Coupling constants and unification

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COUPLING CONSTANTS AND UNIFICATION

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ABSTRACT

The first part of this work gives a general background for the ideas involved in the research presented in this thesis. Coupling constants, Renormalisation Group Equation, Grand Unified Theories (GUTs) and Supersymmetry (SUSY) are briefly introduced.

Following this, we analyse the unification parameters M_{GUT} and $1/\alpha_{GUT}$ as functions of the number of fermion families (F) and Higgs boson multiplets (S). Analytical and numerical solutions to the leading and next-to-leading order evolution equation for the couplings α_i are obtained. This is done in the context of the Standard Model embedded in SU(5), SUSY SU(5) and L-R SO(10). In all these GUTs, the first order analytical approach proves itself a useful probe to examine the structure of M_{GUT} and $1/\alpha_{GUT}$ in terms of the variables F and S. General trends remain the same after including second order corrections to the evolution equations.

Recent precision data for the coupling constants allow more definitive conclusions to be reached. We find that restrictions on the unification parameters constrain F and S in such a way that SU(5) is ruled out by constraints on S (in agreement with previous results), F is severely limited in SUSY SU(5) and, unlike SUSY SU(5), an acceptable unification scenario can still be obtained when Higgs bosons are ignored in L-R SO(10). The structures of the latter two GUTs are found to be very different although some features are common to both.

DECLARATION

I declare that no material in this thesis has previously been submitted for a degree at this or any other University.

The research work in this thesis has been carried out in collaboration with Professor W. J. Stirling. Material in Chapters 2,3 and 4 has been published in

Constraints from unification in SU(5) and SUSY SU(5)
F. Astorga, *J. Phys. G: Nucl. Part. Phys.* **20** (1994) 241.

Unification Parameter Analysis within Left-Right Symmetric SO(10)
F. Astorga and W.J. Stirling, Durham Preprint DPT/94/04

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Contents

Chapter 1

Introduction

Our understanding of how particles interact and of what the fundamental structure of matter is, has progressed, in the course of few years, to the point where a model describes very accurately, it is believed, their behaviour at low ($\leq M_z$) energies. This Standard Model (SM) has indeed stood successfully whenever its predictions have been tested and, furthermore, all the available experimental data can be satisfactorily accommodated within it. No complaints against its efficacy can be made at this stage.

In this theory building process, two elements seem to have played decisive roles: unexplained experimental evidence (evidence, that is, not explained by the current theory), and our own ideas and expectations about the necessary shape and structure of the world of elementary particles. From the former arises a natural mechanism for building physical theories: theoretical descriptions have to be outlined according to the available evidence. Once it is assumed that theories are about objects with independent existence, a criterion is introduced for measuring how close the theory is to the behaviour it tries to describe. This is one of the landmarks of science as we know it now, and it has become an essential part of it.

The latter element has sometimes been expressed in aesthetic terms such as the quest for beauty or simplicity, mathematical symmetries being the realization of some of these intended features of the theory. In this 'pilgrimage' towards the core of the elementary particle world, the searching for the laws of nature has been wrapped about with assumptions about their expected structure. These have proved to be a very important guide in getting theories closer to reality. Without this element, the construction of particle theories might have not progressed to the same level. Theories are many times ahead of experimental development, guided only by adventuring hypotheses on the structure of physical systems to be expected on the grounds of such desirable features as symmetry, derivation from first principles, paucity of free variables, a minimal number of independent principles and some others. Linked to this there seems to be a shift in the question with which nature is approached, from 'how' to 'why'.

However independent these two elements seem to appear at first glance, they are related to each other by their contributory role in this theory building process; and it is the combination of both which allows room for new ideas that change our perception of the particle world, and of the desirable features to aim at in new theories.

Within this stream are contained some of the theories which embed the work we present here. It is now believed that there are 'too many' free parameters in the SM, that is, parameters not fixed within the model but by external (mainly experimental) sources; although for a theory with the predictive power of the SM this may not be a very high number of parameters. New theories have been built in an attempt to reduce this number, among which the couplings for the electromagnetic, weak and strong interactions are included. Grand Unification intends to merge these couplings into one. This would happen at higher energies than those reached in current accelerators, and it would have to reduce to the SM in the corresponding limit. In this process we go from higher to lower symmetry, that is, there is at least one stage in which some symmetry must be broken to leave only the symmetries of the SM. In spite of the beauty of unification, the realization of this idea has to face a major problem. There is a huge gap between the energy scale at which unification occurs and the one at which we obtain the SM. Both of these scales signal the places where symmetry breaking is occurring. If this breaking is accomplished using a Higgs mechanism, radiative corrections to the Higgs boson propagator tend to spoil the SM scale, increasing the mass of this boson at each order in the perturbative approximation. Although this can be fixed order by order, this has always been considered an 'unnatural' way of fixing the theory and, therefore, an undesirable feature within it. This hierarchy problem could be solved by including terms that counterbalance those that make the Higgs boson mass diverge. These terms would arise as contributions from particles of singular characteristics not found in the particle content of the SM. A systematic and very natural way of introducing these particles is to assume a complete symmetry between fermions and bosons in the theory, in such a way that to each fermion (boson) in the SM is ascribed a bosonic (fermionic)-type particle with the same mass. This produces the required sign in the corresponding contributions. The symmetry thus introduced has been called Supersymmetry (SUSY).

One of the striking features of the SM is its 'chirality', that is, the fact that weak interactions distinguish left-helical particle states from right-helical ones. This is a big surprise for our 'symmetry-is-beauty' conceptions, but the experimental evidence forces chirality to be built into the model. Intending to restore the equality of states of both helicities and, in this way, to have a more symmetric theory, various models have been proposed. Left-Right (L-R) symmetric models can be accommodated within some Grand Unified Theories (GUTs) as an intermediate stage on the way to yet higher symmetries.

As this theoretical progress has been achieved, improvements in the experimental data have taken place. In particular, the accuracy of the coupling constant values has increased, allowing for a precise test of the unification hypothesis.

It is the aim of this work to introduce a tool of analysis to explore the relevant parameters of unification as functions of the number of fermion families (F) and Higgs boson multiplets (S) in such a way that, from constraints on those parameters, information about the allowed values for F and S is obtained. As the structure of the unification parameters in terms of these variables is, in general, GUT dependent, the restrictions on F and S will vary from GUT to GUT in spite of having the same set of constraints for all the GUTs. This difference is just a reflection, at the deeper level of the internal (F, S) structure, of the different ways in which unification is occurring in each GUT. This approach is intended to shed some light on the details of this internal structure.

In the rest of this chapter, a minimal background for some of the ideas used in this thesis is given.

1.1 Couplings within the SM

When an attempt is being made to describe the behaviour of a physical system, all the significant information about it has to be put within its Lagrangian function (L). Classically, from here, the evolution equation of the system is obtained, via a Hamilton's Principle which states that the evolution of the system between two points P_1 and P_2 follows the trajectory in its variable space that extremises the integral of L from P_1 to P_2. The solutions to this evolution equation provide the desired precise behaviour of the system. This principle is generalized in Quantum Field Theory [1] to include non-classically-behaved systems. The 'sum over histories' Feynman-Dirac principle, in which one has to integrate over all possible 'paths', is introduced to give account of the quantum behaviour.

There are some principles which shape the structure of a Lagrangian. In the case of Particle Theory, gauge invariance is a must. The invariance of the Lagrangian under infinitesimal transformations of the fields is related to the conservation of probability in the transition amplitude between an initial and a final state of the system. Gauge invariance prevents this amplitude from diverging. This invariance severely restricts the terms allowed within the Lagrangian and gives place, through Nöether's theorem, to some conservation laws. Thus, the physics of the system is outlined and constrained by this required invariance. The symmetry thus obtained finds a faithful and useful representation in Group Theory, which is the suitable language for the symmetries we will be considering here [2].

Gauge bosons are supposed to carry the interactions among the different particles [3]. Once fields have been assigned to particles, the interaction terms in the Lagrangian have the general form:

$$g f_\mu B^\mu$$

where f_μ is the current associated with a fermion and B^μ the field related to the boson. The coefficient g couples the interacting particles and gives a measure of the strength of this coupling. There being three different interactions in the SM (electromagnetic, weak and strong), there are three couplings associated with them in the corresponding Lagrangian. In this sense it is said that the SM is not a unified theory.

Being the ones that contain the information about the strength of the different interactions, these couplings are the *ad hoc* elements in terms of which the idea of unification is expressed and defined. They are deep in the hearts of all GUTs.

The fermion-gauge boson electroweak interaction terms of the SM Lagrangian can be written as follows

$$-gJ_i^\mu W_\mu^i - g'\frac{1}{2}J_Y^\mu B_\mu$$

where

$$J_i^\mu = \overline{\psi}_L \gamma^\mu \frac{1}{2} \tau_i \psi_L$$
$$J_Y^\mu = \overline{\psi} \gamma^\mu Y \psi$$

The g and g' couplings can be expressed in terms of the 'fine structure constant' $\alpha = e^2/4\pi$ and the weak mixing angle θ_w

$$e = \frac{gg'}{\sqrt{g^2 + g'^2}} = g \sin\theta_w = g' \cos\theta_w$$

These are the parameters that are extracted from comparing theoretical expressions with experimental results.

When embedding the SM within a unified model, some normalisation requirements have to be satisfied. The effect of these is to change the U(1) coupling by a factor 5/3. In this sense, it can be said that the SM 'knows' beforehand that it is being unified. There is memory of this event. After embedding, the electroweak couplings are given as follows

$$\alpha_1 = \frac{g_1^2}{4\pi} = \frac{5\alpha}{3\cos^2\theta_w}$$
$$\alpha_2 = \frac{g_2^2}{4\pi} = \frac{\alpha}{\sin^2\theta_w}$$

1.2 The evolution equation

For unification to occur, the strengths of the couplings must be 'comparable'. However, measurements obtained with present accelerators show that this is not the case, but that there is a clear hierarchy among them. How is it then possible to imagine unification within this context? The answer to this lies in one of the most striking features of the couplings: they 'run', that is, they all are functions of the same variable μ. But, what is even more striking is the fact that they become closer as μ increases. It is this behaviour of the couplings what supports the idea of unification. This evolution links the SM to the GUTs' world and provides the connection needed to make the transition.

The equation describing the evolution of the couplings comes from a process that does not seem at first glance to be related at all to them: the renormalisation procedure. Intending to 'cure' the theory from divergences which arise when we consider higher orders in the perturbative expression that approximates the full (all-order) behaviour, a procedure to render all physical quantities (PQ) finite is introduced, and with it a renormalisation scale variable μ. This is only a mathematical tool and it has no physical content. Different procedures could be used to accomplish the same end. Therefore, it is required that any expression corresponding to a physical quantity should be independent of the way we renormalise; that is, there should not be any trace of μ left in it:

$$\frac{\partial(PQ)}{\partial\mu} = 0$$

From this general requirement, the Renormalisation Group Equation (RGE) is obtained [4], one part of which is an evolution equation for the couplings $\alpha(\equiv g^2/4\pi)$:

$$\mu\frac{\partial\alpha}{\partial\mu} = -\beta\alpha^2$$

This is the equation that needs to be solved in order to know the precise behaviour of the couplings as functions of the renormalisation scale μ. Here, β is a function of the couplings, but at lowest order in the perturbation expansion it is a constant.

As a consequence of this μ dependence, the couplings are seen to be non-physical quantities. This is, however, what enables them to fulfil their role as the defining elements for unification. We have here the case of a physical theory whose occurrence is being defined in terms of the convergence of non-physical parameters. Unification is based on an unphysical event.

A complete knowledge of how the actual α evolution is taking place requires an expression for the β-coefficients in the previous equation. These are not fully known. They have to be calculated order by order in an expansion in powers of the coupling. The coefficients for each term in this series form the β-functions of the coupling at each order of approximation. In order to calculate them, all possible loop corrections to the unphysical 'bare' propagators and vertices have to be considered. Through this, the information about the particle content of the model is transferred to the β-functions. This means that the particle content of the model is determining the way the couplings evolve; therefore, any change in this content will affect the coupling evolution.

Since only complete (all-order inclusive) expansions correspond to physical quantities, any truncated expression will carry with it (in general) a renormalisation scheme dependence. That is, the actual values of the coefficients in the expansion will depend on the way we choose to renormalise, although this dependence is cancelled out when the whole series is considered; in fact, these values can be used to label the renormalisation scheme [5]. It is possible to show, however, that the β-functions up to second order in the α expansion are the same for any renormalisation scheme. Even though this is a great advantage in the analysis we will be carrying out in this work, it does not make it free of scheme-dependence. There are other sources which introduce this dependence, namely, the α initial conditions.

After integrating out, initial conditions for the couplings are needed to start the evolution. The quality of these initial values will determine the precision that can be reached in any analysis of unification. Testing GUTs has now been made possible, at an accuracy never seen before, by recent experimental data coming from LEP [6]. This data allows more conclusive results to be achieved.

The values for the couplings are not measured, they are extracted from measurements compared to predictions corresponding to some other parameters. However, these expressions are truncated expansions and, therefore, they depend on the renormalisation scheme. Hence, the initial α values thus obtained are scheme dependent. This opens the door to let the scheme dependence enter the analysis.

1.3 Grand Unified Theories

The idea that at very high energies there is only one interaction (and, hence, just one coupling), from which the ones we know are obtained through successive stages of symmetry breaking, gave birth to a fertile field within Particle Theory [7]. Soon afterwards, efforts were increased in the search for 'the group of the world' and experiments were built in order to test the predictions of these unification theories. An annual meeting was even held to discuss the current status of these theories.

Historically, $\sin^2\theta_w$ has always played an important role within GUTs. Its value can be predicted at the unification scale ($g_1 = g_2$) using previous relationships among the couplings:

$$\sin^2\theta_w = \frac{3}{8}$$

The corresponding $\sin^2\theta_w$ low energy value is obtained evolving this value backwards to the energies at which the couplings have been measured. A criterion to determine the degree of approximation of the GUT considered is provided by comparing this with the $\sin^2\theta_w$ value coming from experimental data. This was the approach taken in most of the early unication analyses (see [7] and references therein).

We will be dealing here with two of the main (and first proposed) GUTs: SU(5) and SO(10). Within these, we will be considering at most one intermediate scale at which some symmetries are introduced, namely, SUSY for SU(5) and L-R for SO(10), although plain SU(5) is also explored. These symmetries involve new particles which will be responsible for changing the evolution of the couplings. In all these cases, no more fermion families are required by the extra fermions coming with the introduced symmetries; these particles can be accommodated within existing families by just adding the needed representations. This only enhances the content of each family and keeps the replication family feature of the SM. Gauge and Higgs bosons find their place in the other group representations. We will now mention some relevant points about these two GUTs.

1.3.1 SU(5)

Grand Unification was first introduced via SU(5). This was presented as 'the group of the world' and it became the paradigm for unified theories. During most of the ten years of the Workshops on Grand Unification (1980-1989), SU(5) was thought to be the GUT. However, towards the end of this epoch, few people kept their hopes on it, due, mainly, to the lack of evidence for the instability of the proton predicted by this theory. The experimental lower bound for this event was soon leaving behind the SU(5) prediction and the current values for the evolved couplings did not give a unique meeting point. Other predictions stemming from unification, although initially claimed to be true at first, were never confirmed either.

The fact that the group structure of the SM fermion content could be fitted so well within SU(5) was one of the attractive features of this GUT. The fundamental 5-dimensional representation can be decomposed as

$$\overline{5} = (\overline{3}, 1) + (1, 2)$$

where the numbers in the brackets correspond to dimensions under SU(3) and SU(2) respectively. The first term in this decomposition is a triplet of colour and a weak singlet, the second is a colour singlet and a weak doublet. The remaining SM group structure can be accommodated in the next higher SU(5) representation, the antisymmetric 10-dimensional, which is decomposed in the following way

$$10 = (3, 2) + (\overline{3}, 1) + (1, 1)$$

This provides exactly the needed group structure to complete the fermion content of the SM, which is one of the main restrictions on the groups and representations intending to go beyond this model.

Proton decay was the inevitable consequence of this embedding. Some of the gauge bosons in this GUT ($24 = 5^2 - 1$, twelve of which correspond to the SM gauge bosons) would mediate quark-lepton transitions among fermions within the same representation, giving rise to the decay process $p \rightarrow e^+\pi^0$.

This larger symmetry group has to be broken down to the SM at some energy scale. The Higgs mechanism, used in the SM to give mass to the weak gauge bosons, provides a very natural candidate to accomplish this transition. Like the other particles, Higgs bosons have to find accommodation within the available representations of the group. Some criteria have to be considered when looking for a place for them. First, the surviving subgroups after breaking the symmetry have to lead in the end to the SM. Second, the fermion content of the SM should not be given mass until the last stage of breaking (this forbids some representations to be used in the first stages and compels some of these to take place in the last one) and, finally, the SU(3) singlet SU(2) doublet Higgs boson of the SM has to be reproduced. All this is satisfied if a 24-dimensional representation is used to break SU(5) down to the SM and a $5 + 45$ representation breaks the SM down to the electrostrong group.

Explanations from first principles of physical features such as charge quantisation, and the relationship between lepton and quark charges, together with desirable characteristics such as the lack of anomalies, all contributed to giving SU(5) the status that it enjoyed for nearly a decade of Workshops on Grand Unification.

1.3.2 SO(10)

Around the same time as SU(5) (some even say half an hour earlier) some other GUTs appeared in the unification arena. SO(10)'s own features made it a very attractive GUT as well. Because of its higher symmetry, even if SU(5) proved to be right, SO(10) could be thought of as the next step in an increasing-symmetry trend context.

The 16-dimensional representation of SO(10) can be decomposed in SU(5) representations as

$$16 = 10 + \overline{5} + 1$$

This gives enough room for the fifteen fermions of each family to be located within it, still leaving one place free. A right-handed neutrino can be ascribed to this singlet. The rank of SO(10) allows intermediate less-symmetric stages which could populate the huge SU(5) desert. One of this is a left-right symmetric extension of the SM, where an extra SU(2) is introduced:

$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

restoring the parity violated by the SM. At this level of symmetry, weak interactions are not 'chiral' anymore, that is, they do not distinguish left from right-handed particle states, and new gauge bosons are required which mediate the interactions with the right helicity states. This extra set of particles modifies the evolution of the couplings on their way to unification.

The way fermions are accommodated within SO(10) again allows proton decay, with the same dominant mode as in SU(5).

Besides parity restoration, L-R models offer an explanation for the smallness of the neutrino mass which, using a 'see-saw' mechanism, is related to the huge Majorana mass of the right handed neutrino. Another novelty found in SO(10) is that anomaly cancellation is derived from first principles from group theory arguments, also making sense of why it happens like that in SU(5) and the SM.

Finally, the Higgs structure needed to spontaneously break this symmetry is much more complex than the one required in SU(5), although the principles used to determine the possible candidates are the same: namely, that the surviving subgroup after the symmetry is broken must give rise to the SM symmetry group, and that SM fermion masses should not appear before the last breaking stage is completed.

1.4 Supersymmetry

The traditional lack of experimental support for supersymmetry [8] within particle theory was strongly shaken three years ago. Using recent data on the initial values of the coupling constants, it was shown that the inclusion of this symmetry within the SM, although not a new idea at all, offered a way out of the proton decay problem posed by the low prediction of this event in SU(5). According to this, some of the new particles predicted by supersymmetry could be 'around the corner'. This would take SUSY from being a beautiful idea on its own and a cure for one of the main problems of any Grand Unification program, to the status of a true symmetry of nature, bringing its eagerly-hoped-for recognition as a physical law.

The huge energy at which unification occurs, although necessary to overcome the lack of evidence for the proton decay prediction, becomes a major troublesome feature of Grand Unification. The energy scales at which the GUT and the SM symmetries break down correspond, if using a Higgs mechanism, to the respective masses of the scalars breaking these symmetries. In order to produce this enormous difference between the two scales, an extremely fine adjustment in the parameters of the Higgs potential is required. The problem here is that, even if this tuning is done by hand (which is considered very unnatural), radiative corrections tend to spoil it and, thus, it becomes necessary to adjust at each order in the perturbative expansion (which is even more unnatural) in order to preserve this gauge hierarchy.

Supersymmetry fits nicely at this point. The fact that, within SUSY, the couplings and masses require no renormalisation other than the one done on the fields, allows this difference to be kept through all orders in perturbation theory, once the initial fine tuning is done at tree level. In this way, even though this symmetry does not solve the problem of the initial adjustment, it avoids all the successive ones. Supersymmetry, that is, remarkably reduces the degree of unnaturalness. This is enough to make GUTs embrace SUSY.

Through this symmetry, fermions and bosons are put on the same footing. To each fermion is assigned a boson with the same mass, and vice versa. This mass degeneracy is maintained as long as the symmetry holds, and, as it has not been detected at low energies, it has to be broken to reproduce the SM particle content. The SUSY partners modify the evolution of the couplings in such a way that the decay of the proton takes place at higher energies, overcoming the experimental limit; and, what is more striking, the required SUSY average scale to achieve this is just around 10^3 GeV.

Chapter 2

SU(5)

2.1 Introduction

The idea of unification came first into being through SU(5) [9]. In this GUT, a huge 'desert' is predicted between the low energy region accurately described by the SM and the region within which unification would occur. This makes the couplings evolve steadily towards their meeting point, since there are no 'new' particles that affect this evolution. In its simplest form, this evolution finds a good representation in the following geometrical scenario.

Let us take two unbroken (i.e. straight) lines that meet at (x_u, y_u) and such that their initial values

$$y_1(x_0) = y_{01}$$

$$y_2(x_0) = y_{02}$$

and slopes m_1 and m_2 are known (See Figure 2.1). If we add a third line with the requirement that it meets the other two at the crossing point, one of the two free and independent parameters of it (either y_{03} or m_3) will be constrained by this. Thus, for each value of m_3 there will be one value of y_{03} (and vice versa) that satisfies 'unification', i.e.,

$$y_{03}(m_3) \text{ and } m_3(y_{03})$$

But the problem could be made more interesting if the slopes had a sort of common 'internal structure', that is, if they depended upon the same set of variables

$$m_1(v_1, \ldots, v_n)$$

$$m_2(v_1, \ldots, v_n)$$

$$m_3(v_1, \ldots, v_n)$$

If so, a change in any v_i $(i = 1, \ldots, n)$ 'component' would modify the three m_i and therefore x_u and y_u would also change. In other words, the effect of taking into account such an internal structure is to make the meeting point a function of the components.

In this way we would have

$$y_{03}(v_1, \ldots, v_n)$$

$$x_u(v_1, \ldots, v_n)$$

$$y_u(v_1, \ldots, v_n)$$

and as a result of this relationship

any restriction on y_{03}, x_u and y_u would impose constraints on the internal structure, that is, on v_i $(i = 1, \ldots, n)$.

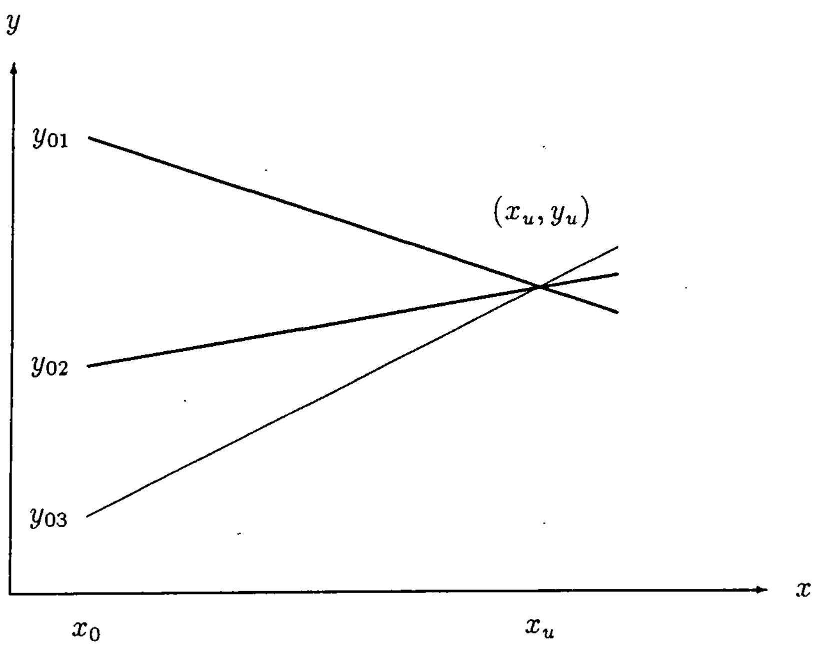

Figure 2.1: A geometrical description of unification in the simplest case

This is the main idea in the work we are presenting here. We intend to analyse the structure of the unification parameters, namely: the value of the coupling at the meeting point ($\alpha = \alpha_G$) and the energy ($\mu = M_G$) at which it takes place, as functions of the corresponding set of internal variables, within the context of the SM embedded in SU(5).

The main equation here comes from the Renormalisation Group Equation (RGE) [15]:

$$\mu \frac{\partial \alpha_i}{\partial \mu} = -\frac{2}{4\pi}\beta_i {\alpha_i}^2 - \frac{2}{(4\pi)^2}\sum_{j=1}^{3}\beta_{ij}{\alpha_i}^2\alpha_j \quad (i = 1,2,3) \qquad (2.1)$$

This gives the evolution of the couplings (α_i, $i = 1,2,3$) as functions of the dimensional variable μ that is introduced with the renormalisation scheme. We will be working in the $\overline{MS}$ scheme [16]. Although μ is an 'arbitrary' variable, we will think of it as a typical energy scale of the process involved.

At 1st order in the approximation (i.e., including only the leading α^2 term on the right hand side) the system of equations (one for each coupling) decouples and can be solved analytically. Further approximations require numerical treatment. A Runge-Kutta-Merson [17] method is used here.

The coefficients in this α expansion have been calculated before [18, 32]. They contain the information about the particle content of the model. Each coefficient is formed by three contributions (see Appendix): one coming from gauge boson loops, another from fermion loops (which can be written in terms of the number of fermion families F) and a third one from Higgs boson loops (in which the number of Higgs boson doublets H appears). These will be playing the role of the internal variables in our analysis and their values will be 'fitted' to the ones allowed by 'general character' restrictions. This is done up to 2nd order in the approximation, i.e., retaining both the $\vartheta(\alpha^2)$ and $\vartheta(\alpha^3)$ terms in Eqn. 2.1.

In order to do this, we keep F and H as variables all the way through the analysis, working out (where possible) expressions for $\alpha_G(F, H)$ and $M_G(F, H)$, on which constraints coming from proton decay and $1/\alpha_G > 0$ are imposed. It turns out that these constraints restrict the values of the number of fermion families (F) and Higgs boson doublets (H).

Proton decay is one of the striking predictions of the GUTs we are considering here and it has become one of the most stringent tests for these theories. The dominant mode $p \rightarrow e^+\pi^0$ predicts a proton lifetime given by $\tau_p \approx {M_G}^4/m_p^5\ {\alpha_G}^2$. Recent experimental data $\tau_p > 5.5 \times 10^{32}$years [14] imply $M_G \approx \vartheta(10^{15} GeV)$. This is the constraint we will be using to decide whether unification is occurring at the 'right' place.

2.2 1st order

At leading order, analytic integration of the evolution equations is possible due to the decoupled character of the system of equations at this order of approximation. The initial conditions are provided by the values of the couplings at the Z^0 mass. They have been obtained with increasing precision in the last few years, in particular from experiments at LEP, and (as they are extracted from experimental data compared to truncated theoretical expressions) carry with them a renormalization scheme dependence. We will be using $\alpha_{01} = 0.016887 \pm 0.000040$, $\alpha_{02} = 0.03322 \pm 0.00025$ [6] and $\alpha_{0s} = 0.107 \pm 0.005$. The reason for the α_{0s} value given here has to do with the range allowed for this coupling at 1st order in the SUSY case, as we will see below. An overview of α_{0s} determinations in different processes is given in Ref. [19], where e^-e^+ annihilation, deep inelastic scattering (DIS) and $p\overline{p}$ collisions are considered. Working in the $\overline{MS}$ scheme, the following average values for $\alpha_s(M_z)$ are reported: 0.120 ± 0.006 (e^-e^+), 0.112 ± 0.005 (DIS) and similar values with higher uncertainties for hadron collisions. It is not clear whether this discrepancy is significant. A world average value $\alpha_s(M_z) = 0.118 \pm 0.007$ is presented. For a critical approach to the treatment of the renormalisation scale μ in this reference see [5]. We will study the effect of choosing a larger α_s starting value in a later section.

The value for the strong coupling α_{0s} can be determined from e^+e^- annihilations. One way to do this is allowed by a precise measurement of the ratio R_z of the hadronic and leptonic partial widths of the Z^0,

$$R_z = \left(\frac{\Gamma_{had}}{\Gamma_{lept}}\right)_{exp} = \left(\frac{\Gamma_{had}}{\Gamma_{lept}}\right)_0 (1 + \delta_{QCD})$$

where δ_{QCD} is a known function of α_{0s} calculated up to third order in powers of this coupling and a expected value for this ratio without QCD corrections has been also calculated. A value for α_{0s} is obtained from this expression using experimental data provided by the four experiments at LEP.

The electroweak mixing angle can be obtained from the relation

$$\sin^2\theta_w = 1 - \frac{M_w^2}{{M_z}^2}$$

using measurements either of the mass ratio or the masses themselves. Some other ways of determining $sin^2\theta_w$ from Z^0 decays are described in [20].

2.2.1 Analysis

In the context of one-loop corrections the RGE for α_i is

$$\mu\frac{\partial\alpha_i}{\partial\mu} = -\frac{1}{2\pi}\beta_i\alpha_i^2 \tag{2.2}$$

from which we obtain

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_{0i}} + \frac{1}{2\pi}\beta_i\ln\frac{\mu}{\mu_0} \tag{2.3}$$

where $\alpha_{0i} \equiv \alpha_i(\mu_0)$ and which

> *allows us to know the value of the coupling at any energy μ if we know it at μ_0 and if β_i is given.*

After assuming that unification occurs, that is,

$$\alpha_1 = \alpha_2 = \alpha_3 = \alpha_G$$

we are left with an overconstrained system: three equations and only two unknowns (namely, M_G and α_G),

$$\frac{1}{\alpha_G} = \frac{1}{\alpha_{01}} + \frac{1}{2\pi}\beta_1 \ln\frac{M_G}{M_z} \tag{2.4}$$

$$\frac{1}{\alpha_G} = \frac{1}{\alpha_{02}} + \frac{1}{2\pi}\beta_2 \ln\frac{M_G}{M_z} \tag{2.5}$$

$$\frac{1}{\alpha_G} = \frac{1}{\alpha_{03}} + \frac{1}{2\pi}\beta_3 \ln\frac{M_G}{M_z} \tag{2.6}$$

Extra freedom can be obtained by thinking of one of the initial conditions as a variable. Since $\alpha_{0s}(=\alpha_{03})$ has the largest experimental uncertainty, it seems to be the best choice for this role. Now, expressions for $\alpha_{0s}(F,H)$, $M_G(F,H)$ and $\alpha_G(F,H)$ are obtained followed by a brief analysis in each case.

$1/\alpha_{0s}(F,H)$

Combining Eqns. 2.4,2.5 and 2.6 by pairs we obtain

$$\begin{aligned} \ln\frac{M_G}{M_z} &= 2\pi\frac{{\alpha_{01}}^{-1} - {\alpha_{02}}^{-1}}{\beta_2 - \beta_1} \\ &= 2\pi\frac{{\alpha_{0s}}^{-1} - {\alpha_{02}}^{-1}}{\beta_2 - \beta_3} \\ &= 2\pi\frac{{\alpha_{0s}}^{-1} - {\alpha_{01}}^{-1}}{\beta_1 - \beta_3} \end{aligned} \tag{2.7}$$

and from here

$$\frac{1}{\alpha_{0s}} = \frac{\alpha_{01}{}^{-1}(\beta_2 - \beta_3) - \alpha_{02}{}^{-1}(\beta_1 - \beta_3)}{\beta_2 - \beta_1}$$

There is an important feature noteworthy in this expression: β_i only appears in differences. As a consequence, the F dependence is cancelled out. This would not happen if F contributions were different for each β_i.

Then, substituting β_i in the previous equation we obtain for the initial value of this coupling

$$\frac{1}{\alpha_{0s}} = \frac{H(-5\alpha_{01}{}^{-1} + 3\alpha_{02}{}^{-1}) - 110(\alpha_{01}{}^{-1} - 3\alpha_{02}{}^{-1})}{2(110 - H)} \quad (2.8)$$

Four remarks follow:

(i) $1/\alpha_{0s}$ does not depend on F. This would not happen if the F contributions were different for each β-function.

(ii) $1/\alpha_{0s} > 0$ constrains the number of allowed Higgs doublets in this unified scenario. As $1/\alpha_{0s}$ decreases when H increases, there is a forbidden region for H. It can not go beyond H_0 (where $1/\alpha_{0s}(H_0) = 0$):

$$H_0 = \frac{110(\alpha_{01}{}^{-1} - 3\alpha_{02}{}^{-1})}{-5\alpha_{01}{}^{-1} + 3\alpha_{02}{}^{-1}} \quad (2.9)$$

$1 \leq H < H_0$ is the first of the restrictions on our 'components'. For α_{01} and α_{02} given as before, we have $H_0 = 16.62$ (see Table 2.1), which means that we need less than 17 Higgs doublets in order to keep $1/\alpha_{0s}$ positive.

(iii) The number of Higgs doublets that we need in order to get unification with α_{0s} given is:

$$H_{\alpha_{0s}} = 110\frac{{\alpha_{01}}^{-1} - 3{\alpha_{02}}^{-1} + 2{\alpha_{0s}}^{-1}}{-5{\alpha_{01}}^{-1} + 3{\alpha_{02}}^{-1} + 2{\alpha_{0s}}^{-1}} \qquad (2.10)$$

This adds one more restriction on H: $H = H_{\alpha_{0s}}$. So, for the α_{0i} values we are working with, we get $H_{\alpha_{0s}} = 7.29$; that is, we need 7 Higgs doublets in order to get unification with the experimental values of α_{01}, α_{02} and α_{0s}.

(iv) If we ignore Higgs loops we are led to the expression

$$\frac{1}{\alpha_{01}} - 3\frac{1}{\alpha_{02}} + 2\frac{1}{\alpha_{0s}} = 0 \qquad (2.11)$$

which can be regarded as a condition on α_{0s} . This gives us $\alpha_{0s} = 0.06433$, which is far away from the experimental value for α_{0s} . Thus, if we assume unification and ignore Higgs loops we are led to an 'unreasonable' value for α_{0s} . Therefore, Higgses can not be neglected.

$\underline{M_G(F, H)}$

The 'structure' of M_G can be seen from Eqn. 2.7. Substituting β_i in this expression we get:

$$\ln\frac{M_G}{M_z} = 2\pi\frac{{\alpha_{01}}^{-1} - {\alpha_{02}}^{-1}}{\frac{22}{3} - \frac{1}{15}H} \qquad (2.12)$$

which, as in the case of α_{0s} , does not depend upon F.

From this expression we see that:

(i) M_G increases when H increases, but this is not fast enough. For the given α_{01} and α_{02}, the values for M_G until $H = 7$ (see Table 2.1) are all forbidden by proton decay. What if we go beyond $H = 7$?

(ii) Solving for H and taking $M_G = 10^{15} GeV$ we get $H = 18.61$ That is, we need 19 or more Higgs doublets in order to agree with constraints coming from proton decay.

But this interval for H does not overlap the one we found previously. This is a fatal impasse for SU(5) at 1st order of approximation.

$1/\alpha_G(F, H)$

Using Eqns. 2.4 and 2.7 the following expression for α_G is obtained:

$$\frac{1}{\alpha_G} = \frac{1}{\alpha_{01}} + \beta_1 \frac{{\alpha_{01}}^{-1} - {\alpha_{02}}^{-1}}{\beta_2 - \beta_1}$$

from where we see that unlike α_{0s} and M_G, α_G does depend upon F. It is only in this case that there is a β_i appearing alone.

Substituting for β_i we have

$$\frac{1}{\alpha_G} = \frac{1}{\alpha_{01}} + (-\frac{4}{3}F - \frac{1}{10}H)\frac{{\alpha_{01}}^{-1} - {\alpha_{02}}^{-1}}{\frac{22}{3} - \frac{1}{15}H} \qquad (2.13)$$

which shows an F dependence.

Some remarks follow from here:

(i) $1/\alpha_G$ depends linearly on F for each value of H. This dependence is such that, for each H, $1/\alpha_G$ decreases when F increases and so it can go negative. As a consequence

$$\exists\ F_0 \text{ such that } \frac{1}{\alpha_G}(F_0) = 0$$

and therefore $1/\alpha_G > 0$ imposes a constraint on the number of fermion families: $F < F_0$.

(ii) The value of F_0 depends linearly on H. From Eqn. 2.13, using the values for α_{01} and α_{02}, we obtain $F_0 = 11.19 - 0.1767H$. This expression tell us that F_0 changes very slowly with H, and, therefore this constraint on the number of fermion families is not too severe.

2.2.2 Conclusions

From looking at the analytical solution in this 1st order SU(5) case, interesting features coming from the constraints on α_{0s} , α_G and M_G, already mentioned, have arisen:

(i) Non-overlapping restrictions on the number of Higgs doublets are obtained. With the usual values for α_{01} and α_{02} we have

$$\begin{aligned} \frac{1}{\alpha_{0s}} > 0 &\implies H < 16.62 \\ M_G > 10^{15} GeV &\implies H > 18.61 \end{aligned}$$

This contradiction seems to rule out this SU(5) 1st order case.

(ii) The number of fermion families is constrained. The fact that $1/\alpha_G$ changes fast enough with F (the slope is around 5) prevents the number of fermion families becoming too large before $1/\alpha_G$ becomes negative, even though the restriction is not too severe in this case ($F < 11$).

(iii) We cannot ignore Higgs loops, otherwise we are led to a value for α_{0s} totally inconsistent with experiment.

In this way, keeping G, F and H as variables all the way through has proved to be a useful method to gain some information about these 'components' within the model we are working with.

2.3 2nd order

At this order of approximation numerical integration is needed. It is carried out here using a Runge-Kutta-Merson method. We follow the same line of the analysis done at 1st order. The novel feature here is that the system of equations is now a coupled one because of the α^3 term included up to this order in the expansion, which mixes the couplings and therefore the equations. We take the same initials conditions as before and analyse the F and H 'structure' of the same parameters. New coefficients are required [18, 32] and a χ^2 criterion is used to determine when unification is occurring. If we remember that the integration method is providing us with three sets of numbers (one

H	$1/\alpha_{0s}$	M_G(GeV)
1	14.74	7.816×10^{12}
2	13.93	9.867×10^{12}
3	13.10	1.251×10^{13}
4	12.25	1.594×10^{13}
5	11.39	2.039×10^{13}
6	10.51	2.622×10^{13}
7	9.609	3.387×10^{13}
8	8.694	4.398×10^{13}
9	7.762	5.740×10^{13}
10	6.810	7.533×10^{13}
11	5.840	9.938×10^{13}
12	4.850	1.319×10^{14}
13	3.839	1.760×10^{14}
14	2.807	2.363×10^{14}
15	1.754	3.193×10^{14}
16	0.6778	4.341×10^{14}
17	-0.4212	5.942×10^{14}

Table 2.1: $1/\alpha_{0s}(H)$ and $M_G(H)$ at 1st order of approximation in SU(5).

for each α_i at each value of μ), and think of unification as occurring at the point (value of μ) where the three α_i's have the 'closest approach' to each other, we will realize that a χ^2-test is a good 'measure' to determine if unification is occurring. Even though we are not comparing experimental points on the same curve, the fact that χ^2 takes error into account makes it good enough for our purposes.

The χ^2 for a set y_i of n experimental data, each one with error σ_{y_i}, is given by

$$\chi^2(x) = \sum_{i=1}^{n} \left(\frac{y_i(x) - \overline{y}(x)}{\sigma_{y_i(x)}} \right)^2$$

where

$$\overline{y}(x) = \frac{1}{n} \sum_{j=1}^{n} y_j(x)$$

Given $\mu \in [\mu_a, \mu_b]$, we will say that unification occurs at $\mu_{min} = M_G$ in this interval, if

$$\chi^2(M_G) \leq \chi^2(\mu) \quad \forall \mu \in [\mu_a, \mu_b]$$

and

$$\frac{d\chi^2}{d\mu}|_{\mu=M_G} = 0$$

this for each set (F, H).

In terms of a diagram, the process we are following is illustrated in Figure 2.2. Numerical results are shown in Tables 2.2 to 2.4.

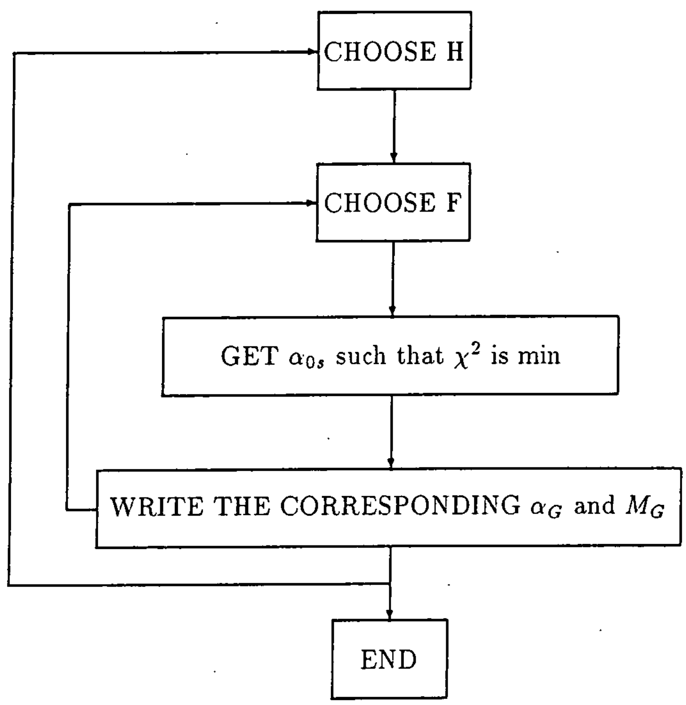

Figure 2.2: Working process

2.3.1 Analysis

$1/\alpha_{0s}(F,H)$

From Table 2.2 we see that:

(i) $1/\alpha_{0s}$ is quasi-independent of F. $1/\alpha_{0s} \approx$ constant as F increases. This agrees with the 1st order case. The largest deviations from this are obtained for high values of F and H.

(ii) $1/\alpha_{0s}$ decreases as H increases. $1/\alpha_{0s}(H)$ is quasi-linear and just below the 1st order values.

(iii) $\alpha_{0s} = 0.107$ is obtained for $6 < H < 7$. Which is slightly smaller than the 1st order value ($H = 7.29$) for the same α_{01} and α_{02}.

(iv) Being much more sensitive to H than to F, $1/\alpha_{0s} > 0$ constrains the number of allowed Higgs doublets. For instance[1], for $3 \leq F \leq 5$, $1/\alpha_{0s} > 0$ implies $H \leq 14$. This is just below the corresponding 1st order value ($H \leq 16$) .

$M_G(F,H)$

From Table 2.3 we see that:

(i) M_G increases with H (as before), and F (unlike the 1st order case). That is, even though $1/\alpha_{0s}$ almost keeps its F-independence at 2nd order, this is not the case for M_G, which does depend upon F at this order of approximation.

[1]We are extrapolating data in Table 2.2

(ii) Again, $M_G \geq 10^{15} GeV$ constrains the number of Higgs doublets. Before H around 12 we do not get M_G values greater than 10^{15}, and for $3 \leq F \leq 5$, $M_G \geq 10^{15}$ implies $H \geq 15$. As in the 1st order case, this lack of overlapping in the intervals for H that comes from the two previous constraints, continues to be the main problem for SU(5).

$1/\alpha_G(F, H)$

From Table 2.4 we have that:

(i) $1/\alpha_G$ has the same behaviour as before, namely: $1/\alpha_G$ decreases as both F and H increase, and is evidently much more sensitive to F than to H. Besides this, it is the most sensitive parameter to F.

(ii) Therefore, $1/\alpha_G > 0$ imposes a constraint on the number of families: $F < F_0$, where $1/\alpha_G(F_0) = 0$. As at 1st order, the value of F_0 depends upon H: F_0 decreases when H increases. It goes from around 10 for $H = 1$, to around 7 for $H = 14$, showing that this 2nd order case is a little bit more restrictive than the 1st order one, but not too much (at 1st order we go from $F_0 \approx 11$ for $H = 1$, to $F_0 \approx 8$ for $H = 18$). Again, F_0 changes very slowly with H.

2.3.2 Conclusions

In summary, all these numerical results (coming from the integration of the set of coupled equations provided by the RGE) give the following features for the 2nd order SU(5) case when the constraints on α_{0s}, α_G and M_G are taken into account:

(i) The behaviour of M_G, α_G and α_{0s} as functions of F and H follows a very similar pattern to the one we found in the 1st order analytical case. There are slight differences concerning the F-dependence of the parameters and a small shift in their values, but the shape of these functions is basically retained at this higher order.

(ii) We have the same main problem as in the 1st order case. With α_{01} and α_{02} as before, we get, for $3 \leq F \leq 5$, that:

$$\frac{1}{\alpha_{0s}} > 0 \implies H \leq 14$$
$$M_G > 10^{15} \implies H \geq 15$$

So, at least for $F \in [3, 5]$, it is not possible to satisfy both constraints simultaneously.

(iii) There is a limit on the allowed number of fermion families. This 2nd order case is a little more restrictive than the 1st order one ($F < 10$ *vs* $F < 11$). Again, the constraint comes from $1/\alpha_G$, which is very sensitive to the change in F.

From these remarks we see that SU(5) 2nd order case is not far from its predecessor in general features, values and troubles (see Fig. 2.3).

It does not seem to be more fortunate than it. This is not surprising because, at the level of the evolution equation (Eqn. 2.1), corrections coming from 2nd order terms are of the order of 3% of the 1st order ones (for α_s) and less for the other two α_i. Therefore, no qualitative difference is introduced by taking into account higher order terms in the expansion.

		F					
		3	4	5	6	7	8
H	1	14.29	14.29	14.29	14.29	14.29	14.49
	2	13.33	13.33	13.33	13.51	13.51	13.51
	3	12.5	12.5	12.5	12.5	12.66	12.66
	4	11.63	11.63	11.63	11.63	11.63	11.76
	5	10.64	10.64	10.64	10.75	10.75	10.87
	6	9.709	9.709	9.709	9.804	9.804	10
	7	8.696	8.696	8.772	8.772	8.849	9.091
	8	7.692	7.752	7.752	7.812	7.936	
	9	6.667	6.711	6.711	6.803	6.944	
	10	5.618	5.650	5.682	5.780	5.988	
	11	4.504	4.545	4.587	4.739	5.050	
	12	3.333	3.390	3.496	3.690	4.202	
	13	2.066	2.174	2.342	2.681		
	14	0.4854	0.7518	1.136	1.818		

Table 2.2: Values of $1/\alpha_{0s}(H, F)$ at 2nd order SU(5).

		F					
		3	4	5	6	7	8
H	1	0.008077	0.009244	0.01107	0.01365	0.01843	0.02682
	2	0.01058	0.01229	0.01472	0.01762	0.02378	0.03843
	3	0.01325	0.01563	0.01871	0.02378	0.03210	0.05509
	4	0.01710	0.02017	0.02451	0.03163	0.04533	0.08016
	5	0.02240	0.02682	0.03259	0.04143	0.06211	0.1220
	6	0.02934	0.03513	0.04334	0.05677	0.08640	0.1913
	7	0.03902	0.04671	0.05763	0.07896	0.1238	0.3486
	8	0.05162	0.06180	0.07857	0.1077	0.1793	
	9	0.06865	0.08343	0.1077	0.1520	0.2729	
	10	0.09128	0.1126	0.1475	0.2179	0.4279	
	11	0.1232	0.1543	0.2083	0.3170	0.7343	
	12	0.1688	0.2146	0.2941	0.4825	1.464	
	13	0.2349	0.3031	0.4344	0.7681		
	14	0.3316	0.4476	0.6813	1.358		

Table 2.3: Values of $M_G(F, H) \times 10^{15} GeV$ at 2nd order SU(5).

		F					
		3	4	5	6	7	8
H	1	42.61	37.07	31.42	25.63	19.58	13.19
	2	42.02	36.41	30.68	24.86	18.73	12.07
	3	41.44	35.76	29.98	24.01	17.80	10.93
	4	40.83	35.10	29.23	23.17	16.78	9.721
	5	40.21	34.40	28.45	22.34	15.80	8.40
	6	39.58	33.69	27.67	21.44	14.76	6.960
	7	38.92	32.97	26.87	20.50	13.65	5.142
	8	38.26	32.23	26.02	19.55	12.50	
	9	37.58	31.47	25.16	18.54	11.22	
	10	36.89	30.69	24.27	17.49	9.824	
	11	36.17	29.87	23.32	16.37	8.184	
	12	35.43	29.02	22.34	15.13	6.086	
	13	34.64	28.11	21.25	13.72		
	14	33.76	27.05	19.92	11.92		

Table 2.4: Values of $1/\alpha_G(F, H)$ at 2nd order SU(5).

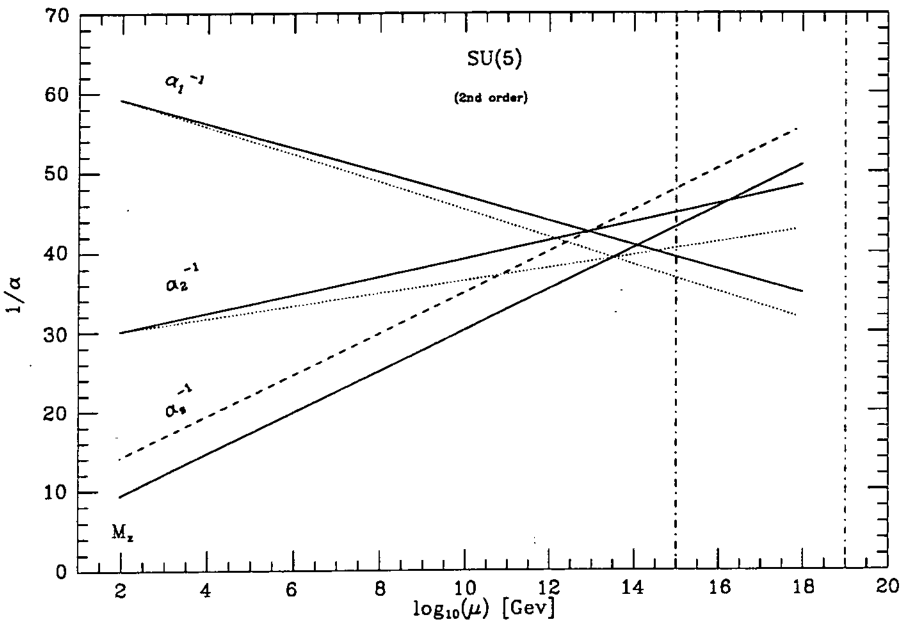

Figure 2.3: Evolution of the inverse of the couplings as functions of μ in the SM embedded in SU(5) at second order of approximation. The solid lines show the evolution in the minimal case ($F = 3$ and $H = 1$) for $\alpha_{01} = 0.016887$, $\alpha_{02} = 0.03322$ and $\alpha_{0s} = 0.107$. Dotted lines correspond to evolving with $H = 6.4$ from the same initial conditions, and the dashed line displays the required value of α_{0s} to satisfy unification in the minimal case if we keep α_{01} and α_{02} as before. The two dot-dash lines show the constraints on M_G imposed by proton decay experimental limits and the Planck mass.

Chapter 3

SUSY SU(5)

3.1 Introduction

Even though the idea of including supersymmetry (SUSY) in a GUT was first suggested some years ago [10], it was only recently, when new data were obtained [11] and a 'precision test' for GUTs was made available, that the importance of SUSY in yielding unification has become clearer [12].

SUSY opens the door to populate the 'desert' once predicted by SU(5), that is, this fermion-boson symmetry allows the possibility of 'new' physics between the dominion of the SM and the unification region. Assuming only one intermediate scale, the picture of the evolution of the coupling constants can be portrayed by the following geometrical set.

Let us consider two uni-broken (at x_s) lines y_1 and y_2, that meet at

(x_u, y_u), and such that their initial values

$$y_{01} \equiv y_1(x_0)$$
$$y_{02} \equiv y_2(x_0)$$

and slopes m_1, m_2 —the slopes between x_0 and x_s— m_{1s} and m_{2s} —the slopes between x_s and x_u— are known (See Fig. 3.1).

If we add a third uni-broken (at x_s) line y_3, and require it to meet the other two at the crossing point, then, of the three free and independent parameters —y_{03}, m_3, m_{3s}— of y_3, one of them will be constrained, leaving two 'degrees of freedom', that is, one more than in the no-broken line case (see previous chapter). This is the first indication of the complexity of the system we are addressing now.

So, given the meeting point of the first two lines, the third one is partially restricted by *'unification'* requirements. We are allowed to choose arbitrarily two of its parameters, but the remaining one is fixed by this choice:

Choosing		Fixes
y_{03}	m_3	m_{3s}
m_3	m_{3s}	y_{03}
m_{3s}	y_{03}	m_3

Therefore, *'unification'* gives place to the following dependence relationship:

$$y_{03}(m_3, m_{3s})$$

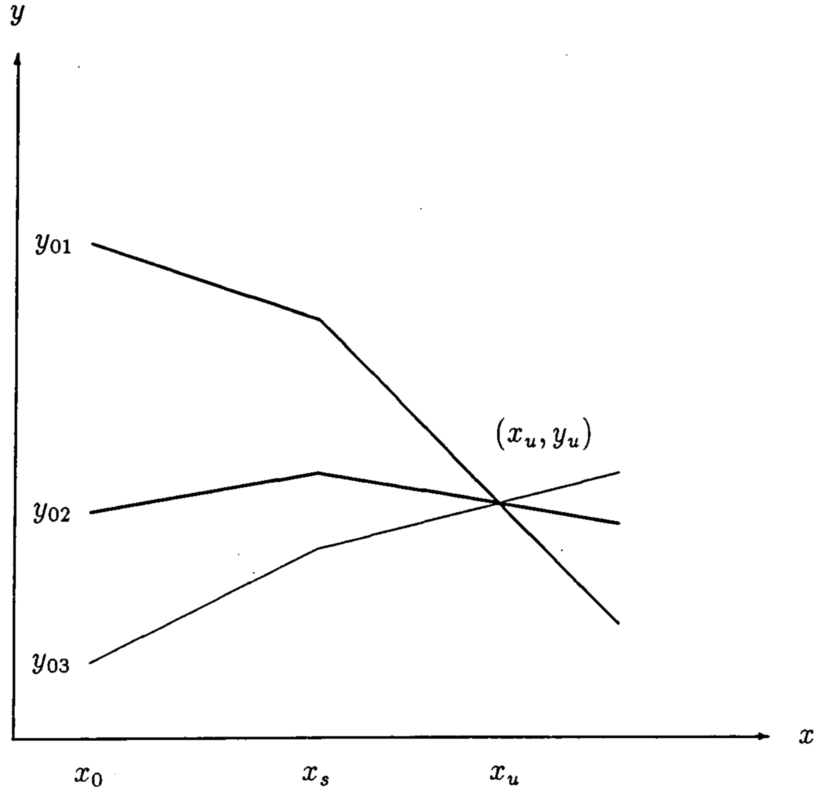

Figure 3.1: A geometrical approach to SUSY SU(5) unification in its simplest form

and the other two that result from inverting this.

Until now, the meeting point has been fixed by y_1 and y_2. This means —if we keep y_{01} and y_{02} constant— that (x_u, y_u) is a function of $(m_1, m_2; m_{1s}, m_{2s})$. Thus, any change on y_3 does not modify the intersection of the other two lines. But, this scenario changes radically if we think of m_i and m_{is} $(i = 1, 2, 3)$ as objects with a sort of 'internal structure', in such way that the m_i's depend on the same set of variables

$(v_1, \ldots, v_n)$, and the m_{is}'s depend on a different —but common among the m_{is}'s— set of variables $(w_1, \ldots, w_n)$:

$$m_i(v_1, \ldots, v_n)$$

$$m_{is}(w_1, \ldots, w_n)$$

If so, (x_u, y_u) becomes a function of these $n + n$ 'basic components':

$$(x_u, y_u)(v_1, \ldots, v_n; w_1, \ldots, w_n)$$

and then, a change in any v_j $(j = 1, \ldots, n)$ will modify all the m_i —the same is true for w_j and m_{is}— and this in turn will affect the meeting point (x_u, y_u).

Doing this we have not only transferred the dependence to a new set of —in principle— independent variables $(v_j; w_j)$, but we have linked the previously independent slope parameters; from now, their values are interconnected. If we want to change any m_i (m_{is}), we have to modify some v_j (w_j), but this will affect the remaining m_i (m_{is}). *They have been tied together by this 'internal structure'.* This is the consequence of introducing such basic components.

In this way, the six slopes are given once we choose a set of values for the components $(v_j; w_j)$ $(j = 1, \ldots, n)$. With these we know the value of the meeting point of the first two lines, and, from here, *unification* fixes the needed value of y_{03} (See Fig. 3.2).

Therefore,

from having introduced an 'internal structure' and assumed that 'unification' occurs, we have ended with:

$$y_{03}(v_j; w_j)$$

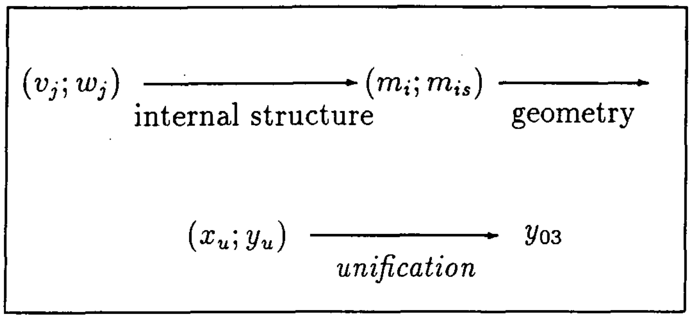

Figure 3.2: Causal chain

$$(x_u, y_u)(v_j; w_j)$$

Thus,

> *constraints on the parameters y_{03} and (x_u, y_u) will restrict the allowed values for the basic components $(v_j; w_j)$ (See Fig. 3.3).*

The final region $(v_j; w_j)_f$ of allowed values will be given by the intersection of the regions coming from each one of the parameters(See Fig. 3.4):

$$(v_j; w_j)_f = (v_j; w_j)_{y_{03}} \cap (v_j; w_j)_{x_u} \cap (v_j; w_j)_{y_u}$$

This provides a criterion to determine whether a GUT is, under the assumptions taken, a 'good' one or not:

$$\text{GUT} = \begin{cases} \text{'not good'} & \text{if } (v_j; w_j)_f = \emptyset \\ \text{'good'} & \text{otherwise} \end{cases}$$

If the former occurs, original model assumptions must be questioned. They would have to be re-thought in order to see whether they are *in-*

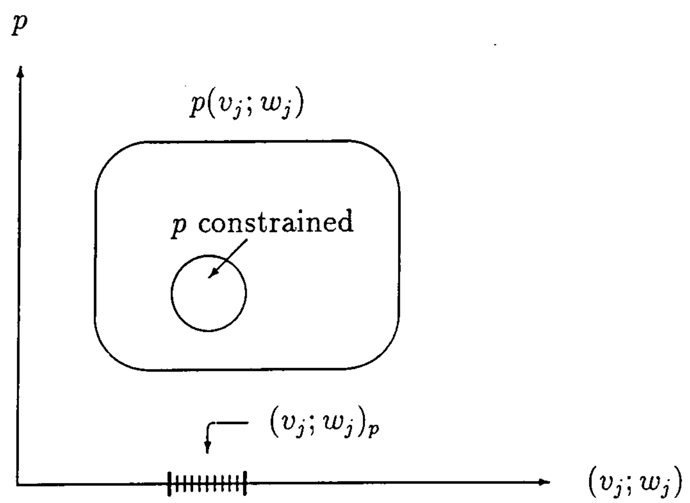

Figure 3.3: Region allowed by constraints on p

trinsic or not to the model, that is, if changing them keeps the previous results or produces very different ones.

Let's talk about x_s, the value of x at which the breaking occurs. Until now, x_s has been kept fixed (This procedure is illustrated in Fig. 3.5). We can release it and take x_s-slices for different values. If we do this, we will be adding a new variable: x_s, and then it will turn out:

$$(x_u, y_u)(v_j; w_j; \underline{x_s})$$

$$y_{03}(v_j; w_j; \underline{x_s})$$

which puts x_s on the same level as our basic variables $(v_j; w_j)$.

There is another thing we can do: *to use x_s as a parameter and not as a variable.*

In order to do this, we can exchange its role with the one of y_{03}. Thus, y_{03} becomes a variable and x_s a function (See diagram in Fig. 3.6). Behind this is the expectation that, given y_{0i}, m_i and m_{is}, $\exists\ x_s$ such that the three lines meet at one point (x_u, y_u). In this alternative way, we have:

$$(x_u, y_u)(v_j; w_j; \underline{y_{03}})$$

$$x_s(v_j; w_j; \underline{y_{03}})$$

or

$$(x_u, y_u)(v_j; w_j)$$

$$x_s(v_j; w_j)$$

if we fixed y_{03}.

Even though these two ways are different as procedures, *they are just alternative approaches to the same set of equations corresponding to*

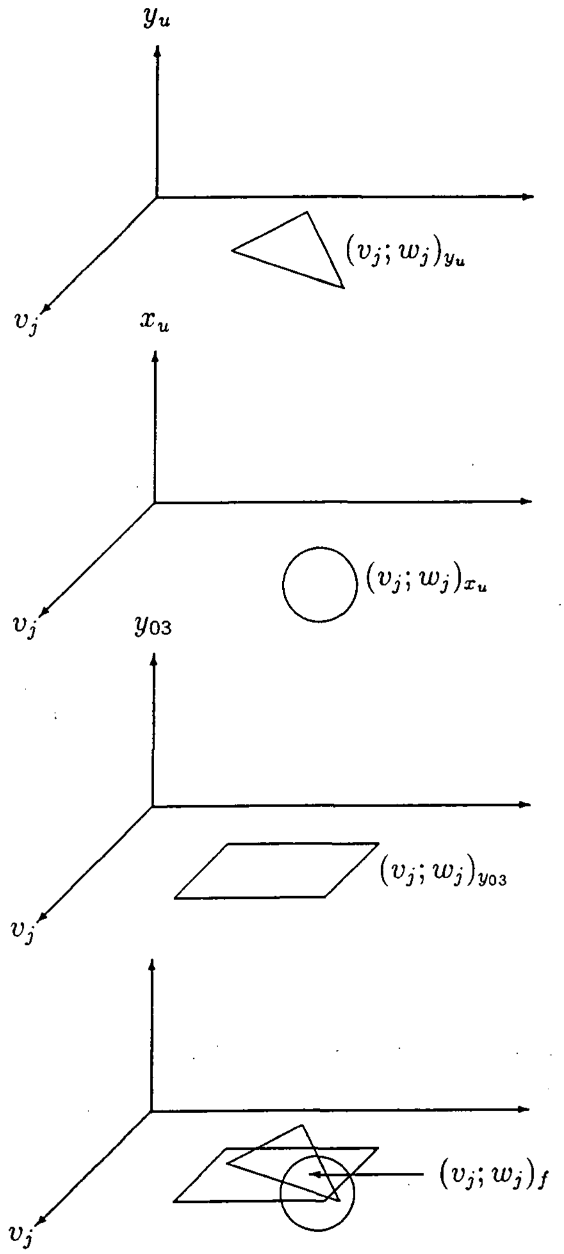

Figure 3.4: Final region allowed by constraints on p

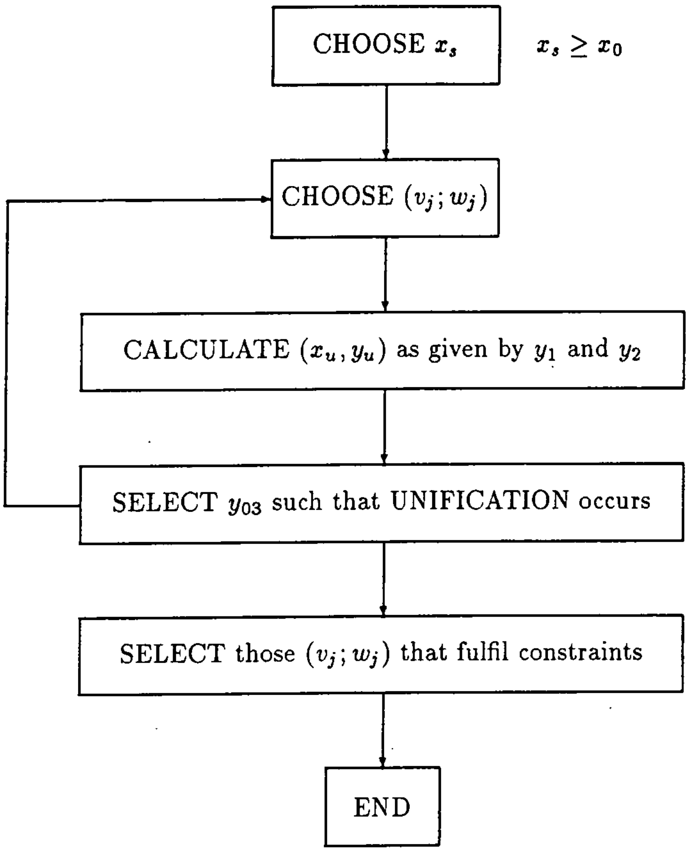

Figure 3.5: x_s as a variable

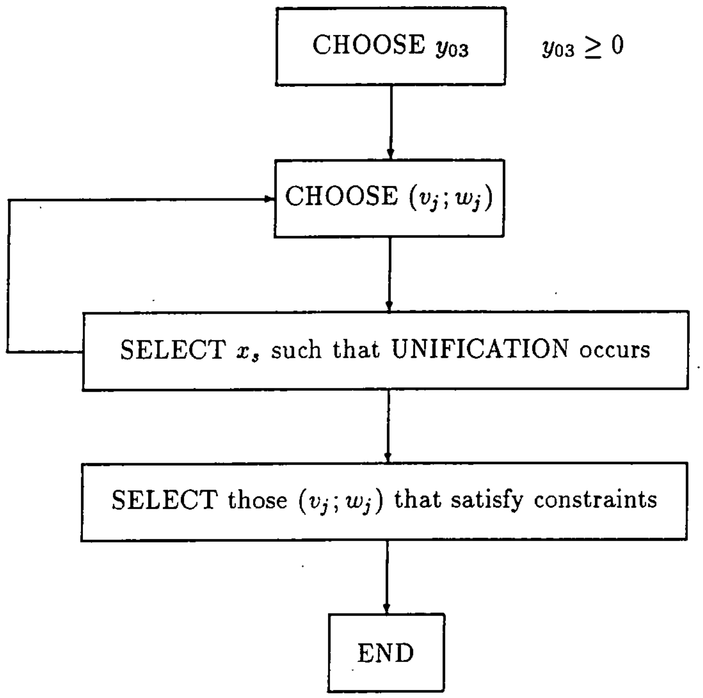

Figure 3.6: y_{03} as a variable

the uni-broken lines y_1, y_2 and y_3. But, looking at the same object from different perspectives, sometimes rewards us in terms of the information that can be obtained.

This geometrical description can be linked to the evolution of the electromagnetic, weak and strong couplings (y_i) in SUSY SU(5) 1st order case if all the SUSY particles have masses around the same value (x_s).

As before, the evolution of the couplings (α_i, $i = 1, 2, 3$) is described by the equation [15]:

$$\mu \frac{\partial \alpha_i}{\partial \mu} = -\frac{2}{4\pi}\beta_i {\alpha_i}^2 - \frac{2}{(4\pi)^2}\sum_{j=1}^{3}\beta_{ij}{\alpha_i}^2\alpha_j \quad (i = 1, 2, 3) \quad (3.1)$$

where μ is a dimensional variable introduced with the renormalisation scheme (we are working in the $\overline{MS}$ scheme [16]) that we will be thinking of as a typical energy scale of the process involved. Again, analytical solution is admitted at 1st order in the approximation but higher orders require a numerical approach. The fact that we are dealing with another GUT is reflected in the actual coefficients of this α expansion [18, 32]. In them, the new SUSY information about the particle content of the model is contained. Once more, there are three contributions to each coefficient (see Appendix): one from gauge boson loops, a second one from fermion loops and a last one from Higgs boson loops.

These coefficients would be the corresponding objects to the slopes. We have now two sets of them: one before x_s, and the other one after.

They will be denoted as follows:

$$\beta\text{-}coefficients = \begin{cases} \beta & \text{if } x_0 \leq x < x_s \\ \beta' & \text{if } x \geq x_s \end{cases}$$

β and $\beta'$are structured by contributions coming from Gauge Boson, Fermion and Higgs boson loops on each side of x_s. *The different particle content —with x_s as the borderline— is reflected in the fact that $\beta \neq \beta'$. This will make the difference.*

In this chapter, we analyse the structure of the unification parameters, namely: the value of the coupling at the meeting point ($\alpha = \alpha_G$) and the energy ($\mu = M_G$) at which it takes place, as functions of the number of fermion families (F) and Higgs boson doublets (H and $H' \equiv H_{SUSY}$), within the context of the SM embedded in SUSY SU(5).

The idea is the same as before: to keep F, H and H' as variables all the way through the analysis, working out (where possible) expressions for $\alpha_G(F, H, H')$ and $M_G(F, H, H')$, on which the following constraints are imposed:

$$\begin{aligned} M_s &\geq M_z \\ \frac{1}{\alpha_{0s}} & \quad \text{within the experimental value} \\ \frac{1}{\alpha_G} &> 0 \\ M_G &\geq 10^{15} \text{ Gev} \end{aligned}$$

From this, limits arise on the region of allowed values for some of our 'basic' variables (F, H, H'), mainly on those which exert a major influence on the parameters.

The decay of the proton is predicted in SUSY SU(5) as well and although other decay modes (highly model dependent) are possible with the inclusion of SUSY [13] (some of them characteristic of this symmetry), we will only consider gauge boson mediated proton decays, where the dominant mode continues being $p \longrightarrow e^{+}\pi^{0}$ implying the same constraint as before: $M_G \approx \vartheta(10^{15}GeV)$.

3.2 1st order

The inclusion of SUSY in the analysis modifies the way the couplings evolve through changing the values of the coefficients in the expansion [32], which, as we have said before, depend upon the particle content of the model. If we assume that all the SUSY particle content of this model occurs at one point: the 'SUSY scale' M_s, only one extra variable is introduced. This leaves us, after integration and having assumed unification, a decoupled system of three equations with three unknowns: M_G, α_G and M_s.

The same initial values for the couplings and restrictions on M_G and α_G apply here, and in addition M_s is required to satisfy $M_z < M_s < M_G$. We will also be considering a more recent set of $\alpha_{0i}{}^{\star}$ values ($\alpha_{01}{}^{\star} = 0.017045 \pm 0.000036$, $\alpha_{02}{}^{\star} = 0.03365 \pm 0.00022$ [21] and $\alpha_{0s}{}^{\star} = 0.110 \pm 0.007$) in order to get an idea of the sensitivity of the results to the initial conditions α_{0i}. The reason for the α_{0s} and $\alpha_{0s}{}^{\star}$ values given here has to do with the range allowed for this coupling at 1st order in this SUSY case, as we will see below. At least one extra Higgs boson doublet is required by SUSY. This increases the number of 'internal variables' to three: F, H and H' (with H' the number of Higgs boson doublets in the supersymmetric regime), one more than in the non-SUSY case, making the analysis slightly more complicated.

Defining

$$\beta_t(\mu) \equiv \begin{cases} \beta & \text{if } \mu_0 \le \mu < M_s \\ \beta' & \text{if } \mu \ge M_s \end{cases}$$

we can write

$$\mu \frac{\partial \alpha_i}{\partial \mu} = -\frac{1}{2\pi}\beta_t(\mu){\alpha_i}^2 \tag{3.2}$$

which includes both sides of M_s. From this we have

$$\frac{1}{\alpha_i} = \frac{1}{\alpha_{0i}} + \frac{1}{2\pi}\beta_t(\mu)\ln\frac{\mu}{\mu_0} \tag{3.3}$$

wich, as before, will play a remarkable role in pointing out general features of the way the system behaves.

3.2.1 Analysis

The basis for our analysis comes from Eqn. 3.3 after imposing the unification condition

$$\alpha_1 = \alpha_2 = \alpha_3 = \alpha_G$$

at $\mu = M_G$.

From here we have

$$\frac{1}{\alpha_G} = \frac{1}{\alpha_{01}} + \frac{1}{2\pi}\beta_1 \ln\frac{M_s}{M_z} + \frac{1}{2\pi}\beta_1' \ln\frac{M_G}{M_s} \quad (3.4)$$

$$\frac{1}{\alpha_G} = \frac{1}{\alpha_{02}} + \frac{1}{2\pi}\beta_2 \ln\frac{M_s}{M_z} + \frac{1}{2\pi}\beta_2' \ln\frac{M_G}{M_s} \quad (3.5)$$

$$\frac{1}{\alpha_G} = \underbrace{\frac{1}{\alpha_{0s}} + \frac{1}{2\pi}\beta_3 \ln\frac{M_s}{M_z}}_{\alpha'_{0i}} + \frac{1}{2\pi}\beta_3' \ln\frac{M_G}{M_s} \quad (3.6)$$

which is, if α_{0i}, β_i and ${\beta'}_i$ are known, a system of three equations with three unknowns: M_s and (M_G, α_G). This means that there is a solution for each set $(\alpha_{0i}; \beta_i; {\beta'}_i)$, or, in terms of the 'internal' structure, for each $(\alpha_{0i}; F, H; F', H')$. *This dependence is the one we are interested in exploring.*

<u>$M_s(F, H, H')$</u>

Combining equations, we get from 3.4 and 3.6

$$\frac{1}{\alpha_{0s}} = \frac{1}{\alpha_{01}} + \frac{1}{2\pi}(\beta_1 - \beta_3)\ln\frac{M_s}{M_z} + \frac{1}{2\pi}(\beta_1' - \beta_3')\ln\frac{M_G}{M_s} \quad (3.7)$$

and from 3.5 and 3.6

$$\frac{1}{\alpha_{0s}} = \frac{1}{\alpha_{02}} + \frac{1}{2\pi}(\beta_2 - \beta_3)\ln\frac{M_s}{M_z} + \frac{1}{2\pi}(\beta_2' - \beta_3')\ln\frac{M_G}{M_s} \quad (3.8)$$

Solving for M_G in 3.8, and substituting in 3.7

$$\frac{1}{2\pi}\ln\frac{M_s}{M_z}[(\beta_2' - \beta_3')(\beta_1 - \beta_3) - (\beta_1' - \beta_3')(\beta_2 - \beta_3)] = \frac{(\beta_3' - \beta_2')}{\alpha_{01}} + \frac{(\beta_1' - \beta_3')}{\alpha_{02}} + \frac{(\beta_2' - \beta_1')}{\alpha_{0s}} \quad (3.9)$$

which gives us $M_s(\alpha_{0i};\beta_i;\beta'_i)$.

Solving for M_s we get:

$$\ln\frac{M_s}{M_z} = \frac{\pi}{-6H + 22H'} \left[30\{\frac{1}{\alpha_{01}} - 3\frac{1}{\alpha_{02}} + 2\frac{1}{\alpha_{0s}}\} + H'\{5\frac{1}{\alpha_{01}} - 3\frac{1}{\alpha_{02}} - 2\frac{1}{\alpha_{0s}}\}\right] \quad (3.10)$$

This equation tells us the value of M_s that is required in order to produce unification with the given initial α_{0i} and for the chosen set of internal variable values. M_s is fixed by these two choices.

Some remarks follow from here:

(i) $\ln M_s$ depends linearly on ${\alpha_{0i}}^{-1}$ and does not depend upon F. The F dependence is cancelled out because of the expansion coefficient subtractions. Since all the fermion contributions are the same, nothing is left in the differences. As a consequence, the fermion contributions are not essential for producing a unification point. So, at 1st order, fermion loops do not contribute to M_s.

(ii) For the minimal SU(5) (H=1, H'=2), this equation becomes

$$\ln \frac{M_s}{M_z} = \frac{4\pi}{19}\left[5\frac{1}{\alpha_{01}} - 12\frac{1}{\alpha_{02}} + 7\frac{1}{\alpha_{0s}}\right] \tag{3.11}$$

From this we see that the SUSY scale is fixed once the initial values α_i are given. However, given the large uncertainty on α_{0s} we can take it as a variable. By doing so, we see that $\ln M_s(1/\alpha_{0s})$ is an increasing linear function (see Table 3.1). As a consequence, there is an maximum allowed value for α_{0s}: the one given by the requirement that M_s does not go below M_z:

$$\frac{1}{\alpha_{0smax}} = \frac{1}{7}\left[-5\frac{1}{\alpha_{01}} + 12\frac{1}{\alpha_{02}}\right]$$

Substituting α_{01} and α_{02}, it turns out that $\alpha_{0smax} = 0.1074$[1]. Therefore, at this order of approximation, unification in the minimal SUSY SU(5) case does not occur for $\alpha_{0s} > \alpha_{0smax}$.

(iii) Ignoring Higgs bosons ($H = 0 = H'$) leads us to the same previous relationship among the α_{0i} :

$$\frac{1}{\alpha_{01}} - 3\frac{1}{\alpha_{02}} + 2\frac{1}{\alpha_{0s}} = 0$$

[1] ${\alpha_{0s}}^{\star}_{max} = 0.1106$, showing that this value is sensitive to the initial conditions ${\alpha_{01}}^{\star}$ and ${\alpha_{02}}^{\star}$, since in this case larger ${\alpha_{0s}}^{\star}$ values are allowed, to produce unification with M_s close to M_z.

Thus, once again, we see that unification makes Higgs boson doublets impossible to ignore. This happens for any three SU(k), SU(n) and SU(m) ($k \neq n \neq m$). Ignoring Higgs bosons we have:

$$\begin{aligned}
\beta_1 &= \frac{11}{3}k - F \qquad \beta'_1 = 3k - F' \\
\beta_2 &= \frac{11}{3}n - F \qquad \beta'_2 = 3n - F' \\
\beta_3 &= \frac{11}{3}m - F \qquad \beta'_3 = 3m - F'
\end{aligned}$$

from where

$$\begin{aligned}
(\beta'_2 - \beta'_3)(\beta_1 - \beta_3) - (\beta'_1 - \beta'_3)(\beta_2 - \beta_3) &= \\
3(n-m)\frac{11}{3}(k-m) - 3(k-m)\frac{11}{3}(n-m) &= \\
11(n-m)(k-m) - 11(k-m)(n-m) &= 0 \;\; \forall k, n, m
\end{aligned}$$

Therefore, cancellation from ignoring Higgs bosons occurs not only for the Standard Model, but for any three SU(i) groups. This cancellation strongly depends upon the form of the 1st order RGE, equal contributions from fermion loops, and the fact that we have the same Casimir factor for the adjoint representation in SUSY and non-SUSY (k, n and m).

(iv) Let us next consider $\ln M_s(H, H')$. Taking α_{01}, α_{02} and α_{0s} as before, and different values for H and H', we get the data in

Table 3.2, from which it follows that although $M_s(H, H')$ is an increasing function of both variables, it is much more sensitive to H' than to H for the values considered here.

The step in M_s when we go from $H' = 2$ to $H' = 3$ is extremely large, taking us from 'light' to 'heavy' SUSY particles. However, the size of the step seems to decrease for higher H'.

$M_G(F, H, H')$

Using equations 3.7 and 3.8 we get the following expression:

$$\frac{1}{2\pi}\ln\frac{M_G}{M_s}\left[(\beta_1-\beta_3)(\beta_2'-\beta_3')-(\beta_2-\beta_3)(\beta_1'-\beta_3')\right] = \frac{(\beta_2-\beta_3)}{\alpha_{01}}+\frac{(\beta_3-\beta_1)}{\alpha_{02}}+\frac{(\beta_1-\beta_2)}{\alpha_{0s}} \quad (3.12)$$

If we use $\ln(M_G/M_s) = \ln(M_G/M_z) - \ln(M_s/M_z)$ together with the expression for M_s we got in previous section, this equation can be written in terms of M_z:

$$\begin{aligned}\frac{1}{2\pi}\ln\frac{M_G}{M_z}\left[(\beta_1-\beta_3)(\beta_2'-\beta_3')-(\beta_2-\beta_3)(\beta_1'-\beta_3')\right] = {}& \frac{1}{\alpha_{01}}\left[(\beta_2-\beta_3)-(\beta_2'-\beta_3')\right]\\ &+\frac{1}{\alpha_{02}}\left[(\beta_3-\beta_1)-(\beta_3'-\beta_1')\right]\\ &+\frac{1}{\alpha_{0s}}\left[(\beta_1-\beta_2)-(\beta_1'-\beta_2')\right] \quad (3.13)\end{aligned}$$

M_G as a function of F, H and H' is given by

$$\ln\frac{M_G}{M_z} = \frac{\pi}{-18H+66H'}\Big[(90-110)\{\frac{1}{\alpha_{01}}-3\frac{1}{\alpha_{02}}+2\frac{1}{\alpha_{0s}}\} \\ +(3H'-H)\{5\frac{1}{\alpha_{01}}-3\frac{1}{\alpha_{02}}-2\frac{1}{\alpha_{0s}}\}\Big] \quad (3.14)$$

This equation gives us the energy scale M_G at which unification occurs for our choices of initial values α_{0i} and set of internal variable values.

From here, the remarks listed below follow:

(i) The behaviour of M_G as a function of the internal variables has many common features with the behaviour of M_s, namely: $\ln M_G$ depends linearly on $1/\alpha_{0i}$ and only subtractions among the coefficients of the expansion appear:

$$\ln M_G(1/\alpha_{0i}; \beta_i - \beta_j; \beta'_i - \beta'_j)$$

Therefore, M_G does not depend upon the number of fermion families, i.e., at 1st order, fermion loops do not influence the value of M_G, and they are therefore irrelevant for determining the energy scale at which unification occurs.

(ii) This equation becomes:

$$\ln\frac{M_G}{M_z} = \frac{5\pi}{114}\left[\frac{1}{\alpha_{01}}+9\frac{1}{\alpha_{02}}-10\frac{1}{\alpha_{0s}}\right] \quad (3.15)$$

for the minimal model ($H = 1$, $H' = 2$). From here, once we know α_{0i}, the scale M_G at which unification occurs is fixed. But, if we allow α_{0s} to change, then M_G is a decreasing linear function of $1/\alpha_{0s}$. This is the reverse of the $M_s(1/\alpha_{0s})$ behaviour, that is, increasing α_{0s} increases M_G (see Table 3.1). Therefore, there is a minimum value of α_{0s} for which $M_G = 10^{15}GeV$. This is given by $\alpha_{0smin} = 0.08911$, which means that any $\alpha_{0s} > 0.08911$ will produce unification with $M_G > 10^{15}GeV$. This gives us a large allowed range [2] for this coupling. This contrasts with the non-SUSY case, in which $1/\alpha_{0smin}$ is so low that it forbids not only the experimental values but also any positive one.

(iii) Ignoring Higgs boson loops takes us to the same contradictory relationship (among the α_{0i}) we had in the corresponding M_s case.

(iv) If we go beyond the minimal Higgs boson content and look at $M_G(H, H')$, we see (Table 3.2) that the lowest values for H and H' are favoured by the constraint $M_G > 10^{15}GeV$, since M_G is a decreasing function of both variables.

<u>$1/\alpha_G(F, H, H')$</u>

Taking either Eqn. 3.4, 3.5 or 3.6, and using 3.10 and 3.13, the following expression for α_G is obtained:

$$\begin{aligned}\frac{1}{\alpha_G} &= \frac{1}{\alpha_{0i}} + \frac{1}{2(-18H+66H')} \\ &\left\{(90\beta_i - 110\beta'_i)\left[\frac{1}{\alpha_{01}} - 3\frac{1}{\alpha_{02}} + 2\frac{1}{\alpha_{0s}}\right]\right. \\ &\left. +(3\beta_i H' - \beta'_i H)\left[5\frac{1}{\alpha_{01}} - 3\frac{1}{\alpha_{02}} - 2\frac{1}{\alpha_{0s}}\right]\right\} \quad \forall i = 1,2,3 \quad (3.16)\end{aligned}$$

[2] $M_G < M_{Planck} \approx 10^{19}GeV$ gives $\alpha_{0s} < 0.2204$

which tells us that the value of the coupling at the unification point is completely determined if we have the initial values α_{0i} and know what values the internal variables take.

From here, we note the following:

(i) Although the β-dependence in this case gets more complicated than in the previous ones, $1/\alpha_G$ depends linearly on $1/\alpha_{0i}$, just as happened with $\ln M_s$ and $\ln M_G$.

(ii) If in this equation we choose $H = 0 = H'$, the same former relation among the α_{0i} is obtained (the coefficient of this relation is $\neq 0 \ \forall F \neq 0$), stressing the statement that Higgs boson doublets cannot be ignored at this order in SUSY SU(5).

(iii) For the minimal model ($H = 1$, $H' = 2$ and $F = 3 = F'$), Eqn. 3.16 becomes:

$$\frac{1}{\alpha_G} = \frac{1}{76}\left[165\frac{1}{\alpha_{01}} - 339\frac{1}{\alpha_{02}} + 250\frac{1}{\alpha_{0s}}\right] \tag{3.17}$$

from which we see that, even though α_G is fixed once α_{0i} are given, as a function of α_{0s} it is increasing like M_G. $1/\alpha_G$ changes slowly enough to avoid restrictions on α_{0s} (see Table 3.1).

(iv) Contrary to M_s and M_G, α_G *does* depend on the number of fermion families. At 1st order, this a unique feature of α_G. The reason for this is that now we have subtractions mixing β_i and β'_i. So even though $F = F'$, their coefficients are not the same. To work out what this F dependence is, we extract from each β the part corresponding to the fermion loops, and call the rest $\beta_{|F}$.

Doing this in Eqn. 3.16, we obtain:

$$\begin{aligned}\frac{1}{\alpha_G} &= \frac{1}{\alpha_{0i}} + \frac{1}{2(-18H+66H')} \\ &\quad \Big\{ (90\beta_{i|F} - 110\beta'_{i|F}) \left[\frac{1}{\alpha_{01}} - 3\frac{1}{\alpha_{02}} + 2\frac{1}{\alpha_{0s}}\right] \\ &\quad +(-120F + 220F') \left[\frac{1}{\alpha_{01}} - 3\frac{1}{\alpha_{02}} + 2\frac{1}{\alpha_{0s}}\right] \\ &\quad +(3H'\beta_{i|F} - H\beta'_{i|F}) \left[5\frac{1}{\alpha_{01}} - 3\frac{1}{\alpha_{02}} - 2\frac{1}{\alpha_{0s}}\right] \\ &\quad +(-4FH' + 2F'H) \left[5\frac{1}{\alpha_{01}} - 3\frac{1}{\alpha_{02}} - 2\frac{1}{\alpha_{0s}}\right] \Big\}\end{aligned}$$

which, grouping terms, can be written as:

$$\begin{aligned}\frac{1}{\alpha_G} &= \frac{1}{\alpha_{0i}} + \frac{1}{2(-18H+66H')} \\ &\quad \Big\{3\beta_{i|F}[A] - \beta'_{i|F}[B] - 4F[A] + 2F'[B]\Big\} \qquad (3.18)\end{aligned}$$

where

$$A = \frac{1}{\alpha_{01}}(30+5H') - 3\frac{1}{\alpha_{02}}(30+H') + 2\frac{1}{\alpha_{0s}}(30-H')$$

$$B = \frac{1}{\alpha_{01}}(110+5H) - 3\frac{1}{\alpha_{02}}(110+H) + 2\frac{1}{\alpha_{0s}}(110-H)$$

Taking $H = 1$, $H' = 2$, $F = F'$ and the values of α_{01}, α_{02} and α_{0s} as before, Eqn. 3.18 becomes:

$$\frac{1}{\alpha_G} = 56.12 - 10.36\,F \quad \forall\, i$$

From this we learn that $1/\alpha_G$ depends linearly on the number of fermion families, and decreases when F increases. Therefore, there is a constraint upon the values of F: $F < F_0$, where

$$\frac{1}{\alpha_G}(F_0) = 0$$

We are therefore not allowed to go beyond F_0, and for $H = 1$, $H' = 2$ and the usual α_{01}, α_{02} and α_{0s}, we find $F_0 = 5.416$, which means that no more than 5 fermion families are allowed for this set of values in this 1st order SUSY case. This is a much stronger constraint than the corresponding constraint in the non-SUSY model.

(v) The value of F_0 depends on H and H'. From Eqn. 3.18, taking $F = F'$ and setting $i = 1$, we get the following expression for $1/\alpha_G(H, H')$:

$$\begin{aligned}\frac{1}{\alpha_G} = & \overbrace{\frac{1}{\alpha_{01}} + \frac{1}{4}\left[\frac{1}{\alpha_{01}} - 3\frac{1}{\alpha_{02}} + 2\frac{1}{\alpha_{0s}}\right]}^{no\ (H,H')\ dependence} \\ & - \frac{F}{6(-3H + 11H')}\left\{-50\left[\frac{1}{\alpha_{01}} - 3\frac{1}{\alpha_{02}} + 2\frac{1}{\alpha_{0s}}\right]\right. \\ & \left. + 2(H' - H)\left[5\frac{1}{\alpha_{01}} - 3\frac{1}{\alpha_{02}} - 2\frac{1}{\alpha_{0s}}\right]\right\}\end{aligned}$$

which tell us that, unexpectedly, there are no terms of the form HH' and only the coefficient of F depends on (H, H'). $F_0(H, H')$ is an increasing, but not linear (in contrast to the 1st order non-SUSY case), function of both variables. However, it is much more sensitive to H' than to H (see Table 3.2). The lower the values of H and H', the stronger the restriction on F. This contrasts with the 1st order non-SUSY case, where the constraint on F was more severe for higher values of H. In both cases the variation with H is slow.

$1/\alpha_{0s}(H, H')$

We saw before that $M_s > M_z$ constrains, in the minimal model, the values of α_{0s}: $\alpha_{0s} < \alpha_{0smax}$. Our intention in this section is to find out what happens with this constraint when we allow the model to go beyond the minimal case. In order to do this, we will take M_s as a variable at the level of the internal structure. Thus, from Eqn. 3.10, we obtain:

$$\frac{1}{\alpha_{0s}} = \frac{30\left[\frac{1}{\alpha_{01}} - 3\frac{1}{\alpha_{02}}\right] + H'\left[5\frac{1}{\alpha_{01}} - 3\frac{1}{\alpha_{02}}\right] - \Delta}{-2(30 - H')} \tag{3.19}$$

where

$$\Delta \equiv \frac{\delta}{\pi}(-6H + 22H'),$$
$$\delta \equiv \ln\frac{M_s}{M_z}.$$

It is important to note that all the non-SUSY internal variables are contained in Δ. Two cases will be considered:

(i) $\delta = 0$ case ($\Rightarrow \quad M_s = M_z$)

In this case, the value of $1/\alpha_{0s}$ that satisfies unification only depends upon the SUSY internal variables. For the current values of α_{01} and α_{02}, the coefficients of the two terms in the numerator have opposite sign. Therefore, $\exists\ H_0'$ such that $1/\alpha_{0s}(H_0') = 0$, i.e., for $M_s = M_z$, we cannot go beyond this number of Higgs boson doublets:

$$H_0' = \frac{-30\left[\frac{1}{\alpha_{01}} - 3\frac{1}{\alpha_{02}}\right]}{5\frac{1}{\alpha_{01}} - 3\frac{1}{\alpha_{02}}}$$

Substituting α_{01} and α_{02}, we get $H'_0 = 4.532$ (see Table 3.3), which means that the value of $1/\alpha_{0s}$ that we need to produce unification in this $\delta = 0$ case is very sensitive to H'. We do not have to go too far in H' before $1/\alpha_{0s}(H'_0) = 0$. However, the fastest growth of α_{0s} takes place for $4 < H' \leq H'_0$ (see Table 3.3), which means that α_{0smax} does not increase so much (from 0.107 to 0.474), but at least gives room for larger values. Thus, going beyond the minimal Higgs boson content in the SUSY sector, gives room for higher α_{0smax}.

(ii) $\delta \neq 0$ case.

Restrictions on H' coming from α_{0s} appear in this case as well. Here,

$$H'_0 = \frac{-30\left[\frac{1}{\alpha_{01}} - 3\frac{1}{\alpha_{02}}\right] - 6\frac{\delta}{\pi}H}{5\frac{1}{\alpha_{01}} - 3\frac{1}{\alpha_{02}} - 22\frac{\delta}{\pi}}$$

is the value of H' for which $1/\alpha_{0s} = 0$, at $\ln(M_s/M_z) = \delta$ and with H Higgs boson doublets in the non-SUSY sector. From this we see that H'_0 depends on the value of M_s. Taking the usual α_{01} and α_{02}, and different values of δ, the data in Table 3.4 are obtained. From here, it follows that H'_0 grows as δ is increased. In fact, it goes to infinity when δ approaches $\delta_\infty = 29.38$ (which corresponds to $M_s = 5.268 \times 10^{14} GeV$). This means that higher SUSY scales allow more Higgs boson doublets in the SUSY sector. For each δ, H'_0 decreases slowly with H. Thus, the constraint on H' moves quickly when larger values of M_s are considered. Beyond δ_∞ there is no longer any constraint.

3.2.2 Conclusions

Summarizing, although we have got more 'internal variables' in this case, the additional complications are not too severe. Some features we obtained in the non-SUSY case are still valid, and some new ones are obtained:

(i) The impasse we reached in non-SUSY SU(5) concerning the lack of overlapping in H intervals coming from different constraints has been overcome with the inclusion of SUSY. This has its source in how M_G depends on the number of Higgs boson doublets. If this had been the case in non-SUSY SU(5), we would have obtained an overlap for low values of H.

(ii) As before, unification makes Higgs bosons impossible to ignore. The same inconsistent relationship among the α_{0i} that was obtained in non-SUSY SU(5), is obtained here. Therefore, SUSY does not make any difference in this respect.

(iii) Again, there is a constraint on the number of fermion families. It comes from $1/\alpha_G$ which, once more, is the only parameter that, at 1st order, depends upon F. The difference is that now the constraint on F is much more severe ($F \leq 5$ for the minimal Higgs boson content and the usual α_{0i} values) than the one we had in the non-SUSY case.

So, the idea of analysing unification through the 'internal structure', seems to prove itself fruitful in this SUSY case as well.

At this point, it is important to recall that we have made an assumption about there being a single SUSY breaking scale M_s. A more realistic approach should consider a non-degenerate supersymmetric spectrum. However, if the masses of the SUSY particles were relatively 'narrow spread', a superposition principle-type (that is, the sum of the individual effects=the effect of the sum of the individuals) could hold. In this case, an 'effective' single SUSY scale would make predictions very close to the 'exact' ones (those obtained considering a SUSY spectrum). This 'effective' scale could be defined as the intersection of straight lines corresponding to the steady evolution of the couplings outside the SUSY spectrum region, that is, far from the SUSY scale. Introducing a realistic spectrum does not affect the slope of these lines although it does change its 'height' and, therefore, the unification point (M_G, α_G). In order to predict masses for the supersymmetric particles, supergravity models have to be considered. These are obtained when promoting SUSY from a global to a local symmetry, that is, local SUSY requires us to include gravity. One of the remarkable features of supergravity grand unification is that the breaking of the SM symmetry can be explained in terms of basic principles of the theory. Within supergravity, the masses of SUSY particles can be written in terms of five parameters, a specific model corresponding to a choice of their values. For a detailed treatment of this point see Ref. [22], where it is shown that "the effect of incorporating the non-degenerate spectrum (is) to increase somewhat the allowed mass of the supersymmetric states for a given value of $\alpha_s(M_z)$". In this reference, the SUSY threshold is found to be restricted to low energies by fine tuning constraints, in agree-

α_{0s}	$\ln M_s$	M_G	$1/\alpha_G$
0.100	3.355	5.390	27.19
0.101	3.156	6.178	26.86
0.102	2.961	7.062	26.54
0.103	2.769	8.052	26.23
0.104	2.582	9.157	25.92
0.105	2.398	10.39	25.62
0.106	2.217	11.76	25.32
0.107	2.040	13.28	25.03

Table 3.1: $\log M_s(\alpha_{0s})$, $M_G(\alpha_{0s}) \times 10^{15} GeV$ and $1/\alpha_G(\alpha_{0s})$ at 1st order SUSY SU(5) in the minimal case.

ment with the results obtained using an 'effective' single SUSY scale approach.

	$H=1$	$H=2$
H'	$M_s(GeV)$	
2	1.096×10^2	1.134×10^2
3	1.840×10^6	5.535×10^6
4	1.670×10^8	5.213×10^8
5	2.250×10^9	6.380×10^9
	$M_G(GeV)\times10^{15}$	
2	13.28	13.17
3	1.528	1.196
4	0.5612	0.4357
5	0.3148	0.2497
	F_0	
2	5.416	5.419
3	6.494	6.644
4	7.155	7.343
5	7.600	7.795

Table 3.2: $M_s(H,H')$, $M_G(H,H')$ and $F_0(H,H')$ at 1st order in SUSY SU(5).

H'	α_{0s}
2	0.1075
3	0.1712
4	0.4745
4.1	0.5820
4.2	0.7541
4.3	1.074
4.53	98.09

Table 3.3: $\alpha_{0s}(H')$ at 1st order SUSY SU(5) with $M_s = M_z$.

3.3 2nd order

As before, including the 2nd order terms in the expansion of the evolution equations prevents the system of equations from decoupling. The corresponding SUSY coefficients have been calculated in reference [32]. The numerical approach used here to analyse the structure at this order of approximation is very similar to the one we employed in the non-SUSY case. The main difference lies in the insertion of the scale M_s, which means a change in the β-coefficients, with the consequent modifications in the program. The process followed is shown in Fig. 3.7. We use the same criterion as before to determine when unification is taking place.

Using this numerical procedure, we will obtain the values of the unification parameters (M_G, M_s and α_G) for $H \in [1, 3]$, $H' \in [2, 5]$ and

	$\delta = 1$	$\delta = 2$	$\delta = 3$
H	H'_0		
1	4.683	4.844	5.017
2	4.673	4.824	4.986
3	4.663	4.804	4.955
4	4.654	4.784	4.924
5	4.644	4.764	4.893
6	4.635	4.744	4.862

Table 3.4: $H'_0(H, \delta \equiv \ln M_s/M_z)$ at 1st order SUSY SU(5).

$F \geq 3$. The initial α_{0i} values we are working with are the same as before, in order to compare with the 1st order case. Results are presented in Table 3.5. Finally, M_G, M_s and α_G are analysed as functions of α_{0s} in the minimal case ($F = 3 = F'$, $H = 1$ and $H' = 2$) for the two sets (α_{0i} and $\alpha_{0i}{}^*$) of initial conditions we are working with (see Table 3.8). All the numbers in the Tables were actually obtained using this numerical method; it is indicated where extrapolations on this basis are made.

3.3.1 Analysis

$M_G(F, H, H')$

At this 2nd order case, M_G as a function of F, H and H' shows the following behaviour:

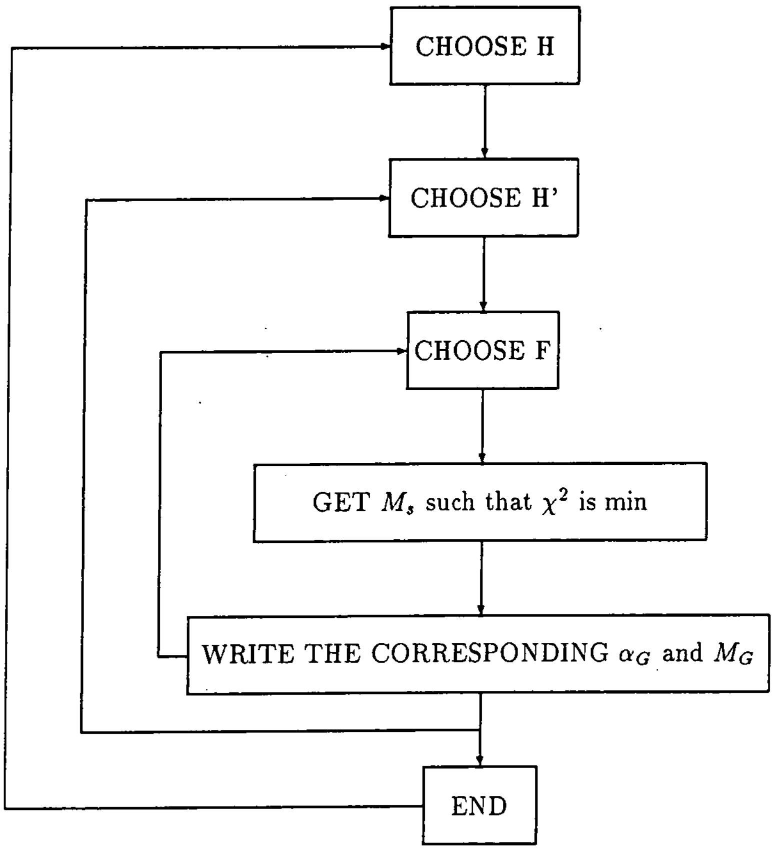

Figure 3.7: Working process

(i) In agreement with 1st order results, M_G decreases with H, for each H' and F, and decreases dramatically with H', for each H and F. But unlike at 1st order, M_G does depend on the number of fermion families: it increases with F, for each H and H'.

(ii) Being more sensitive to H' than to the other two variables, $M_G > 10^{15} GeV$ constrains the allowed number of SUSY Higgs doublets: $H' > 3$ is forbidden by proton decay. For $H' = 3$, low F values are forbidden as well. This order of approximation turns out to be more restrictive for M_G than at 1st order; lower M_G values are obtained here.

Therefore, for the initial α_{0i} values and assumptions taken, minimal Higgs models are favoured by the results of this numerical analysis.

$\underline{M_s(F, H, H')}$

Considering the SUSY scale as a function of the internal variables, we see that:

(i) Again, an F dependence is introduced at this 2nd order level: M_s increases with F, for each H and H'. But in a similar way to what happens at 1st order, M_s increases with H', for each F and H, and increases with H, for each F and H'.

(ii) M_s is much more sensitive to H' than to H and F, and is less sensitive to F than to H: we are taken from light to heavy SUSY particles when we go from $H' = 2$ to $H' = 3$. The same happened at 1st order.

(iii) 2nd order corrections move the SUSY scale further away: $\log M_{s2nd} > \log M_{s1st}$. This changes from around 2 to 3.6 for the minimal case. In this sense, the 2nd order is significant.

$1/\alpha_G(F, H, H')$

As before, interesting features are obtained from considering $1/\alpha_G$:

(i) $1/\alpha_G$ increases with H', for each F and H; increases with H (slower than with H'), for each F and H', and decreases with F, for each H' and H.

(ii) $1/\alpha_G$ is much more sensitive to F than to H and H'. Therefore, $1/\alpha_G > 0$ constrains the number of fermion families. For the minimal Higgs model with the α_{0i} values considered, we are not allowed to go beyond $F = 5$. This is the same restriction we found at 1st order. So we can say that the 2nd order corrections do not significantly alter the $F_{0\,1st}$ value.

(iii) $1/\alpha_G(F)$ shows a quasilinear behaviour.

$M_G(\alpha_{0s})$, $M_s(\alpha_{0s})$ **and** $1/\alpha_G(\alpha_{0s})$

For the minimal case ($F = 3 = F'$, $H = 1$ and $H' = 2$), the following behaviour is obtained (see Table 3.8) for the unification parameters as functions of the coupling α_{0s} (the $\star$ denotes the values that come from using $\alpha_{01}{}^{\star}$ and $\alpha_{02}{}^{\star}$) :

(i) For both sets of initial conditions: M_G increases with α_{0s} and M_s and $1/\alpha_G$ decrease with α_{0s}. Of the three parameters, $1/\alpha_G$ is the least sensitive to the change in α_{0s}, and, therefore, the severity of the constraint on the number of fermion families is not softened when higher values of α_{0s} are considered. On the contrary, because $1/\alpha_G$ decreases with α_{0s}, it could be strengthened if high enough values for α_{0s} are taken. The values of M_s are very sensitive to changes in the initial condition α_{0s} and, as they decrease with this coupling, $M_s > M_z$ imposes a limit on the maximum value allowed for α_{0s}, as happened at 1st order. The difference here is that larger values for this coupling are allowed: $(\alpha_{0s2nd})_{max} > (\alpha_{0s1st})_{max}$, as discussed below.

(ii) Comparing the values obtained for each set: $(1/\alpha_G)^\star$ is slightly larger than $1/\alpha_G$ for each α_{0s}; smaller M_G values are obtained for $\alpha_{0i}{}^\star$ and $(\log M_s)^\star$ is significantly above $\log M_s$ at each α_{0s}. There is a shift ($\Delta \log M_s \approx 0.5$) towards higher values of M_s. The same pattern is found at 1st order. As a consequence, larger α_{0s} values producing unification are allowed for $\alpha_{0i}{}^\star$: $\alpha_{0s}{}^\star{}_{max} \approx 0.122 > \alpha_{0smax} \approx 0.118$[3].

3.3.2 Conclusions

In conclusion, from all these remarks we can see that at 2nd order and with the assumptions and initial values considered, the effects of including these corrections are:

[3]extrapolations based on Table 3.8 are used to obtain these maxima

(i) The shapes and general features of the 1st order case are kept. Shifts and F dependence are the new features introduced with the 2nd order contributions. Some constraints are more severe, but are in any case present in both cases. Perhaps the most important difference lies in the fact that M_s is moved towards higher energy scales when we introduce 2nd order terms in the expansion of the coupling evolution.

(ii) Minimal Higgs models are favoured by proton decay restrictions. The results of this numerical analysis show that this is one of the features kept at the 2nd order level. It has its origin in the M_G internal structure.

(iii) As before, the number of fermion families is constrained. For the case considered, $F \leq 5$. This comes from the strong F dependence of $1/\alpha_G$. Not allowing it to become negative leads us to this restriction on F, which was present at 1st order as well (see Fig. 3.8).

(iv) The unification parameters M_G, M_s and $1/\alpha_G$, are sensitive to the initial conditions α_{01} and α_{02}. For the minimal case ($F = 3 = F'$, $H = 1$ and $H' = 2$), there are significant differences when we change to $\alpha_{01}{}^{\star}$ and $\alpha_{02}{}^{\star}$; namely, there is a shift in M_s at each α_{0s} ($\Delta \log M_s \approx 0.5$) and, therefore, larger α_{0s} values producing unification are allowed ($\approx 0.118 \rightarrow \approx 0.122$, for the maximum allowed). The net effect under this change in the initial conditions seems to be a shift of the behaviour observed using the original α_{01} and α_{02}.

Therefore, at this approximation, the beneficial effects of including SUSY still work; we get unification and, even more, we get it at the 'right' place.

3.4 3rd order

As we said before, the coefficients of the expansion for the evolution equation reflect the particle content of the model currently considered and can be considered as arising from three contributions (see Appendix): one coming from gauge boson loops, another from fermion loops (which is proportional to the number of families F) and a final piece from Higgs loops (proportional to the number of Higgs doublets H and H'). Therefore, we can measure the size of each of these contributions by turning off different pieces of this β- coefficient at the 2nd order level. This will give us an idea of which terms are dominant at this order of approximation. It turns out that the gauge boson contribution is the one that gets closest to the values for M_s obtained using the full 2nd order β-coefficient.

The only non-vanishing pieces in this gauge boson contribution are the diagonal terms corresponding to SU(2) and SU(3) (the one associated with U(1) vanishes due to the absence of photon self-interactions), and so if we assume that at 3rd order of approximation the same pattern is followed, these two terms will give us a good approximation to the real behaviour (the one that comes from using the full 3rd order β-coefficient, that is including the pieces corresponding to fermion and Higgs loops as well).

These three-loop coefficients for SU(N) and SUSY SU(N) can be found in [23] and [24]. If we use them to calculate, for instance, M_s (for different α_{0s} values) at 3rd order of approximation in the minimal case and with the other two couplings fixed, it is found that the 3rd order correction to the 2nd order value of this parameter is really small compared to the 2nd order correction itself (it goes from $2 \to 3.6 \to 3.7$ in $\log M_s$). Therefore, under our assumption, 3rd order corrections do not seem to bring a significant modification to the 2nd order results, compared to effects due to uncertainties on the initial conditions.

		$H = 1$							
		F							
		3	3.5	4	4.5	5	5.5	6	6.5
H'	2	26.70	21.67	16.51	11.35	5.929			
	3	31.36	27.00	22.75	18.20	13.72	8.947		
	4	33.53	29.54	25.70	21.56	17.29	13.13	8.672	
	5	34.81	31.03	27.19	23.521	19.50	15.33	11.24	6.724
		$H = 2$							
H'	2	27.10	22.13	17.06	12.00	6.789			
	3	32.13	27.90	23.79	19.39	15.09	10.57	5.872	
	4	34.32	30.45	26.52	22.50	18.64	14.35	10.11	5.704
	5	35.45	31.78	28.04	24.48	20.59	16.55	12.62	8.380
		$H = 3$							
H'	2	27.61	22.74	17.97	13.09	8.151			
	3	33.17	29.12	24.98	20.99	16.64	12.37	7.744	
	4	35.04	31.32	27.74	23.88	19.92	16.09	12.10	7.748
	5	36.04	32.69	29.08	25.40	21.63	18.02	13.94	9.914

Table 3.5: $1/\alpha_G(F, H, H')$ at 2nd order SUSY SU(5).

		3	3.5	4	4.5	5	5.5	6	6.5
		$H = 1$							
		F							
H'	2	6.535	7.219	8.344	9.775	13.00			
	3	0.9196	1.037	1.134	1.345	1.586	2.036		
	4	0.3702	0.4153	0.4476	0.5501	0.6226	0.7380	0.9524	
	5	0.2179	0.2432	0.2742	0.3001	0.3539	0.4322	0.5306	0.7307
		$H = 2$							
H'	2	5.536	6.087	6.967	8.095	10.19			
	3	0.6678	0.7492	0.8156	0.9620	1.112	1.372	1.843	
	4	0.2688	0.3016	0.3417	0.3950	0.4432	0.5495	0.6847	0.9289
	5	0.1672	0.1857	0.2104	0.2291	0.2688	0.3250	0.3931	0.5178
		$H = 3$							
H'	2	4.442	4.861	5.213	5.907	7.037			
	3	0.4344	0.4873	0.5578	0.6134	0.7417	0.8880	1.181	
	4	0.1962	0.2179	0.2337	0.2688	0.3170	0.3665	0.4454	0.5952
	5	0.1282	0.1355	0.1528	0.1749	0.2042	0.2325	0.2926	0.3739

Table 3.6: $M_G(F, H, H') \times 10^{15} GeV$ at 2nd order SUSY SU(5).

		F							
		3	3.5	4	4.5	5	5.5	6	6.5
		$H=1$							
H'	2	3.6	3.7	3.8	4	4.3			
	3	7.2	7.2	7.3	7.3	7.4	7.5		
	4	8.9	8.9	9	9	9	9.1	9.2	
	5	9.9	9.9	9.9	10	10	10	10.1	10.2
		$H=2$							
H'	2	3.9	4	4.1	4.3	4.6			
	3	7.8	7.8	7.9	7.9	8	8.1	8.3	
	4	9.5	9.5	9.5	9.5	9.6	9.6	9.7	9.9
	5	10.4	10.4	10.4	10.5	10.5	10.5	10.6	10.7
		$H=3$							
H'	2	4.3	4.4	4.6	4.8	5.1			
	3	8.6	8.6	8.6	8.7	8.7	8.8	8.9	
	4	10.1	10.1	10.2	10.2	10.2	10.3	10.4	10.5
	5	10.9	11	11	11	11	11.1	11.1	11.2

Table 3.7: $\log M_s(F,H,H')$ at 2nd order SUSY SU(5).

	α_{01} and α_{02}			$\alpha_{01}{}^{\star}$ and $\alpha_{02}{}^{\star}$		
α_{0s}	$M_G \times 10^{15}$	$\log M_s$	$1/\alpha_G$	$M_G \times 10^{15}$	$\log M_s$	$1/\alpha_G$
0.100	2.684	4.8	28.82	1.450	5.4	29.50
0.101	3.109	4.6	28.47	1.801	5.1	28.98
0.102	3.601	4.4	28.12	2.086	4.9	28.63
0.103	4.171	4.2	27.77	2.237	4.8	28.46
0.104	4.505	4.1	27.59	2.591	4.6	28.11
0.105	5.219	3.9	27.24	3.002	4.4	27.76
0.106	6.045	3.7	26.89	3.477	4.2	27.41
0.107	6.529	3.6	26.71	3.755	4.1	27.23
0.108	7.563	3.4	26.35	4.350	3.9	26.88
0.109	8.760	3.2	26.00	4.698	3.8	26.70
0.110	9.461	3.1	25.82	5.442	3.6	26.34
0.111	10.22	3	25.63	6.304	3.4	25.99
0.112	12.78	2.7	25.10	6.809	3.3	25.81
0.113	13.81	2.6	24.91	7.877	3.1	25.45
0.114	14.91	2.5	24.73	8.518	3	25.27
0.115	16.11	2.4	24.54	9.200	2.9	25.09
0.116	18.60	2.2	24.19	9.937	2.8	24.91
0.117	21.48	2.1	23.99	12.43	2.5	24.37
0.118				13.43	2.4	24.18
0.119				14.50	2.3	24.00
0.120				15.66	2.2	23.81
0.121				17.00	2.1	23.61

Table 3.8: α_{0s} dependence of the unification parameters at 2nd order SUSY SU(5) in the minimal case for α_{01}=0.016887 and α_{02}=0.03322, and for $\alpha_{01}{}^{\star}$=0.017045 and $\alpha_{02}{}^{\star}$=0.03365 (M_G is given in GeV).

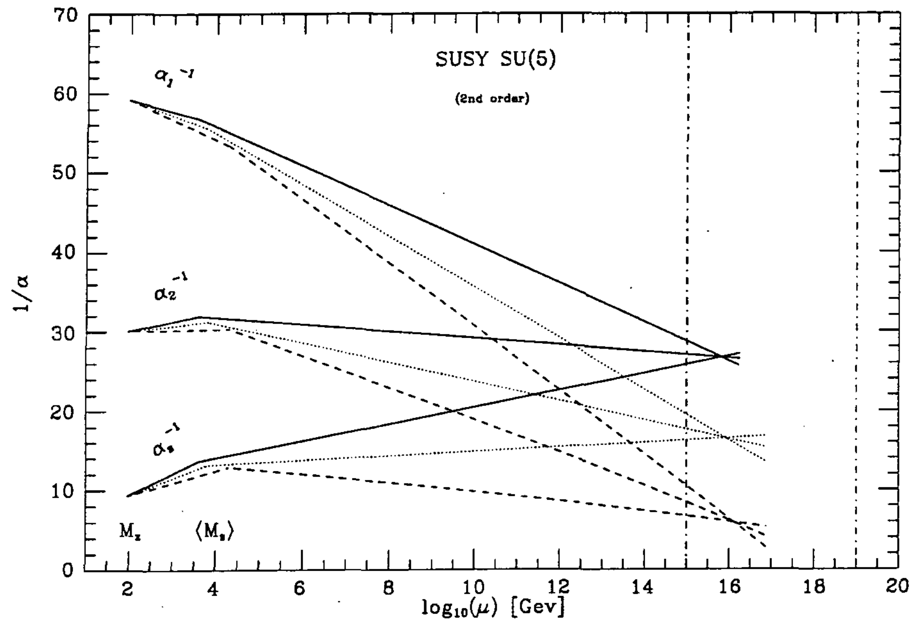

Figure 3.8: Evolution of the inverse of the couplings as functions of μ in the SM embedded in SUSY SU(5) at second order of approximation. The solid lines show the evolution in the minimal case ($F = 3$, $H = 1$ and $H_{SUSY} = 2$) for $\alpha_{01} = 0.016887$, $\alpha_{02} = 0.03322$ and $\alpha_{0s} = 0.107$. Dotted and dashed lines correspond to evolving with $F = 4$ and $F = 5$, respectively, from the same initial conditions. The two dot-dash lines show the constraints on M_G imposed by proton decay experimental limits and the Planck mass.

Chapter 4

L-R SO(10)

4.1 Introduction

In the context of Grand Unification of the strong, electromagnetic and weak interactions of the Standard Model, much attention has been focussed on the SU(5) symmetry group, the simplest model which contains all the known particles and interactions. However, although more complicated, the alternative group SO(10) [25] also has attractive features. In particular, it allows parity (i.e. left-right symmetry) to be restored in a natural way, and provides an explanation of the smallness of the neutrino mass. SO(10)-based Left-Right (L-R) symmetric models (in particular $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$) have been constructed and compared with data [26].

In this chapter, we extend the work of [27] to explore the 'internal structure' of the unification parameters, in particular the energy (M_G) at which unification occurs and the value of the coupling (α_G) at that point, as functions of the number of fermion families (F) and scalar multiplets (S). This is done in the context of the Standard Model (SM) embedded in SO(10) and allowing only one intermediate scale M_R, between M_z and M_G, given by the energy at which the subgroup $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, breaks down to the SM. We proceed as usual keeping F and S as variables throughout the analysis and working out (where possible) expressions for $M_G(F,S)$ and $\alpha_G(F,S)$, on which constraints coming from proton decay and $1/\alpha_G > 0$ are then imposed. We will show that these constraints restrict the number of fermion families and scalars, although not as severely as in SUSY SU(5).

The case we are considering here is only one of the several possible ways of breaking SO(10) down to the SM. Other subgroups give place to more intermediate scales between M_z and M_G. Even the breaking of parity symmetry can be decoupled from the $SU(2)_R$ breaking. Studies on the different chains through which SO(10) can be broken down are found in [27, 28].

As in the previous GUTs, the decay of the proton is predicted also in SO(10) and provides one of the most stringent tests of the model. The dominant decay mode is again $p \rightarrow e^+\pi^0$ [30] implying as before $M_G \approx \vartheta(10^{15}\mathrm{GeV})$. We will use this constraint to decide whether unification is 'acceptable'.

Our basic analysis tool is once more the Renormalization Group Equation (RGE) for the couplings of the theory [15]:

$$\mu\frac{\partial\alpha_i}{\partial\mu} = -\frac{2}{4\pi}\beta_i{\alpha_i}^2 - \frac{2}{(4\pi)^2}\sum_{j=1}^{3}\beta_{ij}{\alpha_i}^2\alpha_j \quad (i=1,2,3) \quad (4.1)$$

Following standard practice, we will work throughout in the $\overline{MS}$ scheme

[16] and although μ is an 'arbitrary' variable, we can regard it as a typical energy scale of the process involved. The analytical approach, possible at first order, gives insight into the way the unification parameters depend on the number of fermion families and scalar multiplets. Including higher-order terms in the evolution equations requires a numerical treatment (a Runge-Kutta-Merson [17] method is used here) and, as we will see, gives only relatively small corrections to the lowest order results.

The coefficients β_i on the right-hand side of Eqn. 4.1 reflect the particle content of the model and keep the same three basic types of contribution as before (see Appendix): one from gauge boson loops, another from fermion loops (which is written in terms of the number of fermion families F) and a third from scalar loops (for each multiplet S of scalars). We can restrict the values of F and S, within the SO(10) model here considered, using the constraints from unification and proton decay mentioned earlier. This is done up to 2nd order in the evolution equations, i.e. retaining both the $\vartheta(\alpha^2)$ and $\vartheta(\alpha^3)$ terms in Eqn. 4.1.

4.2 1st order

Imposition of left-right symmetry adds new particles to the SM. If we assume all of them appear around the same mass scale $\mu = M_R$, then this scale M_R splits the range $[M_z, M_G]$ into two parts, each one with different β-functions and, therefore, different coupling evolutions. The final points in the first stage of this evolution then form the initial conditions for the second part of the evolution. For the GUT we are considering here, α_2 and α_3 continue their evolution through the scale $\mu = M_R$ unchanged, but this is not the case for α_1. There is a discontinuity at M_R coming from normalization requirements [31]. This discontinuity ($\Delta\alpha_1$) depends upon the value of M_R at which it takes place, and is a function of the α_1 and α_2 values just below the discontinuity (M_R^-):

$$\Delta\alpha_1 = f(\alpha_1(M_R^-), \alpha_2(M_R^-))$$

However, the slopes (i.e. the rate of evolution) only depend on the particle content of the model and are independent on the value of M_R.

If we evolve the coupling α_i from M_z to M_R^-, and then from M_R^+ to M_G, we obtain the following equations:

$$\frac{1}{\alpha_i(M_R^-)} = \frac{1}{\alpha_i(M_z)} + \frac{2}{4\pi}\beta_i \ln\frac{M_R^-}{M_z}$$
$$\frac{1}{\alpha_i(M_G)} = \frac{1}{\alpha_i(M_R^+)} + \frac{2}{4\pi}\beta_i' \ln\frac{M_G}{M_R^+}$$

with the 'matching' conditions at M_R:

$$\begin{aligned}
\frac{1}{\alpha_1(M_R^+)} &= \frac{5}{2}\left[\frac{1}{\alpha_1(M_R^-)} - \frac{3}{5}\frac{1}{\alpha_2(M_R^-)}\right] \\
\frac{1}{\alpha_2(M_R^+)} &= \frac{1}{\alpha_2(M_R^-)} \\
\frac{1}{\alpha_3(M_R^+)} &= \frac{1}{\alpha_3(M_R^-)}
\end{aligned}$$

Taking these conditions into account, we find that

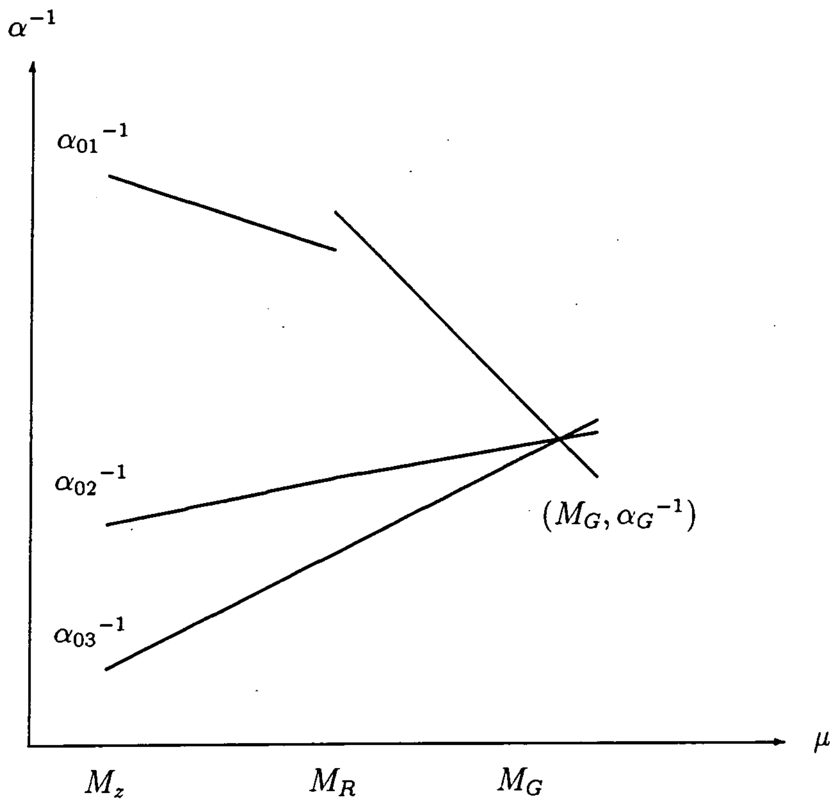

Figure 4.1: α^{-1} evolution in L-R SO(10)

$$\frac{1}{\alpha_3(M_G)} = \overbrace{\frac{1}{\alpha_3(M_z)} + \frac{2}{4\pi}\beta_3 \ln\frac{M_R^-}{M_z}}^{\alpha_i(M_R^+)^{-1}} + \frac{2}{4\pi}\beta_3' \ln\frac{M_G}{M_R^+} \quad (4.2)$$

$$\frac{1}{\alpha_2(M_G)} = \frac{1}{\alpha_2(M_z)} + \frac{2}{4\pi}\beta_2 \ln\frac{M_R^-}{M_z} + \frac{2}{4\pi}\beta_2' \ln\frac{M_G}{M_R^+} \quad (4.3)$$

$$\frac{1}{\alpha_1(M_G)} = \frac{5}{2}\left\{\frac{1}{\alpha_1(M_z)} + \frac{2}{4\pi}\beta_1 \ln\frac{M_R^-}{M_z} - \frac{3}{5}\left[\frac{1}{\alpha_2(M_z)} + \frac{2}{4\pi}\beta_2 \ln\frac{M_R^-}{M_z}\right]\right\} + \frac{2}{4\pi}\beta_1' \ln\frac{M_G}{M_R^+}$$

and from here we can see that the evolution of the couplings from M_z to M_G is (formally) described by the same equations if we define an 'effective' coupling ($\alpha_*(M_z)$) and β-function (β_*) for α_1 as follows:

$$\frac{1}{\alpha_*(M_z)} \equiv \frac{5}{2}\left[\frac{1}{\alpha_1(M_z)} - \frac{3}{5}\frac{1}{\alpha_2(M_z)}\right]$$

$$\beta_* \equiv \frac{5}{2}\left[\beta_1 - \frac{3}{5}\beta_2\right]$$

With this, the evolution of α_i is given by:

$$\frac{1}{\alpha_i(M_G)} = \frac{1}{\alpha_i(M_z)} + \frac{2}{4\pi}\beta_i \ln\frac{M_R^-}{M_z} + \frac{2}{4\pi}\beta_i' \ln\frac{M_G}{M_R^+} \quad (i = *, 2, 3) \quad (4.4)$$

with $\beta_*' \equiv \beta_1'$ and $1/\alpha_*(M_G) \equiv 1/\alpha_1(mg)$. Hence, the discontinuous evolution of α_1 at M_R can be changed into a continuous evolution if we shift the initial condition (from $\alpha_1(M_z)$ to $\alpha_*(M_z)$) and the slope between M_z and M_R (from β_1 to β_*). The rest of the evolution (after M_R) is unaffected.

<u>before</u>		<u>now</u>
M_s	$\longrightarrow$	M_R
α_1	$\longrightarrow$	α_*
β_1	$\longrightarrow$	β_*

An advantage of this change is that it gives a set of evolution equations formally the same as in the lowest order SUSY SU(5) case, and some of the expressions found [29] in that model are also valid here. For example, β_* has the same contribution from fermion loops as β_1 and β_2:

$$\begin{aligned}\beta_* &= \frac{5}{2}\beta_1 - \frac{3}{2}\beta_2 \\ &= \frac{5}{2}\beta_{1|F} - \frac{3}{2}\beta_{2|F} + \frac{5}{2}F_1 - \frac{3}{2}F_2 \\ &\quad \text{with } \beta_{i|F} \equiv \beta_i - F_i \\ &= \beta_{*|F} + F_i \qquad i = 1,2 \\ &\quad \text{if } F_1 = F_2, \\ \Rightarrow &\qquad F_* = F_i\end{aligned}$$

This already reproduces in SO(10) the characteristic feature of F independence obtained in SUSY SU(5) at lowest order.

Within the continuous evolution scheme, we start at a higher point and evolve with a steeper slope ($\beta_* \neq \beta_1$). Comparing $1/\alpha_*$ with $1/\alpha_{01}$

$\alpha_{02}{}^{-1}/\alpha_{01}{}^{-1}$		
$\lVert\beta_2/\beta_1\rVert$		
1/3	2/3	1
2	1.5	1
$\alpha_*{}^{-1}/\alpha_{01}{}^{-1}$		
$\lVert\beta_*/\beta_1\rVert$		

Table 4.1: Values for $\alpha_*{}^{-1}$ and β_*

and β_* with β_1, we have that

$$\frac{1}{\alpha_{01}} > \frac{1}{\alpha_{02}} \Rightarrow \frac{1}{\alpha_*} > \frac{1}{\alpha_{01}}$$
$$\beta_1 < \beta_2 \Rightarrow \beta_* < \beta_1$$

That is, continuous evolution is not just obtained by a simple translation of the ordinary evolution. Rather, it is more of a qualitative change, as is illustrated by the expression for β_*:

$$\beta_* = -11 - \frac{4}{3}F - 0H \tag{4.5}$$

where F is the number of fermion families and H the number of Higgs doublets. This means that the continuous α_1 evolution, obtained by shifting the initial condition and slope for this coupling, corresponds to a self-interacting ($\neq 0$ Gauge Boson contribution) abelian (since $\beta < 0 \Rightarrow$ no asympotic freedom) massless (zero Higgs contribution) gauge theory, which is quite different to the original discontinuous α_1 evolution (non-self-interacting abelian massive gauge theory). Thus, if we want the α_1 evolution equation to have the same form as the other two between M_z and M_R, a different gauge theory for the interaction corresponding to α_1 must be considered.

The Higgs sector of L-R symmetric models is enhanced, since there are new gauge bosons which acquire mass. For the model we are working with here, there is a pair of Higgs doublets (labeled by H') and a pair of Higgs triplets (D) in the region above M_R. For the SM, we will vary the number of Higgs doublets (H) starting from one.

In the following sections we obtain expressions for $M_R(F, H, H', D)$, $M_G(F, H, H', D)$ and $\alpha_G(F, H, H', D)$, obtained after unification is imposed. We use $\alpha_{01} = 0.017045 \pm 0.000036$, $\alpha_{02} = 0.03365 \pm 0.00022$ and α_{0s} within [0.110,0.130]±0.007 [21].

4.2.1 Analysis

$\underline{M_R(H, H', D)}$

The same expression obtained in SUSY SU(5) is valid here:

$$\frac{1}{2\pi} \ln \frac{M_R}{M_z}[(\beta_2' - \beta_3')(\beta_* - \beta_3) - (\beta_1' - \beta_3')(\beta_2 - \beta_3)] = \frac{(\beta_3' - \beta_2')}{\alpha_*} + \frac{(\beta_1' - \beta_3')}{\alpha_{02}} + \frac{(\beta_2' - \beta_1')}{\alpha_{0s}} \tag{4.6}$$

This equation displays the internal structure of M_R, i.e. it gives us the value of M_R that is required in order to produce unification for each set of initial conditions and internal variable values. Substituting for the β-subtractions, we obtain

$$\begin{aligned}\frac{2}{4\pi}\ln\frac{M_R}{M_z}\left[121-\frac{11}{2}H+22H'+11D-\frac{3}{2}HD\right] &= \\ &\left[11\frac{1}{\alpha_*}-33\frac{1}{\alpha_{02}}+22\frac{1}{\alpha_{0s}}\right] \\ &+H'\left[\frac{1}{\alpha_*}-\frac{1}{\alpha_{0s}}\right] \\ &+D\left[2\frac{1}{\alpha_*}-9\frac{1}{\alpha_{02}}+7\frac{1}{\alpha_{0s}}\right] \qquad (4.7)\end{aligned}$$

Some remarks follow from this result:

(i) As was the case for M_s, $\ln M_R$ depends linearly on $1/\alpha_{0i}$, and because of the β-subtractions, M_R does not depend on F or F'. The value of the L-R scale is insensitive to the number of fermion families at this order of approximation.

(ii) However, unlike SUSY SU(5), if Higgs bosons are ignored ($H = 0 = H' = D$) the previous inconsistency between the initial conditions does not occur, i.e. the left-hand side of Eqn. 4.6 does not vanish when we ignore Higgs bosons and therefore unification in this case can take place within the acceptable range for α_{0s}. The values of M_R for which this occurs are given by:

$$\ln\frac{M_R}{M_z} = \frac{2\pi}{11}\left[\frac{1}{\alpha_*}-3\frac{1}{\alpha_{02}}+2\frac{1}{\alpha_{0s}}\right] \qquad (4.8)$$

From which we see that Higgs bosons do *not* seem to be required by unification within a L-R SO(10) GUT.

(iii) For the minimal particle content (H=1=H'=D), the equation becomes

$$\ln\frac{M_R}{M_z} = \frac{4\pi}{21}\left[\frac{1}{\alpha_*} - 3\frac{1}{\alpha_{02}} + 2\frac{1}{\alpha_{0s}}\right] \tag{4.9}$$

which tells us that, once the initial values α_{0i} are given, the L-R scale is fixed. As a function of $1/\alpha_{0s}$, M_R is given by

$$\ln\frac{M_R}{M_z} = 7.744 + 1.197\frac{1}{\alpha_{0s}}$$

which reproduces the main features of the corresponding SUSY SU(5) behaviour, namely, $\ln M_R(1/\alpha_{0s})$ is an increasing linear function. However, the differences (lower slope: 1.2 *vs* 4.6 and positive value [1] at $1/\alpha_{0s}$=0: 7.7 *vs* −41.8) make L-R SO(10) behave in a very different way, since in this case, $M_R > M_z$ does not constrain the allowed values of $1/\alpha_{0s}$, and there is no limit coming from this restriction for α_{0s}. On the other hand, keeping this coupling within the allowed experimental range restricts in a fairly severe way the values that M_R (in GeV) can take:

$$\alpha_{0s} \in [0.110, 0.130] \Rightarrow \log_{10} M_R \in [10.05, 9.322]$$

[1]This comes from the difference between $1/\alpha_{01}$ and $1/\alpha_{02}$, where in this case the 'effective' α_* replaces α_{01}.

This shows that M_R is very far from M_z. Thus there are no 'light' L-R particles ($\log_{10} M_R \approx 9$), and the breaking of the L-R symmetry happens at very high energies.

(iv) The values of M_R for unification without Higgs bosons ($H = H' = D = 0$) are nearly the same as those in the minimal particle content case. Defining the difference between the two cases

$$\begin{aligned}\Delta \ln \frac{M_R}{M_z} &\equiv \left[\ln \frac{M_R}{M_z}\right]_{min\ cont} - \left[\ln \frac{M_R}{M_z}\right]_{no\ H} \\ &= \pi(\frac{4}{21} - \frac{2}{11}) \left[\frac{1}{\alpha_*} - 3\frac{1}{\alpha_{02}} + 2\frac{1}{\alpha_{0s}}\right] \\ &= \ln \frac{[M_R]_{min\ cont}}{[M_R]_{no\ H}}\end{aligned}$$

and using α_{01} and α_{02} as before, we can express this difference as a function of $1/\alpha_{0s}$:

$$\Delta \ln \frac{M_R}{M_z} = 0.3520 + 0.0544\frac{1}{\alpha_{0s}}$$

(v) Different α_{01} and α_{02} values shift the results slightly; however, this does not make any qualitative difference in the M_R 'internal structure', which is not particularly sensitive to the changes in the values of these couplings. This contrasts with the SUSY SU(5) case, where changes in these couplings produced significant changes in M_s.

(vi) For H fixed, M_R is the same whenever $H' = D$. From Eqn. 4.7, setting $E \equiv H' = D$ gives

$$\frac{2}{4\pi} \ln \frac{M_R}{M_z} \left[11(11 + 3E) - \frac{H}{2}(11 + 3E)\right] =$$
$$11 \left[\frac{1}{\alpha_*} - 3\frac{1}{\alpha_{02}} + 2\frac{1}{\alpha_{0s}}\right] + 3E \left[\frac{1}{\alpha_*} - 3\frac{1}{\alpha_{0s}} + 2\frac{1}{\alpha_{0s}}\right]$$

which in turn gives

$$\frac{2}{4\pi}\ln\frac{M_R}{M_z}\left[11-\frac{H}{2}\right] = \frac{1}{\alpha_*}-3\frac{1}{\alpha_{02}}+2\frac{1}{\alpha_{0s}} \tag{4.10}$$

independent of E.

From this equation we see that $H \geq 22$ is forbidden by $1/\alpha_{0s} > 0$:

$$\begin{aligned} 0 &\geq \frac{1}{\alpha_*}-3\frac{1}{\alpha_{02}}+2\frac{1}{\alpha_{0s}} \quad \Rightarrow \\ \frac{1}{\alpha_{0s}} &\leq \frac{1}{2}\left[-\frac{1}{\alpha_*}+3\frac{1}{\alpha_{02}}\right] \quad < 0 \end{aligned}$$

Expressing H in terms of the initial conditions and M_R,

$$\frac{H}{2} = 11 - \frac{\frac{1}{\alpha_*}-3\frac{1}{\alpha_{02}}+2\frac{1}{\alpha_{0s}}}{\frac{2}{4\pi}\ln\frac{M_R}{M_z}}$$

we find that, for $H > 0$ and $1/\alpha_{0s} > 0$, M_R is restricted in this unified $H' = D$ case to $M_R > 1.479 \times 10^5$GeV which means that even for the softest constraints on H and $1/\alpha_{0s}$, M_R is far above present energy scales. If we now require $H \geq 1$ and $\alpha_{0s} = 0.120$, then under the same conditions we obtain $M_R \geq [M_R]_{min\ cont}$ which is a significant change from the previous restriction on M_R. Thus, unification with $H \geq 1$ and α_{0s} within the experimental range, constrains the L-R symmetric particles to be very heavy.

We saw before that $M_R \rightarrow \infty$ as $H \rightarrow 22$, i.e. the number of allowed Higgs doublets in the SM sector cannot go beyond 22. But the actual upper limit is somewhat smaller. Requiring $M_R \leq 1 \times 10^{19}$ implies $H \leq 12.52$, for $\alpha_{0s} = 0.120$. Therefore, $M_R \leq M_{Planck}$ puts an upper limit of about 12 on the number of Higgs doublets in the SM regime in this unified $H' = D$ case. This restriction on H has no counterpart in SU(5) (where restrictions on H come mainly from proton decay). The number of Higgs bosons allowed in the SM regime at each M_R, with the usual values for α_{01} and α_{02}, grows slowly with α_{0s} and M_R, and it is not positive until around $\log_{10} M_R = 10$ (see Table 4.2). Therefore, higher L-R scales lead to more Higgs doublets in the SM, but not more than 12 for the α_{0s} values considered here.

(vii) As a function of H, H' and D, M_R decreases with D and increases (more rapidly) with H and H', see Table 4.3. However, the change of M_R with the non-SM Higgs bosons is not as large as it was for M_s in the SUSY SU(5) case, where SUSY particles are taken from 'light' to 'heavy' scales when H_{SUSY} goes from 2 to 3.

$M_G(H, H', D)$

As in SUSY SU(5), the unification scale is given by

$$\begin{aligned}\frac{1}{2\pi}\ln\frac{M_G}{M_z}[(\beta_2' - \beta_3')(\beta_* - \beta_3) - (\beta_1' - \beta_3')(\beta_2 - \beta_3)] = \\ \frac{1}{\alpha_*}[(\beta_2 - \beta_3) - (\beta_2' - \beta_3')] \\ \frac{1}{\alpha_{02}}[(\beta_3 - \beta_*) - (\beta_3' - \beta_1')] \\ \frac{1}{\alpha_{0s}}[(\beta_* - \beta_2) - (\beta_1' - \beta_2')] \qquad (4.11)\end{aligned}$$

This equation gives the value of M_G that comes from requiring unification for each set of initial conditions and internal variable values. From this, we find

$$\frac{2}{4\pi}\ln\frac{M_G}{M_z}\left[121-\frac{11}{2}H+22H'+11D-\frac{3}{2}HD\right] = 3(11)\left[\frac{1}{\alpha_{02}}-\frac{1}{\alpha_{0s}}\right] +\frac{H}{2}\left[-\frac{1}{\alpha_*}+\frac{1}{\alpha_{0s}}\right] +H'\left[\frac{1}{\alpha_*}-\frac{1}{\alpha_{0s}}\right] +D\left[2\frac{1}{\alpha_*}-9\frac{1}{\alpha_{02}}+7\frac{1}{\alpha_{0s}}\right] \quad (4.12)$$

and therefore:

(i) As for M_G in SUSY SU(5) and M_R, $\ln M_G$ depends linearly on $1/\alpha_{0i}$ and, because of the β-subtractions, is independent of F. The unification scale is not sensitive to the number of fermion families at this order of approximation.

(ii) Unlike SUSY SU(5), there is no inconsistent relationship among the initial conditions if Higgs bosons are ignored ($H = 0 = H' = D$), i.e. in this case unification occurs within the allowed range for α_{0s}. The values of M_G for which this happens are given by

$$\ln\frac{M_G}{M_z} = \frac{6\pi}{11}\left[\frac{1}{\alpha_{02}}-\frac{1}{\alpha_{0s}}\right] \quad (4.13)$$

Thus $M_G(1/\alpha_{0s})$ is a decreasing linear function. For the usual α_{01} and α_{02} values, we find the following expression:

$$\ln\frac{M_G}{M_z} = 50.92 - 1.713\frac{1}{\alpha_{0s}}$$

from which (M_G in GeV),

$$\alpha_{0s} \in [0.110, 0.130] \ \Rightarrow\ \log_{10} M_G \in [17.31, 18.35]$$

which is well inside the safe region allowed for the unification scale M_G. Therefore, Higgs bosons do not seem to be required at all by unification within L-R SO(10). The range allowed for α_{0s} by the constraints from proton decay and the Planck mass is

$$10^{15}\text{GeV} < M_G < 10^{19}\text{GeV} \ \Rightarrow\ 0.082 < \alpha_{0s} < 0.1466$$

which more than covers the range of measured values. In conclusion, the expression obtained (neglecting Higgs bosons) in this unified L-R SO(10) 1st order scheme is consistent with both proton decay ($M_G > 10^{15}$GeV obtains naturally) and experimental bounds on α_{0s}. Thus, ignoring Higgs bosons is allowed by the M_G internal structure, as it was for M_R.

(iii) For the minimal particle content (H=1=H'=D), Eqn. 4.12 becomes

$$\ln\frac{M_G}{M_z} = \frac{\pi}{147}\left[5\frac{1}{\alpha_*} + 48\frac{1}{\alpha_{02}} - 53\frac{1}{\alpha_{0s}}\right] \qquad (4.14)$$

which fixes the unification scale once the initial values α_{0i} are specified. The $1/\alpha_{0s}$ dependence of M_G is given by

$$\ln\frac{M_G}{M_z} = 41.39 - 1.133\frac{1}{\alpha_{0s}}$$

which, as in SUSY SU(5), is a decreasing linear function. The parameters of this function are very close to those of the SUSY SU(5) case (44.93 and −1.378), the main difference being the lower value at $1/\alpha_{0s}$=0, which takes M_G closer, but still above, the proton decay limit (M_G in GeV):

$$\alpha_{0s} \in [0.110, 0.130] \;\Rightarrow\; \log_{10} M_G \in [15.46, 16.15]$$

Note that this is below (by about two orders of magnitude) the corresponding no-Higgs case values. The limits allowed for α_{0s} by proton decay and the Planck mass:

$$10^{15}\text{GeV} < M_G < 10^{19}\text{GeV} \;\Rightarrow\; 0.09970 < \alpha_{0s} < 0.5260$$

are higher than those obtained in the no-Higgs case, but closer to the lower limit for this coupling.

(iv) The values of M_G for unification without Higgs bosons ($H = H' = D = 0$) are very close to those of unification in the minimal particle content case. Defining the difference between the two cases as follows

$$\begin{aligned} -\Delta\ln\frac{M_G}{M_z} &\equiv \left[\ln\frac{M_G}{M_z}\right]_{no\ H} - \left[\ln\frac{M_G}{M_z}\right]_{min\ cont} \\ &= \frac{\pi}{11(147)}\left[-55\frac{1}{\alpha_*} + 354\frac{1}{\alpha_{02}} - 299\frac{1}{\alpha_{0s}}\right] \\ &= \ln\frac{[M_G]_{no\ H}}{[M_G]_{min\ cont}} \end{aligned}$$

and using α_{01} and α_{02} as before, we obtain:

$$-\Delta \ln \frac{M_G}{M_z} = 9.529 - 0.5809 \frac{1}{\alpha_{0s}}$$

From this, we see that the deviation *decreases* with $1/\alpha_{0s}$, contrary to what happens with M_R. For α_{0s}=0.120 we obtain a deviation of 2.036 (in $\log_{10} M_G$), which appears to be large enough to be detectable. This could provide a criterion to discover whether Higgs bosons are there or not.

(v) As a function of H, H' and D (see Table 4.5), M_G is a decreasing function of the three variables, and most sensitive to D. Hence $M_G > 10^{15}$GeV favours the lowest values for these variables. Similar behaviour was obtained in SUSY SU(5).

(vi) $M_G(H, H', D)$ increases with α_{0s}. That is, higher values for this coupling allow more Higgs bosons, while still staying within the 'right' unification range (see Table 4.5).

$\underline{\alpha_G(F, H, H', D)}$

Eqn. 4.4 can be written as:

$$\frac{1}{\alpha_G} = \frac{1}{\alpha_{0i}} + \frac{2}{4\pi}(\beta_i - \beta_i') \ln \frac{M_R}{M_z} + \frac{2}{4\pi}\beta_{i|F}' \ln \frac{M_G}{M_z} - \frac{4}{3}\frac{2}{4\pi} \ln \frac{M_G}{M_z} F \qquad (i = *, 2, 3) \tag{4.15}$$

where all the F dependence in this expression is shown explicitly, since no F contribution remains in the difference $\beta_i - \beta_i'$ (the F terms are the same on both sides of the L-R scale, unlike what happens in SUSY SU(5) which has got a non-vanishing F-dependent contribution coming from this difference). Here we have introduced $\beta'_{i|F}$ as the F-independent part of β'_i.

From this we see that:

(i) As in SUSY SU(5), $1/\alpha_G$ is the only parameter that, at this order of approximation, depends on the number of fermion families. It depends linearly on F.

(ii) This expression attains its simplest form for $i = 3$, since in this case $\beta_3 = \beta'_3$, leaving only the initial condition term and the M_G term. This means that α_3-evolution is 'M_R-blind', which is natural because of the identity of the β-functions. Setting $i = 3$ and substituting for β_3 and M_G we obtain:

$$\begin{aligned}\frac{1}{\alpha_G} &= \frac{1}{\alpha_{0s}} + \frac{11 - \frac{4}{3}F}{[11(11) - \frac{11}{2}H + 22H' + 11D - \frac{3}{2}HD]} \\ &\quad \left\{3(11)\left[\frac{1}{\alpha_{02}} - \frac{1}{\alpha_{0s}}\right] + \frac{H}{2}\left[-\frac{1}{\alpha_*} + \frac{1}{\alpha_{0s}}\right] + H'\left[\frac{1}{\alpha_*} - \frac{1}{\alpha_{0s}}\right]\right. \\ &\quad \left. +D\left[2\frac{1}{\alpha_*} - 9\frac{1}{\alpha_{02}} + 7\frac{1}{\alpha_{0s}}\right]\right\} \qquad (4.16)\end{aligned}$$

This equation exhibits the internal structure of α_G, i.e. it gives the value of the coupling at the unification point for each set of initial conditions and internal variable values.

(iii) For the minimal Higgs content($H = 1 = H' = D$), the above equation becomes:

$$\frac{1}{\alpha_G} = \frac{1}{\alpha_{0s}} + \frac{11-\frac{4}{3}F}{294}\left[5\frac{1}{\alpha_*} + 48\frac{1}{\alpha_{02}} - 53\frac{1}{\alpha_{0s}}\right]$$

which for α_{01}=0.017045, α_{02}=0.03365 and α_{0s}=0.120, is:

$$\frac{1}{\alpha_G} = 64.28 - 6.781\ F$$

showing that $1/\alpha_G$(F) is a decreasing function which becomes negative for $F > 9$. Therefore, again as in SU(5), the number of fermion families is restricted by requiring $1/\alpha_G(F) > 0$. However, in this case the constraint on F is not as severe as it is in SUSY SU(5) (where $F \leq 5$), since now $1/\alpha_G$ starts at a higher value and has an evolution slope which is smaller than in the SUSY SU(5) case.

(iv) For the no-Higgs case (H=0=H'=D) we obtain, from Eqn. 4.16,

$$\frac{1}{\alpha_G} = \frac{1}{\alpha_{0s}} + (3 - \frac{4}{11}F)\left[\frac{1}{\alpha_{02}} - \frac{1}{\alpha_{0s}}\right]$$

which, for the usual initial coupling values, is:

$$\frac{1}{\alpha_G} = 72.49 - 7.776\ F$$

This gives exactly the same constraint on the number of fermion families as the minimal Higss content case, i.e. $F \leq 9$.

(v) Defining the difference between the minimal Higgs content and the no-Higgs cases as

$$-\Delta\frac{1}{\alpha_G} \equiv \left[\frac{1}{\alpha_G}\right]_{no\ H} - \left[\frac{1}{\alpha_G}\right]_{min\ cont}$$

we obtain:

$$-\Delta\frac{1}{\alpha_G} = 8.21 - 0.995\ F$$

which means that the two cases converge and eventually coincide when $F = 8.25$. Ignoring Higgs bosons, therefore, does not have the same effect as in SUSY SU(5).

(vi) Considering the way the coupling at the unification point changes with the internal variables and α_{0s} we find that:

(a) For each α_{0s}: $1/\alpha_G$ is a decreasing function of all the internal variables, being more sensitive to F than to the others. This resembles the behaviour obtained at SUSY SU(5).

(b) $1/\alpha_G$ increases slowly with α_{0s}. This contrasts with the decreasing behaviour obtained in SUSY SU(5).

(c) The maximum number of fermion families (F_0) allowed by the constraint $1/\alpha_G > 0$ decreases slowly with α_{0s} and increases with the rest of the internal variables. This also happens in 1st order SUSY SU(5).

4.2.2 Conclusions

From this first order case analysis the following points emerge:

(i) Even though it is possible to go from a 'jumping' α_1-evolution to a continuous one if we change to the 'effective' coupling α_* and β- function β_*, and in doing this we keep the form of the set of solutions we have before in SU(5) for the 1st order RGE, a quite different behaviour is to be obtained in L-R SO(10).

(ii) M_R seems to have a very different structure compared to M_s(although some features like linear dependence on α_{0i} and F independence are shared) since there is no restriction on the maximum allowable α_{0s} value and α_{0s} within its experimental range requires L-R symmetry to be at very high energies (around $1 \times 10^9 GeV$). Even if we soften the constraint on α_{0s} and only require it to be positive, M_R is beyond M_s. So there seems to be a structural elusiveness in this L-R symmetry. Being so high, a constraint arises from requiring $M_R \leq M_{Planck}$ in the $H' = D$ case: $H \leq 12$.

(iii) M_G does not reveal itself to be very different to its behaviour in SUSY SU(5): linear dependence on $1/\alpha_{0i}$, F independence and 'natural fitness' within the right region allowed by proton decay. Besides this, as it happens in SUSY SU(5) as well, unification within proton decay limit favours low number of Higgses. Luckily, there are no surprises here, except, perhaps, the fact that M_G falls 'naturally' beyond $1 \times 10^{15} GeV$, which is most welcomed. This places L-R SO(10) on the same ground as SUSY SU(5), even though further from close reach.

(iv) α_G seems to be in the middle between M_R and M_G. It keeps previous general features (linear $1/\alpha_{0i}$ dependence, it is the only unification parameter that depends upon F, and linear F dependence) but the actual values diffuse the severity of the restriction that the constraint $1/\alpha_G > 0$ imposes on the number of fermion families in the SUSY SU(5) case. Therefore, even though the restriction seems to be present in 'all' models, its value is model dependent.

(v) One of the main differences of this GUT compared to SUSY SU(5) lies in the role of Higgses. While in the latter they seem to be required by unification, in the former they do not seem to be indispensable at all. Ignoring Higgses in L-R SO(10) does not give rise to any inconsistent relationship among the initial conditions and, furthermore, unification is still 'naturally' achieved within the limits allowed by proton decay and the Planck mass. Although unification with no Higgses ($H = 0 = H' = D$) behaves very much like unification in the minimal Higgs content case ($H = 1 = H' = D$), they may still be distinguished. This could provide a criterion for Higgs existence.

	α_{0s}		
	0.110	0.115	0.120
$\log_{10} M_R$	H		
5	−33.87	−32.45	−31.15
7	−11.70	−10.84	−10.06
9	−2.13	−1.51	−0.95
11	3.21	3.69	4.13
13	6.61	7.01	7.36
15	8.97	9.30	9.61
17	10.71	10.99	11.26
19	12.03	12.28	12.52

Table 4.2: $H(M_R, \alpha_{0s})$ at 1st order in the $H' = D$ case.

H	H'	D	$M_R \times 10^{10}$
0	0	0	0.202
1	1	1	0.451
		2	0.126
		3	0.041
	2	1	1.468
		2	0.451
		3	0.156
	3	1	3.638
		2	1.219
		3	0.451
2	1	1	1.094
		2	0.325
		3	0.109
	2	1	3.321
		2	1.094
		3	0.398
	3	1	7.728
		2	2.792
		3	1.094
3	1	1	2.913
		2	0.936
		3	0.333
	2	1	8.088
		2	2.913
		3	1.135
	3	1	17.43
		2	6.902
		3	2.913

Table 4.3: $M_R(H, H', D)$ (in GeV) at 1st order for α_{0s}=0.120.

α_{0s}	$[M_R]_{minH}$	$[M_R]_{noH}$	M_s
0.110	1.17	0.48	116.1
0.120	0.45	0.20	3.48
0.130	0.21	0.10	0.18

Table 4.4: $M_R(\alpha_{0s}) \times 10^{10}$ (for the minimal and no Higgs content cases) and $M_s(\alpha_{0s})$ in minimal SUSY SU(5) (both at 1st order and given in GeV).

			$\log_{10} M_G$		
H	H'	D	α_{0s}=0.110	α_{0s}=0.120	α_{0s}=0.130
0	0	0	17.3	17.9	18.3
1	1	1	15.5	15.8	16.1
		2	14.6	14.9	15.1
		3	13.9	14.1	14.2
	2	1	15.2	15.5	15.8
		2	14.5	14.7	14.9
		3	13.9	14.0	14.1
	3	1	15.0	15.3	15.6
		2	14.4	14.6	14.8
		3	13.8	14.0	14.1
2	1	1	15.2	15.6	15.9
		2	14.5	14.8	15.0
		3	13.9	14.0	14.2
	2	1	15.0	15.3	15.6
		2	14.4	14.6	14.8
		3	13.8	14.0	14.1
	3	1	14.8	15.1	15.4
		2	14.3	14.5	14.7
		3	13.8	13.9	14.1
3	1	1	15.0	15.4	15.7
		2	14.4	14.6	14.9
		3	13.8	14.0	14.1
	2	1	14.8	15.1	15.4
		2	14.3	14.5	14.7
		3	13.8	13.9	14.1
	3	1	14.6	14.9	15.2
		2	14.2	14.4	14.6
		3	13.7	13.9	14.0

Table 4.5: $M_G(H, H', D)$ for different α_{0s} in the 1st order case.

4.3 2nd order

The effects of introducing the second order terms in the evolution equation (RGE) for the couplings in this GUT are investigated by solving numerically the set of coupled differential equations using a Runge-Kutta-Merson integration method [17]. The values thus found are presented in Tables 4.9 to 4.11 (where the starting values α_{01}=0.017045, α_{02}=0.03365 and α_{0s}=0.120 are used) and the process followed is illustrated in Fig. 4.2. Extrapolations made on this basis are indicated explicitly.

4.3.1 Analysis

$M_G(H, H', D)$, $M_R(H, H', D)$ and $1/\alpha_G(H, H', D)$

Fixing $F = 3$, the following behaviour is obtained for the unification parameters as functions of the number of Higgs bosons (see Table 4.9):

(i) M_G values decrease with these variables, being more sensitive to D. This is the same pattern as the one found at 1st order, where the M_G values are slightly higher than here. Therefore, only low number of Higgses are allowed by the constraint $M_G > 1 \times 10^{15} GeV$. The same happened with 2nd order SUSY SU(5).

(ii) M_R increases with H and H' (this resembles the corresponding M_R 1st order case and M_s in the 2nd order SUSY SU(5) case) and decreases with D. The values here are higher than at 1st order.

α_{0s}	$[M_G]_{minH}$	$[M_G]_{noH}$	$[M_G]_{SUSY}$
0.110	2.9	204.4	10.8
0.120	6.9	748.5	30.8
0.130	14.2	2245	74.6

Table 4.6: $M_G(\alpha_{0s}) \times 10^{15}$GeV at 1st order for the minimal Higgs content and no Higgs cases in L-R SO(10) and for the minimal case at 1st order in SUSY SU(5).

F	$[1/\alpha_G]_{minH}$	$[1/\alpha_G]_{noH}$	$[1/\alpha_G]_{SUSY}$
3	43.93	49.16	22.22
4	37.15	41.38	11.23
5	30.37	33.60	0.24
6	23.59	25.83	−10.76
7	16.81	18.05	
8	10.03	10.28	
9	3.25	2.50	
10	−3.53	−5.27	

Table 4.7: $1/\alpha_G(F)$ at 1st order for the minimal Higgs content and no Higgs cases in L-R SO(10) and for the minimal case at 1st order in SUSY SU(5). We are using α_{0s}=0.120 here.

			$1/\alpha_G$		
H	H'	D	α_{0s}=0.110	α_{0s}=0.120	α_{0s}=0.130
0	0	0	48.5	49.2	49.7
1	1	1	43.7	43.9	44.1
		2	41.6	41.5	41.5
		3	39.8	39.4	39.1
	2	1	43.1	43.2	43.3
		2	41.3	41.1	41.0
		3	39.7	39.3	39.0
	3	1	42.6	43.4	42.6
		2	41.0	41.2	40.6
		3	39.6	39.3	38.8
2	1	1	43.1	43.4	43.6
		2	41.3	41.2	41.2
		3	39.7	39.3	39.0
	2	1	42.5	42.7	42.8
		2	41.0	40.8	40.7
		3	39.6	39.2	38.9
	3	1	42.1	42.1	42.2
		2	40.7	40.5	40.4
		3	39.5	39.1	38.7
3	1	1	42.5	42.7	43.0
		2	40.9	40.9	40.8
		3	39.5	39.2	38.9
	2	1	41.9	42.1	42.2
		2	40.6	40.5	40.4
		3	39.4	39.1	38.8
	3	1	41.5	41.6	41.7
		2	40.4	40.2	40.1
		3	39.3	39.0	38.6

Table 4.8: $\alpha_G(H, H', D)$ at 1st order for $F = 3$ and different α_{0s}.

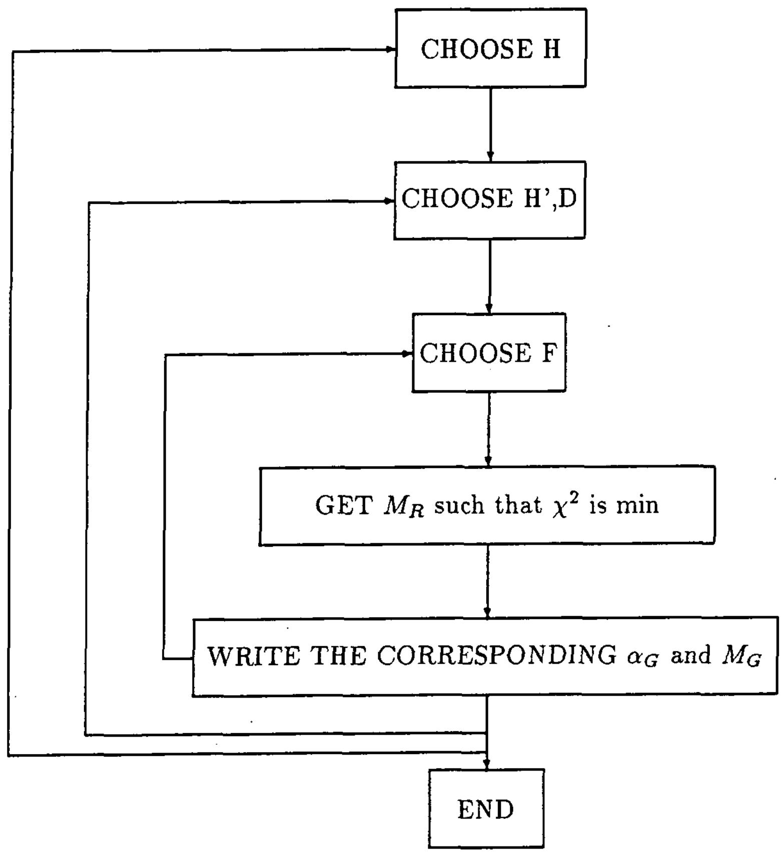

Figure 4.2: Working process

(iii) $1/\alpha_G$ decreases with all the three variables, being D the one that gives the largest change. The same decreasing pattern was obtained at 1st order. However, this contrasts with 2nd SUSY SU(5) where $1/\alpha_G$ increases its values with the number of Higgs doublets.

$M_G(F)$, $M_R(F)$ and $1/\alpha_G(F)$

We consider two cases here: the minimal Higgs content (with $H = 1 = H' = D$) and the No Higgs (where $H = 0 = H' = D$). From Table 4.10 we see that:

(i) Unlike at 1st order, an F dependence is introduced at 2nd order in M_G: it increases with F. This same result was obtained in 2nd SUSY SU(5). The difference between minH and noH is kept about the same size as it was at 1st order.

(ii) M_R nearly keeps its F-independence, as was the case with M_s in 2nd SUSY SU(5). Again, minH and noH are maintained as different as they were at 1st order.

(iii) As at 1st order, $1/\alpha_G$ is a decreasing function of F, although with values slightly lower and with non-linear dependence on F. The same pattern was obtained in SUSY SU(5), even though $1/\alpha_G$ is higher for L-R SO(10) and slightly less sensitive to F than what it was in the case of SUSY SU(5). The constraint on F coming from requiring $1/\alpha_G > 0$ is more severe than at 1st order, now F

is not allowed to go beyond 8 [2]:$F \leq 8$. In this case as well, minH and noH keep nearly the same difference as at 1st order.

$M_G(\alpha_{0s})$, $M_R(\alpha_{0s})$ **and** $1/\alpha_G(\alpha_{0s})$

We work out the results for the minimal and the no Higgs cases with $F = 3$. Looking at Table 4.11 we find that:

(i) M_G increases with α_{0s}(although its values are lower than at 1st order), this means that larger α_{0s} values could allow larger number of Higgses. The same behaviour was obtained in SUSY SU(5), although higher values for α_{0s} are required in L-R SO(10) to get $M_G > 1 \times 10^{15} GeV$. The gap between minH and noH keeps its 1st order value.

(ii) M_R decreases with α_{0s} (as is the case with M_s in 2nd SUSY SU(5)) with higher values than at 1st order. The 1st order difference between the M_R values for min$H$ and no$H$ is retained at 2nd order.

(iii) As was the case at 1st order, $1/\alpha_G$ is an increasing function of α_{0s}, although with slightly lower values. This means that larger α_{0s} values restrict less (not too much) the number of fermion families. This is opposite to the decreasing behaviour found in SUSY SU(5).

[2]extrapolations based on Table 4.10

4.3.2 Conclusions

We summarize below the most important points which emerge from our study of unification in the L-R SO(10) model:

(i) With only small changes in the numerical values, the general pattern shown at 1st order is confirmed at 2nd order. In other words, these higher order terms do not modify significantly the internal structure of 1st order L-R SO(10). This emphasizes, once more, the role of the analytical 1st order study as representative of the all-orders internal structure.

(ii) Ignoring Higgs bosons in L-R SO(10) leaves an acceptable GUT (see Fig. 4.3). The difference between the minimal Higgs content and the no-Higgs cases seems to be large enough to enable us to distinguish between them. In fact, the no-Higgs case seems to be in a more secure position as the experimental bounds on the proton lifetime continue to rise. In contrast to this, the Higgs contribution in SUSY SU(5) is not dispensable.

(iii) Although an F dependence is introduced in the unification parameters that is absent at 1st order, $1/\alpha_G$ continues to be the most sensitive of the variables to F and, since it goes negative as F grows, is the one whose values provide the strongest constraint on the number of fermion families (see Fig. 4.4). This appears to be a universal feature of GUT unification.

(iv) Low values for the variables H, H' and D are favoured by the constraint $M_G > 10^{15}$GeV, since M_G is a decreasing function of these variables. This is a feature shared with SUSY SU(5).

(v) The 2nd order terms do not affect significantly the value of M_R found at 1st order. This contrasts with SUSY SU(5), where a non-negligible change takes place when going from 1st to 2nd orders. Despite this, M_R and M_s are many orders of magnitude (around 6) apart.

As in SUSY, a more realistic approach to unification in this L-R symmetric model would require to consider a mass spectrum for the heavy particles introduced with this symmetry. Similar remarks concerning the L-R threshold apply here, although corrections in this case are expected to be less significant than in SUSY SU(5) since $M_R \gg M_z$.

		$F = 3$		
		$H = 1$		
H'	D	$\log_{10} M_G$	$\log_{10} M_R$	$1/\alpha_G$
1	1	15.4 (15.8)	10.0 (9.6)	43.3 (43.9)
	2	14.6 (14.9)	9.5 (9.1)	41.1 (41.5)
	3	13.8 (14.1)	9.1 (8.6)	39.1 (39.4)
2	1	15.2 (15.5)	10.5 (10.2)	42.6 (43.2)
	2	14.4 (14.7)	10.0 (9.6)	40.7 (41.1)
	3	13.8 (14.0)	9.6 (9.2)	38.0 (39.3)
3	1	15.0 (15.3)	10.9 (10.6)	42.1 (43.3)
	2	14.3 (14.6)	10.4 (10.1)	40.5 (41.2)
	3	13.7 (14.0)	10.0 (9.6)	38.9 (39.3)
		$H = 2$		
1	1	15.2 (15.6)	10.5 (10.0)	42.7 (43.3)

Table 4.9: 2nd order values of the unification parameters as functions of the number of Higgs multiplets for $F = 3$ (M_G and M_R are given in GeV and $\alpha_{0s} = 0.120$). 1st order values are written within brackets.

	$H = 1 = H' = D$			$H = 0 = H' = D$		
F	$\log_{10} M_G$	$\log_{10} M_R$	$1/\alpha_G$	$\log_{10} M_G$	$\log_{10} M_R$	$1/\alpha_G$
3	15.4 (15.8)	10.0 (9.6)	43.3 (43.9)	17.3 (17.9)	9.7 (9.3)	48.2 (49.2)
4	15.5 (15.8)	10.0 (9.6)	36.4 (37.1)	17.5 (17.9)	9.7 (9.3)	40.5 (41.4)
5	15.6 (15.8)	10.0 (9.6)	29.3 (30.3)	17.7 (17.9)	9.7 (9.3)	32.5 (33.6)
6	15.8 (15.8)	10.0 (9.6)	21.9 (23.6)	18.0 (17.9)	9.7 (9.3)	24.1 (25.8)
7	16.1 (15.8)	9.9 (9.6)	13.8 (16.8)	18.5 (17.9)	9.5 (9.3)	14.8 (18.0)

Table 4.10: 2nd order values of the unification parameters as functions of the number of fermion families F (M_G and M_R given in GeV and $\alpha_{0s} = 0.120$). 1st order values are written within brackets.

	$H = 1 = H' = D$			$H = 0 = H' = D$		
α_{0s}	$\log_{10} M_G$	$\log_{10} M_R$	$1/\alpha_G$	$\log_{10} M_G$	$\log_{10} M_R$	$1/\alpha_G$
0.110	15.1 (15.5)	10.4 (10.1)	43.1 (43.7)	16.8 (17.3)	10.0 (9.7)	47.6 (48.5)
0.130	15.7 (16.1)	9.7 (9.3)	43.4 (44.1)	17.8 (18.3)	9.4 (9.0)	48.7 (49.7)

Table 4.11: 2nd order values of the unification parameters (M_G and M_R are given in GeV) as functions of α_{0s}. 1st order values are written within brackets.

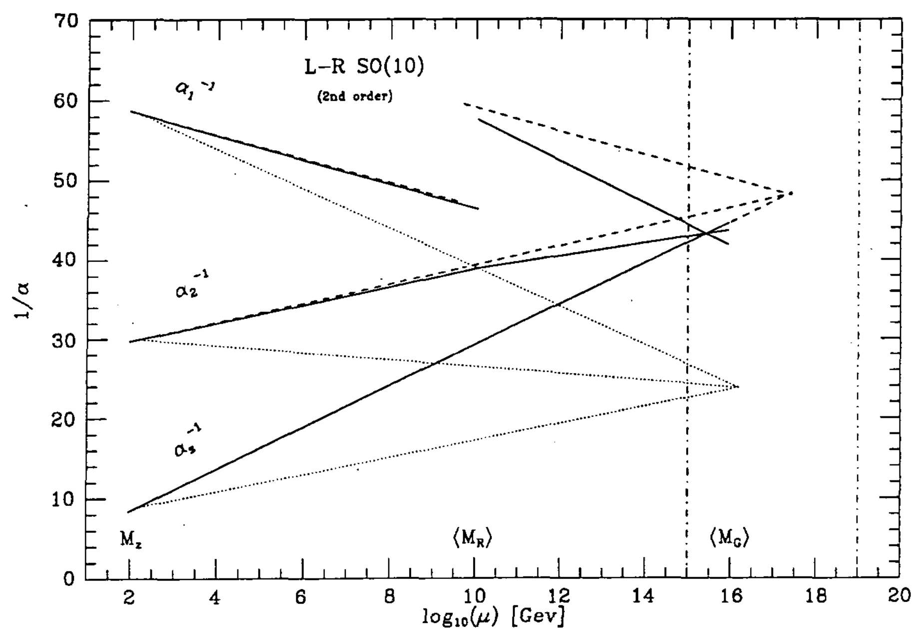

Figure 4.3: Evolution of the inverse of the couplings as functions of μ in the SM embedded in L-R SO(10) at second order with $\alpha_{01} = 0.017045$, $\alpha_{02} = 0.03365$ and $\alpha_{0s} = 0.120$. The solid lines show the evolution in the minimal case ($F = 3$, $H = 1$, $H' = 1$ and $D = 1$). The dashed lines correspond to evolving with no Higgs bosons ($H = 0 = H' = D$), and the dotted lines show the evolution in the minimal SUSY SU(5) case. The two dot-dash lines show the constraints on M_G imposed by proton decay experimental limits and the Planck mass.

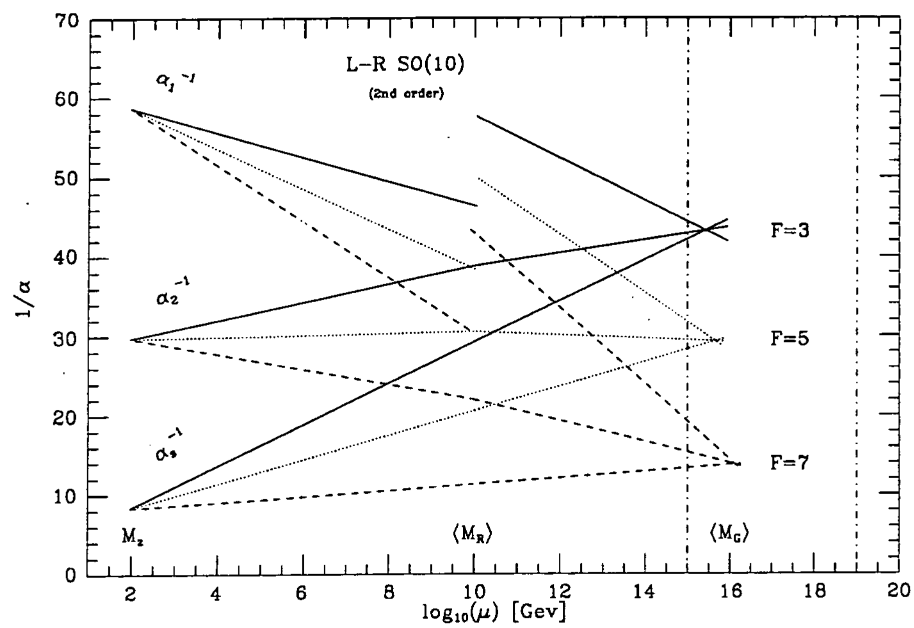

Figure 4.4: Evolution of the inverse of the couplings as functions of μ in the SM embedded in L-R SO(10) at second order with $\alpha_{01} = 0.017045$, $\alpha_{02} = 0.03365$ and $\alpha_{0s} = 0.120$. The solid lines show the evolution in the minimal case ($F = 3$, $H = 1$, $H' = 1$ and $D = 1$). The dotted and dashed lines correspond to evolving with $F = 5$ and $F = 7$, respectively, from the same initial conditions. The two dot-dash lines show the constraints on M_G imposed by proton decay experimental limits and the Planck mass.

Chapter 5

Conclusions

The existence of analytical solutions for the 1st order approximation coupling constant evolution equation, provides a fine tool to analyse any GUT scenario in terms of the 'internal structure' of its relevant unification parameters. This allows for a dynamic situation in which the number of fermion families and Higgs multiplets form the 'basic building blocks' that will structure the parameters of unification. The idea is to gain information about the 'components' from knowledge of the 'compound' objects. This knowledge is a mixture of data (on proton decay) and assumption ($1/\alpha_G > 0$). Analysing unification through the 'internal structure' is made fruitful thanks to recent precision data available mainly from LEP at CERN. This is an essential ingredient within this dynamic analytical approach to unification which we have worked out here.

The main points stemming from this analysis of unification are:

1. The 1st order analytical study has been proved to be a good representative of the all-orders unification parameter internal structure. In general, the trends found at 1st order are confirmed at 2nd order and (in the case of SUSY) the approximated 3rd order contribution seems to be even less significant. This points to the desired convergent behaviour for this approximation and to the *'ad hocness'* of this analytical approach as a fine probe to gain insight into the structure of the unification parameters.

2. Non-overlapping restrictions on the number of Higgs doublets obtained in plain SU(5) makes this GUT an unsatisfactory one in which to realize unification. This agrees with previous claims dismissing SU(5) as a good GUT.

3. The other two GUTs we are considering here, SUSY SU(5) and L-R SO(10), provide systematic ways of modifying the particle content of the SM and, therefore, the evolution of the couplings (normalisation effects in L-R SO(10) are also important). As a consequence, it turns out that unification in these GUTs is naturally achieved within the right region although with significant differences in the corresponding internal structures, as is pointed out below.

4. Although in both cases minimal Higgs models are favoured by proton decay restrictions, Higgs bosons are treated differently in each GUT. While in SUSY SU(5) the Higgs boson contribution is indispensable, ignoring them in L-R SO(10) gives an acceptable GUT and a better situation concerning the increasing proton

decay bound. Therefore, as far as unification is concerned, the status of the Higgs bosons seems to depend on the GUT we unify with.

5. The constraint on the number of fermion families appears to be a universal feature of GUT unification, although its value is model dependent. For the cases we are considering here, SUSY SU(5) is much more severe ($F \leq 5$) than the other GUTs even though this restriction has the same origin in all the cases.

6. In both GUTs there is an intermediate energy scale at which it is assumed all the fermion particle content of the model appears. However, although they share some features like linear dependence on α_{0i} and F independence (at 1st order), these scales, M_s and M_R, present very different internal structures, as is shown by the fact that in L-R SO(10) there is no restriction (from requiring $M_R > M_z$) on the maximum value allowed for α_{0s} and in order to keep this coupling within its experimental range M_R is restricted to very high energies. Even considering softer constraints on α_{0s} there is a huge gap between M_s and M_R. In this sense, SUSY may face first the challenge of the forthcoming experimental results.

In conclusion, although the 'natural' extensions (SUSY and L-R symmetry) of the SM result in very different features in their internal structures, the fact that they both prove effective in achieving unification within the bounds allowed by experimental limits on proton decay, seems inevitably to lead to interesting new physics beyond the SM if the unification hypothesis is accepted.

Appendix

β-coefficients

The coefficients of the α expansion in the RGE at first and second orders of approximation for the SM embedded in SU(5) and SO(10), and for SUSY SU(5) and L-R SO(10) are given below. Up to second order they are renormalization-scheme independent.

1st order

SM

The following expressions are obtained [18, 32] in one loop approximation:

$$\begin{aligned}
\beta_1(F,H) &= 0 \quad -\tfrac{4}{3}F \quad -\frac{1}{10}H \\
\beta_2(F,H) &= \frac{22}{3} \quad -\tfrac{4}{3}F \quad -\frac{1}{6}H \\
\beta_3(F,H) &= 11 \quad -\tfrac{4}{3}F \quad -0H
\end{aligned}$$

SUSY SU(5)

At 1st order, β_i were calculated [32] some years ago with the following result:

$$\begin{aligned}
\beta_1(F', H') &= 0 \quad -2F' \quad -\frac{3}{10}H' \\
\beta_2(F', H') &= 6 \quad -2F' \quad -\frac{1}{2}H' \\
\beta_3(F', H') &= 9 \quad -2F' \quad -0H'
\end{aligned}$$

L-R SO(10)

At 1st order, the β_i were calculated [32, 33] before with the following result:

$$\begin{aligned}
\beta_1(F', H', D) &= \quad 0 - \tfrac{4}{3}F' \quad -0H' - 3D \\
\beta_2(F', H', D) &= \quad \tfrac{22}{3} - \tfrac{4}{3}F' \quad -\frac{1}{3}H' - \frac{2}{3}D \\
\beta_3(F', H', D) &= \quad 11 - \tfrac{4}{3}F' \quad -0H' - 0D
\end{aligned}$$

2nd order

SM

The two-loop β-coefficients have been known for some time now [18, 32]. They have the form:

$$\beta_{ij} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{136}{3} & 0 \\ 0 & 0 & 102 \end{bmatrix} - F \begin{bmatrix} \frac{19}{15} & \frac{3}{5} & \frac{44}{15} \\ \frac{1}{5} & \frac{49}{3} & 4 \\ \frac{11}{30} & \frac{3}{2} & \frac{76}{3} \end{bmatrix} - H \begin{bmatrix} \frac{9}{50} & \frac{9}{10} & 0 \\ \frac{3}{10} & \frac{13}{6} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

SUSY SU(5)

SUSY 2-loop β-functions are given [32] as follows:

$$\beta_{ij} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 54 \end{bmatrix} - F' \begin{bmatrix} \frac{38}{15} & \frac{6}{5} & \frac{88}{15} \\ \frac{2}{5} & 14 & 8 \\ \frac{11}{15} & 3 & \frac{68}{3} \end{bmatrix} - H' \begin{bmatrix} \frac{9}{50} & \frac{9}{10} & 0 \\ \frac{3}{10} & \frac{7}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

L-R SO(10)

The L-R 2-loop β-functions are given [32, 33] as follows:

$$\beta_{ij} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{136}{3} & 0 \\ 0 & 0 & 102 \end{bmatrix} - F' \begin{bmatrix} \frac{7}{6} & 3 & \frac{4}{3} \\ \frac{1}{2} & \frac{49}{3} & 4 \\ \frac{1}{6} & 3 & \frac{76}{3} \end{bmatrix} - H' \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{22}{3} & 0 \\ 0 & 0 & 0 \end{bmatrix} - D \begin{bmatrix} 54 & 72 & 0 \\ 12 & \frac{56}{3} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

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