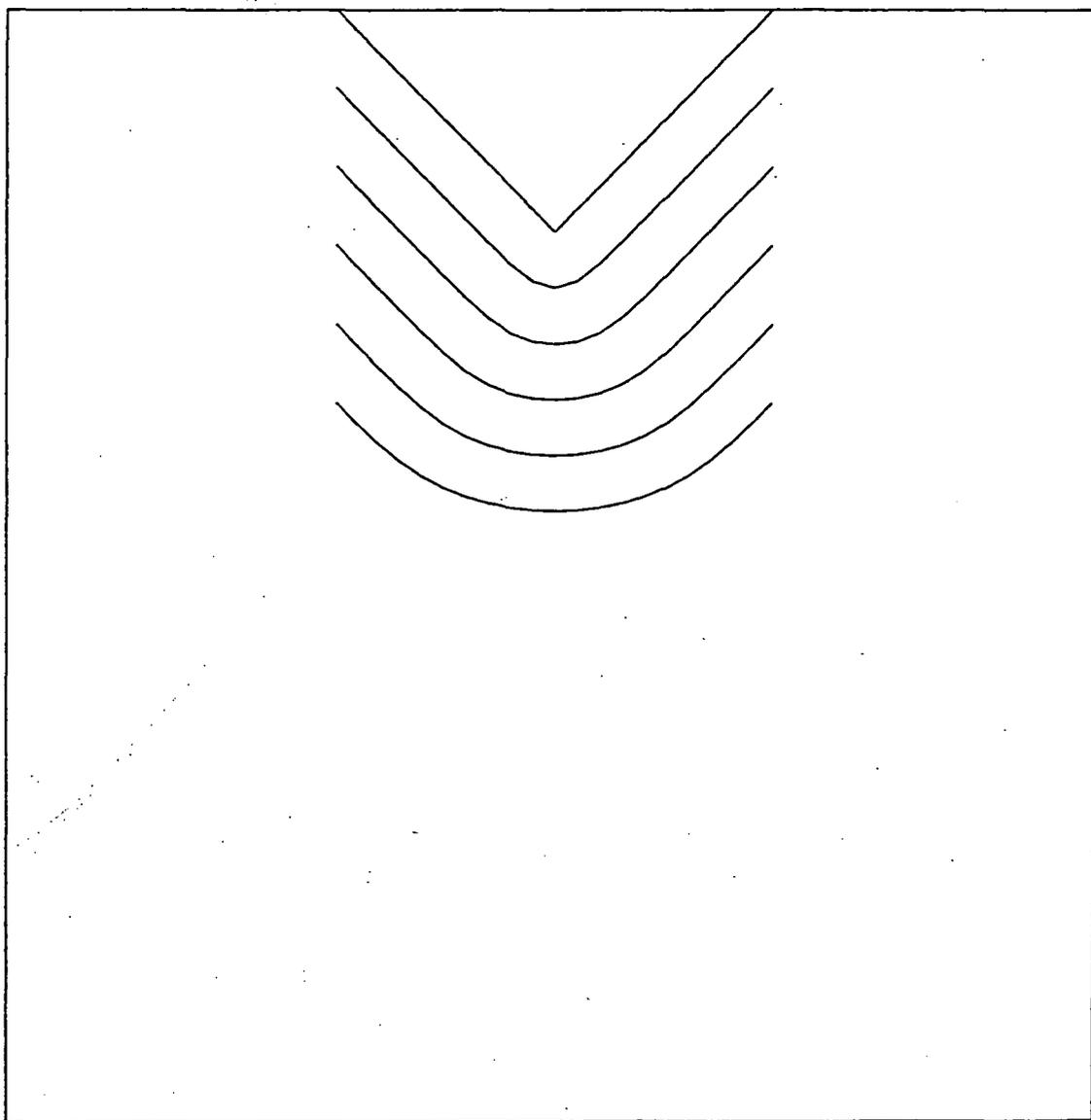


Figure 2.11 Evolution of a bell-shape under constant abrasion using a direct finite-difference formulation of the abrasion equation.

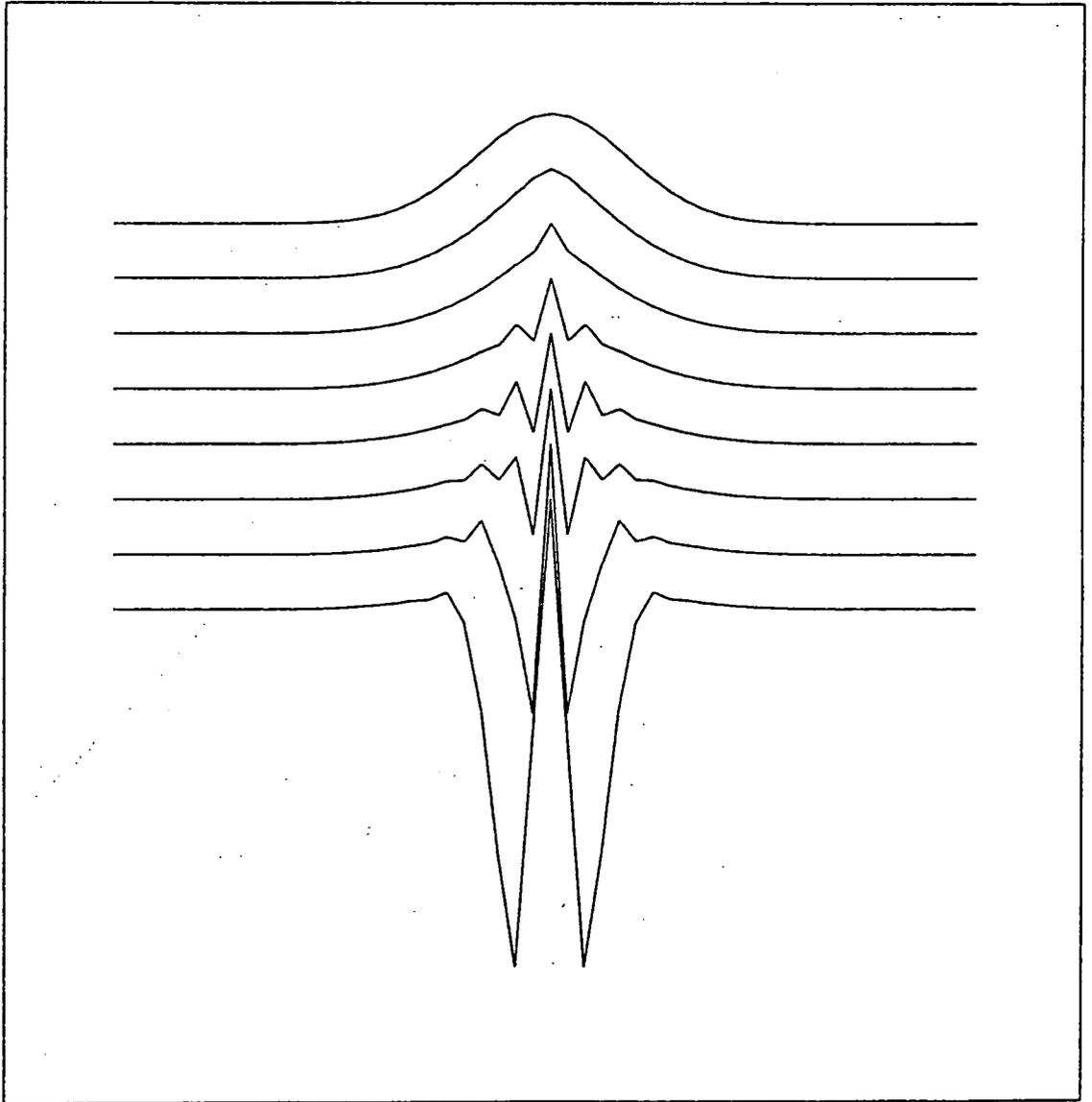


Figure 2.12 Evolution of a V-shaped valley under constant abrasion using the Farmer formulation with a coarse mesh.

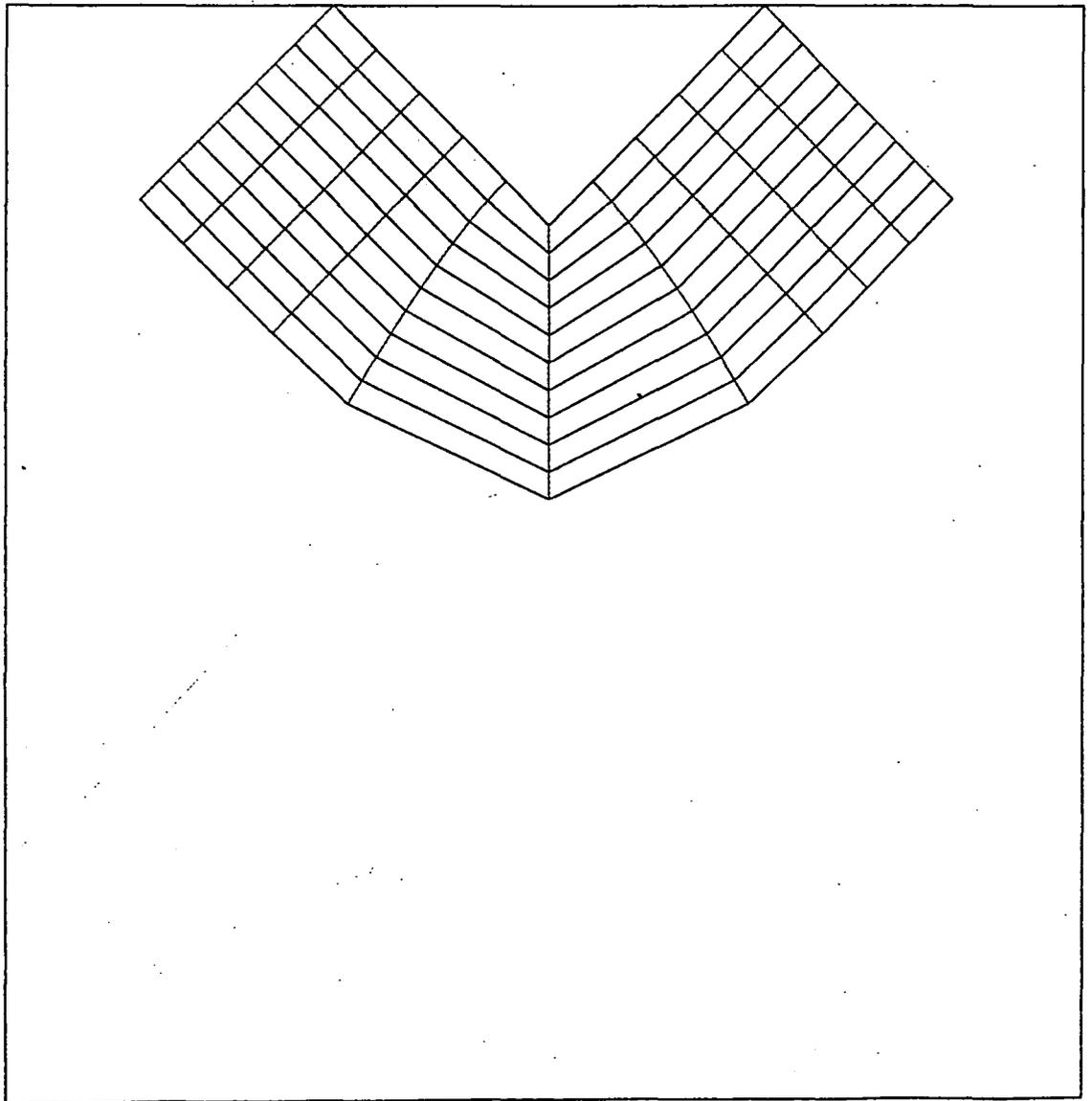


Figure 2.13

Evolution of a V-shaped valley under constant abrasion using the Farmer formulation with a fine mesh.

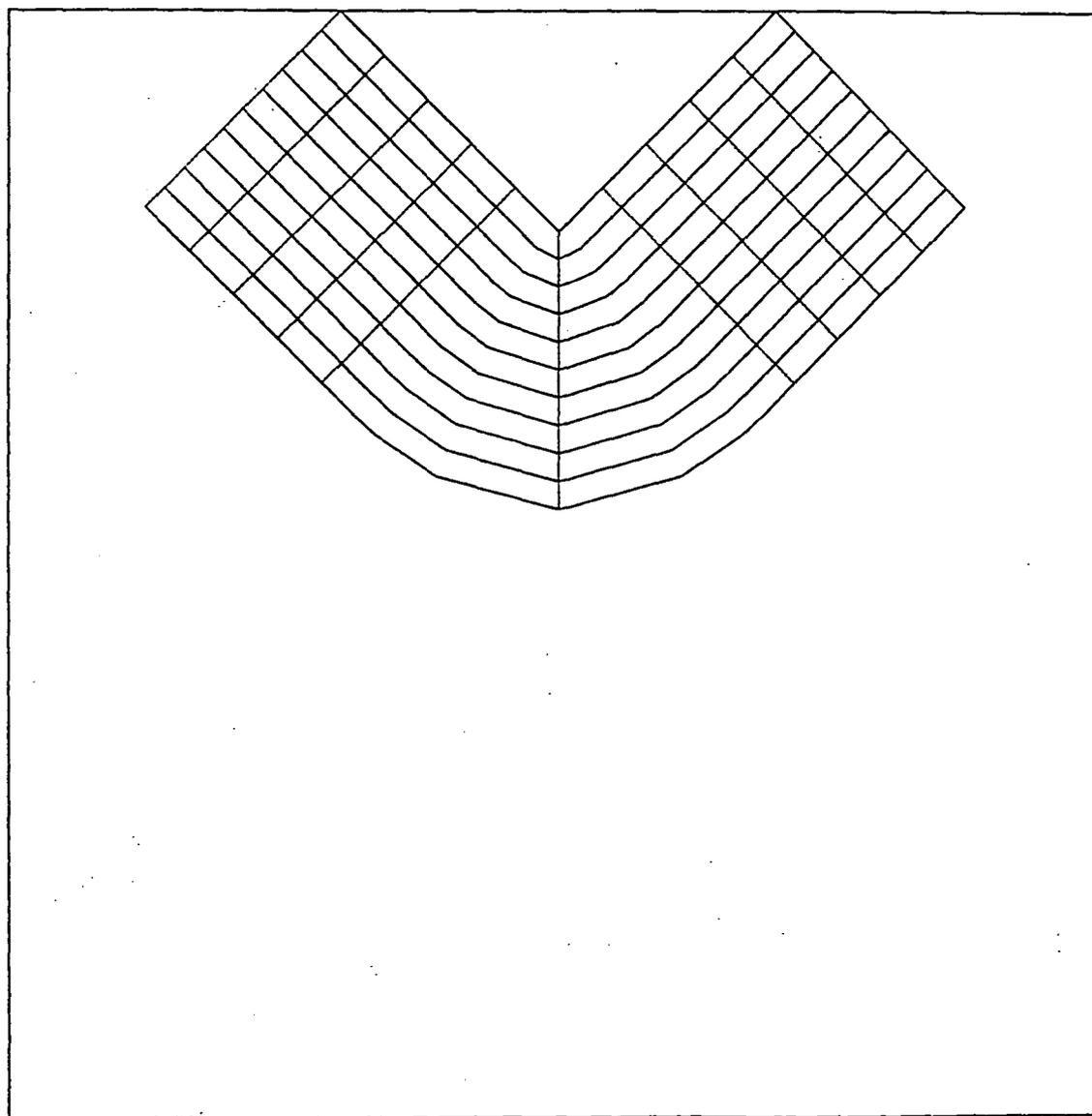


Figure 2.14 Evolution of a bell-shape under constant abrasion using the Farmer formulation.

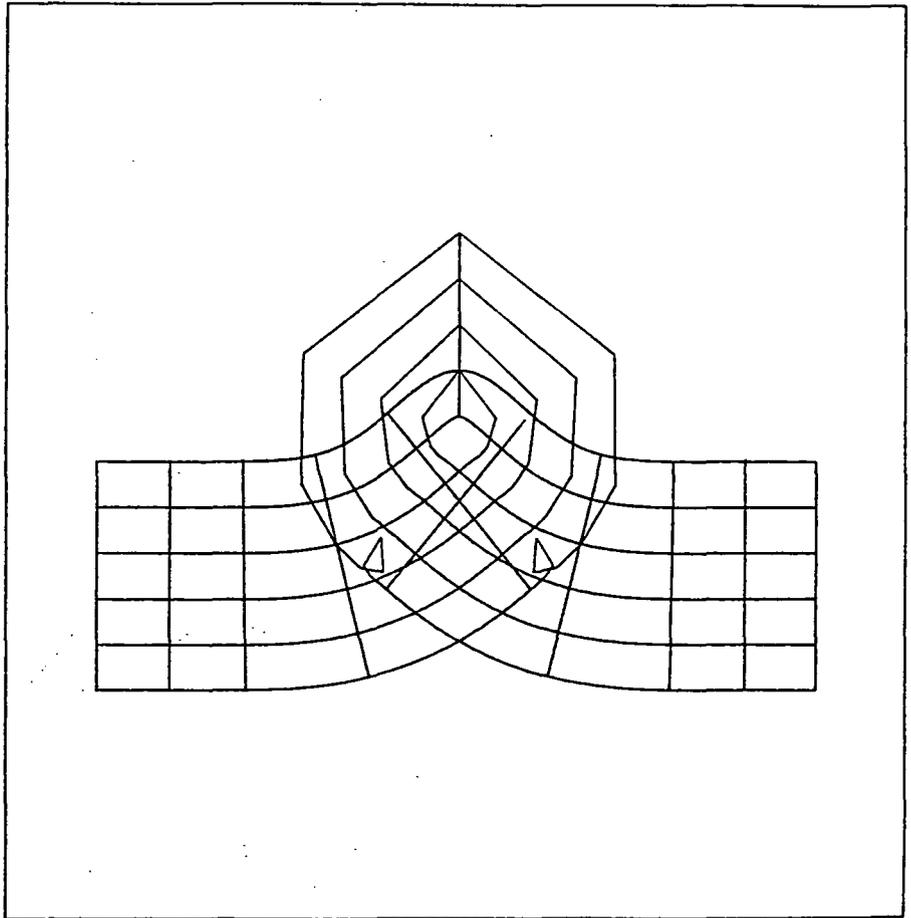


Figure 2.15

Persistence without migration of a bell-shape with lowering.

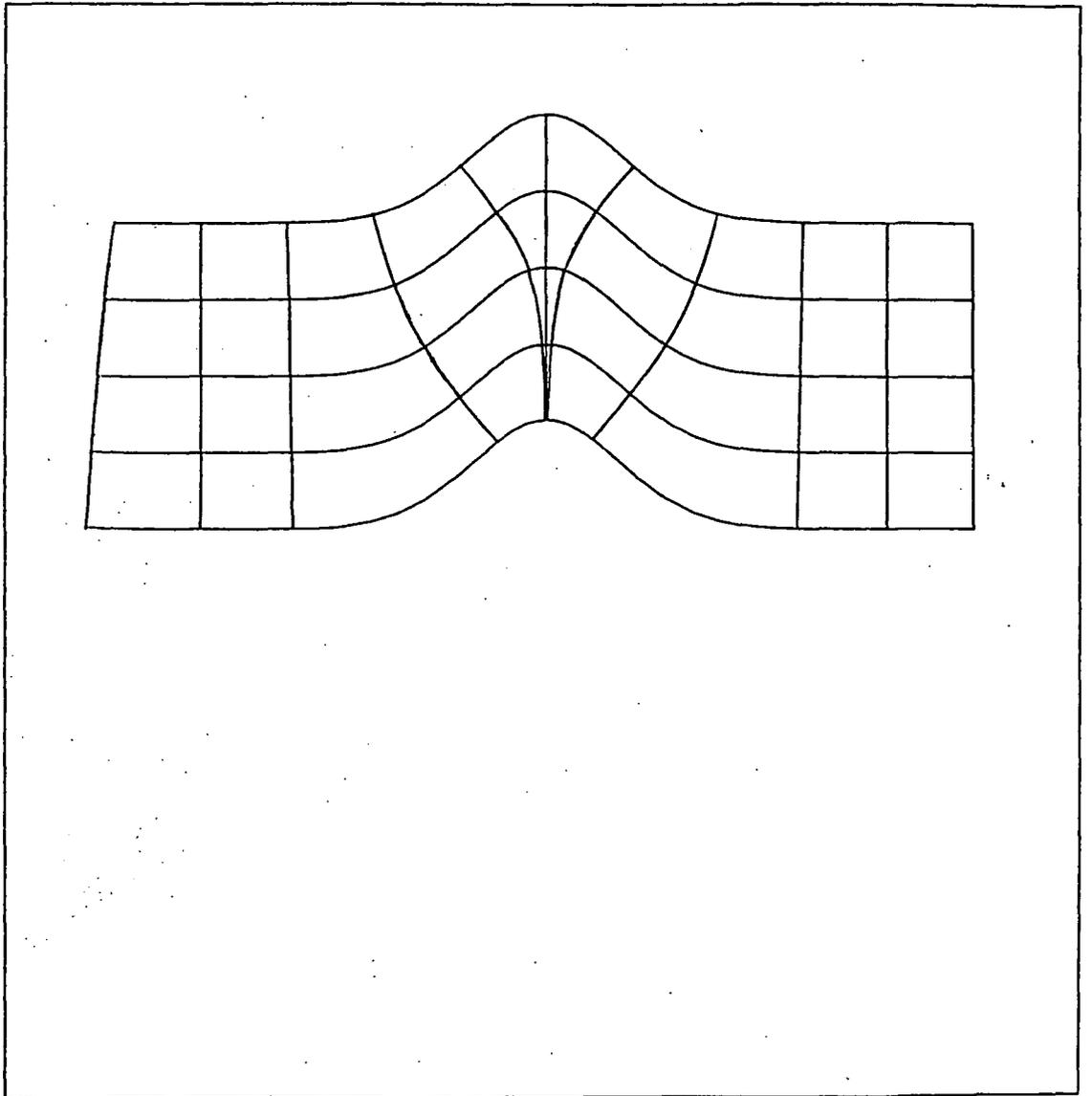


Figure 2.16

Persistence with migration of a bell-shape
with lowering.

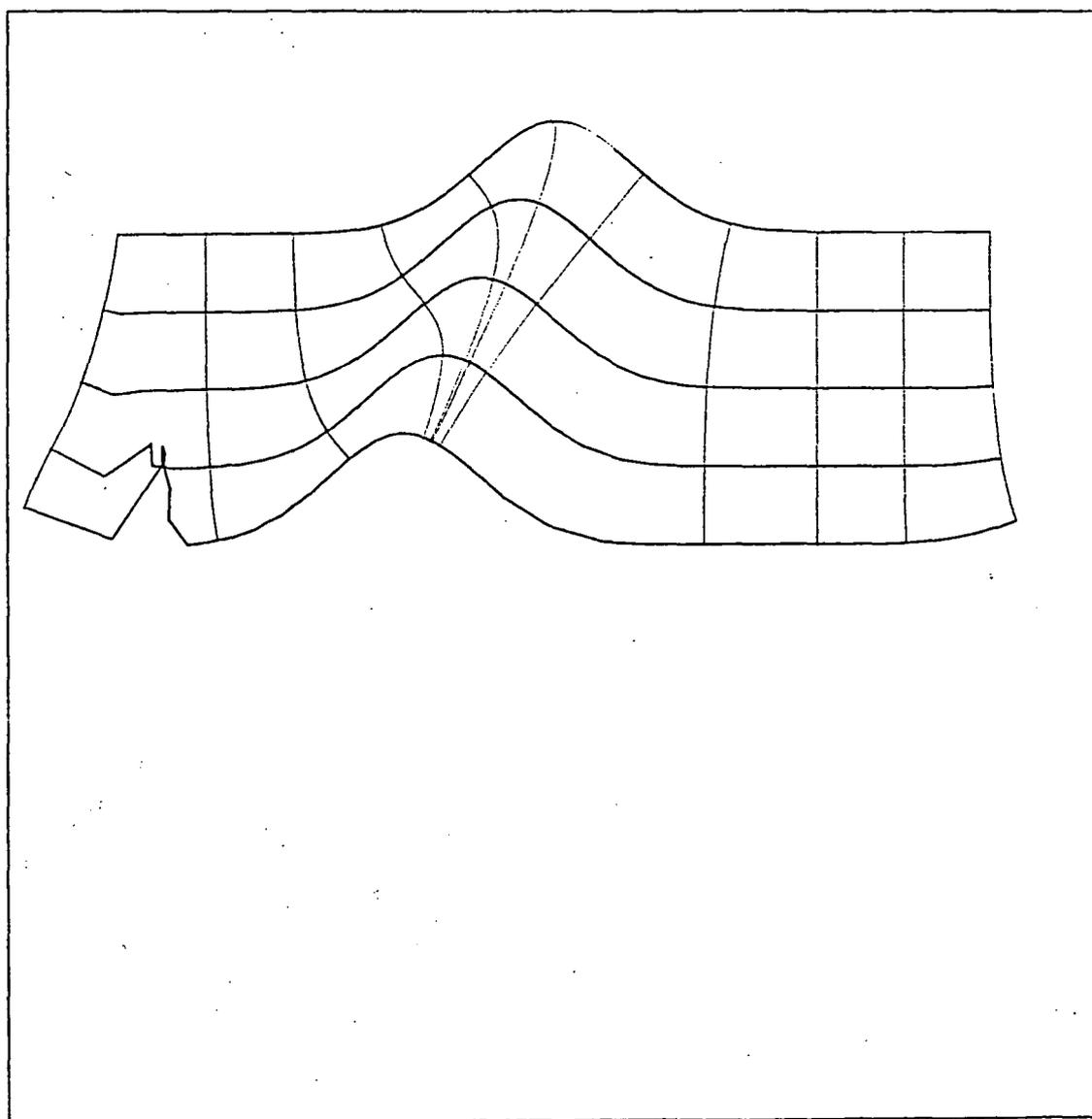


Figure 2.17

Evolution of a V-shaped valley under constant abrasion using the method of characteristics: no definition of point.

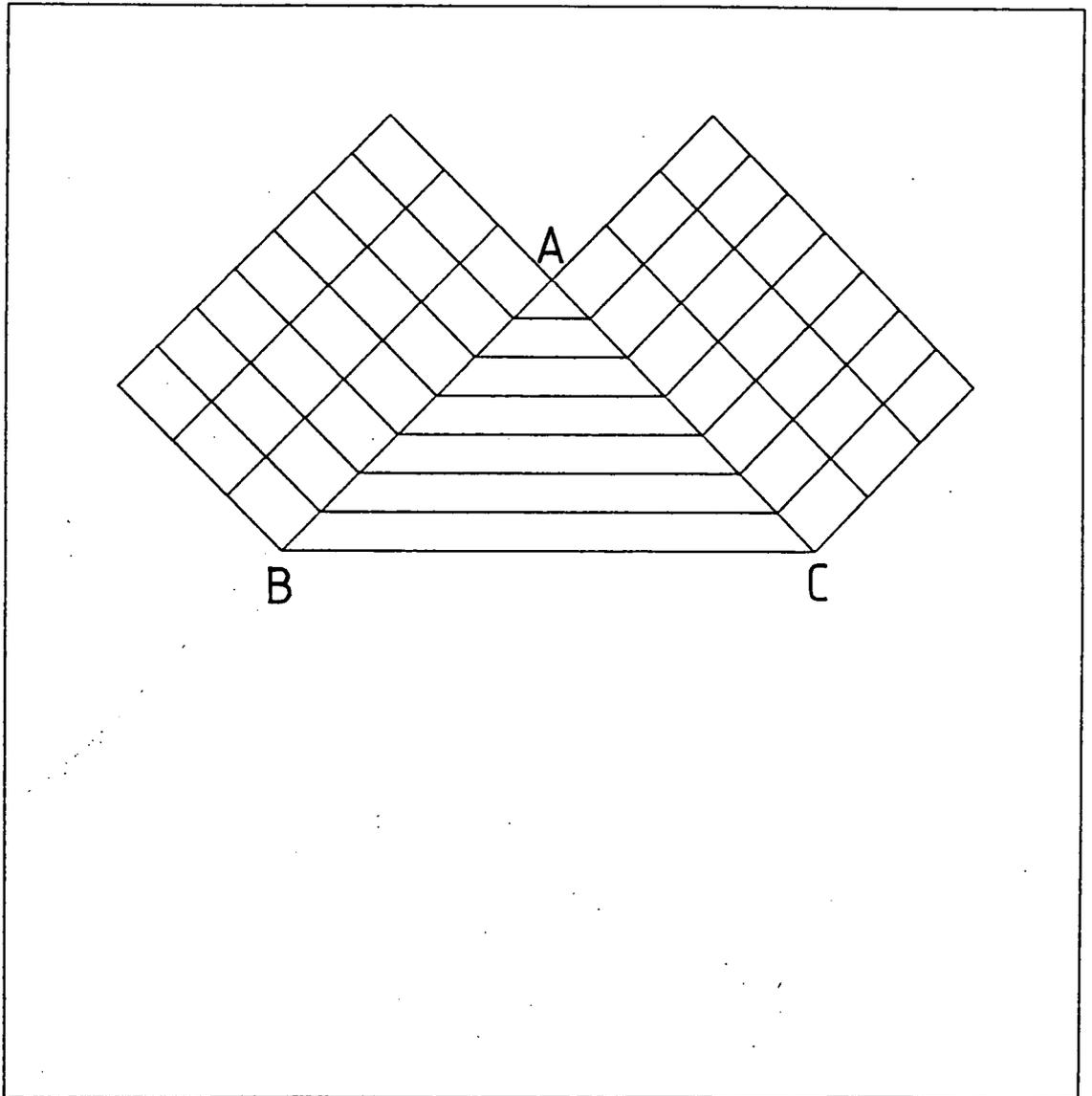


Figure 2.18

Evolution of a V-shaped valley under constant abrasion using the method of characteristics: smooth definition of point.

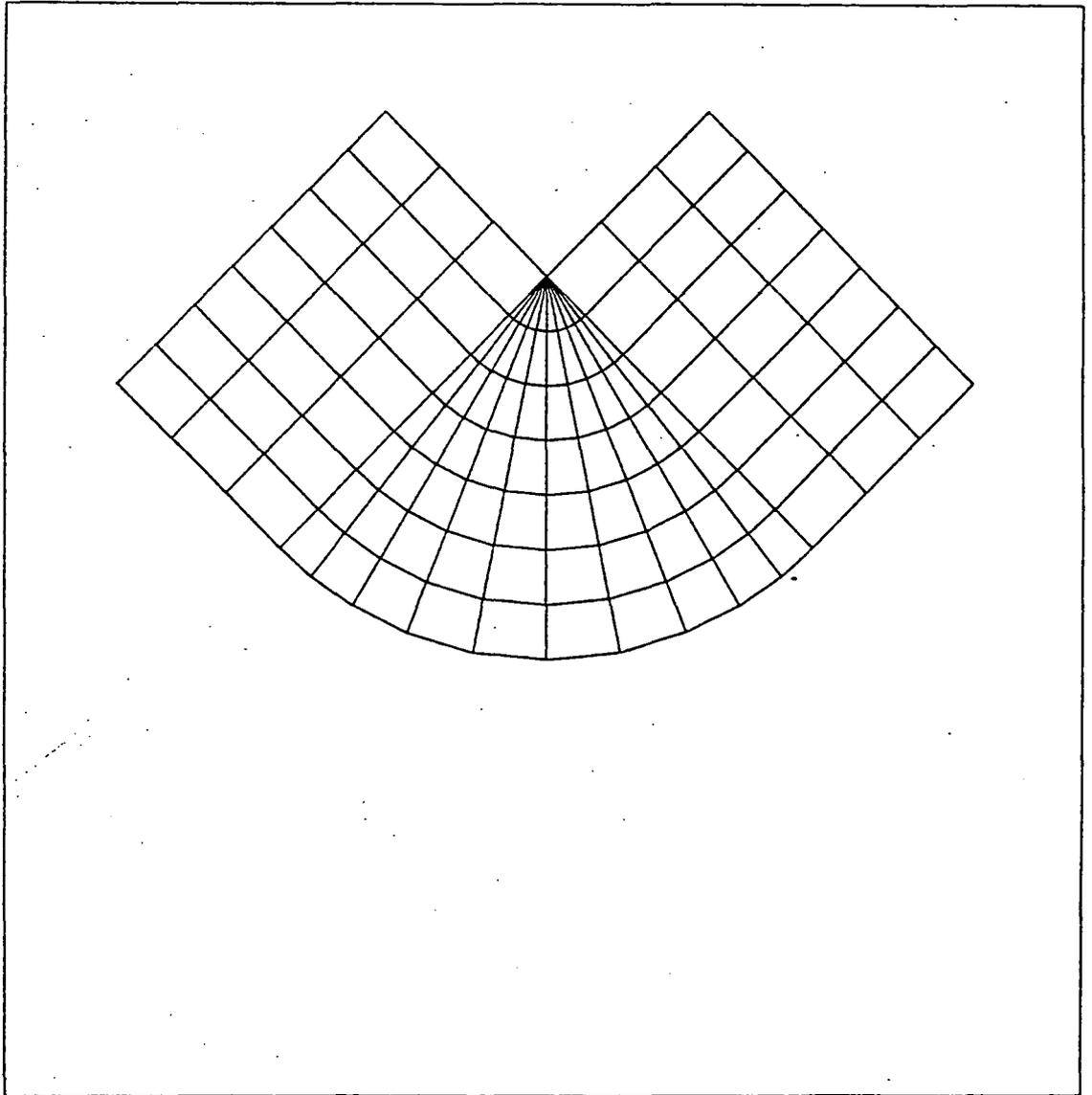


Figure 2.19

Evolution of a V-shaped valley under constant abrasion using the method of characteristics: random definition of point.

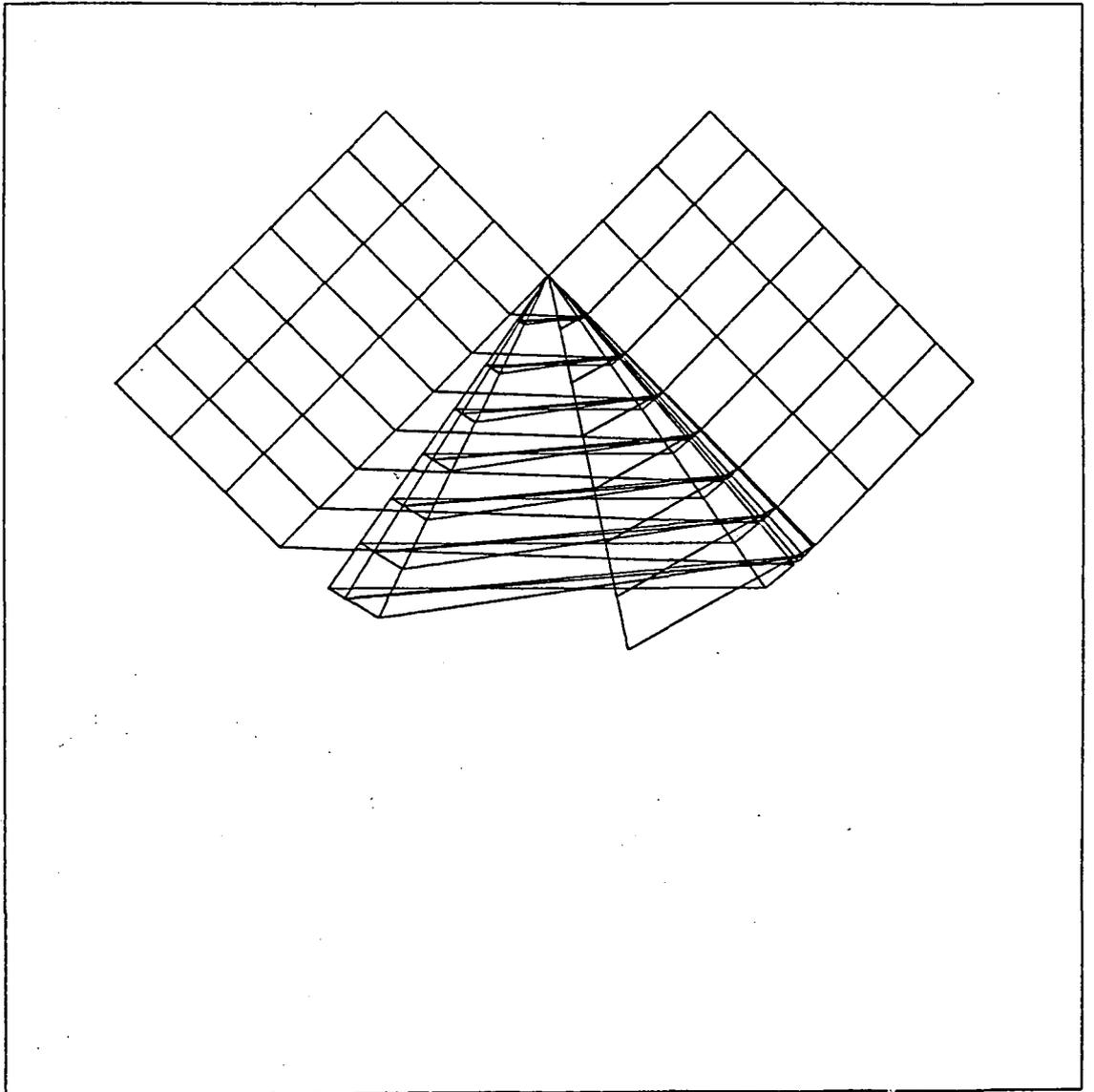


Figure 2.20

Evolution of slope and plateau with curvature dependent abrasion rate using the Farmer formulation.

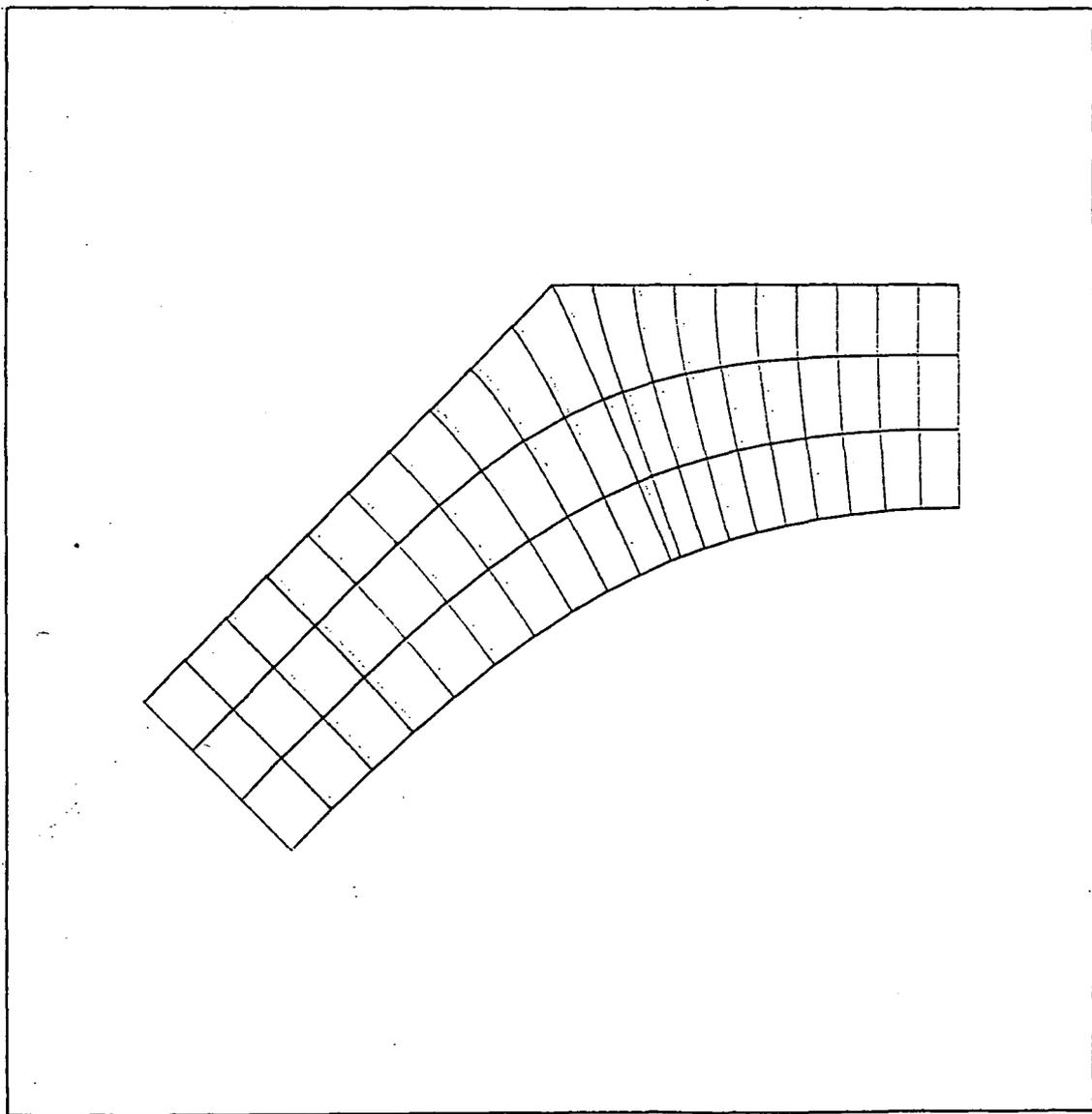


Figure 2.21

Evolution of V-shaped valley with abrasion inhibited by concavity using the Farmer formulation.

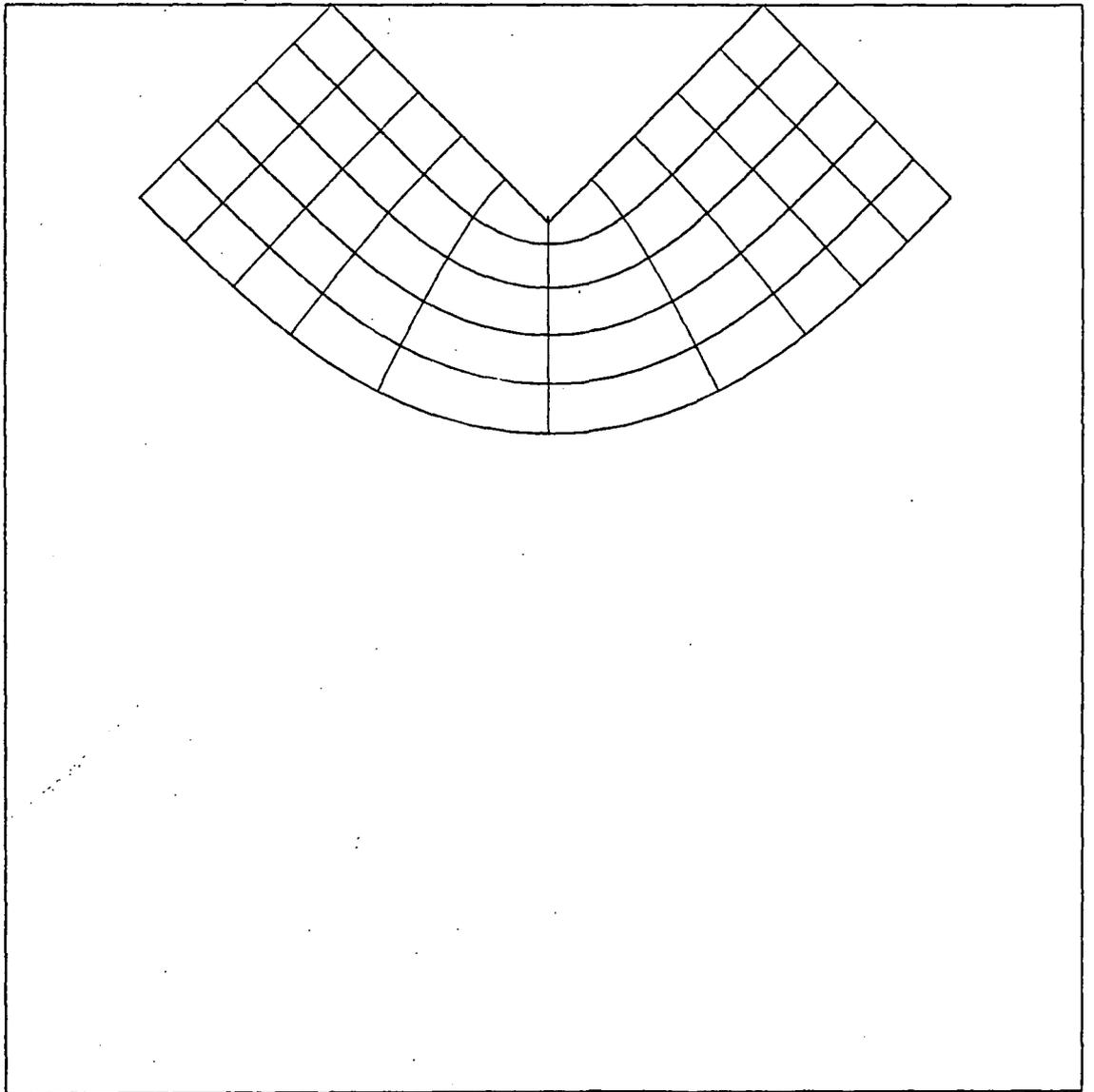


Figure 2.22 Evolution of V-shaped valley with abrasion enhanced by concavity using the Farmer formulation.

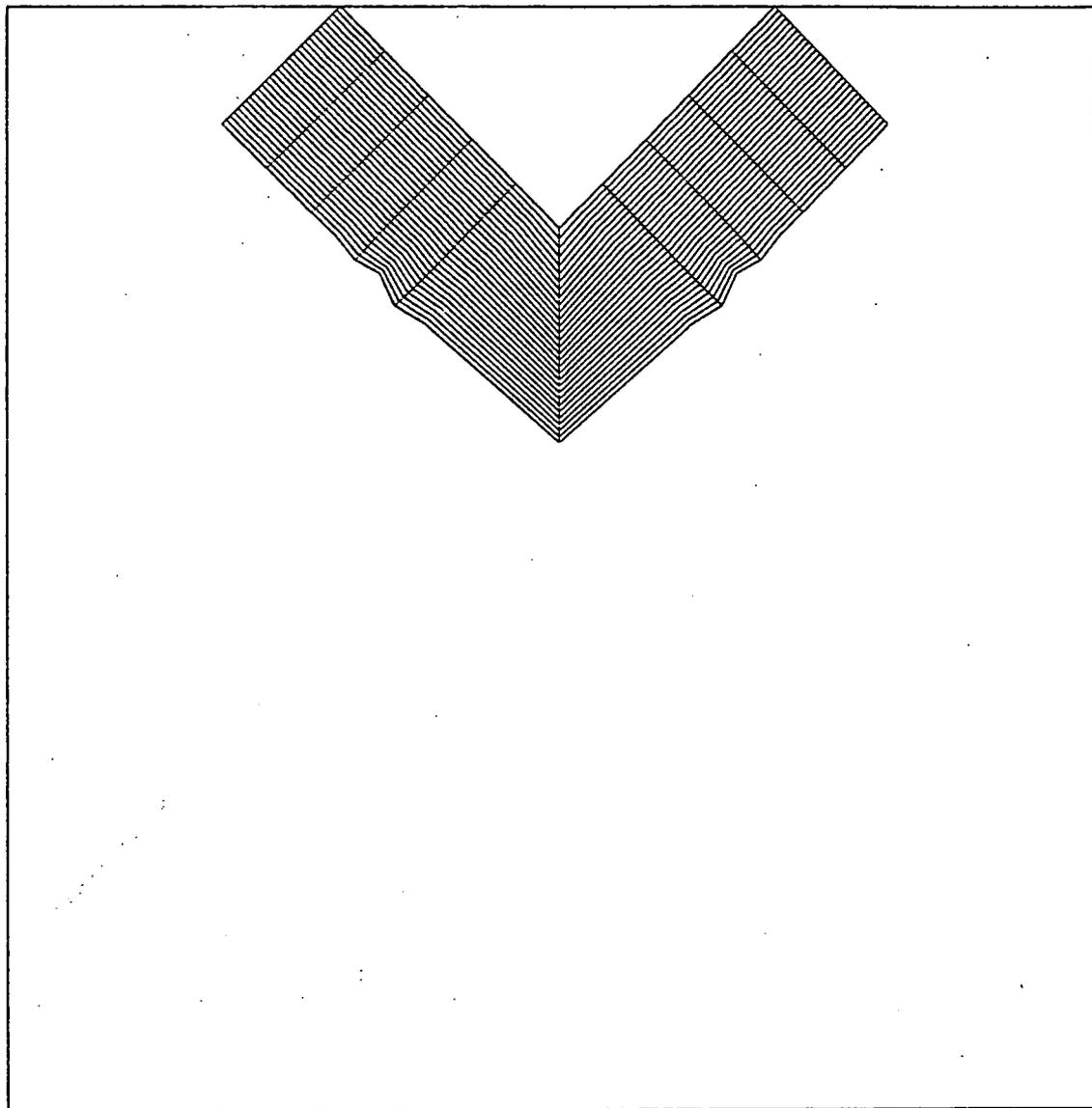


Figure 2.23

Evolution of a V-shaped valley with a Gaussian abrasion function.

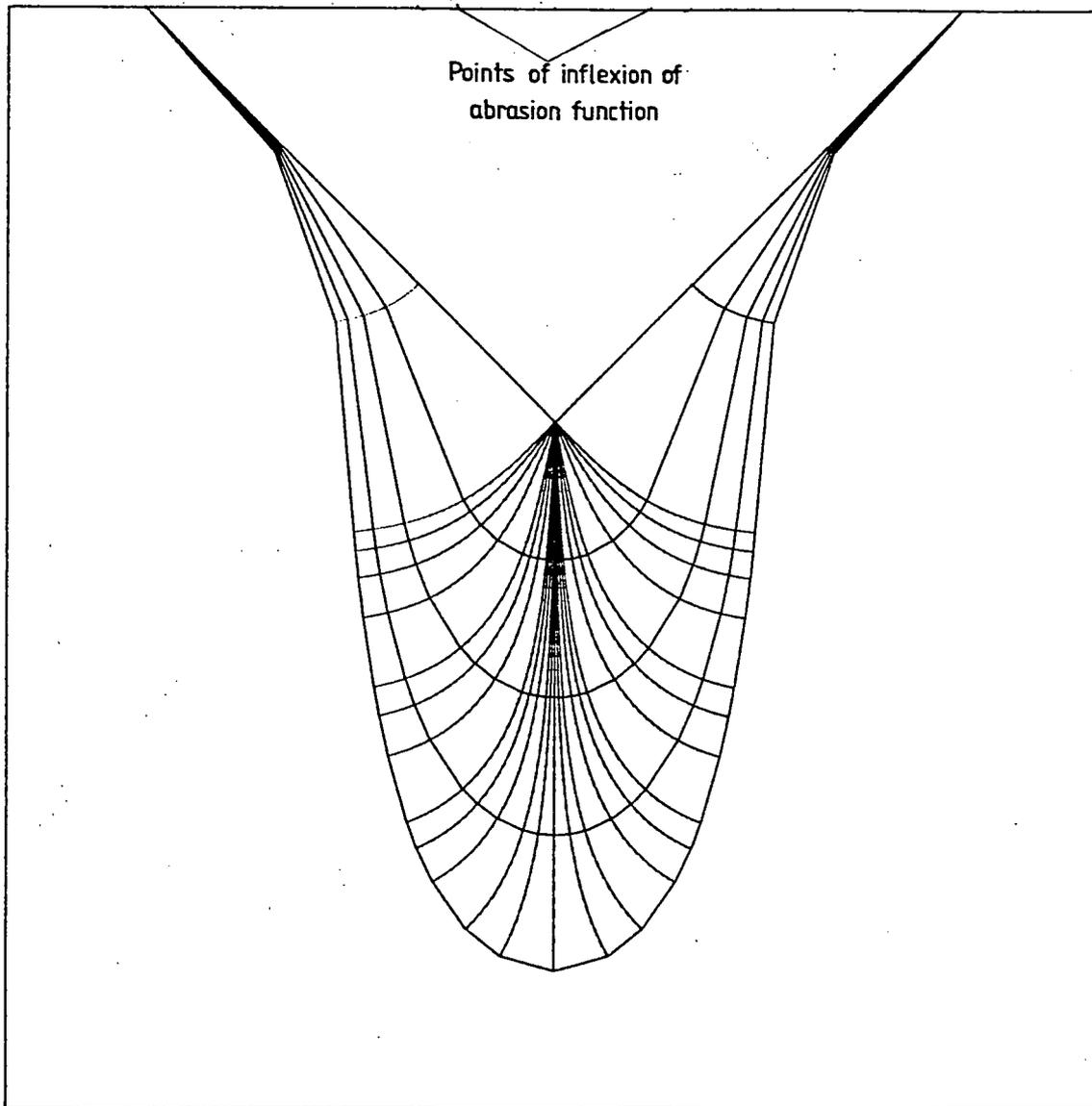


Figure 2.24

Evolution of a V-shaped valley with rocks of different hardness.

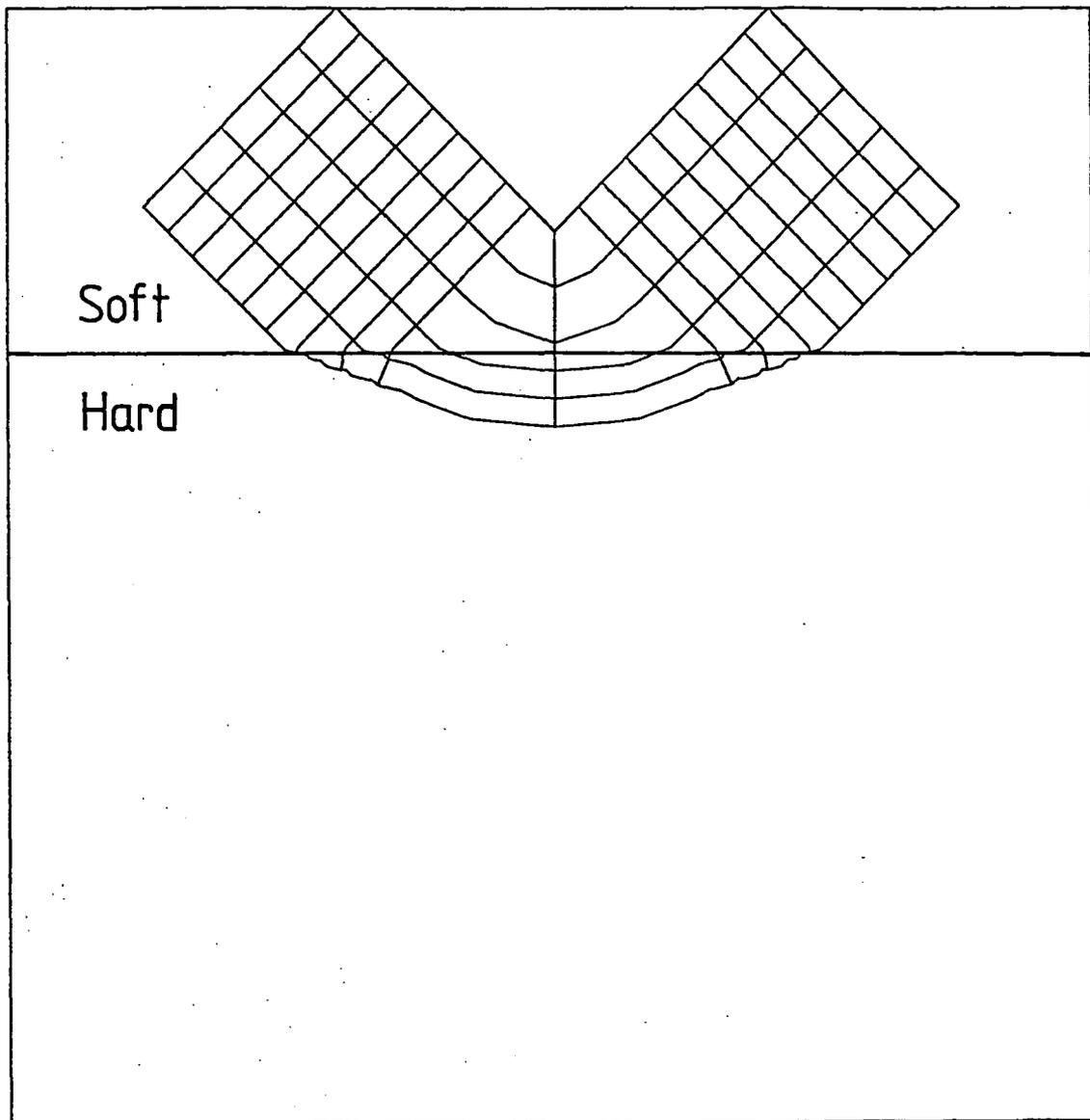


Figure 2.25

Trough persistent under lowering with
 $a \propto 1/h$: hyperbolic cosine.

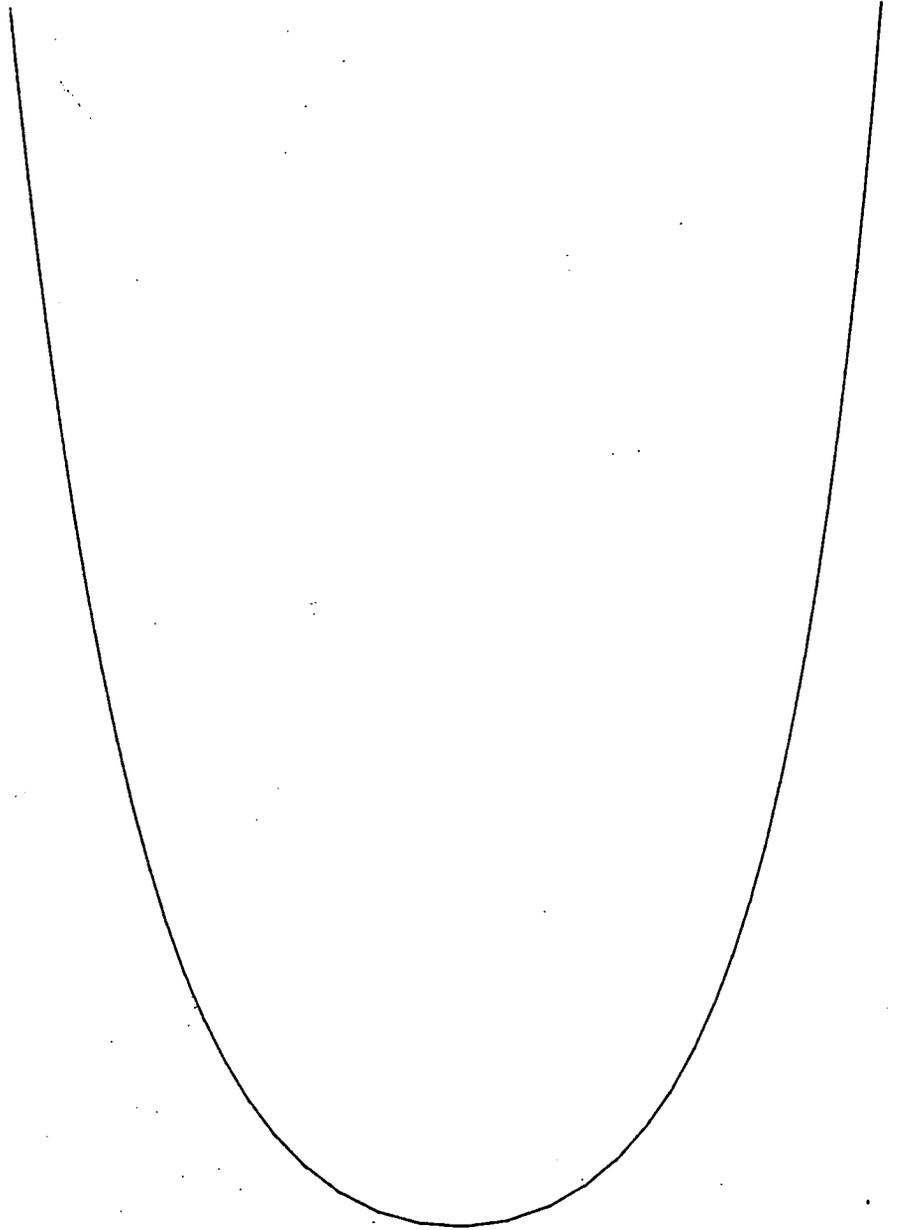


Figure 2.26

Troughs persistent under lowering with abrasion
a function of a curvature:

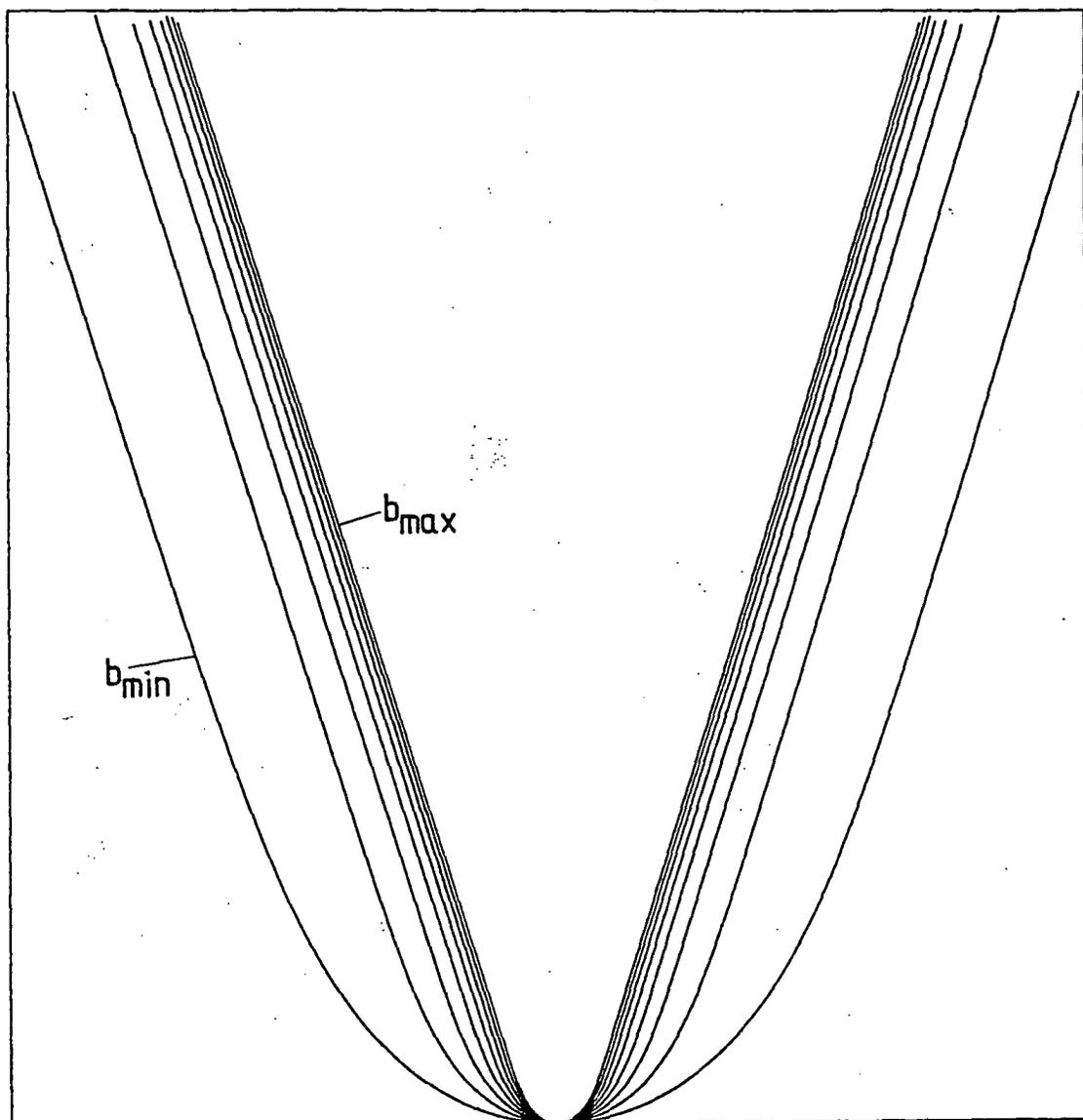


Figure 2.27 Abrasion of a weak zone.

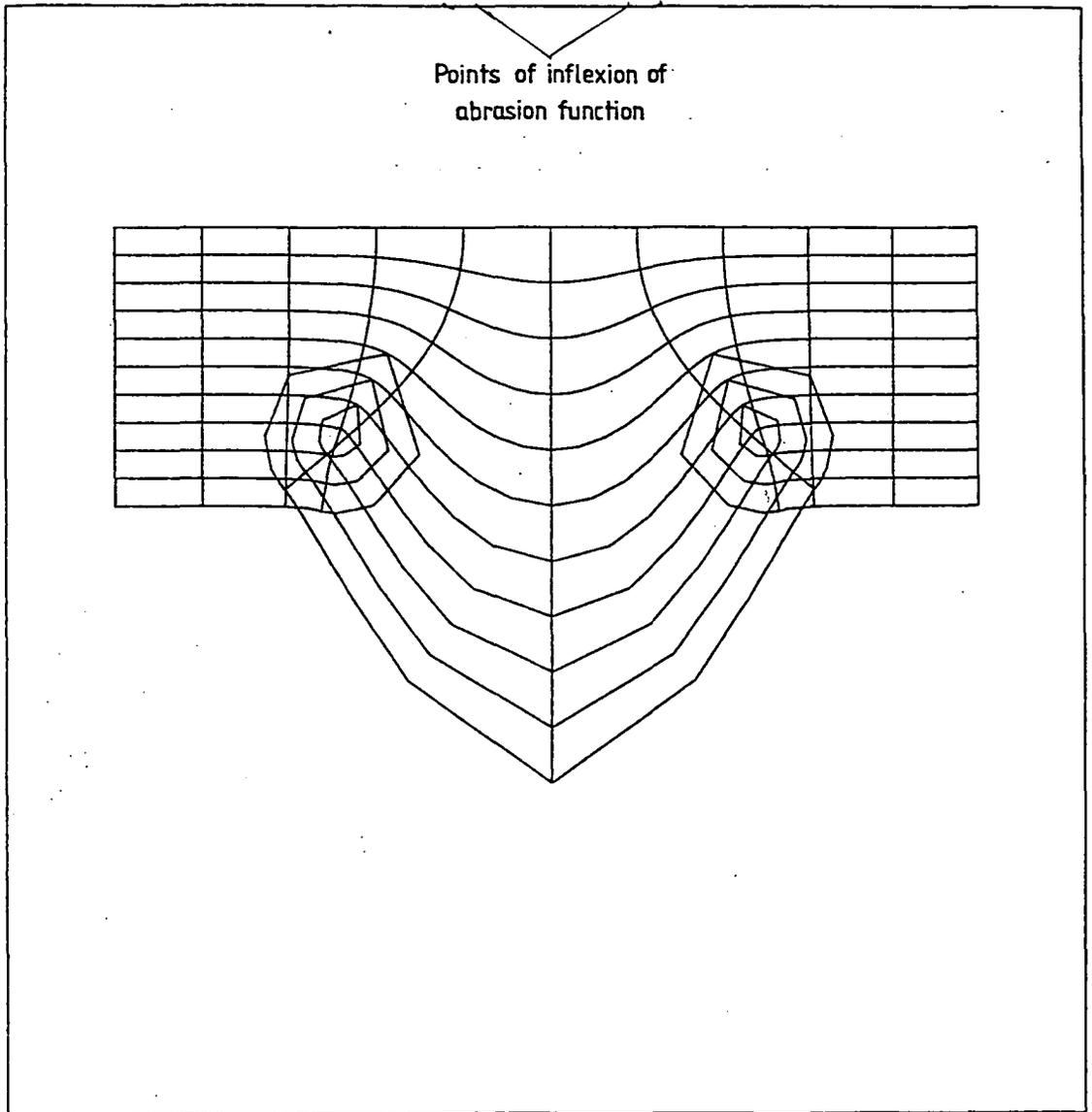
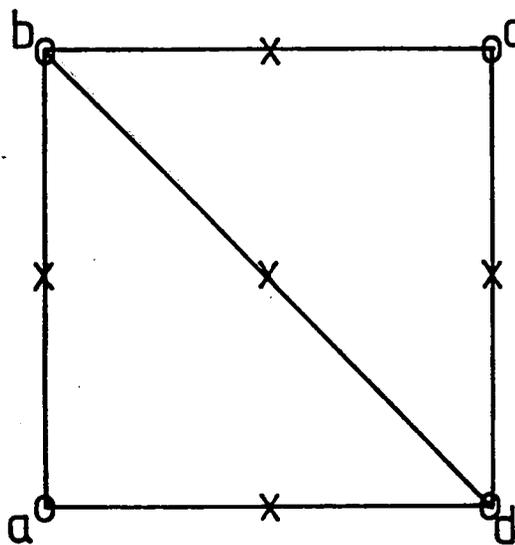


FIGURE 4.1 TWO ELEMENT TEST MODEL

o-Node at which velocity and pressure are solved for

x-Node at which velocity only is solved for



Boundary Conditions

ad - Fixed bed or Weertman

abcd - Prescribed velocity or traction

abd, bcd are the finite elements

Figure 4.2

Inclined plane problem-nine nodes.

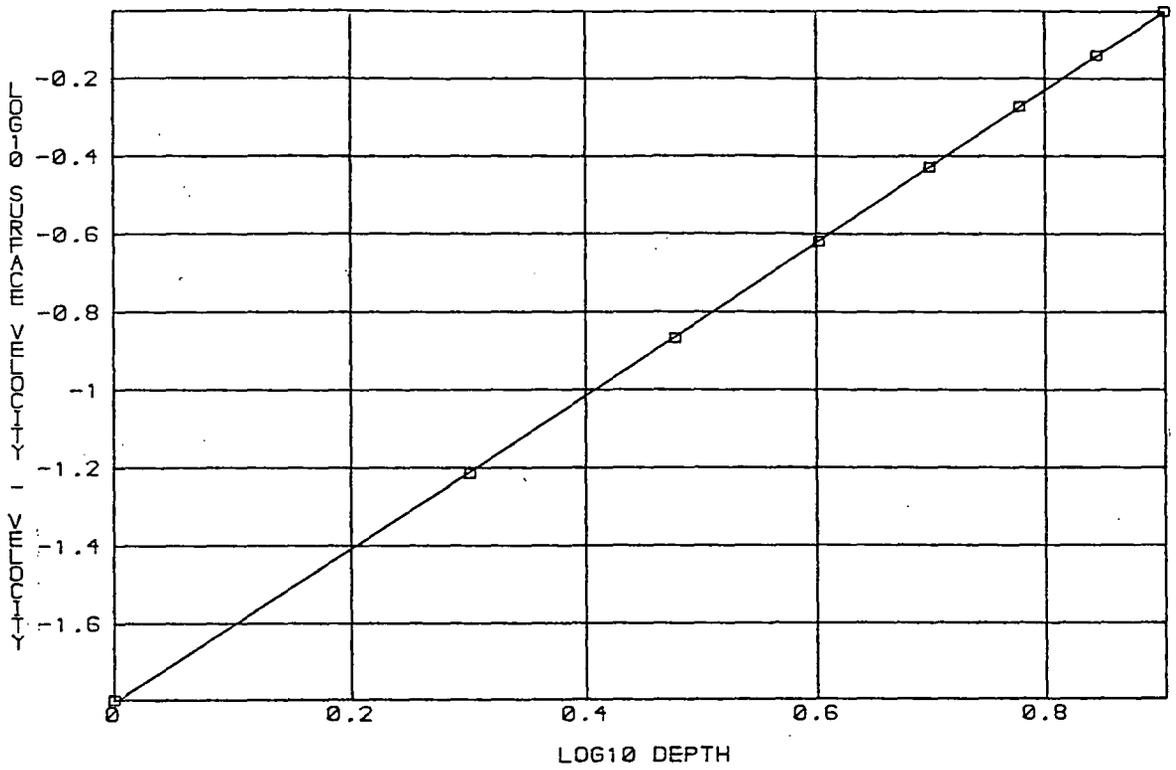


Figure 4.3

Inclined plane problem-seventeen nodes.

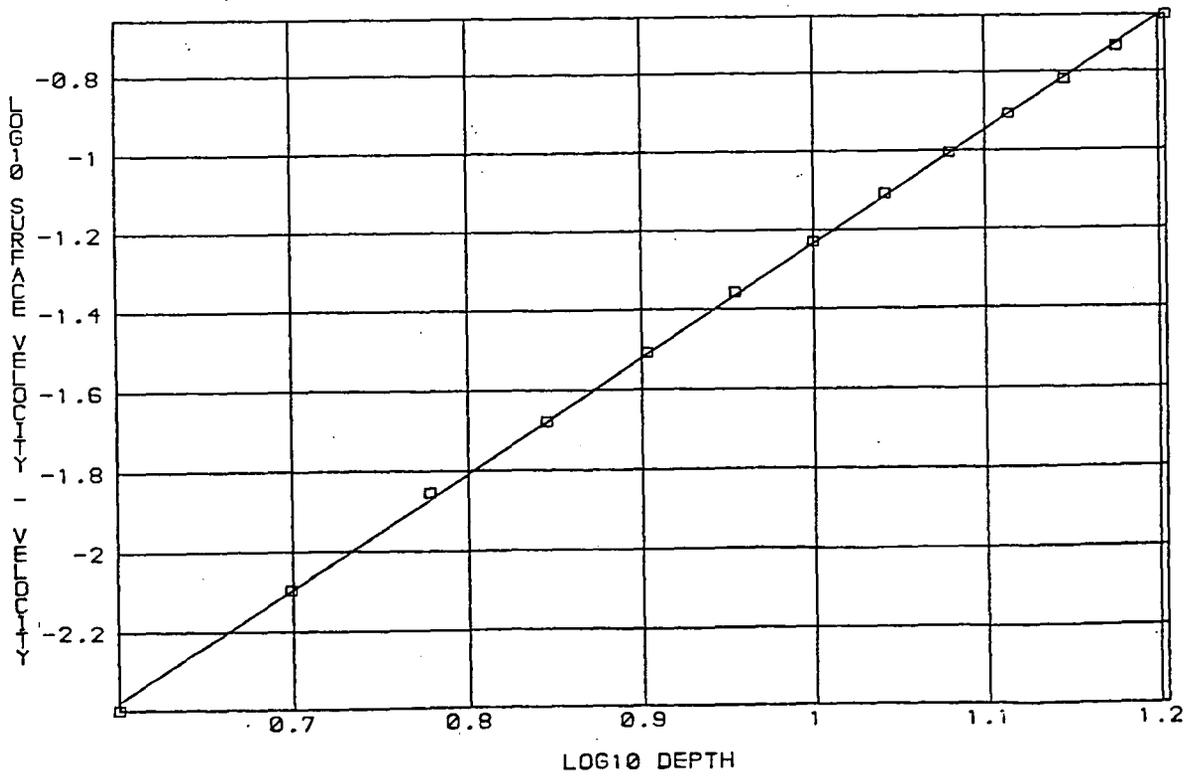


Figure 4.4

Rotating Couette flow.

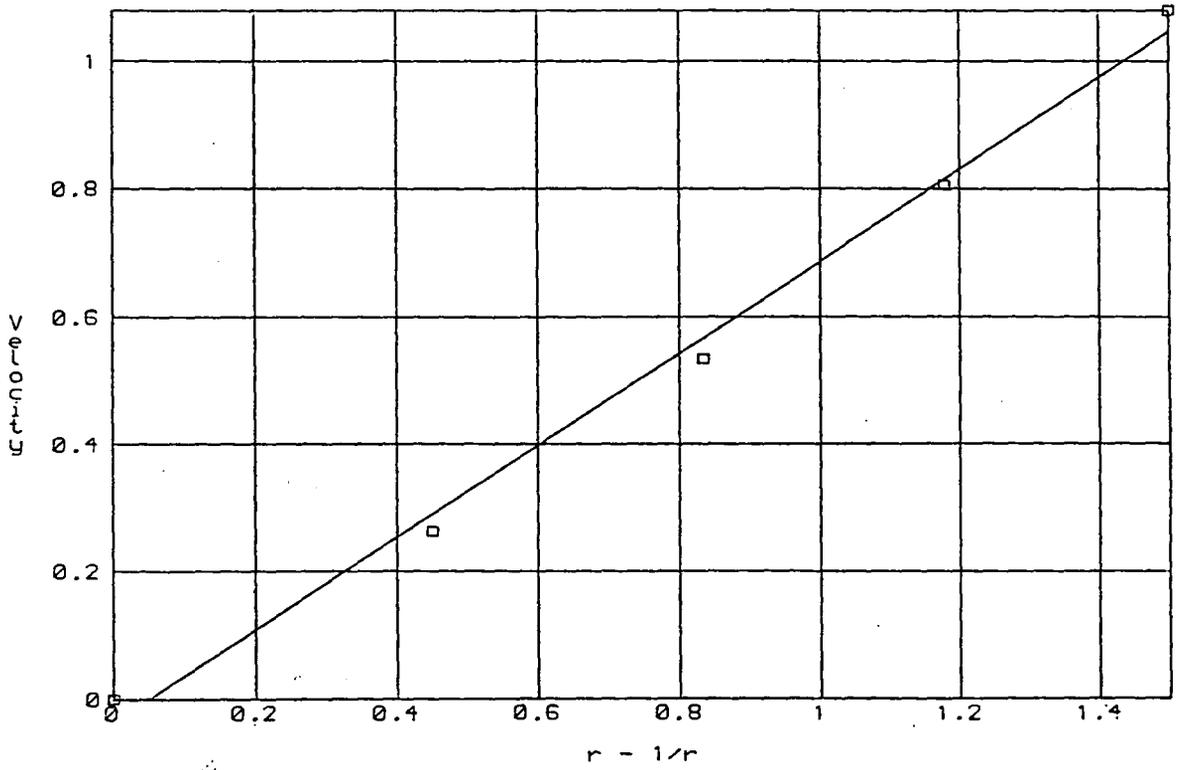


Figure 5.1

View of finite element mesh showing the nesting scheme.

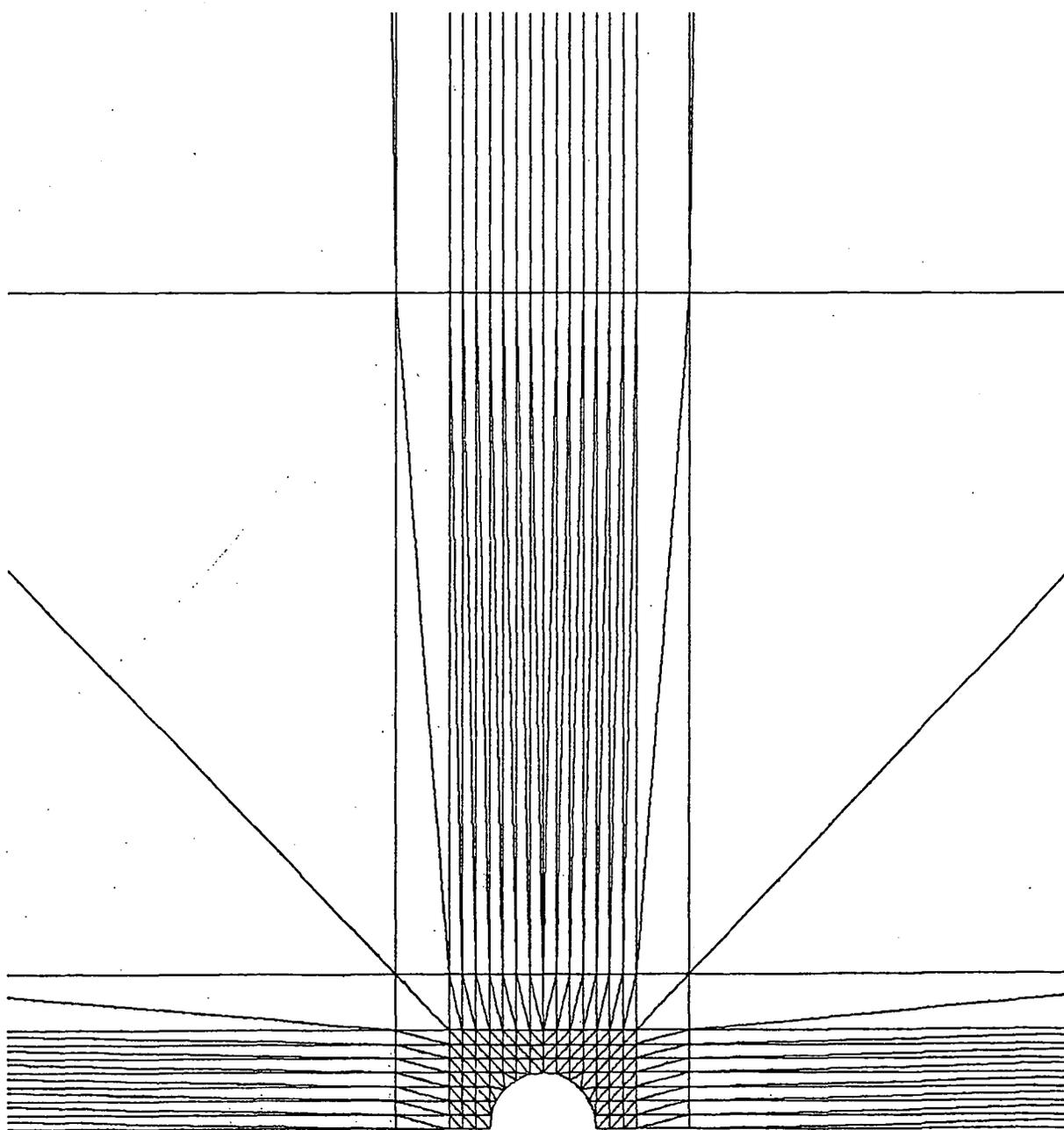


Figure 5.2

Detail from finite element mesh for modelling
flow over a semi-cylindrical ridge.

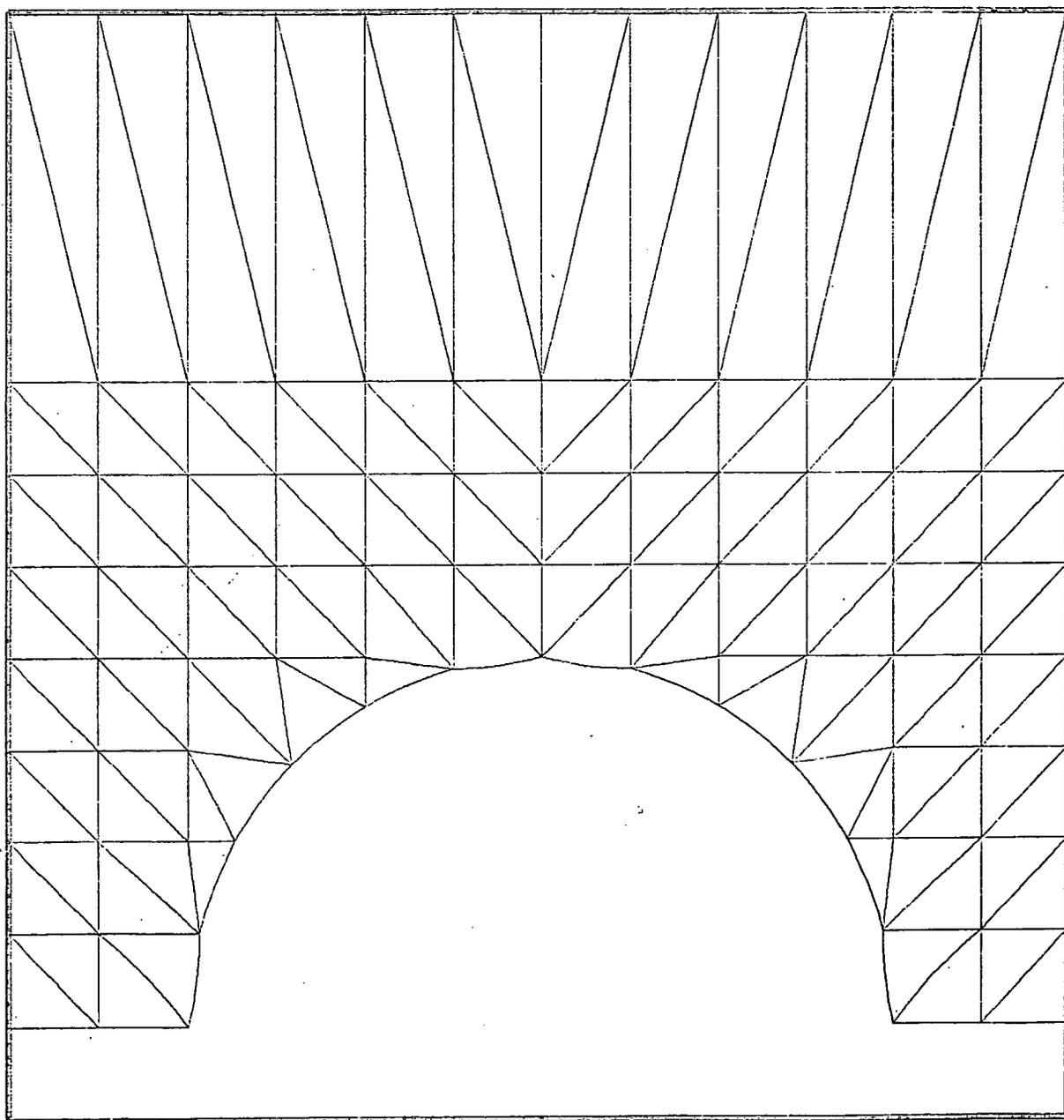


Figure 5.3

Detail from finite element mesh for modelling
flow over a truncated sine ridge.

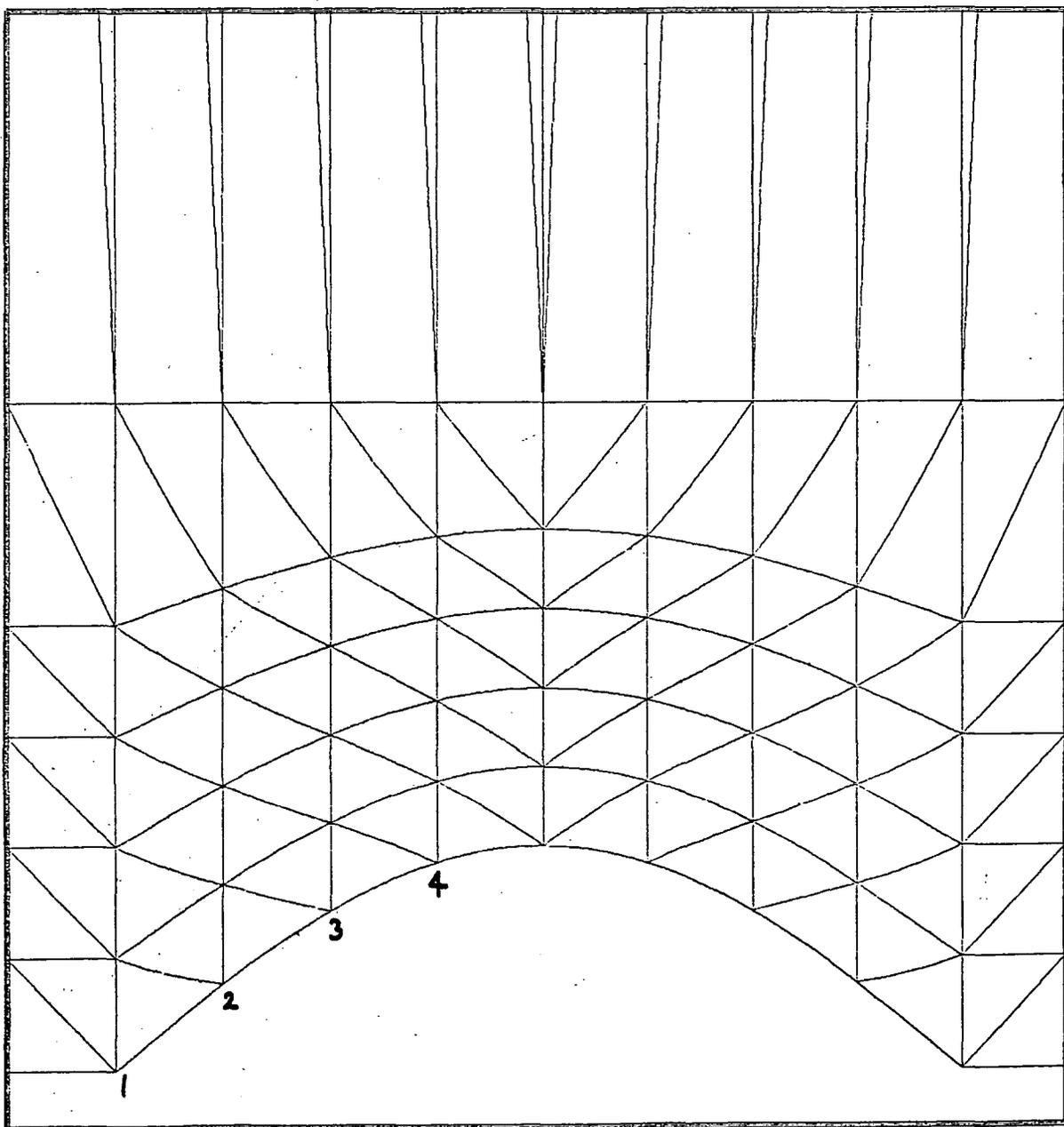
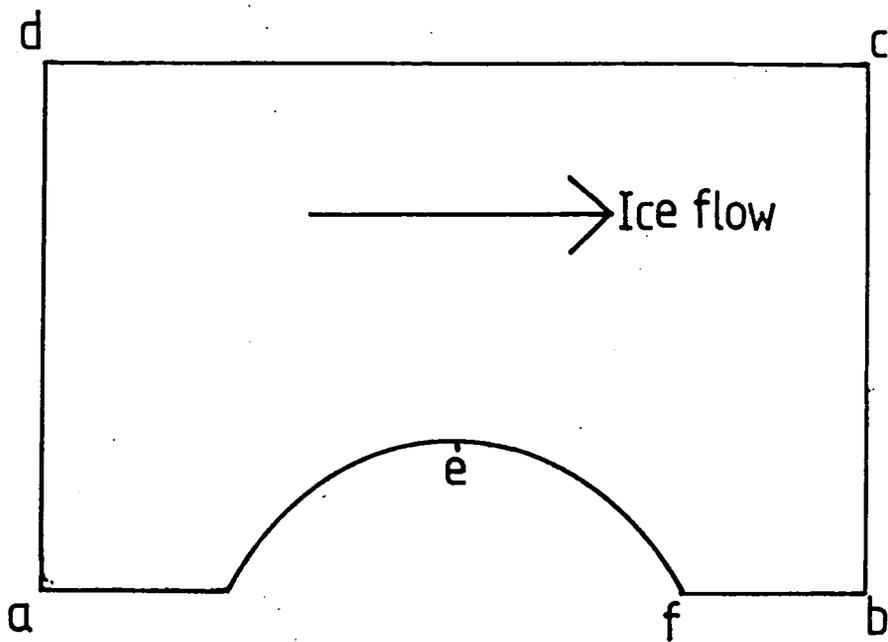


FIGURE 5.4 SCHEMATIC DIAGRAM OF BOUNDARY CONDITIONS FOR PLANE-STRAIN



adcb - Traction at infinity

ab - Zero normal velocity, perfect slip or

Weertman; if cavity exists, then

tractions along ef

a, b - Slip velocity at infinity

Figure 5.5

Velocity vector plot for an incompressible
linear rheology with $v_d = 100 \text{ alu/a}$ (Case 3).

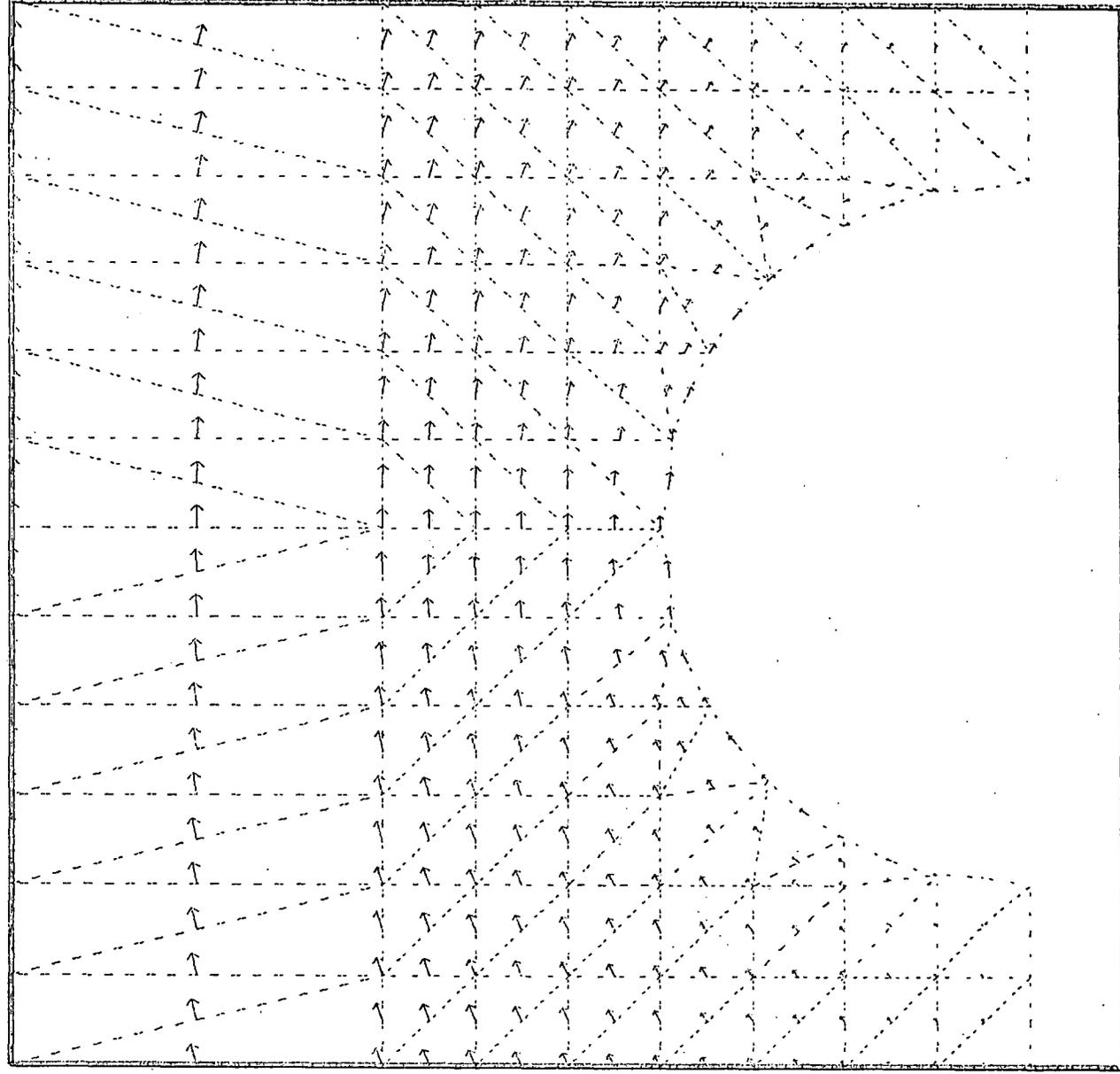


Figure 5.6

Larger view of velocity vector plot for an
incompressible linear rheology with
 $v_d = 100 a\lambda/a$ (Case 3).

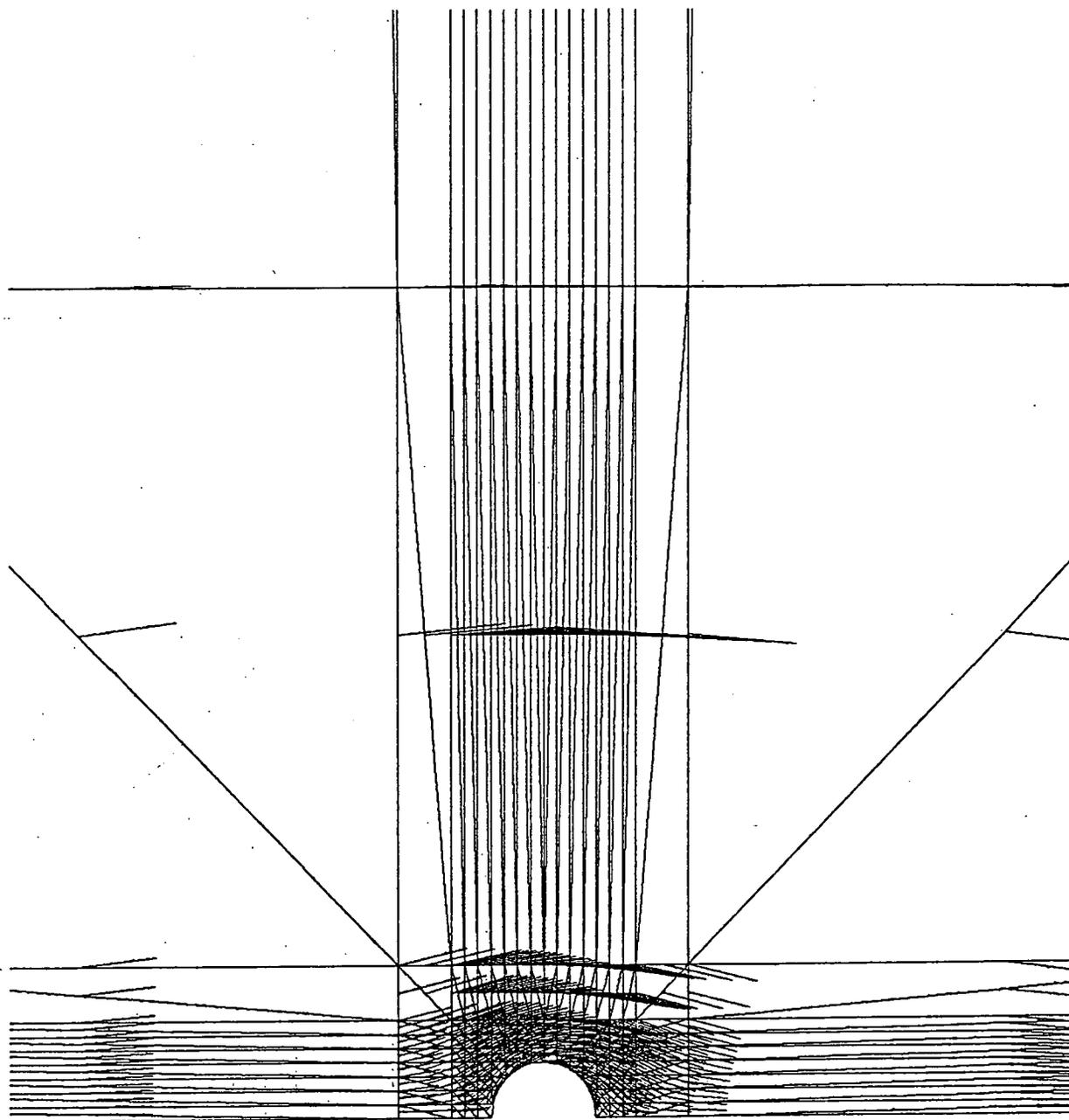


Figure 5.7 v_τ against $\sin\gamma$ for an incompressible linear rheology with $v_d = 10 \text{ alu/a}$ (Case 1).

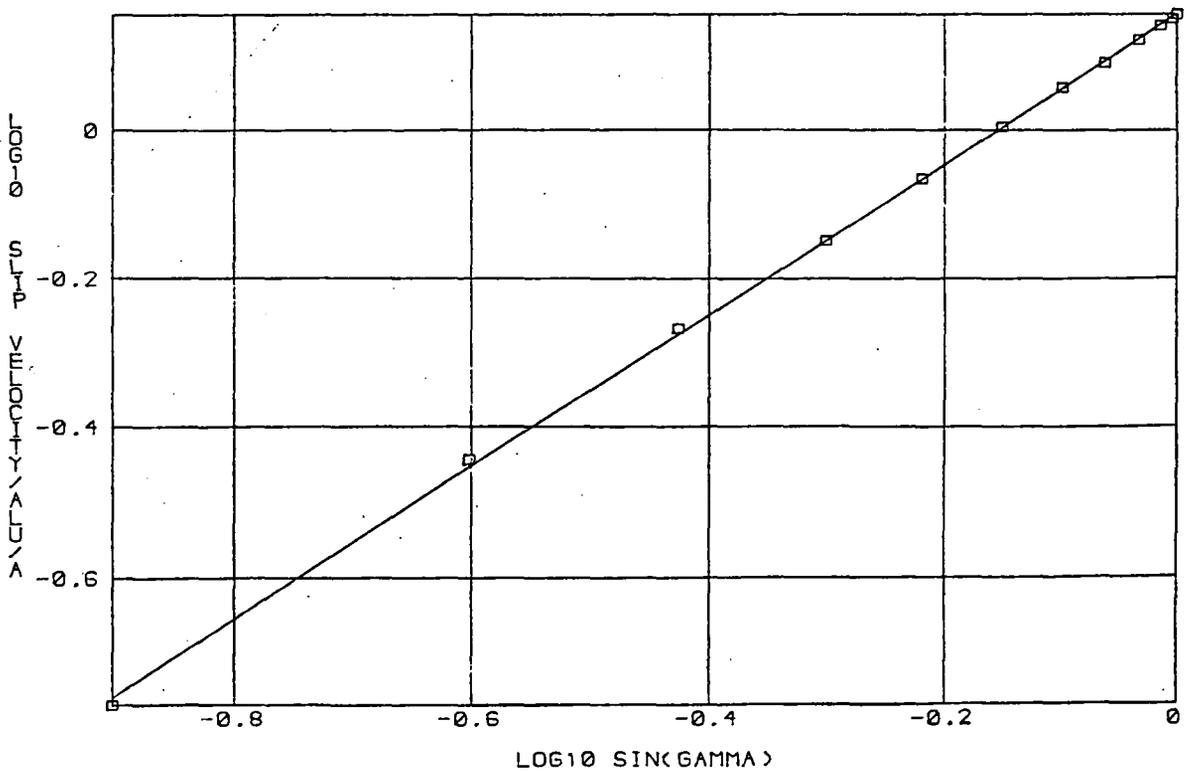
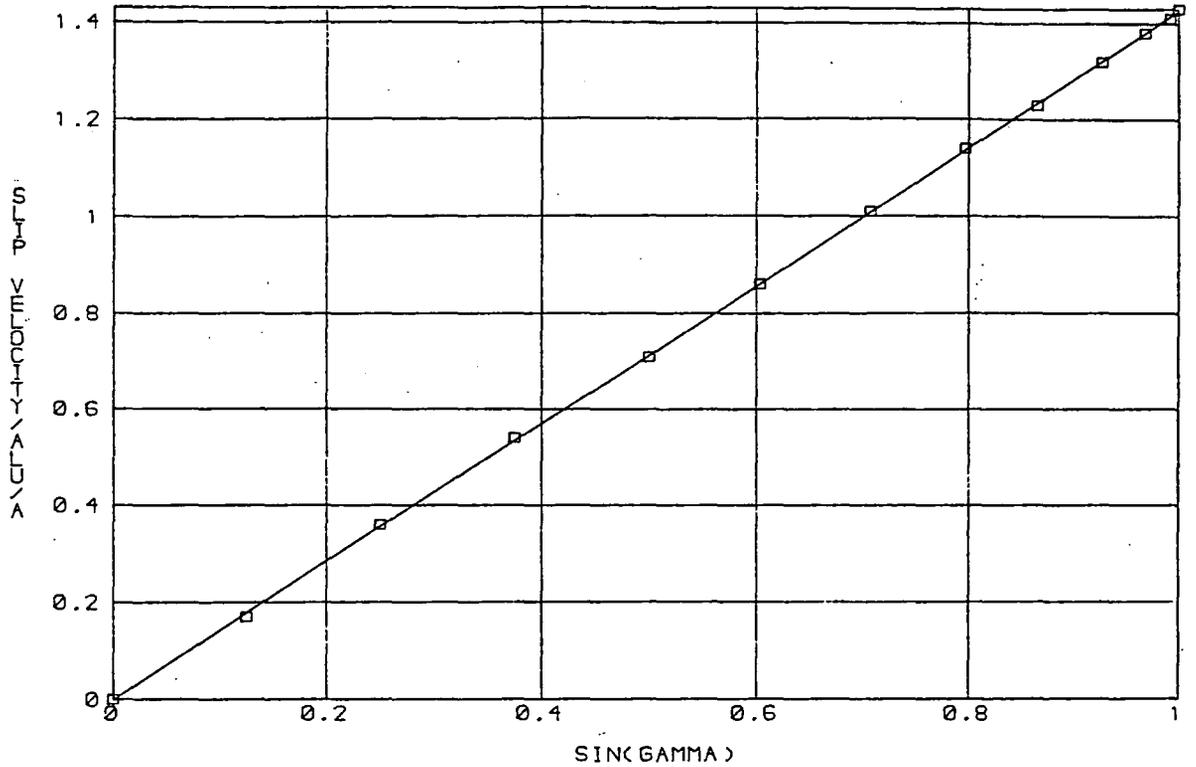


Figure 5.8

v_t against $\sin \gamma$ for an incompressible linear rheology with a cavity and $v_d = 400 a\omega/a$ (Case 7).

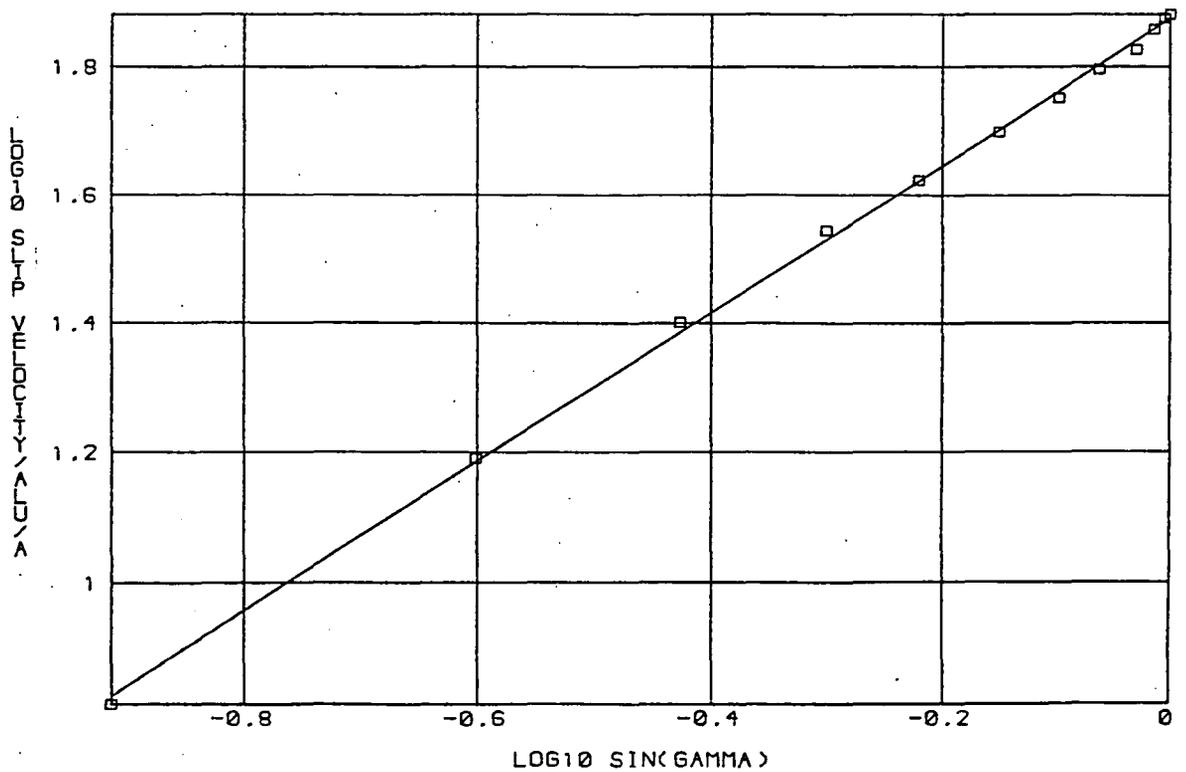
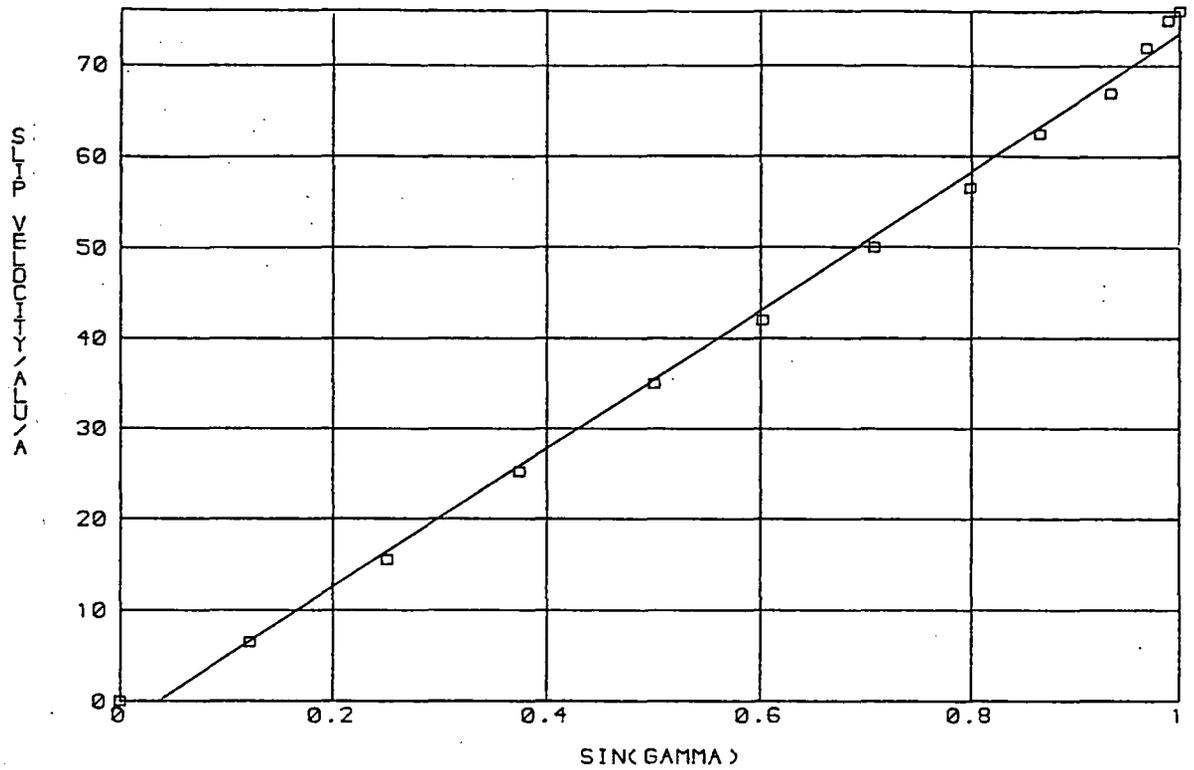


Figure 5.9 v_τ against $\sin\gamma$ for an incompressible linear rheology with a cavity and $v_d = 10 a\mu/a$ (Case 6).

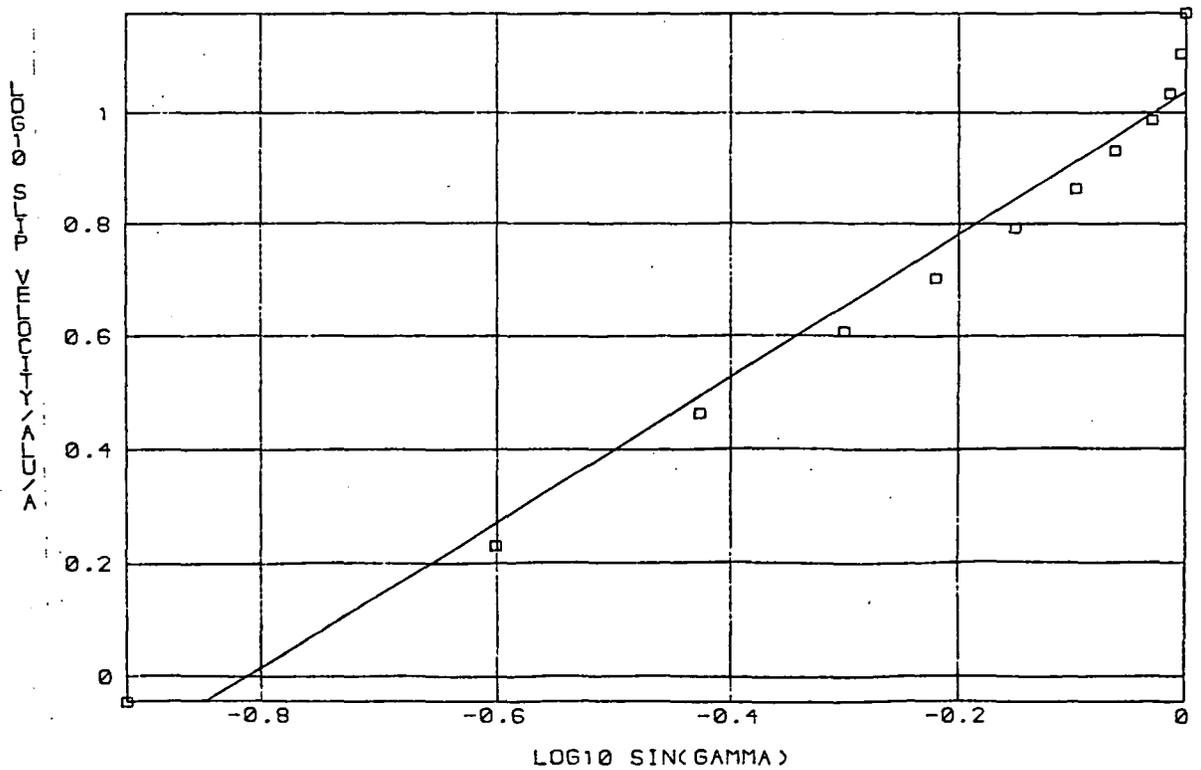
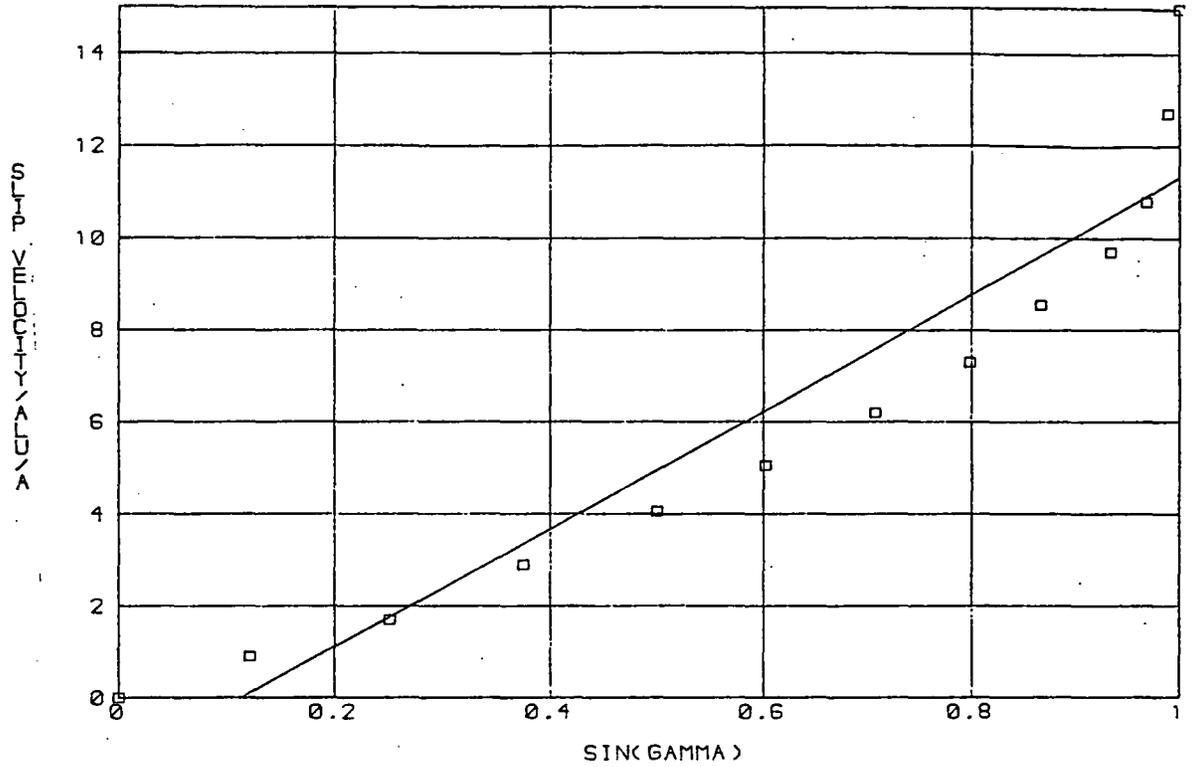


Figure 5.10

v_t against $\sin \gamma$ for a compressible linear rheology with $\chi = 1 \text{ alu}^2/\text{bar}\cdot\text{a}$ and $v_d = 10 \text{ alu}/\text{a}$ (Case 8).

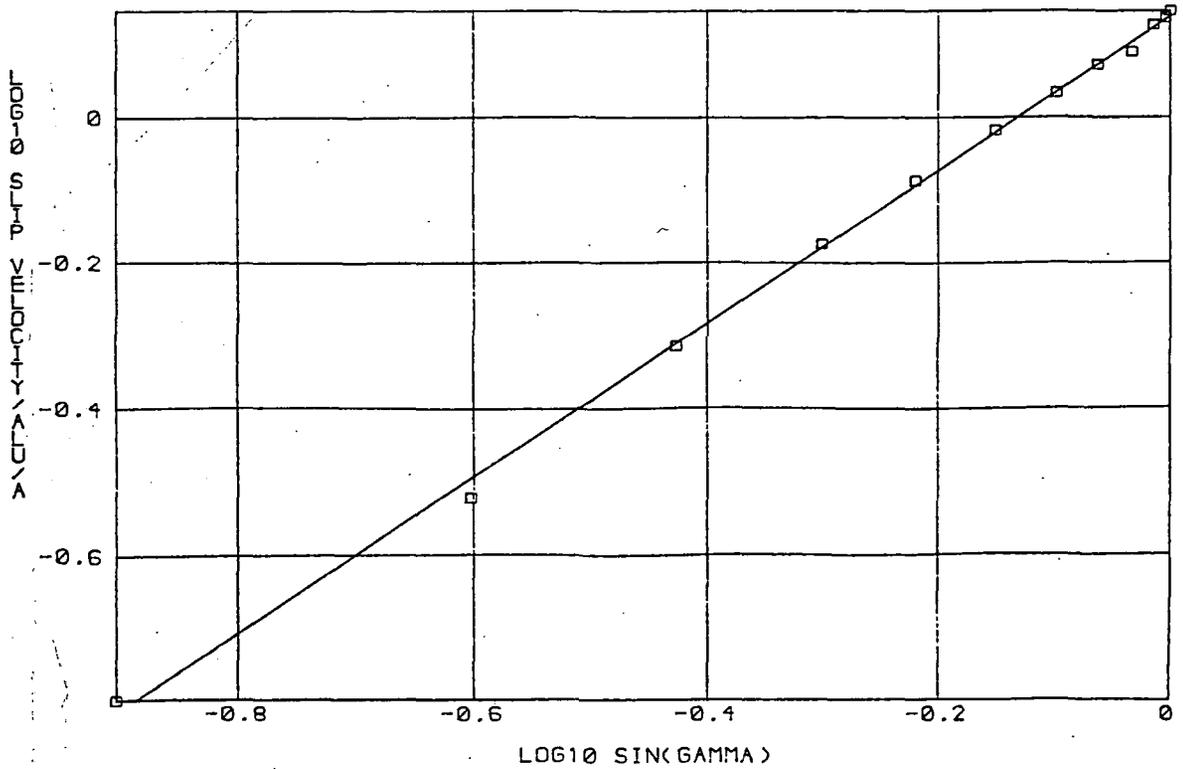
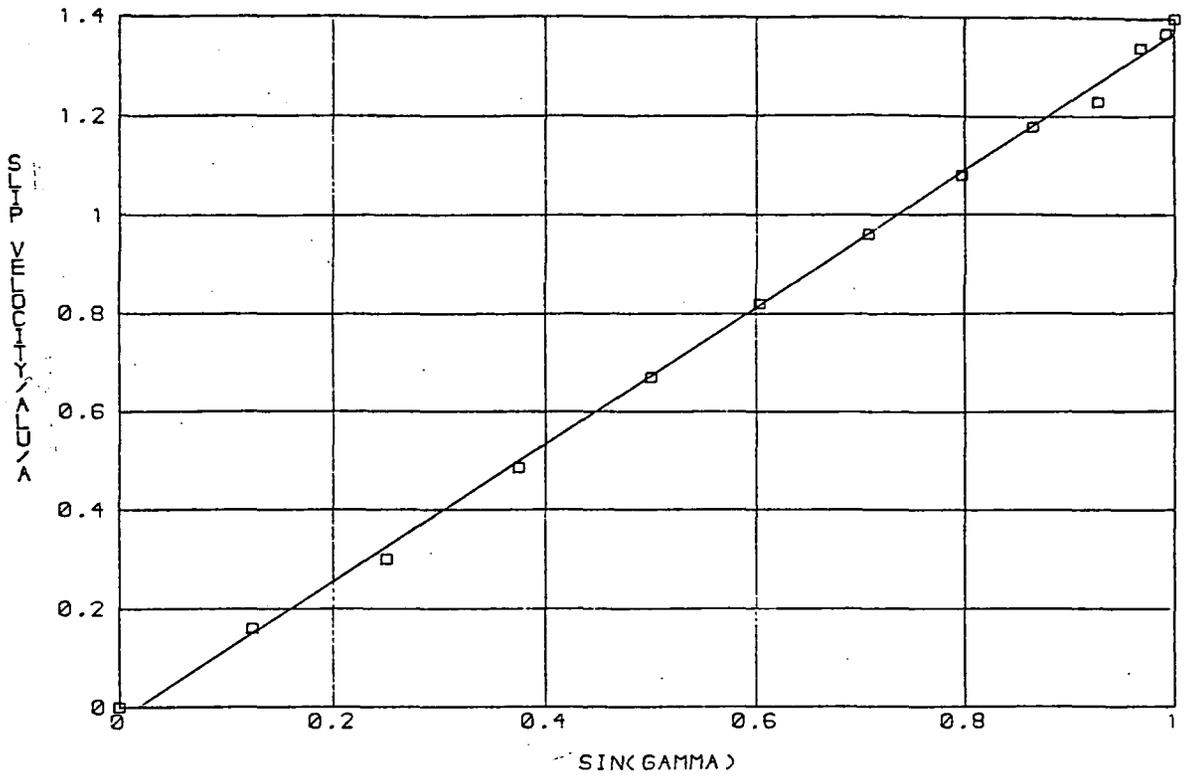


Figure 5.11

Velocity vector plot for a compressible linear rheology with $\chi = 10 \text{ alu}^2 / \text{bar} \cdot a$ and $v_d = 200 \text{ alu}/a$ (Case 16).

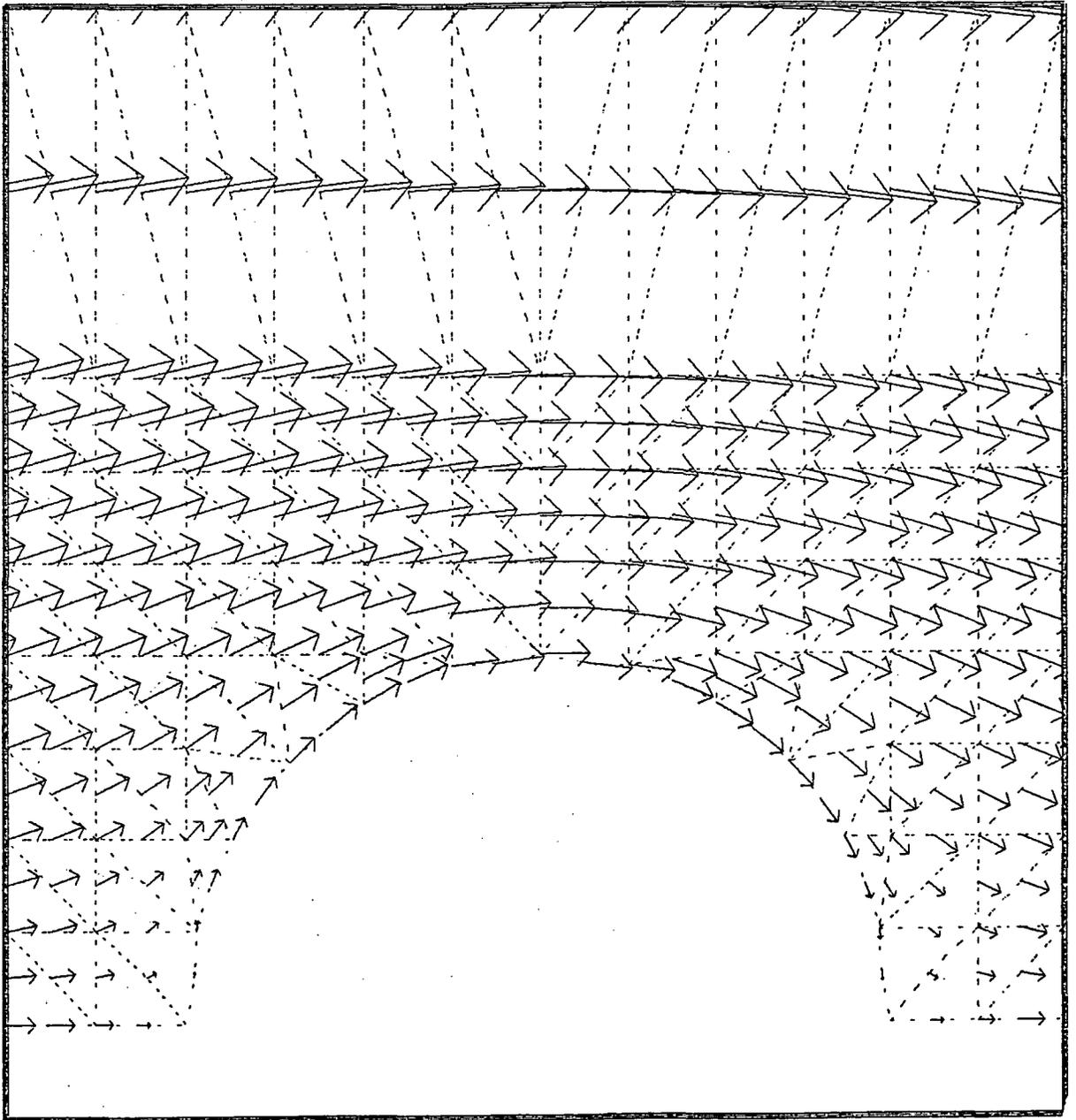


Figure 5.12

v_t against $\sin \gamma$ for a compressible linear rheology with $\chi = 10 \text{ alu}^2/\text{bar}\cdot a$ and $v_d = 200 \text{ alu}/a$ (Case 16).

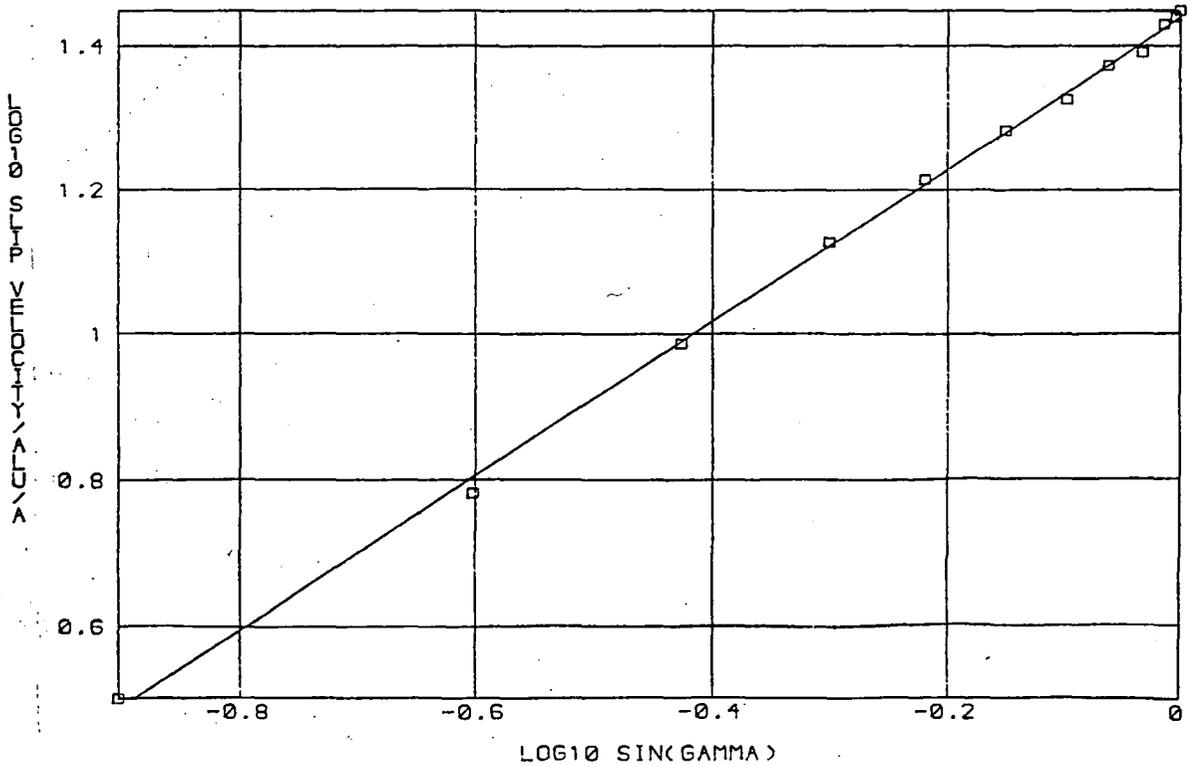
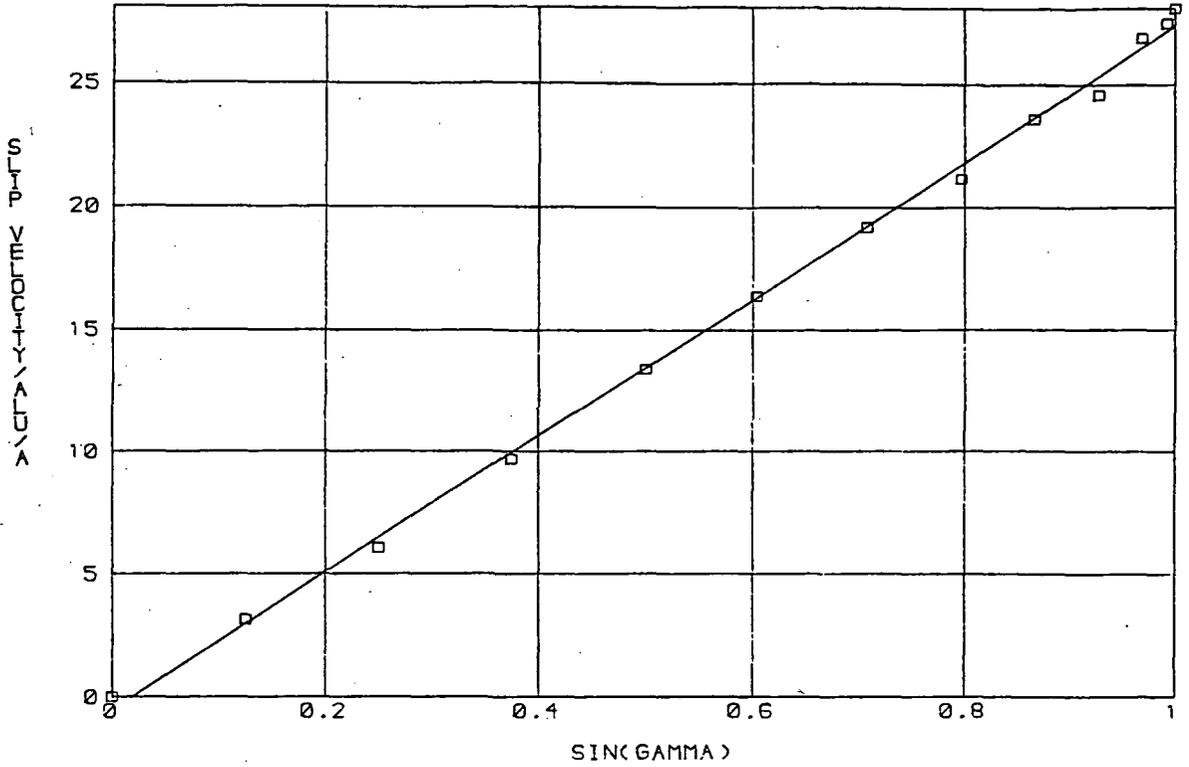


Figure 5.13

Velocity vector plot for an incompressible linear rheology with bed of $S = 100 \text{ alu/bar.a}$ and $v_d = 100 \text{ alu/a}$ (Case 20).

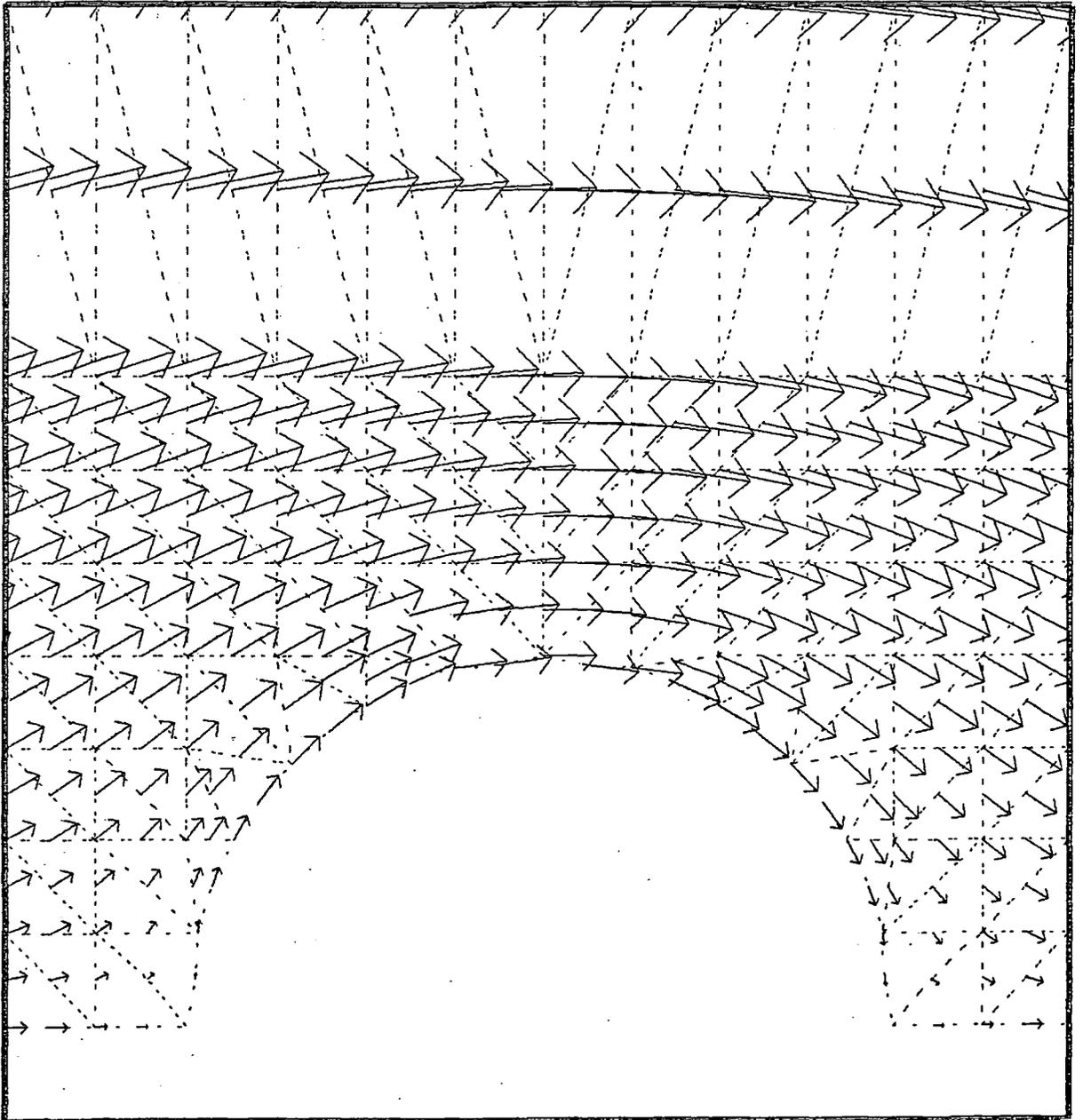


Figure 5.14 v_t against $\sin\gamma$ for an incompressible linear rheology with bed of $S = 10 \text{ alu/bar.a}$, $v_d = 10 \text{ alu/a}$ (Case 18)

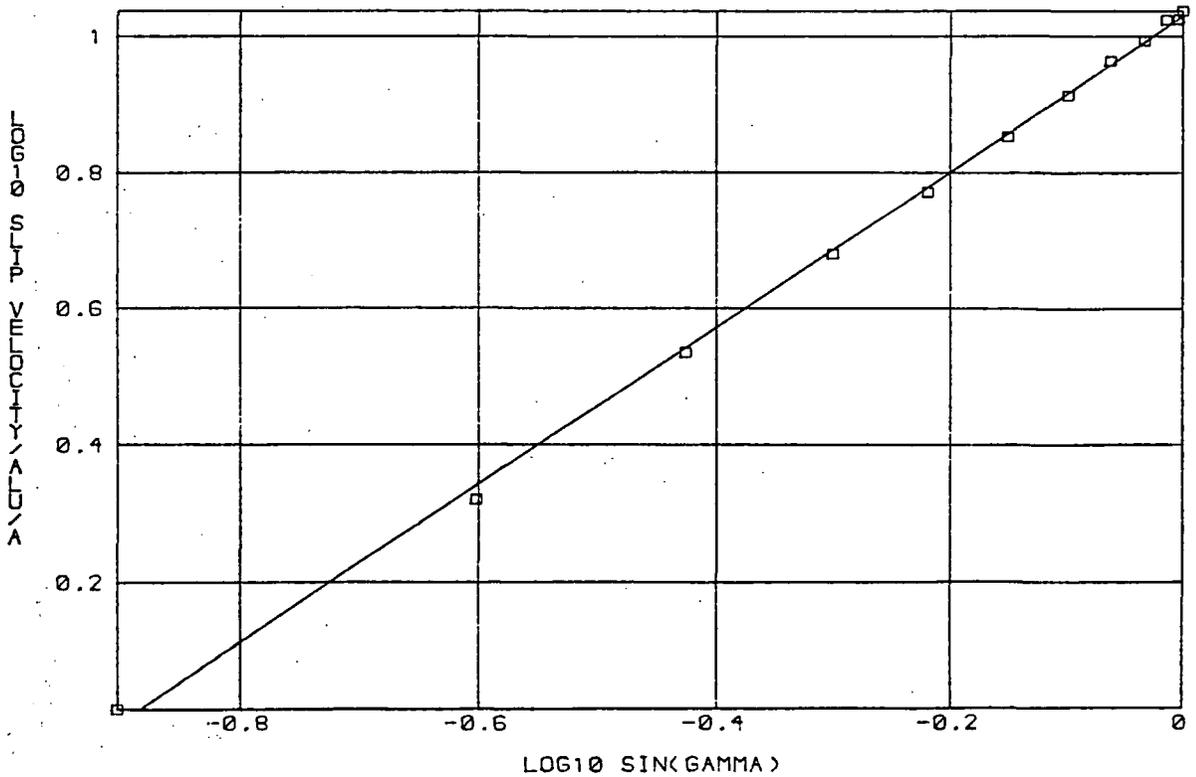
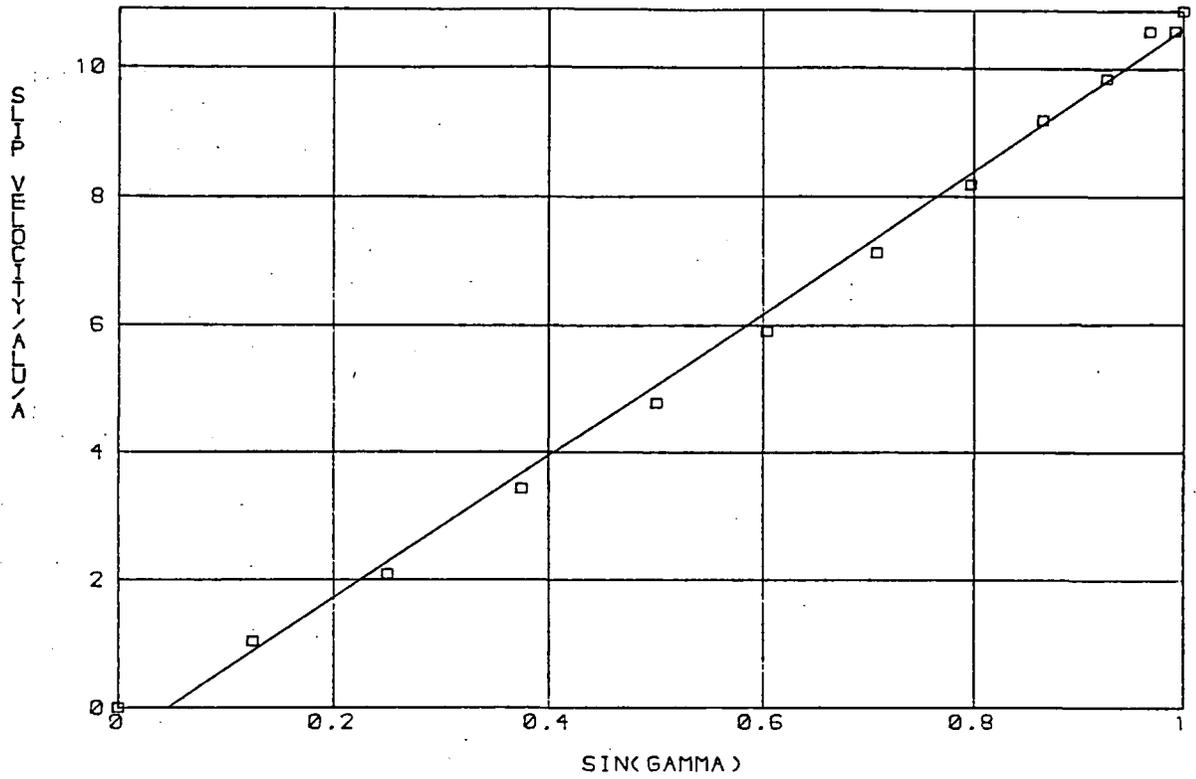


Figure 5.15

v_t against $\sin \gamma$ for an incompressible linear rheology with bed of $S = 400 \text{ alu}/\text{bar}\cdot\text{a}$, $v_d = 400 \text{ alu}/\text{a}$ (Case 22).

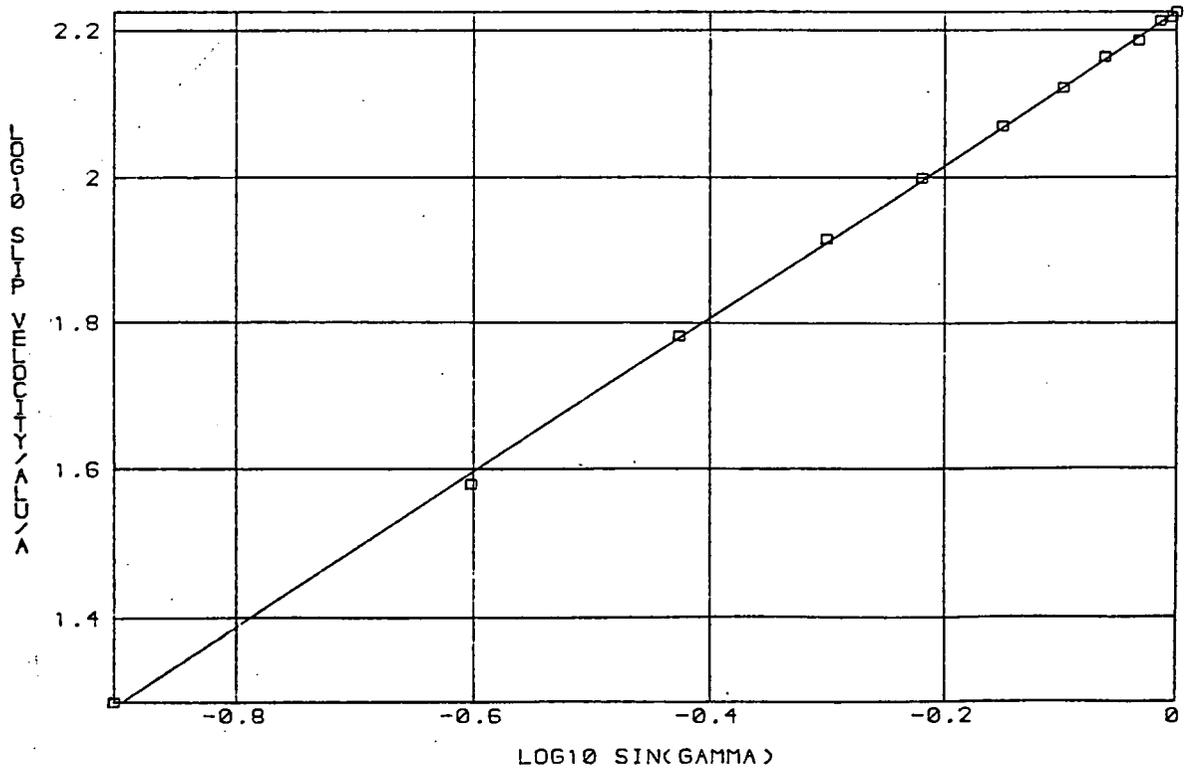
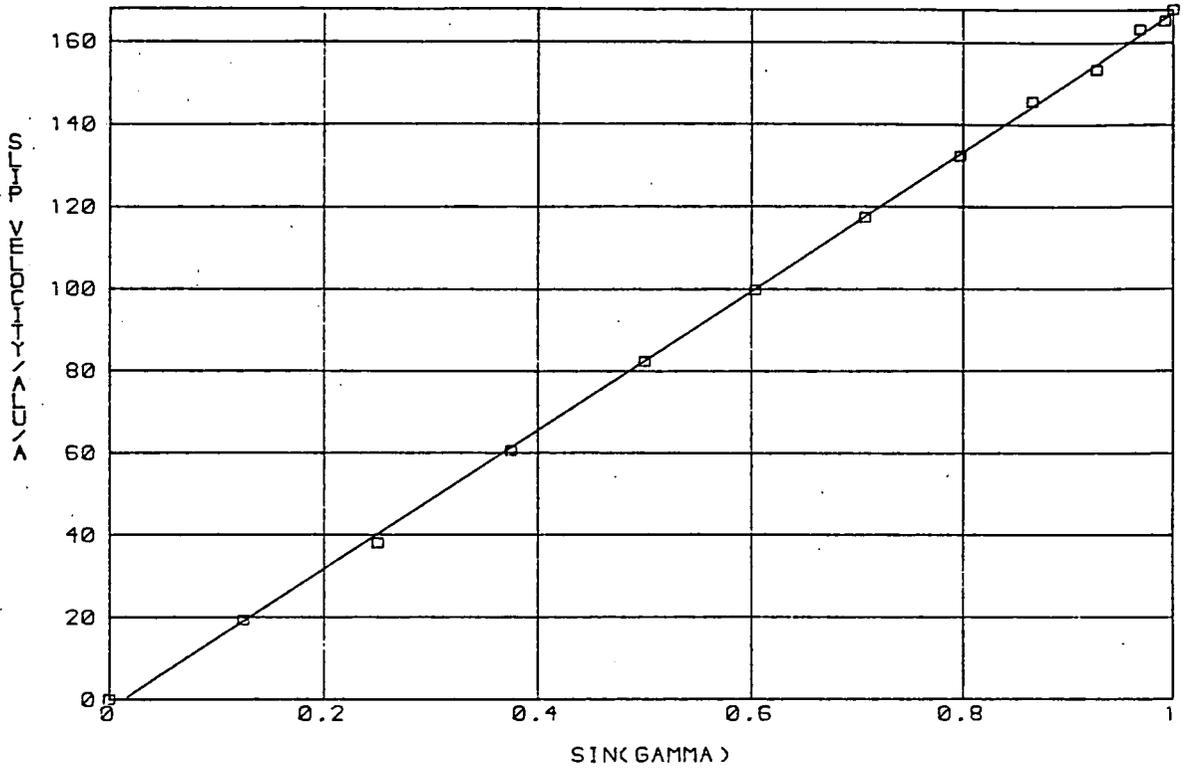


Figure 5.16

Velocity vector plot for an incompressible
Glen rheology with $v_d = 100 \text{ alu/a}$ (Case 25).

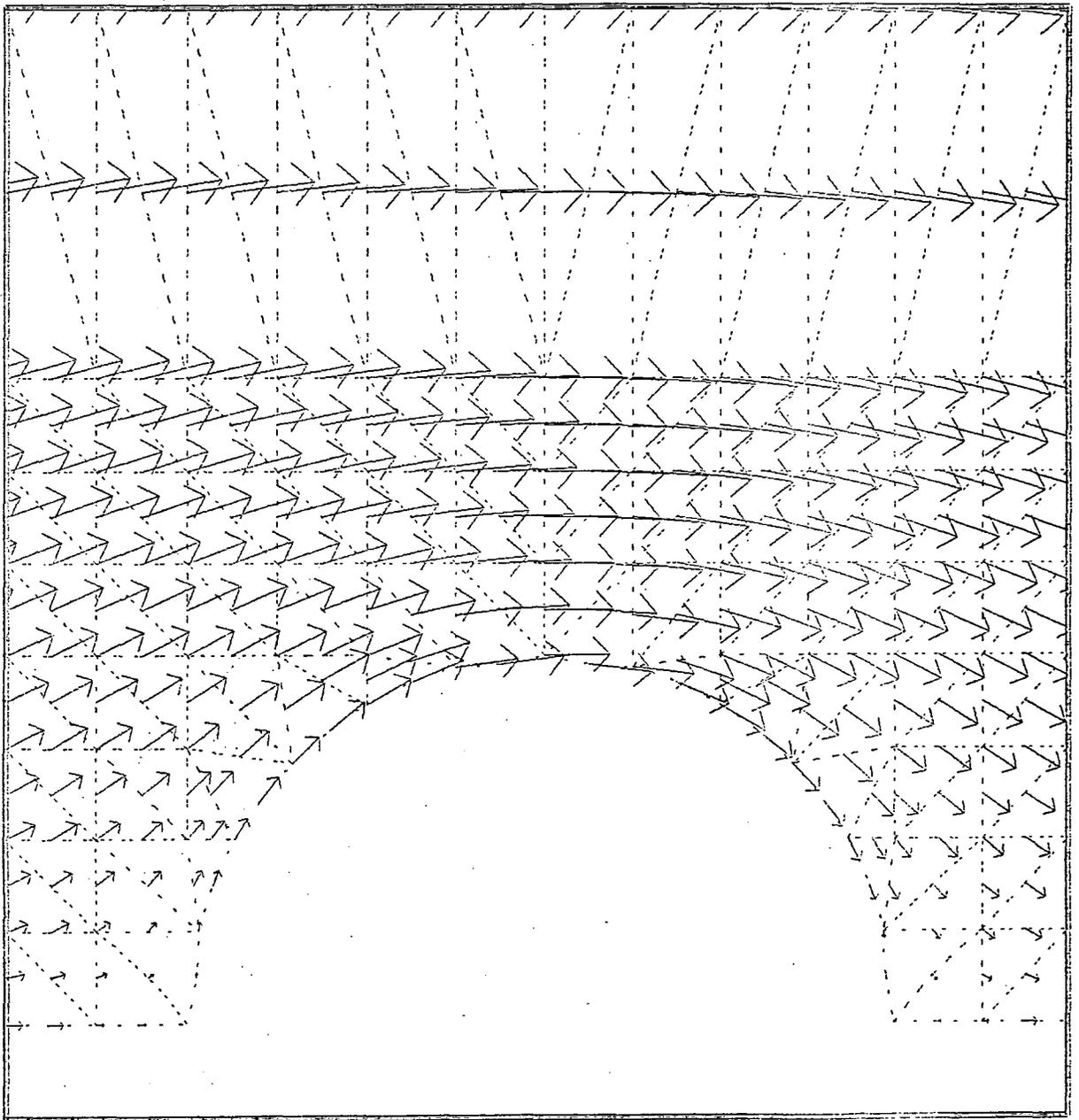


Figure 5.17 v_τ against $\sin\gamma$ for an incompressible Glen rheology with $v_d = 10 \text{ au/a}$ (Case 23).

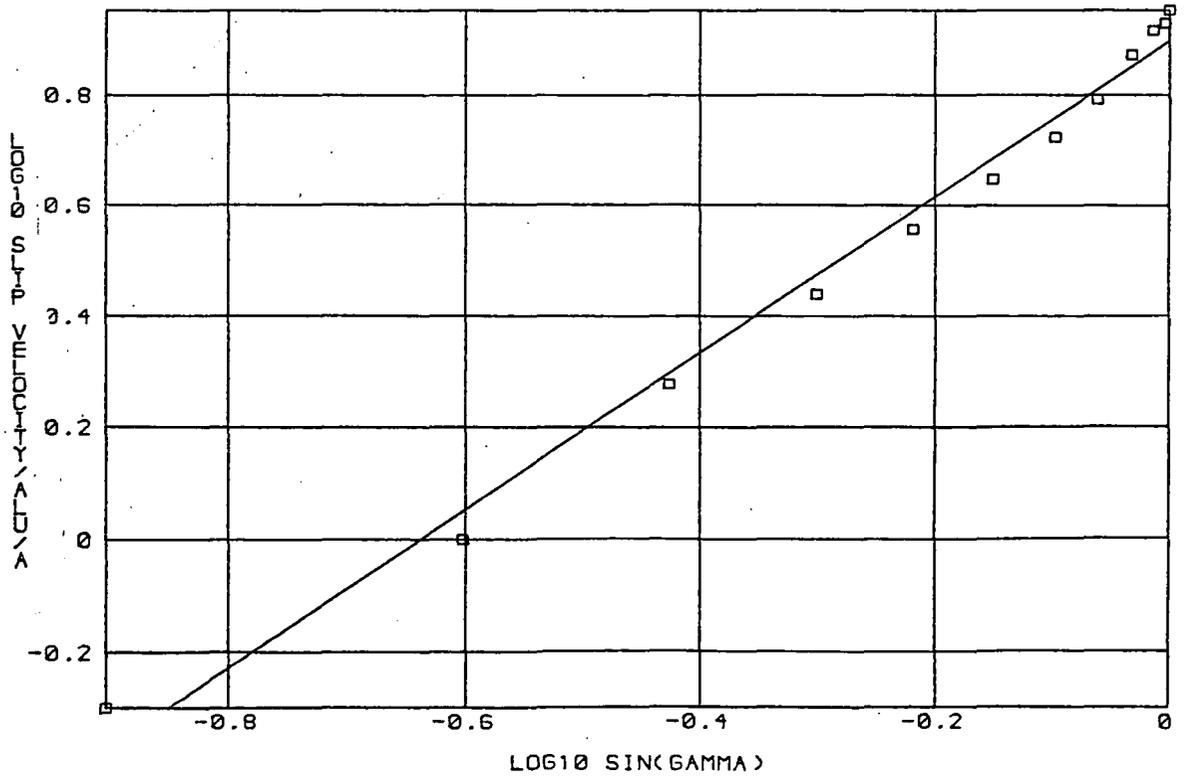
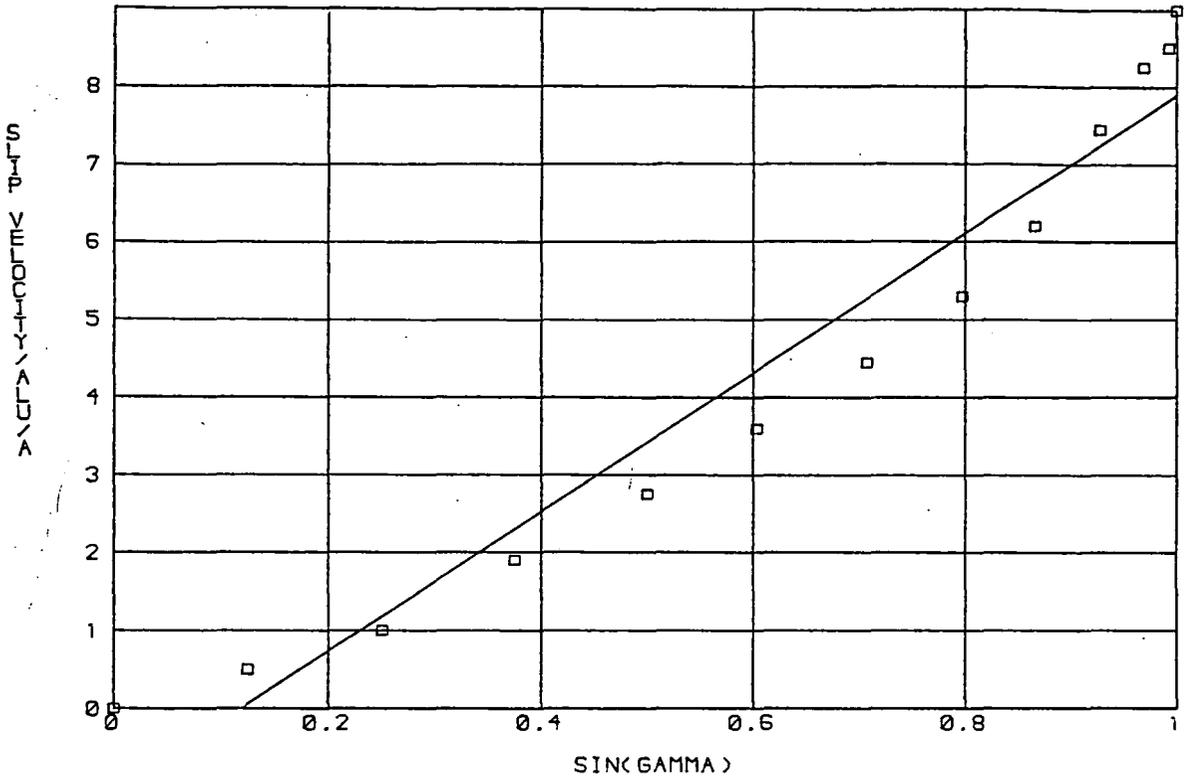


Figure 5.18

v_τ against $\sin\gamma$ for an incompressible
 Glen rheology with $v_d = 400 \text{ alu/a}$ (Case 25).

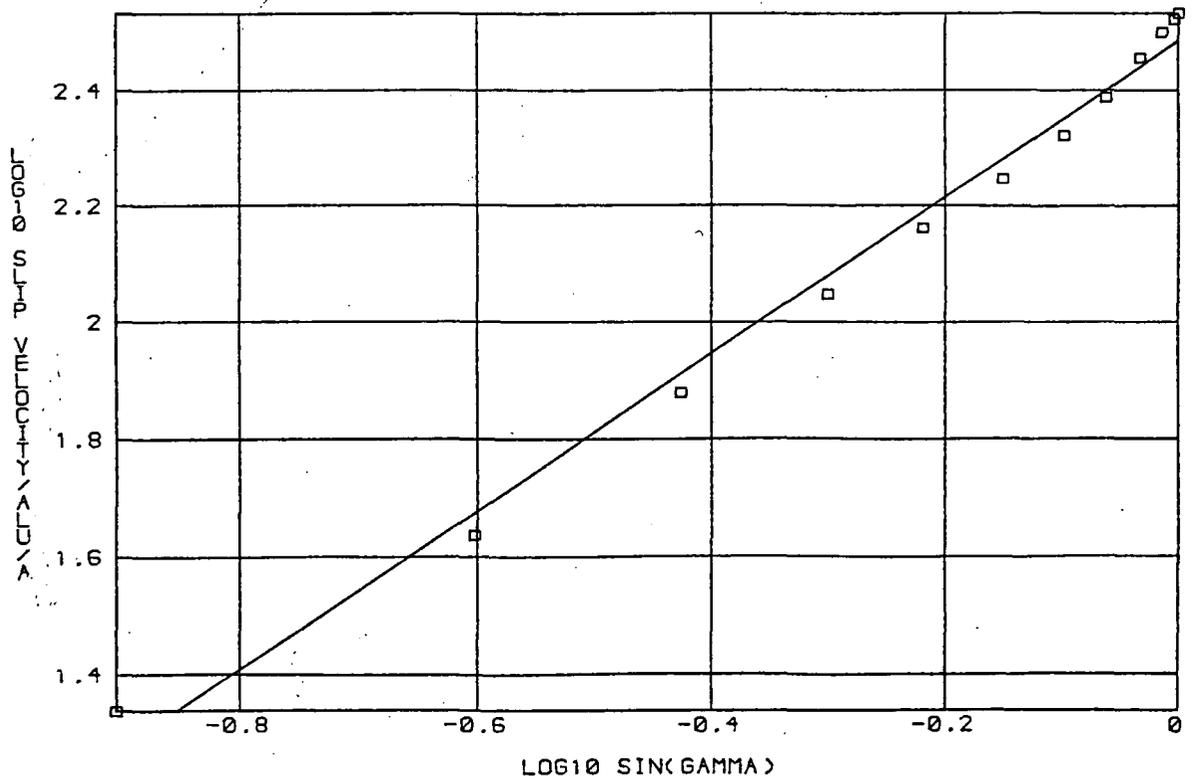
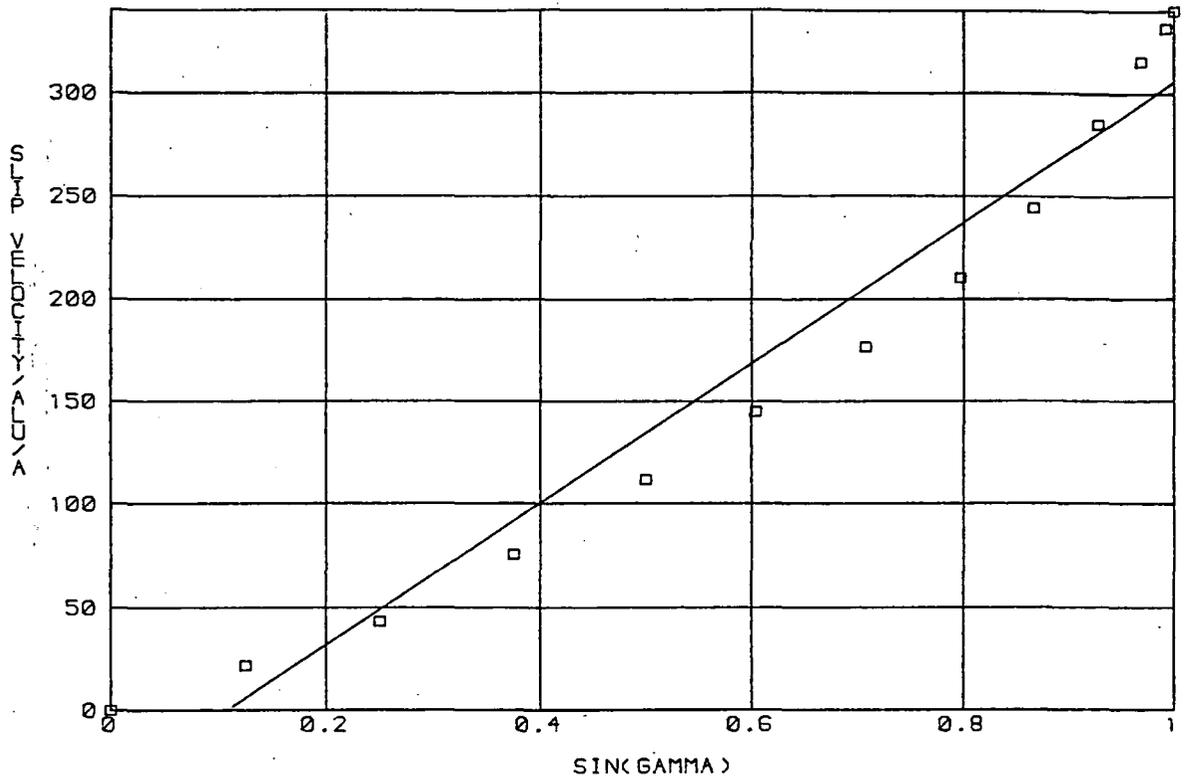


Figure 5.19

Velocity vector plot for an incompressible

Glen rheology with a cavity and $v_d = 400 \text{ alu/a}$

(Case 28).

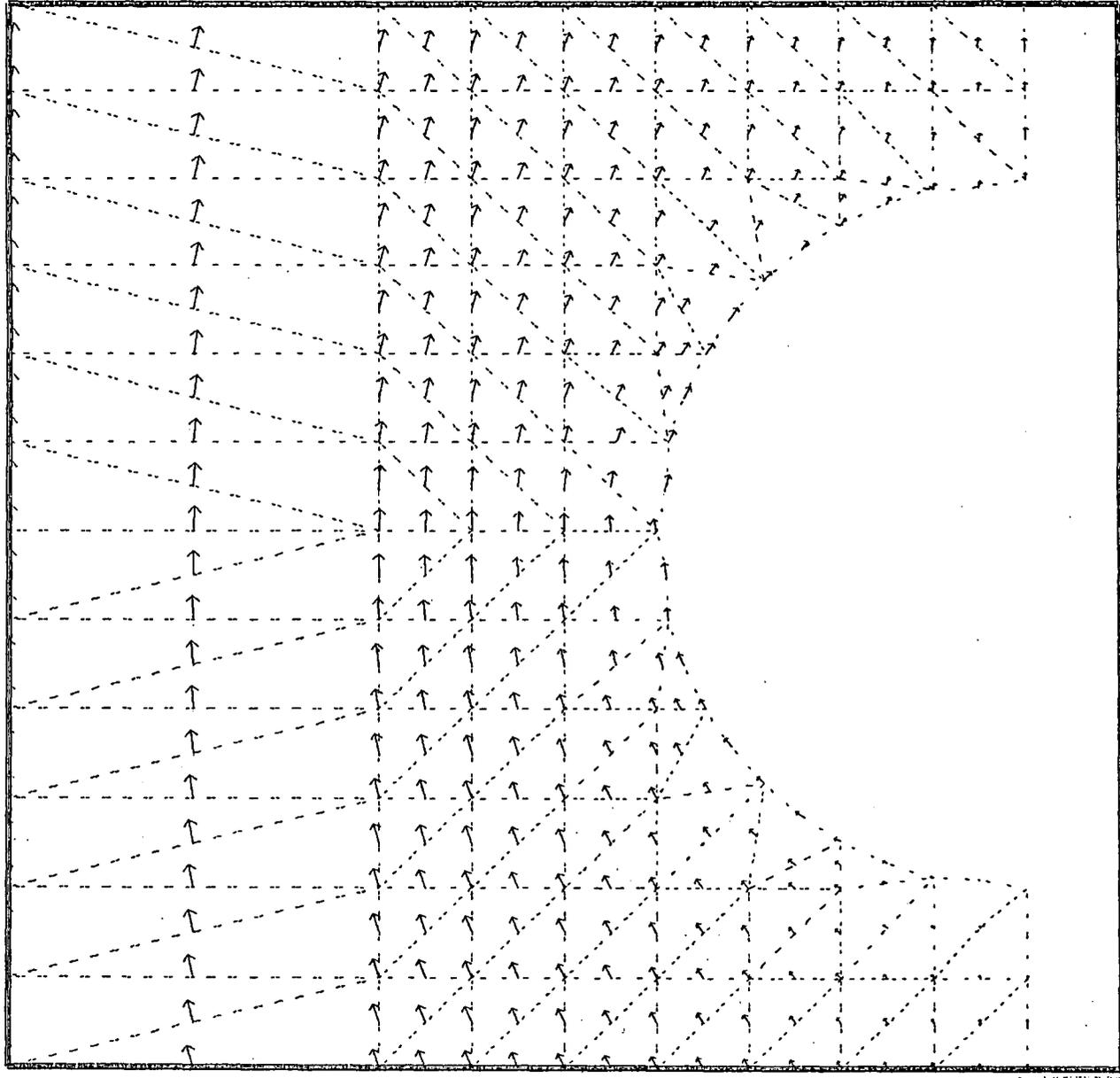


Figure 5.20

v_τ against $\sin \gamma$ for an incompressible
 Glen rheology with a cavity and $v_d = 400 \text{ au/a}$
 (Case 28).

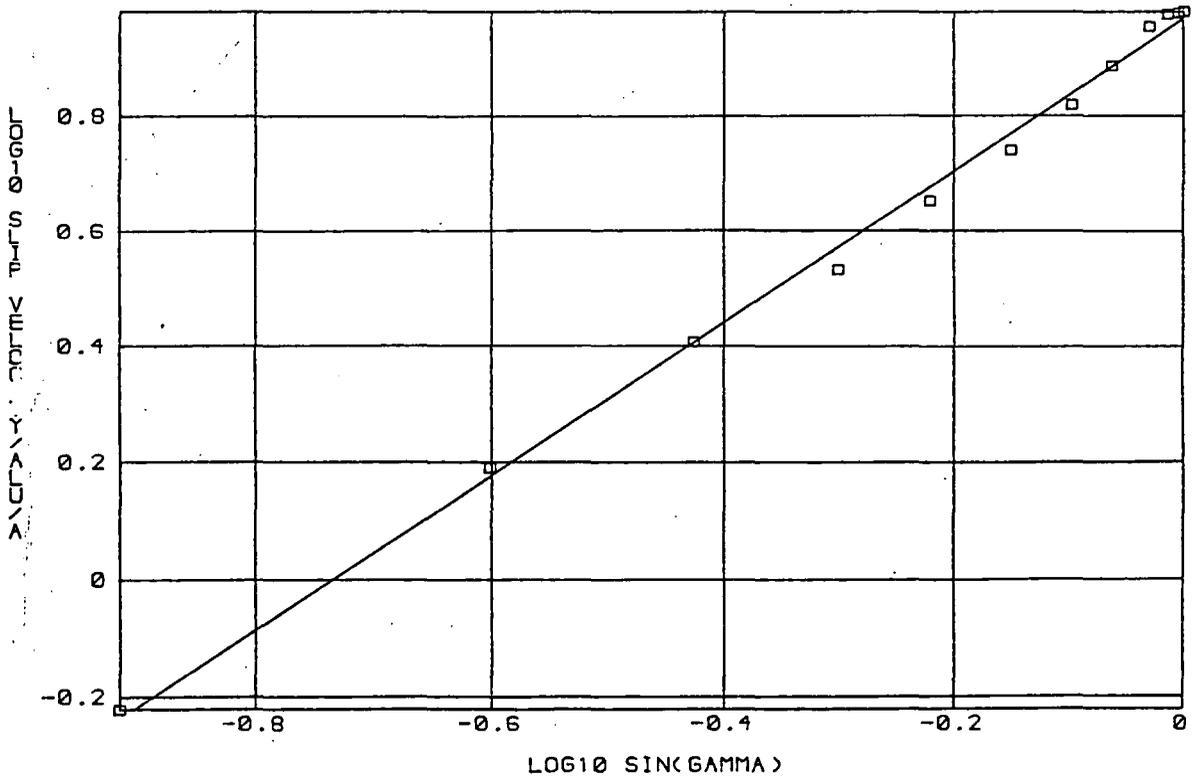
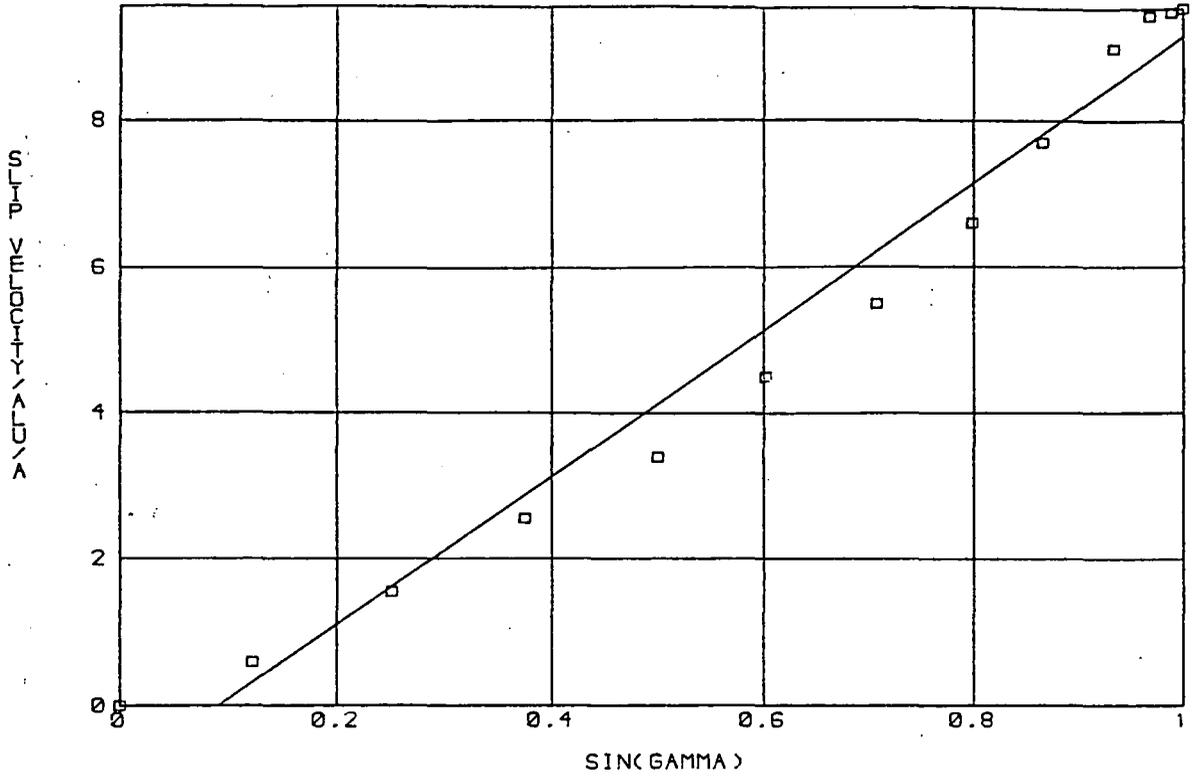


Figure 5.21

Velocity vector plot for a compressible Glen
rheology with $\chi = 1 \text{ alu}^2/\text{bar}\cdot\text{a}$ and
 $v_d = 100 \text{ alu}/\text{a}$ (Case 31).

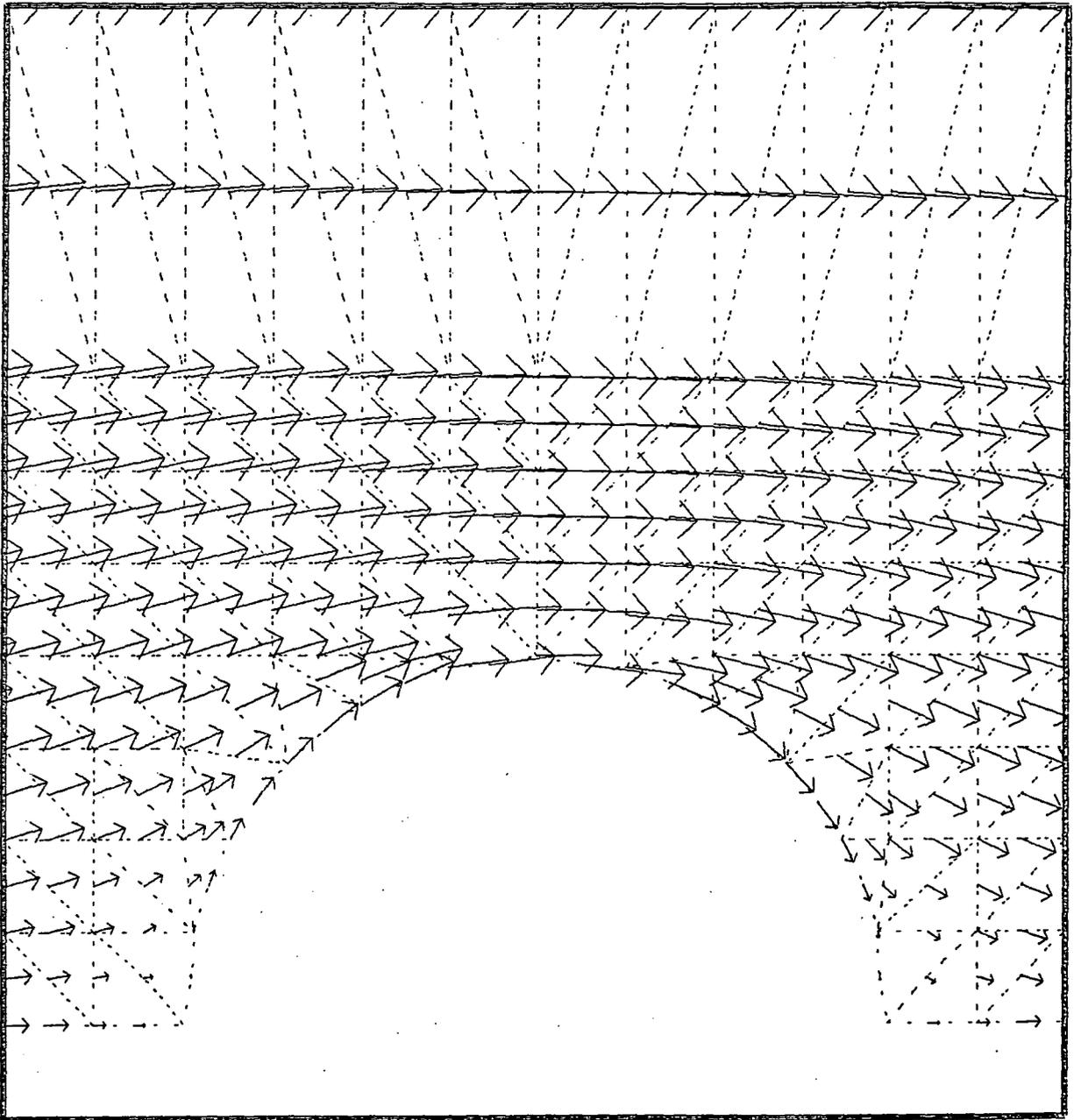


Figure 5.22

v_τ against $\sin \gamma$ for a compressible Glen rheology with $\chi = 1 \text{ alu}^2/\text{bar}\cdot\text{a}$ and $v_d = 200 \text{ alu}/\text{a}$ (Case 32).

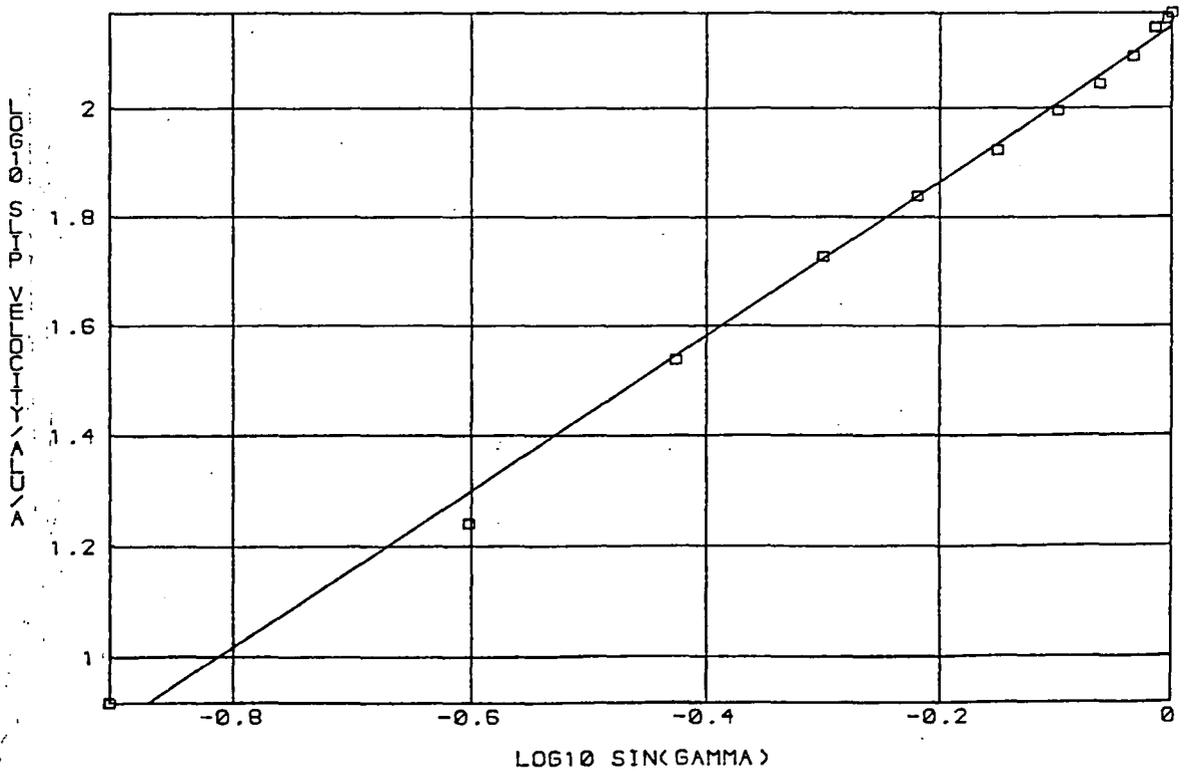
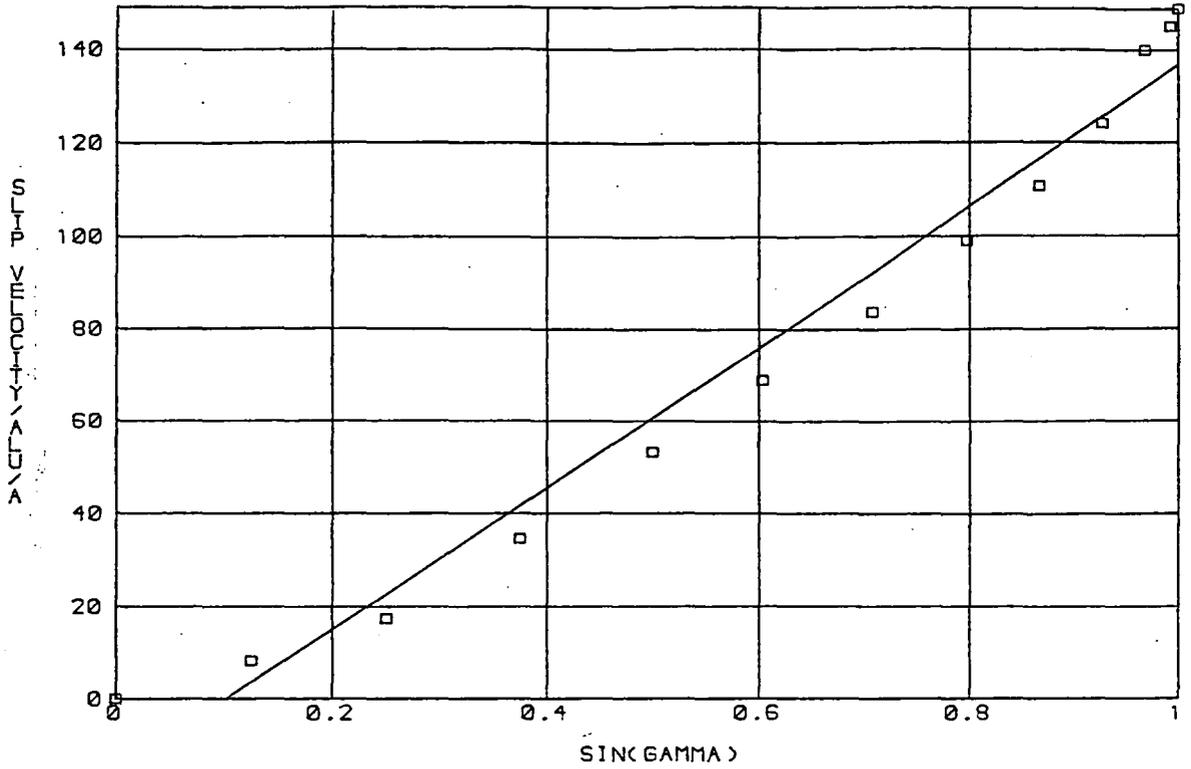


Figure 5.23

v_τ against $\sin\gamma$ for an incompressible
 Glen rheology with bed $S = 10 \text{ alu}/\text{bar}\cdot\text{a}$ and
 $v_d = 10 \text{ alu}/\text{a}$ (Case 34).

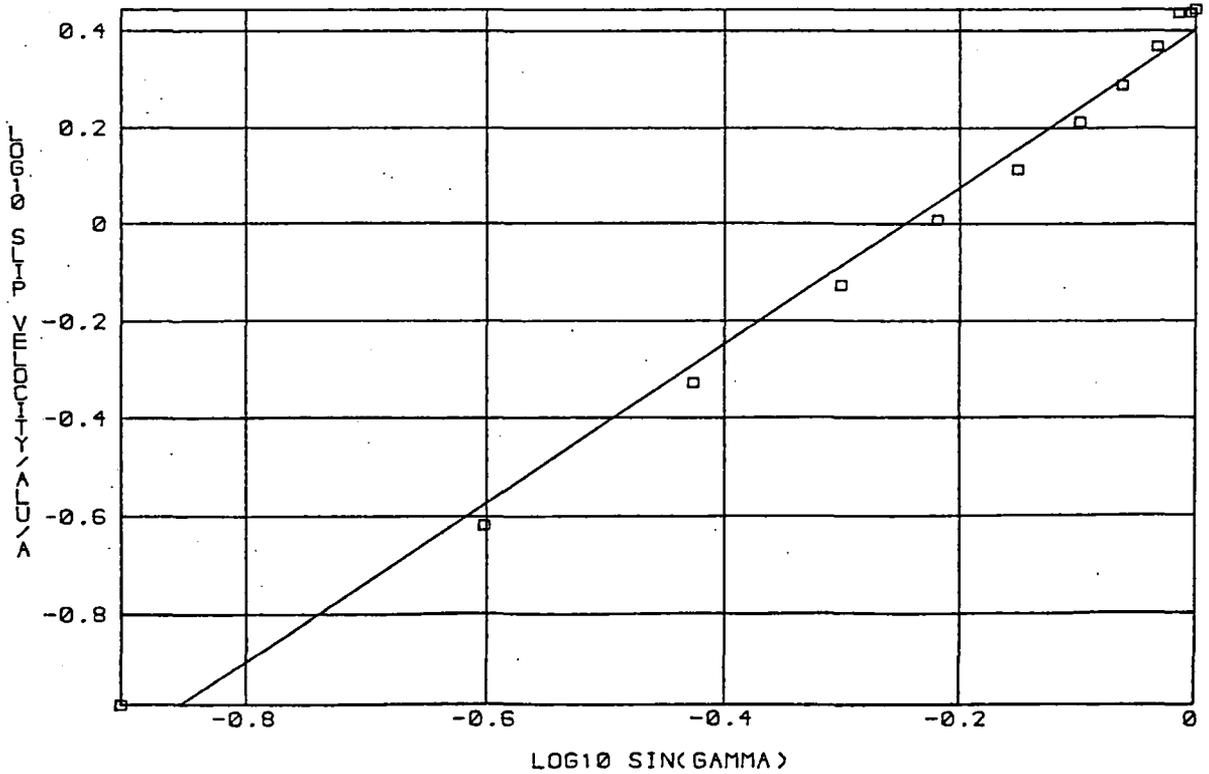
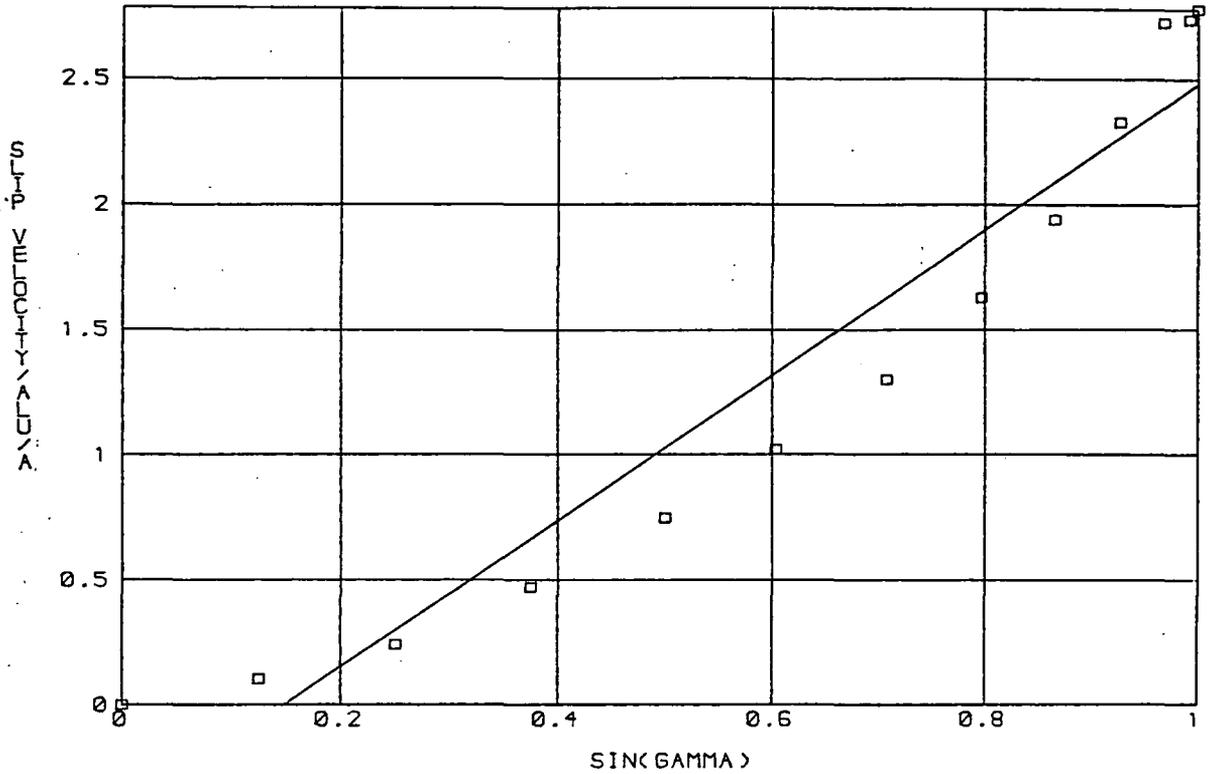


Figure 5.24

Velocity vector plot for an incompressible

Glen rheology over a truncated sine ridge with

$v_d = 100 a u/a$ (Case 47).

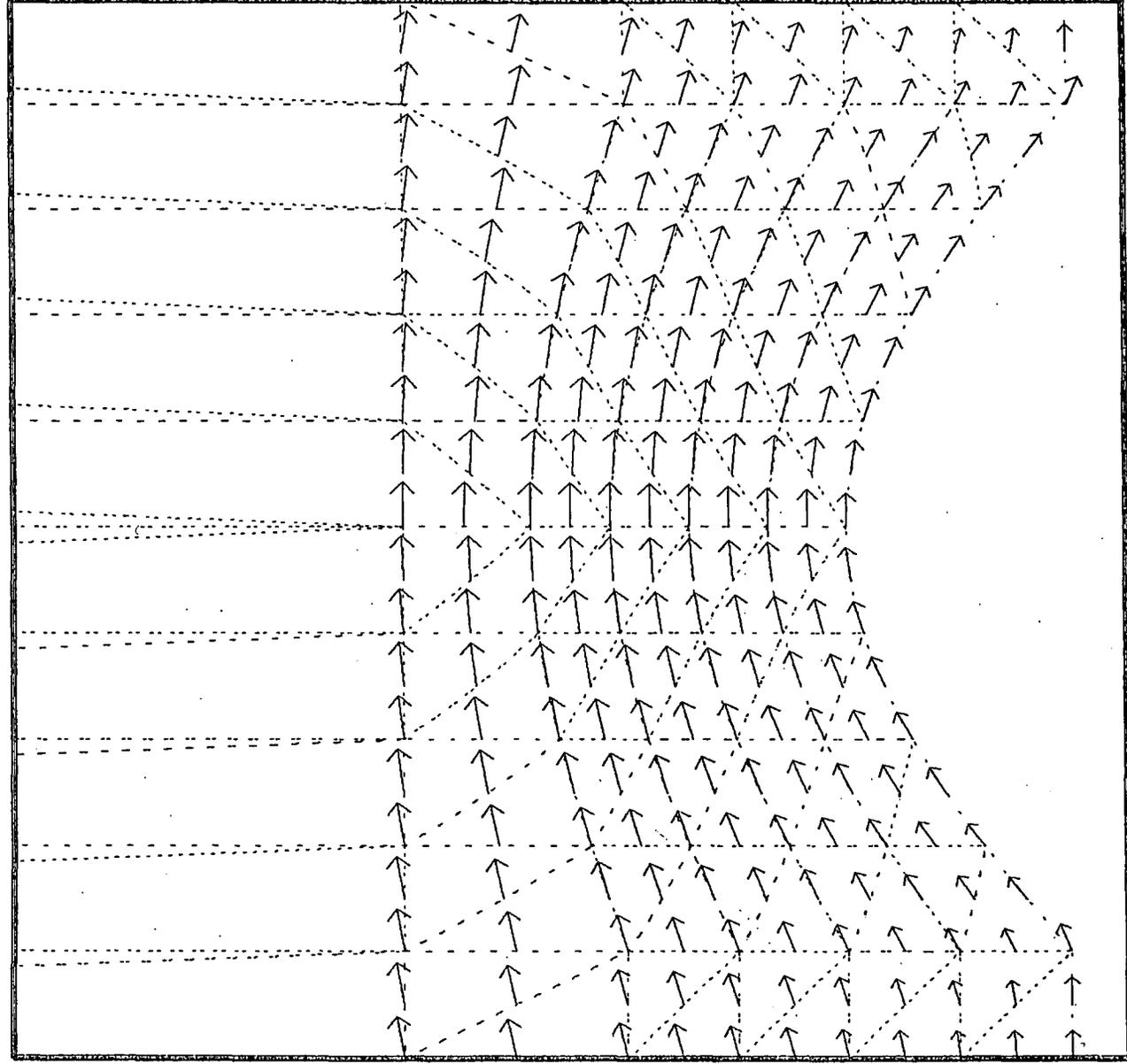
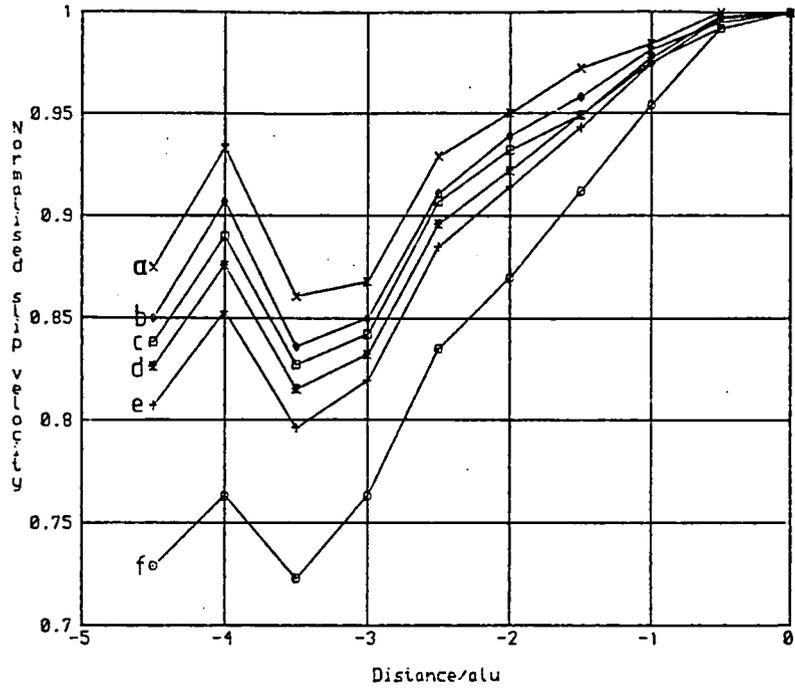


Figure 5.25

v_τ against position for flow over a truncated sine ridge (Cases 40 - 49): Glen rheology



- a-Linear
- b-Glen, $v_g = 400$ alu/a
- c-Glen, $v_g = 200$ alu/a
- d-Glen, $v_g = 100$ alu/a
- e-Glen, $v_g = 50$ alu/a
- f-Glen, $v_g = 10$ alu/a

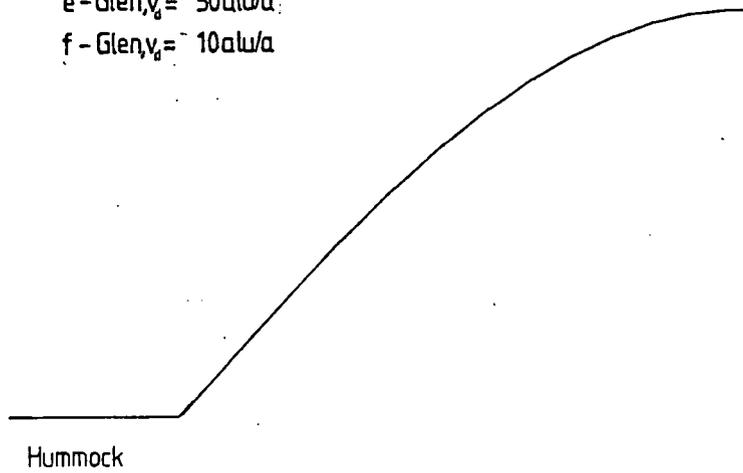


Figure 5.26

Regression coefficients from v_τ against v_d against position for flow over a truncated sine ridge (Cases 45 - 49).

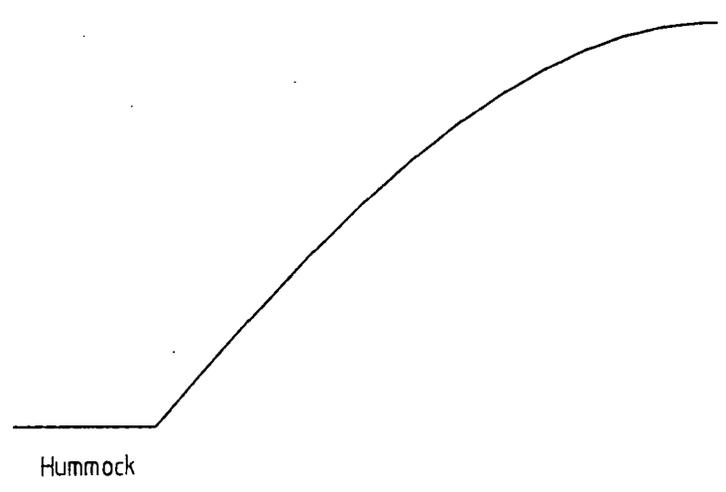
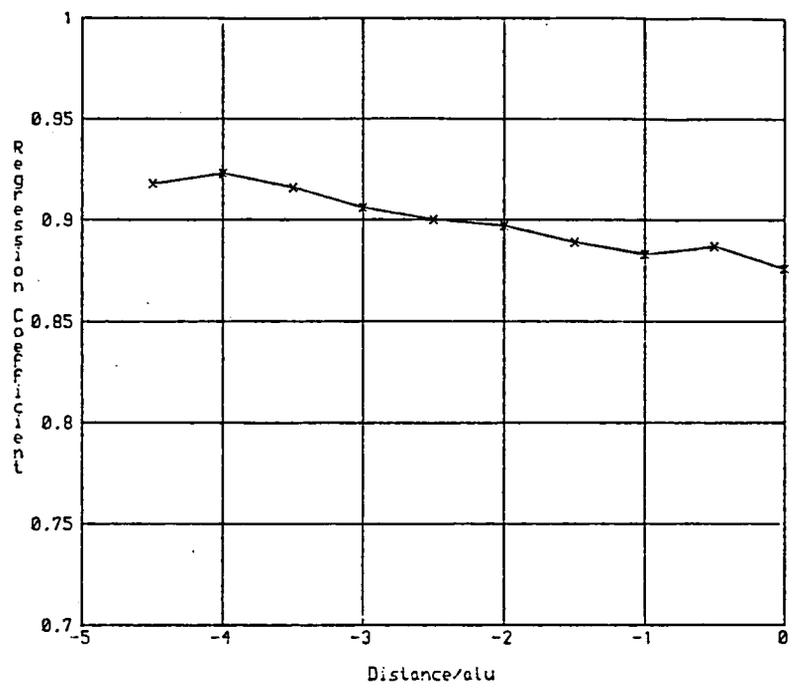


Figure 5.27

Pressure contours in bar for an incompressible linear rheology with $v_d = 100 \text{ alu/a}$ (Case 3).

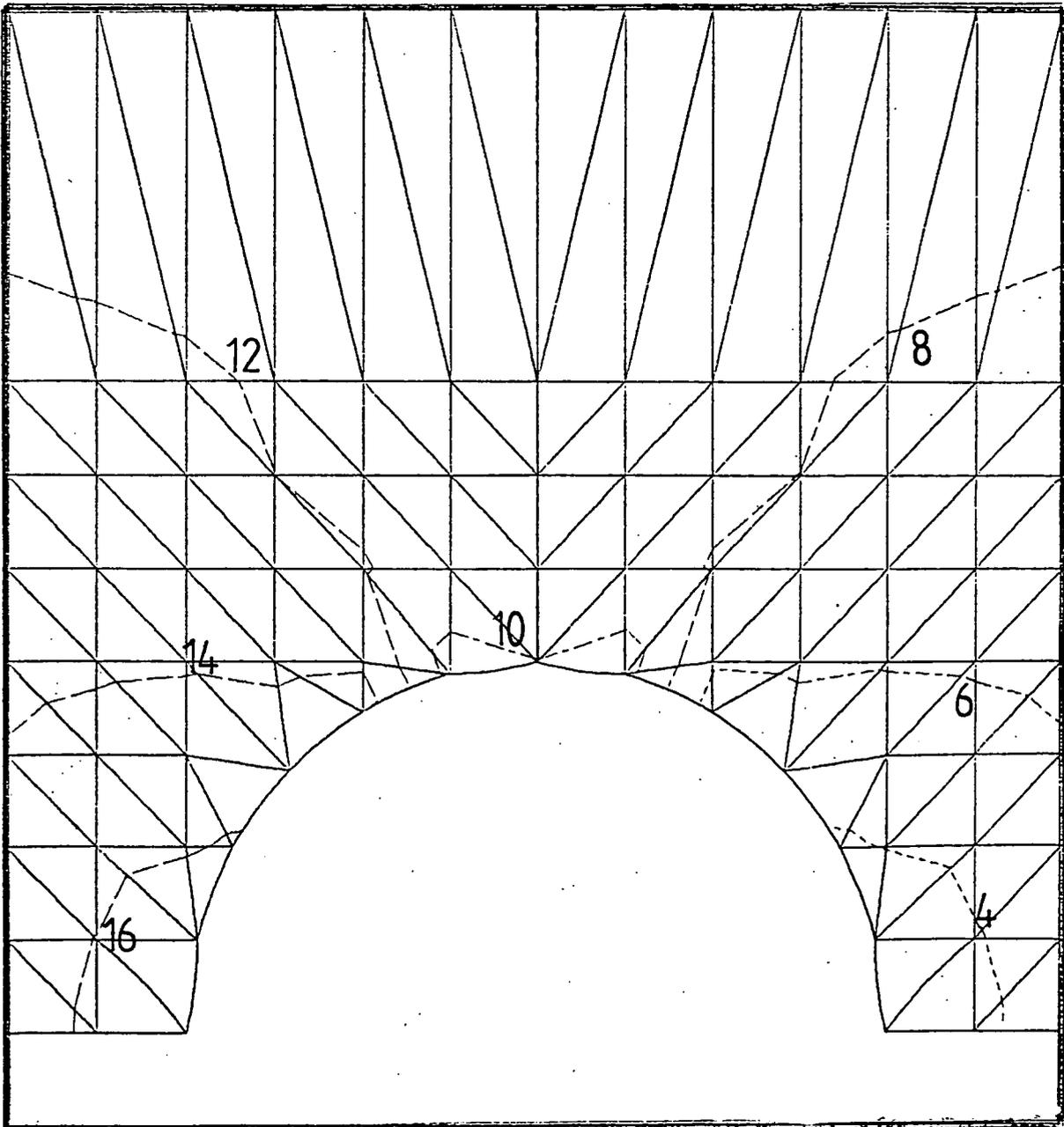


Figure 5.28 p' against $\cos \gamma$ for an incompressible linear rheology with $v_d = 400 \text{ alu/a}$ (Case 1).

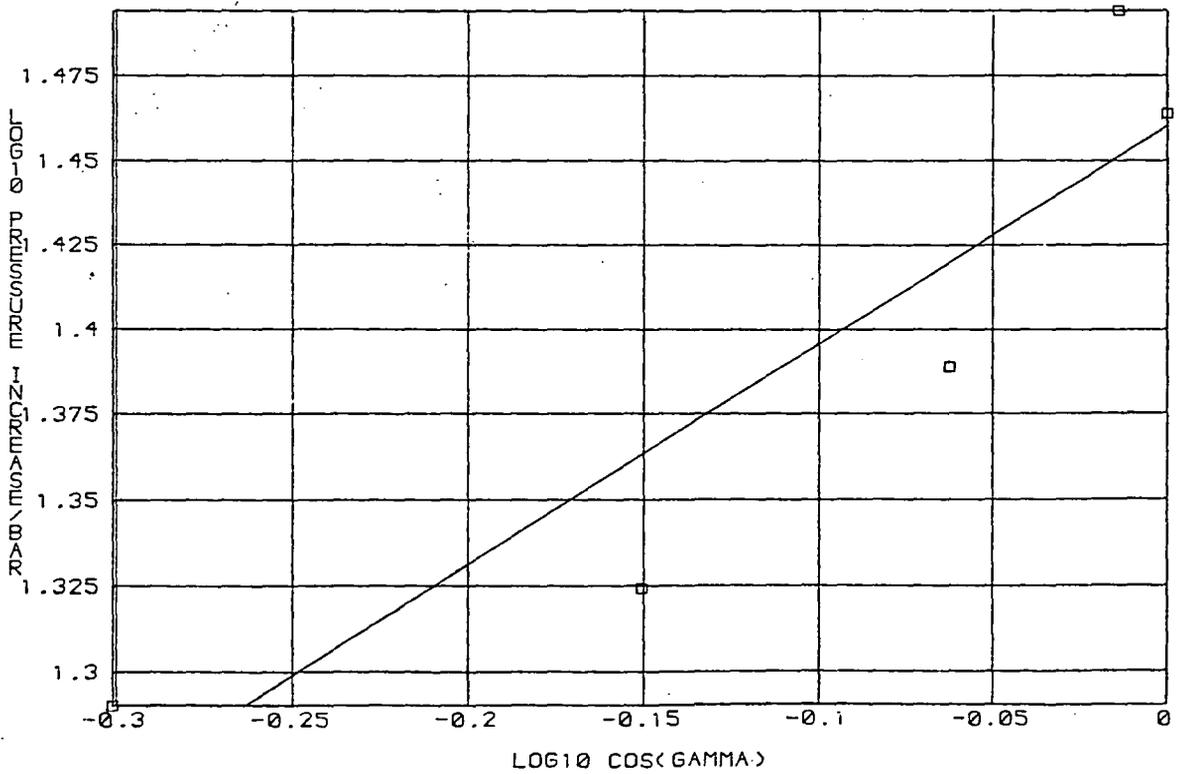
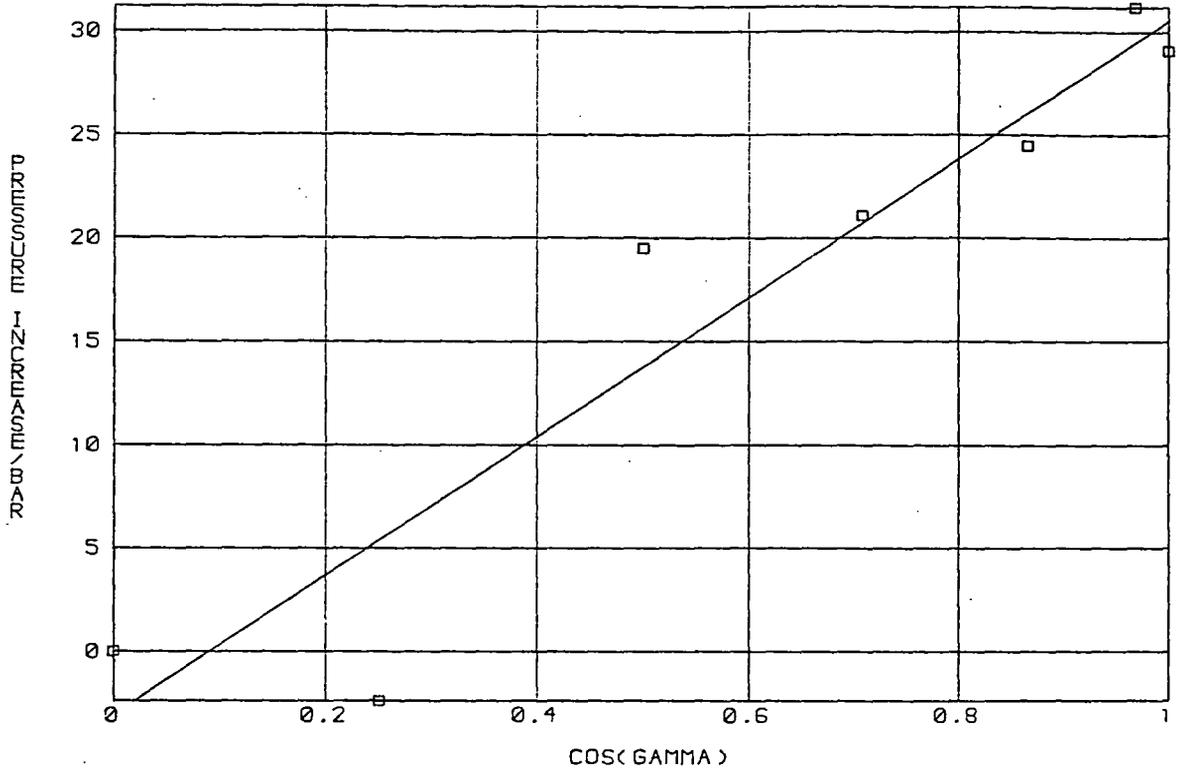


Figure 5.29

p' against $\cos \gamma$ for an incompressible
 linear rheology with a cavity and
 $v_d = 400 \text{ alu/a}$ (Case 5).

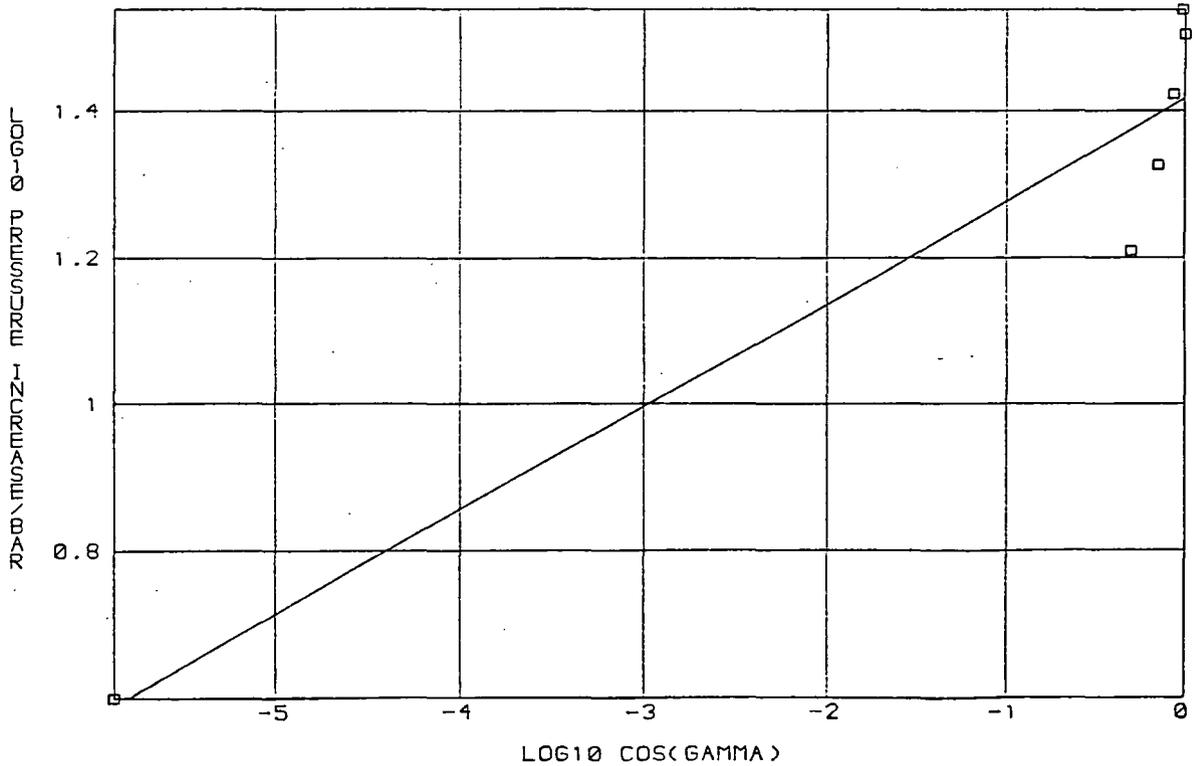
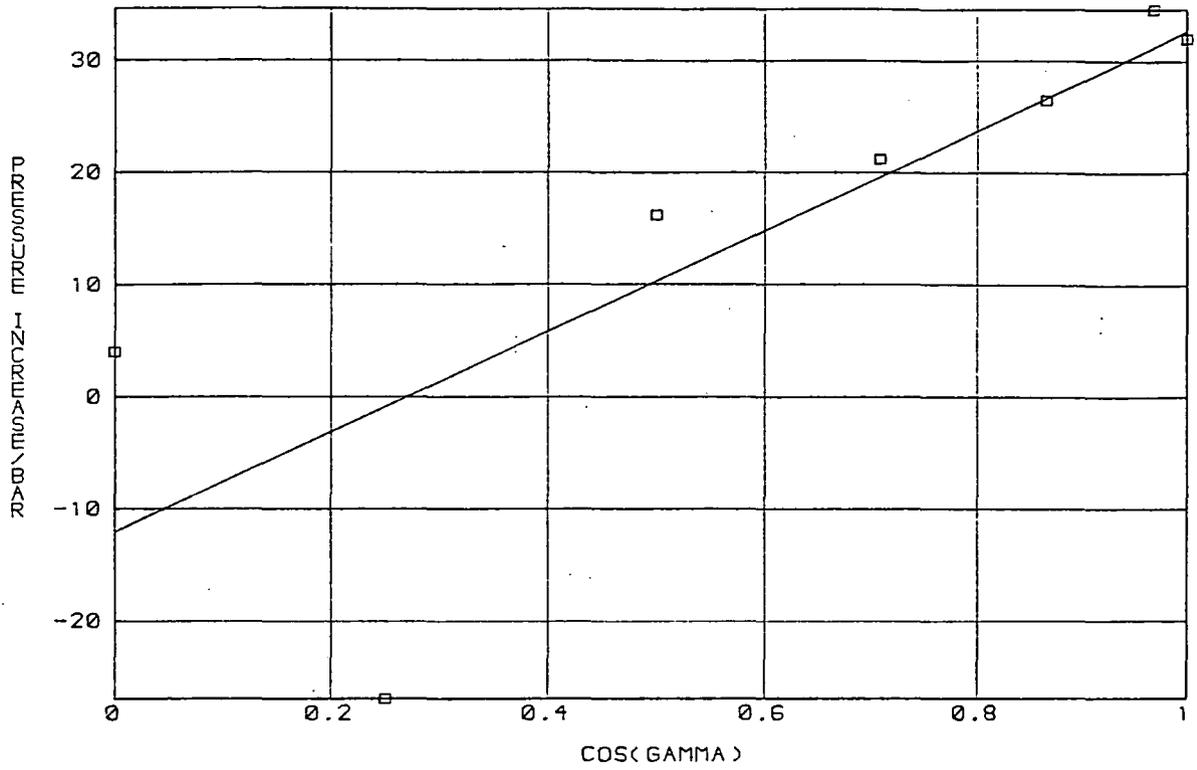


Figure 5.30

p' against $\cos\gamma$ for a compressible linear rheology with $\chi = 1 \text{ alu}^2/\text{bar}\cdot\text{a}$ and $v_d = 400 \text{ alu}/\text{a}$ (Case 12).

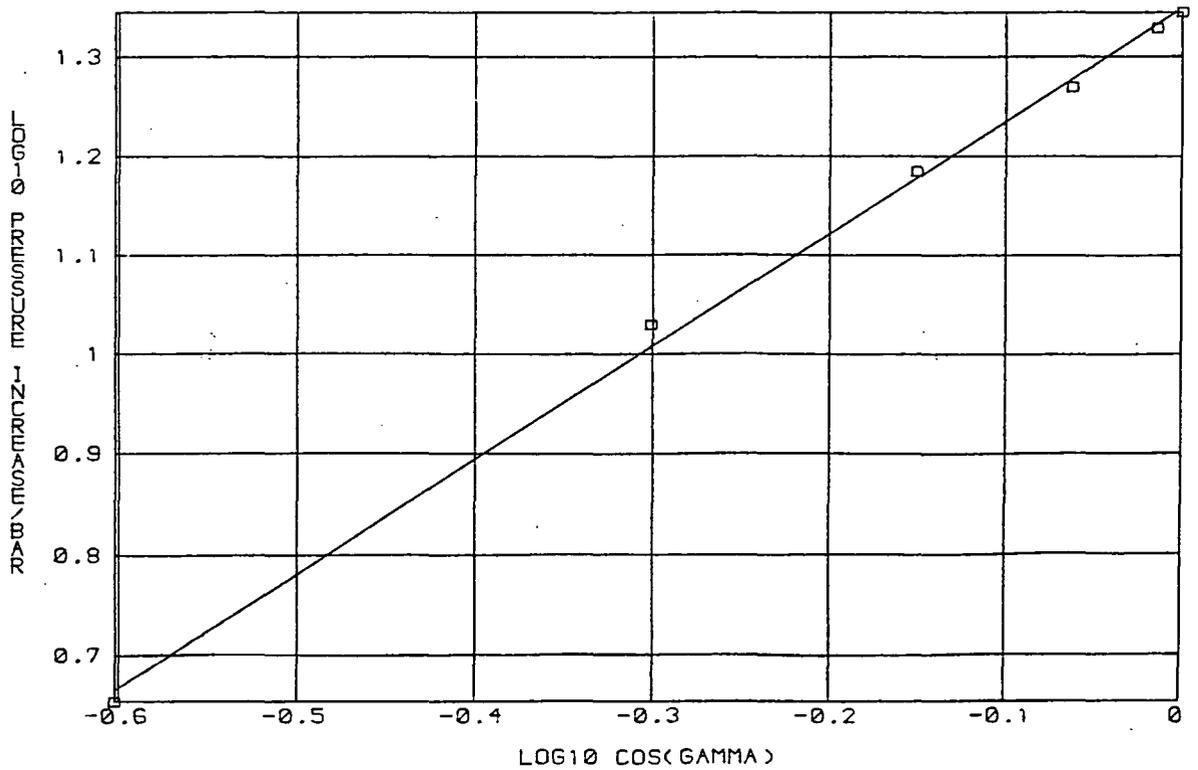
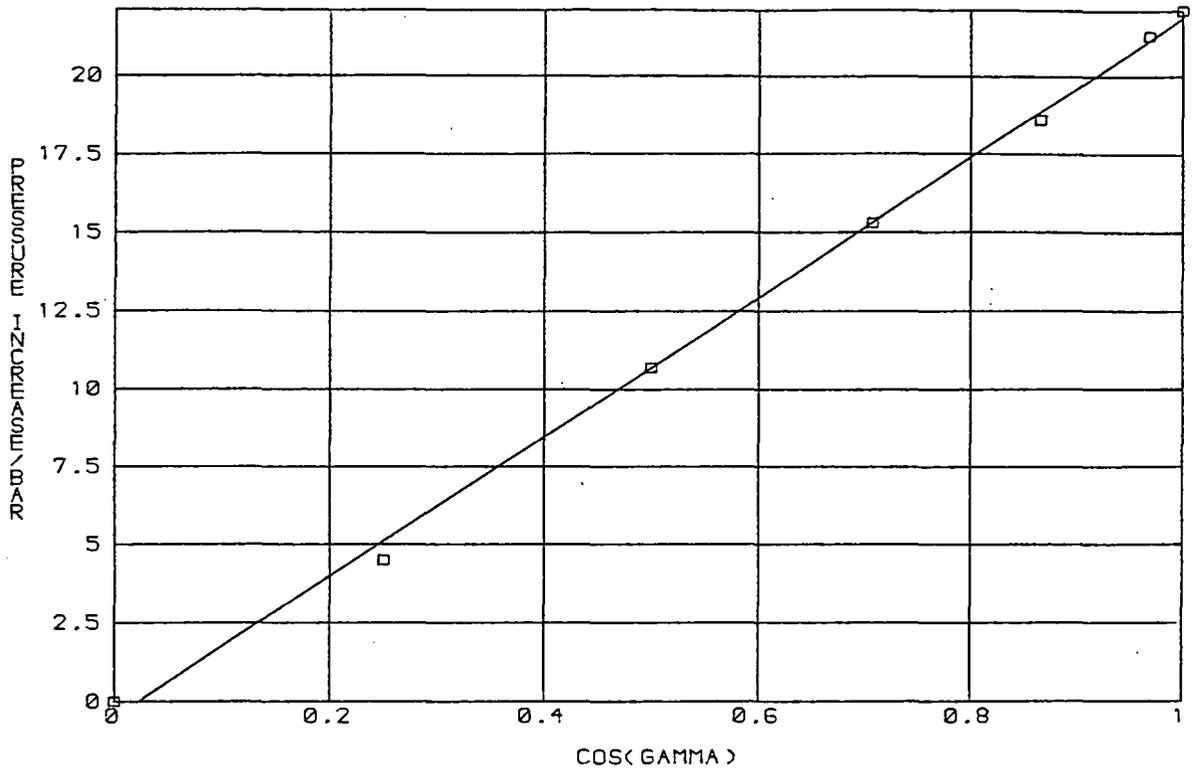


Figure 5.31

Pressure contours in bar for a compressible
linear rheology with $\chi = 10 \text{ alu}^2/\text{bar}\cdot\text{a}$ and
 $v_d = 400 \text{ alu}/\text{a}$ (Case 17).

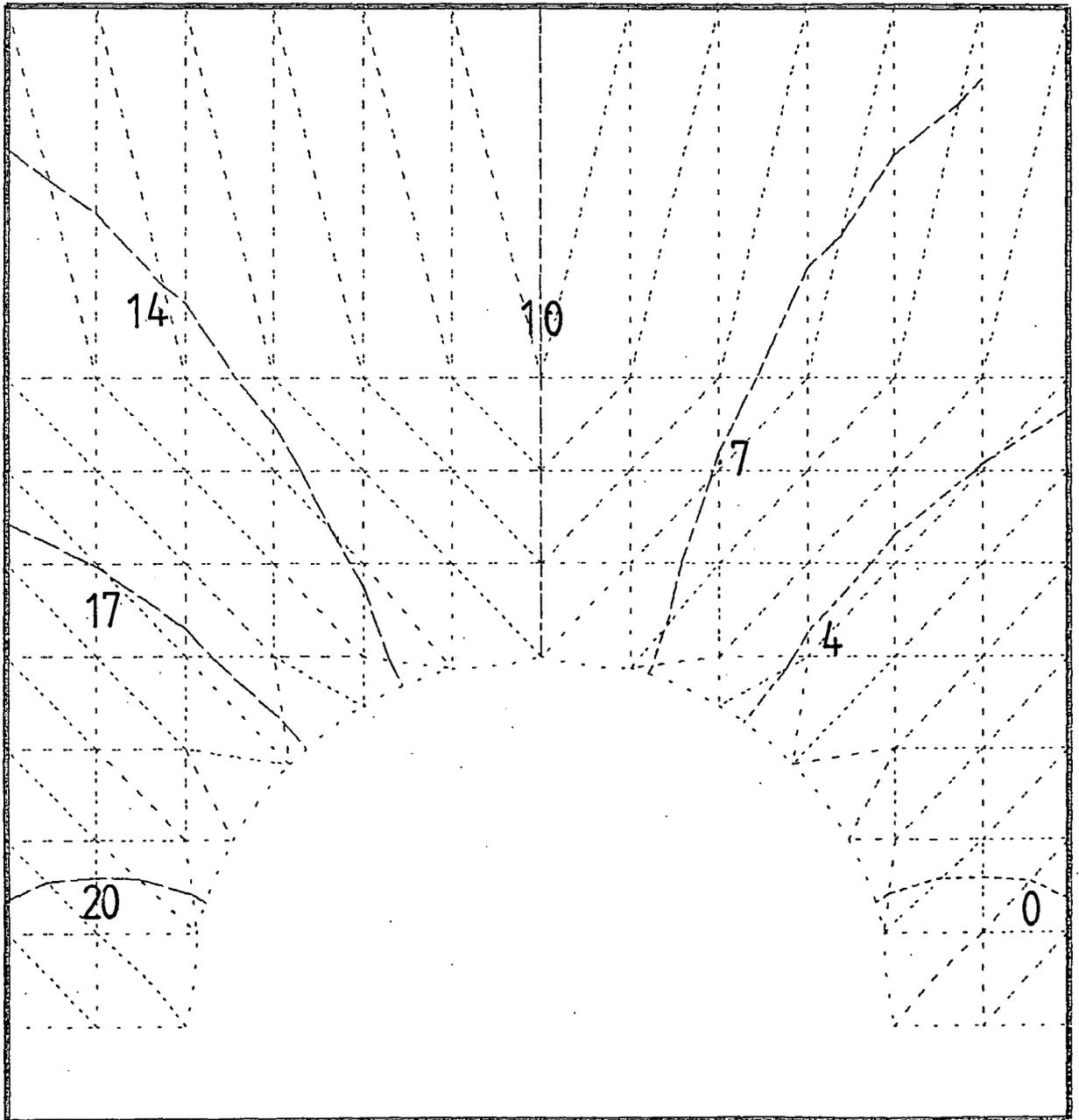


Figure 5.32

p' against $\cos \gamma$ for a compressible linear rheology with $\chi = 10 \text{ alu}^2 / \text{bar} \cdot \text{a}$ and $v_d = 200 \text{ alu} / \text{a}$ (Case 16).

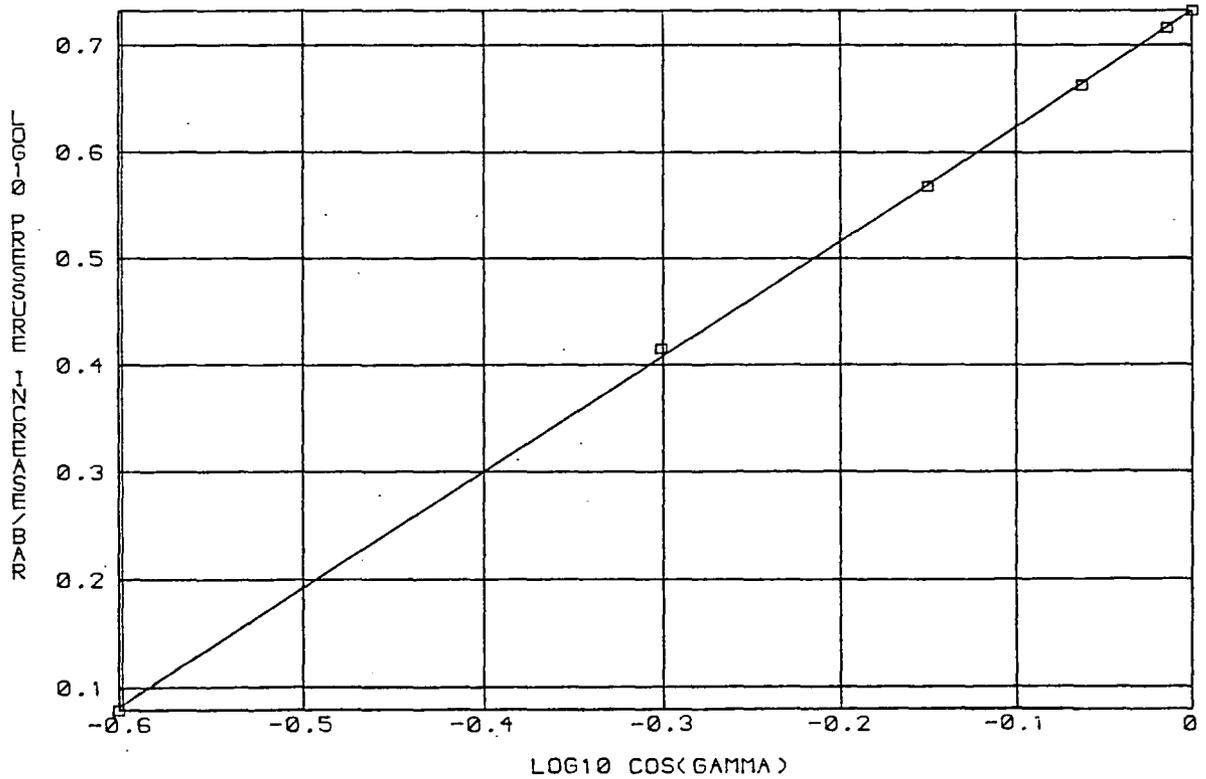
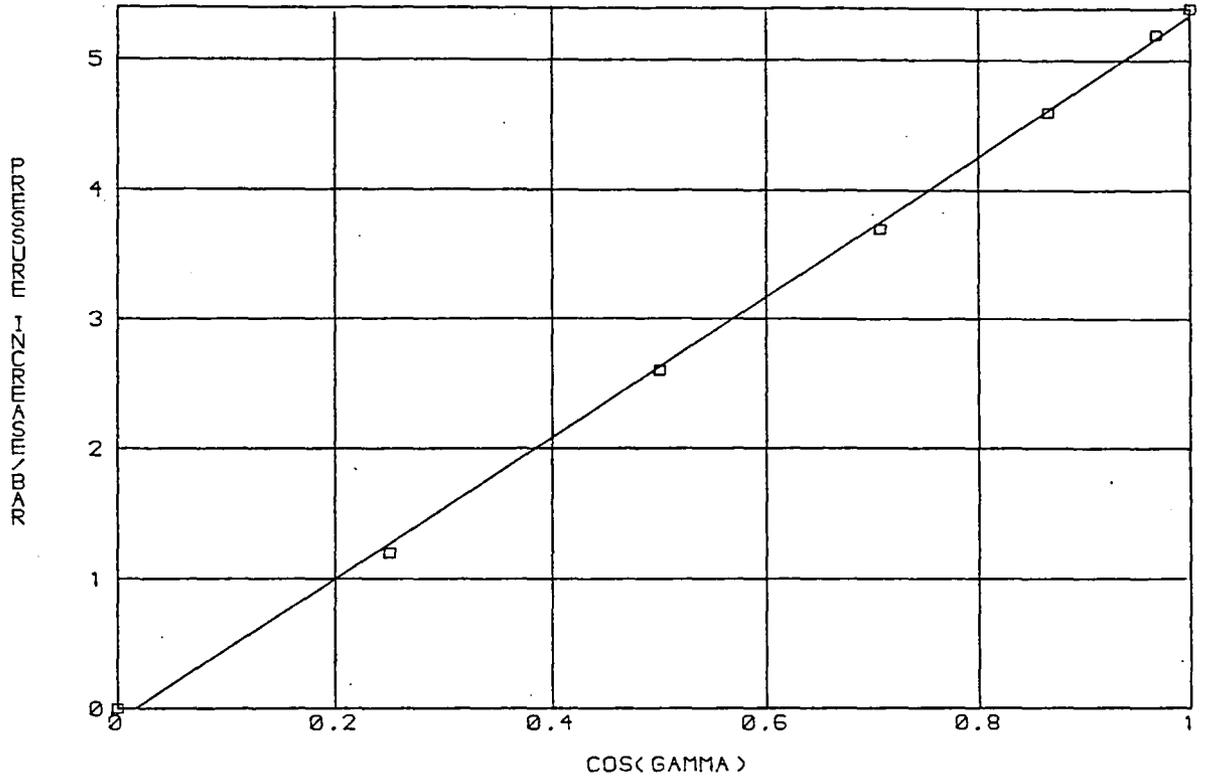


Figure 5.33

Pressure contours in bar for an incompressible linear rheology with bed of $S = 100 \text{ alu} / \text{bar} \cdot \text{a}$ and $v_d = 100 \text{ alu} / \text{a}$. (Case 20).

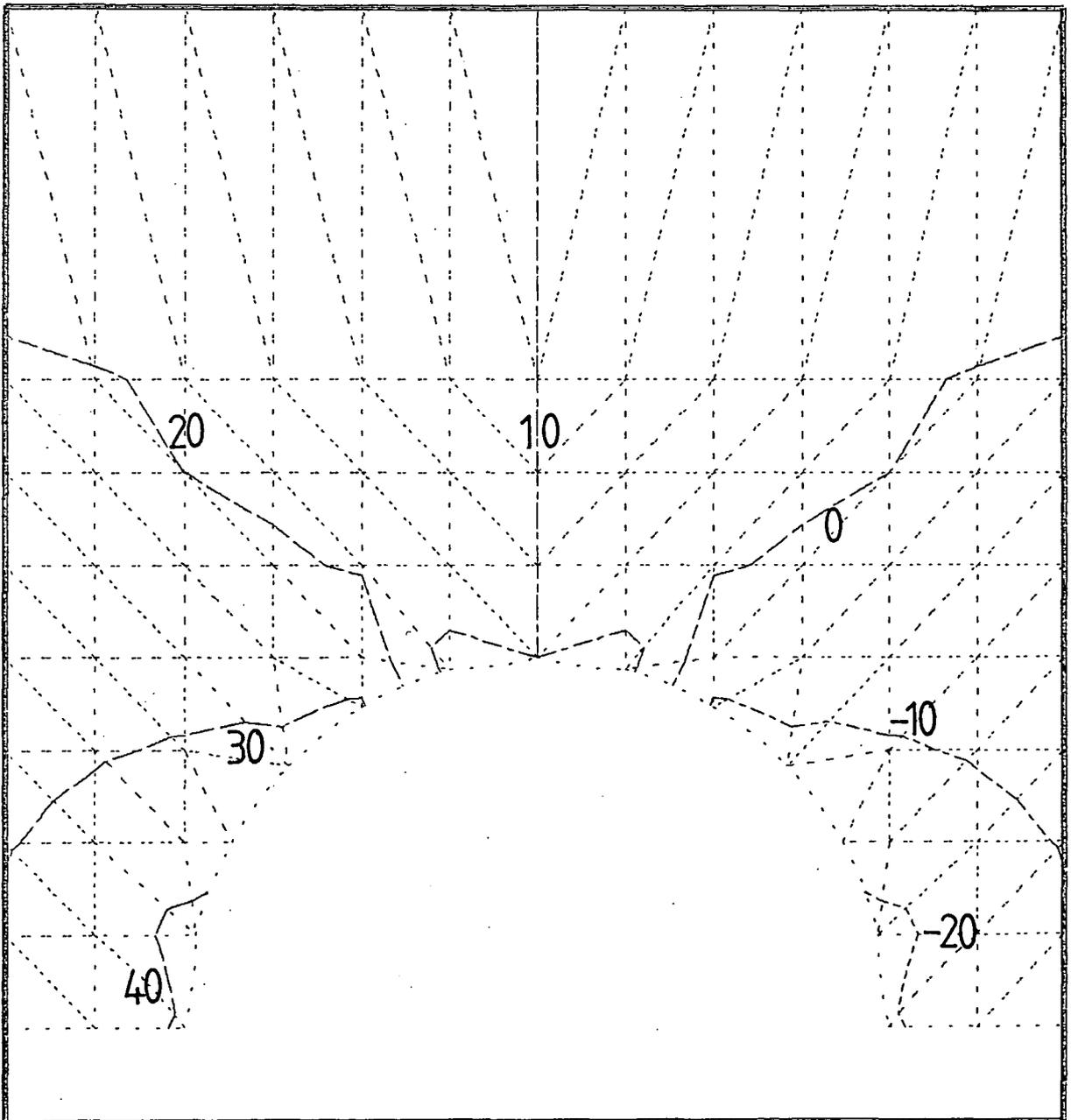


Figure 5.34

p' against $\cos \gamma$ for an incompressible linear rheology with bed of $S = 10 \text{ alu /bar.a}$ and $v_d = 10 \text{ alu /bar}$ (Case 18).

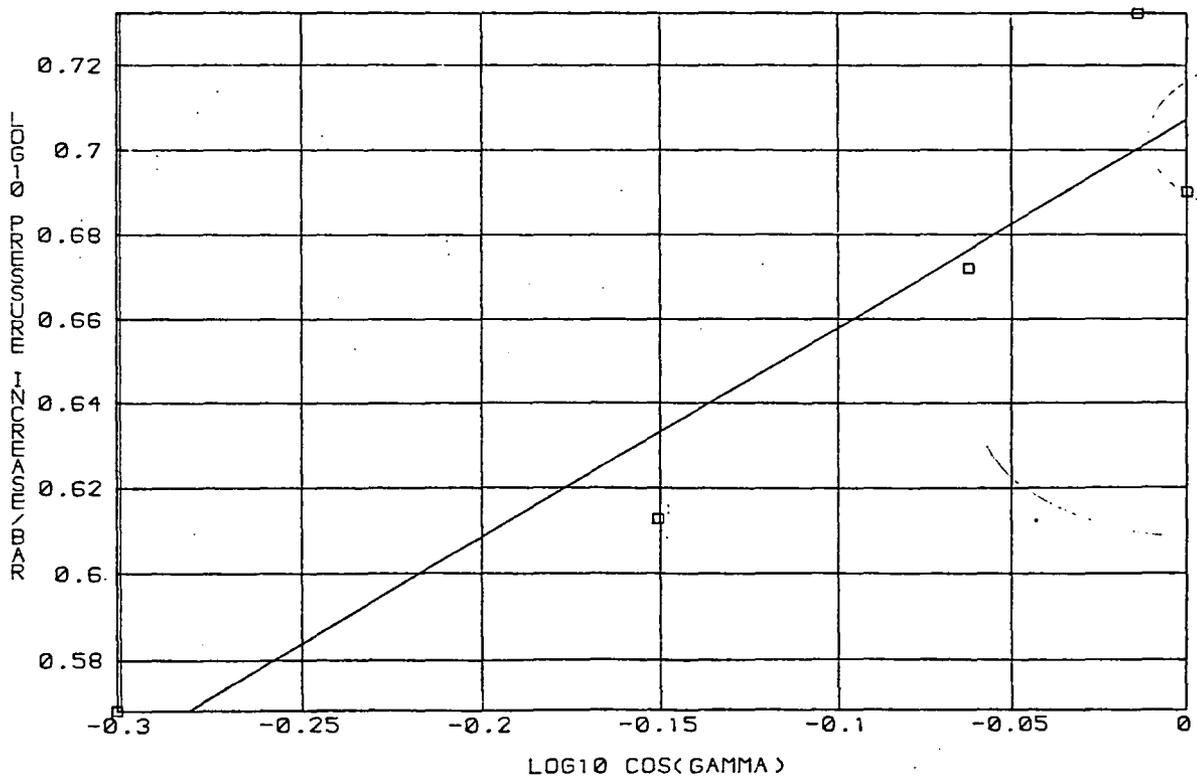
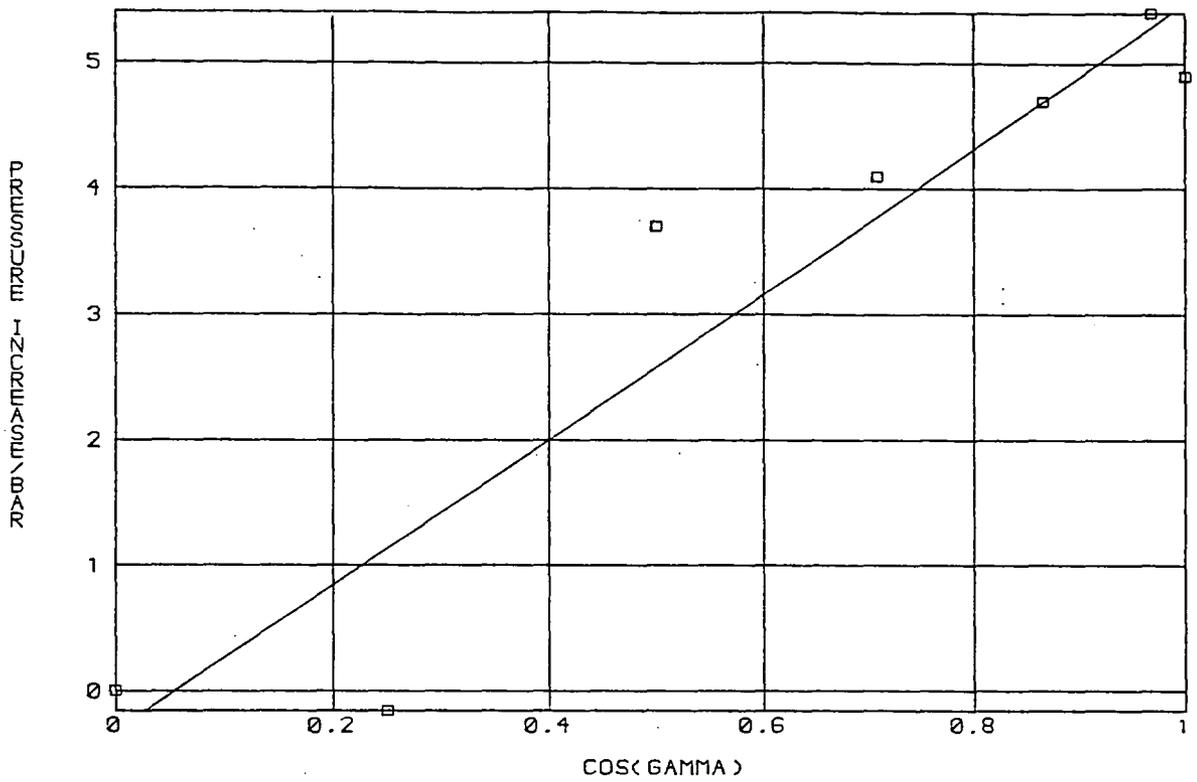


Figure 5.35 p' against $\cos \gamma$ for an incompressible linear rheology with bed of $S = 400 \text{ alu / bar.a}$ and $v_d = 400 \text{ alu / a}$. (Case 22).

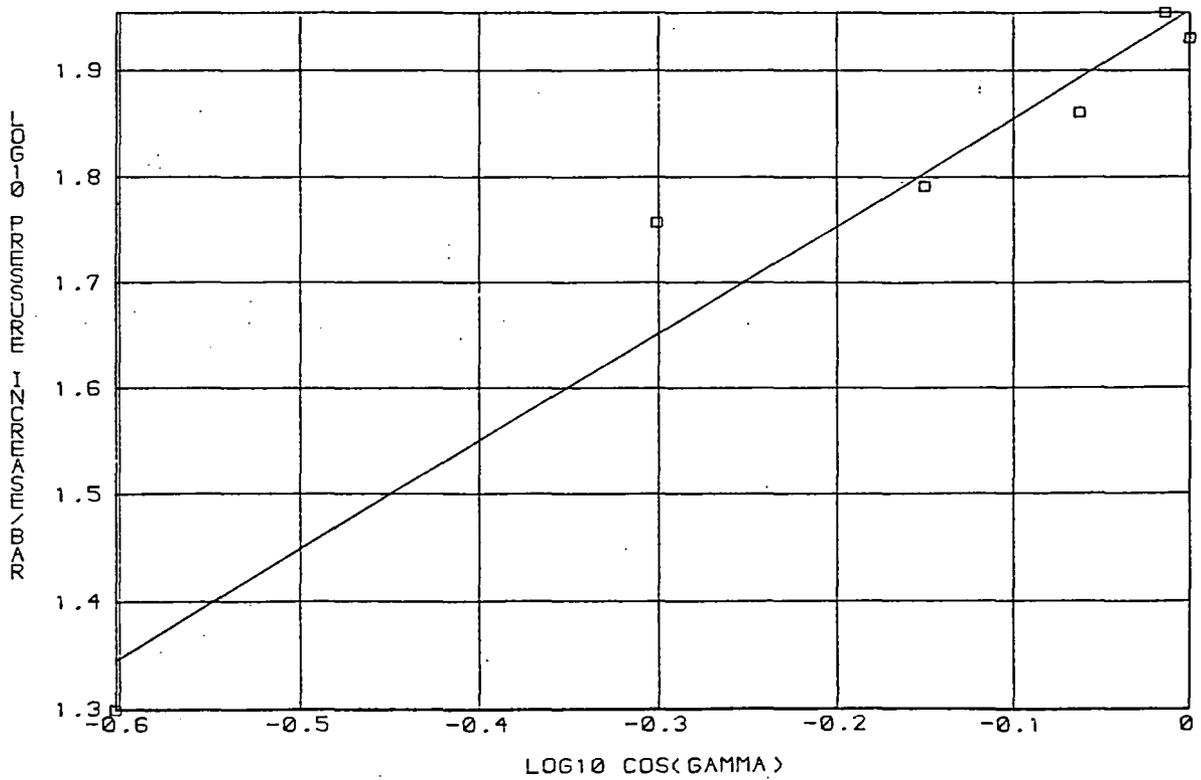
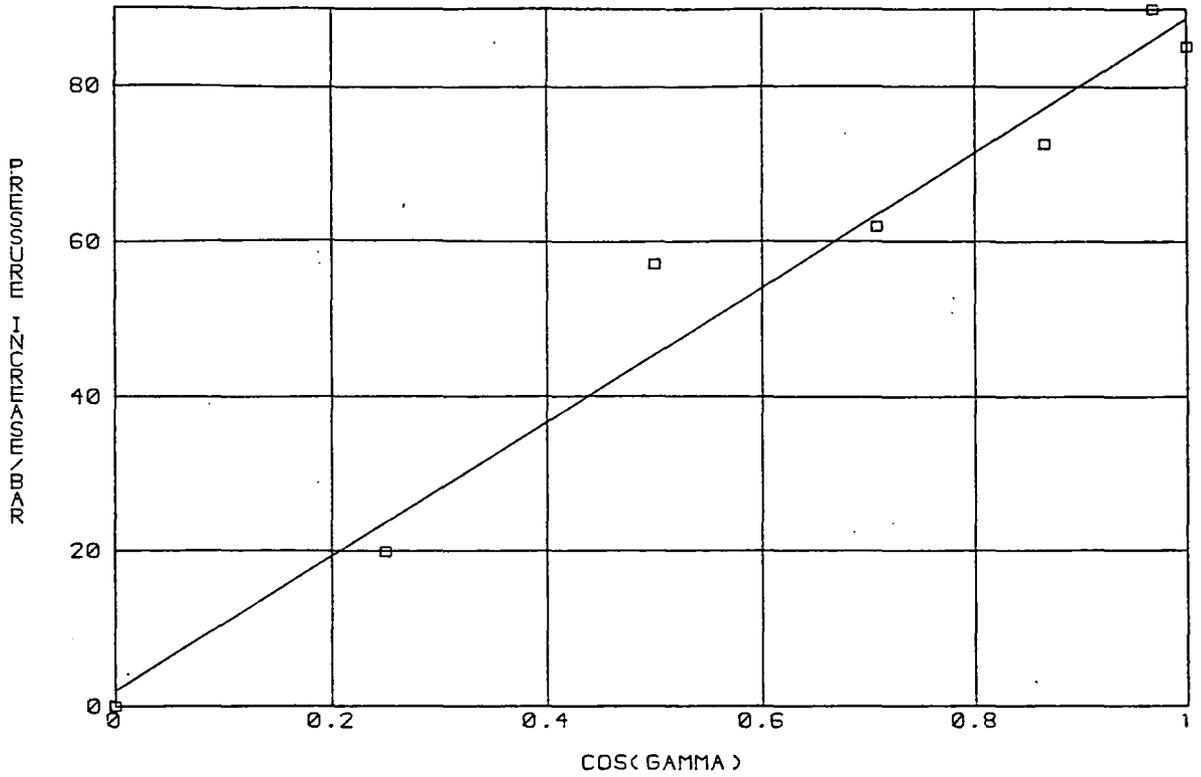


Figure 5.36

Pressure contours in bar for an incompressible
Glen rheology with $v_d = 200 \text{ alu/a}$ (Case 25).

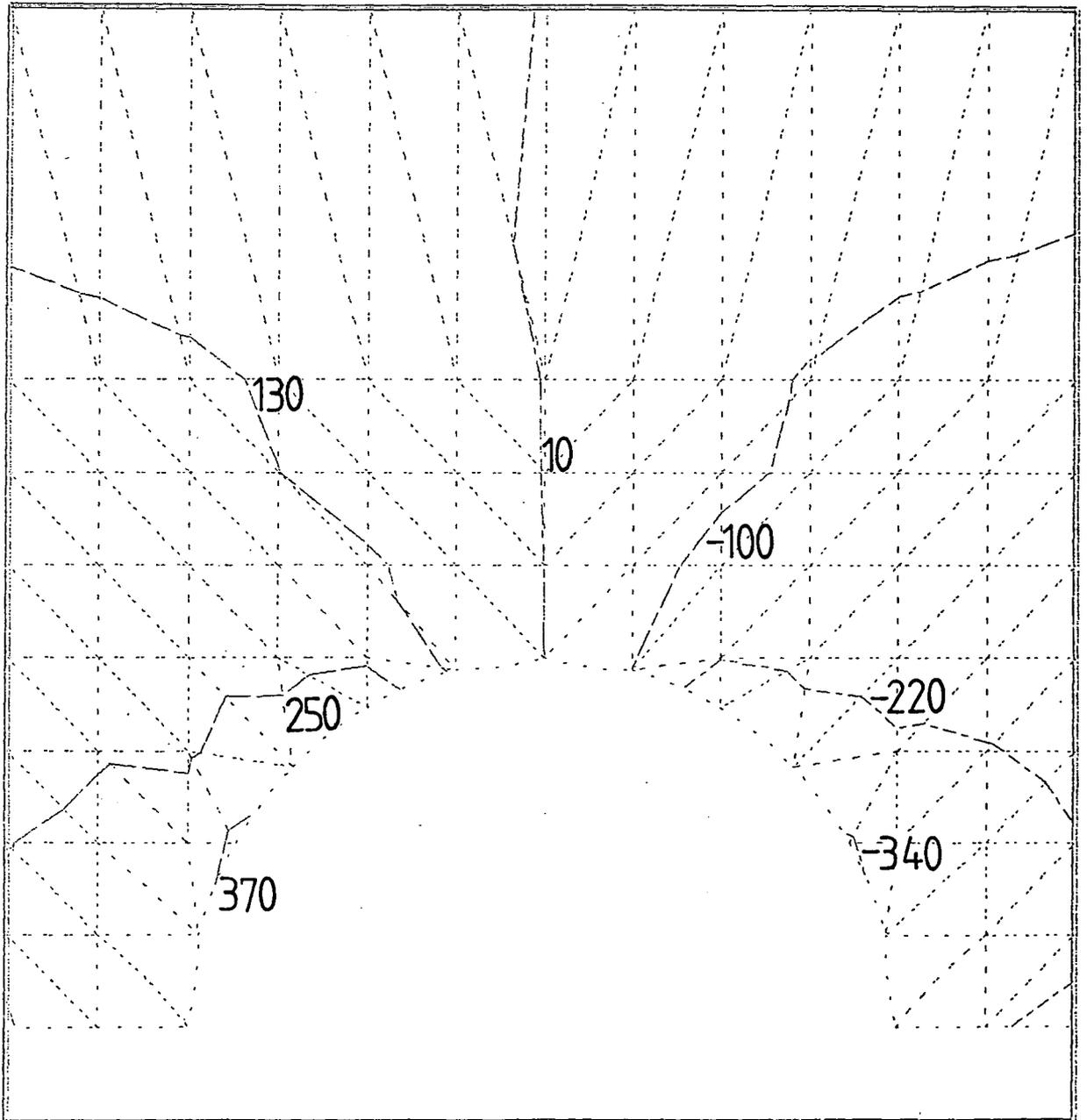


Figure 5.37 p' against $\cos\gamma$ for an incompressible Glen rheology with $v_d = 10 \text{ alu/a}$ (Case 23).

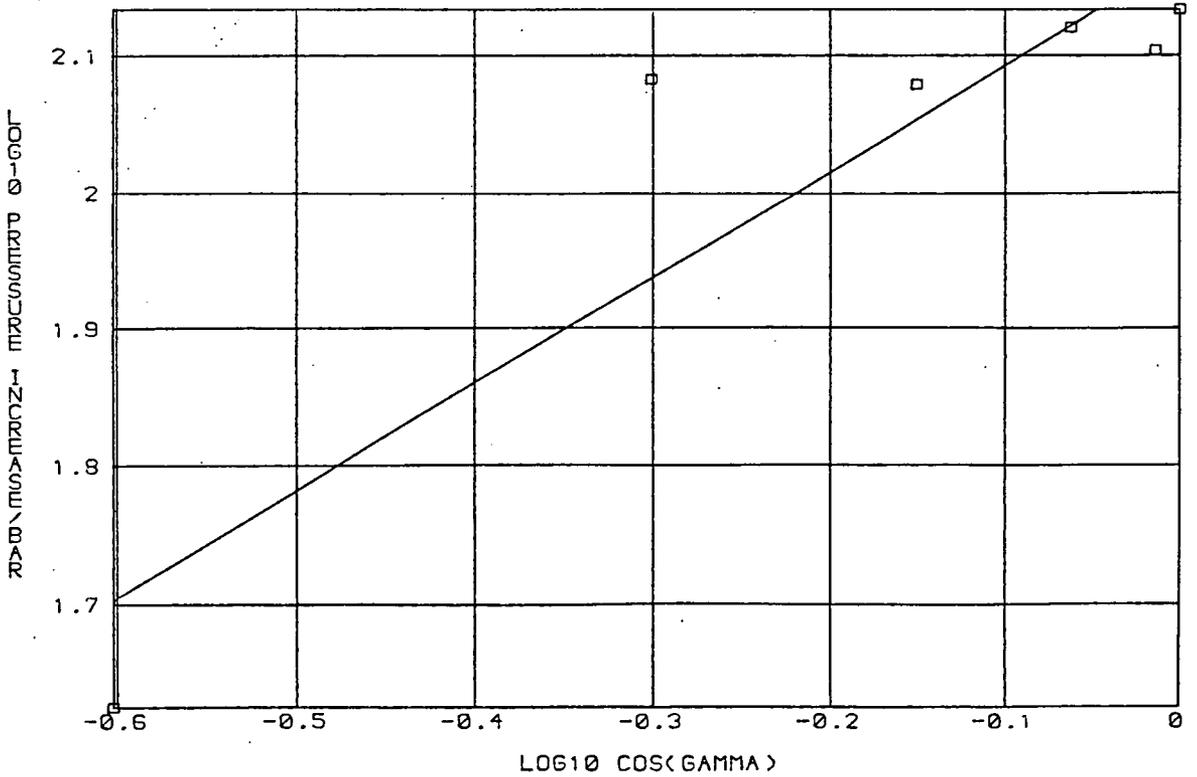
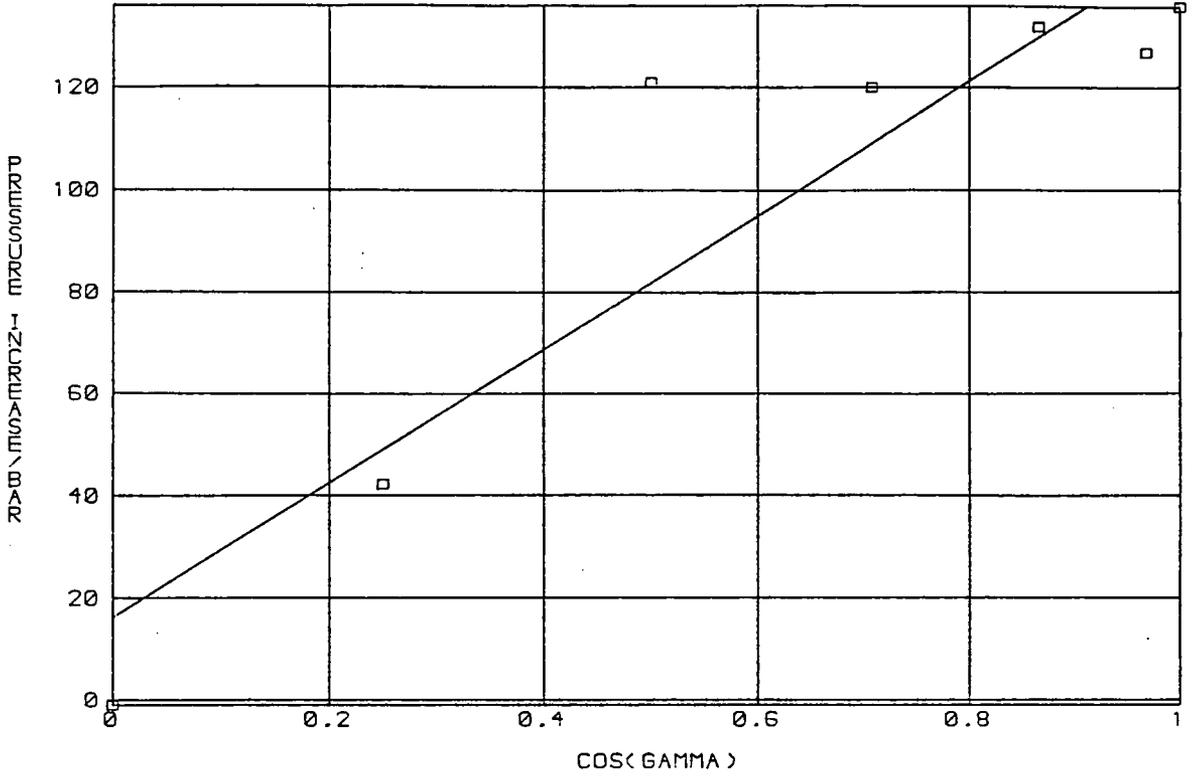


Figure 5.38

p' against $\cos \gamma$ for an incompressible
 Glen rheology with $v_d = 400 \text{ alu/a}$ (Case 27).

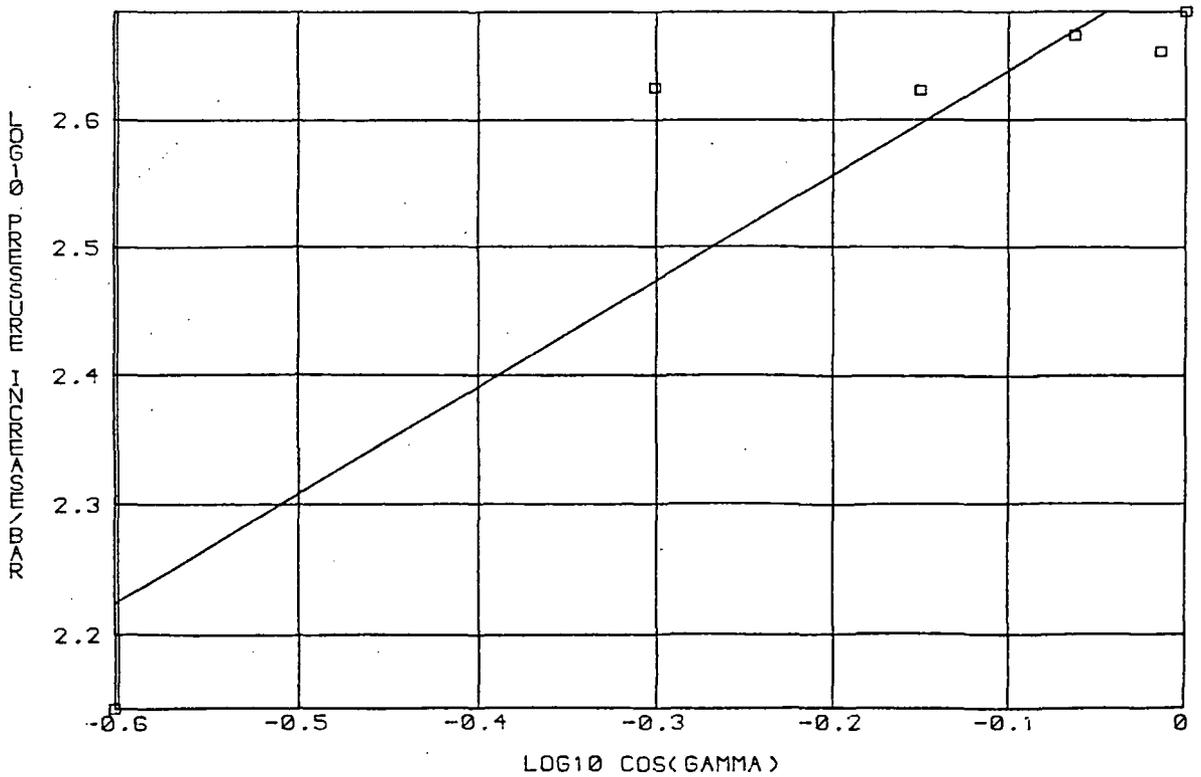
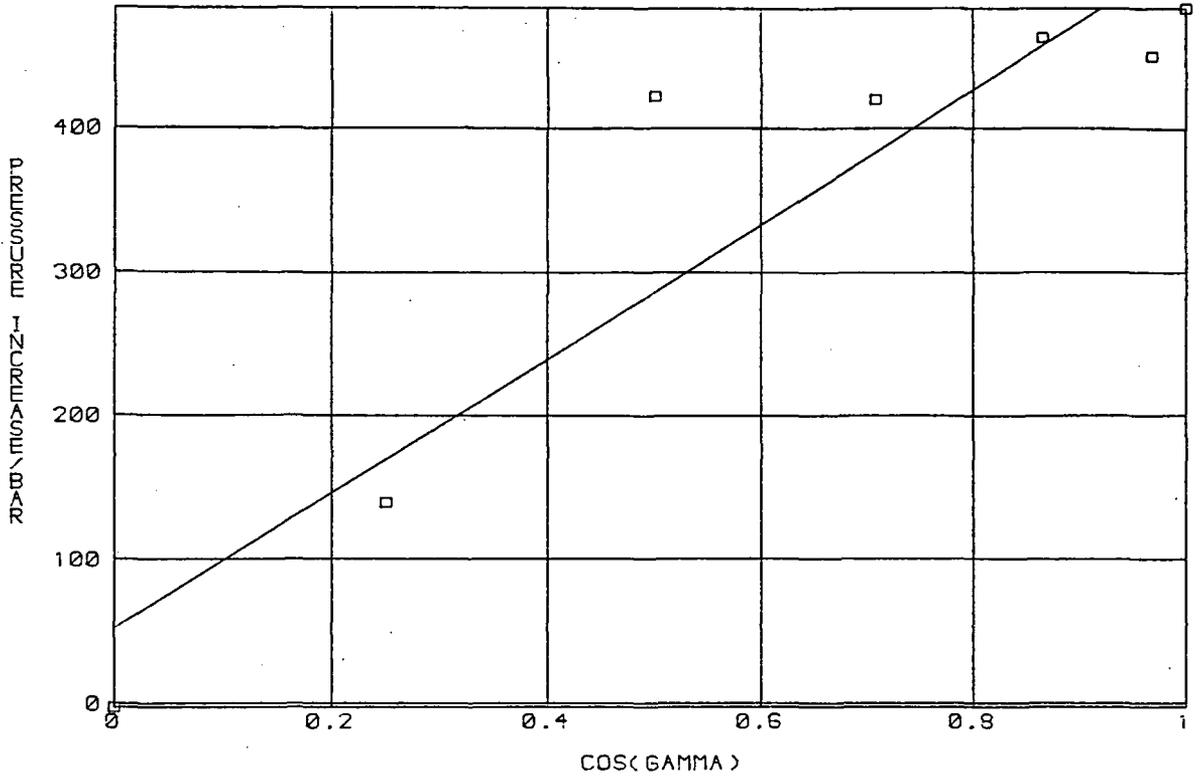


Figure 5.39 Pressure contours in bar for an incompressible Glen rheology with a cavity and $v_d = 10 \text{ alu/a}$ (Case 27).

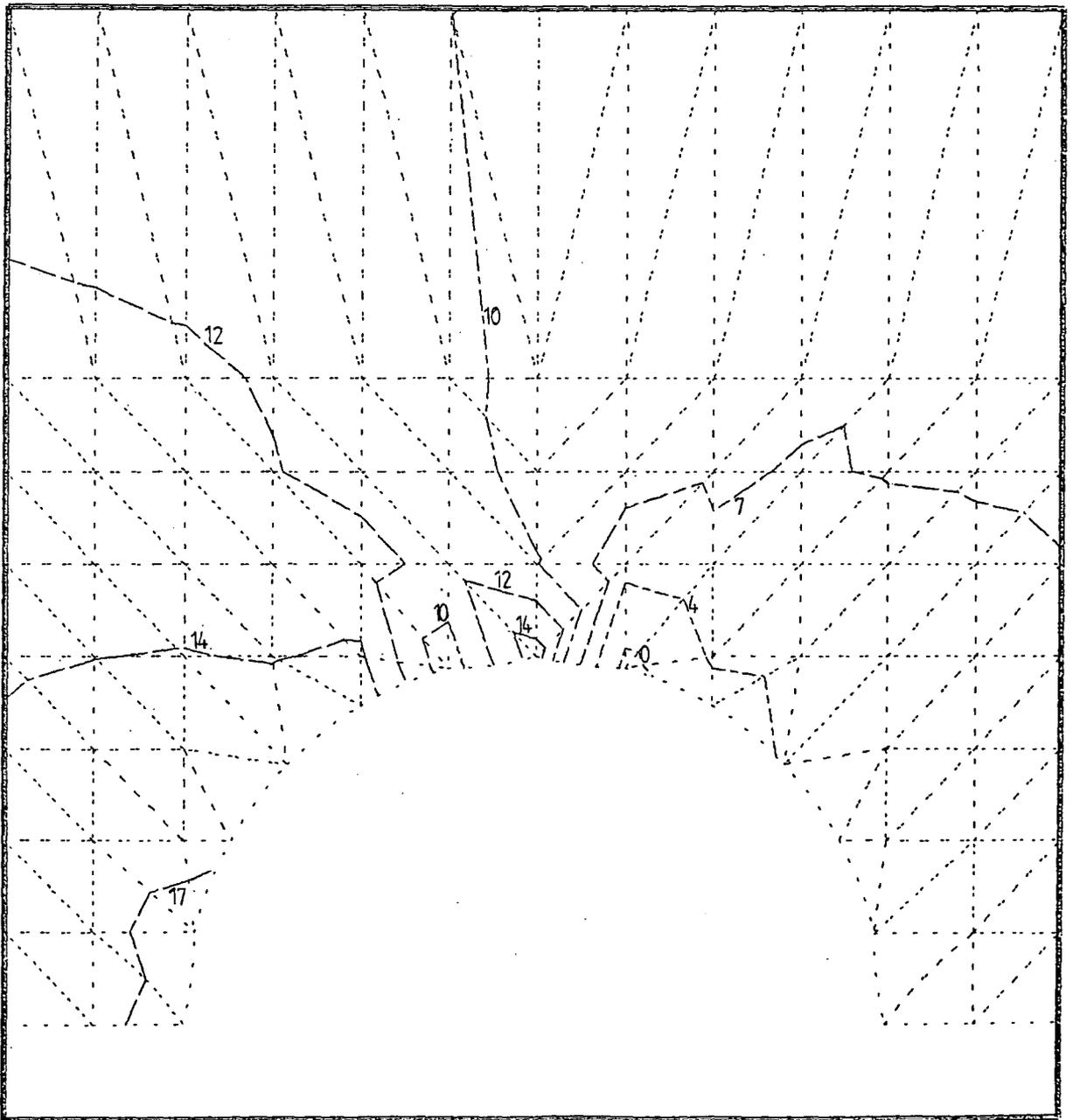


Figure 5.40

p' against $\cos \gamma$ for an incompressible Glen rheology with a cavity and $v_d = 10 a u/a$ (Case 27).

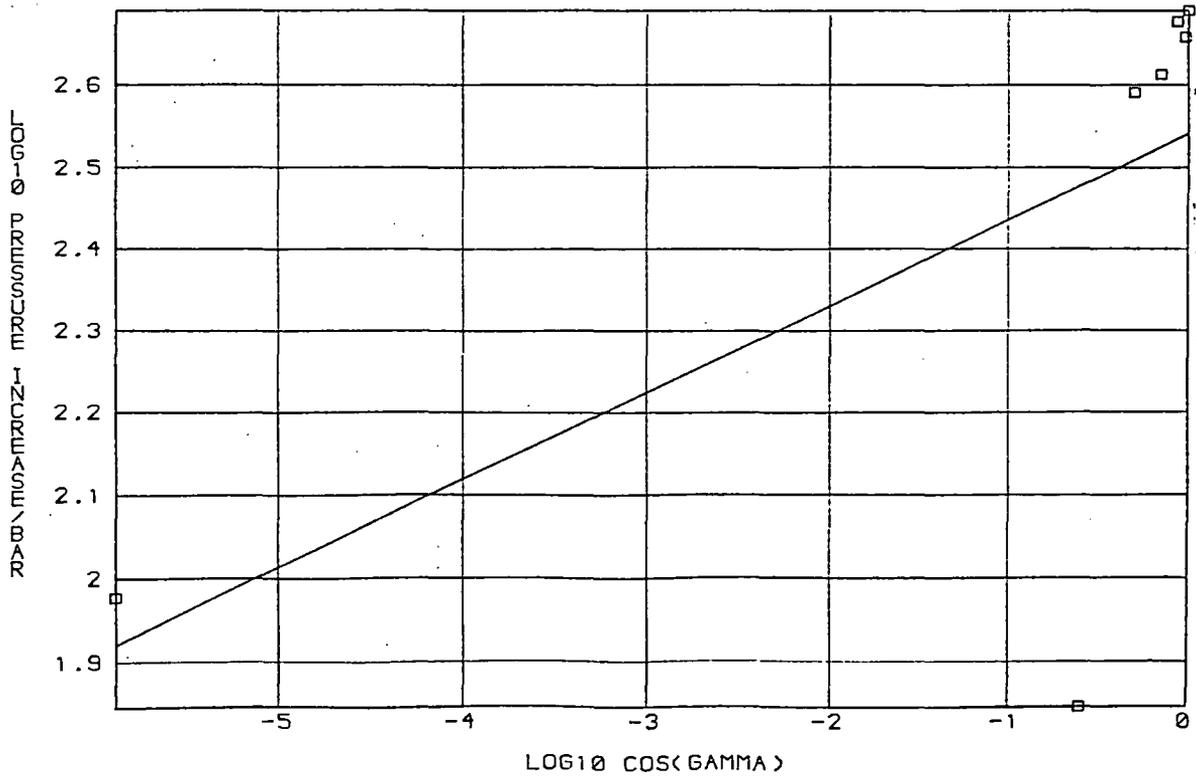
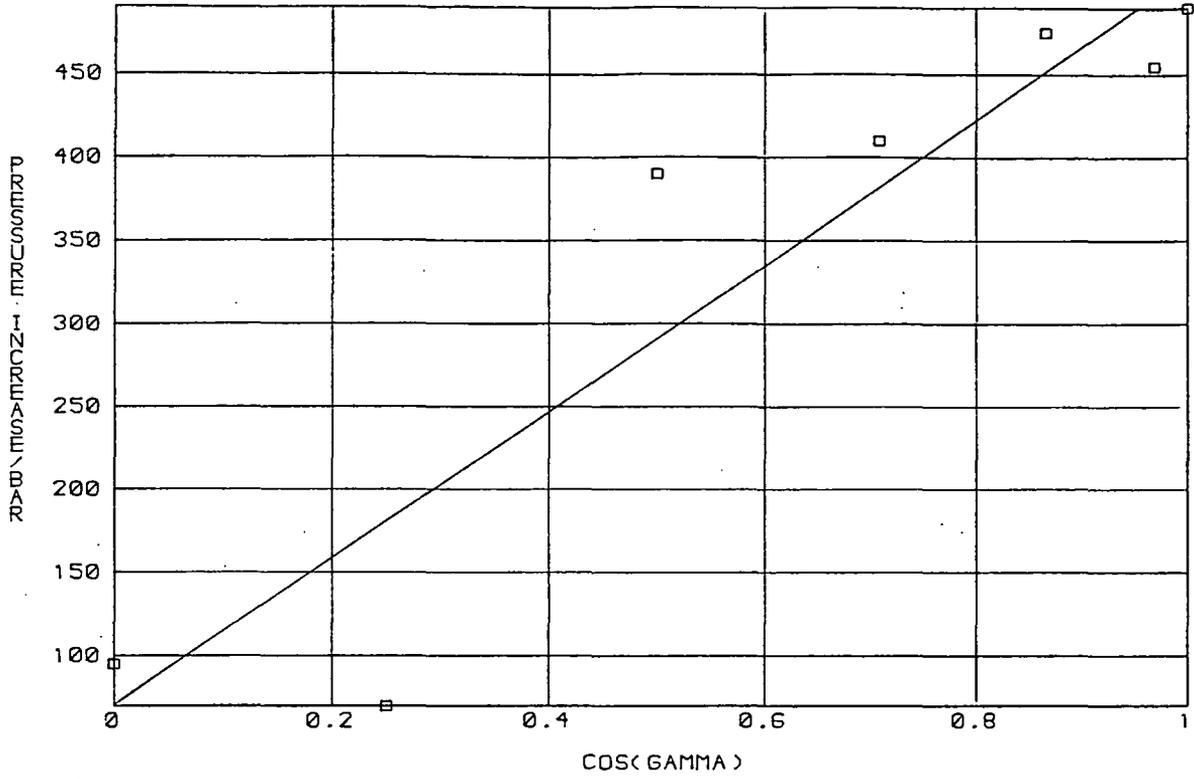


Figure 5.41

Pressure contours in bar for a compressible,
Glen rheology with $\chi = 1 \text{ alu}^2/\text{bar}\cdot\text{a}$ and
 $v_d = 100 \text{ alu}/\text{a}$ (Case 29).

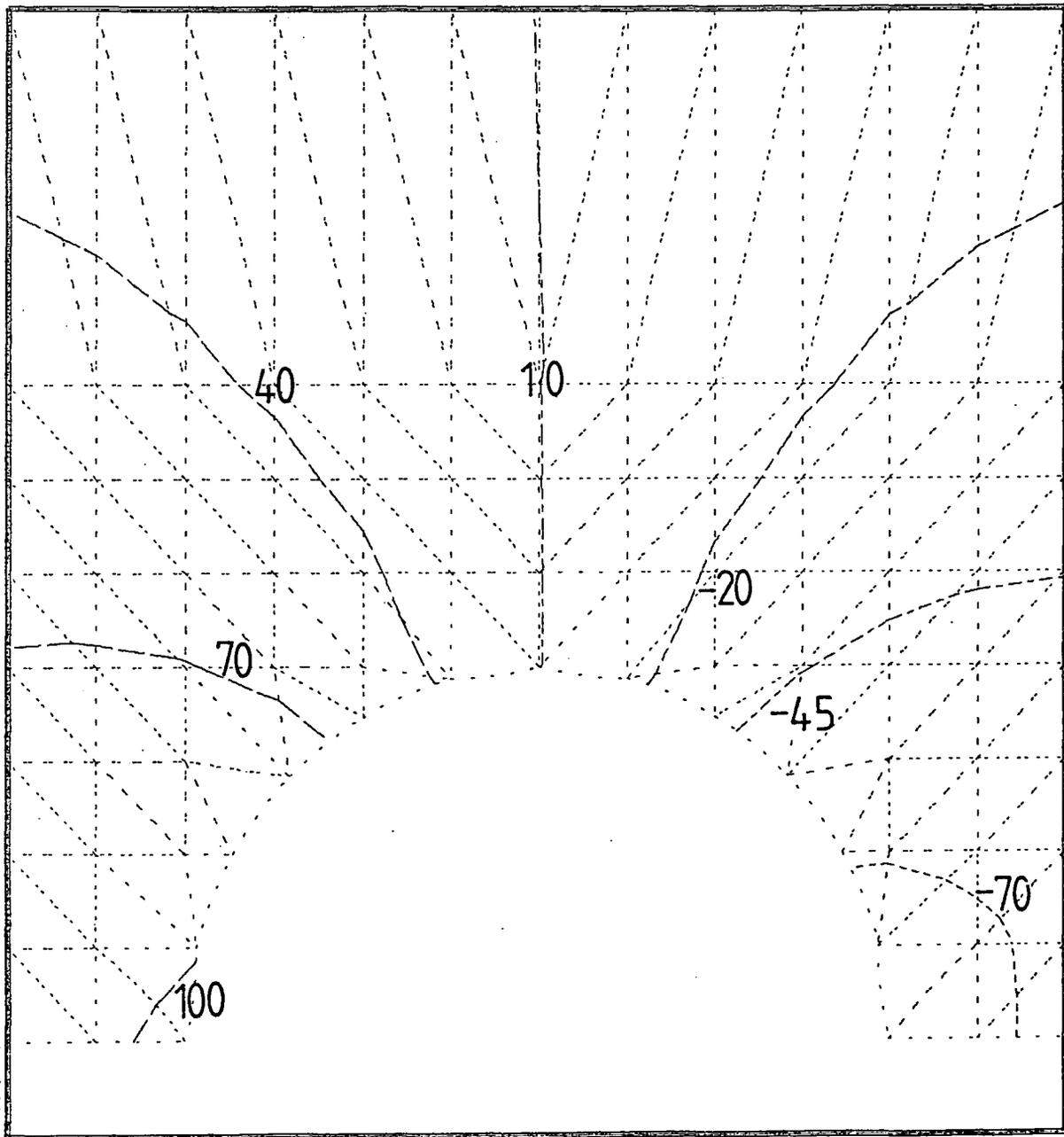


Figure 5.42

p' against $\cos \gamma$ for a compressible Glen rheology with $\chi = 1 \text{ alu}^2/\text{bar}\cdot\text{a}$ and $v_d = 200 \text{ alu/a}$ (Case 30).

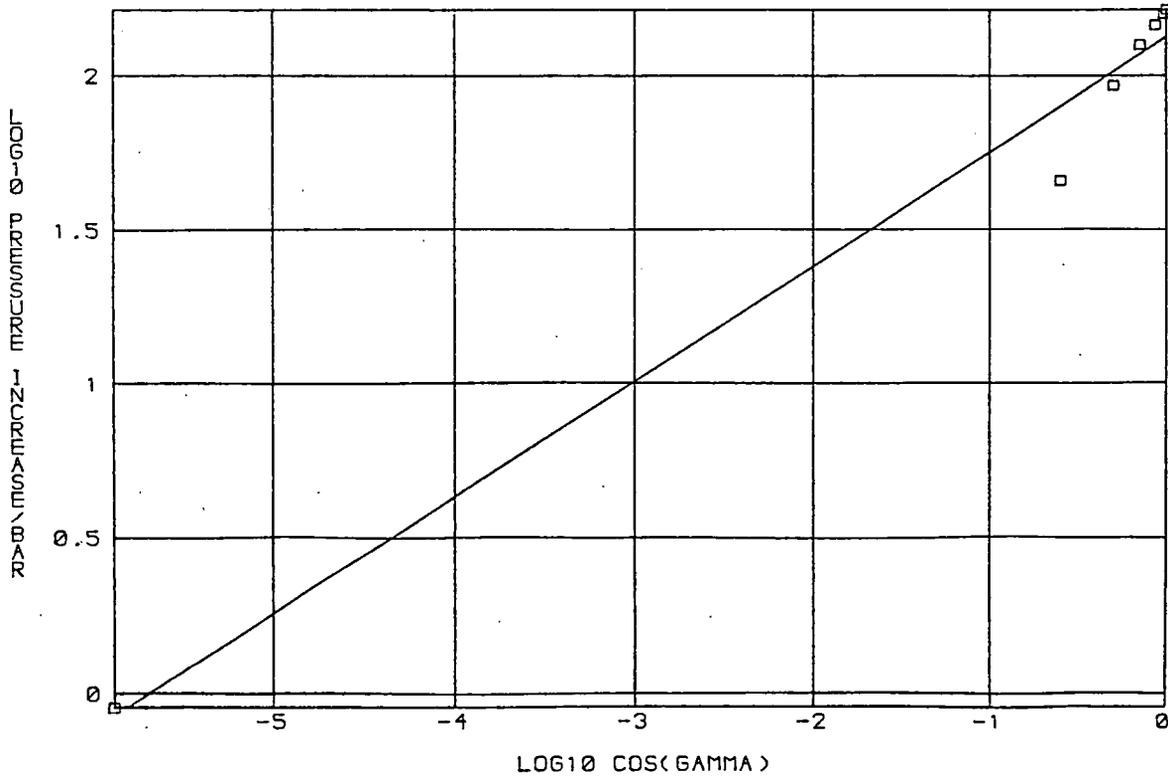
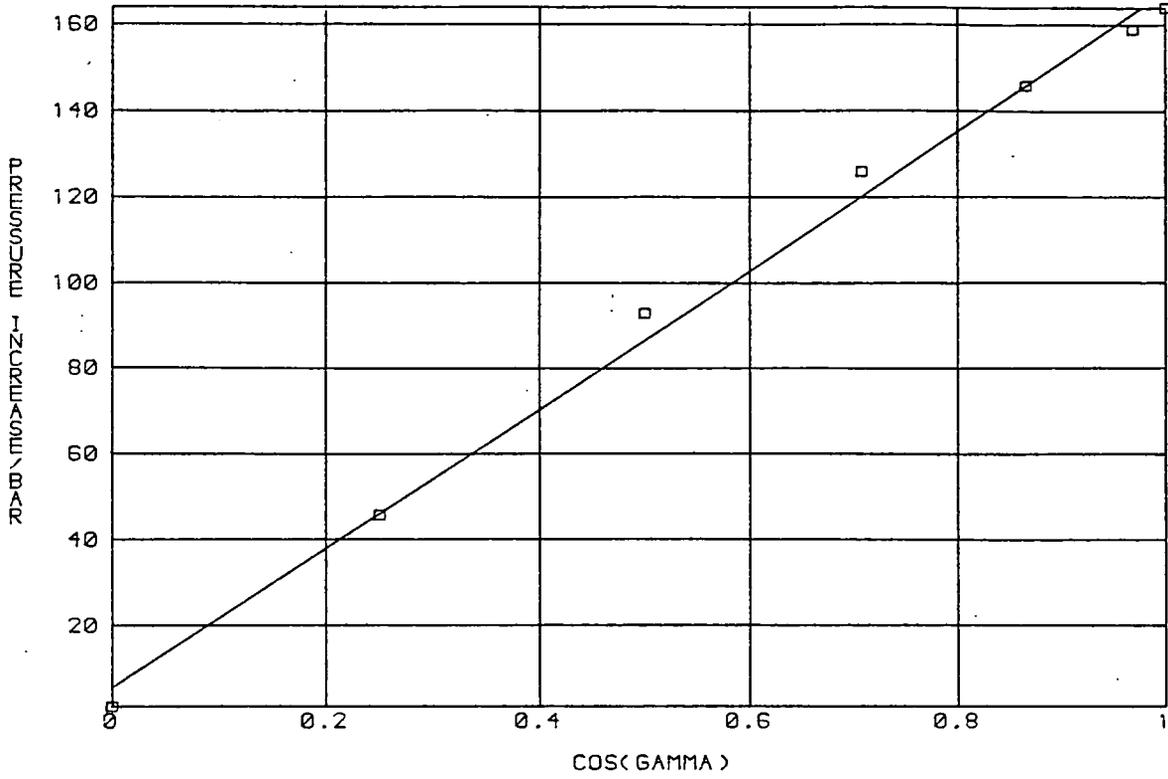


Figure 5.43

p' against $\cos \gamma$ for an incompressible
 Glen rheology with bed of $S = 10 \text{ alu/bar.a}$
 and $v_d = 10 \text{ alu/a}$ (Case 34)

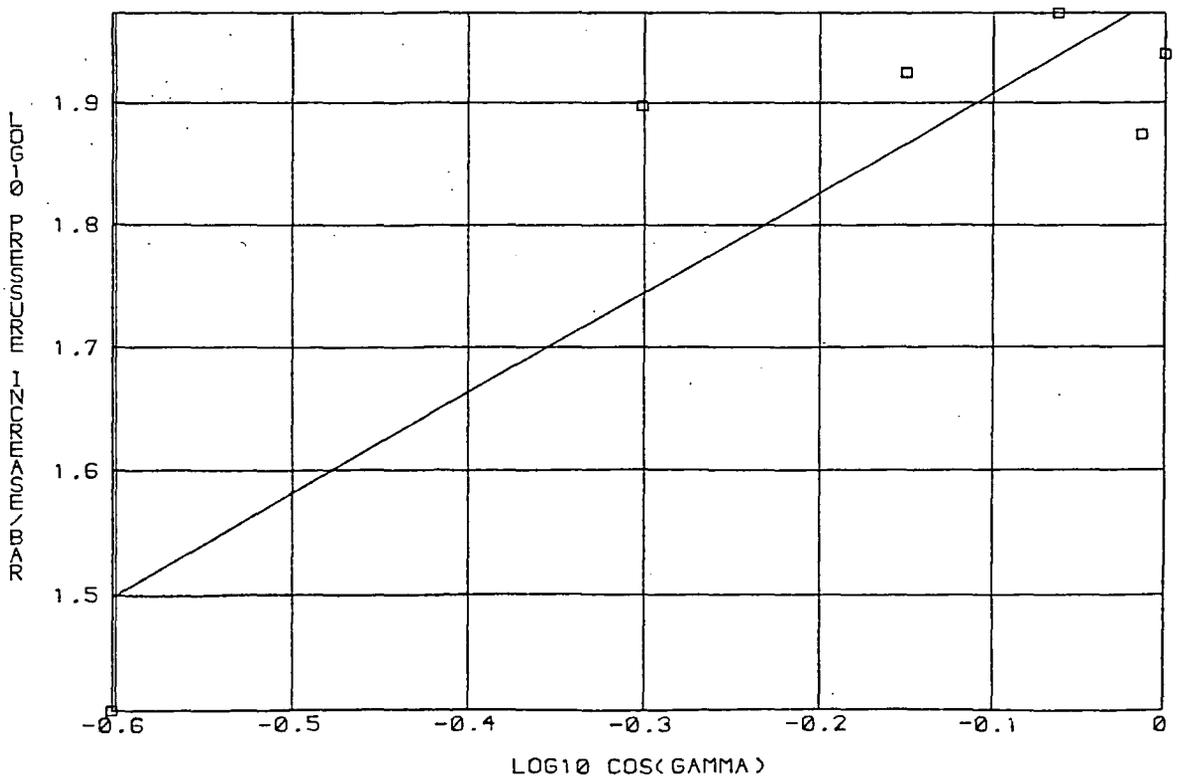
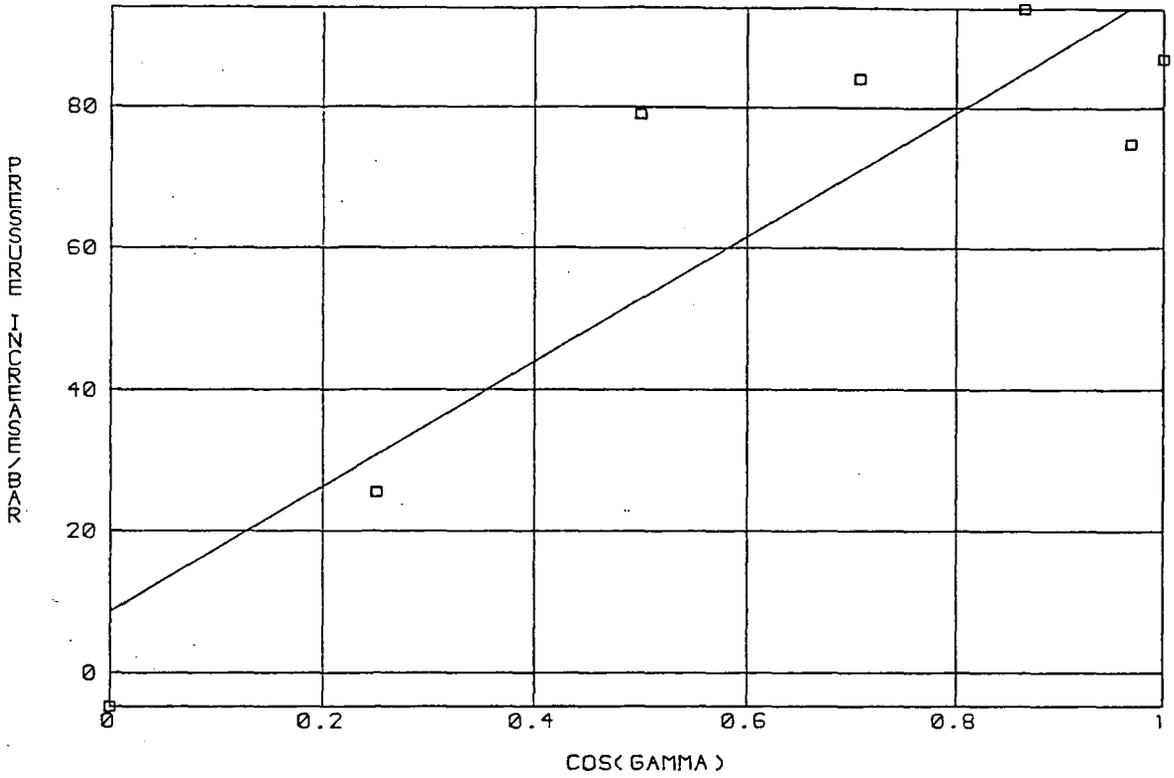
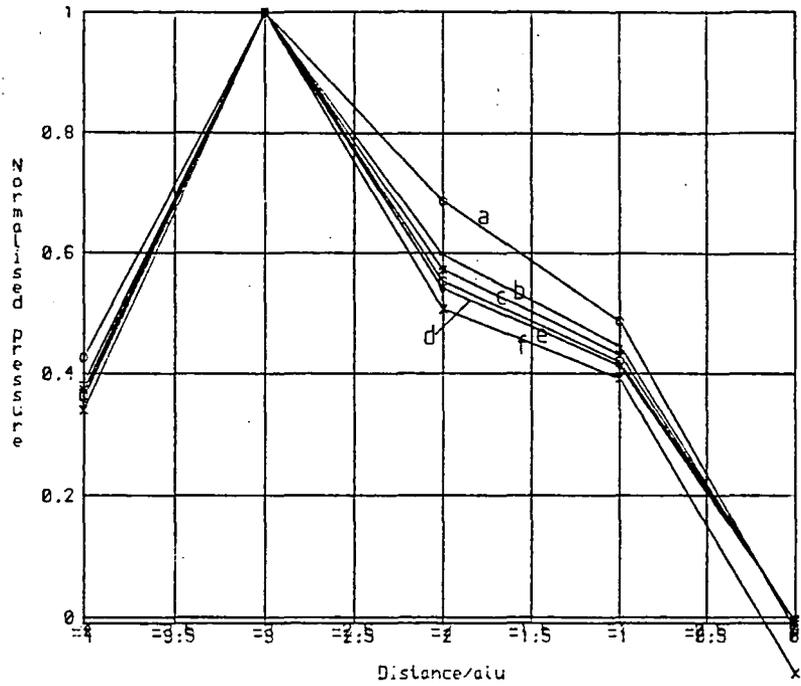


Figure 5.44

p' against position for flow over a truncated sine ridge (Cases 40 - 49).



- a-Linear
- b-Glen, $v_0 = 400au/a$
- c-Glen, $v_0 = 200au/a$
- d-Glen, $v_0 = 100au/a$
- e-Glen, $v_0 = 50au/a$
- f-Glen, $v_0 = 10au/a$

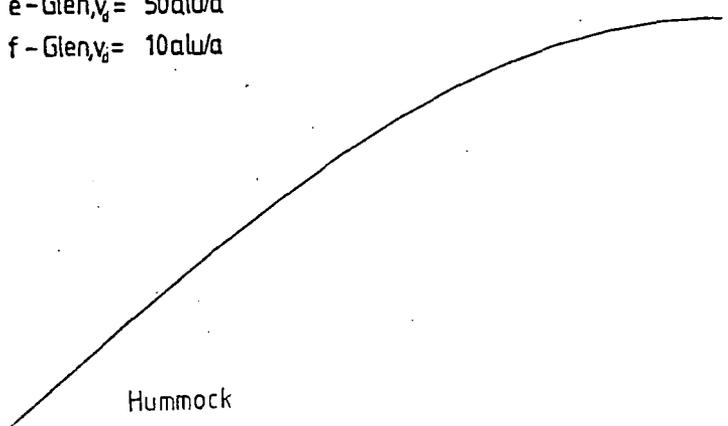


Figure 5.45

Pressure contours in bar for an incompressible Glen rheology with $v_d = 100$ alu/a for flow over a truncated sine ridge (Case 47).

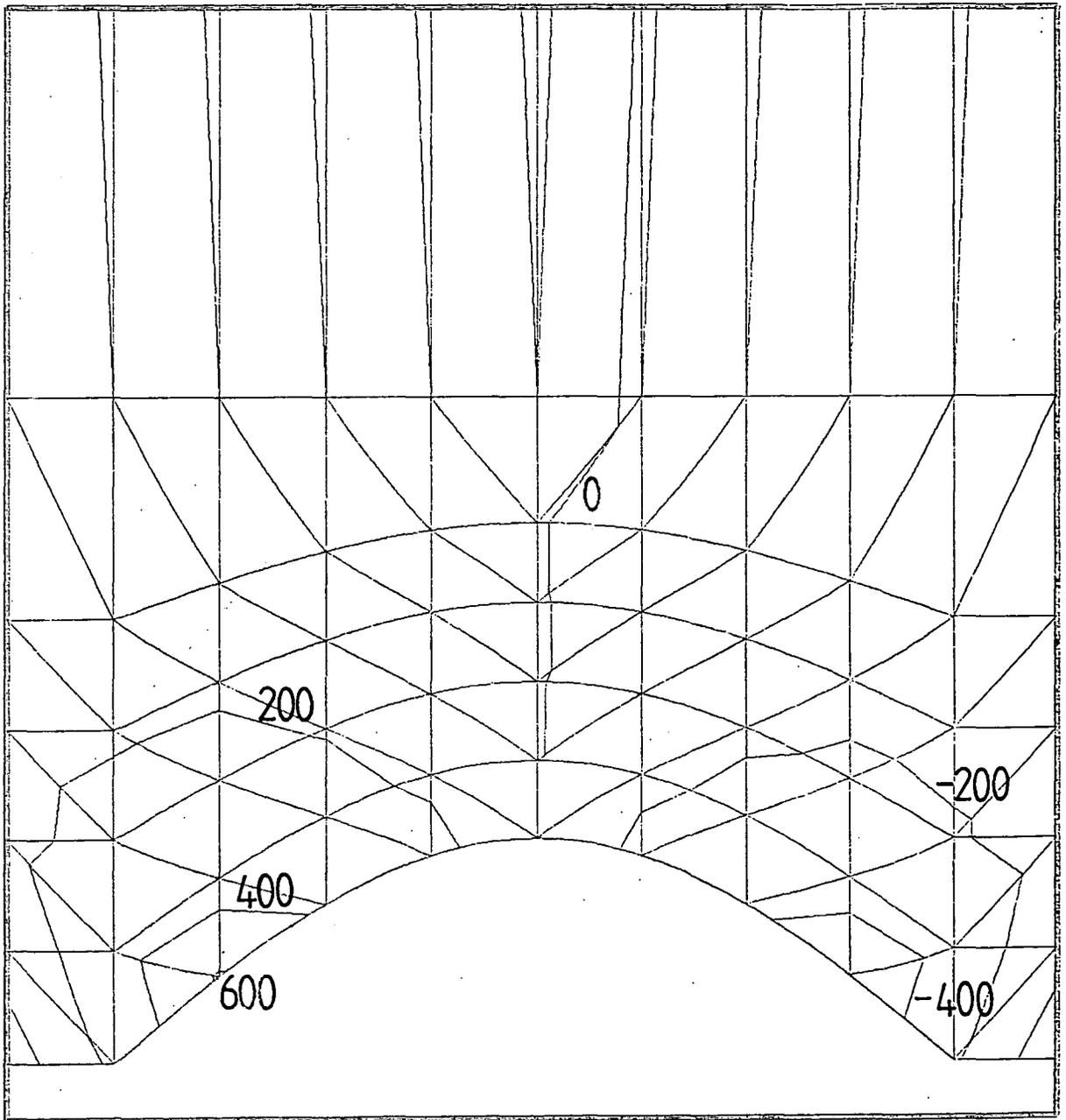


Figure 5.46

$\dot{\epsilon}_v$ in a^{-1} for a compressible linear rheology with $\chi = 1 \text{ alu}^2/\text{bar}\cdot a$ and $v_d = 400 \text{ alu}/a$ (Case 12).

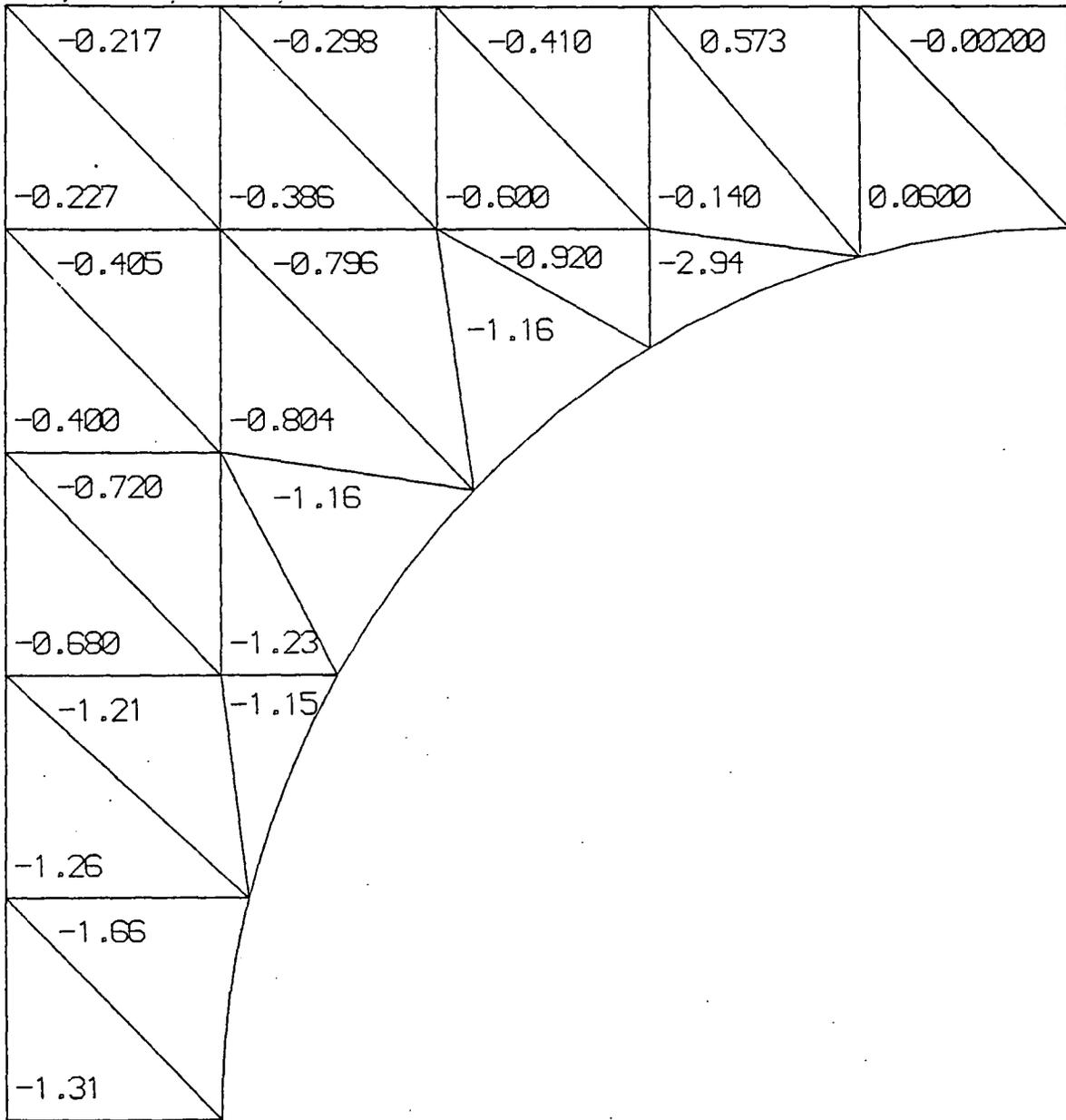


Figure 5.47

$\dot{\epsilon}_v$ in a^{-1} for a compressible linear rheology with $\chi = 10 \text{ alu}^2 / \text{bar} \cdot a$ and $v_d = 200 \text{ alu}/a$ (Case 16).

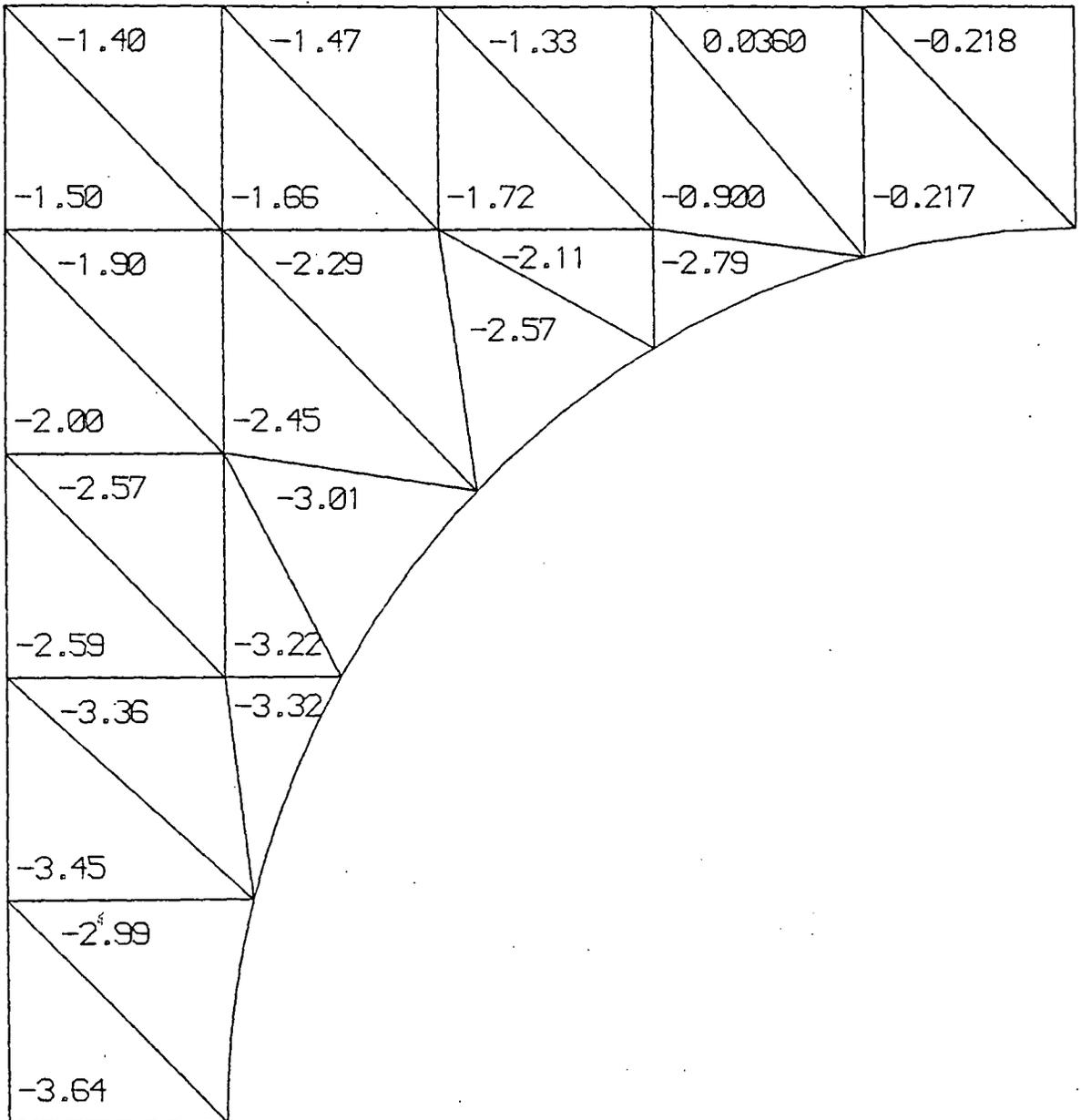


Figure 5.48

$\dot{\epsilon}_v$ in a^{-1} for a compressible Glen rheology with $\chi = 1 \text{ alu}^2/\text{bar}\cdot a$ and $v_d = 10 \text{ alu}/a$ (Case 29).

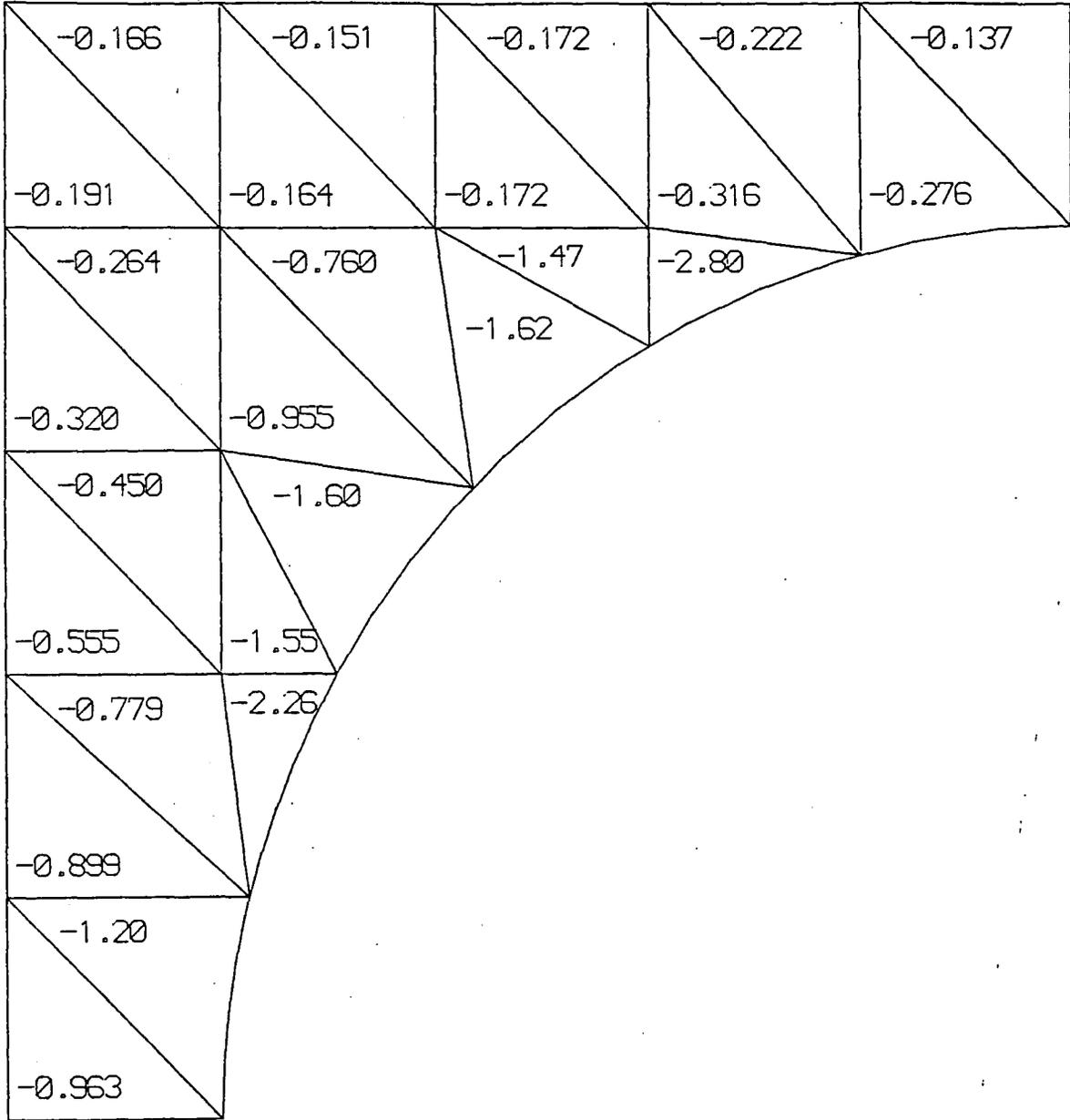


Figure 5.49

$\dot{\epsilon}_v$ in a^{-1} for a compressible Glen
rheology with $\chi = 1 \text{ alu}^2/\text{bar}\cdot a$ and
 $v_d = 400 \text{ alu}/a$ (Case 33).

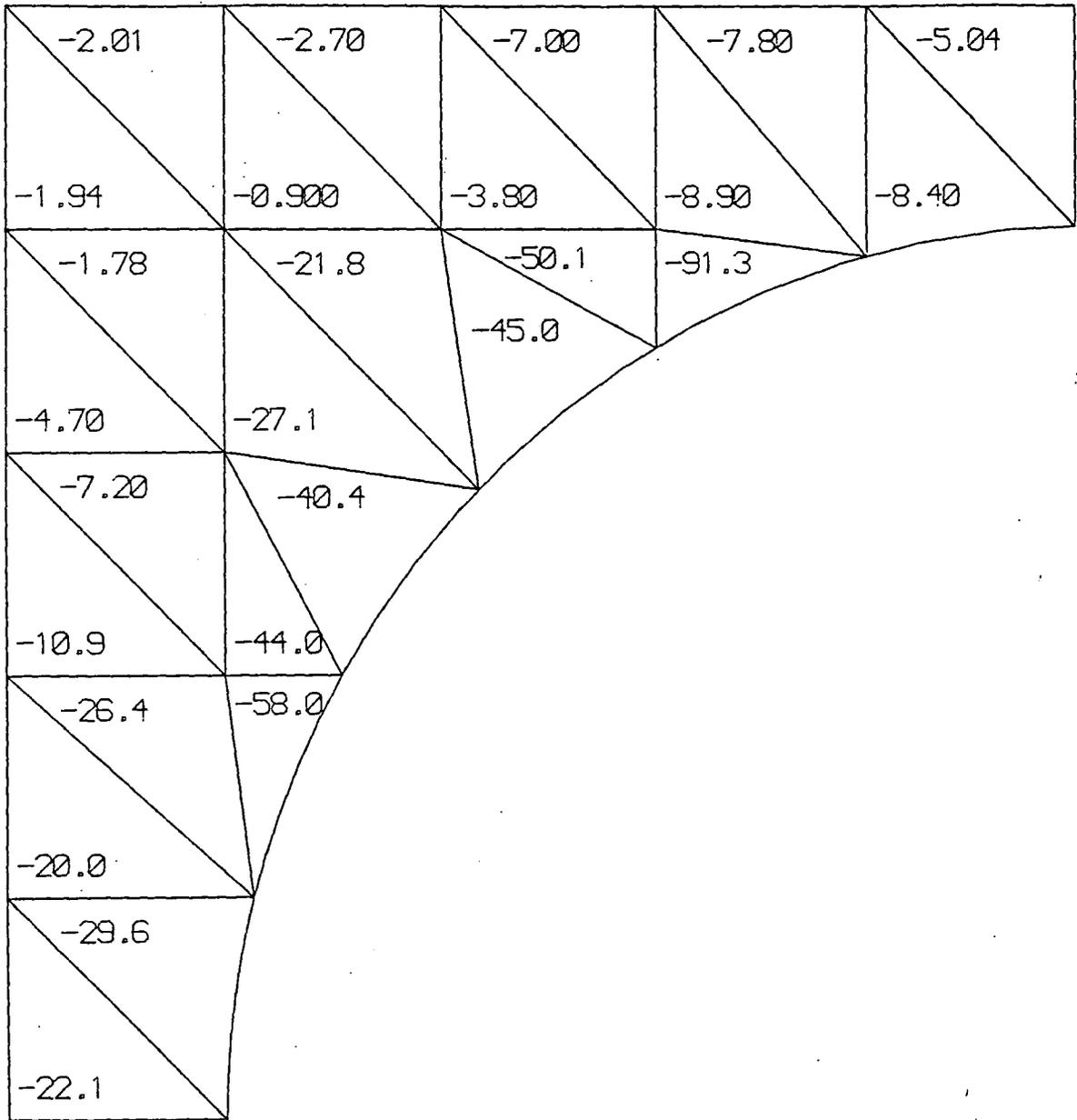


Figure 5.50

Regression exponent for $\dot{\epsilon}_v$ against v_d
 for a compressible Glen rheology with
 $x = 1 \text{ alu}^2/\text{bar}\cdot\text{a}$ (Cases 29 - 33).

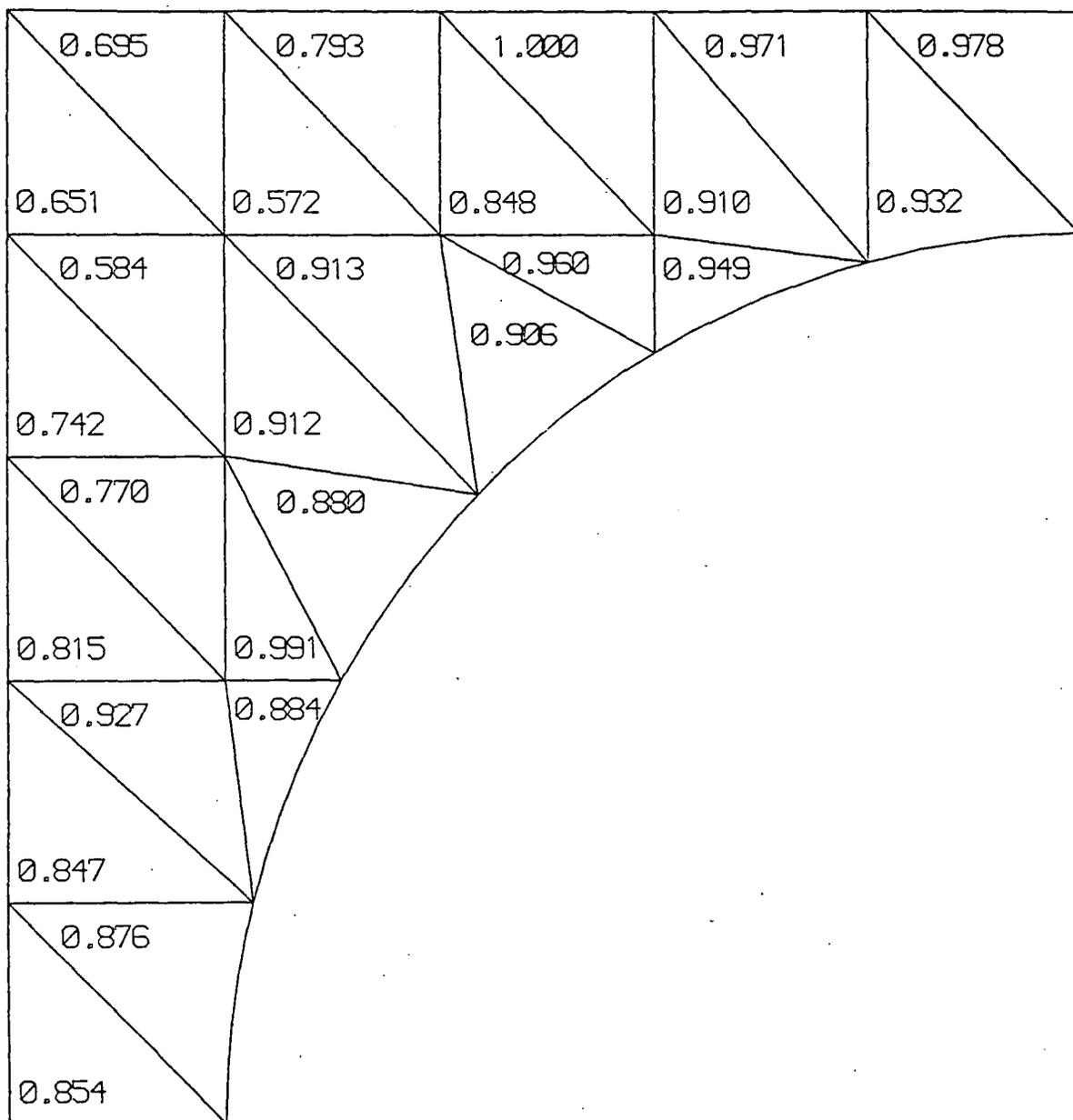


Figure 5.51

μ in bar.a for an incompressible Glen
rheology with $v_d = 10 \text{ alu/a}$ (Case 23).

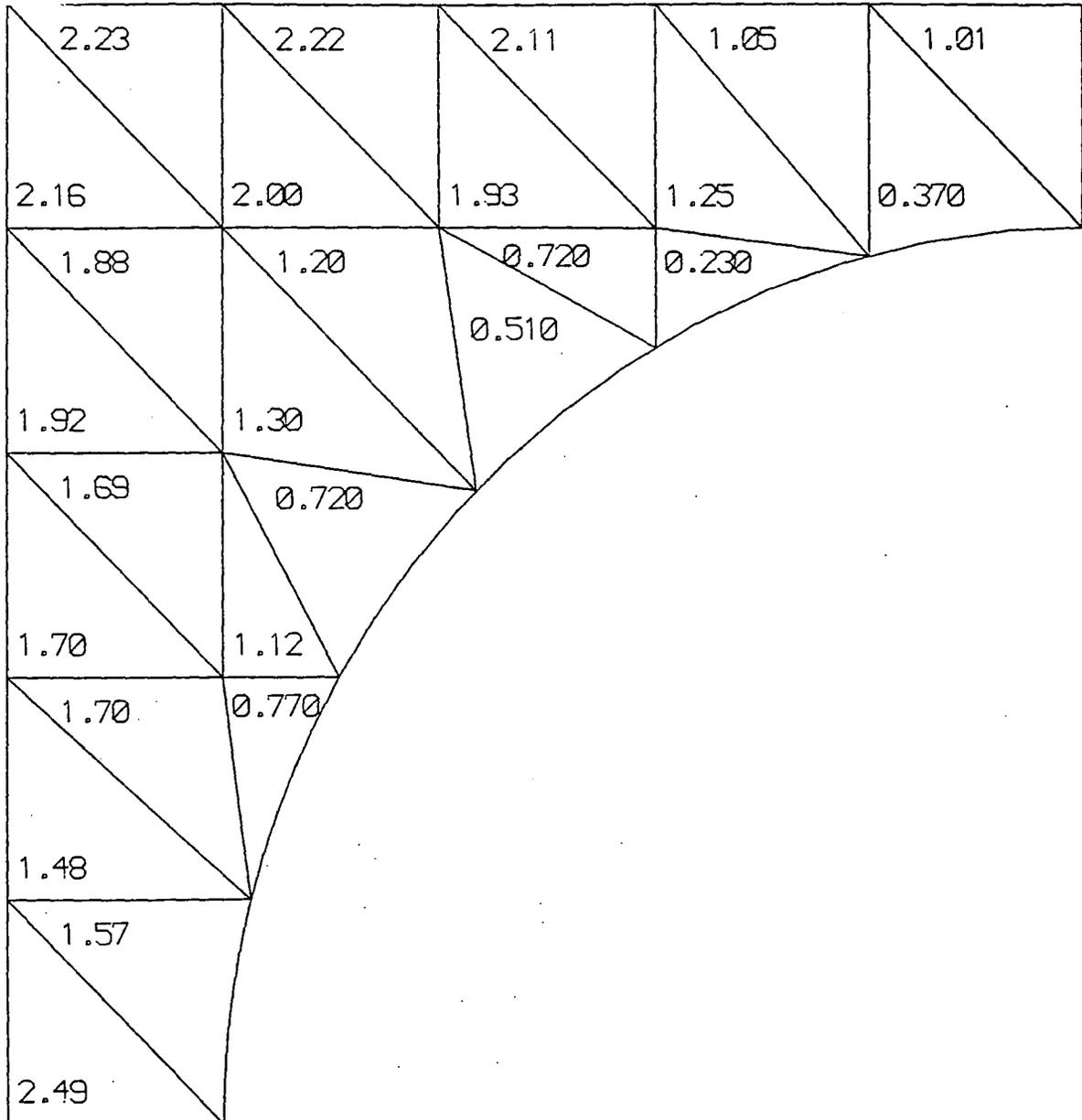


Figure 5.52

μ in bar.a for an incompressible Glen rheology with $v_d = 400 \text{ au/a}$ (Case 26).

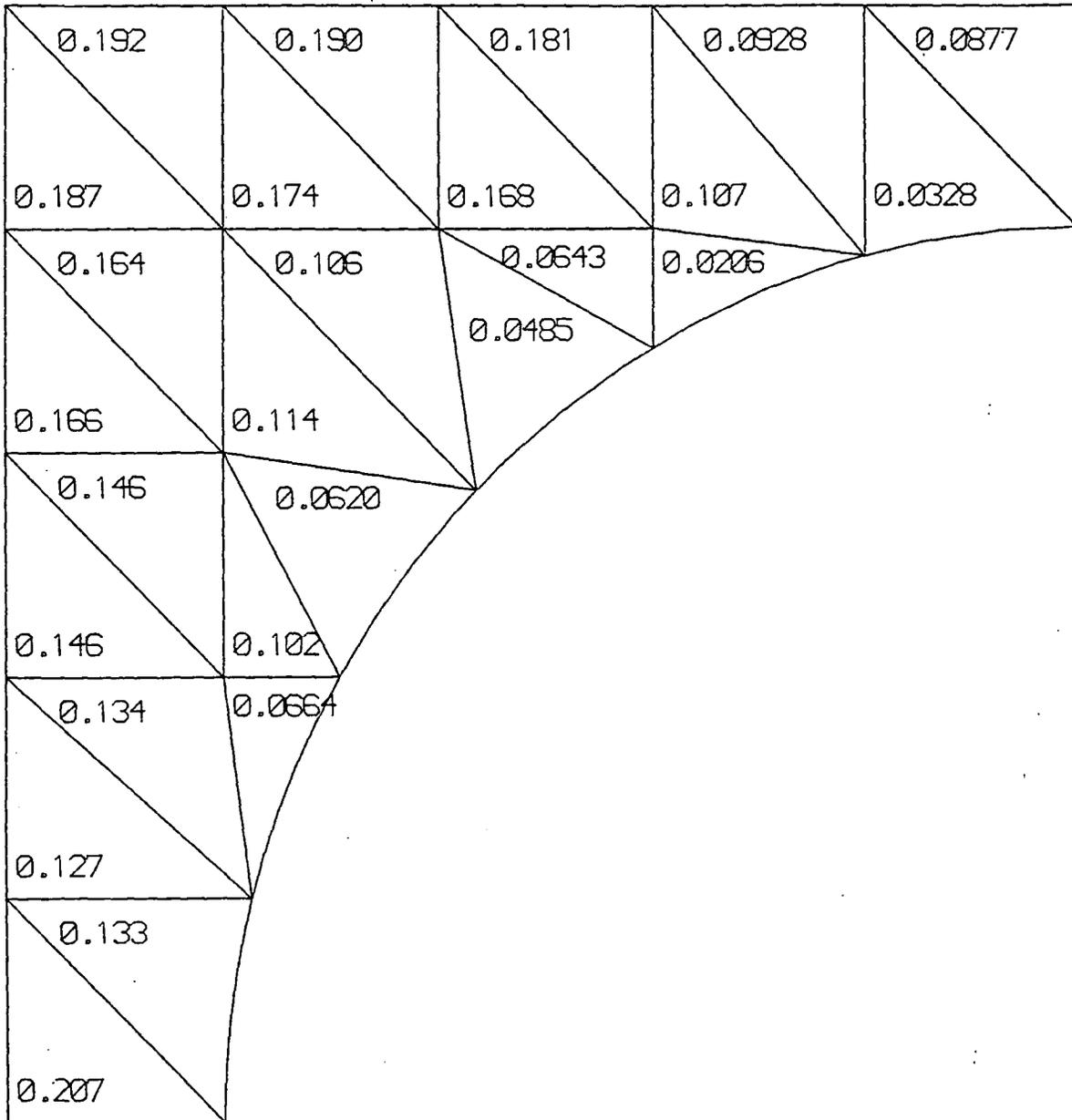


Figure 5.53

μ in bar.a for an incompressible Glen rheology with a cavity and $v_d = 10 a\dot{\mu}/a$ (Case 27).

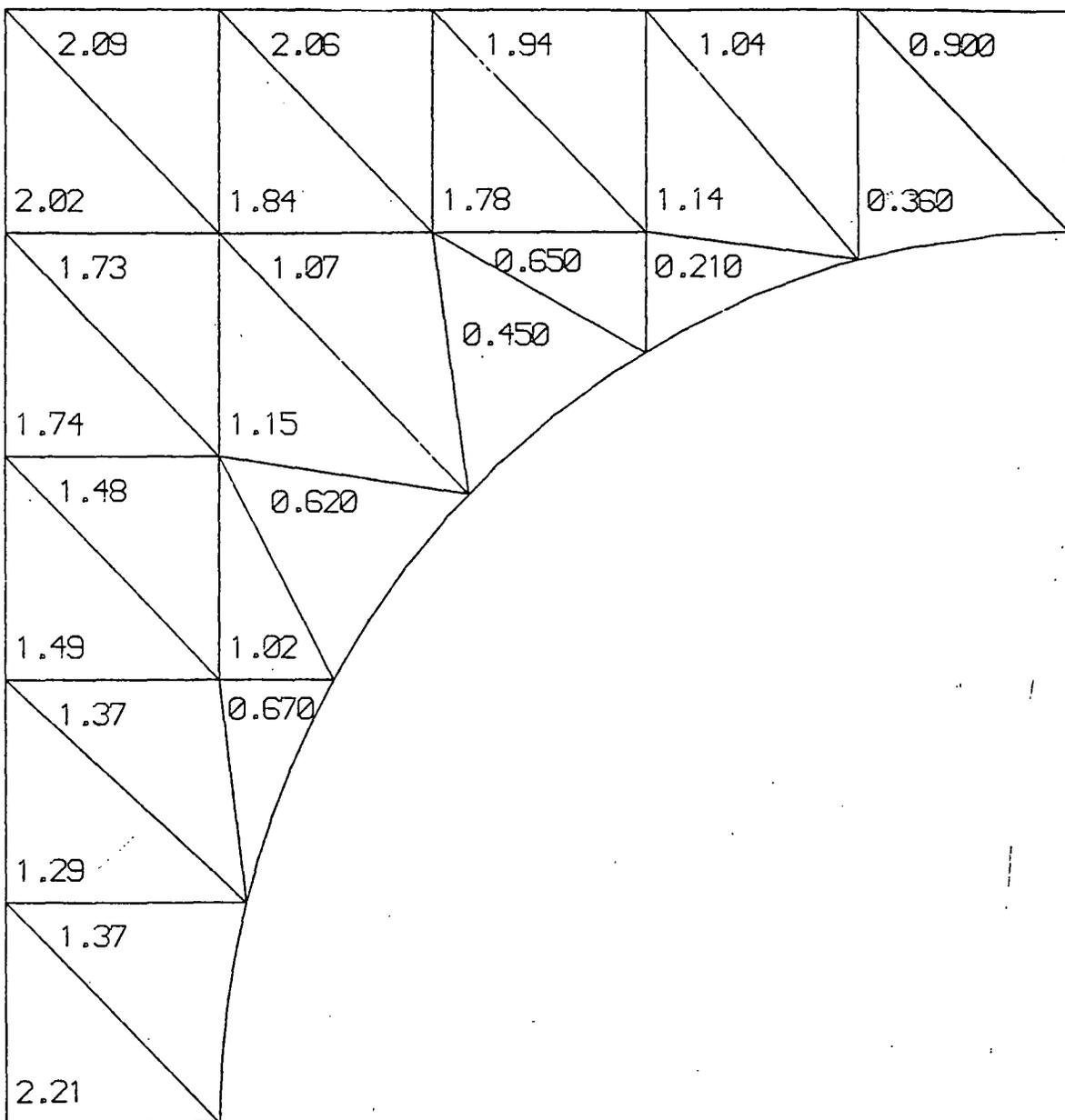


Figure 5.54

μ in bar.a for a compressible Glen rheology with $\chi = 1 \text{ alu}^2/\text{bar.a}$ and $v_d = 10 \text{ alu/a}$ (Case 29).

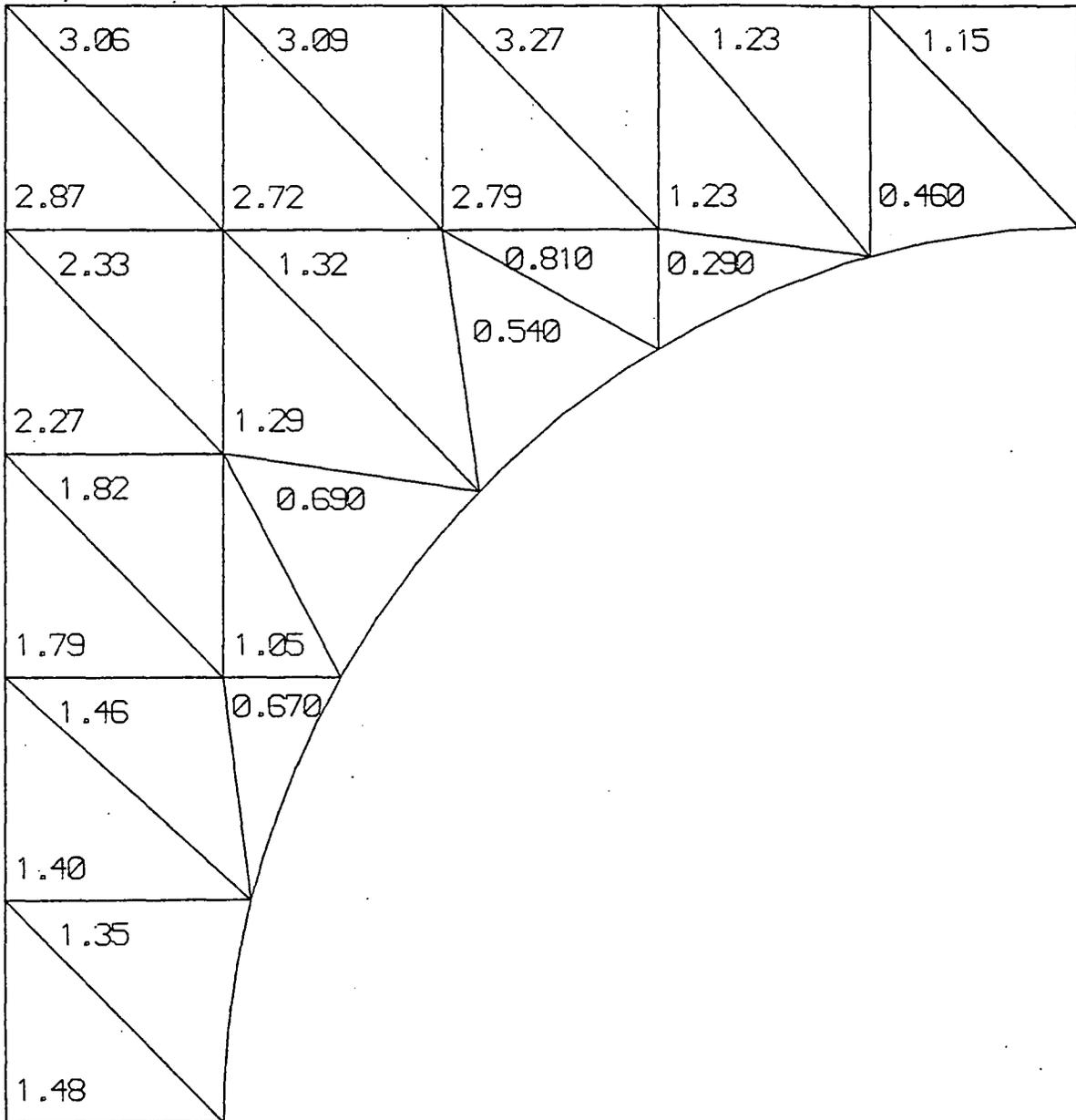


Figure 5.55

μ in bar.a for a compressible Glen
 rheology with $\chi = 1 \text{ alu}^2/\text{bar.a}$ with
 $v_d = 400 \text{ alu/a}$ (Case 33).

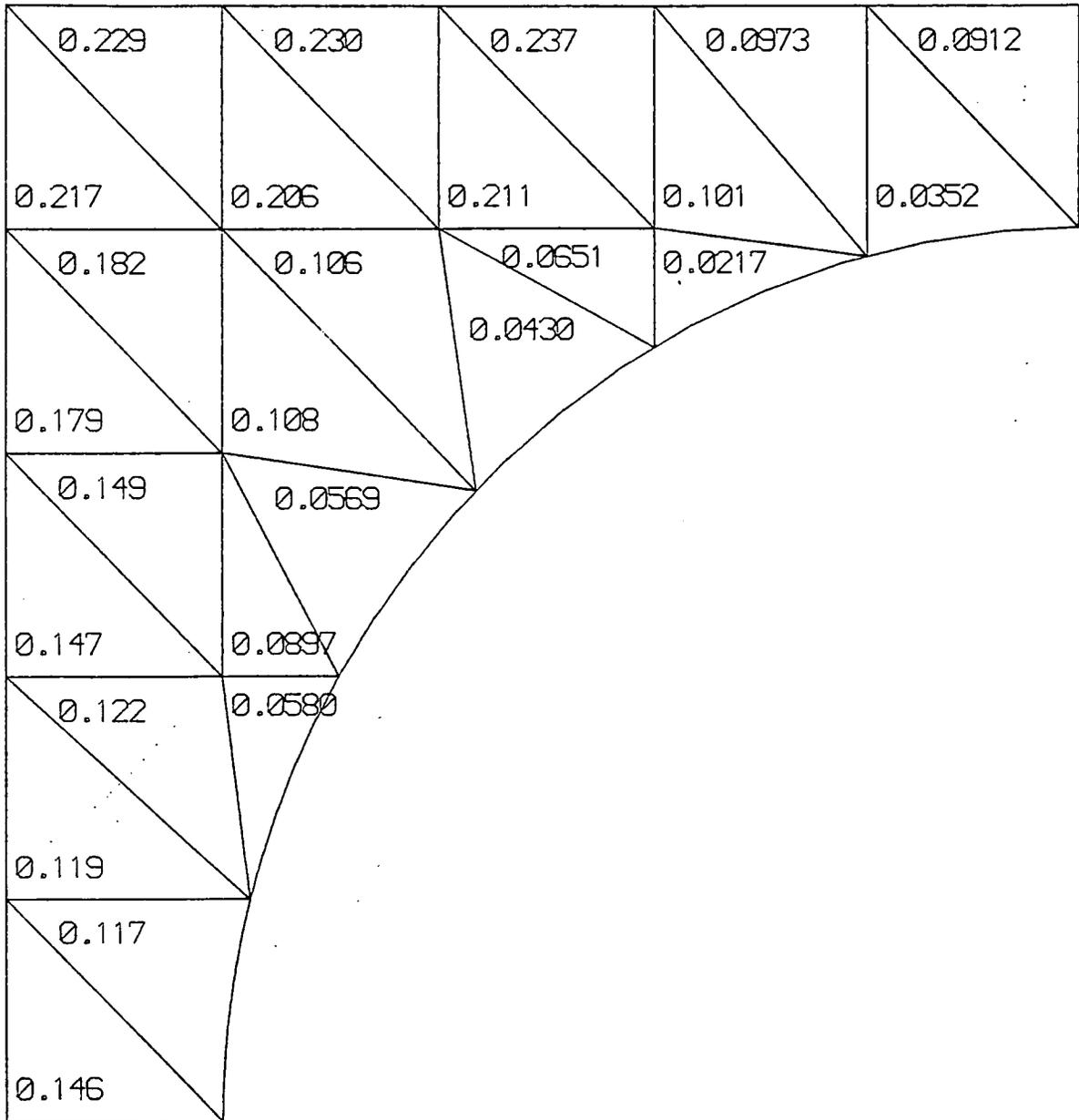


Figure 5.56 μ in bar.a for an incompressible Glen rheology with $S = 10 \text{ alu/bar.a}$ and $v_d = 10 \text{ alu/a}$ (Case 34).

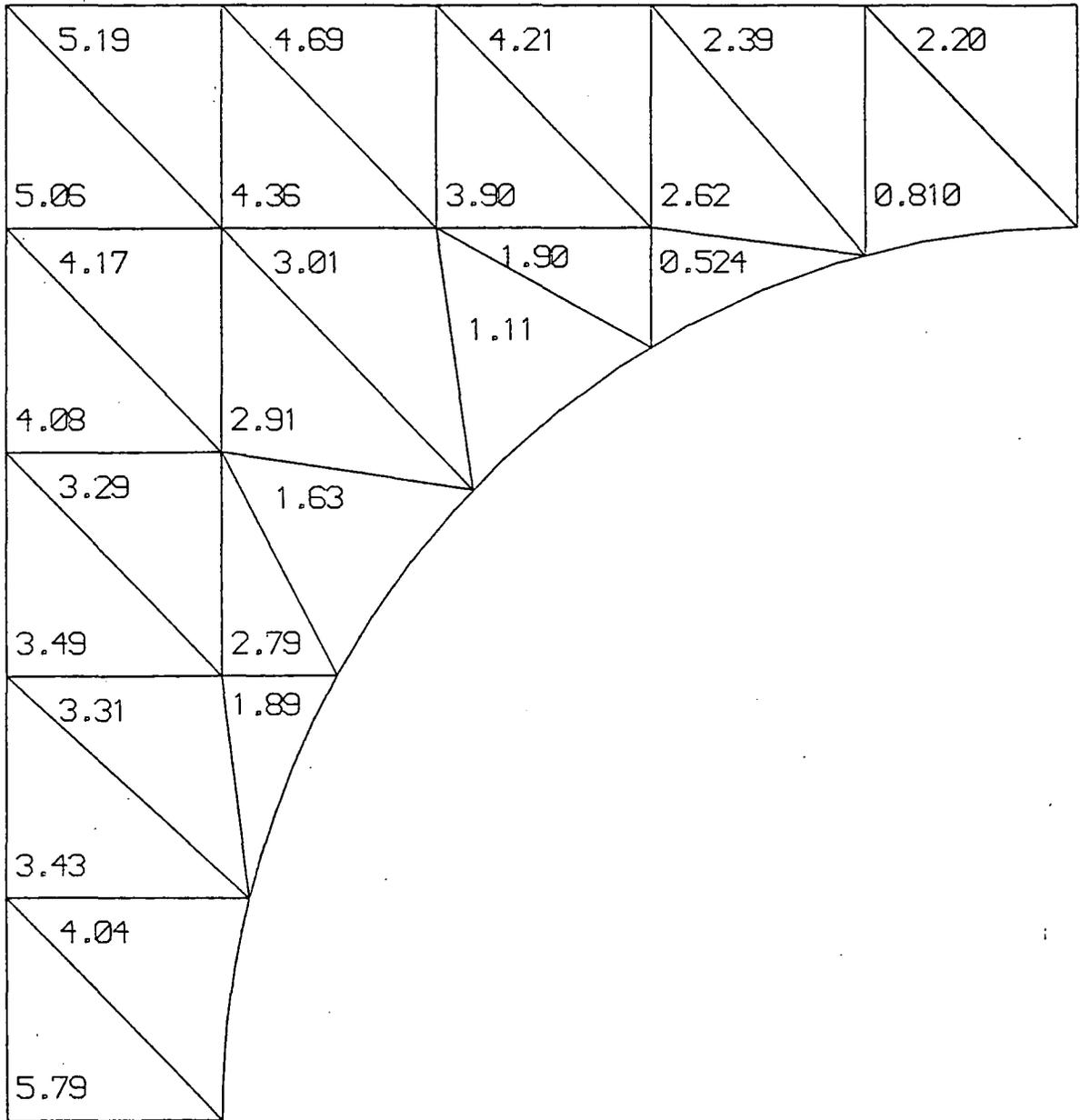


Figure 5.57

μ in bar.a for an incompressible Glen rheology with $v_d = 10$ alu/a for flow over a truncated sine ridge (Case 45).

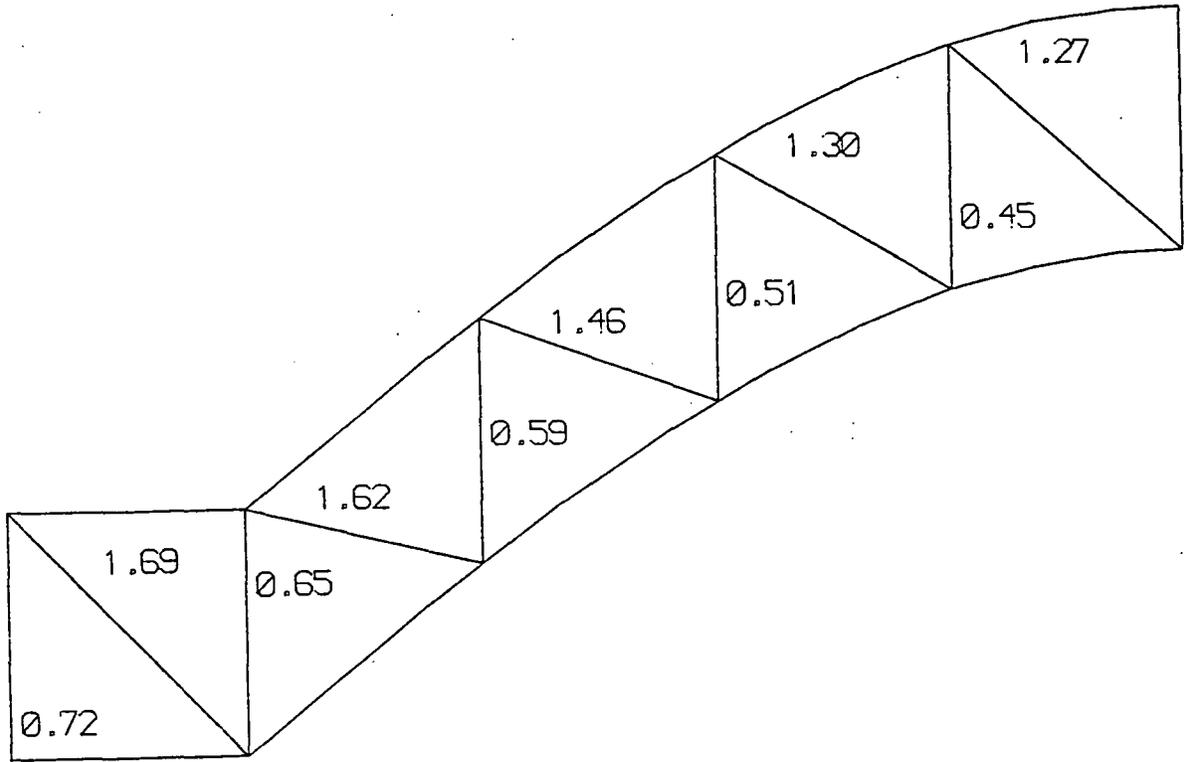


Figure 5.58

μ in bar.a for an incompressible Glen rheology with $v_d = 400$ alu/a for flow over a truncated sine ridge (Case 49).

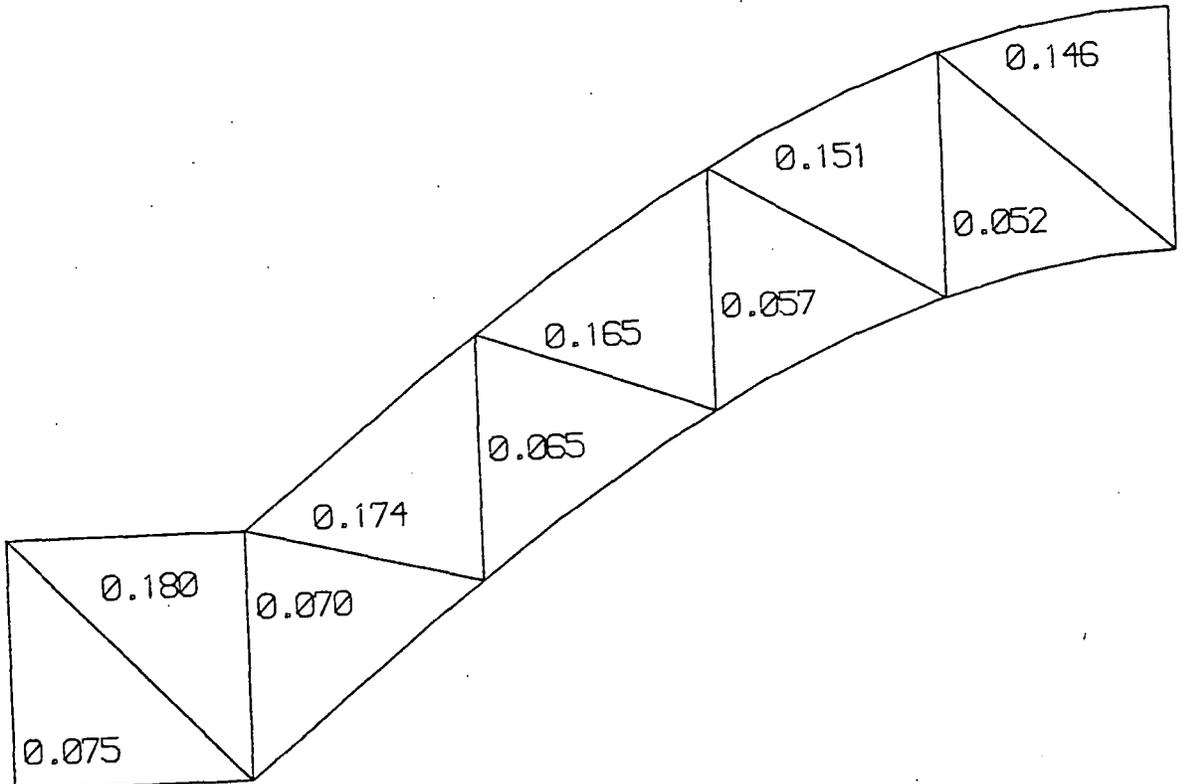


Figure 5.59

Regression exponent of μ against v_d for
flow over a truncated sine ridge
(Cases 45 - 49).

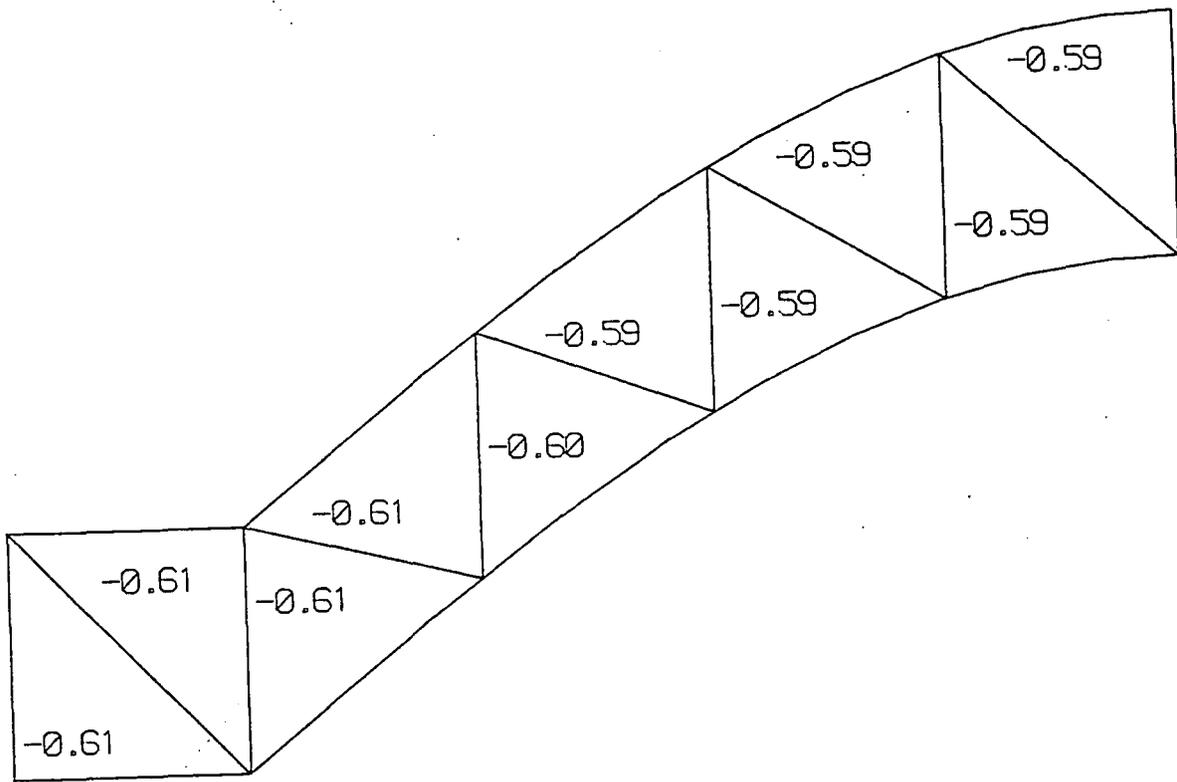


Figure 5.60

Semi-analytic interfacial stress for
incompressible linear rheology with
 $v_d = 10 \text{ alu/a.}$

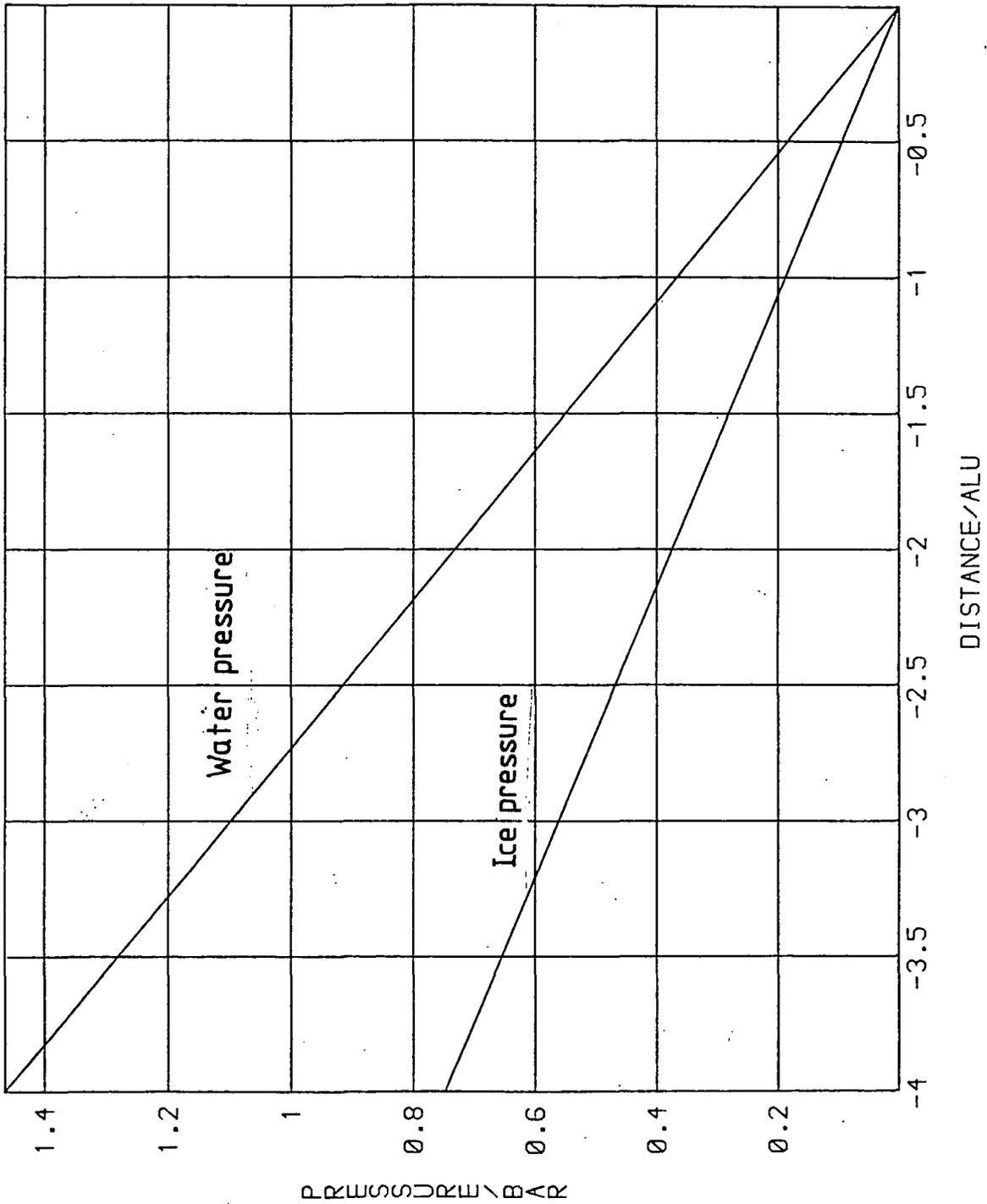


Figure 5.61

Semi-analytic interfacial stress for
compressible linear rheology with
 $\chi = 1 \text{ alu}^2/\text{bar}\cdot\text{a}$ and $v_d = 10 \text{ alu}/\text{a}$.

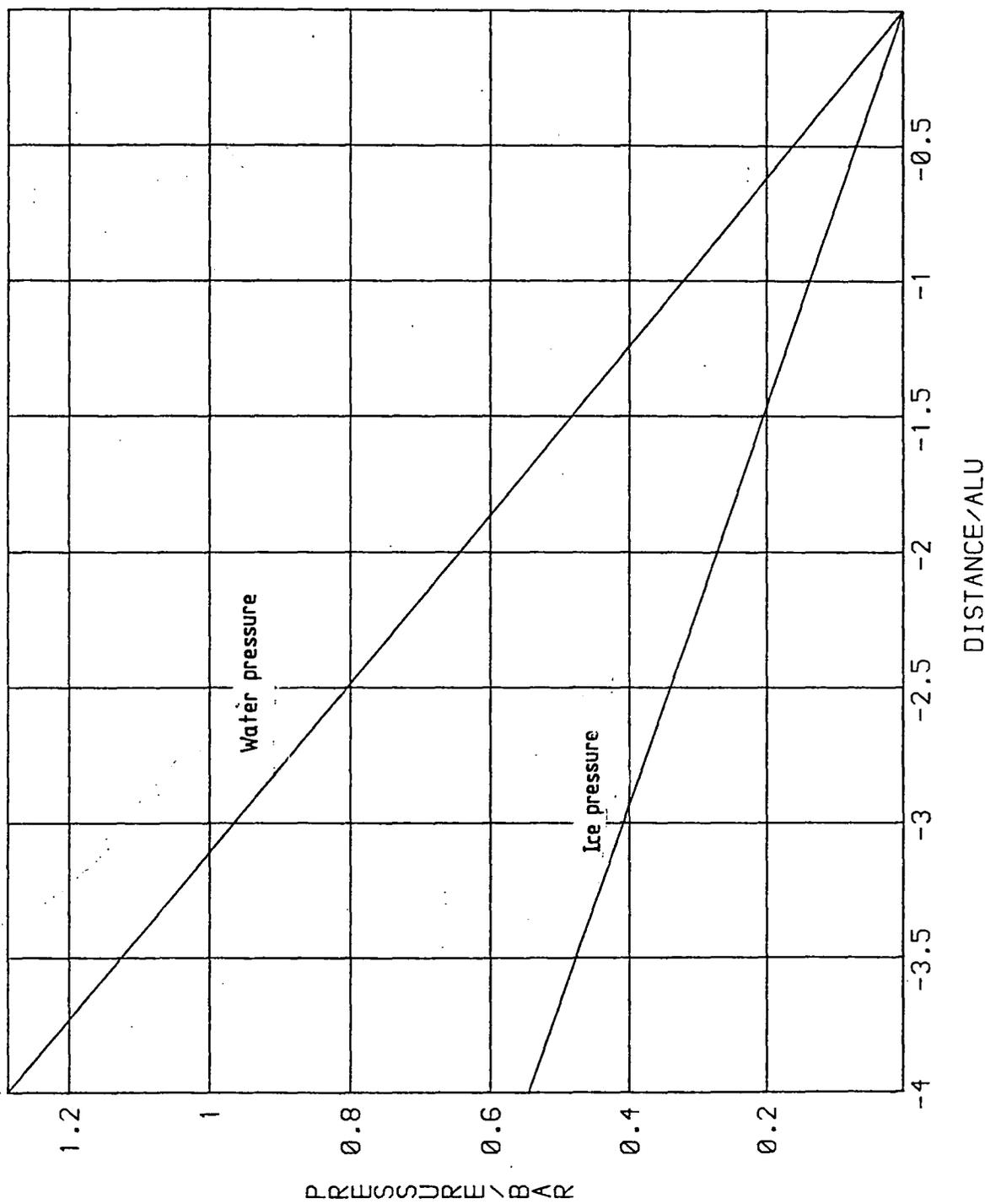


Figure 5.62

Semi-analytic interfacial stress for
compressible linear rheology with

$$\chi = 10 \text{ alu}^2/\text{bar}\cdot\text{a} \text{ and } v_d = 10 \text{ alu/a.}$$

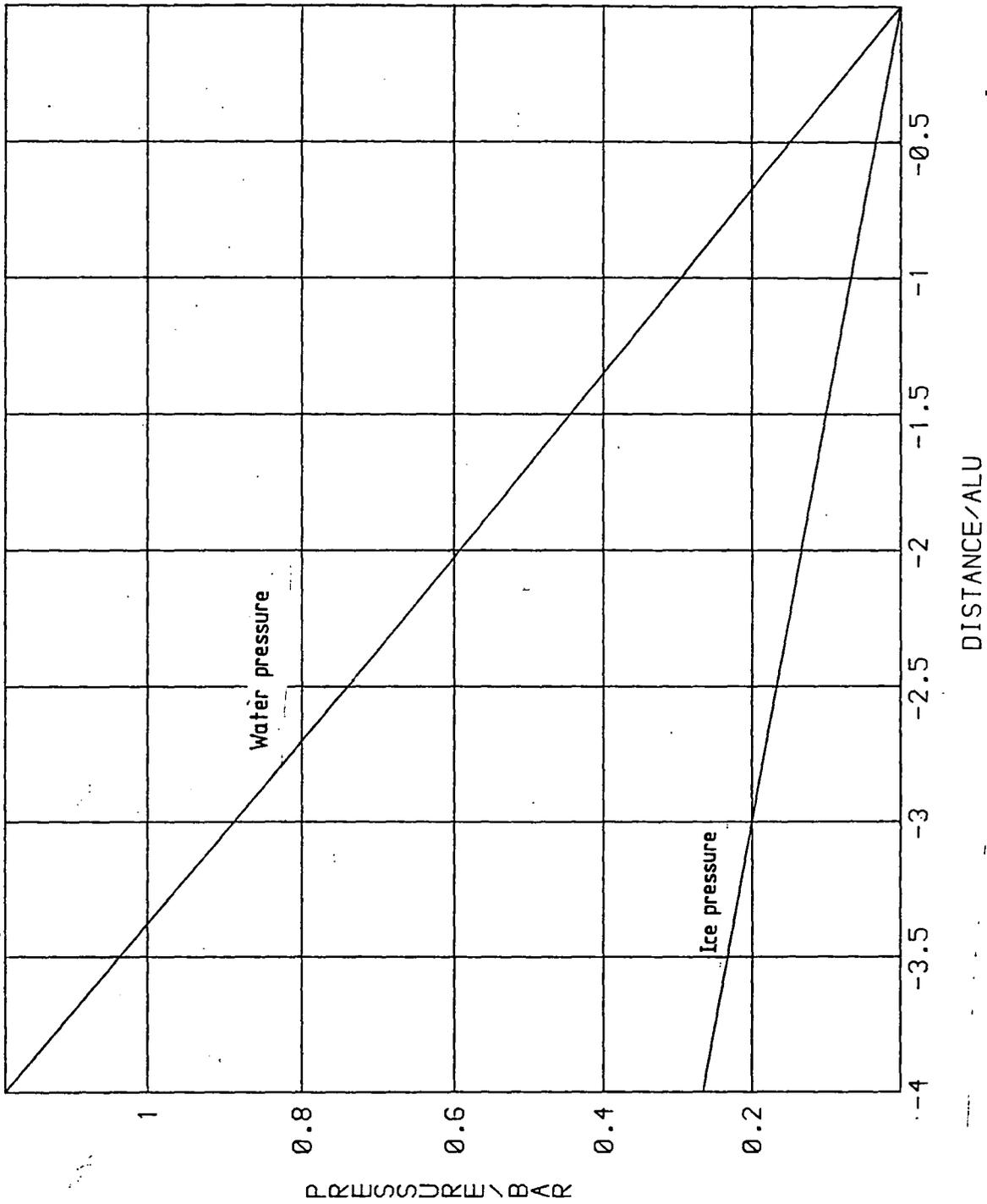


Figure 5.63

Semi-analytic interfacial stress for
incompressible Glen rheology with

$$v_d = 10 \text{ alu/a.}$$

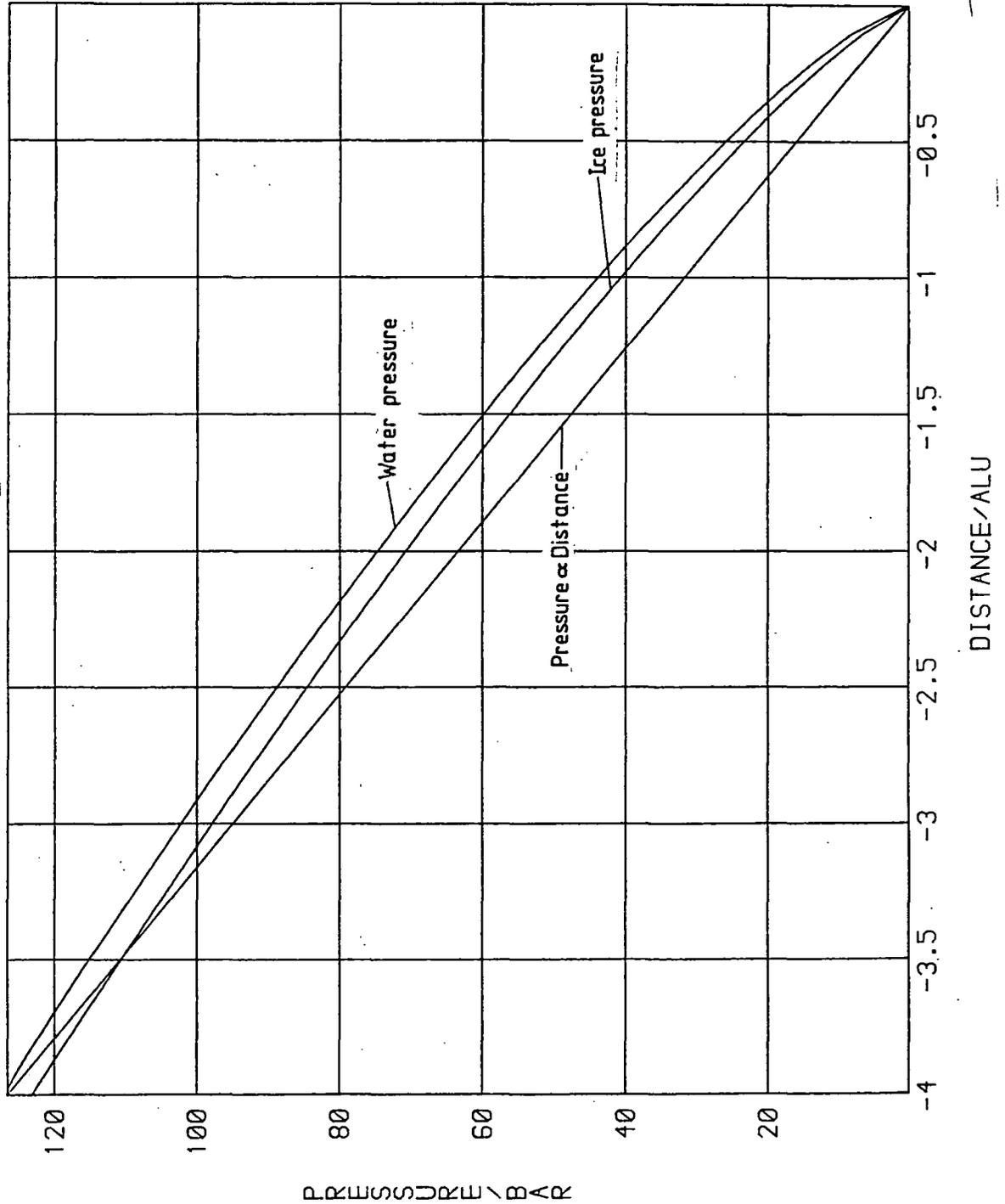


Figure 5.64

Semi-analytic interfacial stress for compressible Glen rheology with

$\chi = 1 \text{ alu}^2/\text{bar}\cdot\text{a}$ and $v_d = 10 \text{ alu/a}$.

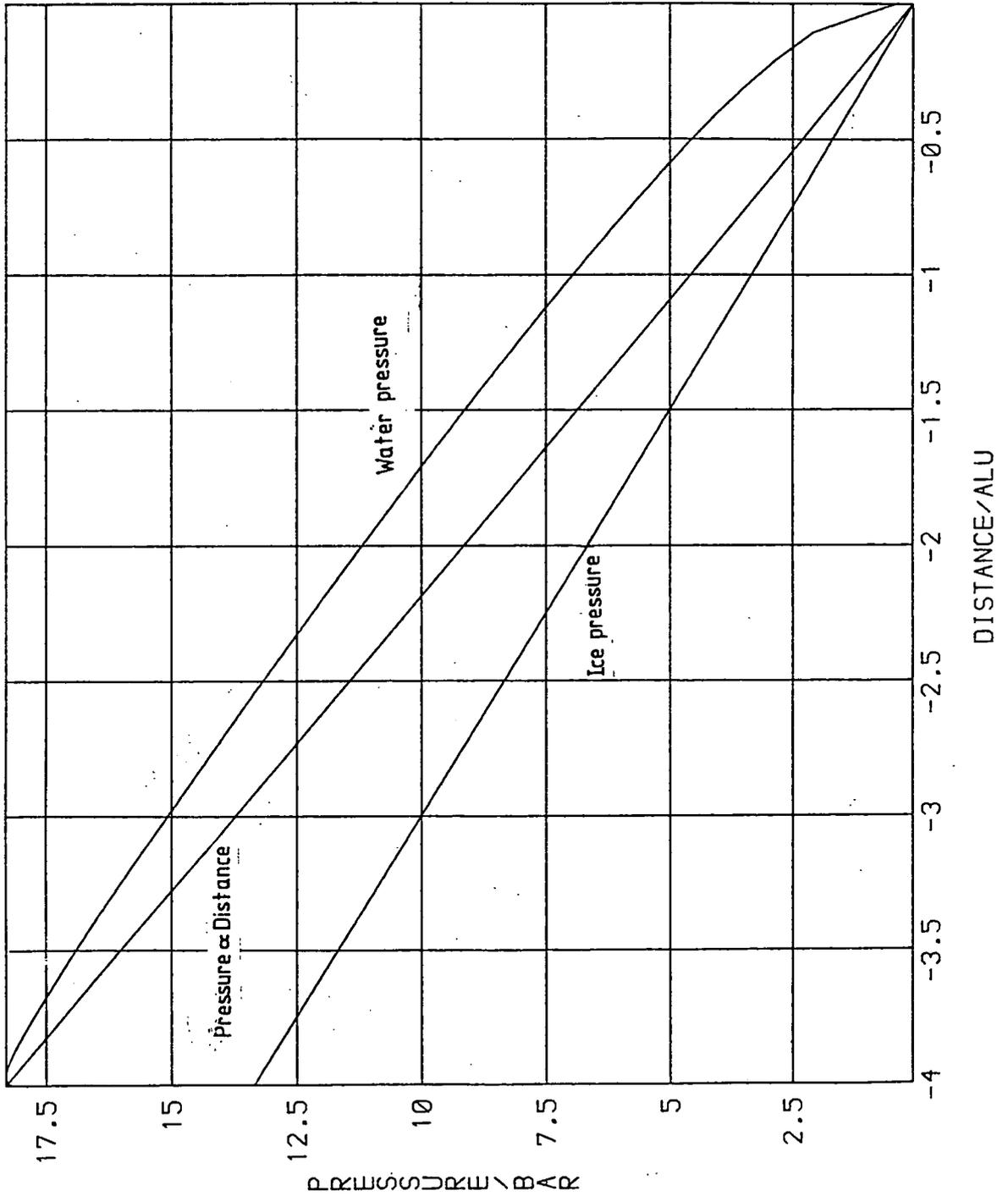


Figure 5.65

Finite element mesh for uni-axial flow modelling.

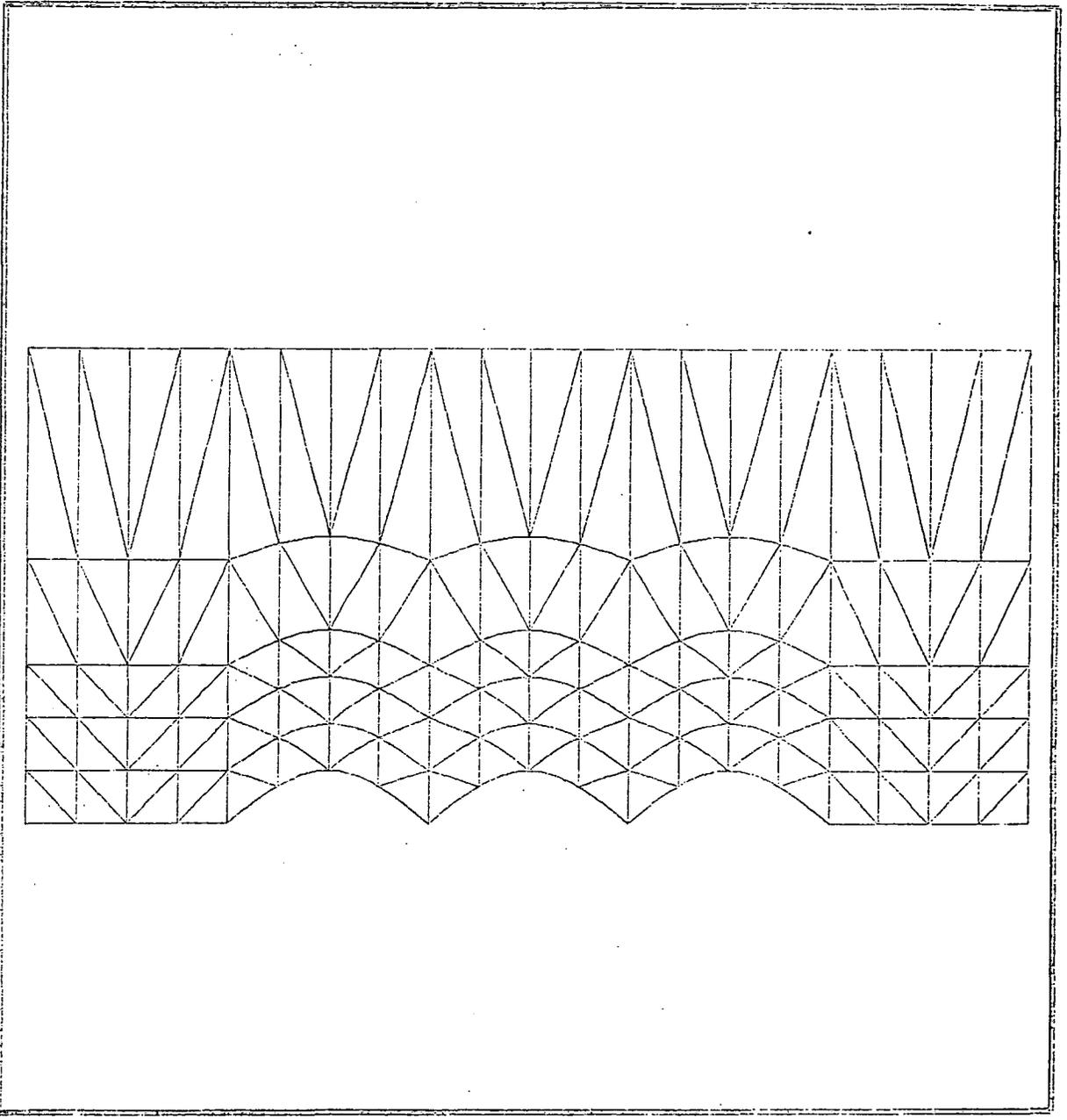


Figure 5.66

Velocity contours in al_u/a for $v_d = 10 al_u/a$
and a bed of $S = 0.002 al_u/bar.a$,
wide-hummocked case.

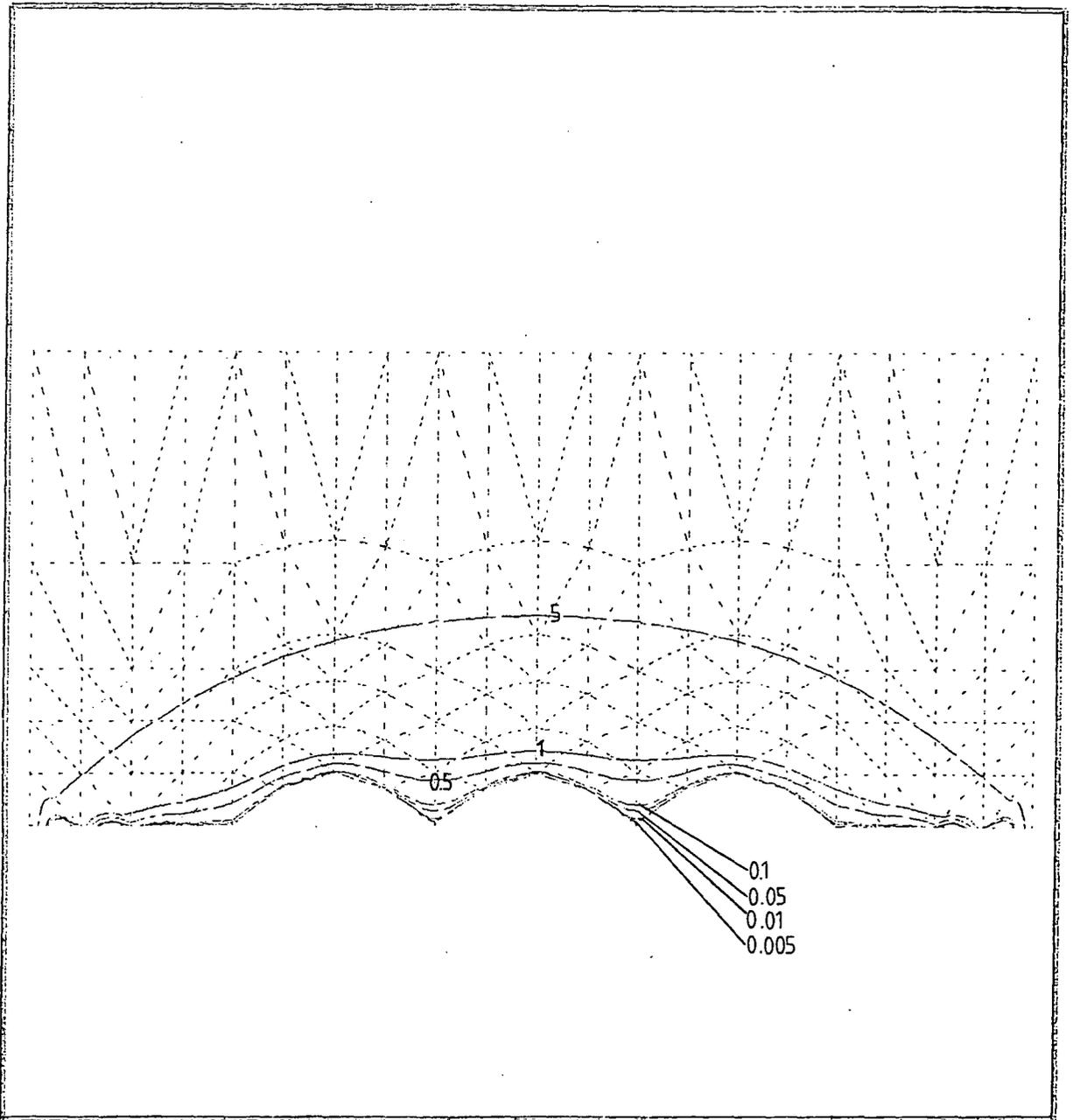


Figure 5.67

Velocity contours in al_u/a for $v_d = 10 al_u/a$
and bed of $S = 0.01 al_u/\text{bar}.a$,
wide-hummocked case.

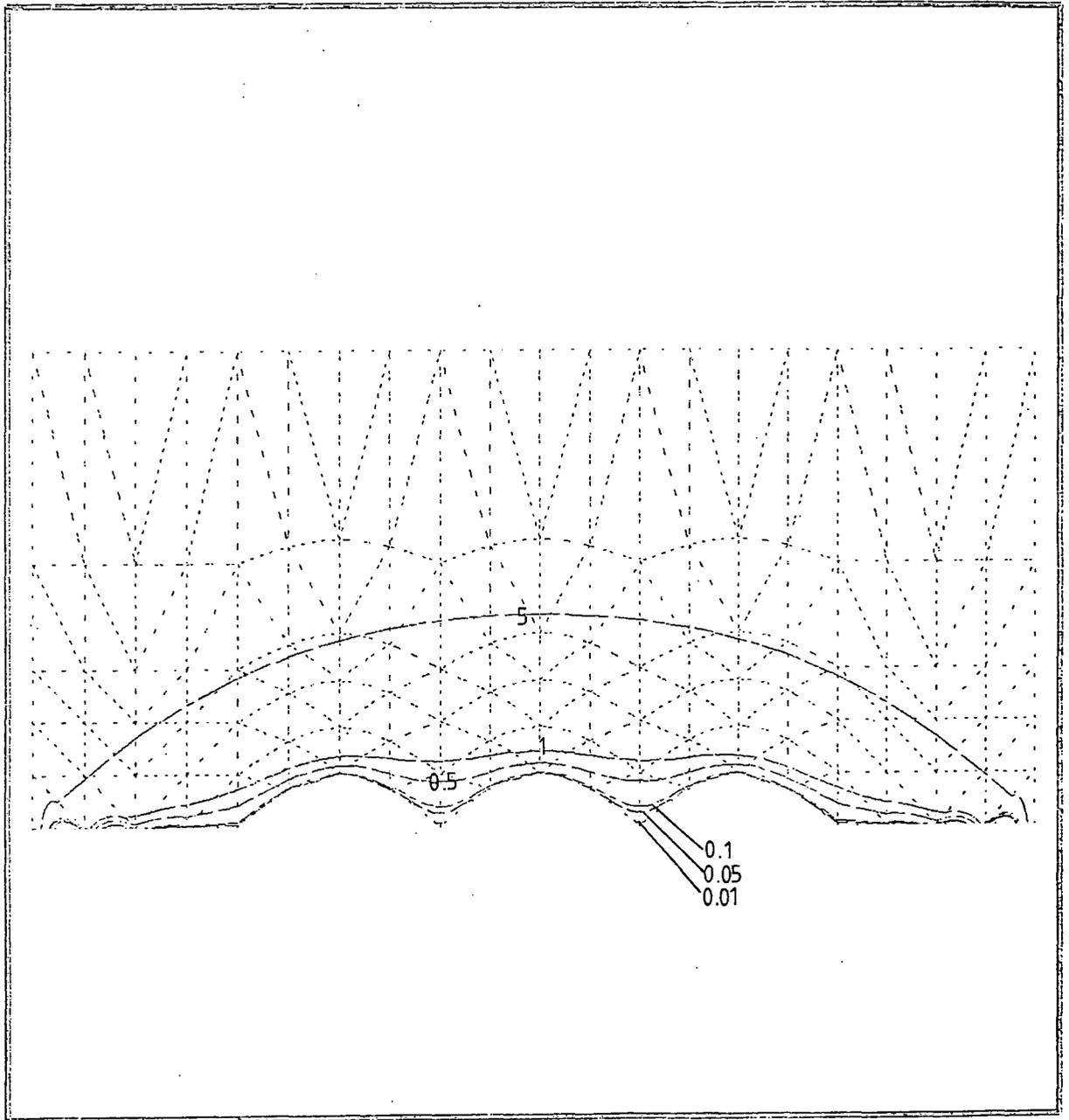


Figure 5.68

Velocity contours in al_u/a for

$v_d = 10 al_u/a$ and a bed of $S = 0.02 al_u/bar.a$,

wide-hummocked case.

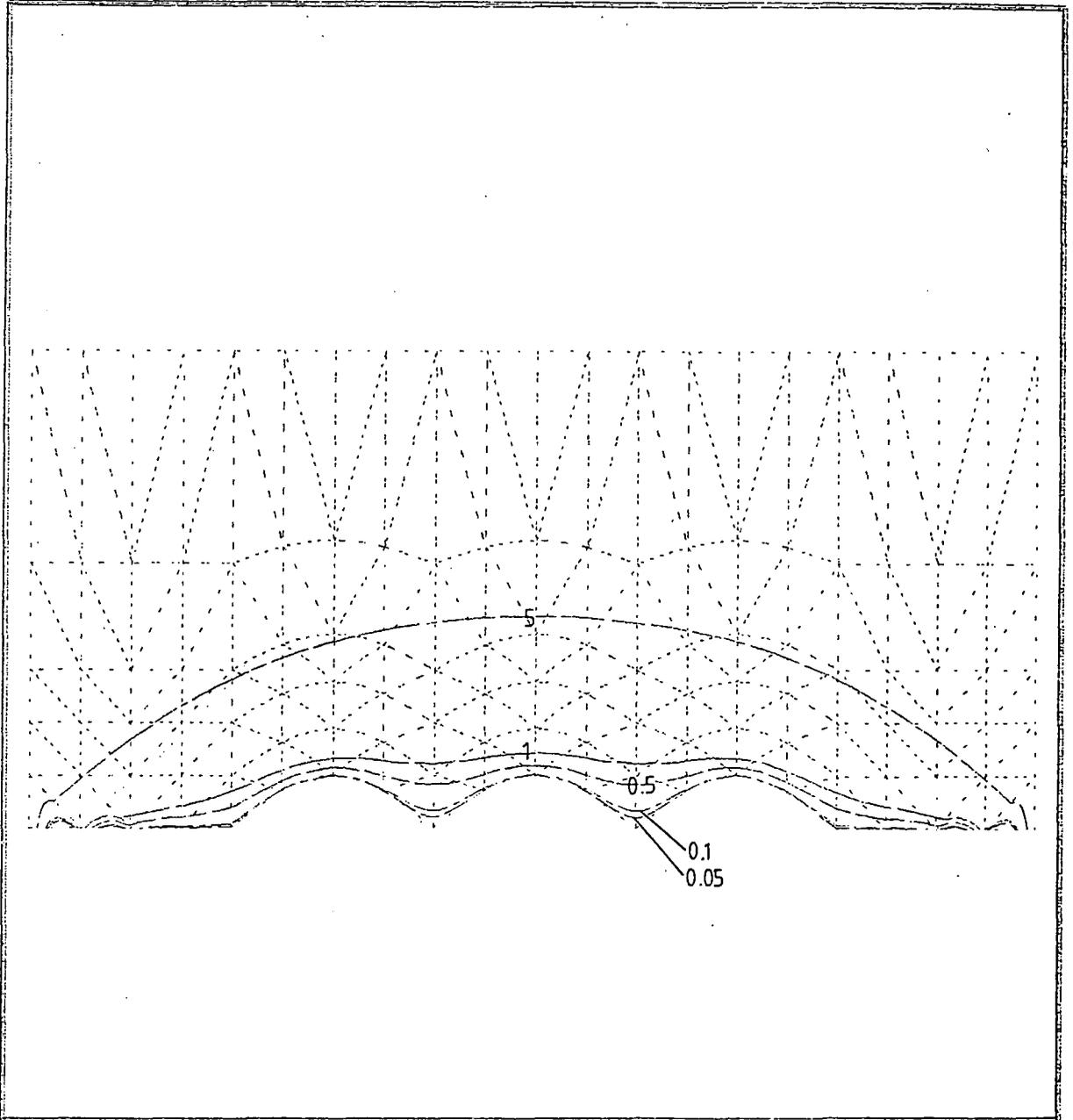


Figure 5.69

Velocity contours in alu/a for

$v_d = 10 \text{ } alu/a$ and a bed of $S = 0.1 \text{ } alu/\text{bar}.a$,
wide-hummocked case.

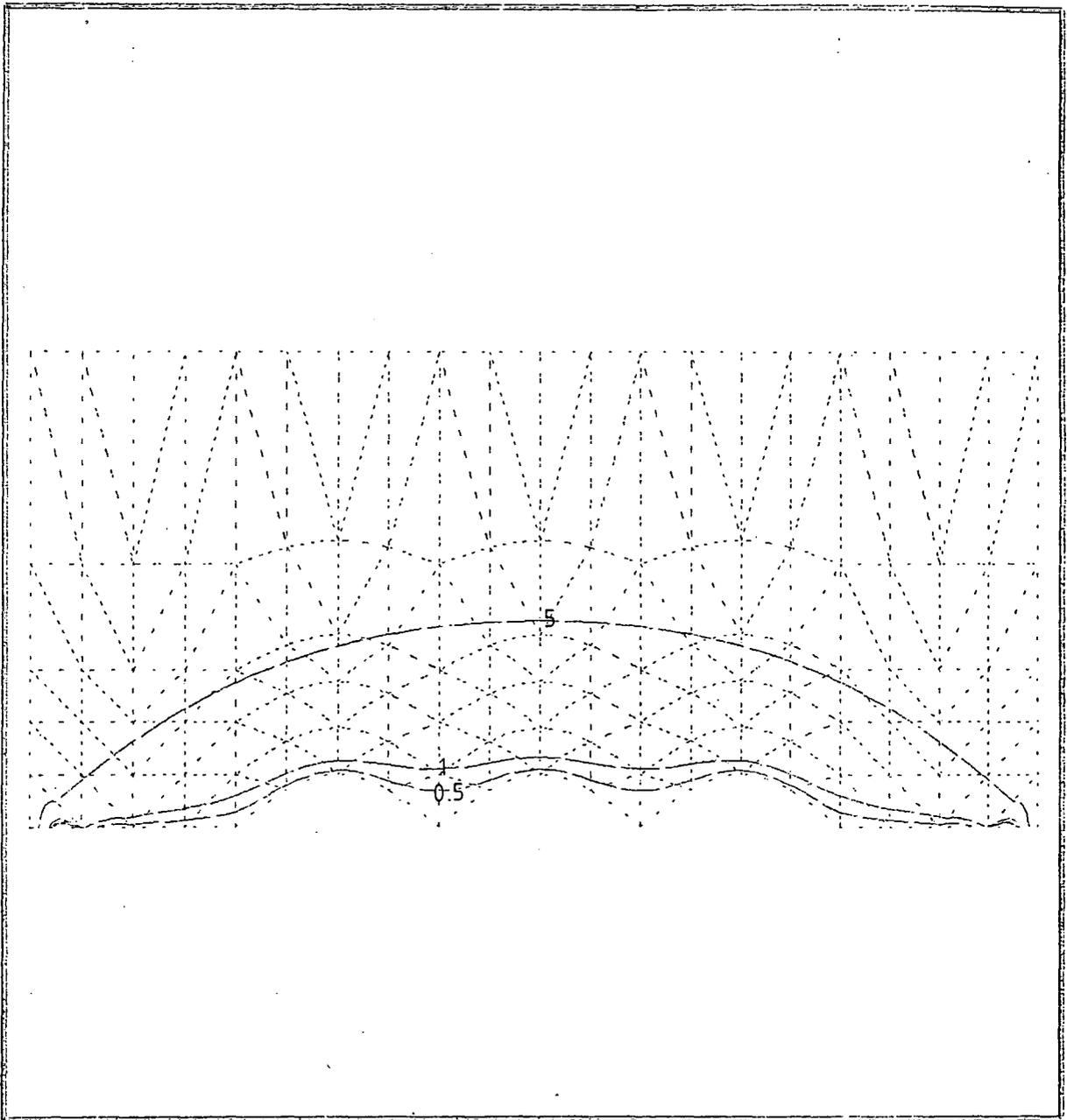


Figure 5.70

Velocity contours in al_u/a for

$v_d = 10 al_u/a$ and a bed of $S = 0.2 al_u/\text{bar}.a$,
wide-hummocked case.

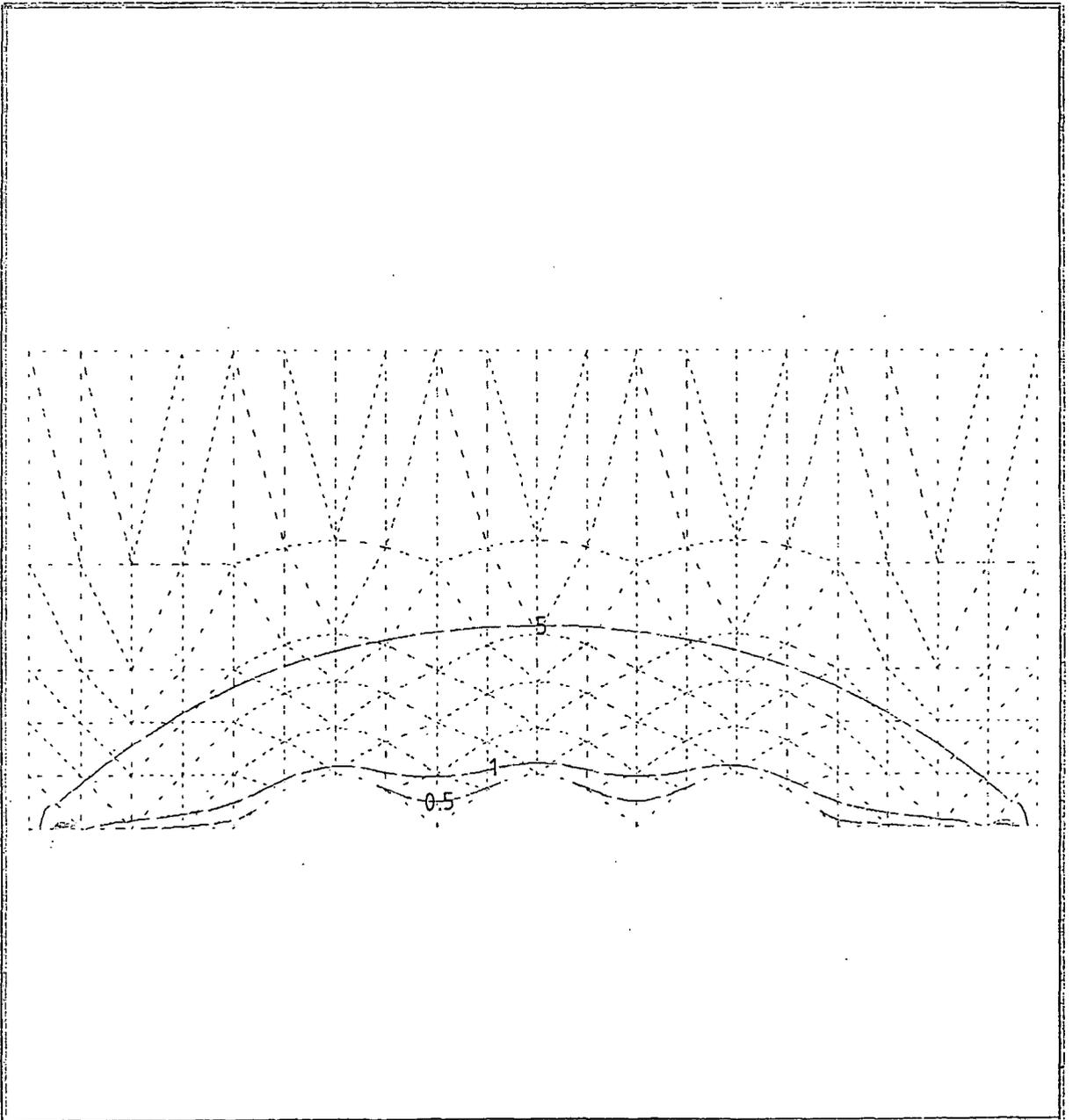


Figure 5.71

Velocity contours in al_u/a for

$v_d = 10 al_u/a$, and a bed of $S = 1 al_u/\text{bar}.a$,
wide-hummocked case.

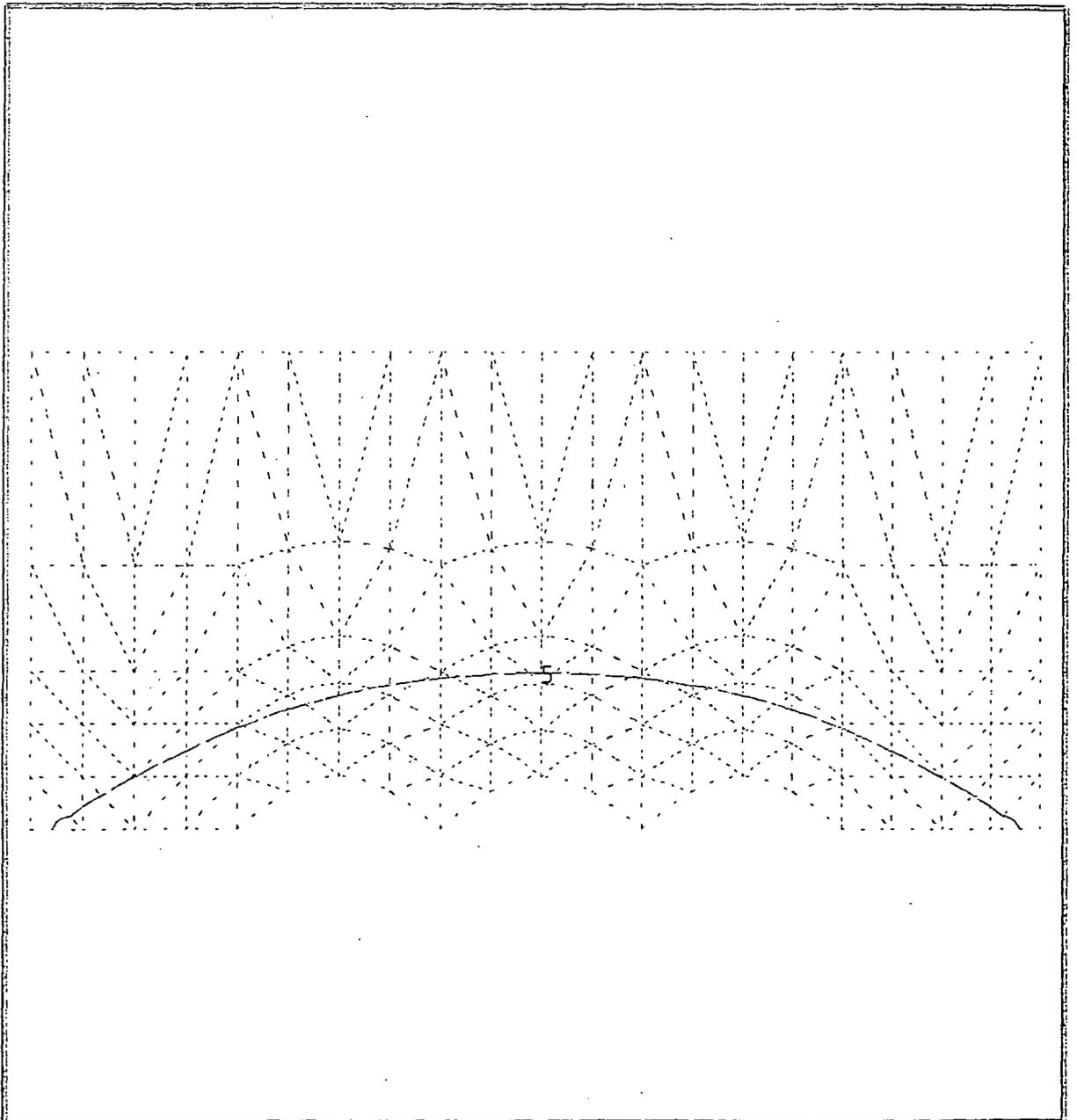


Figure 5.72

Velocity contours in al_u/a for

$v_d = 10 al_u/a$ and a bed of $S = 2 al_u/\bar{b}ar.a$,
wide-hummocked case.

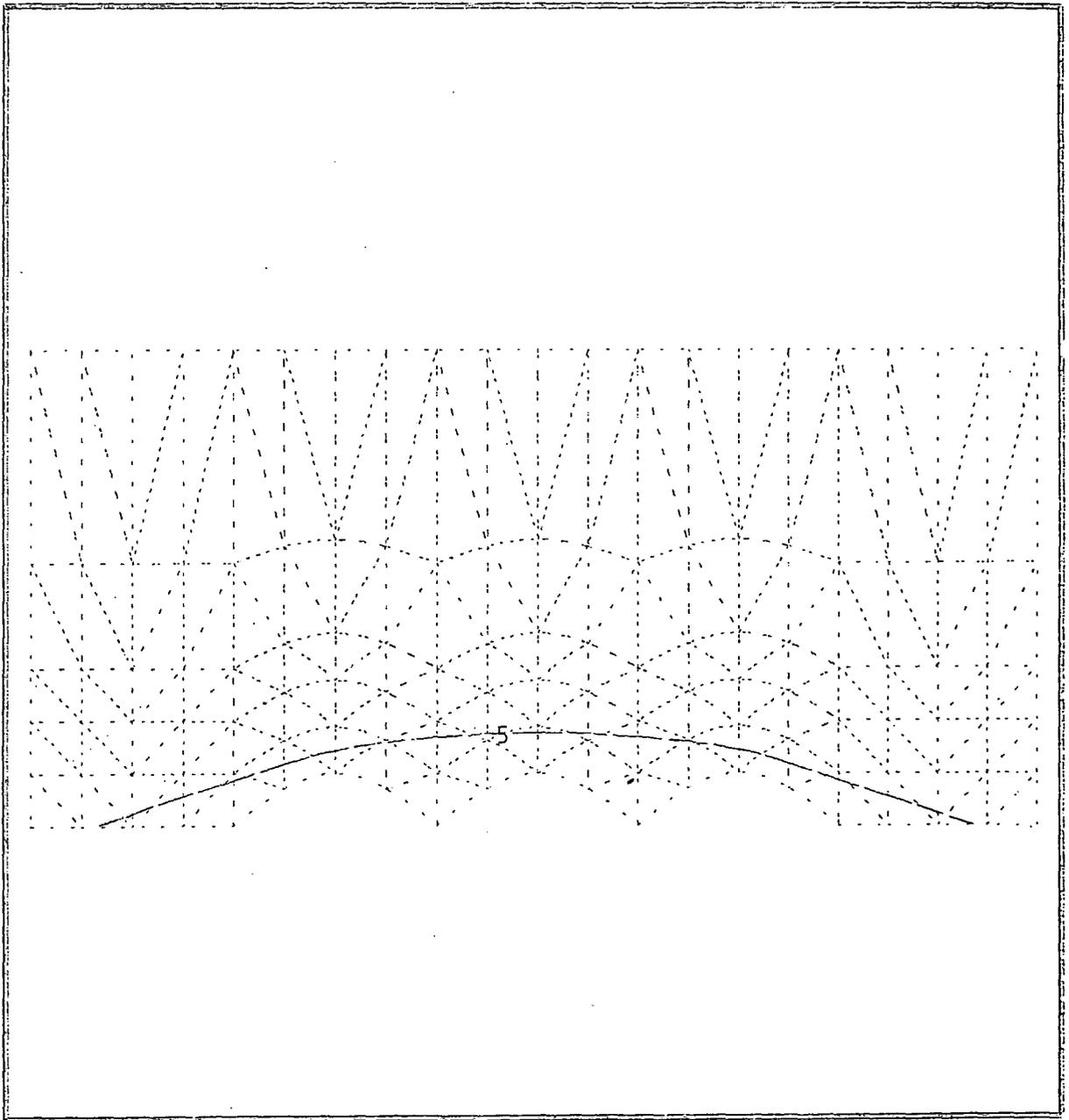


Figure 5.73

Velocity contours in al_u/a for

$v_d = 50 al_u/a$ and a bed of $S = 0.01 al_u/bar.a$,
wide-hummocked case.

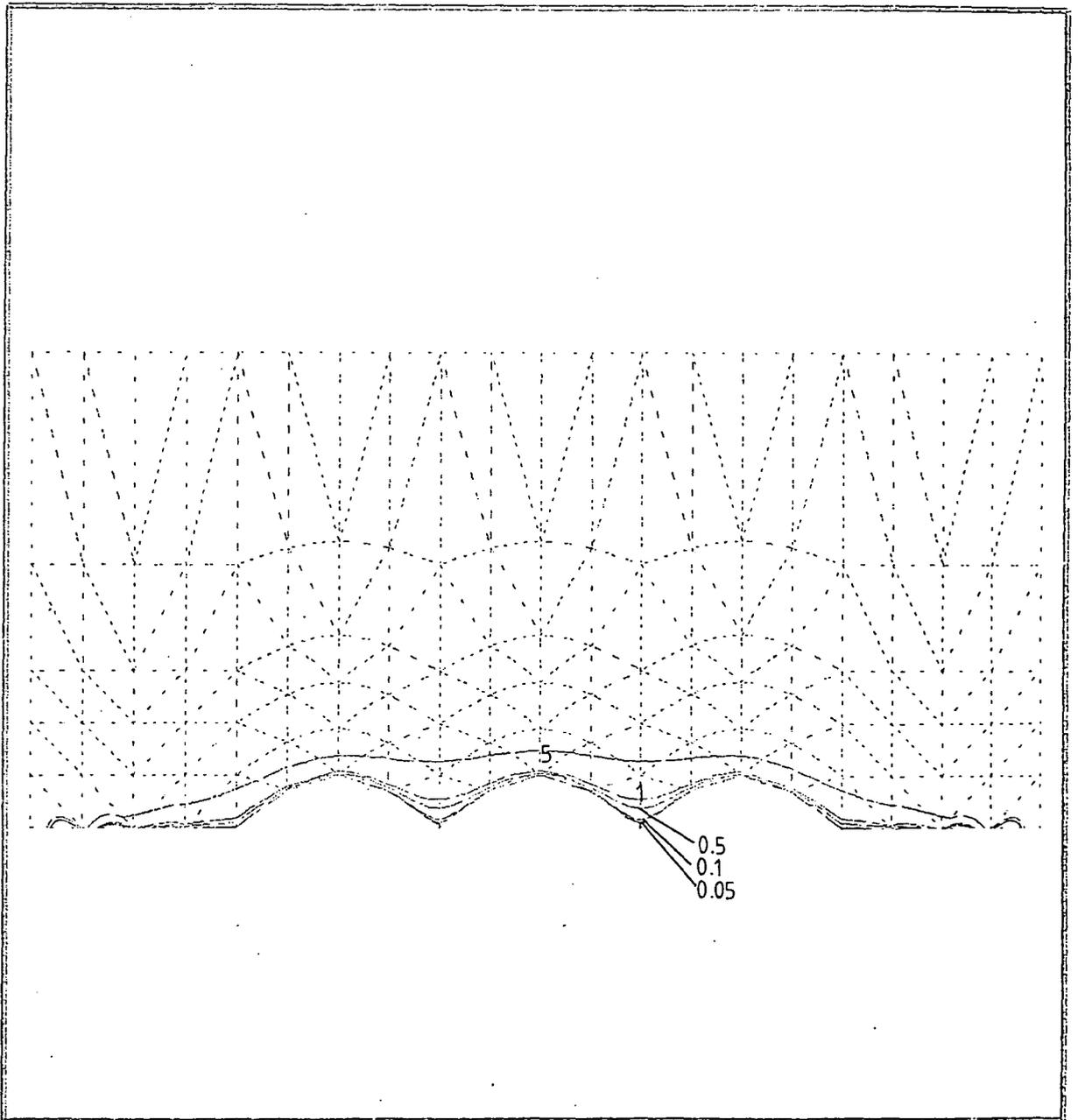


Figure 5.74

Velocity contours in alu/a for

$v_d = 100 \text{ } alu/a$ and a bed of $S = 0.01 \text{ } alu/\text{bar}.a$,
wide-hummocked case.

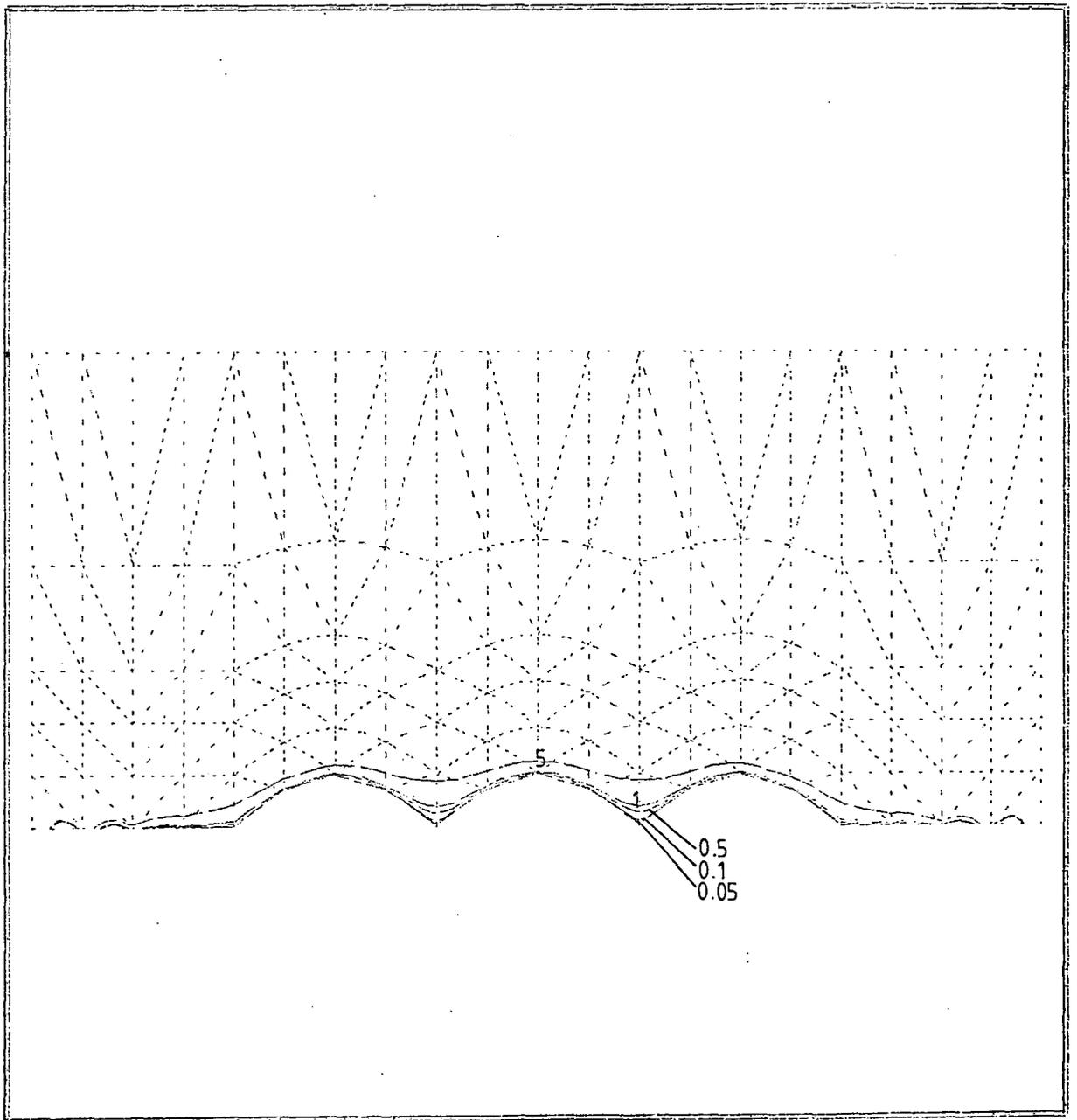


Figure 5.75

Velocity contours in al_u/a for

$v_d = 200 al_u/a$ and a bed of $S = 0.01 al_u/\text{bar}.a$,
wide-hummocked case.

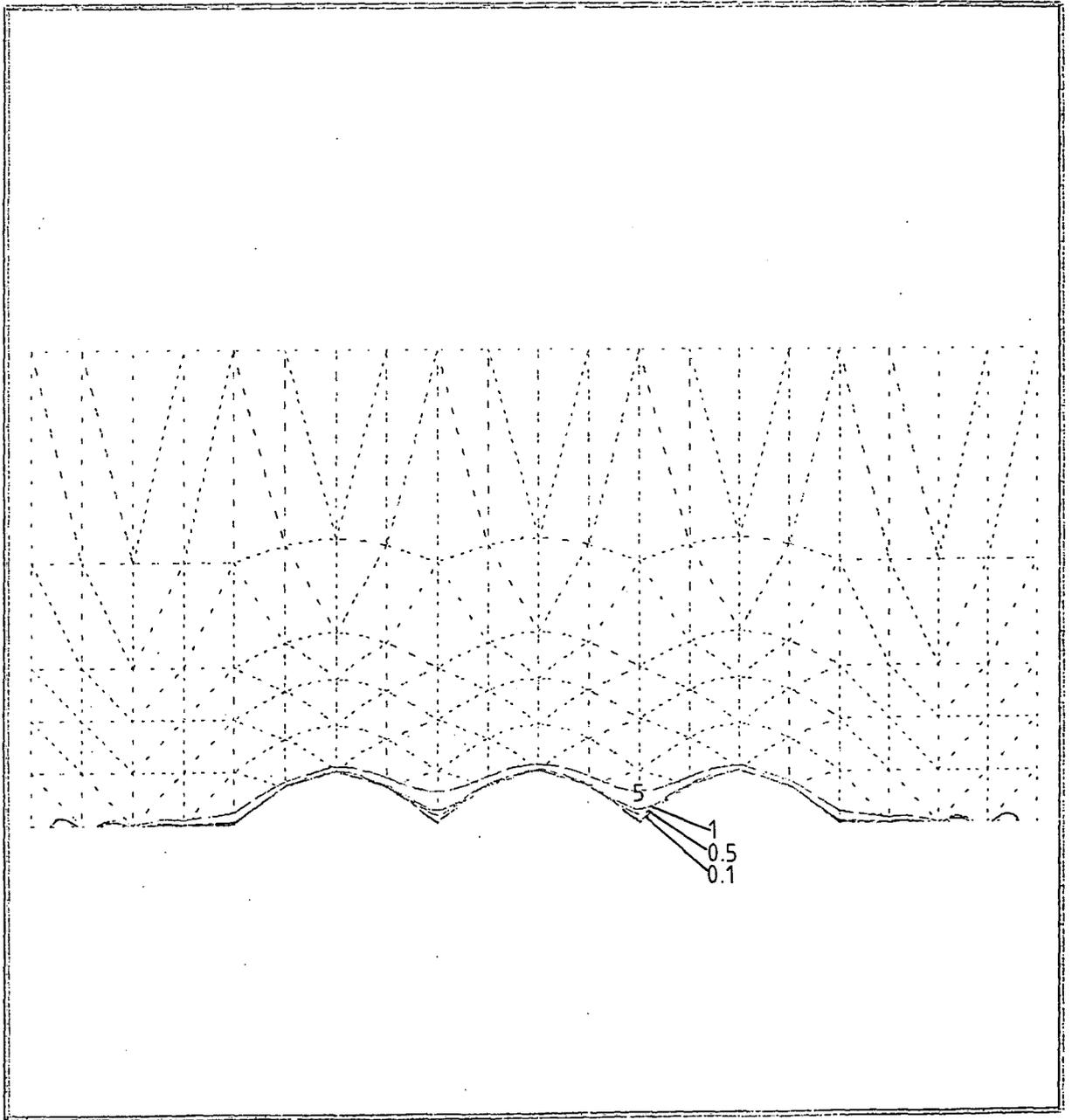


Figure 5.76

Velocity contours in al_u/a for

$v_d = 10 al_u/a$ and a bed of $S = 0.02 al_u/bar.a$,
narrow-hummocked case.

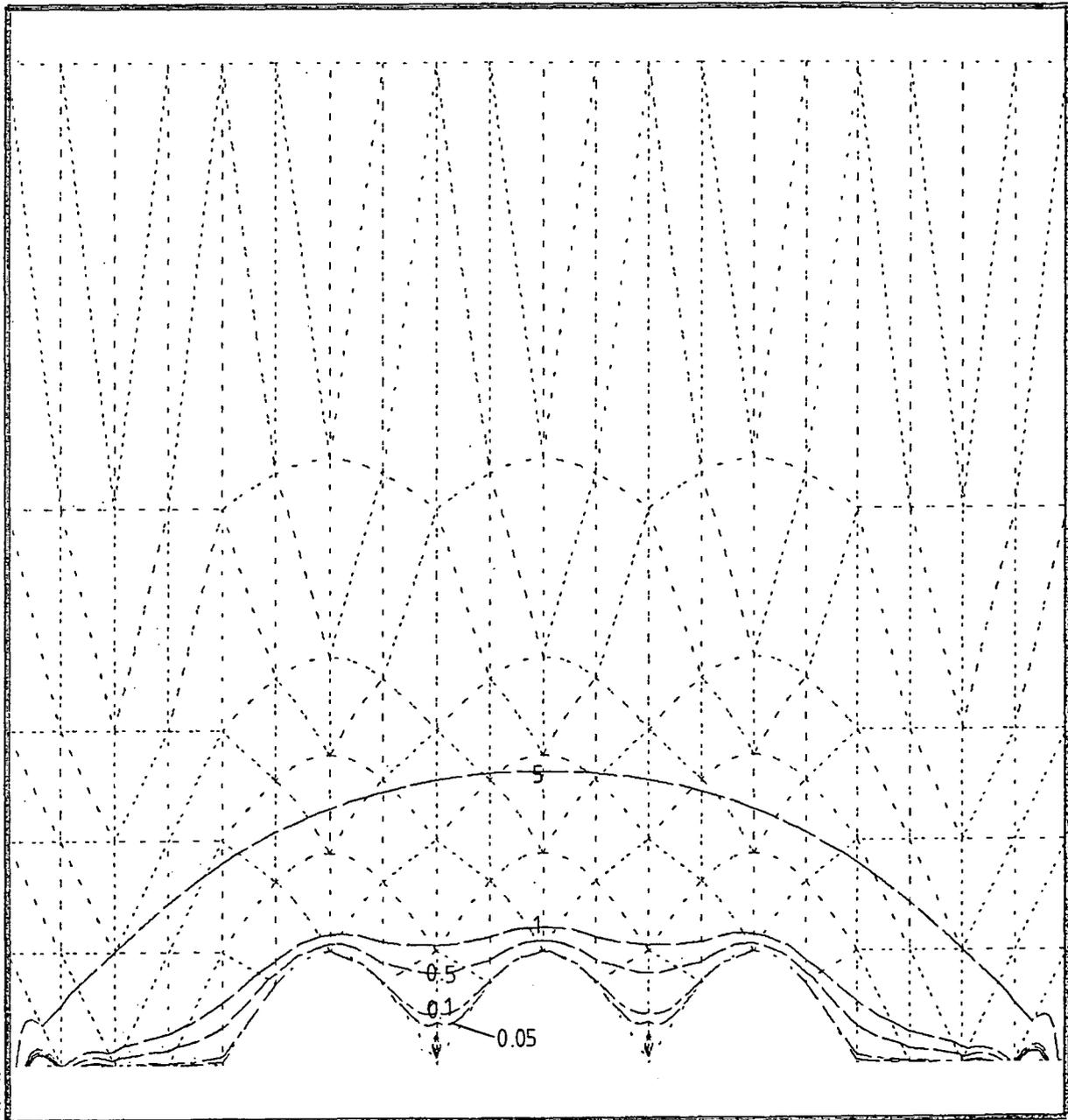


Figure 5.77

Velocity contours in al_u/a for

$v_d = 10 al_u/a$, bed of $S = 0.002 al_u/\text{bar}.a$,

narrow-hummocked case,

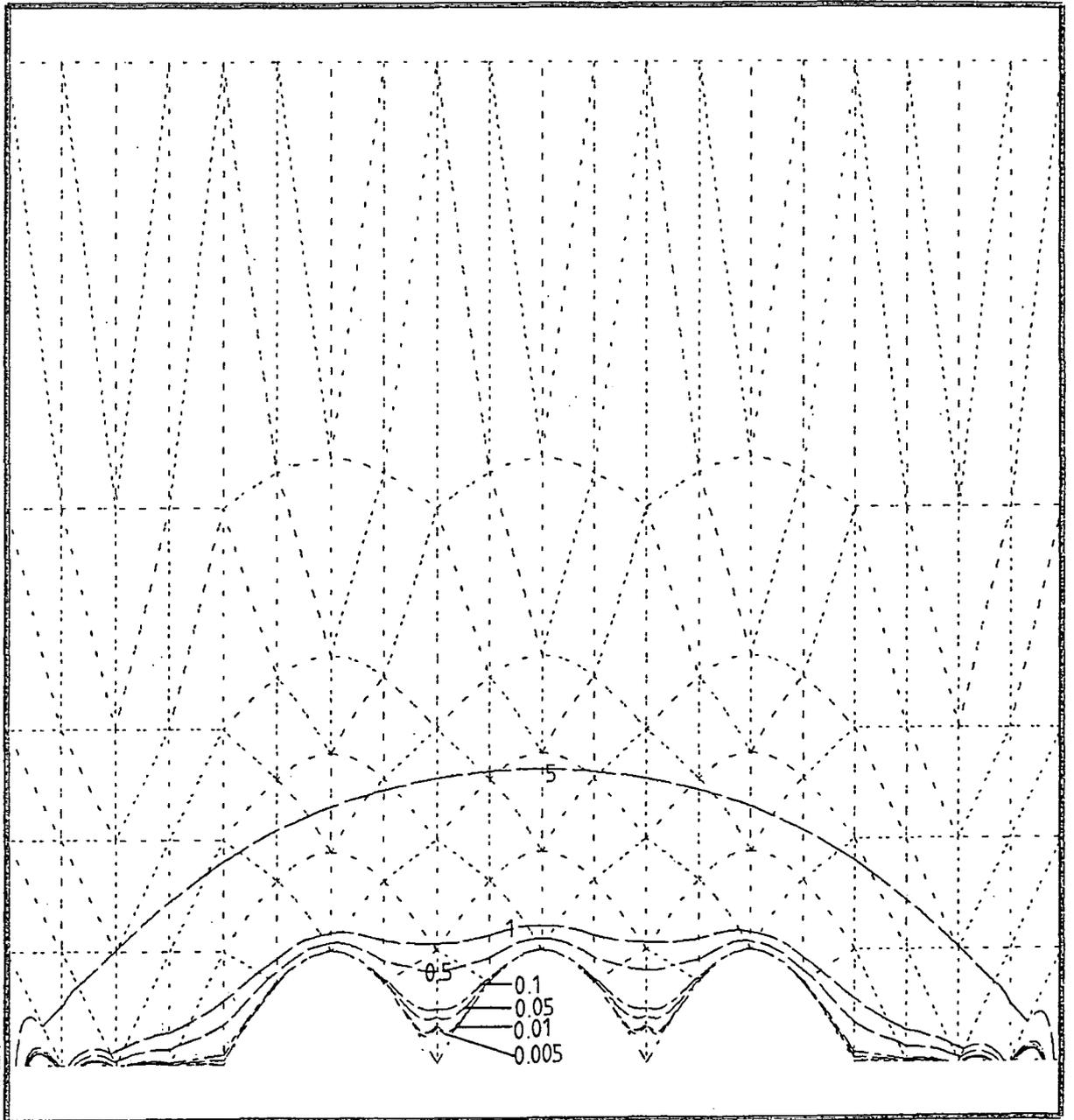


Figure 5.78

Velocity contours in alu/a for

$v_d = 400 \text{ } alu/a$, bed of $S = 0.02 \text{ } alu/\text{bar}.a$,
narrow-hummocked case.

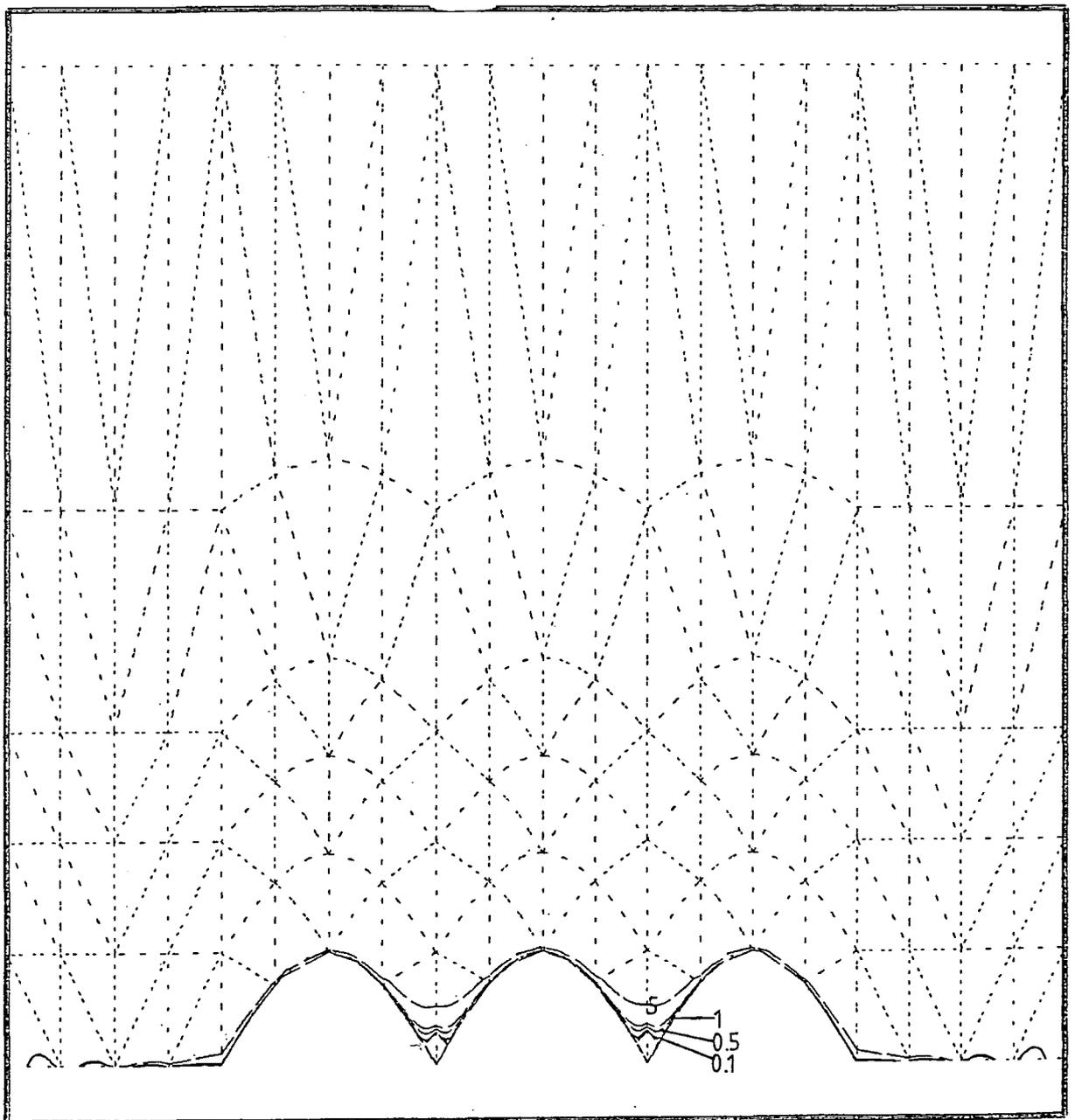
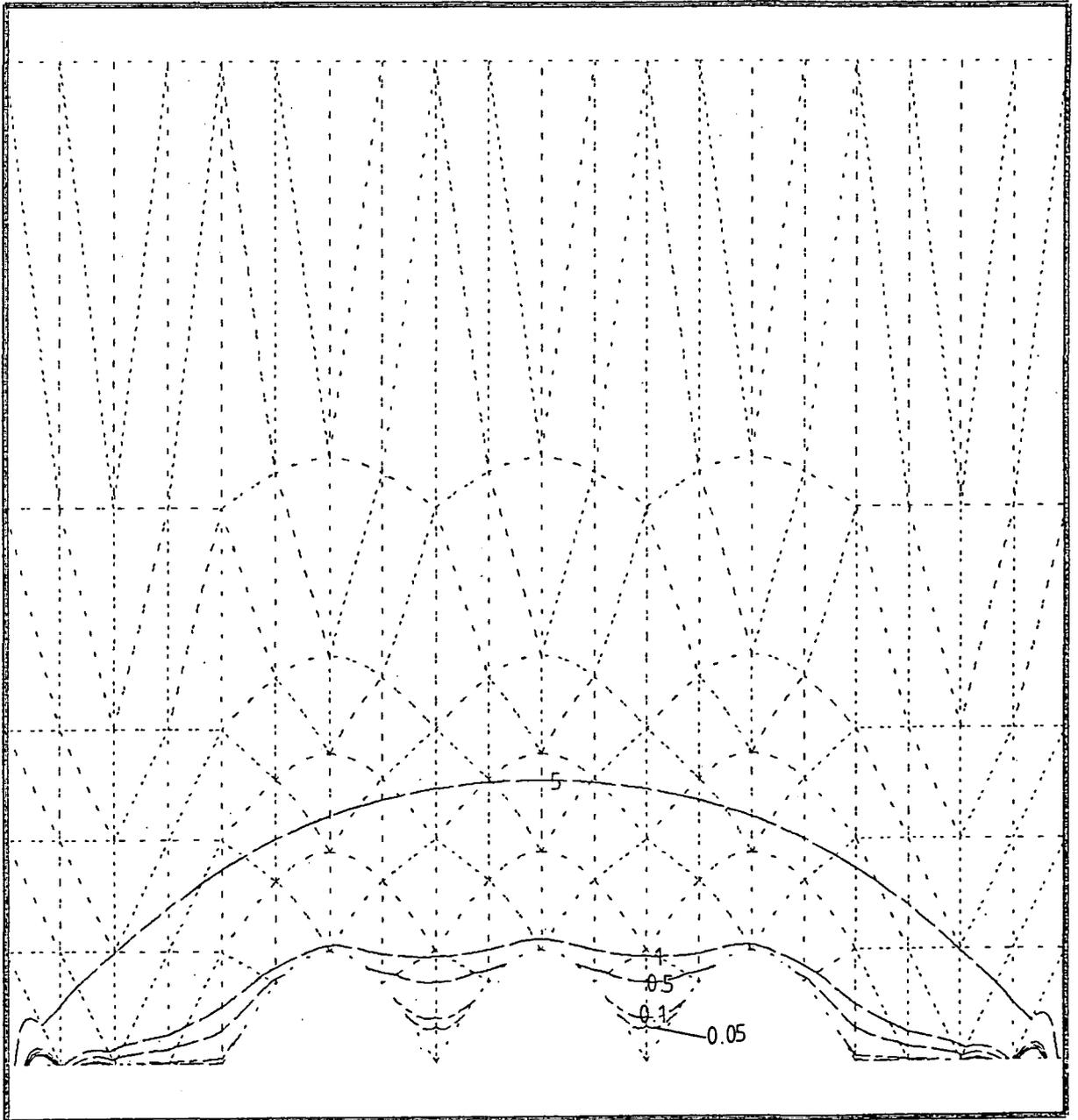


Figure 5.79

Velocity contours in al_u/a for

$v_d = 10 al_u/a$, bed of maximum

$S = 0.02 al_u/\text{bar}.a$, smoothed-crested case.



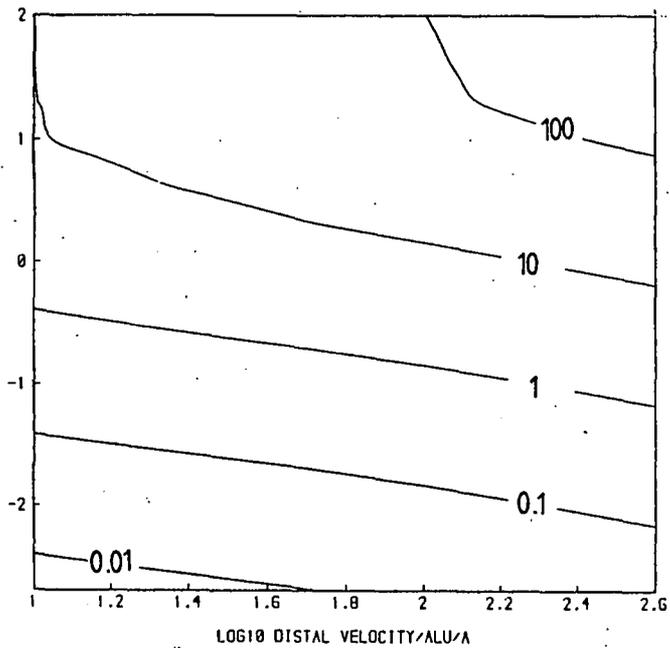


Figure 5.80 Crestal velocities in alu/a for the wide-hummocked case.

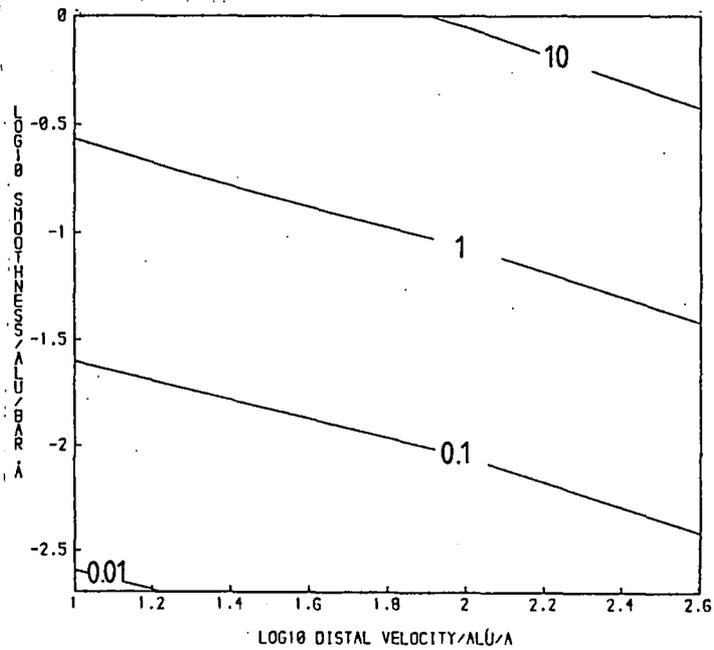


Figure 5.81 Crestal velocities in alu/a for the narrow-hummocked case.

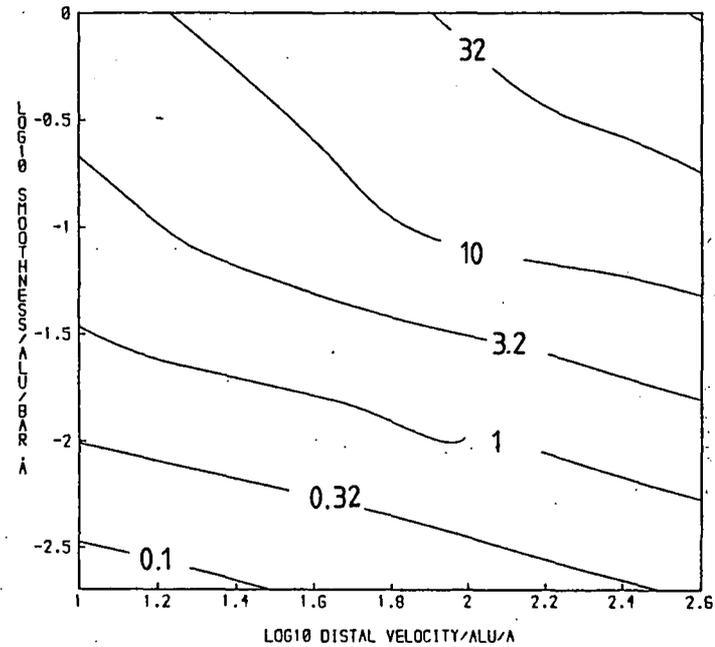


Figure 5.82 Crestal velocities in alu/a for the smooth-crested case.

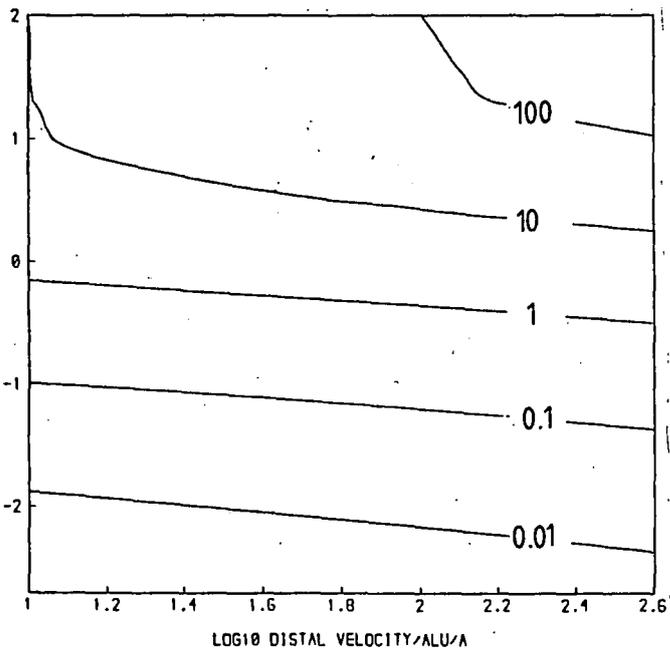


Figure 5.83 Trough velocities in alu/a wide-hummocked case.

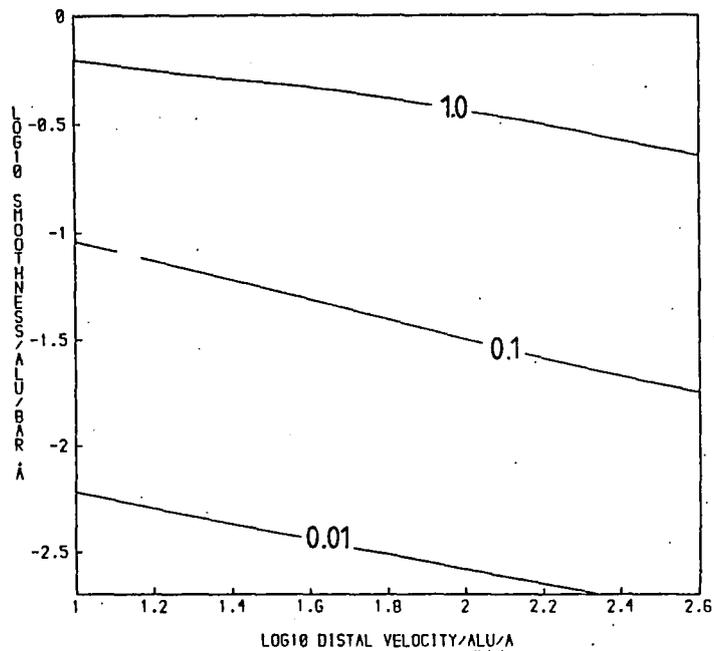


Figure 5.84 Trough velocities in alu/a for the narrow-hummocked case.

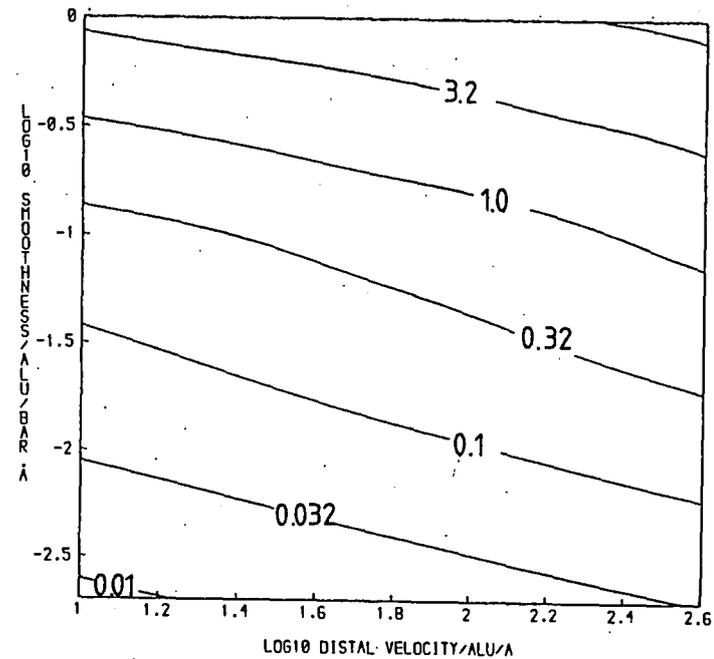


Figure 5.85 Trough velocities in alu/a for the smooth-crested case.

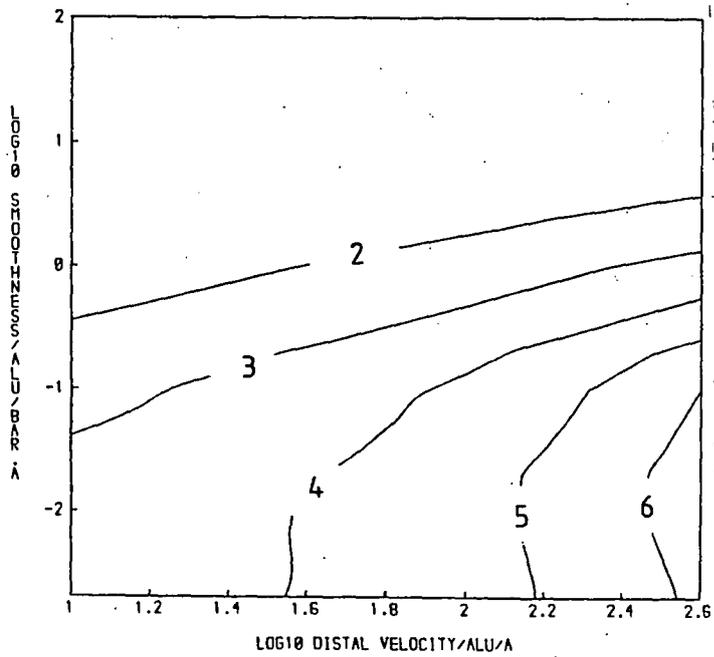


Figure 5.86 Crest/trough velocity ratios for the wide-hummocked case.

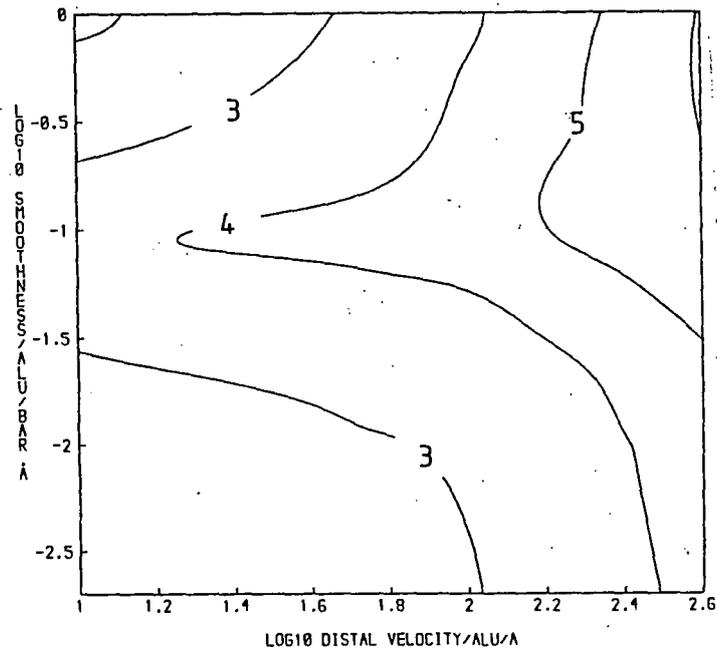


Figure 5.87 Crest/trough velocity ratios for the narrow-hummocked case.

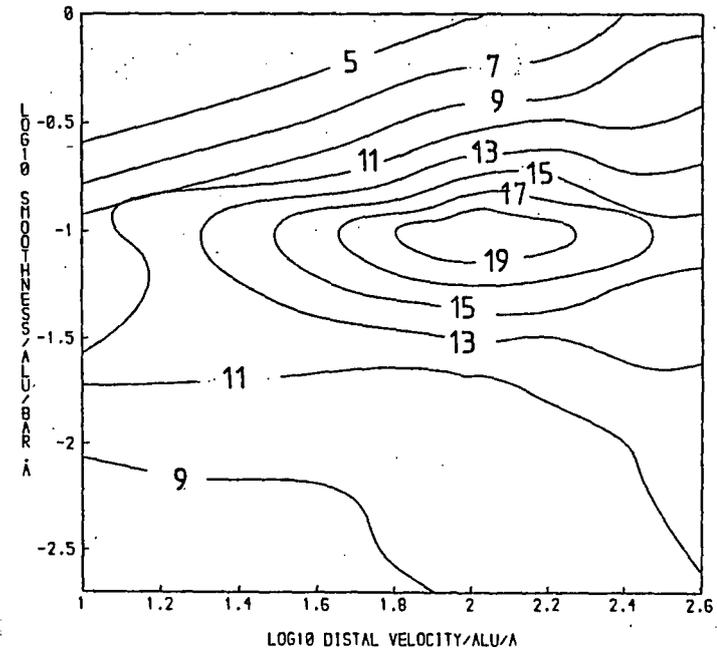


Figure 5.88 Crest/trough velocity ratios for the smooth-crested case.

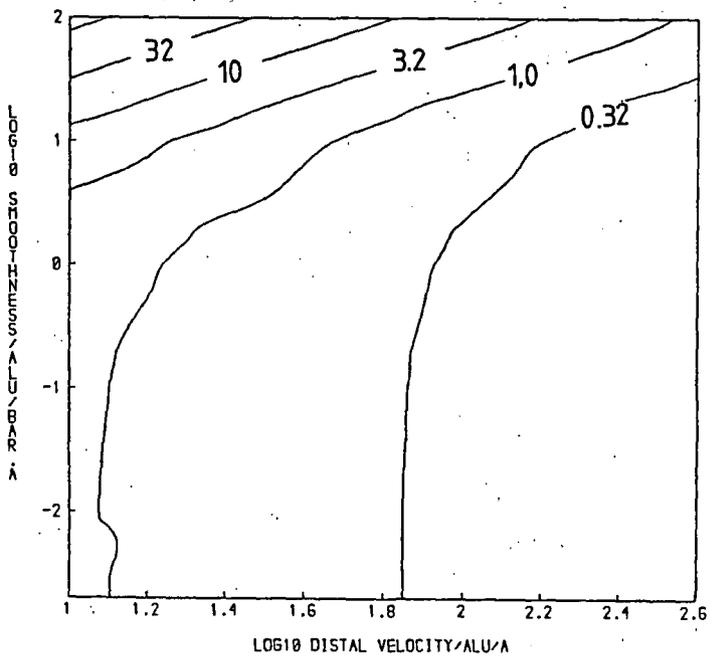


Figure 5.89 Crestal viscosity in bar.a for the wide-hummocked case.

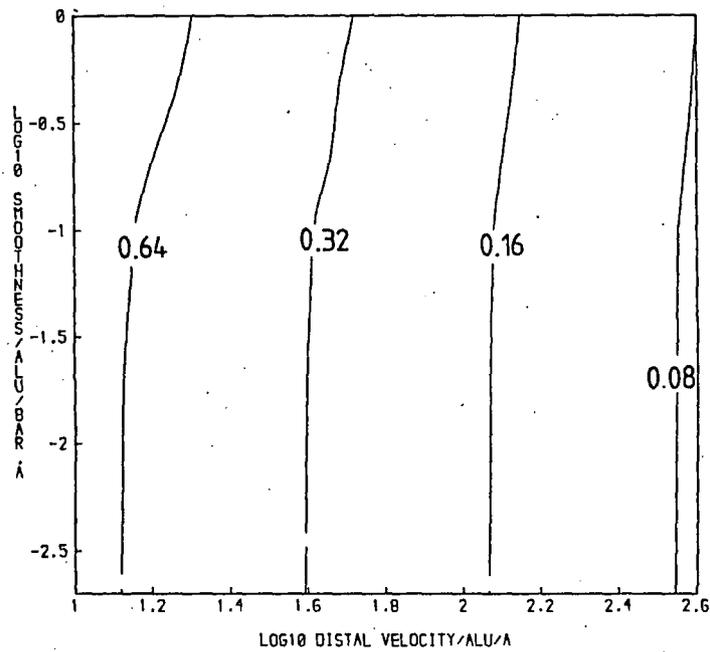


Figure 5.90 Crestal viscosities in bar.a for the narrow-hummocked case.

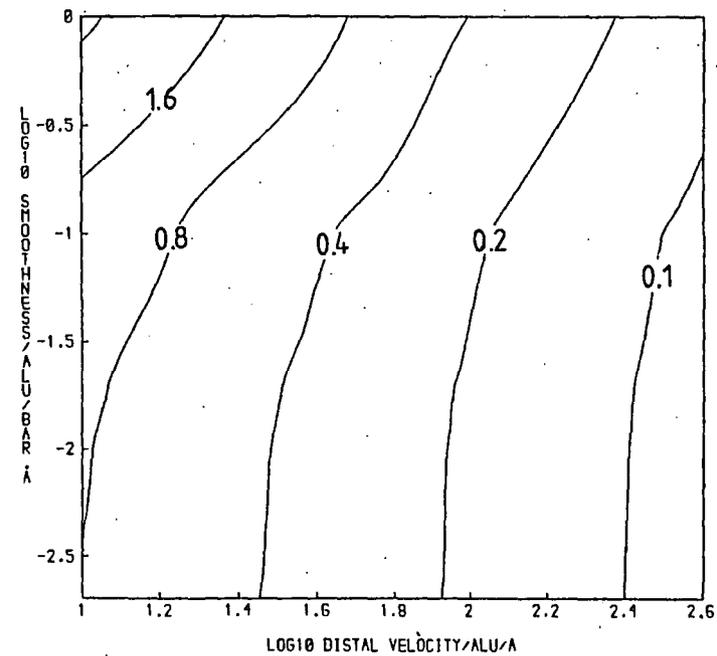


Figure 5.91 Crestal viscosities in bar.a for the smooth-crested case.

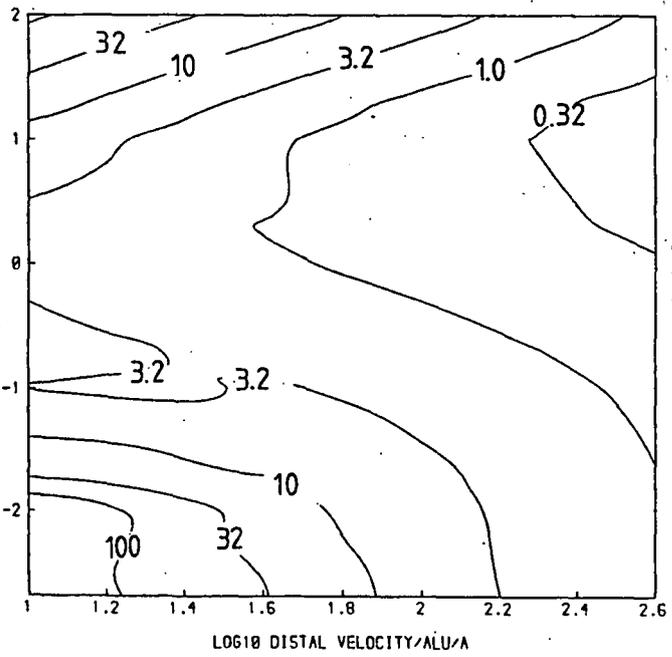


Figure 5.92 Trough viscosities in bar.a for the wide-hummocked case.

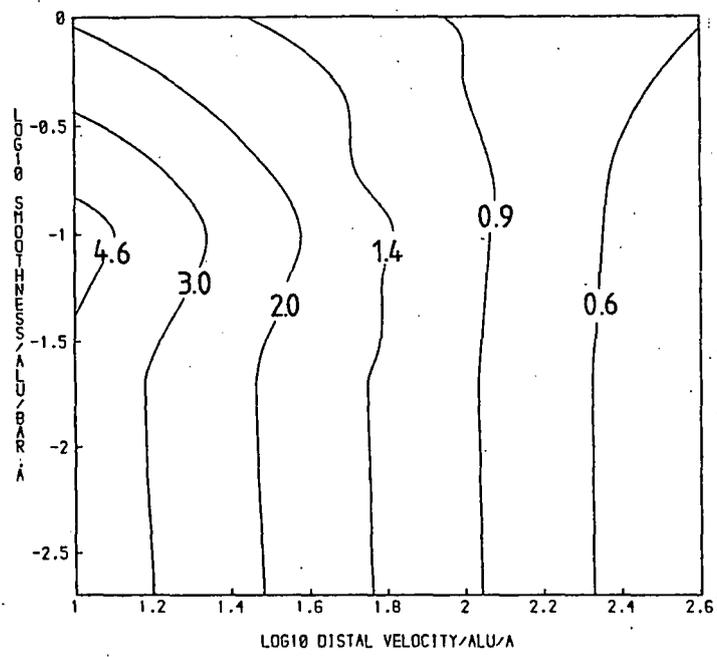


Figure 5.93 Trough viscosities in bar.a for the narrow-hummocked case.

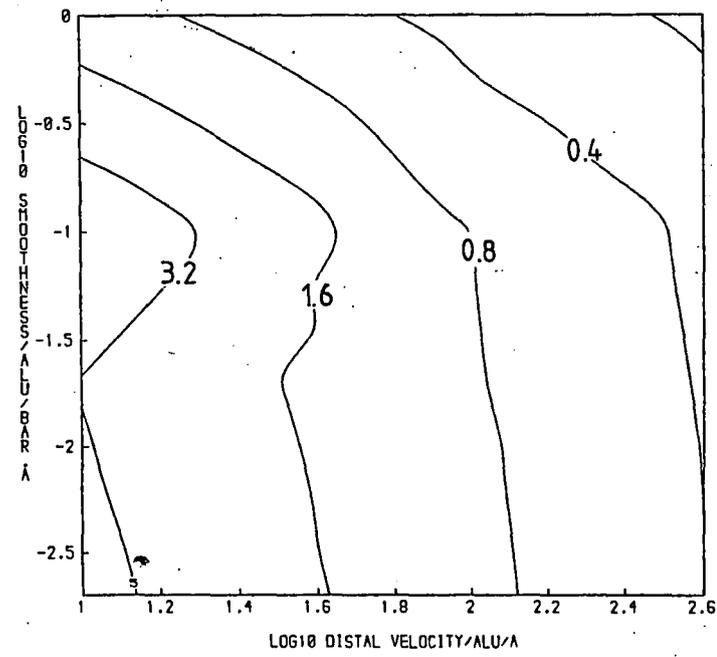


Figure 5.94 Trough viscosities in bar.a for the smooth-crested case.

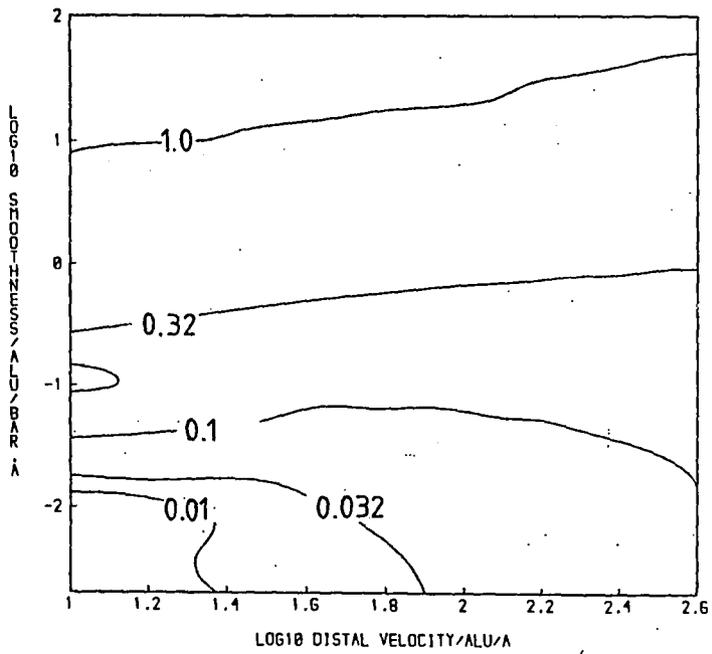


Figure 5.95 Crest/trough viscosity ratios for the wide-hummocked case.

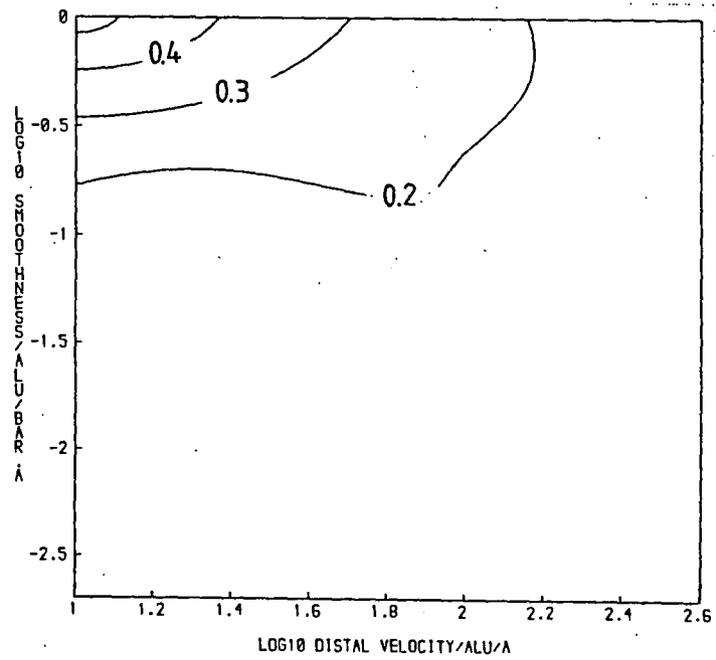


Figure 5.96 Crest/trough viscosity ratios for the narrow-hummocked case.

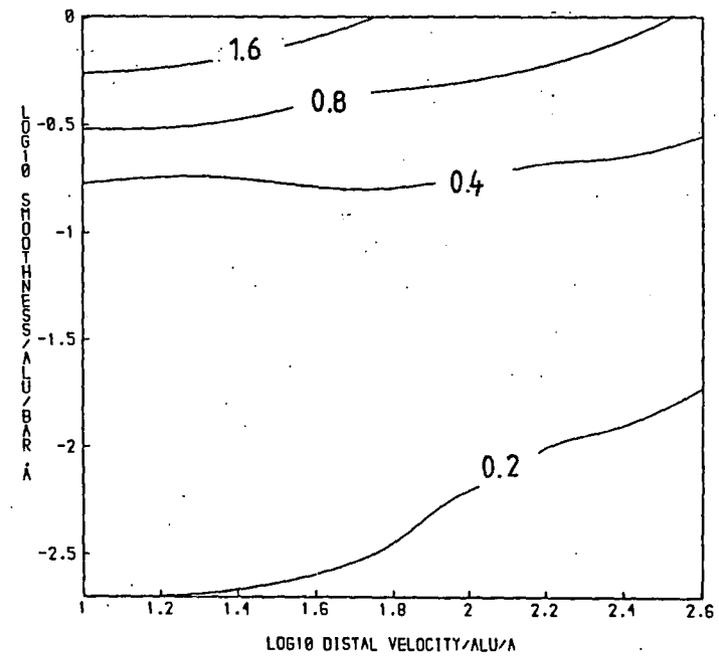


Figure 5.97 Crest/trough viscosity ratios for the smooth-crested case.

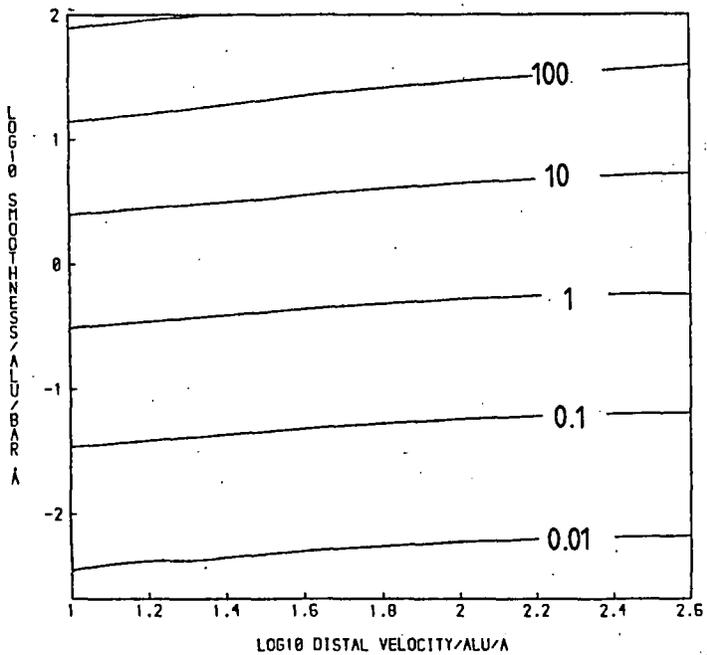


Figure 5.98 Crestal viscosity x velocity in bar.alu for the wide-hummocked case.

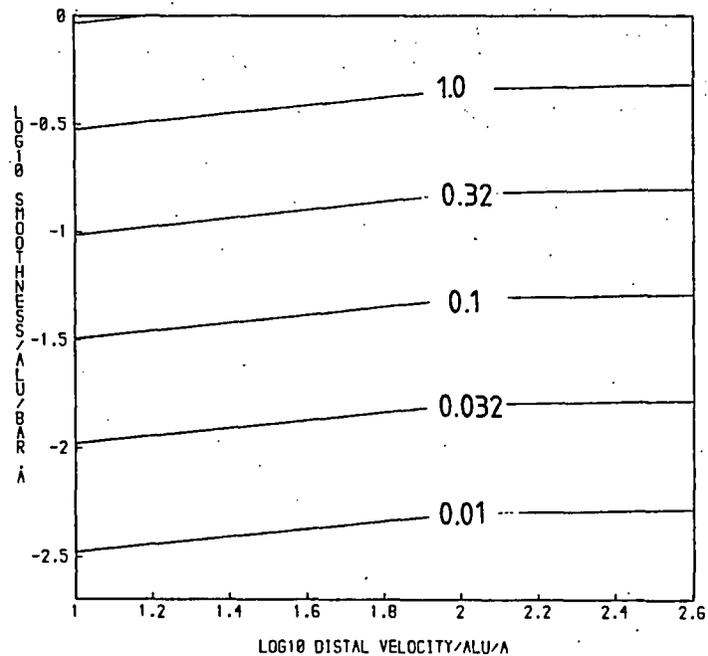


Figure 5.99 Crestal viscosity x velocity in bar.alu for the narrow-hummocked case.

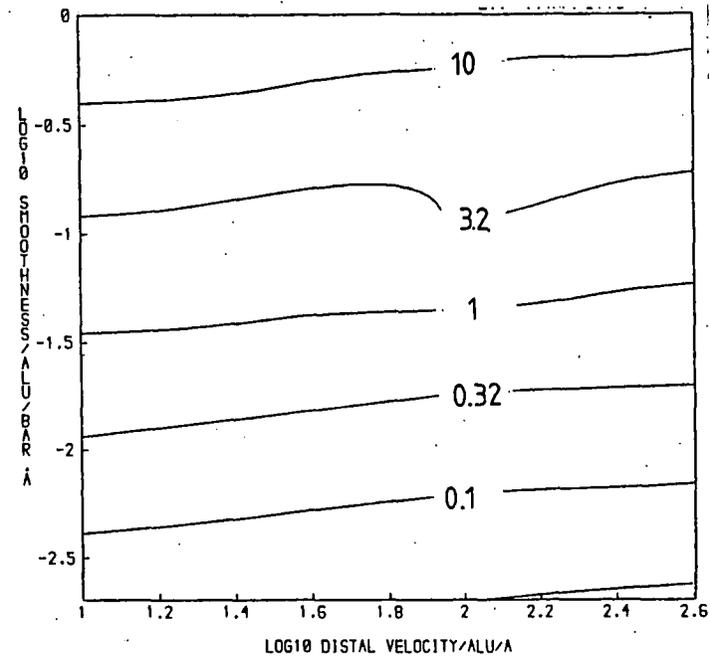


Figure 5.100 Crestal viscosity x velocity in bar.alu for the smooth-crested case.

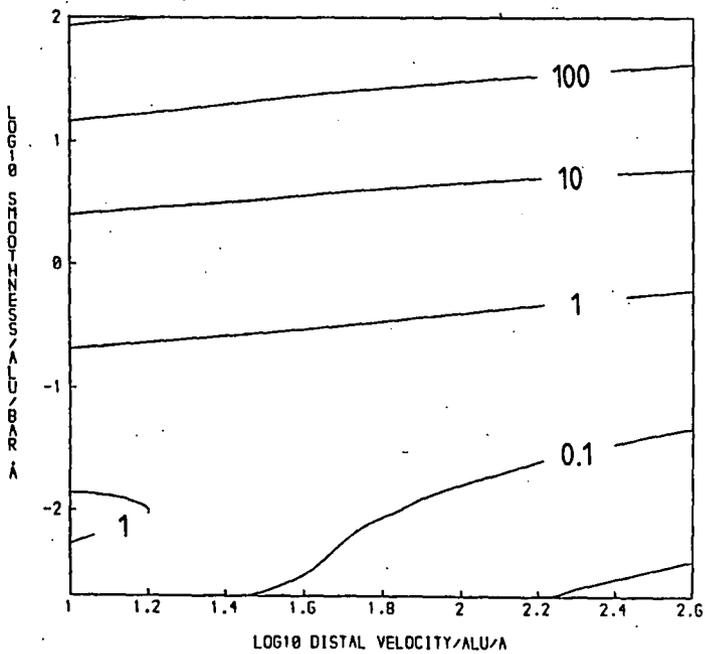


Figure 5.101 Trough viscosity x velocity in bar.alu for the wide-hummocked case.

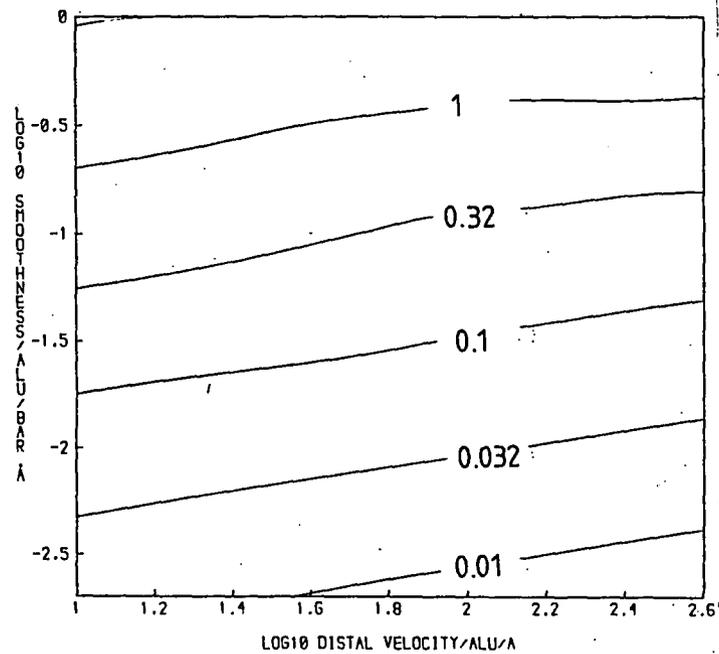


Figure 5.102 Trough viscosity x velocity in bar.alu for the narrow-hummocked case.

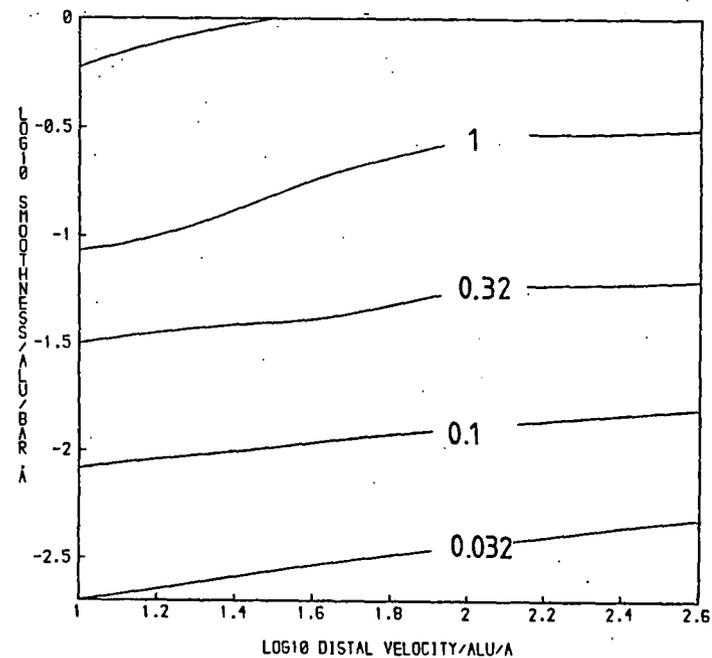


Figure 5.103 Trough viscosity x velocity in bar.alu for the smooth-crested case.

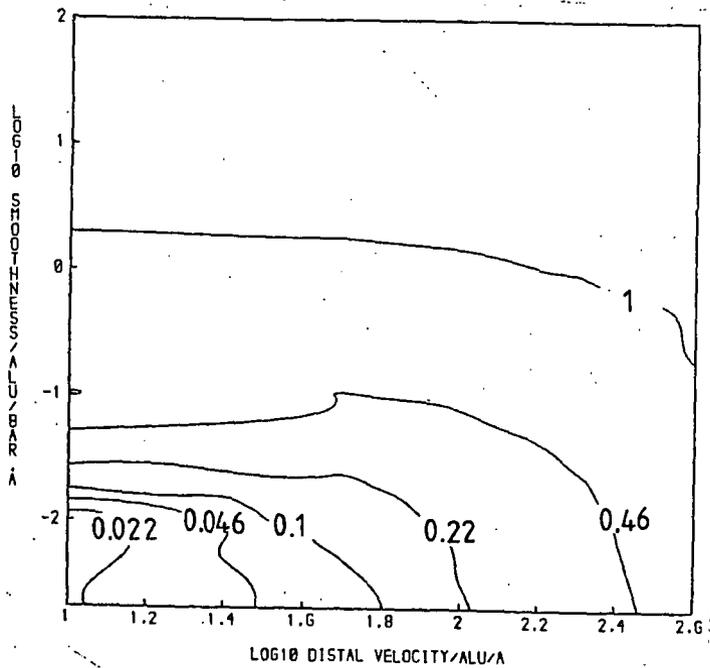


Figure 5.104 Crest/trough viscosity x velocity ratio for the wide-hummocked case.

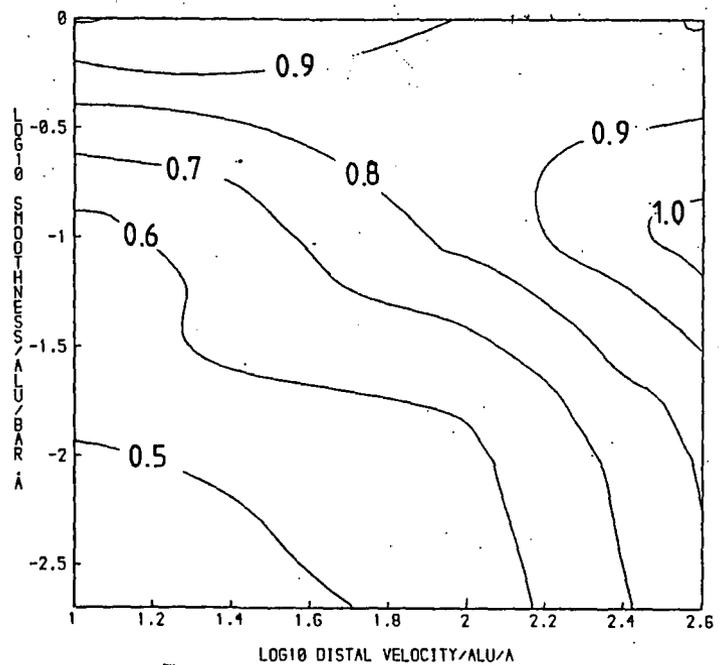


Figure 5.105 Crest/trough viscosity x velocity ratio for the narrow-hummocked case.

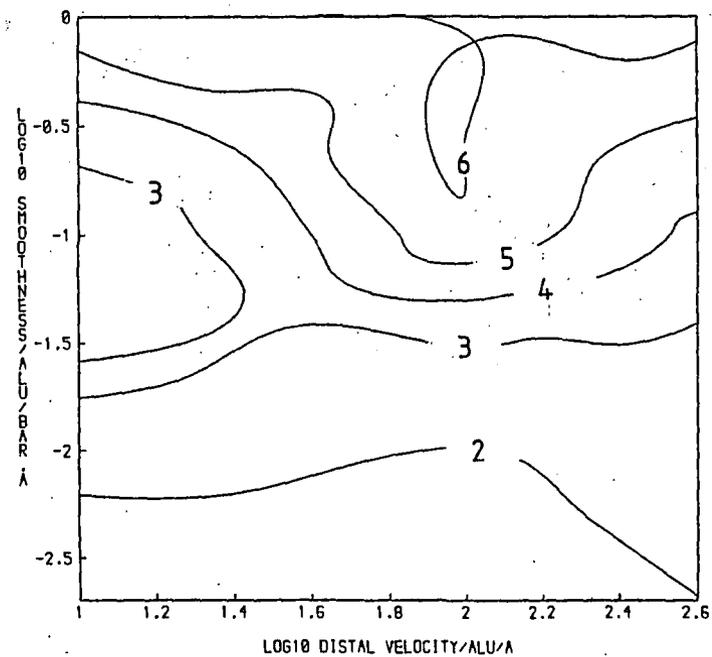


Figure 5.106 Crest/trough viscosity x velocity ratio for the smooth-crested case.

Figure 6.1

Clast at the base of a temperate glacier.

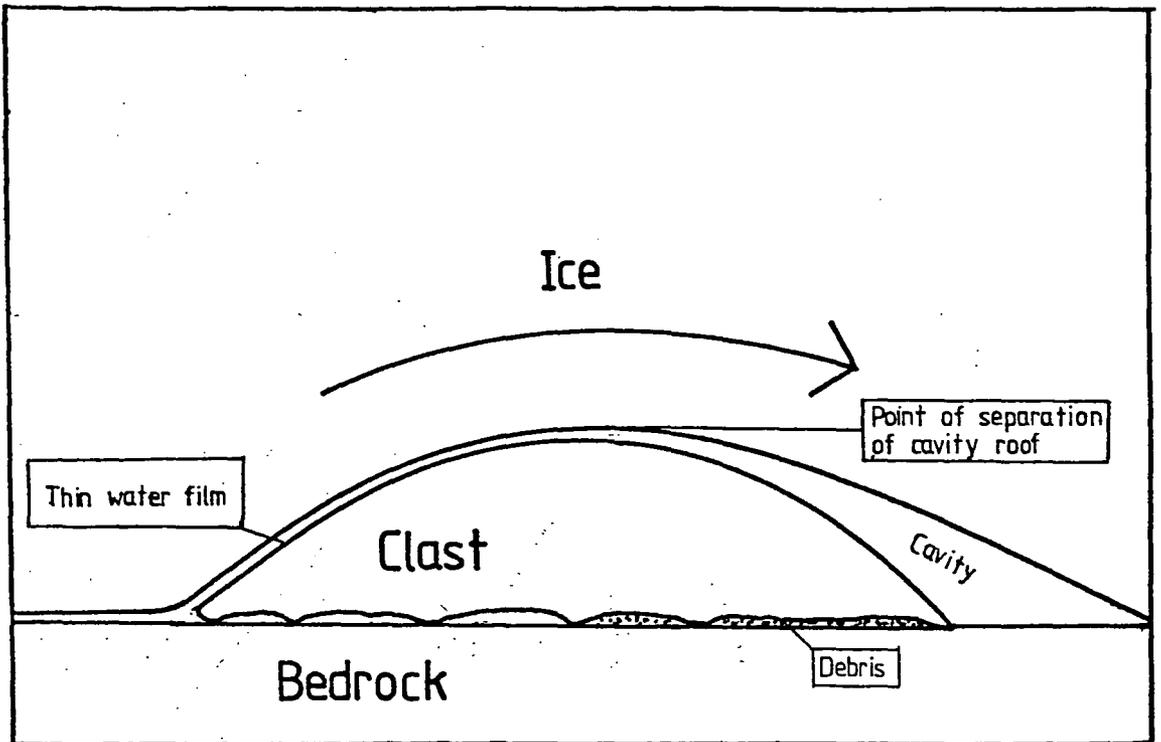
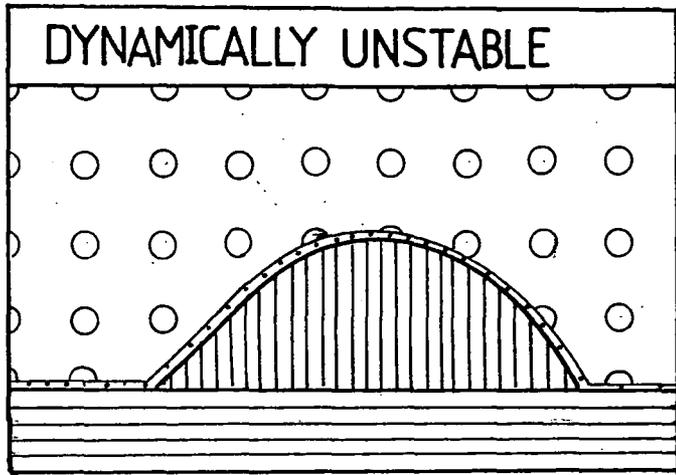
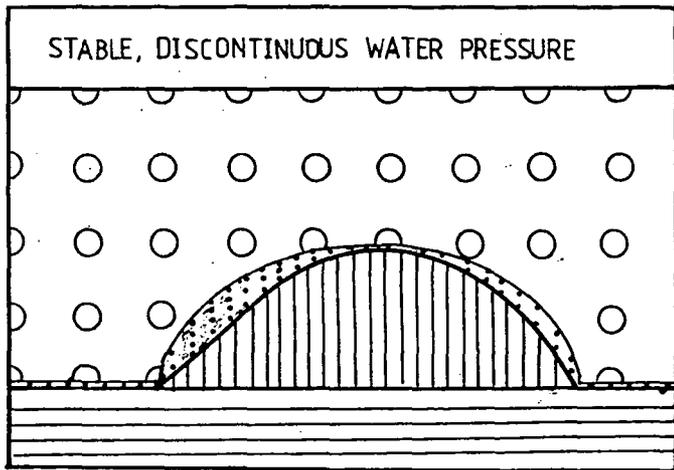


FIGURE 6.2 ICE-CLAST CONTACT GEOMETRIES



←
ICE FLOW DIRECTION



KEY	
○	Ice
	Water-filled cavity of thickness $O(\text{mm})$
	Thin water-film of thickness $O(\mu\text{m})$
	Clast
	Bedrock

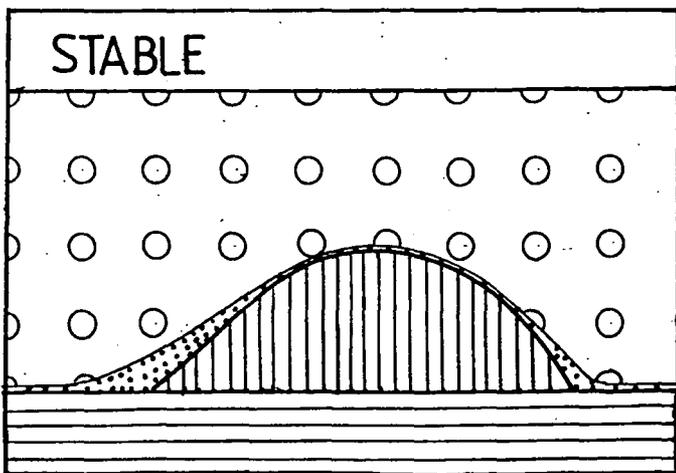


Figure 6.3

The effect of ice-clast contact geometries on the water pressure beneath the clast.

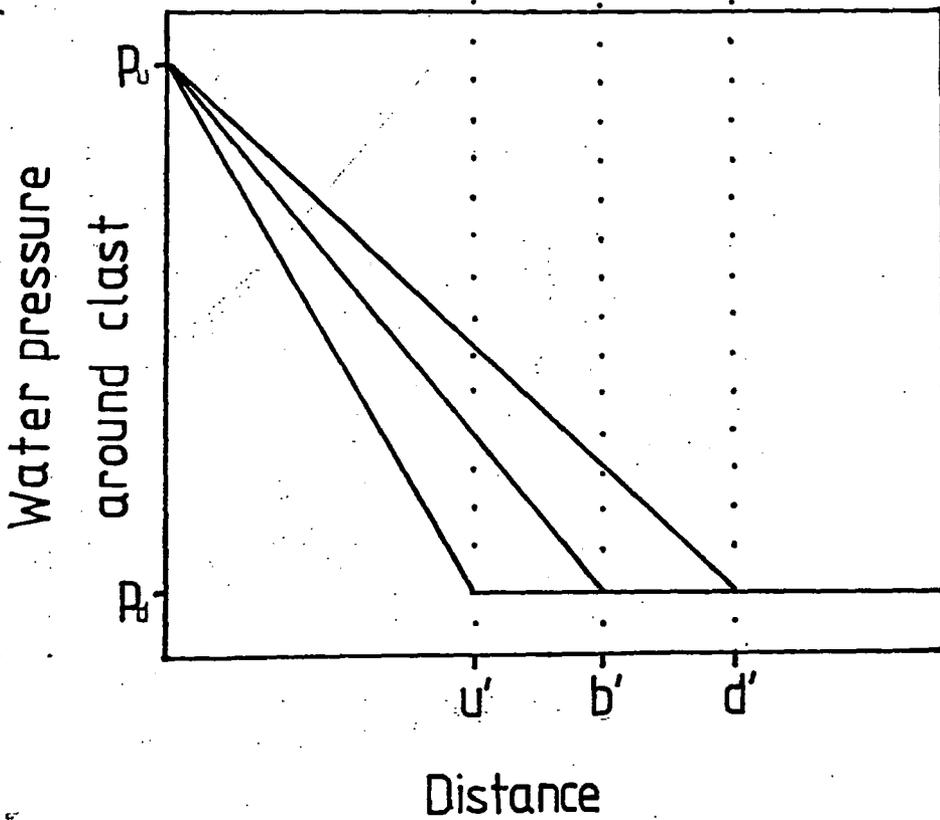
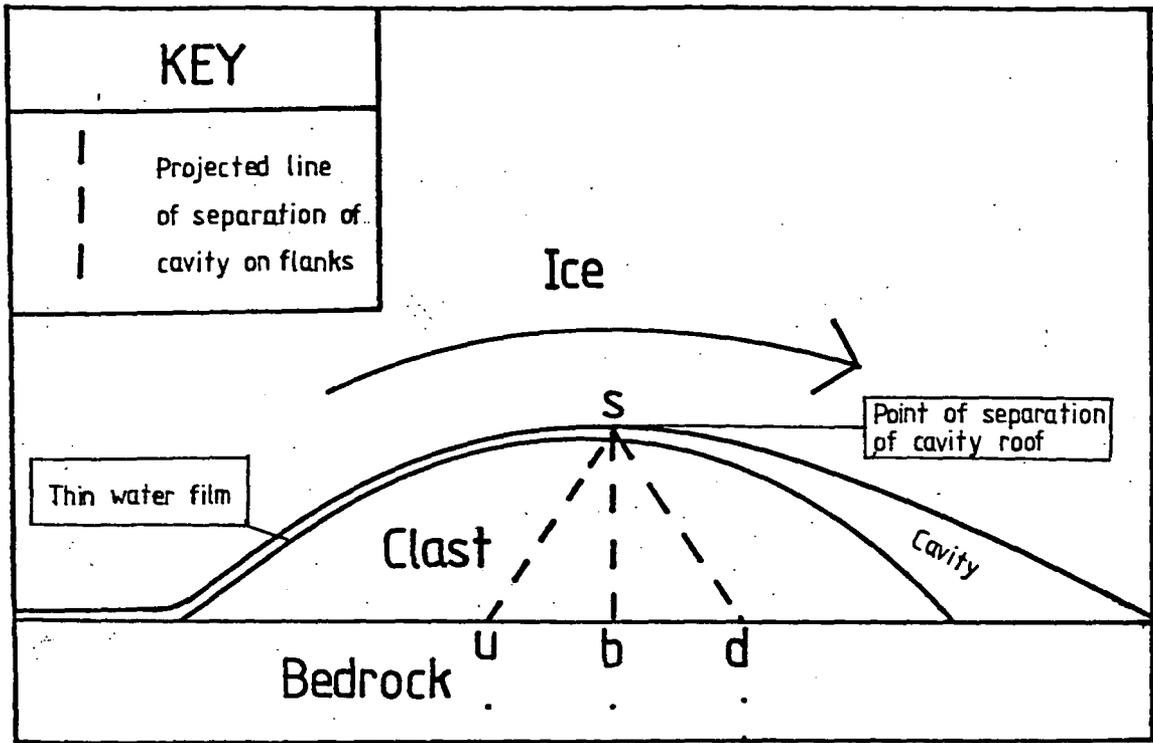


Figure 6.4

The effect of a generalised normal velocity on water pressure.

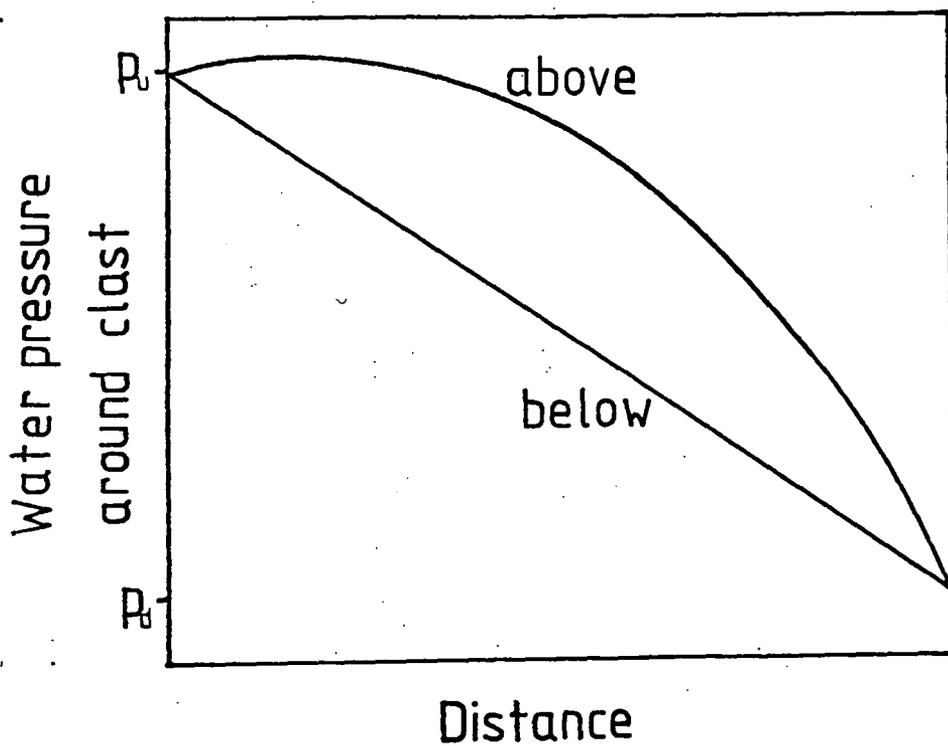
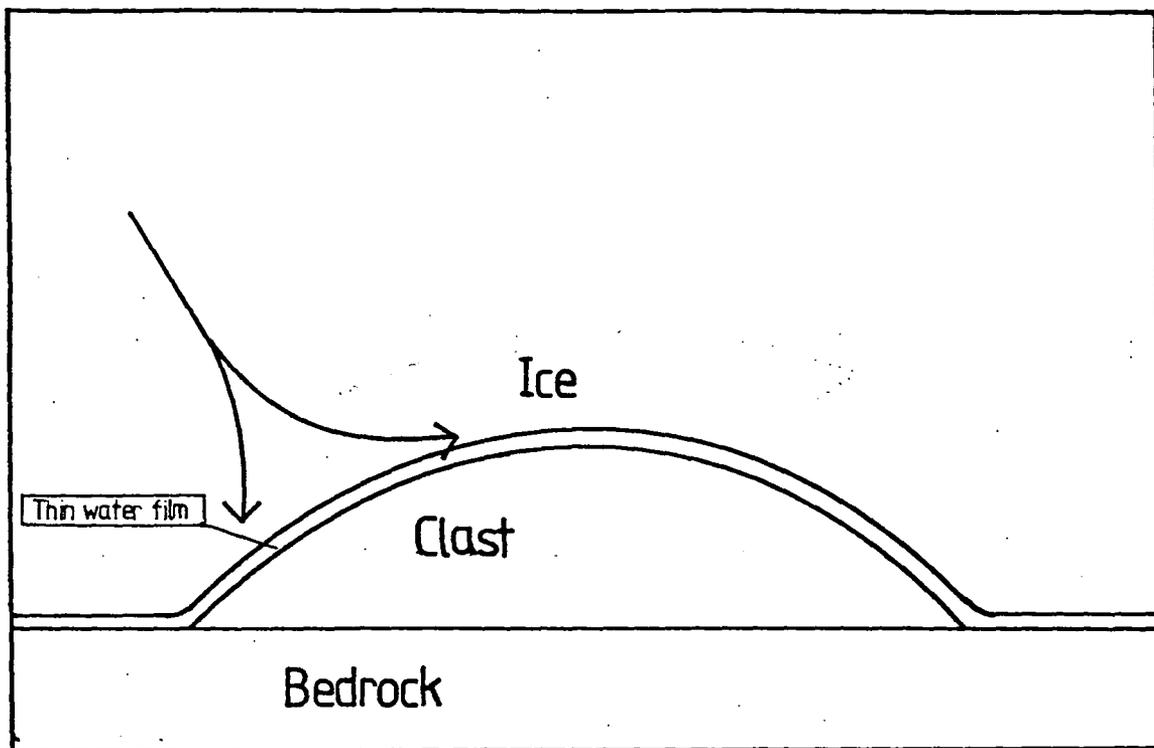


Figure 6.5

The effect of having pressure turning-points away from the clast edge.

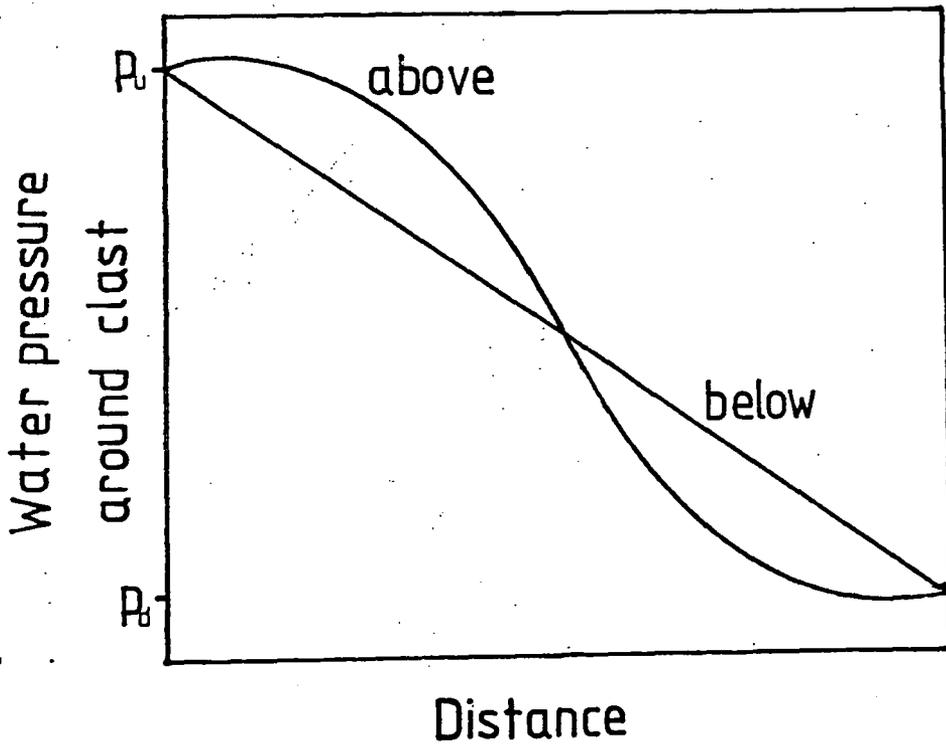
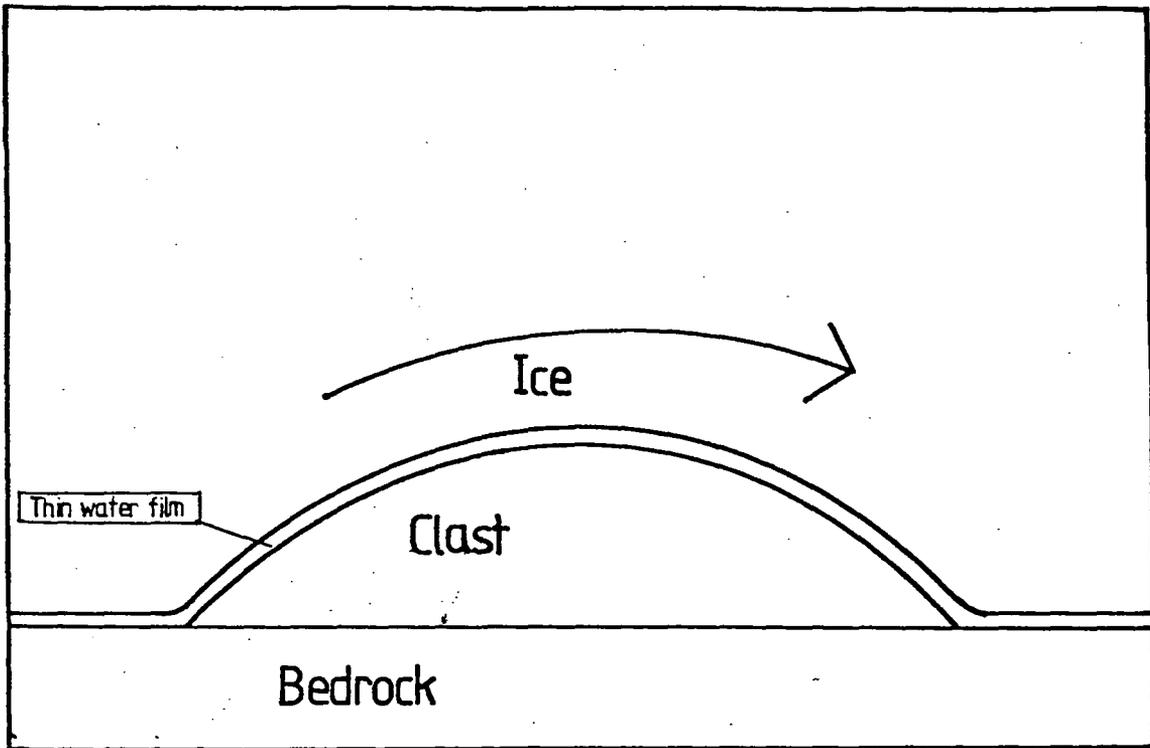


Figure 6.6

The effect of having a pressure turning point away from the clast-edge when a cavity exists.

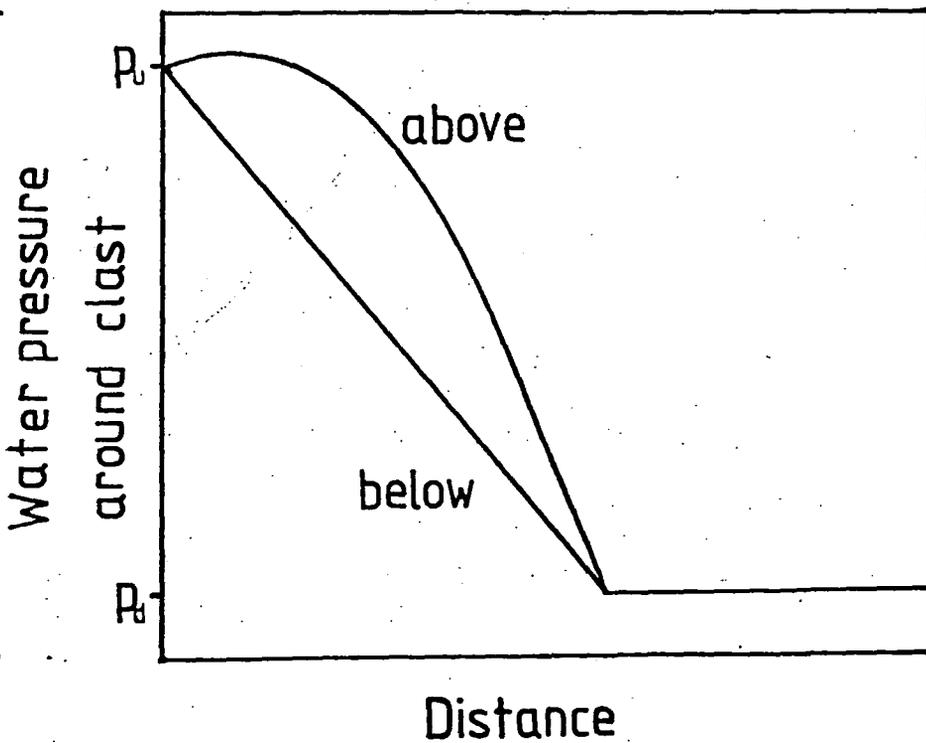
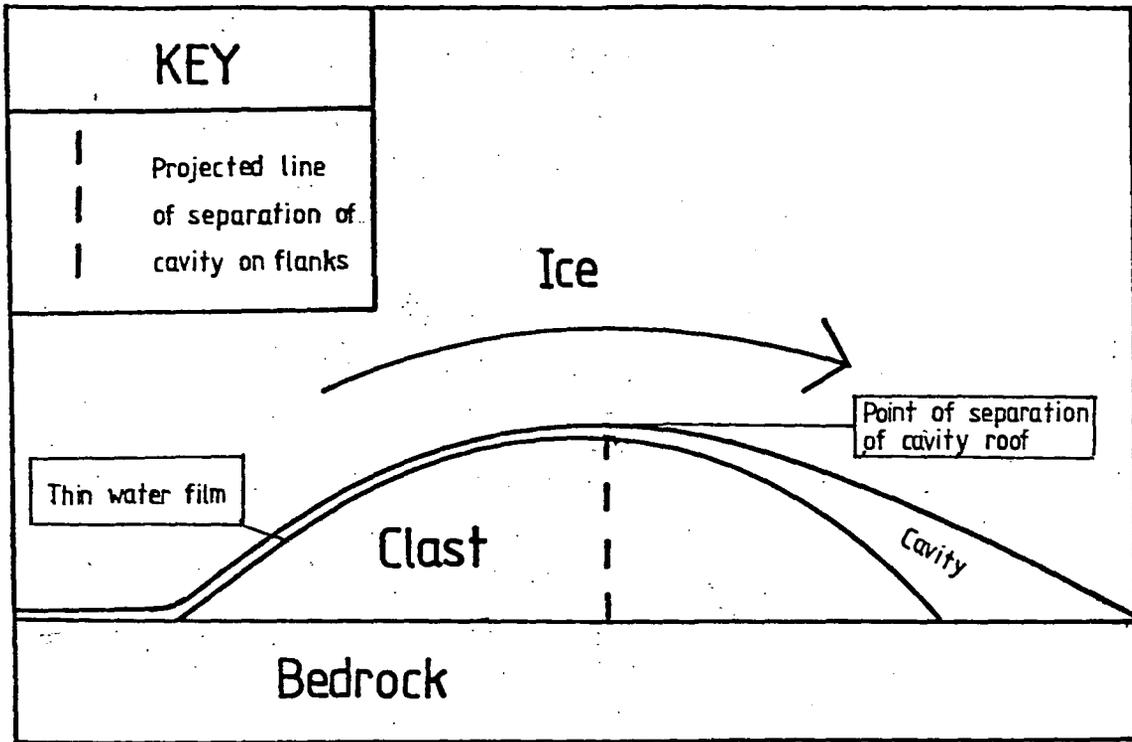


Figure 6.7

Sub-clast water pressures when the upstream end is less transmissible.

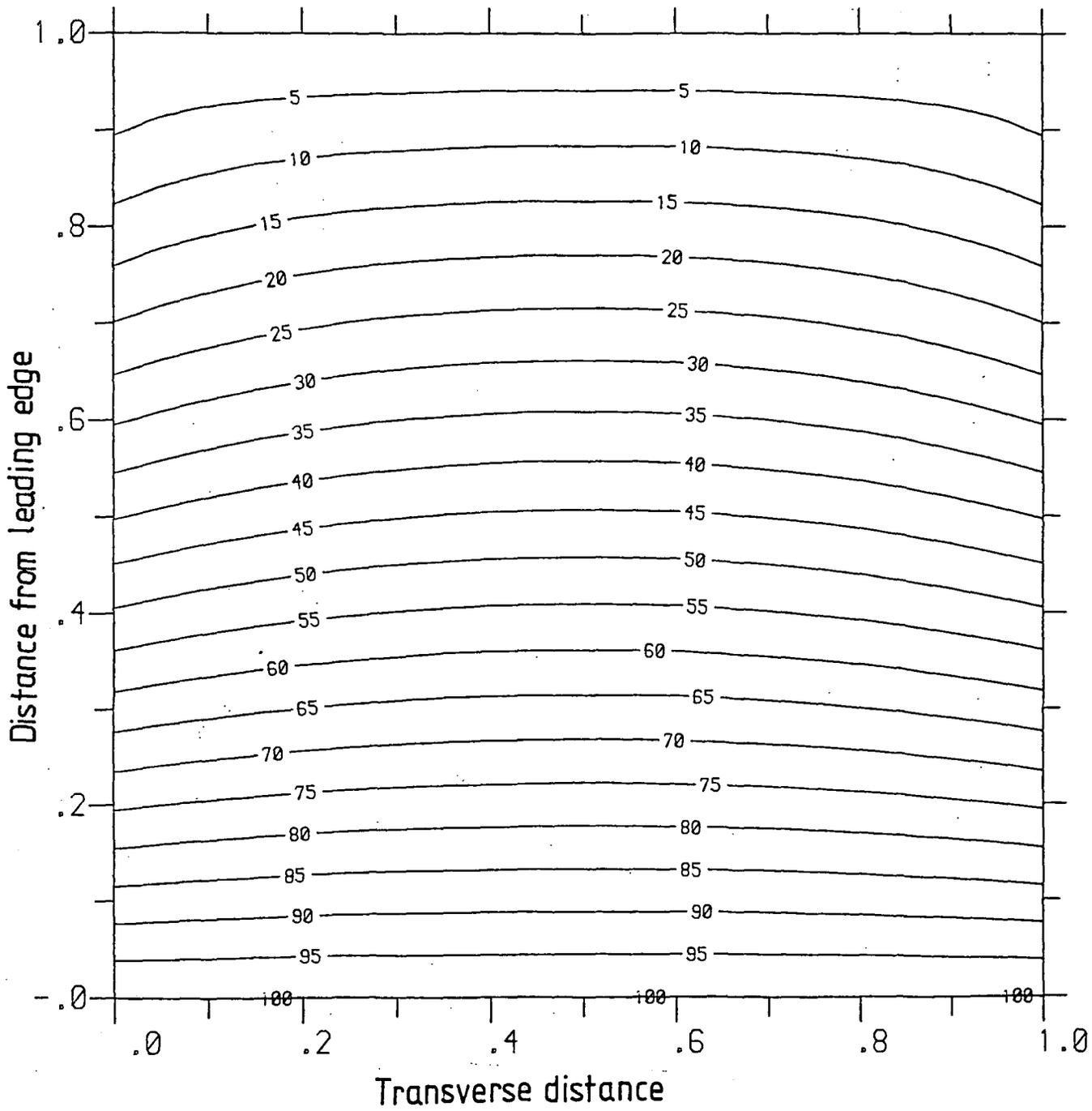


Figure 6.9

Sub-clast water pressures when the boundary pressure decline is slower than linear.

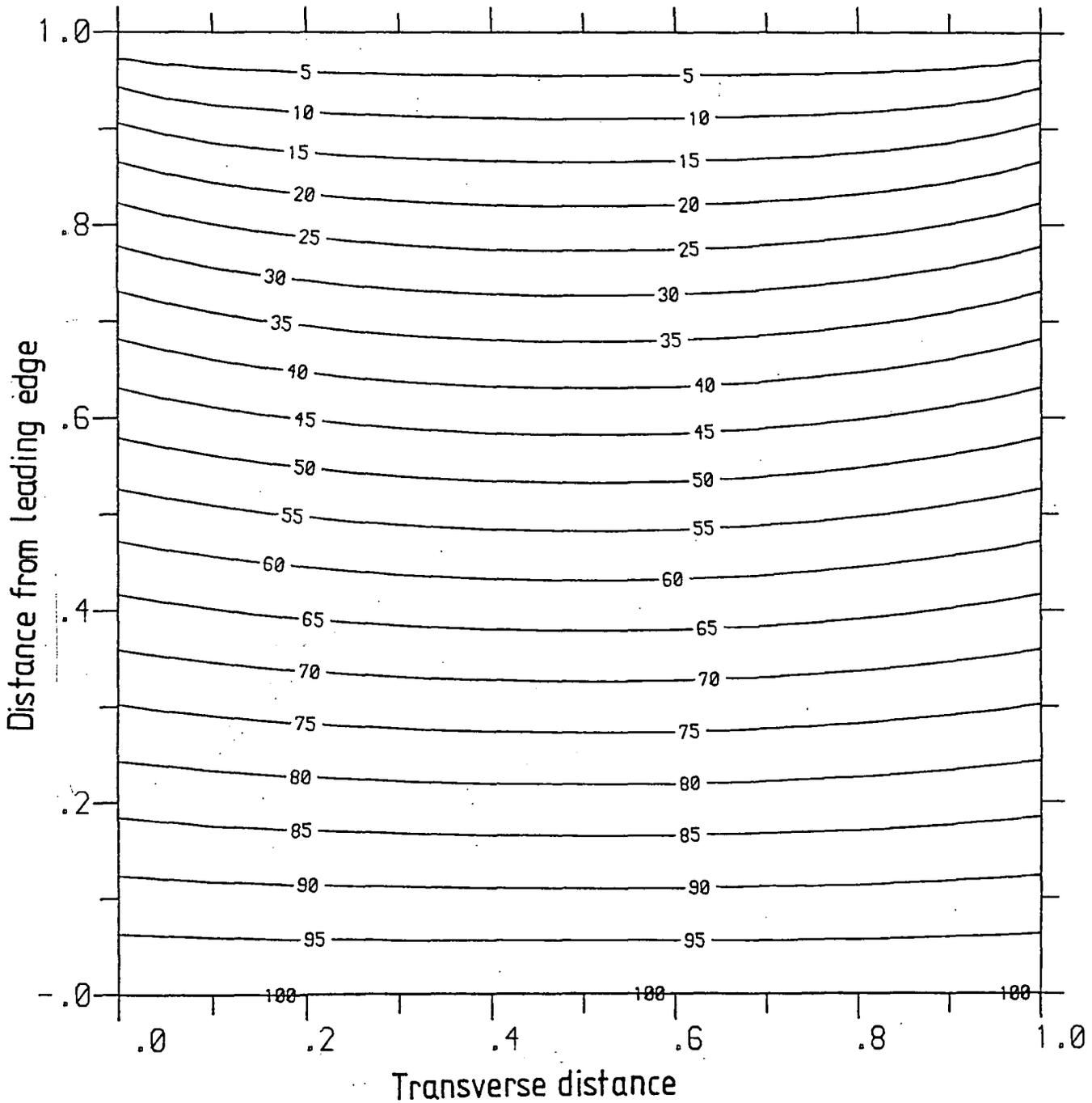


Figure 6.10

Sub-clast water pressures when the boundary pressure decline is faster than linear.

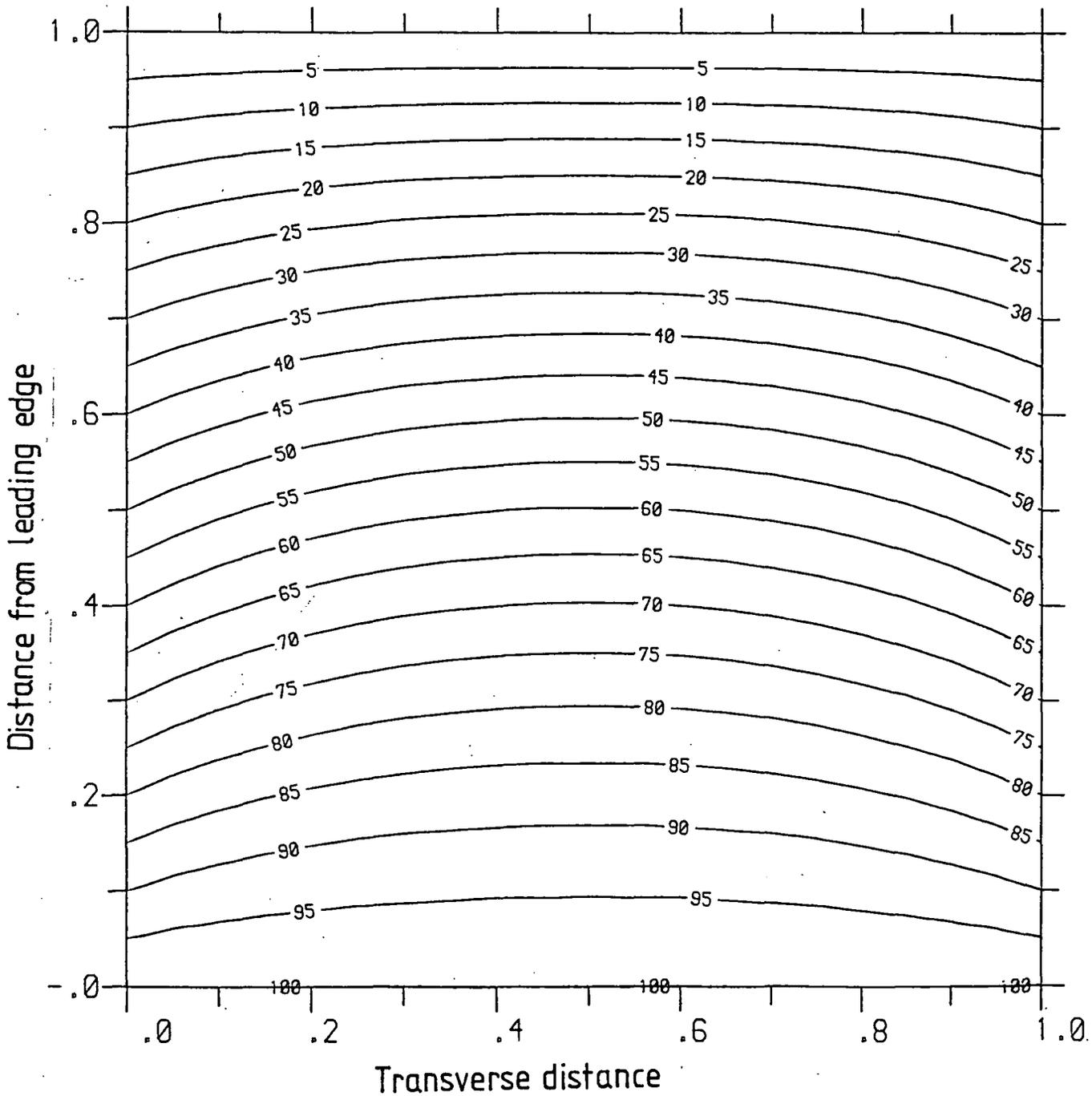


Figure 6.11

Dependence of abrasion rate on velocity and pressure.

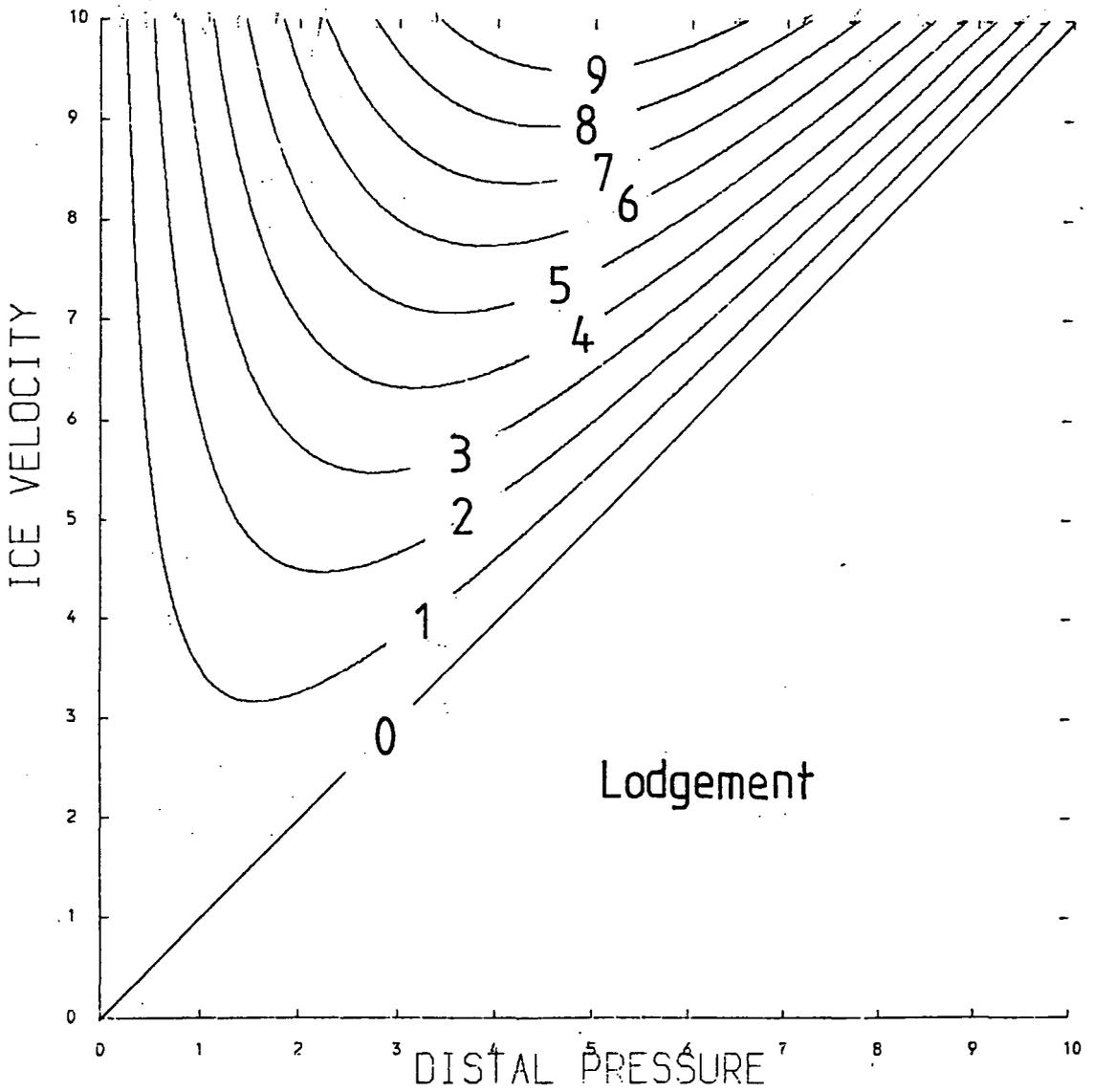


Figure 8.1

Three species mass-balance analysis

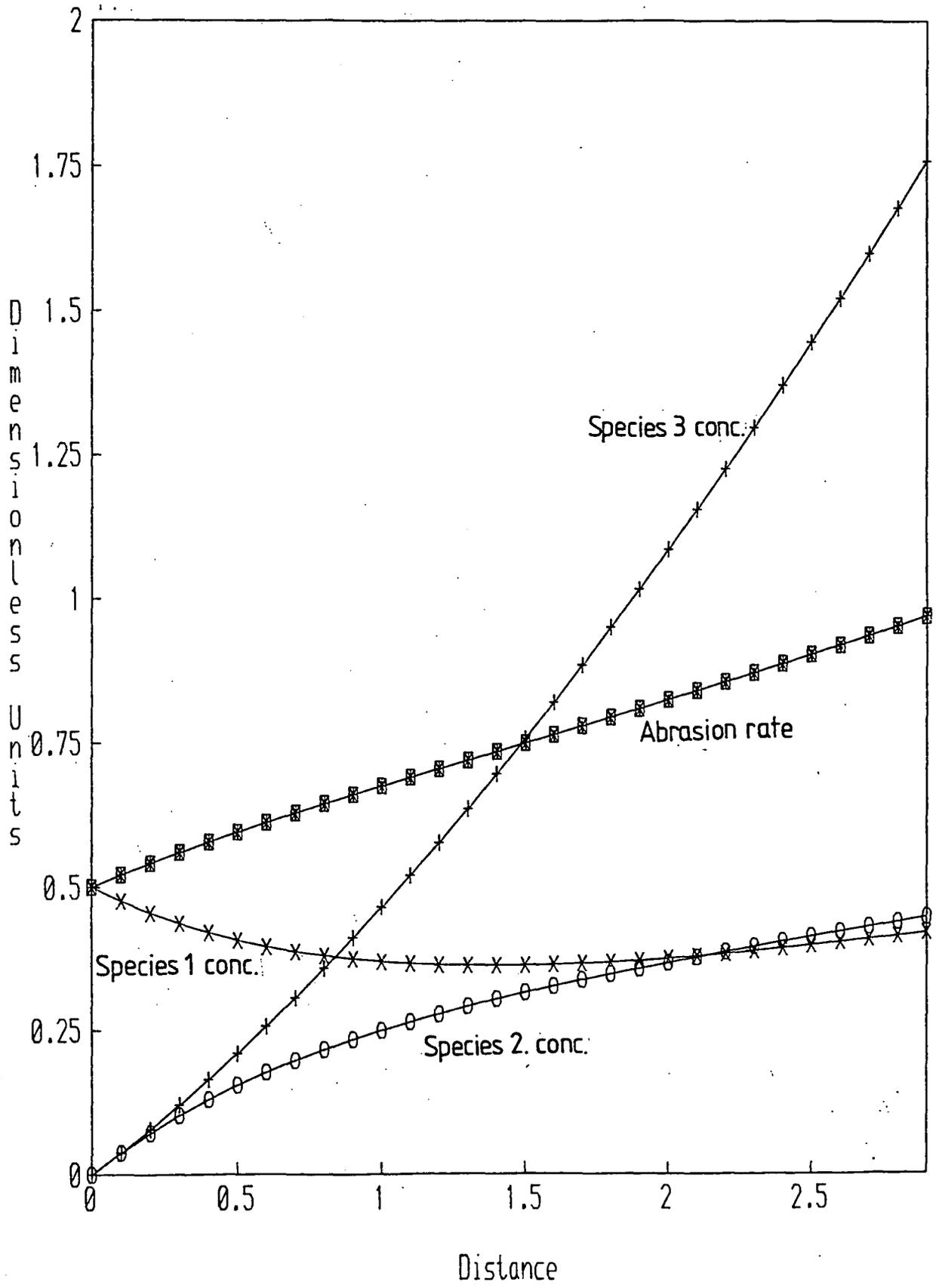


Figure 8.2

Four species mass-balance analysis.

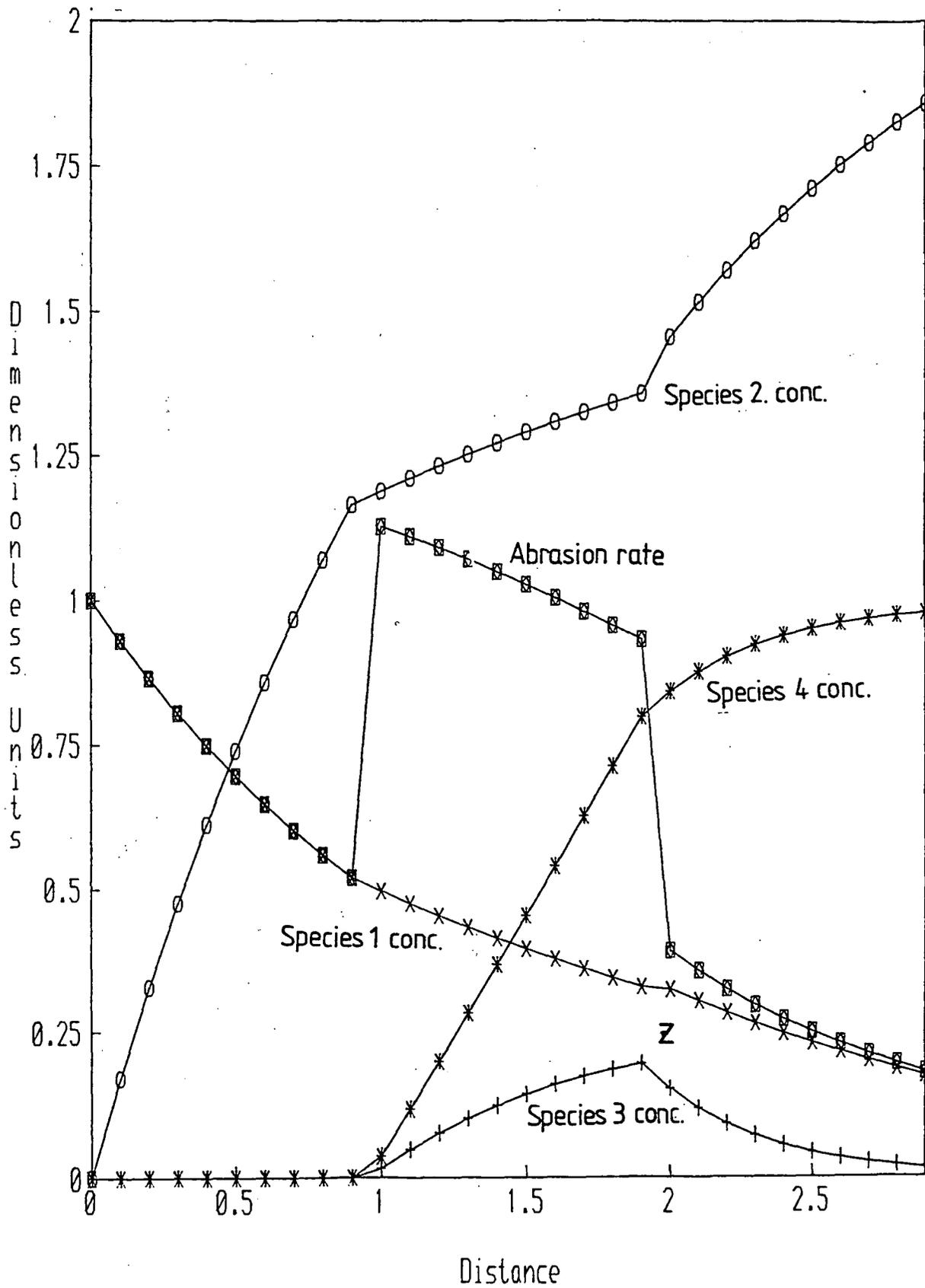


Figure 8.3

Four species mass-balance analysis with tool supply independent of abrasion rate.

