Durham E-Theses

Mathematical models for planning social services resources

Joan Taylor

How to cite:

Taylor, Joan (1986) Mathematical models for planning social services resources. Doctoral thesis, Durham University.

Use policy

The full-text may be used and/or reproduced, and given to third parties in any format or medium, without prior permission or charge, for personal research or study, educational, or not-for-profit purposes provided that:

- a full bibliographic reference is made to the original source
- a https://etheses.durham.ac.uk/id/eprint/6832/ is made to the metadata record in Durham E-Theses
- the full-text is not changed in any way

The full-text must not be sold in any format or medium without the formal permission of the copyright holders.

Please consult the full Durham E-Theses policy for further details.

Academic Support Office, The Palatine Centre, Durham University, Stockton Road, Durham, DH1 3LE
e-mail: e-theses.admin@durham.ac.uk Tel: +44 0191 334 6107
https://etheses.durham.ac.uk

MATHEMATICAL MODELS

FOR

PLANNING SOCIAL SERVICES RESOURCES

by

JOAN TAYLOR, BSc, MBCS

Submitted for Ph.D.

University of Durham

Department of Mathematical Sciences

1986

The copyright of this thesis rests with the author.
No quotation from it should be published without
his prior written consent and information derived
from it should be acknowledged.

Statement of Copyright

The copyright of this thesis rests with the author. No quotation from it should be published without her prior written consent and information derived from it should be acknowledged.

Abstract

This research discusses a number of computer-based mathematical models which are designed to assist planners to make strategic decisions concerning the allocations of social services resources. A new model is postulated which uses current patterns of care to derive a set of alternative modes or packages of care, chooses a suitable set of allocations of clients to packages of care within given resource constraints and can be used to explore the effect on resource requirements of demographic changes, and to explore alternative ways of caring for clients if populations expand and/or resources are reduced. Comparisons are made with the DHSS Balance of Care model and with other models. An exploration is included of the weighting values used in the postulated model's objective function.

Acknowledgements

The author is grateful to Sunderland Polytechnic for assistance with fees and for encouragement to embark upon this research.

Acknowledgements too to County Durham Department of Social Services, for permission to use data relating to social services in the county, and in particular, thanks to Mr G Wade and Mr O Coles for their interest and advice.

To Dr Margaret Greig, her supervisor, special thanks are given for guidance and help throughout the research. Thanks too, to Dr Ed Robson whose questioning persuaded the author to consider more carefully certain aspects of the research.

The author wishes to record her gratitude to her family, friends and colleagues, especially those at Thornhill Park who gave moral support and helped her to sustain the required effort to complete this part-time research, despite the extended timescale.

Finally, the author's unbounded gratitude is extended to Ann, Margaret, Jackie and Sandra whose typing and tabulation skills contributed to the final appearance of this thesis.

CONTENTS

CHAPTER 1

INTRODUCTION

1. Introduction

This thesis is concerned with the use of mathematical models to assist decision-makers who are planning future provision of social service resources. The type of decision is that made at County Hall level and concerns planning within a county rather than national planning.

Chapter 2 discusses the problem of planning the provision of social services and shows how a mathematical model, which represents the care given to clients in the form of social services, can be used in the planning process to suggest to the planners alternative yet acceptable patterns of care for changed client populations and specific resource constraints.

Chapter 3 describes the data on clients and services which was made available by Durham County Council and which was obtained from a census conducted by the Durham University Business School (D.U.B.S.). Details of the analyses of the data are given in Chapter 6.

Chapter 4 discusses mathematical models for planning social services resources. Firstly, the general problem and techniques of multiple-criteria-decision-making are discussed, together with their application to the social services planning problem. Secondly, several mathematical models, which have been proposed to aid the planners of social services, are considered. Among these are the first Balance of Care model; the DUBS goal-programming model; the revised Balance of Care model (SPRAM); a decision algorithm model; and a model proposed within Durham County Council.

Chapter 5 discusses the development of a new model, the author's linear programming model with alternative modes of care (LPAM). Initially the DUBS goal-programming model was investigated and a linear model was developed from this. The linear model proved to be a form of capacitated transportation model and did not model the actual allocation process, so a further development was to include alternative modes of care, fixed in accordance with the caring patterns in use at the time of the DUBS census. Details are given of how the model was developed and constructed for a single client group, then for a random sample of clients and then for the Chester-le-Street district. This model has a utility function which incorporates weights, representing the preferences of social workers for caring for particular categories of clients by particular modes of care. An investigation of the feasibility region for the values of these weights is also described in Chapter 5: a model was developed which represents the constraints upon the values of the weights. This weight-constraints model was used to analyse the sensitivity of the weights for the LPAM model as applied to one client group and as applied to the Chester-le-Street district.

Chapter 6 returns to a discussion of the client-data and service-data and their analyses. Descriptions are given of the DUBS census data and some inconsistencies therein, the summaries of client groups and service usage, the analysis of alternative modes or packages of care in use at that time, the cluster analyses performed to establish a set of alternative modes of care for each client group and finally the selection and analysis of the data for Chester-le-Street district.

Chapter 7 considers some implementations of the Balance of Care model in different regions, viz Wiltshire, East Sussex, West Midlands and Cornwall. Comparisons are made where possible between the surveys in those regions and in Durham.

Chapter 8 discusses and comapres the use of two models (the DHSS/A. Anderson SPRAM model and the author's LPAM model) for planning future allocations of resources when there are changes in demography and in resource provision. Probable demographic changes are discussed for the period up to 1991 and the results of applying the LPAM model to these changes in Chester-le-Street district are shown. A small set of test data is used to make detailed comparisons of the effects of applying the two models under the same conditions of changes in demography and in resource availability. This demonstrates how the SPRAM model plans future allocations which are little changed from current allocations and how the LPAM model's future allocations are often radically different from the current allocations. The opportunity which this latter offers the planners is discussed.

Chapter 9 discusses some limitations of the work done so far with the LPAM model and considers how this model could be applied and extended in practice. Some possible changes in client group definitions, modes of care and service specifications are discussed together with the possibility of using Durham County Council's "priorities and points" system to determine the weighting values needed.

Finally, Chapter 10 presents Conclusions.

References to publications are listed in alphabetical order and are cited within the main text, as are the Appendices.

The Appendices contain lengthy results or details of analyses, calculations and procedures. Figures contained in the main text are positioned immediately after the relevant section.

CHAPTER 2

THE PROBLEM OF PLANNING THE PROVISION OF SOCIAL SERVICES

2. The Problem of Planning the Provision of Social Services

The problem of long-range planning is to recognise long-term needs and to plan the provision of resources to meet those needs. Good planning should help organisations to achieve better performance and to improve efficiency. Planning the provision of social services requires consideration of the needs of individuals for social services, consideration of what should be regarded as "good performance" in providing for those needs and consideration of how resources can be provided to meet those needs. T.A. Booth[7] in "Planning for Welfare" discusses the system for allocating resources to welfare and the workings of the expenditure process through which collective choices are made about the allocation of resources in the personal social services. Booth views the expenditure process as a hierarchy of decision-making, from Whitehall to County Hall and from the Treasury to area social work teams, which encompasses all those rationing and resource decisions which together finally determine who gets how much of which resources.

The DHSS[25] in "Care in Action" has said that statutory responsibility for the personal social services rests with elected local government. The government indicates broad national policies, issues guidance where necessary and has a general concern for standards. There is only a small number of direct controls and these are being reduced as a matter of general government policy towards local government.

This thesis is concerned primarily with the type of decision made at County Hall level. It is concerned with the best ways of meeting local needs for personal social services whilst recognising the constraints upon provision of resources. "Meeting local needs" implies that needs should be satisfied but is insufficiently specific to be used as a guide to planning. Decomposition of this goal shows that satisfaction is viewed differently by the different individuals and groups involved. The apparent differences may not matter in the circumstances when infinite resources are available to fulfil any needs expressed or determined, but in practice resources are never infinite: they are constrained. The constraints may be caused by insufficient availability owing to incorrect provision of resources or, more usually, by insufficient funds to provide all resources which could be desired.

Bevan[5] has discussed the way in which those planners affected by cuts in resources need to be able to accommodate them. There is a need for "decisions robust to an uncertain future" so that actions can be selected now to satisfy immediate pressures for commitment of resources, but these actions nevertheless should enable preferred longer-term strategies to be implemented and still allow flexibility of choice between future strategies. Sir K. Sharp[53], writing in relation to the National Health Service, has said that it is the task of the NHS to utilise the resources at its disposal to produce the most "useful" outputs and that decisions on resource allocation should be taken in the context of the strategic aims of the service.

In Social Services the planning process involves the provision and disposition of resources such as social workers, home-helps, residential homes, day centres, etc i.e. both staff and physical resources or services. Although the individual social worker is concerned with the provision of resources for the needs of each individual client, the planners need to consider groups of clients and types of care rather than care for an individual client.

One aid to planning is to use a mathematical model which represents the conditions being planned (i.e. the needs of clients, within the region under consideration, and the resources available currently and in future) and which includes some measure of performance in respect of meeting client needs. As McDonald[40] says: "A model's contribution is towards estimating the resources needed to achieve a desired position or to indicate the likely position that would result from some planned provision of resources. Whether one situation is better than another is not something a model should decide". The emphasis in model-building is thus to assist the planners, to provide a means by which they can measure the effects of various different policies for service provision or the effects of different changes in needs. The models discussed vary from the purely descriptive type of model which represents what occurs under various conditions to the normative type of model which recommends a specific course of action, i.e. specific allocations of service to clients.

In each of the models discussed, the "clients" are the potential receivers of social services whose needs are assessed by social workers. These clients are individual members of the population at large, who may be, for example, elderly and infirm, or children at risk of abuse, or mentally ill adults, etc. The classification of clients is one aspect of model-building and is considered by Jolliffe[34] et al, who used cluster analysis to derive client groups for the elderly, as well as by each of the model-builders whose work is discussed here. Prentice[49] discusses the choice of client groups for a social services planning system proposed for County Durham by a team from Durham University Business School. Nelson [43,44] introduces the DUBS research and discusses the organisational context for planning decisions.

Another aspect is the classification of services or resources and these may consist of, for example, meals-on-wheels delivered to a client's home, residence in a geriatric home, visits by a health visitor, etc.

The present author was given access to data on clients and services obtained from a census conducted by the Durham University Business School (DUBS). The census data is described briefly in Chapter 3 and the author's analyses of the data are detailed in Chapter 6.

Perhaps the most difficult aspect of modelling in this situation is the definition of what is meant by "best satisfaction of need". One approach to the planning problem is to attempt to minimise the cost of providing resources to meet recognised needs but this assumes that cost minimisation is the objective and that recognised needs can somehow be met. In practice the cost of providing services is what constitutes a major constraint, usually expressed in some form of budget or limitation on the quantities of resources available, and the objective is "to best satisfy recognised needs" within given constraints.

Since the needs of the client groups are many and various, it is apparent that the planners have to meet multiple criteria in order to solve their problem.

Longbottom[37] discusses techniques for resource allocation for public services and the use of "inferred benefit" models such as that proposed in Bayat[4] et al which includes a measure of effectiveness intended to provide the link between client needs and the availability of resources. This measure uses a combination of effectiveness coefficients, representing the contribution of a particular resource to a particular client category, and weightings, representing priorities in respect of caring for different categories of clients.

Prentice[47,48] discusses some fundamental concepts of "inferred worth" planning and Louni[38] gives an account of several published studies on allocating care to the elderly and discusses the problems of resource allocation and the setting of appropriate objectives. Nelson[45] and Wiper, in the DUBS research, chose to consider objectives in the form of targets of allocation, with weightings for categories and services. This attempt to formulate a goal programming model refers to the work of Said[52] who discusses a goal formulation model for public systems which incorporates an interactive procedure including solution of a goal programming model at each stage. Said suggests that the goals themselves become an intermediate factor which the policy makers use to generate an agreeable set of policies, whether or not there is an agreement on goals.

Most researchers have attempted to define some form of goal, for example Mooney[41] attempts to maximise benefit by equating marginal social cost and marginal social benefit in each care location (for the elderly). Torrance[56] et al use multi-attribute utility to measure social preferences (of the client) for a system of health states : this is in relation to cost-effectiveness and benef.ts of intensive care for chronically disabled young children. George[29], Fox and Canvin use weightings in the objective function of a mathematical programming model to find the optimal throughput of hospital patients, giving preference to the categories with higher urgency and giving different weightings to different diagnoses. The values of the weightings were subjectively decided using an iterative process, but George et al comment that if a solution has undesirable characteristics because of the weights used, a new solution can be found by altering the weights in line with the decision maker's judgement.

Algie[1], Hey and Malien discuss ranking the preferences of several decision makers to reach a group preference : they refer to this as "judgement analysis" and have applied it to choose those social services which must be cut in preference to others.

Rousseau[50,51] and Gibbs have used a utility function to represent their objective in relation to a hospital bed usage model. This assumes ever-increasing utility with increases in admission rates and with length of stay. The utility function is similar in concept to that of McDonald[40] as extended by Coverdale[15] and Negrine for the Balance of Care project. This is discussed further in Section 4.2. Aspden[2], Mayhew and Rusnak described a similar utility function for a model of health care resource allocation in Czechoslovakia. This assumes ever-increasing utility with increases in the numbers of clients receiving resources.

The general problem of multiple-criteria-decision-making is discussed in the introductory paragraphs of Chapter 4. Subsequent paragraphs describe a number of mathematical models which have attempted to assist the planners in the particular problem of multiple-criteria-decision-making in the planning of social services resources.

Before this, however, Chapter 3 sets the scene for the specific problem of planning social services resources in County Durham, by describing the data available on clients and services.

CHAPTER 3

THE DURHAM UNIVERSITY BUSINESS SCHOOL (D.U.B.S.) CENSUS

3. The Durham University Business School (D.U.B.S.) Census

3.1 Introduction

A research team at Durham University Business School (D.U.B.S.) had produced a set of papers "Design for a Social Services Planning System", edited by E.G. Nelson[43]. These describe a project sponsored jointly by D.U.B.S. and Durham County Council (D.C.C.) Social Services Department. One of the papers, "Model for a resource allocation decision aid", by L. Wiper[60] describes a goal-programming model proposed for planning purposes. Another," Developmental record of Social Services client grouping", by R.C. Prentice[49] discusses the choice of client groups and services for planning purposes in D.C.C.

The 45 client groups and 99 services chosen by Prentice were subsequently used by the D.U.B.S. team to prepare a questionnaire or census form for completion by social workers in D.C.C. The completed questionnaires provided a census of the client population of D.C.C., with the exception of one district office, Lanchester. For each client, data was recorded at a single point in time in 1977 with the intention of showing a distribution of services in use and some indication of the unsatisfied demand as expressed by social workers. Data was recorded for 8216 clients. Each case has a different life so the census does not show an annual distribution nor a demand in total units of service. This information would be a necessary input to a planning model and could be obtained by applying multipliers to service profiles based on annual throughputs of client types.

The census data was stored on a magnetic tape file and various analyses were performed as described in another of the D.U.B.S. papers, "Census design and Analysis for social services planning and resource allocation", by M. Wheatley.[57]

That magnetic tape file of the 1977 census provided the raw data to be used in the author's research.

3.2 The Classification of Services and Clients

3.2.1 The Complete Data

The raw data from the 1977 census (the "Cendat" data) contains 8216 client records, for each of which is recorded

- a client number
- a code for age group
- a code for sex
- a client group number
- a list of up to ten codes for the actual services received at that time
- a list of up to six codes for the additional services desired by the social worker for the client
- a list of up to eight codes for the alternative services which could be supplied, if the actual services were no longer available
- further comments

Appendix 1 contains a list of the client groups and corresponding group numbers.

Appendix 2 contains a list of the services available and corresponding service code numbers.

The raw data was found to contain a number of internal inconsistencies: the elimination of these, the use of the data to produce client/service analysis, the subsequent reduction of the number of client groups and services is discussed in Chapter 6. The number of client groups was reduced by the present author from 45 to 35 and the number of services was reduced from 99 to 45. This was achieved by eliminating those groups which had very small frequencies of occurrence and including clients from those groups in other appropriate client groups. Similarly, services which had very small frequencies of occurrence were eliminated and an alternative service was substituted. In effect, the service number of the alternative service became a code to designate a resource requirement for that service or for the one eliminated.

The resulting data consisted of 8125 client records belonging to 35 client groups and receiving services of 45 different types.

Appendix 3 contains the revised list of 35 client groups.

Appendix 4 contains the revised list of 45 service codes.

Appendix 5 contains the complete analysis of numbers of clients in each of 35 groups receiving each of 45 services.

3.2.2 Chester-le-Street district

As is described in Chapter 6, the district of Chester-le-Street was chosen for further analysis.

Appendix 10 contains the complete analysis for Chester-le-Street of numbers of clients in each of 35 groups receiving each of 45 services.

CHAPTER 4

MATHEMATICAL MODELS FOR PLANNING SOCIAL SERVICES RESOURCES

4. Mathematical Models for Planning Social Services Resources

4.1 Introduction

4.1.1 The General Problem of Multiple-Criteria-Decision-Making

In the study of decision making, the terms "multiple criteria", "multiple objective" and "multiple attribute" are frequently used to describe decision situations. K.R. Mac-Crimmon[39] in "An overview of multiple objective decision making" (1973) distinguishes between these terms as follows:

(i) Multiple attribute decision problems deal with choosing among a set of alternatives which are described in terms of their attributes. ("Attributes" may be termed characteristics, aspects, factors, performance parameters, components, etc.) An attribute could be, for example, cost, size, or fuel economy.

Most of the techniques for dealing with multiple attributes require information about:

- the decision-maker's preference amongst values of a given attribute (e.g. how much does he prefer 5 mpg fuel saving to 2 mpg fuel saving);
- the decision-maker's preference across attributes (e.g. how much more important is cost than engine size).

Multiple attribute techniques either directly ask the decision maker for an assessment of the strengths of these preferences or they infer them from his past choices.

(ii) Multiple objective decision models recognise that attributes of alternatives are often just means to higher ends (i.e. the decision maker's objectives). Techniques in this case require:

- preference information about the decision-maker's objectives;
- information about the relationship between objectives and attributes.

Preferences among attributes are thus derived from the preferences among objectives and the functions relating attributes to objectives. In multiple objective models an alternative can be described either in terms of its attributes or in terms of the extent to which it achieves the objectives of the decision-maker.

MacCrimmon thus shows that multiple objective decision techniques explicitly treat the means-ends relationship and are more complex than multiple attribute models.

MacCrimmon suggests that multiple-criteria is a term applied most commonly to decisions which involve both multiple attributes and multiple objectives although it can be used for decisions involving either the one or the other. The decision maker may set up criteria directly related to attainment of objectives or to required attribute levels. Despite this, MacCrimmon chooses to use the term multiple objective in general since it encompasses multiple attributes as well.

M.K. Starr[55] and M. Zeleny in "MCDM-State and future of the arts" (1977) discuss the basic concepts and notation of multiple criteria decision-making as follows:

They define a set of potential or feasible alternatives from which a selection of one or more alternatives is to be made or their ranking performed with respect to given criteria.

Each alternative can be characterised by a number of attributes. At any given time the decision-maker considers a finite subset of salient attributes which are scored (measured or assessed) for each alternative.

The attributes are viewed as means or information sources available to the decision-maker for formulating and achieving his objectives. These objectives are closely identifiable with the decision-maker's values and needs : whilst not being the actual attributes they can be viewed as functionally related to or derived from some of the attributes.

Starr and Zeleny suggest that the main reasons for the distinction between attributes and objective are:

(a) attributes are generally numerically measurable while objectives are very difficult to assess by numbers;

(b) trade-offs between attribute levels can be more clearly defined on attributes, while in connection with objectives the very concept of a trade-off is fuzzy;

(c) attributes are more easily characterised through utilities while objectives may require fuzzy linguistic labels instead.

In reality, however, both the attributes and the objectives are often involved in a mixed fashion, so both categories are grouped together as "criteria". Criteria are then both the attributes and the objectives judged to be salient in a given decision situation.

There is clearly no universal agreement on the terminology used. It seems however that "multiple-criteria" is a term which applies to decisions involving both multiple attributes and multiple objectives and thus may be used as a general descriptor.

4.1.2 Techniques for Multiple-Criteria-Decision-Making

MacCrimmon[39] groups the relevant techniques into four main categories of methods:

(i) Weighting;

(ii) Sequential elimination;

(iii) Mathematical programming;

(iv) Spatial proximity.

(i) Weighting Methods

Weighting methods have been applied widely. MacCrimmon suggests that although the methods seem very diverse, they all have the following characteristics:

- a set of available alternatives with specified attributes and attribute values;
- a process comparing attributes by obtaining numerical scalings of attribute values and numerical weights across attributes;
- a well-specified objective function for aggregating the preference into a single number for each alternative;
- a rule for choosing the alternative (or rating the alternatives on the basis of the highest weight).

MacCrimmon identifies three main sub-categories, distinguished by the different bases for preference attainment and different aggregation processes. The sub-categories are where the preferences of the decision-maker are:

(a) Inferred from past choices rather than being obtained by direct query and are inputs to a general linear statistical model;

(b) Obtained by direct questioning and are aggregated additively across all the attributes;

(c) Obtained by direct questioning and specific attributes are taken to represent the whole alternative.

(ii) Sequential Elimination

Sequential elimination methods, says MacCrimmon, are less demanding of the decision maker than weighting methods. These methods are characterised by:

- a set of available alternatives with specified attributes adn attribute values;
- scalings of attribute values (intra-attribute preferences);
- a set of constraints across attributes;
- a process for sequentially comparing alternatives on the basis of attribute values so that alternatives can be either eliminated or retained.

Again three main sub-categories are identified, distinguished by the entities they compare and the processes used for comparison.

The comparisons are:

(a) Across attributes for a given alternative, i.e. comparing the attributes of the given alternative with the attributes of a standard;

(b) Across attributes for two alternatives, i.e. comparing the attributes of one alternative against the attributes of the other;

(c) Across alternatives for a single attribute, i.e. comparing the attribute value of all alternatives.

(iii) <u>Mathematical Programming</u>

These methods have the following characteristics:

- an infinite (or very large) set of alternatives which are inferable from a set description (i.e. constraints specified on the attribute values).
- a set of technolgical or perhaps preference constraints;
- an objective function;
- an algorithm to generate more preferred points in order to converge to an optimum.

Three methods of mathematical programming are described by MacCrimmon. These are:

(a) Linear Programming. The purpose is to design the optimal alternative by putting together the best combination of attribute values.

(b) Goal Programming. Here the decision-maker specifies acceptable or desired levels of single attribute values or of combinations of attributes and these serve as the primary goals. Minimisation of the deviations away from these goal levels becomes the objective. It is necessary to scale the decision-maker's "preferences" for deviations in each direction from these goal levels. The goal deviations are combined to form a global objective.

(c) Interactive, Multi-criterion Programming. Here the decision-maker is required to provide his local trade-offs in the neighbourhood of a feasible alternative. These trade-offs (on the attributes or criterion involving attributes) are used in a local objective function for a mathematical programming algorithm to generate an optimal solution for that objective. The decision-maker then has an opportunity to provide new trade-offs which again serve as inputs to the algorithm. This process continues until the decision-maker no longer wishes to revise his trade-offs and so an optimal solution is reached.

(iv) Spatial Proximity Methods

These are more specialised methods making explicit use of spatial representations. They are characterised by the following:

- a set of identified alternatives, in some cases with vague attribute values;
- a process for obtaining intra- and inter-attribute judgements;
- the construction of a spatial representation;
- the identification of ideal configurations and the choice rule based on the proximity of alternatives to these ideal configurations.

MacCrimmon describes three of these methods:

(a) Indifference Map. Here the decision-maker's preferences can be obtained in the form of indifference surfaces which show the combinations of attribute values that are equally preferred. This method is a more explicit graphical form of the trade-off approach.

(b) Multi-dimensional Scaling with Ideal Points. Here the decision-maker's orderings of the proximities of pairs of alternatives can be used to construct a multi-dimensional spatial representation. The decision-maker is asked to locate his ideal alternative in this space and then the distance from the ideal point is measured in order to rank the alternatives.

(c) Graphical Overlays. This method can be used where the attributes have some obvious diagrammatic interpretation. Each of a number of transparent sheets can be used to reflect the desired way to attain a particular objective and they are then overlaid one on another until all objectives have been incorporated into one visual aggregation. The objectives need to be formed into subsets so that interactions within each subset can be resolved, then the objectives are aggregated in turn, finally producing a consolidated choice.

4.1.3 The application of multiple-criteria techniques to the problem of planning social services resources

From the preceding paragraphs it can be seen that many methods are available to approach decisions involving multiple criteria. In relation to the problem of planning social services resources, it is firstly recognised that multiple criteria exist, i.e. that the alternative means of providing social services to clients include different values of different attributes, namely the amounts of each separate service which are allocated to the clients in the various client groups. Secondly, it is recognised that multiple objectives exist, i.e. the decision-maker has preferences relating to the means of providing resources to different client groups. The preferences may refer to the preference to treat a greater proportion of one client group rather than another, or the preference to treat one client group in a particular fashion.

The development of mathematical models to aid the planning of social services resources is a continuing area of study (see Boldy[6]). Several models have been proposed and the later sections of this Chapter discuss some of these, ranging from the comparatively simple decision algorithm discussed in section 4.5 to the much more complex Balance of Care model discussed in section 4.2.

When the author began this research, there were two particular models of interest: the DHSS Balance of Care model described by McDonald[40] et al (1974) and the D.U.B.S. goal programming model postulated by Wiper[60] (1978). (See Nelson[43]; Volume 3.1). These are described in Sections 4.2 and 4.3 respectively.

McDonald's Balance-of-Care model began life as a linear programming model which minimised the cost of providing resources. Alternative acceptable modes or packages of care were defined by consulting groups of professional workers and administrators. Thus the alternatives were obtained by direct questioning.

Evolution of McDonald's model led to his re-definition of the objective function to an "inferred worth" function, in which the data are inferred from information about what actually happens rather than what ought to happen. Instead of a constraint stating that some number of clients must be treated, there is an incentive contribution to the inferred worth function which provides an incentive to treat this number of clients.

Coverdale[15] and Negrine used McDonald's model (as described in Section 4.2.7) but found that it was difficult to calibrate the parameters in the inferred worth function. Their version of the model describes the provision of care in terms of coverage, modal balance and quotas. Coverage is the ratio of the number of clients receiving care to the number of potential clients in a group. Modal balance consists of the numbers of clients in a group receiving each of the alternative modes or packages of care which could be allocated to clients in that group. Quota is the ratio of the allocation of a particular reducible service to that which would be required if desirable levels of service were to be reached (for that particular modal balance).

The D.U.B.S. goal-programming model described in Section 4.3 is a model which uses ideal levels of service as targets or goals. Target levels of service are set for each client group and "distance from target" is minimised. Since the D.U.B.S. goal-programming model had been postulated with the intention of using the D.U.B.S. census data to provide data for the model, but in fact no model had been constructed, the present author began by considering such a model. The first section of Chapter 5 describes the author's construction of a model along these lines and later sections show the further development to include alternative modes of care.

The D.H.S.S.[25,26] (1981) had meanwhile developed another model. The new model was based conceptually on the Balance of Care model in that it sought to balance the resource availability with the client needs, but it was not based on the mathematical model proposed by McDonald. Instead it used SPRAM (Simple Proportional Resource Allocation Model). This model was developed by Arthur Andersen & Co. in association with the D.H.S.S. and is described in Section 4.4 Implementation of this model has taken place in several different geographical areas, in each case based conceptually on the D.H.S.S. model but varying in the type of analysis undertaken. Some results of these implementations have been made available to the author and these are discussed in Chapter 7 where comparisons have been made with the data for County Durham.

The D.H.S.S. SPRAM model described in Section 4.4 is not a mathematical programming model. Instead a form of weighting method is used where preferences for particular allocations are used to assign clients to modes of care, initial allocations being to the same modes of care as previously, and resources are then allocated in simple proportion to the existing allocation, according to the amounts of each resource available. A form of sequential elimination procedure is also proposed, to take into account the change in resource requirements due to "saturation effects". This is not sequential elimination of the kind described by MacCrimmon[39] but a sequential allocation procedure which allocates the whole of one set of resources to clients, then eliminates this set from further consideration and moves on to another set of resources to be allocated.

The implementation of the Balance of Care model at Calderdale is described in Section 4.5. This implementation is distinctive from implementations in other regions (which are discussed in Chapter 7) in the way that the allocation model has been simplified to a straightforward decision algorithm. Comparing this algorithm with the MacCrimmon classification of methods, this algorithm uses a set of modes of care obtained by direct questioning of the professional workers, thus a form of weighting or priorities is inferred, then a sequential elimination procedure is used to eliminate certain modes of care from consideration for individual clients with attributes of need at certain levels. Sequential consideration of attributes eventually leads to a single mode of care appropriate for the client.

Within Durham County Council (DCC) itself, the planning department has been investigating alternative ways of allocating resources and, partly as a consequence of the D.U.B.S. study and our own discussions with DCC, a simple model based on the SPSS computer package was proposed. This is described in section 4.6 but has not been implemented.

The final section (4.7) of this chapter considers the advantages and disadvantages of all these models.

Since most of the models considered here are mathematical, a consistent symbolic notation has been used throughout. In several cases the models are presented here with different symbols from those used in the original presentations by their authors. The intention has been to avoid the confusion which could be caused by the introduction of a new set of symbols for each model.

Figure 4.1.3.1 shows the notation used.

Figure 4.1.3.1 Symbolic notation

i	=	Subscript for client group (DUBS model) or category within a group (Balance of Care)
j	=	Subscript for DHSS-defined groups (Coverdale)
k	=	Subscript for service resource
l	=	Subscript for mode or package of care
d_i	=	number of clients in group i
x_{il}	=	number of clients in group i to receive mode l of care
u_{ilk}	=	amount of service k allocated to a client in group i receiving mode l of care
B_k	=	total available amount of service k
w_{il}	=	weighting for clients in group i receiving mode l of care
W_{ik}	=	weighting for clients in group i receiving service k
t_{ik}	=	target amount of service k received per client in group i
A_{ik}	=	amount of service k allocated to group i collectively
T_{ik}	=	target amount of service k for group i
a_{ik}	=	amount of service k allocated per person in group i
C_k	=	unit cost of resource k
p_i	=	amount to be spent on prevention for group i
N_i	=	number of cases in group i prevented per unit expenditure on the prevention programme
X_{Lil}	=	minimum number of cases in group i which can be treated by alternative l for medical reasons
X_{uil}	=	maximum number of cases in group i which can be treated by alternative l for medical reasons.
P_i	=	maximum expenditure on prevention for group i
G_k	=	short term marginal cost of resource k
H_k	=	extra(capital) cost of increase in resource k
δ_k	=	increase to be made in resource k
M_k	=	maximum possible increase in resource k

Figure 4.1.3.1 Symbolic notation (continued)

B_{Nk}	=	new amounts of service k
θ	=	general proportion of demand
β	=	general resource use reduction factor
D_i	=	number of clients in group i receiving care (D_i is variable and Coverdale uses D_i to represent the potential number of clients in group i)
g_i	=	a function representing the contribution to inferred worth for group i.
DLi	=	lower bound on demand from group i
DUi	=	upper bound on demand from group i
α_i	=	a constant for group i
E_i	=	elasticity of satisfied demand from group i
$-\pi_i$	=	shadow prices on demand constraints
h_{ilk}	=	a function representing the inferred worth for each client from devoting u_{ilk} units of resource k to each client in group i receiving mode l of care.
U_{Lilk}	=	lower bound on amount of service k allocated to a client in group i receiving mode l of care.
U_{uilk}	=	upper bound on amount of service k allocated to a client in group i receiving mode l of care.
v_{ilk}	=	variable amount of service k allocated to a client in group i receiving mode l of care.
F_{ik}	=	constant elasticity of the actual allocation of resource k to each client in group i with respect to marginal inferred worth.
β_{ilk}	=	a constant for group i, mode l, receiving service k.
c_i	=	coverage in group i, i.e. the ratio of the number of clients in group i receiving care to the number of potential clients in group i.
q_{jk}	=	quota for DHSS - defined group j receiving service k i.e. the ratio of the allocation of service k to that which would be required if desirable levels of service were to be reached for group j (for a certain modal balance).

Figure 4.1.3.1 Symbolic notation (continued)

b_{jk}	=	desirable level of service k for group j.
U_{ilk}	=	desirable level of service k per client in category i receiving mode l of care
ϕ_{jk}, ψ_{jk}	=	parameters for Coverdale's h-function
ξ_i, η_i	=	parameters for Coverdale's g-function
r_{ilk}	=	number of units of resource k allocated to mode l of category i in SPRAM model
q_{ilk}	=	quota for mode l of category i receiving resource k
x_i	=	number of clients receiving care in category i
IPE_i	=	ideal package equivalent for category i.
$B_k^{(1)}$	=	estimated future requirement for resource k
r_{ik}	=	current allocation of resource k to clients in category i
d_i'	=	future size of population in category i
r_{gk}	=	current allocation of resource k to geographic unit g
r_{gk}'	=	future allocation of resource k to geographic unit g
p_g	=	population size of geographic unit g
r_g	=	current allocation of a resource to geographic unit g
r_g'	=	future allocation of a resource to geographic unit g
R	=	current resource availability (units)
R'	=	future resource availability (units)
K	=	a constant used in the SPRAM model
B_k'	=	future availability of resource k
r_{ijgk}'	=	future allocation of units of resource k to clients in category i of group j and geographic unit g
Δl	=	number of units of dominant resource allocated to mode l

Figure 4.1.3.1 Symbolic notation (continued)

M_{kl} = number of units of non-dominant resource k allocated to mode l

λ_k = quota/coverage trade-off parameter for resource k

μ_{ikl} = average amount of resource k allocated currently to clients in mode l of category i

x_{ikl} = number of clients in mode l of category i currently receiving resource k

c_{ik} = coverage of resource k in category i

M_{ikl} = maximum likely use of resource k by a client in mode l of category i

I_i = number of clients in category i allocated to institutional care

ρ_{gk} = maximum amount of resource k available for geographic unit g

r''_{gk} = adjusted allocation of resource k to geographic unit g

μ_{ik} = decision parameters, used in Segment 4 of the SPRAM model, designating the proportional change in allocation of resource k to category i

S_{lk} = saving in other resources by allocating resource k in mode l

A = cost per client of lth mode of care

C = cost per client of cheapest alternative mode not using resource k

a = number of units of resource k used per client in mode l

4.2 Balance of Care

4.2.1 Introduction

In 1974, A.G. McDonald[40], G.C. Cuddeford and EML Beale published a paper "Balance of Care : Some Mathematical Models of the National Health Service". The paper reported on the progress of a series of studies carried out jointly by the Department of Health and Social Services (DHSS) and Scicon Ltd. The term "balance of care" was chosen because the authors regarded the total care delivered with finite resources as a balance between the major types of care and groups of client. The aim was to illuminate the likely consequences of different sets of policy options in terms of balance, both between different resources provided and between various groups of clients cared for at a strategic level.

McDonald et al state that the fundamental contribution from their studies has been the definition of categories of client, the identification of alternative care patterns associated with these categories, the resources involved in the care of clients and the availabilities and costs of these resources.

Their basic concept is that of alternative forms of care for some categories of client.

Categories were defined in such a way that the resources required to care for the same client under different categories are not counted twice. The resources required by one category are regarded as having no effect on those required by another, other than the indirect effect of scarcity. Furthermore it is assumed that some resources will be scarce and that it is adequate to provide clients in a specific category with one of several alternative packages of care. Thus acceptable packages of care could be defined and were obtained by consulting groups of professional workers and administrators.

Figure 4.2.1.1 shows an example of alternative care options for a category of elderly clients. "Demand" for care was estimated in a variety of ways, for example : from existing case-loads, from existing surveys of potential elderly clients, and by extrapolating current demand to estimate future demand.

Data on resources was obtained from DHSS records.

4.2.2 The initial model

Altogether, McDonald et al defined 150 categories and 550 alternative acceptable resource uses for each client. 38 resources were included. Their initial assumptions were:

(i) that resource use for each case should be independent of resource availability - thus maintaining standards,

Figure 4.2.1.1 Alternative care options for a certain category of elderly patients

(Table 1 from Mc Donald[40] et al)

(Category includes those with poor housing, who live with others, have severe to appreciable physical handicap and have severe to moderate dementia).

RESOURCE	ALTERNATIVE CARE OPTIONS							
Location								
Psychiatric bed	X							
Geriatric bed		X						
Special housing place			X	X	X			
Own home						X	X	X
Community support								
District nurse (visits per week)			4	4	5	4	4	5
Part-time domestic help (hours per week)			3	3	4	4	4	5
Meals delivered to patient's own home (number per week)			1	1	4	1	1	4
Attendance at psychiatric day hospital (times per week)			3			3		
Attendance at geriatric day hospital (times per week)				3			3	

(ii) that resource costs are linear with respect to the amount available;

and (iii) that potential demand is not less than presented demand.

The requirement was to choose the best allocation of resources to categories and the objective decided upon was to minimise cost.
The initial model was as follows:

Choose the x_{il} to:

Minimise $\sum_k C_k \sum_i \sum_l u_{ilk} x_{il}$

subject to $\sum_l x_{il} \geq d_i$ for all i,

and $\sum_i \sum_l u_{ilk} x_{il} \leq B_k$ for all k.

This model seeks to find that allocation of resources to categories of client which minimises cost, subject to given case-loads and resources.

4.2.3 Versions of the initial model

In practice, more sophisticated versions of the basic model were used, as follows:

Formulation A: for long-term marginal costs and preventive programmes:

Minimise $\sum_k C_k \sum_i \sum_l u_{ilk} x_{il} + \sum p_i$

subject to $\sum_l x_{il} \geq d_i - N_i p_i$ for all i;

$X_{Lil} \leq x_{il} \leq X_{Uil}$ for all i,l;

$p_i \leq P_i$ for all i;

$\sum_i \sum_l u_{ilk} x_{il} \leq B_k$ for all k.

This model includes expenditure on prevention programmes with assumptions about the consequent reductions in levels of care. The assumptions were so tentative that the preventive version could only be illustrated but not applied.

<u>Formulation B</u>: for short-term marginal costs and capital costs:

Minimise $\sum_k \left[G_k \left(\sum_i \sum_l u_{ilk} x_{il} \right) + H_k \delta_k \right] + \sum_i p_i$

Subject to $\sum_l x_{il} \geqslant d_i - N_i p_i$ for all i;

$X_{Lil} \leqslant x_{il} \leqslant X_{Uil}$ for all i, l;

$p_i \leqslant P_i$ for all i;

$\sum_i \sum_l u_{ilk} x_{il} \leqslant B_k + \delta_k$ for all k;

$\delta_k \leqslant M_k$ for all k.

This model includes the facility for expenditure on prevention programmes as in formulation A above, as well as the facility to permit resources to be increased up to given limits at additional capital cost. This enables a range of combinations of increased resources to be examined in one calculation.

<u>Formulation C:</u> for parametric programming with demand:

Minimise $\sum_k C_k \sum_i \sum_l u_{ilk} x_{il}$

Subject to $\sum_l x_{il} \geqslant \theta d_i$ for all i,

$\sum_i \sum_l u_{ilk} x_{il} \leqslant B_k$ for all irreducible k,

$\beta \sum_i \sum_l u_{ilk} x_{il} \leqslant B_k$ for all reducible k.

This model recognises that available resources cannot provide acceptable cover to satisfy potential demand and introduces two new measures: a "resource usage factor" and a "percentage of potential demand included in the model". The resource usage factor is a hypothetical proportion of the "acceptable" use by each client of some resources. The use of some resources was considered irreducible (eg. hospitals and residential homes) whereas non-residential services (eg domestic help, meals-on-wheels) can rationally be reduced. In the ideal situation both the resource usage factor and the percentage of demand would be 100, but for any given level of resources the model can be used to establish the relationship between the two measures for the minimum cost combination of alternatives.

McDonald shows the relationship obtained using this model for all "elderly" categories : at best, either 40% of the potential demand can be included and 100% of the acceptable domiciliary care for each patient delivered, or 50% of potential demand and 50% of acceptable domiciliary care, or there will be some combination in between.

4.2.4 The predictive model

Development of these early formulations led to construction of a predictive model. Such a model would represent how resources, case-load and care for each client interact. It had been observed that increases in resources revealed further unsatisfied demand. The predictive model would estimate the number of clients drawn into the system and the level of care delivered, corresponding to any planned level of resources. The model is based on the concept of the "inferred worth" of the total care delivered. "Inferred worth" is used to mean a utility function in which the data for it are to be inferred from information about what actually happens rather than what ought to happen. The model has the structure of a normative model but is formulated as a predictive model.

The basic hypothesis is that "the health services allocate various available resources to maximise the net inferred worth of total care delivered, where net inferred worth is a utility function exclusive of cost, which is implicitly defined by the prevailing trends in the average amount of care given to each client and the types of categories of client cared for".

The new model is developed from the initial cost-minimisation model (described in Section 4.2.2 above) in two stages, firstly with fixed resources u_{ilk} devoted to a client receiving each type of treatment and secondly with variable resource uses. The new model permits an expansion of the resources made available.

4.2.5 Predictive model: first stage

The first stage is similar to the cost-minimisation model, except that the number of clients in group i receiving care is now a variable D_i and the new amounts of resource available are denoted by B_{Nk}. Instead of a constraint stating that some number of clients must be treated, the model includes a contribution $g_i(D_i)$ to the inferred worth for each group of clients, which provides an incentive to treat this number of clients. Lower and upper bounds DLi and Dui are defined for the demand from clients in group i.

The new demands can then be calculated by finding the D_i and x_{il} to maximise the net inferred worth as follows:

$$\text{Maximise} \quad \sum_i g_i(D_i) - \sum_i \sum_l \left(\sum_k C_k u_{ilk}\right) X_{il}$$

$$\text{subject to} \quad \sum_l x_{il} - D_i = 0 \quad \text{for all } i,$$

$$\sum_i \sum_l u_{ilk} x_{il} \leqslant B_{Nk} \quad \text{for all } k,$$

$$DLi \leqslant D_i \leqslant Dui \quad \text{for all } i.$$

The functions $g_i(D_i)$ need to be defined.

McDonald et al show that the g_i can be represented as follows:

$$g_i\ (D_i) = \frac{\alpha_i}{1 - 1/E_i}\ D_i^{(1 - 1/E_i)}$$

unless $E_i = 1$, when $g_i\ (D_i) = \alpha_i \ln(D_i)$.

The values of α_i are given by the requirement that $g_i'\ (d_i) = \pi_i$, where π_i are the (negative) shadow prices on the demand constraints in the simpler initial model as formulated in 4.2.2 above. It is assumed that the initial model is solved and that the shadow prices $(-\pi_i)$ on the demand constraints are computed.

The constants Ei, elasticity of satisfied demand from group i, are such that, for all values of D_i, a 1% decrease in the marginal inferred worth of treatment corresponds to an E_i% increase in the number of clients treated. The optimisation model will lead to a solution in which the marginal inferred worth $g_i'\ (D_i)$ from taking extra clients equals the opportunity cost, so Ei is regarded as the elasticity of demand with respect to opportunity cost.

4.2.6 Predictive model : second stage

The second stage, extending the model to allow different allocations of resources v_{ilk} to each client, requires the assumption of another set of constant elasticities, F_{ik}, of the actual allocation of resource k to each client in group i with respect to marginal inferred worth (or equivalently to the opportunity cost of the resource). The inferred worth must be defined so that the acceptable allocations u_{ilk} of resources to each client are optimum when resources are unconstrained.

The model becomes : choose the Di, x_{il} and v_{ilk} to:

$$\text{Maximise} \quad \sum_i g_i(Di) + \sum_i \sum_l \sum_k h_{ilk}(v_{ilk})\, x_{il} - \sum_i \sum_l \sum_k C_k\, v_{ilk}\, x_{il}$$

subject to

$$\sum_l x_{il} - D_i = 0 \text{ for all } i;$$

$$\sum_i \sum_l v_{ilk}\, x_{il} \leqslant B_{Nk} \text{ for all } k;$$

$$D_{Li} \leqslant D_i \leqslant D_{ui} \text{ for all } i;$$

$$U_{Lilk} \leqslant v_{ilk} \leqslant U_{uilk} \text{ for all } i, l, k;$$

where U_{Lilk}, U_{uilk} are lower and upper bounds on v_{ilk}, and $h_{ilk}(v_{ilk})$ are functions representing the inferred worth for each client from devoting v_{ilk} units of resource k to each client in group i receiving mode l of care.

The constant elasticity assumption implies that there is an inferred worth $h_{ilk}(v_{ilk})$, where

$$h'_{ilk}(v_{ilk}) = \beta_{ilk}\, v_{ilk}^{-1/F_{ik}}$$

for some β_{ilk},

the values of β_{ilk} being deduced from the assumption that v_{ilk} is the appropriate allocation of resources to each client when the resources are unconstrained and the opportunity cost of each unit of resource is C_k.

This implies that:

$$h'_{ilk}(v_{ilk}) = C_k \left(\frac{v_{ilk}}{u_{ilk}}\right)^{-1/F_{ik}},$$

and hence by integration,

$$h_{ilk}(v_{ilk}) = \frac{C_k\, u_{ilk}}{(1 - 1/F_{ik})}\left[\left(\frac{v_{ilk}}{u_{ilk}}\right)^{(1 - 1/F_{ik})} - 1\right]$$

or, if $F_{ik} = 1$, $h_{ilk}(v_{ilk}) = C_k\, u_{ilk} \ln (v_{ilk}/u_{ilk})$.

In practice, however, the acceptable levels of care u_{ilk} are inconsistent with the actual set of client numbers d_i and the current resources B_k. Consequently the cost-minimisation model is not a feasible means of calibrating the inferred - worth functions $g_i(d_i)$ in the current situation. Instead the appropriate shadow prices $-\pi_i$ on the demand constraints may be obtained from another model, in which the resource use v_{ilk} for each client is variable but the numbers of clients remain constant.

This model requires choosing x_{il} and v_{ilk} to:

$$\text{Maximise} \quad \sum_i \sum_l \sum_k h_{ilk} (v_{ilk}) x_{il} - \sum_i \sum_l \sum_k C_k v_{ilk} x_{il}$$

$$\text{Subject to} \quad \sum_l x_{il} = d_i \quad \text{for all } i;$$

$$\sum_i \sum_l v_{ilk} x_{il} \leqslant B_k \quad \text{for all } k;$$

$$U_{Lilk} \leqslant v_{ilk} \leqslant U_{uilk} \quad \text{for all } i,l,k.$$

As McDonald states, the elasticity factors Fik are the only undetermined factors in this model and these have to be estimated, with the additional requirement that for the current resource availabilities all resources should be used, so that the standard of care delivered to clients using reducible resources is as high as possible.e.g. Fik = 0 implies that the standard of care for this i,k is fixed at u_{ilk}, since if $v_{ilk} < u_{ilk}$ then $h_{ilk}(v_{ilk}) \rightarrow -\infty$ as $F_{ik} \rightarrow 0$.

4.2.7 Use of the model

I.L. Coverdale[15] and Negrine, in their paper "The Balance of Care Project" discuss the use of the Balance of Care model. They distinguish between "groups" and "categories" of clients where "groups" refer to the types of clients who are normally grouped together for DHSS planning purposes, e.g. elderly, mentally ill, etc. and "categories" refer to those clients who make similar demands on services. In the preceding discussion, the word "group" has been used to mean the sub-classification of a category within a DHSS - defined group. For planning purposes it may be useful to be able to collate the needs of a set of clients belonging to several categories or one DHSS - defined group, but it was not considered necessary for the purposes of the preceding discussion. In this paragraph, however, the nomenclature of group and category will be adopted.

Coverdale and Negrine state that the Balance of Care model has been used to aid planning at national level, to analyse alternative strategies and to examine their consequences. At a local level the model has been used to aid planning in Devon, where it contributed to the joint planning activity between the health authority and the local authority social service department and it has been used to suggest areas where joint financing may be appropriate.

Coverdale and Negrine describe the provision of care in terms of coverage, modal balance and quotas. Coverage, c_i, is the ratio of the number of clients receiving care to the number of potential clients in a group.

Modal balance, x_{il}, is the number of clients in different modes of care. A quota, q_{jk}, is the ratio of the allocation of a particular reducible service to that which would be required if desirable levels of service were to be reached (for that modal balance).

This version of the model is as follows:
Choose that pattern of care (coverage c_i, modal balance x_{il}, and quotas q_{jk}) which will:

$$\text{Maximise} \quad \sum_j \sum_k b_{jk} h_{jk} (q_{jk}) + \sum_i D_i g_i(c_i) - \sum_j \sum_k C_k a_{jk}$$

Subject to:

$$b_{jk} = \sum_{i \in j} \sum_l U_{ilk} x_{il} \quad \text{for all j and k,}$$

$$a_{jk} = q_{jk} b_{jk} \quad \text{for all j and k,}$$

$$\sum_j a_{jk} \leqslant B_k \quad \text{for all k,}$$

$$\sum_l x_{il} = D_k c_i \quad \text{for all i.}$$

In this representation the subscript j is used to represent the DHSS-defined group whilst i is used to represent the category within the group.

Then b_{jk} represents the desired amount of service k for all categories in group j and a_{jk} represents the actual amount of service k supplied to group j.

Thus $a_{jk} = q_{jk} b_{jk}$ implies that the actual service supplied is some quota q_{jk} of the desired service level b_{jk} and $\sum_j a_{jk} \leqslant B_k$ constrains the total actual amount of service k supplied to be within the available amount B_k.

The constraint $\sum_l x_{il} = D_i c_i$ implies that each person receiving care must be in one of the modes. c_i is the coverage within group i and D_i is the number of potential clients in category i.

In the objective function to be maximised,

i.e. $\sum_j \sum_k b_{jk} h_{jk}(q_{jk}) + \sum_i D_i g_i(c_i)$

$- \Sigma\Sigma \; C_k a_{jk}$,

h and g need to be calibrated. The g functions represent the worth of caring for clients at desirable service levels and the h functions represent the penalty of falling short of these levels.

This objective function is directly comparable with McDonald's such that

- the number of clients serviced is expressed in terms of coverage and potential number of clients.
- the level of service supplied is expressed in terms of quotas and number of clients receiving each mode of care.

Coverdale and Negrine describe the calibration of the h function by assuming that it is worth improving quotas up to 100%, at which point service allocations will be at the desirable level and no penalty will be incurred. They show that the h function should reflect diminishing returns as the quota or amount of service supplied to a group increases. Thus a concave curve is required and an inverse power form is what was chosen.

i.e. $h_{jk}(q_{jk}) = \psi_{jk} (1 - q_{jk}^{-\phi_{jk}})$

Where ψ_{jk} and ϕ_{jk} are parameters to be estimated. This defines a curve with diminishing returns to scale whenever $\phi_k > -1$ if ψ_{jk} and ϕ_{jk} have the same sign. This is essentially the same form as that used by McDonald et al.

To calibrate the g function, it is assumed that, up to the number of potential clients, there will be a benefit in treating extra clients and that there will be decreasing returns to scale, the more pressing cases receiving care in preference to others. Once again a diminishing return form is needed. This time, instead of using an inverse power form as did McDonald et al, Coverdale and Negrine used a quadratic form:

$$g_i(c_i) = \xi_i c_i^2 + \eta_i c_i,$$

where ξ_i and η_i are parameters to be estimated.

They comment that calibration of the parameters proved to be difficult and that this may be an inevitable consequence of modelling the choice of service allocation which exists within the Health and Personal Social Services system. (HPSS)

They state however that the model has been useful in clarifying the interrelationships between the different parts of the HPSS and in quantifying the consequences of adopting various strategies.

4.2.8 Predictive model: comments

Although McDonald illustrates in an appendix how his predictive model can be simplified mathematically for computational purposes, it is clear that the complete model (of McDonald or of Coverdale and Negrine) is not easy to use and that it requires a lot of data additional to the basic cost minimisation model.

R.J. Gibbs[30], in his paper "The Use of a Strategic Planning Model for Health and Personal Social Services" discusses the following questions encountered in the Balance of Care study:

(i) Given that the objectives of the service cannot be satisfactorily quantified and final outputs reliably measured, how far can modelling proceed with intermediate or surrogate output measures?

Gibbs suggests that the modeller should be well satisfied if he can construct a model that links inputs to the service with any intermediate surrogate measures of outputs that are available.

(ii) How much should the model take into account the ideal standards which the service would seek to attain in its various activities if resources were not constrained and how much, on the other hand, should the model take account of current practice?

Gibbs suggests that in order that the model should be capable of exploring the consequences of major changes to the status quo rather than simply marginal ones, the modeller must accept the necessity to take some account of the ideal behaviour of the service rather than simply represent current practice. In doing this he should not be reluctant to collect judgemental data for aspects of the service from personnel working in the field.

(iii) Is the use of a comprehensive strategic model by the central authority incompatible with a reasonable degree of devolution of responsibilities to field authorities?

Gibbs suggests that the use of such a model is not incompatible provided that the decision variables of the model correspond to the variables under the central authority's control and that the model represents how the field authorities, within the limits of their autonomy, are likely to respond to different central decisions.

(iv) If both the field authorities and the personnel who are directly concerned with delivering the service have a significant degree of autonomy from the central authority, whose preferences and priorities should be incorporated in the model?

Gibbs suggests that, following from (iii) above, the model should incorporate the de facto priorities and preferences that are employed by service personnel in the field even if these are at variance with those of the central authority.

(v) Should the model be of an optimising nature or of a simulation ("what-if") type?

Gibbs suggests that, again from (iii) above, the model should be of the what-if, simulation type so that it can be used to test out alternative policy options in an interactive mode with central planners.

(vi) Assuming that the planning process is a form of dialogue between the central and field authorities, how should the design of the model take account of the terms in which the dialogue is conducted?

Gibbs suggests that the model may enrich the quality of the planning dialogue between central and field authorities by providing a set of connecting logic between service inputs and intermediate outputs.

4.2.9 Summary

The Balance of Care model, as described by McDonald et al, by Coverdale and Negrine, and by Gibbs, was difficult to calibrate but was useful in quantifying the consequences of adopting various strategies.

4.2.10 Further developments

The Balance of Care model has been developed further by the DHSS in conjunction with Arthur Andersen & Co. Section 4.4 describes this development.

4.3 The D.U.B.S. Goal Programming Model

The D.U.B.S. goal programming model was proposed by L. Wiper[60] in "Model for a Resource Allocation Decision Aid", one of the papers in the set "Design for a Social Services Planning System", edited by E.G. Nelson[43].

The model is defined as follows:

$$\text{Minimise} \quad \sum_i \sum_k (T_{ik} - A_{ik}) W_{ik}$$

$$\text{subject to} \quad \sum_i A_{ik} \leq B_k, \text{ for all } k;$$

where i,k, A,B,T,W are as defined in figure 4.1.1 and

$\sum_k W_{ik} = 1$, for all i, is a condition of the data.

This model is of goal programming type using linear programming to find the optimum solution. Target levels of service are set for each client group and "distance from target" (i.e. the sum of the differences between allocation and target) is minimised.

The procedure is as follows: for each client group i a target "package" of services T_{ik} is defined which will meet all their requirements in terms of "need" for Social Services. Next, a weight W_{ik} is attached to each service k for each client group i indicating the contribution it makes to the package of services for that client group, i.e. the proportion of the clients' needs that is met by the provision of that target level of service. In addition to defining targets and service weights, the resource constraints B_k are also defined. The problem is then stated as being to minimise the sum of shortfalls from targets, weighting for contribution of service, and subject to the resource constraints.

As presented by Wiper, the objective function minimises the sum of weighted differences $(T_{ik} - A_{ik}) W_{ik}$. In theory, these differences could be both positive and negative and a true goal programming model would minimise the sum of the absolute weighted differences $|T_{ik} - A_{ik}| W_{ik}$.

The goal-programming model had been neither tested nor implemented by the D.U.B.S. team.

It was decided that, as part of the current research, the model should be investigated.

It was soon realised that the goal-programming model proposed is in practice identical with a simpler linear programming model. This is because the targets of the goal-programming model are ideals and would never in reality be exceeded, i.e. the actual service allocations will always be less than or equal to the target allocations. Thus minimising the total of the absolute differences between the service allocations and the targets is equivalent to maximising the total service allocations subject to upper bound constraints which are themselves the targets. This linear model is described in Chapter 5.

4.4 The D.H.S.S./A. Andersen Model (SPRAM)

4.4.1 Introduction

The original Balance of Care Model described in Section 4.2 can be described as an "inferred worth" model in which the underlying behavioural assumption is that "field workers act in a way which produces the same results as if they were maximising a joint utility function of a given mathematical form. It is possible to infer the parameters of this function from data on the current proportions of resources received by different sets of clients", (D.H.S.S.[26]/A. Andersen, 1980). The model was initially developed and applied at the national level but was also used at the local level. The consultants Arthur Andersen & Co. were commissioned in 1979 to examine how the Balance of Care approach could be modified to make it more readily applicable at the local level.

The new model was therefore developed jointly by the D.H.S.S. and Arthur Andersen and, whilst retaining the data framework of the original Balance of Care Model, has a completely different mathematical structure. This is both less complex for the planners to understand and more easily implemented at the local level, thus reducing the need for D.H.S.S. Operational Research Service involvement in local application of the Balance of Care Model.

4.4.2 Framework of the D.H.S.S./A. Andersen Model

The data framework of the original Balance of Care Model was retained in the new model : the population of potential and actual clients being divided into client groups (about 6 of these) and categories within client groups (about 15 categories per group). Several alternative equivalent packages or modes of care may be defined for each category. The packages use resources which consist of the various health and personal social services.

Much emphasis is placed (D.H.S.S.[25], 1981) on the broader aspects of the Balance of Care approach : that it is primarily an approach to strategic planning; that it demands a substantial commitment to manage its implementation and that it concentrates on the broader policy issues of the effective use of resources. The Balance of Care approach assumes that at some planning level there are significant amounts of money which the planners can switch from one sector or service to another. Data is organised by the model, professional field workers give advice and planners take the decisions.

A recommendation is made as to the organisational structure which is needed to make best use of the Balance of Care approach. This includes (1) a Joint Management Team (J.M.T.) to provide overall direction and control of the project; (2) the

Professional Advisory Groups (P.A.G.s), who provide professional input, initially agreeing on a specific category structure with which to organise the data, then consolidating the data structure subsequent to the data survey, advising on appropriate alternative modes of care for clients in each category and advising on the ideal levels of service required by each category of client in each mode of care; and (3) the Project Team for the day-to-day running of the project.

4.4.3 Structure of the Model

The Model is described (D.H.S.S.[25,26]) as having six segments, each of which is run independently although an iterative procedure is suggested. The six segments perform the following:

1. Gives current standards of care.
2. Gives resources needed in future to maintain current standards, given expected demographic changes.
3. Gives future patterns of care, given current policies and amounts of resource likely to be available.
4. Gives future patterns of care arising from changes in policies about resource allocation.
5. Analyses which resources are "best-buys".
6. Analyses the "best" ways to allocate a given set of resources.

FIGURE 4.4.3.1 ITERATIVE PROCEDURE FOR D.H.S.S./A. ANDERSEN MODEL

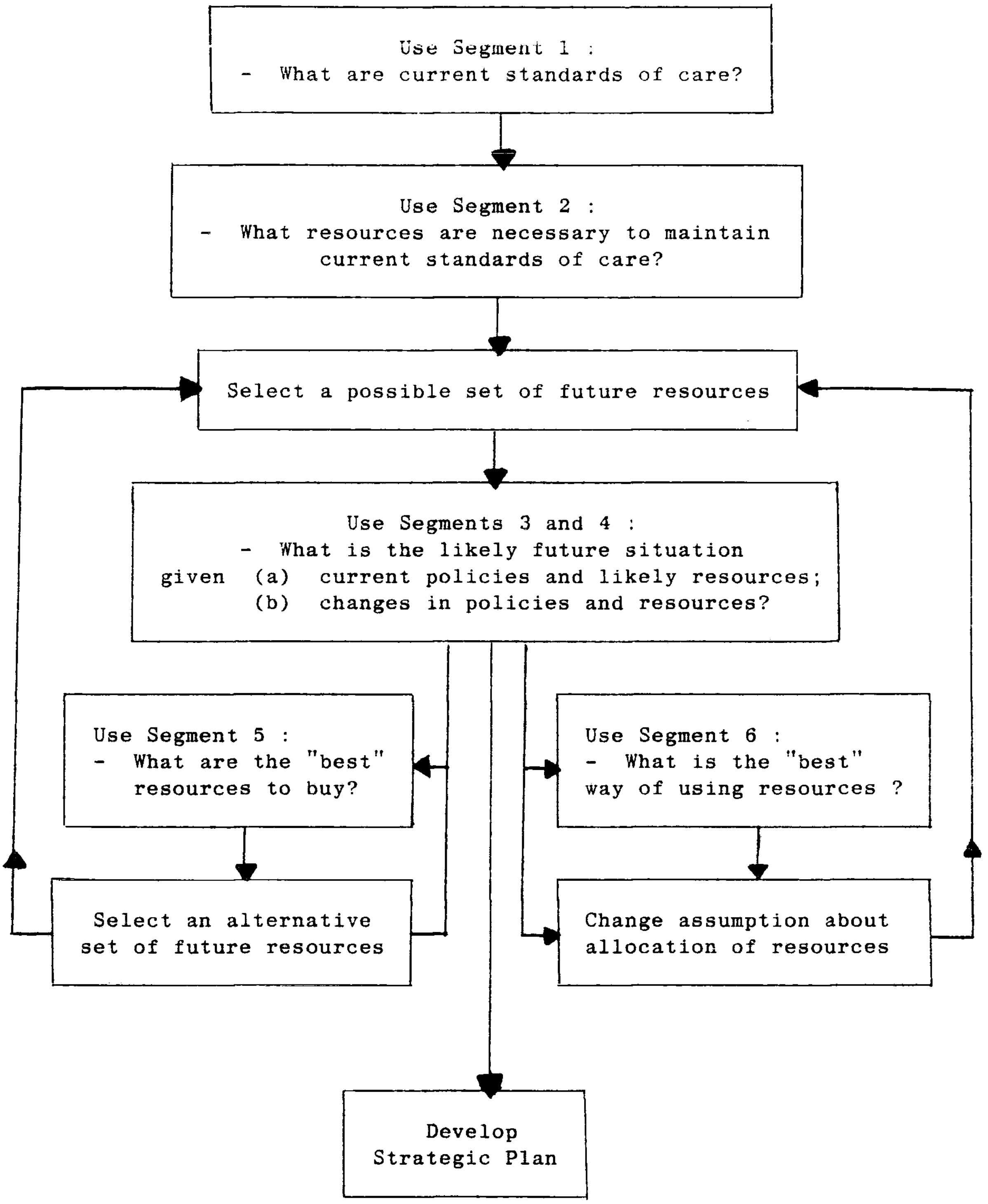


Figure 4.4.3.1 shows the iterative procedure suggested.

Each of the segments is now described.

4.4.4 Segment 1

Segment 1 collates survey data and groups the data according to the categories devised by the P.A.G.'s. Summarises the current allocation of care in terms of the numbers of clients of different types and in different groups and categories receiving care, and the standards of the care they are receiving.

Measures of quota, coverage and ideal package equivalents are used.

Quotas are the average amounts of resources provided to those people receiving care, expressed as percentages of the "ideal" levels set by the P.A.G.

I.e. If the clients in mode l of category i receive a total of r_{ilk} units of resource k and the ideal is U_{ilk} units per person, the quota q_{ilk} is defined by:

$$q_{ilk} = \frac{r_{ilk}}{x_{ilk} U_{ilk}}$$

Here the amounts r_{ilk} and x_{ilk} are obtained from the survey data and the U_{ilk} are defined by the P.A.G.

Coverage is the number of people receiving care expressed as a percentage of the number of potential clients.

I.e. $c_i = \frac{x_i}{d_i}$ where c_i is the coverage in category i,

x_i is the number of clients in category i who are receiving care, and

d_i is the number of clients in category i who are in need of care.

Here the amounts x_i are obtained from the survey data and the d_i are calculated by the project team using available, non-survey data.

"Ideal package equivalents" are used for comparisons of resource provision. They represent the cost of providing a package of care at current standards expressed as a percentage of the cost of providing that package at ideal standards.

I.e. $IPE_i = \sum_l \left(\frac{\sum_k C_k r_{ilk}}{\sum_k C_k U_{ilk}} \right)$ represents the number of ideal package equivalents provided for category i, where C_k is the cost per unit of resource k,

r_{ilk} is the amount of resource k allocated to mode l of category i, and

U_{ilk} is the ideal amount of resource k per client for mode l of category i.

Here the amounts r_{ilk} are obtained from the survey data and C_k, U_{ilk} are defined by P.A.G.

4.4.5 <u>Segment 2</u>

Segment 2 projects forward the current use of resources by increasing them in the proportion by which the population of potential clients is expected to grow in order to give an estimate of the total future requirements by resource type.

Although the calculation method is not detailed in the D.H.S.S.[5,6] publications, it seems likely that the increases are calculated as follows:

$$B'_k = \sum_i r_{ik} \cdot \left(\frac{d'_i}{d_i} \right)$$

where r_{ik} is the total amount of resource k currently allocated to clients in category i,

d_i is the current size of the client population in category i, and

d'_i is the future size of the client population estimated for category i,

and B'_k is the estimated future requirement for resource k.

Quotas, coverage and ideal package equivalents are calculated in the same way as in Segment 1.

4.4.6 <u>Segment 3</u>

Segment 3 simulates the allocation of future resources given the total availability of each resource and information on geographic availability. Allocation is done in hierarchical fashion whereby resources are allocated to geographic units (g), followed by client group (j), followed by categories (i) and finally modes (l). Lastly the effects on quota and coverage are determined.

The S.P.R.A.M. (Simple proportional resource allocation model) method of allocation is used throughout with "pressures" due to population changes.

For each resource, k, the allocation is thus:

$$r'_{gk} = \frac{B'_k \left(\frac{d'_g}{d_g} \cdot r_{gk}\right)}{\sum_h \left(\frac{d'_h}{d_h} \cdot r_{hk}\right)}$$

where r_{gk} is the current allocation of resource k to geographic unit g,

d_g is the current size of the client population in geographic unit g,

and a prime ($'$) denotes the future size of a variable, so that B'_k is the future amount of resource k available.

This allocation is derived from the fundamental assumption that "field-workers' behaviour is such that, if populations of geographic units remain the same but resource levels change, they give a constant proportion of each resource to each geographic unit." If, additionally, populations change, then the further assumption is made that "these above proportions are scaled correspondingly in response to the population 'pressures'."

The proportional allocation is shown as follows:

If d_g geographic units of population (at ideal cover) receive r_g amounts of a resource r and the populations of these geographic units change demographically over a period of time to d'_g and the resource availability changes from $\sum_g r_g = R$ to R', then the amounts of resource received, r'_g will be such that:

$$\frac{r'_g}{r'_h} = \frac{r_g}{r_h} \cdot \frac{(dg'/dg)}{(dh'/dh)} \text{ for all g,h.} \_\_\_\_\_\_ (1)$$

Hence $\frac{r'_g}{r_g} \cdot \frac{d_g}{d'_g} = \frac{r'_h}{r_h} \cdot \frac{d_h}{d'_h}$ = constant, K for all g,h

So $r'_h = r_h \cdot \frac{d'_h}{d_h} \cdot K$ and $\sum_h r'_h = R'$,

hence $K = \frac{R'}{\sum_h \left(\frac{d'_h}{d_h} \cdot r_h\right)}$

Thus $r'_g = \frac{R' \left(\frac{dg'}{dg} \cdot r_g\right)}{\sum_h \left(\frac{dh'}{dh} \cdot r_h\right)}$

Applied to resource k having a future availability $B'k$,

this gives $r'_{gk} = \frac{B'k \left(\frac{dg'}{dg} \cdot r_{gk}\right)}{\sum_h \left(\frac{dh'}{dh} \cdot r_{hk}\right)}$ __________ (2)

The same type of allocation procedure is used for the next level of allocation, i.e. to client groups j. Thus the amount $r'_{g.k}$ of resource k allocated to geographic unit g is now shared out amongst the client groups in that geographic unit.

i.e. $$r'_{jgk} = \frac{r'_{gk} \left(\frac{dj'}{dj} \cdot r_{jgk} \right)}{\sum_{J} \left(\frac{dJ'}{dJ} \cdot r_{Jgk} \right)}$$

Here j represents a particular client group and J represents each client group in turn.

Further allocation of resources to categories and modes is performed only for those client groups of "primary planning interest".

The same allocation procedure is used for client categories, i, so that the amount r'_{jgk} of resource k allocated to client group j within geographic unit g is now shared out amongst the categories in that client group.

i.e. $$r'_{ijgk} = \frac{r'_{jgk} \left(\frac{di'}{di} \cdot r_{ijgk} \right)}{\sum_{I} \left(\frac{dI'}{dI} \cdot r_{Ijgk} \right)}$$

Finally for modes of care, l, the amount r'_{ijgk} of resource k allocated to client category i within group j and geographic unit g is now shared out amongst the modes in that client category. In this case the allocation depends upon the definition of a dominant resource in each mode of care. The selection of the dominant resource must be made by the planners for each mode of care available for each category of client.

If Δl is the amount of the dominant resource currently allocated to mode l for category i and Δ_L is the amount of the same resource currently allocated to mode L, then using the SPRAM method,

$$\Delta'_l = r'_{ijgk} \frac{\Delta l}{\sum_L \Delta_L}$$

Here Δ'_l represents the future allocation of the dominant resource k for mode l of category i, group j and geographic unit g.

As far as possible the other (non-dominant) resources will be allocated to that mode in proportion to their past allocation weighted by the increase in availability of the dominant resource. The intention is to maintain balanced packages of care.

Thus, if M_{kl} is the amount of the K^{th} non-dominant resource currently allocated to mode l, then the future allocation will be:

$$M'_{kl} = \frac{r'_{ijgk}\left(\frac{\Delta l'}{\Delta l} \cdot M_{kl}\right)}{\sum_{L}\left(\frac{\Delta L'}{\Delta L} \cdot M_{kl}\right)}$$

The allocation procedure thus limits the degree of interaction between different components of the model. The model is acknowledged to be inaccurate because it ignores certain types of interactions and pressures (See Sections 4.6.8, 4.6.9), but its modular structure would permit more complex procedures to be incorporated.

4.4.7 Quotas and Coverage

The allocations of resources to geographic units, groups, categories and modes of care have thus far been measured in units of each individual resource. These resources now need to be shared amongst the clients to determine the number of clients treated and the standard of care received. At one extreme is a fixed standard of care with the number of clients treated being calculated from the resource availability so that the coverage depends upon the amount of resource allocated. At the other extreme is a fixed coverage so that a fixed proportion of clients receive the service and the standard of care given depends upon the amount of resource allocated.

A quota/coverage trade-off parameter λ_k is used to specify the required trade-offs for each resource k. A value of $\lambda_k = 1$ implies that all clients in a mode will receive some amount of resource k whereas a value of $\lambda_k = 0$ implies that some clients will receive the full standard of care of resource k whereas the remaining clients in that mode will receive none of resource k. The planners need to specify the value of λk for each resource.

The allocation is thus:

$$\frac{u'_{ikl}}{u_{ikl}} = \left(\frac{r'_{ikl} / r_{ikl}}{d'_i / d_i} \right)^{\lambda k}$$

and

$$\frac{x'_{ikl}}{x_{ikl}} = \left(\frac{r'_{ikl} / r_{ikl}}{d'_i / d_i} \right)^{(1-\lambda_k)} \cdot \left(\frac{d'_i}{d_i} \right)$$

where u_{ikl} , u'_{ikl} are the average amounts of resource k allocated currently and in future respectively to clients in category i and mode l;

x_{ikl} , x'_{ikl} are the numbers of clients in category i and mode l currently and in future respectively receiving resource k;

r_{ikl}, r'_{ikl} are the amounts of resource k allocated currently and in future respectively to clients in category i and mode l;

d_i, d'_i are the population sizes of category i currently and in future respectively;

and $0 \leqslant \lambda_k \leqslant 1$.

From this allocation it can be seen that the coverage of resource k for category i in future will be

$$c'_{ik} = \frac{\sum_{l} x'_{ikl}}{d'_i}$$

The extreme case of $\lambda_k = 0$ (coverage changing at fixed standards) leads to the allocations:

$\frac{u'_{ikl}}{u_{ikl}} = 1.0$ (i.e. Average allocation per client remains fixed)

and $\frac{x'_{ikl}}{x_{ikl}} = \frac{r'_{ikl}}{r_{ikl}}$ (i.e. No. of clients receiving resource k varies in proportion to the amount of the resource allocated.)

At the other extreme, when $\lambda_k = 1$ (standards changing at fixed coverage), the allocations are:

$\frac{u'_{ikl}}{u_{ikl}} = \frac{r'_{ikl}/r_{ikl}}{d'_i/d_i}$ (i.e. Average allocation per client varies in proportion to the amount of the resource allocated and to the change in population.)

and $\frac{x'_{ikl}}{x_{ikl}} = \frac{d'_i}{d_i}$ (i.e. No. of clients receiving resource k varies only with change in population.)

For modes which use many resources there is a problem concerning the likely effect on a mode if the availabilities of its constituent resources change by different amounts. In this case one resource is chosen as a dominant resource and the number, x'_{il} $(= x'_{ilk})$ of clients in category i and mode l allocated this resource k is determined using the chosen quota/coverage trade-off parameter. The same number of clients is then allocated the other resources constituting the mode of care. Then the average allocations per client for the non-dominant resources are obtained from the expression:

$$u'_{ikl} = \frac{r'_{ikl}}{x'_{il}}$$

4.4.8 Resource Interaction

Interactions between resources occur when different resources exist which are substitutes and which tend to compensate for changes in the availability of one another. For example, if a category loses a large part of its allocation of hospital beds it may tend to use more day care such as day centre places.

Since the allocation of resources takes place at the level of categories of clients before the level of mode-of-care, the D.H.S.S. model needs to treat the allocation of resources hierarchically so that resources such as hospital beds are allocated before day centre places. The model is equivalent to assuming that the ideal cover for each category is in effect reduced by the number of people cared for by those groups of resources already allocated. This implies that estimates of ideal cover can be made.

In the D.H.S.S.[26] description, four types of resource interaction are discussed: complete, partial, reverse and other. Methods of dealing with complete and partial resource interactions are discussed which do not need estimates of ideal cover to be made. These methods are extensions of the pro-rata arrangements of SPRAM and in essence average out the results of a series of resource interaction effects. No method is given for reverse interactions since it is unlikely to be a major effect but for "other" interactions, mainly between domiciliary resources, the situation is regarded as extremely complex and difficult to model, so that for the pilot model the whole effect is ignored, on the grounds that "the fairly similar 'shapes' of modes across all the categories of the elderly client group mean that these pressures will have little effect on the allocation of care to categories" and that "it is always possible for planners to override the model and impose their own views".

This separation of resources and partitioning of clients in modes of care at the outset characterise the major differences between the SPRAM model and the mathematical programming models such as the author's linear programming model with alternative modes of care described in Chapter 5.

4.4.9 Saturation Effects

Saturation effects are those which occur when a category receives a resource at close to ideal standards and begins to take a smaller share than previously of any additional resource which is made available. Such effects could become important when a category has close to ideal coverage.

In order to take some account of saturation effects, a simple constraint is used to prevent the resource allocations rising above ideal standards. Each group of resources is allocated in turn, for example institutional resources before domiciliary resources. The additional constraint is included: that cover for categories should not exceed the maxima or "ideal" covers, d_i, firstly for the institutional resources. Allocation of the domiciliary resources then follows, with the constraint that the allocation of any domiciliary resource k should not exceed

$$(\max_{l} M_{ikl}) \cdot (d_i - I_i),$$

where I_i is the number of clients allocated to institutional care, and

M_{ikl} is the maximum likely use of the kth resource by a client in the lth mode of category i (probably this would be set to the ideal standard U_{ikl}.)

Depending on the allocation of resources to modes of care within categories, it is possible that there may still be some excess of a domiciliary resource for a category. In this case the total cover would be constrained to di by scaling down the calculated covers in the domiciliary modes as necessary and allowing the quotas to rise in compensation (there being no evidence of ceilings for quotas coming into effect).

Saturation effects are expected to occur only rarely so more complex models have not been developed. A discussion of a possible method is included in the D.H.S.S.[26] description as the "perceived need" hypothesis, which requires estimates to be made of perceived ideal standards and cover for each client category.

4.4.10 Segment 4

Segment 4 of the model allows exploration of the effects on patterns of care of changes in planners' policies as represented by decision parameters and constraints.

Decision parameters represent the policy of increasing or decreasing the proportional resource share of various groupings of clients. The decision parameters can be entered at various levels of the allocation hierarchy: geographic units, client groups, client categories, modes of care. They provide an extra "pressure" which affect(s) the proportional allocation of the model.

Constraints are used where it is felt that the proportional allocation is not appropriate on its own. Constraints can be entered at each level of the allocation hierarchy and can constrain either amounts of resources or quotas, for example a particular client category should not receive more than 14 home help hours per week in total.

The allocation procedure in Segment 4 is the same as in Segment 3 except to take account of the decision parameters and constraints. At each stage the SPRAM method of allocation is used, for example to geographic units, then these allocations are adjusted to satisfy any user constraints. The D.H.S.S.[25,26] descriptions do not describe in detail how the allocations are adjusted. It may be assumed that an iterative procedure is used to compare each of the allocations, e.g. for geographic units, r'_{gk} , with the maximum constrained amount ρ_{gk}

and then to adjust the amounts as follows:

$$r''_{gk} = \frac{r'_{gk}\left(B'_k - \sum_J (r'_{Jk} - \rho_{Jk})\right)}{B'_k}$$

where r''_{gk} represents the adjusted allocation of resource k to geographic unit g,

B'_k represents the total amount of resource k available in future, and

J represents any geographic unit whose initial allocation r'_{Jk} exceeds the maximum constrained amount ρ_{Jk}.

This example is shown for constraints of the maximum type; similar arrangements could be made for minimum type constraints.

The decision parameters are scaling parameters μ_{ik} which can be used to model the effect of trends in the way in which different resources, k, are used for different categories, i, of clients. It is suggested (D.H.S.S.[26]) that this facility is likely to be used only sparingly since "the planners may need considerable specialist assistance in understanding the implications of alternative estimates of the decision parameters α_{ik} in order to choose 'likely' values".

The way in which the decision parameters are used is to take the basic SPRAM assumption, for example equation (1) of Section 4.4.6 for allocation of resource k to geographic units:

$$\frac{r'_{gk}}{r'_{hk}} = \frac{r_{gk}}{r_{hk}} \frac{(dg'/dg)}{(dh'/dh)}$$

and to apply the scaling parameters μ as follows:

$$\frac{r'_{gk}}{r'_{hk}} = \left(\frac{\mu_{gk}}{\mu_{hk}}\right)\left(\frac{r_{gk}}{r_{hk}}\right)\left(\frac{dg'/dg}{dh'/dh}\right)$$

This leads to the allocation equation as follows, comparable with equation (2) of Section 4.4.6:

$$r'_{gk} = \frac{B_k' \left(\mu_{gk} \cdot \frac{dg'}{dg} \cdot r_{gk}\right)}{\sum_h \left(\mu_{hk} \cdot \frac{dh'}{dh} \cdot r_{hk}\right)}$$

Here a value of $\mu_{gk} = 0$ implies that none of resource k should be allocated to geographic unit g ; $\mu_{gk} = 1$ implies that there should be no change in the way resource k is allocated to unit g; $\mu_{gk} = 2$ would imply a doubling of the resource allocation; $\mu_{gk} = \frac{1}{2}$ would imply halving the resource allocation; and so on. Similar decision parameters are applicable at the other allocation levels of client group, category and mode of care.

Another change in professional practice which is given consideration is that of the introduction of a new resource, but in the initial implementations of this model it is assumed that transformation of the data about the current spread of care to represent the effect of including a new kind of resource would be done manually. It is suggested that it would be possible to include an option in the model such that planners could express their use of new resources in specified ways, for example as some average of other resources.

4.4.11 Segment 5

Segment 5 of the model would analyse which resource mixes are "best-buys". Segments 5 and 6 are regarded as "optimisation" segments in contrast to the "simulation" segments 3 and 4. The D.H.S.S.[25,26] descriptions describe a feasible design for Segments 5 and 6 but the model had not at that stage been constructed. The procedure suggested is to compare the amounts of each resource currently being used for each category of client with the total amounts of resources required to treat each client (currently receiving some care) at "ideal" standards at minimum cost. This should indicate which institutional resource, or package of domiciliary resources, would most increase the quotas or coverage of all resources per unit of cost. More cost-effective patterns of care may then be achieved by altering the mix of resources.

Thus for each category of client, the model would calculate (i) the current cost of providing resources, and (ii) for each alternative mode of care, the cost per client of providing "ideal" standards of each resource required by that mode.

The model would then select and report on that resource mix forming the minimum cost mode of care for each client category.

This cost minimisation is said to lead (D.H.S.S.[26]) to options which are no different in practice from "maximising quotas at fixed cost".

The resource mix forming the minimum cost mode of care for each client category would be found in the following way:

For each category of client which can use a given resource, the "saving" of other resources which can be made by using this resource, k, in a particular mode, l, is $S_{lk} = \frac{G - (H - z\, C_k)}{z}$, as follows,

where H = cost per client of the l^{th} mode of care

C_k = unit cost of resource k,

z = number of units of resource k used per client in mode l,

G = cost per client of the cheapest alternative mode which does not use resource k,

hence $(H - z\, C_k)$ = cost per client of the lth mode, excluding cost of resource k, i.e. cost of all other resources;

and $G-(H-z\, C_k)$ = difference or saving of other resources,

and $S_{lk} = \frac{G-(H-z\, C_k)}{z}$ = saving of other resources per unit of resource k in mode l.

The problem is to maximise the saving S_{lk}, but this is the same as minimising $\left(\frac{H-G}{z}\right)$.

Thus the resource k would be allocated to the mode l for which $\left(\frac{H-G}{z}\right)$ is least.

The actual allocations suggested by the optimisation model would be expected to be frequently unattainable in practice but they would provide ideas for where improvements might be made. The simulation model (Segment 3) would then be used to discover what is practicable.

4.4.12 Segment 6

Segment 6 of the model would analyse which are the "best" ways to allocate a given set of resources, so that clients might be re-distributed amongst the modes of care in order to use the most cost-effective ways of using the available resources.

The proposed procedure is to put the resources into a sequence which is based on the cost of treating clients at "ideal" standards in a typical mode using the resource, the first resource in the sequence being that having the highest cost per client. The resources would be allocated in that sequence to those categories whose cheapest mode using other resources has the highest cost. Thus institutional resources, for example, would be allocated to clients in the most dependent categories, i.e. those who would require most care in domiciliary modes.

It is suggested that results of this procedure could be compared with a run of Segment 3 so that planners might identify any major differences between the results and hence any major opportunities for improving the cost-effectiveness of patterns of care.

4.4.13 Summary

It is clear that this model provides an easily understandable means of allocating resources amongst the various groupings of clients. The method is distinctly different from that used in the original Balance of Care model (see Section 4.2), being based on the Simple Proportional assumption (SPRAM). The D.H.S.S.[25,26] and Arthur Andersen authors are confident that its results are as good as the original model and will be much easier to implement. Since the model is dependent on current care patterns continuing, the allocation of clients to modes of care appears to be restrictive : unless the planners choose to re-assign clients to different modes of care, the proportional allocations of resources to modes of care remain unchanged. This implies a degree of inflexibility which is not present in those models which transfer clients between alternative modes of care when the resource levels are insufficient for care to continue in the initial modes of care.

The amount of resource allocated to a particular mode of care can vary according to the resources available : it is then up to the planners to choose the balance of quota and coverage which results in the individual care given to each client. The implication here is that not only can the levels of care given to a particular mode alter : so also can the level of care between one client and another in the same mode. This is different from the specification in some of the other models (e.g. the linear programming model with alternative modes of care (see Chapter 5) or the decision algorithm model (see Section 4.5) where the definition of each mode of care implies a certain fixed standard of care or allocation of resources to each client placed in that mode. In these specifications the level of care given to a particular mode does not alter : instead the number of clients allocated to a mode may be altered if it is not possible to allocate sufficient resources to care for all clients in the mode at the fixed standard for that mode. Nevertheless, it would still be possible for social workers to vary the amount of care received by individual clients, i.e. to choose an appropriate balance of quota and coverage, since the allocation is made in terms of number of clients to each mode of care together with the required level of resources to treat those clients in that mode.

4.5 The Decision Algorithm Model

4.5.1 Introduction

In parallel with the development of the DHSS Balance of Care models described in sections 4.2 and 4.4 above, the National Health Service Operational Research Group (now known as the Health Operational Research Unit) was investigating the care of the elderly in Calderdale. Results of the investigation are described in "Assessing Care Requirements of Elderly People" published by the Health O.R. Unit[31]. A more detailed description is provided by R.G. Howell[32] and C.J. Parker in "Assessing Care Requirements of Elderly People in Calderdale".

Initially the investigation had included an extensive survey of all elderly people in Calderdale who were receiving some form of health or local authority care at a point of time in 1977. The survey was undertaken by professional staff who were involved in delivering care to the elderly and it was necessary to develop a classification system, for assessing elderly people's requirements for care, which could be used by all the professional staff irrespective of their discipline.

Two aspects of the work are described by Howell and Parker : part 1 describes the use of the classification system in the formulation of policies and plans for the care of elderly people in their own homes and in residential care; part 2 describes the use of the system at the operational level in Todmorden, as a basis for ensuring consistent service allocation and placement decisions.

The classification system was based on seven factors: four factors concerning the physical health of the elderly person, viz.

- mental state,
- ability (to undertake househould and personal care),
- mobility,
- incontinence,

and three factors which describe the home environment, viz.

- housing,
- social environment,
- social support from family/neighbours.

For each factor a range of between two and four possible ratings was defined, for example mental state could be rated as

(a) Normal

or (b) Mild dementia

or (c) Moderate/Severe dementia

Figure 4.5.1.1 shows the range of possible ratings for each factor. It is possible to generate 4608 different groups by combining

Figure 4.5.1.1. Range of possible ratings for each factor

I PHYSICAL FACTORS

Mental State	a.	Normal
	b.	Mild dementia
	c.	Moderate/severe dementia
Ability	1.	Unimpaired
	2.	Impaired and minor handicap
	3.	Appreciable - severe handicap
	4.	Very severe handicap
Mobility	A.	No difficulty
	B.	Can get out of house, with aids, or with difficulty/assistance
	C.	Housebound, but can get around house
	D.	Chairfast or bedfast
Incontinence	1.	Continent
	2.	Slightly incontinent
	3.	Moderately incontinent
	4.	Severely incontinent

II ENVIRONMENTAL FACTORS

Physical Environment	G.	Good housing
	P.	Poor housing
Social Environment	A.	Living alone
	S.	Living with spouse only
	O.	Living with others
Social Support from Family/ Neighbours	E.	Either fit person living in same house or someone prepared to visit every day
	W.	Someone prepared to visit weekly
	L.	Someone prepared to visit less than weekly
	N.	No-one prepared to visit at all

all the ratings on each of the seven factors. Clearly this is too large a number of groups to consider individually, so the work focussed on reducing the number to a reasonable level, whilst retaining sufficient detail to discriminate between the groups, so that an assessment could be made of the care requirements of typical members of each group.

4.5.2 The Decision Tree

A Steering Group had been set up in Calderdale including among its membership staff in the Health and Local Authorities, representing the different professional interests, and technical support staff from the NHS O.R. Group. The professional staff were all involved in providing care for elderly people and included a general practitioner, a housing officer, a community medicine specialist, a geriatrician, a district nursing officer and a health authority administrator as well as social workers.

The Steering Group met to discuss the best way of reducing the 4608 possible combinations to a reasonable number of distinct groups, such that within each group the people would have similar care requirements.

The method chosen was to view the classification system as a decision tree, a collection of paths that branch at various points. Each of the branching points was related to a decision on one of the factors of the classification system. Some factors and their ratings were combined and some were eliminated. The procedure involved identifying those characteristics of greatest significance in discriminating between different care requirements and discarding those of less significance.

Figure 4.5.2.1 illustrates the decision tree produced. The result of producing the decision tree was to reduce the number of groups to just 13. This reduction was achieved by considering, for each factor in turn, the relative importance of individual ratings in relation to the appropriate type of care needed by the elderly person. The intention was to identify the key factors in determining the forms and levels of care that are appropriate to the different groups of elderly people. The easiest way to reduce the number of groups from 4608 was to eliminate all detail that was unnecessary when recommending the appropriate care. For example, mental state could be classified in three ratings, a, b and c but a rating of c would require a person to be cared for in a different setting from a person with a rating of either a or b. Thus three choices were reduced to two: mental state rating "c" and "not c" gave sufficient discrimination in that factor.

For the "ability" factor, all four ratings A,B, C and D were needed for the purposes of discrimination.

For the "mobility" factor, it was found unnecessary to discriminate between ratings B and C, so only three separate ratings were needed : A, B/C, D.

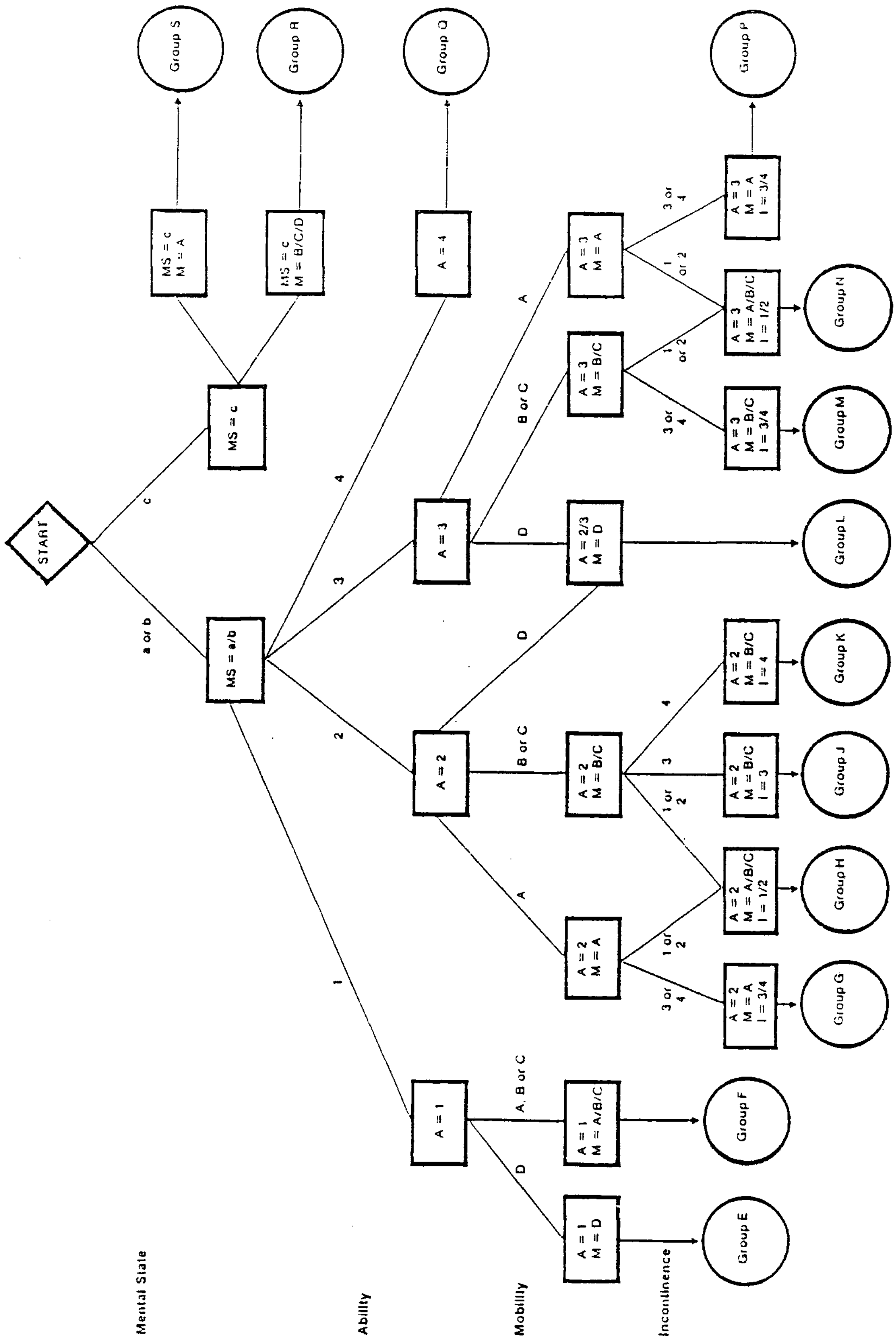
For the "incontinence" factor, only three separate ratings were needed: 1/2, 3, 4.

The four "physical health" factors, with ratings as described, were found to be sufficient to discriminate between the groups requiring different forms of care. The Steering Group considered initially that the "environmental" factors were not necessary to identify appropriate care. It was later decided to introduce one of these, namely Social Support.

When the decision tree was being set up, the most dominant factor was thought to be Mental State. Within the rating C for Mental State the next most dominant factor was considered to be Mobility and within this factor the care required for those with rating A (i.e. unimpaired mobility) would be different from the others. This resulted in two groups with rating C for Mental State and either A or "not A" for Mobility. No other factors were considered to be significant enough to influence the type of care needed and thus groups R and S were identified.

Similarly, the remainder of the decision algorithm was constructed.

Figure 4.5.2.1 The Decision Tree: Allocation of Clients to Groups



4.5.3 The Services Required

The Steering Group was then required to agree the appropriate forms of care for people in each of the defined groups. A number of different schemes of care were considered and in each case the resource "care setting" needed to meet the scheme was calculated and compared with the resource available in Calderdale. "Care Setting" could be the person's own home, an aged people's home, sheltered housing, a geriatric ward or a psychiatric ward. Thus the effect of alternative planning decisions could be assessed.

This initial assessment of "care setting" resources was followed by calculations of the mix of care services that would be required for each of the three schemes. Since members of each group were not all receiving the same services, a package of care services was defined for each group as a whole. An "acceptable package of care" was assumed to be the level of service per capita, received by those members of the group who were already in the care setting considered appropriate for them. For example, only 425 elderly people were living in sheltered accommodation compared with the 2670 recommended for it in one of the schemes. The assumption made was that all 2670 people should receive the same level of service per capita as the 425 received at the time of the survey in 1977.

The services considered were:

- day hospital
- day care
- street warden
- nursing visits
- home help
- meals on wheels

Comparisons of total requirement and actual provision were made for each of three different schemes of care:

Scheme 1 relied heavily on residential care and required more places in geriatric wards and sheltered housing than were actually available.

Scheme 2 was based on the policy that only those people who were living alone would be recommended for residential care and this led to a significant reduction in the required number of places in each residential care setting. In practice the actual number of places available was more than the number required by Scheme 2.

Scheme 3 used a refinement of Scheme 2, taking account of the day to day placement decisions taken by field workers which meant that a proportion of the people would still be recommended for residential care even if they were not living alone. In this case the recommended resources were found to be very close to what was provided at the time of the survey.

4.5.4 A Mathematical Representation

Although Howell and Parker present their model simply as a decision tree and descriptive algorithm it is possible to represent their allocation model mathematically, as follows:

Scheme 1

Let i = group number ($i = 1,\ldots,13$),

di = group size,

l = care setting identifier for current setting,

and of these, L = "appropriate" care setting identifier.

Let x_{il} = number of clients in group i and care setting l;

x_{iL} = number of clients in group i in appropriate care setting L;

a_{ilk} = number of clients in group i, care setting l currently receiving service k;

a_{iLk} = number of clients in group i and the appropriate care setting L who are receiving service k.

Under the "current" caring allocation, if u_{ilk} is the amount of service k allocated per person in group i and care setting l,

$$u_{ilk} = \frac{a_{ilk}}{x_{il}} \quad \text{for all } i,l,k$$

and in particular,

$$u_{iLk} = \frac{a_{iLk}}{x_{iL}} \quad \text{for all } i,k$$

The model assumes that <u>all</u> clients in group i should be allocated to one "appropriate" care setting, L, so that:

$$x_{iL}^{(new)} = di \text{ for all } i$$

and $x_{il}^{(new)} = 0$ for all $l \neq L$ and all i

The model also assumes that the allocation of other services should be in the same proportions as currently allocated to persons in the appropriate care setting.

Thus $u_{iLk}^{(new)} = u_{iLk} = \frac{a_{iLk}}{x_{iL}}$ for all i and k

Thus the total service requirements are $B_k^{(new)}$, where:

$$B_k^{(new)} = \sum_i \sum_l x_{il}^{(new)} u_{ilk}^{(new)}$$

for all k

<u>or</u> $B_k^{(new)} = \sum_i d_i \frac{a_{iLk}}{X_{iL}} = \sum_i d_i u_{iLk}$ for all k.

<u>Schemes 2 and 3</u>

In these schemes the "appropriate" care setting is not the same for all persons in the group. Instead, those persons currently living alone were given priority for residential care in Scheme 2 and about half of those currently living with relatives were given some form of residential care in Scheme 3.

In the mathematical model described above, the "appropriate" care setting, L, represented only one care setting for each group. For schemes 2 and 3 the model must include a range of possible care settings, L_1, L_2, etc.

Then $x_{iL_1}(\text{new}) = \Sigma \quad x_{il}$,

for all l contained in group i being allocated to setting L_1, and for all i,

$$x_{iL_2}(\text{new}) = \Sigma \quad x_{il}, \text{ etc.}$$

and $\sum_{\text{all } L} x_{iL}^{(\text{new})} = di$ for all i

Once again assuming that the allocation of other services should be in the same proportions as currently allocated to persons in the appropriate case setting,

then $u_{iLk}^{(\text{new})} = u_{iLk} = \dfrac{a_{iLk}}{x_{iL}}$ for $L = L_1, L_2, \ldots$ etc. and for all i and k

Thus the total service requirements are $B_k^{(\text{new})}$, where:

$$B_k^{(\text{new})} = \sum_i \sum_{\text{all } L} x_{iL}^{(\text{new})} u_{iLk}^{(\text{new})} \text{ for all k}$$

<u>or</u> $$B_k^{(\text{new})} = \sum_i \sum_{\text{all } L} x_{iL}^{(\text{new})} \frac{a_{iLk}}{x_{iL}}$$

$$= \sum_i \sum_{\text{all } L} x_{iL}^{(\text{new})} u_{iLk} \text{ for all k}$$

4.5.5 Summary

The Calderdale model described by Howell and Parker is clearly a Balance of Care model which uses a small number of groups and which selects a range of modes of care based on the current care setting of each individual. The allocation for planning purposes can be varied by policy decisions which determine what proportion of each group of people should be allocated to a particular care setting and hence to a corresponding mode of care. The quantities of service allocated to each mode of care are based on current practice.

It is necessary for the Steering Group to choose what policy to adopt and the model then calculates the implications of such a policy in terms of the resources needed.

The allocation model is thus of the simulation or "what-if" type and appears to be easy to use but the number of services considered is only eleven (i.e. five care settings and six other resources).

The decision tree appears to give a straightforward reduction in the potential number of client groups to only 13 but these are of course subjectively chosen according to the consensus reached by the Steering Group.

The model is unsophisticated but easy to understand.

4.6 The DCC/SPSS Analysis

4.6.1 Introduction

Within Durham County Council (DCC) itself, the Development and Forward Planning Division has been investigating alternative ways of allocating Social Service resources. The original D.U.B.S. study (see Nelson[43]) was sponsored by DCC, who then agreed to the use of the DUBS Census data for the research presented in this thesis. Subsequent discussions took place between DCC planners, the author and her supervisor about this research and the D.U.B.S. study and about the development of the DHSS Balance of Care model into the SPRAM Model (i.e. the DHSS/A. Andersen model described in Section 4.4). Partly in consequence, DCC proposed a feasibility study of a simple model which uses the SPSS (Statistical Package for the Social Sciences) computer package, this being available on DCC computing equipment.

The proposed model is described in "A model for 'proportioning' available social services resources across a changing client population using the D.U.B.S. Project's client census data," by O. Coles[13].

4.6.2 The Framework of the Model

It is proposed that the model be formed of three modules, in which client numbers are predicted from the size and composition of the County's population, and other local characteristics believed to affect needs. The extent to which such needs would be met by the model reflects the size of the total budget planned, and the relative importance given to individual client groups. The distinctive simplifying assumption of the model is the proportioning of needs as favoured by the DHSS in the DHSS/A. Andersen model (See Section 4.4).

The three modules would constitute:

(i) <u>Equations predicting client totals</u>

This module would take inputs of forecast population totals by age group and of forecast percentage unemployment, on the basis that unemployment will form a potentially major influence on the incidence of at least some forms of client problem. Equations in the model would be used to predict client totals in the various client groups. It is thought that it would be possible to relate the patterns of referrals contained in DCC's computerised client records to the demographic and economic characteristics of the households in the County contained in the 1981 Census.

(ii) <u>Equations predicting departmental budget (client-related) if all needs were met</u>

This module would take inputs of the predicted group totals from module (i), and of the social service packages available, together with the unit costs of individual services. Equations would be used to predict the size (£) of the departmental budget (client-related) which would be needed if all clients were to be cared for with the same package of services as currently used for that client group, i.e. if all needs were met.

(iii) Proportion of required expenditure met from actual budget

This module would take inputs of the predicted size (£) of the departmental budget needed, the actual budget total expected and a specification of "priority" client groups (which would be protected from any cuts in expenditure). Such priority groups would be those whose needs would always be met in full, for example sub-groups of children warranting special treatment on the basis of legal obligations, past neglect, etc. DCC's Caseload Management system provides a means of classifying clients by the urgency of the need for action. The model would then allocate resources to these priority groups and then allocate resources to the remaining groups so that "the resources allocated to each client group fall short of those necessary to meet all its recognised needs by a percentage common to all other client groups".

Output from this module would consist of a breakdown of the budget by service type and, dependent upon an option selected, a list of resource allocations to client groups whose needs are met and to those whose needs are not met. The selection option would permit a reduction either in the intensity of service to be allocated (i.e. reduction in quotas), or provision of the service to only a proportion of eligible clients (i.e. reduction in coverage), or a combination of these two.

4.6.3 Consequences of adopting the model

Two major simplifications are inherent in the model and are discussed by Coles[13]. The first concerns the way in which shortfalls in the budget available for specific services result in corresponding reductions in all the services comprising a particular package of care. It is possible that in reality the policy-makers' response would be to construct new service packages which used more of the lower-cost services and smaller proportions of relatively expensive ones.

The second simplification is the proportioning principle itself. Discussions of the effect of shortfalls in budgets held in Social Services Departments suggest that, far from making "across the board" cuts, Departments consider very selective but major reductions in individual services.

Another aspect is the treatment of resource costs. The proposed model uses unit costs despite conclusions of the D.U.B.S. research that total costs do not vary substantially in response to changes in service levels. Coles suggests that other researchers have concluded that neither approach is particularly realistic, but the advantages of the unit cost approach are that it can be readily incorporated in the model and that it provides some basis for comparison with earlier time periods and with other local authorities.

4.6.4 Problems of using S.P.S.S. to construct this model

With the postulated model described above, the major limitation is that of storage space. By structuring the resource data in the form of transformations of the data on client groups (so that the definition of a client group implicitly defines its package of care and resource needs) it is possible to avoid this difficulty and Coles concludes that such a model could be constructed using SPSS. The main limitations would be those discussed in Section 4.6.3 above.

4.6.5 Progress by D.C.C.

In "The Balance of Care Approach to Resource Allocation" (1982) Coles[12] discussed the application of the general Balance of Care approach within County Durham. He considered three factors of particular relevance. The first was the opportunity for closer co-ordination with the National Health Service (NHS), in particular, alignment of DCC's District boundaries with those of the four Health Districts, and for creation of the organisational framework needed for the day-to-day management of the Balance of Care approach. The second factor was the need to reduce expenditure and the likelihood of being able to improve efficiency of care. The third factor was DCC's investment in computerised records and development of formal decision-making criteria, such as the Caseload Management System.

In Coles[13] (1983) subsequent paper, a proposal was made for a simplified Balance of Care type of model which would use a simple proportioning procedure along the same lines as that of the DHSS/A. Andersen model.

Since 1983 no further developments have taken place on the DCC/SPSS model. Emphasis has instead been on the computer-assisted systems of Client Records and the Caseload Management System. In the Client Records system a set of about 70 client categories within 6 client groups and one "non-client" group has been used.

The seven groups are:

Elderly

Child Care and Adoption

Family Casework

Mental Health

Physically Handicapped

Miscellaneous

Non-clients

These are different groups and categories from both the DHSS and the DUBS sets.

In the Caseload Management System (DCC[17]), each category of client is allocated a number of "points" which indicates a preference or priority identifying which categories are preferred to receive treatment, or to be considered for treatment, if a queue exists. As well as indicating priorities between client categories, the "points" system can be used to equalise the caseload on social workers or to differentiate between the caseloads for different grades of staff. Each social worker is allocated a number of "points" which measures the caseload he/she can be expected to undertake. (Targets and maxima can be fixed).

Progress is also taking place in the identification of expenditure. Where previously it was only possible to identify service costs for a full financial year, it should now, with the imminent introduction of a new computerised financial analysis ("FISCAL"), be possible to monitor monthly expenditure on each social service. Unit costs are not yet identifiable but control of expenditure on individual services should now become feasible.

4.6.6 Summary

Although a mathematical model has been postulated no development of the model has taken place. Emphasis has been on defining client categories and defining priorities for caring for these client categories. Service provision will also be better defined through the analysis of monthly expenditure.

Now that the data on clients and services is more clearly defined, it should be possible for DCC to re-consider the potential of a mathematical model to assist in planning.

4.7 Overview

In the previous sections of this Chapter a number of mathematical models have been described. The original Balance of Care model postulated by McDonald[40] and developed by Coverdale[15] and Negrine and Gibbs[30] was a mathematical programming model with a non-linear objective function representing diminishing returns as the amount of service supplied to a group increases, or as the number of clients receiving service increases.

Such an objective function is difficult to specify completely, since it requires elasticity factors and parameters of curvature to be estimated, preferably by the planners.

The D.U.B.S. goal programming model described in section 4.3 had not been implemented or indeed constructed by the D.U.B.S. researchers. It is considered further in Chapter 5 but is not usable in the form described here.

The D.H.S.S./A. Andersen SPRAM model, although developed from the Balance of Care concept, no longer retained the fixed mode of care which was a feature of the McDonald model and which is part of the LPAM model. Instead the proportional assumption (that care will be allocated in future in patterns which are proportionally similar to current patterns) over-rides the retention of fixed modes of care. Thus the SPRAM model effectively assumes that alternatives modes of care will not be used, rather that care profiles will remain largely unchanged.

This assumption is not sacrosanct, however, since facilities are (in theory) included which permit changes in the proportions of one service or in the type of care given to a particular client group. The SPRAM model is not specified as an optimising model, the main usage being to calculate resources needed for changed populations and to distribute available resources in proportion to the resources needed. This suggests that a form of minimising objective is in force, perhaps of the goal-programming type, viz to minimise the effect of changed (i.e. reduced) resources by sharing these in proportion as before whilst taking account of demographic changes in client group sizes. The "effect" measured may be the total of all deviations from current allocations of service, for all clients.

Evidence suggests, however, (Coles[13]) that planners do not always wish to minimise the effect of change and that radical shifts in resource provision may be regarded more favourably than small adjustments across all resources. Chapter 5 describes the development of a new model, LPAM, a linear programming model with alternative modes of care, which maintains fixed modes of care and which may choose to allocate clients to modes in a very different way from current provision. Such a model can assist planners to consider radical changes in resource provision. This is in contrast to the SPRAM model which tends to minimise the overall change. Chapter 8 makes comparisons between these two models in the ways they plan for future demographic and/or resource changes.

The Calderdale decision algorithm model described in Section 4.5, whilst being based on the Balance of Care concepts, takes a more rigid approach to allocation of resources. Once clients are categorised their allocation to a mode of care is effectively fixed. The planners use a model to calculate the resources required on the basis of different "schemes" of care, i.e. each "scheme" consists of a defined set of modes of care, one mode for each client category. The modes of care are redefined (i.e. different quantities of resources are defined for each mode) to form another "scheme" for which required resources can be calculated. A process of trial and error leads to that scheme being adopted which most closely resembles the amount of resource provision. The exploration of schemes in this way permits the planners to see the effect of different policies, for example, a high intensity of care in residential homes or an increase in the proportion of clients in domiciliary care settings.

The DCC/SPSS postulated model described in Section 4.6 has not been implemented but the planners have made considerable progress in defining client categories for the computerised Client Records System and in defining priorities and "points" for each category of client for the Caseload Management System. The postulated model was based on the DHSS-favoured proportioning of needs as used in the DHSS/A. Andersen SPRAM model and thus it would not maintain fixed modes of care but would tend to minimise the effects of change in resource provision.

From these descriptions, and from the regional implementations of Balance of Care models discussed later (in Chapter 7), it can be seen that the planners need to be able to simulate patterns of care and to explore the effect of policy changes and demographic changes on resource allocation.

The following Chapter (5) describes the development by the present author of an appropriate model which was based initially on a linear programming model similar to the D.U.B.S. goal-programming model and was then extended to include sets of alternative modes of care for each client group. This linear programming model with alternative modes of care (LPAM) is an optimising model which offers the planners true alternatives in patterns of care and can additionally be used as a simulation model to assist the planners in their explorations of different resource allocation policies.

CHAPTER 5

DEVELOPMENT OF A NEW MODEL (LPAM)

5. Development of a New Model (LPAM)

5.1 Introduction

When the author began this research, only two of the models described in Chapter 4 had been postulated: the DHSS Balance of Care model and the D.U.B.S. goal programming model. The DHSS model had been shown to be difficult to specify completely and was not an easy model for planners to comprehend. A great deal of data was required: for example, to specify alternative modes of care, panels of professional workers needed to be set up and agreement needed to be reached on what constituted equivalent acceptable modes of care for clients with particular needs. The objective function was unwieldy and difficult to calibrate, needing estimates of elasticity factors and parameters of curvature.

In contrast, the D.U.B.S. goal programming model was too simplistic. Section 5.2 shows how the goal programming model described in Chapter 4 could be reduced to a linear model, since in practice the goals were targets which would never be exceeded, ideal levels of social service provision being unattainable in practice (and certainly never exceeded). The model itself could then be described as a capacitated transportation model with a very simple allocation procedure dependent upon the weights allocated to the client groups. Each client group i was allocated an amount of a service k such that the targets were upper bounds on the allocations and such that the total allocation of each service did not exceed the amount available. No alternative modes of care were specified. The objective was to maximise the allocations weighted in respect of each group and each service. Such an objective had been postulated by other D.U.B.S. researchers (See Longbottom[37] and Bayat [4], Wade and Longbottom).

Section 5.2 describes the construction of this model for the test data suggested by Wiper[60].

The same "linear equivalent" model was used to try out the 35 client group and 45 service classifications derived by the author from the D.U.B.S. census. This is described in section 5.3, whilst the author's analysis of the D.U.B.S. census data is described in Chapter 6. For this model, group and service classifications were needed, as were the group sizes and the total amounts "currently" available of each service resource (measured in units of clients who receive that service). These total amounts were derived by summing the separate allocations of each service to each client group, which in turn had been aggregated from the individual client records of the census.

By using the known allocations of services to groups, the linear model could be constructed to find the weightings on groups and services which would result in the known allocation. This was done and showed up the transportation form of the linear model.

It was clear that the linear model was inadequate: it was not representing the way in which resources were allocated at the time of the census. The next development was to include packages of care for each client group as had been done in the original Balance of Care model. (See section 4.2) so that more choice would be available for the allocation of resources. Analysis of the census data (see Chapter 6) provided several ways of setting alternative modes of care : this is discussed further in Sections 6.3 and 6.4 and shows that a cluster analysis was used to determine ten alternative modes of care for each of the 35 client groups. Use of the census data in this manner is distinctly different from the Balance of Care method of using groups of professional workers to define the alternative modes of care. Using the census data ensures that all current caring methods are included. Section 5.4 describes the formulation of the linear programming model which included alternative modes of care (LPAM) and Section 5.5. describes the construction of the LPAM Model for a single client group (group 06).

In constructing the group 06 model the weighting factors had been chosen by inference (if somewhat arbitrarily) from the census data, in proportion to the known size of each cluster (i.e. the number of clients allocated by the cluster analysis to a particular mode of care). It was decided that the range of values of weights,which would result in the same optimum solution, should be investigated. This "weight-space" analysis is described in section 5.6 where a weight-constraints model is formulated. This is derived from a consideration of the simplex method of linear programming and sensitivity analysis of its solution. This weight-constraints model provides a definition of a feasibility region which must include the actual sizes of the weights in use at the time when the data was obtained (i.e. at the time of the census). Section 5.7 describes the construction of the weight-constraints model for a small test problem, whilst Section 5.8 describes that for group 06.

The group 06 LPAM model included of course only a single client group. The next stage was to extend the LPAM model to include all client groups. Instead of using all the clients in the census, it was at first decided that a random sample of data should be used, so that results could be compared with results from similar models derived from other random samples of the census data. Section 5.9 discusses the attempt to use a random sample and the problems involved. It was subsequently decided that a geographical district should be selected and all clients from that district would be included. This should mean that the service allocations could be expected to be more homogeneous than those arising from a random sample.

Chester-le-Street district was selected; Section 5.10 describes the construction of the LPAM model for Chester-le-Street and the two sets of weightings used, the first being based on the proportion of clients allocated by the cluster analysis to a particular mode in a particular group and the second being based on the proportion of services allocated. As in the case of the group 06 model, a weight-constraints model was constructed for the Chester-le-Street district, thus giving a feasibility region for the weights. Section 5.11 describes the construction of this model, which, because of its sheer size, required a different treatment from that for the group 06 model. Despite its size (340 constraints), the model could be constructed and a number of maximising runs were performed to find some of the extreme points of the weight-space, in order to demonstrate the practicability of this. It is more likely, however, that the weight constraints would be used to assess the feasibility of a set of postulated weights.

The weight-space problem is discussed further in section 5.12 where other possible approaches are described.

5.2 The Linear Model

The D.U.B.S. goal-programming model described in section 4.3 incorporated target allocations into the objective function to be minimised,

$$\text{i.e. Minimise } \sum_i \sum_k \left| T_{ik} - A_{ik} \right| W_{ik}$$

These target allocations T_{ik} are ideals and would never in reality be exceeded, i.e. the actual service allocations A_{ik} are always less than or equal to the target allocations, so there is no need to consider the instances when allocations exceed targets. Thus minimising the total of the absolute differences between the service allocations and the targets is equivalent to maximising the total service allocations subject to upper bound constraints which are themselves the targets.

The model was therefore constructed as a linear programming model with the goals as upper bounds.

i.e.

Maximise $\sum_i \sum_k A_{ik} W_{ik}$

subject to $\sum_i A_{ik} \leqslant B_k$, for all k

and $0 \leqslant A_{ik} \leqslant T_{ik}$, for all i and k,

and $\sum_k W_{ik} = 1$, for all i, is a condition of the data

The notation used is listed in figure 4.1.3.1 and is the same as that used for the goal-programming model in section 4.3.

Wiper[60] presented the goal-programming model in two ways, and these are both considered overleaf.

The model can be formulated in terms of allocation to groups collectively or allocation to individual persons within each group:

(i) Allocation per group collectively

Maximise $\sum_i \sum_k d_i \, a_{ik} \, W_{ik}$ (1)

subject to $\sum_i d_i a_{ik} \leq B_k$, for all k,

and $0 \leq a_{ik} \leq t_{ik}$, for all i and k,

and $\sum_k W_{ik} = 1$, for all i, is a condition of the data.

(ii) Allocation per person in each group

Maximise $\sum_i \sum_k \frac{di}{\sum_i d_i} \, a_{ik} \, W_{ik}$ (2)

subject to $\sum_i d_i \, a_{ik} \leq B_k$, for all k,

and $0 \leq a_{ik} \leq t_{ik}$, for all i and k,

and $\sum_k W_{ik} = 1$, for all i, is a condition of the data.

Here the factor $\frac{di}{\sum di}$ is a proportioning factor to take account of the group size.

It is clear that, since $\sum_i$ di is a constant, the formulations (1) and (2) are effectively the same.

A small set of test data was presented in Wiper's paper as an illustration of the use of the goal-programming model (equivalent to formulation (2)), but it had not been constructed as a computerised model.

The present author constructed a computer model using the above formulation (2), i.e. the equivalent linear -

programming model with targets as bounds, and used Wiper's test data. Figure 5.2.1 shows the data and results. The IBM package for linear programming, MPSX, was used. No comparisons could be made since no results were presented by Wiper.

Figure 5.2.1 — Test model : data and results
Linear Programming Model with Targets as Bounds

Test data: Two client groups, 1 and 2, sizes 200 and 150 respectively.

Two services, A and B, available amounts 800 and 600 units respectively.

Target units per person

Service (k) \ i	Grp 1	Grp 2
A	4	1
B	2	3

Weightings

Service (k) \ i	Grp 1	Grp 2
A	0.8	0.4
B	0.2	0.6
$\sum_k W_{ik}$	1.0	1.0
$d_i / \sum_i d_i$	200/350	150/350

Model is Max $(0.456\ a_{11} + 0.172\ a_{21} + 0.114\ a_{12} + 0.258\ a_{22})$

Subject to:

$$200a_{11} + 150a_{21} \leqslant 800,$$
$$200a_{12} + 150a_{22} \leqslant 600$$
$$0 \leqslant a_{11} \leqslant 4,$$
$$0 \leqslant a_{21} \leqslant 1,$$
$$0 \leqslant a_{12} \leqslant 2,$$
and
$$0 \leqslant a_{22} \leqslant 3.$$

Computer package MPSX gives solution = 2.6835

When $a_{11} = 4$, $a_{21} = 0$, $a_{12} = 0.75$, $a_{22} = 3$.

5.3 Using the model : deriving the weights

(i) Using the model

The linear model was subsequently set up using the group and service classifications of the D.U.B.S. census data as described in Chapter 3 (and detailed in Chapter 6). The formulation used was formulation (1), which allocates collectively to each group, i.e.

$$\text{Maximise} \quad \sum_i \sum_k d_i \; a_{ik} \; W_{ik} \qquad (1)$$

$$\text{subject to} \quad \sum_i d_i \, a_{ik} \leqslant B_k, \text{ for all } k,$$

$$\text{and} \quad 0 \leqslant a_{ik} \leqslant t_{ik}, \quad \text{for all } i \text{ and } k,$$

and $\sum_k W_{ik} = 1$, for all i, is a condition of the data.

From the census data, the group sizes (d_i) were known, the allocations of each service to each group (ie $d_i a_{ik}$) were known and the total amounts of each service (B_k) were known. Indeed the B_k were derived directly by addition of the allocations, i.e. $B_k = \sum_i d_i \, a_{ik}$ for all k, hence the constraints $\sum_i d_i \, a_{ik} \leqslant B_k$ were necessarily satisfied.

Assuming that the targets were not exceeded, i.e. $0 \leqslant a_{ik} \leqslant t_{ik}$, for all i and k, then the weights W_{ik} in use at the time of the census could be found from the modified model:

$$\text{Maximise} \sum_i \sum_k d_i \, a_{ik} \, W_{ik} \qquad (3)$$

$$\text{subject to} \sum_k W_{ik} = 1$$

This model was constructed and run on the MPSX package, resulting in the following solution:

For each k in turn:

$$\begin{cases} W_{ik} = 1 \text{ for the group } i \text{ where } d_i \, a_{ik} = \text{MAX (all } d_i a_{ik}), \\ W_{ik} = 0 \text{ for all other } i \end{cases}$$

Thus this model (3) was a <u>simple transportation model</u> and did not reflect reality.

(ii) <u>Further considerations</u>

Further consideration of the model (1) showed that this was a <u>capacitated transportation model</u>, and that the solution could be derived as follows:

For each service, the group with largest weighting is allocated as much resource as possible, up to a maximum which is the target appropriate to that group and service. Any remaining resource for that service is allocated,in a similar way, to the group with next largest weighting. This procedure is continued until either the service resource is exhausted or all groups have been allocated their targets.

Considerations of reality suggested that minimum service levels should be taken into account, i.e. for each group i there would be a minimum service allocation A_{ik}^{min} for each service k.

The appropriate modification to the model (1) would be to set these minimum levels,

i.e. $d_i \, a_{ik} \geq A_{ik}^{min}$ for all i and k

However, letting $D_{ik} = d_i \, a_{ik} - A_{ik}^{min}$ for all i and k, and providing that $B_k > \sum_i A_{ik}^{min}$ for all k, which is true in practice,

the model remains a capacitated transportation model for the variable D_{ik}, the D_{ik} being the amounts in excess of the minimum service levels which could be allocated from the remaining amounts of service, $B_k - \sum_i A_{ik}^{min}$, for all k.

The allocation procedure is then:

allocate minimum service levels to all groups, then allocate, for each service in turn, as much as possible (either the target amount or the remainder of the resource if less than the target) to the group with highest weighting, then to the group with next highest, and so on, until either the service resource is exhausted or all groups have been allocated their targets.

(iii) Size of weightings

If the targets are set too high, the weighting which is the maximum weighting of those groups that use a particular service k will be the only one of those weightings which contributes to the solution. This is because the corresponding group, i_x say, will have been allocated all the available service resource, i.e.

$$A_{i_x k} = B_k$$

$$A_{ik} = 0, \; i \neq i_x$$

Thus the sizes of the other weightings are irrelevant, as long as they are smaller than this maximum weighting.

When the targets are more realistic and are set closer to actual values, the weightings are used, in order of size descending from the maximum for any particular service, to determine which groups shall receive allocations of that service.

5.4 The Linear Programming Model with Alternative Modes of Care (LPAM):

Introduction

The linear model described in Sections 5.2 and 5.3 having proved inadequate, a development was proposed which would include alternative modes or packages of care. The new model would permit a choice from several different modes of care for each client group. Once again the objective function included weighting factors and these weights w_{il} would be constants chosen to reflect the relative preferences of caring for clients in group i by allocating mode l of care. The constraints were firstly service constraints such that the amount of each service k allocated to all groups i, in whichever modes of care l were chosen, would not exceed the total amount B_k of service available; and secondly group size constraints such that all the clients in each group i would be allocated to one or other of the modes l of care available.

The model is:

Choose the x_{il} to maximise $\sum_i \sum_l w_{il} x_{il}$

subject to $\sum_i \sum_l x_{il} u_{ilk} \leq B_k$, for all k;

$\sum_l x_{il} = d_i$, for all i;

and $\sum_i \sum_l w_{il} = 1.0$ is a condition of the data.

Here u_{ilk} is the amount of service k (measured in client units) which will be supplied to a client in group i who is cared for in mode l;

B_k is the total amount of service k available (measured in client-units);

d_i is the size of or number of clients in group i;

and x_{il} is the number of clients in group i chosen by the model to be cared for in mode 1.

The modes of care for each group were selected by using the DUBS census data and a cluster analysis as described in sections 6.3 and 6.4 below. Thus for each group a set of ten alternative modes of care was defined. (The decision to use ten alternative modes of care was somewhat arbitrary, being considered neither too few nor too many to handle. It would have been straightforward to set some other number of alternative modes per group and to arrange for a cluster analysis on that basis). For each mode the service allocations per client were extracted from the results of the cluster analysis which showed the service usage as percentages. Each u_{ilk} represented the proportion of clients in group i and mode 1 who would receive service k.

5.5 Test model for group 06 (LPAM)

As a first attempt, the model with alternative modes of care was constructed for a single group, 06, of clients. The computer package for linear programming, MPSX, was used.

The intention of constructing this model was to test whether the "current" service provision could be simulated by an alternative modes of care model.

The model for just one group becomes, for i = 06,

Choose the x_{il} to maximise $\sum_{l} x_{il} w_{il}$

$$\text{Subject to} \quad \sum_{l} x_{il} u_{ilk} \leq B_k, \text{ for all } k$$

$$\sum_{l} x_{il} = d_i,$$

$$\text{and} \quad \sum_{l} w_{il} = 1.0 \text{ is a condition of the data.}$$

In this model the weights w_{il} were calculated in proportion to the known size of each cluster, $x_{il}^{(o)}$

i.e. $w_{il} = \dfrac{x_{il}^{(o)}}{d_i}$ and $d_i = \sum_{l} x_{il}^{(o)}$

The service allocations per client, u_{ilk}, were the proportions of service usage given as percentages in the results of the cluster analysis.

For example, for cluster 6 of client group 06,

$$u_{06,6,20} = 1.0$$

$$u_{06,6,19} = 0.833$$

$$u_{06,6,2} = 0.5$$

$$u_{06,6,17} = 0.333$$

$$u_{06,6,14} = 0.333$$

and $u_{06,6,k} = 0 \text{ for } k \neq 20,19,2,17,14$

The service totals B_k were found from

$B_k = \sum_l x_{il}^{(o)} \, u_{ilk}$, for all k;

i.e. summing the service usage for all clusters in group 06 by using the results of the cluster analysis and multiplying by the known size of each cluster, $x_{il}^{(o)}$.

The client group size d_i was 111 for client group 06. Figure 5.5.1 shows the values of u_{ilk}, B_k and w_{il} used in the model.

When the model was first run the result was an infeasible set and it was realised that this was because the constraints were too strict. Relaxation of the right-hand-sides (i.e. the values of B_k) was tried: two runs were attempted, one having increments of 0.05 to each B_k and one having increments of 0.02. Both runs were feasible; the results using 0.02 were used since these were closer to the actual values.

The results of the linear programming package gave a set of values $x_{il}^{(1)}$ for x_{il}, $l = 1,....10$, and $i = 06$. These were compared with the expected numbers of clients in each cluster, $x_{il}^{(o)}$, derived from the results of the cluster analysis. When rounded to integer values, the sizes were the same. Figure 5.5.2 shows these results.

It was concluded that the model was simulating successfully the service provision for group 06.

The weighting factors w_{il} had been chosen arbitrarily (albeit carefully) and the next stage was to determine the "weight-space" or the range of values of weights which would produce the same result. The following sections describe the analysis of the weight-space and the related linear-programming models.

Figure 5.5.1 Values of u_{ilk}, B_k and w_{il} in Group 06 Model

$i = 06$, $d_i = 111$, $w_{il} = \dfrac{x_{il}^{(o)}}{d_i}$

Values of w_{il}

l =	1	2	3	4	5	6	7	8	9	10
w_{il} =	0.108	0.135	0.099	0.198	0.117	0.054	0.072	0.126	0.054	0.036

Values of u_{ilk} and B_k

k \ l (u_{ilk})	1	2	3	4	5	6	7	8	9	10	B_k
1	1.0		0.182					0.143		1.0	20
2		0.133				0.5					5
3		0.067	0.091					0.071		1.0	7
4			1.0	0.091				0.071	1.0		20
5	0.25		0.091				0.125	0.214		0.25	9
6		0.2	0.091	1.0	0.308						30
7				0.091	0.077		0.125				4
11		0.067			0.077						2
12		0.133			0.077						3
14			0.091			0.333	0.375				6
15	0.083	0.067		0.045				0.071			4
16					0.308						4
17	0.167		0.455	0.136	0.923	0.333	1.0	0.357	0.833		42
18		0.067	0.091	0.091	0.308		0.75	0.071			15
19			0.182	0.045		0.833	0.125	0.286		1.0	17
20	0.417	0.267	0.545	0.455	0.231	1.0	0.125	0.857		1.0	51
21	0.167	0.067	0.182	0.045	0.077			0.214			10
22		0.067	0.182	0.045							4
23	0.083		0.182	0.227				0.429	1.0		20
24			0.091				0.25				3
25								1.0			14
26								0.143			2
27				0.045							1
28		0.467		0.045			0.875		1.0		21
62		0.067									1
73		0.067									1
88		0.067									1
										TOTAL	111

Figure 5.5.2 Results from test model for group 06

First run with $w_{il} = \frac{x_{il}^{(o)}}{d_i}$

Group 06, Size 111 clients

Mode l	Expected $x_{il}^{(o)}$	Model $x_{il}^{(1)}$
1	12	12.3
2	15	15.2
3	11	10.8
4	22	22.0
5	13	13.0
6	6	6.0
7	8	8.0
8	14	14.0
9	6	5.9
10	4	3.8
TOTAL	111	111.0

5.6 Formulation of the Weight-Constraints Model

When using the model with alternative modes of care as described in Section 5.4, the weights w_{il} in the objective function were chosen arbitrarily by choosing weights which were related to the expected group sizes, x_{il}, resulting from the Cluster Analysis. The next stage was to attempt a sensitivity analysis of the weights. The aim was to determine what range of values of weights in the linear programming model would result in the same optimum solution.

When linear programming models are solved by the Simplex Method, some sensitivity analysis can be performed by inspection of values in the final primal tableau, in particular the values of the dual variables. It is also possible to deduce the sensitivity of the coefficients in the objective function as follows:

If the Primal problem is:

Maximise $\underline{c}^T \underline{x}$

subject to $A \underline{x} \leq \underline{b}$, where $\underline{x}$ and $\underline{c}$ are $(n \times 1)$ vectors,

$\underline{b}$ is $(m \times 1)$,

and A is $(m \times n)$,

then the initial Simplex tableau can be written as:

A	I	$\underline{b}$
$\underline{c}^T$	0	0

and the final (solution) tableau as:

R	S	$\underline{x}^*$
$\underline{d}^T$	$\underline{e}^T$	$-f^*$

At the solution point, $\underline{d}^T \leqslant 0$, $\underline{e}^T \leqslant 0$,

and since, by the Simplex Method :

$$\begin{array}{c} m \\ 1 \end{array}\left[\begin{array}{c|c} B & 0 \\ \hline \underline{c}_B^T & 1 \end{array}\right]_{m \quad 1} \cdot \left[\begin{array}{c|c|c|c} R & S & 0 & \underline{x}^* \\ \hline \underline{d}^T & \underline{e}^T & 1 & -f^* \end{array}\right]_{n \quad m \quad 1 \quad 1} = \left[\begin{array}{c|c|c|c} A & I & 0 & \underline{b} \\ \hline \underline{c}^T & 0 & 1 & 0 \end{array}\right]_{n \quad m \quad 1 \quad 1}$$

where B is the basis matrix formed by selecting columns of A or I corresponding to the solution variables and C_B is the corresponding vector of coefficients or weights in the objective function,

then

$$\left.\begin{array}{r} BR = A \\ BS = I \\ B\underline{x}^* = \underline{b} \\ \underline{c}_B^T R + \underline{d}^T = \underline{c}^T \\ \underline{c}_B^T S + \underline{e}^T = 0 \\ \underline{c}_B^T \underline{x}^* - f^* = 0 \end{array}\right] \quad \text{and hence} \quad \begin{array}{l} R = B^{-1} A \\ S = B^{-1} \\ \underline{x}^* = B^{-1}\underline{b} \\ \underline{d}^T = \underline{c}^T - \underline{c}_B^T R \\ \underline{e}^T = -\underline{c}_B^T S \\ f^* = \underline{c}_B^T \underline{x}^* \end{array}$$

and $\underline{d}^T \leqslant 0 \implies \underline{c}^T \leqslant \underline{c}_B^T R$

$\underline{e}^T \leqslant 0 \implies \underline{c}_B^T S \geqslant 0$

] thus defining the feasibility region for the weights $\underline{c}$.

These two inequalities:

$\underline{c}^T \leqslant \underline{c}_B^T R$ (1)

$\underline{c}_B^T S \geqslant 0$ (2)

thus provide a representation of the weight-space within which the values of the weights must lie. If the weights are to be standardised then the equation

$$\sum_i \sum_l w_{il} = 1.0$$

is added to the constraints.

The following paragraphs describe the application of this weight-space to the social services planning problem. Section 5.7 describes a small test problem and its weight-space, Section 5.8 describes the weight-space model constructed for a single client group with alternative packages of care, and Section 5.11 describes the weight-space model constructed for the Chester-le-Street district with alternatives packages of care.

5.7 Test problem : determination of feasibility region for weights

The test problem was a simple alternative mode model :

$$\text{Max } 0.3\, x_{11} + 0.2\, x_{12} + 0.4\, x_{21} + 0.1\, x_{22}$$

subject to

$$0.5\, x_{11} + 0.6\, x_{12} + 5\, x_{21} \leqslant 80$$

$$x_{11} + 3\, x_{21} + 4\, x_{22} \leqslant 50$$

$$0.3\, x_{11} + 0.8\, x_{12} + 7\, x_{22} \leqslant 40$$

$$x_{11} + x_{12} \leqslant 60$$

$$x_{21} + x_{22} \leqslant 40$$

Solving by Simplex, the initial and final tableaux were:

Initial tableau

	x_{11}	x_{12}	x_{21}	x_{22}	s_1	s_2	s_3	s_4	s_5	
s_1	0.5	0.6	5	0	1	0	0	0	0	80
s_2	1	0	3	4	0	1	0	0	0	50
s_3	0.3	0.8	0	7	0	0	1	0	0	40
s_4	1	1	0	0	0	0	0	1	0	60
s_5	0	0	1	1	0	0	0	0	1	40
	0.3	0.2	0.4	0.1	0	0	0	0	0	0
	c_1	c_2	c_3	c_4						

i.e.

$\underline{A}$	I	$\underline{b}$
$\underline{c}^T$	0	0

Final tableau

x_{21}	0	0	1	0.0754	0.1887	0.0188	0	-0.1132	0	9.24
x_{11}	1	0	0	3.7736	-0.5660	0.9434	0	0.3396	0	22.26
s_3	0	0	0	8.8868	-0.2830	0.4717	1	-0.6302	0	3.13
x_{12}	0	1	0	-3.7736	0.5660	-0.9434	0	0.6604	0	37.735
s_5	0	0	0	0.9246	-0.1887	-0.0188	0	0.1132	1	30.753
	0	0	0	-0.3076	-0.0188	-0.1019	0	-0.1887	0	-17.923

i.e.

R	S	$\underline{X}^*$
$\underline{d}^T$	$\underline{e}^T$	$-f^*$

The feasibility region for the weights was defined by the two inequalities:

$$\underline{C}^T \leqslant \underline{C}_B^T R \qquad (1)$$

$$\underline{C}_B^T S \geqslant 0 \qquad (2)$$

In the test problem, $\underline{C}^T = (C_1\ C_2\ C_3\ C_4)$;

$$\underline{C}_B^T = (C_3\ C_1\ 0\ \ C_2\ 0);$$

R was (5 x 4) in the final tableau;

and S was (5 x 5) in the final tableau.

Substituting in (1) gave:

$$C_1 \leqslant C_1$$

$$C_2 \leqslant C_2$$

$$C_3 \leqslant C_3$$

$$C_4 \leqslant 0.0754\ \ C_3 + 3.7736\ \ C_1 - 3.7736\ \ C_2$$

and in (2) gave:

$$0.1887\ c_3 - 0.5660\ c_1 + 0.5660\ c_2 \geqslant 0$$

$$0.0188\ c_3 + 0.9434\ c_1 - 0.9434\ c_2 \geqslant 0$$

$$-0.1132\ c_3 + 0.3396\ c_1 + 0.6604\ c_2 \geqslant 0$$

thus giving four non-trivial inequalities to define the weight-space.

These inequalities were then expressed in terms of the weight ratios $\frac{c_2}{c_1}, \frac{c_3}{c_1}, \frac{c_4}{c_1}$ (providing $c_1 \neq 0$)

i.e. After simplification,

$$\frac{c_3}{c_1} + \frac{3c_2}{c_1} \geqslant 3 \qquad (3)$$

$$\frac{-c_3}{c_1} + \frac{50c_2}{c_1} \leqslant 50 \qquad (4)$$

$$\frac{c_3}{c_1} - \frac{6c_2}{c_1} \leqslant 3 \qquad (5)$$

$$\frac{-c_3}{c_1} + \frac{50c_2}{c_1} + 13.2626\ \frac{c_4}{c_1} \leqslant 50 \qquad (6)$$

Obviously (4) could be subsumed in (6) since in this case $\underline{c}$ was positive.

Using these inequalities (3), (5) and (6), the minimum and maximum for each ratio were found by application of the Simplex method:

$$\left.\begin{array}{l} 0 \leqslant \frac{c_2}{c_1} \leqslant 1.20455 \\ 0 \leqslant \frac{c_3}{c_1} \leqslant 10.22727 \\ 0 \leqslant \frac{c_4}{c_1} \leqslant 3.99620 \end{array}\right] \qquad (7)$$

Comparing with the original coefficients of the test problem, these were

$c_1 = 0.3$

$c_2 = 0.2$

$c_3 = 0.4$

$c_4 = 0.1$

with corresponding ratios $\frac{c_2}{c_1} = 0.67$, $\frac{c_3}{c_1} = 1.33$,

$\frac{c_4}{c_1} = 0.33$, thus verifying that the original

coefficients did lie within the defined ranges (7).

The test problem was used to try out this method of determining the weight-space. In the specification of the alternative modes model another weight constraint can be included because the weights are specified as standardised values such that the sum of all the weights equals one.

$$\text{ie} \sum_i \sum_l w_{il} = 1.0$$

Using this constraint it is then unnecessary to specify the weight constraints as ratios.

The next section describes the application of the weight-constraint model to the test model for group 06.

5.8 Group 06 model : determination of feasibility region for weights

As was described in section 5.5 above, the alternative mode model was first constructed for client group 06 with ten alternative modes of care.

The model was:

Choose the x_{il} to Maximise $\sum_i \sum_l x_{il} w_{il}$

$$\text{subject to} \quad \sum_i \sum_l x_{il} u_{ilk} \leqslant B_k, \text{ for all } k;$$

$$\sum_l x_{il} = d_i, \text{ for all } i;$$

$$\text{and} \sum_i \sum_l w_{il} = 1.0$$ is a condition of the data.

N.B. This model has i = 06 only. There are 10 unknowns (x_{il}) and 47 constraints (45 services and 2 size constraints, $\leqslant$ and $\geqslant$).

This model was run with data from the census and the resulting values for x_{il} were directly comparable with the expected values obtained from the Cluster Analysis (see Figure 5.5.2).

In order to determine the feasibility region for the weights in this model, it was necessary to model the weight constraints (1) and (2) described in Section 5.6 above.

ie $\underline{c}^T \leqslant \underline{c}_B{}^T R$ (1)

$\underline{c}_B{}^T S \geqslant 0$ (2)

In this case the coefficients of the objective function are the weights $\underline{w}$.

In addition, as shown in Section 5.6,

$S = B^{-1}$ (ie the inverse of the Basis Matrix B)

and $R = B^{-1}A$.

When the computer package MPSX is used to solve linear programming problems it is possible to print the inverse matrix B^{-1} at the optimal solution and to print a list of those variables selected for the solution basis. Thus B^{-1} can be found and $B^{-1}A$ can then be calculated. The weights $\underline{w}_B$ corresponding to the variables selected for the solution basis can then be identified and thus all the data is available for substitution in (1) and (2).

ie $\underline{w}^T \leqslant \underline{w}_B^T \; B^{-1}A$ for (1)

and $\underline{w}_B^T \; B^{-1} \geqslant 0$ for (2)

The additional constraint $\sum_i \sum_l w_{il} = 1.0$ was added to the set of constraints to standardise the weights.

For the group 06 model, there were 10 unknowns (w_{il}), A was (28 x 10), B^{-1} was (10 x 10), $\underline{w}^T$ was (1 x 10), $\underline{w}_B^T$ was (1 x 28), thus the weights model contained 28 constraints corresponding to (1) and 10 constraints corresponding to (2) together with one equation to standardise the weights.

The following procedure was used:

(a) The model of the group 06 service provision, described at the beginning of this Section and in Section 5.5, was run with data from the census using the computer package MPSX to solve the model and the MPSX subroutine to print the inverse basis matrix at the optimal solution point and to print the list of variables in the solution basis.

(b) The vector $\underline{w}_B^T$ was constructed using the list of variables in the solution basis.

(c) The inverse matrix B^{-1} was extracted in the correct column sequence, using the inverse printed by MPSX and the list of basic variables in the solution.

(d) The vector $\underline{w}_B^T B^{-1}$ was constructed.

(e) The constraints $\underline{w}_B^T B^{-1} \geqslant 0$ were listed.

(f) The vector $\underline{w}_B^T B^{-1}A$ was calculated using a specially written computer program.

(g) The constraints $\underline{w}^T \leqslant \underline{w}_B^T B^{-1} A$ were listed. These all proved to be of the form $w_{il} \leqslant w_{il}$, for all i and l. (This result is a consequence of the "shape" of the group 06 model, which has more constraints than real variables. Section 5.12.1 discusses an alternative presentation of the weight constraints, obtained by a different partitioning of the Simplex tableau, which eliminates this type of $W_{il} \leqslant W_{il}$ result. Section 5.12.6 discusses the "shape" of the LP problem).

(h) The constraint $\sum_{i} \sum_{l} w_{il} = 1.0$ was added to the set of constraints $\underline{w}_B^T B^{-1} \geq 0$ to form the weight-space model.

These constraints are shown in figure 5.8.1.

(i) These constraints were input to a new MPSX model which maximised and minimised each of the weights in turn.

Results for this model are shown in figure 5.8.2. It can be seen that some of the points identified are maxima or minima for more than one weight.

The initial group 06 model was then tried with six separate sets of weights to discover what effect the values of the weights might have. The first new set was chosen with reference firstly to the original values

$$\frac{x_{il}^{(o)}}{\sum_i \sum_l x_{il}^{(o)}}$$

of the weights, secondly to their minimum and maximum values and thirdly to the actual descriptions of the modes of care and a subjective assessment of preference, giving a set of perturbed weights.

Appendix 11 shows this data, i.e. the ten modes of care for client group 06 together with weights, client numbers and service descriptions.

Subsequent sets of weights were perturbed as shown in Figure 5.8.3, preserving always the relation

$$\sum_l w_{il} = 1.0.$$

Even when some weights were altered to four times their original size, the values of x_{il} changed very little and when rounded to integer values they were unchanged in all cases. Figure 5.8.3 compares the weights w_{il} and results x_{il} for each of these six runs with those for the first run of the model and with the expected values $x_{il}^{(o)}$.

The next stage was to extend the service - allocation model to include all client groups. This is described in the following section.

Figure 5.8.1

Constraints for weight-space of group 06

$$\underline{w}_B^T B^{-1} \geq 0$$

$$\text{and} \quad \sum_i \sum_\ell w_{il} = 1.0$$

$-1.39410\ W_3$		$+0.12686\ W_4$		$+0.15376\ W_7$		$+1.48513\ W_1$		$-0.14025\ W_9$	$-0.23141\ W_{10}$	≥ 0
$-2.78820\ W_3$	$+2.0\ W_6$	$+0.25373\ W_4$		$+0.30752\ W_7$		$+2.97027\ W_1$		$-0.28049\ W_9$	$-2.46282\ W_{10}$	≥ 0
$-1.31022\ W_3$		$+1.11923\ W_4$		$+0.02317\ W_7$		$+1.12632\ W_1$		$-0.07064\ W_9$	$-0.88786\ W_{10}$	≥ 0
$-22.61712\ W_3$	$+4.03030\ W_6$	$-2.97214\ W_4$	$+15.15152\ W_2$	$-6.28911\ W_7$	$+26.17080\ W_5$	$+19.46146\ W_1$		$+12.71247\ W_9$	$-15.34514\ W_{10}$	≥ 0
$10.68221\ W_3$	$-4.03030\ W_6$	$+0.05822\ W_4$	$+15.15152\ W_2$	$+2.75743\ W_7$	$-13.18379\ W_5$	$-11.65738\ W_1$		$-9.49113\ W_9$	$+9.71322\ W_{10}$	≥ 0
$-0.23235\ W_3$		$+0.02114\ W_4$		$+1.35896\ W_7$		$+0.24752\ W_1$		$-1.19004\ W_9$	$-0.20523\ W_{10}$	≥ 0
						$5.98802\ W_1$			$-5.98802\ W_{10}$	≥ 0
$-1.17825\ W_3$		$+0.10722\ W_4$		$+0.03528\ W_7$		$-0.02625\ W_1$	$+1.0\ W_8$	$-0.03570\ W_9$	$+0.09769\ W_{10}$	≥ 0
$-1.39410\ W_3$		$+0.12686\ W_4$		$+0.15376\ W_7$		$+1.48513\ W_1$		$+0.85975\ W_9$	$-1.23141\ W_{10}$	≥ 0
$1.39410\ W_3$		$-0.12686\ W_4$		$-0.15376\ W_7$		$-1.48513\ W_1$		$+0.14025\ W_9$	$+1.23141\ W_{10}$	≥ 0
W_3	$+\ W_6$	$+\ W_4$	$+\ W_2$	$+\ W_7$	$+\ W_5$	$+\ W_1$	$+\ W_8$	$+\ W_9$	$+\ W_{10}$	$= 1.0$

Figure 5.8.2 Maxima and Minima of weights for group 06

		w_3	w_6	w_4	w_2	w_7	w_5	w_1	w_8	w_9	w_{10}
	Max w_3	0.24202		0.05984	0.08910		0.10239	0.22208	0.28458		
(A)	Min w_3				0.46528		0.53472				
(C)	Max w_6		0.78989		0.21011						
(A)	Min w_6				0.46528		0.53472				
	Max w_4	0.05691		0.62539	0.12508		0.19262				
(A)	Min w_4				0.46528		0.53472				
	Max w_2				0.63333		0.36667				
(B)	Min w_2								1.0		
	Max w_7	0.06604		0.06604	0.04862	0.54126	0.22351		0.05153		
(B)	Min w_7								1.0		
(A)	Max w_5				0.46528		0.53472				
(B)	Min w_5								1.0		
(D)	Max w_1	0.07828						0.43011	0.06150		0.43011
(B)	Min w_1								1.0		
(B)	Max w_8								1.0		
(C)	Min w_8		0.78989		0.21011						
	Max w_9			0.01885	0.19564	0.36657				0.41893	
(C)	Min w_9		0.78989		0.21011						
(D)	Max w_{10}	0.07828						0.43011	0.06150		0.43011
(C)	Min w_{10}		0.78989		0.21011						

NOTE: 1. (A), (B) etc. indicate where the same point represents a maximum or minimum for more than one weight

2. Spaces indicate values of 0.0.

Figure 5.8.3 Results from test model for group 06
Six subsequent runs (2) to (7) with perturbed weights
Comparison with first run, $w_{i1}^{(1)}$ and $x_{i1}^{(1)}$
and expected allocations $x_{i1}^{(o)}$

	$x_{i1}^{(o)}$	(1)	(2)	(3)	(4)	(5)	(6)	(7)
w_{i1}		0.108	0.110	0.112	0.106	0.133	0.083	0.115
x_{i1}	12	12.28	12.28	12.28	12.19	12.30	12.12	12.05
w_{i2}		0.135	0.130	0.125	0.140	0.059	0.211	0.060
x_{i2}	15	15.15	15.15	15.15	15.22	15.03	15.22	14.96
w_{i3}		0.099	0.103	0.107	0.095	0.121	0.077	0.060
x_{i3}	11	10.77	10.76	10.76	10.87	10.78	10.97	10.79
w_{i4}		0.198	0.201	0.204	0.195	0.238	0.158	0.080
x_{i4}	22	21.99	21.99	21.99	22.00	22.01	22.03	22.03
w_{i5}		0.117	0.120	0.123	0.114	0.146	0.088	0.100
x_{i5}	13	13.05	13.05	13.05	12.92	13.05	12.82	13.05
w_{i6}		0.054	0.050	0.046	0.059	0.023	0.085	0.130
x_{i6}	6	6.01	6.01	6.01	5.99	6.04	5.99	6.06
w_{i7}		0.072	0.068	0.064	0.077	0.029	0.115	0.100
x_{i7}	8	8.01	8.01	8.01	8.04	7.98	8.07	8.00
w_{i8}		0.126	0.130	0.133	0.122	0.165	0.087	0.080
x_{i8}	14	14.02	14.02	14.02	14.02	14.02	14.02	14.02
w_{i9}		0.054	0.058	0.062	0.050	0.074	0.036	0.145
x_{i9}	6	5.94	5.95	5.95	5.88	6.02	5.86	6.04
w_{i10}		0.036	0.030	0.024	0.042	0.012	0.060	0.130
x_{i10}	4	3.78	3.77	3.77	3.85	3.75	3.90	4.01

CHOICE OF PERTURBATIONS IN WEIGHTING FACTORS

RUN (2) Selected perturbations of between 2% and 16% up or down.

(3) Double the changes of run (2)

(4) Perturb the same amount as run (2) but in the opposite direction

(5) Changes of ten or twelve times the size of those in (2)

(6) As (5) but in the opposite direction

(7) Large perturbations of between 7% and 260%, with increases and decreases in different weightings from the other runs.

5.9 Test model for a random sample including all client groups

It was firstly decided that a random sample of the whole census data should be used so that the results from such a model could be compared with results from similar models derived from other random samples of the census data. Section 6.5.1 describes the extraction of the first random sample of 1015 records and the tests performed to ensure that the sample was a representative sample.

The model used was the same as that postulated in section 5.4.1, i.e.

Choose the x_{il} to maximise $\sum_i \sum_l w_{il} x_{il}$

Subject to $\sum_i \sum_l x_{il} u_{ilk} \leqslant B_k$, for all k;

$\sum_l x_{il} = d_i$, for all i

and $\sum_i \sum_l w_{il} = 1.0$ is a condition of the data.

As in the case of the group 06 test model, the service allocations for the alternative modes of care, u_{ilk}, were selected from the cluster analysis of the whole of the census data and the weights $\bar{w}_{il}$ were calculated in proportion to the known size of each cluster, $x_{il}^{(o)}$.

i.e. $w_{il} = \frac{x_{il}^{(o)}}{d_i}$ and $d_i = \sum_l x_{il}^{(o)}$

The service totals B_k were found from

$B_k = \sum_i \sum_l x_{il}^{(o)} u_{ilk}$, for all k.

When this model was run it was found that only 17 real variables x_{i1} were selected for the optimal solution and no alternative optimal solutions were indicated. Consideration of this result suggested that this sample of data appeared to have a different optimum from the whole data set and that there appeared to be some form of suboptimising occurring. This could happen if the chosen sample was unrepresentative of the whole data or if the assignment of clients to modes for this sample was not adequately represented by the cluster analysis of the whole data set.

Further consideration led to the suggestion that, instead of using a random sample, a geographical "district" should be used. In this case it would be likely that the client service provision, having been selected by a single team of social workers, could be expected to be more homogeneous than that arising from a random sample. Consequently it was then decided to use the data for Chester-le-Street district, the number of clients being 730 and therefore of reasonable size.

5.10 The Chester-le-Street model

The data for Chester-le-Street district was extracted from the whole census data and analysed as described below in Section 6.5. For each client group i the clients were allocated to modes of care l in accordance with the Cluster Analysis of the whole census data. Thus for each i and l the number of clients $x_{il}^{(o)}$ in Chester-le-Street district was known and so was the allocation of a particular service k. Hence the allocations a_{ilk} of current service provision could be used to determine u_{ilk} the allocation per client in mode l of group i receiving service k: $u_{ilk} = \dfrac{a_{ilk}}{x_{il}^{(o)}}$

Similarly, $B_k = \sum_i \sum_l a_{ilk}$, for all k, was used to determine the total amount of service k available in Chester district;

$d_i = \sum_l x_{il}^{(o)}$, for all i, was used to find the size of each group i in Chester district.

The first set of weightings used was:

$$w_{il} = \frac{x_{il}^{(o)}}{\sum_i \sum_l x_{il}^{(o)}}$$

in the model as stated in Section 5.4.1, i.e.

Choose the x_{il} to maximise $\sum_i \sum_l w_{il} x_{il}$

subject to $\sum_i \sum_l x_{il} u_{ilk} \leqslant B_k$, for all k;

$\sum_l x_{il} = d_i$, for all i;

and $\sum_i \sum_l w_{il} = 1.0$ is a condition of the data.

However the results for x_{il} given by this model did not seem to be close enough to the expected values $x_{il}^{(o)}$ to be acceptable, so it was decided to try a different set of weightings. The first set had been chosen in relation to the group size expected in each group and each mode; the new set was related to the service allocations expected in each group and each mode.

Thus:

$$w_{il} = \frac{\sum_{k} x_{il}^{(o)} \quad u_{ilk}}{\sum_{i}\sum_{l}\sum_{k} x_{il}^{(o)} \quad u_{ilk}}$$

The results for x_{il} from this model seemed to be better in their relation to the expected values $x_{il}^{(o)}$.

Appendix 12 compares the expected values $x_{il}^{(o)}$ with those arrived at using the model and the two sets of weightings.

Since the number of constraints was much smaller than the number of real variables in the model (79 constraints arising from 45 service resource constraints and 34 group size constraints for Chester-le-Street district; 340 real variables from the 34 groups, 10 modes in each group) it was apparent that the model had many possible choices of real variables ($340^{C}79$ when all slacks were zero) from which to form a basis. It was therefore not surprising to find that multiple optimal solutions existed. The lists in Appendix 12 show only one of these optimal solutions for each set of weights. It would be possible to arrange for the computer package to move from one optimal solution to another since the package shows which variables are alternative basic variables, but this was not attempted.

At this stage it was decided to investigate the weight-space corresponding to the Chester-le-Street district model. Section 5.11 describes the investigation.

Subsequently the Chester-le-Street model was used to predict service resource needs in the event of demographic changes in the district. This is described in Chapter 8.

5.11 <u>Chester-le-Street district : determination of feasibility region for weights</u>

Once the feasibility region for weights had been constructed for the group 06 model, this showed the viability of the method for determining the feasibility region for weights. (See section 5.8).

As described in Section 5.10 above, an alternative mode model was constructed for Chester-le-Street district. All client groups were included and each group had ten alternative modes of care as defined by the results of the Cluster Analysis. When this model was run, multiple optimal solutions existed. Taking one of these solutions, viz. that printed by the MPSX package, the weight constraints could then be modelled. The size of the Chester weights model was much larger than for group 06 alone, so a modified procedure was used to construct this model. The sizes were: 34 client groups, 10 modes in each group, 340 unknowns (w_{il}), A was (113 x 340), B^{-1} was (113 x 113), $\underline{w}^T$ was (1 x 340), $\underline{w}_B^T$ was (1 x 113), thus the weights model contained 340 constraints corresponding to (1) in Section 5.8 above and 113 constraints corresponding to (2) in Section 5.8 above, together with one equation to standardise the weights.

The modified procedure used was:

(a) The list of basic variables (52 reals and 61 slacks) was extracted from the MPSX solution to the alternative modes model for Chester-le-Street.

(b) The vector $\underline{w}_B^T$ was constructed using this list of basic variables.

(c) The inverse matrix B^{-1} was not extracted from MPSX, its size (340 x 340) being unsuitable for manipulation in the same way as for the group 06 model. Instead B^{-1} was constructed as follows:

(i) The matrix A_s (113 x 113) was constructed. This was formed by selecting 52 columns, corresponding to the real variables in the solution basis, from the initial matrix A (113 x 340) of coefficients in the constraints of the resource allocation model; and by constructing 61 columns to correspond to the slack variables in the solution basis: these 61 columns were the corresponding columns of the unit matrix I (113 x 113).

(ii) A_s was inverted, using the Numerical Algorithms Group (NAG) computer package, to form B^{-1}.

(d) The vector $\underline{w}_B^T B^{-1}$ was constructed.

(e) The constraints $\underline{w}_B^T B^{-1} \geqslant 0$ were listed.

(f) The vector $\underline{w}_B^T B^{-1}A$ was calculated using a specially written computer program.

(g) The constraints $\underline{w}^T \leqslant \underline{w}_B^T B^{-1}A$ were listed.

(h) The constraint $\sum_i \sum_l w_{il} = 1.0$ was added to the two sets of constraints $\underline{w}_B^T B^{-1} \geqslant 0$

and $\underline{w}^T \leqslant \underline{w}_B^T B^{-1}A$

to form the weight-space model for Chester district.

(i) These constraints were input to a new MPSX model which maximised $\sum_{i}\sum_{l} w_{il} \; x_{il}$ for values of x_{il} equal to the Cluster Analysis values $x_{il}^{(0)}$.

The result of this model,

ie Choose the w_{il} to Maximise $\sum_{i} \sum_{l} w_{il} \; x_{il}$

$$\text{subject to } \underline{w}^T \leq \underline{w}_B^T B^{-1} A,$$

$$\underline{w}_B^T B^{-1} \geqslant 0,$$

$$\text{and} \sum_{i} \sum_{l} w_{il} = 1.0.$$

was to choose only one weight,

ie $w_{43,3} = 1.0$, and

all other weights = 0.0.

This is an extreme point of the weight-space.

In order to find other extreme points the model was run with the objective being to Maximise in turn each of the weights for each of the ten modes of care in four client groups, viz. groups 06, 18, 35, 43. Figure 5.11.1shows the maximum value of each weight when that weight was being maximised.

From these runs of the model it could also be deduced that the minimum value of every weight was zero.

Figure 5.11.1 shows, in addition, for client group 18, when the objective was to Maximise the weight given to mode 7 of care, the values of the other weights at the optimum solution point, i.e. the complete specification of an extreme point. This is shown as an example of the results obtained for groups 06, 18, 35 and 43.

It is clear that the specification of weight-space is possible in the manner described. Probably the most useful feature is that if a set of weights is postulated, the model can be used to assess the feasibility of the set.

The next section (5.12) takes the weight-space problem and discusses other possible approaches which could be taken.

Figure 5.11.1

Maximum values of weights w_{il} for Client Groups 06, 18, 35, 43 in Chester-le-Street district

Mode l \ Client Group i	06	18	35	43
1	1.0	0.2	0.5	0.701
2	0.5	0.25	1.0	0.25
3	0.5	0.307	0.757	1.0
4	0.5	0.772	0.5	0.25
5	0.5	1.0	0.5	0.25
6	0.5	0.272	0.5	0.25
7	0.5	0.346	0.5	0.333
8	0.5	0.2	0.5	0.25
9	0.5	0.56	0.5	0.365
10	0.5	0.428	0.757	0.25

Minimum values for these weights were all zero.

Example of Extreme Point Solution:

Maximise Weight for group 18, mode 7

The solution includes the following non-zero weights, all other weights being zero.

Client Group i	Mode l	Weight w_{il}
3	2	0.027
18	2	0.173
18	4	0.058
18	5	0.223
18	7	0.346
25	8	0.173
		1.0

5.12 Other Approaches to the Weight-Space Problem

5.12.1 Alternative Presentation of the Weight Constraints

Instead of presenting the weight constraints as in Section 5.4, an alternative and perhaps preferable presentation can be achieved by a different partitioning of the Simplex tableau.

For the problem $A\underline{x} \leqslant \underline{b}$, $\underline{W}^T\underline{x}$ to max;
where A is (mxn), $\underline{b}$ is (m x 1), $\underline{W}$ is (n x 1); the initial Simplex tableau can be written in terms of the basis matrix as:

$$
\begin{array}{c|c|c|c|c|c}
m & B & A_R & I_R & 0 & \underline{b} \\
\hline
1 & \underline{W}_B^T & \underline{W}_R^T & 0 & 1 & 0 \\
\hline
 & m & n-t & t & 1 & 1
\end{array}
$$

where B represents an (m x m) basis matrix made up of t columns of A representing the real variables in the basis and (m - t) columns of I representing the slack variables in the basis,

A_R represents the (n - t) remaining columns of A,

I_R represents the t remaining columns of I,

W_B^T represents those coefficients of the objective function which correspond to basic variables

and W_R^T represents those coefficients of the objective function which correspond to non-basic variables.

When an optimal solution is reached, this means that

$$
\left[\begin{array}{c|c} B & 0 \\ \hline \underline{W}_B^T & 1 \end{array}\right] \cdot \left[\begin{array}{c|c|c|c|c} G & H & J & 0 & \underline{X}^* \\ \hline \underline{g} & \underline{h} & \underline{j} & 1 & -f^* \end{array}\right] = \left[\begin{array}{c|c|c|c|c} B & A_R & I_R & 0 & \underline{b} \\ \hline \underline{W}_B^T & \underline{W}_R^T & 0 & 1 & 0 \end{array}\right] \qquad (1)
$$

where G is of size (m x m),

H is $(m \times \overline{n-t})$,

J is (m x t),

$\underline{X}^*$ is (m x 1),

$\underline{g}$ is $(1 \times m)$,

$\underline{h}$ is $(1 \times \overline{n-t})$,

and $\underline{j}$ is $(1 \times t)$.

At the optimum, whence f^* cannot be improved, the values in the objective row are all negative or zero except the coefficient of f the objective function. Thus $\underline{g} \leqslant 0$, $\underline{h} \leqslant 0$, $\underline{j} \leqslant 0$.

Expanding the expression (1) above gives:

$BG = B$, hence $G = I$;

$BH = A_R$, hence $H = B^{-1}A_R$;

$BJ = I_R$, hence $J = B^{-1}I_R$;

$B\underline{x}^* = \underline{b}$, hence $\underline{x}^* = B^{-1}\underline{b}$;

$\underline{W}_B^T\, G + \underline{g} = \underline{W}_B^T$, hence $\underline{g} = 0$;

$\underline{W}_B^T\, H + \underline{h} = \underline{W}_R^T$, hence $\underline{h} = \underline{W}_R^T - \underline{W}_B^T\, B^{-1}\, A_R$;

$\underline{W}_B^T\, J + \underline{j} = 0$, hence $\underline{j} = -\underline{W}_B^T\, B^{-1}\, I_R$;

$\underline{W}_B^T \underline{x}^* - f^* = 0$, hence $f^* = \underline{W}_B^T\, \underline{x}^*$.

Thus at the optimum, $\underline{g} = 0$;

$\underline{W}_R^T - \underline{W}_B^T\, B^{-1}\, A_R \leqslant 0$; (ie $\overline{n-t}$ inequalities)

$\underline{W}_B^T\, B^{-1}\, I_R \geqslant 0$. (ie t inequalities)

Thus, altogether there are n inequality constraints for the weights.

If there are multiple solutions, some of these n inequalities will become equal to zero.

So there will be at most n inequalities + some equations } n altogether

In the actual case we can assume that all n real variables can appear in the solution.

So (n - m) variables, which did not appear in the first alternative solution , could be brought into the solution, hence there will be (n - m) equations:

$$\underline{W}_R^T - \underline{W}_B^T \; B^{-1} \; A_R \; = \; 0,$$

thus leaving m inequalities:

$$\underline{W}_B^T \; B^{-1} \; I_R \geqslant 0.$$

Appendix 15 shows an example of this alternative presentation of the weight constraints for a small example having five real variables and two constraints; the actual, known solution having five non-zero variables.

Now take the special case of $A\underline{x} = \underline{b}$, $\underline{W}^T \, \underline{x}$ to max, $m < n$

Now I_R does not exist and the expression (1) simplifies to:

$$\left[\begin{array}{c|c} B & 0 \\ \hline \underline{W}_B^T & 1 \end{array}\right] \cdot \left[\begin{array}{c|c|c|c} G & H & 0 & \underline{x}^* \\ \hline \underline{g} & \underline{h} & 1 & -f^* \end{array}\right] = \left[\begin{array}{c|c|c|c} B & A_R & 0 & \underline{b} \\ \hline \underline{W}_B^T & \underline{W}_R^T & 1 & 0 \end{array}\right] \qquad (2)$$

with sizes B (m x m), G (m x m), A_R (m x $\overline{n-m}$), H (m x $\overline{n-m}$), $\underline{x}^*$ (m x 1), $\underline{g}$ (1 x m), $\underline{W}_B^T$ (1 x m), $\underline{h}$ (1 x $\overline{n-m}$), $\underline{W}_R^T$ (1 x $\overline{n-m}$).

At the optimum, $\underline{g} \leqslant 0$, $\underline{h} \leqslant 0$.

Expanding (2) gives:

$BG = B$, hence $G = I$;

$BH = A_R$, hence $H = B^{-1} A_R$;

$B\underline{x}^* = \underline{b}$, hence $\underline{x}^* = B^{-1} \underline{b}$;

$$\underline{W}_B^T G + \underline{g} = \underline{W}_B^T, \quad \text{hence } \underline{g} = 0;$$

$$\underline{W}_B^T H + \underline{h} = \underline{W}_R^T, \quad \text{hence } \underline{h} = \underline{W}_R^T - \underline{W}_B^T B^{-1} A_R;$$

$$\underline{W}_B^T \underline{x}^* - f^* = 0, \quad \text{hence } f^* = \underline{W}_B^T \underline{x}^*.$$

Thus, at the optimum, $\underline{g} = 0$;

$$\underline{W}_R^T - \underline{W}_B^T B^{-1} A_R \leqslant 0$$

ie (n - m) inequality constraints for the weights.

Since at the actual optimum there must be multiple solutions (all n variables have non-zero values), so that the solution in n variables is a linear multiple of other solutions; then some of these inequalities will be equations, equating to zero.

If there are m basic variables and n real variables then there could be up to (n - m) alternative solutions, each bringing an alternative variable into the solution. In this case all (n - m) inequalities would reduce to equations, equating to zero.

5.12.2 <u>Parametric Programming</u>

An alternative way of defining the weight-space is to find all its extreme points. The maxima and minima for the weights, as found for the small test model in Section 5.7 or for the group 06 model in Section 5.8, lead to only some of the extreme points.

One possibility which was explored was to use parametric programming to perturb the objective function and thus to find another basic feasible solution which then became optimal. This new solution would be assocated with a different extreme point.

The parametric programming facility PARAOBJ in the IBM MPSX package was used to try this method and it proved possible to perturb the objective function as described and to reach a new extreme point, but the specification of the perturbation parameters was arbitrary and unsatisfactory. The perturbation parameters had to be specified as input values, which were therefore pre-set and fixed for each run of the model. In order to use the parametric programming method to find all extreme points some form of automated altering of parameters would be needed.

5.12.3 Extreme Points of the Weight-space

As stated above, an alternative way of defining the weight-space is to find all its extreme points. There may be a very large number of these: if there are n real variables x_{il} with corresponding weights W_{il}, then there are (n - m) constraints on the weights (see Section 5.12.1), where m is the number of resources plus the number of client groups (i.e. the number of constraints on x_{il} in the LPAM model); consequently the maximum number of extreme points is ${}_{n}C_{n-m}$, being the number of ways of choosing (n - m) basic weight-variables from the n available. Each extreme point corresponds to a basic feasible solution, but every selection of a basis may not be feasible, so ${}_{n}C_{n-m}$ represents a maximum. The maximum number of extreme points is thus ${}_{n}C_{n-m}$ or ${}_{n}C_{m}$. For example, if there are 35 client groups, with 10 modes of care in each, and 45 services, then n = 350, m = 80, ${}_{n}C_{m}$ = approx. 59! or 10^{80}.

5.12.4 Extreme Points of the x-space

Another approach is to consider the extreme points, not of the weight space, but of the original problem, i.e. the x-space. The original constraints were $A \underline{x} \leq \underline{b}$, so extreme points of the feasible region can be evaluated by choosing a basis matrix B and calculating $\underline{x} = B^{-1}\underline{b}$. Selection of a basis may not always yield a feasible solution, but the maximum number of extreme points is ${}_{n}c_{m}$, where n is the number of real variables and m is the total number of resource and group size constraints. This takes account of the fact that the actual solution (representing the current situation) occurs where all constraints are fully satisfied and so all slack variables are zero, i.e. $A \underline{x} = \underline{b}$. There may thus be a very large number of extreme points, as in the previous section : if n = 350 and m = 80 then there can be up to 10^{80} extreme points.

Supposing it were possible to enumerate these, the weight constraints could be derived from these:

Let E_1, E_2, ... be the extreme points of the x-space. Then the actual solution, which contains more non-zero real variables (i.e. n of these) than can be included in a basis (of size m), $\underline{x}^*$ can be expressed as a linear combination of some of the extreme points, i.e.

$$\underline{x}^* = \sum_{\text{some } i} \alpha_i E_i$$

So if x^* and E_1, E_2 are known, α_i can be found.

Since $\underline{x}^*$ is optimal, so also are the extreme points being combined, i.e. there are multiple optima, and the objective function has the same optimal value at these points.

i.e. $f^* = \sum_j W_j X_{ij}$ at point E_i $(x_{i1}, x_{i2}, \ldots, x_{in})$,

for all i representing those extreme points being combined.

Hence $\sum_j W_j X_{i_1 j} = \sum_j W_j X_{i_2 j} = \sum_j W_j X_{i_3 j} = \ldots$

for all such i $(i_1, i_2, i_3, \ldots)$

These result in a set of equations which correspond to the equations for the weights:

$\underline{W}_R^T - \underline{W}_B^T B^{-1} A_R = 0$ as derived in section 5.12.1.

In addition, any of these extreme points must be a maximum, so $f^* \geqslant 0$.

At the extreme point for the basis B, $\underline{x} = B^{-1} \underline{b}$

and $f = \underline{W}_B^T \underline{x} = \underline{W}_B^T B^{-1} \underline{b}$

Now $\underline{b} \geqslant 0$

Hence $\underline{W}_B^T B^{-1} \geqslant 0$, being the set of inequalities for the weights corresponding to those derived in section 5.12.1.

Appendix 16 shows an example of this method applied to a small example having 5 real variables and 2 constraints. In this example there were ${}_5C_2 = 10$ possible extreme points of which only 6 were feasible. The known solution point (in 5 variables) could be expressed as a linear combination of the feasible extreme points and from these the weight constraints could be deduced.

5.12.5 Multiple Solutions

In the methods described in the preceding sections the weight constraints have been derived by choosing a basis and developing the constraints from consideration of that basis (or corresponding extreme point).

Since the actual solution point contains more real variables than are contained in a basis, multiple solutions exist. The actual solution point can be expressed as a linear combination of extreme points so some of the bases lead to optimal extreme points.

In the examples given so far, a single choice of basis has been made. It remains to show whether the choice of basis influences the constraints derived for the weights.

Appendix 17 takes the same small example as used in Appendices 15 and 16 and shows that the choice of basis is not significant : that whatever basis is chosen, if $n > m$, the eventual set of weight constraints will be the same.

5.12.6 "Shape" of the LP Problem; Degeneracy

Sections 5.8 and 5.11 showed the derivation of the weight-constraint models for Client Group 06 and for the Chester-le-Street district. There is a significant difference between these two models because of the "shape" of the constraint matrix A in the original linear programming (LP) problem in each case. The problem is to Maximise $\sum_i W_i x_i$

subject to $A\underline{x} = \underline{b}$

In the Group 06 model, the number of constraints, m, is greater than the number of real variables, n, whereas in the Chester-le-Street model n is greater than m.

This affects the simplex tableau as follows. Using the notation of Section 5.12.1, the final simplex tableau is:

G	H	J	0	x^*
$\underline{g}$	$\underline{h}$	$\underline{j}$	1	$-f^*$

where $\underline{g} = 0$, $\underline{h} = \underline{W}_R^T - \underline{W}_B^T B^{-1} A_R$, $\underline{j} = -\underline{W}_B^T B^{-1} I_R$, corresponding to an initial tableau:

B	A_R	I_R	0	$\underline{b}$
$\underline{W}_B^T$	$\underline{W}_R^T$	0	1	0

For group 06, m $>$ n so B always contains some columns of I as well as all the columns of A. The solution must always include some slack variables in the basis and these must be zero at the actual solution point (since $A\underline{x} = \underline{b}$). The simplex solution is thus degenerate. Aucamp[3] and Steinberg have shown that when degeneracy is present in an optimal basic solution, the optimal values of the dual variables do not necessarily correspond to shadow prices and that commercial LP packages, such as IBM's MPSX, may provide misleading information about the shadow prices.

In the construction of the weight constaints, the shadow prices are not used explicitly, the relationships being derived from the basis matrix and its inverse.

Appendix 18 shows the construction of weight constraints for another small problem, in this case with $m > n$. In such a problem, where $A\underline{x} = \underline{b}$ and $m > n$, there can be a solution to all the equations only if some equations are dependent on others and the equations are consistent. A unique solution exists if $m = n$ or if there are $(m - n)$ equations linearly dependent on the other n independent and consistent equations. Multiple solutions exist if $m < n$ or if more than $(m - n)$ equations are linearly dependent on the other independent and consistent equations.

To illustrate the choice of basis and construction of weight constraints the small problem has been chosen with $m = 4$, $n = 3$ and two equations linearly dependent on the other two. A solution exists at two extreme points and at points which are linear combinations of these two.

The example shows that the construction of the weight constraints is the same whichever optimal basis is chosen and despite the presence of degeneracy.

5.12.7 "Efficient" Solutions to Multiple Objective Linear Programming (MOLP)

Kornbluth[36] in "Duality, indifference and sensitivity analysis in multiple objective linear programming" (1974) suggests that the construction of a utility function by a decision maker may be an unrealistic expectation and that it may be more appropriate to consider each performance measure separately rather than combining these to form a composite utility function. There is then no single optimal solution to the problem since it is unlikely that all performance functions will be optimised simultaneously.

"Efficient" solutions can be defined which are as good as other solutions, in respect of some performance measures, and better than the other solutions for at least one performance measure. Algorithms exist to find efficient solutions and the problem can be solved by allowing the decision maker to choose between these on a subjective basis.

Kornbluth goes on to show that each efficient solution to a MOLP problem is associated with a whole set of possible weights and that these sets are bounded by linear constraints, similar to the weight constraints discussed previously.

This method may offer an alternative approach to the construction of an objective function and computation of weight space.

5.13 Conclusions

This chapter has demonstrated the development of a new resource planning model (LPAM), which has a linear objective function, similar to the linear equivalent of the D.U.B.S. goal programming model, and which can choose allocations of clients to alternative modes of care. Unlike the modes of care used in Macdonald's Balance of Care model, these modes were not chosen by a professional group. Instead an analysis of the Census data was performed using the Cluster Analysis software package (see Section 6.4). The modes of care constructed were thus derived from the existing patterns of care at the time of the Census. In this model too, weighting factors were used in the objective function. These weightings could be obtained from the field workers by direct questioning, in order to reflect the relative preferences of caring for clients in a particular group by allocating a particular mode of care. In practice the weightings were calculated from the Census data, as described in Section 5.10, and thus the preferences were inferred from the resource allocations made by the social workers at the time of the Census. An alternative approach which could be used to set the values of the weightings would be to build upon the recent work of Durham County Council in their Caseload Management System (briefly described in Section 4.6) which has identified priorities in respect of groups of clients and a "points" system which identifies priorities of categories within groups. This possibility is considered further in Chapter 9.

Thus the modes of care and the weightings in the objective function are inferred from the census data, as indeed are the totals of service availabilities. Furthermore, the services allocated to each mode of care are maintained unchanged in the model, although the number of clients assigned to each mode may vary.

A weight-constraints model has also been developed which uses the known resource usage and resource availability at the time of the census to derive a set of constraints for the weighting values used in the objective function of the LPAM model. This shows the range of possible weights which would produce the same resource allocation to clients. The weight-constraints model can be used to test the feasibility of a set of weighting values postulated by the planners.

Following this Chapter, Chapter 6 details the analyses of the census data including the cluster analysis to derive alternative modes of care; Chapter 7 makes some comparisons of the Durham survey with other regional models; Chapter 8 shows how the LPAM model can be used to plan for demographic and/or resource changes and makes comparisons with the DHSS SPRAM model; and Chapter 9 discusses limitations, applications and extensions of the LPAM model, with a view to its future usage in the context of County Durham.

CHAPTER 6

ANALYSIS OF THE CENSUS DATA

6. Analysis of the census data

6.1 The census data and internal inconsistencies

As mentioned in section 3.2, the Cendat data contained 8216 client records, for each of which was recorded

- a client number
- a code for age group
- a code for sex
- a client group number
- a list of up to ten codes for the actual services received at that time.
- a list of up to six codes for the additional services desired by the social worker for the client.
- a list of up to eight codes for the alternative services which could be supplied, if the actual services were no longer available.
- further comments.

Appendix 1 contains a list of the client groups and corresponding group numbers.

Appendix 2 contains a list of the services available and corresponding service code numbers.

The first requirement was to vet the data for internal consistency: a computer program was written to do this and at the same time to summarise the requirements of all the clients in the census in terms of actual services supplied, those services additionally desired and alternative services which could be supplied.

105 "errors" were detected in the 8216 client records, consisting of

- records with no services recorded
- records with group numbers outside the valid range

- records with apparent inconsistencies in the services listed.

5 records were found to have two errors in each, so the number of records in error was 100.

Of these, 36 records, having group numbers outside the valid range, could not be corrected and were excluded from further consideration.

Similarly, 55 records, having no actual services supplied, could not be corrected and were excluded from further consideration of "current" services.

The remaining 9 errors were spacing errors and these were easily corrected, thus leaving 8125 valid client records for further analysis.

It was subsequently found that many of these records contained duplications in the services listed, but these duplications were eliminated when the client groups were merged.

6.2 Group and service summaries

An analysis was produced, from the validated client records, of the usage of services by clients : the analysis showed, for each client group, the number of clients in the group and, for each service, the number of clients in each group receiving that service currently, the number of clients in each group for whom the service was additionally desired by the social worker and the number of clients in each group for whom the service could be regarded, by the social worker, as an alternative acceptable service.

This first analysis was based on the 45 client groups and 99 services chosen by the DUBS team for use in collecting the census data. However, 45 groups and 99 services were considered to be too many for the author's research, so an examination of the analysis client groups and services was undertaken in the hope of reducing the number of client groups and service codes.

(i) Client Groups

The number of clients in each of the 45 groups (i.e. group size) was considered. See Figure 6.2.1

There were 8125 clients in total, so group sizes were then classified as follows:

A :	Size $\geq$ 80 clients	: 31 groups
B :	40 $\leq$ size $<$ 80 clients	: 6 groups
C :	Size $<$ 40 clients	: 8 groups

Subsequent examination of service requirements for the 14 groups classified as B or C showed that for any individual client group the highest percentage of any service used was of negligible size, except for groups 5, 12 and 21.

This suggested that the other 11 groups of small size might be merged with other groups of greater size.

The description and service requirements of each of the 11 groups were inspected in order to decide which remaining groups could reasonably include the clients from the groups to be discarded. Figure 6.2.2 gives a list of these eleven groups, their descriptions and which groups they were merged with.

Group 35 with 52 clients was considered to be so unlike any other group that it was decided to retain group 35 as a separate client group.

Thus the 45 client groups were reduced to 35, Figure 6.2.3 gives a list of the sizes of the merged groups.

Appendix 3 contains a complete list of the 35 client groups and their revised descriptions.

(ii) <u>Service codes</u>

The number of clients actually receiving each service was considered. See Fig. 6.2.4.

There were 8125 clients but each client receives several services so the total number of 19,448 "Service Receipts" includes the same clients several times. However, for each individual service, a client is counted only once. Thus it was possible to classify the 99 service codes as follows:

A : Service received by 80 or more clients : 38 service codes.

B : Service received by between 40 and 79 clients : 6 service codes

C : Service received by less than 40 clients : 55 service codes

Hence each one of 55 services was supplied to less than half a percent (½%) of all clients. This suggested that these service codes should be merged with other service codes.

The description of each of the 55 services was inspected in order to decide which remaining service codes could reasonably include the services to be discarded. Figure 6.2.5 gives a list of these 55 services codes, brief descriptions and which service codes they were merged with. In effect, some service codes would then represent a resource of more than one service, but for planning purposes this would be acceptable.

Inspection of the service descriptions showed that it would not be appropriate to exclude all 55 service codes because the remaining 44 service codes could not reasonably include all 55 services. It was therefore decided that one of the 55 service codes would be re-designated, so as to include some of the excluded service codes. Code 49 was therefore renamed as "additional services for mentally handicapped and/or mentally ill clients". Some of the 44 service codes retained were, in effect, renamed; for example code 28 was originally described as "group living schemes for the elderly" and now includes "additional services for the elderly/physically handicapped".

Thus the 99 service codes were reduced to 45. Appendix 4 contains a list of the 45 service codes and their revised descriptions, whilst figure 6.2.6 shows the number of clients receiving each of the 45 revised services.

Appendix 5 contains the complete analysis of numbers of clients in each of 35 groups receiving each of 45 services.

Figure 6.2.1 Numbers of clients in each of 45 groups

GROUP	SIZE	GROUP	SIZE
1	83	24	33
2	169	25	86
3	291	26	69
4	101	27	14
5	39	28	157
6	97	29	241
7	14	30	280
8	216	31	300
9	226	32	37
10	190	33	41
11	301	34	276
12	64	35	52
13	39	36	150
14	125	37	86
15	20	38	92
16	365	39	169
17	250	40	139
18	550	41	282
19	22	42	320
20	360	43	645
21	49	44	536
22	39	45	247
23	263	TOTAL	8125

Figure 6.2.2 Client groups of size less than 80 clients and having very small amounts of service requirements

GROUP	NUMBER OF CLIENTS	DESCRIPTION	MERGE WITH	
			GROUP	DESCRIPTION
7	14	Adult clients unable to cook adequately unaided.	6	Adult clients unable to clean house and/or go out unaided.
13	39	Elderly clients unable to bathe and/or use toilet unaided.	16	Frail elderly clients.
15	20	Elderly clients unable to cook adequately unaided.	16	Frail elderly clients.
19	22	Severely mentally ill adult clients.	18	Mentally ill adult clients.
22	39	Adult clients violent within relationships with other adult(s)	23	Adult clients with non-violent but inadequate relationship with partner, other adults or children.
24	33	Adult victims of violence within relationships with other adult(s)	29	Adult clients with multiple relationship, environmental and financial problems.
26	69	Adult persons non-accidentally injuring or otherwise abusing children.	30	Other adult clients with relationship, environmental and financial problems.
27	14	Homeless adults	30	"
32	37	Prospective playgroup leaders and child-minders.	34	Other adult clients.
33	41	Ante- and post-natal unmarried mothers and women requesting abortions.	34	Other adult clients.
35	53	Physically handicapped children and young persons.	-	(not merged)

Figure 6.2.3 Numbers of clients in each of 45 groups after merging 10 of the original groups with the other 35

GROUP	SIZE	GROUP	SIZE
1	83	24	0
2	169	25	86
3	291	26	0
4	101	27	0
5	39	28	157
6	111	29	274
7	0	30	363
8	216	31	300
9	226	32	0
10	190	33	0
11	301	34	354
12	64	35	52
13	0	36	150
14	125	37	86
15	0	38	92
16	424	39	169
17	250	40	139
18	572	41	282
19	0	42	320
20	360	43	645
21	49	44	536
22	0	45	247
23	302	TOTAL	8125

Figure 6.2.4 Number of clients receiving each service

Service Code	No. of Clients	Service Code	No. of Clients	Service Code	No. of Clients	Service Code	No. of Clients
1	887	26	481	51	4	76	94
2	339	27	162	52	1	77	9
3	341	28	819	53	4	78	3
4	2749	29	14	54	3	79	85
5	1403	30	3	55	0	80	25
6	2562	31	0	56	0	81	10
7	55	32	0	57	8	82	0
8	119	33	0	58	16	83	303
9	6	34	1	59	51	84	300
10	3	35	0	60	4	85	12
11	80	36	0	61	7	86	3
12	79	37	23	62	339	87	4
13	9	38	107	63	4	88	76
14	261	39	26	64	15	89	23
15	216	40	16	65	4	90	118
16	48	41	20	66	0	91	164
17	907	42	216	67	16	92	1
18	453	43	215	68	1	93	0
19	355	44	295	69	8	94	0
20	944	45	30	70	568	95	0
21	127	46	17	71	194	96	1
22	99	47	1	72	560	97	2
23	606	48	1	73	637	98	0
24	255	49	0	74	46	99	0
25	198	50	2	75	175		

Total "Service receipts" 19,448

Figure 6.2.5 Services such that each is supplied to less than ½% of all clients

Service Code	Brief description	No. of Clients	Merged with Code
09	Hairdressing	6	16
10	Laundry	3	16
13	Homes for physically handicapped	9	11
29	Mobile day care for elderly	14	28
30	Night sitting	3	28
31	Homes for psychogeriatrics	0	28
32	Assessment centres for elderly	0	28
33	Support groups for relatives	0	28
34	Fostering of elderly	1	28
35	Weekday residential care	0	28
36	Para-medic halfway care	0	28
37	Homes for M.H. adults	23	11
39	Group living : M.H.	26	11
40	Group living : M.Ill	16	12
41	Adult training centres	20	49
45	Hospital based social work	30	27
46	Homes for M.Ill	17	12
47	Special care : M.H.	1	49
48	Special care : M.Ill	1	49
49	Day Centres : M.H.	0	49
50	Day Centres : M.Ill	2	49
51	Occup. th. in homes for M.H.	4	49
52	Support groups, M.H. relatives	1	49
53	Pre-school groups : M.H.	4	49
54	Holiday playschemes : M.H.	3	49
55	Short-term fostering : M.H.	0	49
56	Convalescent home	0	56

Figure 6.2.5 Services such that each is supplied to less than ½% of all clients (Continued)

Service Code	Brief description	No. of Clients	Merged with Code
57	Family training unit	8	56
58	Family advice centre	16	56
59	Family service unit	51*	56
60	Hospital based social work	4	56
61	Preventive, supportive services	7	56
63	Hostel for battered wives	4	62
64	Mother and baby home	15	62
65	Alcoholic treatment	4	62
66	Abortion counselling	0	62
67	Home helps: family training	16	62
68	Support groups : single parents	1	62
69	Statutory care: ch & yp	8	71
77	Residential nursery	9	79
78	Day nursery	3	79
80	Childminders	25	79
81	Salaried childminding	10	79
82	School based social work	0	27
85	Professional foster parents	12	83
86	Intermed. trtmt, residential	3	73
87	Intermed. trtmt, other	4	73
89	Child guidance	23	73
92	Intermed. treatment centre	1	73
93	Domic. obs. and assess.	0	73
94	Flats for adolescents	0	73
95	Children leaving care	0	73
96	Short term resid. I.T.	1	73

Figure 6.2.5 Services such that each is supplied to less than ½% of all clients (Continued)

Service Code	Brief description	No. of Clients	Merged with Code
97	Resid. trtmt : dist. adol.	2	73
98	Peripatetic houseparents	0	73
99	Mobile day nursery	0	73

*Although service 59 has 51 clients, it was decided to group all service codes 56 to 61 together as code 56.

Figure 6.2.6 Number of clients receiving each of 45 revised services

Service Code	No. of Clients	Service Code	No. of Clients
1	887	27	192
2	339	28	835
3	341	38	107
4	2749	42	216
5	1403	43	215
6	2562	44	295
7	55	49	35
8	119	56	85
11	133	62	375
12	112	70	568
14	261	71	202
15	216	72	560
16	55	73	670
17	907	74	46
18	453	75	175
19	355	76	94
20	944	79	130
21	127	83	315
22	99	84	300
23	606	88	76
24	255	90	118
25	198	91	164
26	481	Total "Service receipts"	19,430

6.3 Alternative modes of care : preliminary analyses

The original Cendat data contained, as well as the list of actual services received by each client, a list of up to eight codes for alternative services which could be supplied if the actual services were no longer available.

The data thus offered two means of analysing alternative modes of care as specified by the social workers who completed the census. The first was to analyse the actual services received and to count the number of distinct "packages of care" actually being supplied to each client group. A "package of care" in this context meant a set of services supplied to a client. The quantity of each service supplied was not considered. For example, a client might receive a package which included day care, meals on wheels and technical officer services for the blind.

The second means available directly from the data was to analyse the alternative service provision selected by the social worker. Again the number of distinct, alternative "packages of care" was counted for each client group.

Results of these analyses follow:

(i) Analysis of actual service provision

Computer programs and sort routines were written:

(a) to sort the actual services into numerical sequence within each record of the census data,

(b) to sort the records into client groups,

and (c) to recognise and count the number of clients in each client group who were actually receiving the same package of care.

Figure 6.3.1 shows the client groups, the number of clients in each group and the number of packages actually received by each group. The number of packages ranged from 6, supplied to group 32 with 37 clients, to 184, supplied to group 16 with 378 clients. Altogether there were 3206 different packages of care actually being supplied to 8125 clients. (The table shows a total of 8180 clients but only 8125 clients were actually receiving services).

Clearly there were too many packages for planning purposes.

(ii) <u>Analysis of alternative service provision</u>

Analysis of the alternative service provision as specified by the social workers was carried out for the 35 merged client groups, subsequent to the merging of groups described in section 6.2. Alternative packages had been specified for 1135 clients. Computer programs and sort routines were again written :

(a) to extract the relevant records from the census data,

(b) to sort the alternative services into numerical sequence within each record,

(c) to recognise and count the number of clients in each client group who were potential clients for each alternative service package,

and (d) to recognise those alternative packages which were identical to packages actually supplied to a client group.

Figure 6.3.1 shows the number of alternative service packages specified for each client group by the social workers, alongside the numbers of actual service packages. Also shown is the number of alternative packages in each client group which are identical to packages of actual services supplied. The "total" column shows the total number of alternative modes of care for each client group, however specified.

The number of alternative packages specified by the social workers ranged from none at all for some client groups to 35 for group 44, which has 536 clients. In addition, only three of these 35 alternatives for group 44 were the same as any of the 138 packages actually in use for these clients. Altogether there were 257 different packages of care specified as possible alternatives to the actual services supplied. Of these, only 29 were identical to the actual packages supplied.

Since the alternative packages specified by the social workers had to be considered in conjunction with the actual packages supplied, giving a total of 3434 alternative modes of care for 8180 clients, it was decided that another means of choosing modes of care must be sought.

Figure 6.3.1 Analysis of actual and alternative service provision

Client Group	No. of Clients	No. of packages actually received	No. of Alternative Packages	Inc.*	Total
1	83	70	0		70
2	171	116	2		118
3	291	140	1		141
4	102	27	2		29
5	40	22	0		22
6	97	72	4		76
7	14	11	N/A		11
8	219	120	2		122
9	229	110	5	1	114
10	190	119	3		122
11	302	132	5	1	136
12	64	46	3		49
13	39	25	N/A		25
14	126	88	5		93
15	20	14	N/A		14
16	378	184	11	1	194
17	267	110	5		115
18	551	166	16	4	178
19	22	18	N/A		18
20	361	99	15	2	112
21	50	31	3	1	33
22	39	19	N/A		19
23	264	54	10		64
24	33	23	N/A		23
25	86	30	5	1	34
26	69	31	N/A		31
27	14	9	N/A		9
28	157	51	4	1	54
29	242	71	10		81
30	280	63	8	1	70
31	300	37	5	1	41
32	37	6	N/A		6
33	41	23	N/A		23
34	280	54	7		61
35	53	45	N/A		45
36	150	41	2		43
37	87	60	2		62
38	92	40	5		45
39	169	82	12		94
40	139	80	7		87
41	282	116	15	2	129
42	320	157	8	2	163
43	646	184	28	6	206
44	536	138	35	3	170
45	248	72	12	2	82
Totals	8180	3206	257	29	3434

N/A indicates that a group had been merged with another and separate figures are not available.

*Inc. indicates the number of alternative service packages specified by the social workers, which are the same as packages of actual services supplied, for each client group.

6.4 Alternative modes of care : cluster analysis

6.4.1 Cluster analysis and packages of care

Another way of choosing modes of care was to use the technique of cluster analysis to analyse the service provision for each client group. Cluster analysis is a method used in classification problems. Its objective is to sort a sample of cases under consideration into groups such that the degree of association is high between members of the same group and low between members of different groups. "It can be used to reveal associations and structure in data which though not previously conceived, are nevertheless sensible and useful when found".(Wishart[61])

With reference to the problem of choosing fewer modes or packages of care for each client group than are currently provided by the social workers, cluster analysis can be used to group together current packages which call for similar service provision. For planning purposes this reduction in the number of different packages of care makes possible easier recognition of alternative means of service provision, whilst at the same time providing a way of collating all the service requirements across all packages of care supplied to all clients. The result of cluster analysis is a "chosen set" of packages of care for each client group. Each package chosen contains a number of different resources and a measure of the proportion of each resource supplied per client.

For example, consider four clients currently receiving services as follows:

Client A receives services 15,16

Client B receives services 15,17,20

Client C receives services 15,16,17

Client D receives services 15,17

Each client is receiving a different "package of care". The result of a cluster analysis designed to form only one package of care to serve all these clients could be a package which contains:

1 unit of service 15,

0.5 units of service 16,

0.75 units of service 17, and

0.25 units of service 20.

Thus this package could be supplied to each of the four clients A,B,C and D and the total requirement for services 15,16,17 and 20 would be exactly as before. For the purposes of long-range planning, such a package is equivalent to the original four packages as supplied to clients A,B,C and D. Furthermore, the new package has a degree of in-built flexibility because the allocation of services by the social workers can be based on the proportions of units of service for each client as specified, or can be spread across the client group to provide different services in appropriate quantities to each client. The latter would be more practical for day-to-day provision but the former can be regarded as equivalent for long-term planning purposes.

6.4.2 The appplication of cluster analysis

The computer package CLUSTAN was used to apply cluster analysis to the problem of choosing fewer modes of care for each client group than are currently provided by the social workers.

Initially an investigation was made of the results of each of four clustering methods, viz. Ward's hierarchical method, the relocation method using a random start, the relocation method using Ward's classification, and the density method. Appendix 6 gives a brief description of each of these methods.

The density method was rejected because of the difficulty of printing results in the required form and because of the massive volume of unnecessary printout which could not be suppressed.

Ward's method and the two relocation methods were compared: the results were similar but Ward's method appeared to group modes into clusters more nearly in the way that a manual, qualitative assessment would do. Ward's hierarchical method was therefore chosen for further use.

An example showing the manual application of Ward's method is shown in Appendix 7.

Ward's method being decided upon, it was then necessary to construct a computer file of the client/current service data in a form suitable for use by the CLUSTAN package.

The first client group analysed with CLUSTAN was group 06, with 111 clients (including clients originally in group 07) receiving 83 different modes of care. Using Ward's hierarchical method and choosing ten as the desired number of clusters or modes of care, the results were as shown in Figure 6.4.2.1. For each cluster the allocation of each service is given as a percentage of the whole cluster. Using these percentages together with the number of clients allocated a particular cluster or mode of care, a measure is given of the total service usage for that mode of care.

e.g. Cluster 6, with 6 clients:

Services	20	19	2	17	14
%	100	83.3	50	33.3	33.3
Service measure ("client" units)	6	5	3	2	2

Thus cluster 6 represents a service package consisting of

service 20 allocated to 6 clients,
and service 19 allocated to 5 clients,
and service 2 allocated to 3 clients,
and service 17 allocated to 2 clients,
and service 14 allocated to 2 clients.

Another interpretation of service usage is to regard the package as an allocation per client so that each client receives

100% of a unit of service 20
and 83.3% of a unit of service 19
and 50% of a unit of service 2
and 33.3% of a unit of each of services 17 and 14.

The results of the cluster analysis could now be used in an alternative modes model for client group 06.

Appendix 11 includes a full description of the services allocated to each of the ten modes of care for group 06.

Subsequently the CLUSTAN package was used to analyse each of the 35 client groups in turn, resulting in a set of ten modes of care for each client group.

An analysis was then produced of the usage of services by clients when allocated to the modes of care selected by the Cluster Analysis. This showed, for each mode of care within each client group, the number of clients receiving that mode of care and, for each service, the number of clients currently receiving that service.

An incidental aspect of the CLUSTAN analyses was the problem of estimating computer times for each analysis. Over-estimating the time required resulted in low priorities being given to the task and hence longish delays in receiving results; under-estimating the time resulted in premature abortion of the task by the computer operating system and consequently no results at all. The CLUSTAN manual suggests that the central processor time required is proportional to the square of the number of cases in the data : whilst this is approximately true for small numbers of cases it was found that the true relationship is more nearly exponential and a useful estimator (see also graph in Appendix 8) was found to be

$$t = 2.157\ (1.0087)^{x}$$

where x is the number of cases

and t is the central processor time required on the

University's NUMAC computer.

This relationship erred on the side of over-estimation but was a useful estimator, particularly for some of the client groups with large numbers of clients.

Figure 6.4.2.1

Results of Ward's method of cluster analysis

Current services, group 06 (and clients from group 07)

Total No. of cases = 111

Cluster 1 Total 12 cases

Services	1	20	5	17	21	23	15
%	100	41.7	25.0	16.7	16.7	8.3	8.3
% x Total	12	5	3	2	2	1	1

Cluster 2 Total 15 cases

Services	28	20	6	2	12	3	73
%	46.7	26.7	20	13.3	13.3	6.7	6.7
% x Total	7	4	3	2	2	1	1

and

Services	62	22	21	18	11	88	15
%	6.7	6.7	6.7	6.7	6.7	6.7	6.7
% x Total	1	1	1	1	1	1	1

Cluster 3 Total 11 cases

Services	4	20	17	1	19	23	22	21
%	100	54.5	45.5	18.2	18.2	18.2	18.2	18.2
% x Total	11	6	5	2	2	2	2	2

and

Services	3	5	6	24	14	18
%	9.1	9.1	9.1	9.1	9.1	9.1
% x Total	1	1	1	1	1	1

Cluster 4 Total 22 cases

Services	6	20	23	17	18	7	4	27
%	100	45.5	22.7	13.6	9.1	9.1	9.1	4.5
% x Total	22	10	5	3	2	2	2	1

and

Services	28	22	21	19	15
%	4.5	4.5	4.5	4.5	4.5
% x Total	1	1	1	1	1

Figure 6.4.2.1 (Continued)

Cluster 5 — Total 13 cases

Services	17	18	16	6	20	7	21	12	11
%	92.3	30.8	30.8	30.8	23.1	7.7	7.7	7.7	7.7
% x Total	12	4	4	4	3	1	1	1	1

Cluster 6 — Total 6 cases

Services	20	19	2	17	14
%	100	83.3	50	33.3	33.3
% x Total	6	5	3	2	2

Cluster 7 — Total 8 cases

Services	17	28	18	14	24	7	20	19	5
%	100	87.5	75	37.5	25	12.5	12.5	12.5	12.5
% x Total	8	7	6	3	2	1	1	1	1

Cluster 8 — Total 14 cases

Services	25	20	23	17	19	5	21	26	1
%	100	85.7	42.9	35.7	28.6	21.4	21.4	14.3	14.3
% x Total	14	12	6	5	4	3	3	2	2

and

Services	3	4	18	15
%	7.1	7.1	7.1	7.1
% x Total	1	1	1	1

Cluster 9 — Total 6 cases

Services	28	23	4	17
%	100	100	100	83.3
% x Total	6	6	6	5

Cluster 10 — Total 4 cases

Services	20	3	19	1	5
%	100	100	100	100	25
% x Total	4	4	4	4	1

6.5 Analysis of the data for Chester-le-Street district

6.5.1 The use of a single district

As part of the initial modelling investigation it was decided to construct a model which would be derived from a random sample of the original census data. The results from such a model could then be compared with results from similar models derived from other samples of the census data.

The first random sample ("KSI") of 1015 records was extracted from the original data using a randomising function available on the NUMAC computer system. Since it was required that the sample KSI should be a representative sample, a chi-squared test was performed to test the hypothesis that there was no significant difference between the distribution of client group sizes in the sample and the distribution of client group sizes in the original data. Appendix 9 shows the comparison of client group sizes and the application of the chi-squared test. The conclusion was that there was no significant difference and so the sample could be regarded as representative of the original data.

Another chi-squared test was applied to test that the sample records had been drawn uniformly from the original data and again the sample data passed the test.

Subsequently, when the model was constructed from this random sample, it was found that the results did not appear to be suitable for use with the whole county and it seemed likely that the sample optimum was at a different point from the optimum for the whole county.

Some form of sub-optimisation appeared to be occurring Consideration of this result led to the suggestion that, instead of using a random sample, a geographical "district" should be used. The consequence of selecting clients from the same district would be that their service provision would have been selected by a single team of social workers and hence could be expected to be more homogeneous than that arising from a random sample of clients.

The data for Chester-le-Street district was therefore extracted from the Cendat data. There were 730 client records for this district.

6.5.2 Group and service summaries for Chester-le-Street

In the same way as for the complete set of data, an analysis was produced, from the Chester-le-Street client records, of the current usage of services by clients. The analysis is shown in Appendix 10 and gives, for each client group, the number of clients in the group and, for each service, the number of clients in each group currently receiving that service.

6.5.3 Alternative modes of care for Chester-le-Street

The Chester-le-Street data, having been extracted from the Cendat data, was then matched with the cluster analysis. Each client record on Cendat had been allocated, using the CLUSTAN package, to a particular mode of care for the client group to which that client belonged. An analysis was then produced, for the Chester-le-Street district, of the usage of services by clients when allocated to the modes of care. This showed, for each mode of care within each client group in Chester-le-Street, the number of clients receiving that mode of care and, for each service, the number of clients currently receiving that service.

CHAPTER 7

COMPARISONS WITH OTHER REGIONAL MODELS

7. Comparisons with other Regional Models

7.1 Introduction

The DHSS Balance of Care approach has been attempted in several different regions and some of these results have been made available for purposes of comparison. The Wiltshire Area Health Authority [58] and Wiltshire County Council [59] have followed the DHSS approach quite closely, as has East Sussex [27,28]. In the West Midlands a pilot programme was begun in the district of Dudley (see Nicholls [46]) and in Cornwall [14] too the Balance of Care approach has been used.

In all cases the models have been used for the elderly client group alone. Only in Dudley was the model to be extended to include other client groups. The present author's linear programming model with alternative modes of care (LPAM) was constructed (see Chapter 5) to include all client groups but the comparisons in this chapter necessarily relate only to the group of elderly clients.

Borley [8], Taylor and West have discussed the implementations of the Balance of Care approach in Wiltshire and in East Sussex: their discussions focus on the need to set up Joint Management Teams (JMT) and Professional Advisory Groups (PAG). Klemperer [35] and McClenahan have discussed the choice of groups and categories of clients and the selection of suitable modes of care when the Balance of Care approach is to be applied.

The following sections describe the Wiltshire, East Sussex, Dudley and Cornwall implementations and make some comparisons with the Durham study.

7.2 Wiltshire

Wiltshire Area Health Authority[58] and Wiltshire County Council (1982) worked with the DHSS Operational Research Service consultants Arthur Andersen and Co. on a pilot project which set out to establish a computer-based model for care of the elderly which could be used to examine the consequences of different policy options for the future.

In the first instance a survey was carried out of elderly clients receiving care, both in hospital institutions, in residential homes and in their own homes. The survey results published include analysis of length of stay and home circumstances of those in geriatric or psychiatric hospitals; the home circumstances and disability levels of those in residential homes or attending day hospitals (psychiatric or geriatric) or day centres; the pattern of visits of community nursing or community psychiatric nursing; home help allocations to those in their own homes; disability levels of and social services supplied to those in sheltered housing, private and voluntary homes.

A Professional Advisory Group (PAG) was set up and this group used the survey results to divide the elderly into 31 categories and to set a range of alternative acceptable forms of care. The categories were grouped by using the client's degree of physical disability-severe, moderate, minor or none; the degree of incontinence - doubly, regularly or not at all; the mental state -dementia, behaviour disorders or normal; and the environmental circumstances - housing adequate or inadequate, supportive neighbours or relatives or adverse support.

Appendix 14 shows the 31 categories of elderly clients devised for Wiltshire together with the ideal alternative modes of care for each category. The P.A.G. then gave each mode of care a score ranging from 3 (preferred) to 0 (unacceptable for the future but currently being used) and these are also shown in Appendix 14. The P.A.G. next considered service levels and have indicated minimum standards for day and domiciliary services. These are used outside the computer model to give each service an acceptable range of allocation from the minimum to the ideal for each client.

Comparisons have been made by the present author between categories of elderly clients used in Wiltshire, East Sussex and Durham, and between some of the modes of care in use in Wiltshire and Durham.

(i) Categories of elderly clients

Both Wiltshire and East Sussex used the degree of disability as an attribute for classification purposes and the Durham groups have been combined by the author into similar categories.

Thus, "Severely disabled" includes group 09
"Moderately disabled" includes groups 10,11,13,15
"Minor disability" includes groups 12,14,16
"None" includes group 17

The numbers and percentages of clients surveyed are shown in the following table, classified according to the degree of disability.

	Severe		Moderate		Minor		None		
	No.	% of tot	No.	%	No.	%	No.	%	Total
Durham	226	14	491	31	613	39	250	16	1580
Wiltshire	1486	9	3285	20	6966	43	4297	27	16034
E. Sussex	2169	14	4014	26	5740	37	3754	24	15677

These figures are not truly comparable since the Wiltshire and East Sussex figures include clients in hospital institutions whereas the Durham figures do not. In addition the Durham survey included only "active" cases on the social workers' caseload and the Wiltshire and East Sussex surveys included all (or a sample of all) clients receiving care. Many of the Wiltshire and East Sussex clients would not be "active" cases since their needs would have been assessed in the past and the care they receive decided at that time.

Nevertheless, the proportions of the clients in each of the disablement categories could be expected to be similar.

Assuming the Durham distribution to be the "expected" distribution, tests of goodness of fit (χ^2) were applied to the Wiltshire and East Sussex classifications in turn.

Expected numbers were thus:

	Severe	Moderate	Minor	None	Total
Wiltshire	2293	4983	6221	2537	16034
E. Sussex	2242	4872	6082	2481	15677

Test statistic $\chi^2 = \sum \frac{(O-E)^2}{E}$ with 3 degrees of freedom.

At 5% significance level, $\chi^2_{0.05,3} = 7.815$,

Critical Region $\chi^2 > 7.815$.

For Wiltshire, $\chi^2 = 2173$.

Thus, χ^2 is in the Critical Region and there is a significant difference between the Wiltshire distribution and the Durham proportions.

For East Sussex, $\chi^2 = 826$.

Thus, χ^2 is again in the Critical Region and there is a significant difference between the East Sussex distribution and the Durham proportions.

Since the Wiltshire and East Sussex surveys were using the same categories there may be expected to be similarities in the proportions of clients for these two counties

Using Wiltshire as the "expected" distribution, the expected numbers in East Sussex are :

	Severe	Moderate	Minor	None	Total
E. Sussex	1453	3212	6811	4201	15677

Then $\chi^2 = 769$

Thus χ^2 is again in the Critical Region and there is a significant difference between the East Sussex distribution and the Wiltshire proportions.

It is apparent from this analysis that there are sufficient differences in client categories between the counties, whether through how the clients are categorised or through how the clients are distributed, to make comparisons very difficult.

(ii) Modes of Care

The Wiltshire survey results showed how different categories of the elderly were cared for at that time (1981). The present author has attempted to make some comparisons of the care given in Wiltshire and in County Durham. For the "completely dependent" elderly group in County Durham and the equivalent "severely disabled" elderly group in Wiltshire, care was given in the following proportions:

	WILTSHIRE			DURHAM	
	No.	%	% exc. Inst.	No.	%
Institutional	484	32.6	-	-	-
Residential	154	10.4	15.4	49	21.7
Day Care	93	6.3	9.3	60	26.5
Domiciliary	752	50.7	75.3	117	51.8
TOTAL	1483	100	100	226	100

N.B. The Wiltshire survey included hospital institutions but the Durham survey did not.

When clients in institutional domiciles are excluded from the Wiltshire figures, it appears that twice the proportion of the severely dependent group are in residential homes in Durham than in Wiltshire and more than twice the proportion are given some form of day care. This comparison may be misleading, however, since the Durham group shown does not include those clients in the category "more than 3 physical handicaps" - these clients not being considered to be " completely dependent". It is possible that in Wiltshire such clients would be classified in with the "severely disabled" group. If these clients are included, the comparison becomes:

	WILTSHIRE		DURHAM	
	No.	% exc inst	No.	%
Residential	154	15.4	62	14.9
Day Care	93	9.3	93	22.4
Domiciliary	752	75.3	261	62.7
	999	100	416	100

When these figures are compared, a statistical test of difference between the proportion in residential homes in Durham and the proportion in residential homes in Wiltshire shows that there is no significant difference.

(The test statistic $z = \frac{p_1 - p_2}{\sqrt{p(1-p)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = 0.25$ and this is outside the Critical Region $|z| > 1.96$ at the 5% significance level).

A statistical test of difference between the proportions in day care in Durham and in Wiltshire shows that there is a significant difference. (The test statistic $Z = 6.62$ is inside the Critical Region $|Z| > 1.96$ at the 5% significance level).

Similarly, there is a significant difference between the proportions in domiciliary care in Durham and in Wiltshire (The test statistic $Z = -4.76$ and is in the Critical Region).

It is apparent that, for these severely disabled clients, although the two counties have a similar proportion domiciled in residential homes, more County Durham clients receive care in day centres than do the Wiltshire clients.

At the other end of the elderly client spectrum, clients categorised as having no disabilities were cared for in the two counties in the following proportions by domicile, excluding institutional care:

	WILTSHIRE		DURHAM	
	No.	%	No.	%
Residential	73	1.7	5	2.0
Day Care	158	3.7	28	11.2
Domiciliary	4026	94.6	217	86.8
	4257	100	250	100

A statistical test of significance between the proportions cared for in residential homes in the two counties shows that there is no significant difference.

(The test statistic $Z = 0.34$ and is not in the Critical Region).

However, similar tests on the proportions cared for by day care and those cared for in domiciliary mode show that there are significant differences between the two counties.

In each of the elderly classes compared (ie the most disabled and the least disabled) there is a similar proportion cared for in residential homes in Wiltshire and Durham, but the proportion receiving some form of day care (at a day centre or day hospital) is considerably greater in Durham than in Wiltshire. (Between twice and three times the proportion of each group receive day care in Durham than in Wiltshire).

The Wiltshire project had continued by projecting the resource requirements on the basis of recommendations by the P.A.G. These were in turn based on the view that there would be a move towards community based forms of care and away from more institutional types. Amounts of resources were projected on the basis of 22% increase in elderly population by 1991 and patterns of care moving towards the preferred or "best" (ie cheapest) forms. The method used to calculate the amounts of resources which would be needed was not described but it is likely that the DHSS/A. Andersen SPRAM model was used since the Wiltshire project is described as a pilot project in conjunction with the DHSS and A. Andersen & Co. The main conclusions of the Wiltshire project appeared to be that there were many inadequacies in the present services; that operation of the services needed to be improved before clear choices could be made for strategic decisions about which services to spend finance on;and that the Balance of Care analysis could be used to establish directions for change and priority areas for further resources.

Recent (1986) enquiries by the present author have found that in Wiltshire, development of the model continued in conjunction with the DHSS and Arthur Andersen & Co. The intention was to change the original regional model, which was based on a mainframe computer, into a district-oriented model which would be developed for a micro-computer. The regional model had been based on the simple proportional resource allocation method (SPRAM) and the planners had found it difficult to understand. It had taken a lot of resources during development but had been found not to be particularly useful. Although the model could propose plans for resource usage, it was more difficult to control the usage of resources in line with the plans.

The new district-based model was expected to become available during the next six months (1986) and would be used to give more effective continuous control to the use of resources. It was expected that the new model would contain more features such as control parameters to permit the planners to exercise more direct control over the plans.

7.3 <u>East Sussex</u>

In East Sussex [27,28] another pilot study took place in conjunction with the DHSS Operational Research Service and Arthur Andersen & Co. The mathematical model used to aid planning is not described in detail but is referred to as a simplified computer-based planning model in a portable version. "This model takes data and assumptions and from this produces the implications of different methods of care". Like the Wiltshire project, the East Sussex study was to develop a strategy to improve the care of the elderly: reference is made to previous use of the model to balance the care between acute and long stay services. In East Sussex a major aim was to achieve joint planning between Health, Social Services and Housing. A joint Management Team (JMT) and a Professional Advisory Group (PAG) were set up along the lines recommended by DHSS/A. Andersen (see section 4.4 and Borley[8], Taylor and West's paper "Balance of Care - a User's View of a New Approach to Joint Strategy Planning"). A survey of clients was conducted; clients were classified based on degree of disability, mental state, degree of incontinence, social circumstances and environmental circumstances. 32 client categories were used. Resources included in the model were those regarded as playing a significant part in the long term care of the elderly and those having major implications for strategic planning. 17 resources were included. The PAG defined packages of care which were regarded as preferred, acceptable or unacceptable for each client category. (These are described further in Klemperer[35] and McClenahan's paper "Joint Strategic Planning between health and local authorities").

The Balance of Care model was then used as described in Section 4.4 to produce:

(i) Current provision of care.

(ii) Resources needed to maintain the current standards for the forecast elderly population.

(iii) Likely consequences if amounts of resources are changed.

(iv) Resources required to provide the best form of care.

(v) Given a pattern of resources what is the most effective way to allocate them so that the greatest number of people can receive the best care.

Results given relate to Brighton, Eastbourne and Hastings areas and are:

(a) those relating degree of disability to how clients are cared for by domicile,

(b) estimates of numbers of those elderly clients with each degree of disability currently receiving care and those estimated to be in need of care,

(c) a subjective comparison rating current levels of services supplied with those considered advisable by the PAG.

The results in (a) were used in section 7.2 above and have been compared with the Wiltshire project and the Durham survey. It was found that there were significant differences in client categories between the three counties: Wiltshire, East Sussex and Durham, so that comparisons are difficult to make.

An attempt was made, however, by the present author to make comparisons for the severely disabled group, as was done for Wiltshire. In the East Sussex results available, only percentages were presented for clients currently receiving care in different domiciles, and these were separated into Brighton, Eastbourne and Hastings areas. By taking the total number of severely disabled clients in each of the three areas, it was possible to derive the actual numbers in each domicile and to aggregate these to give the total number of East Sussex clients receiving care in institutions, in residential homes, by day care or by domiciliary care.

There was however another category "inappropriate care" and it is not clear whether all clients being inappropriately cared for (in the view of the PAG) are grouped in this category. For comparative purposes this group and the institutional clients were excluded, so the numbers of severely disabled clients in East Sussex were cared for by domicile as follows:

	Number	%
Residential	344	28.9
Day Care	103	8.6
Domiciliary	744	62.5
Total	1191	100

These were compared with the Wiltshire and Durham groups, which were as follows:

	Wiltshire		Durham	
	No.	% exc.Inst	No.	%
Residential	154	15.4	62	14.9
Day Care	93	9.3	93	22.4
Domiciliary	752	75.3	261	62.7
	999	100	416	100

Statistical tests of difference between the various proportions in residential homes, day care or domiciliary care in Durham and the proportions in East Sussex show that there is no significant difference between the proportions of clients in domiciliary care in Durham and in East Sussex ($Z = 0.10$) but that there are significant differences between the proportions in residential care in the two counties ($Z = -5.7$) and between the proportions in day care in the two counties ($Z = 7.4$).

Comparisons between the Wiltshire and East Sussex proportions show that there is no significant difference between the proportions of clients in day care in Wiltshire and in East Sussex ($Z = 0.76$) but that there are significant differences between the proportions in residential care in the two counties ($Z = -7.5$) and between the proportions in domiciliary care in the two counties ($Z = 6.4$).

Thus on the basis of the figures available and with the assumptions already stated, it appears that, for the most severely disabled of elderly clients: Durham and Wiltshire care for similar proportions in residential homes, Durham and East Sussex care for similar proportions by domiciliary care whilst Wiltshire and East Sussex care for similar proportions by day care. The proportion of clients in residential homes in East Sussex is considerably greater (ie. about twice) than the proportions in the other two counties; the proportion of clients in domiciliary care in Wiltshire is greater than in the other two counties; and the proportion of clients receiving day care in County Durham is considerably greater (ie. more than twice) than in the other two counties.

Since the distributions of categories of clients are not the same (see section 7.2), the reasons for the above differences may be that clients with the same disabilities would be allocated to different categories in the three counties. Further examination of the severely disabled categories for East Sussex and Durham show that the proportions (of all elderly clients under consideration) being allocated to the severely disabled category are not significantly different between these two counties, so the similarity in domiciliary care in these two counties is comparable, as is the greater proportion in residential homes in East Sussex. Since the proportion receiving day care is greater in County Durham this suggests that some clients who would receive residential care if they lived in East Sussex are being given day care in County Durham.

Recent (1986) enquiries by the present author have found that in East Sussex, although the Balance of Care Model was initially used to produce information for planning purposes, there was almost a surfeit of information so that there was too much for the planners to digest. Since then the model has been little used and the database has not been updated but recently work has begun again and the model is "on the verge of running". In the meantime the emphasis had been, as in County Durham, on other planning systems such as a dependency scoring assessment system. A management package based on a micro-computer was being developed to generate reports on alternative packages of care and indicators of performance in respect of resource usage.

7.4 Dudley, West Midlands

For the West Midlands, I G Nicholls[46] has described in "Joint Planning in Dudley - the Role of the Balance of Care" an application of the Balance of Care (BOC) approach in a single-district area, Dudley. It had not been possible to convince both the West Midlands Health Authority and the Social Services Department of the value of the Balance of Care model but Directors in Dudley district believed it might be possible to proceed with BOC in their area. Instead of going ahead and developing a model specific to Dudley it was necessary to begin by demonstrating the potential of the BOC model and so the original Devon framework had to be used together with data for Dudley. This was successful enough to convince the Joint Care Planning Team that a tailor-made model should be constructed for Dudley. That model was being constructed but no results were presented.

Recent (1986) enquiries by the present author have found that, since Nicholls' paper was published, the model was developed and implemented in the West Midlands region. A mainframe computer version of the DHSS/A. Andersen SPRAM model is now (1986) in use for several different client groups as well as the elderly, viz. the younger physically handicapped, the mentally handicapped, the mentally ill and (currently being implemented) children, each of these groups having its own separate model. The operational research workers have found that the plans produced by the models "were not particularly interesting to the planners", and in particular that no strategic plan for the elderly (or indeed the other groups) has yet been established.

Of all the client groups, the mentally handicapped model has been the least successful : the Balance of Care Model in its SPRAM version is based on the assumption that field workers will continue to care for clients in similar fashion to current care, but the philosophy of caring for the mentally handicapped has been changing considerably lately, so that previous patterns of care are no longer considered appropriate.

From these recent (1986) observations, two appear to be of most interest:

(a) the plans produced by the models "were not particularly interesting to the planners", and

(b) where the philosophy of care was changing, the model was "least successful".

These observations, in relation to the DHSS/A. Andersen model, suggest that a model which produced "more interesting" plans and which could develop different caring patterns might be more useful to the planners. The author's linear programming model with alternative modes of care (LPAM) may therefore be a more useful model than the SPRAM model since it has been shown (See Chapter 8) that the LPAM model may propose quite radical changes in caring patterns when changes in population and changes in resource availability are input. The alternative modes of care are defined as being acceptable alternatives and the LPAM model may choose an unexpected (and hence more interesting) combination of modes of care for the planners to consider. In order to handle changing philosophies of care the LPAM model would need to be provided with an appropriate new range of alternatives modes of care (within the changed philosophy) and weights would then be allotted so that the new modes were given greater weighting than the old modes. The model could then select from the range of modes of care whatever combination of allocations of clients to modes which best fitted both the resources available and the preferences for certain patterns of care.

7.5 Cornwall

Cornwall County Council [14] has used the Balance of Care approach to determine which of the services for the elderly should have priority for development. In Cornwall, surveys were carried out to find out the numbers and types of clients receiving each type of service. Professional Advisers (a sub-group of the Care Planning Group for the Elderly in Cornwall) considered how to group clients into categories and used the following factors to do so: ability to look after themselves, social circumstances, level of incontinence, mental state. They then described the type of care that would be appropriate to each category. Constraints on resources were set by the Project's Management Group.

The model was used to provide information on:

(i) Current use of resources.
(ii) Future use of resources over the next 5 years assuming increases in population and that a constant proportion of the total would receive care.

The results available do not divide the elderly clients into categories so it is not possible to make comparisons with the other implementations.

Some difficulties are discussed, such as the problems in providing transport to day care facilities for clients residing large distances away, and the different levels of contribution to care made by individual private and voluntary homes.

Recent (1986) enquiries by the present author have found that, since the 1982 publication the Cornwall study has come to a halt. A considerable amount of development of the model took place but it did not reach the stage of being accepted by the planners.

Priorities changed and the budget was cut so that no further development could take place.

CHAPTER 8

PLANNING FOR CHANGES IN DEMOGRAPHY AND IN RESOURCE AVAILABILITY

8. Planning for Changes in Demography and in Resource Availability

8.1 Introduction

The two major influences on the future allocations of social services are the availability of the service resources themselves, dependent mainly on the size of budget available to finance these resources, and the needs of the client populations. These needs are themselves proportional to the sizes of the client population in the various client groupings having similar needs. Other influences may be policy decisions on the way in which the budget should be spent, for example increasing the expenditure on domiciliary services and decreasing the expenditure on institutional resources. (See D.H.S.S.[24] (1981) : Report of a Study on Community Care, D.H.S.S.[22] (1981) : Care in Action, and Mooney[41] : Planning for Balance of Care of the Elderly), or decisions on the ways in which clients should be cared for. (See D.C.C.[20] Social Services Department Position Statement 1980 : "When the 'ideal' service is not available alternatives are used. It must be, and is, assumed that the services do meet needs, because they are not rejected by clients. It is not assumed that the current range and extent of the services provided is ideal".)

This chapter discusses some of the methods which can be used to plan for these changes. Applications of the author's Linear Programming Model with Alternative Packages (L.P.A.M.) (as described in Chapter 5) are shown for demographic changes in the Chester-le-Street district, from the population at the time of the

D.U.B.S. census in 1977 to known changes in 1979 and 1983 and to projected changes in 1987 and 1991. Comparisons are also made with the D.H.S.S./A. Andersen model (using the simple proportional resource allocation method SPRAM as described in Section4.4) for a small test set of data which is used on both LPAM and SPRAM models to show the effect of changes in population only, changes in resource availability and changes in both population and resource availability.

A discussion is then given of the use of the LPAM model as a simulation rather than an optimising model, so as to assess resource needs if patterns of care were maintained for future client populations and to assess resource needs for specific allocations of clients to modes of care.

Finally, other planning decisions are discussed in relation to the planners' use of the LPAM model.

8.2 Demographic Changes

In order that realistic population figures could be used for the demographic changes in the Chester-le-Street district, the Central Statistical Office[9] "Annual Abstract of Statistics" 1979-1985 editions and the CSO[10] "Regional Statistics" 1979 and 1981 editions together with the Durham County Council publications[18, 20, 19] "Computerised Client Information System", "Position Statement 1980" and "County Durham in Figures" were consulted.

The age structure within which the client groups in the LPAM model were derived was simply the three ages : children-and-young-persons, adults and elderly (i.e. over 65). The CSO publications grouped ages 15 to 64 together so it was not possible to distinguish "Young Persons" from "Adults".

The same statistics were not available for all the years being considered so figures from the Regional Statistics (1979 and 1981) were used to provide indicators of population changes in the period 1977 to 1979 and 1977 to 1983. Comparison of the Regional (North) figures showed the population changes to be slightly different from those in the whole of the U.K., there being a greater decline in the numbers of children and young persons and a lesser increase in the numbers of adults and elderly people. Consequently the Regional figures were used for the 1977 to 1979 changes and, since Regional figures were not produced for later years, the Annual Abstract figures were adjusted slightly to provide the 1977 to 1983 changes in line with the

Regional differences noted above.

The following table shows the population changes from the CSO publications together with the data used in the LPAM model:

		% change 1977-1979 (Regional Statistics)	% change 1977-1979 Model data	% change 1977-1983 (Annual Abstract)	% change 1977-1983 Model data
Total population	(North)	-0.93	0	(UK) +0.36	0
Age under 14	(North)	-5.25	-5	(UK) -8.50	-10
Age 15-64	(North)	+0.05	0	(UK) +3.67	+3
Age 65+	(North)	+1.85	+2	(UK) +3.70	+3

Projections were then made, using the DCC Position Statement 1980 and DCC County Durham in Figures 1981, for the changes in age structure by 1987 and 1991. The figures used in the model were as follows:

Age	% change 1977-1987	% change 1977-1991
Under 14	-14	-20
15-64	+10	+15
65+	+10	+15

8.3 Use of the LPAM model to plan for demographic changes

8.3.1 The LPAM model (Linear Programming Model with Alternative Modes of Care)

The LPAM model, as described in Chapter 5, is re-stated here for ease of reference.

The model chooses the x_{il} to:

$$\text{Maximise } \sum_i \sum_l w_{il}\, x_{il}$$

$$\text{subject to } \sum_i \sum_l x_{il}\, u_{ilk} \leqslant Bk, \text{ for all } k;$$

(i.e. Service Constraints)

$$\text{and } \sum_l x_{il} = di, \text{ for all } i$$

(i.e. Population Constraints)

where x_{il} = number of clients in group i allocated mode l of care,

w_{il} = weighting given to care by mode l in group i $\left(\text{and } \sum_i \sum_l w_{il} = 1.0\right)$,

u_{ilk} = amount of service k in client-units allocated per client in mode l of group i,

B_k = total available amount of service k in client-units,

and d_i = number of clients in group i.

All runs of the LPAM model were done using the IBM[33] MPSX mathematical programming package.

8.3.2 Using the LPAM model with population changes in Chester-le-Street

The future changes in client population need to be described by a new set d_i' for the expected future size of each client group i.

If the resource availability remains unchanged, then the LPAM model is the same as stated in Section 8.3.1 above, except that the bounds of the population constraints are changed from d_i to d_i' .

Figure 8.3.2.1 shows the original (1977) group sizes di, the new sizes di$'$ for 1979, 1983, 1987 and 1991 (calculated from the population changes discussed in Section 8.2) and the resulting choice of numbers of clients to be allocated to each mode of care within each client group in Chester-le-Street district. The initial allocation is shown for comparison.

It can be seen that the allocations to modes of care do not change a great deal but that when a group increases in size additional modes of care may be brought into use. Conversely, when a client group decreases in size the preferred mode can be used to care for a proportionately greater number of the group.

Figure 8.3.2.1 Allocations of Modes of Care for Chester-le-Street district initially (1977) and projected for 1979, 1983, 1987, 1991

N.B. Projections are based on population changes as follows:-

Groups		1979	1983	1987	1991
1-8, 18-34	Adult	0	+3%	+10%	+15%
9-17	Elderly	+2%	+3%	+10%	+15%
35-45	Children and Young Persons	-5%	-10%	-14%	-20%

$x_{i\ell}$ is allocation to mode ℓ of group i

Group i	Size d_i	1977		1979		1983		1987		1991	
		ℓ	$x_{i\ell}$	ℓ	$x_{i\ell}$	ℓ	$x_{i\ell}$	ℓ	$x_{i\ell}$	ℓ	$x_{i\ell}$
1	1	7	1.0	7	1.0	7	1.0	7	1.0	7	1.0
2	8	4 5 7	2.6 5.3 0.03	4 5 7	2.6 5.1 0.3	4 5 7	1.7 4.9 1.4	4 7	1.3 7.7	4 7	1.1 7.9
3	23	2 8	18.8 4.2	2 8	18.8 4.2	2 8	18.7 5.3	2 8 9	18.6 5.4 1.0	2 4 8	19.6 2.0 4.4
4	13	3 9	1.0 12.0	5 9	1.0 12.0	5 9	1.2 11.8	5 9	2.6 11.4	5 9	4.0 11.0
6	6	1	6.0	1	6.0	4	6.0	4	7.0	4	7.0
8	23	7	23.0	7	23.0	7	24.0	7	25.0	7	27.0
9	3	6	3.0	6	3.0	6	3.0	6	3.0	6 10	1.9 1.1
10	5	4	5.0	4	5.0	4	5.0	4	6.0	4	6.0
11	3	1 3	0.4 2.6	3	3.0	3 10	1.4 1.6	7	3.0	9	3.0
12	4	3	4.0	3 8	3.6 0.4	3 8	2.6 1.4	3	4.0	3	5.0

Figure 8.3.2.1 (continued)

Allocations of Modes of Care for Chester-le-Street district initially (1977) and projected for 1979, 1983, 1987, 1991

Group i	Size d_i	1977		1979		1983		1987		1991	
		ℓ	$x_{i\ell}$	ℓ	$x_{i\ell}$	ℓ	$x_{i\ell}$	ℓ	$x_{i\ell}$	ℓ	$x_{i\ell}$
14	5	2	5.0	2 3	4.5 0.5	2 3	4.0 1.0	2	6.0	2	6.0
16	21	2	21.0	2	22.0	2	22.0	2	23.0	2	24.0
17	16	9	16.0	9	16.0	9	17.0	2 9	1.8 16.2	2 9	1.0 17.0
18	56	2 4 5 7	20.8 2.1 28.3 4.9	2 4 5 7	16.2 3.0 28.3 8.6	2 4 5 7	18.4 2.6 28.3 8.8	2 4 5 7 9	32.1 0.2 26.7 1.9 1.2	2 4 5 7 9	33.8 0.7 19.1 3.3 7.2
20	10	1	10.0	1	10.0	1	10.0	1 7	10.3 0.7	1	12.0
21	1	9	1.0	9	1.0	9	1.0	9	1.0	9	1.0
23	58	3	58.0	3	58.0	3	60.0	3	64.0	3	67.0
25	7	8 9	5.6 1.4	8 9	2.5 4.5	9	7.0	1 9	1.4 6.6	9	8.0
28	13	2	13.0	2	13.0	2	13.0	2	14.0	2	15.0
29	49	3	49.0	3	49.0	3	51.0	3	54.0	3 7	52.7 3.3
30	46	4	46.0	4	46.0	4	47.0	1 4	3.5 47.5	1 4 7	6.0 44.9 2.1
31	32	6	32.0	6	32.0	6	33.0	6	35.0	6	37.0
34	52	1	52.0	1	52.0	1	54.0	1	57.0	1	60.0

Figure 8.3.2.1 (continued) Allocations of Modes of Care for Chester-le-Street district initially (1977) and projected for 1979, 1983, 1987, 1991

Group i	Size d_i	1977		1979		1983		1987		1991	
		ℓ	$x_{i\ell}$	ℓ	$x_{i\ell}$	ℓ	$x_{i\ell}$	ℓ	$x_{i\ell}$	ℓ	$x_{i\ell}$
35	4	2	4.0	4	4.0	7	4.0	4	4.0	4	3.0
36	6	1	6.0	1	6.0	1	5.0	1	5.0	1	5.0
37	5	7 10	3.3 1.7	7	5.0	7	5.0	1	4.0	1	4.0
38	4	1 3 6	0.2 2.8 1.0	1 3 6	2.0 1.0 1.0	1 6	3.98 0.02	1	3.0	1	3.0
39	19	3 6	4.1 14.9	3 6	0.5 17.5	6	17.0	6	16.0	6	15.0
40	10	3	10.0	3	9.0	2 3	2.3 6.7	2 3	5.3 3.7	2	8.0
41	8	10	8.0	10	8.0	10	7.0	10	7.0	10	6.0
42	27	6 8	5.8 21.2	6 8	8.7 17.3	6 8	13.5 10.5	6 8	13.8 9.2	4 6 8	1.9 14.2 6.0
43	81	3 7 9	74.0 3.4 3.6	3 9	74.0 3.0	3	73.0	3	70.0	3	65.0
44	65	5 8	38.0 27.0	5 8	39.1 22.9	5 8	38.6 20.4	5 8	36.9 19.1	5 8	32.6 19.4
45	46	2 7	18.8 27.2	2 7	16.8 27.2	2 7	13.7 27.3	2 7	11.1 27.9	2 7	8.2 28.8

8.4 Resources needed : use of the LPAM model for Chester-le-Street

It has been shown above that allocations to modes of care are altered if the client population changes but resources are unchanged. It may be helpful to the planners to know what resources would be needed to care for the new client population if existing patterns of care were to be maintained. This has been done for the Chester-le-Street district by entering as data a new set x_{il} of allocations of clients to modes of care which was derived as follows:

If, in the initial allocations, a group i had m_i modes of care allocated to clients in that group, then the new set of allocations would reflect the population change from di to di′ by increasing or decreasing the allocation to each mode of care used by the group in the same amounts.

Thus

$$x_{il}^{new} = x_{il}^{old} + \frac{(di' - di)}{m_i} \quad \text{for each mode of care used}$$

e.g. If a client group increased by 12 clients and there had been 3 modes of care used for that group, the allocation to each mode would be increased by 4 clients.

As well as fixing the allocations to each mode of care, it is necessary to permit the service capacities to expand as necessary for the new client population. This is done by specifying the service constraints as unbounded and simply monitoring the required size of each service.

The LPAM model is thus being used as a simulation model rather than an optimising model. This use is discussed further in Section 8.9 where other possible approaches are given to the specification of the new allocations to modes of care.

Figure 8.4.1 shows the effect on resources needed if the future population projected for Chester-le-Street district were to be cared for in the same modes of care as in the initial allocations, proportionately increased or decreased as described above. For each service, the resource is measured in client-units and the initial (1977) amount is shown together with the projected requirements for 1979, 1983, 1987 and 1991, based on the population changes described in Section 8.2 above. These population changes were included as data in the model, each run having its data (allocations to modes of care) calculated by using the expression already described, i.e.

$$x_{il}^{new} = x_{il}^{old} + \frac{(di' - di)}{m_i} \quad \text{for each mode of care } l \text{ used in group } i.$$

These data values are shown in Figure 8.4.2 for the initial (1977) allocations and the projected allocations in 1979, 1983, 1987 and 1991.

Figure 8.4.3 shows a graph of the resultant effect for those services having a noticeable change of requirements.

It is clear that the LPAM model can readily be used as a simulation model in this way. Section 8.9 describes alternative approaches to the specification of data for such a model.

Figure 8.4.1 Effect on Service Resources in Chester-le-Street district initially (1977) and projected for 1979, 1983, 1987, 1991

The table shows the service resources B_k in client-units required for each service k, for the projected population changes.

B_k	Initial	Projected			
	1977	1979	1983	1987	1991
k = 1	112	111.9	114.8	121.7	125.8
2	12.5	12.2	11.8	11.9	11.7
3	39	38.5	39.2	40.7	41.4
4	288	283.2	282.7	289.2	288.9
5	111	109.2	107.3	110.3	110.8
6	173	169.3	166.1	167.7	168.3
7	0	0	0	0	0
8	0.4	0.4	0.4	0.6	0.7
11	0	0	0	0	0
12	3.3	3.3	3.5	4.0	4.2
14	13	13.2	13.5	14.6	14.9
15	25	25	25.7	26.5	27.2
16	0	0	0	0	0
17	42	42.2	43.5	47.0	47.6
18	12.1	12.1	12.4	13.9	14.0
19	1.4	1.4	1.5	1.6	1.7
20	39.6	39.5	39.9	41.7	42.5
21	1.4	1.4	1.4	1.6	1.7
22	3.6	3.6	3.6	4.0	4.0
23	50.3	50.2	51.1	53.5	54.6
24	3	3	3.1	3.4	3.5
25	13	13	13.4	13.7	14.1
26	24	24	25.0	25.6	26.6
27	23	22.7	22.5	22.9	23.4
28	5	5	5	4.6	5.6
38	0	0	0	0	0
42	0.1	0.1	0.1	0	0
43	6	6	6.5	7.3	7.7
44	38	38.1	39.2	41.8	43.0
49	2	2	2.1	2.1	2.2
56	0	0	0	0	0
62	33.1	33.0	33.9	36.2	37.2
70	77	74.8	72.4	70.5	65.4
71	18.8	18.0	16.1	15.0	13.5
72	94.1	89.6	84.7	82.1	76.4
73	90	86.5	82.4	80.5	77.7
74	0	0	0	0	0
75	3.2	3.0	2.9	2.8	2.6

Figure 8.4.1 (continued) Effect on Service Resources in Chester-le-Street district initially (1977) and projected for 1979, 1983, 1987, 1991

B_k	Initial	Projected			
	1977	1979	1983	1987	1991
k = 76	9	8.9	8.7	8.5	7.9
79	8.2	7.5	7.4	7.3	6.6
83	0	0	0	0	0
84	23.4	21.6	18.7	16.9	15.2
88	12	11.6	11.0	10.5	10.0
90	5.5	5.3	5.0	4.8	4.6
91	11	10.7	10.2	9.9	9.4

Figure 8.4.2. Data for client population in Chester-le-Street district allocated to modes (1977) and changes in group sizes and mode allocations projected for 1979, 1983, 1987, 1991

Group i	Mode ℓ	1977 d_i	1977 $x_{i\ell}$	1979 $d_i\pm$	1979 $x_{i\ell}\pm$	1983 $d_i\pm$	1983 $x_{i\ell}\pm$	1987 $d_i\pm$	1987 $x_{i\ell}\pm$	1991 $d_i\pm$	1991 $x_{i\ell}\pm$
1	7	1	1								
2	4	8	2.6					+1	+0.3	+1	+0.3
	5		5.3						+0.3		+0.3
	7		0.03						+0.3		+0.3
3	2	23	18.8			+1	+0.5	+2	+1	+3	+1.5
	8		4.2				+0.5		+1		+1.5
4	3	13	1.02					+1	+0.5	+2	+1
	9								+0.5		+1
6	1	6	6					+1	+1	+1	+1
8	7	23	23			+1	+1	+2	+2	+4	+4
9	6	3	3								
10	4	5	5					+1	+1	+1	+1
11	1	3	0.4								
	3		2.6								
12	3	4	4							+1	+1
14	2	5	5					+1	+1	+1	+1
16	2	21	21	+1	+1	+1	+1	+2	+2	+3	+3
17	9	16	16			+1	+1	+2	+2	+2	+2
18	2	56	20.8			+2	+0.5	+6	+1.5	+8	+2
	4		2.1				+0.5		+1.5		+2
	5		28.3				+0.5		+1.5		+2
	7		4.9				+0.5		+1.5		+2
20	1	10	10					+1	+1	+2	+2
21	9	1	1								
23	3	58	58			+2	+2	+6	+6	+9	+9
25	8	7	5.6					+1	+0.5	+1	+0.5
	9		1.4						+0.5		+0.5
28	2	13	13					+1	+1	+2	+2
29	3	49	49			+2	+2	+5	+5	+7	+7
30	4	46	46			+1	+1	+5	+5	+7	+7
31	6	32	32			+1	+1	+3	+3	+5	+5
34	1	52	52			+2	+2	+5	+5	+8	+8
35	2	4	4							-1	-1
36	1	6	6			-1	-1	-1	-1	-1	-1
37	7	5	3.3					-1	-0.5	-1	-0.5
	10		1.7						-0.5		-0.5
38	1	4	0.2					-1	-0.2	-1	-0.2
	3		2.8						-0.4		-0.4
	6		1.0						-0.4		-0.4
39	3	19	4.1	-1	-0.5	-2	-1	-3	-1.5	-4	-2
	6		14.9		-0.5		-1		-1.5		-2
40	3	10	10	-1	-1	-1	-1	-1	-1	-2	-2
41	10	8	8			-1	-1	-1	-1	-2	-2
42	6	27	5.8	-1	-0.5	-3	-1.5	-4	-2	-5	-2.5
	8		21.2		-0.5		-1.5		-2		-2.5
43	3	81	74.0	-4	-1.3	-8	-2.7	-11	-4.0	-16	-9
	7		3.4		-1.3		-2.7		-3.4		-3.4
	9		3.6		-1.3		-2.7		-3.6		-3.6
44	5	65	38.0	-3	-1.5	-6	-3	-9	-4.5	-13	-6.5
	8		27.0		-1.5		-3		-4.5		-6.5
45	2	46	18.8	-2	-1	-5	-2.5	-7	-3.5	-9	-4.5
	7		27.2		-1		-2.5		-3.5		-4.5

Figure 8.4.3 (i) Graphs of effect on services in Chester-le-Street

Graph showing services where increased need is projected.

eg. S_{23} shows projected increase in service 23.

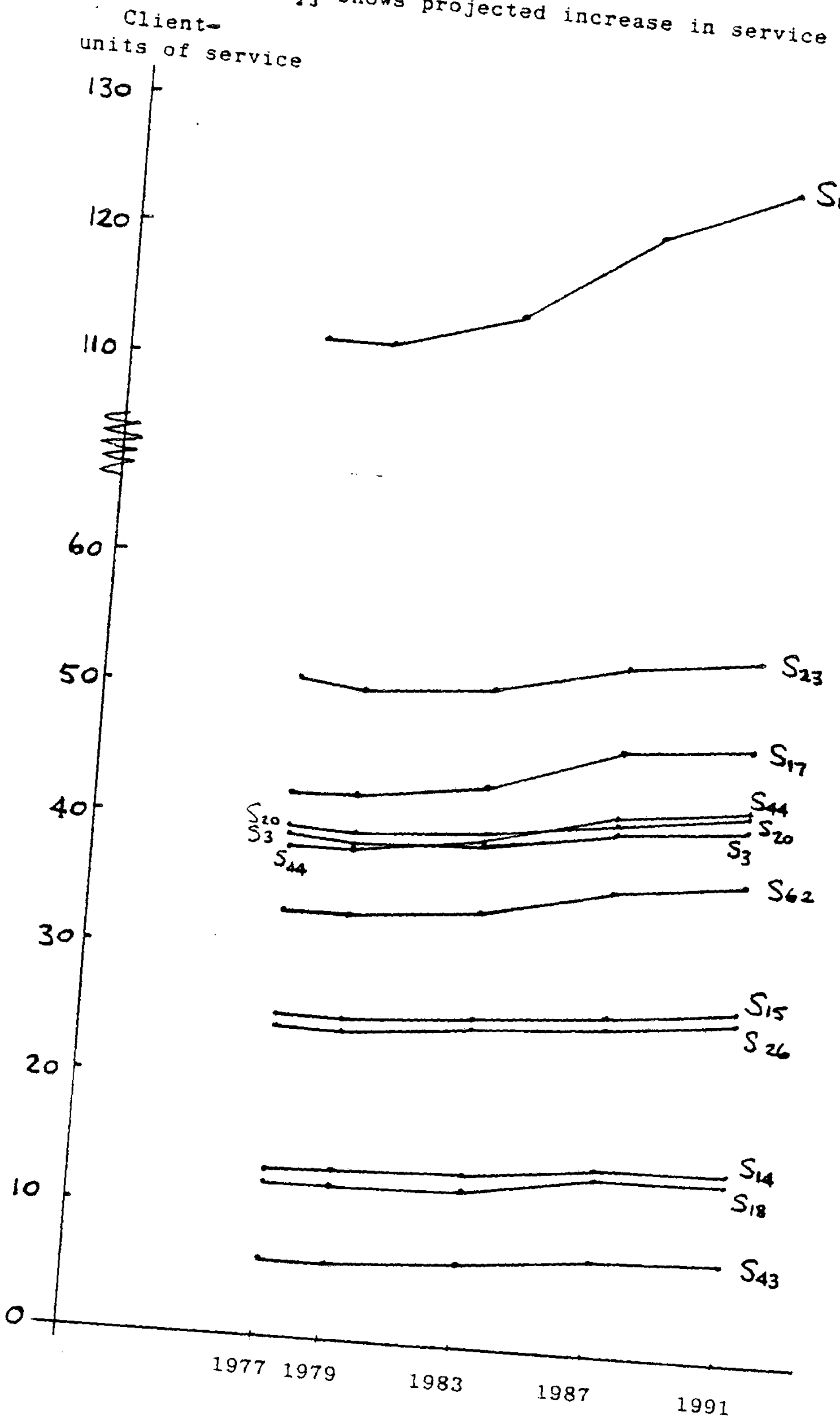
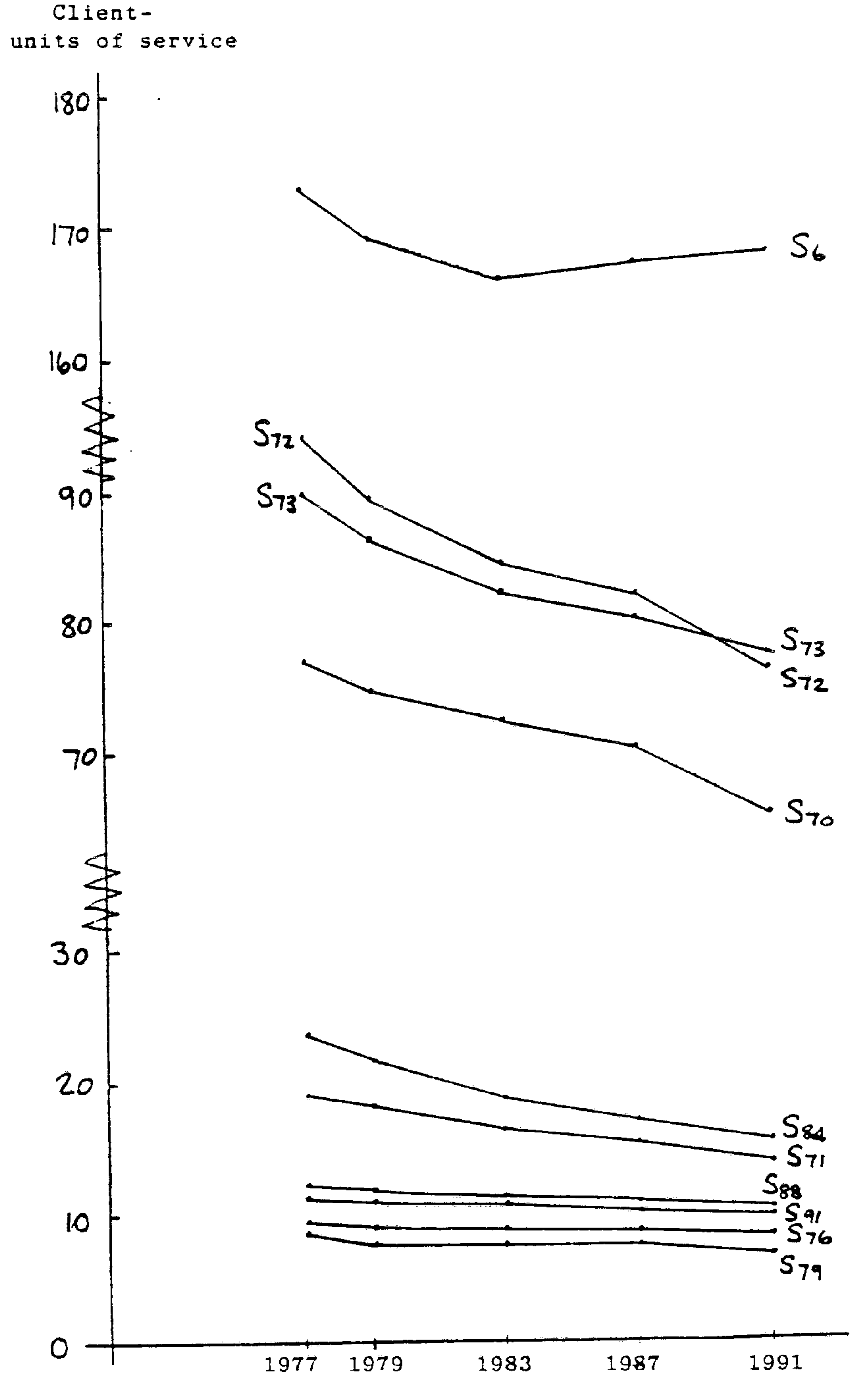


Figure 8.4.3 (ii) Graphs of effect on services in Chester-le-Street
Graph showing services where reduced need is projected eg. S_{70} shows projected reduction in service 70



8.5 Comparisons between the LPAM and SPRAM models

The previous sections have demonstrated the use of the LPAM model for planning purposes but it is appropriate to make comparisons between the LPAM model and other models. For detailed comparison the D.H.S.S./A. Andersen SPRAM (Simple proportional resource allocation method) model has been used as it represents the present evolution of the Balance of Care model.

A small test set of data was created in order to illustrate the use of the two models. Figure 8.5.1 shows this data and the x_{i1} values chosen by the LPAM model for the initial (i.e. "current") numbers of clients and amounts of resources.

The following Sections (8.6 - 8.8) describe the ways in which the LPAM model can be used by the planners to suggest alternative ways of caring for clients when there are future changes in resource availability, changes in client population or changes of both these types. In each case, a comparison is made with the SPRAM model in respect of both the method used and the results obtained. Appendix 13 details the calculations performed using the SPRAM model for future changes in both resources and population size; calculations in respect of the separate changes were performed in a similar manner.

Section 8.9 discusses the ways in which the LPAM model can be used as a simulation model to assist planners to explore intermediate allocations between the current situation and the LPAM model's optimum allocations; or to assess the resources needed if patterns of care were to remain unchanged although the client population changed in size.

Figure 8.5.1 Small Test Set of Data

Client Group, i		1			2			3			Service amounts	
Group size (current)		40			35			30				
Group size (future)		44			38			27				
Mode of care, l		1	2	3	1	2	3	1	2	3	B_k	B_k^1
Services used	k = 1	1	0.5	0	0	0	0	0.3	0.3	0	27	26
per client	k = 2	0	1	1	1	1	0	1	0	1	73	69
in mode l of	k = 3	0	0	1	.5333	1	1	0.5	0.5	0.5	55	52
group i, i.e. U_{ilk}	k = 4	0	0	1	.5333	.6667	1	0	1	0.5	51	49
Weighting w_{il}		.0680	.1019	.1748	.1505	.1553	.0777	.0874	.0874	.0971		
"Dominant" resource		1	1	2	2	2	3	1	1	2		
Allocation x_{il}		21	0	19	30	0	5	14	6	10		

B_k = current amount of service k

B_k^1 = future amount of service k

LPAM Model is: Choose the x_{il} to:

$$\text{Maximise } 0.0680\, x_{11} + 0.1019\, x_{12} + 0.1748\, x_{13} + 0.1505\, x_{21} + 0.1553\, x_{22} + 0.0777\, x_{23} + 0.0874\, x_{31} + 0.0874\, x_{32} + 0.0971\, x_{33}$$

subject to resource constraints:

$$x_{11} + .5\, x_{12} + .3\, x_{31} + .3\, x_{32} \leqslant 27$$

$$x_{12} + x_{13} + x_{21} + x_{22} + x_{31} + x_{33} \leqslant 73$$

$$x_{13} + .5333\, x_{21} + x_{22} + x_{23} + .5\, x_{31} + .5\, x_{32} + .5\, x_{33} \leqslant 55$$

$$x_{13} + .5333\, x_{21} + .6667\, x_{22} + x_{23} + x_{32} + .5\, x_{33} \leqslant 51$$

and group size constraints:

$$x_{11} + x_{12} + x_{13} = 40$$

$$x_{21} + x_{22} + x_{23} = 35$$

$$x_{31} + x_{32} + x_{33} = 30$$

The x_{il} chosen are shown in the last line of the table above.

8.6 Changes in Resource Availability

Any future changes in resource availability are defined by a new set B'_k representing the expected total available amount of each service k.

If there are no other changes then the LPAM model remains unchanged, with only the bounds of the service constraints changing from B_k to B'_k. The model then produces a new set, x_{il}, of allocations of clients to modes of care.

Figure 8.6.1 shows the new set of allocations produced by the LPAM model for the test data of Figure 8.5.1 when the service amounts are reduced by approximately 5%. The allocations of services are shown as well as the allocations of clients to modes of care. These service allocations were obtained by multiplying the client numbers in each mode by the service allocations per client for that mode of care (i.e. by the corresponding u_{ilk}).

The same change in service amounts, when used with the SPRAM model, results in a different set of allocations and these are also shown in Figure 8.6.1, together with the original allocations.

The two models perform the allocations differently : LPAM allocating clients to modes of care which have fixed packages of resources and SPRAM allocating resources firstly to client groups, then to different modes according to the change in the "dominant" resource for each mode.

Figure 8.6.2 shows a comparison of the "service profiles" for each client group and for each mode of care. The "service profiles" represent the amounts of each service, measured in client-units, allocated to each client group or mode of care. The figure shows the service profiles of each client group and mode of care for the original data together with the new profiles produced by the SPRAM and LPAM models when the resource availability is changed. The profiles are shown for each client group (where the total amount of the resource allocated to all three modes in the group is accumulated) and for each separate mode of care.

It is apparent that the SPRAM model does indeed retain a similar service profile to the original profile in each case, yet there are discrepancies such as when the number of client units of two resources allocated to a group are the same in the original data, they may not remain exactly the same as one another in the SPRAM new profile. This is the case for services 3 and 4 allocated to groups 1 and 2 and is a consequence of the different changes in the total amounts of the two services.

The LPAM model diverges considerably from both the original allocation and the SPRAM allocation. Although the LPAM service profile per client in a given mode of care remains fixed, the number of clients allocated to a mode is changed so that the allocation of resources is quite different from the SPRAM choice. This is because the alternative modes of care are selected according to weighting priorities and according to the resources available. Reduction in a resource may mean that fewer clients can be allocated to a particular mode of care, but the LPAM optimising procedure will choose an alternative mode of care with due reference to the weightings attached to each mode for each client group. Thus the LPAM model offers the planners radical alternatives in the ways whereby resources can be allocated, whereas the SPRAM model is very conservative, simply reducing resources by small amounts across all groups and modes of care.

It was suggested (Coles[13]) that "far from making 'across the board' cuts, (Social Services) Departments consider very selective but major reductions in individual services" and this leads the author to believe that the LPAM model may be a useful tool in the way that it can suggest to the planners alternative yet acceptable ways of allocating services. The LPAM model may recommend a particular allocation of clients to modes of care which is considerably different from current allocations yet which makes use of reduced amounts of resources.

Figure 8.6.1 Comparison of results of LPAM and SPRAM models, for changes in resource availability from B_k to B_k'

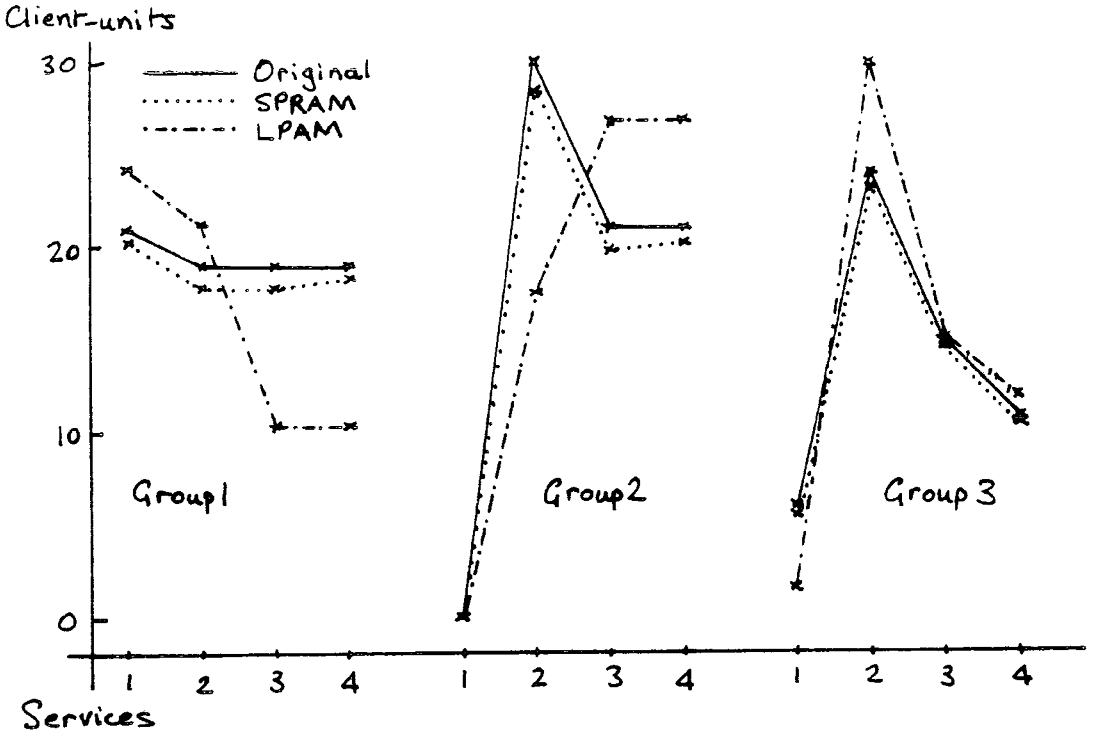
Client group i	1				2				3				
Size d_i	40				35				30				
Mode of care, ℓ	1	2	3		1	2	3		1	2	3		
LPAM Allocations	B11k	B12k	B13k	B1k	B21k	B22k	B23k	B2k	B31k	B32k	B33k	B3k	Bk'
Services, k = 1	18.6	5.55	0	24.2	0	0	0	0	1.8	0	0	1.8	26
2	0	11.1	10.2	21.3	17.6	0	0	17.6	6	0	24	30	69
3	0	0	10.2	10.2	9.39	0	17.4	26.8	3	0	12	15	52
4	0	0	10.2	10.2	9.39	0	17.4	26.8	0	0	12	12	49
Allocations, $x'_{i\ell}$	18.6	11.1	10.2		17.6	0	17.4		6	0	24		
SPRAM Allocations	B11k	B12k	B13k	B1k	B21k	B22k	B23k	B2k	B31k	B32k	B33k	B3k	Bk'
Services, k = 1	20.22	0	0	20.22	0	0	0	0	4.04	1.73	0	5.77	26
2	0	0	17.96	17.96	28.36	0	0	28.36	13.33	0	9.45	22.78	69
3	0	0	17.96	17.96	15.13	0	4.73	19.86	6.66	2.85	4.67	14.18	52
4	0	0	18.25	18.25	15.37	0	4.80	20.17	0	5.81	4.75	10.56	49
Original Allocations	B11k	B12k	B13k	B1k	B21k	B22k	B23k	B2k	B31k	B32k	B33k	B3k	Bk
Services, k = 1	21	0	0	21	0	0	0	0	4.2	1.8	0	6	27
2	0	0	19	19	30	0	0	30	14	0	10	24	73
3	0	0	19	19	16	0	5	21	7	3	5	15	55
4	0	0	19	19	16	0	5	21	0	6	5	11	51
Original $x_{i\ell}$	21	0	19		30	0	5		14	6	10		
Weighting $w_{i\ell}$	0.0680	0.1019	0.1748		0.1505	0.1553	0.0777		0.0874	0.0874	0.0971		

Comparison of Allocations to Client Groups, i.e. B_{ik}

	LPAM			SPRAM			Original		
Client group, i	1	2	3	1	2	3	1	2	3
Services, k = 1	24.2	0	1.8	20.22	0	5.77	21	0	6
2	21.3	17.6	30	17.96	28.36	22.78	19	30	24
3	10.2	26.8	15	17.96	19.86	14.18	19	21	15
4	10.2	26.8	12	18.25	20.17	10.56	19	21	11

Figure 8.6.2 Comparison of LPAM and SPRAM models, when future resources change

Service profiles for each group



Service profiles for each mode in group 1

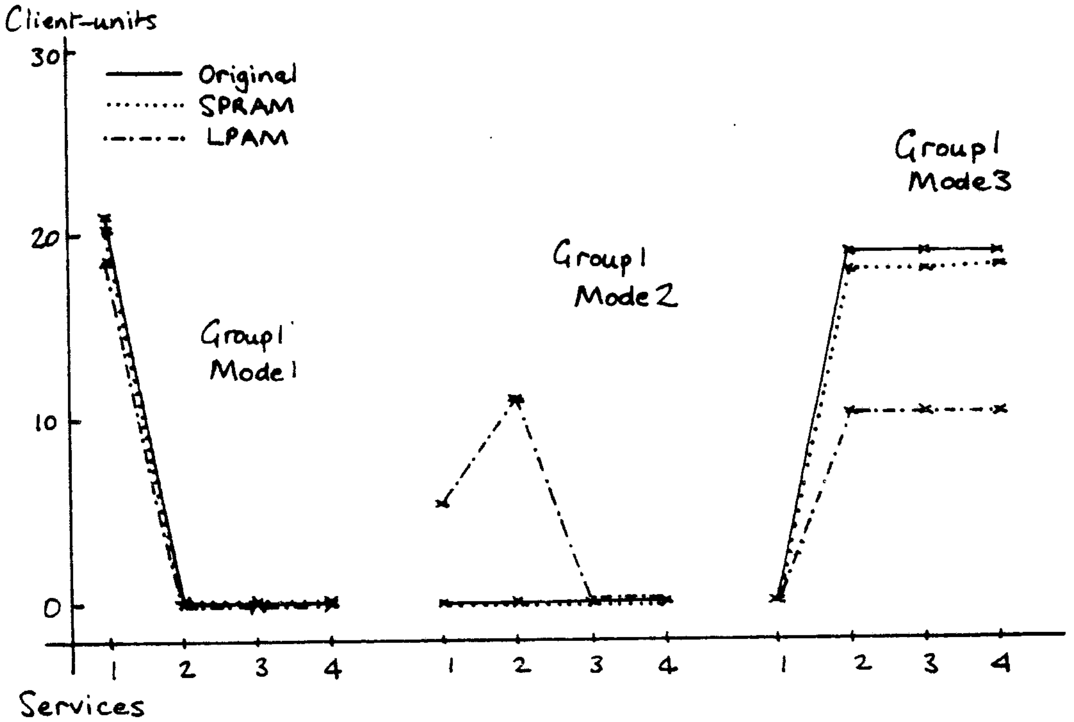
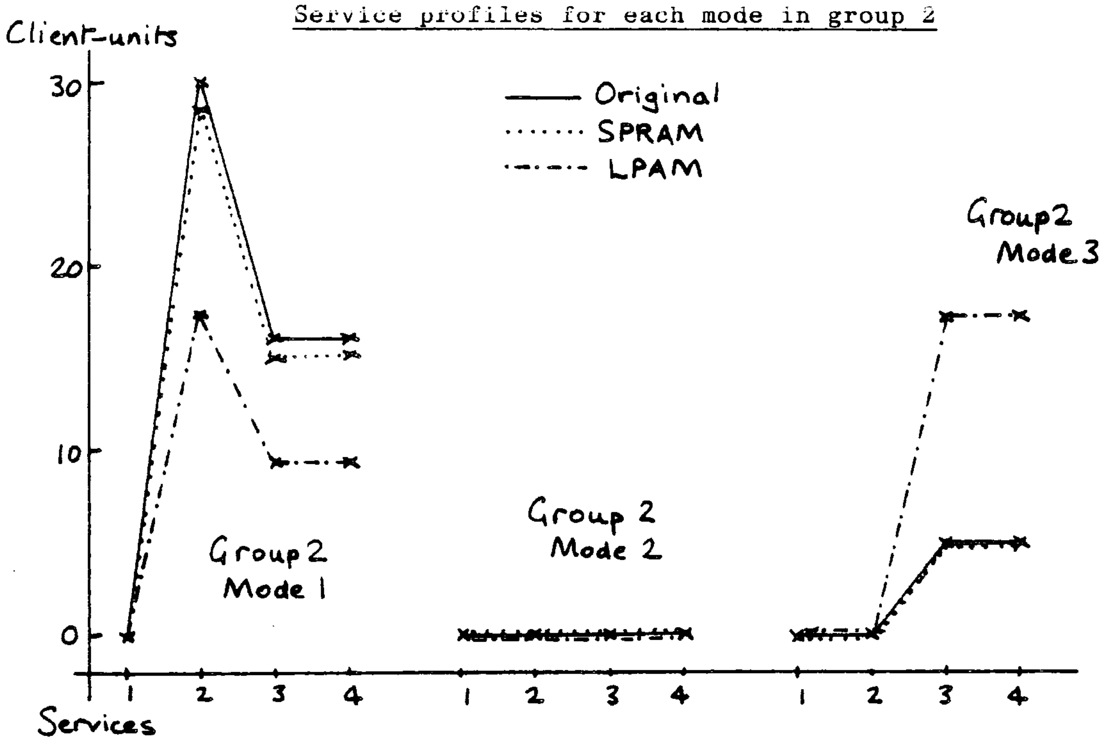


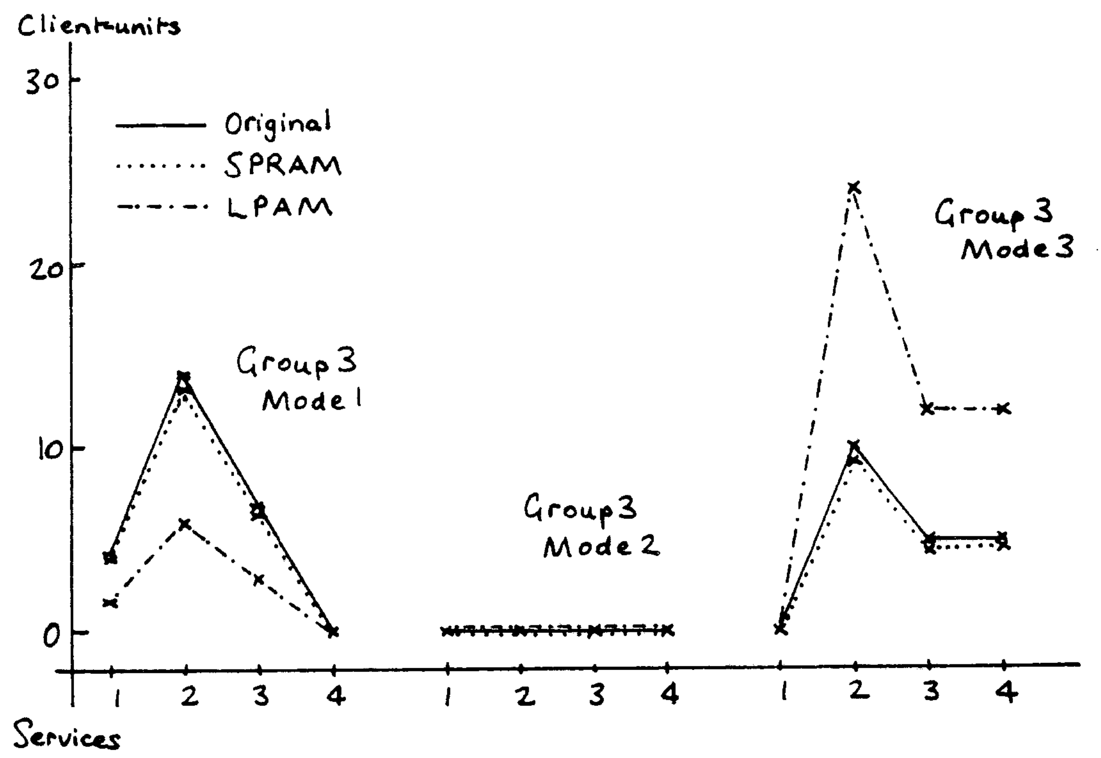
Figure 8.6.2 (Continued)

Comparison of LPAM and SPRAM models, when future resources change

Service profiles for each mode in group 2



Service profiles for each mode in group 3



8.7 Demographic Changes in Client Population

Any future changes in client population are described by a new set d_i' for the expected future size of each client group i.

If there are no other changes and in particular if the resource availability remains unchanged, then the LPAM model itself remains unchanged, with only the bounds of the population constraints changing from d_i to d_i'. The model then produces a new set x_{il}' of allocations of clients to modes of care. Once again, since the alternative modes of care are selected according to weighting priorities and according to the resources available, the optimising procedure will choose alternative modes of care for the clients in a particular group so that if a group size were to increase, some clients could be allocated to different modes of care from those modes already in use, and if a group size were to decrease, fewer clients could be allocated to a particular mode of care.

Figure 8.7.1 shows the new set of allocations produced by the LPAM model for the test data of Figure 8.5.1 when the client population is increased by about 10% in two of the client groups (1 and 2) and decreased by 10% in client group 3. The results are presented in the same form as those in Figure 8.6.1 (which showed changes in resource availability).

The same changes in client population, when used with the SPRAM model, result in a different set of allocations and these are also shown in Figure 8.7.1, together with the original allocations.

Once again, the two models perform the allocations differently : LPAM allocating clients to modes of care which have fixed packages of resources and SPRAM allocating resources firstly to client groups, then to different modes according to the change in the "dominant" resource for each mode.

Figure 8.7.2 shows a comparison of the service profiles for each client group and for each mode of care, similar to the comparisons of service profiles for changes in resource availability which were shown in Figure 8.6.2. The service profiles represent the amounts of each service, measured in client-units, allocated to each client group or mode of care. The figure shows the service profiles of each client group and mode of care for the original data together with the new profiles produced by the SPRAM and LPAM models when the client population is changed.

It is apparent that the SPRAM model again retains a fairly similar service profile to the original profile in each case, with the amount of resource allocated to each client group or mode increasing or decreasing in accordance with the increase or decrease in group size and in accordance with the resource availability.

The LPAM model again diverges considerably from both the original allocation and the SPRAM allocation. Where a client group decreases in size, proportionately more clients may be allocated to a preferred mode of care because of the consequent reduction in resource requirements of the original allocation. This is evident in group 3 where only 10 clients were originally allocated to mode 3, which had the highest weighting, but the LPAM model allocates 19 clients to this mode when the group size decreases from 30 to 27.

Conversely, where a client group increases in size, fewer clients may be allocated to a preferred mode of care because of the consequent increase in resource requirement of the original allocation. This is evident in group 1 where 19 clients were originally allocated to the preferred mode 3 but the LPAM model allocates only 13 clients to this mode, sharing the remaining clients of this group between the other two modes of care.

The LPAM model thus encourages the planners to consider quite different allocations of clients to modes of care than the original allocations, whilst still using the same amounts of resources.

Figure 8.7.1 Comparison of results of LPAM and SPRAM models, for changes in client population from d_i to d'_i

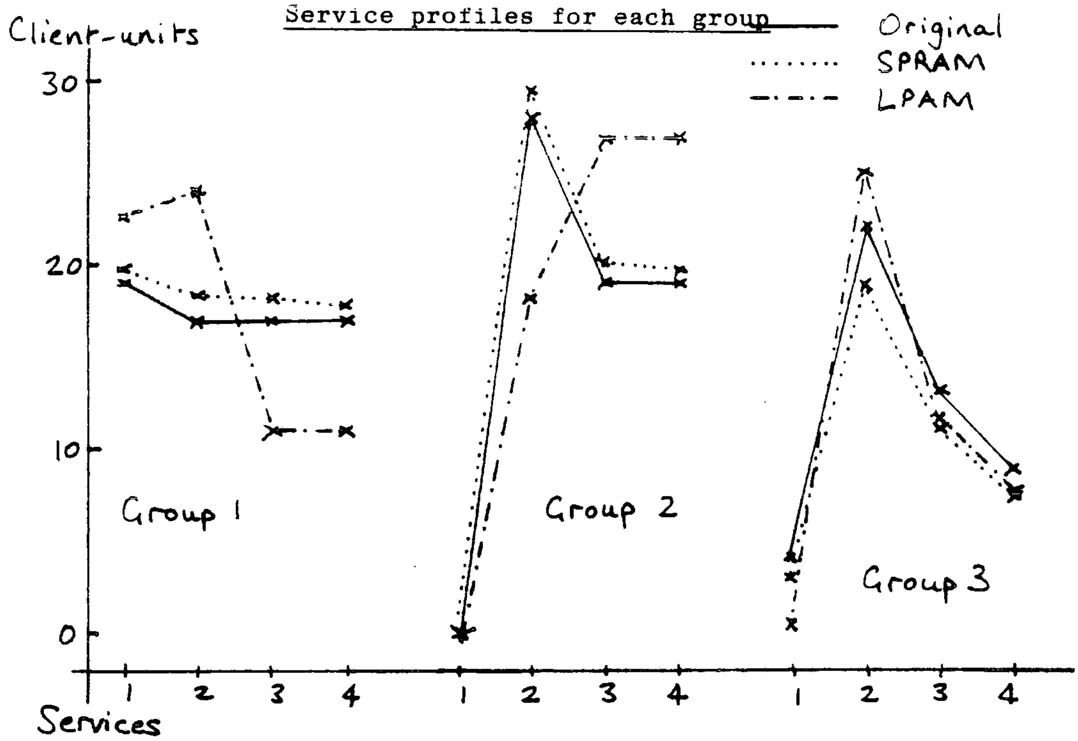
Client group i	1				2				3				
Size d_i (d'_i)	40(44)				35(38)				30(27)				
Mode of care, ℓ	1	2	3		1	2	3		1	2	3		
LPAM Allocations	B11k	B12k	B13k	B1k	B21k	B22k	B23k	B2k	B31k	B32k	B33k	B3k	Bk
Services, k = 1	18	6.5	0	24.5	0	0	0	0	2.40	0	0	2.4	27
2	0	13	13	26	20	0	0	20	8	0	19	27	73
3	0	0	13	13	10.67	0	18	28.67	4	0	9.5	13.5	55
4	0	0	13	13	10.67	0	18	28.67	0	0	9.5	9.5	51
Allocations, $x'_{i\ell}$	18	13	13		20	0	18		8	0	19		
SPRAM Allocations	B11k	B12k	B13k	B1k	B21k	B22k	B23k	B2k	B31k	B32k	B33k	B3k	Bk
Services, k = 1	21.88	0	0	21.88	0	0	0	0	3.58	1.54	0	5.12	27
2	0	0	20.32	20.32	31.67	0	0	31.67	12.07	0	8.75	20.82	73
3	0	0	20.10	20.10	16.75	0	5.22	21.97	5.99	2.57	4.43	12.99	55
4	0	0	19.89	19.89	16.60	0	5.09	21.69	0	5.06	4.36	9.42	51
Original Allocations	B11k	B12k	B13k	B1k	B21k	B22k	B23k	B2k	B31k	B32k	B33k	B3k	Bk
Services, k = 1	21	0	0	21	0	0	0	0	4.2	1.8	0	6	27
2	0	0	19	19	30	0	0	30	14	0	10	24	73
3	0	0	19	19	16	0	5	21	7	3	5	15	55
4	0	0	19	19	16	0	5	21	0	6	5	11	51
Original $x_{i\ell}$	21	0	19		30	0	5		14	6	10		
Weighting $w_{i\ell}$	0.0680	0.1019	0.1748		0.1505	0.1553	0.0777		0.0874	0.0874	0.0971		

Comparison of Allocations to Client Groups, i.e. B_{ik}

	LPAM			SPRAM			Original		
Client group, i	1	2	3	1	2	3	1	2	3
Services, k = 1	24.5	0	2.4	21.88	0	5.12	21	0	6
2	26	20	27	20.32	31.67	20.82	19	30	24
3	13	28.67	13.5	20.10	21.97	12.99	19	21	15
4	13	28.67	9.5	19.89	21.69	9.42	19	21	11

Figure 8.7.2 Comparison of LPAM and SPRAM models, when future populations change

Service profiles for each group



Service profiles for each mode in group 1

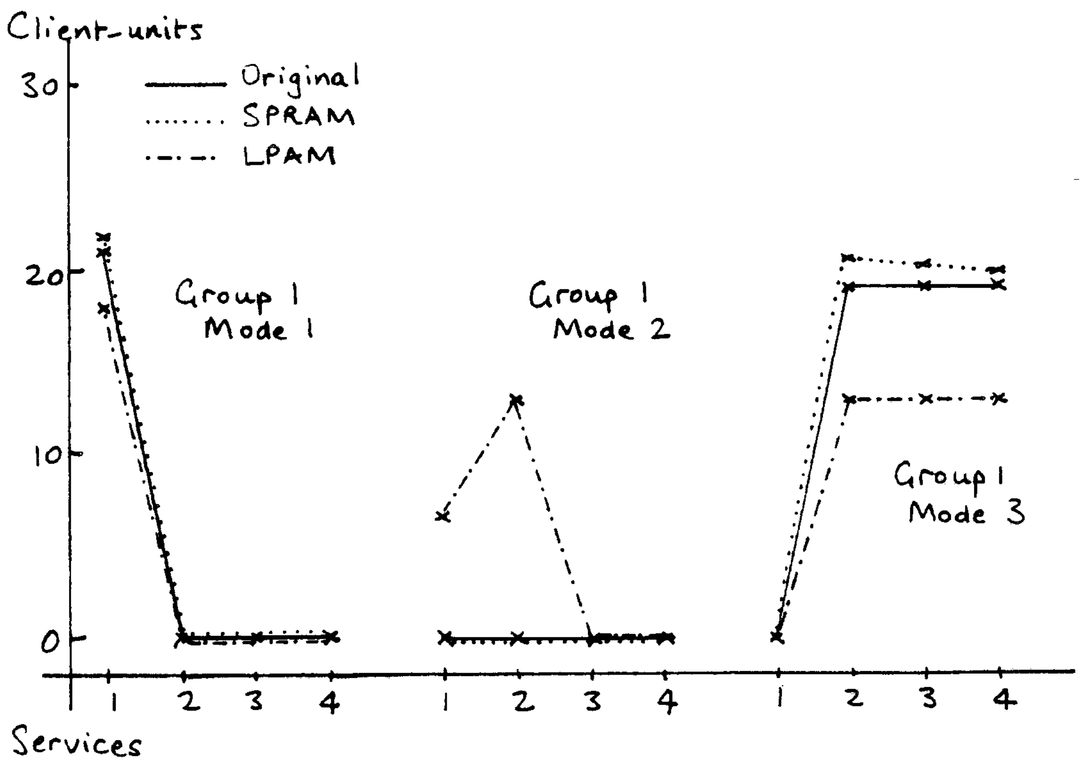
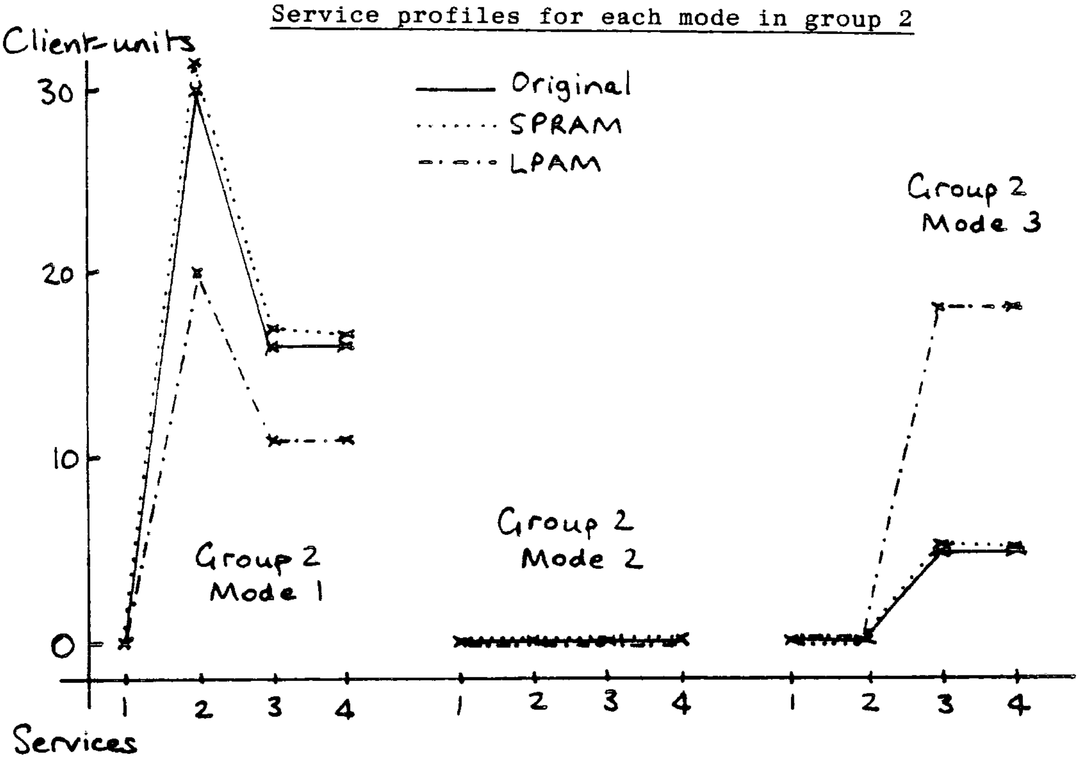


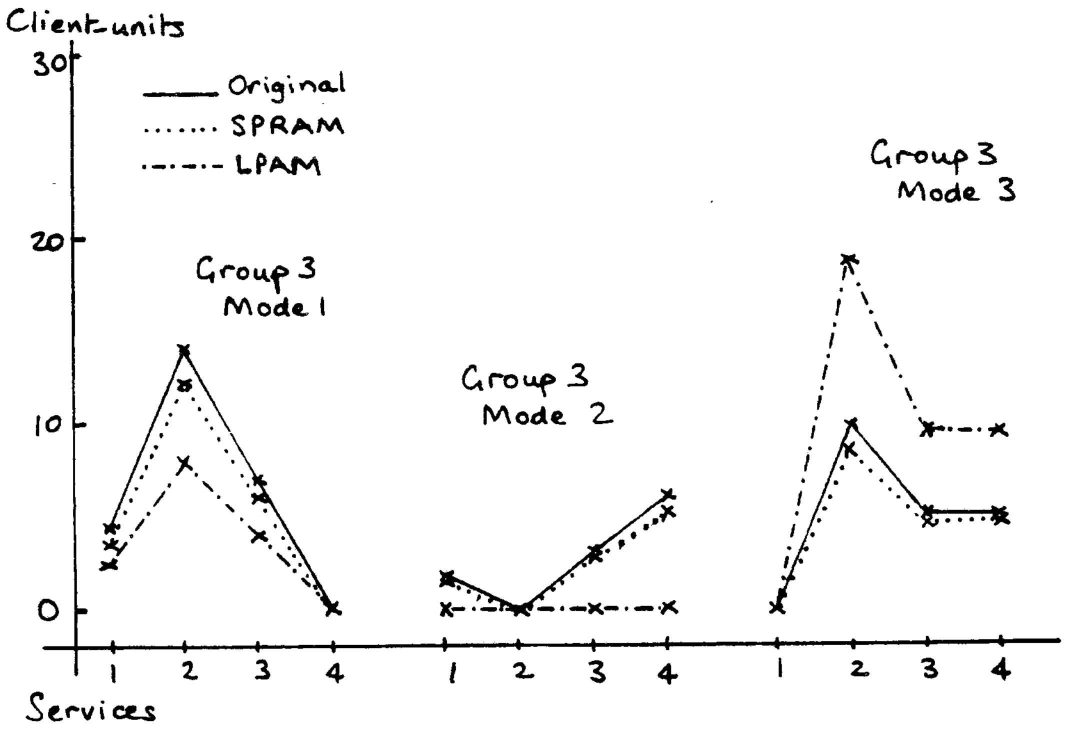
Figure 8.7.2 Comparison of LPAM and SPRAM models,

(Continued) when future populations change

Service profiles for each mode in group 2



Service profiles for each mode in group 3



8.8 Changes in both Resource Availability and Client Population

As before, future changes in resource availability are defined by a new set Bk' and future changes in client population are described by a new set di'.

The LPAM model itself remains unchanged, only the bounds of the service and population constraints being altered. The model then produces a new set x'_{il} of allocations of clients to modes of care within the new constraints.

Figure 8.8.1 shows the new set of allocations produced by the LPAM model for the test data of Figure 8.5.1 when the resource availability is reduced by approximately 5% across all services and the client population is increased by about 10% in client groups 1 and 2 and decreased by 10% in client group 3.

The same changes in resource availability and client population, when used with the SPRAM model, result in a different set of allocations and these are also shown in Figure 8.8.1, together with the original allocations.

Figure 8.8.2 shows a comparison of the service profiles for each client group and for each mode of care for the original data with the new service profiles produced by the SPRAM and LPAM models when there are future changes in both the resource availability and the client population.

Although the SPRAM model allocations differ slightly more in this case from the original allocations, the relative proportions of service allocations to client groups and modes remain similar, thus illustrating once again the SPRAM assumptions that:

> "field-workers' behaviour is such that, if populations of sets of clients remain the same but resource levels change, they give a constant proportion of each resource to each set of clients.
> If, additionally, populations change then these proportions (of resources) are scaled correspondingly in response to the population pressures" (DHSS[26])

The LPAM model again diverges considerably from both the original allocation and the SPRAM allocation. Similarities are apparent, however, between the LPAM allocations for the three types of changes illustrated in Figures 8.6.1, 8.7.1 and 8.8.1 or Figures 8.6.2, 8.7.2 and 8.8.2, which show gradual changes in the allocations as firstly the resources change (Figures 8.6.1, 8.6.2), secondly the populations change (Figures 8.7.1, 8.7.2) and finally both types of changes take place (Figures 8.8.1 and 8.8.2).

It is clear once again that the LPAM model could be used by the planners to suggest quite radical changes in allocations from current allocations and to show that such changes are feasible within the stated resource constraints and demographic changes. The movement of clients between alternative modes of care would not be expected to take place instantly but the planners would be able to see that such movements over the longer term would meet client needs at an acceptable level (the modes being true alternatives) within given resources.

It is thought that planners would wish to explore intermediate possibilities, i.e. allocations of clients to modes somewhere between the current allocations and the model's suggested allocations. This could be done by using the LPAM model as a simulation model rather than as an optimising model. This and other uses of LPAM as a simulation model are discussed in the next section.

Figure 8.8.1 Comparison of results of LPAM and SPRAM models, for changes in both resource availability and client population

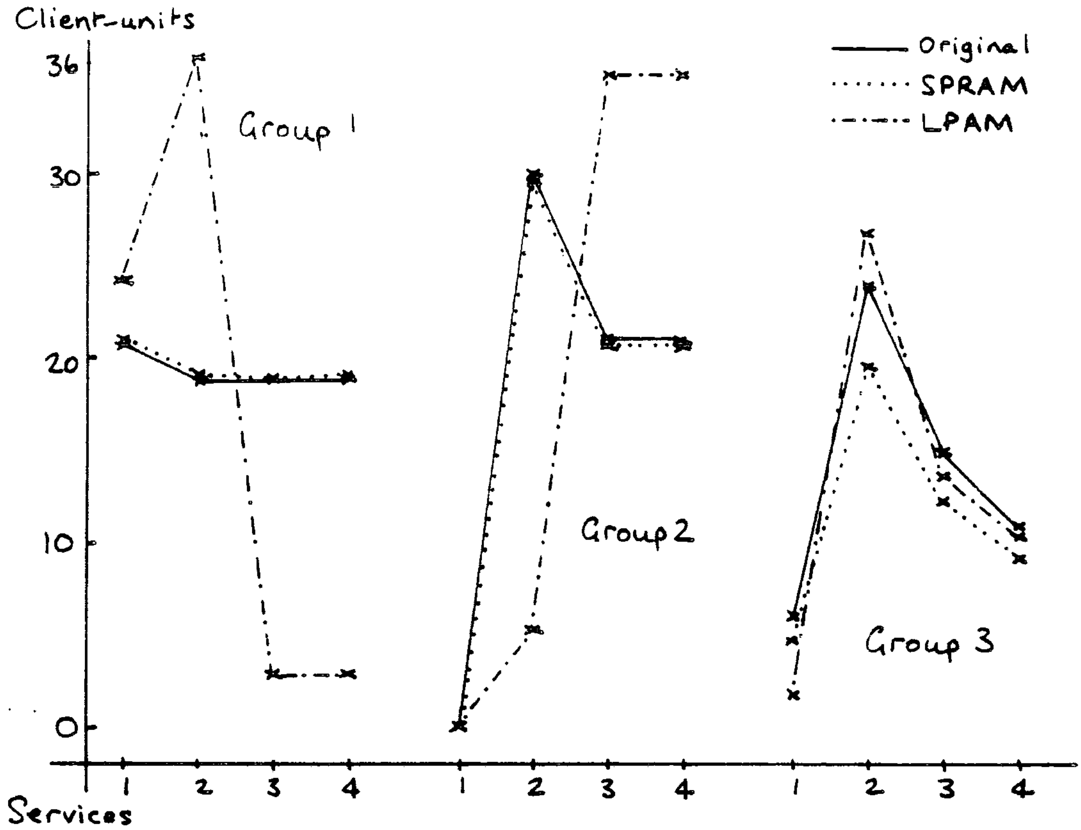
Client group i	1				2				3				
Size d_i (d'_i)	40(44)				35(38)				30(27)				
Mode of care, ℓ	1	2	3		1	2	3		1	2	3		
LPAM Allocations	B11k	B12k	B13k	B1k	B21k	B22k	B23k	B2k	B31k	B32k	B33k	B3k	Bk'
Services, k = 1	7.45	16.75	0	24.2	0	0	0	0	1.8	0	0	1.8	26
2	0	33.5	3	36.5	5.45	0	0	5.45	6	0	21	27	69
3	0	0	3	3	2.91	0	32.5	35.41	3	0	10.5	13.5	52
4	0	0	3	3	2.91	0	32.5	35.41	0	0	10.5	10.5	49
Allocations, $x'_{i\ell}$	7.45	33.5	3		5.45	0	32.5		6	0	21		
SPRAM Allocations	B11k	B12k	B13k	B1k	B21k	B22k	B23k	B2k	B31k	B32k	B33k	B3k	Bk'
Services, k = 1	21.07	0	0	21.07	0	0	0	0	3.45	1.48	0	4.93	26
2	0	0	19.21	19.21	29.94	0	0	29.94	11.55	0	8.27	19.82	69
3	0	0	19.00	19.00	15.83	0	4.94	20.77	5.71	2.45	4.11	12.27	52
4	0	0	19.11	19.11	15.92	0	4.92	20.84	0	4.92	4.13	9.05	49
Original Allocations	B11k	B12k	B13k	B1k	B21k	B22k	B23k	B2k	B31k	B32k	B33k	B3k	Bk
Services, k = 1	21	0	0	21	0	0	0	0	4.2	1.8	0	6	27
2	0	0	19	19	30	0	0	30	14	0	10	24	73
3	0	0	19	19	16	0	5	21	7	3	5	15	55
4	0	0	19	19	16	0	5	21	0	6	5	11	51
Original $x_{i\ell}$	21	0	19		30	0	5		14	6	10		
Weighting $w_{i\ell}$	0.0680	0.1019	0.1748		0.1505	0.1553	0.0777		0.0874	0.0874	0.0971		

Comparison of Allocations to Client Groups, i.e. B_{ik}

	LPAM			SPRAM			Original		
Client group, i	1	2	3	1	2	3	1	2	3
Services, k = 1	24.2	0	1.8	21.07	0	4.93	21	0	6
2	36.5	5.45	27	19.21	29.94	19.82	19	30	24
3	3	35.41	13.5	19.00	20.77	12.27	19	21	15
4	3	35.41	10.5	19.11	20.84	9.05	19	21	11

Figure 8.8.2 Comparison of LPAM and SPRAM models, when future resources and populations both change

Service profiles for each group



Service profiles for each mode in group 1

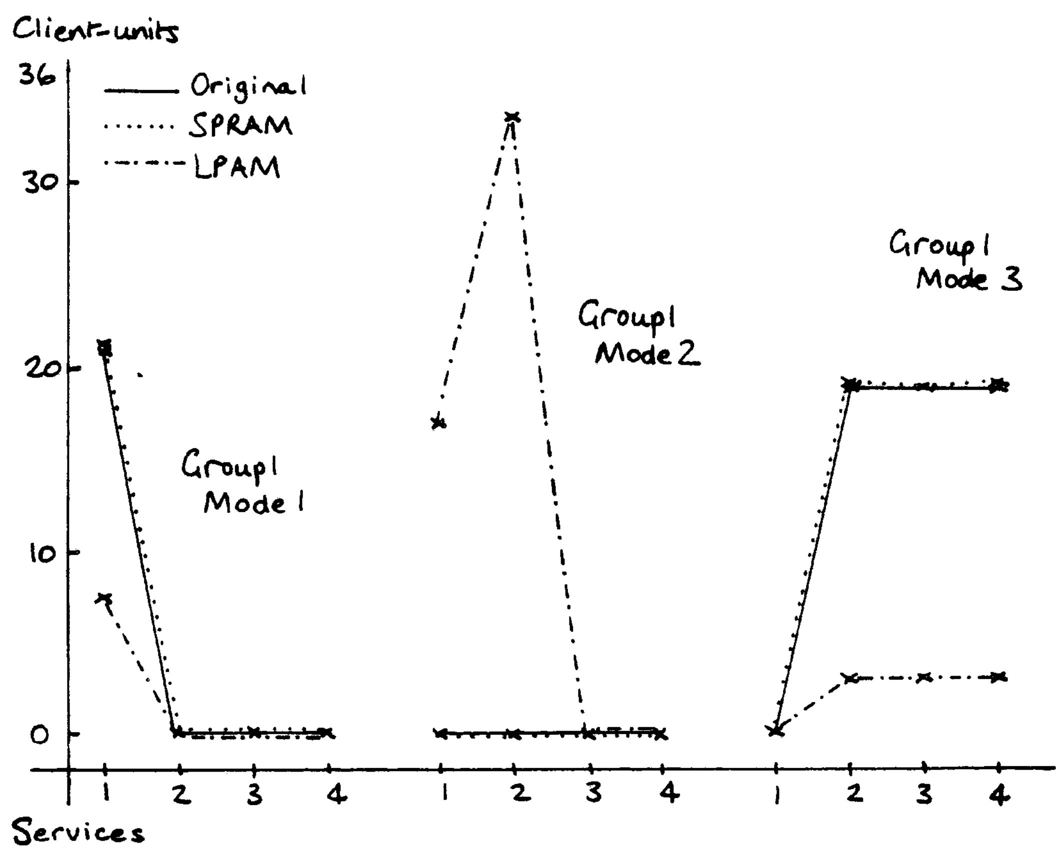
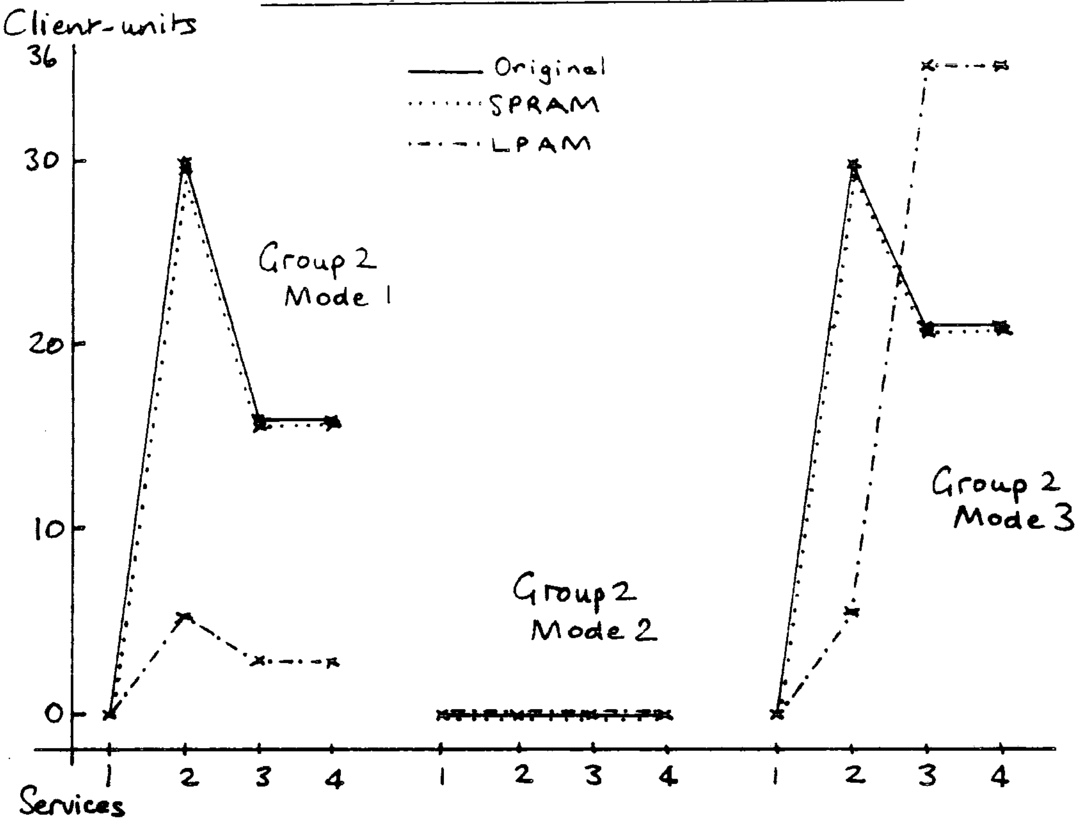


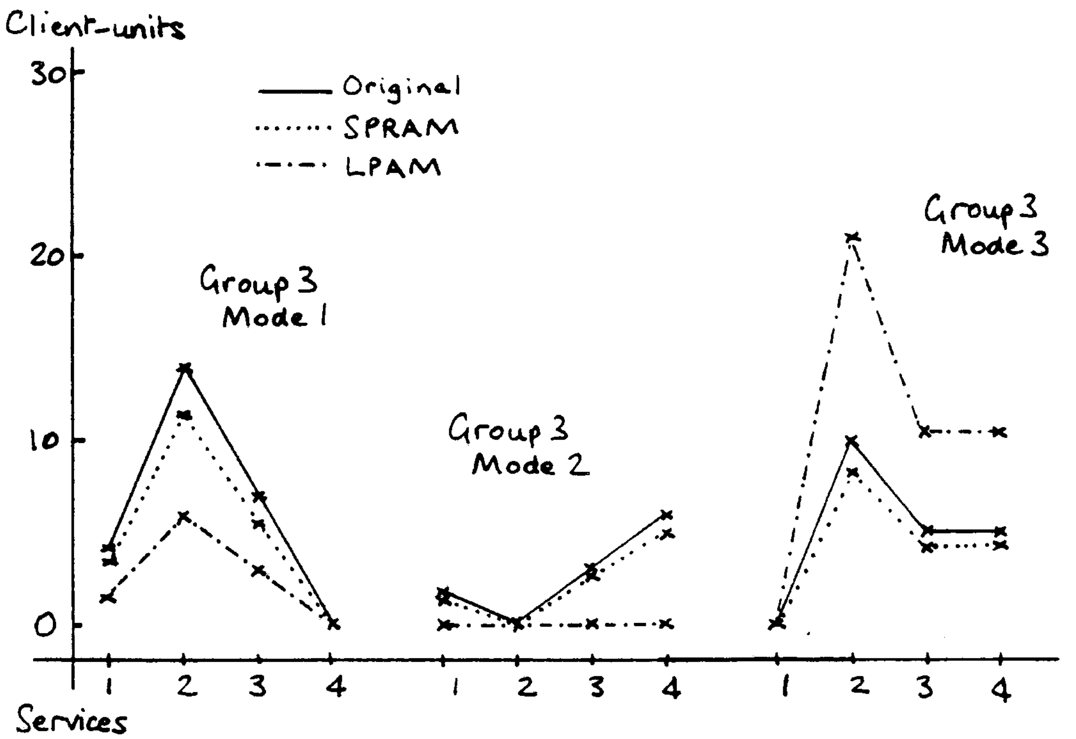
Figure 8.8.2 Comparison of LPAM and SPRAM models,

(Continued) when future resources and populations both change

Service profiles for each mode in group 2



Service profiles for each mode in group 3



8.9 Use of the LPAM Model as a Simulation Model

8.9.1 Introduction

In Section 8.4 a description was given of the use of the LPAM model as a simulation model to assess the resource needs in Chester-le-Street district for a changed client population if existing patterns of care were to be maintained.

In order to assess the resources it was necessary to make assumptions about the allocation of clients to modes of care so that, instead of allowing the LPAM model to choose the allocations of clients subject to resource constraints, the model would simulate the resource allocation and monitor the amounts of resource needed rather than constraining the resources.

It is possible to use the LPAM model as a simulation model in several ways, depending upon how the data is specified. The following paragraphs discuss these methods.

8.9.2 Assessment of Resources; Equal Increments in Modes of Care

The requirement here is to assess the amounts of resources needed in future to care for a changed client population if existing patterns of care were to be maintained. One possibility, as used in Section 8.4 with the Chester-le-Street district, is to assume that the population changes are described by a new set di' of client group sizes and to assume further that the same modes of care will continue to be used and that the changes in numbers of clients to be allocated to modes will be derived by sharing the increase or decrease equally between these modes.

i.e. $$x_{il}^{new} = x_{il}^{old} + \frac{(di' - di)}{m_i}$$

Where x_{il}^{old} = number of clients currently allocated to mode l in group i,

x_{il}^{new} = number of clients to be allocated in future to mode l in group i,

di = current size of group i,

di' = future size of group i,

and m_i = number of modes of care currently used for group i.

The LPAM model is then used to find the amounts of service resources needed in future to care for the changed client population.

8.9.3 Assessment of Resources; Proportional Increments In Modes of Care

The requirement is again: to assess the amounts of resources needed in future to care for a changed client population if existing patterns of care were to be maintained.

Another possibility is to assume that the population changes are described by a new set di' of client group sizes and to assume further that the same modes of care will continue to be used, but that the numbers of clients to be allocated to modes will be derived by increasing or decreasing the current allocations in proportion to the change in group size.

i.e.

$$x_{il}^{new} = x_{il}^{old} \cdot \frac{di'}{di}$$

This is similar to the assumption of the SPRAM model.

The LPAM model is then used as before to find the amounts of service resources needed in future to care for the changed client population.

8.9.4 Assessment of Resources; Unconstrained Choice of Modes of Care

It is possible to use the LPAM model to assess resources needed in future to care for a changed client population by allowing the model itself to choose the allocation of clients to modes of care. If no assumptions are made with respect to the number of clients x_{il} allocated to modes of care and if changes in the client population are described by changes in group sizes from di to di', then the LPAM model would choose a set of x_{il} with reference to the weightings w_{il}, the only constraints being the group size constraints:

$$\sum_{l} x_{il} = di', \text{ for all } i.$$

The resource constraints would be unbounded and the size B_k of each resource would be monitored.

i.e.

Choose the x_{il} to Maximise $\sum_i \sum_l w_{il} \ x_{il}$

Subject to $\sum_l x_{il} = di'$, for all i,

and calculate $Bk = \sum_i \sum_l u_{ilk} \ x_{il}$, for all k.

This approach is, however, unacceptable because, when the resource constraints are removed, the model chooses the most preferred mode of care (i.e. for which w_{il} is largest) for each client group i and the resulting allocations are, for each i, $x_{il} = di'$ for that mode l having the largest w_{il}, and $x_{il} = 0$ otherwise.

Whilst this would reflect an "ideal" situation, all clients being allocated to the most preferred modes of care, it is not considered to be acceptable.

8.9.5 <u>Assessment of Resources; Pre-Defined Choice of Modes of Care</u>

This approach assumes that changes in the population are defined by changes in the client group sizes from di to di$'$ and in addition that the planners define the numbers of clients in each group who will receive each mode of care, i.e. x_{il}^{new}.

The LPAM model is then used as before to find the amounts of service resources needed in future to care for the changed client population.

This is a very straight-forward approach but it requires the planners to supply data for every group and mode of care. In order to reduce the amount of data needed, an "incremental" or "difference" model could be built which used data only on those groups and modes where a change takes place.

Thus, if j = client group number

and m = mode of care

for those groups and modes where a change takes place,

and x_{jm}^{old} = current allocation,

x_{jm}^{new} = future allocation,

then the new service resources required would be:

$$Bk^{new} = Bk^{old} - \sum_{j}\sum_{m} u_{jmk}\, x_{jm}^{old} + \sum_{j}\sum_{m} u_{jmk}\, x_{jm}^{new},$$

for all j and m where a change takes place, and for all k.

This set of equations could be included in the LPAM model.

8.9.6 Exploration of Intermediate Allocations

Since the LPAM model provides planners with a set of future allocations of clients to modes of care which may be radically different from the initial allocations, it may be appropriate for the planners to explore the effect of intermediate allocations somewhere between the initial state and the LPAM optimal allocations. This may be necessary because the movement of clients between alternative modes of care would not be expected to take place instantly and the planners may wish to consider re-allocation of fewer clients from one mode to another as an intermediate stage.

In this case the planners would wish to specify as data the numbers of clients allocated to each mode of care in each client group, i.e. the x_{il}. This implies that the requirement is to simulate what resources would be needed to care for the future client population for the planners' choice of modes of care - and this is identical with the model discussed in Section 8.9.5 above. As before, if it were simpler to specify data only for those groups and modes where a change was to take place, an "incremental" model could be constructed by extension of the LPAM model.

8.10 Other Planning Changes

Other planning changes may be policy decisions on the way in which the resource budget should be spent or decisions on the ways in which clients should be cared for.

The former implies that there may be reduced expenditure in some areas and perhaps increased expenditure in others. Because of the way in which the planning process operates, these reductions or increases are effected by corresponding reductions or increases in social service resources. It is not usual (and may be impossible) directly to increase or reduce expenditure on a particular client group : instead, increases or reductions take place directly in resources and hence indirectly affect the client group. Consequently such changes in expenditure will be defined by changes in the amounts of each resource available, i.e. by changes from Bk to Bk′ for all resources k. Sections 8.4, 8.6, 8.8 and 8.9 above describe the ways in which LPAM, the linear programming model with alternative modes of care, caters for such changes.

Decisions on the ways in which clients could be cared for could be of various types. In the D.H.S.S./A. Andersen SPRAM model (see Section 4.4), provision is made for decisions relating to the choice of mode of care, the choice of amounts of resources available in future, the specification of new modes of care and the choice of proportions of quota and coverage.

These decisions can all be included in the LPAM model. The choice by the planners of modes of care for individual client groups has been discussed in Section 8.9.6; the choice of amounts of resources available in future is assumed to be an input to the LPAM model; the specification of new modes of care can be an input to the model. The choice of proportions of quota and coverage is now discussed.

In the SPRAM model, resources are firstly allocated to each client group, then the allocation to individual clients within each mode of care is made on the basis of a pre-defined trade-off between the quota of resource allocated to a single client and the coverage or allocation across all clients in the group. This may result in a high quota of resource being allocated to some clients and none to others or conversely in a lower quota of resource being allocated to all clients. In this model the allocation of resources within modes of care is not fixed.

In the LPAM model, however, the allocations of clients to modes of care within a group, i.e. x_{il}, are chosen by the model, thus giving a fixed "coverage" of clients within a mode of care; and the modes of care are themselves fixed at particular levels of resource per client, thus giving a fixed "quota" of resource to each client. The very definitions of the modes of care in the LPAM model imply certain allocations of resources available per client, whereas in the SPRAM model this is not so. If larger "quotas" were desirable then new modes of care ought to be created for the LPAM model with appropriate weighting factors.

Nevertheless, the LPAM model still permits flexibility of allocation of resources to an individual client, since the model only allocates how many clients should be cared for in a particular mode of care and thus the average amount of each resource per client is allocated, but the total amount, i.e. $x_{il}\ u_{ilk}$, of each resource k may be shared out by the social workers evenly or unevenly. Thus within each mode of care, each client could receive the same amount of a resource or some clients could receive more and some could receive less.

The LPAM model thus allows the planners and social workers to determine the precise allocation of resources to individual clients.

CHAPTER 9

LIMITATIONS, APPLICATIONS AND EXTENSIONS: FUTURE WORK

9. LIMITATIONS, APPLICATIONS AND EXTENSIONS: FUTURE WORK

9.1 Limitations: the "Snapshot" Census

The linear programming model with alternative modes of care (LPAM) was constructed using data made available by Durham County Council (DCC) from the D.U.B.S. census taken in 1977. That census constituted a "snapshot" taken at a point in time of the "active" social work cases in the majority of County Durham (ie. excluding Lanchester district). The active case load represents those cases currently needing attention by social workers and excludes clients who are currently receiving social services, but whose cases are no longer active since their needs for services have been assessed and those services are now being supplied.

As a consequence of the survey being restricted to "active" cases, the data in the LPAM model does not reflect the whole picture of social service allocation or demand. Other clients who are receiving packages of care on a regular basis but whose packages do not include social worker involvement are not at present included. Each case has a different "active" life. Estimates of demand in total units of service would be needed in order to model the allocations completely. M. Wheatley[57], when discussing the census design for the D.U.B.S. research, suggested that estimates of demand "would be obtained by applying multipliers to service profiles based on annual through-puts of client types".

The present author has made comparisons between the service levels and the numbers of clients as derived from the census data and as available currently (1986) and there appears to be a multiplication factor of about 10 in respect of the majority of services supplied. The number of active cases currently is about the same as it was at the time of the census; the amounts of most services have not changed greatly; the proportion of most services supplied to "active" cases is about a tenth of the total amount allocated of each service. If the LPAM model were to be implemented in practice, more careful estimates would be needed in respect of non-active cases and their demand for services. The total amount of each service available is known; the modes of care for non-active cases is not known at present. Estimates of care packages could be made from existing information on services supplied or a survey could be carried out.

9.2 Limitations: Validation of Model

Although the model has been tested, for the Chester-le-Street district and some simple test data, to ensure that demographic and resource changes can be included and plans can be produced, the model has not been rigorously validated. Validation of consistency could take place in a number of ways, for example:

- comparing current DCC plans with what the model would allocate based on the 1977 data;
- comparing allocations for Chester-le-Street with those for other districts within County Durham;
- comparing allocations for Durham with those for other regions.

The first of these is feasible although it depends to some extent upon the second; it would be appropriate if a practical model were to be implemented. The second requires certain assumptions to be made concerning the ways in which social workers choose to care for their clients. Comparisons of allocations in two districts would be appropriate providing the social workers do care for their clients using similar packages of care. Discussions with the planners and evidence from the census suggests that there can be quite wide variations in the care packages chosen by different social workers for apparently similar clients. Caution would be needed to ensure that "similar" districts (in terms of social workers' packages of care) were chosen for purposes of comparison. The differences in choice of care by individual social workers would not affect the model itself since the requirements of the planners are for aggregated amounts of services needed, rather than the individual client allocations. The third means of validating consistency would be to compare the allocations for County Durham with those for other regions. This is probably least satisfactory, since, as shown in chapter 8, there can be quite wide divergence in the proportions of client groups cared for with similar services in different counties. Yet there is encouragement from the Government, in the form of broad policies and national priorities, to treat clients consistently across the country. (See for example, DHSS[22]: Care in Action and DHSS[23]: Community Care). Guidelines are from time to time issued, for example priority groups identified in

1981 (DHSS[22]) were: elderly people, the mentally ill, the mentally handicapped and the physically and/or sensorily handicapped. Similarly, guidelines on priority services listed in 1981 (DHSS[22]): maternity services and neonatal care, primary care services and services related to the care of young children at risk and to the care and treatment of juvenile offenders.

At present, however, each County has freedom to decide how to pursue these policies and priorities in its own locality. There may be locally one particular group or service within the priority list which requires most attention and so the emphasis may vary from one county to another.

9.3 Applications and Extensions

The LPAM model as postulated includes client groups as chosen in the D.U.B.S. research. Since the D.U.B.S. client groups were proposed, Durham County Council (DCC) has developed the Computerised Client Records System and Caseload Management System. A model which used the same client groups and categories as these systems would be more appropriate and would be easier for the planners to use now that they are familiar with those groupings. To include these groups in the model is simply a matter of definition and data gathering: the basic framework of the model would not alter.

The current LPAM model uses alternative modes of care derived by the author from Cluster Analysis of the original (1977) DUBS census data. These modes, whilst reflecting precisely the care given to active cases at that time, may be considered to be too detailed and in any case may not be appropriate if new definitions of client groups were to be used. At least three possible courses of action exist: firstly, to conduct another survey of resource usage along the lines of the DUBS census but using the new client groups; secondly, to make use of reports of resource allocations which are now produced by the Social Services Department; or thirdly, to decide upon a range of alternative packages of care for each client group: this latter possibility would require professional advice such as that employed in the Balance of Care projects.

The current LPAM model defines packages of care in terms of the resource allocated, measured in numbers of client-units, i.e. without defining precisely what is meant by a client-unit for each resource. It would be possible to construct the model using realistic units of measure for services, so that, for example, meals-on-wheels would be measured in terms of financial cost.

The weights used in the objective function of the current LPAM model are derived (somewhat arbitrarily) from the current (1977) allocation of services to client groups. The opportunity now exists to make use of the "priority and points" system operated in DCC as part of the Caseload Management System. It should be possible to define weights for client groups and modes of care based on these priorities and "points" since they are now used to assist social workers in ranking their workload order of priority.

The definition of a suitable objective function has been a major preoccupation of researchers in this field. Whilst the LPAM model requires an objective function to be defined in order to select a suitable allocation of clients to modes of care, it has been shown in Chapter 5 that selection of the weights may be within quite large ranges without significantly altering the optimal solution point. The main advantage of the LPAM model rests in its ability to select a new set of allocations of clients to modes of care which are within the resource constraints and which may be considerably different from current modes of care. Thus the main advantage is that of exploration of the feasible region: allowing planners to consider quite different allocations from those currently in use, and to explore intermediate allocations which may be "optimal" but which are feasible and which may be used as intermediate stages on the route to optimality. Since the definition of an "optimum" in this context is extremely difficult, the precise form of the objective function may not be too significant.

One possible extension to the LPAM model would be to include costs, so that any chosen allocation can be measured in terms of its cost as well as in terms of the objective function value of effectiveness. Unit costs of resources are not at present available in DCC but, with the imminent implementation of the "FISCAL" computer model, they may become available in the near future. Such an extension to the model would improve its means of presenting information to the planners for comparative purposes.

CHAPTER 10

CONCLUSIONS

10. Conclusions

From the preceding discussions it is clear that some form of mathematical model can be helpful to the planners in evaluating appropriate strategies for future allocation of social services resources, particularly when demographic changes are taking place and resources are scarce.

Strategies for future allocation of resources may be determined using a model, but individual assessment of client needs will continue to be made by social workers and other professional experts. It is not anticipated that a rigid set of allocations would be imposed upon the field workers, rather that they would be encouraged to consider alternative acceptable packages of care and to move towards those packages of care which are more desirable in terms of their usage of resources or in terms of cost.

The preceding chapters have described and compared a number of mathematical models for such long range planning of social services. The author has postulated a linear programming model with alternative modes of care (LPAM) and has shown how this model can be used to plan for future demographic changes and resource changes.

Analysis of a large survey of client care in County Durham has been undertaken in order to ascertain the patterns of care at the time of the survey, to ascertain the resource usage for all services offered and to define a range of alternative modes of care which reflect the care patterns at the time of the survey.

A discussion of the weighting values required in the LPAM model has shown that a feasibility region can be defined for the weights which reflects existing patterns of care.

The author's model has been compared with other models, in particular the DHSS/A. Andersen SPRAM model. Regional attempts at implementing the DHSS model have been described and, where possible, contrasted with results from the Durham survey. The ways in which the author's LPAM model and the DHSS SPRAM model undertake planning when there are future demographic and/or resource changes have been discussed and compared. Limitations in respect of the LPAM model have been discussed, as have ways of extending and applying the model in the context of County Durham.

The advantage offered by the LPAM model lies in the way that radically different methods of care can be chosen by the model and be shown to make acceptable use of resources (i.e. within resource constraints) whilst providing acceptable modes of care, even though the client population may increase or decrease differently according to the age group of the clients. Although such radical changes could not be immediately implemented, the planners can use the model to explore the effects of intermediate allocations of care between current allocations and the model's proposals, so that in the longer term these radical changes could be effected. Since planners are nowadays required to make major reductions in public expenditure, this model offers a means of considering radical alternatives to existing methods of care, whereas the SPRAM model takes an incremental or decremental approach and its proposed allocations are therefore little changed from the current allocations.

The West Midlands experience has shown that the plans produced by the SPRAM model were not particularly interesting to the planners - probably because they were little different from the current position. Furthermore, the flexibility offered by the LPAM model, in terms of the alternative modes which can be specified and possibly chosen, permits planners to incorporate different packages of care to reflect changed philosophies of care. Preferences for certain types of packages can be expressed through the weighting system.

With the introduction of the priority and points system in the Durham caseload management system there is an implicit weighting in favour of certain types of care for certain clients groups and this would offer a realistic means of setting the weights for the LPAM model. In addition, the defining of client groups for the computerised Client Record System together with the improved reporting of resource usage in County Durham would all contribute to make the application of the LPAM model in County Durham a practical proposition.

CHAPTER 11

REFERENCES

11. References

1. Algie, J.; Hay, A. and Malien, G. "Cuts : getting your priorities right". Community Care 1981.

2. Aspden, P.; Mayhew, L. and Rusnak, M. "DRAM: a model of health care resource allocation in Czechoslovakia". Omega Vol. 9 No. 5, 1981.

3. Aucamp, D.C. and Steinberg, D.I. "The computation of shadow prices in linear programming". Jnl. Opl. Res. Soc. Vol. 33, 1982.

4. Bayat, G.R.; Wade, G. and Longbottom, D.A. "Effectiveness as a guide to planning in the social services". Management Decision Vol. 13 No. 5, 1975.

5. Bevan, R.G. "O.R. and Public Expenditure Planning and Control". Jnl. Opl. Res. Soc. Vol. 31, 1980.

6. Boldy, D. "A review of the application of mathematical programming to tactical and strategic health and social services problems". Opl. Res. Q. Vol. 27, 1976.

7. Booth, T.A. "Planning for Welfare". Basil Blackwell and Martin Robertson, 1979.

8. Borley, R.G.; Taylor, S.H. and West, C.R. "Balance of Care - a user's view of a new approach to joint strategic planning". Omega Vol. 9 No. 5, 1981.

9. Central Statistical Office. "Annual abstract of statistics". HMSO 1979, 1980, 1981.

10. Central Statistical Office. "Regional statistics". HMSO 1979, 1980, 1981.

11. Cochrane, J.L. and Zeleny, M. (eds). "Multiple criteria decision making". Univ. of South Carolina press. Conference proceedings 1973.

12. Coles, O. "The Balance of Care approach to resource allocation : its possible implications for the Department". Internal report. DCC 1982.

13. Coles, O. "A model for 'proportioning' available social services resources across a changing client population using the D.U.B.S. project's client census data". Internal paper. DCC 1983.

14. Cornwall County Council. "Balance of care - extract from report". Internal paper, received 1982.

15. Coverdale, I.L. and Negrine, S.M. "The Balance of Care project : modelling the allocation of health and personal social services". Jnl. Opl. Res. Soc. Vol. 29 No. 11, 1978.

16. Cuthbertson, J. "Cost analysis for resource allocation". Vol. 3.2.3 of D.U.B.S. "Design for a social services planning system". ed. E. Nelson, 1978.

17. Durham County Council. "Caseload management and a priority system for social work in Durham", 1981.

18. Durham County Council. "Computerised client information system-outline specification". 1981.

19. Durham County Council. "County Durham in figures". 1981.

20. Durham County Council. "Position statement 1980". 1980.

21. Durham County Council. "Trends in the residents of homes for the elderly". 1982.

22. DHSS "Care in action". 1981.

23. DHSS "Care in the community". 1981.

24. DHSS "Report of a study on community care". 1981.

25. DHSS with Arthur Andersen & Co. "A user's guide to the Balance of Care report. Vol. 1. Non-technical user manual". DHSS 1981.

26. DHSS with Arthur Andersen & Co. "Wider applications of the Balance of Care approach". Vol 1 DHSS 1979, Vol. 2 DHSS 1980.

27. East Sussex County Council. "The Balance of Care in East Sussex". Internal paper, received 1982.

28. East Sussex County Council. "Review of the Balance of Care exercise in East Sussex". Internal paper, received 1982.

29. George, J.A.; Fox D.R. and Canvin R.W. "A hospital throughput model in the context of long waiting lists". Jnl.Opl. Res. Soc. Vol. 34, 1983.

30. Gibbs, R.J. "The use of a strategic planning model for health and personal social services". Jnl. Opl. Res. Soc. Vol 29, No. 9, 1978.

31. Health O.R. Unit. "Assessing care requirements of elderly people". Broadsheet No. 8.

32. Howell, R.G. and Parker, C.J. "Assessing care requirements of elderly people in Calderdale". NHS O.R. Group Report No. 79/8, 1979.

33. I.B.M. "Introduction to Mathematical Programming System" (Ref. GH20-0849-3) 4th edition 1973.

34. Jolliffe, I.T.; Jones, B. and Morgan B.J.T. "Utilising clusters : a case-study involving the elderly". Jnl. Royal Stat. Soc. (A) Vol. 145, part 2 1982.

35. Klemperer, P.D. and McClenahan, J.W. "Joint strategic planning between health and local authorities" Omega Vol. 9, No. 5 1981.

36. Kornbluth, J.S.H. "A survey of goal programming" Omega Vol. 1, No. 2, 1973.

37. Longbottom, D.A. "Resource allocation techniques for public services" Vol. 1, No. 3 of D.U.B.S. "Design for a social services planning system" ed. E. Nelson, 1978.

38. Louni, G.R. "Alternative objectives in allocating care to the elderly in Durham". University of Durham, MSc Thesis, 1979.

39. MacCrimmon, K.R. "An overview of multiple objective decision making" in Cochrane and Zeleny's "Multiple criteria decision making", 1973.

40. McDonald, A.G.; Cuddeford, G.C. and Beale, E.M.L. "Balance of Care : some mathematical models of the National Health Service". Brit. Med. Bull. Vol. 30, No. 3, 1974.

41. Mooney, G.H. "Planning for Balance of Care of the elderly". Scot. Jnl. of Political Economy, Vol. 25, No. 2, 1978.

42. Murdoch, J. and Barnes, J.A. "Statistical tables". Macmillan, 2nd edition, 1972.

43. Nelson, E.G. (ed) "Design for a social services planning system". Durham University Business School, 1978.

44. Nelson, E.G. "Development of a planning process". Ibid. Vol. 1, No. 2, 1978.

45. Nelson, E.G. and Wiper, L. "Weighting concepts for resource allocation". Ibid. Vol. 3.1, No. 2 1978.

46. Nicholls, I.G. "Joint planning in Dudley - the role of Balance of Care". Omega Vol. 9, No. 5, 1981.

47. Prentice, R.C. "The determination of worth : a review of concepts and techniques". Vol. 2, No. 1 of D.U.B.S. "Design for a social services planning system" ed. E. Nelson, 1978.

48. Prentice, R.C. "Concepts of inferred worth planning in the social services". Ibid. Vol. 2, No. 2, 1978.

49. Prentice, R.C. "Development record of social services client grouping". Ibid. Vol. 3.2.1, 1978.

50. Rousseau, J-M. and Gibbs, R.J. "A model to assist planning the provision of hospital services". IIASA Collaborative papers, Ref. CP-80-3, A-2361, 1980.

51. Rousseau, J-M. and Gibbs, R.J. "A model to assist planning the provision of hospital services". Jnl. Opl. Res. Soc. Vol. 32, 1981.

52. Said, K.E. "A policy-selection/goal formulation model for public systems". Policy Sciences Vol. 5, 1974.

53. Sharp, K "Management budgeting". Hospital and Health Services Review, September 1984.

54. Starr, M. and Zeleny, M. (eds) "Multiple criteria decision making" TIMS Studies in the Management Sciences, Vol. 6, North-Holland 1977.

55. Starr M.K. and Zeleny M. "MCDM - state and future of the arts" Ibid. 1977.

56. Torrance G.W., Boyle, M.H. and Horwood, S.P. "Application of multi-attribute utility theory to measure social preferences for health states". Operations Research, Vol. 30, No. 6, 1982.

57. Wheatley, M. "Census design and analysis for social services planning and resource allocation". Vol. 3.2.2 of D.U.B.S. "Design for a social services planning system" ed. E. Nelson 1978.

58. Wiltshire Area Health Authority and Wiltshire County Council "Balance of Care project for the elderly - area survey report". 1982.

59. Wiltshire County Council. "Long term planning for the Wiltshire health area - report on the Balance of Care pilot project". 1981.

60. Wiper, L.E. "Model for a resource allocation decision aid". Vol. 3.1, No. 1 of D.U.B.S. "Design for a social services planning system", ed. E. Nelson, 1978.

61. Wishart, D. "CLUSTAN user manual". Edinburgh University, 3rd edition, 1978.

62. Zeleny, M. (ed) "Multiple Criteria Decision Making". Springer-Verlag 1975.

APPENDICES

APPENDIX 1 : CLIENT GROUPS

ADULT PHYSICALLY HANDICAPPED CLIENTS BELOW 65 YEARS OF AGE

(including impaired hearing, blind and partially sighted clients).

GROUP	CODE
Completely dependent adult clients	01
Adult clients with at least three physical handicaps	02
Blind or partially sighted adult clients	03
Adult clients with impaired hearing	04
Adult clients unable to bathe and/or use the toilet unaided	05
Adult clients unable to clean their house and/or go out of their house unaided	06
Adult clients unable to cook adequately unaided	07
Other physically handicapped adult clients. Social workers specify their problem(s) etc., on the census form	08

CLIENTS ABOVE 65 YEARS OF AGE ('ELDERLY CLIENTS')

(including impaired hearing, blind and partially sighted clients).

GROUP	CODE
Completely dependent elderly clients including psychogeriatric clients	09
Elderly clients with at least three physical handicaps	10
Elderly blind or partially sighted clients	11
Elderly clients with impaired hearing	12
Elderly clients unable to bathe and/or use the toilet unaided	13
Elderly clients unable to clean their house and/or go out of their house unaided	14
Elderly clients unable to cook adequately unaided	15
Frail elderly clients	16
Other elderly clients. Social workers specify their problem(s) etc., on the census form	17

APPENDIX 1 : (continued) - CLIENT GROUPS

MENTAL HEALTH ADULT CLIENTS

GROUP	CODE
Mentally ill adult clients	18
Severely mentally ill adult clients	19
Mentally sub-normal adult clients	20
Severely mentally sub-normal adult clients	21

ADULT CLIENTS WITH RELATIONSHIP, ENVIRONMENTAL AND FINANCIAL PROBLEMS ('FAMILY CASEWORK')

GROUP	CODE
Adult clients violent within relationships with other adult(s)	22
Adult clients with a non-violent but inadequate relationship with partner, other adults or children	23
Adult victims of violence within relationships with other adult(s)	24
Adult persons hindering, neglecting or impairing the development and/or health of their children (excluding persons non-accidentally injuring their children or otherwise abusing their children)	25
Adult persons non-accidentally injuring or otherwise abusing children	26
Homeless adults	27
Adult clients unable to pay bills etc.	28
Adult clients with multiple relationship, environmental and financial problems. Social workers specify their problems on the census form	29
Other adult clients with relationship, environmental and financial problems. Social workers specify their problem(s) etc. on the census form	30

OTHER ADULT CLIENTS

GROUP	CODE
Prospective and approved foster parents or persons proposing to adopt children	31
Prospective playgroup leaders and childminders	32

OTHER ADULT CLIENTS (Continued)

GROUP	CODE
Ante and post natal unmarried mothers and women requesting abortions	33
Other adult clients. Social workers specify their problem(s) etc., on the census form	34

CHILDREN AND YOUNG PERSONS

GROUP	CODE
Physically handicapped children and young persons	35
Educationally handicapped children and young persons	36
Mentally handicapped children and young persons	37
Severely mentally handicapped children and young persons	38
Emotionally disturbed or emotionally impaired children and young persons	39
Abused children and young persons NAI	40
Children or young persons whose development and/or health is being or would be avoidably prevented or neglected in their home situation other than children assigned to groups 35, 37, 38 or 40.	41
Children or young persons who are beyond, or would be beyond parental control or who irregularly attend school other than children assigned to groups 35, 37, 38 or 40	42
Children or young persons under Local Authority Supervision or Care for reasons other than listed in 35, 37, 38, 40, 41, or 42. Social workers specify these reasons on the census form	43
Children and young persons suffering parental or environmental inadequacy insufficient to justify the terms of groups 40, 41, 42 or 43.	44
Other children. Social Workers specify their problem(s) etc., on the census form	45

APPENDIX 2 : SERVICE GROUPS

1. FIELDWORK SERVICES

	CODE
Preliminary assessment/investigation by fieldstaff	01
Immediate short-term crisis intervention by fieldwork staff to protect the client	02
Exploratory casework by fieldstaff for further assessment	03
Supportive role for the longer term protection of the client	04
Active involvement of fieldstaff with the client with a positive end in view	05
A watching brief to ensure that no crisis develops or recurs	06

2. VOLUNTARY SERVICES

Visiting by directly recruited volunteers	07
Visiting by members of private Voluntary agencies	08

3. OTHER SERVICES

Hairdressing	09
Laundry Service for the incontinent	10

4. SERVICES FOR ELDERLY AND/OR PHYSICALLY HANDICAPPED CLIENTS

EXISTING SERVICES

Residential

Homes for the elderly - long stay	11
Homes for the elderly - short stay	12
Homes for physically handicapped	13

Day Care

Day Centres	14
Day care within homes for the elderly	15

APPENDIX 2 : (Continued) - SERVICE GROUPS

SERVICES FOR ELDERLY AND/OR PHYSICALLY HANDICAPPED CLIENTS (Continued)

EXISTING SERVICES

	CODE
Domiciliary	
Home Help	16
Meals on Wheels	17
Adaptations, Structural Alterations	18
Aids	19
Telephones - Rentals	20
Telephones - Installations	21
Domiciliary Health Services e.g. Health Visitor, Chiropody	22
Specialist Fieldwork Staff Support	
Occupational Therapy	23
Craft Instructors	24
Technical Officer Services - Blind	25
Technical Officer Services - Deaf	26
Hospital based Social Work	27
ADDITIONAL SERVICES	
Group living schemes for the elderly	28
Mobile Day Care Unit for the elderly	29
Night Sitting Schemes for elderly/handicapped	30
Homes for Psychogeriatrics	31
Assessment Centres for the elderly	32
Support Groups for relatives of physically handicapped clients	33

APPENDIX 2 : (Continued) - SERVICE GROUPS

SERVICES FOR MENTALLY HANDICAPPED AND/OR MENTALLY ILL CLIENTS (Continued)

ADDITIONAL SERVICES

	CODE
Homes for Mentally Ill	46
Special Care Units for M.H. attached to Adult Training Centres	47
Special Care Units for M.I.	48
Day Centres - Mentally Handicapped	49
Day Centres - Mentally Ill	50
Occupational Therapy within Homes for M.H.	51
Support Groups for M.H. clients' relatives	
Pre-School Playgroups for Mentally Handicapped Children	53
Holiday Playschemes for Mentally Handicapped Children	54
Intermittant Short-term Fostering for Mentally Handicapped Children (to provide relief for caring parents)	55

6. SERVICES FOR CLIENTS WITH RELATIONSHIP, ENVIRONMENTAL AND FINANCIAL PROBLEMS (FAMILY AND INDIVIDUAL)

EXISTING SERVICES

	CODE
Convalescent Home	56
Family Training Unit	57
Family Advice Centre	58
Family Service Unit	59
Hospital Based Social Work	60
1963 Preventive and Supportive Family Services (money and/or material aid)	61
Home Helps	16

APPENDIX 2 : (Continued) - SERVICE GROUPS

SERVICES FOR ELDERLY AND/OR PHYSICALLY HANDICAPPED CLIENTS (Continued)

ADDITIONAL SERVICES

	CODE
Fostering of elderly clients	34
Weekday Residential Care only for elderly/handicapped	35
Para-medic Half Way Care Units between Part III and Geriatric Care	36

5. SERVICES FOR MENTALLY HANDICAPPED AND/OR MENTALLY ILL CLIENTS

EXISTING SERVICES

	CODE
Residential	
Homes for Mentally Handicapped Adults	37
Homes for Mentally Handicapped Children	38
Group Living Schemes - M.H.	39
Group Living Schemes - M.I.	40
Day Care	
Adult Training Centres	41
Home Help	16
Meals on Wheels	17
Domiciliary Health Services e.g. Health Visitor, Chiropody.	22
Hospital In-Patient Treatment for M.I.	42
Hospital Out-Patient Treatment for M.I.	43
Psychiatric Clubs	44
Specialist Fieldwork Staff Support	
Hospital based Social Work	45

APPENDIX 2 : (Continued) - SERVICE GROUPS

SERVICES FOR CLIENTS WITH RELATIONSHIP, ENVIRONMENTAL AND FINANCIAL PROBLEMS (FAMILY AND INDIVIDUAL)

ADDITIONAL SERVICES

	CODE
Welfare Rights Service	62
Hostel for Battered Wives	63
Mother and Baby Home	64
Treatment Units for Alcoholics	65
Abortion Counselling Service	66
Peripatetic Home Helps for Domiciliary Family Training	67
Support Groups for Single Parent Families	68

7. SERVICES FOR CHILDREN AND YOUNG PERSONS

EXISTING SERVICES

	CODE
Residential	
Statutory care	69
Voluntary care	70
Statutory support	71
Voluntary support	72
Reception/Observation and Assessment Centre	73
Family Group and Community Homes	74
Community Homes with Education on premises	75
Hostel for Children over School Age	76
Residential Nursery	77
Day Care	
Day Nursery	78
Pre-School Playgroups	79
Childminders	80
Salaried Childminding Scheme	81

APPENDIX 2 : (Continued) - SERVICE GROUPS

SERVICES FOR CHILDREN AND YOUNG PERSONS (Continued)

EXISTING SERVICES

	CODE
Specialist Fieldwork Staff Support	
School Based Social Work	82
Hospital Based Social Work	27
Other Services	
Foster Homes	83
Assessment/Recruitment of Foster Parents	84
Professional Foster Parent Scheme	85
Intermediate Treatment - Residential	86
Intermediate Treatment - other	87
Adoption	88
Child Guidance	89
Special Residential Schooling	90
ADDITIONAL SERVICES	
Secure Accommodation in C.H.E's	91
Intermediate Treatement Centre	92
Domiciliary Observation and Assessment	93
Flats for Adolescents	94
Programmes for Children Leaving Care	95
Short Term Residential I.T. Projects	96
Residential Treatment Units for Disturbed Adolescents	97
Peripatetic Houseparents	98
Mobile Day Nursery	99

APPENDIX 3 : CLIENT GROUPS (Revised to 35 groups)

ADULT PHYSICALLY HANDICAPPED CLIENTS BELOW 65 YEARS OF AGE

(including impaired hearing, blind and partially sighted clients)

GROUP	CODE
Completely dependent adult clients	01
Adult clients with at least three physical handicaps	02
Blind or partially sighted adult clients	03
Adult clients with impaired hearing	04
Adult clients unable to bathe and/or use the toilet unaided	05
Adult clients unable to clean their house and/or go out of their house unaided OR Adult clients unable to cook adequately unaided	06
Other physically handicapped adult clients. Social workers specify their problem(s) etc., on the census form	08

CLIENTS ABOVE 65 YEARS OF AGE ('ELDERLY CLIENTS')

(including impaired hearing, blind and partially sighted clients).

GROUP	CODE
Completely dependent elderly clients including psychogeriatric clients	09
Elderly clients with at least three physical handicaps	10
Elderly blind or partially sighted clients	11
Elderly clients with impaired hearing	12
Elderly clients unable to clean their house and/or go out of their house unaided	14
Frail elderly clients, clients unable to bathe and/or use the toilet unaided, and clients unable to cook adequately unaided	16
Other elderly clients. Social workers specify their problem(s) etc., on the census form	17

APPENDIX 3 : (Continued) - CLIENT GROUPS (Revised to 35 groups)

OTHER ADULT CLIENTS (Continued)

GROUP	CODE
Other adult clients. Clients include prospective playgroup leaders and childminders and ante- and post-natal unmarried mothers and women requesting abortions	34

CHILDREN AND YOUNG PERSONS

GROUP	CODE
Physically handicapped children and young persons	35
Educationally handicapped children and young persons	36
Mentally handicapped children and young persons	37
Severely mentally handicapped children and young persons	38
Emotionally distrubed or emotionally impaired children and young persons	39
Abused children and young persons NAI	40
Children or young persons whose development and/or health is being or would be avoidably prevented or neglected in their home situation other than children assigned to groups 35, 37, 38 or 40.	41
Children or young persons who are beyond, or would be beyond parental control or who irregularly attend school other than children assigned to groups 35, 37, 38 or 40	42
Children or young persons under Local Authority Supervision or Care for reasons other than listed in 35, 37, 38, 40, 41, or 42. Social workers specify these reasons on the census form	43
Children and young persons suffering parental or environmental inadequacy insufficient to justify the terms of groups 40, 41, 42 or 43.	44
Other children. Social Workers specify their problem(s) etc., on the census form.	45

APPENDIX 3 : (continued) - CLIENT GROUPS (Revised to 35 groups)

MENTAL HEALTH ADULT CLIENTS

GROUP	CODE
Mentally ill adult clients	
OR	
Severely mentally ill adult clients	18
Mentally sub-normal adult clients	20
Severely mentally sub-normal adult clients	21

ADULT CLIENTS WITH RELATIONSHIP, ENVIRONMENTAL AND FINANCIAL PROBLEMS ('FAMILY CASEWORK')

GROUP	CODE
Adult clients violent within relationships with other adult(s)	
OR	
Adult clients with a non-violent but inadequate relationship with partner, other adults or children	23
Adult persons hindering, neglecting or impairing the development and/or health of their children (excluding persons non-accidentally injuring their children or otherwise abusing their children)	25
Adult clients unable to pay bills etc.	28
Adult clients with multiple relationship, environmental and financial problems. Clients include adult victims of violence within relationships with other adult(s).	29
Other adult clients with relationship, environmental and financial problems. Clients include homeless adults and adult persons non-accidentally injuring or otherwise abusing children.	30

OTHER ADULT CLIENTS

GROUP	CODE
Prospective and approved foster parents or persons proposing to adopt children	31

APPENDIX 4 : SERVICE GROUPS (Revised to 45 codes)

1. FIELDWORK SERVICES

	CODE
Preliminary assessment/investigation by fieldstaff	01
Immediate short-term crisis intervention by fieldwork staff to protect the client	02
Exploratory casework by fieldstaff for further assessment	03
Supportive role for the longer term protection of the client	04
Active involvement of fieldstaff with the client with a positive end in view	05
A watching brief to ensure that no crisis develops or recurs	06

2. VOLUNTARY SERVICES

Visiting by directly recruited volunteers	07
Visiting by members of private Voluntary agencies	08

3. SERVICES FOR ELDERLY AND/OR HANDICAPPED CLIENTS

Residential

Homes for the elderly, long stay; for physically and mentally handicapped adults; group living for M.H.	11
Homes for the elderly, short stay; for mentally ill; group living for M.I.	12
Homes for mentally handicapped children	38

Day Care

Day Centres	14
Day care within homes for the elderly	15

Domiciliary

Home help, hairdressing and laundry	16
Meals on wheels	17
Adaptations, structural alterations	18
Aids	19

SERVICES FOR ELDERLY AND/OR HANDICAPPED CLIENTS (Continued)

Domiciliary	CODE
Telephones - rentals	20
Telephones - installations	21
Domiciliary health services, e.g. health visitor, chiropody	22
Specialist Fieldwork Staff Support	
Occupational Therapy	23
Craft Instructors	24
Technical Officer Services - Blind	25
Technical Officer Services - Deaf	26
Hospital based social work, for adults, children and young persons	27
Day care for mentally handicapped/ill	
Hospital in-patient treatment for M.I.	42
Hospital out-patient treatment for M.I.	43
Psychiatric clubs	44
Additional Services	
Additional services for elderly/handicapped	28
Additional services for mentally handicapped/ill	49

4. SERVICES FOR CLIENTS WITH RELATIONSHIP, ENVIRONMENTAL AND FINANCIAL PROBLEMS (FAMILY AND INDIVIDUAL)

Existing services, except home helps	56
Additional services	62

APPENDIX 4 : SERVICE GROUPS (Revised to 45 codes) (Continued)

5. SERVICES FOR CHILDREN AND YOUNG PERSONS

	CODE
Residential	
Voluntary care	70
Statutory care and support	71
Voluntary support	72
Family group and community homes	74
Community homes with education on premises	75
Hostel for children over school age	76
Nurseries, playgroups and childminding	79
Other Services	
Foster homes and foster parent scheme	83
Assessment/recruitment of foster parents	84
Adoption	88
Special residential schooling	90
Secure accommodation in C.H.E's	91
Reception/observation and assessment centre, intermediate treatment, child guidance and additional services	73

APPENDIX 5 Complete analysis of numbers of clients in each of 35 groups receiving each of 45 services

(i) Client groups 1 to 6

Client group / Service	1	2	3	4	5	6
Size	83	169	291	101	39	111
1	14	25	15	0	20	21
2	3	3	17	1	0	5
3	2	3	5	0	8	7
4	20	29	36	73	1	19
5	8	6	9	4	2	9
6	15	33	113	16	3	30
7	3	3	0	0	0	4
8	0	2	11	0	0	0
11	4	2	5	1	1	2
12	1	1	2	1	1	3
14	6	7	29	4	0	6
15	7	27	46	1	2	4
16	0	0	2	0	0	4
17	17	37	62	6	2	42
18	4	8	29	3	1	15
19	23	68	17	0	21	16
20	46	101	78	49	28	51
21	9	13	7	0	0	10
22	4	12	7	1	0	4
23	22	39	19	0	2	21
24	10	55	17	0	10	3
25	6	28	34	2	1	14
26	3	21	242	1	0	2

Client group / Service	1	2	3	4	5	6
27	1	6	1	90	1	1
28	13	18	9	1	5	21
38	1	0	1	0	0	0
42	1	0	1	0	0	0
43	0	2	1	0	0	0
44	1	0	1	1	0	0
49	0	0	1	0	0	0
56	0	0	1	0	0	0
62	1	1	1	2	0	1
70	0	0	0	0	0	0
71	0	0	0	0	0	0
72	0	0	1	0	1	0
73	0	0	0	0	0	1
74	0	0	0	0	0	0
75	0	0	0	0	0	0
76	0	0	0	0	0	0
79	0	0	0	0	0	0
83	0	0	0	0	0	0
84	0	0	0	0	0	0
88	0	0	1	0	0	1
90	0	0	0	0	0	0
91	0	0	2	0	0	0
Total "Service receipts"	245	550	823	257	110	317

APPENDIX 5 Complete analysis of numbers of clients in each of 35 groups receiving each of 45 services

(ii) Client groups 8 to 12 and 14

Client group / Service	8	9	10	11	12	14
Size	216	226	190	301	64	125
1	67	11	24	10	1	23
2	16	6	9	49	1	21
3	3	10	4	27	1	3
4	62	46	51	46	25	11
5	14	8	9	9	5	7
6	47	29	20	147	25	23
7	5	3	2	3	1	3
8	3	1	5	3	0	1
11	4	42	4	23	4	6
12	5	7	9	5	1	4
14	8	53	16	29	15	8
15	24	7	17	18	10	9
16	2	10	3	5	2	3
17	38	52	88	97	23	78
18	9	69	35	56	12	35
19	36	20	64	4	2	8
20	72	49	106	110	26	42
21	16	5	18	5	0	13
22	20	4	16	5	0	6
23	22	23	31	28	11	41
24	14	24	50	5	2	6
25	11	54	7	13	0	4
26	0	3	2	194	1	3

Client group / Service	8	9	10	11	12	14
27	0	0	0	4	47	0
28	16	96	27	5	4	38
38	0	1	0	0	0	0
42	1	0	0	0	0	0
43	0	5	1	2	0	1
44	0	1	0	0	0	1
49	0	0	0	0	0	0
56	0	0	0	1	0	1
62	2	1	0	0	0	1
70	0	0	0	0	0	0
71	0	0	0	0	0	0
72	0	0	0	0	0	0
73	2	0	0	0	0	1
74	0	0	0	0	0	0
75	0	0	0	0	0	0
76	0	0	0	0	0	0
79	1	0	0	0	0	0
83	0	0	0	0	0	0
84	0	0	0	0	0	0
88	0	0	0	0	0	0
90	0	0	0	0	0	0
91	0	0	0	0	0	0
Total	520		618		219	
"Service receipts"		640		903		401

APPENDIX 5 Complete analysis of numbers of clients in each of 35 groups receiving each of 45 services

(iii) Client groups 16, 17, 18, 20, 21, 23

Client group / Service	16	17	18	20	21	23
Size	424	250	572	360	49	302
1	48	89	56	4	0	29
2	18	15	33	3	2	11
3	7	5	32	7	1	24
4	79	59	162	125	19	103
5	26	31	57	24	2	88
6	142	52	188	190	15	116
7	7	3	3	1	0	0
8	11	7	2	0	0	2
11	16	0	1	14	2	0
12	24	5	5	5	4	0
14	47	17	10	2	0	0
15	21	11	8	1	0	1
16	11	4	6	2	0	0
17	232	70	30	6	2	1
18	128	39	6	2	1	0
19	38	7	2	1	4	0
20	107	28	6	2	6	0
21	19	5	4	0	1	0
22	9	5	2	0	1	0
23	84	26	42	5	1	2
24	31	9	3	0	1	1
25	4	4	14	1	0	0
26	2	0	0	1	0	0

Client group / Service	16	17	18	20	21	23
27	1	2	24	5	1	1
28	90	26	191	50	8	14
38	0	0	0	82	16	0
42	0	0	4	178	19	0
43	0	2	111	13	9	0
44	1	1	217	22	2	6
49	0	0	24	2	1	0
56	0	2	2	2	0	6
62	0	0	14	5	0	22
70	1	0	0	3	1	0
71	0	0	0	5	0	0
72	0	0	1	2	0	7
73	0	1	17	19	0	19
74	0	0	0	1	0	0
75	0	0	0	1	0	0
76	0	0	0	0	0	0
79	0	0	0	1	1	1
83	0	1	2	3	0	1
84	0	0	0	1	0	0
88	0	0	0	0	0	0
90	0	0	0	0	0	2
91	1	0	0	0	0	0
Total "Service receipts"	1205	526	1279	791	120	457

APPENDIX 5 Complete analysis of numbers of clients in each of 35 groups receiving each of 45 services

(iv) Client groups 25, 28 - 31, 34

Client group / Service	25	28	29	30	31	34
Size	86	157	274	363	300	354
1	3	48	20	51	50	64
2	3	28	18	17	0	5
3	8	3	10	9	16	15
4	30	31	138	157	71	75
5	34	37	82	80	127	51
6	51	46	103	121	70	114
7	1	0	5	2	0	2
8	3	3	3	8	4	4
11	0	0	0	1	0	0
12	0	0	0	0	0	0
14	1	1	0	1	0	0
15	0	0	0	0	0	0
16	0	0	0	0	0	0
17	1	0	6	5	1	3
18	0	0	0	0	0	0
19	0	0	0	0	0	0
20	0	0	0	1	0	1
21	0	0	1	0	0	0
22	0	0	0	0	0	0
23	6	7	15	29	7	12
24	0	0	0	0	0	0
25	0	0	0	1	0	0
26	1	1	1	0	0	0

Client group / Service	25	28	29	30	31	34
27	0	0	0	0	0	0
28	0	4	4	11	1	68
38	0	0	0	2	0	0
42	0	0	0	0	0	0
43	0	0	1	0	0	0
44	1	0	6	9	0	1
49	0	0	0	0	0	0
56	3	7	8	14	0	7
62	10	23	72	59	2	22
70	0	0	0	0	9	0
71	0	0	2	7	0	3
72	0	4	6	1	43	13
73	9	6	36	56	16	30
74	0	0	0	0	0	0
75	0	0	0	0	0	0
76	0	0	0	0	0	0
79	0	0	1	2	0	3
83	0	0	24	2	5	12
84	0	0	0	0	12	0
88	0	0	0	0	0	0
90	0	0	1	2	6	0
91	0	0	0	1	0	0
Total	165		563		440	
"Service receipts"		249		649		505

APPENDIX 5 Complete analysis of numbers of clients in each of 35 groups receiving each of 45 services

(v) Client groups 35 - 40

Client group	35	36	37	38	39	40	Client group	35	36	37	38	39	40
Size	52	150	86	92	169	139							
Service							Service						
1	15	4	5	1	6	3	27	2	0	0	1	0	0
2	0	0	1	3	5	5	28	8	2	13	54	21	1
3	5	8	1	1	16	5	38	0	1	3	0	0	0
4	13	119	29	12	85	80	42	1	0	7	3	0	0
5	3	54	12	5	72	39	43	0	0	8	46	13	0
6	14	26	39	15	40	46	44	0	1	6	10	2	0
7	0	0	1	0	0	1	49	0	0	1	5	0	1
8	1	0	1	3	18	1	56	0	1	0	0	2	0
11	0	0	1	0	0	0	62	1	4	3	0	4	19
12	0	1	18	6	3	0	70	1	6	4	5	16	43
14	1	0	0	0	0	0	71	1	5	5	2	7	4
15	1	0	0	0	0	0	72	3	10	3	3	6	15
16	0	0	0	0	0	0	73	3	66	9	6	37	39
17	0	0	0	1	0	2	74	0	1	1	0	4	2
18	0	0	0	1	0	0	75	0	1	0	2	11	11
19	16	0	3	4	0	0	76	0	2	0	0	2	0
20	22	0	5	3	0	0	79	3	3	1	2	3	17
21	0	0	1	0	0	0	83	1	111	2	0	46	4
22	2	0	1	0	0	0	84	1	3	4	4	4	23
23	6	0	3	5	1	22	88	1	0	1	1	3	0
24	12	0	1	1	0	0	90	2	5	3	1	30	6
25	0	0	0	0	0	0	91	9	63	9	2	32	2
26	2	0	0	1	0	0	Total	150		205		489	
							"Service receipts"		497		209		391

APPENDIX 5 Complete analysis of numbers of clients in each of 35 groups receiving each of 45 services

(vi) Client groups 41-45 and service totals for all clients

Client groups / Size / Service	41	42	43	44	45	Totals for all clients	Group / Service	41	42	43	44	45	Totals for all clients
Size	282	320	645	536	247	8125							
1	10	31	9	60	50	887	27	0	0	0	1	3	192
2	3	10	5	19	4	339	28	1	1	1	6	7	835
3	6	33	28	21	7	341	38	0	0	0	0	0	107
4	135	109	376	243	80	2749	42	0	0	1	0	0	216
5	76	130	180	74	29	1403	43	0	0	0	0	0	215
6	60	93	177	247	96	2562	44	1	1	3	0	0	295
7	0	0	1	1	0	55	49	0	0	0	0	0	35
8	7	1	7	5	2	119	56	3	0	1	24	0	85
11	0	0	0	0	0	133	62	13	14	15	41	22	375
12	0	0	0	1	0	112	70	129	110	212	18	10	568
14	0	0	0	0	0	261	71	35	25	66	24	11	202
15	0	0	1	0	0	216	72	32	57	304	31	17	560
16	0	0	0	1	0	55	73	13	39	18	162	65	670
17	1	0	0	2	2	907	74	5	19	12	0	1	46
18	0	0	0	0	0	453	75	61	31	50	5	2	175
19	0	0	1	0	0	355	76	5	35	49	1	0	94
20	0	0	1	1	3	944	79	20	5	12	51	3	130
21	0	0	0	0	0	127	83	25	26	17	31	2	315
22	0	0	0	0	0	99	84	100	8	110	18	12	300
23	13	0	7	41	13	606	88	1	24	34	9	0	76
24	0	0	0	0	0	255	90	6	15	21	15	3	118
25	0	0	0	0	0	198	91	22	6	8	3	4	164
26	0	0	0	0	0	481	Total	783		1727		448	
							"Service receipts"		823		1156		19430

N.B. The discrepancy of 18 "Service receipts" between the total here and that in Figure 5.2.4 is due to duplications in services, caused by the merging of services from 99 to 45 codes.

APPENDIX 6 Cluster Analysis methods using CLUSTAN package

Four clustering methods are described in turn:

(i) Ward's hierarchical method

(ii) relocation method using Ward's classification

(iii) relocation method using a random start

(iv) density method

(i) Ward's hierarchical method

The method starts with N clusters, each of which is a single individual (i.e. receiving a package of care), numbered according to the input order. A similarity matrix is produced by routine CORREL which calculates similarity coefficients S(i,k) for all possible pairs of clusters i and k as follows:

$$S(i,k) = \frac{B + C}{A+B+C+D}$$

where A = No. of attributes (or services) common to both clusters i and k,

B = No. of attributes (or services) present in i, absent from k,

C = No. of attributes (or services) absent from i, present in k,

D = No. of attributes (or services) absent from both

and A+B+C+D = total No. of attributes.

Then, in each of (N-1) fusion cycles, the two clusters which are most similar (i.e. whose similarity coefficient is smallest) are fused, the resultant union cluster being labelled with the lesser of the two codes of its constituent clusters. The fusion hierarchy is produced by means of a combinatorial transformation of the similarity coefficients from the similarity matrix, as follows:

When P and Q are the clusters to be fused, then the similarity S(R,P+Q) between any cluster R and the new cluster (P+Q) is given by

$$S(R,P+Q) = \frac{(NR + NP)}{(NR+NP+NQ)} \cdot S(R,P) + \frac{(NR + NQ)}{(NR+NP+NQ)} \cdot S(R,Q) - \frac{NR.}{(NR+NP+NQ)} \cdot S(P,Q)$$

where NR,NP,NQ are cluster sizes of R,P and Q respectively. The method completes all (N-1) fusions and the user may select which of these are to be printed, i.e. how many clusters are to be constructed.

(ii) Relocation method using Ward's classification

The method starts with a classification of the population of N individuals (i.e. receiving packages of care) into k clusters and then considers each individual in turn, computing its similarities with all k clusters. If the similarity between an individual X and its parent cluster P is less than the similarity between X and another cluster Q then X is moved from cluster P to cluster Q. The population is repeatedly scanned until no objects are relocated during one full scan, when a local optimum solution for k clusters in terms of the similarity function S will have been obtained. (S is calculated as in routine CORREL, described in (i) above). This local optimum may not be the global optimum, but if the same result is achieved from several different starting classifications then the probability is high that the global solution has been obtained.

Next, similarities between all pairs of clusters are computed and those two clusters which are most similar are fused, giving (k-1) clusters. Again a local optimum is found for these (k-1) clusters, then fusion to (k-2) clusters, etc.

When Ward's classification is used as the starting point, the computer package requires the HIERARCHY procedure to be completed before the RELOCATE procedure is used.

(iii) Relocate method using a random start

The method is exactly the same as described in (ii) above exept that instead of requiring the procedure HIERARCHY to precede RELOCATE, the RELOCATE procedure itself collects k clusters at random from the initial population of N individuals (i.e. packages of care).

(iv) Density method

This method also uses the similarity matrix produced by routine CORREL, i.e. similarity coefficients (called "distance coefficients") for all possible pairs of clusters. CORREL then computes k-linkage lists, i.e. lists of the nearest neighbours for all N individuals. The DENSITY procedure selects monotonic increasing radii R to define spherical neighbourhoods of points which, for a fixed value of k, have monotonic increasing density. Any cluster that is isolated is taken to represent the nucleus of a mode, and such groupings of points are permitted to grow through the addition of new points as the density threshold decreases. The density estimate is the average of the (2k-1) least distances. This assumes that the true density in the local neighbourhood of the point is either changing linearly or uniformly; at a mode the estimate is too large, while at a saddle it is too small. Hence the effect is to level off the modes and saddles of the distribution thus providing a smooth range of estimates for increasing values of k. The resulting density estimates are printed in decreasing order.

APPENDIX 7. Example of use of Ward's hierarchical method of cluster analysis

See Appendix 6 (i) for a brief description of Ward's hierarchical method.

The example used contains five clients, each receiving a different package of care made up of a selection from five services, codes 15,16,17,20 and 22.

Thus, client 1	receives	services	15,16,17,20;	i.e. initially 5 clusters
client 2	"	"	15,17,22;	
client 3	"	"	15,16;	
client 4	"	"	16,22;	
client 5	"	"	17,20	

Step 1

Similarity coefficients for all possible pairs of clusters are calculated:

$S(1,2) = \frac{2+1}{5} = \frac{3}{5}$ $S(2,3) = \frac{2+1}{5} = \frac{3}{5}$ $S(3,4) = \frac{1+1}{5} = \frac{2}{5}$

$S(1,3) = \frac{2+0}{5} = \frac{2}{5}$ $S(2,4) = \frac{2+1}{5} = \frac{3}{5}$ $S(3,5) = \frac{2+2}{5} = \frac{4}{5}$

$S(1,4) = \frac{3+1}{5} = \frac{4}{5}$ $S(2,5) = \frac{2+1}{5} = \frac{3}{5}$ $S(4,5) = \frac{2+2}{5} = \frac{4}{5}$

$S(1,5) = \frac{2+0}{5} = \frac{2}{5}$

The smallest similarity coefficient is $\frac{2}{5}$, represented by S(1,3) or S(1,5) or S(3,4).

Taking the first of these, fuse clusters 1 and 3.

The new cluster will contain services	15	16	17	20
in percentages	100	100	50	50

which shows that all clients in the original clusters 1 and 3 received services 15 and 16 but only half received services 17 and 20.

APPENDIX 7 (Continued)

Step 2

New similarity coefficients are calculated:

$$S(2,1+3) = \frac{2}{3}.\ S(2,1) + \frac{2}{3}.\ S(2,3) - \frac{1}{3}.\ S(1,3)$$

$$= \frac{2}{3}.\frac{3}{5} + \frac{2}{3}.\frac{3}{5} - \frac{1}{3}.\frac{2}{5}$$

$$= \frac{2}{3}$$

$$S(4,1+3) = \frac{2}{3}.\ S(4,1) + \frac{2}{3}.\ S(4,3) - \frac{1}{3}.\ S(1,3)$$

$$= \frac{2}{3}.\frac{4}{5} + \frac{2}{3}.\frac{2}{5} - \frac{1}{3}.\frac{2}{5}$$

$$= \frac{2}{3}$$

$$S(5,1+3) = \frac{2}{3}.\ S(5,1) + \frac{2}{3}.\ S(5,3) - \frac{1}{3}.\ S(1,3)$$

$$= \frac{2}{3}.\frac{2}{5} + \frac{2}{3}.\frac{4}{5} - \frac{1}{3}.\frac{2}{5}$$

$$= \frac{2}{3}$$

The smallest similarity coefficient is now $\frac{2}{3}$, represented by S(2,4) or S(2,5) or S(2,1 + 3) or S(4,1 + 3) or S(5,1 +3).

Taking the first of these, fuse clusters 2 and 4.

The new cluster will contain services	15	16	17	22
in percentages	50	50	50	100

which shows that all clients in the original clusters 2 and 4 received service 22 but only half received services 15, 16 and 17.

APPENDIX 7 (Continued)

Step 3

New similarity coefficients are calculated:

$$S(1+3,\ 2+4) = \frac{3}{4}.\ S(1+3,2) + \frac{3}{4}.\ S(1+3,4) - \frac{2}{4}.\ S(2,4)$$

$$= \frac{3}{4}.\frac{2}{3} + \frac{3}{4}.\frac{2}{3} - \frac{2}{4}.\frac{3}{5}$$

$$= \frac{7}{10}$$

$$S(5,2+4) = \frac{2}{3}.\ S(5,2) + \frac{2}{3}.\ S(5,4) - \frac{1}{3}\ S(2,4)$$

$$= \frac{2}{3}.\frac{3}{5} + \frac{2}{3}.\frac{4}{5} - \frac{1}{3}.\frac{3}{5}$$

$$= \frac{11}{15}$$

The smallest similarity coefficient is now $\frac{2}{3}$, represented by S(5,1 + 3), so fuse clusters 5 and (1 + 3).

The new cluster will contain services	15	16	17	20
in percentages	67	67	67	67

which shows that two-thirds of the clients in the original clusters received each of these services.

Thus, the sequence of merging in this example is:

first clusters 1 and 3,

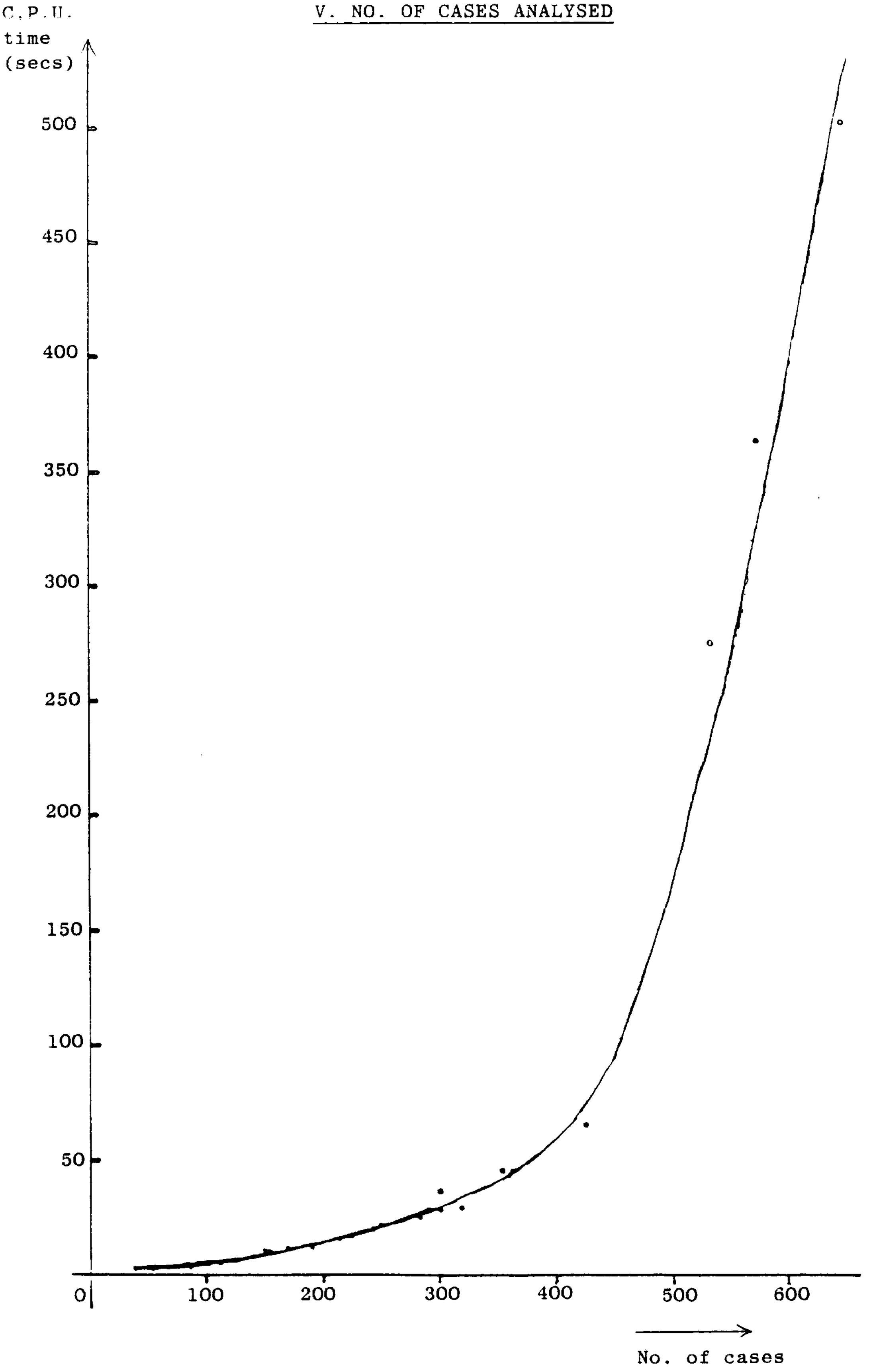
then clusters 2 and 4,

then 5 and 1 and 3 etc.

APPENDIX 8

CLUSTAN C.P.U. TIMES (SECONDS)

V. NO. OF CASES ANALYSED



APPENDIX 9 Sample KS1 : comparison of client group sizes with original Cendat data. Chi-squared test.

(i) Client group sizes

(a) In sample KS1, i.e. "Observed" values, Oi

(b) In original Cendat data, di

(c) "Expected" values, $Ei = di \times \frac{1015}{8125}$

Group i	Oi	di	Ei	Group i	Oi	di	Ei
1	9	83	10.37	25	11	86	10.74
2	15	169	21.11	28	18	157	19.61
3	38	291	36.35	29	35	274	34.23
4	11	101	12.62	30	41	363	45.35
5	6	39	4.87	31	40	300	37.48
6	18	111	13.87	34	50	354	44.22
8	26	216	26.98	35	5	52	6.50
9	24	226	28.23	36	20	150	18.74
10	21	190	23.74	37	12	86	10.74
11	37	301	37.60	38	9	92	11.49
12	10	64	8.00	39	23	169	21.11
14	13	125	15.62	40	17	139	17.36
16	57	424	52.97	41	43	282	35.23
17	35	250	31.23	42	45	320	39.98
18	81	572	71.46	43	78	645	80.58
20	41	360	44.97	44	63	536	66.96
21	10	49	6.12	45	21	247	30.86
23	32	302	37.73	Total	1015	8125	1015.02

(ii) Chi-squared test

Ho: Distribution of group sizes in sample KS1 is the same as distribution of group sizes in Cendat data.

H1: Distributions different.

Sample size = 35: 34 degrees of freedom

Choose significance level of 5%.

$\chi^2_{0.05,34}$ is between $\chi^2_{0.05,30} = 43.8$

and $\chi^2_{0.05,40} = 55.8$ (Reference 42: Murdoch & Barnes Statistical Tables)

Calculate $\chi^2 = \sum_i \frac{(O_i - E_i)^2}{Ei} = 19.98$

This is not in the critical region

∴ Accept Ho,

and Conclude that the distribution of group sizes in sample KS1 is the same as the distribution of group sizes in Cendat data.

APPENDIX 10 Analysis of Chester-le-Street district

Numbers of clients in each of 35 groups receiving

receiving each of 45 services

(i) Client groups 1 to 6

Client group / Service	1	2	3	4	5	6
Size	1	8	23	13	0	6
1	1	1	2			2
2			1	1		
3		1				
4		2	18	4		
5		1		4		1
6		2	2	3		1
7						
8						
11						
12						
14			3			
15		1	13			1
16						
17		2	6	2		1
18			2	1		1
19		3				1
20	1	4	14	5		3
21						
22		2				
23	1	3	4			
24		1				
25		1	10			1
26			21	1		

Client group / Service	1	2	3	4	5	6
27				13		
28						
38						
42						
43						
44			1			
49						
56						
62	1	1		2		
70						
71						
72						
73						1
74						
75						
76						
79						
83						
84						
88						1
90						
91						
Total	4		97		0	
"Service receipts"		25		36		14

APPENDIX 10 Analysis of Chester-le-Street district

Numbers of clients in each of 35 groups receiving each of 45 services

(ii) Client groups 8 to 12 and 14

Client group / Service	8	9	10	11	12	14	Client group / Service	8	9	10	11	12	14
Size	23	3	5	3	4	5							
1	18		1	1	1	2	27					2	
2			1				28		1			1	
3						1	38						
4	3			1	1	1	42						
5	1	1	2				43						
6	1	1	1	1	2	3	44						
7							49						
8							56						
11		1				1	62						
12							70						
14	1	1		1		1	71						
15	4						72						
16							73						
17	1	2	3	2		3	74						
18	1	1	2	2		3	75						
19	2						76						
20	4	1	2		2	1	79						
21	1						83						
22	1						84						
23	1	2	1		1		88						
24							90						
25	2						91						
26				1			Total	41		13		10	
							"Service receipts"		11		9		16

APPENDIX 10 **Analysis of Chester-le-Street district**

Numbers of clients in each of 35 groups receiving each of 45 services

(iii) Client groups 16,17,18,20,21,23

Client group / Service	16	17	18	20	21	23
Size	21	16	56	10	1	58
1	2	10	5			6
2		1	2			2
3	1		3			6
4	3		18	6		18
5	6	1	5		1	13
6	9	4	22	4		13
7	1					
8		1	1			
11	1					
12	2		3			
14	2	1	3			
15	1		3			1
16	1					
17	9	6	4			
18	8	3	2			
19			1			
20	9	2	2			
21			1		1	
22			1		1	
23	9	2	8			
24			1			1
25						
26						

Client group / Service	16	17	18	20	21	23
27	1		3			1
28			1			1
38				2		
42				2		
43			6			
44	1	1	27			2
49			2			
56						
62			1			6
70						
71						
72						2
73			1	4		1
74						
75						
76						
79						
83						
84						
88						
90						
91						
Total	66		126		3	
"Service receipts"		32		18		73

APPENDIX 10 Analysis of Chester-le-Street district

Numbers of clients in each of 35 groups receiving each of 45 services

(iv) Client groups 25,28-31,34

Client group / Service	25	28	29	30	31	34
Size	7	13	49	46	32	52
1		3	5	10	9	15
2	1	1	3	6		
3			6		4	1
4	2	2	25	13	13	6
5	2	7	8	7	6	5
6	2	1	9	10	1	25
7						
8						
11						
12						
14						
15						
16						
17				1		
18						
19						
20						
21			1			
22						
23	2		4	1		1
24						
25						
26						

Client Group / Service	25	28	29	30	31	34
27						
28						1
38						
42						
43						
44	1		1	2		
49						
56				1		1
62	2	1	16	11		4
70						
71						
72		2	2		2	
73			3	2		1
74						
75						
76						
79						3
83					1	
84					1	
88						
90						
91						
Total	12		83		37	
"Service receipts"		17		64		63

APPENDIX 10 Analysis of Chester-le-Street district

Numbers of clients in each of 35 groups receiving each of 45 services

(v) Client groups 35-40

Client group / Service	35	36	37	38	39	40
Size	4	6	5	4	19	10
1			1		1	
2						
3	1				9	
4	1	4	3	2	3	6
5	2	2			3	
6			1	2	6	4
7						
8	1					
11						
12						
14						
15	1					
16						
17						
18						
19				1		
20	1			1		
21						
22						
23				1	1	
24						
25						
26				1		

Client group / Service	35	36	37	38	39	40
27						
28				1	1	
38						
42				1		
43						
44			1		1	
49						
56						
62						
70					1	3
71	1	3	1	1	1	1
72	1	3				2
73		2	3		8	3
74					2	
75		1				
76						
79					1	2
83		1			1	
84		1	1	1		1
88			1	1		
90	1	1	1		3	
91	2	3			1	
Total "Service receipts"	12	21	13	13	43	22

APPENDIX 10 Analysis of Chester-le-Street district

Numbers of clients in each of 35 groups receiving each of 45 services

(vi) client groups 41-45 and Service Totals for all clients

Client Group / Size / Service	41	42	43	44	45	Totals for all clients	Group / Service	41	42	43	44	45	Totals for all clients
Size	8	27	81	65	46	730							
1	4	1		6	5	112	27					3	23
2		2				21	28						7
3				6		39	38						2
4	3	7	75	28	20	288	42						3
5		14		12	8	112	43						6
6	1	4	6	17	15	173	44						38
7						1	49						2
8						3	56						2
11						3	62			1			46
12				1		6	70	1	6	63	2	1	77
14						13	71	3	2	14	3		30
15						25	72	2	4	67	6	4	97
16						1	73	1	9	1	28	22	90
17						42	74		1	1			4
18						26	75	1	1	7	1		11
19						8	76		2	7			9
20					2	54	79			1	1	2	10
21						4	83			1			4
22						5	84	3		23	4		35
23				6	3	51	88		6	2	1		12
24						3	90			5	4	1	16
25						14	91			2		3	11
26						24	Total	19		276		89	
						"Service receipts"			59		126		1563

APPENDIX 11 Group 06 modes of care and weighting values

For each mode or package of care the number of clients $x_{il}^{(o)}$ in group 06 assigned by the cluster analysis is shown. In addition the original value of the weighting $w_{il}^{(1)} = \frac{x_{il}^{(o)}}{\sum_i \sum_l x_{il}^{(o)}}$ is given for each mode together with the maximum value of the weighting (resulting from the weights model) and the first selected perturbed weighting value.

For each mode of care the services selected by the cluster analysis of the original CENDAT data are indicated by the service code number and a description of the service. For each service selected, that proportion of the client group which receives the service is shown as a percentage of the total group.

APPENDIX 11 Group 06 modes of care and weighting values (continued)

MODE 1 12 CLIENTS Original w_1 = 0.108 Maximum = 0.430

Service	%	Description
1	100	Prelim assessment
20	41.7	Telephone rental
5	25.0	Active involvement of fieldstaff, +ve end in view
17	16.7	Meals on wheels
21	16.7	Telephone installation
23	8.3	Occupational therapy
15	8.3	Day care within homes for elderly

MODE 2 15 CLIENTS Original w_2 = 0.135 Maximum = 0.633

Service	%	Description
28	46.7	Group living scheme for elderly
20	26.7	Telephone rental
6	20	Watching brief
2	13.3	Short-term crisis intervention
12	13.3	Short stay home for elderly
3	6.7	Exploratory casework/further assessment
73	6.7	Receiption/observation & assessment centre
62	6.7	Welfare rights service
22	6.7	Domiciliary Health Services: chiropody etc.
21	6.7	Telephone installation
18	6.7	Adaptations, Structural alterations
11	6.7	Home phys/eld. longstay
88	6.7	Adoption
15	6.7	Daycare within home for elderly

MODE 3 11 CLIENTS Original w_3 = 0.099 Maximum = 0.242

Service	%	Description
4	100	Supportive role
20	54.5	Telephone rental
17	45.5	Meals on wheels
1	18.2	Prelim assessment
19	18.2	Aids
23	18.2	Occupational therapy
22	18.2	Domiciliary health services
21	18.2	Telephone installation
3	9.1	Exploratory casework for further assessment

APPENDIX 11. Group 06 modes of care and weighting values (continued)

MODE 3 (Cont.) 11 CLIENTS Original w_3 = 0.099 Maximum = 0.242

Service	%	Description
5	9.1	Active involvement of fieldstaff, +ve end in view
6	9.1	Watching brief
24	9.1	Craft instructors
14	9.1	Day centres
18	9.1	Adaptations, structural alterations.

MODE 4 22 CLIENTS Original w_4 = 0.198 Maximum = 0.625

Service	%	Description
6	100	Watching brief
20	45.5	Telephone rental
23	22.7	Occupational therapy
17	13.6	Meals on wheels
18	9.1	Adaptations, structural alterations
7	9.1	Visiting by recruited volunteers
4	9.1	Supportive role
27	4.5	Hospital based social work
28	4.5	Group living scheme for elderly
22	4.5	Domiciliary health service
21	4.5	Telephone installation
19	4.5	Aids
15	4.5	Day care within homes for elderly

MODE 5 13 CLIENTS Original w_5 = 0.117 Maximum = 0.535

Service	%	Description
17	92.3	Meals on wheels
18	30.8	Adaptations, structural alterations
16	30.8	Home help
6	30.8	Watching brief
20	23.1	Telephone rental
7	7.7	Visiting by recruited volunteers
21	7.7	Telephone installation
12	7.7	Homes for elderly - short stay
11	7.7	Homes for elderly - long stay

APPENDIX 11 Group 06 modes of care and weighting values (continued)

MODE 6 — 6 CLIENTS — Original w_6 = 0.054 — Maximum = 0.790

Service	%	Description
20	100	Telephone rental
19	83.3	Aids
2	50	Short term crisis intervention
17	33.3	Meals on wheels
14	33.3	Day centres

MODE 7 — 8 CLIENTS — Original w_7 = 0.072 — Maximum = 0.544

Service	%	Description
17	100	Meals on wheels
28	87.5	Group living schemes for elderly
18	75	Adaptations, structural alterations
14	37.5	Day centres
24	25	Craft instructors
7	12.5	Visiting by recruited volunteers
20	12.5	Telephone rental
19	12.5	Aids
5	12.5	Active involvement, +ve end in view

MODE 8 — 14 CLIENTS — Original w_8 = 0.126 — Maximum = 1.0

Service	%	Description
25	100	Technical officer services - Blind
20	85.7	Telephone rental
23	42.9	Occupational therapy
17	35.7	Meals on wheels
19	28.6	Aids
5	21.4	Active involvement, +ve end in view
21	21.4	Telephone installation
26	14.3	Technical officer services - Deaf
1	14.3	Preliminary assessment
3	7.1	Exploratory casework for further assessment
4	7.1	Supportive role
18	7.1	Adaptations, structural alterations
15	7.1	Day care within homes for elderly

APPENDIX 11. Group 06 modes of care and weighting values (continued)

MODE 9 — 6 CLIENTS — Original w_9 = 0.054 — Maximum = 0.419

Service	%	Description
28	100	Group living schemes for elderly
23	100	Occupational therapy
4	100	Supportive role
17	83.3	Meals on wheels

MODE 10 — 4 CLIENTS — Original w_{10} = 0.036 — Maximum = 0.430

Service	%	Description
20	100	Telephone rental
3	100	Exploratory casework for further assessment
19	100	Aids
1	100	Preliminary assessment
5	25	Active involvement, +ve end in view

APPENDIX 12 — Comparison of allocations x_{il} for Chester-le-Street district

$x_{il}^{(o)}$ = Number of clients in group i allocated to mode 1 by applying the Cluster Analysis results to Chester-le-Street district.

$x_{il}^{(1)}$ = Number of clients in group i allocated to model 1 by the LP model with weights $w_{il} = \frac{x_{il}^{(o)}}{\sum_i \sum_l x_{il}^{(o)}}$

(rounded to integer values)

$x_{il}^{(2)}$ = Number of clients in group i allocated to mode 1 by the LP model with weights $w_{il} = \frac{\sum_k x_{il}^{(o)} u_{ilk}}{\sum_i \sum_l \sum_k x_{il}^{(o)} u_{ilk}}$

(rounded to integer values)

APPENDIX 12 (cont.)

GROUP 1	MODE 1	2	3	4	5	6	7	8	9	10
$x_{il}^{(o)}$							1			
$x_{il}^{(1)}$							1			
$x_{il}^{(2)}$							1			

GROUP 2	MODE 1	2	3	4	5	6	7	8	9	10
$x_{il}^{(o)}$	1			2	2		2			1
$x_{il}^{(1)}$				4	4					
$x_{il}^{(2)}$				3	5					

GROUP 3	MODE 1	2	3	4	5	6	7	8	9	10
$x_{il}^{(o)}$		13			1	2	1	4		2
$x_{il}^{(1)}$		19						4		
$x_{il}^{(2)}$		19						4		

APPENDIX 12 (cont.)

GROUP 4	MODE 1	2	3	4	5	6	7	8	9	10
$x_{il}^{(o)}$		2		2		1	2	1	4	1
$x_{il}^{(1)}$									13	
$x_{il}^{(2)}$			1						12	

GROUP 6	MODE 1	2	3	4	5	6	7	8	9	10
$x_{il}^{(o)}$	3	1	1	1						
$x_{il}^{(1)}$	6									
$x_{il}^{(2)}$	6									

GROUP 8	MODE 1	2	3	4	5	6	7	8	9	10
$x_{il}^{(o)}$	1		1	2	4		14			1
$x_{il}^{(1)}$							23			
$x_{il}^{(2)}$							23			

APPENDIX 12 (cont.)

GROUP 9	MODE 1	2	3	4	5	6	7	8	9	10
$x_{il}^{(o)}$	1					1				1
$x_{il}^{(1)}$										3
$x_{il}^{(2)}$						3				

GROUP 10	MODE 1	2	3	4	5	6	7	8	9	10
$x_{il}^{(o)}$			1	4						
$x_{il}^{(1)}$				5						
$x_{il}^{(2)}$				5						

GROUP 11	MODE 1	2	3	4	5	6	7	8	9	10
$x_{il}^{(o)}$	1		1	1						
$x_{il}^{(1)}$	3									
$x_{il}^{(2)}$	0.4	2.6								

APPENDIX 12 (cont.)

GROUP 12	MODE 1	2	3	4	5	6	7	8	9	10
$x_{il}^{(o)}$			1					1	2	
$x_{il}^{(1)}$			4							
$x_{il}^{(2)}$			4							

GROUP 14	MODE 1	2	3	4	5	6	7	8	9	10
$x_{il}^{(o)}$		2	2				1			
$x_{il}^{(1)}$			5							
$x_{il}^{(2)}$		5								

GROUP 16	MODE 1	2	3	4	5	6	7	8	9	10
$x_{il}^{(o)}$	2	6	1	4	4		2		2	
$x_{il}^{(1)}$		21								
$x_{il}^{(2)}$		21								

APPENDIX 12 (cont.)

GROUP 17	MODE 1	2	3	4	5	6	7	8	9	10
$x_{il}^{(o)}$	1	7	3					1	4	
$x_{il}^{(1)}$		16								
$x_{il}^{(2)}$									16	

GROUP 18	MODE 1	2	3	4	5	6	7	8	9	10
$x_{il}^{(o)}$	3	8	2	6	14	2	8		11	2
$x_{il}^{(1)}$		28			28					
$x_{il}^{(2)}$		21		2	28		5			

GROUP 20	MODE 1	2	3	4	5	6	7	8	9	10
$x_{il}^{(o)}$	5	2		1		1	1			
$x_{il}^{(1)}$	10									
$x_{il}^{(2)}$	10									

APPENDIX 12 (cont.)

GROUP 21	MODE									
	1	2	3	4	5	6	7	8	9	10
$x_{il}^{(o)}$									1	
$x_{il}^{(1)}$									1	
$x_{il}^{(2)}$									1	

GROUP 23	MODE									
	1	2	3	4	5	6	7	8	9	10
$x_{il}^{(o)}$	13	11	16	2	6		3	6		1
$x_{il}^{(1)}$			58							
$x_{il}^{(2)}$			58							

GROUP 25	MODE									
	1	2	3	4	5	6	7	8	9	10
$x_{il}^{(o)}$	1						2	2	2	
$x_{il}^{(1)}$							1	6		
$x_{il}^{(2)}$								6	1	

APPENDIX 12 (cont.)

GROUP 28	MODE 1	2	3	4	5	6	7	8	9	10
$x_{il}^{(o)}$	1	6		2	3					1
$x_{il}^{(1)}$		13								
$x_{il}^{(2)}$		13								

GROUP 29	MODE 1	2	3	4	5	6	7	8	9	10
$x_{il}^{(o)}$	3	5	10	2		7	9	11		2
$x_{il}^{(1)}$			33					16		
$x_{il}^{(2)}$			49							

GROUP 30	MODE 1	2	3	4	5	6	7	8	9	10
$x_{il}^{(o)}$	6	10		10	4	7	9			
$x_{il}^{(1)}$		18		28						
$x_{il}^{(2)}$				46						

APPENDIX 12 (cont.)

GROUP 31	MODE 1	2	3	4	5	6	7	8	9	10
$x_{il}^{(o)}$	6	4	2	1		11	8			
$x_{il}^{(1)}$						32				
$x_{il}^{(2)}$						32				

GROUP 34	MODE 1	2	3	4	5	6	7	8	9	10
$x_{il}^{(o)}$	23	1	1	5		5		2	15	
$x_{il}^{(1)}$	52									
$x_{il}^{(2)}$	52									

GROUP 35	MODE 1	2	3	4	5	6	7	8	9	10
$x_{il}^{(o)}$			1			2				1
$x_{il}^{(1)}$			1						3	
$x_{il}^{(2)}$		4								

APPENDIX 12 (cont.)

GROUP 36	MODE 1	2	3	4	5	6	7	8	9	10
$x_{il}^{(o)}$	1	1	1			2				1
$x_{il}^{(1)}$	6									
$x_{il}^{(2)}$	6									

GROUP 37	MODE 1	2	3	4	5	6	7	8	9	10
$x_{il}^{(o)}$			1				1	1	2	
$x_{il}^{(1)}$							5			
$x_{il}^{(2)}$							3			2

GROUP 38	MODE 1	2	3	4	5	6	7	8	9	10
$x_{il}^{(o)}$	2		1			1				
$x_{il}^{(1)}$	4									
$x_{il}^{(2)}$	0.2		2.8			1				

APPENDIX 12 (cont.)

GROUP 39	MODE 1	2	3	4	5	6	7	8	9	10
$x_{il}^{(o)}$	1	2	5		1	5		4	1	
$x_{il}^{(1)}$			6			13				
$x_{il}^{(2)}$			4			15				

GROUP 40	MODE 1	2	3	4	5	6	7	8	9	10
$x_{il}^{(o)}$	1	2	2		1	3		1		
$x_{il}^{(1)}$						10				
$x_{il}^{(2)}$			10							

GROUP 41	MODE 1	2	3	4	5	6	7	8	9	10
$x_{il}^{(o)}$	1					4			1	2
$x_{il}^{(1)}$						8				
$x_{il}^{(2)}$										8

APPENDIX 12 (cont.)

GROUP 42	MODE 1	2	3	4	5	6	7	8	9	10
$x_{il}^{(o)}$	1	2	3	2		8	5	4	1	1
$x_{il}^{(1)}$						10	17			
$x_{il}^{(2)}$						6		21		

GROUP 43	MODE 1	2	3	4	5	6	7	8	9	10
$x_{il}^{(o)}$	7		41	5		2	2		17	7
$x_{il}^{(1)}$			74				4		3	
$x_{il}^{(2)}$			74				3		4	

GROUP 44	MODE 1	2	3	4	5	6	7	8	9	10
$x_{il}^{(o)}$	5		12	11	12			12	5	8
$x_{il}^{(1)}$					21			44		
$x_{il}^{(2)}$					38			27		

APPENDIX 12 (cont.)

GROUP 45	MODE 1	2	3	4	5	6	7	8	9	10
$x_{il}^{(o)}$		9		3	7	2	11	5	3	6
$x_{il}^{(1)}$		23					23			
$x_{il}^{(2)}$		19					27			

Appendix 13   Details of Calculations using the SPRAM model

This appendix details the calculations performed to apply the DHSS/A. Andersen SPRAM model to the small test set of data used in Chapter 8, i.e. to show the use of the SPRAM model when there are future changes in both resource availability and population sizes.

Original Data

Group i / Size d_i (d_i^1)	1 / 40(44)				2 / 35(38)				3 / 30(27)						
Mode l	1	2	3		1	2	3		1	2	3				
No. in mode x_{il}	21	0	19		30	0	5		14	6	10				
Resource Allocation	B_{11k}	B_{12k}	B_{13k}	B_{1k}	B_{21k}	B_{22k}	B_{23k}	B_{2k}	B_{31k}	B_{32k}	B_{33k}	B_{3k}		B_k	B_k^1
Service k = 1	21	0	0	21	0	0	0	0	4.2	1.8	0	6		27	26
k = 2	0	0	19	19	30	0	0	30	14	0	10	24		73	69
k = 3	0	0	19	19	16	0	5	21	7	3	5	15		55	52
k = 4	0	0	19	19	16	0	5	21	0	6	5	11		51	49
"Dominant" resource	1	1	2		2	2	3		1	1	2				

Changes

Changes in resources are decreases of approximately 5%. Thus, for services 1,2,3,4 respectively, the amounts of resource (in client-units) change from 27 to 26, 73 to 69, 55 to 52 and 51 to 49.

Changes in population are increases of approximately 10% in the sizes of client groups 1 and 2 and a decrease of 10% in the size of client group 3, i.e. sizes change from 40 to 44, 35 to 38 and 30 to 27 for groups 1,2,3 respectively.

(i) Allocate resources to groups

$$B_{jk}^1 = \frac{B_k^1 \, (d_j^1/d_j) \, B_{jk}}{\sum_i \left(d_i^1/d_i \cdot r_{ik} \right)}$$

$$\frac{d_1^1}{d_1} = \frac{44}{40} = 1.1, \quad \frac{d_2^1}{d_2} = \frac{38}{35} = 1.0857$$

$$\frac{d_3^1}{d_3} = \frac{27}{30} = 0.9$$

Resource 1

Group 1 j = 1, k = 1, $B_{11}^1 = \frac{26\ (1.1)\ (21)}{(1.1)\ (21) + (1.0857)\ (0) + (0.9)\ (6)} = \frac{26\ (1.1)\ (21)}{28.5}$

$= 21.07$

Group 2 j = 2, k = 1, $B_{21}^1 = 0$ since $B_{21} = 0$

Group 3 j = 3, k = 1, $B_{31}^1 = \frac{26\ (0.9)\ (6)}{28.5} = 4.93$

$\sum$ = :1.07 + 4.93 = 26 (Correct)

Resource 2

Group 1 j = 1, k = 2, $B_{12}^1 = \frac{69\ (1.1)\ (19)}{(1.1)\ (19) + (1.0857)\ (30) + (0.9)\ (24)} = \frac{69\ (1.1)\ 19}{75.0710}$

$= 19.21$

Group 2 j = 2, k = 2, $B_{22}^1 = \frac{69\ (1.0857)\ (30)}{75.0710} = 29.94$

Group 3 j = 3, k = 2, $B_{32}^1 = \frac{69\ (0.9)\ (24)}{75.0710} = 19.85$

$\sum$ = 19.21 + 29.94 + 10.85 = 69 (Correct)

Resource 3

Group 1 $j = 1,\ k = 3,\ B_{13}^{1} = \dfrac{52\ (1.1)(19)}{(1.1)\ (19) + (1.0857)\ (21) + (0.9)\ (15)} = \dfrac{52\ (1.1)\ (19)}{57.1997}$

$= 19.00$

Group 2 $j = 2,\ k = 3,\ B_{23}^{1} = \dfrac{52\ (1.0857)\ (21)}{57.1997} = 20.73$

Group 3 $j = 3,\ k = 3,\ B_{33}^{1} = \dfrac{52\ (0.9)\ (15)}{57.1997} = 12.27$

$\sum = 19.00 + 20.73 + 12.27 = 52$ (correct)

Resource 4

Group 1 $j = 1,\ k = 4,\ B_{14}^{1} = \dfrac{49\ (1.1)\ (19)}{(1.1)\ (19) + (1.0857)\ (21) + (0.9)\ (11)} = \dfrac{49\ (1.1)\ (19)}{53.5997}$

$= 19.11$

Group 2 $j = 2,\ k = 4,\ B_{24}^{1} = \dfrac{49\ (1.0857)\ (21)}{53.5997} = 20.84$

Group 3 $j = 3,\ k = 4,\ B_{34}^{1} = \dfrac{49\ (0.9)\ (11)}{53.5997} = 9.05$

$\sum = 19.11 + 20.84 + 9.05 = 49$ (correct)

(ii) **Allocate resources to modes within groups**

I **Identify dominant resource K in each mode of group i,**

then $B_{ilK}^{1} = \dfrac{B_{iK}^{1}\ B_{ilK}}{\sum_{L} B_{iLK}}$ for this i, l, K

(& $\sum_{L} B_{iLK} = B_{iK}$)

& $\Delta q^{1}/\Delta q = B^{1}_{ilK}/B_{ilK}$

$i = 1,\ l = 1,$ Dominant $K = 1;\ B_{111}^{1} = \dfrac{B_{11}^{1}\ B_{111}}{B_{11}} = \dfrac{21.07(21)}{21} = 21.07$

$\dfrac{\Delta q^{1}}{\Delta q} = \dfrac{21.07}{21} = 1.0033$

$i = 1,\ l = 2,$ Dominant $K = 1;\ B_{121}^{1} = 0$ since $B_{121} = 0;\ \dfrac{\Delta q^{1}}{\Delta q} = 1$

$i = 1,\ l = 3,$ Dominant $K = 2;\ B_{132}^{1} = \dfrac{B_{12}^{1}\ B_{132}}{B_{12}} = \dfrac{19.21\ (19)}{19} = 19.21$

$\dfrac{\Delta q^{1}}{\Delta q} = \dfrac{19.21}{19} = 1.0111$

$i = 2,\ l = 1,$ Dominant $K = 2;\ B_{212}^{1} = \dfrac{B_{22}^{1}\ B_{212}}{B_{22}} = \dfrac{29.94\ (30)}{30} = 29.94$

$\dfrac{\Delta q^{1}}{\Delta q} = \dfrac{29.94}{30} = 0.9980$

$i = 2,\ l = 2,$ Dominant $K = 2;\ B_{222}^{1} = 0$ since $B_{222} = 0;\ \dfrac{\Delta q^{1}}{\Delta q} = 1$

$i = 2,\ l = 3,$ Dominant $K = 3;\ B_{233}^{1} = \dfrac{B_{23}^{1}\ B_{233}}{B_{23}} = \dfrac{20.73\ (5)}{21} = 4.94$

$\dfrac{\Delta q^{1}}{\Delta q} = \dfrac{4.94}{5} = 0.9880$

$i = 3,\ l = 1$, Dominant $K = 1$, $B_{311}^{\,1} = \dfrac{B_{31}^{\,1}\, B_{311}}{B_{31}} = \dfrac{4.93\ (4.2)}{6} = 3.45$

$\dfrac{\Delta q^1}{\Delta q} = \dfrac{3.45}{4.2} = 0.8214$

$i = 3,\ l = 2$, Dominant $K = 1$, $B_{321}^{\,1} = \dfrac{B_{31}^{\,1}\, B_{321}}{B_{31}} = \dfrac{4.93\ (1.8)}{6} = 1.48$

$\dfrac{\Delta q^1}{\Delta q} = \dfrac{1.48}{1.8} = 0.8222$

$i = 3,\ l = 3$, Dominant $K = 2$, $B_{332}^{\,1} = \dfrac{B_{32}^{\,1}\, B_{332}}{B_{32}} = \dfrac{19.85\ (10)}{24} = 8.27$

$\dfrac{\Delta q^1}{\Delta q} = \dfrac{8.27}{10} = 0.827$

II For non-dominant resources in mode l, group i

$$B^1_{ilk} = \frac{B_{ik}^{\,1}\left(\dfrac{\Delta^1_1}{\Delta^1}\, B_{ilk}\right)}{\sum_q \left(\dfrac{\Delta q^1}{\Delta q}\, B_{ilk}\right)}$$

q = mode in group i

Δq = amount of dominant resource in mode q of group i

$i = 1,\ l = 1$, Non-dominant $k = 2, 3, 4$

$B_{112}^{\,1} = 0$ since $B_{112} = 0$

$B_{113}^{\,1} = 0$ since $B_{113} = 0$

$B_{114}^{\,1} = 0$ since $B_{114} = 0$

$i = 1,\ l = 2$, Non-dominant $k = 2, 3, 4$

$B_{122}^{\,1} = 0$ since $B_{122} = 0$

$B_{123}^{\,1} = 0$ since $B_{123} = 0$

$B_{124}^{\,1} = 0$ since $B_{124} = 0$

$i = 1,\ l = 3$, Non-dominant $k = 1, 3, 4$; $\Delta q^1/\Delta q = 1.0111$

$B_{131}^{\,1} = 0$ since $B_{131} = 0$

$$B_{133}^{\,1} = \frac{B_{13}^{\,1}\ (1.0111)\ B_{133}}{(1.0033)\ B_{113} + (1)\ B_{123} + (1.0111)\ B_{133}} = 19.00$$

$$B_{134}^{\,1} = \frac{B_{14}^{\,1}\ (1.0111)\ B_{134}}{(1.0033)\ B_{114} + (1)\ B_{124} + (1.0111)\ B_{134}} = 19.11$$

$i = 2,\ l = 1$, Non-dominant $k = 1, 3, 4$; $\Delta q^1/\Delta q = 0.998$

$B_{211}^{\,1} = 0$ since $B_{211} = 0$

$$B_{213}^{\,1} = \frac{B_{23}^{\,1}\ (0.998)\ B_{213}}{(0.998)\ B_{213} + (1)\ B_{223} + (0.988)\ B_{233}} = \frac{20.73\ (0.998)\ (16)}{20.9080}$$

$= 15.83$

$$B_{214}^{\,1} = \frac{B_{24}^{\,1}\ (0.998)\ B_{214}}{(0.998)\ B_{214} + (1)\ B_{224} + (0.988)\ B_{234}} = \frac{20.84\ (0.998)\ (16)}{20.9080}$$

$= 15.92$

$i = 2,\ l = 2$, Non-dominant $k = 1, 3, 4$; $\Delta q^{1}/\Delta q = 1$

$B_{221}^{1} = 0$ since $B_{221} = 0$

$B_{223}^{1} = 0$ since $B_{223} = 0$

$B_{224}^{1} = 0$ since $B_{224} = 0$

$i = 2,\ l = 3$, Non-dominant $k = 1, 2, 4$; $\Delta q^{1}/\Delta q = 0.988$

$B_{231}^{1} = 0$ since $B_{231} = 0$

$B_{232}^{1} = 0$ since $B_{232} = 0$

$$B_{234}^{1} = \frac{B_{24}^{1}\ (0.988)\ B_{234}}{(0.998)\ B_{214} + (1)\ B_{224} + (0.988)\ B_{234}} = \frac{20.84\ (0.988)\ (5)}{20.9080}$$

$$= 4.92$$

$i = 3,\ l = 1$, Non-dominant $k = 2, 3, 4$; $\Delta q^{1}/\Delta q = 0.8214$

$$B_{312}^{1} = \frac{B_{32}^{1}\ (0.8214)\ B_{312}}{(0.8214)\ B_{312} + (0.8222)\ B_{322} + (0.827)\ B_{332}} = \frac{19.85\ (0.8214)\ (14)}{19.7696}$$

$$= 11.55$$

$$B_{313}^{1} = \frac{B_{33}^{1}\ (0.8214)\ B_{313}}{(0.8214)\ B_{313} + (0.8222)\ B_{323} + (0.827)\ B_{333}} = \frac{12.27\ (0.8214)\ (7)}{12.3514}$$

$$= 5.71$$

$B_{314}^{1} = 0$ since $B_{314} = 0$

$i = 3,\ l = 2$, Non dominant $k = 2, 3, 4$; $\Delta q^{1}/\Delta q = 0.8222$

$B_{322}^{1} = 0$ since $B_{322} = 0$ $\qquad B_{323}^{1} = 2.45$

$$B_{324}^{1} = \frac{B_{34}^{1}\ (0.8222)\ B_{324}}{(0.8214)\ B_{314} + (0.8222)\ B_{324} + (0.827)\ B_{334}} = \frac{9.05\ (0.8222)\ (6)}{9.0682}$$

$$= 4.92$$

$i = 3,\ l = 3$, Non-dominant $k = 1, 3, 4$; $\Delta q^{1}/\Delta q = 0.827$

$B_{331}^{1} = 0$ since $B_{331} = 0$

$$B_{333}^{1} = \frac{B_{33}^{1}\ (0.827)\ B_{333}}{(0.8214)\ B_{313} + (0.8222)\ B_{323} + (0.827)\ B_{333}} = \frac{12.27\ (0.827)\ (5)}{12.3514}$$

$$= 4.11$$

$$B_{334}^{1} = \frac{B_{34}^{1}\ (0.827)\ B_{334}}{(0.8214)\ B_{314} + (0.8222)\ B_{324} + (0.827)\ B_{334}} = \frac{9.05\ (0.827)\ (5)}{9.0682}$$

$$= 4.13$$

Summary of Results of SPRAM Method

Group i	1				2				3				
Size d_i^{1}	44				38				27				
Mode l	1	2	3		1	2	3		1	2	3		
Resource allocation	B^{1}_{11k}	B^{1}_{12k}	B^{1}_{13k}	B^{1}_{1k}	B^{1}_{21k}	B^{1}_{22k}	B^{1}_{23k}	B^{1}_{2k}	B^{1}_{31k}	B^{1}_{32k}	B^{1}_{33k}	B^{1}_{3k}	B_k^{1}
Service k = 1	21.07	0	0	21.07	0	0	0	0	3.45	1.48	0	4.93	26
k = 2	0	0	19.21	19.21	29.94	0	0	29.94	11.55	0	8.27	19.82	69
k = 3	0	0	19.00	19.00	15.83	0	4.94	20.77	5.71	2.45	4.11	12.27	52
k = 4	0	0	19.11	19.11	15.92	0	4.92	20.84	0	4.92	4.13	9.05	49

APPENDIX 14 Wiltshire Balance of Care Project

Categories and Care for Elderly Clients

The following table provides a general illustration of the framework of the 31 categories of elderly people, and the ideal alternatives for providing care for each category.

The packages of care and levels of service provide only a benchmark against which future policies may be considered. The numbers provided relate to a monthly basis, so that the number 8 under community nursing refers to a service level of 8 visits per month. The units of service are :

Hospital	-	bed days per month
Residential Home	-	place days per month
Sheltered Housing	-	unit days per month
Day Care	-	attendances per month
Nursing	-	visits per month
Home Help	-	hours per month
Meals	-	number of meals per month

The Advisory Group identified a range of acceptable alternatives for each category, and also gave each mode a 'score'. These should be interpreted as :

3	-	Preferred
2	-	Good
1	-	Acceptable

Where a score of 0 is given, this means that the mode is one by which a number of people are presently cared for, but one which the Advisory Group feel is an unacceptable form for future planning.

The Advisory Group have also indicated, in general terms a form of minimum standard for day and domiciliary services. These are used outside the model, and give the standards of service a range of acceptable levels, to complement the ideal levels provided.

Appendix 14 (continued)

Wiltshire Balance of Care Project
Categories and care for elderly clients

CATEGORIES AND PREFERRED MODES OF CARE

* B.D. = Behaviour Disorder

Category	Ability to undertake Household & Personal Care	Incontinence	Mental State	Environmental Circumstances	Mode Number	Geriatric Hospital Bed	Short Stay Care	Psychogeriatric Hospital	Short Stay Care	Residential Home	Short Stay Care	Home Nursing	Community Psychiatric Nursing	Home Help	Meals on Wheels	Day Hospital	Psychiatric Day Hospital	Day Centre	Sheltered Housing	Score
1	Severe	Doubly, regularly or continent	Dementia ± B.D.*	Adverse	1	28														3
2	Severe	Doubly, regularly or continent	No Dementia	Adverse	1	28														3
3	Severe	Doubly, regularly or continent	Dementia + B.D.	Supportive	1	28														3
4	Severe	Doubly, regularly or continent doubly, regularly	Dementia No Dementia	Supportive	1 2 3		4 4 4					28 16 20		36 20 28		8		8	28 28 28	3 3 1
5	Severe	Continent	No Dementia	Supportive	1 2 3	4 4 4						28 8 12		20 12 12		8		8	28 28 28	3 3 1
6	Moderate	Doubly or regularly	Dementia ± B.D.	Adverse	1 2 3 4 5			28				55 55 56 56	2 2	56 36 44 44	20 8 8 8	12	20	12	28 28 28 28	3 1 1 1 1
7	Moderate	Doubly or regularly	No Dementia	Supportive	1 2 3 4		2 2 2 2					20 8 8 12		32 16 16 20		12	12	8	28 28 28 28	3 3 3 3
8	Moderate	Continent	No Dementia	Supportive	1 2 3		2				2 2	4		8		4		4	28 28 28	3 2 2
9	Moderate/ Minor	Doubly or regularly	Dementia + B.D.	Supportive	1 2 3 4			28	9 14 9			48 24	4 2 2	32 12 20		12		12	28 28 28	3 1 1 1
10	Moderate/ Minor	Doubly or regularly doubly	Dementia	Supportive	1 2 3 4 5 6 7	28		28	7 7 7 7	28		4 16 16 28	4 4 4 4	12 20 20 28		8	20	8	28 28 28 28	3 1 1 1 1 1 1
11	Moderate	Doubly or regularly	No Dementia	Adverse	1 2 3 4					28		56 56 56		32 32 48	8 8 20	12		12	28 28 28	3 2 1 1

Appendix 14 (continued) — Wiltshire Balance of Care Project

Categories and care for elderly clients

CATEGORIES AND PREFERRED MODES OF CARE (Page 2)

Category	Ability to undertake Household & Personal Care	Incontinence	Mental State	Environmental Circumstances	Mode Number	Geriatric Hospital Bed	Short Stay Care	Psychogeriatric Hospital B	Short Stay Care	Residential Home	Short Stay Care	Home Nursing	Community Psychiatric Nursing	Home Help	Meals on Wheels	Day Hospital	Psychiatric Day Hospital	Day Centre	Sheltered Housing	Score
12	Moderate	Continent	Dementia +R.D.	Adverse	1			28												3
					2							56	2	36	20				28	1
13	Moderate	Continent	Dementia	Adverse	1			28												3
					2					28										3
					3							56	2	32	20				28	1
					4							56	2	20	8			12	28	1
14	Moderate	Continent	Dementia +R.D.	Supportive	1				7				4	12			20		28	3
					2				7			12	2	12				12	28	2
					3				7			16	2	16					28	1
15	Moderate	Continent	Dementia	Supportive	1						2	8	2	8				12	28	3
					2				2			4		8			12		28	3
					3				2			12	2	12					28	2
					4				2			4		8		12			28	1
16	Moderate	Continent	No Dementia	Adverse support, inadequate housing	1					28										3
					2							56		24	12			8	28	2
					3							56		32	20				28	1
17	Moderate	Continent	No Dementia	Adverse support, adequate housing	1							56		20	20				28	3
					2							56		16	8			8	28	3
					3					28										1
18	Minor	Doubly or Regularly	Dementia	Adverse	1			28												3
					2					28										3
					3							28	4	32	20				28	1
19	Minor	Regularly	Dementia	Supportive	1		7						8	12			12		28	3
					2		7					8	4	16					28	2
					3		7					4	4	8				8	28	2
20	Minor	Regularly	No Dementia	Adverse	1					28										3
					2							8		16	8			12	28	2
					3							8		16	8	12			28	2
					4							12		24	20				28	1
21	Minor	Regularly	No Dementia	Supportive	1					2				8				8	28	3
					2					2				12					28	2
22	Minor	Continent	Dementia +R.D.	Adverse	1							2	2	24	20				28	3
					2			28												2
					3					28										1
23	Minor	Continent	Dementia +R.D.	Supportive	1		7						4	12			20		28	3
					2		7					4	4	16				8	28	2
					3		7					2	2	20					28	2

Appendix 14 (continued) Wiltshire Balance of Care Project

Categories and care for elderly clients

CATEGORIES AND PREFERRED MODES OF CARE (Page 3)

Category	Ability to undertake Household & Personal Care	Incontinence	Mental State	Environmental Circumstances	Mode Number	Geriatric Hospital Bed	Short Stay Care	Psychogeriatric Hospital Bed	Short Stay Care	Residential Home	Short Stay Care	Home Nursing	Community Psychiatric Nursing	Home Help	Meals on Wheels	Day Hospital	Psychiatric Day Hospital	Day Centre	Sheltered Housing	Score
24	Minor	Continent	Dementia	Adverse support, inadequate housing	1							4		20	20				28	3
					2							4		12	8			12	28	2
					3					28										2
					4			28												
25	Minor	Continent	Dementia	Adverse support, adequate housing	1					28										3
					2								4	12	20				28	2
					3								4	12	8			12	28	2
					4									12	20				28	1
					5			28												1
26	Minor	Continent	Dementia	Supportive	1						1			8				4	28	3
					2		1							8			4		28	2
					3		1					4		8					28	1
					4						1			8					28	1
27	Minor	Continent	No Dementia	Adverse support, inadequate housing	1					28										3
					2									12	8			12	28	3
					3							4		20	20				28	2
					4									12	12	8			28	1
					5									16	20				28	1
28	Minor	Continent	No Dementia	Adverse support, adequate housing	1					28										3
					2									12	12			8	28	3
					3							4		16	20				28	3
					4									12	12	8			28	1
					5									16	20				28	1
29	Minor	Continent	No Dementia	Supportive	1						1								28	3
					2		1					2		4		4			28	1
					3		1					2		4				4	28	1
					4						1	4		4					28	1
					5									4					28	1
					6														28	1
30	None	Regularly	No Dementia	Adverse	1									8				8	28	2
					2							4		12					28	2
31	None	Regularly or Continent	No Dementia	Any	1							2		4					28	3
					2									4				4	28	2
					3									4					28	1
					4														28	1

APPENDIX 15 : ALTERNATIVE PRESENTATION OF WEIGHT CONSTRAINTS

As described in Section 5.5.1, the weight constraints could be derived slightly differently from the method used in the preceding sections of Chapter 5. The resultant constraints are the same, only the partitioning of the matrices differs. This Appendix shows the alternative presentation of the weight constraints for a small example having five real variables and two constraints, the actual, known solution having five non-zero variables.

The constraints are:

$$x_1 + x_2 + x_3 + x_4 + x_5 = 6$$

$$x_1 - 2x_2 - 3x_3 + 2x_4 + 3x_5 = 4$$

$$\underline{x} \geqslant 0$$

and the known solution occurs at $\underline{x}^* = (1,1,1,1,2)^T$ for an objective function $f = \sum_i w_i x_i$ where the weights $\underline{w}$ are unknown.

The simplex tableau is partitioned as follows:

B	A_R	I_R	0	$\underline{b}$
$\underline{w}^T$	$\underline{w}^T$	0	1	0

so that B is a (2 x 2) basis matrix.

At an optimal solution, the following is true:

$$\left[\begin{array}{c|c} B & 0 \\ \hline \underline{w}_B^T & 1 \end{array}\right] \cdot \left[\begin{array}{c|c|c|c|c} G & H & J & 0 & \underline{x}^* \\ \hline \underline{y} & \underline{h} & \underline{j} & 1 & -f^* \end{array}\right] = \left[\begin{array}{c|c|c|c|c} B & A_R & I_R & 0 & \underline{b} \\ \hline \underline{w}_B^T & \underline{w}_R^R & 0 & 1 & 0 \end{array}\right]$$

In the example given, taking x_1, x_2 as the basis gives

$$B = \begin{pmatrix} 1 & 1 \\ 1 & -2 \end{pmatrix}, \quad B^{-1} = \begin{pmatrix} 2/3 & 1/3 \\ 1/3 & -1/3 \end{pmatrix},$$

$$A_R = \begin{pmatrix} 1 & 1 & 1 \\ -3 & 2 & 3 \end{pmatrix}, \quad I_R = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

$$\underline{w}_B^T = (w_1, w_2), \qquad \underline{w}_R^T = (w_3, w_4, w_5)$$

By the method shown in section 5.5.1 the weight constraints are:

$$\underline{w}_R^T - \underline{w}_B^T B^{-1} A_R = 0 \quad \text{(3 of these)} \qquad (1)$$

$$\underline{w}_B^T B^{-1} \geqslant 0 \quad \text{(2 of these)} \qquad (2)$$

<u>Hence</u> $(w_3, w_4, w_5) - (w_1, w_2) \begin{pmatrix} 2/3 & 1/3 \\ 1/3 & -1/3 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ -3 & 2 & 3 \end{pmatrix} = 0$ from (1)

i.e $(w_3, w_4, w_5) - (w_1, w_2) \begin{pmatrix} -1/3 & 4/3 & 5/3 \\ 4/3 & -1/3 & -2/3 \end{pmatrix} = 0$

or

$$\left.\begin{aligned} w_3 &= -\tfrac{1}{3} w_1 + \tfrac{4}{3} w_2 \\ w_4 &= \tfrac{4}{3} w_1 - \tfrac{1}{3} w_2 \\ w_5 &= \tfrac{5}{3} w_1 - \tfrac{2}{3} w_2 \end{aligned}\right\} \quad (3)$$

<u>And</u> $(w_1, w_2) \begin{pmatrix} 2/3 & 1/3 \\ 1/3 & -1/3 \end{pmatrix} \geqslant 0$ from (2)

or

$$\left.\begin{aligned} \tfrac{2}{3} w_1 + \tfrac{1}{3} w_2 &\geqslant 0 \\ \tfrac{1}{3} w_1 - \tfrac{1}{3} w_2 &\geqslant 0 \end{aligned}\right\} \quad (4)$$

All the weights are non-negative and they sum to 1.

i.e. $\underline{w} \geq 0$ and $w_1 + w_2 + w_3 + w_4 + w_5 = 1$

So (3) can be expressed as:

$$\left.\begin{array}{l} -\frac{1}{3}w_1 + \frac{4}{3}w_2 \geq 0 \\ \frac{4}{3}w_1 - \frac{1}{3}w_2 \geq 0 \\ \frac{5}{3}w_1 - \frac{2}{3}w_2 \geq 0 \end{array}\right\} \quad (5)$$

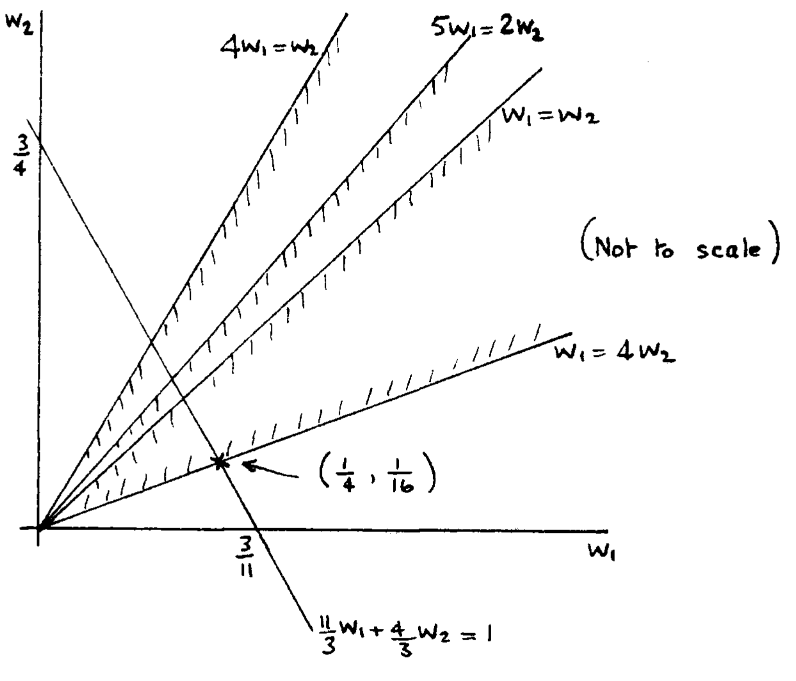
and $\sum w = 1$ implies that $\frac{11}{3}w_1 + \frac{4}{3}w_2 = 1$ (6)

The solution occurs at $\max \sum_i w_i x_i$ where $\underline{x} = (1,1,1,1,2)$

so the effect is of maximising $w_1 + w_2 + w_3 + w_4 + 2w_5$ subject to the weight constraints (4), (5) and (6).

This is shown below and results in

$\underline{w} = (1/4,\ 1/16,\ 0,\ 5/16,\ 3/8)^T$



APPENDIX 16 : DERIVATION OF WEIGHT CONSTRAINTS FROM EXTREME POINTS

As described in Section 5.5.4, the weight constraints could be derived from knowledge of the extreme points which could be combined to form the actual solution point. This Appendix shows the derivation of the weight constraints for a small example having five real variables and two constraints and the actual solution having five non-zero variables. This is the same problem as used in Appendix 15.

The constraints are:

$$x_1 + x_2 + x_3 + x_4 + x_5 = 6$$

$$x_1 - 2x_2 - 3x_3 + 2x_4 + 3x_5 = 4$$

$$\underline{x} \geqslant 0$$

and the known solution occurs at $\underline{x}^* = (1,1,1,1,2)^T$
for an objective function $f = \sum_i w_i x_i$ where the weights $\underline{w}$ are unknown.

Firstly the extreme points are found:

Taking x_1, x_2 as the basis, $B = \begin{pmatrix} 1 & 1 \\ 1 & -2 \end{pmatrix}$, $B^{-1} = \begin{pmatrix} 2/3 & 1/3 \\ 1/3 & -1/3 \end{pmatrix}$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \underline{x} = B^{-1}\underline{b} = \begin{pmatrix} 16/3 \\ 2/3 \end{pmatrix} \text{ i.e. } E_{12}$$

Similarly, feasible extreme points can be found for five other bases:

$$\underset{E_{13}}{\begin{pmatrix} x_1 \\ x_3 \end{pmatrix} = \begin{pmatrix} 11/2 \\ 1/2 \end{pmatrix}}, \quad \underset{E_{24}}{\begin{pmatrix} x_2 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}}, \quad \underset{E_{25}}{\begin{pmatrix} x_2 \\ x_5 \end{pmatrix} = \begin{pmatrix} 14/5 \\ 16/5 \end{pmatrix}}, \quad \underset{E_{34}}{\begin{pmatrix} x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 8/5 \\ 22/5 \end{pmatrix}}, \quad \underset{E_{35}}{\begin{pmatrix} x_3 \\ x_5 \end{pmatrix} = \begin{pmatrix} 7/3 \\ 11/3 \end{pmatrix}}$$

The actual solution is written as a combination of these extreme points, i.e. $x^* = \sum_i \alpha_i E_i$,

$$\underline{x}^* = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 2 \end{pmatrix} = 3/16 \begin{pmatrix} 16/3 \\ 2/3 \\ 0 \\ 0 \\ 0 \end{pmatrix} + 1/4 \begin{pmatrix} 0 \\ 2 \\ 0 \\ 4 \\ 0 \end{pmatrix} + 15/112 \begin{pmatrix} 0 \\ 14/5 \\ 0 \\ 0 \\ 16/5 \end{pmatrix} + 3/7 \begin{pmatrix} 0 \\ 0 \\ 7/3 \\ 0 \\ 11/3 \end{pmatrix}$$

Then $f^* = \sum_i w_i x_i$ is the same at each of these extreme points,

i.e. $\frac{16}{3} w_1 + \frac{2}{3} w_2 = 2w_2 + 4w_4 = \frac{14}{5} w_2 + \frac{16}{5} w_5 = \frac{7}{3} w_3 + \frac{11}{3} w_5$

<u>or</u> $w_3 = -\frac{1}{3} w_1 + \frac{4}{3} w_2$

$w_4 = \frac{4}{3} w_1 - \frac{1}{3} w_2$

$w_5 = \frac{5}{3} w_1 - \frac{2}{3} w_2$

Also, any of these extreme points must be a maximum,

so $\underline{w}_B^T B^{-1} \geqslant 0$, i.e. $(w_1 \; w_2) \begin{pmatrix} 2/3 & 1/3 \\ 1/3 & -1/3 \end{pmatrix} \geqslant 0$

or $\frac{2}{3} w_1 + \frac{1}{3} w_2 \geqslant 0$

$\frac{1}{3} w_1 - \frac{1}{3} w_2 \geqslant 0$

and in addition $\underline{w} \geqslant 0, \sum w = 1$

<u>Thus the constraints are identical with those derived in Appendix 15.</u>

APPENDIX 17 : CHOICE OF BASIS, n > m

As described in Section 5.5.5, the choice of a basis needs to be made in order to derive weight constraints, by any of the methods shown.

This Appendix shows the effect of choosing different bases for the same small problem as that used in Appendices 15 and 16.

The constraints are:

$$x_1 + x_2 + x_3 + x_4 + x_5 = 6$$

$$x_1 - 2x_2 - 3x_3 + 2x_4 + 3x_5 = 4$$

$$\underline{x} \geqslant 0$$

and the known solution occurs at $\underline{x}^* = (1,1,1,1,2)^T$ for an objective function $f = \sum_i w_i x_i$ where the weights $\underline{w}$ are unknown.

Using the method shown in Appendix 15,

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & -2 & -3 & 2 & 3 \end{pmatrix}, \quad \underline{b} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$$

The first choice of basis had been x_1, x_2.

Now choosing x_2, x_4 as the basis:

$$B = \begin{pmatrix} 1 & 1 \\ -2 & 2 \end{pmatrix}, \quad B^{-1} = \begin{pmatrix} 1/2 & -1/4 \\ 1/2 & 1/4 \end{pmatrix},$$

$$A_R = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -3 & 3 \end{pmatrix}, \quad I_R = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

$$\underline{w}_B^T = (w_2, w_4), \quad \underline{w}_R^T = (w_1, w_3, w_5)$$

By the method shown in Section 5.5.1 the weight constraints are:

$$\underline{w}_R^T - \underline{w}_B^T B^{-1} A_R = 0 \quad \text{(3 of these)} \qquad (1)$$

$$\underline{w}_B^T B^{-1} \geqslant 0 \quad \text{(2 of these)} \qquad (2)$$

<u>Hence</u> $(w_1, w_3, w_5) - (w_2, w_4) \begin{matrix} 1/2 & -1/4 & 1 & 1 & 1 \\ 1/2 & 1/4 & 1 & -3 & 3 \end{matrix} = 0$ from (1)

i.e. $$(w_1, w_3, w_5) - (w_2\ w_4) \begin{pmatrix} 1/4 & 5/4 & -1/4 \\ 3/4 & -1/4 & 5/4 \end{pmatrix} = 0$$

or
$$\left.\begin{aligned} w_1 &= \tfrac{1}{4} w_2 + \tfrac{3}{4} w_4 \\ w_3 &= \tfrac{5}{4} w_2 - \tfrac{1}{4} w_4 \\ w_5 &= -\tfrac{1}{4} w_2 + \tfrac{5}{4} w_4 \end{aligned}\right\} \quad (3)$$

<u>And</u> $$(w_2, w_4) \begin{pmatrix} 1/2 & -1/4 \\ 1/2 & 1/4 \end{pmatrix} \geqslant 0 \quad \text{from (2)}$$

or
$$\left.\begin{aligned} \tfrac{1}{2} w_2 + \tfrac{1}{2} w_4 &\geqslant 0 \\ -\tfrac{1}{4} w_2 + \tfrac{1}{4} w_4 &\geqslant 0 \end{aligned}\right\} \quad (4)$$

For purposes of comparison, constraints are required in terms of w_1 and w_2. Rearranging the above leads to the following in terms of w_1 and w_2:

$$w_3 = -\tfrac{1}{3} w_1 + \tfrac{4}{3} w_2$$

$$w_4 = \tfrac{4}{3} w_1 - \tfrac{1}{3} w_2$$

$$w_5 = \tfrac{5}{3} w_1 - \tfrac{2}{3} w_2$$

$$\tfrac{2}{3} w_1 + \tfrac{1}{3} w_2 \geqslant 0$$

$$\tfrac{1}{3} w_1 - \tfrac{1}{3} w_2 \geqslant 0$$

These are identical constraints with those derived in Appendix 15 with x_1, x_2 as the basis.

Choices of other bases (corresponding to basic feasible solutions) have shown that in all cases (six possible bases for this small example) the same set of weight constraints are derived.

APPENDIX 18 : CHOICE OF BASIS, $n < m$

This Appendix shows the effect on the weight constraints of choosing different bases, for a small problem having fewer real variables, n, than constraints, m. (See Section 5.5.6), but with some equations dependent on others so that more than one extreme point leads to an optimal solution.

The constraints are:

$$4x_1 + 3x_2 + 4x_3 = 36$$

$$2x_1 + 3x_2 + 2x_3 = 24$$

$$x_1 + x_2 + x_3 = 10$$

$$5x_1 + 4x_2 + 5x_3 = 46$$

Solutions exist at (6,4,0) and (0,4,6)

and also at (3,4,3) which is a linear combination of these.

No basis can be formed using both x_1 and x_3 since the columns of their coefficients are linearly dependent.

Choosing the basis (x_1,x_3,S_1,S_2) and using the method shown in Section 5.5.1, the weight constraints can be derived as follows:

$$B = \begin{pmatrix} 4 & 3 & 1 & 0 \\ 2 & 3 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 5 & 4 & 0 & 0 \end{pmatrix}, \quad B^{-1} = \begin{pmatrix} 0 & 0 & -4 & 1 \\ 0 & 0 & 5 & -1 \\ 1 & 0 & 1 & -1 \\ 0 & 1 & -7 & 1 \end{pmatrix}$$

$$A_R = \begin{pmatrix} 4 \\ 2 \\ 1 \\ 5 \end{pmatrix}, \quad I_R = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \underline{w}_B^T = (w_1, w_2, 0, 0), \quad \underline{w}_R^T = (w_3).$$

Then $\underline{w}_R^T - \underline{w}_B^T B^{-1} A_R = 0$ (1 of these)

$\underline{w}_B^T B^{-1} \geqslant 0$ (2 of these)

<u>Hence</u>

$$w_3 - (w_1, w_2, 0, 0) \begin{pmatrix} 0 & 0 & -4 & 1 \\ 0 & 0 & 5 & -1 \\ 1 & 0 & 1 & -1 \\ 0 & 1 & -7 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \\ 1 \\ 5 \end{pmatrix} = 0$$

i.e.

$$w_3 - (0, 0, -4w_1 + 5w_2, w_1 - w_2) \begin{pmatrix} 4 \\ 2 \\ 1 \\ 5 \end{pmatrix} = 0$$

i.e. $w_3 - w_1 = 0$ <u>or</u> $w_3 = w_1$

<u>and</u>

$$(w_1, w_2, 0, 0) \begin{pmatrix} 0 & 0 & -4 & 1 \\ 0 & 0 & 5 & -1 \\ 1 & 0 & 1 & -1 \\ 0 & 1 & -7 & 1 \end{pmatrix} \geqslant 0$$

i.e. $(0, 0, -4w_1 + 5w_2, w_1 - w_2) \geqslant 0$

or $-4w_1 + 5w_2 \geqslant 0$

$w_1 - w_2 \geqslant 0$

So constraints are
$$\left.\begin{aligned} w_3 &= w_1 \\ -4w_1 + 5w_2 &\geqslant 0 \\ w_1 - w_2 &\geqslant 0 \end{aligned}\right\}$$

Choosing a different basis, (x_2, x_3, S_1, S_2),

$$B = \begin{pmatrix} 3 & 4 & 1 & 0 \\ 3 & 2 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 4 & 5 & 0 & 0 \end{pmatrix}, \quad B^{-1} = \begin{pmatrix} 0 & 0 & 5 & -1 \\ 0 & 0 & -4 & 1 \\ 1 & 0 & 1 & -1 \\ 0 & 1 & -7 & 1 \end{pmatrix},$$

$$A_R = \begin{pmatrix} 4 \\ 2 \\ 1 \\ 5 \end{pmatrix}, \quad I_R = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$\underline{w}_B = (w_2, w_3, 0, 0), \quad \underline{w}_R^T = (w_1).$

Then $\underline{w}_R^T - \underline{w}_B^T B^{-1} A_R = 0$ (1 of these)

$\underline{w}_B^T B^{-1} \geqslant 0$ (2 of these)

Hence $w_1 - (w_2, w_3, 0, 0)\begin{pmatrix} 0 & 0 & 5 & -1 \\ 0 & 0 & -4 & 1 \\ 1 & 0 & 1 & -1 \\ 0 & 1 & -7 & 1 \end{pmatrix}\begin{pmatrix} 4 \\ 2 \\ 1 \\ 5 \end{pmatrix} = 0$

i.e. $w_1 - (0, 0, 5w_2 - 4w_3, -w_2 + w_3)\begin{pmatrix} 4 \\ 2 \\ 1 \\ 5 \end{pmatrix} = 0$

i.e. $w_1 - (5w_2 - 4w_3) - 5(-w_2 + w_3) = 0$

i.e. $w_1 - w_3 = 0$ as before

And $5w_2 - 4w_3 \geqslant 0$

$-w_2 + w_3 \geqslant 0$

Substituting $w_3 = w_1$, $\left.\begin{matrix} 5w_2 - 4w_1 \geqslant 0 \\ -w_2 + w_1 \geqslant 0 \end{matrix}\right\}$ as before

Thus for these two bases the weight constraints derived are the same.

Another basis could be selected using (x_1, x_2) and different slack variables, or using (x_2, x_3) and different slack variables. It can be shown that these bases also result in the same weight constraints.

No basis can be selected using only one of the real variables and three slack variables because the original equations can not be solved using just one real variable.