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Mathematics Problem Solving Instructional Program
on the achievement of Grade Five Students in Belize,
Central America.*

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ABSTRACT

Analysis of the effects of a Constructivist-Based Mathematics Problem Solving Instructional Program on the achievement of Grade Five Students in Belize, Central America.

This thesis examined whether social constructivist activities can improve the mathematical competency of grade five students in Belize, Central America. The sample included 342 students and eight teachers from two rural and urban schools. A switching replication design was employed enabling students in the experimental groups to be taught using social constructivist activities for 12 weeks and the controls exposed to similar instructional practices from weeks 7 to 12. Students' performance was assessed using Pre-test, Post test 1 and 2 with an internal consistency of 0.89, 0.90 and 0.93 respectively. As revealed by the repeated measures ANOVA within subject analysis, there were significant differences among the pre-test and post test 1 and 2 results. That is, students in the control groups, who were instructed using a procedural approach from weeks 1 to 6, demonstrated higher gains than the experimental groups who were immersed in social constructivist activities. Furthermore, when the control groups became immersed in similar activities from weeks 7 to 12, they continued to outperform the experimental groups who were exposed to social constructivist activities alone. Hence, due to this unexpected result, the aim of this thesis became to explain why these results came about and what implications for teaching were highlighted by the consideration.

Besides the quantitative results highlighted above, qualitative data was also obtained as part of the study. For example, students were videoed within constructivist math groups and their performance analyzed using Pirie and Kieren's (1994) Model of Growth for Mathematical Understanding. The data from the video recording revealed that use of one step math problems did not enabled students to restructure their thinking to solve innovative problems. Data from semi-structured interviews also revealed that some students lacked basic math skills and were not exposed or guided to solve complex problems. Besides the need for careful examination of social constructivist activities on performance, this thesis underscores the importance of relevant teaching and learning activities, the important role of teachers during social constructivist activities and the need to identify suitable forms of assessment to measure performance.

Key words:

social constructivism, higher order thinking, effective pedagogical practices

An analysis of the effects of a Constructivist-Based Mathematics Problem Solving
Instructional Program on the achievement of Grade Five Students in

Belize, Central America

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A thesis submitted for the Degree of Doctorate in Education

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DECLARATION

I solemnly declare that this thesis is my own work and has not been offered previously in candidature for any other degree in this or any other university.

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DEDICATION

This work is dedicated to my mother,

Jane Leonel Brown (1929 - 1988)

Thank you for believing in me and instilling that the possibilities lie within.

Part One

Research Context

This dissertation is an analysis of the effects of a Constructivist-Based Mathematics Problem Solving Instructional Program on the achievement of Grade Five Students in Belize, Central America. Attempts to identify instructional practices to improve mathematical competencies remain central to teaching and learning in Belize. The need to identify “best practices” arose from the concern that students consistently perform poorly in math examinations. For example, the mean average in mathematics in the 2009 Primary School Examination, administered to students at the end of eight years of elementary schooling, was 46% (Ministry of Education Examination Report, 2009). Noteworthy is that the average performance in mathematics in 2007 and 2008 was below 48% (ibid.: 16). In addition to poor performance at the end of primary schooling, the results of a standardized math test administered to all Grade Five students also revealed that their performance was below 50% (Ministry of Education, 2007). Based on the results of national examinations, which illustrate that many students lack basic math skills, this study aims to assess the effects of social constructivist activities among students who ordinarily were taught using traditional teacher directed activities.

To analyze the challenges and limitations of immersing grade five students in social constructivist math activities in Belize, Central American, this thesis is organized into three parts. In the first, the research context, literature, design and research procedures, and quantitative results are presented. In the second, issues pertaining to the use of social constructivist activities to solve one step math problems are analyzed. Finally, in the third section, the limitations of the study, summary of the findings, and conclusions are stated.

Chapter 1

Introduction and Background

1.1 Introduction

The purpose of this first chapter is to introduce the context, the research questions, hypotheses, and rationale for the study. Hence, this chapter begins with an overview of Grade 5 students' performance in math word problems. It also introduces Belize's educational system, curriculum development, and national examinations. In addition to contextual background, the research significance, research questions, hypotheses, and rationale provide a comprehensive overview of the phenomenon being studied.

In an era of change and development, mathematical competence, particularly in word problems, is a necessary requirement to function in a literate society. Encompassed in mathematical word problems is the ability to analyze and critically seek solutions to varied situations within formal education, leisure, and employment (National Council of Teachers of Mathematics [NCTM], 2000). Besides the belief that math word problems promote higher order thinking, traditionally it has been perceived that:

Skill in application of problem-solving skills can be equated with quality of life, in that those of us who are adept at problem solving, can apply the principles of problem solving to everyday life, may make better decisions about common problems.

Kilpatrick, 1925:249

Therefore, even if problem solving skills are applied in everyday life, students must develop the requisite analytical skills (Patton et al, 1997). While it is apparent that

competencies must be developed, the 2007 Primary School Examination (P.S.E.) results in Belize, as revealed in Table 1, revealed that the average percent achieved was 50% or less in math (Ministry of Education Statistical Report, 2007).

Table 1:

Primary School Examination (PSE) 2007 Results

Subjects assessed by the P.S.E Examination				
Districts in Belize	English	Math	Science	Social Studies
Belize	59.9	44.0	58.8	61.6
Cayo	60.7	45.6	60.2	60.6
Corozal	59.6	49.7	59.9	59.4
Orange Walk	62.9	50.0	60.3	62.0
Stann Creek	58.8	43.1	56.6	56.8
Toledo	58.9	42.4	57.7	55.9
National Mean	60.1	45.8	58.9	59.4

As illustrated in the third column, the highest percentage in the National Primary Examination was 50% and the lowest 42.4%. When compared to passes in other subjects such as Social Studies, Science and English, students performed poorly in Mathematics. Clearly, if students are to become competent problem solvers, measures must be taken to ensure that they develop the skills to succeed.

1.2. Background and Research Context

To provide an overview of the research context and mathematical performance in Belize, this subsection begins with a brief introduction of Belize's education system. It also outlines the two national examinations administered at grades five and eight. Ultimately, an overview of the national performance in mathematics and implications for mastery are presented.

Belize, located on the Caribbean coast of Northern Central America, has an educational system comprising of three levels of schooling: 6-14 years for primary, 14-18 years for secondary and 18+ for post secondary (Thompson, 1999).

Students pursuing primary education are required to complete eight years of schooling and sit two national examinations. The first is the Belize Junior Achievement Test (BJAT) to assess numeracy and literacy skills at the end of the first five years of primary schooling (Ministry of Education, 2005). While it is hoped that the results of the BJAT would reveal that students can read, write, and demonstrate mastery of basic mathematics skills, as illustrated in Table 2, approximately 50% of all grade five students continuously performed poorly in the word problem section of this examination (Davis, 2003).

Table 2:

Belize Junior Achievement Test 2007 Average Performance

Districts	Composition	Problem Solving	Language Arts Multiple Choice	Math Multiple Choice
Belize	42.4%	53.3%	69.0%	47.5%
Cayo	41.9%	46.2%	63.1%	46.4%
Corozal	41.4%	48.0%	59.4%	45.9%
Orange Walk	40.2%	45.6%	60.5%	46.2%
Stann Creek	38.0%	39.8%	53.4%	39.9%
Toledo	40.9%	39.3%	54.1%	42.8%
Total	41.1%	46.6%	61.5%	45.3%

The average performance for 2007 as indicated in Table 2 revealed that the percentage achieved in problem solving for all district was below 50% except for the Belize District with 53.3% (Ministry of Education, 2007). In addition to poor performance in problem solving, percentages in computation skills was below 50% in all districts. Generally, the data illustrate that at the midpoint of primary school in Belize, students performed poorly in math word problems and multiple choice items to assess computation skills.

Besides weak performance in mathematics, as revealed by the 2007 BJAT results, the second national examination is the Primary School Examination (P.S.E) which is administered to all students at the end of the eighth year of primary school and is comprised of four main subject areas: Mathematics (Multiple Choice and word problems), Language Arts, Social Studies, and Science (Ministry of Education, 2006). Worth noting is that since the inception of this examination in 2001, the national performance in Mathematics word problems remains at or below 40% (ibid.: 10). Despite efforts to train teachers, in 2006, the annual result in mathematical word problems declined to 31.5%

(Williams, 2006). While a number of students in Belize are unable to meet national mathematical standards, studies conducted in 40 countries also indicated that students in the United States and other countries are performing poorly in comparison to students living in Asia (National Centre for Education Statistics, 1998). This suggests that there are prevailing factors that impede mastery of mathematical word problems. Therefore, the underlying causes of poor performance must be analyzed to ensure that children are provided with the adequate educational support and mathematical competencies (Lauren and Adam, 2000).

1.2.1 Curriculum Development in Belize and the Context of Schooling

Besides a brief examination of mathematical performance in the previous subsection, a clear understanding of the structure of schooling and the effects of curriculum development on teaching and learning are paramount. Consequently, factors governing educational change and development in Belize are presented herein.

In Belize, schools are managed by a church state system comprising 284 primary schools, 44 secondary schools and 6 tertiary institutions (Pineda, 2006). While students in Belize are afforded the opportunity to pursue education from early childhood to tertiary, according to the National Human Development Advisory Committee (1999) only 75% of students, ages of 5-14, attend primary school and less than 60% of those attending primary school pursue secondary or higher education. Also worth noting is that the enrolment at the secondary level represents less than 50% of students within the 14-18 age range who ordinarily should be enrolled in some form of educational training. This illustrates high

attrition rates in secondary schools and that a significant number of young children are not acquiring the skills and competencies to survive within their social milieu.

Although low attendance may be attributed to social factors such as limited classroom space and weak community relations, according to Barrow (2001), many students fail to pursue secondary education because of rigorous standardized examinations. The prevailing view is that the BJAT and the PSE standardized examinations are used to select those who will pursue further education. This means that national standards actually serve as the yardstick to judge performance and to stigmatize students (Gipps, 1994; Apple, 2001). What is apparent is that while effort has been made to select those most able to pursue academic training, those who fail to demonstrate competencies are ignored or abandoned. While many arguments can be used to analyze the effects of standardized examinations, there is need to review the curriculum structure and instructional practices to ensure that students are provided with the competencies to succeed.

Besides the aforementioned which indicate that many students are not measuring up to National Standards in Belize, according to Bennett (1997), between 1964 and 1970, efforts were made to improve the delivery of education and to eliminate inefficiency. For example, as noted in the History and Development of Education in Belize, when the Curriculum Development Unit was first established, primary teachers were provided with foreign curriculum material from developed countries (*ibid.*: 37). A decade later, suitable instructional materials in core areas such as Mathematics, Language Arts, Science and Social Studies became available. Although the Ministry of Education should be applauded for its efforts, foreign textbooks and resources are still utilized in all schools (Lopez,

2002). Undoubtedly, students were not provided with adequate learning resources in areas such as mathematics instruction; instead, instructional materials were far removed from students' background and experiences. Inadequate teaching and learning can be described thus:

The primary and often exclusive focus on quite a narrow spectrum The norms and cultures of this narrow spectrum were seen as archetypes of "tradition" for everyone.

Apple 1996: 68:69

Apple's use of the term 'archetypes' refers to inadequate materials within textbooks and that is the reality in Belize. For example, for decades, a number of the books and math problems referred to snow, people travelling in trains, and situations that were far removed from the realities of persons living in Belize. What educational planners failed to realize is that mathematics is "a process of enculturation" (Cobb, 1994). Therefore, unless math problems and teaching and learning resources are relevant to students' background and experiences, they will experience difficulty in solving them. Despite the fact that a number of studies have identified the need for relevant and meaningful contexts (Cobb, 1994; Sfard, 1998; Wood et al, 1992), students in Belize are still exposed to inappropriate instructional resources and are expected to excel in national examinations.

Besides inappropriate instructional resources, changes within Belize's educational system progressed at a very slow pace. However, the inception of the Belize Primary Education Development Project (BPEDP) in 1992 gave rise to a revitalized curriculum and educational resources (Ministry of Education, 2005). According to the Educational Statistic Digest (1992), the purpose of the BPEDP project was to improve the quality of

instructional inputs and educational achievement of all primary students. This initiative resulted in the development of the National Primary Curriculum linked to national goals to govern instruction, acquisition of adequate resources, instructional monitoring, and the development of curriculum materials for the first two years of secondary education. Even as effort has been made to improve Belize's national curriculum, a survey to examine the perceptions and effectiveness of 150 principals and teachers revealed that unless teachers are trained to teach and assess performance, no significant change would occur (Lopez, 2002). The importance of skilled instructors aligns with Fullan's (2001) belief that students are successful when provided with relevant, engaging, and worthwhile experiences. Hence, there is need to also explore whether mastery is dependent on instructional practice.

1.3 Factors affecting Belize's Education System

There are a number of factors which may affect students' ability to develop mastery of math problems. In this subsection, the effects of untrained teachers and attrition rates are explored to examine the likely impact of instructional practices on students' response to learning.

School is a major factor in a child's education both in terms of the number of hours spent and the quality of teaching and learning (Eggen et al., 2003). Therefore, teachers play an important role in promoting the required knowledge, skills and attitudes for active participation in a society (Ministry of Education, 2005). While teachers play an important

role in students' growth and development, education statistics in Belize denote that less than 50% of all primary teachers are untrained (Mason and Longworth, 2005). Although teachers should be equipped to skilfully design instruction and engage students in meaningful learning tasks (Danielson, 1996), many have not acquired the professional training to do so (Brown-Lopez et al., 2009; Magana, 2010).

Coupled with limited cohort of trained teachers, students in Belize continuously perform poorly in standardized examinations. For example, as previously stated, the results of the Primary School Examination which is administered at the end of elementary schooling illustrates that the average performance in core subjects is below 50% (Ministry of Education, 2007). This illustrates that at the end of elementary schooling, a number of students are not performing at the expected standards. While the results of the Primary School Examination suggest that many students lack essential knowledge and skills, similar results have been identified in the Belize Junior Achievement Test which is administered to grade five students at the midpoint of elementary school to assess math and English skills. A four year report of students' performance in the Belize Junior Achievement Test is illustrated in Table 3.

Table 3:

2000 – 2004 Average Performance in Language Arts and Mathematics out of a total of 100%

Year	National Average Performance in Language Arts	National Average Performance in Mathematics
2000	46.3	47.6
2001	46.7	45.8
2002	42.6	46.8
2003	44.6	48.2

Revealed in Table 3 is that from 2000 to 2004, a number of young children between age 8 and 11 consistently performed poorly in Language Arts. This illustrates that many students in Belize lack written and communication skills, (Lo Bianco & Freebody 1997:28). In addition to the inability to read and use written information in a range of contexts, the data in Table 3 also illustrate that grade five students also consistently performed poorly in mathematics (Ministry of Education, 2004).

Besides poor performance at the midpoint of elementary schooling, identified is that many of the primary children are continuously dropping out of school as illustrated in Table 4 on the next page.

Table 4:

Average Primary School Dropout rates 2000 – 2003

Sex	Dropouts	Enrolment	Dropout Rates
Total	620	59,930	1.0 %
Male	336	30,613	1.1%
Female	284	29,317	1.0%

Illustrated in the Ministry of Education 2004 report is that between 2000 and 2003, a total of 620 primary students discontinued their primary education. Although factors such as migration, lack of interest in school, and socio-economic conditions contribute to school dropout (Darling- Hammond et al., 2007), the fact that many students continuously perform poorly on national examinations and many teachers are untrained underscore the need to assess instructional practices and to identify ways to improve performance. Therefore, based on poor performance in mathematics as illustrated in the Belize Junior Achievement Test results, the Primary School Examination, and the attrition rates of primary students in Belize, the researcher embarked on a twelve week study to assess whether exposing grade five students to interactive social constructivist activities would improve their ability to interpret and solve math problems.

1.4 Rationale for promoting social constructivist activities

According to Shugert (1979), a rationale is the articulation of the purpose or use of a particular theory or theories. It also provides the reason or principle underlying a particular course of action (Shavelson et al., 2000). Based on Shugert and Shavelson's views, this subsection provides a brief justification of theoretical views used as part of the implemented intervention and the design of the study.

Firstly, researchers such as Fosnot (1989) and Brooks and Brooks (1999) suggest that a constructivist approach to learning builds on the natural innate capabilities of the learner. From this perspective, the learner is viewed as actively constructing understanding through the use of authentic resources and social interaction (Eggen et al., 2003). Therefore, the focus is on cognitive development and deep understanding in which learning is nonlinear and students are encouraged to freely and actively search for solutions. Consequently, for significant changes to occur, students must be provided with skills to construct understanding. Hence, this study was designed to explore this educational perspective and to provide instructional guidelines to enable students to become more engaged in math instruction.

Based on Eggen et al., (2003), Brooks and Brooks (1999) and Fosnot's (1989) proposition that students construct their own understanding by using prior knowledge to interpret information, the view that effective use of authentic resources can aid knowledge construction, and that peer discussion and negotiation is critical to the constructive process, it is the researcher's belief that these are feasible guidelines to be implemented among grade five students to assess whether social constructivist activities will improve

performance and enhance students' problem solving skills. It is hoped that the following conditions can also support the implementation of a social constructivist teaching experiment in Belize:

1. Grade five students in Belize Central America have been taught using the Mathematics National Curriculum from grades 1 to 4 (Ministry of Education Report, 2007) . Hence, it is expected that grade five students have acquired some background knowledge that can be used to construct “understanding”.
2. Teachers can be provided with guidelines to scaffold students to discuss and interpret math word problems within social learning groups (Vygotsky, 1986).

Therefore, to assess whether social constructivist activities will improve performance among grade five students, the quantitative analysis and the data from pupils' performance on word problems will provide valuable insights on whether use of social constructivist activities can improve mathematical competencies. Additionally, the qualitative analysis of students' experiences will also be used to identify whether they developed mathematical skills and competencies.

1.5. Significance of the Study

As previously mentioned, the aim of this study was to examine whether a Constructivist Based-Math Problem Solving program would result in improved performance for pupils. If this intervention was successful, it could be adopted in similar schools to assist students to develop problem solving skills and improve performance.

It was also envisaged that this study would contribute to the existing body of knowledge by:

1. increasing our understanding of students' perceptions, understandings and opinions as they "tackle" difficult problem situations;
2. identifying whether the constructivist approach is a viable strategy to improve performance;
3. providing educators with insights on appropriate instructional guidelines to teach math word problems.

In addition to providing pertinent information on how best to engage students in meaningful constructivist-based activities, this study would also provide insights on:

1. the use of relevant assessment strategies to analyze students' interactions in constructivist learning groups;
2. factors that impact the extent to which students use prior knowledge to make meaningful connections and to extend their thinking to solve a range of problems.

1.6 Research Questions

The following general and specific questions were therefore used to explore the impact of the constructivist-based mathematics problem-solving on achievement.

<i>General Questions</i>	<i>Specific Questions</i>
1. What is the impact of the Constructivist-Based Math Problem-Solving Instruction on achievement?	1. Will use of social constructivist activities assist students to interpret and identify solutions to mathematics word problems?
2. Will fifth graders develop problem solving skills?	2. What are Grade Five students' experiences as they interpret, analyze, and solve mathematics word problems?

1.7 Research Hypothesis

The following hypotheses regarding the impact of constructivist-based instructional program on student achievement were therefore tested quantitatively:

1. The math problem solving achievement test scores of fifth graders who have completed six weeks of the constructivist-based instruction for math problem solving would show a significant gain over pre-test scores.

2. The post test scores of fifth graders who have completed the constructivist-based instruction for math problem solving would show significant gain over pre-test scores at weeks six and twelve.
3. The control groups who were taught using the constructivist-based instruction for math problem solving, from weeks six to twelve, would show significant gains having been taught using this approach.

To analyze whether a constructivist- based instructional program provided students with successful experiences while identifying solutions to math word problems, the fourth hypothesis was tested qualitatively.

4. Unlike regular instructional methods, the constructivist-based approach would provide fifth graders with deep insights of successful experiences and difficulties as they sought solutions to Math word problems.

1.8 Summary

This chapter presents the research context, questions and rationale for use of social constructivist activities among students who customarily are taught using traditional teacher centred instruction in Belize, Central America. In the subsequent chapters, the literature review and a detailed description of the process for implementing 12 weeks of social constructivist-based activities are described.

Chapter 2

Designing the Intervention: A Literature Review

2.1 Introduction

School-based interventions are often used to assess students' response to a particular mode of instruction within specific time frames (Fuch, Fuch & Vaughn, 2008). That is, as stated previously, the purpose of this study was to assess the effects of twelve weeks of social constructivist activities on the performance of grade five students. Therefore, while school-based interventions entail "the systematic application of research-validated procedures to change behaviours through either teaching new skills or through the manipulation of antecedents and consequences" (Bowen, Jenson, and Clark, 2004:7), there is need to also analyze pertinent literature related to the phenomenon being studied.

Consequently, to outline various theoretical views underpinning the effect of instructional practices and mastery of math word problems, the literature has been organized into three main sections. In the first section of this chapter, causes for poor performance and guidelines for improvement are reviewed. In the second, the theoretical views relating to Pólya's (1954) Steps to Solving Word Problems provide insights on systematic guidelines to improve problem solving skills. In addition to an extensive review of underachievement and the effects of linear instructional approaches on performance, arguments in support of student-centred approaches such as Problem-Based Learning and Constructivist-based instruction are analyzed and implications for practice noted.

2.2 Difficulties associated with Math Word Problems

Similar to the results of standardized tests and research studies in Belize which denote poor performance in mathematics, longitudinal research also illustrate that students in many countries lack basic math skills (Gonzales & Miles, 2001). Consequently, to examine the effects of instructional practice on performance, this section is organized into three further components. In the first, pertinent literature on the use of algorithms and systematic guidelines are analyzed to identify the effects of linear instructional strategies on performance. In addition to an extensive review of the effects of systematic instructional guidelines, the influence of teachers' knowledge and experiences are reviewed to determine whether poor performance can be attributed to classroom instruction. Finally, literature on the effects of reading and comprehension skills are reviewed.

2.2.1 Algorithm and Math Word Problems

According to Montague et al., (2000), disappointing levels of mathematical performance are deeply rooted in an overemphasis on mastery of algorithms. That is, students are often provided with systematic procedures and specific guidelines such as: (1) read the problem, (2) decide what to do, (3) compute, and (4) check your answer (Montague, 2003). While clear structural guidelines can be useful, strict adherence to step-by-step approaches fails to promote critical thinking skills (ibid.: 02). Implied is that instructional practice affects the extent to which students develop and apply problem solving skills. As Hiebert puts it:

Most children enter school with reasonably good problem-solving strategies. A significant feature of these strategies is that they reflect a careful analysis of the

problems they are given. However, after several years, many children abandon their analytic approach and solve problems by selecting a memorized algorithm based on a relatively superficial reading of the problem.

1984:497-513

While this position suggests that systematic approaches can affect performance, also noted is that:

Many students see little connection between the procedures they use and the understandings that support them. This is true even for students who demonstrate in concrete contexts that they do possess important understandings.

(ibid.: 479:513)

The belief that students often fail to accurately interpret math problems implies that instructional practices influence students' learning (De Corte et al., 1996). Added to this view is that, "school mathematics involves the mechanical learning and the mechanical use of facts - adaptations to a system that are unencumbered by the demands of consistency or even common sense" (Baroody and Ginsburg, 1986: 237-354).

Seemingly, students are often engaged in tasks which fail to challenge their cognitive skills to effectively analyze and solve problems; therefore, unless students are provided with opportunities to think critically, they may never gain mastery. While it is perceived that overemphasis on algorithms may be unproductive, according to NCTM (1989), students' mathematical power increases with clear structural guidelines. The prevailing view is that algorithms provide students with specific procedures to solve math word problems.

Furthermore, when students are presented with word problems such as:

You want to purchase a bookcase to hold 37 books. If each shelf can hold 7 books, how many shelves should the bookcase you purchase have?

these problems require application of formal procedures and understanding of the context of the problem (Allen, 1985). Specifically, students will need to compute that 37 divided by 7 is 5 with a remainder of 2. They also need to deduce that 6 shelves are required and not 5. This suggests that mastery of mathematical word problems is dependent on algorithms and background knowledge to interpret and identify solutions (NCTM, 1989). Although algorithms are useful in the problem solving process, knowledge of algorithms alone will not result in mastery. For example, studies conducted with ten year olds in several elementary schools in the United States to examine the use of algorithms to solve subtraction word problems revealed that only 60% demonstrated mastery using standard “borrowing” (Carroll et al., 1997). While it appears that students applied skills from their math classrooms, a study conducted in Japan with four hundred and sixty-six fifth and eighth graders also illustrated that 56% of the eighth graders and 74% of the fifth graders achieved mastery of math word problems using algorithms (Reys et al., 1991). Both Carroll and Reys’s findings indicate that although more than 50% of the students were successful, a significant number were also unsuccessful. Hence, while algorithms provide systematic solutions, there is no guarantee that all students will succeed. Therefore, in addition to an inability to use algorithms, there are instances when students fail to develop conceptual understanding of math problems. This highlights that “mathematics is a way of understanding, a thinking process, and not a collection of detached procedures to be learned and applied separately” (Missouri Department of Higher Education, 2008:01).

Although the importance of thinking and mathematical skills has been recognized, (Carpenter & Lehrer, 1999; Hiebert *et al.*, 1997; National Research Council [NRC], 2001), many young children are unable to comprehend basic addition facts such as $(6+7 = 6+6+1)$ or $(7+9 = 8+8)$ (Putnam *et al.*, 1986). Furthermore, problems requiring logical reasoning such as subtract x from y , 24 divided by 8 and 8 divided by 24 are also challenging for most students (Dale and Cuevas, 1992). Worth noting is that a study to establish a causal link between logical reasoning and mathematical learning in the United Kingdom revealed that primary school children who received training in logical reasoning made more progress in mathematics than the control group (Nunez *et al.*, 2007). This indicates that problem solving skills is highly dependent on innovative strategies, critical thinking, and analytical skills (Fosnot, 2005).

While the importance of critical thinking skills has been acknowledged, (Lockwood, 2007; Torres *et al.*, 1995; Ennis, 1989; Zoller *et al.*, 2000), a three year study conducted with 177 students in a rural high school in the North of Israel illustrated that only two or 20% of the teachers purposefully utilized strategies to promote higher ordered thinking skills (Barak *et al.*, 2007). While it is perceived that teachers seldom engage students in higher order math activities, to what extent do problems within textbooks and those currently used in schools promote higher order cognitive skills? An analysis of a sample problem type is presented below.

Nancy is hemming a trim around a tablecloth. If the tablecloth is 108 inches long and 72 inches wide, how many inches of trim does Nancy need?

Chapman 2004:38

According to Chapman, this problem requires more than a systematic approach. Firstly, students must have the background knowledge of finding the distance around an object and the procedures for identifying the perimeter of a rectangle (ibid.: 01). They also need to understand that the term trim refers to the outer edge of the tablecloth and finally background knowledge of how to measure length and width are required. Essentially, students' thinking skills in mathematics include the ability to analyze, interpret, and use some specific method to solve. Hence, this calls for more than simple memorization of sequential steps and basic skills. Instead, students are required to apply background knowledge within various contexts and to interpret mathematical symbols and their use (NCTM, 1989). While there is the notion that background knowledge and algorithms are essential to the problem solving process, we are reminded that knowledge of mathematical facts or algorithm is no guarantee that students will succeed (Prawat, 1979). In fact, students must competently interpret and use various mathematical operations; otherwise, chances of error will increase considerably (Hiebert, 1986). In essence, teachers need to ensure that students learn to apply their mathematical background and procedures to solve real life problems (DeVaries et al., 1994).

2.2.2 Effects of Didactic Strategies on Performance

In addition to difficulties associated with using algorithms and the need for students to acquire the necessary background skills and experiences to interpret math problems, worth examining are the effects of the lecture-based method which includes long monologues in which the teacher talks and the students listen, (Goodlad, 1984; Cuban, 1984). In a study to assess instructional effectiveness of teachers who have completed two years of initial

teacher training in Belize, it was illustrated that although teachers were provided with professional guidelines for using a repertoire of instructional strategies, lessons were very teacher-centred with little or no student involvement (Brown-Lopez et al., 2009).

Illustrated is that students are often encouraged to absorb and apply rudimentary guidelines instead of using real life experiences to interpret and solve problem (Meier et al., 1989).

To ensure that students assume a more active role, it is hoped that instead of explaining how to solve a problem including 37 and 14, it would be more meaningful to ask, for example in an American Indian community where horses are an important part of life, to add 14 horses to 37 horses (Saxe, 1982). The prevailing view is that mastery is dependent on authentic examples and clear representations (ibid.: 12).

Even as it is apparent that schematic representations are crucial, as noted by Woodward et al., (2001), providing opportunities for conceptual understanding is vital. That is, opportunities should be provided for students to move away from memorization of steps and using “quick fix” approaches with searches for key words such as “more” or “gave away” (ibid.: 80-83). Instead, students should construct “knowledge of the idea, and how it relates to already acquired ideas” (ibid.: 80-83). Although conceptual understanding has to do with if a child knows that to add 50 to 40 they can take off the zeros and add 5 and 4 and then put back the zeros, this does not illustrate “real” understanding. Conversely, if the child can explain that my answer is correct because I am adding 5 tens and 4 tens, this illustrates moving from mechanical application to deep conceptual understanding (ibid.: 01). Therefore, to construct understanding, students must have knowledge of the idea and how it relates to other related facts (Hiebert, 1986). This means that when students are taught to memorize that the phrase “take away” means to subtract, they do not ask, “What

do I know, what do I need to know, what do I need to learn, and how do I measure or describe the results” (Gallagher et al., 1992:195-200). Hence, procedural approaches and lectures fail to foster deep processing and analytical interpretation of the underlying factors of the problem (Tschudi and Lafer, 1996). Secondly, conceptual understanding, which is an integral part of the deep processing of the problem, has to do with understanding of the contexts of the problem and its limitations (U.S. Department of Education, 2007).

The points aforementioned which emphasizes the need to engage students in meaningful problem solving strategies, aligns to Carpenter *et al.*'s (1999) notion that instead of looking for key terms such as “take away” or “more”, students should analyze and apply more meaningful problem solving strategies. For example students can examine problems and identify “start-change-results”. This means that students must first read and interpret the problem, examine whether a change has occurred and determine how to proceed. Hence, students may be presented with a problem such as:

“Thomas had 8 cookies. Then he ate 6. How many cookies did he have left?”

(Cougar, 1995:178 cited in Carpenter *et al.*, 1999).

To solve this, students must first determine if a greater amount is reduced and then subtract, else if something is included, the numbers are added. This highlights that conceptual understanding entails “an understanding of the contexts within which the idea is applicable, as well as its limitations” (U.S. Department of Education, 2007:01).

Secondly, students are required to use analytical skills and experiences to recreate and to construct logical mathematical thought (Fosnot, 2005). In essence, mathematical thought is best developed when:

Children in the primary grades invent their own arithmetic without the instruction they are now receiving from textbooks and workbooks

(Kamii and Dominick, 1998, 130-140)

While lived experiences often enhance students' ability to construct mathematical ideas, "if you ask children to make up problems about everyday math, they will not make up problems about their experienced lives, they will invent examples of the genre; they too know what a word problem is" (Lave, 1993: 77).

Undeniably, children do not automatically reflect on their experiences to interpret and solve problems. Instead, they practice and model those very "lived classroom experiences." That is, students observe and imitate what they observe in their classrooms (Bandura, 1997). Therefore, if students are not provided with opportunities to reconstruct ideas and become fully immersed, they may never critically analyze and "tackle" culturally rich and relevant word problems (Weist, 1996/1997). Restated is that mastery is dependent on effective teaching and learning experiences (Baroody & Hume, 1991; Carroll et al., 2000). That is, teachers can reword the problems and students guided to read, reconstruct and devise their own strategies (Carpenter, Fennema, & Franke, 1992; Ross and Morrison, 1991).

Even as teachers are encouraged to use a repertoire of instructional strategies, a word of caution is that rewording the problem to facilitate comprehension may be beneficial to younger students who have "less developed schemata for standard textbook problems" (Davis-Dorsey et al., 1991:66). This means that experienced problem solvers may require

less scaffolding and guidelines than less experienced problem solvers (ibid.: 67).

Therefore, students' learning styles and prior knowledge must be considered if they are to become competent problem solvers. Furthermore, students need to become exposed to various instructional strategies and engaged in meaningful learning tasks (Baroody & Hume, 1991; Hiebert, 1986; Hiebert & Behr, 1988; Skemp, 1987; Doyle, 1988; Schoenfeld, 1988).

Whilst the previous two paragraphs indicate that teachers should be encouraged to use real life experiences and appropriate instructional strategies, authentic word problems are not always easy to generate (Graves, 1983). This suggests that although students are encouraged to use their rich cultural experiences to generate meaningful problems, culturally relevant problems are seldom used. Limited use of culturally rich problems is illustrated in Woodward and Baxter's (2000) study of "real life" word problems with middle school students, which showed that what was initially thought to be appropriate real world problems, had little relevance to 12 and 13 year olds. Clearly, teachers need to become aware of the multiple experiences in their classroom and skilfully incorporate these within authentic learning task (Cohen et al, 1994; Resnick et al., 1992). These views illustrate that mathematical competency is dependent on a number of factors such as instructional strategies, learning styles, and background (Hiebert, 1986).

2.2.3 Effects of Classroom Experiences on Performance

Besides a review of the process for developing mathematical competencies, it is important to examine the effects of classroom experiences on performance. At the onset, Eggen *et al.*, (2003) notes that students' performance is highly dependent on teacher effectiveness

and classroom instruction. Furthermore, Salvin (1989) posits that effective teaching and learning experiences is dependent on the teachers' ability to skilfully present information in ways in which learners will understand. Therefore, if skills such as reflecting on the key attributes of the problem, identifying knowns and unknowns and to devise a plan to solve a problem are effectively fostered, students' problem solving competencies can be enhanced (Rubenstein and Thompson, 2001; Steinbring, 1998). While teachers play an important role in the development of problem solving skills, students are most successful when they select the most appropriate strategies (Montague, 2003). That is, students should be encouraged to use a number of problem solving strategies (Hiebert, 1988). For example, in a problem such as,

Jessica and Joy baked 36 cookies. Then, Joy dropped 12 cookies on the floor. How many were left? (Couger, 1995:273)

Students could use prior knowledge and analytical thought to form mental representations of basic addition and subtraction problems (Riley, Greeno, & Heller, 1983; Nathan, Kintsch, & Young, 1992; Halford, 1993; English, 1997; Pape, 2004). That is, students can count the number of cookies that were not dropped or if they are unfamiliar with cookies, beans or pebbles can be used as a concrete model of the situation (Resnick et al., 1987). Furthermore, to interpret problems students can draw pictures or symbols such as $36 - 12 = \square$ (Cougar, 1995:274). Therefore, educators need to be aware that when students are provided with meaningful representations, they are better able to interpret and solve complex problems (Carpenter et al., 1993).

In addition to providing opportunities for students to interpret and identify solutions, they should be provided with multiple ways to solve word problems, (NCTM, 2000). That is,

When different methods yield the same answer, students gain confidence both in the answer and in the various methods. This can be particularly useful for students learning more abstract and powerful methods: if a more powerful method gives the same answer as a more familiar method, then the powerful method is more likely to be understood and trusted. Carpenter et al., 1993: 276

In addition to the view that students should be guided to use various problem solving strategies, we are reminded that when students socially construct understanding they are better able to compare points of views and examine multiple solutions (Vygotsky, 1986; NCTM, 2000). That is, word problems are best solved when students share, negotiate, and discuss with others. The effect of social interaction is best described in the following interview conducted in a study to examine experiences in social learning groups:

We got to work with partners and so if one of us did not know the answer then maybe the other one did and because we could use each other's ideas and put them together to help us we found many ways to solve the problem.

Gray, 2004:83

Identified is that peer interaction is a decisive element in the problem solving process.

While this is seemingly true, many teachers lack skills to facilitate group interaction and to represent mathematical content (Payne, 1976; Fendel 1987; U.S Department of Education, 2007). They often also lack skills to engage students in meaningful learning experiences.

Limited background skills to engage students in meaningful math activities have been identified in a School and Staffing Survey conducted in the U.S. which illustrated that 21

percent of secondary and 65 percent of middle school math teachers did not have an undergraduate or graduate major in mathematics (Seastrom et al, 2002). Similarly, an examination of the mathematical competency of elementary teachers in Belize, Central America, also illustrated that many lack the background skills to teach elementary mathematical content (Petzold, 2007). Furthermore, studies on teacher effectiveness have also revealed that some were unclear and unable to explain key mathematical terms such as minuend, exponent, rational, and odd (Zevenbergen, 2000). In addition to research findings which indicate that teachers often lacked knowledge of mathematical concepts, in a study with one hundred and forty-seven elementary prospective teachers, it was revealed that twenty-six mathematics majors and one hundred and twenty-one teachers held primitive views about multiplication and division (Tirosh, 1998). Tirosh further noted that 60% of the sample argued that “multiplication always makes bigger” and 51% argued that “division always makes things smaller” (ibid.: 04). This literal interpretation of mathematical operations highlights that if students are to excel, teachers must develop clear interpretations (Ball, 1993b; Even 1990). Secondly, students are often taught wrong or partially accurate mathematical concepts (Graeber et al., 1989; Reys, 1985; Simon, 1993; Wheeler, 1983).

Besides research findings which suggest that many teachers lack the requisite math skills, a case study conducted with six Filipino Secondary School Mathematics Teachers to identify the use of effective instructional skills revealed that teachers also used limited ways to prepare lessons to address mathematic deficiencies. There were also instances when teachers taught in a clear manner, yet, they lacked depth in content (Vistro-Yu, 2000). This restates that teachers’ background and instructional practices must be addressed if

students are to succeed (Eggen and Kauchak, 2002).

While teachers should acquire requisite background pedagogical skills, of equal importance is their belief about mathematics (Charalambos, Philippou & Kyriakides, 2002; Ernest, 2000; Karp, 1991). As Bandura (1977) rightly notes, students are vicariously affected by teachers' actions. That is, teachers' background and motivation can influence students' response to instruction. This has been supported in an empirical study of the role of examples in schema-based problem solving, which revealed that in addition to motivating students, skilled teachers effectively presented students with thorough demonstration of specific problems (Cooper & Sweller, 1987). Noteworthy is that teachers' knowledge and belief about mathematics are closely linked to their instructional decisions and actions (Brown, 1985; National Council of Teachers of Mathematics, 1989; Wilson, 1990a, b; Brophy, 1990; Thompson, 1992; Doyle, 1988; Raymond, 2000). These perspectives indicate that instructional practice can have far-reaching effects and can be damaging to students. Furthermore, mastery of word problems is dependent on an array of factors such as motivation, background, and the learning environment (Vygotsky, 1986; Cooper and Sweller, 1987; Fosnot, 2005).

Although the prevailing view is that performance is highly dependent on pedagogical skills, background knowledge, and teacher motivation, earlier studies on the relationship between teachers' knowledge of mathematics and students' success showed a lack of correlation between the two (School Mathematics Study Group, 1972; Eisenberg, 1977; and General Accounting Office, 1984 as cited in Fennema and Franks, 1992). While the blame cannot be placed solely on teachers, Boaler (2003) posits that teachers are

responsible for using appropriate instructional practices to scaffold learners. Therefore, teachers must work assiduously to ascertain that the necessary competencies are developed. Students must also value and respond positively to mathematics instruction (Crespo, 2003). This calls for recognizing the importance of students' response to mathematics instruction, as has been cited in studies to analyze the behavioural attitudes of Hispanic and Native Americans in the U.S., which indicated that:

There is a negative image of math, whereby mathematicians are perceived as remote, sloppy, obsessive and (no pun intended) calculating. No matter what your socioeconomic status happens to be, it is not likely that you will strive hard to become an unscrupulous, calculating, cold, sloppy, and obsessive number cruncher.

Cocking and Chapman: 1988:32

This dismal view illustrates that unless students develop positive attitudes toward mathematics instruction, many initiatives and new approaches to learning may be futile.

2.2.4 Effect of Comprehension Skills on Math Word Problem

Although the points in the previous subsection suggest that lack of mastery can be attributed to problems associated with instructional practices, also opined is that mastery is based on language and socio-cultural background (Durkin and Shire, 1987; Orton & Wain, 1994). That is, many students are unable to solve math word problems due to language difficulty, cultural background, and learning modality (Saxe, 1992; Davison and Schindler, 1988). Also noted in a study conducted by the International Students Assessment Program

(2003) is that 15 year olds in the U.S. scored below other developed countries in math word problems and lower than 25 of 38 developing countries. Similar research in the United States with ninth grade Chinese students who usually excelled in mathematics also revealed that students experienced difficulty solving open-ended word problems (Li, 1998a). While it is presumed that these students were unable to solve the foreign word problems in their American classrooms, it appears that language, culture, and learning styles can affect students' performance. Notwithstanding that experiences and culture can affect performance, Secada (1993) warns that it is too simplistic to attribute difficulties to any one factor, hence the need to analyze factors which influences students' ability to read and analyze word problems.

Based on the perspective that there are many reasons why students experience difficulty, worth noting is that good problem solvers read and use comprehension skills to translate language and numbers into meaningful mathematical notations. They also read the problem several times to identify possible solutions (Montague, 2003). This restates that solutions to word problems are dependent on students' ability to read and to interpret written words (Fosnot, 1989; Chamot & O'Malley, 1988a).

Although students are required to read and comprehend words, (MacGregor & Price, 1999), the length of sentences affects the extent to which students interpret the problem (Cragg and Nation, 2006). Furthermore, word problems often include a number of words that are very distracting including prepositions with multiple meanings (Smith, 1994). For example, the phrase "in the box" and "on the box" may be difficult to translate in some languages. Additionally, phrases such as three-fourths of sixteen pizzas and terms such as

reduce by five inches appear complex to most students (Weist, 1996/1997). Furthermore, the same preposition can also point to different actions. For example, 3 multiplied by 10 versus 3 increased by 10 (Dale and Cuevas, 1992). Clearly, “prepositions in general and the relationships they indicate are critical lexical items in the mathematics register that can cause a great deal of confusion” (ibid.: 333). Hence, if students are expected to comprehend and solve word problems, these must be written in very clear terms and free of jargons.

The aforementioned indicate that if students are expected to read and interpret written information, it must be written clearly and concisely. Secondly, educators must provide students with the competencies to critically examine and tackle math word problems (Swanson & De La Paz, 1998). Consequently, teachers need to respond to questions such as can students read and interpret information, and to what extent are they able to do so (Cragg and Nation, 2006). Word problems must also be meaningful and relevant to students’ culture and background. Therefore, educators must consider the “social tradition or culture, and how these social goals influence student learning” (NCTM, 2001a: 7). Hence, students’ culture and background must be taken into account if they are to become skilled problem solvers. In short, “an important component of mathematics education should be to reaffirm, and in some instances, to restore the cultural dignity of children” (D’Ambrosio, 2001: 308). Undeniably, if children are to interpret and solve problems, it must be meaningful and relevant to their culture and background.

2.3 Problem Solving Skills

To further examine factors which may impact on the extent to which social constructivist-based activities are successfully implemented in Belize, Central America, literature pertaining to problem solving skills and strategies are examined in this subsection.

Problem solving is one of the five fundamental mathematics standards aligned with competencies such as the ability to reason, identify proof, effectively communicate mathematical thoughts, and to accurately represent mathematical concepts (National Council of Teachers of Mathematics, 2000). Central to mathematical teaching and learning is linking mathematical concepts to real life experiences which is an integral part of the problem solving process (Kilpatrick, Swafford, & Findell, 2001; Swartz & Parks, 1994). This process which includes “visualization, association, abstraction, comprehension, manipulation, reasoning, analysis, synthesis, generalization” (Garofalo & Lester, 1985: 169), involves use of multiple cognitive skills to make meaningful connections among and within mathematical concepts (Hiebert, 1986). Therefore, problem solving skills include the ability to use prior knowledge to interpret information (Mayer & Wittrock, 1996). That is, to solve problems students must make meaningful connections between what they know and the new problem (Hiebert and Carpenter, 1992). Indicated is that, “students must learn mathematics with understanding, actively building new knowledge from experiences and use prior knowledge” (NCTM, 2000: 20).

Although the importance of prior knowledge and meaningful interpretation has been acknowledged, (Good and Brophy, 2000; Thompson, 2000; Pirie and Kieren, 1994;

Vygotsky, 1986), as previously noted, students are often presented with problems that are totally unrelated to their experiences. For example, if students are presented with a problem such as:

Two ponies and three elephants are racing through a jungle. The ponies' speed is twice as fast as the elephants' speed. If the ponies are running at a speed of 20 miles an hour, what is the elephants' speed?

This problem can prove challenging to students who have no cultural experiences of elephants and ponies (Hines III 2008:27). Shown is that transfer, which refers to “when a skill acquired in one setting can be applied in a different setting” (Eggen and Kauchak, 2003:296), is an important aspect of the problem solving process. In addition to transferring information and appropriately applying math skills, “good problem solving calls for using efficiently what you know: if you don't have a good sense of what you know, you may find it difficult to be an efficient problem solver” (Schoenfeld, 1987a: 190). Shown is that problem solving skills entails more than drawing on ones background knowledge; instead, information must be effectively applied to new problem situations (Fosnot, 2005; Thompson, 2000; Pirie and Kieren, 1994). While these skills may appear simplistic, students are often engaged in routine problem solving activities and not exposed to meaningful problem solving tasks (Boaler, 1989). This has been identified in a study of teacher effectiveness among teachers in urban schools in Belize, Central America, which illustrated that classroom instruction was routine and students often engaged in activities which failed to promote critical thinking skills (Brown Lopez et al., 2009). Therefore, if students are not exposed to effective problem solving skills, it is unlikely that they will demonstrate mastery. Undoubtedly, poor math performance as revealed in the 2007 Primary School Leaving Examination results in Belize can be attributed to ineffective classroom practices (Ministry of Education, 2007). While there are multiple factors which

affect performance, it is important that students are exposed to appropriate teaching and learning conditions and guided to develop critical thinking skills (Hines III, 2008).

Besides the notion that problem solving skills are promoted when students transfer background skills, and are exposed to contextually rich and relevant problems, problem solving skills has to do with self awareness and regulations. This includes:

1. making sure that you understand what a problem is all about before you hastily attempt a solution;
2. plan how to solve a problem;
3. monitoring, or keeping track of how well things are going during a solution; and
4. allocating resources, or deciding what to do, and for how long, as you work on the problem

Schoenfeld, 1987: 190-191

These processes refer to students consciously thinking about their thinking to solve a problem. Wilson and Clarke (2004) describe this reflective cognitive process as metacognition which occurs in three main parts: 1. Awareness of one's thinking; 2. Evaluation of that thinking; and 3. Regulating thought. The view is that during the problem solving process, students consciously interpret a problem, analyze the important attributes and decide on a solution (Simon, 1997). Shown is that unless students are able to engage in these higher order thinking activities, they may experience difficulty solving math problems (Hiebert and Carpenter, 1992).

Although it is apparent that problem solving skills require higher order and metacognitive skills, in many countries, students' "ability to solve word problems falls far below their ability to compute" (Burns, 2000:02). This is due to the fact that in most situations, classroom instruction is relegated to imitation and procedures (Goldberg, 2000). That is, teachers provide students with sequential steps without providing opportunities to develop personal representation and understanding of math problems (Hines III, 2008). Clearly, these linear teacher-directed problem solving processes fail to promote thinking skills as Cockcroft rightly notes that "problem solving is a means of developing mathematical thinking tools as a form of daily living, saying that problem-solving ability lies 'at the heart of mathematics'" (1982:73). The prevailing view is that effective problem solving skills and strategies include opportunities for formulating key questions, analyzing and conceptualizing problems, defining problems and goals, discovering patterns and similarities, seeking out appropriate data, experimenting, transferring skills and strategies to new situations (NCTM, 1989).

While guidelines to enhance problem solving skills have been identified, restated is that teachers play an important role in the development of students' thinking skills (Eggen and Kauchak, 2003). Therefore, although proponents such as Cobb et al., suggest that problem solving involves meaningful learning activities to "encourage the interiorization and reorganization of the involved schemes as a result of the activity", (1991:187), these will not become evident unless teachers provide opportunities for students to acquire problem solving skills (Thompson, 2000).

2.4 Analysis of appropriate use of Pólya's Steps to Solving Word Problems and Problem-Based Learning Strategies

Having discussed problems solving strategies in the previous subsections, three commonly used mathematical procedures will be analyzed. Firstly, Pólya's Problem Solving Approach will be reviewed to examine whether this approach is suitable to teach math problems. Secondly, the effects of reconstructing the learning environment to enable students to work in a problem-based learning environment will be explored. Finally, the Constructivist Approach and its implications will form the basis for determining whether authentic resources, social interaction and current understanding are suitable guidelines to develop mastery of math problems.

Fundamental to analyzing solutions to word problems is the work of Pólya (1945) in his text entitled "How to solve it". In Pólya's view, four sequential steps can be used to facilitate clear interpretation and solutions to word problems. At the onset, Pólya emphasized that students must interpret and understand the problem (Bradford and Stein, 1984). While students are expected to read and interpret mathematical ideas, this is not an automatic process (Gavora, 1992). This requires competency to identify the elements of the problem and to represent information in a clear and systematic way. In view of the fact that word problems require that students read and interpret written information, levels of understanding vary among students (Skemp, 1987). That is, some students have strong prior knowledge to comprehend and interpret word problems while others lack the basic skills to do so (Eggen and Kauchak, 2003). To overcome these difficulties, students' development and prior knowledge must be considered.

The second essential step in Pólya's (1954) Problem Solving Method includes devising a clear plan to solve the problem. In this step, students are asked to respond to key questions such as what should I do to find a solution to the problem and to select formal mathematical techniques (Nickerson, 1987). The problem is that although there is a need to devise a plan to solve the problem, most teachers often fail to teach students how to think and to use appropriate strategies to do so (deBono, 1985). As a result, many students may not acquire the requisite fundamental skills during their formative years of development. This has been noted in research studies conducted by the National Commission on Excellence in Education in the United States which identified that:

Many 17 year olds do not possess the 'higher-order' intellectual skills we should expect of them. Nearly 40 percent cannot draw inferences from written material; only one-fifth can write a persuasive essay; and only one-third can solve a mathematics problem requiring several steps.

Tanner, 1993:288-297

Indicated is that many students lack critical thinking skills or are able to identify key attributes of a problem (deBono, 1985). Therefore, if students are to develop mathematical competency, critical thinking and analytical skills must be taught. Even as it is implied that math problems can be challenging for students with weak background skills, the saturation of textbooks and curricula which emphasize memorization over understanding also stifled students' creativity and analytic skills (Norris, 1985). This shows that if students are to devise a plan to solve a problem, they need to be exposed to relevant resources, analyze information, and become active thinkers in their quest to develop problem solving skills (Fosnot, 1989).

The views presented thus far suggest that mastery of Pólya's (1954) first two steps is dependent on prior knowledge and innovative instructional strategies. Additionally, the final two steps suggest that students must also execute a plan and evaluate the results. What is troubling is that these final steps call for a range and variety of critical thinking skills such as to justify, infer, hypothesize, and to draw conclusions (Webb et al., 1986). Despite the fact that students are expected to acquire critical thinking skills, many cannot solve word problems with multiple steps even if they know facts and procedures (Lauren and Adam, 2000). Hence, if students are to apply Pólya's four step approach to solving word problems, teachers also need to scaffold and develop students' analytical and critical thinking skills (Brown et al., 1989). This indicates that mastery of word problem is not solely dependent on students' performance; instead, teachers play a critical role in shaping the learning environment and guiding students to mastery (Eggen et al., 2002).

2.5 Problem Based Learning

In addition to Pólya's (1954) approach in the previous subsection, the effects of Problem-Based Learning in which authentic resources are used to construct and recreate real life problems are examined. At the onset, Ha (2003) notes that Problem-Based Learning fosters creative and innovative solutions to math problems. This means that the learning environment is structured to enable students to examine multiple resources such as pictures and word games and to detect patterns and solutions (Mason, 1990).

Noted in this approach is that learning begins with a problem to be solved and students are required to gain new knowledge before they can solve it (ibid.: 01). However, if many

students are unable to interpret word problems, presumably they will also experience difficulty identifying additional information and suitable resources (Skemp, 1987). Notwithstanding the difficulties associated with interpreting and identifying solutions, according to Boaler (1998), in Problem-Based Learning, mastery is dependent on the organization of the learning environment and use of multiple resources. This means that mathematical instruction is organized around mathematical activities (Krulik and Rudnick, 1999). Therefore, students must be provided with resources and diagrams to visualize problems (Van Garderen & Montague, 2003; Ha, 2003). The importance of visual representation has been documented in a study to explore the effectiveness of interactivity on learning. In this study, as a student became engaged in designing a plan for a learning environment, a second student listened to the explanation for doing so. The findings illustrated that the student who interacted with the environment remembered more information than the student who listened passively (Moreno et al., 2001). While it is apparent that interactivity and authentic resources can positively impact student learning, worth noting is that over 90% of mathematics class time in the United States is spent on practicing routines and limited time spent on interactive authentic learning experiences (Schmidt, Mcknight and Raizen, 1997). Contrary to this view is that Grade Eight Japanese students spend approximately 60% of academic learning time applying and inventing new procedures (ibid.: 69-102). Evidently, resources and instructional practices affect the extent to which students become engaged in meaningful discourse and solve problems. In addition to the abovementioned, according to Willis & Fuson (1988), if young children are provided with adequate resources, they are better able to understand and solve math word problems. Similarly, resources such as tiles can be used to ease the transition to the abstract level in algebra among young students (Howden, 1986). This study illustrated that

using tiles and drawing pictures encouraged children to form mental representations and solutions to word problems. While it is important to form mental representations and use manipulatives, we are reminded that long-term use of manipulatives is far more effective than short-term use (Sowell, 1989). Implied is that consistent use of adequate resources can assist students to interpret and solve problems (Fosnot, 2005). While it is perceived that resources can impact the extent to which students interpret and solve problems, a meta-analysis of 60 studies to examine the effectiveness of a range and variety of manipulatives from kindergarten to postsecondary revealed that most teachers do not use manipulatives (Sowell, 1989; Jitendra, 2002). Furthermore, a study to recognize the use of manipulatives among primary teachers in 11 states in the U.S. revealed that inexperienced teachers use manipulatives more often than experienced teachers (Gilbert and Bush, 1988). While it is reasonable to assume that the reverse would be true, it appears that experienced teachers often resort to using traditional systematic approaches requiring less resources and limited student interaction (Eggen and Kauchak, 2003). Based on these perspectives, we are reminded that students are expected to manipulate concrete objects, engage in small group interactions, and have a strong mathematical background (Kalaian and Mullan, 1996). Therefore, if learners are to successfully apply approaches such as Problem-Based Learning, there is also need to ensure that appropriate resources are used to critically analyze and to actively identify solutions (Boaler, 1998).

Even as it is apparent that manipulatives can and must be used to help students to understand the critical attributes of a problem, we are cautioned that careful attention should be placed in use of manipulatives (Howden, 1986). For example,

You can have students in cooperative groups working on trivial tasks. You can use manipulatives to do rote and meaningless procedures. A teacher can walk around encouraging students but never check their work or their thinking.

Burrill, 1997:03

This indicates that while manipulatives can be used to assist students to interpret math problems, there are instances when these are not used effectively. While these perspectives suggest that students should engage in meaningful activities, it also reinforces the idea that students must be introduced to manipulatives in small sequential steps. Hence, teachers play a crucial role in the selection and appropriate use of manipulatives. This calls for scrutiny of instructional practices to ensure that students are engaged in meaningful learning experiences.

Besides the extensive discourse on authentic learning resources, it is worth noting that Problem-Based Learning is centred on inquiry and reconstructing word problems. Therefore, two critical issues must be considered. The first is that students do not automatically become creative and develop skills to solve math problems (Carpenter et al., 1992). In fact, teachers are also expected to guide students to develop the requisite skills to solve (Stigler & Hiebert, 1997). Illustrated is that while some approaches suggest that learners should construct their own understanding, the role of the teacher in guiding and supporting students is also crucial (Fosnot, 2005).

2.5.1 Implications of Pólya's approach and Problem Based Learning

It is important to question whether instructional methods such as Pólya's or Problem-Based Learning are suitable for children of varying abilities or can be used to enhance students' math skills in Belize, Central America. According to Kalaian and Mullan (1996), the Problem-Based Learning approach is not appropriate for students with learning disabilities who are unable to perform basic commutative operations such as $2+9$ and $9+2$. While lack of basic mathematical facts may affect performance, it is perceived that in Problem-Based Learning, the classroom resources can be organized to enable students to develop conceptual understanding and to facilitate enquiry (Schoenfeld, 1989; Boaler, 1998). This suggests that mastery is not solely dependent on knowledge of mathematical facts but on the use of resources to interpret, construct and to model key elements of the problem (Fosnot, 2005). As Montague (2003) rightly notes, successful problem solvers are those who draw a picture or diagram, use verbal translation and visual representation to solve the problem. Hence, the literature on Pólya's (1954) approach and Problem-Based Learning illustrates that Problem Solving skills are not easily acquired. Furthermore, educators need to continuously identify creative ways to assist students to develop and refine these skills (Brown et al., 1989). The literature also illustrates that while teachers play a crucial role in guiding students to identify solutions, it is apparent that some students lack the necessary critical thinking and enquiry skills (Schoenfeld, 1988).

In addition to the literature on Pólya's and Problem-Based Learning, it is important to assess whether these approaches are suitable to enhance problem solving skills among students in Belize who are customarily taught using traditional teacher-directed instruction. Firstly,

illustrated in the Pólya's problem solving methods is that students become engaged in a number of specified steps to solve problems. While structural guidelines provide explicit steps to be followed, students can benefit from a less rigid problem solving process (Eggen et al., 2006). For example, Schoenfeld notes that although Pólya's approach is widely known, use of heuristics to solve problems has been disappointing. That is,

“Despite all the enthusiasm for the approach, there was no clear evidence that the students had actually learned more as a result of their heuristic instruction or that they had learned any general problem-solving skills that transferred to novel situations.”

Schoenfeld, 1987a:41

Lester (1994) also identified that use of explicit problem solving steps does not enhance problem solving skills. Therefore, an important aspect of the problem solving process is to promote the development of thinking skills enabling students to apply higher order cognitive process (Desoete, Roeyers & De Clerq, 2003; Ginsburg-Block & Fantuzzo, 1998; Pape & Smith, 2002).

In addition to insights on how students may become engaged in problem solving activities and the importance of higher order cognitive activities, an examination of the appropriate use of Problem-Based Learning revealed that for this approach to be successful, teachers must have in-depth understanding of mathematical concepts and have the requisite pedagogical skills (Eggen & Kauchak, 2006). Clearly, an instructional strategy which requires strong background knowledge and instructional skills will be challenging for many teachers in Belize who are not professionally trained (Ministry of Education, 2007).

Neufeld and Barrows (1974) also note that teachers who often teach by directing

instruction will experience difficulty adopting the role of a Problem-Based Tutor. Even after training, teachers who have taught in traditional ways “mistakenly develop the belief that tutoring is nothing more than observation of the process and tutorial dynamics” (Neville, 1999:01). Therefore, if teachers are unskilled, it is very likely that they would not choose appropriate tasks and effectively guide students using Problem-based activities (Prawat & Smith III, 1997). Based on this limitation and the need to identify a feasible instructional approach to improve students’ problem solving skills in Belize, in the subsequent subsection, the constructivist approach will be analyzed to determine whether social interaction and the use of authentic resources are viable strategies to acquire skills to solve a range of math word problems.

2.6 Constructivist Approaches to solving Mathematical Word Problems

According to Eggen and Kauchak (2003), constructivism “is a view of learning in which learners use their experience to create understanding rather than having understanding delivered to them in already organized forms” (2003:230). While Eggen and colleague describe constructivism as a process in which the learner explores and develops meaning, according to Cobb et al.,(1992), the interchange between the learner and his environment is described thus:

learning would be viewed as an active, constructive process in which students attempt to resolve problems that arise as they participate in the mathematical practices of the classroom. Such a view emphasizes that the learning-teaching process is interactive in nature and involves the implicit and explicit negotiation of mathematical meanings. In

the course of these negotiations, the teacher and students elaborate the taken-as-shared mathematical reality that constitutes the basis for their ongoing communication

Cobb, Yackel, & Wood, 1992: 2-33

Although Cobb et al., describe constructivism as explorations guided by the teacher, what does the term “constructivism” really imply? Firstly, if the term constructivism is broken into sub parts, the word construct is defined as an image, idea, or theory, especially a complex one formed from a number of simpler elements (Von Glasersfeld, 1987). This indicates that constructivism has to do with formulating ideas based on experiences. It also goes beyond how the brain stores and retrieves information but fosters “meaning making” based on students’ personal experience (Savery et al., 1994). Notwithstanding these views, it is also important to question whether learning can be constructed. In Fosnot’s (2005) view, learning is constructed when students generate their own knowledge through active participation. This suggests that students can be provided with opportunities to examine information and to “make sense” of it. The prevailing view is that the learner is guided towards acquiring knowledge or skill rather than being told or explained (Cobb et al., 1992).

Even as Fosnot and colleagues describe knowledge construction as those experiences that are personally constructed, in Ausubel’s view, understanding occurs only when “the content is potentially meaningful and the learner relates it in a meaningful way to his or her prior knowledge” (1968:02). The prevailing view is that understanding has to do with connecting information to prior knowledge or experiences (Eggen et al., 2003). Similarly,

Rogers (2003) notes that understanding is the result of using ones prior knowledge to bring about a change in behaviour as illustrated in the vignette below:

“No, no, that's not what I want; Wait! This is closer to what I am interested in, what I need; Ah, here it is! Now I'm grasping and comprehending what I *need* and what I want to know!”

Rogers et al., 1983: 18-19

This implies that constructing understanding is illustrated by students interpreting information and vocalizing their thinking as they synthesize, analyze and critically evaluate information (Eggen et al., 2003). Therefore, “to know is to represent accurately what is outside the mind; so to understand the possibility and nature of knowledge is to understand the way in which the mind is able to construct such representations” (Cobb, Yackel, and Wood, 1992:03). Furthermore, constructing understanding is evident when learners are able to read and interpret word problems and to accurately represent their thought process with others. Therefore, to construct understanding, students must have the prerequisite mathematical background to read and interpret the written text (Weist, 1987).

In addition to the aforementioned, Good and Brophy posit that in the constructivist approach, “Learners construct their own understanding; new learning depends on current understanding; learning is facilitated by social interaction and meaningful learning occurs with the use of authentic learning tasks” (2000:231). At the onset, it is important to examine the first two characteristics, which has to do with constructing understanding based on experiences. Firstly, these two characteristics suggest that students must use their rich background to interpret problems (Scardamalia and Bereiter, 1991). While students

can use their own experience to interpret and identify solutions, of note is that students frequently develop understandings that are immature and incomplete (Eggen and Kauchak, 2003). Parallel to this view is that:

Even with what is taken as good instruction, many students, including academically talented ones understand less than we think they do. Their understanding is often limited or distorted, if not altogether wrong.

Rutherford and Ahlgren, 1990:185

This illustrates that if students are left to develop their own understanding, they may misinterpret essential components of the problem. This has been noted in a video clip of Grade 5 students in the U.S. to observe how they worked in cooperative learning groups to balance objects on a beam. This video illustrated that although students were provided with clear guidelines to discuss and construct understanding, teachers prompted and used multiple verbally clues to guide students to reflect and identify a solution (Eggen et al., 2003). Also illustrated is that although students discussed ways of balancing the beam in their cooperative learning groups, individual students were steadfast in their views about how the problem should be solved. What this really showed is that students' background influenced how they perceived and interpreted word problems. Thus, to assist students to interpret and solve, they must be guided to reflect on the critical aspects of the problem and provided authentic resources for this to occur (Eggen et al., 2003; Vygotsky, 1986). For example, the importance of instructional scaffolding and authentic resources has been documented in a research involving multiplication and division situations. This study revealed that when provided with manipulatives to solve the problem, students gained and

retained problem-solving skills and were able to transfer these skills to other subject areas (Jitendra et al., 2002). Implied is that, students learn best when they manipulate resources to interpret math problems.

Despite the perception that the constructivist approach is a viable strategy to solve math problems, teachers are often faced with two major challenges. The first is that during the problem solving process, a great deal of expertise is required to scaffold students (Brown, 1993). Secondly, most teachers use didactic instructional methodologies which fail to promote deep interpretations (Kukla, 2000). This indicates that constructing understanding is a highly intricate process which relies on the experiences of the learner and the teachers' ability to guide students to use background experiences to generate new information (Fosnot, 2005). Therefore, if teachers are untrained and unskilled, students may never develop the skills to solve math problems (Brownstein, 2001).

Having identified implications of prior knowledge in constructing understanding, it is also important to critically examine Good and Brophy's (2000) final two characteristics which entail constructing understanding in social learning groups. The thinking is that optimal learning can occur if there is dynamic interaction between the instructor and learners (Nelson, 1994). The prevailing view is that knowledge is not passively received but actively developed through use of authentic resources, socialization and ongoing small group discussions (Ernest, 1991). This also means that unlike traditional instructional approaches, students are encouraged to vocalize their thinking and to work in doer-listener pairs to successfully develop mastery (Leinhardt, 1989). The effects of social interaction on students' learning has been documented in a seven week empirical study with one

hundred and four low achieving, low income 3rd and 4th graders in an urban elementary school in the United States. In this study, students who were randomly assigned to experimental groups to interact with peers to solve word problems, outperformed students who worked alone (Ginsburg-Block and Fantuzzo, 1998). This denotes that interactivity and dialogue among peers can be beneficial (Cox et al., 1992). That is, “joint attention and shared problem solving is needed to create a process of cognitive, social, and emotional interchange” (Hausfather, 1996:10). The term ‘shared problem solving’ implies that social interaction occurs best if students have equal opportunity to voice opinions and identify solutions. Essentially, successful interaction is also highly dependent on meaningful collaboration among all parties (Driscoll, 1994).

In addition to the notion that the constructivist approach emphasizes social interaction, similarly as in Problem-Based Learning, teachers are expected to use their initiative to guide students through the inquiry process to design and structure the learning environment (Rhodes and Bellamy 1999; Cox et al., 1992). Although strong advocates of the constructivist approach promote organization and meaningful interaction, Savery (1994) contends that the more structured the learning environment, the harder it is for the learners to construct meaning. In essence, students should be given the autonomy to reconstruct their own environment and to select resources to “tackle” problems (Vygotsky, 1986). This also shows that the constructivist approach requires detailed planning, skilled instructors and relevant resources if students are to develop the skills to analyze and identify meaningful solutions (Eggen and Kauchak, 2003).

While it has been echoed that social interaction and peer instruction can result in enriched learning experiences, in these learning groups, misconceptions are often not intentionally instructed; yet, learners often misunderstand problems repeatedly (Leinhardt et al., 1990). Restated is that while it is paramount for students to collaborate and construct understanding, this should be done under the auspices of the teacher. As Vygotsky (1986) rightly notes, during social interaction there are instances when teachers must bridge the gap with what the students comprehend and the task to be learnt. Clearly, the role of the teacher in scaffolding the student is vital. Therefore, students are required to think beyond the ordinary and teachers are expected to use appropriate strategies to scaffold students to do so (Eggen and Kauchak, 2003).

Besides Good and Brophy's belief that constructing understanding requires use of prior knowledge and social interaction, an important final component is the use of authentic resources to aid in knowledge construction. Identified is that if students are provided with real life examples, they are better able to interpret and solve math problems (Lach, 2005). That is, "objects that appeal to several senses and that can be touched, moved about, rearranged, and otherwise handled by children" (Kennedy, 1986:06) can contribute to students' problem solving abilities. The importance of meaningful authentic resources has also been noted in a study conducted with five classes of eight graders which revealed that use of authentic resources resulted in greater performance in pre-algebra classes (Raymond and Leinenbach, 2000). While manipulatives can result in higher levels of achievement, students often use manipulatives in a routine manner without clearly interpreting or developing conceptual understanding of the problem (Wearne and Hiebert, 1988). This illustrates that although manipulatives should provide students with concrete

representations, (Hall, 1998), these must be appropriately utilized. Therefore, use of manipulatives alone is no guarantee that students will automatically interpret problems. Essentially, “manipulatives alone cannot—and should not—be expected to carry the burden of the many problems we face in improving mathematics education” (Ball, 1992:47). What is more is that performance is highly dependent on teachers’ experience and skilful use of manipulatives (Raphael, Wahlstrom and Sowell, 1989).

Based on the perspective that authentic resources alone may not enhance mathematical competency, in Thompson and Lambdin’s (1994) view authentic resources can be used as follows:

1. To enable teachers and students to have discourse about something concrete—discussing how to think about materials and the meanings of various actions
2. To provide something upon which students can act.

Therefore, authentic resources can be used to present mathematical ideas and to assist students to interpret concepts (Hiebert, & Carpenter, 1992). Another important consideration is that authentic resources should be aligned to teaching activities and contribute to conceptual understanding of math problems (Eggen et al., 2003).

Therefore based on the guidelines presented for constructing understanding and as identified in the rationale of the study, teachers can be provided with information on how to guide students to construct understanding in social learning groups. Given that grade five students have been taught basic computational skills from grades 1 to 4, and authentic resources can be acquired in Belize to assist students to represent and interpret math problems, it is perceived that this approach is most suited to assess whether immersing

students in social constructivist activities will improve their critical thinking and mathematical skills.

2.7 Summary

The literature in this chapter denotes that while some approaches can provide students with guidelines to solve word problems, students should be immersed in activities to apply and develop critical thinking skills. Furthermore, background experiences, meaningful social learning context and visual representations must form the basis for knowledge construction. Having reviewed various theoretical and empirical studies aimed at providing guidelines for improving competencies and identifying that the constructivist approach is suitable to assess grade five students performance, in the subsequent chapter, the research design which outlines the scope of a study conducted is presented. A detailed account of efforts to analyze whether social constructivist-based activities will result in higher achievement is also described.

Chapter Three

Design of the Study

3.1 Introduction

Having described the research purpose, context and questions in Chapter One and the Review of Literature in Chapter Two, the purpose of this chapter is to describe the methods and theoretical framework of the study. In so doing, a restatement of the research questions and hypothesis precedes a description of the research design. Thereafter, Bruner (1986) and Vygotsky's (1986) Social Learning Theory provide the theoretical framework to analyze the effects of a constructivist-based instruction on the achievement of grade five students. Finally, efforts to secure threats to internal validity and a description of the sampling procedures are described.

3.2 Restatement of the Research Questions and Hypothesis

This study attempts to answer these questions regarding the impact of constructivist-based math problem solving on achievement:

1. Will use of the constructivist approach assist students to interpret and identify solutions to math word problems?
2. What are Grade Five students' experiences as they interpret, analyze, and solve math word problems?

Research Hypothesis

1. The mathematics problem solving achievement test scores of fifth graders who have completed six weeks of the constructivist-based instruction for mathematics problem solving will show a significant gain over pre-test scores.
2. The post-test scores of fifth graders who have completed the constructivist-based instruction for mathematics problem solving will show a significant gain over pre-test scores six weeks and twelve weeks after treatment.
3. The control/second-experimental group who will be taught using the constructivist-based instruction for mathematics problem solving from weeks six to twelve will show a significant gain over pre-test scores.
4. Unlike regular instructional methods, the constructivist-based approach will provide fifth graders with deep insights into successful experiences and difficulties as they seek solutions to Math word problems.

3.3 Design of the Study

Social science researchers such as Lincoln et al. (1985) and Schwandt (1989) posit that qualitative and quantitative approaches are incompatible; that is, total immersion into quantitative or a qualitative method provides a more focused approach rather than a

combination of both. While mixed methods have found their application in various social and behavioural researches, use of a purely quantitative or qualitative research is suitable in some instances (Gay et al., 2000). Contrary to Gay's perspective is that skilled researchers can successfully combine these approaches (Patton, 1990; Reichardt & Cook et al., 1979). The prevailing view is that mixed methods designs, which are an integration of different research procedures, can provide meaningful insights that neither analysis can provide alone (Green et al., 1989; Strauss and Corbin, 1990). Hence, a synthesis and justification of the qualitative and quantitative approaches are presented.

Firstly, quantitative methods, which often include experimental and control groups, allow the researcher to manipulate at least one independent variable and to control and observe the effect on one or more dependent variables (Gay et al., 2000). This means that to analyze the effectiveness of a constructivist-based instruction approach on the achievement of grade five students, treatment can be applied to the experimental group and comparisons made with the control groups to determine whether significant changes occurred. Use of true experimental designs are supportive of Cook and Campbell's (1979) and Huck et al.'s (1996) position, that these designs are more trustworthy than other pre-experimental designs since they reduce potential threats to internal validity such as experimenter bias, maturation, history and selection. In addition to the notion that quantitative research can be used to measure the relationships between various elements, we are reminded that the data gathering process should be free of bias and reflective of all research protocols (Shulman, 1986; Baxter Magolda, 2001). Hence, to investigate the effects of a Constructivist-Based Mathematics Problem Solving Instructional Program on the achievement of Grade Five Students, teachers were provided with guidelines for engaging

students in social interaction to construct understanding. Lesson plans were also reviewed to ensure that these were reflective of the constructivist approach to learning and pre-tests and post-tests were administered to the control and experimental groups. The pre test and post test 1 and 2 are illustrated in Appendix A, B and C respectively.

In addition to the quantitative procedures, qualitative designs provide deep understanding and multiple realities of the phenomenon being studied (Strauss and Corbin, 1992; Gosling and Edwards, 1995). In so doing, researchers learn more about the participants and the research setting (Bogdan & Biklen, 1998; Eisner, 1991; Patton, 1990). Therefore, qualitative methods were employed to examine how students constructed understanding in social learning groups (Vygotsky, 1986). Video recordings of students' interaction and semi-structured interviews were also utilized to gather "multiple perspectives as they emerged" (Ely et al., 1991:55).

While qualitative methods such as interviews allow for in-depth insights into students' thinking, it is often conceived as one person asking questions of another (Babbie, 2001). Despite this perspective, in qualitative research, interviews are used to "understand the subject's point of view to unfold the meaning of people's experiences and to uncover their lived world prior to scientific explanations" (Kvale, 1996:122- 130). Although interviews are used to "find out what is in and on someone else's mind," (Patton, 1990: 278), the interviewer maintains "double attention" by listening to the informant's responses and ensuring that questions are answered at the level of depth and detail" (Wengraf, 2001: 194). Therefore, unless interviewers skillfully elicit information and focus on the key elements of the interview guide, limited information may be obtained. In addition to

ensuring that questions are used to elicit desired responses, disturbing interviewer effects such as guiding the interview in a special direction must be avoided. Instead, interviewers must effectively gather the required information without interference or distortion (Opdenakker, 2006). These perspectives indicate that while interviews provide a rich source of information, appropriate research protocol must be employed. Incorporating the guidelines aforementioned, a semi-structured interview guide was also used to gather data on challenges and successes as student solved problems in social learning groups.

Besides using semi-structured interviews to gather information on strengths and weaknesses, worth noting is that students were also videoed and their actions recorded as they interacted with peers to solve math problems. While researchers such as Hendriks-Jansen (1996) propose that video recording provide opportunities to observe real events, the following advantages and disadvantage must be considered. Firstly, one advantage of using video recording was identified in a social research aimed at identifying whether career health workers effectively implemented training guidelines. Revealed is that video recordings enabled researchers to identify skills and other training possibilities (Leap et al., 2009). Similarly, use of video recordings to identify women's coping skills having undergone surgical treatment for cancer illustrated that video recordings enabled the researcher to pin point emotional and social well being that could not be documented by other research methodology (Lammer, 2009). In addition to these advantages, also worth noting is that video recordings also allow researchers to extensively review video footage (Fetterman, 2002). While these recordings provide rich contextual information, video recording can only capture what is observed. That is, "unspoken thoughts and feelings of a participant cannot be seen or heard on the tape" (DuFon, 2002:45). This implies that while

videos can be used to capture students' actions, as revealed in this study, other methods such as interviews or questionnaires were employed.

3.4 Theoretical Framework

According to Gay et al., (2009), a theoretical framework identifies the predominant theory and concepts of a study. It also provides information about, “what is being studied and how we are studying it” (ibid.: 429). Therefore, in this subsection two theoretical models of cognitive constructivism will be reviewed to provide deeper insights on the underlying principles governing the social constructivist intervention in Belize.

Firstly, in Bruner's (1986) Constructivist Theory, learning is an active social-process in which students construct new ideas based on their current knowledge. Bruner also proposed that while child-centred activities are important, authentic learning resources enable students to form meaningful representations in mathematics. Thus, Bruner cites that:

The concept of prime numbers appears to be more readily grasped when the child, through construction, discovers that certain handfuls of beans cannot be laid out in completed rows and columns. Such quantities have to either be laid out in a single file or in an incomplete row-column design in which there is always one extra or one too few to fill the pattern. These patterns, the child learns, happen to be called prime. It is easy for the child to go from this step to the recognition that a multiple table, so called, is a record sheet of quantities in completed multiple rows and columns. Here is factoring, multiplication and primes in a construction that can be visualized.

(1973:02)

Hence, in Bruner's view, high quality examples and representation of content is vital to learning. While Bruner proposed that interactivity, authentic resources and a well-structured curriculum are paramount, this view is supported in Vygotsky's (1986) Social Learning Theory, which suggests that constructing understanding provides the medium for students to share ideas and achieve common goals. That is, constructive activity has to do with individual achievement embedded and enabled by social interaction. As Vygotsky puts it, "the educational influence of the environment the child is immersed in is the only tool for adaptation of....reactions" (1997:02). This indicates that activity-based learning and students' experiences are paramount.

While prior knowledge and authentic resources are vital, the important role of the teacher is emphasized as follows: "In short, in some way or another, I propose that the children solve the problem with my assistance" (Vygotsky, 1978: 86). This suggests that although social learning groups provide the medium for students to generate thought and construct understanding, learning is also highly dependent on the teachers' ability to scaffold students during these constructive activities (Raymond, 2000).

Notwithstanding the theoretical foundation provided by Bruner and Vygotsky, this design is reflective of research studies on inquiry learning which indicated that constructing understanding is valued more than memorizing algorithms and using them to solve problems. One such study conducted in Massachusetts, U.S.A. with 40 constructivist kindergarten teachers, 40 traditional non-constructivist teachers, and 40 teachers with

beliefs falling in the middle, revealed that children in constructivism classroom performed significantly higher than students taught using traditional instruction (Pfannenstiel et al., 1997). Similarly, a study conducted with 17 fifth graders who received six weeks of constructivist-based problem solving instruction illustrated that there were significant gains in problem solving ability in post-test over pre-test achievement scores (Gray, 2004). The prevailing view is that when students share experiences and work collaboratively, their performance improve (Fosnot, 2005). Hence, it is worth examining whether the use of the constructivist approach will yield similar results among fifth graders in urban and rural schools in the Belize District, Belize, Central America.

3.5 The Importance of the Training of Teachers

In this subsection, the importance of adequately preparing teachers to implement educational innovations and the process for training teachers in the control and experimental groups are discussed. Firstly, worth noting is that there are multiple perspectives and guidelines on how to “best” provide teachers with requisite skills to engage students in meaningful learning tasks (Eggen et al., 2003). One important belief is that changes in instructional practice have to do with whether teachers perceive that these will have positive effects on students’ achievement (Hawkins, Stancavage, & Dossey, 1998; McCormick et al., 1995). Indicated is that unless teachers believe that educational intervention will result in greater educational gains, it is unlikely that they will “buy into” or adapt these changes. Therefore, teachers must be convinced that these innovations will result in greater academic achievement or will increase students’ knowledge base. While it

may be wise to provide information on the benefits of educational innovations, a word of caution is that “teachers with more sympathetic attitudes toward reform used inquiry-based practices significantly and had more sympathetic attitudes toward reform” (Supovitz and Turner 2000:97). Noting this consideration, teachers were also provided with professional development training on what is constructivism and the possible impact on learning. For example, teachers became aware that the term constructivism is often described as, “a process of constructing meaning; it is how people make sense of their experience” (Merriam, Caffarella, & Baumgartner, 2007: 291). Teachers also identified that their role during constructivist-based activities is to facilitate the process of knowledge construction to enable students to use their experiences to interpret math concepts and to interact with peers to develop new meaning (Prouix, 2006). That is, teachers became aware that in constructivist-based activities, students are the focus of the lesson and the teachers’ role is to guide students to discover new information (Henson, 2003). In addition to ensuring that teachers were cognizant that during constructivist-based activities students are guided to interpret and analyze information, they were reminded that students’ lived experiences and their ability to openly communicate and analyze information are critical to the problem solving process (Lerman, 1993). While teachers can be provided with training and sensitized to the pros and cons of educational innovations, according to Cohen and Ball:

“school improvement interventions rarely create adequate conditions for teachers to learn about or develop the knowledge, skills, and beliefs needed to enact these interventions successfully in the classroom” (1999:01).

Noting these shortcomings, the researcher continuously reviewed teachers' instructional plans and obtained authentic resource materials such as play dough, crayons, papers, and models to assist them to actively engage students in meaningful learning activities. In addition to the training information described, a systematic outline of the procedures for the training of teachers is further described in the subsection below.

3.5.1 Procedures for the Training of Teachers

During August 20 to 31, 2007, teachers who were randomly selected and assigned to the experimental groups attended two weeks of training and were informed of the purpose of the study and provided with guidelines for implementation. Consequently, at the beginning of the training, teachers were informed that the purpose of the study was to assess whether social engagement and use of authentic resources would enable students to interpret and identify solutions to word problems to develop critical thinking skills. Teachers were also informed that they were required to facilitate this process by providing students with selected math problems and to guide students to read, interpret, discuss and use resources to identify strategies and solutions.

Having discussed the guidelines for engaging students in social learning groups, teachers were presented with copies of a consent form which indicated that their participation was voluntary and they were free to withdraw from the study at any time. The importance of providing participants with explicit guidelines and information on the protection of right is consistent with Ethical Research Principles which denotes that participants should be informed of the:

- purpose of the research, expected duration and procedures
- rights to decline to participate and to withdraw from the research once it has started, as well as the anticipated consequences of doing so
- prospective research benefits

American Psychological Association 2008:12

In addition to reviewing the contents of the consent form, teachers were also informed that the information obtained from semi-structured interviews would be confidential and that at no time would they be exposed to harm.

Besides a description of the research procedures and ethical guidelines, teachers were provided with resource readings on social constructivism which indicate that knowledge can be acquired through meaningful interactions among peers (Simons, 1995).

Additionally, helpful hints to scaffold students were also discussed. That is, teachers were presented with the following ways to guide and support students during social engagement:

1. Provide opportunities to practice and discuss with peers
2. Provide ongoing and relevant feedback
3. Offer flexible opportunities to demonstrating skill

Mawhinney et al., 2004: 12

Furthermore, possible courses of action for breaking complex skills into sub-skills, use of effective questioning strategies and utilizing authentic resources to represent math concepts were presented.

Besides a thorough review of instructional guidelines to promote meaningful social interactivity, the importance of recognizing cognitive and cultural differences among students was discussed. Finally, at the end of the two weeks of training, the procedures for administering the pre-test, post test 1 and 2 and the consent forms were once again reviewed.

Worth noting is that from October 22 to November 30, 2007, teachers who were randomly assigned to the control groups received similar training and instructional guidelines. This enabled these teachers to acquire the necessary skills to engage the students who were initially assigned to the control groups to also be taught using social constructivist guidelines from weeks 7 to 12.

3.5.2 Securing Internal Validity

Besides the theoretical framework in the previous sub-section, it is necessary to examine factors which may affect this intervention. Therefore, efforts to secure threats to internal validity are described.

Firstly, according to Trochim (2001) threats to internal validity prevent researchers from establishing the real causal relationships of a study. Gay and Airasian (2000) also proposed that random selection of participants and random assignment to treatment and control groups are powerful approaches to overcoming threats to internal validity. Based on this perspective, grade five students in rural and urban Belize District were randomly

assigned to treatment and experimental groups to examine whether a constructivist-based instruction can improve performance. In addition to random assignment, to ascertain that the observed changes are attributed to the constructivist approach and not by other possible causes, the following three threats to internal validity were also controlled.

3.5.3 Equivalence

To validate that the groups were equivalent or probabilistically equal, students from each of the four randomly selected schools were taught using the National Primary Curriculum and used similar textbooks. To ascertain that the students were equivalent, comparison were also made among the mean of the pre-test scores across each of the four schools. These results of the independent sample t-test as illustrated in Appendix D revealed that the p value for each of the eight items on the pre-test was $p > 0.05$. According to Trochim (2001), when a confidence interval of 95% is used, values above 0.5 provide a clear indication that there are no significant differences between the means of the groups. Hence, the random selection of students in the control and experimental is probabilistically equal.

3.5.4 Instrumentation threats

To control for instrumentation threats, twenty-four two-step math word problems of similar difficulty were randomly selected from the Belize Junior Achievement (BJAT) item bank obtained from the Ministry of Education in Belize. These items were written by

teachers and principals across the six districts and aligned to the National Comprehensive Primary Curriculum. Eight of these items were used for the pre-test and eight for each of the two post-tests. To determine whether the items were reliable, a pilot test was conducted with 323 Grade Five Students from March to April 2007. The reliability measures were analyzed using the Statistical Package for the Social Sciences (SPSS) Version 10. Therefore, the pre-test, which consisted of eight word problems revealed an internal consistency of Cronback alpha 0.89; a text output is illustrated in Appendix E. Secondly, the reliability of the first post test items as illustrated in Appendix F was alpha 0.90. The reliability of the second post-test items, which consisted of eight mathematical problems shown in Appendix G, was alpha 0.93.

Notwithstanding the fact that researchers such as Gay et al., (2000) hypothesized that randomization is the single best way to simultaneously control for many extraneous variables, environmental effects were also controlled by:

- Seeking parental assistance to ensure that students did not receive any additional tutoring during the implementation phase.
- Convening with the teachers in the experimental and control groups to review their lesson plans to ensure that the content and level of difficulty were similar across the four schools.

3.5.5 Social Interaction Threats

As described in the design, only students in the first experimental groups were taught using a constructivist-based instruction during the first six weeks. To avoid contamination, the researcher sought permission from the Ministry of Education and school administrators to monitor and encourage teachers assigned to the control and experimental groups to adhere to the research guidelines.

3.6 Duration

This intervention took place from September to December 2007. This time span allowed the researcher to control for maturation threats, pre-test, apply the instructional approach, and to administer a post-test at Week 6 and a second, at the end of Week 12. The administration of the pre-test, two post-tests and short interview did not exceed two and a half hours.

Furthermore, from September to December 2007, teachers assigned to the first experimental group applied the constructivist strategies to solve word problems for five days per week and one hour and fifteen minutes per day. This meant that students were placed in small groups and provided with a math problem. They were then encouraged to read, discuss and to underline key aspects of the problem. Views were shared in whole class discussion and students were then asked to identify strategies to solve the problem. Thereafter, procedures were discussed and clarified among students. Students were also

required to apply the procedure and discuss the step-by-step approach for doing so.

Finally, with assistance of the teacher, the results were discussed and evaluated in small groups and in whole group discussion.

3.7 Sampling

The monthly summary of the number of fifth graders enrolled in the Belize District is approximately 800 with each classroom comprising of 33 to 35 students (Ministry of Education Monthly Report September – November 2006). Given that a combination of confidence level and confidence interval provides 95% certainty that the true percentage of the population is between 43% and 51%, the sample size was tabulated using a sample size calculator (Berkner et al., 2005). Hence, using a confidence interval of $\pm 4\%$ and a 95% confidence level, the calculated sample size was 341. This calculated sample size is aligned to Krejcie and Morgan's (1970) Table of Random Numbers which also indicate that 341 is a representative sample from a given population of 800. It is worth noting that while 341 is an appropriate sample size from a population of 800, the number of students in the four control and experimental groups are 342.

3.8 Sampling Procedures

According to the Ministry of Education School Report (2007), there are 65 schools in the Belize District. Therefore, stratified random sampling procedures were used to ensure that students in urban and rural schools had equal chance of participating. As a result, schools in urban and rural Belize District were divided into two strata. Thereafter, the following random sampling procedures were used to select the two schools from each stratum:

- The names of the school in each strata, urban and rural, were placed in a bag;
- Two schools were randomly drawn from each strata

After selecting the two urban and two rural schools, a number was assigned to each of the four Grade 5 classrooms. Thereafter, two numbers were randomly drawn from each school. The first number to be drawn for a school was used to represent the group of students to receive constructivist instruction during the first six weeks of the study. The second number to be drawn represented the group who served as the control group during this same period. Therefore, a random selection of one experimental and one control group was selected from each of the four schools.

3.9 Demographic Profile

The target population is limited to Grade 5 students between 8 to 10 years who are enrolled in urban and rural schools in the Belize District. Based on the requirements stipulated in the Mathematics Curriculum and support resources available in each school, Grade 5 students are expected to have the requisite skills to compute using the four mathematical

operations. Hence, it is perceived that difficulties associated with students' inability to solve mathematical word problems due to lack of mathematical skills did not affect this intervention.

In addition to students' demographic profile, the Government of Belize School Analysis report 2006 indicates that a number of Grade 5 teachers in the Belize District have not received teacher training above the Associate Degree level. Similarly, their average teaching experience ranged from 3-5 years. Therefore, teachers also have similar background and teaching experience.

3.10 Summary

This chapter described the methodology of the study. In so doing, a restatement of the hypothesis and research questions were presented along with how teachers were trained. Additionally, this chapter examined Bruner and Vygotsky's (1986) models of constructivist. Furthermore, efforts to secure threats to internal validity such as accounting for equivalence, selection and instrumentation threats are included. Finally, the duration of the study and stratified and random sampling procedures to select a representative sample of 342 students were outlined. Having described the design of the study, the quantitative procedures and analysis are presented in the subsequent chapter.

Chapter Four

Quantitative Methodology

4.1 Introduction

Having presented the design of the study in Chapter Three, this chapter describes the quantitative procedures used to assess whether the use of social constructivist activities will improve performance and thinking skills among eight grade students in Belize, Central America. Firstly, the procedures for training teachers and the administration of pre-tests and post-tests 1 and 2 are explained. Thereafter, a detailed description of the methods for analyzing the quantitative data and assumptions governing the use of Repeated Measure ANOVA are presented.

4.2 Design

According to Campbell and Stanley (1963), pre-test and post-test comparisons provide robust assessment of a pedagogical intervention by detecting possible changes before or after treatment. Therefore, the switching replication design was used to detect the real effects of the program on the experimental group and to control for threats to internal validity (*ibid.*: 04). In this design, the experimental and control groups often alternate as either the treatment or the control group. To assess the extent to which social constructivist activities resulted in improved performance, during the first six weeks, the experimental groups received constructivist-based instruction and a test was administered to the experimental and control groups. The test results were then compared to examine

whether there were significant differences over pre-test scores. Thereafter, the control groups received similar instruction and differences between the experimental and control group results were also compared. Instructing the experimental groups for 12 weeks with constructivist-based activities and the controls from weeks 7 to 12, enabled both groups to receive constructivist guidelines at different intervals thereby allowing the researcher to assess the effect of the instructional approach on the experimental and control groups.

4.3 Procedures

This subsection provides a detailed account of the step-by-step procedures to gather the quantitative data. Firstly, a letter was sent to the Chief Education Officer seeking permission to conduct this research in four schools in the Belize District. Upon receipt of written consent, written permission was sought from school managers, principals, and teachers. Additionally, a letter and a consent form were sent to all parents requesting signed consent. Parents were also informed of the procedures of the study and their autonomy to withdraw their child at any time. Copies of the letters and consent forms are included in Appendix H, I, J, K, L.

4.3.1 Sampling Procedures and Program Implementation

Quality of research studies are marked by systematic application of robust procedures to provide trustworthy information about educational problems (Heneman et al., 2000). To identify whether social constructivist activities resulted in higher cognitive processing and improved performance among grade five students in Belize, Central America, the procedures for the administration of the pre-test to the experimental and control groups, and the program implementation are described in this subsection.

As previously mentioned, one procedure for selecting the schools to be included in the study included placing rural and urban schools into different stratum and randomly selecting the schools from each stratum. Having randomly selected two rural and urban schools, the names of the grade five teachers from each school was placed in a bag and two names were randomly drawn from each school. The first name to be drawn from a school was used to identify the teacher to instruct the experimental group and the second name drawn was used to identify the teacher assigned to the control groups. Observing research protocol which indicates that participants are informed of their right to withdraw from the study and to be protected from harm (Scott-Jones, 2000; Raphael, Wahlstrom and Sowell, 1989; and Eyde, 2000), the random selection of eight teachers from two rural and urban schools were provided with the research guidelines. Having reviewed the process for administering pre-test and post test 1 and 2, and instructing students using constructivist-based instruction, each teacher willingly signed the Teacher Consent form included in Appendix M.

4.4 Methods for Analyzing the Quantitative Data

This subsection describes the quantitative analysis and also clarifies the procedures and accounts for the assumptions governing the use of the t-test and Repeated Measures ANOVA.

4.4.1 Quantitative Analysis: Pre-test Results

Having administered a pre-test to the two experimental and control groups, a t-test for independent samples was used to analyze whether the means of the randomly assigned groups were statistically different. As illustrated in Appendix D, the probabilistic equivalence of the groups were greater than 0.05. According to Siegle (2005), $p > 0.5$ provides clear indication that the groups were "probabilistically" equivalent.

4.4.2 Quantitative Analysis: Post tests results

In addition to using a pre-test, two post-tests were administered to the experimental and control groups to analyze whether there were significant gains over pre-test scores. These pre-test and post-test results were analyzed using Repeated Measures ANOVA, which is a pre-test, post-test experimental design to measure the same subject on an interval-scaled variable (Stevens, 1996).

4.5 Repeated Measure ANOVA

While the Repeated Measures ANOVA is a robust statistical design, it relies on certain assumptions about the variables used in the analysis. Therefore, Pedhazur (1997) warns that if these assumptions are not met, the results are often unreliable resulting in a Type I or Type II error, or overestimation of the significance of the effect size(s). This indicates that "knowledge and understanding of the situation when violations of assumptions lead to serious biases and when they are of little consequence are essential for meaningful data analysis" (ibid.: 1997: 33). Consequently, the paragraphs below describe efforts to account for the assumptions governing the Repeated Measures ANOVA.

4.5.1 Assumption One: Normality

As outlined in the design of the study, the researcher ensured that students were equivalent or probabilistically equal by randomly selecting students taught using the same National Curriculum and textbooks in Belize. Additionally, to account for normality, which dealt with the assumption that variables have a normal distribution, the expected normal quartiles were calculated using Blom's proportional estimation.

While Blom's proportional estimate indicates normality, Chambers et al. (1983) posit that there is no statistical tool as a well-chosen graph to illustrate normal distribution.

Consequently, the Q-Q plots, which is the most commonly used for checking the normality

of data illustrated an approximate normal distribution, which revealed that the plot falls near the $y=x$ line. Therefore, the straightness of the normal Q-Q plot also illustrates that the data were normally distributed.

4.5.2 Assumption Two: Sphericity

Sphericity, which is a mathematical assumption in Repeated Measures ANOVA, is the equality of the variance for each set of scores (O'Brien and Kaiser, 1985). In this study, the assumption governing the use of sphericity for within subject designs were accounted for by conducting an ANOVA with repeated measures factor using SPSS. As revealed in the Mauchly's test, the significance level was $p > 0.05$. According to Field (2005), if Mauchly's test is not significant ($p > 0.05$), then it is reasonable to conclude that the variances of differences are not significantly different. Therefore, this assumption has not been violated.

4.6 Summary

The chapter described the quantitative methods, program implementation, and efforts to secure internal validity. The sample consisted of 342 students from two urban and rural schools in the Belize District, Belize, Central America. Students from each of the four schools became engaged in social constructivist-based activities for a six weeks period incorporating the switching replication designed proposed by Campbell and Stanley (1963). The switching replication design was utilized to ensure that the experimental and control groups were exposed to constructivist instruction and to compare performance. While the quantitative methodology has been described, the process for administering the semi-structured interviews and use of the video transcripts to analyze the extent to which students constructed understanding are described in the subsequent chapter.

Chapter Five

Qualitative Methodology

5.1 Introduction

This chapter presents the methods and procedures for analyzing the qualitative data.

Firstly, the procedures for analyzing the semi-structured interview and videos are described. Ultimately, the procedures to collect and analyze data from teachers' semi-structured interviews are explained.

5.2 Procedures for collecting data from videos

As indicated in the previous chapter, a switching replication design was used to examine whether the constructivist approach improved test scores among Grade 5 students in urban and rural Belize, Central America. In addition to analyzing differences between pre- and post-test results, it is also essential to provide holistic descriptions of students' thought processes as they interpreted math word problems (Stainback and Stainback, 1988).

Hence, to select a suitable sample of students to be interviewed, group size of not more than four or five students were identified (Salvin, 1990). As a result, four groups with four students from the experimental and control groups were randomly selected. That is, the names of the students in each of the two control and experimental groups were also placed in a bag. Four names were then randomly drawn from the two control and experimental groups to form four groups with four students in each group.

Prior to recording students' interactions in social learning groups, parental permission was sought. Upon receipt of written consent, randomly selected groups of students from the control and experimental group were seated in groups of four around a table. Each student was then provided with a word problem from either post test 1 or 2. Students were then encouraged to read, discuss, represent the problem, and to identify solutions. This enabled the researcher to record students as they sought solutions to math problems. Having recorded these interactions, two days later, students observed the video recordings of their interactions in social learning groups. After viewing the recordings, a semi-structured interview was used to encourage students to describe their thought processes and interactions as observed on the recording.

5.3 Coding and Analysis of Video Tape Group Sessions

The verbatim derived from the videos were coded using selective coding. This includes selecting a code category and relating it to other categories (Strauss and Corbin, 1990). To differentiate between information obtained from the video and interviews, the data was coded as follows: the letter of the alphabet V was used to represent data derived from the video. Thereafter, either 1 or 2 was used to distinguish information obtained from the videoing of students from the first two schools. Similarly, 3 and 4 were used to distinguish information obtained from video recordings of students from the third and fourth school. For example, data obtained from videoing the students from the first school was coded as V1 and information obtained from videoing students from the second, third and fourth

schools was coded as V2, V3, and V4 respectively. A detailed description of this process is further described in subsection 5.5 of this chapter.

5.4 Procedures for analyzing Data Obtained from the video recording of Students' Performance

In this subsection, the fourth research question that aims to identify whether constructivist-based instruction enabled students to engage in critical analysis is restated. The process for gathering and analyzing the video transcripts and students' reflections are also described. Furthermore, the procedures for gathering and presenting students' reflective thoughts are outlined. This chapter also describes the procedures to examine whether students constructed understanding in social learning groups.

5.4.1 Restatement and review of hypothesis Four and Procedures for analyzing data from the semi-structured interviews

Hypothesis Four: Unlike regular instructional methods, the constructivist-based approach provides fifth graders with deep insights of successful experiences and difficulties as they sought solutions to Math word problems.

The fourth hypothesis refers to whether the constructivist-based approach has resulted in deep cognitive processing of ideas and the skills in order to solve math word problems. Hence, the data in this subsection is aimed at identifying whether students were able to interpret the problem and construct understanding. While the word “understanding” is a

mental construct, this will be analyzed based on guidelines provided by Eggen et al. who noted that, “constructivism is a view of learning in which learners use their own experience to create understanding that makes sense to them rather than having understanding delivered to them in an already organized form” (2003: 230). The use of the term constructivism will also be based on the notion that constructing understanding involves using background knowledge to interpret new information, engaging students in small group discussion and authentic learning task (Good and Brophy, 2000). To examine whether the constructivist approach resulted in active engagement and thoughtful reflection, as previous stated, random groups of students from each of the four selected schools were videoed as they interacted in social learning groups to solve math word problems. In accordance with Beresin’s (1993) guidelines for transcribing video recording, the tapes were reviewed by the researcher and transcripts for speech, gestures, and patterns of movement for each student in each of the four groups were reviewed. The information obtained from the video was also transferred to an optical disc, allowing the researcher to observe segments promptly and to make comparisons with the transcripts (Ball & Smith, 1992).

To ensure that the students’ responses and reflections were not influenced by the researchers’ perception, as previously stated, two days after the recording, students were randomly selected to observe and to describe their thoughts and actions (Collier & Collier, 1986). Furthermore, to reduce interview bias, students were encouraged to formulate their own answers and to disclose their own opinions and feelings without interference from the researcher (Strauss & Corbin, 1990; Alston and Bowles, 1998). Having ensured that all research protocols were addressed, students were interviewed and responses recorded

using audio tape. These recordings were then transcribed and compared with the video transcriptions. Thereafter, the transcripts and students' observations were reviewed and final emerging themes were identified and reported.

5.5 Procedures and Justification of the use of Semi-structured interviews to analyse the effects of constructivist-based instruction

While other forms of interviews such as a structured interview schedule could be utilized, semi-structured interviews allow for in-depth discussions and reflection (Hitchcock and Hughes, 1989). Furthermore, in semi-structured interviews, sample sizes are not determined by hard and fast rules but by the nature of the study (Morse, 1991).

Consequently, the emphasis is on the examination of processes and meaning and not in terms of quantity (Labuschagne, 2003). Therefore, the name of each student who was videoed was placed in a bag and two names randomly drawn from students who were initially assigned to the control and experimental groups. Hence, eight interviews provided a broad overview of students' experiences as they sought solutions to math problems using a constructivist-based approach.

Of note is that at the beginning of the interview, each student was asked to reread the math problem used during the video recording. Thereafter, students were asked to respond freely to each of the four questions on the interview guide as illustrated in Appendix N. Additionally, students also observed video clippings of their interaction and described why procedures or diagrams were used and discussed their problem solving strategies.

5.6 Procedure for Analyzing data obtained from the Semi-structured Interviews

Patton (1990) posits that in qualitative analysis, emerging themes, thoughts and beliefs should be analyzed at a very early stage. Based on this perspective, after the first group of four students was interviewed, the data was transcribed using MS Word. Next, the information obtained via interviews was coded using “I” to represent interviews and the numbers 1 to 8 were assigned to each of the randomly selected students. For example, I1 represents an interview conducted with first student and, I2, I3, I4, I5, I6, I7, and I8 represented information obtain via interviews from the second to the eighth student respectively. The researcher then read the transcripts and reviewed the audio tapes and similar responses were grouped. After transcribing and organizing the information obtained from the first set of interviews, the researcher proceeded to interview the remaining students. Subsequently, the students from each school were interviewed and the data transcribed, reviewed and compared to review the tentative codes derived from students’ responses (Strauss and Corbin, 1990). Subsequently, each line was read and the categories reviewed and coded (Bogdan and Biklen, 1998).

Having described and coded the data from the semi-structured interviews, students’ reflection as observed on the video recordings were also analyzed. Firstly, students were asked to observe the video recording. The researcher then asked students to focus on the video recordings and to describe their thought processes. Therefore, the structured/unstructured continuum also referred to as guided conversations allow interviewees to direct their own thoughts processes and clarify views (Mischler, 1991; Rubin & Rubin, 1995). Responses were then recorded on audio tapes and transcribed

using Microsoft Word. The researcher then reviewed the transcripts for main themes in order to highlight students' metacognition and to identify successful and unsuccessful strategies.

In addition to labeling and discovering categories, axial coding was employed in order to analyze intervening conditions and interactional strategies. According to Bogdan and Biklen (1998), this type of coding allows the researcher to analyze the data for a broader structural context pertaining to the category. For example, the transcripts were analyzed to examine whether students' perception of their ability to solve word problems was as a result of their interaction with peers and/or having acquired the ability to represent, analyze and to solve math word problems. Furthermore, the data was also examined to analyze whether context such as resources, teachers' support and relevance of problems affected performance. In the end, the categories were compared and connections between various categories of data were reviewed to discover emergent themes (Patton, 1990; Bogdan and Biklen 1998). Finally, a report was developed with verbatim from the transcripts to describe students' experiences as they sought solutions to math word problems.

In addition to interviews with students, semi-structured interviews as illustrated in Appendix O were used to analyze teachers' experiences and to determine whether they observed peer interaction and knowledge construction in social learning groups. Similar to the data obtained from students', these were also transcribed, labelled and coded. That is, the interview conducted with the first teacher was labelled T for Teacher and 1. Consequently, the interviews conducted with the remaining three teachers were labelled T2, T3, and T4 respectively. Thereafter, the main themes were identified and analyzed.

5.7 Summary

The qualitative procedures and data analysis were described. This included the procedures for conducting the semi-structured interviews and a description of the data analysis. Additionally, a sequential outline of the video recordings, transcriptions and coding procedures were described. Besides the quantitative and qualitative procedures in chapters four and five, in the subsequent chapter, the analyses of the quantitative data from pre-test, post test 1 and 2 are presented.

Chapter 6

Quantitative Results

6.1 Introduction

In this chapter, the quantitative findings are presented. In the first section, the pre-test and post-test scores of students who received six weeks of instruction using a constructivist-based instruction were compared to the control groups who were taught using step-by-step procedures or systematic guidelines to solve problems. In addition to a comparison of the pre-test and post test 1 result using Repeated Measures ANOVA, after the first control groups were taught using a constructivist-based instruction from weeks 7-12, their performance was also compared to the experimental groups. Having analyzed the pre-test and post test 1 and 2 results, this chapter ends with a summary of the findings and implications for using social constructivist activities.

6.2 Comparison of the Pre-test and Post test Scores

Table 5 overleaf compares the results of the pre-test and post test 1 using Two Way Repeated Measures ANOVA to examine the interaction within subject effects of students in the experimental and control groups.

Table 5:Pre-test and Post test 1 Results**Tests of Within-Subjects Effects**

Measure: MEASURE_1

Source		Type III Sum of Squares	df	Mean Square	F	Sig.
FACTOR1	Sphericity Assumed	75082.690	1	75082.690	599.091	.000
	Greenhouse-Geisser	75082.690	1.000	75082.690	599.091	.000
	Huynh-Feldt	75082.690	1.000	75082.690	599.091	.000
	Lower-bound	75082.690	1.000	75082.690	599.091	.000
FACTOR1 * GROUPS	Sphericity Assumed	5900.880	1	5900.880	47.084	.000
	Greenhouse-Geisser	5900.880	1.000	5900.880	47.084	.000
	Huynh-Feldt	5900.880	1.000	5900.880	47.084	.000
	Lower-bound	5900.880	1.000	5900.880	47.084	.000
Error(FACTOR1)	Sphericity Assumed	30078.641	240	125.328		
	Greenhouse-Geisser	30078.641	240.000	125.328		
	Huynh-Feldt	30078.641	240.000	125.328		
	Lower-bound	30078.641	240.000	125.328		

The interaction between the pre-test and post test 1 results (factor 1) and the group, as illustrated in the Table 5, was found to be significant, with $F(59,00) = 47.08$ $p < .001$.

In addition to the between subject effects, the Profile Plots depicting the Estimated Marginal Means of Measure overleaf also illustrate the interaction between factor 1 (pre and post test 1) and the group. To interpret the difference in performance between the pre-test and post test 1 results, Figure 1 is summarised in Table 6 below.

Figure 1:

The Estimated Marginal Measure of Means Plot 1

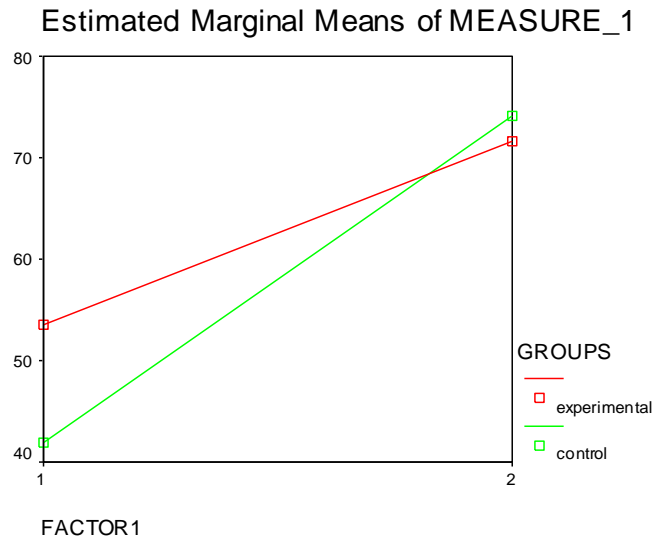


Table 6:

The Experimental and Control Groups Pre-test and Post test 1 Mean Performance

Groups	Pre-test Mean Performance	Standard Deviation for Pre-test score	Post test 1 Mean Performance	Standard Deviation for Post test 1 scores
Experimental	48.4	16.7	72.7	12.6
Control	41.9	17.2	74.1	10.6

The data in Table 6 show that the mean performance of the experimental group before instruction was 48.4 while the mean performance of students in the control was 41.9. This indicates that the difference between the pre-test results of the experimental and control groups was 6.5. Hence, although students in the experimental and control groups were

taught using similar textbooks and national curriculum and their teachers had similar educational background, there were observable differences in performance.

In addition to the difference between the pre-test and post test 1 results, a further analysis of the data in Table 6 revealed that after six weeks of constructivist instruction, the mean performance of students in the experimental group was 72.7. While a mean of 72.7 suggests that the constructivist approach resulted in gains between pre-test and post test 1 scores, it is also worth noting that there were increases among the controls from a mean of 41.9 to 74. Furthermore, a comparison between the post test 1 performance of the control and experimental groups in Table 6 also illustrated that the mean performance of the controls was greater than the experimental groups.

6.3 Comparison of the Post test 1 and 2 scores

In addition to a comparison of the pre-test and post test 1 result of students in the control and experimental group, in this subsection the post test 1 and 2 scores are also compared to determine within subject effects using Repeated Measures ANOVA.

Table 7:

Pos test 1 and 2 Results

Tests of Within-Subjects Effects

Measure: MEASURE_1

Source		Type III Sum of Squares	df	Mean Square	F	Sig.
FACTOR1	Sphericity Assumed	15644.003	1	15644.003	279.822	.000
	Greenhouse-Geisser	15644.003	1.000	15644.003	279.822	.000
	Huynh-Feldt	15644.003	1.000	15644.003	279.822	.000
	Lower-bound	15644.003	1.000	15644.003	279.822	.000
FACTOR1 * GROUPS	Sphericity Assumed	543.275	1	543.275	9.717	.002
	Greenhouse-Geisser	543.275	1.000	543.275	9.717	.002
	Huynh-Feldt	543.275	1.000	543.275	9.717	.002
	Lower-bound	543.275	1.000	543.275	9.717	.002
Error(FACTOR1)	Sphericity Assumed	13417.661	240	55.907		
	Greenhouse-Geisser	13417.661	240.000	55.907		
	Huynh-Feldt	13417.661	240.000	55.907		
	Lower-bound	13417.661	240.000	55.907		

The interaction between post test 1 and 2 results (factor 1) and the group, as illustrated in Table 7 was found to be significant with $F(54,27) = 97.17$ $p < .001$.

Besides the between subject effects, the Profile Plots depicting the Estimated Marginal Means of Measure overleaf also illustrate interaction between factor 1 (post test 1 and 2) and the group. The graph in Figure 2 was analyzed in Table 8 overleaf.

Figure 2:

The Estimated Marginal Measure of Means Plot 2

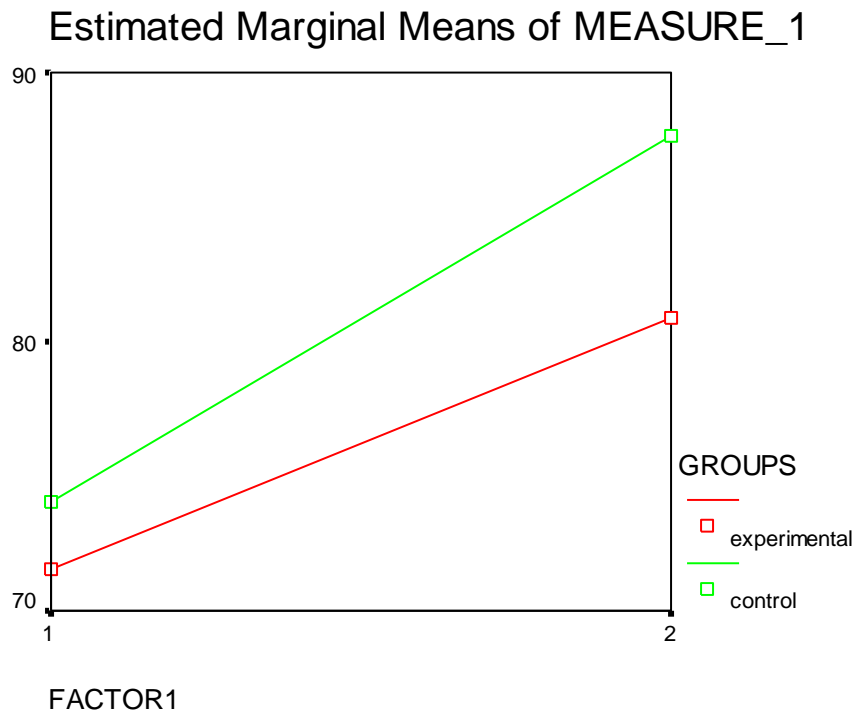


Table 8:

The Experimental and Control Groups Pre-test and Post test 1 mean performance

<i>Groups</i>	Post test 1 Mean Performance	Standard Deviation for Post test 1 scores	Post test 2 Mean Performance	Standard Deviation for Post test 2 scores
First Control Group	72.7	12.2	84.1	8.5
First Experimental	74.1	13.9	80.9	10.3

The data in Table 8 illustrate that the post test 1 mean performance for the first control group was 72.7. However, after receiving six weeks of instruction using a constructivist-based approach, the post test 2 mean performances was 84.1. The increase between post test 1 and post test 2 was significantly higher for the control group than for the

experimental group. However, having found that the control group initially out performed the experimental group at the start of the intervention, we cannot draw any clear conclusions about what this second significant increase implies.

6.4 Summary of Findings

In this chapter, the quantitative data was presented. Revealed in the results of the Repeated Measures ANOVA is that the interaction between post test 1 and 2 was found to be significant.

It was found that against expectations, the control group that was not engaged in constructivist-based activities out-performed the control group. This unexpected result requires an explanation as to why the control group did better than the experimental group in the intervention. Therefore, in the second part of the thesis, the ideas of constructivist teaching, social constructivism and the assessment of understanding in order to deconstruct these issues and to provide clear guidelines for a teaching experiment in the context of Belize will be examined more closely. In deconstructing these issues, qualitative results obtained in the study will be reviewed thoroughly look closer at what occurred for this particular teaching experiment.

Part 2

Deconstructing the issues

Having described the context, research design, and quantitative findings of the study, in part two of the thesis, the possible reasons behind the quantitative findings will be examined; namely that students in the control group outperformed those in the experimental group. In order to do so, students' thinking whilst tackling the word problems will be examined, through the video recordings of students' interactions in social learning groups. Teachers' and students' semi-structured interviews will be analyzed in order to get further insight into the reasons behind the quantitative findings. However, before deconstructing the issues behind the present findings, theoretical ideas associated with constructivism and the research findings of two constructivists teaching experiments will firstly be analyzed and compared to the teaching experiment conducted in Belize.

Consequently, the chapters in part two of the thesis are organized as follows:

In Chapter Seven, the general constructivist framework is reviewed. Additionally, the use of the term social constructivism in this study is operationally defined as the cognitive process of using prior knowledge and experiences to interpret new information (Simon, 1995; Pirie and Kieren, 1994; Piaget, 1977) and the social process of interacting with peers to develop shared meaning (Vygotsky, 1986; Von Glaserfeld, 1987, Fosnot, 2005; Yackel and Cobb, 1994; Steffe, 1994). In addition to an examination of the term constructivism, an important component of Chapter Seven is the analysis of the results of two constructivists teaching experiments from existing research to identify considerations for implementing a teaching experiment in Belize, Central America.

- Chapter Eight focuses on the concept of social constructivism, critical thinking and Pirie and Kieren's (1994) layers of understanding as a means for assessing students' thinking in social learning groups. Also analyzed are teachers' semi-structured interviews to assess their perceptions of the use of social constructivist activities to enhance problem solving competencies.
- Finally, in Chapter 9, factors impacting on the teaching experiments are examined including the effects of one-step problems and the limitations of using traditional tests to assess knowledge construction.

Chapter Seven

Implications for Implementing a Constructivist Teaching Experiment

7.1 Introduction

To begin to explain the reasons behind the quantitative findings of this study, this subsection commences with a re-examination of the general definition of the term constructivism and the term social constructivism. Additionally, two constructivist teaching experiments are analyzed in order to identify factors which may have impacted on the quantitative results from the present constructivist teaching experiment conducted with grade five students in Belize, Central America. Therefore, it is envisioned that the review of the teaching experiments will provide responses to the following key questions:

1. What factors impact on knowledge construction and transfer during social constructivist activities?
2. What specific teaching and learning conditions should be considered when implementing a constructivist experiment?

It is anticipated that the responses to these questions and a review of the implications for immersing students with limited math skills to constructivist based activities will provide vital information for deconstructing students' interactions with peers as they solved one-step math problems in the subsequent chapter.

7.2 Constructivism

In this subsection the term constructivism is defined and the use of the term social constructivism explained. Hence, various definitions of the term constructivism are analyzed to identify the cognitive and psychological nature of this theory.

The range of research and theoretical views on the impact of constructivist-based instruction has “led to a plethora of different meaning” (Simon, 1995:115). For example, Jonassen refers to constructivism as “reality that is constructed by the knower upon mental activity” (1991:10). In Bednar et al.’s view, constructivism occurs when, “the learner is building an internal representation of knowledge, a personal interpretation of experience” (1991:92). In the first definition, constructivism is perceived as an active mental process in which students engage in reflective thought to analyze information. An important consideration is that this personal reflective process, which often involves restructuring one’s thinking to interpret new information, is often termed as “radical constructivism” (Thompson, 2000; Von Glaserfeld, 1995). Furthermore, “the radical constructivist position focuses on the individual’s construction, thus taking a cognitive or psychological perspective” (Simon, 1995:116), described as:

The work that radical constructivism does for me is that it serves as a constant reminder that I must question my interpretations of how others understand what I take as a common setting while included in the latter is the importance of personal experiences to construct understanding.

Thompson 2000:421

Identified is that personal knowledge construction is an internal reflective process resulting in a critical examination of one's thinking. While radical constructivism refers to personal reflection, worth examining is, "what is the work that a radical constructivist view of knowledge does for the person having it" (ibid.: 420)? Therefore, of importance is what occurs during these internal thought processes and how these are used to interpret and advance one's thinking (Pirie and Kieren, 1994).

While it is important to consider how these various facets of constructivism can be used to deconstruct the issues associated with implementing a constructivist experiment, Van De Walle noted that, "the basic tenet of constructivism is simply this: children construct their own knowledge" (2007: 22). He further notes that, "different learners will use different ideas to give meaning to the idea" (ibid.: 22). Van De Walle's belief highlights two major components of constructivism. The first is that individuals use prior knowledge to interpret information (Simon, 1995; Piaget, 1979, Good and Brophy, 2000; Vygotsky, 1986; Bruner, 1986; Pirie and Kieren, 1994). Therefore, if students are presented with a math problem, they make connections with the problem and their background knowledge of mathematics. Implied is that, "mathematics is understood if its mental representation is a part of network of representations" (Hiebert and Carpenter, 1992:67). Although the importance of meaningful connections has long been acknowledged, important to this process is the extent to which personal experiences are used to interpret information (Thompson, 2000).

In addition to background knowledge and experiences, Van De Walle's second point indicates that "different learners will use different ideas to give meaning to the idea" (2007:22). Shown is that an important aspect of constructivism is using ones' experiences

to engage in meaningful dialogue and to develop shared understanding (Bruner, 1986). While social constructivism will be discussed in more depth in the subsequent chapter, worth noting is that “social interaction is an important context for learning, the focus is on the reorganization of individual cognition” (Simon, 1995:116). Hence, of importance is not whether students discussed within these learning groups, but whether the result is meaningful reflection and cognitive shifts (Steffe and D’Ambrosio, 1995). Consequently, use of the term social constructivism in this study refers to an examination of students’ internal processing and how these are utilized within social settings to interpret problems and to restructure thinking. Furthermore, use of the term social constructivism is aligned to findings by Wood, Cobb & Yackel (1995) which denote that personal cognition is an important aspect of knowledge construction and Vygotsky’s (1986) sociocultural theory which emphasizes the importance of social interaction and using experiences in knowledge construction.

Besides identifying how social constructivism is used in this study, worth noting is that,

1. Knowledge is actively constructed by the learner, not passively received from the environment.
2. Coming to know is a process of adaptation based on and constantly modified by a learner's experience of the world.
3. It does not discover an independent, pre-existing world outside the mind of the knower

Jaworski, 1997:02

Restated is that background experiences and meaningful social interactivity are important components of knowledge construction (Eggen et al., 2003; Good and Brophy, 2000). In

addition to providing opportunities for students to interpret, discuss and engage in meaningful discourse, the result is that “children grow into the intellectual life of those around them”, (Vygotsky,1978: 88), and “the individual’s environment undergoes change when it expands to participation in societal production” (ibid.: 43). Suggested in these perspectives is that critical reflection and meaningful interaction can bring about meaningful personal and societal changes (Fullan, 2000).

Also restated in the second and third of Jaworski’s (1997) positions is that one of the outcomes of knowledge construction is using background experience to interpret new information and to modify one’s thinking (Piaget, 1979). Therefore, an important question is whether changes in thinking occur by pure chance or whether these must be nurtured and developed. Evidently,

Teachers must....construct a form of practice that fits with their students’ ways of learning mathematics.....We have to reconstruct what it means to know and to do mathematics in schools and thus what it means to teach mathematics.

Wood, Cobb and Yackel, 1995: 401-422

That is, teachers must understand how students learn within different social settings and consciously organize learning activities to enhance mathematical thinking (Steffe, 1994).

Despite the fact that many teachers in Belize are untrained, for social constructivist activities to be successful, these teachers must skilfully plan and organize activities to facilitate “real knowledge” construction.

7.3 What is Constructivist Teaching?

In this subsection, two constructivist teaching experiments from research are outlined in order to identify factors which may impact on the implementation of a teaching experiment in Belize. Therefore, this subsection begins with a brief analysis of whether constructivism is a theory of learning. Next, Steffe (1994) and Simon's (1995) teaching experiments are outlined and both interventions critiqued. Additionally, Steffe and D'Ambrosio's (1995) reaction to Simon's teaching experiment are reviewed. Finally, implications for using similar teaching activities among grade five students in Belize are noted. Besides reviewing the components of constructivism, it is important to also assess what constructivist teaching is.

Worth noting is that the term *constructivist teaching* is not well defined, and it contradicts, philosophically the meaning of constructivism as I understand it. In fact constructivism is not about teaching at all. It is about knowledge and learning. So I believe it makes sense to talk about a *constructivist view of learning*

Jaworski, 1993:01

Aligned to this view is that,

Constructivism is not a theory about teaching...it is a theory about knowledge and learning... the theory defines knowledge as temporary, developmental, socially and culturally mediated, and thus, non-objective.

Brooks and Brooks, 1993:07.

Identified is that key components such as using background knowledge to interpret information and engaging students in meaningful learning groups do not provide explicit guidelines on how to teach. The point is that constructivism is a theory of learning and not a description of teaching. It is not, “a cookbook teaching style or pat set of instructional techniques that can be abstracted from the theory and proposed as a constructivist” (Fosnot and Perry, 2005:34). Perceived is that constructivism is simply a set of propositions about how individuals may use what they know to interpret new information (Piaget, 1979) and how they may refine their thinking to develop greater understandings (Pirie and Kieren, 1994; Steffe and D’Ambrosio, 1995, Simon, 1995).

Even as there is the notion that constructivism cannot straightforwardly be adopted into a pedagogy of teaching, “on the one hand, constructivism seems to suggest a pedagogy based on openness, empathy, and tolerance” (Thompson, 2000:418). Illustrated is that while it is not a description of how to teach, there are some explicit guidelines about what constitutes “constructivist teaching”. While this is seemingly true, “to pronounce constructivism as a background theory is not to announce a commitment to a particular theory of learning or pedagogy” (ibid.: 418). Shown is that although it is reasonable to adhere to general constructivist views, it is hoped that this does not create a narrow perspective or a singular view of how persons come to know and to develop conceptual understanding. To be noted is that although teachers in Belize were provided with some possible ways to include students in social constructivist activities, there are no specific instructional guidelines for doing so.

7.4 Constructivist Teaching Experiments

In addition to the belief that there are no explicit constructivist teaching guidelines, in this subsection, teaching experiments conducted by Steffe (1994) and Simon (1995) are presented to identify considerations for engaging students in Belize, Central America in a constructivist teaching experiment.

Despite the discord on whether constructivism can be applied as an instructional pedagogy, in Steffe's (1991) teaching experiment, the term is used to describe attempts to investigate how students respond to mathematical teaching and to identify guidelines to inform how mathematics may be taught. Therefore, in this experiment, the researcher assumed the role of a participant observer; that is, the researcher collected the data and also became immersed in the research activities (Myers, 1999). Hence, the researcher was required to organize instructional tasks on the basis of the current classroom events and students' response to instruction (Steffe and Tzur, 1994). Also identified was that during these constructivist experiments, the researcher establishes a flexible open responsive environment in which students were encouraged to share views, and negotiate with peers (Cobb & Steffe, 1983).

Besides a brief outline of the framework of Steffe's teaching experiment, identified in the research design is that one purpose of this experiment was to "analyze the knowledge involved in the teaching" (Steffe, 1995: 178), and to "understand the progress children make over an extended period of time...to formulate a model of learning and the particular context involved" (ibid.: 178). Therefore, to examine children's operative

scheme, or level of thinking, Steffe examined an eight year old child named Maya's interpretation of multiplication and division facts. Shown is that when presented with 21 cards and asked to visualize and place into groups of 3s, Maya grouped the cards in 3s starting with the number 21, 20, 19 as one group. She then continued to identify groups of 3s by counting backwards until 7 groups were obtained. Steffe identified that Maya was never instructed to count backwards and to group numbers as illustrated in her response to this activity. Revealed is that often students devise their own mathematical strategies although provided with alternative instructional guidelines.

In addition to the notion that Maya developed her own strategies, also hypothesized was that she counted by 3s beginning with the number 21 because she had an awareness of unit of 3s included within 21 individual objects. Although this showed that students used background knowledge to interpret and solve a problem (Vygotsky, 1986, Piaget, 1979, Fosnot, 1989), Steffe also identified that when Maya was asked to use her understanding of the division scheme to interpret a multiplication problem, she shared her experience as illustrated in the verbatim below. Note that Maya's responses are labelled M and the teacher's responses T.

T: Can you give me a multiplication problem for that

M: Twenty-one times three

T: What does twenty-one time three means?

M: Twenty-one and take three out of twenty-one

T: Is that twenty-one times three or twenty-one divided by three

M: Twenty-one divided by three!

T: Can you give me a multiplication problem

M: (sits silently for a minute)

T: What are you doing?

M: I am trying to figure out how many threes equal seven

Shown is that although Maya used her experience of dividing 21 in groups of 3s, she was unable to identify the inverse, 7×3 is 21. Furthermore, she could not abstractly perceive that there are 7 groups of 3s in 21 which shows that while students can recognize numbers they often lack the skill to conceptualize their property. Maya's responses highlight several considerations for implementing a constructivist teaching experiment in Belize. This includes the need for teachers to probe students, assess their understanding, tailor instruction to address needs, and to challenge their thinking.

In addition to difficulties with inverse operations, identified was that Maya had a strong grasp of dividing by 3s but was unable to work with organizing numerical ideas abstractly. For example, when Maya was asked to identify how many groups of 3s were in 18 and 15 she was able to do so. However, when the 18 blocks and 15 were combined in a container, she was unable to identify that it consisted of 11 groups of 3s. Shown is that instead of adding the 5 groups of 3s obtained by grouping 15 objects and the 6 groups of 3s by grouping 18 objects, she suggested that the 5 and 6 should be multiplied. That is, if 15 and 18 objects are divided by 3s, Maya stated that there would be a total of 30 groups.

Identified in this response is that there are instances when students respond in a haphazard manner without meaningfully reflecting or interpreting mathematical ideas. Restated is the

need to closely examine students' thinking and to organize tasks to strengthen their understanding.

Besides organizing instruction to improve mathematical competencies, Steffe's experiment has led to a number of important observations. For example, based on Maya's inability to identify that there were 11 groups of 3s when 18 and 15 blocks are combined, Steffe posits that:

If a child has not associated the action appropriate to remove a disequilibrium with the assimilated situation that triggered the schema, the child's search for the action would be unsuccessful.

Ibid.: 182

Shown is that if students are unable to make meaningful connections with their previous knowledge and the attributes of the problem, they cannot solve. Therefore, to be considered is that,

educators we have a choice between determining the mathematics for children through interaction communication, or on the other hand, through the conventional meaning of terms like borrowing, quotative division or distributive property. Choosing the former is requirement for constructivism in mathematics

(ibid.: 182).

Suggested is that teachers play a critical role in facilitating knowledge construction; this means that teachers must skilfully guide students to explore mathematics ideas. In addition to the important role of teachers, Steffe refers to the importance of assimilation which involves making meaningful connection with what one knows to the new situation

(1994). Steffe noted that only when a problem brings about a perturbation or “difficulty” and students are required to identify a solution to neutralize their thinking is when assimilation occurs. Schifter in her research of two teachers’ response to mathematics instruction also identified that,

The construction of new concepts is provoked when those settled understandings do not satisfactorily accommodate a novel circumstance and that this constructive activity is not simply an individual achievement but one embedded in and enabled by contexts of social interaction

1996:03

Indicated is that the process of thinking about and clarifying difficulties occurs through meaningful dialogue with others.

Besides the importance of meaningful social interaction, noted so far is that, constructivism is perceived as a mental process in which individuals “build an internal representation of knowledge, a personal interpretation of experience” (Bednar et al., 1991:92). Further identified is that there are no specific teaching guidelines on how to engage students in constructivist activities. That is, many proponents are of the view that constructivist teaching is about how students come to learn and structure their thinking and less on how to teach (Brooks and Brooks, 1993; Fosnot and Perry, 2005; Jaworski, 1993). Other than distinctions between constructivism and constructivism teaching, also emphasized is that teachers play a critical role during social constructivists’ activities. That is, teachers are expected to choose appropriate activities to foster higher levels of thinking and to prompt students to critically reflect on problem situations (Simon, 1995). The notion

that students should be provided with opportunities to critically reflect on mathematical concepts to extend their thinking is critical to the teaching experiment in Belize for two reasons. The first is that observations of instructional practices in Belize revealed that students are not accustomed to deciphering solutions on their own without being told what to do (Brown- Lopez et al., 2009). That is, customarily, the instructional practices in Belize do not lead to students pondering upon or critically analyzing mathematical concepts. Classroom observations by educational personnel also revealed that when students experienced difficulty, teachers often re-explained concepts, restates steps to solve, or provided individual support (Ministry of Education, 2009). Clearly, if students are “spoon fed” to identify responses to math problems, this can pose significant problems when less structured activities requiring personal reflection are presented.

Although the thinking is that activities aimed at facilitating independent thinking may be a challenge for many students in Belize, an important point in Steffe’s experiment is that perturbations, or the need to search for solutions, only occur when students are unable to use their background knowledge and engage in critical thinking and analysis. That is, a second important point is that trivial or non challenging activities will not foster higher levels of thinking and cognitive shifts. In fact, no learning will occur if students are not engaged in meaningful activities to challenge their thinking. This aligns to the behaviourist perspective that learning is a “permanent change in behaviour” (Ormrod, 1999:02). That is, unless students in Belize make meaningful connections and extend their mathematical thinking, it may be questionable whether actual learning took place.

7.4.1 Simon's Constructivist Teaching Experiment

While several implications for constructivist teaching have been identified, this subsection provides a brief review of Simon's (1995) teaching experiment to examine additional factors which may impact on the experiment conducted in Belize.

Of note is that Simon's teaching experiment was part of a Construction of Elementary Mathematics Project and included 26 prospective teachers. The focus was to explore multiplicative relationships to assess how teachers interpreted the area of rectangles. In so doing, in the first situation, groups of perspective student teachers were provided with a small cardboard rectangle and asked to solve the following:

Determine how many rectangles, of the same size and shape you were given, could fit on the top surface of your table. Rectangles could not be overlapped, cannot be cut nor can they overlap the edges of the table. Be prepared to describe to the class how you solved this problem.

Simon, 1995:125

When asked to describe how to solve, student teachers consistently stated that the easiest way was to multiply the length and width of the rectangle. Therefore, Simon concluded that:

1. Cognitively, a number of my students were employing a procedure that was well practiced but not well examined conceptually;
2. Socially, they did not have a view of mathematical activity in general and of appropriate activity for our discussion; in particular this included the type of

relational thinking and development of justification in which I was attempting to engage them.

(ibid.: 125)

This observation highlights three essential points. The first is that student teachers interpreted mathematical ideas based on prior learning experiences. That is, if teachers have been exposed to weak or routine mathematical practices, it is very likely that they may adopt these within their own teaching experience. Secondly, unless teachers are aware that their mathematical beliefs and practices are inadequate, the result would be poor instructional practice. Also identified is that while it may be perceived that students can interact with peers and generate meaningful discussions, often students have not been exposed to meaningful social activity. What can be implied from this third consideration is that if teachers in Belize are untrained and appear to have a weak background, engagement in these social learning groups must be closely monitored; otherwise, the constructivist teaching experiment may be hampered.

In addition to the importance of focusing on quality teaching and students' interaction, also identified is that there were instances when problems were unsuited to facilitate meaningful discourse and reflective thinking. This fourth consideration, also identified in Steffe's experiment, restates that if students are not exposed to appropriate learning tasks, this can affect whether learning occurs. Besides the need for relevant problems, a fifth important consideration is that teachers must structure activities to facilitate reflective thinking. This is, revealed in Simon's second teaching situation in which he attempted to represent the problem as illustrated below:

Rectangle problem 2: Bill said, “If the table is 13 rectangles long and 9 rectangles wide, and if I count 1, 2, 3.... And then again, 1, 2, 3...9 and then I multiply, 13×9 , then I have counted the corner rectangle twice.” Respond to Bill’s comment.

Shown is that representing the problem using a real life scenario resulted in more meaningful dialogue and deeper reflections. Reemphasized is the important role of teachers to skillfully guide social interactivity and knowledge construction (Vygotsky, 1986). While teachers are expected to competently guide students during these social activities “a particular modification of a mathematical concept cannot be caused by a teacher any more than nutrients can cause plants to grow” (Steffe, 1990: 392). Therefore, besides the “skillfulness” of the teacher, students’ readiness and ability to engage in these meaningful thinking experiences is of equal importance.

While teachers should skillfully structure activities to promote reflective thinking, there are instances when it is difficult to create disequilibrium. Disequilibrium, which is a term coined by Piaget (1979), refers to instances when students are unable to use prior knowledge to interpret information but must engage in reflective thought processes to modify their thinking. In this regard, Simon denotes that changes in the “learning trajectory” or current thinking can be challenging. That is, illustrated in Simon’s experiment is that if students believe that the area of a rectangle is determined by applying a mathematical formula, to shift students’ thinking to focus on the units within the rectangle can be challenging. Identified is that prior learning can affect the extent to which students critically reflect and solve problems. Therefore, a sixth consideration is that exposure to traditional instructional practice and limited critical thinking skills can affect

the extent to which students interpret and engage in constructivist thinking activities.

Clearly, a challenge for mathematics teachers is to move students from their “comfort zone of thinking” to reflective analysis.

Besides difficulties associated with guiding students to modify their thinking, also revealed in Simon’s experiment is that after he reworded and restructured the problem, most students “understood the relationship between the linear measures and the area measures in this context”; however, a few did not clearly interpret the concept of units within a plane figure (ibid.: 131). This illustrates that teaching and learning activities must be tailored to meet students’ needs. Furthermore, to analyze the teaching experiment, Simon reflected on the “hypothetical trajectory” which refers to the teachers’ beliefs about how students may respond to a particular problem situation. Highlighted is that when teachers reflect on students’ performance and on the extent to which they have benefitted from instructional practice, teachers are then better able to assess and tailor activities to address students’ needs. Shown in this seventh consideration is that effective math teachers reflect on students’ responses and continuously modify teaching and learning situation to enhance “understanding”.

In addition to the seventh consideration, the eighth is that higher levels of cognitive process can only occur if students have the requisite skills to interpret and participate in activities to refine their thinking. Worth noting is that this consideration, also identified in Steffe’s experiment, begs the question, how will students who are ordinarily taught in teacher-directed classrooms in Belize respond to activities requiring high levels of thinking? In this regard, Simon notes that students must be exposed to novel problems and

learning situations to “drawing out” and to stimulate their thinking.

Revealed in Simon’s teaching experiment is that instructional practices can affect whether students benefit from social constructivist activities. That is, if teachers are skilled to effectively restructure problems to facilitate higher order thinking, students are then able to reflect critically on problem situations and refine their mathematical skills. Also identified is that although teachers are expected to structure, rephrase and guide students to think critically, this can be challenging. Further noted is that teachers must continuously reflect on their practices to identify how best to structure tasks to cater to students’ needs.

7.5 Reaction to Simon’s teaching experiments

While the focal points from Steffe’s and Simon’s teaching experiments can provide meaningful considerations for implementing a social constructivist experiment in Belize, in this subsection, reactions to Simon’s teaching experiment by Steffe and D’Ambrosio (1995) are analyzed to re-examine factors which may impact on the teaching experiment implemented in Belize.

Firstly, Steffe and D’Ambrosio’s reaction highlights that while there is the perception that there is no such thing as a model for constructivist teaching, there is what is known as “constructivist teaching”. Proposed is that “constructivist teaching” refers to implementing teaching activities that are aligned to tenets of constructivism such as learning is an active social process, reflective thinking and restructuring thought processes. While this premise

is disregarded in Simon's reaction to Steffe and D'Ambrosio's proposition, one mutual agreed upon idea is that the notion of "constructivist teaching" does not offer a prescription of how to teach.

In addition to the belief that constructivist teaching activities are non prescriptive, according to Steffe and D'Ambrosio, Simon's proposition that teachers did not respond adequately because of their understanding of multiplication is untrue. Alternatively, Steffe and D'Ambrosio suggest that when students do not respond adequately to a math problem it is likely that, "they did not spontaneously activate the necessary multiplication schemes" (1995:150). What this means is that it is difficult to actually assess what is inside the "learners' head"; instead, teachers must work assiduously to stimulate students' thinking. Therefore, although educators may predict that students in Belize who seemingly perform poorly in mathematics lack the background skills to facilitate knowledge construction, this should not deter teachers from creating the medium to "activate learning schemes" or what Steffe and D'Ambrosio coined as "zone of potential development". While this legitimate observation cannot be ignored, if students lack the requisite skills, it will be challenging to activate a nonexistent cognitive scheme. That is, "a teacher may pose a task; however, it is what the students make of the task and their experience with it that determines their potential for learning" (Simon, 1994:133).

A second important reaction to Simon's teaching experiment is the need to "generate different specimens of teaching" (Steffe and D'Ambrosio, 1995:147). Perceived is that the three situations identified in Simon's experiments focused on one main teaching activity. Instead, it is proposed that researchers use various teaching situations in their analysis of

students thinking as, “an essential source of novelty” (ibid.: 147). Derived from this perspective is that to meaningfully analyze students’ thinking in Belize, there is need to examine multiple data. Hence, it is hoped that students’ semi-structured interviews and video recordings with peers will provide a rich data source to assess whether use of constructivist-based activities enhanced students’ problem solving capabilities.

An important point also identified in the reaction to Simon’s experiment is that teachers must skilfully pose problems to facilitate knowledge construction. Essentially, it is expected that teachers would foster,

a problem-posing and problem-solving environment in which developing an approach to thinking about mathematical issues should be valued more highly than memorizing algorithms and using them to get right answers

Schifter, 1996:03

Restated is that an important role in the teaching experiment is to guide students to become engaged in high order thinking activities rather than a routine response to mathematics instruction.

While multiple data sources and teachers’ ability to effectively engage students in meaningful learning is critical, a fourth critical point identified in Steffe and D’Ambrosio’s reaction is the need for distinction between the teacher and students’ knowledge. This means that it is important to distinguish between the subject matter knowledge of the teacher and students’ background knowledge (Salvin, 1990). The view is that while teachers’ and students’ knowledge-base can affect the extent to which knowledge

construction occurs, efforts must be made to examine these separately and to identify how they contribute or hinder construction. Also worthy of consideration is that,

We believe that students have a mathematics reality of their own, independent of us teachers. We do not deny students their own mathematics reality. In fact, we find that we are constrained by the students' mathematical language and cannot do just anything we want to do when teaching.

Steffe and D'Ambrosio, 1995:152

Identified is the importance of acknowledging students competencies and using it as the basis to promote knowledge construction.

In conclusion, the points noted in the reaction to Steffe and D'Ambrosio's reaction highlights the need to recognize students' knowledge base and consciously create meaningful learning situation to stimulate students' thinking. Secondly, distinguishing between students and teachers' understanding of mathematics is important if we are to accurately interpret the effects of teaching and learning experiments. While the reaction to Simon's teaching experiment reinforces the importance of identifying meaningful learning goals, we are reminded of the importance to vary teaching experiments to provide an abundance of data from which valid conclusions can be drawn.

7.6 Simon's response to Steffe and D'Ambrosio' Reaction

The teaching experiments in the previous subsections illustrate that Steffe's initial teaching experiment has sparked a response from Simon and a rebuttal from Steffe and D'Ambrosio. Despite varying perspectives, the suggestions and research findings have provided valuable insights to the teaching experiment conducted in Belize. For example, identified in Steffe's (1994) teaching experiment is that there are instances when students develop their own mathematical strategies. Also shown is that often students respond without carefully deliberating on the problem to be solved. Furthermore, students' background can significantly affect their mathematical interpretations. In addition to Steffe's experiment and Simon's response, Steffe and D'Ambrosio's (1995) reaction to Simon's experiment has highlighted the need for teachers to activate students' cognitive schemes and the need for multiple data sources from which valid conclusions can be drawn. Having reviewed major findings in these research experiments, in this subsection, Simon's final responses to Steffe and D'Ambrosio's are summarized to identify additional considerations for implementing a teaching experiment in Belize.

Several important ideas can be derived from Simon's (1995) reaction to Steffe and D'Ambrosio positions. Firstly, there is no discord that constructivism, "does not stipulate a particular model of teaching" (Simon, 1995:160). Instead, proposed is that "constructivist teaching" should be clearly defined; otherwise, it may be regarded as a mathematical pedagogy. Secondly, Simon conceded that Steffe and D'Ambrosio's view that teachers may inappropriately predict students' background knowledge is a worthwhile and valid conclusion. Shown is that although it is perceived that students' background should form

the basis for knowledge construction, it is difficult to accurately assess their knowledge about a mathematical idea or concept (Noss, 2002; White and Mitchelmore, 2003; Sierpiska, 2003). Suggested is that teachers must reflect on the problem to be solved and cautiously form judgment about what students know or are able to do.

Additionally, Simon agrees with Steffe and D'Ambrosio proposition that:

1. More teaching experiments would provide a rich data source and more accurate prediction of students' mathematical competency.
2. the hypothetical learning trajectory or zone of potential construction which refers to students' knowledge of mathematics has been clearly described; however, students should be encouraged to reflect on their mathematical practice and engage in reflective thinking. Implied is that "creating a teaching practice guided by constructivist principles requires a qualitative transformation of virtually every aspect of mathematical teaching" (Schifter, 1996:05). Therefore, teachers will need to structure the learning environment and also modify their teaching practice to effectively guide students to interpret, analyze and solve problems.
3. teachers play a critical role in guiding students through "perturbations" or reflective thought to assess innovative problems. While teachers play a crucial role during social constructivist activities, there are instances when are unable to accurately prompt, guide and assist students to analyze difficult situations. Identified is that even as it is hoped that teachers can activate students' thinking, they may experience difficulty doing so. Suggested is that teachers in Belize who ordinarily

have taught using teacher centered approaches, may require additional training and support if they are to effectively engage students using constructivist teaching guidelines. Also emphasized is that the belief that teachers should create situations to bring about perturbation has made “the role of mathematical teachers problematic” (ibid.: 162).

Resoundingly clear is that structuring activities to promote higher level thinking can be challenging. Therefore,

teachers who begin to based their practice on principles of constructivism should not expect to develop a finish repertoire of behaviours There is no point of arrival, but rather a path that leads on to further growth and change

Schifter, 1996: 08

Indicated is that although initial attempts have been made in Belize to assess whether engaging students in social constructivist activities will improve their mathematical competencies, changes among teachers and students will occur over time (Martin and LaCroix, 2008).

Although there are agreement with most of Steffe and D’Ambrosio’s views, Simon highlights that Steffe and D’Ambrosio’s suggestion that students’ cognitive scheme were not activated is unclear and requires additional research. Notwithstanding that Simon perceived that there is need for additional data to assess how cognitive schemes are activated, it is apparent that teachers must continuously reflect on their instructional practice and assist students to analyze math problems.

Simon is also of the view that Steffe and D'Ambrosio overly emphasized that teachers often focus on the social aspect and less on the psychological processes. The thinking is that teachers focus more on the interactions within social learning groups and less on processes such as use of language and students' thinking skills. Despite this assertion, Simon posits that teachers must "create psychological models as a basis for their psychological decisions" (ibid.: 1995:161). Proposed is that teachers should use research data to continuously reflect and refine their pedagogical skills. While the information from research studies can be used to inform practice, research on mathematical pedagogy is virtually nonexistent in Belize (Brown-Lopez et al., 2009). Regardless that research studies have not been conducted to assess students' mathematical competencies, teachers in Belize must continuously reflect and identify how to best improve mathematical skills (Simon, 1995).

Revealed in Simon's reaction to Steffe and D'Ambrosio experiment is agreement among researchers that there are no specific steps or constructivist teaching guidelines. Teachers also need to assume the role of reflective practitioners and to continuously examine their instructional practices so as to effectively develop students' competencies. Teachers must also work assiduously to activate students' background knowledge and to promote higher order thinking skills. In addition to the main points noted in Simon's reactions, there are two recurrent themes in Steffe (1994), Simon (1995), and Steffe and D'Ambrosio (1995) teaching experiment. The first is the important role of teachers during social constructivist activities. Secondly, students' background and experiences remain the focal point of mathematics instruction. Hence, to further examine how these two prevailing views may impact the teaching experiment in Belize, teachers' role and implications for implementing

social constructivist activities among grade five students, who ordinarily are exposed to less interactive activities, are explored in the subsequent section.

7.7 Considerations for Implementing a Teaching Experiment in Belize

In this subsection, two major observations from the teaching experiments will be further reviewed to examine how these may have impacted on the experiment conducted with grade five students in Belize, Central America. Firstly, identified in the teaching experiments is that during social constructivist activities, teachers played a vital role rephrasing math problems to foster critical thinking skills (Steffe, 1995; Simon, 1995). Therefore, it has to be determined whether teachers who are untrained and customarily teach by asking students to routinely apply step-by-step arithmetic algorithms can effectively scaffold students to improve their problem solving competencies.

Secondly, revealed in Steffe's and Simon's teaching experiments is that students' background and experience can affect the extent to which they interpret, analyze and solve non routine math problems. Consequently, also reviewed are implications for engaging students who lack basic math skills in social learning groups to enhance their mathematical competencies. The use of one step word problems are also critiqued to analyze whether this may hinder the development of critical thinking skills.

7.7.1 Teachers' role in the Teaching Experiment

Noted in Steffe and Simon's teaching experiments is that teachers play a critical role during social constructivist activities. That is,

“the teacher has the dual role of fostering the development of conceptual knowledge among his or her students and of facilitating the construction of shared knowledge in the classroom.”

Simon, 1995: 119

Implied is that teachers are expected to guide students' thinking as they reflect, discuss, and analyze math problems (Lentell, 2003). Teachers must also “intentionally pose situations of learning that serve in defining the students' zone of potential construction” (Steffe and D'Ambrosio, 1995:158). That is, teachers do not simply explain a concept or observe students as they interact with peers; instead, they are required to create learning situations enabling students to critically reflect and broaden their mathematical understanding (Steffe, 1994; Treffers, 1987; Cobb, 1987; Cobb, Wood and Yackel; 1990). For this to occur among teachers in Belize, they must have

A bifocal perspective perceiving mathematics through the mind of the learner while perceiving the mind of the learner through the mathematics

Ball, 1993: 159

That is, teachers must consciously interpret students' thinking and identify how best to structure activities to develop competencies and mathematical thinking skills. Hence, to be considered is whether teachers in Belize who have acquired training to prompt and to

rephrase mathematical problems can effectively assist students to enhance their mathematical skills. While it is hoped that teachers in Belize will promptly assume the role of facilitators of learning it must be noted that,

the role of the mathematics teacher is a very demanding one. Teachers will need access to relevant research on children's mathematical thinking, innovative curriculum materials, and ongoing professional development in order to meet the demands of this role

Simon, 1995:142-143

In addition to the need to refine instructional skills, the complex role of the teacher can be compared to an athletic coach who employs a variety of practice activities that challenge the athlete's strength and skill, often beyond what is required of the athlete in competition (dribbling two basketballs while blindfolded, playing a soccer game where each player may not touch the ball two consecutive times, performing a figure eight skating program three times in a row with only a 2-minute rest in between). These activities are not aimed at constant success, but rather at increase competence

ibid.: 139

While teachers are expected to multitask and to guide students, more than half of the primary teachers in Belize are untrained (Central Statistics office, 2009), or are seldom exposed to innovative teaching activities. Therefore, teachers in Belize may experience difficulty guiding students during social constructivist activities. Also to be considered is that teachers often lack skills to,

1. Attend to the mathematics in what students are saying and doing.

2. Assess the mathematical validity of students' ideas.
3. Listen for the sense in students' mathematical thinking – even when something is amiss.
4. Identify the conceptual issues the students are working on and providing the basis for an interesting mathematical conversation with the student.

Schifter, 2001: 109

Identified in Schifter's four points is that teachers are often unskilled to analyze students' thinking or to create conditions to develop their mathematical skills. Although teachers in Belize appear ill equipped to effectively scaffold students during social constructivist activities, there is need to assess students' response to social constructivist activities and teachers' readiness to guide this process.

Even as there is need to effect changes in pedagogical practices in Belize, three important factors must be considered. The first is that teachers teach based on the status quo or the observed practices within the school environment (Stigler and Hiebert, 1999). That is, The most significant and most deeply embedded influences that operate on us are the image, models and conception of teaching derived from our own experiences as learners.

Brookfield, 1995: 49-50

Implied is that changes in pedagogy may be challenging for many teachers in Belize whose instructional practices may be reflective of their teaching and learning experiences. In addition to tension between teachers' experiences and the need for changes in pedagogical practices, many elementary teachers experience difficulty solving mathematical concepts and dislike the subject (Ball et al., 2005; Ma, 1999; Boaler, 1998). Therefore, to be considered is whether teachers in Belize who have not received professional training may also lack the subject matter knowledge and may not respond positively to math instruction.

Also to be reflected upon is the extent to which the actual two weeks training in Belize assisted teachers to cope with the challenge of guiding students during social constructivist activities. That is, did providing teachers with background readings on constructivism, enforcing the use of effective questioning strategies, and providing guidelines for grouping students sufficiently prepared them to effect the required changes? While teachers were providing students with skills to use alternative questions, to focus on key attributes of the problem, and to prompt students to critically reflect on problem situations, providing opportunities to practice these skills with real subjects or pupils could be beneficial. That is, to be learnt from this experiment is that even in instances when teachers are provided with explicit guidelines, they may require practical application of the skill. Even as it is assumed that prior to the experiment, skills should be applied within practical school settings, we are reminded that even expert teachers have reported that these skills are challenging to implement and are refined over an extended period of time (Simon, 1995; Martin and LaCroix, 2008).

7.7.2 Considerations for immersing students in social constructivist activities

Although it is apparent that teachers play a critical role in guiding social constructivist activities, to be considered are the effects of students' prior knowledge and experience within social constructivist groups. Hence, this subsection examines the implications of limited background knowledge on the development of mathematical skills.

The importance of prior knowledge as an important element in knowledge construction has been noted by many researchers (Simon, 1995; LaCroix, 2008; Steffe and D'Ambrosio, 1995; Good and Brophy; 2000). Background knowledge as the basis for interpreting concepts is embedded in cognitive constructivist theoretical views which denote that "meaning occurs as a result of the interaction between the readers' or listeners' prior knowledge about the world and the text or speech" (Chiang & Dunkel, 1992:350). Also perceived is that prior knowledge, which refers to students' background and experiences is the building block of thinking (Woolfolk, 1987). While the primary view is that background knowledge and experience is the basis upon which knowledge is constructed, students in Belize often lack background mathematical skills. Therefore, to be examined is the extent to which limited background knowledge and skills can affect grade five students' ability to analyze and solve challenging word problems. Even as it is important to consider the effects of background knowledge on achievement, recent findings show that if students rely heavily on background knowledge, this inhibits their ability to solve algebraic problems (Khng and Lee, 2009). Hence, in addition ensuring that students acquire the requisite math skills, they should be encouraged to reflect on problem situations and to decipher how best to solve.

While students should also be encouraged to reflect on problem situations, any discourse on prior knowledge and its effect on students' cognition cannot be divorced from actual classroom practices. Essentially, knowledge construction occurs when teachers activate students' cognitive schemes or create situations to stimulate their prior knowledge and skills. For example, Steffe and D'Ambrosio (1995) explained that to activate students' thinking, teachers must identify their zone of potential construction. That is,

The zone of potential knowledge construction is determined by the teacher as she interprets the schemes and operations available to the student and anticipates the students' actions when solving different tasks in the context of interactive mathematical communication

Ibid.: 154

Illustrated is that teachers must make reasonable judgement about what students should know and do. The teacher then uses the zone of potential development as the basis for structuring activities to engage students in higher levels of mathematical thinking. This implies that "effective mathematics teaching requires understanding of what students know and need to learn and then challenging and supporting them to learn it well" (NCTM, 2000: 16). Hence, it must be considered that if teachers are unskilled and lack background mathematical skills, can they make reasonable judgement about what students know and are able to do? A greater challenge is that even as teachers are expected to determine students' background knowledge and skills, it is difficult and at times impossible to predict (Simon, 1995). Instead, teachers can only develop hypothetical trajectories or presumptions about students' skills or competencies. In Simon's view, "it is hypothetical

because the actual learning trajectory is not knowable in advance. It characterizes an expected tendency” (ibid.: 139).

In addition to presuming what students may know about a particular mathematical concept, teachers must also stimulate students’ prior knowledge about the topic or concept to be learnt. In so doing, the teacher acts as

an athletic coach who employs a variety of practice activities that challenge the athletes’ strength and skill, often beyond what is required of the athlete in competition.

ibid.: 139

Two major considerations are highlighted in the aforementioned. The first is that even as it is hoped that immersing students in activities to interact and discuss will improve problem solving competencies, background skills and experiences are important to this process. In addition to the hope that students would have acquired the requisite skills, teachers must structure meaningful tasks to stimulate students’ thinking. Hence, to be observed in this teaching experiment is the extent to which students who have limited background skills will become engaged in meaningful tasks to stimulate their thinking.

7.8 Summary

In this chapter, the term constructivism and factors relating to its use were discussed. Additionally, Simon's (1994) and Steffe's (1995) teaching experiments and Steffe and D'Ambrosio response to Simon's experiment were reviewed. Hence, the following key considerations for implementing a teaching experiment in Belize were identified.

1. Interactions among peers should result in knowledge construction and the development of mathematical skills.
2. Teachers play a critical role in guiding students' thinking during social interactivity.
3. If students are exposed to non-challenging task, it is unlikely that higher levels of thinking will occur.
4. Teachers are required to restructure problems to facilitate greater analysis and cognitive shifts.
5. Students respond more positive to teachers' probes and questions if they are highly motivated and can interpret the mathematical idea or concept.
6. Teachers often experience difficulty prompting and guiding students during social interactivities.
7. Teachers must skillfully rephrase and modify instructional task to promote meaningful knowledge construction.
8. There are instances when students develop their own problem solving strategies or respond spontaneously without deeply reflecting on the attributes of the problem.

9. There are instances when students have limited or inaccurate understanding or mathematical concepts.
10. If students' prior knowledge is not properly activated, they may respond inaccurately to problem situations.
11. To appropriately assess students' mathematical understanding, there is need to gather multiple sources of evidence from which valid conclusions can be drawn.
12. There must be clear distinction between teachers' subject matter knowledge and their ability to engage students in social constructivist activities and students' level of understanding.
13. Constructivism does not provide explicit guidelines on how to teach.
14. While it is hoped that the result of these teaching experiments will provide immediate results, this is a gradual process.

The main points identified in the above will be considered and reviewed in the subsequent chapter to identify implications for implementing social constructivist activities in Belize, Central America.

Chapter Eight

Qualitative Analysis

8.1 Introduction

In this chapter, the qualitative data consisting of two video recordings of students' interactions in social constructivist learning groups are analyzed. Also assessed are students' responses to the semi-structured interview guide to identify whether the description of their thought processes aligns to the observable behaviours on the video recordings. Ultimately, teachers' responses to the semi-structured interview guide are also assessed to identify factors which assisted or impeded students' ability to discuss, analyze, and solve math word problems. Prior to analyzing the qualitative data five critical points arising from Steffe (1994), Simon (1995) and Steffe and D'Ambrosio (1995) teaching experiments were considered. Therefore, to assess the extent to which students in Belize benefitted from social constructivist activities, to be reflected upon are:

1. Whether the interactions among peers resulted in knowledge construction and the development of mathematical skills.
2. If students are exposed to non-challenging tasks, whether higher levels of thinking will occur.

Therefore, to be examined in the video recordings is whether social interactivity enabled students to not only identify a solution to a problem but to refine and advance their mathematical thinking skills. Also to be determined is whether the use of one-step problems enabled students in Belize to engage in analytical thinking skills, to structure their thinking and to develop new conceptual understanding.

Also identified in the previous teaching experiments is that the role of the teacher is to bring about a “perturbation” or opportunities for critical reflection. Hence, to be considered is whether teachers in Belize,

3. guided students through the use of prompts and clues to critically reflect on problem situations.
4. restructured problems to facilitate greater analysis and cognitive shifts.
5. experienced difficulty prompting and guiding students during social constructivist activities.

Hence, to be examined is whether teachers in Belize used prompts, alternative questions, or rephrased problems to enable students to make meaningful connections with other problem types and to interpret authentic situations.

In addition to the five important points to be considered, this chapter begins with a critical analysis of the term social constructivism to highlight that this term is used to refer to students’ engagement in social activities where knowledge is shared and understanding is developed (Vygotsky, 1986). Additionally, given that problem solving skills entails deep cognitive processing and use of analytical thinking skills (Boaler, 1998), the term critical thinking is analyzed to examine its relevance to the development of mathematical competencies. Having examined factors related to social constructivism and critical thinking, the term understanding is critiqued to highlight that mathematical competency requires more than identifying solutions to problems; instead it entails reasoning, analyzing and structuring one’s thinking (NCTM, 2000).

Having described the framework for assessing grade five students' mathematical thinking, various models of understanding are reviewed, including Sierpeniska (1994), Skemp (1986), Hiebert and Carpenter (1992) and Pirie and Kieren's (1994) layers of mathematical thinking, to determine the extent to which these can be used to assess students' cognition during social constructivist activities. Having identified that Pirie and Kieren's model has been used in a number of research studies, this model is critiqued and observable behaviours depicting each of the eight layers of thinking are aligned to grade five students' responses during social constructivist activities to assess whether their thinking depicted the eight layers of this model. Teachers' actions as revealed in the two video recordings are also assessed to identify the extent to which they scaffolded students during the teaching experiment. Finally, the qualitative data is compared to the main considerations from the Steffe, Simon and Steffe and D'Ambrosio's teaching experiments and implications for implementing teaching experiments in Belize are discussed.

8.2 Social Constructivism

In addition to a brief review of the term 'social constructivism' in the previous chapter, to reflect on the process of the teaching experiment in Belize, in this subsection, the term social constructivism is further critiqued to highlight factors which affected students' interactions in social learning groups as they discussed and developed shared understanding of mathematical word problems. The effects of knowledge construction, as described by research studies, are also examined to identify factors which may have impacted on the teaching experiment conducted in Belize, Central America.

For decades, the term *Social Constructivism* has been at the forefront of educational discourse. As identified in Chapter Seven, social constructivism in this study refers to students' cognition and how these are utilized within social settings to negotiate and to develop shared meaning. Fundamentally, the use of the term social constructivism highlights that "words and ideas do not have inherent meanings apart from those created and negotiated by people in particular contexts" (Hughes & Sears, 2004:260). Also emphasized is that prior knowledge and social interaction is the basis for knowledge construction (Derry, 1999). That is, "... all knowledge is tied to action, and knowing an object or an event is to use it by assimilating it to an action scheme..." (Piaget, 1967:14-15). The prevailing view is that learning as a social interactive process is highly dependent on students' background and experiences.

Besides the importance of background knowledge, social constructivism is rooted in a number of beliefs. Firstly, new knowledge develops when teachers organize learning situations to facilitate meaningful interpretations with more accomplished peers (Vygotsky, 1986). Also identified is that there are instances when "the subject....generates a perturbation relative to some expected result" (Glaserfeld, 1989:163). In essence, during these social interactions, students often encounter a problem which requires internal cognitive processing to critically assess an idea or concept. There are also instances when students are at their "zone of proximal development", which refers to a particular moment when they are unable to develop meaningful interpretations unless they are guided by more competent individuals. Shown is that within these social learning groups, meaningful learning occurs when students are guided to critically analyze an issue or problem.

While engaging students in meaningful activities is a vital aspect of knowledge construction, during these social exchanges, there are instances when students' perspectives differ (Lerman, 1993). Illustrated is that, within these social learning groups, students often have different backgrounds and beliefs about the problem to be solved. This means that these engagements require

serious mathematical thinking....a genuine respect for others' ideas, a valuing of reason and sense-making, pacing and timing that allow students to puzzle and to think, and the forging of a social and intellectual community.

NCTM, 1991:01

Therefore, “meaning making” is a highly intricate experience involving reflective thinking and organizing schemes of thought. Knowledge is also constructed when students draw on their experiences to discuss, negotiate and to develop shared meaning (Vygotsky, 1978). Explained is that “social activity is conducted jointly - collaboratively - by a community, rather than by individuals who happen to be co-locate” (Stahl, 2003: 523). Therefore, shared meaning entails developing socially agreed ideas about a given problem or situation (Ernest, 1999).

Although shared meaning is an important element of knowledge construction, these social interactions must occur within meaningful contexts (Boaler, 1993). Hence, the problem to be solved or situation to be addressed must be relevant to students' experiences. The importance of contextually meaningful learning has been identified by Lave (1998) who denotes that adequate mathematical context provides a more suitable condition for learning

and enhances performance. For example, a study by Taylor (1989) revealed that when students were asked to solve a problem requiring dividing a cake and loaf into equal parts, they developed a clearer interpretation of fraction parts using the familiar concept of a cake. The view is that if problems are relevant, students are better able to interpret and make meaningful connections. Therefore, the word problems to be presented to students in Belize should be aligned to their experiences and milieu.

While relevant and meaningful context are vital, clear goals should be identified and aligned to teaching and learning activities. What this means is that, “to infer the context within which an individual is operating is to infer the overall goals that specify the framework within which the action and thought is carried out” (Cobb, 1986: 04). Hence, in addition to ensuring that activities are relevant and aligned to students’ experiences, instructional goals must be identified. Indicated is that within these social constructivist situations, teachers need to develop what Simon (1995) refers to as a Hypothetical Learning Trajectory. That is, prior to engaging students in social constructivist activities teachers must identify the,

Learning goal that defines the direction, the learning activities, a prediction of how students thinking, and the understanding that will evolve in the context of the learning activities.

Ibid.: 1995:136

Besides identifying goals and deciphering how students may respond and how best to guide a particular activity, teachers need to continuously analyze students’ responses and

modify problem situations to ensure that they advance their thinking and mathematical skills (ibid.: 136).

In addition to setting goals and to skilfully scaffold students during social constructivist activities, the effects of social interactivity on students' learning remain central to mathematical teaching. For example, researchers have identified that social constructivist activities have enabled students to work collaboratively and to build strong learning communities (Reznitskaya, Anderson & Kuo, 2007). Furthermore, a study conducted with 100% white elementary and 92% African American students to examine the effects of social interactivity within these cultural groups revealed that when students became immersed in relevant interactive activities, their performance improved (Pigott and Cowen, 1998). Seemingly, within these social settings, students' motivation, collaborative skills and problems solving skills are enhanced (Matsumara, Slater & Crosson, 2008).

Even as social constructivist activities are perceived as beneficial, students are often not engaged or accustomed to these social engagements (Corden, 2001; Nystrand, 1996). For example, in a three year study with 2,400 students in 60 different classrooms, it was revealed that on average teachers spent less than three minutes in an hour engaging students in social learning groups (Nystrand, 1996). Furthermore, students in low socioeconomic schools are allowed even fewer opportunities for discussion (Corden, 2001; Nystrand, 1996; Weber, Maher, Powell & Lee, 2008). Implied is that while there are benefits to be derived from structuring activities to facilitate social interaction and knowledge construction (Kukla, 2000), some teachers may be unskilled, unaware, or hesitant to engage students in these learning tasks. While it is perceived that social

constructivist activities foster higher achievement and higher order thinking, educators continue to question whether constructivism and social constructivism in particular, represent explicit guidelines on how to teach. In this regard, Brooks and Brooks posit,

Constructivism is not a theory about teaching...it is a theory about knowledge and learning... the theory defines knowledge as temporary, developmental, socially and culturally mediated, and thus, non-objective.

1993:07

That is, the belief that students construct understanding rather than to be presented with information in an already organized form (Eggen, 2003), or the premise that knowledge is constructed using prior knowledge and experiences (Vygotsky, 1986, Bruner, 1986, Piaget, 1979) does not provide explicit guidelines on how to teach. Instead, it provides the basis upon which learning may occur (Lorsbach and Torbin, 1997). That is, as noted in Chapter Seven, the dynamic process involving students' experiences, culture and the ability to meaningfully construct shared meaning does not constitute a specific instructional approach (Steffe, 1994; Simon, 1995; Steffe and D'Ambrosio, 1995).

Notwithstanding that no explicit guidelines have been identified, teachers are often required to organize learning tasks to stimulate and promote students' thinking (Richards, 1991). Therefore, "the challenge is how can mathematics foster students' construction of powerful mathematical ideas that took the community of mathematicians thousands of years to develop" (Simon, 1995:118). Revealed is that although research studies suggest that there are benefits to be derived from meaningful social interactivity (Steffe, 1995;

Simon, 1995; Vygotsky, 1996), mathematicians continue to grapple with best practices for applying social constructivist activities.

8.3 Knowledge Construction and Critical Thinking Skills

The importance of meaningful learning activities to enhance problems solving competencies has been identified in several teaching experiments (Steffe, 1994; Simon, 1995; Steffe and D'Ambrosio, 1995; Martin and LaCroix, 2008). Meaningful activities refer to learning tasks which allow students to explore complex problems and to develop higher order cognitive skills (Tsui, 2000). Hence, in this subsection, the term 'critical thinking' is reviewed to identify factors which contribute to the development of thinking skills during social constructivist activities. Also identified are ways to promote critical thinking skills among students who ordinarily are taught using traditional instructional practices.

The term critical thinking is generally described as the ability to use evidence to form reasonable judgment (Ennis et al., 2004). The phrase 'reasonable judgment' refers to forming plausible conclusions based on evidence. Therefore, critical thinking entails more than recalling basic information; instead, it has to do with mental processes such as synthesizing, analyzing and evaluating evidence. It also entails,

that mode of thinking - about any subject, content, or problem - in which the thinker improves the quality of his or her thinking by skillfully taking charge of the structures inherent in thinking and imposing intellectual standards upon them.

Two important cognitive processes are identified in Scriven & Paul's view. The first is that critical thinking is a mental process resulting in improvement in cognition through careful reflection on a given problem or idea (Schoenfeld, 1983). Secondly, critical thinking has to do with intellectual skills which are "a particular kind of activity of the mind which enables its possessor to arrive at sound judgment about something proposed to him for action or belief" (Johnson et al., 1994:01). Illustrated is that critical thinking activities involve careful analysis of the attributes of a problem.

While critical thinking refers to analyzing, synthesizing and evaluating information, mathematical thinking involves "discovering relationships among ideas ... to solve problems" (Lutifyya, 1998:55). Therefore, for mathematical thinking to occur, students must engage in activities to identify, analyze, and assess mathematical concepts (Schoenfeld, 1992). For example, in a study with 49 engineering undergraduates to examine whether the teaching of differentiation promoted mathematical thinking, shown is that mathematical thinking required clear definition and interpretation of mathematical concepts, use of relevant examples and opportunities to analyze and form generalizations (Roselainy, Yudariah and Mason, 2002). Therefore, if students are to accurately interpret, analyze and apply math concepts, innovative strategies and active engagement must become an integral component of math instruction (Clark & Biddle, 1993). Despite the fact that appropriate pedagogical skills must be employed, traditional school settings often rely on rote memorization and didactic instructional strategies and less on thinking skills (Kennedy, 1991; Schrag, 1988; Nickerson, 1987). Undoubtedly, if students are to become

competent problem solvers, they must be exposed to relevant activities to promote mathematical thinking.

8.4 Mathematical Understanding

Identified in the previous subsection is that an important aspect of social interactivity is whether students are engaged in critical thinking activities. In addition to the need to ensure that students synthesize, analyze and evaluate problem situations, of importance is whether they internalize and “make sense of their observations and practical engagement with mathematical symbols and concepts” (Haylock and Cockburn, 2008:06). Hence, in this subsection, the term understanding and various models of understanding are reviewed to assess the importance of engaging students in non-routine tasks to improve their cognitive skills. Students’ levels of thinking and areas of strengths or weaknesses are also assessed to examine how best to structure activities to improve problem solving competencies.

“One of the most widely accepted ideas within the mathematics education community is the idea that students should understand mathematics” (Hiebert & Carpenter, 1992:65). While the term “understanding” is often used to describe students’ interpretation of mathematical concepts, there are discrepancies about how this occurs. For example, in Webster’s view, understanding entails “to comprehend and to know thoroughly” (2007:501). That is, understanding involves making reasonable judgment and responding intelligently to given situations (Brownell and Sims, 1946 in Weaver & Kilpatrick, 1972).

Also perceived is that understanding occurs when students connect their thoughts and experiences to interpret a novel idea (Piaget, 1977). Therefore, “mathematical understanding is a process, grounded within a person, within a topic, within a particular environment” (Pirie and Kieren, 1994:39). The belief is that understanding entails using experiences and background knowledge to interpret mathematical ideas. That is, mathematics is understood if its mental representation is part of a network of representations. The degree of understanding is determined by the number and strength of its connections. A mathematical idea, procedure, or fact is understood thoroughly if it is linked to existing networks with stronger or more numerous connections. Hiebert and Carpenter, 1992: 67

Although knowledge is constructed when meaningful linkages are made, students’ experiences and background differ (Chi, Feltovich, & Glaser, 1981; Glaser, 1984). That is, if a group of students are presented with a mathematical problem, they will interpret the problem using their individual experience and background skills (Schoenfeld, 1988). Shown is that understanding, which has to do with the ability to interpret information, is dependent on an individual’s background knowledge and skills.

While prior knowledge and experience form the basis for understanding to occur, four components are embedded in this process: the understanding subject, the object of understanding, the basis of understanding, and the mental operations that link the object of understanding (Sierpinska, 1994). Firstly, the “understanding subject” refers to students’ cumulative experience about a particular mathematical concept. These cumulative experiences which include using prior knowledge and skills to interpret information can

result in “barriers to changes in frame of mind” (ibid.: 1994:121). Perceived is that instructional practices and students’ experience can affect their mathematical understanding. For example, if students have been taught to add mixed fractions, when presented with a problem requiring use of this concept, their background knowledge can either aid or hinder understanding (Simon, 1995).

In addition to the first component in Sierpinska’s theory which refers to students’ cumulative knowledge, the second, third and fourth entail interpretation of the concept and the formation of mental operations to produce an act of understanding. The thinking is that for understanding to occur, students must first identify the object or concepts, followed by the formation of mental images or internal interpretation of the problem to be solved. While these mental operations often include searching for a “common link” within one’s cognitive scheme (ibid.: 60), differences are also identified and generalizations formed. Hence, understanding also occurs when students make meaningful associations or discriminate between problem situations.

Besides the fact that during understanding students identify, synthesize, discriminate and form generalizations, there are various layers of mathematical understanding. Therefore, based on curriculum reform efforts that advocate for problem solving, critical thinking, and mathematical understanding, (NCTM, 2000; Nelson, 1997; Brosnan, et al., 1996), Pirie and Kieren (1994b) proposed a theory describing the process for constructing understanding and applying mathematical concepts to new situations. Pirie and Kieren’s effort to describe students’ growth in understanding is also in response to the ongoing dissonance on whether understanding is an act or emotional experience (Sierpinska, 1990).

Therefore, through classroom observations of students' response to math instruction, they developed a model comprised of eight potential non-linear layers of knowledge construction.

The first layer described as Primitive Knowledge is the initial stage, consisting of students' background knowledge and experiences forming the basis for knowledge construction.

Thereafter, in the second layer known as "Image Making", learners use their background knowledge to develop meaning. This denotes that while Primitive Knowledge is the basis for knowledge construction, in Image Making, these fundamental principles and ideas are used to interpret information (Greeno, 1991; Wearne and Hiebert, 1988).

While background knowledge and meaningful connections are vital for knowledge construction, Pirie and Kieren noted that in the third layer known as "Image Having," students develop logical mental representations of mathematical concepts (Viholainen, 2008:03). Therefore, having used background knowledge to interpret math problems, students then "think about" and process the information. Thereafter, students make "distinctions and combinations between multiple mental images" in the fourth layer known as "Property Noticing" (Meel, 2003: 145). Hence, Property Noticing involves processing and identifying patterns, properties and forming reasonable conclusions (Pirie and Kieren, 1994).

In addition to the first four initial stages, in the fifth layer, "Formalizing", students develop generalized descriptions of mathematical concepts. Therefore, in this stage, as a result of comparison between multiple representations and noting differences, students develop 'full

mathematical definition', and holistic interpretation of mathematical concepts (Meel, 2003:145). Besides "full mathematical definition", in the subsequent Observing Layer, students "consider and reference their own thinking" (ibid.: 146). Therefore, having reflected on the generalized concepts, students also restructure their thinking.

Besides engaging in the metacognitive process of reflecting and organizing, in the seventh layer, "Structuring" students compare and contrast axiom and prototypical examples. Therefore, having developed conceptual understanding, students extend their thinking to identify broader relationships. Additionally, in the final layer, "Inventizing", students question, analyze, and apply their "understanding" from other layers to critically examine a broad range of concepts. Worth noting is that although Inventizing is depicted as the outermost layer, it also occurs within the other layers of knowledge construction.

Therefore, Pirie and Kieren's model of knowledge construction indicates that in the first two stages, learners use background knowledge to interpret information. Thereafter, mental images and attributes of the concepts are identified and compared in the third and fourth stages. Having used experiences to interpret the information, in the fifth stage, students develop generalized ways to examine concepts, and reflect and reorganize their thinking in the sixth. Finally, in the seventh layer, they develop greater understanding of the underlying structures of the problem and in the eighth students begin to identify relationships between and across various mathematical concepts. Ultimately, students transcend their thinking through further exploration and analysis.

This model illustrates that there are eight potential layers of mathematical thinking.

Therefore, to be assessed in the subsequent subsection is the extent to which this model has been used in other teaching experiment to assess mathematical thinking. Also to be examined is the extent to which the layers of understanding can be used to assess grade five students' thinking in Belize.

8.5 Considerations for use of Pirie and Kieren's Layers of Mathematical Thinking

Prior to assessing the video recordings and students' semi-structured interviews, implications for using Pirie and Kieren's layers of understanding are identified. Firstly, noted in a study conducted by Lyndon Martin is that the notion of folding back in Pirie and Kieren's model, which refers to students' thinking progressing between layers as they construct mathematical understanding, presents a limited view of how thinking occurs. Martin describes this limitation as follows,

there was a lack of substantial data, examples demonstrating folding back and limited understanding how and why it occurred and its relationship to subsequent mathematical actions

1999:04

Therefore, using case studies to assess the mathematical understanding of first year to post graduate students, Martin suggests that folding back has to do with awareness of one's thinking and resorting to a lower layer of understanding to critically reflect and analyze a problem. The prevailing view is that students' thinking does not just fold back to lower

layers but within these layers they engage in a number of metacognitive thought processes such as thinking about the structures of the problem and analyzing their interpretation of the problem before returning to a higher or lower layer of thinking. In addition to the beliefs that during folding back students critically analyze their thought processes, Martin suggests that if students are unable to interpret or synthesize problems posed by teachers, they may “go backwards” instead of folding back. This means that when students experience a “perturbation” or challenge, unless they are able to interpret the problem, they return to a lower layer and experience difficulty reanalyzing their thinking. Identified is that while Pirie and Kieren refer to folding back as moving back through different layers, in Martin’s view, folding back is a conscious re-examination of one’s thinking and thought processes.

In addition to varying perspectives about how folding back occurs, Martin perceived that, “an observer can never know exactly what Primitive Knowledge a learner might have, and he or she can therefore only make assumptions about what the learner already knows” (ibid.: 05).

The view is that it is difficult to accurately assess students’ background knowledge or understanding of a concept. Illustrated is that although teachers may be inclined to make judgments about students’ knowledge, it is difficult to accurately assess their experiences and prior knowledge. For example, Simon’s (1995) teaching experiment revealed that while it was perceived that teachers had background knowledge of the units in a rectangle, they did not. Steffe and D’Ambrosio (1995) also identified that in instances when there was the belief that students lacked primitive or background knowledge, what really

occurred is that teachers did not accurately activate students' cognitive schema. Shown is that it is difficult or even impossible to accurately assess students' background knowledge and experiences.

In addition to the view that folding back and primitive knowledge in Pirie and Kieren's model are inconclusive, Martin also noted that the continuous broken lines illustrating shifts in thinking from cross layers of understanding to another shows that, "growth is occurring, albeit at a particular layer, rather than from it to another" (Martin and LaCroix, 2008:124). Described is that the graphic illustration of Pirie and Kieren's layers of thinking which is depicted with a series of small lines illustrates that at each and every layer, students are refining their mathematics skills. This means that while it may be perceived that thinking within the lower layers does not constitute real mathematical understanding, some level of cognitive processing and refining of one's thinking occurs. Also emphasized is that "it is important to recognize that growth of understanding of any concept occurs over time" (ibid.: 136). Therefore, even as it is hoped that the video transcripts of students' interactions in social learning groups in Belize will illustrate that their thinking is reflective of Pirie and Kieren's highest layer, mathematical growth and development is a gradual process.

Besides the limitations of folding back and primitive knowledge, further emphasized is that in any given teaching situation, it is difficult for students to progress through all of Pirie and Kieren's layers of understanding. Explained is that,

To attempt to trace the growth of understanding of a learner for a concept such as fractions (in the way offered by the Pirie and Kieren's Theory) would in the

apprenticeship classroom, be impossible (save in a few rare cases where a session may focus on a particular concept). However, this is not to say that during any classroom session where the task has a mathematical component that the mathematical understanding of the apprenticeship is not growing and developing just that this growth will not simply be about the understanding of a single, isolated, and identifiable concept, and that it will occur within a growing understanding of the task being undertaken.

Martin and LaCroix, 2008:125

Restated is that it is challenging, if not impossible, to observe all the layers of understanding in a single class period; instead, what can be observed are students' thinking during a particular task or mathematical activity.

Although longitudinal research by Martin and LaCroix (2008) and Martin (1999) illustrates that there are important points to be considered when using Pirie and Kieren's Model, also identified is that

Pirie and Kieren's Theory allows the observer to generate a very detailed account of the mathematical actions of an individual learner and to offer a level of specificity that is perhaps not possible through more general socio-cultural perspectives.

(ibid.: 2008:126)

That is, even as it is questionable that folding back in Pirie and Kieren's model has to do with returning to a lower layer and that students' thinking may not reflect all eight layers,

Martin and LaCroix (2008) posit that this model can be used to assess students' thinking within social constructivist learning groups.

8.6 Experimental Studies depicting Pirie and Kieren's Layers of Mathematical Understanding

In addition to the review of Pirie and Kieren's layers of understanding which suggests the need to reconsider that within a teaching situation, students' thinking did not transcend from the first to the eighth layer and the perception that folding back is a much more in-depth reflective process, teaching experiments using this model are reviewed to identify the extent to which it has been used to assess students' thinking. These teaching experiments are also examined to identify observable behaviors aligned to each of the eight layers. The specific actions reflective of each of the eight layers will be used to assess grade five students' interactions during social constructivists' activities.

At the onset, it must be noted that Pirie and Kieren's (1994) Layers of Mathematical Understanding has been applied in numerous research studies (Martin, Towers and Pirie, 2006; Thom, 2004; Towers and Davis, 2002). For example, Pirie and Kieren's layers of mathematical understanding was used by Droujikova, et al., (2005) to identify a conceptual framework for studying prospective middle and high school teachers' collective understanding. Revealed in this research is that individual differences affected how decisions were made and mathematics ideas were presented. Martin and LaCroix (2008) have also drawn extensively from Pirie and Kieren's model to examine the growth and

understanding of mathematics within the context of workplace training. This experiment illustrated that mathematical thinking within the workplace can occur at the Image Making and Image Having layers.

Additionally, Pirie and Kieren's model has been used to explore the growth of mathematical understanding in construction trades which revealed that "it cannot be assumed that the images held by adult apprenticeship for basic mathematical concepts are flexible or deep and that folding back to modify or make new images", facilitates mathematical understanding (Martin, LaCroix and Fownes, 2006:01). Emphasized is that mathematical understanding may differ among adults and reflecting on one's thinking does not necessarily result in greater mathematical understanding or the ability to interpret challenging situations.

In addition to an overview of the use of Pirie and Kieren's layers of mathematical understanding, to be examined are how these layers of mathematical thinking were identified within various teaching experiments. That is, teaching experiments by Martin and LaCroix (2008) and Martin, LaCroix and Frownes (2006) illustrated examples of specific actions aligned to the first three layers of Pirie and Kieren's Model. For example, identified in Martin and LaCroix's (2008) experiment to assess "the nature and growth of mathematical understanding", is that the first layer known as Primitive Knowledge can be observed in students' ability to use their prior knowledge to interpret problems. Besides using prior skills and experiences, Martin and LaCroix posit that the second layer or Image Making can be observed when students demonstrate understanding of a concept verbally or through pictorial representations. That is, Image Making was evident when the workmen

observed the drawn elevations and reviewed the measurements for constructing a building. When the workmen no longer needed to observe the drawn elevations and formed a mental representation of the task, this action was labelled as Pirie and Kieren's third layer known as Image Having. Therefore, Image Having refers to forming a mental idea or plan without referring to a representation of it.

While Martin and LaCroix's (2008) experiments provided explicit examples of Primitive Knowledge, Image Making and Having, similarly in a teaching experiment by Droujikova, Bereson, Slaten, and Tombes (2005) observable behaviours illustrating the first three layers were evident. Revealed in this research to examine the growth of collective understandings among mathematics teachers was that Primitive Knowledge consisted of "previous knowledge brought to the learning context" (2005:290), which can also be described as "knowledge that is actively built up by the cognizing subject" (Von Glaserfeld, 1995:51). Besides the notion that Primitive Knowledge is evident when students draw on prior knowledge to interpret problem situations, also shown is that Image Making can be observed in verbal interpretation or use of pictorial representations a mathematical idea or problem. For example, in this experiment students used real objects, number and symbols to represent math assignments; thereafter, Image Having was illustrated as "using the idea of multiple representations without actions of creating them" (ibid.: 293). That is, the third layer, or Image Having was evident when students used information from their math assignments during small group and whole class discussions.

Besides explicit actions illustrating Image Making and Having, examples of the fourth and fifth layers were also identified. For example, the fourth layer, Property Noticing was

manifested in students' ability to identify connections between one concept and another such as seeing the connections between ratios and proportions. Observable behaviours depicting the fifth layer, Formalizing, was also evident when students "abstracted the noticed idea of differences or similarities of a concept" (ibid.: 294). For example, one student's formalized idea was that, "using different types of representations to explain a concept will give students a chance to see the lesson from a different perspective" (ibid.: 295). Hence, revealed in these examples is that specific actions can be aligned to each of Pirie and Kieren layers of mathematical thinking. Therefore, examples of observable actions in teaching experiments by Martin and Pirie (2003); Martin and LaCroix, (2008) and Droujikova, Bereson, Slaten, and Tombes (2005) were aligned to grade five students' interactions during social constructivist activities in Belize. Extensive reviews of Pirie and Kieren (1996), Martin, Towers and Pirie (2000) and Meel's (2003) description of the eight layers were also used to identify responses reflective of the eight layers of mathematical thinking. A summary of observable behaviours/actions aligned to Pirie and Kieren's (1994) layers of mathematical understanding are illustrated in Table 9 on the next page.

Table 9:

Examples of Pirie and Kieren's layers of Mathematical Understanding

Layers of Mathematical Understanding	Examples illustrated in Teaching Experiments and review of Pirie and Kieren's layers of mathematical understanding
First Layer -Primitive Knowledge	Anything that the learner may know about a topic (Martin and La Croix, 2008:05; Meel, 2003)
Second Layer- Image Making	Activities aimed at helping students to develop an idea of what the concept is about. For example, pictorial representation, expression in language or action (Martin and La Croix, 2008:05)
Third Layer- Image Having	Carry out a mental plan. No need for a particular action or examples (ibid.: 06)
Fourth Layer- Property Noticing	Examine properties, connections or distinctions. Include self reflection, questioning one's understanding (Martin and LaCroix, 2008:06; Martin and Pirie, 2003:174)
Fifth Layer- Formalizing	images the learner now has are examined for properties, connections, or distinctions (Martin and Pirie, 2003:174)
Sixth Layer- Observing	Observe, structure and organize personal thoughts. Combines definition, example, theorems and demonstrations to identify essential components (Meel, 2003:146)
Seventh Layer- Structuring	Explain the interrelationship of the observations by an axiomatic system. Understanding transcend a particular topic; conceive proofs or properties of a concept (ibid.: 146)
Eight Layer- Inventizing	Break free of structured knowledge which represents complete understanding and to create totally new questions that will result in the development of a new concept (ibid.: 146)

Therefore, as illustrated in Table 9, Pirie and Kieren’s layers of understanding can be identified using specific behaviours drawn from research studies to examine students’ growth and understanding. Hence, in the subsequent subsection, the video transcripts will be analyzed to assess the extent to which grade five students’ thinking in Belize are aligned to these layers of mathematical thinking.

8.7 Analysis of the Video recordings and Semi-structured Interviews

To assess grade five students’ understanding, this subsection begins with a review of the video transcripts of the experimental group of students who were continuously immersed in social constructivist activities for the twelve weeks of the teaching experiment. Next, two students are randomly selected and their sequential responses outlined to examine their sequential thought processes as they collaborated with peers to interpret and solve one step math problems.

Post test 2: Item 5

Use the diagram below to solve Item 5

Movie Ticket Prices	
Before 6 p.m.	
Children under 12.....	\$ 5
Adults.....	\$ 10
After 6 p.m.	
Children under 12.....	\$ 7
Adults.....	\$ 15

Item 5:

14 year old Matt is inviting his 10 year old brother to the movie before 6:00 p.m. How much will it cost them to see the movie?

Students' action, as noted on the video recording, is as follows:

All students are reading silently.

Response 1: (Student 3)

This problem is asking us to find the cost of movie tickets and there are different prices for different times.

Response 2: (Student 1)

This shows that children pay one price and adults pay another price. It says here, children under 12 pay \$5.00 and adults pay \$10.00 before 6 p.m. But if you go to the movies after six, then children pay \$7.00 and adults pay \$15.

Response 3: (Student 2)

The problem also states that 14 year old Matt is inviting his 10 year old brother to the movie before 6 p.m. How much will it cost?

Response 4: (Student 3)

So, we have to look at our price list and decide how much to charge.

Response 5: (Student 1)

14 years is young so the cost for Matt is the same as his brother.

Response 6: (Student 2)

What if Matt looks big and he is taking his brother as an adult?

Response 7: (Student 1)

14 year olds are still not adults. So, let's check how many people are going to the movies.

(Students pause for a brief moment and observe the diagram)

Response 8: (Student 2)

Matt and his brother are two. So, they will pay $5 + 5$; that makes 10 dollars.

Response 9: (Student 3)

You can also say $5 \times 2 = 10$. This is a shorter way to do it.

Response 10: (Student 1)

Let me draw a picture to show this problem.

(The student is observed drawing a sketch to represent Matt and his brother. The other students observe and compare the diagram with the price of the tickets.)

Response 11: (Student 1)

Yes, the answer is \$10 if it is \$5.00 for each.

To assess students' layer(s) of understanding as depicted in this video transcript, Student Three and One's thinking are reviewed to analyze their responses to this problem.

Therefore, Student Three sequential responses are as follows:

1: This problem is asking us to find the cost of movie tickets and there are different prices for different times.

2: So, we have to look at our price list and decide how much to charge.

3: You can also say $5 \times 2 = 10$. This is a shorter way to do it.

The first and second responses revealed that Student Three read and interpreted the problem which is reflective of the second layer. The third response, "You can say $5 \times 2 = 10$, this is a shorter way to do it," shows that the properties of the problem were reviewed. The student then used this information to make connections that the problem could be solved in a shorter way depicting the fourth layer or Property Noticing. Therefore, shown is that Student Three's thinking depicted Pirie and Kieren's second and fourth layer of understanding. Shown is that there are instances when students' thinking may not move from one layer to another but move across layers. That is, having interpreted the problem at the second layer, Student Three formed a mental image, examined the properties, and made connections reflective of the fourth layer of thinking.

To examine the extent to which grade five students in Belize constructed understanding, a second student's sequential responses are also outlined and subsequently analyzed:

Student One's sequential responses are analyzed below:

1. This shows that children pay one price and adults pay another price. It says here, children under 12 pay \$5.00 and adults pay \$10.00 before 6 p.m. But if you go to the movies after six, then children pay \$7.00 and adults pay \$15.
2. 14 years is young so the cost for Matt is the same as his brother.
3. 14 year olds are still not adults. So, let's check how many people are going to the movies.
4. Let me draw a picture to show this problem.
5. Yes, the answer is \$10 if it is \$5.00 for each.

Student One's first response shows efforts to interpret the problem by identifying specific prices for children and adults at different time periods which depicts the second layer of understanding. In addition to using background knowledge to interpret and identify key elements, the second response also denotes Image Making which illustrates that this student reviewed the problem to identify that the cost for Matt and his brother were similar. Noted in the third response is that the student reflected on the problem and concluded that "14 year olds are not adults." The ability to examine properties, make connections or distinctions is identified in teaching experiments by Martin and LaCroix, (2008) and Martin and Pirie (2003) as examples of the fourth layer known as Property Making. Furthermore, revealed in the fourth response is that this student folded back to the second layer by reviewing the problem and drawing a pictorial representation. At the

end, the student reflected on the fact that the price for Matt and his brother was \$10.00 if the cost for one child is \$5.00. To make connections and to reflect on ones' understanding also aligns to Property Noticing. Therefore, shown is mathematical thinking at the second layer and transcending to the fourth. The student then folded to the second and returned to the fourth layer. Again illustrated in this response is that there are instances when students' thinking move across layers, in this instance, from the second to the fourth.

8.8 Reflections on the Video Recordings

As previously noted, the video transcript in section 8.7 revealed that the students' interactions were aligned to the second and fourth layers of Pirie and Kieren's Model. To analyze the research question, "What are grade five students' experiences as they interpret, analyze, and solve math problems," the two students who were randomly selected to observe the video recording were also interviewed using a semi-structured interview guide to assess whether there is alignment between the actions on the video recordings and students' personal account of their thought process and experience. Hence, the verbatim of Student Two's reflections are presented and analyzed in this subsection.

The responses to the semi-structured interviews are as follows:

Question 1: Can you explain what you were thinking when you said, "The problem says that 14 year old Matt is inviting his 10 year old brother to the movie before 6 p.m. How much will it cost?"

Student 2: O.k. I was thinking about what we needed to do to solve this problem. So I looked at the time and the prices and I wrote it on paper.

Question 2: You drew a lot on a paper in the video. We saw you adding numbers and then drawing a person for Matt and one for his younger brother. Can you explain what you were thinking?

Student 2: I was just thinking about what the problem was asking. So, I drew Matt and his brother to compare the cost for both of them.

Question 3: Why did you compare the price for Matt with the price for his brother?

Student 2: We had different prices for different times and age, so I needed to know if the prices would be the same or different.

Question 4: The video shows that you also drew a picture. Was this helpful?

Student 2: This is one of the new things I learnt to do. My teacher gives us little blocks, sticks, and pictures to work with and she said we could also draw the problem. Drawing a picture helped me to see clearly the price for the children and how much to charge at what time. So, when I drew the picture, I could check when children are charged one price and when they are charged another.

Question 5: What were you thinking when you said, “What if Matt looks big and he is taking his brother to the movie as an adult?”

Student 2: Some problems are tricky. I was thinking that if Matt is as big as I think he is, he could be charged as an adult. Also, when I go to the movies, I have to go with an adult.

So, I was thinking Matt can be charged as an adult because of his size. Children do not go to movies alone.

Question 6: Is it important to think about the problem using your experiences or things you did before?

Student 2: I think so. My teacher also said that the best way to understand something is to think about what it means.

Question 7: So, do you do this to solve math problems?

Student 2: Sometimes; for this problem, the age was important. My friends helped me to understand that size was not important. This was an easy problem. In other problems, you have to think about all kinds of things.

Question 8: Explain what you meant by “it was easy”.

Student 2: I did not have to do a lot of things. Some problems ask for more than 2 steps to find the answer.

Several conclusions can be drawn from these responses. The first is that this student’s reflections also showed that initially his thinking was aligned to the second layer of understanding. Revealed in the third and fourth responses is that the student’s reflective views also describes thinking at the fourth and fifth layers. Identified in the fifth is that the student acknowledged that background knowledge assisted him to interpret the problem. To be also noted is that in the sixth and seventh response, Student Two conceded that the problem was not difficult and students were only required to identify the specific attributes to solve.

Therefore, as noted in Pirie and Kieren's Model, the ability to effectively use background knowledge is crucial to the problem solving process. That is, students will experience difficulty unless they effectively use background knowledge and experiences to interpret problems (Simon, 1995).

While background knowledge is the basis for knowledge construction, also implied is that problems should be aligned to students' experiences (Nathan et al., 2003). That is, the student used the phrase, "new things I learnt to do" to emphasize the importance of making meaningful connections with the problem to be solved. Furthermore, Student Two's belief that problems requiring multiple steps or procedures are difficult to solve highlights the need to expose students to problems with varying degrees of difficulty to improve and refine problem solving skills.

In addition to a review of Student One and Three's verbal responses and semi-structured interview, the overall responses to Post Test 2: Item 5 are illustrated in the Table 10 on the subsequent page:

Table 10:

Overview of Responses and Layers of Knowledge Construction of students who were taught using Constructivist-based Instruction

<i>Response</i>	<i>Respondents</i>	<i>Layers of Knowledge Construction</i>
1	Student Three	Second Layer
2	Student One	Second Layer
3	Student Two	Second Layer
4	Student Three	Second Layer
5	Student One	Second Layer
6	Student Two	Second Layer
7	Student One	Fourth Layer
8	Student Two	Fifth Layer
9	Student Three	Fourth Layer
10	Student One	Second Layer
11	Student One	Fourth Layer

Illustrated in Table 10, seven of the responses depicted Image Making. This was revealed in activities which showed forming pictorial representations or students' oral interpretation of the problem. Three of the responses also illustrated the fourth layer or Property Noticing. That is, Student One and Three's interactions depicted forming connections, distinctions, and reflecting on the problem. Also revealed is an example of Formalizing as shown in this response, "What if, Matt looks big and he is taking his brother as an adult". This response showed that Student Two formed a mental image of the size of an adult and assessed the likelihood that Matt could be charged \$10.00 instead of \$5.00. Hence, shown

in the first video recording is that, in most instances, students' interactions reflected the second layer of thinking. However, there is also evidence that students' thinking transcended to the fourth and fifth layers. Also revealed is that there was no evidence of Observing, Structuring or Inventizing which reflects the sixth, seventh and eight layers.

While reasons to justify why students' thinking did not extend beyond the first five layers will be assessed in the subsequent chapter, to be noted is that in Droujkova, et al.,'s (2005) experiment is that students' thinking also did not extend beyond this layer. One similarity is that in the teaching experiment in Belize, grade five students were provided with one step problems within social constructivist learning group. In Droujkova et al.'s study, prospective teachers were provided with metaphors which are figures of speech to be used as a primary mechanism for thinking (Lakoff & Nunez, 2000). That is, students were provided with situations in which they were required to examine, identify similarities and to form conclusions (Droujkova et al., 2005: 290). Shown is that in both instance students were provided with a single issue to be discussed or resolved. Therefore, it appears that thinking beyond the sixth layer occurs if students are provided with situation which requires greater analytical thinking and application. This means that if students in Belize are to engage in thinking activities at the seventh and eight layers, appropriate problems must be identified. Also illustrated in the video transcripts is that the teacher observed students and did not probe or attempted to restate the problem to bring about a "perturbation" or deeper analysis. Worth emphasizing is that for higher levels of mathematical thinking to occur, teachers must also stimulate students' thinking by rephrasing problems and providing opportunities to engage in critical analysis within social constructivist learning groups (Simon, 1995; Vygotsky, 1986).

8.9 Video Transcripts and Reflections of the Second Random Selected Group

In this subsection, the video transcripts of students who were taught using procedures and became engaged in constructivist-based activities from weeks 7 to 12 are also reviewed to examine whether their responses depicted Pirie and Kieren's layers of mathematical thinking. Therefore, two students were also randomly selected and their sequential responses reviewed to assess their mathematical thinking. Data obtained from the semi-structured interview guide used to interview students after they were videoed during social constructivist activities are also examined to assess students' description of their thoughts processes and to identify strengths and weaknesses. Finally, this subsection ends with a summary of the responses to Post test 1 Item 4.

Post-test 1 Item 4:

Each week you are allowed to play a video game for 3 hours. How many hours of game would you have played after 6 weeks?

The transcripts illustrating students' interactions are as follows:

(Students read the problem for approximately one and a half minute.)

Response 1: (Student 4)

It says that this person plays video games 3 hours every week. So how many hours will be played in 6 weeks?

Response 2: (Student 1)

It is asking to find the number of hours.

Response 3: (Students 2)

If we draw a picture, we could show the hours and days. Let me draw it.

(The student draws a large square and separates the square into six spaces. The student then writes 3 in each space)

Response 4: (Student 2)

O.k. this is it!

Response 5: (Student 1)

So, it is showing that for 6 weeks the person played video games 3 hours every week.

Response 6: (Student 3)

So let us all add to get the answer.

Response 7: (Student 2)

Remember it is 3 hours for 6 weeks. So, that means that we have to find out how many hours by multiplying.

Response 8: (Student 3)

So, we have to multiply the number of weeks by 3 hours to find how many hours of video games were played. I mean we need to check how much time the game is played and for how long.

Response 9: (Student 1)

Okay, so it's like checking if one of us is playing video games for 3 hours every week and someone asks us how many hours of video games we played in 6 weeks. Hmm.... Let me see... from week one to week six ...

(The student remains pensive for a brief moment and begins to count her fingers)

Response 10: (Student 2)

We can add or we can multiply. Remember Miss said that if we have a number so many times, we can write, $3+3+3+3+3+3$ which is the same as 3×6 .

Response 11: (Student 1) Let us see if we can solve it?

(Students write on small sheets of paper and the discussion continues)

Response 12: (Student 3)

I got 18 hours, what did you get?

Response 13: (Student 2)

I have 18 too!

Response 14: (Student 1)

I got 14. How did you get 18?

The students all gather around Student 1 looking carefully at his paper.

Response 15: (Student 2)

You added but you have a wrong answer. What did you do?

Response 16: (Student 3)

Let's add the number in small groups to check, $3+3=6$; $3+3=6$; $3+3=6$. All these 6's are equal to 18. How did you get 14?

Response 17: (Student 1)

Oh my! I did not count correctly!

The teacher looks at Student One's paper and says:

Teacher: Okay, let's talk about what you just did.

Response 18: (Student 3)

First I read the problem. Then I noticed that we had to multiply the 6 weeks by 3.

Teacher: How are you sure that this is what you needed to do?

Response 19: (Student 3)

When we solve multiplication facts, I remembered that we multiplied the sums to find the answer.

Response 20: (Student 4)

That is true. I mean we had to multiply 6 weeks by 3 and got 18 hours. I also checked that it is only 3 hours every week and not 3 hours daily for 6 weeks. Because if you do not check and read over the problem, you can multiply 3×7 days for 1 week and then multiply by 6. But, this is not what the problem is asking.

Teacher: Okay, so is it clear how to solve problems like these?

Response 21: (Student 3)

Miss, it means that we can find the amount of something by reading the problem and checking to see what we need to do to get our answer. The way to solve is sometimes different for some problems.

Teacher: Any other questions?

(All students shake their heads indicating that there were no more questions)

As stated previously, to assess students' thinking and layers of understanding, two students who were engaged in the social constructivist learning group were randomly selected. Their sequential responses were then reviewed to assess how they processed information during social constructivists' activities and their levels of mathematical thinking. To examine whether these students' interactions were a true reflection of the layer(s) of Pirie and Kieren's mathematical thinking, each student was also interviewed using the semi-structured interview guide in Appendix N. The first randomly selected student's response is as follows:

Student Three's Sequential Responses:

1. So let's all add to get the answer
2. So, we have to multiply the number of weeks by 3 hours to find how many hours of video games were played. I mean we need to check how many times the game was played and for how long.
3. I got 18 hours, what did you get?

4. Let's add the number in small groups to check, $3+3 = 6$, $3+3 = 6$, $3+3 = 6$. All these 6's are equal to 18. How did you get 14?
5. First I read the number then I noticed that we had to multiply the 6 weeks by 3
6. When we solve multiplication facts, I remembered that we multiplied the sums to find the answer
7. Miss, it means that we can find the amount of something by reading the problem and checking to see what we need to do to get out answer. The way to solve it sometimes is different for some problems.

Shown in the first three responses is Student's Three's attempt to interpret the problem which is aligned to the second layer of thinking or Image Making. In the fourth response, this student makes the connection that multiplication is a short cut for addition illustrating that " $3+3 = 6$, $3+3 = 6$, $3+3 = 6$. All these 6's are equal to 18." To examine properties of numbers and to identify that groups of six can be added to the sum of 18 illustrates mathematical thinking at the fifth layer, Formalizing. Shown in the fifth and sixth responses is that to describe the process for interpreting the problem, this student folded back to the second layer, Image Making. Illustrated so far is that Student Three's thinking was reflective of the second layer, then the fifth, and folding back to the second.

Revealed in the seventh response is that this student structured personal thoughts to explain that different problems may require different solutions which is aligned to the sixth layer or Observing. Therefore, revealed in these sequential responses are that, during social constructivist activities, Student Three's thinking did not extend beyond the sixth layer.

Furthermore, this student's thinking moved across layers and did not progress in a linear fashion.

Having reviewed the first randomly selected student's sequential responses, the second is outlined below.

Student Two's Sequential Responses

1. If we draw a picture, we could show the hours and day. Let me draw it.
2. Ok, this is it.

Remember it is 3 hours for 6 weeks. So, that means that we have to find out how many hours by multiplying.

3. We can add or we can multiply. Remember Miss said that if we have a number many times, we can write $3+3+3+3+3+3$, which is the same as 3×6 .
4. I have 18 too!
5. You added but you have a wrong answer; what did you do?

Student Two's first two responses illustrate Image Making which is reflective in this student's use of pictures to represent the problem and thinking about the number of hours and weeks of video games played. In the third, Student Two's ability to state that they needed to find out how many hours by multiplying, illustrates the ability to carry out a mental plan which is reflective of the third layer, Image Having. Revealed in the fourth response is that this student recollected on that previous instructional guidelines and successfully described how it can be used to solve this problem. To be noted is that to form mental images and to examine properties are similar the fifth layer of understanding known as Formalizing. Further revealed in the fifth and sixth responses is Student Two's attempt to examine answers to the problem to assess accuracy. The use of language to interpret an idea or concept is reflective of the second layer of thinking, Image Making. Therefore,

revealed in Student Two's sequential responses is thinking at the second, third, fifth and folding back to the second layer. This again indicates that students' thinking often does not transcend from one layer to another but can move across layers. Furthermore, even in instances when students were provided with procedural guidelines followed by immersion in social constructivist learning groups, their thinking did not extend beyond the sixth layer. Shown is that unless students are scaffolded to extend their thinking and provided with the appropriate context for doing so, this may never occur.

8.10 Reflections on the Video Recordings

Besides assessing students' mathematical thinking, to identify areas of difficulty and the extent to which they constructed understanding, as previously mentioned, the two students whose sequential responses were reviewed in subsection 8.9 were also interviewed to cross examine whether the actions observed on the video recording are aligned to the students' description of their thought processes. Secondly, these students were also interviewed to identify strengths and areas of difficulty.

Consequently, this subsection begins with a review and summary of Student Three's responses.

The transcript obtained from the semi-structured interview is as follows:

Question 1: You mentioned in the video, let us just add to get the answer, what were you thinking at that time?

Student 3: At first, I quickly thought about the numbers in the problem and I knew that we needed to know how many hours of video games could be played in 6 weeks if students played 3 hours every week.

Question 2: Can you explain why you said, “So, let’s add”.

Student 3: My teacher says that if we want to find out the total amount of something, we can add. So, I thought maybe we should just add the 3 hours, six times.

Question 3: We just saw that you later agreed that it was best to multiply. Can you explain why you thought you should do so?

Student 3: I was thinking at that time that $3+3+3+3+3+3$ is the same as 3×6 . I was also looking at the problem again to really check what I needed to do.

Question 4: I saw you writing on a paper as you listened to the others discussed the problem. What were you writing?

Student 3: I wrote down 3×6 and I was also flipping the pages of my book to see if this problem is the same as other problems. Then, after I looked at this problem and the ones we solved last week, I figured out how to solve it.

Question 5: Why did you need to check the problem in your book?

Students 3: It helps me to check if this problem is the same or different. Last week we looked at multiplying things like if someone has 4 sacks of oranges and each sack has 15,

how many oranges are there in all? Now, we are also multiplying but by looking at how many hours and weeks.

Question 6: I noticed that you were able to solve this problem. Was this easy?

Student 3: It was not very easy. I had to know that the problem was asking to solve 3 hours for 6 weeks and nothing else. After I read the problem and understood it, then it was easy to solve.

Question 7: Can you use the same steps to solve other problems?

Student 3: If there are problems that ask how many or the time spent to do something, I can use the same steps to solve.

Illustrated in the first question is an attempt to guide Student Three to focus on her actions and thought processes. Therefore, noted in this first response is that this student acknowledged that initially she reflected on the problem to identify how it can be solved which aligns to the Second Layer, Image Making. In the second response, this student conceded that she formed a mental image of additional facts and identified a possible solution to the problem reflective of the fifth layer or Formalizing. Noted in the third response is that although Student Three's thinking was reflective of the fifth layer, she folded back to the second to examine specific attributes of the problem. This shows there often students fold back to a lower layer to reflect on a previous idea or problem (Martin and LaCroix, 2008).

Further revealed in the fourth and fifth responses is that Student Three continuously developed her mathematical understanding by reflecting on similar problem types. Finally, in the sixth response, this student conceded that the problem was not challenging and could be easily applied to other problem situations which suggests that students are not always challenged to extent their mathematical thinking or to engage in critical thinking activities. Undoubtedly, if students are to think at the two higher levels of Pirie and Kieren's model they must engage in activities by,

formulating key questions, analyzing and conceptualizing problems, defining problems and goals, discovering patterns and similarities, seeking out opportunities and transferring skills and strategies to new situations.

NCTM, 1980:02-03

It is apparent that students stand to benefit from organized goal-directed activities to facilitate knowledge construction beyond the first six layers of understanding. The fundamental point is that if students are to structure their thinking and to develop new conceptual understanding, they must be exposed to structured activities to promote higher levels and analytical thinking skills (Carlson and Bloom, 2005)

The overall response to Post Test 1 Item 4 is summarized in Table 11.

Table 11:

Overview of Responses and Layers of Knowledge Construction of students who were taught using Procedures followed by Constructivist-based Instruction

<i>Response</i>	<i>Respondents</i>	<i>Layers of Knowledge Construction</i>
1	Student Four	Second Layer
2	Student One	Second Layer
3	Student Two	Second Layer
4	Student Two	Second Layer
5	Student One	Second Layer
6	Student Three	Second Layer
7	Student Two	Fourth Layer
8	Student Three	Second Layer
9	Student One	Fourth Layer
10	Student Two	Fifth Layer
11	Student One	Second Layer
12	Student Three	Second Layer
13	Student Two	Second Layer
14	Student One	Second Layer
15	Student Two	Second Layer
16	Student Three	Fifth Layer
17	Student One	Second Layer
18	Student Three	Second Layer
19	Student Three	Second Layer
20	Student Four	Sixth Layer
21	Student Three	Sixth Layer

As illustrated in Table 11, of the students who were taught using procedures from weeks 1 to 6 and were instructed using constructivist-based instruction from weeks 7 to 12, fifteen of the responses reflected the second layer of thinking illustrated in activities to

interpret Post Test 1: Item 4 the problem. Two of the verbal responses illustrated Image Having which is the third layer, as shown in specific actions or verbal responses, illustrating that a mental plan was formed.

Furthermore, illustrated in Table 11 is that two of the responses illustrated that students examined properties and made connections and distinctions as illustrated in the fourth layer, Property Noticing. Also revealed is that two students' thinking reflected the fifth layer as shown in Student Three's ability to form a vivid recollection of the process for reviewing a problem to extract specific components. Ultimately, two students' observed, structured and organized personal thoughts as illustrated in the sixth layer known as Observing. Also revealed is that at no time did their thinking extent beyond the sixth layer.

This highlights that one step problem which requires identifying a specific response to a given problem did not facilitate thinking at the Structuring layer which entails thinking beyond the context of the problem to explain interrelationship or to conceive proofs and properties requires. So, to be considered is that if students are to engage in activities to reflect the eight layer, Inventizing, teachers must structure activities to enable students to use their current understanding to create new ideas about a concept of problem (Meel, 2003). What this means is that, "rather than behaviours or skills as the goal of instruction, concept development and deep understanding are the foci" (Fosnot, 1996:10). Secondly, teachers should skilfully guide these activities to assist students to become engaged in "real mathematical thinking" and knowledge construction (Steffe, 1994; Simon, 1995; Steffe and D'Ambrosio, 1995). Identified is that during social constructivists' activities,

The role of the authority figure has two important components. The first is to introduce new ideas or cultural tools where necessary and to provide the support and guidance for students to make sense of these for themselves. The other is to listen and diagnose the ways in which the instructional activities are being interpreted to inform further action. Teaching from this perspective is also a learning process for the teacher.

Driver, Aasoko, Leach, Mortimer & Scott, 1994:11

Revealed is that if students are to extend their mathematical thinking and develop new conceptual understanding, teachers must accurately assess students during social constructivist activities and appropriate problem types and activities must be used to develop mathematical competencies.

8.11 Teachers' responses and perceptions of the Teaching Experiment

To further explore the belief that teachers play a critical role during social constructivist activities and to assess whether they skilfully guided students to analyze problem situations, in this subsection one teacher's response, as illustrated in the second video transcript, is analyzed. Next, teachers' responses to the semi-structured interview guide are reviewed to identify:

1. What were the teachers' roles during the teaching experiment?
2. Do teachers' believe that social constructivist activities can be used to enhance students' mathematical competency?

Firstly, identified in the first video transcript, is that when students were asked to solve a problem involving finding the cost of movie tickets, the teacher observed students' interactions and only made a few non-verbal gestures. That is, at no time did the teacher restate the problem or prompted students to critically assess the essential attributes.

Highlighted is that even in instances when teachers are provided with guidelines on how to improve instructional practices, these are not always adopted. One possible justification is that teachers who customarily taught using traditional activities experienced difficulties facilitating knowledge construction (Driscoll, 2005). Also implied is that shifts in teaching modality is not easy to achieve. That is, the teachers' thoughts, beliefs and background experiences greatly influence the extent to which specific teaching guidelines are employed (Fenstermacher & Soltis, 1992). In essence, teachers' response to instruction affects the extent to which an innovation is utilized (Koehler & Grouws, 1992). The prevailing view is that although teachers received instructional guidelines to foster higher levels of thinking, the video transcript in Section 8.7 did not illustrate use of these skills as illustrated in the teacher's responses below.

1. Okay, let's talk about what you've just did.
2. How are you sure that this is what you needed to do?
3. Okay, so is it clear how to solve problems like these?
4. Any other questions?

The first response shows that this teacher asked students to reflect on the solution to the problem. While this is somewhat consistent with Driscoll's (2005) belief that students should be guided to reflect on their actions, of importance is whether they were prompted to critically reflect on the problem. Also shown is that the focus seems to be more on

finding a solution and less on fostering critical thinking. Revealed is that “the teacher’s actions are manifestations of unconsciously held views of expressions of verbal commitments to abstract ideas that may be thought as part of a general ideology of teaching” (Thompson, 1984:112). Indeed, a common belief among teachers in Belize is that mathematical competency has more to do with finding solutions and less about promoting thinking skills (Lopez-Brown et al., 2009). Therefore, “one must recognize that their beliefs about mathematics teaching and learning are part of a larger system of beliefs that also includes beliefs about teaching generally” (Ambrose, 2004: 96). Implied is that teachers’ experiences and beliefs can greatly affect the extent to which they respond to educational innovations.

While beliefs about teaching and learning are critical, the teaching context can also affect the extent to which students are meaningfully engaged (Simon, 1995). Hence, the need to also examine whether one step word problems were suited to facilitate the layers of thinking proposed in Pirie and Kieren (1994) model. While the effects of one-step word problems on the development of critical thinking skills will be discussed in the subsequent chapter, to be noted is that the students’ thinking did not extend beyond the sixth layer. This aligns to the notion that at any given time, it is unlikely that students’ thinking can depict the eight layers of Pirie and Kieren’s model (Martin and LaCroix, 2008).

8.12 Teacher's perception of the effectiveness of the social constructivist experiment

Prior to analyzing perceptions of the effective use of social constructivist activities to enhance mathematical competency, worth noting is that if teachers believe that a particular teaching strategy is beneficial, it is likely that it will be adopted (Burkhardt Fraser & Ridgway, 1990). Implied is that teachers can be agents or obstacles of change (Prawat, 1990). Therefore, to assess teachers' perceptions of the use of social constructivist activities to improve problem solving competencies, transcripts of the semi-structured interviews conducted with two teachers who were randomly selected to teach students in the two experimental groups and those assigned to teach the control groups are analyzed. Hence, the views of one teacher who taught the first experimental group are as follows,

This is different. I mean, students are asked to really think about the math problems. This meant that they had to work together to understand and solve. This was not easy. A lot of these students don't always work well together and some do not like math. So, to ask students to think about a problem and to apply this information, their mathematical skills will take some time. This is very new to all of us.

(T1: January 17, 2008)

Several themes are identified in this response. The first has to do with the view that students should be guided to engage in critical thinking activities. Also identified is that the teacher perceived that guiding students to think and to restructure their thinking requires major instructional shift. This means that although "the basic responsibility of constructivist teachers is to learn the mathematical knowledge of their students and how to harmonize their teaching methods with the nature of that mathematical knowledge" (Steffe

& Wiegel, 1992:17), this can be challenging for teachers who, for the first time, are exposing students to meaningful mathematical activities. Suggested is that it is not easy to facilitate or guide students' thinking during social constructivist activities. Instead, this is a gradual developmental process as described in this reflective account below:

I had taught essentially this way for many years. This particular teaching experience led to further elaboration of my teaching; the teaching-experiment design led to a new level of analysis of the teaching.

Simon, 1995:132

Revealed is that while it is hoped that teachers will automatically adopt a particular teaching strategy, this occurs through careful reflection and over an extended period of time. Furthermore, shown in the response to the semi-structured interview is that students in Belize are often taught using teacher-directed approaches. Therefore,

telling students what they should understand (a lecture approach) is relatively straightforward, developing situation a-didactiques or representative context is more complex and uncertain (ibid.: 122).

Indeed, if teachers customarily expose students to traditional teaching practices, they may experience difficulty guiding students to solve complex problems or situations.

In addition to the first teacher's perceptions, the second teacher is of the view that,

This was a very good experience because if a child can explain and draw the problem, this shows that the child understands what the problem is about. But, I noticed that

students take a very long time to draw a picture and write in the numbers and others just write the first thing that comes to mind; they are not always careful!

(T2: January 17, 2008)

Shown is that while diagrams can be used to assist students to interpret mathematical problems, some students experienced difficulty forming pictorial representations. While Pirie and Kieren's second layer, Image Making, often involves "drawing of diagrams and working through specific examples or playing with numbers", it also include the ability to verbally interpret a problem (Martin and LaCroix, 2008: 122). Undoubtedly, opportunities to read, interpret and to express ones' views about the situation to be solved is an important step in the problem solving process. While the resounding view is that teachers play a critical role during these social constructivist activities, "a particular modification of a mathematical concept cannot be caused by a teacher any more that nutrients can cause plants to grow" (Steffe, 1990:392). Consequently, even if attempts are made to engage students in critical thinking activities, this may never occur if they lack the requisite math skills. Despite that mastery seem to be dependent on thinking skills, "no one has succeeded in demonstrating that understanding improves algorithmic performance" (Nesher, 1986b:16). Hence, the need for additional research to also examine causes for limited competency and mathematical thinking skills.

In addition to the above mentioned, also revealed in this verbatim is that the teacher perceived that students are "not always careful". The notion of not being careful has also been noted in Steffe's teaching experiment with a student named Maya who appeared to have a good grasp of division facts; however, when asked to restate the problem using

multiplication, she promptly gave an inaccurate response. Shown is that although it is hoped that students can perform certain task, there are instances when they fail to carefully analyze a problem or situation.

Besides the views expressed by teachers who were assigned to the experimental groups, a teacher assigned to the control group perceived that:

I think it is good to have them share their ideas in groups but I am worried that when I ask them to think more about the problem or why they are thinking in a certain way, they seem blank. I keep asking the question over and over and I am not making any progress. I also noticed that students who are weaker in math benefit from working with children who are doing much better. I will continue working with the students but it is so hard to get them to think and to talk about their thinking.

T3: January 17, 2008

Revealed is that this teacher concedes that group interactivity is beneficial; however, she experienced difficulty facilitating deeper analysis and reflection. Illustrated is that “teachers who take this path must work harder, concentrate more, and embrace larger pedagogical responsibilities than if they only assigned text chapters and seatwork” (Prawat, 1992: 357). Also implied is that if teachers are to effectively prompt students to extend their thinking, there is need for continuous teacher training and support (Ward, 2001).

Further shown is that often students remained silent and did not respond to the teacher’s prompts. This observation aligns to Warrington’s (1997) belief that constructivist

activities work best for highly motivated students and competent problem solvers but may be challenging for students who seem less competent or skilled. In addition to difficulties associated with eliciting required responses, a fourth teacher also perceived that,

I like to guide students but it is a hard work and can be discouraging. Yesterday, I asked a question and the students gave me an answer that made no sense. I had to explain what I meant and realized that some students did not know how to divide.

Then, I had to go to the board and discussed how to solve problems with long division.

T4: January 14, 2008

Revealed is the need to provide opportunities for students to think about their thinking and to carefully examine the problem to be solved. Also restated is that lack of mathematical skills can affect the extent to which students interpret and solve problems.

8.13 Effects of the teaching experiment on the Students assigned to the Control Groups and Experimental Groups

Identified in the previous subsection is that teachers did not scaffold or assisted students to reflect and to engage in higher order thinking. While lack of effective scaffolding can be attributed to limited exposure to social constructivist activities, it is apparent that the performance of students who were assigned to the control groups differed from those who were continuously engaged in social constructivist activities. Hence, to be examined in this subsection are differences in cognitive thinking and ability to solve problems among students in the control groups who were taught using procedures from weeks 1 to 6 and

became engaged in social constructivist activities from weeks 7 to 12. The performance and levels of thinking of students who were continuously immersed in social constructivist activities are also examined to assess whether exposure to continuous social constructivist activities were more beneficial. Consequently, this subsection begins with a review of the pre test and post test 1 and 2 results. Next, the control and experimental groups' levels of thinking are critiqued to identify whether use of procedures during the first six weeks assisted students in the control to engage in deeper reflection and critical analysis of word problems. Ultimately, this subsection culminates with additional guidelines for implementing social constructivist activities in Belize.

To assess the impact of social constructivist activities on students who were assigned to the experimental and control groups, the mean performance is presented in Table 12 below

Table 12:

Mean performances of Pre test, Post test 1 and 2 Results

Groups	Pre-test mean Performance	Post test 1 mean performance	Post test 2 mean performance
Experimental	48.6	72.7	80.9
Control	41.9	74.1	84.1

Shown in Table 12 is that the mean performance of students who were instructed using procedures was 74.1 and those in the experimental groups who were immersed in social constructivist activities mean post test 1 scores was 72.7. This illustrates that even as students in the experimental groups were provided with opportunities to dialogue with

peers to interpret math problems, they did not solve as many problems as those taught using systematic guidelines. Also revealed is that when students in the control groups became immersed in social constructivist activities from weeks 7 to 12, their mean performance was 84.1 while students in the experimental groups performance was only 80.9. This again illustrates difference in performance among students who were only exposed to social constructivist activities during the final six weeks and those who were continuously engaged throughout the twelve weeks teaching experiment.

Besides that students who were exposed to procedures during first six weeks followed by social constructivist activities were able to solve more problems, noted in a summary of these students' layers of thinking in Table 11 is that there were 21 exchange of ideas among students. Also shown in Table 10 is that there were only 11 exchanges or dialogue among students who were continuously engaged in social constructivist activities. While factors such as students' motivation could affect the extent to which they discussed and analyzed math problems, it is apparent that exposure to systematic guidelines enabled students who were assigned to the control groups to quickly reflect, discuss and identify solutions. This aligns to the view that,

Mathematics... naturally includes knowledge of algorithms and heuristics, but it also includes a person's awareness of strategies to aid comprehending problem statements, organizing information or data, planning solution attempts, executing plans, and checking results

Garofalo and Lester 1985: 168

Suggested is that if students are to collaborate with peers to interpret math problems they must acquire the necessary background knowledge and skills. Essentially, students cannot be expected to accurately solve problems such as adding and subtracting decimals if they have not been taught to align the decimal points while adding and subtracting (Hiebert & Lefevre, 1986). Clearly, “mathematics relies upon specific representations and tools which plays a role in the structure and role of mathematics thinking” (Schiemann and Carraher, 2002:242). Restated is that unless students have the requisite math skills, they may be unable to accurately interpret and solve given problem situations.

Even as it appears that background knowledge assisted students in control groups to more accurately identify responses to math problems, research studies have shown that many students arbitrarily used procedures without understanding the underlying components math problems (Nagel and Swingen, 1998). This view is consistent with a study by Cobb (1992) to assess students’ procedural and conceptual understanding of double digit addition which revealed that Grades 1 and 2 students successfully used procedures to solve mathematical sums; however, when provided with problem situations, they were unable to solve. This denotes that while background knowledge can form the basis for knowledge construction, efforts must be made to ensure that students understand how and why a particular algorithm was used.

Furthermore, revealed in Tables 10 and 11 is that two of the students who were taught using procedures followed by immersion in social constructivist activities thinking reflected the 6th layer of Pirie and Kieren’s model. To be noted is that none of the students from the experimental group’s thinking reflected the 6th layer which entails observing,

structuring and organizing personal thoughts. Suggested is that strong systematic guidelines enabled students who were assigned to control groups to strengthen their mathematical skills and to more accurately solve math problems. Secondly, ability to use background knowledge and skills enabled these students to engage in more meaningful dialogue to interpret math problems. Although immersion in social constructivist activities enabled students in the experimental groups to identify some accurate responses to math problems, none of the students were able to think beyond the fifth layer of Pirie and Kieren's model. These findings suggest that for students to fully benefit from social constructivist activities, effort must be made to ensure that they develop conceptual understanding of mathematical concepts and to engage in activities to enhance their mathematical skills.

8.14 Summary

In this subsection, the salient points identified in students' video recordings and semi-structured interviews are summarized. Teachers' responses to the semi-structured interview guide are also restated. Next, the main findings are compared to Steffe (1994) and Simon's (1995) teaching experiments. In the end, Martin and LaCroix's (2008) considerations for implementing a teaching experiment are identified. Therefore, illustrated in the data from the video recordings and students' responses to the semi-structured interviews is that:

1. Most students did not construct understanding beyond the sixth layer or Observing which refers to combining definition, examples theorems and demonstrations to

identify essential components (Meel, 2003: 146)

2. Students who lacked background skills experienced difficulty interpreting and constructing understanding.
3. Students who were first taught using procedures followed by exposure to social constructivist activities appeared to be able to use algorithms to identify solutions and to engage in more critical analysis of math problems.
4. Students perceived that problems with multiple steps were difficult to solve.
5. Teachers need to skillfully guide students to think at the higher layers of thinking.
6. Students' thinking did not progress in a linear fashion from one layer to another. Often their verbal responses and actions revealed that their thinking moved across layers.

The video recording of one teacher interactions illustrated that:

1. Even in instances when teachers communicated with students, they did not prompt, rephrase math problems, or guided students to think and engage in critical thinking activities.

Teachers' responses to semi-structured interview questions revealed that:

1. Social constructivist activities were beneficial; however, it was challenging to engage students in critical thinking activities.
2. Some students were not accustomed to solving problems in social learning groups.
3. Using prompt and clues to engage students in critical thinking activities was challenging.

4. Students in Belize are often taught using teacher-directed activities.

The findings in this chapter are similar to the Simon and Steffe's experiment as follows:

As identified in Steffe's 1994 teaching experiment:

1. There are instances when students respond in a haphazard manner without reflecting critically on the problem. Similarly, shown in the video transcripts is that there are instances when students' responses illustrate that little effort was made to carefully analyze mathematical problems.

As identified in Simon's (1995) teaching experiment

1. Background experiences can affect students' response to math instruction. For example, identified in the teacher's responses is that often students did not respond to questions or clues. That is, students were accustomed to traditional teaching practices.
2. Engaging students in critical thinking activities can be challenging. That is, as identified in Simon's experiment, only after careful reflection was he able to decisively plan and engage students in activities to activate their cognitive schemes.

Therefore, in the final chapter, these main findings will be further discussed to identify how to improve future teaching experiments in Belize.

Having identified the main findings factors which impacted the teaching experiment, such use of one-step word problems to engage students in critical thinking activities, and the limitation of using traditional test to assess performance are analyzed in the subsequent chapter.

Chapter Nine

Effects of One Step Word Problems and Traditional Assessment within Social Constructivist Activities

9.1 Introduction

In Chapter Eight, the video recordings of students' interaction in social constructivist learning groups revealed that while a number of students successfully interpreted and solved one step word problems, they did not restructure their thinking nor did they apply skills to further interpret problem situations. Therefore, to be examined in this chapter is whether one step math problems can facilitate thinking beyond Pirie and Kieren's (1994) sixth layers of understanding. In addition to the effects of one step problems, analyzed are whether tests such as Pretest and Post Test 1 and 2 were suitable to assess students' mathematical thinking. Ultimately, this chapter aims to critically analyze factors which impacted on the teaching experiment to identify best practices for implementing future educational initiatives in Belize.

9.2 Effects of One Step Math Problems on Mathematical Understanding

According to Hiebert and Carpenter (1992), learning context affects the extent to which students construct understanding; consequently, in this subsection, the effects of using one step problems to construct understanding during social constructivist activities are assessed. To be noted is that permission was obtained from the Chief Education Officer in

Belize, as illustrated in Appendix A, to conduct this study. This included permission to use item types similar to the Belize Junior Achievement Test (BJAT), which assesses grade five students' ability to solve basic mathematical facts and one step problems. Therefore, this subsection begins with a brief recap of the importance of problem solving skills. Next, the effects of one step problem on the development of problem solving competencies are assessed. Finally, this subsection culminates with considerations for enhancing further teaching experiments in Belize.

Important to mathematics instruction is the development of problem solving skills, which entails identifying goals and using strategies to solve problems (Szetula, 1992). Problem solving is also a highly cognitive process which includes use of,

interesting and well-selected problems to launch mathematical lessons and to engage students. In this way, new ideas, techniques and mathematical relationships emerge and become the focus of discussion. Good problems can inspire the exploration of important mathematical ideas, nurture persistence, and reinforce the need to understand and use various strategies, mathematical properties, and relationships

National Council for Teachers of Mathematics, 2000:182

Therefore, a wide array of skills are required if students are to become competent problem solvers. This includes skills to interpret, analyze, and solve a range of problems (Charles et al., 1997). These skills are developed when students are exposed to,

good problems that inspire the exploration of important mathematical ideas, nurture persistence, and reinforce the need to understand and use various strategies, mathematical properties and relationships

Undoubtedly, problem solving skills are enhanced when students are exposed to relevant and meaningful problem solving activities (Ernest, 1991). This means that students “must ...not simply be expected to repeat different permutations of work they have clearly mastered. (New Zealand Curriculum Framework Ministry of Education, 1992:12); instead, they should analyze problem situations and engage in tasks to promote mathematical thinking (Boaler, 2001; Clarke, 2008).

Even as it is evident that students should be exposed to relevant activities to enhance skills(NTCM, 2000), an abundance of research studies have shown that a number of students are performing poorly in mathematics (Hembree, 1992; Pace, 1986; Stigler, Fuson, Ham & Kim, 1986; Carpenter et al., 1980). For example, in Belize many students lack basic mathematical skills and continuously perform below 50% in national mathematics examination (Ministry of Education, 2010). In the U.S., a significant number of students also lack skills to compute and solve word problems (Muller and Mercer, 1997). Also revealed is that despite efforts to develop a National Numeracy Strategy and standardized curriculum in the UK, gaps between the percentiles on standardized mathematics among 10 to 11 year olds are consistently low (Cockcroft, 1982; Brown, Askew, Rhodes et al., 2002). This shows many young children in developed countries appear unskilled or unable to demonstrate mastery of mathematical concepts. Therefore, if mathematical skills and competencies are to improve, factors affecting mathematics teaching and learning must be addressed.

The aforementioned denotes that mathematical competency develops when students are engaged in activities to stimulate their thinking skills. Also revealed is that while it is hoped that students will acquire skills to analyze and solve problems, many are unable to do so. Hence, there is a need to assess factors which may have impacted on the development of mathematical skills among primary students in Belize.

Firstly, identified in a countrywide study to assess teacher effectiveness in Belize is that students are often exposed to one step math problems and are taught in a routine traditional manner (Brown-Lopez et al., 2009). These one step word problems involve use of one operation to solve (Yan et al., 2004). One step problems have also been described as a “process constrained problem to be solved by applying a standard algorithm where students figure out what algorithm to apply and actually carry out a procedure or sets of procedures” (Cai; 2002:02). For example, students assigned to the control and experimental groups were required to solve,

Post Test 2: Item 1:

A man leaves his home in Belize City at 7:00 a.m. and arrives in Punta Gorda at 12:45 p.m. How long did it take him to travel from Belize City to Punta Gorda?

This problem which required identifying the distance from one location to another has two complete components conveying the numerical information and one component, “in which the numerical information is missing but the description of the set is given (Nesher et al., 1994:03). The three components identified by Nesher et al., are evident in Post Test 2:

Item 1 as follows:

- (1) A man leaves his home in Belize City at 7:00 a.m. (complete component)

- (2) Arrives in Punta Gorda at 12: 49 p.m. (complete component)
- (3) How long did it take him to travel from Belize City to Punta Gorda? (incomplete component)

Illustrated is that one step problems often consist of one or two statements and a question to be solved. For example, this problem required that students to read, comprehend and add the number of hours between the two locations. Evidently, counting the number of hours and minutes from one location to another does not allow “for the emergence of consciousness and critical intervention in reality” (Friere, 1998:62). Explained is that one step problems do not allow students to “evaluate reasons, including the tendency and ability as well as alertness for the needs to demand justification, weigh and to assess reasons” (Trishman, Jay & Perkins, 1992:06). Further implied is that one step problems such as “Connie had 13 marbles; She gave 5 marbles to Jim. How many does she have left” (Carpenter and Moser, 1983:16), require, “an action on and transformation of a single set” (Rowland, 2004:01). This includes removing a sum from a given number to obtain a result. Even as it has been identified that these basic problem types do not promote thinking at Pirie and Kieren’s seventh and eight layers, there is the notion that the location of the unknown affects the level of difficulty (Bermejo et al., 2002; Carpenter, 1986). That is, if the unknown is at the beginning of the problem, the level of difficulty is greater than if the unknown is placed at the end (Grouws, 1992; Hiebert, 1982). Suggested is that if the problem is reworded, the level of difficulty increases. For example, to foster higher levels of thinking, Post Test 2: Item 1 can be rephrased as follows

What time would a man arrive in Punta Gorda if he leaves Belize City at 7:00 a.m. and arrives in Punta Gorda at 12:45 p.m?

Although it is perceived that teachers can restructure or reword problems to facilitate higher levels of thinking, as noted in the video recordings in Chapter Eight, unless students are prompted to critically reflect on problem situations, they may never structure their thinking or apply skills (Simon, 1995). Reemphasized is that when students are exposed to challenging mathematical activities and are prompted to critically analyze problems, their cognitive skills develop (Alston, Davis, Maher & Martino, 1994; Davis & Maher, 1997; Kiczek & Maher, 1998; Davis & Maher, 1990). Therefore, to be considered is that mathematical competencies and higher order thinking among students who lack basic mathematical skills in Belize will require incremental modification in instructional practice and exposure to other problem types (Clarke, 2008).

Even as it is presumed that alternative problem types may result in high order thinking, illustrated in one student's response to the semi-structured interview guide is that,

Problems with more than one step are hard to solve.

November 27, 2008

In the Belizean context, the term "hard" is synonymous with terms such as difficult and challenging. Hence, the phrase "hard to solve" illustrates that this student perceived that problems with more than one mathematical step are challenging. In addition to the notion that non routine problems may be challenging, to be also noted is that a study to assess the

results of exposing students to more challenging problems showed that 80.6% of fifth grade students who successfully solved one step problems experienced difficulty solving two step problems (Quintero, 1980). This study also illustrated that to ensure success, students were gradually exposed to different problem types. Further revealed in a case study is that students who were exposed to routine problems did not apply background skills to solve challenging problem types; instead, they ignored the relationship among quantities and arbitrarily selected numbers and performed arithmetic calculations (Lee, 2002). Shown is that if students are consistently exposed to routine problems, it is unlikely that they will acquire or use analytical thinking skills. Consequently, if problem solving competencies are to improve, instructional practices must be assessed to identify whether students are adequately prepared to “tackle” challenging problems (Darling-Hammond, 2003). These changes will also require “a qualitative transformation of virtually every aspect of teaching” (Schifter, 1996:05). This includes modifying instructional practices to scaffold students to interpret, analyze and solve problems (Clarke, 2008; Simon, 1995). For example, Renkl and Stern (1994) identified that students who were exposed to challenging tasks effectively solved intricate word problems. Furthermore, revealed in a study to assess the effects of procedural or conceptual understanding during constructivist based activities is that,

From a cognitive constructivist view, in the long run, if there are also adequate opportunities for application and practice, learning arithmetic based on conceptual understanding is more advantageous than learning based on drill and practice only.

Staub and Stern 2002:02

This restates that meaningful problem solving instruction is one in which students interpret, analyze and solve various problem types.

Two important points can be drawn from these perspectives. The first is that traditional instructional practices appear to have affected the extent to which students in Belize are exposed to meaningful problem types and are scaffolded by teachers. Secondly, while some background mathematical skills are required to solve one step problems, these problem types do not foster restructuring of one's thinking or application of skills. The overarching view is that unless there are modifications in instructional practices and more relevant problems are utilized, many students in Belize will remain unskilled and unable to improve their problem solving competencies.

9.2.1 Re-examination of the effects of one step problems on thinking skills

Besides an examination of whether one step problems enabled students to restructure and apply mathematical skills, in this subsection, four main criteria for distinguishing between elementary and challenging tasks will be reviewed to further assess the extent to which the one step problems promoted higher order thinking skills.

Firstly, to be examined is whether exposure to one step problems promotes Wertsch's summary of Vygotsky's (1978) four criteria below.

1. shift of control from the environment to the individual, that is, the emergence of voluntary regulation;

2. emergence of conscious realization of mental process;
3. the social origins and social nature of higher mental functions; and
4. use of signs to mediate higher mental functions.

Wertsch, 1985: 25

Emphasized in the first criteria is the importance of self regulated learning which entails creating learning situations allowing students to set goals and to monitor their progress (Panaoura & Philippou, 2003; Pape, Bell & Yetkin, 2003). Self regulated learning, in this regard, calls for “an individual conscious use of mental strategies designed to improve thinking and learning” (Bruning et al., 2004; In Eggen and Kauchak, 2006: 189). This process which empowers students to make decisions and to assess their own thinking was not evident in students’ interactions within social constructivist groups. That is, problem types such as Post Test 2: Item 1 did not allow students to monitor and regulate their own thinking (Rheinberg et al., 2000). The prevailing view is that use of basic addition and subtraction facts did not foster deep reflection or structuring of thought. It also did not “allow students to reflect on what was done or why a difficulty occurred” (Schoenfeld, 1987:01).

Furthermore, reemphasized in the second and fourth criteria is that thought processes develop when students are immersed in tasks which promote critical reflection (Tanner and Jones, 2003; Pintrich et al., 1991; Schoenfeld, 1992). Therefore, problems which require arbitrary use of algorithms do not facilitate,

awareness of how one learns; awareness of what one does and does not understand;

knowledge of how to use available resources to achieve a goal; ability to judge the

cognitive demands of a particular task; knowledge of what strategies to use for what purposes; and assessment of one's progress both during and after performance

Flavell, 1979:906

The resounding view is that one step problems cannot be used to effectively facilitate restructuring of one's' thinking or application of skills.

In addition to assessing whether self regulation and reflective thinking occurs when students are exposed to one step problems, revealed in the third criteria is "the social origins and social nature of higher mental functions" (Wertsch, 1985: 25). Implied is that knowledge develops through the use of interactive activities to improve thinking skills (Brophy, 1992). To be noted is that in the current study, there were instances when students engaged in thinking activities; however, their interactions did not result in any modification in thinking. This emphasizes that use of one step problems did not promote the development of "real" mathematical thinking and learning. Therefore, it is likely that students did not really learn a new skill or were challenged to broaden their thinking. In actual fact, grade five students in Belize are often exposed to traditional instructional practices; hence, their involvement or response to social constructivist activities is new (Brown Lopez et al., 2009). This means that even as it is hoped that these students would automatically demonstrate all eight layers of thinking, they seem unable to rapidly "engage in aversive learning activities, even if the consequence of the learning outcomes are important" (Rheinberg et al., 2000: 516). That is, as previously stated, for maximum benefits to be derived from social constructivist activities, exposure to new innovative

teaching and learning task must be done in a timely fashion. As Martin and LaCroix puts it,

It is important to recognize that growth of understanding of any concept occurs over timeit is likely that the timescale for growth will be a lengthy one.

2008:136

This means that teachers must also monitor and guide students to master essential cognitive skills.

9.3 Relevance of Traditional Tests to assess Constructivist-Based Instruction

Previously noted is that when students are exposed to one step problems, these problem types did not facilitate structuring of thinking or application of skills. Hence, in this subsection, the importance of appropriate measurements is reviewed to identify whether pre-test and post-test 1 and 2 can be used to adequately assess students' thinking. The importance of alternative ways to assess mathematical understanding is also discussed to identify suitable means of analyzing interactions within social constructivist learning groups.

Worth noting is that the term assessment comes from the Latin word "assidere" referring to actions such as talking with or shared learning experiences (Conrad, 1995). Assessment is also described as gathering information on students' progress (Eggen et al., 2003), and providing meaningful information about the result of instruction and what students know and are able to do (National Council of Teachers of Mathematics [NCTM], 1989). Besides

the notion that assessment entails identifying students' response to learning, according to the National Council for the Teaching of Mathematics, assessment entails,

gathering evidence about students' knowledge of ability to use and disposition toward mathematics and of making inference from that evidence for a variety of purposes

NCTM, 1995:03

While this denotes that assessment is used to make decisions on students' progress, the term 'testing' is defined as the act of measuring students' ability to perform a particular task or skill (Liskin-Gasparro, 1997). Even as it is perceived that tests provide information on specific skills and abilities, "any individual task can be performed correctly without understanding" (Hiebert & Carpenter, 1992:89). Implied is that selecting the right responses on written tests is no guarantee that students have developed conceptual understanding. Hence, although tests can be used to measure performance, they cannot assess "real understanding."

9.4 Implications for Traditional and Alternative Assessment

In addition to identifying the importance of assessment, to be discussed in this subsection is the use of traditional tests such as Post test 1 and 2 to assess performance. Thereafter, implications for alternative forms of assessment in mathematics are reviewed.

Firstly, traditional assessment is described as the act of testing right and wrong responses. Therefore, a traditional paper and pencil test does not assess a range of skills and provides limited information about what students know and understand (Eggen et al., 2003).

Implied is that the quantitative analysis of the Post Test 1 and 2 scores does not

demonstrate students' understanding of math problems; instead, this represents a quantitative analysis of numerical responses. Therefore, even as this study aims to assess the effects of social constructivist activities on performance, traditional paper and pencil tests are used to assess gains on math word problems. Clearly,

Instructional designers are wrong to assume that they can base instructional strategies on the analysis of an objective, standard world. Evaluation of learning can only tell us what students appear, or pretend to know, not what they really know.

Winn, 1993:12

This means that traditional assessments such as Post Test 1 and 2 may be inappropriate to assess students' thinking in social constructivist activities since it:

1. fails to measure a range of critical thinking skills or students' ability to process and apply information in real life settings;
2. provides little insight into how students learn or their level of thinking;
3. fails to analyze students' ability to "apply their understanding" to real life task.

Herman et al., 1992 and Marzano et al., 1993:18

Even as it is perceived that the traditional test provides limited insights on real learning, contrary to this view is that:

traditional tests can't be beaten when it comes to reliability, not to mention efficiency. When responses are obviously right or wrong, there is little chance that the scores on a test will vary between one rater and another, or if the student takes two parallel versions of the same test. Hence, traditional tests lend themselves to a wide range of

statistical analyses and comparisons because we can be fairly confident that the true score on a test is very close to the reported score

Liskin-Gasparro, 1997:01

Even as there is the notion that traditional tests provides robust assessment of students' progress, of paramount importance is whether these tests are congruent with the teachers' goal and instructional practices (Cohen, 1986). Hence, while traditional assessment is often utilized, it must be instructionally aligned to classroom practice. Implied is that if students are immersed in social constructivist activities, a suitable assessment should be identified to assess instructional goals. Therefore, while traditional tests can be used to provide reliable results, often they do not measure what we think they are measuring.

9.4.1 Implications for Alternative forms of Assessment in Mathematics

Two main points are illustrated in the previous subsection. The first is that it appears that there is weak alignment between instructional practice and assessment. Secondly, while it appears that traditional tests fail to measure “understanding,” it serves to assess numerical responses and consistency in performance. To identify whether alternative assessments can be used to assess students' thinking, this subsection begins with an overview of alternative assessment such as written explanations, oral interviews and video recordings. Hence, this subsection examines the importance of meaningful measurements to assess how students tackle word problems.

Worth noting is that alternative assessment refers to measuring students' performance on tasks when the solutions is not so obvious or requires more than a numerical response (Worthen, 1993; Gronlund, 1993). That is, unlike traditional tests, alternative assessment in constructivist classrooms facilitates assessing the quality of students' discussion and discoveries (Lochhead, 1985, 1988; Minstrell, Stimpson & Hunt, 1992). Hence, as noted in Chapter Eight, assessing how students reflected upon and solved problems provide a "crucial link between assessment carried out in the classroom and learning and teaching (Assessment Reform Group, 1999: 01).

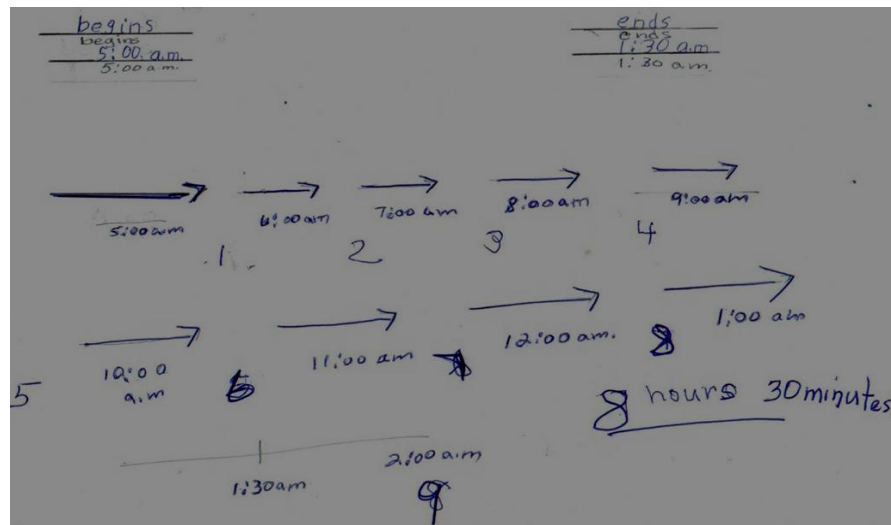
Therefore,

Instead of giving the children a task and measuring how well they do or how badly they fail, one can give the children the task and observe how much and what kind of help they need in order to complete the task successfully. In this approach, the child is not assessed alone; rather, the social system of the teacher and child is dynamically assessed to determine how far along it has progressed.

Newman, Griffin, & Cole, 1989: 87

Clearly, assessing students' reflection and solutions provide more information about their thinking rather than soliciting numerical responses. Suggested is the need to also assess students' questions and use of language during social constructivist activities. To develop greater understanding of students' thinking as they represented math problems, one response is reviewed.

Summer Camp activity begins at 5:00 a.m. and ends at 1:30 p.m. Calculate how much time this is?



As illustrated in this response, an outline of the worked example provides a clearer representation of students' interpretation than a numerical response of 8 hours 30 minutes. This illustrates a move away from numerical responses to analyzing why certain processes provide vital information on students' thinking, strengths or needs. While there is need to interpret students' response to learning, a move towards alternative forms of assessment does not imply that teachers become less engaged or accountable for students' learning; instead, learning and assessment become a shared process whereby students also assess and reflect on their own learning (Gipps, 1999).

While the information in this subsection denotes that assessing students' mathematical responses provide in-depth information on performance, according to Herman et al., (1992), it is not easy to identify alternative forms of assessment. Firstly, more time is required to assess why a solution was used or how a problem was solved (Perkins, 1993). This means that unlike traditional responses where right responses are ticked and tabulated, alternative forms of assessment require that:

1. Teachers help students understand from the outset the standards by which their work will be judged.
2. Students document their progress for the duration of the project or unit.
3. Through performance and feedback, students come to understand the complex nature of judging and improving one's work

Simmons, 1994:124

This denotes that alternative assessment requires greater participation and suitable means to assess performance (Black and William, 2003).

9.4.2 Analysis of the use of Post Test 1 and 2 to measure performance in Social Constructivist learning groups

Having reviewed varying perspectives on assessment and testing, this subsection examines the appropriateness of Post test 1 and 2 to assess knowledge construction in social learning groups. To ascertain whether these tests were suitable to assess performance during constructivist-based instruction, this subsection begins with a review of the reliability of the measures and tests used. Also examined in this subsection is the relevance of reliable and valid tests to assess inquiry-based instruction in mathematics. Prior to discussing the reliability of Post test 1 and 2, worth noting is that the term is often described as:

“the degree to which test scores for a group of test takers are consistent over repeated applications of a measurement procedure and hence are inferred to be dependable and repeatable for an individual test taker” (Berkowitz, Wolkowitz, Fitch, and Kopriva, 2000).

Firstly, the phrase, “inferred to be dependable and repeatable for an individual test taker” suggests that reliable tests consistently provide meaningful results. Therefore, as noted in Chapter Three, Post test 1 and 2 were tested with students of similar background and age resulting in an internal consistency of 0.89 and 0.93 respectively. While an internal consistency of 0.89 and 0.93 denote that these tests can provide reliable results, all that this illustrates is that Post test 1 and 2 can reliably assess the achievement scores of grade five students in urban and rural, Belize District, Belize C.A. This perspective warrants response to two critical questions. The first is whether Post Test 1 and 2 are suitable to assess students’ interactions within social constructivist learning groups or to assess a more traditional approach to teaching mathematics? The prevailing question is whether Post test 1 and 2 can be used to assess students’ ability to think critically and abstract the important aspects or to routinely solve problems? In essence, are these tests valid? In this regard, the American Psychological Association notes that valid tests are important since they focus on “the degree to which evidence and theory support the interpretations of test scores entailed in the purpose and use of tests” (A.P.A. 1999). Implied is that if the tests were designed to measure performance, then it represents a valid assessment for that purpose only.

While the previous paragraph indicates that validity refers to appropriate test use, as described in Chapter Three, Pre-test and Post test 1 and 2 were reviewed for content validity. This means that these tests were reviewed to ensure alignment to Belize’s National Curriculum and the national educational goals governing the teaching of Mathematics in Belize. While it is apparent that efforts were made to ensure that these tests were valid, Gay et al. posit that “a test cannot accurately reflect student’s achievement

if it does not measure what the student was taught and was supposed to learn (2000: 163). Restated is that if Post test 1 and 2 were intended to assess the number of right and wrong responses, it is a valid test for this purpose only. Although it appears that validity has to do with verifying whether tests served their intended purpose, “it is often impossible to prove that a test is really measuring what it is intended” (Messick, 1989; Kane, 2006). In fact, “there is no formula by which it can be computed and there is no way to express it quantitatively” (Gay et al., 2000:164). Therefore, validity is often determined by “expert judgement” or by test constructors. Despite these assertions, a critical point is that tests should be aligned to instructional practices and expectations. In short, examinations should reflect students’ experiences (Apple, 1993). The validity of Post tests 1 and 2 to assess the experimental and control groups’ performance highlights several critical points. One is that students must be assessed to examine the extent to which they constructed understanding in social learning groups. Explicit in this view is that while Post-test 1 and 2 provided information on whether scores improved, relevant assessment such as observing students’ interactions and conducting interviews to identify whether they constructed understanding are more valid. This signals that when assessing students in constructivist-based classroom, rather than saying, “No” when a student does not give the exact answer being sought, the constructivist teacher attempts to understand the student's current thinking about the topic” (Brooks and Brooks, 1993). Thus, in constructivist-based classrooms, teachers are to not only seek numerical responses but also assess students’ interpretations of mathematical facts (Neimeyer & Raskin, 2000). Also implied in these perspectives is that if traditional assessment is used to tabulate improvement scores, this must be supplemented with valid assessment.

Notwithstanding that a combination of alternative and traditional assessment appear to be more suitable, students are often required to sit examinations that are weakly aligned to the school curriculum or instructional practices (Apple, 1993). Suggested is that the assessment experience for many students is still based on a behaviourist approach where discrete facts and skills are tested, and grading and ranking are the primary goals (Niss, 1993). This means that students often sit examinations that are perceived to be “reliable and valid” (Apple, 1993). Clearly, test makers fail to understand that, "right answers don't necessarily signal understanding and wrong answers don't necessarily signal the absence of understanding" (Kohn, 2000). Restated is that a test which seeks right or wrong responses is unsuited to assess understanding in social learning groups.

9.4.3 Implications for using alternative measure to assess understanding in Social Constructivists Learning Groups

Based on the perspective that traditional tests are unsuitable to assess mathematical understanding, resoundingly clear is that knowledge construction cannot be measured directly (Hiebert and Carpenter, 1992). Therefore, alternative assessments, which refer to assessing students' abilities using real life tasks, are considered to be more suited to assess whether students constructed understanding in social constructivist learning groups. Given that alternative assessment can provide in-depth information on how students construct understanding, in this subsection, the importance of relevant tests and alternatives means of assessment are discussed and summarized.

An important starting point to discussing the relevance of tests and alternative assessment has been noted by the Curriculum and Evaluation Standards for School Mathematics (NCTM 1989), which posits that besides using traditional paper and pencil computation in mathematics, students should be exposed to other forms of instruction to assess critical thinking skills. Consequently, assessment in mathematics requires more than applying formulas or arbitrarily solving problems; instead, students should become exposed to innovative and meaningful assessment (Wong, 1991). Parallel to this view is that students possess a wealth of background knowledge and experiences which can be integrated in the teaching, learning and assessment process (Jones & Brader-Araje, 2002). In addition to meaningful alternative assessment, “the mathematical knowledge being assessed, the characteristics of the individuals or groups who are to respond, and the purpose for assessment must be in alignment” (Webb, 1992: 668). In essence, if students are taught using a constructivist approach, the assessment should reflect the manner in which students were taught. Therefore, assessment should reflect the purpose and objectives of instruction (Thompson & Briars, 1989).

As noted in the above, the resounding view is that if Post test 1 and 2 were unsuited to assess students who were instructed using a constructivist-based instruction, alternative measures must be used if insights are to be obtained on how they interpreted and solved problems (Boritch and Tombari, 1997). The importance of identifying alternative assessment was noted in a study conducted with 80 teachers in Singapore, which illustrated that both teachers and students benefitted from innovative assessment. This indicates that alternative assessment enabled teachers to accurately assess students’ needs. Secondly, students demonstrated a more positive disposition about mathematics (Fan et al., 2000).

Based on the belief that alternative assessment can provide deep insights on students' thinking, three alternative measures to assess knowledge construction are discussed.

Firstly, proponents of alternative means of assessment proposed that there are several ways to assess students' thinking (Adams, 1998). One viable option is observing students' interactions and recording specific skills and behaviours. For example, as noted in Chapters Eight, in addition to using Post test 1 and 2 to identify whether performance improved, students' interactions were observed, videoed and transcribed. While direct observations provided insights on how well the experimental groups interpreted and solved word problems, both teachers and students conceded that this process was time consuming. This aligns to a study with 10 secondary school teachers in Turkey which revealed that observations were time consuming and there is need for trained personnel to observe and document students' progress (Aksu, 2008). Therefore, while teachers are required to use alternative means of assessment, they must be skilled and supported to do so.

Although observations can be used to document progress, another viable option is using portfolios to observe, record, and analyze performance. This includes compiling and continuously reviewing worksheets and other resources to interpret the value of students' work (Adams, 1998). While portfolios can be used to examine students' progress, a study of the use of portfolios conducted with 206 special education students revealed that success was enhanced when students reflected on their progress and consciously worked to improve weaknesses. While students benefited from continuously reflecting and developing mastery, teachers complained that this process was time consuming and administrators provided limited support (Kampher et al., (2001). This shows that even as

efforts are made to use alternative assessment, difficulties such as limited expertise, training and institutional support affect implementation.

Besides observations and portfolio, strengths and weaknesses can be documented in journals to provide vivid accounts of students' mathematical understanding (Adams, 1998). In addition to written records of strengths and weaknesses, students are also required to solve problems to demonstrate conceptual understanding of mathematical facts (Chapman, 1990). The use of journals to provide explicit information on students' progress has been described in research with more than 20 students in an advanced mathematical course which revealed that reflective journals enabled students to describe how prior knowledge was used to interpret math concepts (White, 1999). This illustrates that while journals are often used in other disciplines, it can be used to provide a reflective account of students' competencies. This aligns to a review of a teacher education program at South Eastern University which showed that reflective journals served as a permanent record of students' thoughts and progress and allowed teachers to interpret students' thinking and provide individual support (Spaulding and Wilson, 2002). Two main points are noted in these studies. The first is that alternative assessment provides valuable information on students' understanding. Also restated is that innovative measures are time consuming and teachers must be skilled to develop and use alternative assessment.

While the scope of this study does not allow for an in depth discussion of the other alternative assessments, self assessment provides opportunities for students to reflect on their progress and rubrics can be also used to assess specific areas of mastery. That is, as described in Chapters Eight, while observations and interviews provided insights on

students' thinking, a number of other viable options exist. Even as it has been acknowledged that students should be assessed using alternative forms of assessment, there is the belief that this can be supplemented by traditional tests. Therefore, while traditional tests and alternative assessment serve specific purposes, there are instances when a combination of both can provide deeper insights on students' performance and abilities (Mueller, 2008).

9.5 Summary

Two main points were discussed in this chapter. The first is that one step problems did not foster restructuring of one's thinking or application of mathematical skills. Hence, students must be exposed to relevant problems and should be guided to gradually refine their mathematical skills. Secondly, reviewed in this chapter is the appropriateness of Post test 1 and 2 to assess students' thinking during social constructivist activities. To identify the importance of valid and relevant testing in mathematics, alternative means of assessment such as portfolios, journals and observations to assess knowledge construction were also described. Finally, discussed in this chapter is the importance of a combination of alternative and traditional tests and implications for practice.

Part Three

Summary, Conclusion and Recommendations

The second part of this thesis has analyzed the effects of immersing grade five students in social constructivist activities in Belize Central America. To assess the effects of social constructivist activities, teaching experiments by Steffe (1994), Simon (1995) and Steffe and D'Ambrosio's (1995) were analyzed and their main findings provided the framework for identifying factors which impacted on the students' interactions within social constructivist learning groups. To also provide a precise description of students' mathematical thinking, a number of research experiments utilizing Pirie and Kieren's (1994) model for growth of mathematical thinking were critiqued including the work of Martin, LaCroix and Frownes (2006), Droujikova, Berson, Slaten and Tombes (2005), Martin and LacCroix (2008), Pirie and Kieren (1996) and Martin, Towers and Pirie (2000). Having reviewed a range of research studies and using Pirie and Kieren's layer of understanding to assess students' mathematical thinking, the qualitative data were analyzed. Hence, in Chapter Ten of this study, the qualitative and quantitative findings, conclusion, and recommendations are presented.

Chapter 10

Summary, Conclusions, and Recommendations

10.1 Introduction

In this chapter, the main findings and implications for immersing students in social constructivist activities are summarized. Consequently, this chapter begins with a review of the purpose and the specific research questions. Thereafter, four main issues are summarized and compared to research in mathematics to identify factors which have impacted on the teaching experiment in Belize. Finally, limitations of the study, general recommendations, and areas for further research are identified to examine how best to implement additional teaching experiments in Belize.

10.2 Purpose of the Study

As stated in Chapter One, students in Belize, Central America, consistently performed poorly in mathematics word problems which has been noted in two National Examinations. The first is the Belize Junior Achievement Test (BJAT) administered to Grade Five students to assess their numeracy and literacy skills during the first five years of primary schooling. The 2000-2010 BJAT report revealed that less than 50% of all grade five students obtained passes in mathematics (Ministry of Education, 2010). While students are performing poorly at the mid stage of primary education, a similar trend has been observed on the Primary School Examination (PSE) administered at the end of elementary school in Belize. The PSE 2000-2005 report also revealed that less than 40% of all Grade Eight Students obtained passes in Mathematics (Ministry of Education, 2007). Therefore, to

examine whether immersing students in social constructivist activities resulted in greater gains among Grade Five students, the switching replication design was used to assess the effects of social constructivist activities on students assigned to the control and experimental groups. Thus, it was hoped that this study would provide insights into best practices for teaching word problems and the use of constructivist-based activities in mathematics.

10.3 Restatement of the Research Questions

To examine whether social constructivist activities could result in improved performance among Grade Five Students in Belize, this study was guided by two specific questions:

1. Will use of social constructivist activities assist students to interpret and identify solutions to mathematics word problems?
2. What are Grade Five students' experiences as they interpret, analyze, and solve mathematics word problems?

10.4 Main Issues and Recommendations

In this subsection, four major issues that impacted on the teaching experiment in Belize are described to identify factors which affected the extent to which students constructed understanding and developed mathematical skills. Additionally, recommendations for improving mathematical pedagogy to enhance further teaching experiments in Belize are described.

Issue 1:

Identified in the responses to the items in Pre-test and Post test 1 and 2 is that a number of students in this study lacked basic mathematical skills.

This observation aligns to Educational Reports in Belize which indicate that more than 70% of all grade five students are unable to obtain passes on the Junior Achievement Test administered to all grade five students in Belize (Ministry of Education, 2007-2010). Also identified in the quantitative data is that students who were taught using systematic guidelines from weeks 1 to 6 solved more problems than those in the experimental groups who were not provided with background mathematical skills and became immersed in social constructivist group activities.

Highlighted in this issue is that if students are to successfully interact with their peers to discuss, analyze, and extend their cognitive skills, students must have the prior knowledge and experiences to do so. The use of background skills as a “spring board” upon which thinking skills can develop aligns to the view that “all students must learn to think mathematically, and they must think mathematically to learn” (Kilpatrick, Swafford, Findell, 2001:221). Undoubtedly, if students are unaware of the underlying attributes of math problems, it is unlikely that they will accurately interpret, solve, or improve their thinking skills. While it is important to ensure that students acquire the requisite background skills, an ethnographic study by Boaler (1998) indicates that back-to-the-basics textbook approach did not yield high levels of performance when compared to continuous open-ended activities. Using research procedures such as interviews and quantitative assessment, Boaler also identified that those who were taught in a systematic manner responded in limited ways while those in an open environment solved various

types of problems. This illustrates that while a more structured approach provided students in Belize with “tools” to interpret the attributes of the problem and to engage in reflective thought, it is important that students also benefit from less structured teaching and learning activities.

Based on the above mentioned, it is recommended that:

1. In addition to reinforcing mathematical concepts using systematic guidelines, efforts are made to develop conceptual understanding. The need for conceptual understanding has been noted in numerous research studies which suggest that for students to accurately interpret word problems, they must be provided with opportunities to understand the language of mathematics and to become engaged in activities to improve skills (Webb and Romberg 1992; Wilson et al., 2004; Wilson, 1992). Teaching mathematics with more in-depth understanding of the principles of mathematics also encourages students to become competent problem solvers and transfer skills to a variety of situations (Grows and Cebulla, 2000; Schifter, 1991; Simon, 1995).

2. Suitable pedagogical approaches are identified to address the needs of learners. To be noted is that constructivism works well with motivated, high achieving students, but not as well with less motivated students who have trouble grasping things quickly and experience difficulty working with others (Hiebert and Carpenter, 1992). Therefore, a repertoire of instructional strategies must be employed to address students’ learning and motivational needs (Morse, 2005).

Issue 2:

Although some students identified solutions to math problems, they did not restructure their thinking or applied the newly acquired knowledge to solve novel problems.

As illustrated in Chapter Eight, to analyze the video recording of students' interactions in social learning groups, Pirie and Kieren's (1994) model of growth and understanding was used to assess students' thinking. Revealed is that some students' thinking illustrated the sixth layer; however, their thinking did not extend beyond the seventh and eighth layers. Identified is that use of one-step problems did not enable students to accommodate new information or to apply skills. While it is apparent that this teaching experiment did not enable students to think at Pirie and Kieren's seventh and eighth layers, it seems unlikely that in any given situation, this would occur. As Martin and LaCroix explain,

to trace the growth of understanding of a learner for a concept... (in the way offered by Pirie and Kieren's theory) would, in the apprenticeship classroom, be impossible (save a few rare cases where a session may focus on a particular concept).

2008: 125

Suggested is that although it is hoped that students' thinking would reflect all eight layers, this may occur through a series of lessons or follow-up teaching activities and not within a particular instructional period. Furthermore, given that students in Belize are often exposed to traditional type instruction and routine problems, changes in thinking may occur over an extended period of time (ibid.: 136).

Also worth noting is that teaching experiments by Droujikova, Bereson, Slaten, Tombes (2005), Martin La Croix and Frownes (2006), and Martin and La Croix (2008) did not illustrate thinking at all eight layers of Pirie and Kieren's (1994) model. Notwithstanding that a number of teaching experiments did not illustrate thinking across all layers, to improve pedagogical practices and mathematical skills in Belize, it is recommended that:

1. teachers are trained to identify and to expose students to activities to enhance their mathematical thinking skills. For example Lutiyya posits that

mathematical thinking involves using mathematically rich thinking skills to understand ideas, discover relationships among the ideas, draw or support conditions about the ideas and their relationships and solve problem involving the ideas.

1998: 55

Therefore, unless teachers are trained to engage students in meaningful learning tasks, it is unlikely that students will be guided by competent teachers to improve their mathematical skills.

2. Hence, it is recommended that effective pedagogical strategies are employed so that students can enhance their mathematical thinking and reflect on their thought processes as they tackle and solve challenging word problems. That is, "good problem solving calls for using efficiently what you know: if you don't have a good sense of what you know, you may find it difficult to be an efficient problem solver" (Schoenfeld, 1987:190).

Recommended is that in addition to ensuring that teachers are properly trained, students

should be guided to become reflective thinkers who can critically examine problem situations and devise strategies to solve.

Issue 3:

Teachers were unable to effectively scaffold students during social constructivist activities to critically reflect on problem situations and to deepen their understanding of mathematical concepts.

Revealed in the video transcripts depicting students' interactions during social constructivist activities is that although teachers were provided with training to use questions to stimulate thought and assist students to reflect on the critical attributes of math problems, they did not effectively scaffold students during social constructivist activities. Illustrated is that in instances where teachers lack pedagogical training or have not been exposed to social constructivist activities, it is likely that they will experience difficulty in scaffolding and promoting higher order thinking. Although teachers are required to effectively guide students to analyze math problems and to extend their thinking, Simon (1995) reminds us that this process is challenging. In fact, "teachers who take this path must work harder, concentrate more, and embrace larger pedagogical responsibilities than if they only assigned text chapters and seatwork" (Prawat: 1991:321). Even as it may be challenging for teachers to effectively scaffold students during social constructivist activities, it is recommended that teachers in Belize acquire skills to,

1. become reflective practitioners. For example, Simon (1995) describes that his ability to structure meaningful activities to improve mathematical skills resulted from continuous

reflections on classroom events and designing adequate learning tasks to bring about “perturbations” in students’ thinking. Suggested is that teachers in Belize must be cognizant of the need to reflect on students’ responses and their own instructional practices to identify how best to structure learning tasks to improve mathematical skills.

2. make informed decision about how to teach and which methodologies are most appropriate to promote knowledge construction (Carpenter and Fennema, 1991). As Yackel, Cobb and Wood (1991) and Ball (1987) have identified, students can significantly benefit from well planned and meaningful learning activities.

Issue 4:

Use of one-step problems limited the extent to which teachers guided students to restructure their thinking and to apply mathematical skills.

Described in Chapter 9 is that one-step math problems, which are often used in Belize to assess students’ mathematical thinking, require that students perform a single mathematical operation to solve (Yan et al., 2004). Additionally, these one-step math problems do not promote the development of critical reflection or provided the medium for students to extend their mathematical thinking (Trishman, Jay and Perkins, 1992). Hence, it is recommended that,

1. the mathematics curriculum and standards for teaching learning in Belize are revised to depict high quality mathematics activities aimed at fostering the development of critical thinking skills (Asku, 2008; Hiebert et al., 2003; Tomlinson, 1999).

2. students are exposed to relevant and meaningful problem types. For example, Bennett et al., (2002) identified in a study with 28 Grade Five children in New South Wales, Australia that when students were provided with challenging problems that were contextually meaningful, they were better able to interpret, discuss, and broaden their mathematical thinking skills.

10.5 Limitations of the Study

According to Gay et al., (2009), the limitation of a study refers to factors that the researcher is unable to control or those factors that may impact on the extent to which the findings can be effectively utilized. Therefore, limitations pertaining to the use of social constructivist activities are as follows:

1. This study may not be an accurate representation of the total population of Fifth Grade students in Belize, Central America, due to varying geographic and ethnic dimensions. The students in the control and experimental groups reside in the main metropolitan area of Belize, are from varied ethnic background and have greater access to educational resources than students in other parts of the country. Therefore, the results may differ if a similar study is conducted with students from different ethnic background or experiences who live in remote communities in other districts.
2. The one step problems used in this study were stipulated by the Ministry of Education in Belize as the problem types to be used to assess grade five students'

math performance. In fact, The Belize Junior Achievement Test (BJAT), which is a standardized examination to assess grade 5 students' mathematical skills, is made up of a number of one step math problems. Hence, the findings of the study have illustrated that use of math problems with a single operation did not foster thinking beyond Pirie and Kieren's sixth layer. This has been identified in students' responses to one-step math problems and the responses to the semi-structured interview guide. Therefore, while students are often exposed to routine problems, they need to be provided with more relevant problem types to foster higher levels of thinking and to enhance their mathematical skills.

3. Less than 50% of all primary teachers in Belize are untrained (Ministry of Education, 2009). A recent study on perceptions of effectiveness of primary teachers illustrated that more than 40% seldom or never attend professional development training (Flores-Alvarez, 2010). If teachers are not adequately trained or exposed to meaningful pedagogical practices, this will adversely affect the extent to which they teach or guide students during social constructivist activities. Hence, if these innovative approaches are to be successfully implemented in Belize, teachers must be adequately trained and provided with opportunities to practice, reflect on progress, and to scaffold students.
4. noted is that traditional forms of assessment such as Pre-test and Post test 1 and 2 provided limited information on students mathematical skills and competencies. In fact, these tests assess basic facts and are unsuited to analyze students' interactions in social learning groups. Hence, to assess students' thinking during social constructivist activities, self assessment instruments which allow students to

assess their progress and take responsibility for their own learning can be utilized. In this regard, Ross et al. (2000) noted that self assessment instrument enable students to reflect on strengths and weaknesses, set personal goals and be responsible for their own learning. Therefore, teachers need to be cognizant that reflecting on one's own progress promotes "participation, equality, inclusiveness and social justice" (Hargreaves & Fullan, 1998:14). Revealed is that in an era of change and development, assessment should be participatory thereby allowing students to assume active roles in assessing their own progress.

5. In addition to opportunities for students to assume a more participatory role in the assessment of their progress, it is recommended that alternative assessment which provides substantial feedback on students' strengths and weakness should also be utilized (Mertler, 2001; Nitko, 2001). While it may be time consuming to assess individual progress, teachers can use analytic rubrics to develop profiles of students' competencies and to modify instruction to address needs.

10.6 General Recommendations

Despite the limitations in the aforementioned, this study has provided meaningful insights into factors which can impact the use of social constructivist activities in mathematics. This includes ensuring that word problems are relevant and that students are engaged in activities to enhance mathematical competencies. Hence, in addition to the main issues

and recommendations in subsection 10.4, the following general recommendations are proposed to improve pedagogical practices and mathematical performance in Belize.

- a. Students need to be guided to read, interpret, analyze and solve various types of mathematical problems.
- b. Students must be provided with skills to effectively analyze non-routine problems.
- c. Students can benefit from more goal-oriented activities which promote higher order thinking.
- d. Students need to acquire skills and be guided to consciously apply newly formalized concept.
- e. Through continued professional development, more teachers should be sensitized about the strengths and limitations of social constructivist and procedural teaching activities.
- f. Teachers should be encouraged to assumed a more active and meaningful role during social constructivist activities.
- g. Teachers need to be supported as they become acquainted with and use innovative teaching and learning activities.
- h. There is need for suitable assessment to measure performance within social constructivist-based activities.

10.7 Recommendations for further study

The following recommendations for further study are proposed:

1. To examine the effects of social constructivist activities to teach math problems, this study should be replicated with students of diverse demographic profiles. That is, further research can be conducted with students in multi-grade schools to assess whether social interactivity and authentic resources can provide heterogeneous learning groups with relevant math skills.
2. Further study should be conducted to assess the extent to which social interactivity within constructivist learning groups can promote thinking beyond Pirie and Kieren's seventh and eighth layers. The need to examine the extent to which social constructivist learning groups promote thinking at Pirie and Kieren's seventh and eighth layers also stems from studies such as an ex-post facto research by Reid (1992) which identified that performance improved when Grade 7 students interacted in social learning groups; however, he did not assess whether social interactivity enabled students to restructure their thinking or to apply skills. Furthermore, studies by Droujikova, Bereson, Slaten, Tombes (2005), Martin La Croix and Frownes (2006), and Martin and La Croix (2008) did not illustrate thinking at all eight layers of Pirie and Kieren's (1994) layers of mathematical understanding. Hence, the need for more extensive research to identify the conditions required for students to engage in activities reflective of Pirie and Kieren's seventh and eighth layers.

3. While the twelve weeks study provided valuable information on the effects of social constructivist activities, it can be conducted over a longer time period of time. For example, studies to assess instructional effectiveness by Belz & Kinginger (2002) revealed that a four month bi-weekly meeting to assess the benefits of interactive study groups provide more meaningful results than studies conducted within shorter time spans. While additional resources, commitment from the researcher, and extensive tracking of students' progress is required for longitudinal studies, (Gay et al., 2009), a more extended study may provide deeper insights on whether goal directed constructivist activities can result in higher order thinking to improve mathematical competencies.

10.8 Summary

This chapter reviewed the purpose, research questions, the four main issues, limitations of the study, general recommendations and areas for further research. Revealed in this chapter is that while students can benefit from social constructivist activities, they must acquire the requisite background knowledge to critically analyze problem situations, apply mathematical skills, and develop changes in thinking. Further revealed is that these mathematical competencies do not occur by pure chance but must be supported by teachers who are skilled and have acquired the practical experiences and “know how” to effectively engage students in activities to enhance their cognitive thinking skills. Ultimately, appropriate learning context such as relevant math problems must be utilized to provide enriching and rewarding social constructivist experiences.

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APPENDIX A : ITEMS FOR PRE-TEST

Student Number: _____

Date: _____

1. Summer Camp activity begins at 5:00 a.m. and ends at 1:30 p.m. Calculate how much time this is?

2. Marla bought 3.5 of cucumber, 1.5 pounds of lettuce, 2.5 pounds of tomatoes, and 1.4 pounds of carrots. How many pounds of vegetables did she buy?

3. One loaf of bread makes 10 sandwiches. How many loaves do you need to make 54 sandwiches?

4. During the summer, your brother earns extra money mowing lawns. He mows 6 lawns in an hour and has 21 lawns to mow. How long will it take him?

Use the diagram below to solve numbers 5 and 6.





Movie Ticket Prices	
Before 6 p.m.	
Children under 12	\$ 5
Adults	\$ 10
After 6 p.m.	
Children under 12	\$ 7
Adults	\$ 15

Mr. and Mrs. Riley want to take their two children to the movies. Their children are 5 and 9 years old.

5. How much will it cost them to see a movie before 6:00 p.m.?

6. How much will it cost them to see a movie after 6:00 p.m.?

Use the drawings to solve numbers 7 and 8

<u>Dina's Toys</u>	
 \$1.99	 \$2.50 $\frac{1}{2}$ off
 \$3.20 $\frac{1}{2}$ off	 \$1.25

7. What is the cost of the duck and the jacks?

8. What is the cost of the top and the bat and ball?

APPENDIX B : ITEMS FOR POST TEST 1

Student number: _____

Date: _____

1. A bus leaves Belize City at 7:15 a.m. and arrives in Chetumal at 10:30 a.m. How far was the journey in time from Belize City to Chetumal?

2. A man bought the following food items at a store: 3.5 pounds of rice, 2 pounds of beans and 3.5 pounds of flour. How many pounds of items did he buy altogether?

3. A pack of ham has 6 slices. If you need 42 slices of ham, how many packs of ham should you purchase?

4. Each week you are allowed to play a video game for 3 hours. How many hours of game would you have played after 6 weeks?





Use the diagram below to solve numbers 5 and 6.

Movie Ticket Prices	
Before 6 p.m.	
Children under 12	\$ 5
Adults	\$ 10
After 6 p.m.	
Children under 12	\$ 7
Adults	\$ 15

5. 14 year old Matt is inviting his 10 year old brother to the movie before 6:00 p.m. How much will it cost them to see the movie?

6. After 6:00 p.m. Matt's mom and dad decided that they will go to the movie and invited their 7 year old daughter. What is the cost for the three persons?

Use the drawings to solve numbers 7 and 8

<u>Dina's Toys</u>	
 <p>\$1.95</p>	 <p>\$3.98</p> <p>$\frac{1}{2}$ off</p>
 <p>\$2.76</p> <p>$\frac{1}{2}$ off</p>	 <p>\$2.79</p>

7. What is the cost of the duck and the jacks and ball?

8. What is the cost of the top and bat and ball?

APPENDIX C: ITEMS FOR POST TEST 2

Student Number: _____

Date: _____

1. A man leaves his home in Belize City at 7:00 a.m. and arrives in Punta Gorda at 12:45 p.m. How long did it take him to travel from Belize City to Punta Gorda?

2. A farmer sold 5.5 pounds of potatoes, 3 pounds of onions, 7.5 pounds of beans and 12.25 pounds of cabbage. How many pounds of items did he sell in all?

3. A teacher needs 50 crayons for an art project. If each pack of crayons has 12 pieces, how many packs of crayons does she need?





4. During the summer, you earn extra money running errands. How many errands will you run in 9 hours if you run 4 errands per hour?

Use the diagram below to solve numbers 5 and 6.

Movie Ticket Prices	
Before 6 p.m	
Children under 12	\$ 5
Adults	\$ 10
After 6 p.m.	
Children under 12	\$ 7
Adults	\$ 15

5. Mrs. Barrow is taking 5 boys between the age of 8 to 11 to the movies before 6 p.m. How much will the ticket cost?
6. At 8 p.m., the Robinsons purchase tickets for two adults and a 4 year old. How much will the tickets cost?

Use the drawings to solve numbers 7 and 8

<u>Dina's Toys</u>	
 <p style="text-align: center;">\$1.69</p>	 <p style="text-align: center;">\$3.88</p> <p style="text-align: center;">½ off</p>
 <p style="text-align: center;">\$2.94</p> <p style="text-align: center;">½ off</p>	 <p style="text-align: center;">\$2.76</p>

7. What is the cost of the top and the jacks and ball?
8. What is the cost of the duck and bat and ball?

**APPENDIX D: INDEPENDENT SAMPLES T-TEST PRE-TEST AND
POST TEST 1 RESULTS**

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
PRETEST	Equal variances assumed	3.371	.068	5.692	240	.000	11.58	2.035	7.576	15.594
	Equal variances not assumed			5.575	204.944	.000	11.58	2.078	7.488	15.682
POST1	Equal variances assumed	11.535	.001	-1.531	240	.127	-2.49	1.627	-5.695	.715
	Equal variances not assumed			-1.583	239.835	.115	-2.49	1.573	-5.590	.609

APPENDIX E : RELIABILITY ANALYSIS PRE TEST

***** Method 2 (covariance matrix) will be used for this analysis *****

—

RELIABILITY ANALYSIS - SCALE (ALPHA)

N of Cases = 107.0

Item Means	Mean	Minimum	Maximum	Range	Max/Min	Variance
	1.5923	1.2991	1.9439	.6449	1.4964	.0705

Analysis of Variance

Source of Variation	Sum of Sq.	DF	Mean Square	F	Prob.
Between People	87.5841	106	.8263		
Within People	119.1250	749	.1590		
Between Measures	52.8213	7	7.5459	84.4455	.0000
Residual	66.3037	742	.0894		
Nonadditivity	9.9811	1	9.9811	131.3152	.0000
Balance	56.3226	741	.0760		
Total	206.7091	855	.2418		
Grand Mean	1.5923				

Tukey estimate of power to which observations
must be raised to achieve additivity = 3.1639

Reliability Coefficients 8 items

Alpha = .8919 Standardized item alpha = .8855

APPENDIX F: RELIABILITY ANALYSIS POST TEST 1

***** Method 2 (covariance matrix) will be used for this analysis *****

—

RELIABILITY ANALYSIS - SCALE (ALPHA)

N of Cases = 120.0

Item Means	Mean	Minimum	Maximum	Range	Max/Min	Variance
	1.4865	1.2000	1.8667	.6667	1.5556	.0722

Item Variances	Mean	Minimum	Maximum	Range	Max/Min	Variance
	.1882	.1165	.2515	.1349	2.1581	.0031

Analysis of Variance

Source of Variation	Sum of Sq.	DF	Mean Square	F	Prob.
Between People	105.6990	119	.8882		
Within People	134.1250	840	.1597		
Between Measures	60.6156	7	8.6594	98.1271	.0000
Residual	73.5094	833	.0882		
Nonadditivity	2.0839	1	2.0839	24.2739	.0000
Balance	71.4255	832	.0858		
Total	239.8240	959	.2501		
Grand Mean	1.4865				

Tukey estimate of power to which observations
must be raised to achieve additivity = 1.8306

Reliability Coefficients 8 items

Alpha = .9006 Standardized item alpha = .8976

APPENDIX G: RELIABILITY ANALYSIS POST TEST 2

***** Method 2 (covariance matrix) will be used for this analysis *****

—

RELIABILITY ANALYSIS - SCALE (ALPHA)

N of Cases = 112.0

Item Means	Mean	Minimum	Maximum	Range	Max/Min	Variance
	1.5815	1.3214	1.8304	.5089	1.3851	.0339

Item Variances	Mean	Minimum	Maximum	Range	Max/Min	Variance
	.2156	.1421	.2523	.1101	1.7748	.0017

Analysis of Variance

Source of Variation	Sum of Sq.	DF	Mean Square	F	Prob.
Between People	30.4275	111	1.1750		
Within People	87.6250	784	.1118		
Between Measures	26.5971	7	3.7996	48.3759	.0000
Residual	61.0279	777	.0785		
Nonadditivity	1.3514	1	1.3514	17.5722	.0000
Balance	59.6765	776	.0769		
Total	218.0525	895	.2436		
Grand Mean	1.5815				

Tukey estimate of power to which observations
must be raised to achieve additivity = 1.9343

Reliability Coefficients 8 items

Alpha = .9332 Standardized item alpha = .9320

APPENDIX H: PARENTAL CONSENT FORM

TITLE OF PROJECT: Analysis of the effects of a Constructivist Based Mathematics Problem Solving Instructional Program on the achievement of Grade Five Students

Note: The participant should complete the whole of this sheet himself/herself

*Please cross out
as necessary*

Have you read the Participant Information Sheet? YES / NO

Have you had an opportunity to ask questions and to discuss the study? YES / NO

Have you received satisfactory answers to all of your questions? YES / NO

Have you received enough information about the study? YES / NO

Do you agree for your child to be video taped for a period not exceeding 15 minutes YES/ NO

Who have you spoken to Mrs. Priscilla Brown Lopez? YES/ NO

Do you consent to participate in the study? YES/NO

Do you understand that you are free to withdraw from the study:

- * at any time and
 - * without having to give a reason for withdrawing and
 - * without affecting your position in the University?
- YES / NO

Signed **Date**

(NAME IN BLOCK LETTERS)

Signature of witness **Date**

(NAME IN BLOCK LETTERS)

APPENDIX I: LETTER TO PARENTS

Priscilla B. Lopez, M. Ed

P.O Box 1137,
Belize City, Belize



Tel: 501-227-0688 / 501-660-8247

Fax: 501-223-3941

www.ub.edu.bz

Email: plopez@ub.edu.bz

p.a.m.brown-lopez@durham.ac.uk

Faculty of Arts & Education

August 2007

Dear Parents/Guardian:

The information provided in letter and accompanying cover is presented in order to fulfil legal and ethical requirements in complying with the University of Durham, School of Education guidelines which ensures that all educational research are conducted within an ethic of respect for persons, knowledge, democratic values and quality of educational research

The main objective of this study is to identify whether the constructivist approach can be used to assist students to interpret and identify solutions to mathematical word problems. In so doing, the study will observe interaction skills within cooperative learning groups and the procedures used by grade five students as they solve mathematical word problems using any of the four mathematical operations.

Your child will be involved in this study by way of the following:

1. completion of a pretest on mathematical word problems.
2. engage in cooperative learning groups to solve mathematical word problems.
3. completion of two posttest on week 6 and 12
4. briefly share how they analyze and identify solutions to word problems

All of these activities will take place within the classroom under the guidance of the class teacher and researcher. These activities should not take more than two and a half hours per student. Please note that the parent or researcher may remove the student from the study with just cause.

Specific information about individual student will be kept strictly confidential but can obtainable from the school principal if desired. If the results are published, no reference will be made of individual students since only relations among groups of data will be analyzed.

The purpose of this form is to seek your permission to allow your child to participate in the study. Parental consent for this research study is strictly voluntary without due influence

or penalty. The parent signature below also assumes that the child understands and agrees to participate cooperatively.

If you have additional questions regarding the study, the rights of subjects or any potential problems, please call the principal, _____ or the researcher Mrs. Priscilla Lopez at 501-822-8240, Faculty of Education and Arts, University of Belize.

Yours in education,

Priscilla B. Lopez
Doctoral Student
University of Durham
England

APPENDIX J: LETTER TO THE CHIEF EDUCATION OFFICER

Priscilla B. Lopez, M. Ed

P.O Box 1137,
Belize City, Belize



Tel: 501-227-0688 / 501-660-8247

Fax: 501-223-3941

www.ub.edu.bz

Email: plopez@ub.edu.bz

p.a.m.brown-lopez@durham.ac.uk

Faculty of Arts & Education

March 5, 2007

Re: Permission to conduct Research Study

Dear Mrs. Maud Hyde,

I am a post graduate student currently enrolled in the Doctor of Education (EdD.) program at the University of Durham, England. Through this medium, I humbly request your permission to conduct a research entitled: “Analysis of the effects of a Constructivist-Based Mathematics Problem Solving Instructional Program on the Achievement of Fifth Grade Students”.

The duration of this study is from September to December 2007. During the initial phase, 8 teachers from 4 randomly selected schools in urban and rural Belize District will be provided with resources and receive specialized training on the use of the constructivist approach to teach mathematical word problems. Thereafter a pre test will be administered to students prior to the implementation of the program followed by a post test on week six and twelve to the experimental and control groups respectively.

Parental consent will also be sought from the parents/guardian for each Grade Five students in the four selected schools. The researcher will also ensure that the identities of the students are protected and that not more than two and a half hours is utilized to administer the pre and post test. All participants will also receive information

regarding the results of this study.

Finally, it is hoped that this research will provide teachers, the Ministry of Education, teacher trainers and teachers with guidelines for using interactive instructional approaches to improve performance.

If you have additional questions on the rights of subjects or potential problems please contact me at 501-227-0688 or email plopez@ub.edu.bz.

Thanks for your cooperation.

Sincerely,

Priscilla Brown Lopez (Mrs.)
Lecturer,
Faculty of Education,
University of Belize,

APPENDIX K: CHIEF EDUCATION OFFICER APPROVAL LETTER



Ministry of Education & L

To Educate! To Transform! To Liberate!

Ref: 10/05/18/07(81)

April 5, 2007

Priscilla Brown Lopez, M.Ed
Doctoral Student
School of Education
University of Durham
England

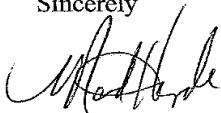
Dear Mrs Lopez:

I wish to acknowledge receipt of your letter requesting permission to collect data from eight schools in the Belize District.

Please be informed that approval is granted for you to undertake this project in the schools requested. Kindly inform the Local Manager and Principal of the nature of your visits and your intended schedule.

I wish you every success with your programme of studies

Sincerely



Maud Hyde (Ms)
Chief Education Officer

West Block, City of Belmopan
Belize
Central America

Tel: (501) 822-2329/2380,0385
Fax: (501) 822-3389
E-mail: moeducation@moes.gov.bz

APPENDIX L: LETTER FOR SCHOOL MANAGERS AND PRINCIPALS

Priscilla B. Lopez, M. Ed

P.O Box 1137,
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Faculty of Arts & Education

April 5, 2007

Angel Lane,
Belize City, Belize

Dear Sir/Madam:

I am a post graduate student currently enrolled in the Doctor of Education (EdD.) program at the University of Durham, England. Through this medium, I humbly request your permission to conduct a research entitled: “Analysis of the effects of a Constructivist-Based Mathematics Problem Solving Instructional Program on the Achievement of Fifth Grade Students” in _____ school.

The purpose of this research is to identify whether the constructivist approach is a viable instructional strategy to assist students to develop analytical skills and to improve their ability to solve two-step mathematical word problems. The duration of this study is from September to December 2007. During the initial phase, 8 teachers from 4 randomly selected schools in urban and rural Belize City will be provided with resources and receive specialized training on the use of the constructivist approach to teach mathematical word problems. Thereafter a pre test will be administered to students prior to the implementation of the program followed by a post test on week six and twelve to the experimental and control groups respectively.

Parental consent will also be sought from the parents/guardian for each Grade Five student in the four selected schools. The researcher will also ensure that the

identities of the students are protected and that not more than two and a half hours is utilized to administer the pre and post test. All participants will also receive information regarding the results of this study.

Finally, it is hoped that this research will provide teachers, the Ministry of Education and teacher trainers with guidelines for using interactive instructional approaches to improve performance in Math Problem Solving.

Please find attached a copy of a permission letter from the Chief Education Officer granting permission for this research to be conducted in Schools in Belize. **If you have additional questions on the rights of subjects or potential problems please contact me at 501-227-0688 or email plopez@ub.edu.bz.**

Thanks for your cooperation.

Sincerely yours,

Priscilla Brown Lopez (Mrs.)

cc: class teachers

APPENDIX M: CONSENT FORM FOR TEACHERS

TITLE OF PROJECT: Analysis of the effects of a Constructivist Based Mathematics Problem Solving Instructional Program on the achievement of Grade Five Students

Note: The participant should complete the whole of this sheet himself/herself

*Please cross out
as necessary*

Have you read the Information Letter sent to your manager and principal? YES / NO

Have you had an opportunity to ask questions and to discuss the study? YES / NO

Have you received satisfactory answers to all of your questions? YES / NO

Have you received enough information about the study? YES / NO

Do you agree to instruct students using the constructivist approach to Mathematical word problems during the duration of this research YES/ NO

Who have you spoken to Mrs. Priscilla Brown Lopez? YES/ NO

Do you consent to participate in the study? YES/NO

Do you understand that you are free to withdraw from the study:
* at any time and
* without having to give a reason for withdrawing and
* without affecting your position in the University? YES / NO

Signed **Date**

(NAME IN BLOCK LETTERS)

Signature of witness **Date**

(NAME IN BLOCK LETTERS)

APPENDIX N: STUDENTS' SEMI STRUCTURE INTERVIEW GUIDE

Research Question:

What are grade five students' experiences as they interpret, analyze and solve mathematical word problems?

Research Hypothesis:

Unlike regular instructional methods, the constructivist approach provides fifth graders with deep insights of successful experiences and difficulties as they sought solutions to Math word problems.

Interview Guide

1. What was it like working in groups discussing and solving math word problems?
2. What did you like most? Explain.
3. **Initial question:** What was difficult?
Modified question after pilot: What was not easy?
4. What can be done to help you to improve your problems solving skills?

APPENDIX O: TEACHERS' SEMI STRUCTURE INTERVIEW GUIDE

1. What do you do to help or assist students to solve math problems?
2. Do students benefit from group interactions? Why or why not?
3. To what extent were students able to interact with peers to solve problems?