Opportunities for Non-Routine Problem-Solving within a Teaching for Mastery Approach in Primary Mathematics

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Abstract

Non-routine problem-solving (NRPS) has long been recognised as a crucial component of mathematics education and has been associated with a wide range of benefits for pupils. However, educators invariably find this difficult to implement successfully in the mathematics classroom, partly due to a lack of understanding in the education world regarding what activities constitute a 'problem'. In recent years, the focus on NRPS in England has taken place in the context of the Teaching for Mastery (TfM) reform, which was 'borrowed' from nations with high mathematical outcomes in International large-scale assessments in hopes of replicating their success. This has come at a significant cost to UK taxpayers and has caused significant upheaval for schools, however, it is unclear from the current research base whether this has culminated in any substantial benefits for pupils. Furthermore, no research has examined opportunities for NRPS within a TfM approach.

This study aimed to contribute to this gap in the research by exploring opportunities for NRPS within a TfM-aligned programme (White Rose Maths [WRM]). A mixed methods approach involving content analysis was utilised, supplemented by a case study comprising interviews in one school in North-East England. The findings indicate that there are few opportunities for pupils to solve non-routine problems within the WRM materials, and where these are provided, teachers may 'scaffold away' challenges, rendering tasks fairly straightforward. The participants held incomplete understandings of NRPS and TfM, and also reported low fidelity to WRM materials in their school, reducing the opportunities for teaching with variation (as one aspect of a TfM approach).

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Statement of Copyright

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Dedication

This one's for you, Aunty Lynn. What I wouldn't give to have just one more car ride through to university together, chatting about maths, education, research, and life. I miss you every day.

Chapter 1: Introduction

1.1 Background

Problem-solving has been a key focus for research in mathematics education for over 50 years (Liljedahl & Cai, 2021), and the vast quantity of research conducted over this period has culminated in a shared belief that problem-solving is a crucial component of teaching and learning mathematics (Borthwick, 2019; Foster, 2015; Liljedahl & Cai, 2021; Liljedahl, Santos-Trigo, Malaspina & Bruder, 2016; Schoenfeld, 2014). As a result, it is not surprising that problem-solving has gradually attracted widespread global attention:

> *…problem solving has woven itself into curricula around the world both as a skill to be taught and a vehicle through which mathematics is learned.*
> *(Xu & Qi, 2022, p.723)*

Accordingly, the National Curriculum in England (Department for Education [DfE], 2013) holds problem-solving as one of three key aims of mathematics education. However, despite the high status accorded to problem-solving in the education world, there have long been difficulties in 'pinning down' what kinds of activities constitute a 'problem', especially since the term is often used indiscriminately to describe a myriad of different task types (Baumanns & Rott, 2022).

This dissertation focuses on opportunities for non-routine problem-solving (henceforth NRPS) specifically, which has been associated with a wide range of benefits for pupils, including (but not limited to): the development of a deeper understanding of mathematical concepts (Hiebert, Carpenter, Fennema, Fuson, Human, Murray, Olivier & Wearne, 1996; Hiebert & Grouws, 2007; Woodward, Beckmann, Driscoll, Franke, Herzig, Jitendra, Koedinger & Ogbuehi, 2012); improved mathematical attainment (Li, Liu, Zhang & Liu, 2020; Woodward et al., 2012); the development of problem-solving skills (Burkhardt & Schoenfeld, 2019; Merrienboer, 2013; Woodward et al., 2012); an array of affective benefits (Doherty, 2019; Hiebert et al., 1996; Rattan, Savani, Chugh & Dweck, 2015; Sun, 2018; Thente, 2019); and the capacity to foster mathematical creativity in pupils (Gregoire, 2016; James, Houston, Newton, Daniels, Morgan, Coho, Ruck & Lucas, 2019).

Whilst the importance of NRPS cannot be understated, schools and educators invariably find this critical aspect of mathematics education the most difficult to implement successfully in the classroom (Barmby, Bolden & Thompson, 2014; Ofsted, 2012; Xenofontos, 2014). Consequently, problem-solving is often cited as a key area for development in school development plans ([SDPs] Barmby et al., 2014). Furthermore, the focus on NRPS takes place in the context of significant change for teachers, brought about by a new education policy for mathematics. The Teaching for Mastery (henceforth TfM) initiative is based on elements of pedagogy from East-Asian regions including Shanghai, Singapore and Hong Kong (henceforth SSHK) and was heavily influenced by these nations' high mathematical achievement in the Organisation for Economic Co-operation and Development's (OECD) Programme for International Student Assessment (PISA).

Since its inception in England, the TfM initiative has received substantial government investment and has also come at a high cost in terms of time and resources for schools (Clapham & Vickers, 2018; Cole & Helme, 2022). However, this is not the first educational reform to be implemented in England (Pawlik, 2020), nor is it the first to be 'borrowed' from nations with higher attainment on international large-scale student assessments ([ILSAs] Auld & Morris, 2014, p.138). A plethora of research highlights significant problems inherent in large-scale educational reform and policy-borrowing (Aloisi & Tymms, 2017; Bolden & Tymms, 2020; Burner, 2018; Scott, Terano, Slee, Husbands & Wilkins, 2016), and despite numerous attempts at improving educational attainment through policy reform, this goal has proven incredibly difficult to achieve (Bolden & Tymms, 2020; Aloisi & Tymms, 2017). Furthermore, while *pedagogy* in SSHK has been credited with the high mathematical achievement of its pupils, many academics have argued that other factors outside of the education system may have a more profound influence on educational outcomes, such as contextual and cultural factors.

1.2 Research initiatives

Whilst TfM is increasingly used as an underpinning approach in primary mathematics classrooms (Marks, Barclay & Barnes, 2023), research studies exploring the effect of

TfM on mathematical achievement have reported mixed results, with large-scale studies yielding only small effects (for Key Stage 1 pupils only) and it is unclear which particular aspects of TfM are most effective at improving mathematical outcomes (Boylan, Maxwell, Wolstenholme, Jay & Demack, 2018). The limited research evidence available is presented in the literature review of this dissertation, however, there are significant gaps in the research, particularly with regard to NRPS. To date, no research studies (that this author is aware of) have examined NRPS activities within a TfM approach, despite the prominence of problem-solving in curricular documents (DfE, 2013) and the importance accorded to NRPS in the field of education research and the education community. This gap poses an opportunity to explore opportunities for NRPS within a TfM-aligned mathematics programme, to determine whether pupils can reap the many benefits associated with this crucial component of mathematics education.

Besides the lack of research focused on NRPS within a TfM approach, there is minimal documented research to date that explores teachers' perspectives about opportunities for NRPS within the TfM approach. Teachers and school leaders are at the forefront of educational reform as drivers of change in the classroom (Harries & Jones, 2019), therefore it is crucial that teacher voice is not overlooked when evaluating the effectiveness of large-scale changes.

This study used a mixed methods approach involving quantitative and qualitative content analysis of TfM-aligned teaching materials, supplemented by a case study comprising semi-structured interviews with decision-makers for mathematics in one school. Combining different methods allowed for a deeper understanding of the opportunities for NRPS within a TfM approach to be captured and described, to contribute towards several gaps in the research literature.

This dissertation begins with an extensive review of the research literature, which explores the existing evidence base surrounding problem-solving and TfM. Following this, the methods utilised to gather empirical data in three stages of the research are described, and justification for the use of mixed methods is provided. The results chapters are structured to present the findings of each stage of the research, after which a discussion of the key findings in relation to each of the three research questions is presented, and the discussion chapter concludes by outlining the key implications for future research.

Chapter 2: Problem-solving

Problem-solving has long been considered a crucial component in mathematics teaching and learning and has been the subject of education research studies and academic debate for several decades (English & Sriraman, 2010; Lester, 2013; Liljedahl & Cai, 2021; Liljedahl, et al., 2016; Posamentier & Krulik, 2015). In England, the national curriculum for mathematics (Department for Education [DfE], 2013) aims to ensure that all pupils:

> *…can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions. (p.3)*

Acknowledging the critical role of this aspect of mathematics, problem-solving is presented as one of three key aims for mathematics in the National Curriculum (DfE, 2013) alongside fluency and reasoning. However, it is widely acknowledged that the ultimate goal of mathematics education is to develop children's problem-solving proficiency (Foster, 2015). Therefore, fluency and reasoning should be viewed as a means of supporting children in achieving this crucial objective, rather than as ends in themselves (Foster, 2015). As Posamentier & Krulik (2015) argue, it is of little use knowing *how* to do something if you do not know *when* to do it. Put simply, the real point of doing mathematics is to become a mathematical problem solver (Borthwick, 2019; Foster, 2015), and when provided with rich and regular opportunities to develop problem-solving proficiency, children are simultaneously provided with,

> *…a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics and a sense of enjoyment and curiosity about the subject. (DfE, 2013).*

A plethora of research in the field of mathematics education has linked problem-solving with a wide range of additional benefits for children (see for example, Liljedahl et al., 2016; Schoenfeld, 2014), which will be discussed in detail later in this paper. The importance of problem-solving in primary mathematics education cannot be overstated and is reflected in several guidance reports from Ofsted (2012; 2021) and the Education Endowment Foundation ([EEF] Henderson, Hodgen, Foster &

Kuchemann, 2022). Furthermore, a common characteristic of schools which were identified as ‘outstanding’ by Ofsted in their good practice survey (2012) was that they provided ‘extensive’ opportunities for pupils to apply their mathematical learning to solve problems (p.34).

Whilst the critical nature of problem-solving in mathematics education is widely acknowledged, developing a shared understanding of what problem-solving actually entails has proven to be problematic. Although various definitions have been put forward over many decades, there is no universally accepted definition of a mathematical problem (English & Gainsburg, 2015; Sriraman & English, 2010; Lester, 2013). The wide variation between definitions (even within the sole domain of mathematics) can result in difficulties in distinguishing problem-solving from other learning experiences (Rhodes, 2019). The next section aims to evaluate some definitions which are pertinent to this study to identify some key features and operationalise a definition of problem-solving.

2.1 What is a problem?

Traditionally, definitions of problem-solving have focused on the idea that the route to a solution is blocked in some way (Lester, 2013; Polya, 1962; Posamantier, 2015; Schoenfeld, 1985). One of the most influential writers on the topic of problem-solving, research mathematician George Polya, explained that,

> *Solving a problem means finding a way out of a difficulty, a way around an obstacle, attaining an aim which was not immediately attainable. (1962 p.v)*

Problem-solving is somewhat unpredictable in that no fixed strategy can be readily drawn on to overcome these obstacles (Mulholland, 2021). Following logically from this, a problem can be defined as a task which engenders this situation for the solver. For example, Lester (2013) defines a problem as,

> *…a task for which an individual does not know (immediately) what to do to get an answer. (p. 247).*

In the absence of any obvious way to approach the problem, the solver is required to construct their own solution method (Barmby et al., 2014; English & Gainsburg, 2015; NCTM, 2000), and there are usually multiple possible approaches

(Posamantier, 2015). When children are exposed to such situations and actively involved in applying their mathematical knowledge and innovative thinking to solve problems, this provides opportunities for the development of problem-solving skills (Polya, 1973).

Another crucial aspect of problem-solving is the element of cognitive challenge: English & Gainsburg (2015) argue that a problem must engender ‘high-level thinking and reasoning’ for the individual engaging in a problem-solving task (p. 326), and the notion of challenge is also highlighted in the National Curriculum (DfE, 2013) stipulation that students must have opportunities to ‘persevere in seeking solutions’ (p. 3). Pupils experience a period of ‘productive struggle’ as they engage in problem-solving tasks (p.91), which is characterised by a period of perplexity that continues as pupils make sense of the problem and devise and deploy a strategy to solve it. Since each individual will have different levels of competence and previous experience to draw on (Barmby et al., 2014; Schoenfeld, 1985), this infers that what may be a problem for one student may not be a problem for another, which causes significant difficulties for educators. Whilst a problem must sufficiently challenge the solver, it is also important that it is accessible to them, in that they possess the prerequisite skills and knowledge to allow for a reasonable expectation of success in solving it (Leavy & O’Shea, 2011; Lester, 2013). As Leavy & O’Shea (2011) point out, problem-solving tasks must,

> *… build on knowledge that the learners already have while at the same time engaging and drawing on contexts and situations that are new to them. (p.9)*

This highlights the importance of the teacher’s role in selecting problems which will meet the above criteria for their particular cohort of pupils (Walsh, 2016).

The terms ‘problem’ and ‘non-routine problem’ (henceforth NRP) are often used interchangeably. However, a distinction must be made between these and ‘routine problems’, which differ significantly (Jader, Lithnerband & Sidenvall, 2020; Kolovou, 2011). A routine problem is a task for which there is an obvious solution procedure available (Sánchez-Barbero, Chamoso, Vicente & Rosales, 2020; Kolovou, 2011). This may be obvious from some characteristic of the specific task, or because it is presented after a solution template has already been provided (Jader et al., 2020

Lithnerband & Sidenvall,2020). Routine problems are therefore only useful in providing the student with opportunities to practise a learned mathematical procedure or technique and do not require the high-level of cognition needed for problem-solving (English & Gainsburg, 2015). The emphasis in routine problems is on imitation and memorisation to produce the correct answer, but it is the *process* by which an individual arrives at an answer which is the focus of NRPS (Posamantier, 2015). Exposure to routine problems, therefore, does not develop problem-solving skills or competence (Schoenfeld, 1985). As Drury (2019) points out,

> *...once a problem becomes 'routine', it ceases to be a problem at all. (p.2)*

In this dissertation, the term 'problem-solving' is used to delineate NRPS activities as described above, and the term 'problem' is used to refer to NRPs specifically, as opposed to routine tasks.

Having outlined the important features of non-routine problems (NRPs), the next section draws on the seminal works of mathematician, George Polya (1973), who proposed a four-step problem-solving process to support individuals as they engage with and attempt to solve a mathematical problem. Despite the age of Polya's work, the steps that he described continue to be used as a key model to represent the problem-solving process in current research (Rott, 2012). Furthermore, this model can be used to support understanding of the stages and thought processes involved in finding solutions to mathematical problems (Siswono, Kohar & Hartono, 2019).

2.2 The problem-solving process

Much of the research in mathematical problem-solving is based on the seminal works of Polya (1973), who outlined a linear, four-stage process through which problem solvers attempt to understand the problem, devise a plan to solve it, implement the plan, and finally reflect on the process. Each one of these stages is described below:

Understand the problem - In the first stage, the solver extracts the important information to decide what needs to be done to find the unknowns and solve the problem. The solver may interpret the problem by representing it in a different format, perhaps by drawing a picture or diagram or rephrasing the problem.

Devise a plan - In the second stage, the solver decides on an approach or strategy for solving the problem using the information they have found in the first stage. Polya (1973) put forward a range of problem-solving strategies which can be drawn upon in different problem-solving scenarios, such as draw a picture/diagram, make a list, solve smaller versions of the problem, look for a pattern, guess and check, and work backwards. Children should be encouraged to draw on strategies that have been effective for them in solving similar problems. Since students' skills in choosing appropriate strategies are developed most effectively through experiences in problem-solving, it follows that children need to be exposed to many different problems to allow them to become increasingly proficient in selecting appropriate strategies. As their bank of prior experiences increases, children's autonomy in choosing a suitable strategy will increase.

Implement the plan - The solver then attempts to find a solution using their plan from the previous stage, checking each step carefully and refining the plan if needed.

Look back - Finally, in the looking-back stage, the solver reflects on whether their answer is reasonable, explores alternative strategies, considers whether the same approach could be applied to a different type of problem (O'Shea, 2010), and poses new or related problems. Polya (1957) argued that good teachers should adopt a viewpoint that a problem is never completely exhausted, even when a solution procedure has been found, as there are always opportunities to improve upon a solution or develop a deeper understanding of the solution procedure.

These four stages form Polya's (1973) heuristic which can be used to help pupils become efficient mathematical problem solvers. According to Polya (1973), individuals who are given opportunities to practise solving a variety of appropriate problems using the heuristic model will internalise these skills, and this will allow them to solve new, unfamiliar problems independently (Polya, 1973).

Over the years, different approaches based on Polya's (1973) heuristic problem-solving model have been put forward (eg. Borgersen, 1994; Mason, Burton & Stacey, 1982; OECD, 2014a; Schoenfeld, 1985; Wilson & Clarke, 2004), with many placing greater focus on self-monitoring and regulation during problem-solving activities (eg. Mason et al., 1982; Schoenfeld, 1987; Wilson & Clarke, 2004). For example, Schoenfeld's (1985) model of the problem-solving process (see Figure 1) included an additional element of exploration linked with Polya's (1973) 'devise a plan' phase (labelled design/planning in Schoenfeld's [1985] model), which represents how problem-solvers go through cycles of analysis, exploration and planning until they identify a feasible plan to implement, exerting control over their decisions to ensure their actions are beneficial in solving the problem.

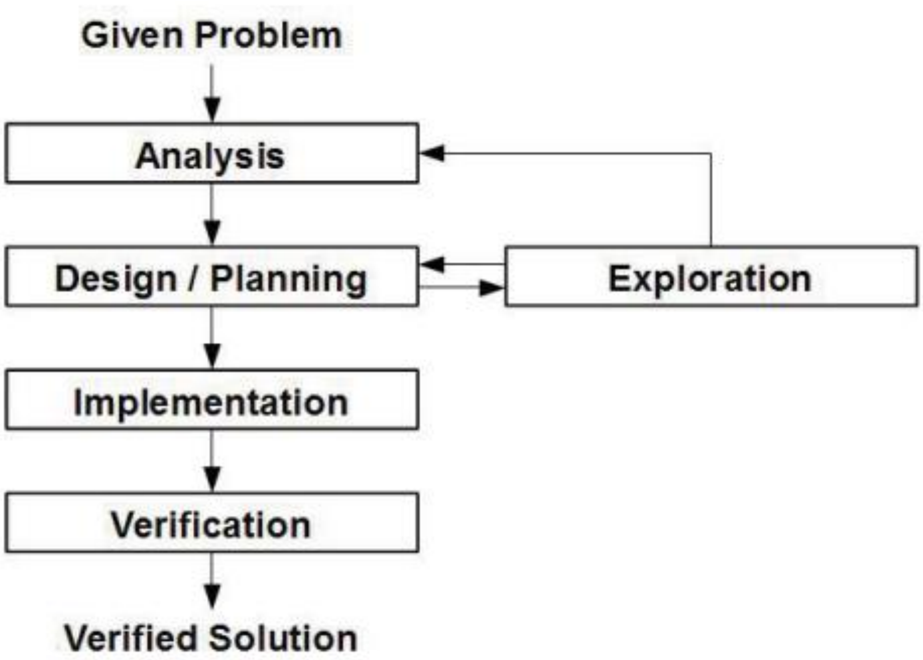


Figure 1: *Schoenfeld's (1985) model of the problem-solving process (reprinted from Rott [2012] p.97)*

Mason et al. (1985) also emphasised the importance of self-monitoring and regulation in the problem-solving process. In their model (see Figure 2), they described a three-stage process of Entry, Attack and Review: the Entry phase focuses on accessing the problem through questions such as 'what do I know?'/ 'what do I want?'; the Attack phase involves taking different approaches and making plans in order to find a solution; and (after a solution has been found) the Review phase incorporates critical opportunities for reflection on their key ideas and extension of these ideas to a wider context (Rott, 2012,p.99). Mason et al. (1985) asserted that the period of reflection on thought processes and solution strategies is the most important element because this is when the most learning takes place.

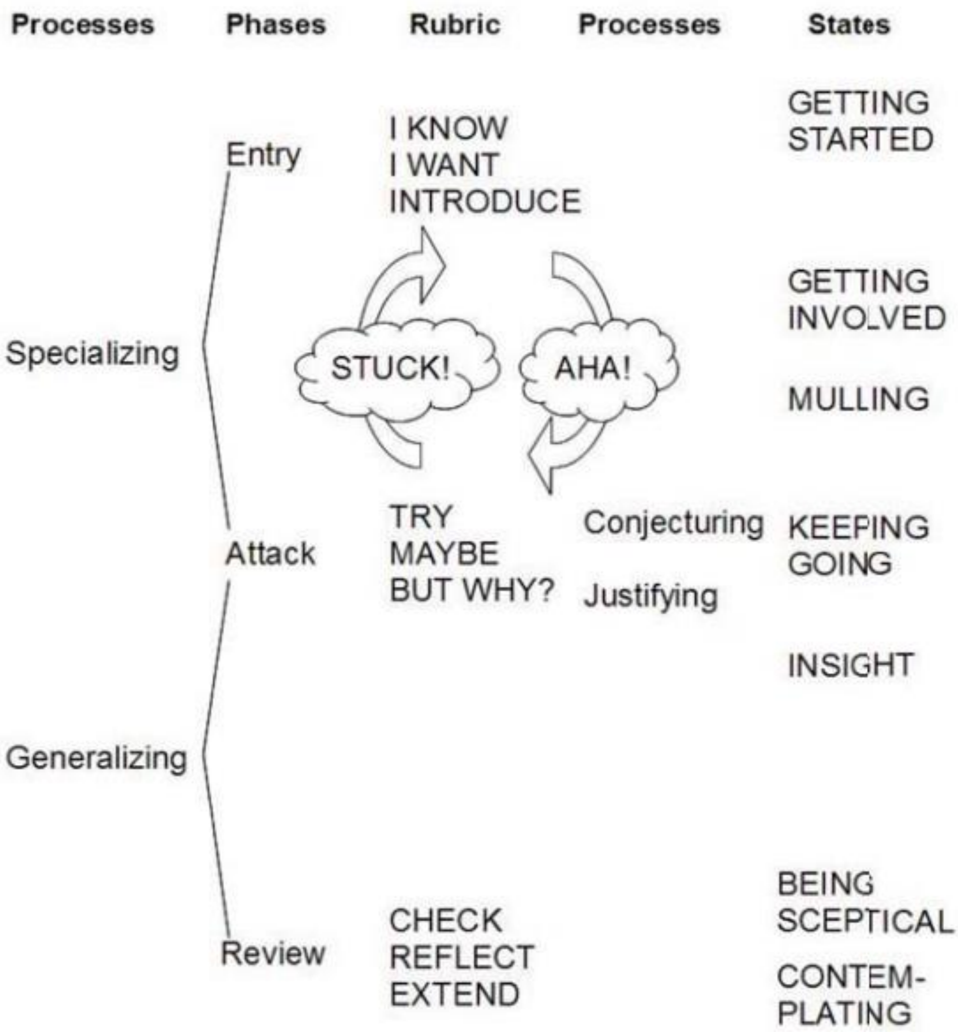


Figure 2: Mason, Burton & Stacey's (1982) model of the problem-solving process (reprinted from Rott [2012] p.99)

Although there are many similarities between the models put forward by Polya (1972), Shoenfeld (1985) and Mason et al. (1982), the latter two do not follow the linear structure of Polya's (1973) model, and place greater importance on activities which encourage self-regulatory behaviours.

This section has explored problem-solving as a process, drawing on the seminal works of Polya (1973), as well as other models of the problem-solving process (Mason et al., 1982; Schoenfeld, 1985). However, the research literature indicates that the explicit teaching of heuristics is not sufficient to develop pupils' problem-solving ability and that children may require a greater range of skills to experience success in problem-solving activities (Schoenfeld, 1985, Sweller, Clark & Kirschner, 2010). One skill which has been closely associated with problem-solving proficiency at the primary level is metacognition, which can be linked with the self-regulatory behaviours emphasised by Schoenfeld (1985) and Mason et al. (1982). Therefore, the next section will explore the importance of metacognitive processes in the problem-solving process.

2.3 The importance of metacognition

Metacognition is often simply described as ‘thinking about thinking’ or ‘learning to learn’ (Quigley, Muijs & Stringer, 2018, p.4), however, this definition fails to capture the complex nature of the construct and its crucial role in problem-solving situations. The term was first coined by American psychologist John H. Flavell in the 1970s, who defined metacognition as:

> *One’s knowledge concerning one’s own cognitive processes and products or anything related to them … [and] refers, among other things, to the active monitoring and consequent regulation and orchestration of these processes … usually in the service of some concrete goal or objective.* (*1976, p. 232*).

This definition highlights two important dimensions of metacognition: metacognitive knowledge refers to an individual’s knowledge of their own cognitive capacity, their knowledge and experience with a particular task type, and appropriate strategies available to them; and metacognitive regulation describes how learners monitor and control their cognitive processes as they engage with a task.

Metacognition, therefore, involves active monitoring of cognition and conscious decision-making to direct learning, and this involves phases of planning, monitoring and evaluating (Flavell, 1979; Schoenfeld, 1987). In the planning phase, learners consider how they will approach the task and select appropriate strategies, drawing on prior knowledge of similar tasks and effective strategies they have used previously. The monitoring phase involves the learner monitoring their progress as they put their plan into action, refining and making changes to the plan where necessary. During the evaluation phase, the learner appraises their strategy in terms of how effective it was in helping them achieve their goal. Each of these phases occurs in an ongoing cycle, with a core component of reflection throughout, whereby students keep checks on their own decisions and strategies as they progress towards the intended goal.

In exploring the metacognition phases, parallels can be drawn with Polya's heuristics in the problem-solving process, such as devising a plan and evaluating or 'looking back' (Sevgi & Karakaya, 2020). When the phases of metacognition are considered in the context of mathematical problem-solving, this could involve students drawing on their previous problem-solving experiences and strategies to help them to understand the problem and make strategic choices based on this knowledge. They may monitor and make judgements about the appropriateness and efficiency of their ideas and strategies as they work towards a solution, identifying the next steps and adapting their strategy where necessary (Clark, 1998). In the evaluation phase, students may look back over their solution process and review its effectiveness in solving the problem, thinking about whether they chose the most efficient strategy and if this strategy could be useful in solving other problems (Sevgi & Karakaya, 2020, p.262). Barmby et al. (2014) suggest that this act of reflection involves 'problematising' the results of an activity; whereby students develop a deeper understanding by searching for problems with their approach and methods (p.52).

Metacognitive strategies are therefore particularly useful to children during problem-solving activities since a wide range of metacognitive processes is needed during the problem-solving process (Kuzle, 2018). Several studies in the research literature suggest a relationship between metacognitive skills and problem-solving achievement (Divrik, Pilten & Tas, 2020; Hollingworth and McLoughlin, 2001; Kuzle, 2018; Quigley et al., 2018; Sevgi & Karakaya, 2020). Furthermore, while some research has indicated that children are unable to develop metacognitive skills before the age of 8 to 10 (Veenman & Spaans, 2005), more recent studies have proven that metacognitive strategies can be used to improve problem-solving performance at the primary level: the EEF guidance report on metacognition and self-regulation (Quigley et al., 2018) synthesised a wealth of international research to conclude that children as young as three demonstrate metacognitive behaviours such as goal-setting and checking their own understanding.

The EEF guidance report for metacognition (Quigley et al., 2018) included recommendations for educators to explicitly teach primary pupils effective strategies to plan, monitor and evaluate their own learning in relation to a specific task, which may involve the use of metacognitive questions at each stage (Quigley et al., 2018). For example, questions for the planning phase would aim to activate prior knowledge and memories of similar experiences to make strategic choices; monitoring questions would support children in determining whether their strategy needs refining/changing or whether it is working as they progress towards a solution; and evaluation questions would focus the student's attention on the efficiency of their chosen strategy, highlight alternate strategies and allow them to reflect on their performance in solving the problem. Additionally, Mulholland (2022) recommends that primary teachers should model metacognition through a 'Think Aloud' strategy (para 6), whereby they describe their own metacognitive processes to demonstrate how an expert problem-solver tackles a new problem.

The EEF guidance report on metacognition (Quigley et al., 2018) highlighted the importance of ensuring pupils are adequately challenged (at an appropriate level) and that purposeful classroom dialogue is utilised to develop metacognitive skills (Quigley et al., 2018). Additionally, Schneider & Artelt (2010) argued that pupils must acquire knowledge about a range of different tasks and efficient strategies for coping with these types of tasks to develop 'executive skills' (p.1), which support children in monitoring their cognitive activities. This suggests that children should be exposed to a wide range of different types of problems so that they have a wealth of experience to draw on in a new problem-solving situation, allowing them to make appropriate choices based on their prior experiences.

This section has explored the role of metacognition in supporting children in mathematical problem-solving situations. The use of metacognitive strategies to support problem-solving in the primary classroom would necessitate exposing children to a wide range of problem-solving experiences with different types of NRPs. The next section explores different types of problems which are relevant to the primary mathematics classroom and can be used to develop problem-solving abilities in primary pupils.

2.4 Types of problems

In a recent review based on literature relating to the field of mathematics education, Ofsted (2021) suggested that it is not sufficient to teach pupils problem-solving strategies (such as guess and check or work backwards) without also teaching pupils to recognise the different *types* of problem and the specific strategies that are appropriate in solving each problem type. As part of the Primary National Strategy, the Department for Education and Skills ([DfES] 2004) outlined five different types of problems relevant to primary schools, which are briefly described below with examples drawn largely from the 'NRICH' resources developed by the University of Cambridge (2023).

2.4.1 Finding all possibilities problems

Finding all possibilities problems (henceforth FAP) aim to provide opportunities for pupils to work in a "methodical and efficient way" (Woodham, 2018, para 2) which is often referred to as 'working systematically' (para. 2). For example, children might use a systematic approach by working in ascending or descending order and using a list or a table to ensure answers are not repeated and to help them identify when they have found all possible answers. An example of this type of problem can be seen in Figure 3, which requires children to find what the half-time score of a hockey match could have been when the final score is given. Developing children's skills in working systematically through the use of FAP problems is also thought to be beneficial to pupils in structuring effective methods to tackle other problem types (Barmby et al., 2014; Woodham, 2018).

Half Time

Age 5 to 11
Challenge Level ★

When Spain played Belgium in the preliminary round of the men's hockey competition in the 2008 Olympics, the final score was 4 - 2.

What could the half time score have been?
Can you find all the possible half time scores?
How will you make sure you don't miss any out?

Figure 3: *AN FAP problem (University of Cambridge, 2023)*

2.4.2 Logic problems

Logic problems (henceforth LG) are used to develop children's skills of deductive reasoning. As explained by Barmby et al. (2014), this involves a sequential approach whereby "…each step and decision follows logically from the previous step" (p. 11). An example of an LG problem can be seen in Figure 4, which involves arranging digits in multiplication squares so that the given products are correct. To solve this problem, pupils must draw upon their knowledge of common factors to support their deductive reasoning. For example, students might recognise that the only factor combination for 15 is 1, 3 and 5 and the only factor combination for 8 is 1, 2 and 4, therefore the 1 must be placed in the top middle column. This might lead students to place the 5 in the top right column since 5 is a factor of 315 but not 144. In this way, children work logically towards further deductions to complete the multiplication square.

Multiplication Squares

Age 7 to 11
Challenge Level ★

In the 2×2 multiplication square below, the boxes at the end of each row and the foot of each column give the result of multiplying the two numbers in that row or column.

7	5	35
3	4	12
21	20	

The 3×3 multiplication square below works in the same way. The boxes at the end of each row and the foot of each column give the result of multiplying the three numbers in that row or column.

			15
			108
			224
144	8	315	

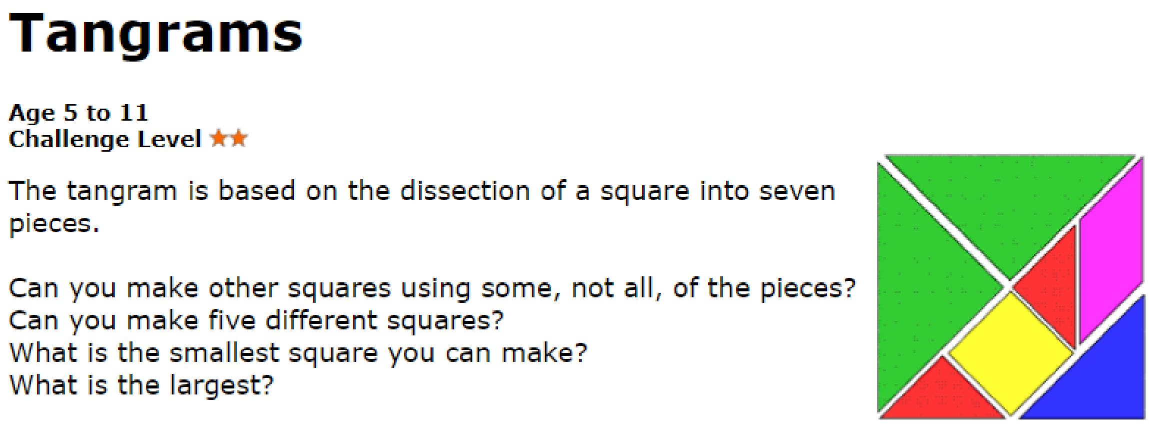
The numbers $1 - 9$ may be used once and once only.

Can you work out the arrangement of the digits in the square so that the given products are correct?

Figure 4: *An LG problem involving multiplication squares (University of Cambridge, 2023)*

2.4.3 Diagrams/Visual problems

Diagrams and visual problems (henceforth DV) are presented visually, often without words and sometimes without mathematical symbols, with the majority of the information presented as a diagram or visual representation (Barmby et al., 2014). An example of a visual problem, taken from the NRICH website, can be found in Figure 5, which requires the solver to use their knowledge of squares and use visualisation skills to support their attempts in making new squares of varying sizes.



Tangrams

Age 5 to 11
Challenge Level ★★

The tangram is based on the dissection of a square into seven pieces.

Can you make other squares using some, not all, of the pieces?
Can you make five different squares?
What is the smallest square you can make?
What is the largest?

Figure 5: *A visual problem involving tangrams (University of Cambridge, 2023)*

2.4.4 Finding rules and describing patterns problems

Barmby et al. (2014) argue that finding rules and describing patterns problems (henceforth RP) are crucial for developing children's ability to make generalisations in mathematics and use inductive and deductive reasoning. They use the example of an exploration into what happens when any two odd numbers are added together. Starting with specific examples, children may surmise that since the answers they have found in their calculations are always even, two odd numbers will always sum to an even number. This movement from the particular case towards forming wider conclusions and generalisations about mathematical rules is known as inductive reasoning, which may not be wholly accurate but should provide a coherent argument based on specific examples (Simon, 1996). Following this, pupils can use deductive reasoning to prove their argument or prediction is mathematically sound (Barmby et al., 2014).

2.4.5 Word problems

Mathematical word problems (henceforth W problems) are commonly defined as problem situations given in a verbal format, where the answer can be obtained via the application of a mathematical operation/s (Verschafel, Greer, & De Corte, 2000, as cited by Verschafel, Schukajlow, Star & Van Dooren, 2020). A common misconception surrounding problem-solving in primary mathematics is that problems are always presented in a worded, rather than numerical, format. Descriptions of the various other problem-types given above demonstrate that this is not the case, and word problems are just one of many types of problem-solving activity. These come in various forms, such as single-step problems (which require only one calculation) and multi-step problems (which require more than one calculation). Closed word problems represent a kind of routine problem, as they require the simple application of a method learned previously (Barmby et al., 2014), while open-ended word problems have more than one solution and can be used to engender higher-order mathematical thinking due to an increased focus on mathematical relationships (Way, 2017). An example of this is given in Figure 6, in which children are required to find abundant numbers.

Abundant Numbers

Age 7 to 11
Challenge Level ★

To find the **factors** of a number, you have to find **all** the pairs of numbers that multiply together to give that number.

The factors of 48 are:

1 and 48

2 and 24

3 and 16

4 and 12

6 and 8

If we leave out the number we started with, 48, and add all the other factors, we get 76:

$1 + 2 + 3 + 4 + 6 + 8 + 12 + 16 + 24 = 76$

So .... 48 is called an **abundant** number because it is less than the sum of its factors (without itself). (48 is less than 76.)

See if you can find some more abundant numbers!

Figure 6: *An open-ended word problem involving abundant numbers (University of Cambridge, 2023)*

Open-ended word problems like the example given in Figure 6 are sometimes confused with investigations (discussed in the next section), which are not problems in themselves but share some similarities with mathematical problems.

Another type of word problem is real-world problems, which simulate scenarios that may arise in the real world, outside of school contexts. These types of problems are given various other labels in the research literature, including context/contextualised problems, realistic problems, authentic problems, or modelling problems (Smith & Morgan, 2016) and have been credited with many benefits for pupils:

> *Contextualising mathematics is also widely seen as a means of motivating students, by reducing its abstractness, by relating to student interests, and by showing them that mathematics is useful in the world outside the classroom and may have relevance for their current and future lives. (Smith and Morgan 2016, p.27)*

However, many studies have shown that real-world word problems and open-ended word problems are often under-represented in curricular materials, particularly when compared with routine word problems (Yang, Tseng & Wang, 2017).

Although it is not possible to provide an exhaustive list, this section has outlined the main problem-types that should be used regularly in the primary classroom (Barmby et al., 2014; DfES, 2004; Way, 2017). Ofsted (2021) argue that teachers must explicitly teach their pupils how to recognise these different problem types and make links with effective strategies, such as those suggested by Polya (1973) to help children to experience success in problem-solving. However, some other types of mathematical tasks, such as investigations and problem-posing, have been closely related to problem-solving. The next section briefly discusses investigations and problem-posing in terms of their benefits for problem-solving in the primary mathematics classroom.

2.5 Other task types linked with problem-solving

Previous sections have outlined the importance of exposing children to a wide range of experiences with a variety of NRPs so that they can become more experienced and develop problem-solving proficiency through the use of metacognitive processes. However, while they cannot provide a substitute for solving NRPs, engaging in other types of mathematical tasks, such as investigations and problem-posing activities, has been associated with the development of problem-solving abilities in children (Yeo & Yeap, 2010). This section discusses the features of investigations and problem-posing activities and draws on the research literature to outline their relative benefits for the development of problem-solving skills in the primary mathematics classroom.

2.5.1 Investigations

Mathematical investigations provide opportunities for students to use multiple pathways to investigate a particular mathematical situation (Marshman, Clark & Carey, 2015). This often involves multiple entry points and a period of extended exploration for pupils (Marshman, Clark & Carey, 2015). Southwell (2001) provides an example of a mathematical investigation involving the exploration of patterns in digital sums, which is shown in Figure 7:

Complete a standard multiplication table.
Write down the digital sum of each product in the table, e.g.

Products: (×7)	7	14	21	28	35	42	49	56	63	70
Digital Sums:	7	5	3	10	8	6	13	11	9	7
				1			4	2		

Investigate the patterns in the digital sums.

Figure 7: *A mathematical investigation involving the exploration of patterns in digital sums (Southwell, 2001 [adapted from Bonsangue, Gannon & Watson, 2000])*

As can be seen in Figure 7, there are many possible avenues for exploration. Children may notice that in the example given using products of 7, the digit sum begins with the first product of 7, and decreases by 2 each time giving odd numbers until it reaches 1, then starts with the largest one-digit even number and decreases by 2 each time until it reaches the lowest possible 1-digit even number. After this, the pattern seems to repeat from the first digit sum. Children may choose other pathways for investigation. For example, they may choose to investigate whether this pattern will be the same for multiplication tables for other odd numbers or 2-digit odd numbers, or they could explore whether the pattern in the digit sums changes with even numbers or when the product increases to become a 3-digit number.

While mathematical investigations are closely related to problem-solving activities, Barmby et al. (2014) note differences in the intended outcome of each activity:

> *What distinguishes investigations from problems is that the end-points for the mathematics activity are less constrained. (p.90)*

In problem-solving activities, the intended outcome or goal of the problem is given in the problem itself, and once an answer (or multiple possible answers) has been reached, the problem is finished. Investigations, on the other hand, are more open-ended than problems and can result in a wider range of different approaches and outcomes. The investigators must define what aspect of the investigation they will focus on, what approach to use and what options should be explored (Marshman, Clark & Carey, 2015). This leads to pupils posing their own mathematical problems to determine what the outcome of the investigation should be (Barmby et al., 2014; Way, 2017).

Yeo & Yeap (2010) compared the problem-solving process with the cognitive processes involved as individuals engage in a mathematical investigation. They found several similarities between the two processes, with mathematical investigations involving an additional phase of problem-posing (or goal-setting) before they attempt to solve the problems they have posed from the information in the investigation (Yeo & Yeap, 2010). Therefore, although there are fewer constraints placed upon the outcome for the learner, they are likely to draw on problem-solving skills during investigative tasks in the mathematics classroom. However, unlike problem-solving situations, the level of challenge involved in the task is, at least to a certain degree, decided by the students and the nature of the problems they pose themselves (Quinnell, 2010).

2.5.2 Problem-posing

Problem-posing has been referred to above as part of the process involved in mathematical investigations, however problem-posing often constitutes a mathematical activity in itself, and has been described as the younger sibling of problem-solving (Liljedahl & Cai, 2021). As with problem-solving, problem-posing has been an area of interest in the field of mathematics education research for many decades (Liljedahl & Cai, 2021), particularly since Brown & Walter's (1990) seminal works on ‘The art of problem-posing’ was published, in which they described pedagogical practices to support pupils in posing their own problems.

Although the concept of problem-posing is a broad one, it is usually associated with students generating new problems or reforming given problems (Karp & Wasserman, 2015). Generation of a new problem may take place either before or after a problem-solving activity (Silver, 1994) and involves the construction of new conditions to an existing problem or investigation, whereas reformulation problem-posing occurs during or after a problem-solving activity and involves adapting various conditions of an existing problem or omitting certain aspects to give a different version of the given problem (Baumanns & Rott, 2022). Additionally, problem-posing can arise during the problem-solving process: either during the ‘devise a plan’ phase, as students alter the conditions of a given problem to support them in solving it (Baumanns & Rott, 2022); or during the ‘look back’ phase, when students can pose new or related problems (Cansiz Aktas, 2022).

When combined with problem-solving activities, opportunities for students to pose their own mathematical problems can engender a deep understanding of the problem-solving process (Baumann & Rott, 2022; Karp & Wasserman, 2015) and support the development of problem-solving skills. The research also suggests that problem-solving and problem-posing may support one another in a symbiotic relationship, with problem-solving experiences and capabilities supporting the development of problem-posing skills (Liljedahl & Cai, 2021). Karp & Wasserman (2015) list several benefits of problem-posing activities for promoting problem-solving proficiency, such as improved understanding of underlying structures of mathematical problems, improved attitudes towards problem-solving in mathematics and greater flexibility with problem-solving strategies. However, gaps in the research literature mean that the exact nature of the relationship between problem-posing and problem-solving is not fully understood (Aktas, 2022). Furthermore, Liljedahl & Cai (2021) argue that teaching materials for mathematics educators lack both problem-posing tasks and guidance for teachers for implementing problem-posing in the classroom.

This section has explored and discussed task types which are closely related to problem-solving, including mathematical investigations and problem-posing activities. Although these cannot be considered a substitute for engagement in NRPS, they may provide additional experiences to supplement problem-solving activities, increase knowledge of the problem-solving process, and help develop problem-solving skills. Of course, the extent to which these benefits can be cultivated in students is dependent on the quality of the mathematical experiences and how the teacher utilises these tasks in the classroom. Different approaches to problem-solving instruction may culminate in different educational outcomes for students. Therefore, the next section explores some different approaches to problem-solving instruction in the context of the primary mathematics classroom.

2.6 Instructional approaches to problem-solving in mathematics

In terms of problem-solving instruction in mathematics, the research literature describes three distinct approaches: teaching *for* problem-solving; teaching *about* problem-solving; and teaching *through* problem-solving (Bostic, Pape & Jacobbe, 2016; Lester, 2013; Schroeder & Lester, 1989). Each of these approaches is described below:

Teaching *for* problem-solving – this approach involves teaching students mathematical procedures or techniques before allowing opportunities for them to apply the skills they have learned to routine problems (Bostic et al., 2016). Although this approach is useful in providing practice for pupils in using a mathematical technique in different contexts, it does not involve NRPS and does not support children in developing problem-solving abilities (Bostic et al., 2016).

Teaching *about* problem-solving - Teaching *about* problem-solving typically involves explicit heuristic instruction, or teaching pupils the steps involved in the problem-solving process (as outlined previously) to improve their problem-solving abilities (Lester, 2013). This often involves the use of cognitive and metacognitive strategies, as well as mnemonic devices to support children during problem-solving activities (Myers, Witzel, Powell, Li & Pigott, 2022). Although Polya (1973) positioned heuristics as central to the development of problem-solving skills in schoolchildren, a plethora of research studies over subsequent decades have found little evidence that the use of heuristic strategies in the classroom leads to pupils developing transferable problem-solving skills or achieving success in novel situations (Schoenfeld, 1985, Sweller et al., 2010). Chapman (2015) suggested that the failure of explicit teaching of heuristics may reflect the multi-dimensional nature of problem-solving and the wide range of skills required to solve problems efficiently. In addition to knowledge about heuristics and specific problem-solving strategies, pupils must have a deep conceptual understanding of mathematical concepts, the ability to monitor their progress during problem-solving activities and they must also hold positive beliefs about themselves as problem solvers to succeed in new problem-solving situations (Chapman, 2015). This implies that the use of heuristics may not be worthwhile unless accompanied by strategies to improve students' capacities in the other elements discussed by Chapman (2015).

Teaching *through* problem-solving - Teaching *through* problem-solving (henceforth TTPS) is an approach whereby mathematical concepts are taught in the context of problem-solving activities. This usually involves the teachers' careful selection of an NRP that can be used to draw out key aspects of the mathematical idea (Rhodes, 2019). In this way, the mathematical knowledge or skill to be learned emerges as students grapple with an unfamiliar problem (Bostic et al.,2016; Hiebert & Wearne, 2003). In this approach, there is a focus on problems which engender high levels of cognitive demand in students, as well as the use of collaborative reasoning with peers as they make sense of the problems and the underlying mathematical concepts they convey (Bostic et al., 2016).

This section has briefly outlined the main instructional approaches associated with problem-solving in primary mathematics. A more detailed comparison of the different approaches is beyond the scope of this research; however, this dissertation supports the view of academics in the field who argue that problem-solving should be seen as both an outcome of learning mathematics and a vehicle through which mathematics can be learned (DiMatteo & Lester, 2010; Lester, 2013; Stein, Boaler, & Silver, 2003). Therefore, a combination of different methods may be needed to ensure primary pupils are exposed to a wide range of different problem-solving experiences so that they reap the full range of benefits of problem-solving in mathematics. With this in mind, the next section explores the benefits which have been associated with problem-solving in primary mathematics education.

2.7 Benefits of problem-solving

Providing opportunities for mathematical problem-solving in the primary mathematics classroom has been associated with a plethora of different benefits. Although this section does not attempt to provide a comprehensive description of the many advantages associated with NRPS, some of the key benefits (those which were a recurring theme within the research literature) are outlined in the sections below.

2.7.1 Understanding of mathematical concepts

Hiebert & Carpenter (1992) explain that developing an understanding of mathematics involves connecting new information to prior knowledge and existing mental representations in a cohesive network. Mental representations of mathematical concepts might include 'facts about that concept, pictures or procedures we might draw on to explore the concept' (Barmby, Harries, Higgins & Suggate, 2007, p.43). As connections are made between these different representations, an understanding of the concept develops (Barmby et al., 2007). A deeper understanding would entail stronger and more numerous connections, and teaching should therefore highlight relationships between mathematical concepts. As Levav-Wayneberg & Leiken (2012) assert,

> *One of the measures of mathematics knowledge is its connectedness. (p.314).*

Where mathematical knowledge is fragmented, the individual is forced to rely on rote memorization of isolated concepts and procedures (Levav-Waynberg & Leiken, 2012) which is likely to be quickly forgotten (Li & Schoenfeld, 2019).

While primary mathematics lessons are all too often focused on demonstrating progress against fragmented goals (Povey, Boylan & Adams, 2019), NRPS takes learning beyond this, allowing pupils to connect new ideas to related ones and also draw on existing knowledge to help them in a new situation (Hiebert et al.,1996). When given opportunities to engage in rich problem-solving experiences in mathematics, pupils create new connections within their existing mental structures and also strengthen old ones to generate a deep and connected understanding of mathematical concepts (Hiebert et al, 1996; Hiebert & Wearne, 1993). Furthermore, while many pupils struggle to link conceptual knowledge with procedural skill, problem-solving opportunities require pupils to link the two together by drawing on their conceptual understanding to inform their procedural advances (Hiebert et al, 1996). In this way, problem-solving supports pupils in forming connections between mathematical procedures, ideas and representations, which leads to a deeper understanding of the mathematics (Hiebert & Grouws, 2007; OECD, 2014a; Woodward et al., 2012).

There is a great deal of empirical research which lends weight to these theories. For example, in a synthesis of meta-analyses, Hattie, Fisher, Frey, Gojak, Moore, Delano & Mellman (2016) found that teaching through problem-solving had a significantly positive effect on mathematical learning. Their results demonstrate a clear link between opportunities for problem-solving and developing deep understanding of mathematical concepts, however, the authors called attention to the importance of selecting appropriate problems with the correct level of challenge to achieve positive learning outcomes: when problems were introduced too early in the learning cycle – that is, when the pupils lacked the knowledge base required to attempt the problem – the effect size dropped from 0.61 to 0.15. This would suggest that problems must be carefully selected to build on earlier understandings (Hattie et al., 2016) without following immediately from exercises which reveal a solution method to the pupil.

2.7.2 Creativity

Another beneficial aspect of NRPS is its ability to foster mathematical creativity in children, providing them with opportunities to use creative approaches in mathematics (Gregoire, 2016). Creativity is often linked with innovation and the production of something which significantly changes or advances a particular domain, usually by an expert in the field (Simonton, 2017). This level of creativity is also known as a ‘big-C’ view of creativity (Simonton, 2017, p.6). However, in a ‘little-C’ view of creativity (p.6), the focus shifts towards the production of something new and meaningful on a personal level to that particular individual, rather than something ground-breaking in the field (Pham & Cho, 2018; Sriraman, 2009). Therefore, everyone is capable of being creative to some degree (Gregoire, 2016), even young children (Pham & Cho, 2018).

Creativity at a school level can be defined as ‘the capacity to imagine, conceive, express, or make something that was not there before’ (James et al., 2019, p.2), and promoting creativity in schools has increasingly been recognised as an area of particular interest in the education community (Elgrably & Leikin, 2021; Khalid, Saad, Abdul, Ridhuan, Ibrahim & Shahrill, 2020; Singer, Sheffield & Leiken, 2017), forming the basis for a great deal of education research and reports. The recent Durham commission on creativity (James et al., 2019), for example, emphasises the importance of creativity for students’ personal, social and academic development, and contains recommendations for nurturing creativity in education through changes to school systems. The report highlighted that teaching for creativity is becoming a ‘global phenomenon’ (James et al., 2019, p.6), with many countries adopting important changes to education systems and curricula to ensure a focus on creativity in schools and protect future prosperity.

Following closely in this pursuit, Ofsted’s (2019) revised inspection framework for UK schools refers to ‘culture capital’, which is the knowledge that is essential to prepare children for success in later life, and which engenders ‘appreciation of human creativity and achievement’ (Ofsted, 2019. p.10). Furthermore, the OECD has also recognised the importance of creativity in education and carried out its first (non-compulsory) assessment of creativity in PISA 2022 (Lucas, 2019), although the results have yet to be published.

In mathematics, creativity can be viewed as the process whereby pupils apply their knowledge of mathematical concepts to construct their own novel, original or imaginative method of solving a problem (Haylock, 1987; Liljedahl & Sriraman, 2006) or formulate new questions to consider the problem from a different perspective (Liljedahl & Sriraman, 2006). A growing body of evidence in education research demonstrates that problem-solving is an effective method of fostering mathematical creativity in school (Khalid et al., 2020). For example, in a quasi-experimental study involving 172 students aged twelve to thirteen, Khalid et al. (2020) found a large increase in post-test scores of student creativity after only one week of an intervention focused on teaching through problem-solving. Unfortunately, the researchers used a very small control group and were subsequently unable to determine whether the improvements at post-test were due to the intervention. However, Khalid et al. (2020) also reported an increase in evidence of creative thinking during observations of mathematics lessons, increasing the reliability of the results through triangulation of the data.

It is important to point out that creativity is not synonymous with the ability to solve problems, however authentic opportunities to be mathematically creative are generated from engagement in NRPS (Bolden, 2012; Khalid et al., 2020). Consequently, most interventions aimed at promoting mathematical creativity are based on NRPS activities (Bishara & Hui, 2016). In addition to the clear links with NRPS, creativity also goes hand-in-hand with critical thinking skills, whereby pupils evaluate and improve upon innovative ideas through logical chains of reasoning (Lau, 2011). This, in turn, supports further creative approaches, as well as proficiency in identifying efficient strategies in the face of unfamiliar problems, and the cycle continues in this way, with each of these three elements supporting each other in a symbiotic relationship (Lau, 2011).

2.7.3 Problem-solving skills and 21st century skills

As well as the advantages described for mathematical understanding and creativity, problem-solving is important in its own right (Woodward, 2012). Not only does problem-solving ability support pupils in national standardised tests in primary and secondary education (Woodward et al., 2012), experience and skill with solving NRPs are crucial if pupils are to adapt to the many problems they will face in different aspects of their lives (Merrienboer, 2013). After all, the true meaning of numeracy is the ability of the individual to use mathematics to solve problems confidently in everyday life (National Numeracy, 2017). However, one in four adults in England do not believe that their mathematics education prepared them adequately for this crucial goal (National Numeracy, 2014), which raises many questions about the level of opportunity for students to engage in NRPS activities during their school career. Burkhardt & Schoenfeld (2019) argue that if pupils are to be adequately prepared for mathematics in the outside world, a ‘substantial proportion’ of the tasks they engage with must be NRPs (p.37). However, this kind of ‘balanced diet’ remains rare in primary classrooms (Burkhardt & Schoenfeld, 2019, p.37).

Teachers are also in a position at this time where there is a need to develop 21st-century skills in pupils, to fully prepare them for a fast-moving world (English & Gainsburg, 2015). Scott (2015) defines 21st-century skills as, ‘The knowledge, skills and attitudes necessary to be competitive in the twenty-first-century workforce’ (p.8); in other words, the competencies students must develop to succeed in their future careers. Although there is a lack of consensus between academics about the particular skills attributed to this construct, almost all conceptualisations refer to problem-solving skills as a key component, alongside other dimensions associated with problem-solving such as critical thinking, metacognition and creativity (Joynes, Rossignoli & Fenyiwa Amonoo-Kuof, 2019). Collaborative problem-solving is also frequently cited as a 21st-century skill, which involves pupils sharing ideas, explaining their thought-processes, and justifying their solution strategies with peers during an NRPS activity (Liljedahl & Cai, 2021).

In terms of mathematics education, the skills which are easiest to teach are also the easiest to digitise and automate, meaning that pupils will need more than knowledge about mathematical concepts and procedures (Csapo & Funke, 2017). As Csapo & Funke (2017) point out,

> *Put simply, the world no longer rewards people just for what they know – Google knows everything – but for what they can do with what they know. (p.3)*

As computers are utilised more and more in workplaces to perform routine and manual tasks, the need for workers with routine skills is replaced with a greater demand for individuals who can deal with unfamiliar and unexpected situations and think flexibly to overcome obstacles and find solutions to unfamiliar problems (Csapo & Funke, 2017; English & Gainsburg, 2015). If schools only prepare pupils for employment prospects which exist in society today, their knowledge and skills may be outdated by the time they need to draw upon them (Csapo & Funke, 2017), which will undoubtedly hinder human potential and economic opportunities.

Despite general agreement in education systems around the globe regarding the importance of problem-solving in developing 21st-century skills, there is little evidence to show how effective they are in developing these (Csapo & Funke, 2017). In 2017, a survey conducted by Nesta found that over two-thirds of English teachers did *not* think England education system equipped young people with 21st-century skills such as collaborative problem-solving. Furthermore, 79% said they could not remember receiving training in how to incorporate collaborative problem-solving in their lessons. However, since the sample was derived from teachers who were subscribed to a teaching app and was largely comprised of secondary teachers (almost two-thirds), the results may not be generalisable to a larger population.

2.7.4 Affective benefits and growth mindset

Arslan, Yavuz, & Deringol-Karatas (2014) argue that the link between problem-solving and mathematical understanding has advantages in improving pupils' attitudes towards mathematics. Students adopt a negative attitude towards constructs that they do not understand, but by building a good understanding by engaging pupils in problem-solving tasks, attitudes towards mathematics can be improved (Arslan et al., 2014; Rahayuningdewi & Faradillah, 2020).

Pupils' attitudes towards mathematics are shaped by the way the subject is treated in the classroom and the types of tasks pupils engage in (Hiebert et al. 1996). For example, in a case study conducted in a secondary school in Norway, Thente (2019) found that pupils who were exposed to more opportunities for NRPS reported increased motivation to learn mathematics and increased interest in the subject, and this was also true of pupils who had reported a dislike for mathematics at the beginning of the study. Drawing upon the findings from this study, Thente (2019) suggested that pupils enjoy opportunities to think in a new way and experience feelings of success after struggling with challenging problems. Problem-solving provides pupils with exhilarating experiences which increase their motivation to learn mathematics and empowers them to be more actively involved in their learning (Apino & Retnawati, 2009).

Although Thente's (2019) study took place over only 2 weeks, which may not be adequate to judge the impact of increasing pupils' exposure to problem-solving opportunities, other studies have also found affective benefits associated with problem-solving in mathematics. For example, Doherty (2019) found that Irish primary pupils who had been exposed to more problem-solving experiences during the school year had higher self-efficacy beliefs about their problem-solving ability, with 60% agreeing that they were good at working out problems compared with only 42% of children who had only been exposed to infrequent problem-solving experiences from the class textbook. Doherty (2019) pointed out that the quality of the problems may also have been an important factor in improving pupils' attitudes to mathematics and problem-solving, which suggests that teachers play a vital role in preparing and choosing suitable problems for pupils.

Recent studies have also suggested that providing opportunities for pupils to engage in challenging problem-solving activities could promote a growth mindset - the belief that intelligence or ability can be developed through effort and perseverance, which has been associated with higher mathematical achievement (Rattan, Savani, Chugh & Dweck, 2015; Sun, 2018). A growth mindset is juxtaposed with a fixed mindset, which denotes the belief that intelligence and ability are fixed traits which cannot be improved (Dweck, 2006). Worryingly, pupils are more likely to have a fixed mindset about their abilities in mathematics than for any other school subject (Jonsson Beach, Korp & Erlandson, 2012), especially when they lack opportunities to engage in challenging problems, and this can have a negative impact on their mathematical achievement (Yeager & Dweck, 2012).

Although the outcomes from empirical studies linking growth mindset with academic performance seemed promising (Burnette, O'Boyle. Van Epps, Pollack & Finkel, 2013), the results of a recent meta-analysis study suggested that the association is weak (Burnette et al., 2013). Additionally, Bahnik & Vranka (2017) conducted a study involving university students and found that mindset did not predict any change in test performance between administrations. The researchers concluded that contrary to the findings of other studies (Burnette et al 2013), a growth mindset is not positively associated with test results. Despite the inconsistent findings concerning growth mindset and academic performance in the research literature, the premise seems an interesting one which is worthy of further investigation in future studies, which could focus on the development of a growth mindset in primary pupils through the use of challenging problem-solving activities.

2.7.5 Challenge, cognitive activation and mathematical achievement

This dissertation has already positioned challenge as a fundamental feature of problem-solving, and it is also crucial to high-quality mathematics education (Carter, 2009). Research has consistently highlighted a relationship between challenging tasks and improved mathematical achievement: In a synthesis of meta-analyses of educational research, Carter (2009) found that quality teaching involves challenging pupils to think more deeply. Furthermore, in a report commissioned by the DfE, Burge, Lenkeit & Sizmur (2015) drew on evidence from PISA to argue that NRPS not only provided a challenge for pupils but also opportunities for 'cognitive activation' (p.4), which has been associated with higher levels of mathematical attainment. Cognitive activation is about helping pupils to make links with prior knowledge, think deeply about solutions and focus on the solution strategy used to arrive at an answer, rather than the answer itself (Burge, et al., 2015).

In a large-scale survey in China involving over 8,000 ten and eleven-year-old children and their mathematics teachers, Li et al., (2020) found that pupils who had greater opportunity for cognitive activation in mathematics lessons, particularly through problem-solving activities, performed better in mathematics tests, and the relationship may be even stronger for pupils with low social economic status (SES). This survey relied on students' perceptions of teacher behaviour rather than direct observation, which may have resulted in biased results due to differing interpretations (Desimone & Le Floch, 2004). However, similar findings have been made in other studies (Burge et al., 2015; Kunter, Klusmann, Baumert, Richter, Voss & Hachfeld, 2013; Baumert, Kunter, Blum, Brunner, Voss, Jordan & Tsai, 2010), increasing the credibility of the conclusions.

Cognitive activation is fostered through high-quality problem-solving tasks which must be implemented effectively by the teacher to cultivate the benefits associated with it (Walshaw & Anthony, 2008). In addition, higher levels of cognitive activation are achieved when pupils have opportunities for purposeful discussion about different solution strategies and for reflection on their learning after solving a problem (Liu et al., 2020).

2.7.6 Formative assessment

Whilst this section has focused on the benefits of problem-solving for pupils, Charlesworth & Leali (2012) point out that there are also benefits for mathematics educators in terms of formative assessment during problem-solving activities:

> *Problem-solving provides a window into children's mathematical thinking and thus is a major vehicle for assessment. (p.373).*

Engaging pupils in NRPs allows teachers to uncover more of the understanding held by the pupil (Barmby, 2007; Charlesworth & Leali, 2012), especially where teachers engage their pupils in conversations during problem-solving activities and observe their actions and creations in their attempt to solve the problem (Glanfield, Bush & Stenmark, 2003, as cited by Nabie, Akayuure & Sofo, 2013).

This section has demonstrated the wide range of benefits that result when pupils are given opportunities to engage in NRPS activities. However, schools and teachers face a great many challenges in providing high-quality problem-solving opportunities for their pupils (Barmby et al., 2014; Leong et al., 2011; Xenofentos, 2014). Barmby et al. (2014) point out that problem-solving is frequently included as a key area for development in school action plans, and Ofsted (2012) emphasised that while schools are largely aware of the need to develop pupils' problem-solving skills, many teachers struggle to achieve this. The next section explores the difficulties faced by educators in implementing this crucial component of the mathematics curriculum.

2.8 Challenges of teaching problem-solving

In the previous section, the multiple benefits of problem-solving were outlined and discussed. However, several studies point out potential difficulties in implementing problem-solving successfully in the mathematics classroom to reap the benefits for pupils (eg. Hattie et al., 2016; Walshaw & Anthony, 2008). This section outlines the challenges teachers face in creating rich opportunities for problem-solving in the primary classroom.

2.8.1 Teacher knowledge

Schoenfeld (1985) contends that the teacher's approach to problem-solving in the classroom determines, to a large degree, the quality of pupils' experiences in mathematical problem-solving. Walsh (2016) further argues that the teacher plays a crucial role in supporting children to choose appropriate strategies and resources, monitor their progress during problem-solving tasks and develop positive dispositions towards problem-solving activities. This dissertation has previously discussed the wide range of knowledge and skills that must be developed for an individual to become an effective mathematical problem solver (Chapman, 2015). What logically follows is that teachers must also have a specific knowledge base and skill set to teach problem-solving effectively.

Chapman (2015) categorised the types of knowledge needed for effective teaching of problem-solving as content knowledge of problem-solving, pedagogical problem-solving knowledge and affective factors and beliefs about mathematical problem-solving. Figure 8 provides a summary of the knowledge required for each of these three categories (Kohar, Hartono, Rosyidi, Kurniasari, Karim & Siswono, 2019). A more comprehensive description of problem-solving knowledge for teachers is beyond the scope of this dissertation but can be found in Chapman (2015) and Kohar et al. (2019).

Type of knowledge	Knowledge	Description
Problem-solving content knowledge	Mathematical problem-solving proficiency	Knowledge of what is needed for successful mathematical problem-solving
	Mathematical problems	Knowledge of the nature of meaningful problems; structure and purpose of different types of problems; the effectst of problem characteristics on learners
	Mathematical problem solving	Being proficient in problem-solving Knowledge of mathematical problem-solving as a way of thinking; problem-solving models and the meaning and use of heuristics; how to interpreting students' unusual solutions; and implications of students' different approaches
	Problem Posing	Knowledge of problem posing before, during and after problem-solving
Pedagogical problem-solving knowledge	Students as mathematical problem solvers	Knowledge of what a student knows can do, and is disposed to do (e.g. characteristics of good problem solvers; students' difficulties with problem-solving; students' problem-solving thinking)
	Instructional practices for problem-solving	Knowledge of how and what it means to help students to become better problem solvers
Affective factors and beliefs		Knowledge of nature and impact of productive and unproductive affective factors and beliefs on teaching and learning problem-solving

Figure 8: *A summary of Chapman's (2015) categories of problem-solving knowledge for teachers (Kohar et al., 2019, p. 3-4)*

It is clear from Chapman's (2015) summary that teachers must have a high level of proficiency in problem-solving to implement it successfully in the mathematics classroom. However, a large body of evidence suggests that pre-service teachers often lack problem-solving skills themselves (Walsh, 2016; Silver & Marshal, 1990). For example, Berenger (2018) analysed 179 Australian pre-service teachers' solutions to the 'fashion warehouse problem' (p.164) aimed at upper primary school pupils. He found that despite taking part in weekly mathematical problem-solving activities and discussions (which focused on heuristics and were modelled by university tutors), 60% could not provide a correct solution to the problem (see appendix E for problem and examples of correct solutions), and most struggled to describe their thinking processes or strategy (Berenger, 2018).

Similarly, Walsh (2016) found that less than 10% of second-year student-teachers at one university in Ireland could achieve a PISA level 6, which is used to demarcate the highest level of mathematical proficiency for 15-year-olds (OECD, 2019). Considering that pre-service teachers struggle in the face of NRPS, and lack the skills in describing their solution strategies, this raises serious questions about whether they can *teach* problem-solving effectively (Silver & Marshal, 1990). Furthermore, Walsh (2016) argued that many teachers'/pre-service teachers' experiences as students were based largely on routine word problems, which inhibits their understanding of NRPs and effective strategies for solving them. Therefore, this could represent a vicious cycle in mathematics education: a lack of experience with NRPS in their own schooling restricts their ability to demonstrate effective problem-solving behaviours due to a lack of confidence and expertise in this area. Without concerted efforts to improve this situation in initial teacher training (ITT) programmes, this could lead to ineffective methods of teaching problem-solving skills in the primary classroom.

In addition to an apparent lack of problem-solving proficiency in pre-service teachers, Xenofontos & Andrews (2014) found that pre-service teachers held different beliefs about what problem-solving actually entails in the primary classroom and what constitutes an NRP, which relates to Chapman's 'mathematical problem-solving proficiency' and 'mathematical problem-solving' categories in Figure 8. Xenofontos & Andrews (2014) suggested that trainee teachers' incomplete understandings of problem-solving may be due to a lack of information and ambiguous guidance from policymakers:

> *…little information seems to have been provided with respect to what constitutes a mathematical problem and how problem-solving is systemically perceived… if problem-solving is to be a core objective of mathematics teaching and learning, we believe it is incumbent on curriculum writers … to develop appropriately researched explications of the systemic construal of mathematical problem and problem-solving. (p. 294)*

Additionally, Baumanns & Rott (2022) point out that the term 'problem' is used widely in the education world to refer to a range of different task types, including routine exercises, which could create further confusion for educators. This may be exacerbated by research literature and educational resources which use the term indiscriminately, resulting in misinterpretations and false conclusions about problem-solving (Schoenfeld, 1992). Without a comprehensive knowledge of problem-solving, schools may revert to more traditional practices such as over-reliance on routine problems (especially routine word problems) as consolidation activity or plenary at the end of a lesson (Barmby et al., 2014). Consequently, teachers may focus more on achieving the correct answer rather than the problem-solving process and development of problem-solving skills (Owens & Nolan, 2022).

2.8.2 Access to NRPs

A further difficulty for teachers lies in their ability to select or pose their own problems effectively: without explicit knowledge of the attributes of a high-quality mathematical problem, teachers may struggle to select problem tasks for their students that will support them in developing problem-solving proficiency (Leavy & Hourigan, 2019; Walsh, 2016; Kohar et al., 2019). For example, in a study involving over 400 prospective primary teachers, Leavy & Hourigan (2019) found that the participants were not able to pose high-quality problem-solving tasks for students, select appropriate NRPs for a specific year group from a selection of problems, or even reformulate a problem to make it better.

Even where teachers have sufficient knowledge about problem-solving to select appropriate NRPs for their students, Walsh (2016) suggests a further challenge in that teachers may not have sufficient access to high-quality problem-solving tasks. Mathematics textbooks traditionally neglect NRPS tasks and focus on routine (single and multi-step) word problems (Schoenfeld, 2007). Zhu & Fan (2006) found that more than 96% of tasks within Chinese and American textbooks were routine, and few opportunities for NRPS were provided. Similarly, Kolovou's (2011) analysis of six Dutch mathematics textbooks revealed that NRPs represented less than 3% of all tasks across 5 different textbook series. Unfortunately, more recent studies suggest that the dominance of routine tasks in textbooks has remained largely the same (Hidayah & Forgasz, 2020; van Zanten & van den Heuvel-Panhuizen). For example, van Zanten & van den Heuvel-Panhuizen (2018) developed an analysis framework to analyse textbook tasks based on a range of different criteria which affect the amount of challenge required for solving them. Based on their analysis of four Dutch mathematics textbooks using this framework, they determined that NRPs accounted for only 0-5% of the total tasks within each textbook. Although there is a lack of research surrounding the opportunity for NRPS within mathematics textbooks in England (specifically), it could be reasonably assumed that the situation would not likely be very different to that found in other countries. With the research suggesting a scarcity of NRPs within curricular materials, it is not clear how teachers manage to source NRPS activities for mathematics lessons.

Zhu & Fan (2006) argue that the way in which teachers use NRPs in mathematics lessons can change the nature of the problem itself and the potential impact on pupils, however, many teachers do not have enough knowledge about the curricular resources they are using and how they should be used, which limits their capacity to use such resources effectively in the classroom. The next section discusses some common teaching practices which restrict pupils' opportunity to engage in NRPS.

2.8.3 Using worked examples and 'teaching to the test'

Leong, Tay, Toh, Quek & Dindyal, (2011) argue that teachers face significant pressure to prepare their students for high-stakes examinations. In England, children sit Standard Assessment Tests (SATs) in the summer term of their final year in primary school, the results of which determine school rankings in league tables, and this can lead to increased 'teaching to the test' to achieve government targets (Leong et al., 2011, p.162). Until recently, children in Key Stage 1 were also required to sit SATs papers for mathematics in Year 2, however, this is now optional following the introduction of the reception baseline assessment, which replaced Key Stage 1 SATs from 2023 (DfE, 2020).

The mathematical items included in standardised examination papers are juxtaposed with the stated aims of the National Curriculum and largely assess proficiency in mathematical procedures and facts rather than problem-solving skills (Jones & Inglis, 2015; Noyes, Wake, Drake, & Murphy, 2011). Furthermore, teachers may attempt to prepare their students for standardised tests by exposing them to many similar problems from past tests in hopes to 'routinise' the problems (Leong et al., 2011, p.161). Since schools are not usually required to formally assess problem-solving processes, this crucial area of mathematics education is often neglected (Leong et al., 2011).

Besides issues of ‘teaching to the test’, Cox (2022) suggests that recommendations in guidance reports aimed at improving mathematics education can be misconstrued by educators and have a negative impact on problem-solving in the primary classroom. For example, the EEF report, ‘Improving Mathematics at Key Stages 2 and 3’ (Henderson et al., 2022) encourages the use of worked examples, which is thought to provide opportunities for pupils to focus on effective strategies for solving particular problems without the need for them to carry out the required procedures (Henderson et al., 2022; Mulholland, 2022). However, an overemphasis on worked examples during problem-solving activities can easily become counterproductive, as Cox (2022) warned:

> *If we are not careful, our worked examples could …[be] providing too much structure and guidance, and removing the ‘problem’ from the ‘problem-solving’. (para 2)*

Many studies have found evidence that worked examples have a positive impact on mathematical achievement (Barbieri, Miller-Cotto, Clerjuste & Chawla, 2023; Henderson et al., 2022). However, few of these studies focus on the impact of worked examples on problem-solving performance (specifically in relation to solving NRPs). One study which did explore the impact of worked examples on subsequent problem-solving performance was conducted by Adeniji & Baker (2023), who found that while worked examples were effective in improving pupils’ performance in solving *routine* mathematics questions, the benefits did not extend to *NRPs*. Furthermore, children gradually forgot the strategies explored in worked examples over time, and instruction using worked examples had no effect on higher-attaining pupils (Adeniji & Baker, 2023). The authors theorised that worked examples are useful in supporting lower-attaining pupils to develop proficiency in using mathematical procedures but may not lead to long-term retention of mathematical learning and have a negligible impact on children’s problem-solving skills (Adeniji & Baker, 2023).

2.8.4 Teacher responses to struggle

Earlier sections of this dissertation have highlighted the fundamental aspect of challenge in problem-solving activities, as well as the benefits of cognitive challenge for pupils. Dingman et al. (2019) explained that pupils experience ‘productive struggle' as they engage in problem-solving tasks (p.91), which is characterised by a period of perplexity as pupils make sense of the problem and devise and deploy a strategy to solve it (Dingman et al., 2019). Other researchers have used the term 'cognitive activation' to describe processes by which pupils make links with prior learning and partake in high levels of thinking and reasoning as they persevere to form a solution strategy (Burge et al., 2015, p.4; English & Gainsburg, 2015). However, this presents difficulties for teachers in selecting problems with sufficient levels of challenge whilst still being accessible to the learner (Leavy & O'Shea, 2011). Additionally, teachers must maintain a delicate balance in terms of the support they provide to pupils during problem-solving activities; allowing them to experience success in problem-solving without diminishing opportunities for them to experience productive struggle or cognitive activation (Dingman et al., 2019). A plethora of research and guidance reports support the use of 'scaffolding' in education, which is defined as,

> *The process that enables a child or novice to solve a problem, carry out a task, or achieve a goal which would be beyond his unassisted efforts. (Wood, Bruner & Ross, 1976, as cited by Bakker, Smit & Wegerif, 2015, p.1048)*

Scaffolding involves the supports put in place by the teacher to allow pupils to succeed in a particular task (Wood, Bruner & Ross, 1976, as cited by Bakker, Smit & Wegerif, 2015). However, the ultimate goal of scaffolding is that the student becomes independent, therefore supportive structures must be gradually removed so the student can work without assistance (Anghileri, 2006).

Warshauer (2011) found that many teachers provided too much scaffolding to pupils during problem-solving activities and responded to pupils who were experiencing struggle in a way that significantly reduced the level of cognitive demand required to find a solution, often by directing them to focus on one prescribed strategy. Many teachers feel a responsibility to intervene as soon as a pupil is struggling, often suggesting or modelling a solution strategy or even providing the answer in some cases, rather than allowing pupils time to form their own solutions (Dale & Scherrer, 2015). Too much scaffolding severely limits opportunities for pupils to persevere through productive struggle, while too little scaffolding leads to frustration in pupils

(Dale & Scherrer, 2015). Dale & Scherrer (2015) proposed an approach of 'Goldilocks discourse' (p.60), whereby just the right amount of scaffolding can be provided to pupils through probing questions during problem-solving activities:

> *Asking a question that gets students to reflect on their work — rather than just giving them answers — maintains the cognitive demand of the task and lets students struggle with ideas that they otherwise could not resolve on their own. (p.61)*

Dale & Scherrer's (2015) recommendations demonstrate the fine line that teachers must walk in terms of scaffolding problem-solving activities in mathematics, and this highlights the challenges for teachers in implementing problem-solving effectively in the classroom.

2.9 Conclusion

This chapter began by identifying some important features of NRPs: the lack of an obvious solution strategy; sufficient challenge to elicit a period of productive struggle for the solver; and multiple possible approaches. Additionally, high-quality NRPs should offer opportunities for students to be creative in their solution strategies since these two constructs are closely linked and have a mutually beneficial relationship. This dissertation has argued that providing opportunities for NRPS in the primary mathematics classroom is of critical importance for students' mathematical achievement and success as part of the future workforce, and research demonstrates that engaging in NRPS culminates in a wide range of important benefits for pupils. However, many teachers encounter difficulties in implementing these opportunities successfully, as they typically lack the complete skill set required to develop and maintain effective problem-solving practices.

In recent years, the focus on NRPS in primary classrooms in England has taken place in the context of significant change in mathematics education, brought about with the introduction of the TfM initiative. The next chapter outlines the different components of the TfM approach, including its roots in pedagogy from SSHK, and explores the existing evidence-base regarding its impact on mathematical achievement in primary schools in England as well as some problems surrounding the TfM reform identified in the research.

Chapter 3: Teaching for Mastery

3.1 Introduction

The most recent incarnation of a 'Teaching for Mastery' approach to mathematics instruction in England began with a desire from government ministers in England to emulate the high mathematical achievement of SSHK in International Assessments (Boylan et al., 2018). In 2012, England ranked only 26th in the world PISA tests in mathematics (OECD, 2014b), and government ministers expressed concern that these results were 'stagnating' (Coughlan, 2016). In contrast, SSHK consistently outperformed England in terms of the mathematical performance of 15-year-olds in PISA tests (Cantley, 2019) and the OECD described the difference as being equivalent to approximately three years of additional schooling (2012). In addition to this, the 2012 PISA results indicated a significantly wider attainment gap in England, with over 21% of England's 15-year-olds labelled as low-achievers in mathematics compared with under 4% of Shanghai's students (OECD, 2014b).

In order to address England's comparative underperformance and reduce the attainment gap between students in England, the government identified teaching methods from SSHK as the inspiration for educational reform in England (You & Morris, 2016), and policymakers embarked upon an initiative to borrow and implement what was referred to as 'Mastery' (Department for Education [DfE], 2013) - an approach based on mathematics education policies from SSHK (Clapham & Vickers, 2018). Adopting these 'world class' teaching practices was expected to lead to improved pedagogy in England (Boylan et al., 2018; Ginsburg, Leinwand, Anstrom & Pollock, 2005, p.133) and crucially, higher mathematical achievement in students (Jerrim, Vignoles & Cowan, 2015).

Although Mastery has been portrayed as an innovative new pedagogy, the term actually has a long history in mathematical education research and is often traced back to the 'Mastery Learning' model proposed by American Psychologist Benjamin Bloom (1968; Almond, 2020; Guskey & Jung, 2011). Bloom (1968) believed that wide variation between pupils with regard to their learning was due to a lack of variation in teachers' instructional practices, with all pupils receiving the same amount of time to learn and being taught in exactly the same way. He was interested in how beneficial aspects of individualised learning and tutoring could be harnessed to support children's learning in the classroom (Guskey & Jung, 2011), and subsequently proposed two key principles of his learning model:

***Feedback, corrective and enrichment process* –** Through frequent formative assessment, students receive specific feedback about their learning progress to show them what they have learned well and where they need to improve (Bloom, Hastings & Medaus, 1971; Guskey, 1997). This should come at a point in their learning where they have opportunities to work on areas of difficulty, and the teacher should provide 'correctives' targeted to each student's identified learning needs (Guskey & Jung, 2011, p.250). Students who excel in initial assessments are given opportunities to deepen their understanding through enrichment activities (Bloom, 1968). In this way, although students will not all reach the same level of understanding, all pupils will have the chance to achieve their learning goals (Bloom, 1968).

***Instructional alignment* –** this is the extent to which instructional processes, learning outcomes and assessments correspond with one another, and is described by Cohen (1987) as,

> *The precise match among what is taught, what is measured, and what is intended to be learned. (1987, p.16)*

In practice, this means that teachers' planning must begin with the end goal, and then the important concepts that must be learned are broken down and organised into small instructional units (Guskey & Jung, 2011).

Bloom (1981) believed that with increased time, additional support and adaptable teaching strategies, all pupils could achieve a similar level of attainment as their peers, resulting in reduced variation in learning outcomes. However, Guskey (2007) argued that many features of Bloom's model have been misinterpreted over the years, resulting in strategies which focus too much attention on lower-level skills. In fact, Bloom (1968) emphasised the need for higher-level learning goals, such as problem-solving and creativity, rather than an over-emphasis on basic skills. He argued that these higher cognitive processes supported children in applying their knowledge in unfamiliar situations and prepared them for a rapidly moving world (Guskey, 2007).

A large body of evidence over many decades suggests that when implemented faithfully, Mastery Learning approaches can have a positive impact on pupil achievement (Anderson 1994; Kulik, Kulik & Bangert-Drowns 1990), without the need for drastic changes to teaching practices (Guskey, 1989). More recently, the Education Endowment Foundation (EEF, 2021) summarised the evidence on approaches based on Bloom's (1968) Mastery Learning and reported a positive impact on achievement, leading to (on average) five months of additional progress, based on moderate evidence from several meta-analyses (EEF, 2021). However, Mastery learning seems to be more effective for lower-attaining students when used for less than 12 weeks (EEF, 2021).

Although the current reform based on pedagogy from SSHK is also often referred to as a 'Mastery' approach, it is not synonymous with Bloom's (1968) model, although the two approaches do share some characteristics (Boylan, 2019). The official term for the model based on SSHK pedagogy is 'Teaching for Mastery' (TfM). The next section outlines key features of this approach as described by the National Centre for Excellence in Teaching Mathematics (NCETM), to which the government have allocated responsibility for promoting the large-scale roll-out of the TfM approach in English schools (Boylan, Wolstenholme, Maxwell, Demack, Maxwell, Jay, Adams & Reaney, 2019).

3.2 What is Teaching for Mastery?

The NCETM (2018) states that, similar to Bloom's (1968) mastery learning model, TfM is rooted in the belief that every child is capable of being successful in their mathematical learning, regardless of their prior attainment. The TfM approach also involves breaking learning content down into smaller units, which are covered over a longer period. The majority of students are expected to progress through curriculum content at broadly the same pace, with differentiation achieved only through the level of support or use of concrete or visual resources (NCETM, 2014). Higher-attaining pupils, who may have progressed to new content under previous English teaching practices, are instead given opportunities to explore the same mathematical concepts in greater depth (NCETM, 2014).

The NCETM (2023) outlines five ‘big ideas’ underpinning TfM and demonstrates how these components are inter-related in Figure 9. The five big ideas are described below:

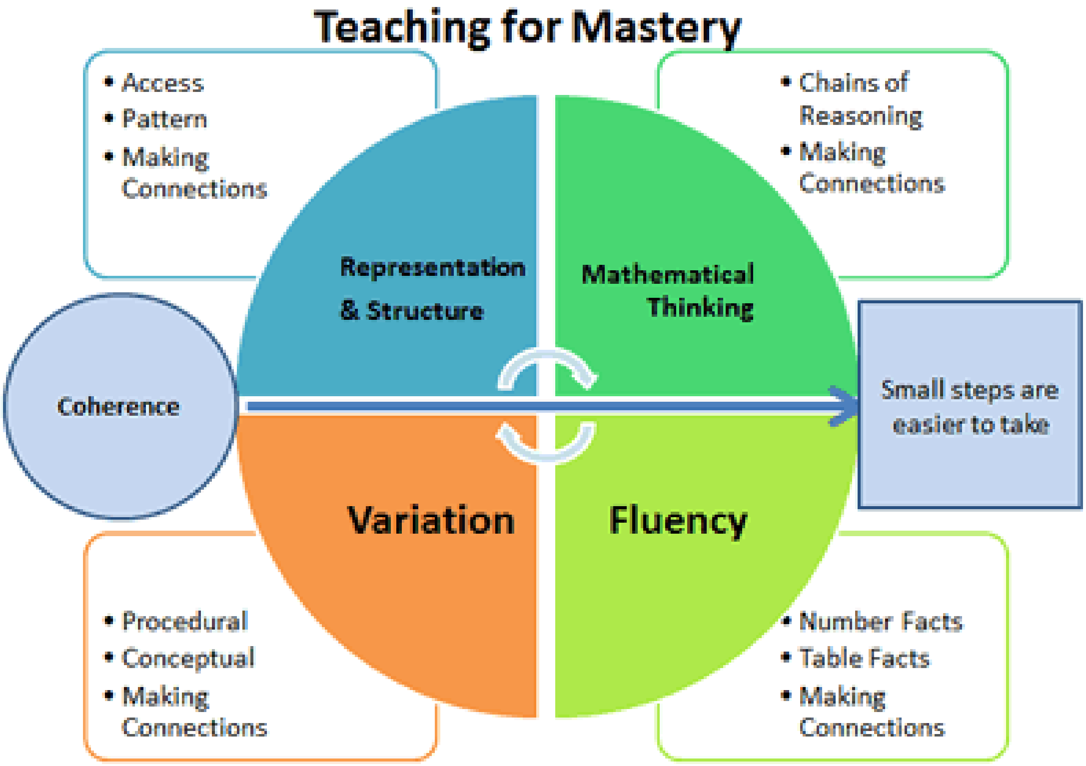


Figure 9: *The NCETM (2023) provides a visual representation to demonstrate how the five ‘big ideas’ are inter-related*

***Coherence* –** Instruction is organised into a logical sequence of small steps which allow all children to access the content, make connections with prior learning and form generalisations.

***Representation and Structure* –** these include concrete and pictorial resources which reveal underlying mathematical structures and support the development of a deeper conceptual understanding of mathematical ideas. The aim is that children will eventually be able to do the mathematics without the need for external representations.

Mathematical Thinking – Children have opportunities to be actively involved in their learning as they think about, reason with and discuss mathematical ideas by looking for patterns and relationships and connecting ideas.

Fluency – Children learn important mathematical facts and procedures for automaticity, such as number bonds, doubling, or multiplication tables. With fast recall of these facts, the mind is freed up to work on the mathematical concepts. Pupils also develop flexibility in applying their understanding in different mathematical contexts, by making connections, recognising mathematical relationships and choosing appropriate strategies to solve problems.

Variation – This idea has two aspects: conceptual and procedural variation. Conceptual variation refers to the way in which the teacher uses a range of different representations of the same mathematical ideas, which may each demonstrate a different idea underpinning the same mathematical concept. Procedural variation supports pupils' understanding of a procedure by introducing subtle changes in the careful sequencing of calculations, questions or exercises to expose particular structures or relationships, which is sometimes referred to as intelligent practice (NCETM, 2023).

Other features of a TfM approach include a focus on mathematical talk and high expectations with regard to pupils' use of mathematical vocabulary and reasoning skills, as well as an emphasis on whole-class teaching which is dominated by teacher-led instruction (Boylan et al., 2019).

In exploring the NCETM's (2023) 'big ideas' associated with TfM, it is surprising that problem-solving is referred to very infrequently in their explanation, being mentioned only briefly in the 'fluency' dimension. This might indicate that problem-solving is not a key focus of the TfM approach. However, as is discussed in the next section of this paper, pedagogies in SSHK (on which TfM is based) seem to assign considerable importance to problem-solving. While Ofsted (2021) found that "*Some* mastery approaches place a greater emphasis on problem-solving and on deepening pupils' understanding" ([emphasis added] para 12), it is unclear whether this comment alluded to TfM approaches or Mastery approaches based on Bloom's (1968) model.

Having defined the underlying principles of the TfM approach as outlined by the NCETM (no date), the following section explores the key aspects of TfM that have been attributed to the success of the approach in SSHK and have also been identified as distinguishing features of this approach in the research literature.

3.3 Links with teaching practices from SSHK

While there were many similarities between the research literature on teaching practices in SSHK and the NCETM's 'big ideas' of TfM, a more detailed explanation of the key features is warranted to ensure a deeper understanding of the fundamental aspects of the TfM approach, particularly for problem-solving, which received little attention in NCETM documents, website materials and reports. A review of the research literature provided further explanation of the more specific pedagogical practices which have been attributed to the success of the approach in SSHK. This section attempts to identify the most important aspects of a TfM approach (as practised in SSHK) and compare this with pedagogical practices which were more commonplace in England before the roll-out of the TfM approach.

3.3.1 Small steps and success for all

Within a TfM pedagogy, curriculum content is taught at a slower pace with a greater emphasis on developing a deeper understanding before progressing to new mathematical topics (Boylan et al., 2019). Learning is also broken down into smaller units which are taught over a longer period (Jerrim & Vignoles, 2015). In contrast, common teaching practice in England prior to the TfM reform was for higher-attaining pupils to progress to new mathematical topics before their lower-attaining peers (Jerrim & Vignoles, 2015). In China, however, higher-attaining pupils are given opportunities to develop a deeper conceptual understanding of the same mathematical idea through more challenging problem-solving tasks (Boylan et al., 2019; Jerrim & Vignoles, 2015). This is thought to maximise opportunities for all children in the class to access the learning content, regardless of prior attainment (Boylan et al., 2019) and reduce the need to periodically revisit content matter (Jerrim & Vignoles, 2015).

3.3.2 Variation

Teaching with variation has been an important feature in Chinese pedagogies for approximately 40 years and became popular in mainland China after a teaching experiment in Shanghai found a positive effect on student learning (Gu, Hunag & Marton, 2004; Mok, 2017). Also referred to as 'bianshi' in China (which derives from 'bian', meaning variation, and 'shi', meaning style [Wong, 2017]), variation provides opportunities for students to create a deep understanding with more numerous connections within and across mathematical topics.

Hiebert & Carpenter (1992) theorised that learning is most effective when it allows connections to be made between different representations of mathematical ideas, to form 'networks' of internal representations which can be called upon in the face of unfamiliar mathematical situations such as in solving problems (p.69). In China, teaching with variation is lauded as a teaching strategy which supports children in forming generalisations in mathematics, a skill which accorded great value by educators in China (Cai, 2002; Gu et al., 2004; Zhang, Wang, Huang & Kimmins, 2017). Gu (1999) described the fundamental purpose of variation in mathematics instruction as,

> *To illustrate essential features of a concept by demonstrating various visual materials and instances, or to highlight essential characteristics of a concept by varying non-essential features. The goal of using variation is to help students understand the essential features of a concept by differentiating them from nonessential features and further develop a scientific concept. (Gu, 1999, as cited by Zhang et al., 2017 p.215)*

In other words, non-essential features can be varied in different examples and exercises to support children to identify the critical features of a mathematical concept (Gu et al., 2004). For teachers, this involves careful planning and sequencing of activities (called pudian) which focus on the effect of changing specific aspects to draw attention to underlying mathematical structures and build an understanding of mathematical concepts (Jacques, 2018).

As defined above as part of the NCETM's big ideas (2023), teaching with variation incorporates both conceptual variation and procedural variation. Conceptual variation in China relies on the careful selection of different representations, including non-standard representations, to expose children to the same mathematical construct from different perspectives (Wong, Lam & Chan, 2012). Wong et al. (2012) provide an example of investigating isosceles triangles of a variety of shapes, sizes, materials and colours so that pupils can distinguish the defining feature of side length from non-essential features. Non-conceptual variation is also commonly used in Chinese teaching practices by sharing non-examples with children, which encourages deeper cognition about mathematical concepts (Marton & Häggström, 2017).

In Chinese teaching methods, procedural variation involves the design of a series of tasks which change slightly each time to help students uncover mathematical patterns and identify mathematical rules (Jacques, 2018; Wong et al., 2012). The first of these tasks will be familiar to the pupils and is referred to by Gu et al. (2004) as the 'anchoring point' (p.310). Subsequent questions are chosen to allow children to move between the anchoring point and the intended learning goal (Gu et al., 2004). Figure 10 gives an example of procedural variation in a Chinese textbook which focuses on multiplication by a multiple of 10 (Crozier & Gerrard, 2016). In this example, children can be encouraged to explore how the answer changes when the multiplicand or multiplier is ten or one hundred times greater. This may also support children in deriving the commutative law of multiplication due to variation in the presentation of the calculations. As children progress through the questions, they have opportunities to make connections between each task by exploring the relationships between them and also use the answer to one question to work out the answer to subsequent questions (Crozier & Gerrard, 2016).

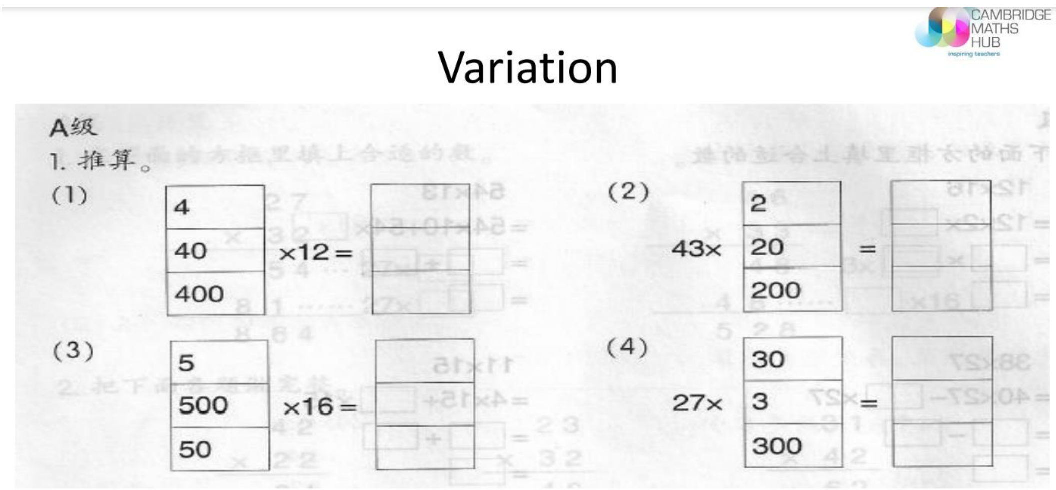


Figure 10: *An example of procedural variation from a Chinese textbook (Crozier & Gerrard, 2016)*

A typical mathematics lesson in China involves the use of both conceptual and procedural variation, which are thought to collectively contribute to a deeper understanding of mathematical concepts (Kullberg, Runesson & Marton, 2017). However, Wong et al. (2012) outline a more thorough description of variation within a TfM approach, whereby four types of variation are used in sequence to promote depth of understanding in pupils:

Inductive bianshi – This involves using a variety of realistic examples to support children in deriving rules and concepts. For example, pupils can be supported in deriving a formula for finding the area of a rectangle through exploration of arrays, rather than being provided with the formula from the outset.

Broadening bianshi – This provides further tasks with slight variations to highlight the same underlying mathematical construct. This may also involve real-life questions and worded contextual questions.

Deepening bianshi – After exploring questions with the same underlying structure, pupils attempt tasks in which significant changes to the underlying structure are introduced to support children in making connections between different mathematical constructs. As an example, children may explore the similarities and differences between the division of integers and the division of fractions.

Applying bianshi – At this stage, pupils are exposed to a variety of mathematical problems which allow them to apply their learning in different situations (Wong et al., 2012).

Chinese teachers believe that by actively involving pupils in concept formation, rather than emphasising mechanical calculation, cognitive performance improves, and pupils are more able to make connections within and between mathematical ideas (Wong et al., 2012).

Considering the long prevalence of teaching with variance in China, it is not surprising that variance is a fundamental feature of mathematical textbooks in Chinese regions (Zhang et al., 2017). In contrast to the West, where textbooks are typically thought of as an 'encyclopaedia' of various tasks for teachers to pick and choose from (Wang & Fan, 2021, p.686), this is not the case in China: mathematics textbooks are regarded as mandatory documents, closely aligned with the mathematics curriculum, which are relied upon heavily by Chinese teachers (Park & Leung, 2006; Zhang, Wong, Huang & Kimmins, 2017). The aspect of variance in mathematical textbooks has also been associated with increased problem-solving ability in Chinese students: Zhang et al. (2017) argue that students develop more flexible approaches and a greater capacity for applying mathematical knowledge in problem-solving tasks when they have previously experienced mathematical concepts in a range of different contexts. Although there are few recent studies to support this claim, a quasi-experimental study carried out by Wong, Lam, Sun & Chan (2008) investigated the impact of bianshi on problem-solving ability in over 1000 children in Hong Kong schools. They found a significant difference at post-test on routine and NRPS performance. However, since participants included both primary and secondary children, it is not clear whether the same results could be replicated with a primary cohort alone.

Hunag & Leung (2004) suggested that teaching with variation may provide a scaffold which allows children to make connections between different mathematical concepts, and this supports them in accessing unfamiliar problems. However, they also acknowledged that without a sufficient evidence-base, it is not possible to establish a clear connection between variance and problem-solving ability. Furthermore, Wong

(2017) warns that having a deep understanding of mathematical concepts may not necessarily culminate in high performance in problem-solving tasks.

3.3.3 Conceptual understanding and forming connections

In 1976, Skemp's seminal paper distinguished between 'instrumental understanding' and 'relational understanding' (p.9). The former involves pupils' knowledge of mathematical rules and methods, including *how* to perform calculations, which is useful in calculating quickly but is easily forgotten as children move on to new mathematical topics. Relational understanding, on the other hand, concerns an understanding of *why* rules and methods work, which results in greater flexibility in applying mathematical knowledge to new contexts and problems (Skemp, 1976). In China, teachers place greater emphasis on developing an interconnected and well-structured understanding of mathematical concepts in their pupils (Cai, Ding & Wang, 2013), and explicitly plan opportunities to make links within and across mathematics lessons (Chen & Li, 2010). Chinese teachers also use more mathematical models and representations to expose mathematical structures with the aim of developing conceptual understanding in students (Boylan et al., 2019).

With regard to mathematics textbooks in China, a number of comparative studies have found that Chinese textbooks promote a deeper understanding of mathematical concepts compared to those commonly used in Western countries (Ding, 2016; Ding & Li, 2014; Wang, Barmby & Bolden, 2015), often through carefully considered use of representations (Ding, 2016). Additional features which differ from traditional mathematics textbooks in the West include a greater emphasis on exposing underlying mathematical structures (Bao, 2004), and supporting students to progress to more abstract levels of understanding (Wang et al., 2015).

3.3.4 Mathematical communication

Pupil engagement in high-quality mathematical communication has been associated with a range of beneficial mathematical outcomes for students in both the East and the West: These benefits include the development of a deep understanding of mathematical concepts *(*Hiebert & Carpenter, 1992) and improved problem-solving ability (Webb, Franke, Ing, Wong, Fernandez, Shin, & Turrou, 2014). Although the West often associates mathematics instruction in China with transmission-style instruction, there is evidence to suggest that pupil engagement in mathematical communication is highly valued by mathematics teachers in China (Xu & Clarke, 2019), who routinely incorporate opportunities for mathematical talk in their lesson planning (Boylan et al., 2019).

Peng & Li (2020) conducted a naturalistic inquiry-based study to explore Canadian teachers' perceptions of effective teaching practices in China. Following lesson observations, the Canadian teachers reported frequent use of higher-order questioning to elicit mathematical responses in Chinese classrooms, including explicitly asking pupils to give a reason to support their answer (Peng & Li, 2020). Participants also noticed a common feature of mathematics lessons included opportunities for pupils to reflect upon, consolidate and discuss what they had learned at the end of the lesson. Similar observations were made by Dong, Seah, Cao & Clarke (2019) and Miao, Reynolds, Harris & Jones (201), although Dong et al. (2019) found that pupils may not have *equal* opportunities to engage in mathematical communication during lessons, with one group of children often dominating classroom discussion.

In classrooms in Chinese regions, the emphasis of mathematical communication is on students' use of accurate and precise mathematical language (Axbey, 2020; Boylan et al., 2019). The NCETM (2023) identify precise mathematical language in full sentences as part of 'thinking mathematically' and consider this to be a key element of TfM, which may engender a deeper understanding of mathematical concepts and increase mathematical attainment in pupils (Jerrim & Vignoles, 2015). However, Xu & Clarke (2019) conducted over one hundred lesson observations in China, Australia and the US and noted that while students in Chinese classrooms demonstrated significantly more use of mathematical terms and phrases, almost all student talk was in response to the teacher and there was very little paired or group talk. Similarly, Zhou, Bao & He (2023) found that the fast pace of mathematics lessons in Chinese regions constrained opportunities for students to express their mathematical ideas or understanding of a concept. When children were able to communicate their ideas, these were rarely explored or discussed further (Zhou et al., 2023).

3.3.5 Problem-solving

Problem-solving is a major focus in mathematics curricular materials in China (Chan, Clarke & Cao, 2018), and Chinese teachers hold the development of problem-solving skills in pupils as an important goal of mathematics education (Gu, Huang & Gu, 2017). In informal conversations with mathematics teachers in a primary school in Shenzhen, China, (during a teaching placement opportunity offered to undergraduate students at Durham University), problem-solving was frequently referred to in conversations regarding effective mathematics instruction, with the head teacher at the same school emphasising problem-solving as a crucial component of the TfM approach as practised in China (personal communication, 2019 [for a more detailed explanation, see Appendix G]). Furthermore, while problem-solving activities have been traditionally used as a consolidation activity at the end of a lesson in England (Barmby et al., 2014), mathematics lessons in Chinese regions often begin with an NRP as a starting point for further learning (Boylan, 2019; Lopez-Real, Mok, Leung & Marton, 2004; Miao et al., 2015; Zhang & Siegler, 2022).

Research suggests that the high priority accorded to problem-solving in China is also evidenced in their curricular materials, including mathematics textbooks (Ding & Li, 2014): compared with mathematical resources used in western countries, Chinese textbooks feature more opportunities for NRPS (Sun, 2011) and typically involve a higher level of challenge for pupils (Zhu & Fan, 2006). However, Zhu & Fan (2006) found that Chinese textbooks lacked word problems involving real-world scenarios, with the majority of NRPs being puzzle-like logic problems (almost 85%). In comparison, American textbooks had a more even distribution of different problem types and made more use of real-world situations (Zhu & Fan, 2006).

Considering the apparent emphasis placed on problem-solving in regions from which TfM originated, it is strange that the NCETM's (2019) progress report on TfM in English primary schools makes only one mention of problem-solving in the entire document. Furthermore, there are only two brief references to problem-solving within the explanation of the five 'big ideas' on the NCETM website, which are made with reference to the importance of developing pupils' fluency skills so that their memory is freed up to, ‘think deeply about concepts and problems’ and ‘choose appropriate methods and strategies to solve problems’ (NCETM, 2023, para 6).

Some academics have questioned whether Chinese pupils are actually given regular opportunities to engage in solving NRPs due to the heavily structured manner of implementation of these experiences in the classroom. Zhou et al. (2023) analysed video records of prize-winning mathematics lessons from national teaching competitions in eight different regions of China to identify the crucial features that are most highly valued. They found that teachers tended to ‘scaffold away’ challenges, rendering the task relatively straightforward following teacher input and resulting in much lower cognitive demand for pupils (Zhou et al., 2023, para 22). In 23 episodes analysed, only 4 episodes demonstrated tasks involving cognitive struggle during the lesson without intense scaffolding of the problem by the teacher. The researchers acknowledged the possibility that the Hawthorne effect may have influenced these findings, whereby participants alter their behaviour due to their awareness of being observed (Cohen, Manion & Morrison, 2018), which may have caused the teachers to adapt their usual classroom behaviours because they were being observed and judged for the competition. However, the fact that these lessons each received the top prize in their jurisdictions demonstrates that they were perceived as being exemplar lessons in the teaching community and therefore demonstrated features that are highly valued in China.

This section has outlined the key features of teaching practices in SSHK and SSHK-inspired textbooks (Clapham & Vickers, 2018). The following section discusses each of these initiatives in turn, focusing on the impact of each with regard to implementing TfM in English schools.

3.4 Implementing TfM in England

In the early stages of reform initiatives, the DfE focused on three key initiatives to promote the implementation of the TfM approach: teacher exchange visits between England and Shanghai, professional development opportunities (through regional Maths Hubs led by the NCETM) and SSHK-inspired textbooks (Clapham & Vickers, 2018). The following sections discuss each of these initiatives in turn, focusing on the impact of each with regard to implementing TfM in English schools.

3.4.1 The Mathematics Teacher Exchange

The government-funded Mathematics Teacher Exchange (MTE) was launched following two initial study visits to Shanghai, organised by the National College for School Leadership (2013/2014). Led by the NCETM and coordinated through the Maths Hubs, this proved instrumental in bringing about change in English schools and providing the impetus for many schools to adopt a TfM approach in their own settings (Boylan et al., 2019).

In 2014, teachers and school leaders from 48 primary schools in England formed the first group to visit Shanghai schools to observe and learn from their mathematics teaching practices first-hand (Boylan et al., 2018). Following this visit, mathematics teachers from Shanghai were also hosted in English schools to provide workshops and offer further opportunities for English teachers to observe their planning and professional development (Boylan, 2018). Schools that were part of the MTE attempted to incorporate key features of the teaching practices they had observed in Shanghai and were expected to share their experiences and learning with other schools (Boylan et al., 2018). In 2016 and 2017, a further visit involving teachers from 70 English primary schools was organised to expand the reach of the MTE programme to a larger number of English schools (Boylan et al., 2018).

In their Longitudinal evaluation of the MTE, Boylan et al. (2019) argued that this had supported the roll-out of the TfM initiative and had led to significant changes in pedagogical practices in schools whose teachers were involved. The report also highlighted small gains in mathematical achievement for Key Stage 1 pupils after the MTE, although causality could not be established due to the quasi-experimental research design, and there was no improvement for Key Stage 2 students. The authors acknowledged the need for further research to ascertain the impact of the TfM approach on mathematics attainment, and also to identify which components of the initiative are most effective.

3.4.2 Maths Hubs

In the early stages of the TfM reform, the NCETM established regional Maths Hubs (DfE, 2014a) - a network of schools charged with promoting TfM implementation. Following the first round of the MTE, a larger scale roll-out of the TfM reform in schools was promoted through the expansion of these Maths Hubs (of which there are now forty nationally [NCETM, no date]). The Maths Hubs were key in bringing about changes endorsed by the NCETM in schools within their respective regions of England (Boylan et al., 2019), and professional development opportunities were offered to teachers. The latest annual report of the Maths Hubs programme (NCETM, 2022) states that over 9,000 schools are now involved with their regional Maths Hub in some capacity.

3.4.3 SSHK-inspired textbooks

In Chinese regions such as SSHK, schools are only permitted to use mathematics textbooks which have been approved by the Ministry of Education, and these textbooks undergo a strict review process involving a panel of experts (Marks et al., 2023). Although this kind of strict regulation has not been imposed on English schools, the DfE appeared to partially follow suit by publishing a detailed list of criteria for TfM-aligned textbooks in 2017, which included mathematical coherence, mathematics structure/language and teacher guidance. Of the many TfM-aligned textbooks on the market, only two met the published criteria: Maths – No Problem! and Power Maths, and schools investing in these DfE-approved textbooks could access matched-funding of up to £2,000.

Since 2016, the DfE has invested £76 million towards subsidising approved TfM mathematics textbooks for primary schools in England (Barclay, Barnes & Marks, 2022). However, the campaign to implement TfM textbooks necessitated significant organisational, cultural, and pedagogical changes for English schools (Marks et al., 2023): In his influential policy paper, Oates (2014) spoke of an ‘anti-textbook ethos’ in England, with many English teachers demonstrating strong opposition to the use of mathematics textbooks (p.8). He suggested that this was particularly true in primary schools, which may compromise government attempts to improve mathematical attainment through the use of high-quality textbooks.

Haggerty & Peppin (2002) suggest that negative attitudes towards textbooks by teachers may stem from a history of low-quality textbooks in England, which traditionally presented mathematical concepts in a disjointed manner, with limited use of mathematical language and an over-emphasis on repetition of mathematical procedures rather than investigation and problem-solving. Furthermore, in inspection reports between 2005 and 2009, Ofsted also indicated disparaging views about over-reliance on textbooks (Bokhove & Jones, 2014). Consequently, primary schools have tended to avoid adherence to one textbook, and controversy surrounding the use of textbooks continues to be prevalent (Marks et al., 2023), with some teachers viewing textbooks as a threat to their professional autonomy (Boyd & Ash, 2018; Turvil, 2021), and many pupils seeing textbooks as boring (Ni Shuilleabhain Cronin & Prendergast, 2021).

Considering poor attitudes towards textbooks, it is not surprising that almost all primary teachers in England (both experienced and newly qualified) prefer to select tasks and activities from a range of different sources, or create their own (Polly, 2017; Silver, 2022), which culminates in a 'patchwork' of curricular materials (Marks et al., 2023, p. 15). In recent years, the need to adapt to changes may have resulted in teachers becoming familiar with a wider range of sources for curricular materials. For example, during the Covid-19 pandemic, teachers had to seek out resources which would be suitable for home and online learning (Marks et al., 2023).

Until very recently, there was a notable lack of research to demonstrate how TfM textbooks were used in primary schools, However, in March 2023, research conducted by the Nuffield Foundation provided some long-awaited answers (Marks et al., 2023). This study, which involved a survey of over 600 schools in England and almost 2,000 teachers, found that only 3% of primary schools were using one scheme exclusively, and only 1% used a DfE-approved textbook scheme in a way that emulated teaching practices in SSHK, where textbooks are followed in a systematic and sequential manner (Marks et al., 2023). This is a salient finding since TfM-aligned materials are designed to be used with fidelity (in the way that the author intended) because they are underpinned by variation theory involving the careful sequence of tasks to uncover particular mathematical concepts (Marks et al., 2023). By adapting or supplementing TfM resources, teachers may be inadvertently reducing the potential for the materials to impact mathematical understanding and achievement.

In terms of the rationale behind the choice of curricular materials in English primary schools, Marks et al., (2023) noted that cost was the most influential factor, and while 33% of eligible schools had made use of funding for DfE-approved textbooks, over a third of these had subsequently abandoned them (to a large degree or entirely), partially due to continuing demands on the schools' budgets.

3.4.4 The current situation with TfM

Although the evidence suggests that wide-scale implementation of TfM-aligned textbooks does not appear to have transpired in the way that was intended by the DfE, Marks et al. (2023) pointed out that,

> *… mastery (in its many interpretations) is being seen as the (or a) underpinning approach in a growing number of primary mathematics classrooms… (p.76)*

TfM has now become a central element of education policy in England (Blausten et al., 2020), and the revised primary mathematics curriculum (while not explicitly outlined as such) is often regarded as a Mastery curriculum. For example, stipulations that 'the majority of pupils will move through the programmes of study at broadly the same pace' (p.3) emulate teaching practices in SSHK and is outlined as a key feature of TfM by the NCETM (no 2023). Furthermore, the revised National Curriculum (DfE, 2013) demonstrated similarities with Chinese TfM curricular materials in their expectation that higher-achieving children will be given more opportunities to develop depth of understanding through problem-solving, rather than moving on to new curriculum content:

> *Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content. (p.3)*

The TfM approach was also advocated by Ofsted, providing a greater incentive for schools to adopt a TfM approach. As a result of changes to mandated government documents and encouragement from influential stakeholders, following a TfM approach is now commonplace in England, particularly in primary schools (Almond, 2020).

3.4.5 White Rose Maths Programme

In response to the increasing popularity of the approach, a range of TfM-aligned programmes and schemes (both government-funded and private enterprises) are now available to schools (Boylan et al., 2018), which provide additional access to resources and training for schools. One particularly popular TfM-aligned scheme is White Rose Maths (henceforth WRM). WRM resources are central to mathematics provision in many schools, and their lesson resources share many features of textbooks, although the materials are internet-based, rather than physical textbooks (Marks et al., 2019). WRM materials are purported to be aligned with the National Curriculum and break learning goals down into a series of small steps (Bessemer, 2023; Master the Curriculum, 2020; WRM, no date). Many of these resources are free, and WRM claim that these resources are used (to some extent) by around 90% of English primary schools (Marks et al., 2023; WRM [personal correspondence], 2022), therefore it could be surmised that these internet-based resources, compared with physical textbooks, have not met with such resistance from teachers and schools.

WRM originally operated as one of the government-funded Maths Hubs, but was forced to separate from the NCETM and become a private company in 2017 (Staufenberg, 2017). Staufenberg (2017) reported that this rift was due to disagreements about how government funding was being used: The NCETM decided that only resources that had been created in collaboration with (and approved by) the entire Maths Hub network (and had undergone the proper trials) should be made widely available to schools.

There are many more TfM-aligned programmes available to schools, and it could be said that in terms of widespread implementation and prevalence of the approach in mathematics instruction, the reform has been successful (Boylan et al., 2019). However, there is limited evidence to suggest that this implementation has culminated in substantial gains with regard to mathematical attainment for English pupils (Boylan, 2019; Clapham & Vickers, 2018). The next section draws on the available research evidence to evaluate the impact of the TfM reform on mathematical achievement in England.

3.4.6 Impact of TfM

It is difficult to determine the impact that the TfM reform has had on mathematics achievement in England: few TfM programmes have been independently evaluated, and to date, there have been only two randomised controlled trials (RCTs) of TfM approaches (although neither was designed or implemented by the NCETM). The first was based on a 'Mathematics Mastery' approach, which incorporates many elements of the NCETM's (2023) big ideas but places increased emphasis on problem-solving. The RCT of Mathematics Mastery involved 90 primary schools and found a small positive impact for year 1 pupils (aged 5-6) over one academic year. The researchers speculated that longer-term evaluations of the teaching method may yield greater cumulative effects on student achievement, but conceded that,

> *…it is unlikely that widespread introduction of this particular programme would springboard Western countries like England to the top of the PISA educational achievement rankings. In other words, it cannot be seen as a 'silver bullet' that will guarantee a country success in mathematics. (Jerrim & Vignoles, 2016, p.42)*

However, the positive results found by Jerrim & Vignoles (2016) in Key Stage 1 are substantiated by a further mixed methods RCT of the 'Inspire Maths' programme, which also found a small but positive impact on mathematical achievement for children aged 5-6 (Hall, Lindorff & Sammons, 2016). This approach differed from Mathematics Mastery in that it incorporated textbooks (not DfE approved), based on Singaporean textbook translations, but since a relatively small sample of only 12 schools was used, this decreases the external validity of the conclusions. More recently, a longitudinal evaluation of the outcomes of primary schools that participated in the first MTE initiative reported no effect for Key Stage 2, and only a small positive effect at Key Stage 1 (Boylan et al., 2019).

To date, there have been no large-scale evaluations of WRM materials in terms of their impact on mathematical attainment in primary schools. However, in a small-scale case study involving four English schools, Barclay et al. (2019) investigated additional benefits beyond pupil achievement resulting from the use of textbook-based TfM approaches, including 'Maths No Problem!', 'Inspire Maths' and WRM (which were identified as the predominant materials used in primary schools in

South-East England). Through teacher interviews and lesson observations, the researchers suggested that Mathematics subject leaders and NQTs were enthusiastic about using the various TfM textbooks/materials and concluded that these programmes could support NQTs' mathematical instruction by providing structure and logical progression without reducing teacher autonomy (Barclay et al., 2019). However, since the research was funded by one of the programmes involved in the study and was largely based on the researchers' interpretations of qualitative evidence, there is potential for bias in the conclusions.

It is important to recognise that there may be large variations in the extent to which TfM is implemented in different schools, which causes difficulties in measuring the impact of the approach (Blausten et al., 2020). Furthermore, TfM approaches are multifaceted, and some components may be more effective in raising achievement than others. For example, Boylan et al. (2018) summarised the research evidence on separate components of TfM and concluded that increased dialogue and the use of representations had the potential to improve students' mathematical attainment. Furthermore, adopting TfM approaches have tended to bring greater opportunities for professional development, which may have contributed to the improvements in mathematical performance at Key Stage 1 (Boylan et al., 2018; Boylan, 2019; Hall et al., 2016). However, the policy reforms brought about in England in favour of TfM seem unlikely to have the desired impact on achievement that was anticipated by politicians (Boylan, 2019).

This section has described various government initiatives which were used to promote the implementation of the TfM approach in English primary schools with the hopes of raising mathematical attainment of pupils. Although a lack of large-scale research in this area creates difficulties in evaluating the true impact of the TfM reform to date, it does not seem to have culminated in the gains that were anticipated by the DfE (Boylan, 2019). The next section discusses some possible reasons for this, as outlined in the research literature.

3.5 Problems surrounding the TfM reform in England

A review of the available research literature revealed some possible reasons why the TfM approach does not appear to have produced the mathematical outcomes that were anticipated by the DfE. Firstly, the rushing through of any educational reform without consideration for crucial components of education is commonplace in England due to the political system: reforms are brought about with the election of a new political party every 5 years, who attempt to make significant improvements during their short term in office (Bolden & Tymms, 2020). Bolden & Tymms (2020) argue that in order for careful consideration and trialling of new initiatives to take place prior to wide-scale roll-out, decisions about educational reform should be transferred to a non-political body, so that policy change is not tied to the short terms of political parties. Other prevalent arguments concerning the TfM reform (specifically) are based on a lack of consideration for the impact of context and culture on mathematical achievement, the difficulties of replicating pedagogical practices in another setting and the importance of the teacher's role in education reform. These issues are discussed in the following sections.

3.5.1 Considerations for the impact of context and culture on mathematical achievement

When the TfM initiative was being introduced in England, government officials cited *teaching methods* in SSHK specifically as the reason for their success in international mathematics tests (Jerrim & Vignoles, 2016). Under this presumption, research conducted by the DfE to better understand their success was based mainly on reviews of their curricula (DfE, 2012) and observations of teaching practices (Boylan et al., 2019; DfE, 2014b). However, many academics in the field of comparative education have called attention to the influence of factors outside of an education system's control, such as contextual and cultural conditions (Boylan et al., 2018; Clapham & Vickers, 2018; Elliott, 2014; Harris & Jones, 2017; Jerrim, 2015). A great deal of evidence suggests that these are far from 'irrelevant background noise' in the conversation regarding policy-borrowing (Harris & Jones, 2017, p.636). However, these factors seem to have been disregarded by policymakers in their arguments for the introduction of the TfM reform. Elliott (2014) describes this as a 'pick and mix' approach to policy-borrowing (p.36), with policymakers showing a tendency to focus on specific aspects of education systems that they can readily adopt while downplaying more challenging factors which may not fit with the borrowing country's governmental vision.

As convenient as it may be to focus solely on in-school factors, context and culture has been proven to play a fundamental role in educational outcomes (Harris & Jones, 2017), and this is no different in the case of SSHK's success in ILSAs. Chinese students are strongly influenced by Confucianism, which focuses on the individual's role in learning through hard work and perseverance (Shi & Jain, 2021). Confucian culture shapes, to a large degree, how Chinese education systems prioritise the acquisition of mathematical knowledge and skills and place a great deal of importance on a rigorous examination procedure in schools (Shi & Jain, 2021). Furthermore, Confucian values also shape student behaviour outside of school, with Chinese elementary students spending a substantial amount of time completing mathematics homework each week.

In addition to high levels of homework assigned to Chinese students, Chinese parents are more likely to have high levels of involvement in their child's education, including investment in after-school tuition (Jerrim, 2015; Kim, 2020). Tutoring for academic subjects is commonplace in China, with high-performing regions seeing up to 90% engagement in extra tuition from elementary and middle school pupils (Zheng, Wang, Shen & Fang, 2020). In contrast, 73% of 11-16-year-olds surveyed in England and Wales in the Ipsos MORI Young People *Omnibus* Survey reported never having had private/home tuition (Sutton Trust, 2019). Such high levels of out-of-school tuition could function to both raise academic standards in mathematics and also allow lower-achieving students to catch up with their higher-achieving peers and therefore minimise gaps in achievement within the classroom (Jerrim, 2015; Ma, Jong & Yuan, 2013; OECD, 2014b).

Further differences in terms of recruitment, training and typical daily routines and structures for primary teachers in China may also contribute to their students' high mathematical performance (Boylan et al., 2018; Clapham & Vickers, 2018; Jerrim & Vignoles, 2016). Ng (2013) described the rigorous selection of teachers from the top 30% of the graduate cohort as a key factor in Singapore's success in ILSAs, and Axbey (2020) pointed out that the majority of teachers in China are educated to master's degree level in their particular field (Axbey, 2020). Furthermore, teachers in China complete a five-year-long NQT (Newly Qualified Teacher) qualification which involves ongoing support from an experienced teacher as a mentor (Clapham & Vickers, 2018), as well as a reduced timetable to allow for hundreds of hours of subject-specific professional development during the NQT period (Boylan et al., 2018). Until recently, teachers in England completed one year's NQT (although this was extended to two years in September 2021 [DfE, 2021]).

Additionally, mathematics teachers in China only teach their specialist subject, whereas English teachers are generalists and teach across the curriculum (Boylan et al., 2018; Clapham & Vickers, 2018). Furthermore, teachers in China spend less time teaching than teachers in England (Axbey, 2020; Boylan et al., 2018; Clapham & Vickers, 2018), with more time being spent planning and preparing for lessons, accessing subject-specific professional development opportunities and research into best practice, and providing same-day intervention for struggling students (Axbey, 2020; Clapham & Vickers, 2018). Conversely, English primary teachers have only a few hours each week dedicated to planning and preparation (Axbey, 2020), and continuing professional development is typically generic and accessed on an ad-hoc basis (Cordingley, Higgins, Greany, Buckler, Coles-Jordan, Crisp, Saunders & Coe, 2015). Finally, there are also sharp contrasts in the value accorded to the teaching profession in English and Chinese society (Jerrim & Vignoles, 2016; Norton & Zhang, 2018): Chinese teachers are seen to hold a privileged status which attracts the very best candidates to the profession (Norton & Zhang, 2018), whereas teachers in England do not hold such a privileged position in society (Boyd & Ash, 2018), and the profession has low retention rates, particularly for early-career teachers (Fullard & Zuccollo, 2021).

Interestingly, where context and culture seem to be largely ignored by English policymakers, it has been accorded great importance in China (Elliott, 2014; You, 2020b). For example, Ng (2013) attributed the success of Singapore in ILSAs to five key factors: government-prescribed goals for education and stringent monitoring of standards; generous government investments for educational improvement including research into best practice; rigorous selection of teachers and a high status for the teaching profession in society; parental support in terms of educational resources, extra tuition and strong desire for their child's educational achievement; and ensuring a safe school environment. Similarly, Tan (2013) conducted interviews with head teachers and students in Shanghai to understand their beliefs regarding the reasons for their high performance in PISA. One factor which was accorded the greatest significance included a strong work ethic in students, which was underpinned by

Confucian values and reinforced by parents. In addition, the rigorous and highly competitive examination system was frequently cited as a contributing factor to the success of the school system in producing high mathematical results in ILSAs (Tan, 2013).

Although it would be unreasonable to assume teaching methods have an insignificant influence on mathematical performance, it seems there are many factors outside the classroom which are at play here, and these seem to be of critical importance to understanding the mathematical outcomes in SSHK. In terms of policy-borrowing, research demonstrates that what works best may very well be context-specific (Clapham & Vickers, 2018). Therefore, the most effective practices in country A may not necessarily prove effective in country B (Clapham & Vickers, 2018; Elliott, 2014).

Two key studies have provided evidence that culture is a more consequential factor than teaching methods or education systems: Feniger & Lefstein (2014) compared mathematical outcomes from PISA data for students living in Shanghai, Chinese students whose parents had immigrated to New Zealand and Australia, and native New Zealand and Australian students. They found that students of East-Asian descent living in New Zealand and Australia achieved similar PISA scores in mathematics to students living in Shanghai (Feniger & Lefstein, 2014), despite the fact that their school career had been based in a western education system deemed average by PISA standards. The authors concluded that parental cultural factors have a significant influence on mathematical outcomes (Feniger & Lefstein, 2014).

Subsequently, Jerrim (2015) expanded on this study by including data from the PISA background survey, in an attempt to identify observational characteristics which may be at play in determining why mathematical achievement in Australian children of East-Asian heritage was higher than that of their native Australian peers. He determined that second-generation East-Asian students outperformed students with two Australian parents by 100 test points on average, achieving scores consistent with highest-ranking PISA countries. In terms of causality, while no single feature could account for the difference in attainment between second-generation immigrant students from East-Asian regions (such as SSHK) and native Australian students, a combination of cultural factors appeared to have had a statistically significant effect, including a strong work ethic, out-of-school tuition, high value accorded to education and high aspirations for the future (Jerrim, 2015).

Although there are limitations in these studies in terms of drawing causal influences (due to the possibility of selection bias), these studies demonstrate the pitfalls of using PISA test scores as a measure of the quality of a country's education system and highlight the need for policymakers to recognise the importance of contextual and cultural factors that are outside of schools' control. Furthermore, these studies provide proof that high mathematical competency can be developed in countries that do not achieve highly in ILSAs (Jerrim, 2015). Therefore, implementing changes within a school system may be insufficient to replicate achievement from high-performing nations (Jerrim, 2015).

This section has considered the causal factors outside of school systems' control which may contribute in a significant way to mathematical achievement in high-performing PISA countries. However, even if we assume that pedagogies and teaching methods from these nations do have the greatest impact on student performance in high-achieving countries, the question of whether it can be successfully replicated in another context persists. The next section considers whether pedagogies can be successfully transplanted into a different context.

3.5.2 Can it be replicated in another setting?

Previous sections have examined the research evidence regarding causality and the impact that factors outside of the school's control could have on students' mathematical achievement. Considering the vast differences in contextual and cultural features between SSHK and England, as well as the findings from Feniger & Lefstein (2014) and Jerrim (2015), it raises the question of whether pedagogies from these regions can actually be replicated in a western country with similar outcomes. Clapham & Vickers (2018) argued that borrowed policy could not simply be parachuted into another culture and that the most conducive conditions for successful policy-borrowing are those where contextual and cultural ideologies are similar between borrower and lender. However, as has already been highlighted in previous sections, there are significant cultural and systemic differences which may impede the successful transplantation of the TfM reform (Axbey, 2020; Boylan et al., 2018; Clapham & Vickers, 2018; Jerrim & Vignoles, 2016; Marks et al., 2023; Ng, 2013; Tan, 2013; You, 2020b). As an example, the prevalence of an ‘anti-textbook’ culture in English schools has been discussed in previous sections, with teachers valuing professional autonomy in being able to choose from various resources, which contrasts sharply with Chinese teachers’ positive attitudes towards the exclusive use of one core textbook (Park & Leung, 2006; Zhang et al., 2017).

With regards to systemic changes, there seem to have been few attempts by policymakers and government officials promoting the TfM approach in England to make any changes to the structure and management of the education system to replicate conditions in SSHK (Boylan, 2019; Clapham & Vickers, 2018). For example, Chinese teachers’ reduced teaching timetable allows them to plan and prepare their lessons more carefully, access more continuing professional development (henceforth CPD), conduct research to improve their practice, and also carry out small-group interventions for struggling students. However, there have been no allowances or changes implemented to allow English teachers any time for this (Boylan, 2019). This has led both teachers and academics to worry that without the culture of parental involvement, extra tuition and strong work ethic in this country, coupled with a lack of time during the school day for teachers to help struggling students catch up, weaker pupils may be left behind (Axbey, 2020; Clapham & Vickers, 2018).

In addition, despite the NCETM’s promotion of a slower pace for coverage of mathematical concepts, there have been no changes made to the National Curriculum in England to reduce the vast amount of mathematical content that must be covered each year (DfE, 2013). In fact, the revised curriculum, which contains many fundamental features of a TfM approach, actually mandated that many mathematical topics be introduced at an earlier stage and at an accelerated pace, particularly in Key Stage 1 (Clarke, 2012).

Another problem with transplantation is the various interpretations of what TfM actually means and what it entails for teaching practice (Pawlik, 2020; Simpson & Wang, 2023). Simpson & Wang (2023) examined interpretations of ‘Mastery’ in policy makers, curriculum designers, researchers and secondary teachers and found that there was no consensus in the mathematics education community about what TfM means. They pointed out that neither the DfE nor the NCETM provide an unambiguous definition of the key elements that a TfM approach would encompass, making it extremely difficult for teachers to understand the changes needed to implement the approach consistently or in a way that is faithful to the borrowed model (Simpson & Wang, 2023). Adding to the confusion, Shearman (2021) found that different TfM programmes used by schools provided different definitions of TfM and emphasised different components as being key to implementing a TfM approach.

Confusion surrounding what TfM means in terms of teaching practices may have led to a mismatch between the changes implemented in UK primary schools and the core ingredients of teaching methods in SSHK (Boylan, 2019), and also caused a great deal of variation between schools in the way in which they have translated the TfM approach into their own school context. In evaluations of the MTE, Boylan (2019) noted that teachers adapted what they had seen and understood from their observations of the TfM approach in Shanghai to fit with their own practices AND other elements promoted by the NCETM.

Further evidence that the English adaptation of TfM lacks fidelity to SSHK teaching practices was found in a qualitative study by Axbey (2020). She conducted email interviews with English trainee teachers and qualified primary teachers following a two-week-long visit to China to observe mathematics lessons. She found that what the participants observed in mathematics lessons in Chinese primary schools did not match their previous understandings of the TfM approach as practised in England. Boylan (2019) argues that TfM is a ‘mashup’ of previous teaching practices from England (and elsewhere) that were deemed effective and later refined to correspond with teaching practices in SSHK, rather than a faithful representation of any whole pedagogical model (p.14), and this notion is corroborated by Simpson & Wang (2023), who pointed out the that the term Mastery is used to delineate teaching methods used in both Singapore and Shanghai, despite the fact that they do not share the same pedagogical approach.

This section has considered whether successful transplantation of pedagogical models into a different context is possible. It has outlined some of the barriers which may have prevented a faithful reproduction of TfM as practised in SSHK and has examined the research evidence about whether England’s interpretation of TfM is a faithful representation of practices used in high-performing PISA regions. One important factor that deserves a more detailed description is the role of teachers in implementing educational change. The next section explains the role of teachers in implementing educational reform and provides some reasons why teachers may resist policy changes.

3.5.3 Teachers and educational reform

Coles & Helme (2022) argue that teachers should be viewed as active partners who are integral to the translation of education policy into practice. The way in which teachers and school leaders construct and drive change is central to the success (or failure) of educational reform (Harris & Jones, 2019), therefore it is crucial that teachers are motivated to implement change. A review of the research literature outlines some important criteria that determine whether teachers decide to adapt their teaching practices and implement a new reform in full, and these are discussed below:

***Clarity of information provided* –** Many academics acknowledge the importance of providing a clear rationale and explanation of the theory underpinning educational reform, as well as demonstrations of any new procedures (Donnell & Gettinger, 2015). Providing clarity of information for educators often takes the form of CPD, and this is an essential factor which influences teachers' decisions about whether the policy change will be carried out faithfully in schools (Donnell & Gettinger, 2015).

***Alignment with the teacher's own teaching philosophies and practices* –** Pawlik (2020) argues that teachers hold strong perceptions about their teaching practices and have their own preferred way of teaching pupils. It is in terms of these personal beliefs about the nature of teaching and learning that teachers make sense of reform and make judgements about its relevance in their school context (Hill, 2001, as cited by Donnell & Gettinger, 2015) Where these perceptions differ significantly with recommendations from policymakers, teachers may resist the proposed change (Pawlik, 2020).

***The amount of time and effort needed to implement the change* –** Teachers are already required to juggle a range of demands on their performance, workload and professional development (Heijden, Beijaard, Geldens & Popeijus, 2018). However, with policy change comes the need for teachers to learn new methods and make adaptations to normal teaching practices (Bolden & Tymms, 2020). If the perceived gains are not seen to be worth the increased effort, teachers may resist the reform (Bolden & Tymms, 2020; Harris & Jones (2019).

Whether they view the change as important – Teachers are likely to question whether there is a legitimate need for the proposed change to be implemented (Bolden & Tymms, 2020; Burner, 2018), particularly when they view reform as a criticism of their professional practice (Bolden & Tymms, 2020). Teachers may also doubt whether the reform is relevant if their existing practices are deemed successful, particularly when this view is supported by attainment data and inspection reports (Clapham & Vickers, 2018). This problem may be exacerbated by the tendency of policymakers to focus on *how* changes can be introduced and implemented in the classroom, rather than on *why* the change is actually needed (Biesta, 2010). Furthermore, teachers may view reforms as being contentious with existing structures such as standardised tests (Drury, 2018).

There are therefore many reasons why teachers may resist changes involved in educational reform in general, and many of these reasons are relevant to the TfM reform. Previous sections have also discussed a variety of challenges for teachers in implementing a TfM approach in their mathematics instruction. Although a more in-depth examination of teacher resistance to the TfM reform (specifically) is beyond the scope of this dissertation, a summary of the factors which may have resulted in resistance to change for teachers is provided in Table 1, which is organised in relation to the criteria listed above.

Table 1: *Factors which may cause resistance to reform (TfM approach)*

Criteria	Factors which may cause resistance to reform (TfM approach)
Clarity of information provided	• Lack of clear definition provided by DfE or NCETM and no shared consensus of what TfM means in the education community (Simpson & Wang, 2023). • Different TfM-aligned programmes give different definitions of TfM (Shearman, 2021). • Confusion caused by a previous incarnation of Mastery which had a different meaning (Bloom, 1968; Boylan, 2019). • Large variations in how TfM is implemented in different schools, (Blausten et al., 2020; Marks et al., 2023).

	• Uncertainty about which components of TfM are responsible for mathematical success in SSHK (Boylan et al., 2018)
Alignment with the teacher's own teaching philosophies and practices	• Concerns about borrowing policy from countries with poor human rights records (Clapham & Vickers, 2018). • Wide range of contextual and cultural differences between China and England (Boylan, Maxwell, Wolstenholme, Jay & Demack, 2018; Clapham & Vickers, 2018; Elliott, 2014; Harris & Jones, 2017; Jerrim, 2015) • Differences in teaching timetables restrict English teachers' opportunity to carry out small-group interventions for struggling students (Boylan, 2019) and concerns that lower-attaining pupils will fall behind (Axbey, 2020; Clapham & Vickers, 2018). • High levels of curriculum content for mathematics compared with reduced content in China (DfE, 2013). • Teachers are often reluctant to use textbooks (Blausten et al., 2020; Oates, 2014). • Possible tensions between requirements of TfM and preparing children for standardised tests (Drury, 2018).
The amount of time and effort needed to implement the change	• Difficulties disseminating feedback from MTE to schools not involved in the Shanghai teacher exchange (Boylan et al., 2018). • Need for teachers to take part in CPD related to the TfM approach and familiarise themselves with new textbook materials (Boylan et al., 2018).
Whether they view the change as important	• Limited evidence to suggest that the TfM approach improves mathematical attainment for English pupils (Boylan, 2019; Clapham & Vickers, 2018). • Many teachers have seen reforms come and go without any perceived impact on attainment (Bolden & Tymms, 2020).

3.6 Conclusion

The TfM initiative represents the latest incarnation of a Mastery approach in mathematics, and widespread implementation of this model through regional Maths Hubs has resulted in this being the predominant approach underpinning mathematics teaching in English primary schools today. However, there are wide variations between schools in terms of implementation, with a range of different TfM-aligned programmes and textbooks now available on the market. Since the majority of research evidence on TfM is based on experimental methods, few studies have explored the opinions of teachers regarding the compatibility of different TfM-aligned programmes and resources with key requirements of the National Curriculum (DfE, 2013), such as problem-solving. Additionally, no research explores the motivation behind schools' choices of one particular programme or curriculum resource over another. Additionally, with confusion in English schools surrounding what particular elements make up a TfM approach, it would also be advantageous to explore how teachers interpret and make sense of the initiative, particularly since some important features of pedagogy in SSHK (such as problem-solving) appear to be neglected in guidance from the NCETM.

This chapter has explored some difficulties in implementing and sustaining changes through policy reform, with a particular focus on cross-cultural policy-borrowing. While the role of context and culture has been largely ignored by English policymakers in the roll-out of the TfM reform, the research evidence clearly illustrates that these factors have a pronounced impact on educational outcomes, and this is exemplified by studies which have investigated high achievement in children of Chinese immigrants in western education systems. According to the research, out-of-school tuition, parental involvement, strong work ethic and high aspirations for the future appear to be particularly consequential to educational outcomes.

The research points to teachers as being key to the success or failure of policy reform, and their willingness to adapt their teaching practices is based largely on the clarity of information they are provided with and their beliefs about whether the changes will be worth the amount of time and effort needed to implement them. However, very few studies investigate teachers' understandings of the TfM approach or their beliefs about its efficacy in the context of English schools.

3.7 Research questions

The literature review outlined in Chapters 2 and 3 explored the current knowledge base regarding NRPS and the TfM approach. Despite the high status accorded to NRPS in the National Curriculum (DfE, 2013) and within the education community, there are gaps in the research literature concerning opportunity for NRPS in the context of the TfM initiative. Furthermore, few studies have explored teachers' opinions of this reform, and none have investigated their perceptions about NRPS within a TfM approach.

Following this evaluation of the research literature, key research questions for this study were identified:

1) What opportunities for NRPS exist within the WRM TfM materials?
2) What are the perceptions of school senior managers about problem-solving opportunities within the WRM TfM materials?

The next chapter outlines the methods used to address the research questions listed above, providing justifications for the research design as well as details about each stage of the study.

Chapter 4: Methodology

Having completed a comprehensive review of the literature on NRPS and TfM, this chapter discusses the methods used to address the overarching research questions. Here, the methodological stance and research design are described first, and then the two stages of the research are described. An explanation of the measures taken to address ethical issues at each stage of the research process is presented before the chapter concludes with a brief explanation of where this study fits within the current research-base.

4.1: Methodological stance

In all empirical research, the researcher's views about the world will almost inevitably influence their choice of research approach. The term 'paradigm' refers to a set of beliefs which determine what type of data will be collected and the methods that should be used to carry out and interpret the results (Bryman, 2016, p.636). Historically, the most prevalent paradigms were positivist and interpretivist, which are directly juxtaposed: Positivism is guided by the belief that objective accounts of a phenomenon can be made and focuses on the collection of quantitative data (Newby, 2014). In contrast, interpretivism focuses on the different meanings that individuals bring to a given situation and involves the collection of qualitative data (Denscombe, 2017). However, Cohen, Manion & Morrison (2018) argue that researchers should use all the means and data at their disposal to understand the research issue, rather than restricting themselves to either qualitative or quantitative approaches.

Mixed methods research rejects the idea of an exclusive affiliation to either qualitative or quantitative approaches and instead encourages the view that there are different ways of looking at the world which help, rather than hinder, our understanding of it (Cohen et al., 2018). The rationale for using this approach is based on the idea that combining qualitative and quantitative methods can generate a deeper understanding of the research issue, and that using one type of data may limit the scope of the research (Cresswell & Plano Clark, 2017). Furthermore, mixing methods can enhance findings by providing alternative perspectives that give a more complete picture of the matter being studied (Denscombe, 2017).

A mixed methods approach is guided by a pragmatist paradigm, which is based on the belief that the methods used to gather data should be aligned with the needs of the research issue, rather than the allegiances of the researcher (Cohen et al., 2018; Denscombe, 2017; Punch, 2009). Unlike positivism and interpretivism, pragmatism acknowledges that multiple versions of reality exist, both objective and socially constructed, and adopts an eclectic methodology based on the criteria of fitness for purpose in answering the research questions (Johnson & Onwuegbuzie, 2004; Punch, 2009).

An alternative methodology which was explored in the initial stages of the research was that of 'grounded theory' which is fairly common in research projects which focus on the analysis of teaching resources (Rezat & Strasser, 2015, p.259). Originating with the work of Glaser & Strauss (1967), grounded theory is concerned with the generation of theory through an inductive process of systematic analysis and comparison between emerging ideas and existing data (Denscombe, 2017). Researchers following a grounded theory approach do not set out with a specific focus or framework for their investigation, instead allowing theory to emerge from the data (Cohen et al., 2018). However, this research project was focused specifically on exploring the opportunities for NRPS within WRM TfM materials using a framework drawn from the existing research base. This kind of objective approach is antithetical to grounded theory, therefore this approach was dismissed in favour of a mixed methods design.

Having discussed and justified the choice of methodological stance in terms of its appropriateness in approaching the research topic to be investigated, the next section describes the research design and outlines the sequence of methods implemented during the research.

4.2 Research design

Although the importance of NRPS is well-documented in the research literature, no empirical studies exist to date (that the author is aware of) which explore opportunities for NRPS within the context of a TfM approach to mathematics. Similarly, no research has yet investigated the highly popular WRM materials in terms of alignment with teaching practices in SSHK or our own National Curriculum (DfE, 2013), including requirements for NRPS.

This study aims to understand the opportunities for NRPS in the context of a TfM approach to mathematics using a mixed methods approach. First, quantitative data was collected through content analysis to give a broad picture of opportunities for NRPS and determine the frequency of these opportunities. The results from the quantitative analysis were then used to inform the qualitative aspect of the content analysis by identifying specific tasks to be analysed in depth and detail. Subsequently, additional qualitative methods were used to provide alternative perspectives on NRPS within the WRM materials, which involved face-to-face interviews with a headteacher and mathematics subject leader in England. This process involved an explanatory sequential design, whereby quantitative data is collected and analysed first, with the results informing the focus of qualitative data collection and analysis (Denscombe, 2017). This was intended to ensure that qualitative methods would produce greater depth and detail about the findings from quantitative analysis, provide alternative perspectives regarding the WRM materials and allow for meanings in the data to be corroborated and triangulated (Cohen et al., 2018; Denscombe, 2017).

The sequencing of the stages of the research is intended to allow for modifications to be made in later stages where appropriate. Therefore, each stage of the research complements and informs the next, so that the research questions can be answered as fully as possible. Table 2 illustrates the methods that will be used at each stage of the research procedure to address the two research questions.

***Table 2**: Data collection methods used to answer research questions*

Stage	QUAN/QUAL	Method	Research question
1	QUAN	Content analysis	1) What opportunities for NRPS exist within the WRM TfM materials?
	QUAL		
2	QUAL	Interview (face-to-face)	2) What are the perceptions of school senior managers in England about problem-solving opportunities within the WRM TfM materials?

The following sections describe each stage of data collection and the research approaches and methods employed at each stage.

4.3 Stage 1: Analysing WRM materials

The first stage of data collection within this study involved content analysis, which was undertaken by the author in the summer of 2021 to address the first research question: '*What opportunities for NRPS exist within the WRM TfM approach?'* Quantitative and qualitative methods were combined (intramethod triangulation) as a means of providing both breadth and depth of information concerning the opportunities for NRPS within WRM materials as a TfM-aligned resource (Denscombe, 2017). The findings from quantitative content analysis were intended to inform the qualitative aspect of content analysis by identifying tasks within the materials which would be explored in greater detail.

Quantitative methods are useful in measuring the frequency with which particular categories occur, to produce nominal data (Denscombe, 2017). As a first step in addressing the first research question, quantitative content analysis was used to ascertain how many opportunities there were for NRPS within the WRM materials and explore the proportion of NRPs in relation to other task types. Following this, qualitative methods were used to explore the nature of NRPs within the TfM-aligned materials. Since WRM materials claim to include opportunities for problem-solving in each lesson, it is important to identify what proportion of the tasks represent NRPs, whether this is sufficient to satisfy the requirements of the National Curriculum (DfE, 2013), and what the nature of these problems are in terms of their potential for developing a range of problem-solving skills.

This section begins by discussing textbooks as a feasible focus for educational research. A brief description of the WRM materials is provided as well as the rationale behind the sampling method utilised. Content analysis is discussed as a means of analysing problem-solving opportunities within TfM mathematics textbooks, and a revised analysis framework (based on an existing model) for quantitative content analysis is explored. Finally, considerations for content analysis and measures taken to secure high levels of reliability and validity are described.

4.3.1 Textbooks as a focus of educational research

Textbooks are increasingly recognised as a valuable indicator of students' opportunity to learn or engage in a particular aspect of a subject, including NRPS (van Zanten & van den Heuvel-Panhuizen, 2018). Although in England, textbooks have been traditionally viewed as one of many resources from which teachers can choose appropriate mathematical tasks (Hodgen, Foster, Marks & Brown, 2018; Wang & Fan, 2021), the recent TfM reform (with its emphasis on textbooks and mathematical coherence) has brought about a shift in how textbooks are used in many English schools, resulting in a more comprehensive approach to textbook use (Blausten et al., 2020; Hodgen et al., 2018). Where textbooks are adopted in this manner, they both indicate and influence the weight accorded to different aspects of mathematics in the classroom (Howson, 2013).

Fan, Zhu & Miao (2013) describe textbooks as 'a major conveyor of the curriculum' in mathematics (p. 635), as they are often seen as a reflection of the intended curriculum for schools (Yang, Tseng & Wang, 2017), which can promote a distinct pedagogical model and influence mathematics instruction (Hodgen et al., 2018; Marks et al., 2019). Furthermore, textbooks often provide the main source of mathematical activities for students (Banilower, Smith, Malzahn, Plumley, Gordon & Hayes, 2018). Consequently, textbooks have a significant influence on the teaching and learning of mathematics and can therefore provide valuable information to educational researchers (Fan et al., 2013).

Considering the significant influence of textbooks reported in the research literature, it is surprising that studies investigating mathematics textbooks represent a rather small area in the field (Marks et al., 2019). Fan (2013) argued that theoretical frameworks and research methods relating to mathematics textbook research remain 'fundamentally underdeveloped' (p.766), which may explain why this area of mathematics research has been somewhat neglected in recent studies. By offering a clear and transparent explanation and overview of the methods utilised for analysis of the WRM materials, this study aims to contribute towards the gaps in the research literature and provide a basis for further investigation to occur.

In their systematic review of existing research on mathematics textbooks, Fan et al., (2013) identified four areas of research focused on mathematics textbooks:

1) *Role of textbooks* – reflects philosophical or non-empirical works focusing on the role of textbooks in the teaching and learning of mathematics.
2) *Textbook analysis and comparison* – involves analysis of specific features of textbooks or comparing similarities and differences between two or more textbooks.
3) *Textbook use* – focuses on how textbooks are used by teachers and students and how they influence the teaching and learning of mathematics.
4) *Other areas* – includes studies which seek to describe a relationship between textbooks and students' mathematical achievement.

This study focuses on the second area of research focusing on mathematics textbooks outlined by Fan et al. (2013) and involves the analysis of NRPS within a TfM approach. Since textbooks have been promoted in recent years by government ministers in England as part of the TfM reform (Boylan et al., 2019), they may have an even greater influence on the available opportunities for NRPS than they might have had prior to the TfM initiative.

Since NRPS has been identified as an area of critical importance to students' mathematical education (Drury, 2018; Khalid et al., 2020; Merrienboer, 2013), there is a crucial need to investigate whether sufficient opportunities for NRPS exist within TfM lesson materials, such as textbooks and online materials. The next section outlines the sampling technique used for this stage of the research procedure involving quantitative content analysis.

4.3.2 Sample

The research question to be addressed in the first stage of the study involved the investigation of the opportunity for NRPS in the WRM TfM materials. WRM materials were chosen using purposive sampling – a non-probability sampling strategy whereby the choice is determined based on the characteristics of the sample itself (Denscombe, 2017), usually due to the unique or valuable attributes of the sample. WRM materials were particularly suitable for this study due to the following characteristics:

1) *Representativeness of a TfM approach* – WRM (2018; WRM, personal correspondence, 2021) claim to follow a TfM approach, therefore their materials provide a suitable sample to investigate the opportunities for NRPS within a TfM approach.
2) *Popularity* – WRM state that approximately 80% of primary schools (and 40% of secondary schools) in England use their resources to some degree (White Rose Maths, personal correspondence, 2021).
3) *Availability* – While many TfM textbooks and teaching resources require the purchase of physical textbooks, White Rose Maths' main resources are available online, reducing any potential problems in accessing the materials to be analysed.

Quantitative content analysis relies on large amounts of data (Denscombe, 2017), therefore lesson materials for three year-groups for a full year were included in the sample. Although analysing lesson materials for *all* primary year groups would have provided a more comprehensive analysis, this study was restricted in terms of time and resources available to incorporate a larger sample. Therefore, lesson materials for Years 2, 4 and 6 were selected (for pupils aged from 6 to 11) so that Key stage 1 as well as lower and upper Key Stage 2 materials were represented in the final sample.

Although Polikoff, Zhou & Campbell (2015) suggest that the results of content analyses are unlikely to be affected in any significant way if only every fifth item is coded, rather than the entire set of tasks, the purpose of this study was to ascertain the *maximum* number of NRPS opportunities available in the TfM materials. Therefore, the free-access materials (henceforth FA) and paid subscription materials (henceforth SUB) for year groups 2,4 and 6 comprised the units of analysis, and the term ‘task’ is used to denote the smallest unit that requires an answer from a student, as in van Zanten & van den Heuvel-Panhuizen’s (2018) study. Since some mathematical activities require several steps to be completed (for example, requiring a pupil to calculate the answer mentally and then explain the strategy they used to work out the answer), the number of units analysed may not correspond with the quantity of numbered tasks within the materials.

This section has described the decision-making process regarding the sample utilised as the focus of analysis and justified the use of purposive sampling in selecting the WRM materials (as a TfM-aligned resource). The next section provides a description and explanation of the quantitative content analysis, which was used to analyse the opportunities for NRPS within the sample.

4.3.3 Content analysis

Content analysis was chosen to address the first research question regarding opportunities for NRPS within the WRM TfM materials. This method involves the examination of documents, which may be printed or virtual, and often makes use of both quantitative and qualitative methods (Bryman, 2016). Krippendorff (2018) describes content analysis as ‘a research technique for making replicable and valid inferences from texts …to the contexts of their use’ (p. 24 as cited by Drisko & Maschi, 2016), and having established the importance of textbooks in mathematics education, content analysis of WRM materials was considered likely to provide an understanding of pupils’ opportunities for NRPS.

Content analysis has advantages in that it allows for careful exploration of a particular issue in an unobtrusive way (Bryman, 2016; Newby, 2014), and is also a well-established method of analysing the features of school textbooks (Bryman, 2016). However, it is important to note that content analysis involving textbooks can only measure *opportunities* to learn, and no inferences about the impact of textbooks on classroom instruction can be drawn without further investigation via additional research methods (Rezat & Strasser, 2015). Therefore, content analysis was used to identify the opportunities for NRPS within WRM materials, while additional methods were used in further stages of this study to gain further information about how the TfM materials were used in the classroom.

Charalambous, Delaney, Hsu & Mesa (2010) identified two different approaches to content analysis of textbooks: Horizontal analysis involves the examination of general characteristics of the textbook and the range of content included, while vertical analysis explores how one aspect of mathematics is treated within the textbook (Charalambous et al., 2010). This study aims to examine the opportunities for NRPS within a TfM approach, therefore vertical analysis was most appropriate for this research.

Drisko & Maschi (2016) refer to the quantitative approach to content analysis as basic content analysis, and this involves the use of a coding system to analyse and reduce the data to descriptive statistics. Interpretive content analysis, on the other hand, relies on a qualitative approach and draws on the interpretations of the researcher about key areas of interest within a text (Ginger, 2006, as cited by Drisko & Maschi, 2016). In this study, basic content analysis was used to gather objective data about the frequency of opportunities for NRPS in the TfM materials, while interpretive content analysis was used to obtain qualitative, descriptive data about the nature of problem-solving opportunities available in the materials. This entailed a mixing of methods to achieve both objectives.

Quantitative content analysis often involves the categorisation of data based on a predetermined framework, where the aim is to demonstrate objectiveness and transparency in describing coding procedures to allow for greater replicability (Bryman, 2016; Denscombe, 2017). To fulfil these aims, Bryman (2016) argues that content analysis should include the following aspects:

- Mutually exclusive categories – to ensure there is no overlap between different dimensions.
- An exhaustive list of categories – so that every possible dimension can be categorised.

- Clear instructions – which outline the factors to be taken into account during the coding process.
- A clear unit of analysis – with clearly defined parameters for the investigation.

These measures reduce the possibility of bias in the coding process and increase the reliability of the results (Bryman, 2016). Quantitative content analysis also allowed for the identification of specific tasks which would be explored in greater depth through qualitative content analysis, which was hoped to provide a deeper understanding of the nature of NRPS opportunities offered to pupils within TfM-aligned materials.

Qualitative content analysis involves examining underlying themes in the data to allow for a thorough description of an aspect presented in the materials (Drisko & Maschi, 2016). Hsieh and Shannon (2005) outlined three approaches to qualitative content analysis based on the varying degrees of induction involved:

***Conventional qualitative content analysis* –** the coding categories emerge inductively from raw data.

***Directed content analysis* –** begins with existing research and, through immersion in the data, additional codes emerge to supplement existing ones.

***Summative content analysis* –** relies on counting and comparing content (such as keywords) to make interpretations about the underlying context.

During the literature review, an existing framework for categorising tasks within textbook materials was identified as an appropriate tool for quantitative analysis, therefore a directed approach to content analysis was chosen for the purposes of this study. Van Zanten & van den Heuvel-Panhuizen's (2018) framework is based on the degree to which higher-order cognitive thinking is required to provide an answer and whether a known solution procedure was available to the solver. In order to make these judgements, the framework takes into account a number of different criteria which affect the degree to which cognitive challenge is required to solve it, such as the number of conditions that the unknown must meet, the interdependency of the data provided to the solver, the order in which data is provided, and in the case of tasks with multiple solutions, whether all possible correct solutions need to be provided in the answer. To be classified as an NRP, the task must meet at least two of the criteria listed under that category (Van Zanten & Van den Heuvel-Panhuizen, 2018). In contrast, straightforward tasks (henceforth STs) are tasks which can be solved by a straightforward calculation or are given after an example/explanation demonstrating how to solve them (van Zanten & van den Heuvel-Panhuizen, 2018).

Zhu & Fan (2006) point out that classifying problems as NRPs (or otherwise) can be difficult since a great deal depends on the prior knowledge and capability of the pupil attempting to solve it. However, to mitigate this limitation, van Zanten & Van den Heuvel-Panhuizen's (2018) framework includes a category labelled 'grey-area tasks' (henceforth GATs) which may be an NRP depending on the features of the solver (p.827). The use of this category also satisfies Bryman's (2016) argument for an exhaustive list of categories to reduce the risk of error in the coding process.

This framework was based on extensive research and had been subject to reliability checks by an independent expert. Furthermore, the methods utilised in the development of this framework were peer-reviewed, providing further reassurance of its credibility and appropriateness for the aims of this study. Due to the limitations inherent in small-scale research, it was beneficial both in terms of time and resources to use a 'tried and tested' framework rather than devise a completely new framework or coding schedule.

The next section explains the procedures and outcomes of the pilot study, which took place in July 2021, and describes the decision process that occurred during this phase to respond to difficulties with using the original framework.

4.3.4 Pilot

Research involving quantitative content analysis typically involves a pilot study using a small sample of the materials before the main coding is carried out (see for example Cannon, 2021; Dalton, 2017; Ercan, 2020; Zhang & Qi, 2019). This allows for any potential problems with the coding frame to be identified at an early stage (Schreier, 2012) and provides an opportunity for the researcher to become more familiar with the categories (Ercan, 2020). Furthermore, conducting a pilot study can help researchers to determine the amount of time needed to carry out the final analysis (Cohen et al., 2018), and it was expected that the pilot study would also inform decisions about the final sample selection.

In total, just over 10% of the units of analysis were coded in the pilot study (10% for each subscription type [FA and SUB] and year group). Appropriate sample sizes for pilot studies involving quantitative content analysis range from 10-50% (Cohen et al., 2018), and although some similar studies using analysis of textbooks have used a greater proportion of tasks for the pilot study (see, for example, Ercan, 2020), the high number of tasks within the materials and lack of time available for one researcher to analyse the materials restricted opportunity for a more comprehensive pilot phase. Tasks were selected for the pilot study at random using a random number generator (tasks within the FA materials were numbered to allow for this), to ensure there would be sufficient variability and to increase the level of opportunity to identify a range of different task types.

In the pilot phase, a coding sheet was used to keep a record of how each task had been categorised according to van Zanten & van den Heuvel-Panhuizen's (2018) framework – as either an NRP, GAT, or ST. However, difficulties arose in using the analysis framework due to the presence of tasks within the WRM materials that did not fall perfectly within the given categories. Although these task types could have been discounted, Denscombe (2017) warned that researchers should avoid neglecting data that does not fit the analysis in mixed methods research, so that any explanations for the additional categories can be investigated. Subsequently, the decision was made to amend van Zanten & van den Heuvel-Panhuizen's (2018) framework to incorporate all task-types present within the WRM materials, to increase instrument validity and ensure the analysis framework contained sufficient categories to categorise all tasks correctly.

Table 3 shows the revised analysis framework, which is an amalgamation of the analysis framework from van Zanten & van den Heuvel-Panhuizen (2018), and task descriptors for different task types found within the sample TfM materials (based on the literature review). Although this was not an anticipated part of the data collection process, the most appropriate means of obtaining an exhaustive list of categories seemed to involve an inductive or open coding process in the initial stages of content analysis, whereby additional categories emerged from the data.

Table 3: *Revised analysis framework (Adapted from van Zanten & van den Heuvel-Panhuizen, 2018)*

NRPs	**The task meets two or three of the following features:** The unknown has to meet three or more conditions. The data provided are interdependent. The data are provided in another order than needed for solving the task. **In case the task has multiple correct solutions:** All possible correct solutions have to be given. ***or*** The total number of all possible correct solutions has to be given.
GATs	**The task meets one of the following features:** The unknown has to meet three or more conditions. The data provided are interdependent. The data are provided in another order than needed for solving the task. **In case the task has multiple correct solutions:** One possible correct solution has to be given. ***or*** Some but not all possible correct solutions have to be given.
STs	The task is solvable by straightforward calculation, or the task is offered after an explanation or an example which demonstrates how it can be solved.
Reasoning tasks	Require pupils to explain their solution method. ***or*** Require pupils to diagnose misconceptions (e.g., explain Annie's mistake). ***or*** Require pupils to justify assertions. (e.g., explain how they know an answer is correct) ***or*** Require pupils to say what is the same and what is different about two different mathematical procedures. ***or*** Require the pupil to give an opinion (e.g., which solution method is easiest/more efficient?).
Representing tasks	Require the pupil to represent a mathematical concept in another way, such as representing a given value on a place value chart using counters.
Mathematical investigations	Can result in a wide range of different approaches and outcomes. Allows pupils the freedom to pose their own mathematical problems.
Problem-posing	An activity involving the generation of a new problem, or a reformulation of a particular given problem.

Although amending the analysis framework may reduce the reliability of the results and objectivity of the research since there was no opportunity in this small-scale study to implement reliability checks as conducted by van Zanten & van den Heuvel-Panhuizen (2018), it was decided that accommodating a more comprehensive list of categories was an important step in providing a more complete picture about NRPS solving within a TfM approach. For example, the inclusion of all task types created opportunities to ascertain the importance accorded to NRPS in the materials, through comparison of their frequency in relation to other task types.

During this phase, the majority of tasks which were categorised as GATs or NRPs according to the analysis framework were finding all possibilities problems (see Chapter 2 for a description of this type of problem). As a result, it was decided that each task that had been categorised as an NRP would be given a code according to the type of problem it represented. In order to create a coding frame to enable the problems to be categorised in this way, descriptions for the different problem types were taken from Barmby et al's., (2014) book on problem-solving, and a brief version of this description was developed to assist with the coding process (see table 4).

Table 4: *Coding sheet for different problem types.*

Problem Type	Description	Code
Finding all possibilities	• Has multiple possible answers. • One or more possible answers have to be given *or* the total number of possible correct solutions has to be given.	**FAP**
Logic	• Necessitates a step-by-step approach. • Each step follows logically from the previous step.	**LG**
Word	• Presented as written text. • May be single step (requires one calculation) or multi-step (requires more than one calculation).	**W**
Diagrams/visual	• Almost all of the information is presented as a diagram or visual representation.	**DV**
Finding rules and describing patterns	• Involves the investigation of a particular case. • Allows opportunities for pupils to make generalisations about mathematical rules	**RP**

4.3.5 Analysis of data

The WRM materials were analysed following a mixed methods research design incorporating both quantitative and qualitative content analysis. Quantitative measures were used to quantify the opportunities for NRPS within the TfM materials using the revised analysis framework (appendix D), during which time particular tasks were identified for in-depth, qualitative description to investigate the nature of problem-solving opportunities in the materials.

While categorising tasks within the WRM materials, particular care was taken that guidance for using the analysis framework (van Zanten & van den Heuvel-Panhuizen, 2018) was referred to frequently during content analysis, in order that different tasks were classified accurately. Due to the nature of problem-solving being relative to an individual's prior experience and knowledge, making judgements about whether a task is classified as a NRP or not is extremely difficult – particularly since the exact features of the students solving the task is not known by the researcher (Zhu & Fan, 2006). Van Zanten & van den Heuvel-Panhuizen (2018) further acknowledged the inherent difficulties in distinguishing between NRPs and GATs, and provided some exemplification of the decision-making process in making this distinction.

In Figure 11, the task on the left is a magic frame in the form of a triangle which the solver must fill in with the numbers 1-9 so that each side of the triangle sums to 17. In following van Zanten & van den Heuvel-Panhuizen's (2018) analysis framework (see Table 3), a task must satisfy two or three of the listed criteria if it is to be classified as an NRP. Below is an explanation of how the features of this task can be used to classify it as an NRP:

- **The unknown has to meet three or more conditions –** each side of the triangle must sum to 17, numbers 1-9 must be used, no numbers can be repeated/omitted.
- **The data provided are interdependent –** the three numbers in the corners of the triangle are involved in two combinations of numbers of numbers which sum to 17.

Van Zanten & van den Heuvel-Panhuizen (2018) reasoned that pupils are unlikely to identify a starting point for solving the problem without some exploration and investigation, and that this task requires creative thinking and a high level of cognitive demand in combining numbers which sum to 17 while also taking into account that the three numbers in the corners of the triangle are involved in two combinations of numbers which sum to 17. Therefore, they classified this task as an NRP.

The task on the right (Figure.11), which is also in the form of a magic frame, could not be classified as an ST since it requires the solver to identify the appropriate starting point and the subsequent steps to be taken to fill in all of the missing values. However, the level of cognitive demand and opportunity for creative thinking are significantly reduced compared to the task on the left, since the upper left empty cell can be derived based on the numbers already filled in in the top row. After finding one missing value, the solution process can continue in a similar way, where empty cells can be filled in based on two values that have been given or previously filled in values from the same row, column or diagonal. Following the analysis framework (van Zanten & van den Heuvel-Panhuizen, 2018) the task met only one of the listed criteria:

The data provided are interdependent – since each value that was inputted affected the subsequent missing value in each row, column and diagonal within the magic frame.

Therefore, this task was classified as a GAT by van Zanten & van den Heuvel-Panhuizen (2018).

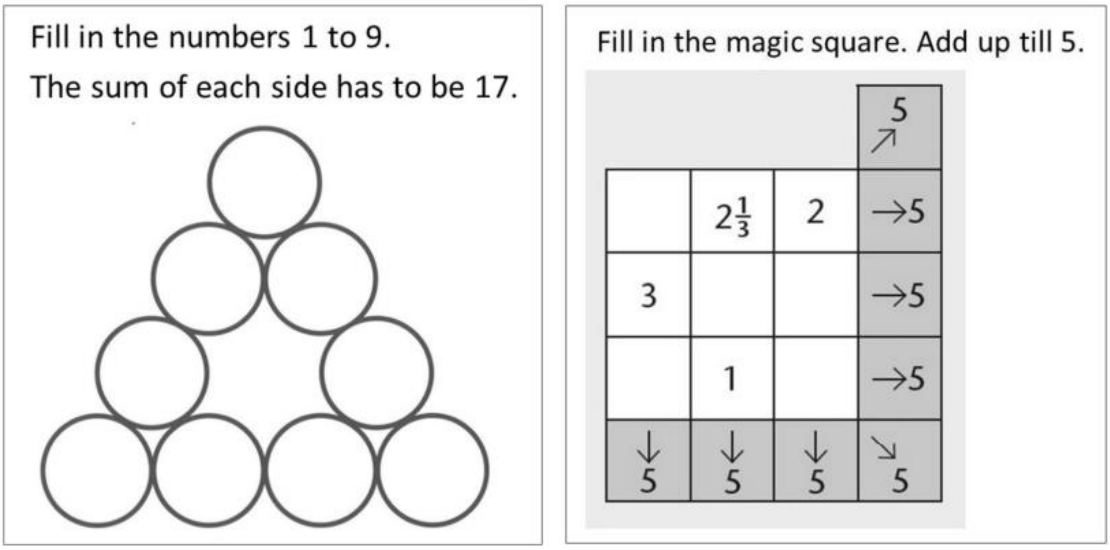


Figure 11: *Van Zanten & van den Heuvel-Panhuizen (2018) used two examples to exemplify the distinction between an NRP (left) and a GAT (right).*

Whilst recognising the limitations of this kind of interpretation of individual tasks, this research aimed to replicate the classification process carried out by van Zanten & van den Heuvel-Panhuizen (2018) using an adaptation of their original analysis framework. For example, Figure 12 (taken from the Year 4 materials) was classified as an NRP since it satisfied the following criteria:

The data provided are interdependent – since each person's age is expressed only in terms of its relationship with another person's age.

The data are provided in another order than needed for solving the task – since the solver is required to move back and forth between the different clues in order to identify relationships between them.

These features of the task are likely to engender a high level of cognitive demand for Year 4 pupils who may struggle to identify the starting point to solve this problem. In addition, pupils must draw on their knowledge of multiplying and dividing by 10 to support their deductive reasoning in order to solve it. Students may recognise that Jack's number must be 350 as his number is ten times smaller than Mo's but ten times larger than Dora's. Once they have established this, it logically follows that Mo's number is 3,500 and Dora's number is 35, leaving the number 53 which must belong to Alex.

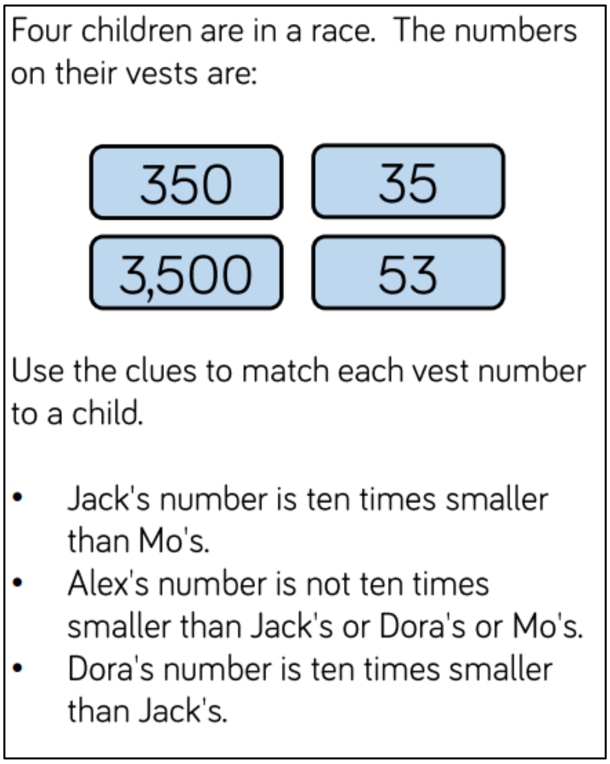


Figure 111: *An LG problem within the Year 4 sample materials (WRM, 2023)*

As an example of a task that was categorised as a GAT, Figure 13 (taken from the Year 6 sample materials) is similar to the example provided by van Zanten & van den Heuvel-Panhuizen (2018). In this task, although the data provided are interdependent, students can start with either the horizontal or the vertical calculation. Although technical mathematical difficulty may arise due to the need to find a common denominator, earlier fluency tasks where students were required to do this mean that cognitive struggle is unlikely to be significantly increased when the calculations are shown in a different representation.

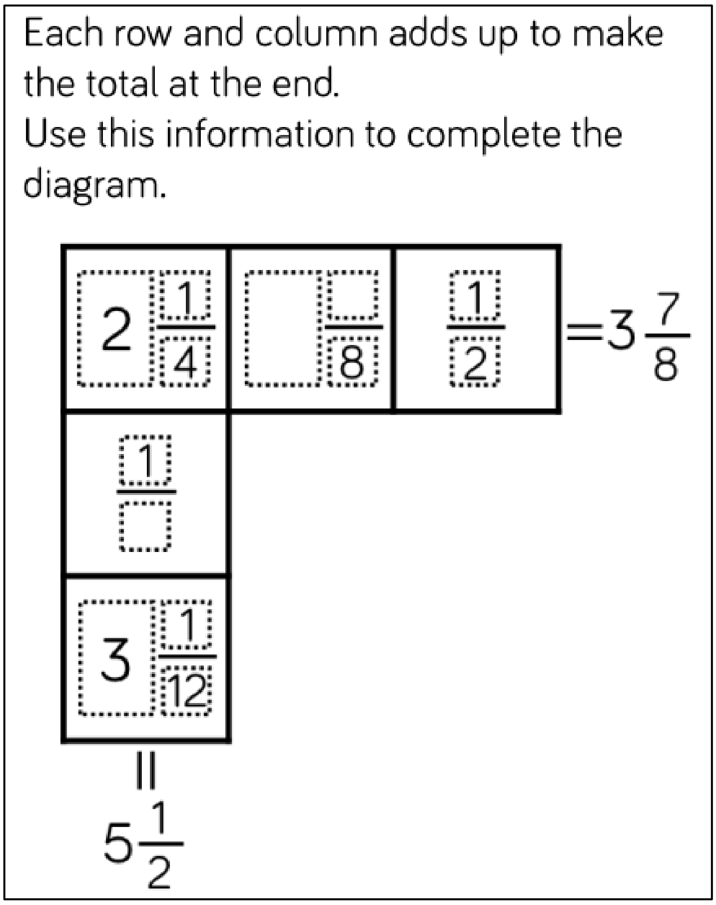


Figure 13: *A further LG GAT within the Year 6 sample materials (WRM, 2023)*

Other guidance for classification of tasks provided by van Zanten & van den Heuvel-Panhuizen (2018) involved the stipulation that for worded tasks, the property of having multiple steps was not a defining feature of an NRPS activity:

> …we consider multiple steps not a distinctive feature as such - when multiple steps in solving a task involve nothing more than just straightforward calculation, we consider that task still to be a straightforward one. (p.833)

Therefore, multi-step worded questions were categorised as routine tasks unless specific indicators of GATs or NRPs within the framework were evident.

Due to the mixed methods design of this research project, there are many different ways in which the results could be presented; in either a quantitative or qualitative style (Schreier, 2012). Owing to the nature of the data collected (being drawn from both quantitative analysis and qualitative methods), there was a need for the results to be presented in different ways. Therefore, it was decided that the results from the quantitative analysis of the TfM sample materials would be presented in a manner which focused on the categories rather than specific cases (Schreier, 2012), while the data collected via qualitative content analysis would be reported using rich description and explanation of particular tasks.

Schreier (2012) describes 3 strategies that can be used for presenting quantitative data: absolute frequencies, descriptive group comparisons, and inferential statistics. Analysis of textbooks can only describe the *opportunity* to learn specific aspects of mathematics (van Zanten & van den Heuvel-Panhuizen, 2018), therefore absolute frequencies were suitable for describing the number of NRPs within the TfM sample. However, since the different data sets from WRM varied in terms of the number of units of analysis, descriptive group comparisons were also useful to report percentages alongside absolute frequencies (Schreier, 2012). For example, the FA materials typically had fewer tasks than the SUB materials (1513 on average compared to 2216 for SUB materials). Therefore, it was expected that the results would be more easily interpreted where proportions were given alongside absolute frequencies.

4.3.6 Considerations for content analysis

Several considerations must be made when conducting content analysis to ensure high measurement validity and reliability. Validity in the context of a content analysis method concerns the ability of the analysis framework or coding frame to capture the issue under investigation (Schreier, 2012). Due to the relative and personal nature of NRPS, there are inherent difficulties in judging whether a task within a textbook (or online lesson resource), is an NRP or not, as the prior experience and features of the problem-solver are unknown (Zhu & Fan, 2006). However, in this study, the main analysis framework was based on an existing one which had undergone validity and reliability checks by an independent expert in mathematics education. Although this framework was adapted to meet the needs of the particular sample and represent all tasks appropriately in terms of categorisation, the criteria for identifying and categorising NRPs and GATs remained unaltered, therefore it can be reasonably assumed that the framework was a valid measure of the opportunities for NRPS within the TfM materials.

Validity can be negatively impacted if the analysis framework does not account for all dimensions of the issue under investigation, however, van Zanten & van den Heuvel-Panhuizen's (2018) framework included an additional category (GATs) for items which did not satisfy the criteria for NRPs (due to more limited opportunities for high levels of cognitive demand and creativity), but could not be categorised as straightforward due to particular features (e.g., the solver must identify a suitable starting point). Furthermore, when the pilot study found tasks within the TfM materials which did not fit any of the categories on van Zanten & van den Heuvel-Panhuizen's (2018) framework, additional categories were created and an amended version of the framework was developed so that all tasks could be categorised accurately, and this new framework was revised systematically during the pilot study to ensure high levels of validity.

Bryman (2016) also pointed out that validity can be negatively impacted by errors in the implementation of the coding framework or schedule if the researcher does not understand fully how it works and therefore does not administer it as intended. To ensure the framework was being used precisely, the researcher periodically returned to van Zanten & van den Heuvel-Panhuizen's (2018) research article, which provided detailed explanations of the decision-making process and examples of how different items should be categorised using the analysis framework.

High reliability in content analysis refers to whether the measurement instrument (or analysis framework) produces the same results at different times or with different coders (Cohen et al., 2018). Some researchers assess the reliability of their coding framework using Cronbach's alpha, which measures internal consistency via correlation. However, Cronbach's alpha is more suited to determining the reliability of scales used in questions from surveys or questionnaires and is not particularly suited to determining the reliability of coding frameworks used in content analysis (Lombard, Snyder-Duch & Bracken, 2002). Furthermore, the original analysis framework, which included the indicators used to identify NRPs within the TfM materials, had already undergone reliability checks as part of van Zanten & van den Heuvel-Panhuizen's (2018) study. Therefore, the procedures implemented to ensure high levels of internal consistency in this study were focused on intra-rater reliability, which is the extent that the categorisation of data by one coder is consistent over time (McHugh, 2012). Ideally, an inter-rater reliability check using a second independent coder would have been conducted, but the large sample size and lack of time available during this study meant that this was not possible.

Bryman (2016) argued that intra-coder reliability may be threatened when the investigator categorises the items in slightly different ways over time with greater familiarity with the framework. To reduce the risk to intra-coder reliability, the researcher returned periodically to the materials and used a comparison coding sheet (as used by Ercan, 2020) to check for stability of coding over time. There are a variety of statistical methods commonly used in content analysis for reliability testing. For example, Cohen's kappa measures the amount of agreement in the way the data has been coded by different individuals or on different occasions, factoring in the possibility of random chance or guessing (McHugh, 2012). However, drawbacks of this method include difficulties with interpreting kappa values, and McHugh (2012) argues that while kappa statistic may be appropriate if coder/s are likely to make guesses when categorising the data,

> *…if raters are well trained and little guessing is likely to exist, the researcher may safely rely on percent agreement to determine interrater reliability. (p.282)*

Percent agreement is the percentage of the coding decisions which match over time or between different coders, which is a widely used index of intra-rater reliability (Vevea, Zelinsky, Orwin, Cooper, Hedges & Valentine, 2019). Although this method fails to account for the amount of agreement that would occur by random chance, significant efforts had been made to ensure that a reasoned decision had been made for all items within the TfM materials, including returning periodically to van Zanten & van den Heuvel-Panhuizen's (2018) research paper as a reminder of how various task types should be categorised. Furthermore, percent agreement has advantages in that it is easy to calculate and can be interpreted relatively easily (McHugh, 2012).

To calculate the percent agreement, the number of units that were categorised in the same way is divided by the total number of units of coding, then the answer is multiplied by 100. The closer the percent agreement is to 100%, the more the analysis can be considered reliable, with 80% agreement being the minimum acceptable intra-rater reliability specified in many texts (McHugh, 2012). During the pilot study, the percent agreement was 92%. Units which were coded differently were returned to retrospectively, to identify why a different decision was made on different occasions. This provided opportunities to become more familiar with the analysis framework, refine the indicators for different task types where necessary (for task types that were added to the original analysis framework) and improve understanding of it before conducting the main content analysis. For the main analysis, the percent agreement was 94%, which can be considered reliable. Where there was disagreement between the first and second categorisation, I reviewed the explanation of the decision-making process in van Zanten & van den Heuvel-Panhuizen's (2018) research paper and then returned to the unit to make a final decision about how the unit should be coded.

4.4 Stage 2: Perceptions of school senior managers in England about problem-solving opportunities within the White Rose TfM materials

The second stage of data collection within this study comprised a case study, which took place in the summer of 2022 and was focused on the second research question: '*What are the perceptions of teachers in England and China about problem-solving opportunities within the WRM TfM materials?'*

This section begins by explaining the reasoning behind the decision to carry out a case study of one primary school in England and explores the advantages and drawbacks of a case study as a means of investigating the second research question. Following this, the reasoning and justification for methodological decisions pertaining to interviews are provided, then the pilot study is described, and the sampling technique is outlined. Finally, an explanation of data analysis methods is provided and considerations for case study and interview are discussed.

4.4.1 Case Study

As has been discussed earlier in this paper, analysing textbooks can provide valuable insights regarding students' opportunities to engage in particular mathematics activities such as problem-solving (van Zanten & van den Heuvel-Panhuizen, 2018). However, textbook analysis cannot provide information about how teachers implement these materials in their regular teaching practices, or why school leaders choose to adopt particular materials for mathematics instruction (Fan et al., 2013). Case study research involves gathering rich, qualitative data and investigating the features of a unique, bounded case in relation to the research issue (Bryman, 2016). This approach recognises the importance of context, holism, and natural settings in determining cause and effects and allows for a deeper understanding of the research problem in all its complexity (Cohen et al., 2018; Punch, 2009). Therefore, a case study approach was used in this stage to explore the perceptions of decision-makers in one school regarding WRM materials, including their reasons for using the materials. The purpose of this approach was to obtain a deeper understanding of how and why these materials were used within a school which had chosen to focus on NRPS in their school development plan.

4.4.2 Interview

Interviews are a common method in case study research, including within mixed method designs (Denscombe, 2017). Although conducting interviews is time-consuming, they are extremely valuable in allowing the researcher to obtain privileged information about a complex case from individuals, based on their position and experience (Denscombe, 2017). Face-to-face interviews were carried out with the headteacher (henceforth HT) and subject leader for mathematics (henceforth SLM) in the case study school (henceforth School A). Although interviews may not completely capture the reality of classroom practices (Cohen et al., 2018), the emphasis of this study was to understand the perceptions of decision-makers for mathematics about the WRM TfM materials and NRPS in their school, including the reasons behind their decisions to adopt a TfM approach and follow WRM materials, and to focus on NRPS in the school development plan.

The interviews used open-ended questions to allow informants to give more detailed answers (James, 2012), and each interview lasted approximately 30 minutes. To reduce the possibility of bias, care was taken during the design of interview questions to ensure that they did not reflect any potential preconceptions of the researcher. The interviews were semi-structured, using a set of initial guidance questions (Table 5) as well as relevant impromptu follow-up questions during the interviews to allow for clarification and elaboration of responses to engender a deep understanding between the researcher and informant (Denscombe, 2017).

Table 5: *Interview guidance questions used in the pilot study*

Question 1	What does Teaching for Mastery mean to you?
Question 2	Can you tell me what influenced your decision to incorporate a Teaching for Mastery approach in mathematics in your school?
Question 3	Can you tell me what influenced your decision to use White Rose materials in your school?
Question 4	Are White Rose materials used exclusively as mathematics resources in your school? Why/why not?
Question 5	Can you tell me about your reasons for focusing on NRPS in your school development plan?
Question 6	Do you think the White Rose materials provide sufficient opportunities for NRPS in mathematics? Why/why not?
Question 7	What do you think of the problems included in White Rose materials?

4.4.3 Pilot interview

Before collecting qualitative research data through interviews, it is best practice to carry out a pilot interview. This can provide valuable insights regarding the appropriateness of the guidance questions in eliciting answers to the research questions and ensure the questions can be easily understood by informants (Denscombe, 2017). Conducting a pilot interview can lead to the generation of new questions or an improved interview schedule to ensure a deep understanding of a phenomenon (Malmqvist, Hellberg, Mollas, Rose & Shevlin, 2019). Furthermore, a well-conducted pilot interview can encourage greater methodological rigour (Ismail, Kinchin & Edwards, 2018) and places the researcher in a better position to conduct high-quality research in the final study (Malmqvist et al., 2019).

Although the pilot interview should ideally replicate the conditions intended for the final interviews as closely as possible (Cohen et al., 2018; Newby, 2014), Covid-19 regulations restricted opportunities for face-to-face meetings earlier in 2021 and therefore differences between the pilot interview and final interviews were unavoidable during this study. The pilot study was carried out in April 2021 over Microsoft Teams and was not audio-recorded, although written field notes were taken throughout, whereas the final interviews were conducted face-to-face in School A. During the process of the pilot interview, the need for a thorough description of the attributes of both the school and interviewees was highlighted. Developing an understanding of the contexts and backgrounds that influence participants' interview responses is an important aspect of case study research (Cohen et al., 2018; Punch, 2009). Therefore, to obtain this information without creating additional workload for the participants involved in this study, details about the school were acquired from a recent Ofsted report (see table 7).

It was also realised during the pilot interview that a greater mutual understanding of the interviewees' definitions of problem-solving and NRPS was crucial to the validity of the results obtained through interviews. Therefore, two more questions were added to the interview schedule to ascertain exactly what the terms mean to each participant. Table 6 shows the final interview schedule used for the main research study (see questions 6 and 7).

Another difficulty that presented during the pilot interview was that complex ideas were put forward very quickly which could cause the interviewees to become uncomfortable and reduce the quality of their answers. Powney & Watts (2018) argue that the interviewer effect may cause teacher interviewees to focus on giving 'correct' answers rather than describing their own ideas and experiences if they are questioned on a topic they are not confident about. To put interviewees at ease at the start of the interview, it was decided to include a starter question about the interviewees' teaching philosophy (question 1, Table 6) in the interview schedule. This was hoped to 'break the ice' at the very start of the interview by allowing participants the freedom to express themselves freely and demonstrate that their opinions and perceptions are valued.

Table 6: *Final interview guidance questions used for the main research study*

Question 1	Can you tell me about your teaching philosophy?
Question 2	What does Teaching for Mastery mean to you?
Question 3	Can you tell me what influenced your decision to incorporate a Teaching for Mastery approach in mathematics in your school?
Question 4	Can you tell me what influenced your decision to use White Rose materials in your school?
Question 5	Are White Rose materials used exclusively as mathematics resources in your school? Why/why not?
Question 6	Can you tell me what ‘problem-solving’ means to you?
Question 7	Can you tell me what ‘NRPS’ means to you?
Question 8	Can you tell me about your reasons for focusing on NRPS in your school development plan?
Question 9	Do you think the White Rose materials provide sufficient opportunities for NRPS in mathematics? Why/why not?
Question 10	What do you think of the problems included in White Rose materials?

4.4.4 Sample

School A was chosen using a form of non-probability sampling called purposive sampling, which involves selection based on desirable characteristics of the particular case and its capacity to provide rich data regarding the research issue (Punch, 2009). The desirable characteristics of the school are outlined below:

- The school had been using WRM materials for a reasonable amount of time to allow them to draw conclusions about them (over four years).
- Teachers used WRM resources for long and medium-term planning as well as textbook-style lesson resources.
- The headteacher had chosen to focus on developing NRPS in mathematics in the SDP in the last year.

The third feature of the school raises the question of why such action should be needed since WRM materials are purported to align with the National Curriculum (DfE, 2013), which holds NRPS as a key area of mathematics education. This is therefore likely to be worthy of exploration and to provide rich and detailed information about the research issue. Some additional data about the school is provided in table 7.

Table 7: *Attributes of School A*

Location	North-East England
School size	Slightly larger than average-sized primary school
School category	Voluntary aided
Form entry	Single-form entry
Pupil premium	Low proportion of pupils eligible for support from pupil premium funding
SEN (special educational needs) and disability	Below-average proportion of pupils who have SEN or a disability
Ethnicity and EAL (English as an additional language)	Very high proportion of pupils from minority ethnic backgrounds and with EAL. However, most pupils speak fluent English
Most recent Ofsted rating	Good (October 2019)

Interview participants were also chosen using purposive sampling due to their position as decision-makers about mathematics in the school. As a consequence of their relative positions in School A, these participants were responsible for the decision to adopt a TfM approach to mathematics, to follow WRM materials as a representative of this approach, and to focus on NRPS in the SDP. Furthermore, the participants, having made these decisions for School A, were in a position to explain their motivation and the underlying reasons behind each of these decisions. Additional information about interview participants is shown in Table 8.

Table 8: *Attributes of interview participants in School A*

Participant	Gender	Year group	Position in school	Years of teaching experience	Years of experience in the role
A	Male	-	Headteacher	20+	20+
B	Female	5	Mathematics subject leader	20+	10+

4.4.5 Analysis of data

Three methods of data analysis were considered for this stage of the research study: grounded theory, content analysis and thematic analysis. However, as discussed earlier in this chapter, grounded theory requires the corresponding methodological values to be embedded in the research study, and this would therefore be inappropriate for mixed methods research (Bennett, Barrett & Helmich, 2019). Content analysis and thematic analysis have similar processes and involve coding of data (Cohen et al., 2018), however, content analysis focuses on the systematic classification of data, whereas thematic analysis involves identifying common themes, which is useful in determining opinions, knowledge or values from a data set (Clarke, Braun & Hayfield, 2015). Therefore, it was decided that thematic analysis would be more appropriate to describe particular themes that emerged in interviews.

An inductive approach to thematic analysis was used, which allows for themes to emerge naturally from the raw data, rather than restricting the analysis to preconceived ideas (as with deductive analysis). Since each school's experience with introducing and using TfM materials is likely to be different, inductive analysis was more appropriate to ensure that important themes which may not fit with preconceived ideas were fairly represented in the reporting of results. Clarke et al., 2015) outline six steps involved in thematic analysis:

1) Familiarisation – this involves getting to know the data by reading carefully to obtain a broad overview of the data and making initial notes.
2) Coding – phrases of interest are highlighted and given codes or shorthand labels to describe their content. Other phrases which match these codes are also highlighted and more codes are added as further items of interest occur. As the process progresses, some codes may be discarded or merged with other codes, and new codes may emerge. Subsequently, the highlighted data is collated into corresponding groups by their final code, which provides a basic overview of the recurring points in the data.
3) Generating themes – patterns between the codes that were created in step 2 are examined and similar ones combined to identify themes, which reveal important information about the issue under investigation.
4) Reviewing themes – the researcher returns to the data and compares the themes with the raw data, ensuring the themes accurately reflect the meanings in the interview data and do not neglect important points.
5) Defining and naming themes – succinct names which adequately describe the themes are created, and a brief definition for each theme is developed.
6) Writing up – each theme is explored in a logical sequence, with examples of the data provided as evidence.

These steps were followed carefully during the thematic analysis of interview data in this study. Additionally, Braun & Clarke (2006) outlined another consideration for researchers regarding the level at which themes are identified. For example, with a semantic approach, the researcher identifies themes based on the surface meaning of the data (Braun & Clarke, 2006). In contrast, thematic analysis at the latent level involves an additional examination of underlying meanings that can be inferred from the data (Braun & Clarke, 2006). Therefore, a latent approach was applied so that underlying or implicit meanings were explored as well as explicit statements by interviewees.

The next section describes the considerations made for carrying out interviews with participants in School A, as well as the thematic analysis method used to analyse the raw data from interviews.

4.4.6 Considerations for case study, interview and thematic analysis

While case study research is used frequently in education research as a means of acquiring a rich understanding of a situation, this method is not unproblematic: due to the large number of variables inherent in education research, the results cannot be generalised in the way that this concept is used in the natural sciences. While no claims of generalisability are made in this dissertation, Bassey's (2001) concept of 'fuzzy' generalisations, used frequently in education research, may be a possible outcome of this study (p.5). These can be described as 'particular events may lead to particular consequences' (p.6), meaning that where the researcher endeavours to thoroughly explain the features of the particular case under investigation, some predictions can be made about the extent to which the findings would be replicated in other similar contexts. Therefore, efforts were made during this study to obtain and report appropriate and relevant details about School A and its features, so that sufficient information is provided on which to base potential comparisons.

Further difficulties with the interview method involve the possibility that interviewees' responses are interpreted by the researcher in a way that does not reflect their original meaning, which would reduce the credibility of the findings (Denscombe, 2017). To check the accuracy of the interpretations drawn from interview data, respondent validation was sought via email. Although ideally, this validation technique would involve both participants, The HT at School A took a new job shortly after the interview took place. However, obtaining respondent validation from the SLM provided some assurance that the data obtained via interview were interpreted accurately.

Shifting the focus towards considerations for thematic analysis, Clarke et al. (2015) warned about similar issues regarding the tendency of the researcher to be subjective in the creation of themes during thematic analysis. With this in mind, close attention was given to the data to ensure opinions and statements were represented fairly and that none of the data was in any way obscured. Furthermore, in reporting the findings, the risk of decontextualising the data was considered, since interview data reported in a thematic style can result in the integrity and wholeness of participants' responses being lost (Cohen et al., 2018). Although it is not possible to eliminate the risk of researcher bias completely (Cohen et al., 2018), efforts were made to mitigate this risk as far as possible by periodically returning to the raw interview transcripts to ensure that responses were not reported out of their original context wherever this could cause ambiguity.

4.5 Ethical considerations

The methods and procedures implemented during the course of this study complied with ethical guidelines issued by the English Educational Research Association (BERA, 2018), and ethical approval for this study was granted by Durham Education Ethics Committee on 26th March 2021. This section describes the measures taken to address ethical issues at each stage of the research process.

The first stage of this research study involved content analysis of TfM materials which were available on the WRM website. Some of these materials were published in the public domain and were intended to be disseminated to a wide audience, negating any threat with regard to privacy (Denscombe, 2017). Other resources (which formed part of the sample in this study) required a school subscription to gain access to the materials, therefore no images or direct quotes were taken from these resources, to comply with copyright law.

The second and third stages of this study involved human participation via face-to-face and email interviews. Each participant received an information sheet (appendix A) which specified the purposes of the research, outlined participants' right to withdraw, and detailed the procedures implemented to maintain confidentiality and anonymity in accordance with General Data Protection Regulation (BERA, 2018). Interview participants received a participant information sheet prior to interviews via email and were given opportunities to gain clarification for any items which they did not understand. These measures ensured that participants could make informed decisions about their involvement in the study (Brooks, te Riele & Maguire, 2014; Cohen et al., 2018).

Informed consent was given by all participants (appendix B), and prior to face-to-face interviews taking place, verbal consent for audio recording was obtained from the participants in School A. These interviews took place in School A to ensure convenience for the participants, and care was taken to procure a quiet space to minimise distractions. Audio recordings were transcribed in the same week and anonymised, and transcriptions were stored on the researcher's personal computer, which was password-protected (Brooks et al., 2014).

4.6 Conclusion

This chapter has provided an explanation and justification of the methodology used in the process of study. The research base surrounding opportunities for NRPS within TfM-aligned mathematics textbooks is small, and no existing research (that the author is aware of) combines methods of content analysis with interviews of senior managers in an English school to explore opportunities for NRPS within a TfM approach. Therefore, this mixed-methods study contributed towards a gap in the research literature. The research project involved two stages: Stage 1 involved content analysis of TfM materials, with data obtained via quantitative analysis informing subsequent qualitative analysis of the materials; and Stage 2 comprised a case study of one school in England involving semi-structured interviews with decision-makers for mathematics.

The following chapters provide a detailed description of the findings from the research. Chapter 5 aims to answer the first research question: *What opportunities for NRPS exist within WRM TfM materials?* The findings from the quantitative and qualitative content analysis are combined in order to provide an in-depth explanation of the opportunities for problem-solving within the WRM materials. Chapter 6 addresses the second research question: *What are the perceptions of school senior managers about problem-solving opportunities within the WRM TfM materials?* which draws on findings from interviews to explore perceptions of decision-makers in one school in England. Each of these chapters concludes with a summary of the results in relation to the corresponding research questions.

Chapter 5: Results - Opportunities for NRPS within WRM TfM materials

In Chapter 4, the procedure and steps which were followed to address the three research questions were described, which involved a mixed methods design. This chapter presents the results of the quantitative and qualitative content analysis which were used to address the first research question: *What opportunities for NRPS exist within the WRM TfM materials?* In the first section, results from the quantitative content analysis are presented using absolute frequencies and descriptive comparison groups to outline the number of opportunities for NRPS within the TfM materials and draw comparisons with proportions of other task types (as used by Dalton, 2017; Ercan, 2020; van Zantan & van den Heuvel Panhuizen; Yang, Tseng & Wang, 2017). Following this, the results from the qualitative analysis are presented alongside detailed explanations of the different tasks, with a focus on the types of NRPs provided within the TfM materials. In the final section, a summary of the combined content analysis is provided.

5.1 Quantitative analysis – quantifying opportunities for NRPS within a TfM approach

In this section, the results of the quantitative content analysis are outlined. This part of the research aimed to examine and quantify opportunities for NRPS within lesson materials which were aligned with the TfM approach, in order to address the first research question. As explained in chapter 4, van Zanten & van den Heuvel-Panhuizen's (2018) original categories (STs, NRPs and GATs) were supplemented with four additional categories: reasoning tasks, representing tasks, mathematical investigations and problem-posing tasks. Indicators for each of these new categories were drawn from the research literature, however, there were no changes made to the criteria for the categories from the original framework.

5.1.1 NRPs

The adapted version of van Zanten & van den Heuvel-Panhuizen's (2018) framework (appendix D) was used to conduct a quantitative content analysis of the TfM materials. Table 10 shows the results of the analysis for each data set, which includes materials for two subscription types (FA and SUB) for three different year groups (2,4 and 6). In all of these data sets, the percentage of NRPs ranges from ≈1-2%, with the lowest proportion found in the Year 2 SUB materials. The SUB materials typically had lower proportions of NRPs than the FA materials, with ≈1-1% compared with 2% in the FA materials. However, the number of NRPs provided within each data set was fairly similar, providing between 24 and 36 opportunities for NRPS during the academic year. The exception to this was the Year 2 SUB materials, in which only 4 tasks could be classified as NRPs.

Table 9: *Distribution of task types by year group and subscription type*

Task types	**Year 2 (Subs.)**		**Year 2 (Free access)**		**Year 4 (Subs.)**		**Year 4 (Free access)**		**Year 6 (Subs.)**		**Year 6 (Free access)**	
	N	%	N	%	N	%	N	%	N	%	N	%
Straightforward task (ST)	1311	72	1066	72	2147	84	1248	77	1892	83	1073	75
Grey-area task (GAT)	48	*3*	31	*2*	41	*2*	52	*3*	39	*2*	54	*4*
Non-routine problem (NRP)	**4**	***≈1***	**30**	***2***	**29**	***1***	**36**	***2***	**29**	***1***	**24**	***2***
Reasoning	242	*13*	264	*18*	257	*10*	201	*12*	217	*10*	198	*14*
Representing	208	*12*	93	*6*	78	*3*	84	*5*	94	*4*	69	*5*
Investigation	0	*0*	0	*0*	1	*≈1*	2	*≈1*	3	*≈1*	6	*≈1*
Problem-posing	0	*0*	0	*0*	6	*≈1*	4	*≈1*	1	*≈1*	5	*≈1*
Total tasks	1813		1484		2559		1627		2275		1429	

(Due to rounding off, some percentages do not add up correctly).

The Year 4 FA materials provided the most NRPs (36), followed by the Year 2 FA materials (30), while the SUB materials for Years 4 and 6 both provided 29 opportunities for NRPS. The lowest frequencies of NRPs were found in the Year 6 FA materials (24) and the Year 2 SUB materials. However, the Year 2 SUB materials had significantly fewer opportunities for NRPS, with only 4 examples of NRPs within this data set (see Figure 14). It seems strange that the materials targeted towards older children do not provide the most NRPs, since it could be expected that older children are better able to deal with NRPS activities, however, the proportion of NRPs is similar for each data set. With 39 academic weeks in the academic year, the number of NRPs found within each data set would translate to less than one opportunity per week for NRPS (4-36 opportunities within a year) if one subscription type was used for a particular year group.

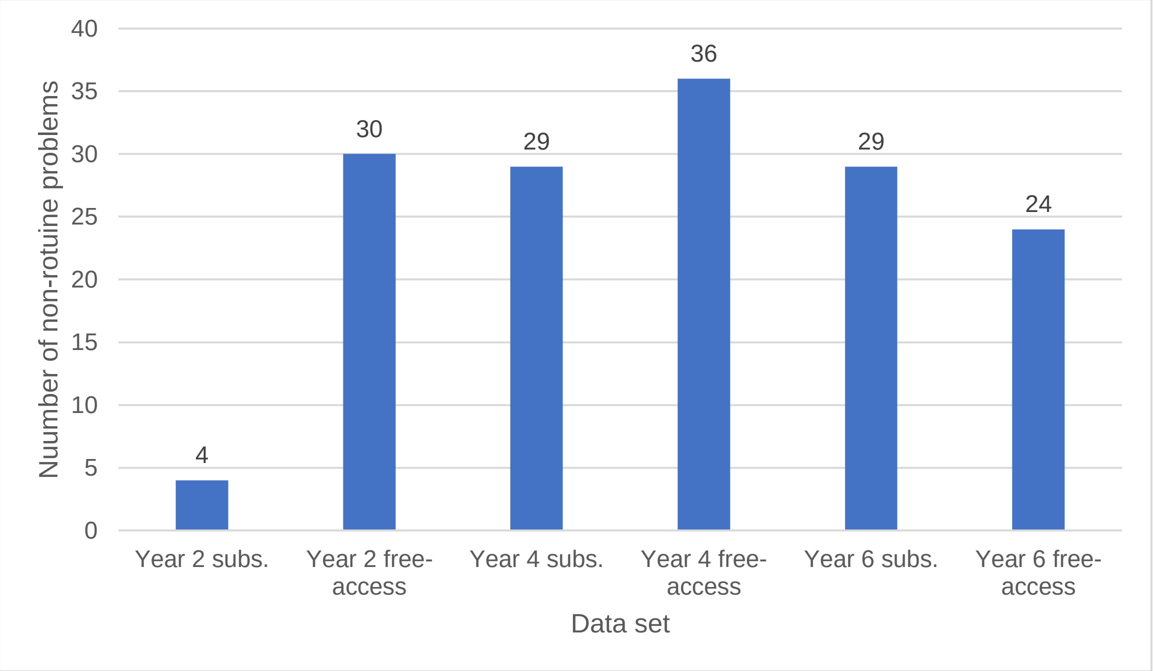


Figure 14: *Number of NRPs for each data set*

When the SUB and FA materials are combined for each year group, the frequency of NRPs rises to between 34 and 65 opportunities (see Figure 15). The greatest number of opportunities for NRPS is found in the combined Year 4 materials (65), while the combined Year 6 materials provide fewer opportunities (53) and the combined Year 2 materials offer the fewest opportunities for NRPS (34). Where all NRPs from both subscription types are used for a particular year group, this would translate to less than 2 opportunities per week (on average) for Year 4 and Year 6 children to engage in NRPS during the academic year, and less than 1 opportunity per week for Year 2 children.

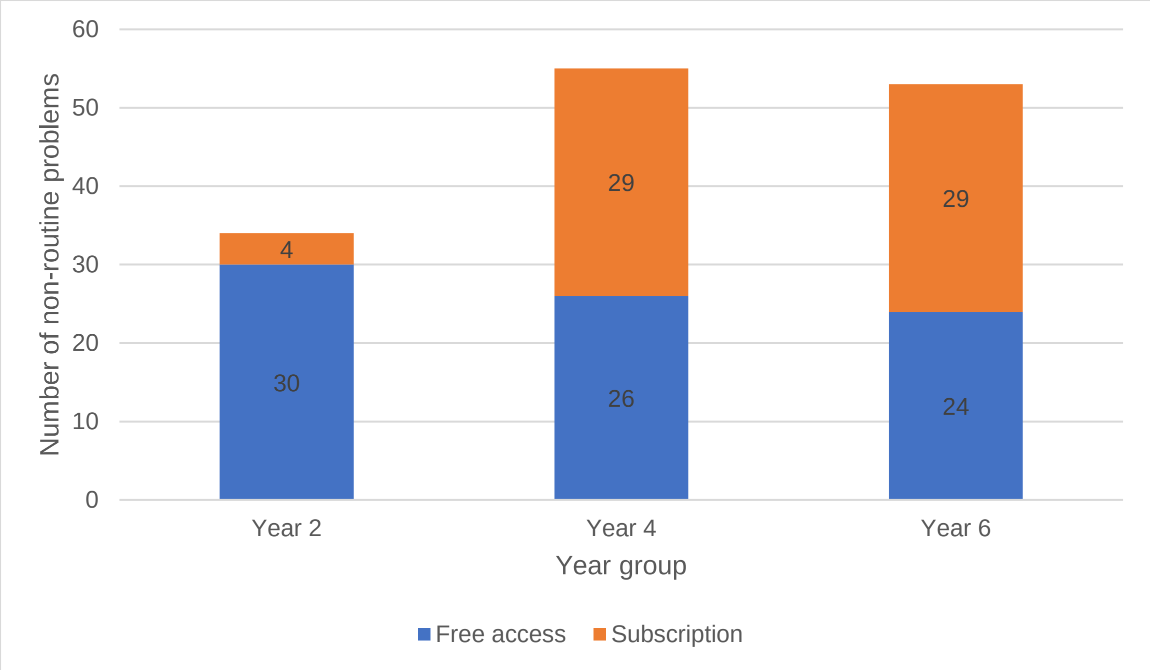


Figure 15: *Number of NRPs when SUB and FA materials are combined for each*

The next section outlines the findings in relation to GATs, which are more limited in their capacity to elicit high levels of cognitive demand and creativity than NRPs, but share some features associated with NRPS, such as the lack of an obvious starting point (van Zanten & van den Heuvel-Panhuizen, 2018). Therefore, GATs could constitute a problem for some learners, depending on the relative experience of the student.

5.1.2 GATs

As with NRPs, the number of GATs is low: the Year 6 FA materials provided the most GATs (54) followed by the Year 4 FA materials (52). The Year 2 and 4 SUB materials provided 48 and 41 GATs, respectively, while the Year 6 SUB materials and Year 2 FA materials had the fewest GATs (39 and 31 respectively). In comparison with NRPs, there were more GATs within each data set than there were NRPs, although the difference varied significantly for different data sets: there was only one more GAT than NRPs within the Year 2 FA materials, while the number of GATs was 12 times that of NRPs within the Year 2 SUB materials (48 GATs compared with 4 NRPs). There was also a larger difference between the number of GATs and NRPs within the Year 6 FA materials (a difference of 30, with 54 GATs compared with 24 NRPs. For all other data sets, the difference ranged from 10-16, with more GATs than NRPs in each case (see Figure 16).

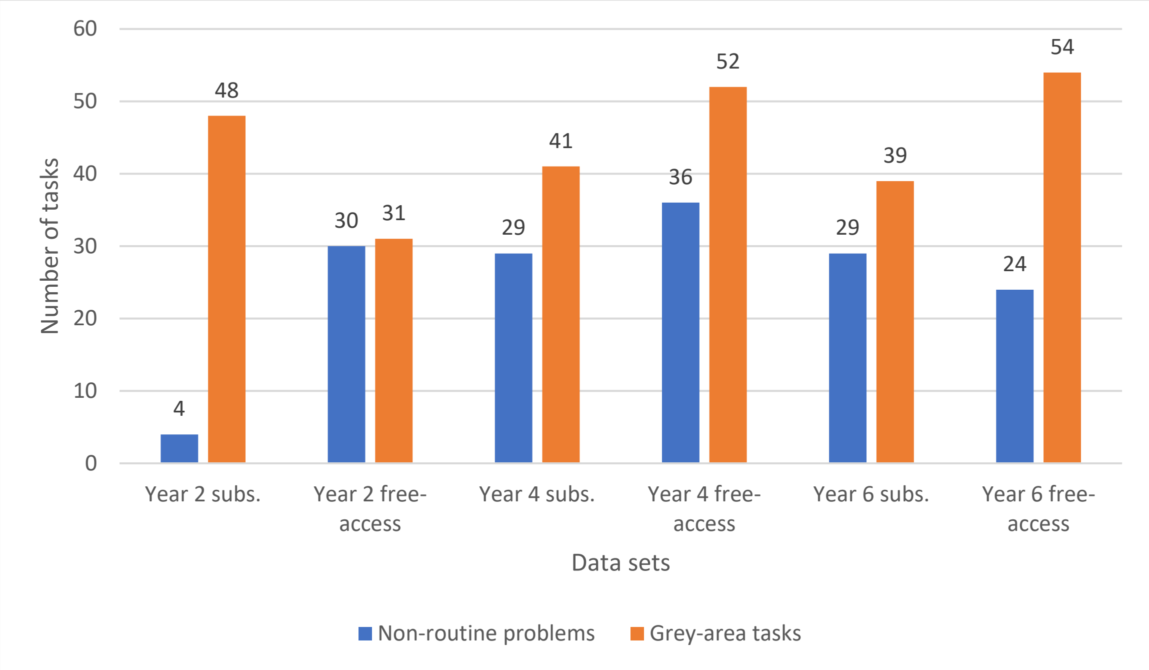


Figure 126: *Number of GATs compared with NRPs in each data set.*

In general, there was also a greater proportion of GATs than NRPs within the data sets, with the percentage ranging from 2-4% depending on the year group and subscription type, compared with ≈1-2% for NRPs. GATs represented 4% of the total tasks within the Year 6 FA materials, 3% of the Year 4 FA and Year 2 SUB materials, and 2% of all tasks within each of the remaining data sets. With more GATs than NRPs in all of the materials, this could create difficulty for teachers in selecting NRPS activities for their pupils.

5.1.3 Combined NRPs and GATs

Previous sections have specified that GATs will not usually constitute a problem for all pupils within a given year group, however, it is useful to explore the maximum possible opportunities provided for NRPS including GATs, as these may be problematic for children with lower levels of problem-solving proficiency (van Zanten & van den Heuvel-Panhuizen, 2018). Figure 17 shows the total number of opportunities provided for NRPs and GATs combined for one academic year, which ranges from 52 (Year 2 SUB materials) and 88 (Year 4 FA materials). The Year 6 FA materials provided the second highest frequency of NRPs and GATs combined with a total of 78, and the SUB materials for Years 4 and 6 offered a combined total of 70 and 68, respectively. There was a combined total of 61 for the Year 2 FA materials.

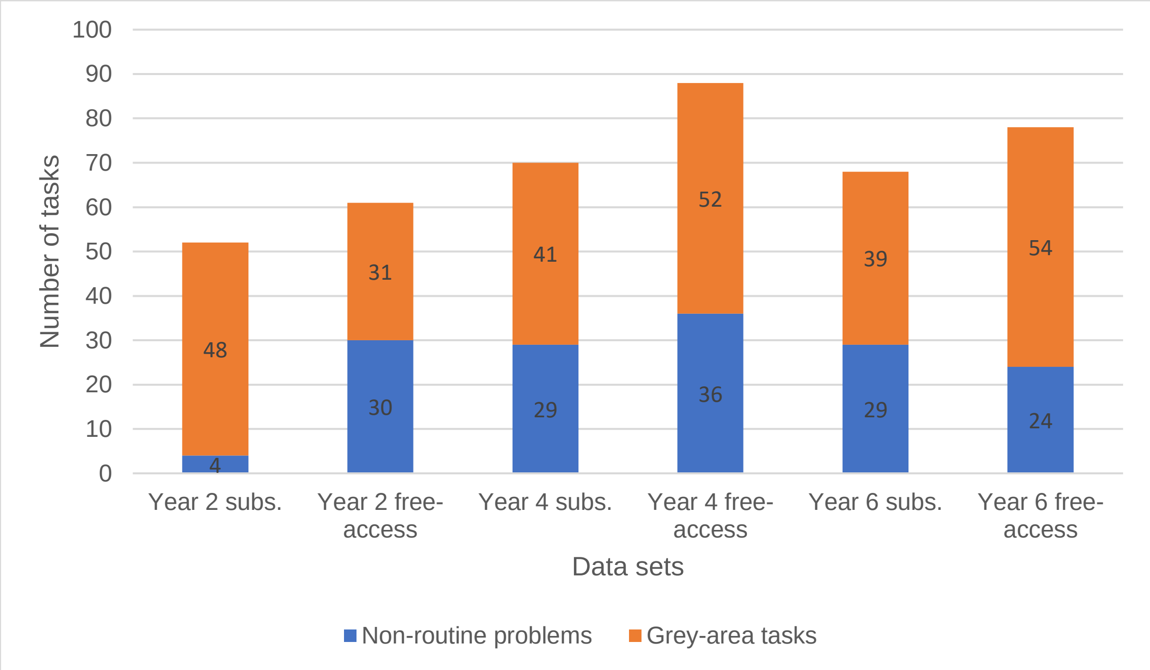


Figure 17: *Number of NRPs and GATs combined for each data set.*

The proportion of NRPs and GATs combined accounted for between 3-5% of the total tasks for each subset of the materials. The Year 4 FA materials had the highest proportion of NRPs and GATs combined (5%), followed by the Year 6 FA materials and Year 2 FA materials (with 4% for each). All other data sets had a proportion of under 3% (due to rounding figures, this may not match the combined percentages presented in Table 10).

<u>5.1.4 Other task types</u>

Within the WRM materials, STs (see Chapter 5 for description) accounted for between 72% (Year 2 SUB and FA materials) and 84% (Year 4 SUB materials) of each subset of the WRM materials. Although this proportion was significantly lower than that found by van Zanten & van den Heuvel-Panhuizen (2018) and Hidayah & Forgasz (2020) of over 90%, the incorporation of additional categories in the amended framework is likely to have contributed to this difference. Reasoning and representing tasks, for example, could not be classified as STs within the original framework as often there was no calculation needed to provide an answer (see original and revised analysis frameworks – appendix C/D), however, these tasks were usually fairly straightforward in nature and unlikely to require high levels of cognitive demand or creative thinking (for example, requiring the student to explain how they worked out the answer to a previous task or representing a two-digit number by drawing counters in a place value chart).

Although these task types were not the focus of this study, examining the proportions of each category of task individually did allow for inferences to be drawn about the importance placed on NRPS in comparison to other task types. The proportion of representing tasks, for example (which focused on the representation of numbers and values in different ways), was higher than that for NRPs, making up between 3-6% for most of the materials. However, the proportion was higher for the Year 2 SUB materials, where 12% of tasks were representing tasks, compared with under 1% for NRPs. Reasoning tasks also formed a greater proportion of the total tasks than NRPs on average, forming between 10-18% of the tasks for each subset of the TfM materials, compared with between ≈1-2% for NRPs. If reasoning and representing tasks were combined with STs, these would account for between 94-97% of the tasks in the materials (see Figure 18).

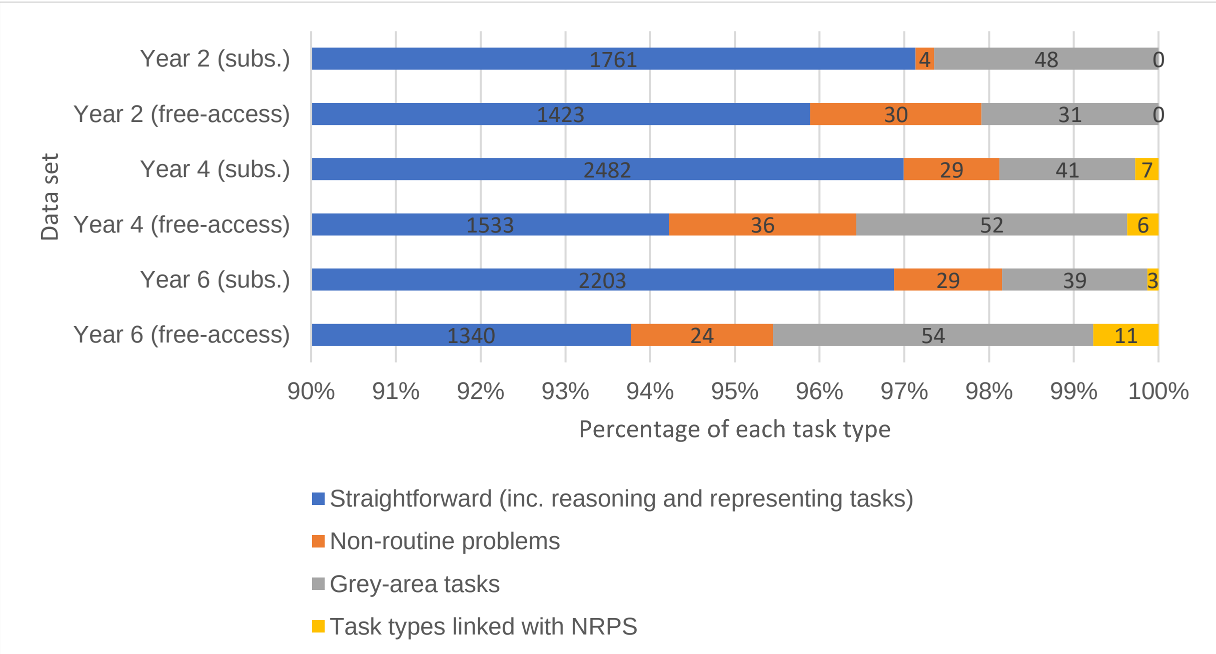


Figure 138: *Distribution of task types in the WRM materials*

In contrast, tasks which link more closely to problem-solving, such as investigations and problem-posing, made up a much smaller proportion of the total tasks in each data set. Investigations, for example, accounted for less than 1% of the total tasks within each data set: There were no opportunities within any of the Year 2 materials for children to carry out mathematical investigations and 1 to 6 opportunities within each set of materials for Years 4 and 6, with the Year 6 FA materials offering most opportunities for mathematical investigation. Problem-posing also formed a small proportion of between 0 and ≈1% of tasks within the materials in the different subsets. There were no opportunities for Year 2 pupils to pose problems in either the SUB or FA materials and between 1 and 6 opportunities within each set of materials for Year 4 and Year 6, with the greatest number of opportunities for problem-posing provided in the Year 4 SUB materials (see Table 10/Figure 18).

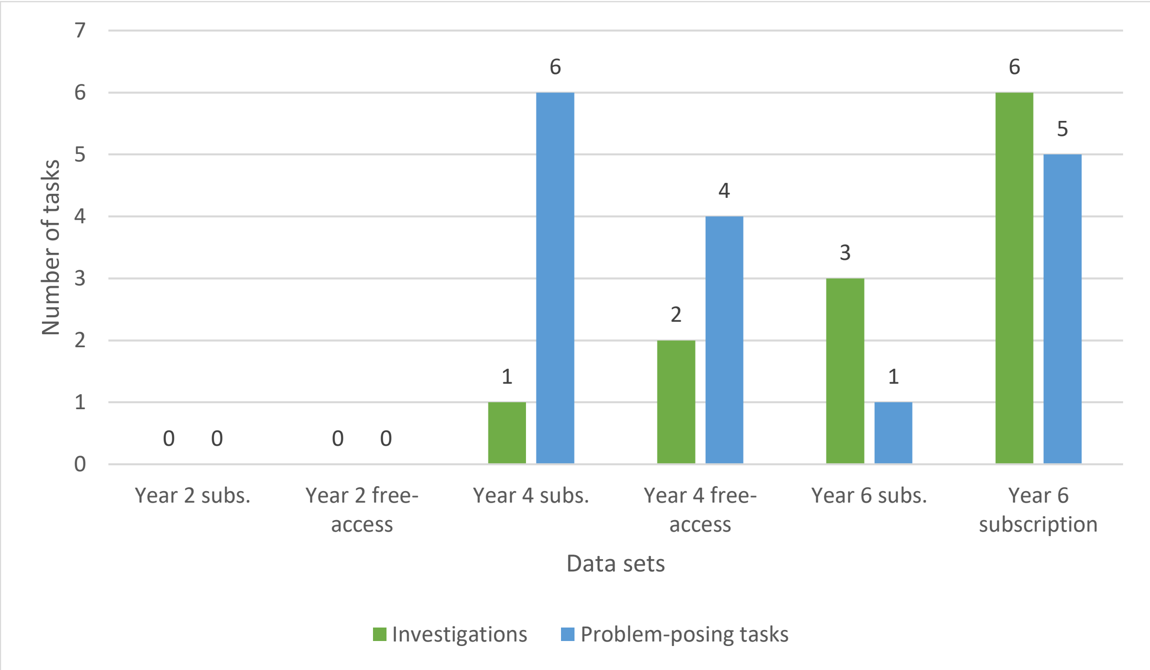


Figure 148: *Number of tasks linked with problem-solving in each data set.*

5.2 Qualitative analysis - types of problems

The previous section demonstrated via quantitative content analysis, that there are few opportunities for NRPS with the sample TfM materials. Following this, the next step would be to explore the nature of those opportunities in greater detail. The quantitative content analysis allowed for specific tasks to be identified which could be explored in greater depth and detail, and this stage also involved categorisation of NRPs (identified during quantitative analysis) into their problem type. The following sections describe the WRM TfM materials in general, before investigating the type of NRPs included within the sample and exploring specific problems of interest.

5.2.1 Structure of the WRM materials

The sample provided between 129 and 143 lessons to be used during the school year (see table 11), with additional assessments for each topic provided in the SUB materials.

Table 10: *Number of lessons within the sample materials for each year group.*

Year group	SUB materials	FA materials
2	134	129
4	143	143
6	130	130

The structure of the materials was slightly different for the FA and SUB materials: the tasks within the FA materials were bullet-pointed, rather than numbered (as with the SUB materials) and included two subheadings of ‘varied fluency’ followed by ‘reasoning and problem-solving’, which were always presented in this order. However, many of the tasks which came under the ’reasoning and problem-solving’ subheading were actually STs or GATs rather than NRPs, which could cause some confusion for teachers in identifying appropriate NRPs for mathematics lessons. Although the SUB materials did not use subheadings to delineate the different types of tasks, nevertheless they followed a similar structure, with fluency tasks presented first, followed by worded tasks, reasoning activities, and (in some cases) NRPs.

Both the SUB and FA materials included a title at the top of all resources, which was linked to the learning objective for that particular lesson. The FA materials also provided a small section entitled ‘notes and guidance’ which gave an overview of the success criteria relating to the learning objective. Below this was a section for ‘mathematical talk’, which outlined key questions for the teacher to use during the lesson. In contrast, the format of the SUB materials resembled that of a textbook for children to work through sequentially, with no notes or guidance for teachers presented on the printable sheets.

Both the FA and SUB materials include a variety of pictorial representations for different tasks for all year groups, including mathematical representations such as dienes, hundred squares, place value counters, number lines, bar models and part-whole models (among others) as well as more generic images such as pictures of cartoon characters with speech bubbles for reasoning tasks, which were used on all resources in both the SUB and FA materials.

5.2.2 Distribution of problem types

The distribution of problem types within the WRM sample can be seen in Table 12, which demonstrates that the majority of NRPs within the TfM materials were finding all possibilities problems (FAPs). These made up over two-thirds of all problems within the sample, while the second most popular problem type (logic problems [LG]) accounted for only 15% of all NRPs within the materials. Word problems (W) and diagram/visual problems (DV), made up between 6% and 7%, respectively, while finding rules and describing patterns problems (RP) were the least represented, forming just 4% of the combined sample materials. Some problem types were not present at all within the sample materials for particular year groups; for example, the Year 2 materials contained no W or DV problems within either the SUB or FA materials.

Table 11: *Distribution of different problem types by year group and subscription type*

Materials	**Finding all possibilities (FAP)**	**Logic (LG)**	**Word (W)**	**Diagrams/ Visual (DV)**	**Finding rules and describing patterns (RP)**
Year 2 SUB	2	2	0	0	0
Year 2 Free access	26	3	0	0	1
Year 4 SUB	19	8	1	0	1
Year 4 Free access	30	3	1	1	1
Year 6 SUB	14	3	4	6	2
Year 6 Free access	13	4	3	3	1
Total	**104**	**23**	**9**	**10**	**6**
%	**68%**	**15%**	**6%**	**7%**	**4%**

For most year groups, FA materials provided more opportunities for solving FAP problems, except the Year 6 materials, while SUB materials typically provided slightly more opportunities than the FA materials for children to engage in solving problems other than FAP. Figure 19 shows the distribution of different problem types for each year group when the SUB and FA materials are combined. Where teachers have access to both sets of materials for their year group (and select all of the NRPs within each set), this would provide between 28-49 opportunities for pupils to engage in FAP problems throughout the academic year. In comparison, engaging in LG problems would be restricted to between 5 and 11 opportunities within the school year, and opportunities to engage in other problem types are even lower.

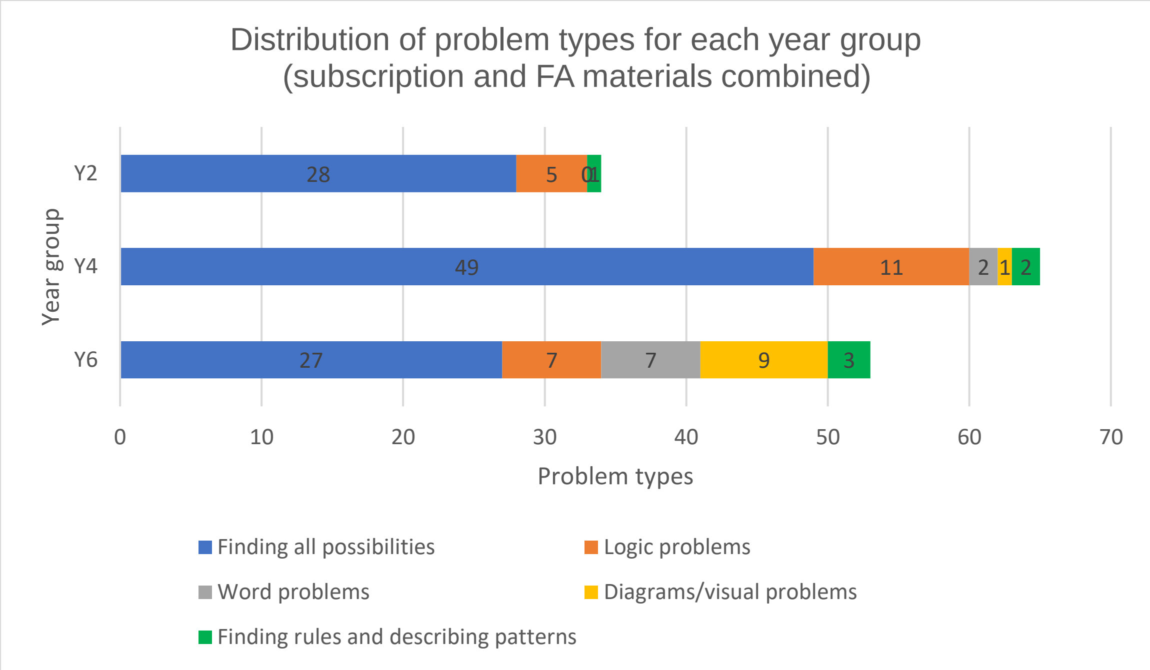


Figure 159: *Distribution of task types in the WRM materials*

As can be seen in Figure 19, the combined Year 6 materials have a more even distribution of problem types compared to that for younger year groups, and all problem types are represented, although there are fewer NRPs overall than in the combined Year 4 materials. In the Year 2 materials, not only are there fewer NRPs overall, but W and DV problems are not represented within the materials, and the distribution of problem types is uneven. However, these findings are dependent on teachers selecting *all* of the NRPs from within the sample materials and does not account for any NRPs teachers may source elsewhere to supplement the WRM materials.

In all of the sample materials, worded tasks were largely routine or were given after an example of how to solve it, particularly in the SUB materials. In the majority of instances, word problems were directly linked with earlier fluency tasks so that children could easily identify the calculation required to solve it. Similarly, DV problems (where these were provided) were usually preceded by an example of how to solve them. However, there were slightly more opportunities for solving DV NRPs within the Year 6 SUB materials, particularly within the 'properties of shape' topic. In the Year 2 SUB materials, NRPs were usually given after an example of how to solve them were provided, reducing the level of cognitive demand required for the task as well as the opportunity for creativity in solving. In the SUB materials for all year groups, there were some instances where the examples which preceded problem tasks were identical to them and provided not only the method but also the correct answer to the problem.

The following sections provide a more detailed explanation of the problems provided in the WRM materials and provide examples of the different problem types within the sample. For copyright reasons, the examples provided are derived only from the FA materials, which are freely available to view on the WRM website.

<u>5.2.3 Finding all possibilities problems</u>

The most popular type of NRP within the WRM sample materials was FAP problems. Within the Year 2 SUB and FA materials, there were a total of 28 opportunities for this kind of NRPS. An example of this kind of problem is given in Figure 20, which is taken from a lesson on 'fact families' within the addition and subtraction topic.

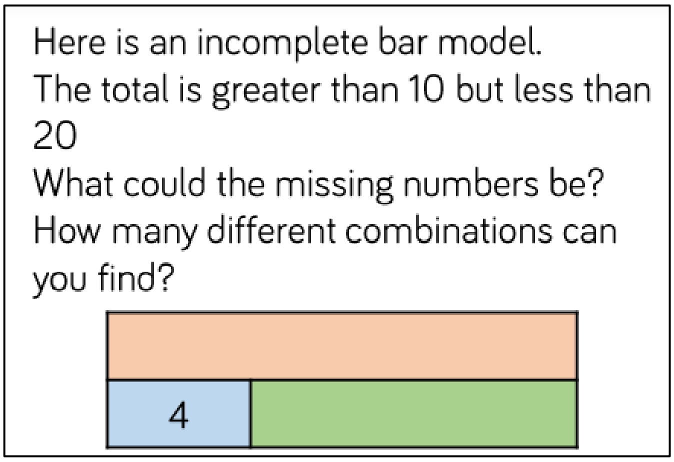


Figure 20: *An FAP problem within the Year 2 sample materials (WRM, 2023)*

Year 2 children are likely to encounter a range of difficulties with this problem, partly due to the difficulties with language such as 'incomplete' or 'combinations', as well as mathematical vocabulary 'greater than', 'less than' and 'total'. In addition, children need to be able to draw on prior knowledge of how bar models work, with the total in the top row being the sum of the two values beneath. In terms of problem-solving skills, since the task requires the solver to find all possible combinations, this may encourage children to use a systematic approach, possibly by starting with the smallest possible total of 11 and working up to 19, finding the value of the part by subtracting 4 from the total each time. Although the task makes no mention of finding rules and describing patterns, there is also the opportunity for children to notice a pattern emerging, with the missing part increasing by 1 every time the total increases by 1.

The Year 4 sample materials provided the greatest number of non-routine FAP problems, with a combined total of 49. As with the Year 2 materials, the FA materials provided more opportunities for solving non-routine FAP problems, however, the difference between the different subscription types was smaller, ranging from 19 for the SUB materials to 30 in the FA materials. An example of this problem type from the Year 4 FA materials is provided in Figure 21.

> The width of a rectangle is 2 metres less than the length.
> The perimeter of the rectangle is between 20 m and 30 m.
> What could the dimensions of the rectangle be?
> Draw all the rectangles that fit these rules.
> Use 1 cm = 1 m.

Figure 16: *An FAP problem within the Year 4 sample materials (WRM, 2023)*

In the task shown in Figure 21, the solver must draw on their knowledge of perimeter being the distance around the outside of a closed shape. They may identify the smallest possible length as 6 (6+6+4+4=20) and increase the length by 1 until the perimeter exceeds 30m, subtracting 2 from the length to find the width each time. Due to the number of conditions that need to be met (width = length – 2, perimeter ≥ 20 and perimeter ≤ 30), coupled with the interdependency of the length and width, this task was classified as an NRP which is likely to require a higher level of cognitive demand for the solver and encourage a systematic approach to identify all possible correct solutions.

The Year 6 materials had fewer opportunities for solving FAP problems than the Year 2 or Year 4 materials when the SUB and FA materials were combined for each year group. Unlike the materials for Years 2 and 4, the number of opportunities for this type of problem-solving was very similar for the SUB and FA materials. An example of an FAP problem in the Year 6 FA materials is given in Figure 22.

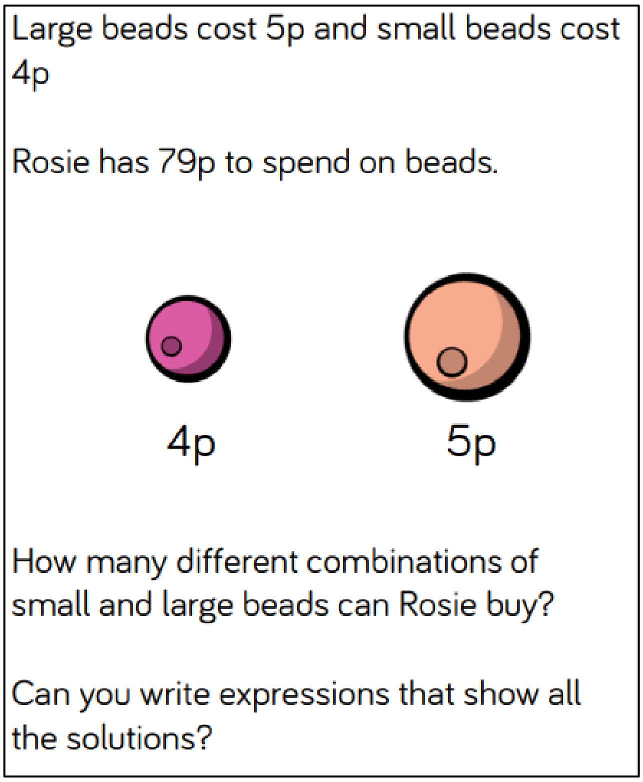


Figure 22: *An FAP problem within the Year 6 sample materials (WRM, 2023)*

This problem could possibly be reworded to make clear to pupils that the combinations of small and large beads should sum to *exactly* 79p (rather than any combination that sums to less than the amount Rosie has to spend on beads) since this would engender a higher level of cognitive demand due to greater restrictions placed on the possible outcomes. When used in this way, pupils may recognise that the price of small beads must sum to a value ending in 4, since multiples of 5 end in 5 or zero and no multiple of 4 ends in 9. From there, children can subtract each possible cost of the small beads from 79 and divide the answer by 5 to find the number of large beads for each combination. Furthermore, this task encourages the use of a systematic approach, possibly starting with the smallest possible number of small beads and working up.

As has been outlined previously, GATs were more numerous within the sample materials than NRPs. The most pronounced difference between the number of GATs and NRPs was seen in the Year 2 SUB materials, which contained twelve times the number of GATs than NRPs. However, in the Year 2 FA materials, there was only one more GAT than NRPs. For FAP problems, a task was categorised as a GAT rather than an NRP if only one or some of the possible answers were required, or the task met only one of the features outlined in the analysis framework (see appendix C/D). An example which was categorised as a GAT is given in Figure 23.

Jack has 2 p.

Eva has 10 p.

Both of them have a 2 p coin.

What **other** coins could Eva have?

Figure 23: *A GAT with only one possible answer required (WRM, 2023).*

Here, children must draw upon their prior knowledge of coin denominations that sum to 8p. Some children may use eight 1p coins, while others may attempt to use a variety of coins with different denominations, such as 5p + 2p + 1p. Children may also use multiplication table knowledge to determine that Eva could have another four 2p coins. However, since only one answer is required, the cognitive demand is reduced and since there is no need to work systematically, there is little opportunity to develop problem-solving skills. Many children are likely to choose the easiest solution method to complete this task. However, there is an opportunity for the teacher to challenge children to find all possible answers, even though this is not stated in the question. Therefore, this task was categorised as a GAT which could potentially provide an opportunity for problem-solving, at the discretion of the teacher. Pupils may work systematically by starting with the smallest coin denominations and gradually replacing these with coins with greater denominations:

1p x 8

(1p x 6) + 2p

(1p x 4) + (2p x 2)

(1p x 2) + (2p x 3)

2p x 4

5p + (1p x 3)

5p + 2p + 1p

In other examples of GATs, all possible answers needed to be given, however, the task only met one of the features in the framework, reducing the level of cognitive demand placed on the solver. For example, Figure 24 shows a GAT taken from the Year 4 FA materials lesson on the topic of time.

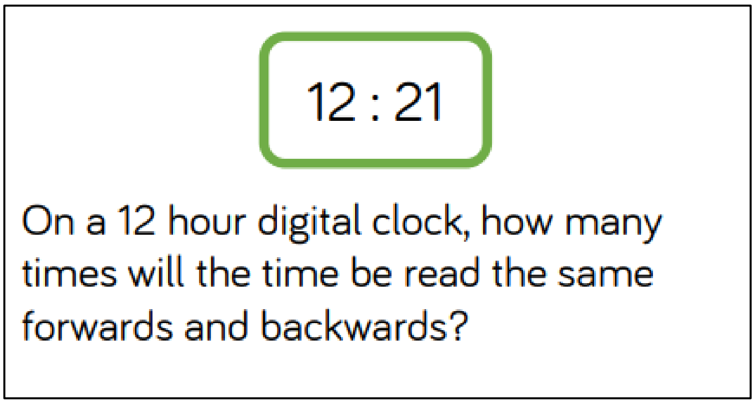


Figure 24: *A ‘FAP GAT within the Year 4 sample materials (WRM, 2023)*

To find all possible solutions, pupils must draw upon their knowledge of the number of minutes within an hour and work systematically, possibly by starting with the smallest hours value and reversing the digits to find the minutes (01:10). Difficulties may arise once the hours value passes 5, however, most Year 4 pupils should recognise that there are 60 minutes in an hour and therefore 06:60 is not a legitimate answer (especially since this subject matter was the basis of prior fluency tasks). However, the number of conditions that must be met is low (12-hour clock, minutes figure is the reverse of the hours figure, minutes figure <60), reducing the level of challenge for the solver.

A similar example is taken from the Year 6 free access materials (see Figure 25), where all possible answers need to be given but the task meets only one of the features in the framework (data provided are interdependent), reducing the cognitive demand for the solver. Considering the age and experience of typical Year 6 children, most are likely to identify that the task requires them to find factor pairs with a product of 48. Therefore, while the GATs shown in Figures 24 and 25 provide practice for pupils in working systematically, which could be drawn upon in more challenging problem-solving situations, these tasks could not be categorised as NRPs themselves.

$$c \times d = 48$$

What are the possible integer values of c and d?
How many different pairs of values can you find?

Figure 175: *An FAP GAT within the Year 6 sample materials (WRM, 2023)*

5.2.4 Logic problems

The second most popular problem type was LG problems, although there were significantly fewer opportunities to solve LG problems than FAP problems: The number of LG problems was less than a quarter of that for FAP problems, with the majority of these provided within the Year 4 SUB materials (see Figure 19). In contrast, there were only 2 opportunities for Year 2 children to solve logic problems in the SUB materials. Figure 26 is taken from the Year 2 free access materials for a lesson based on comparing volume.

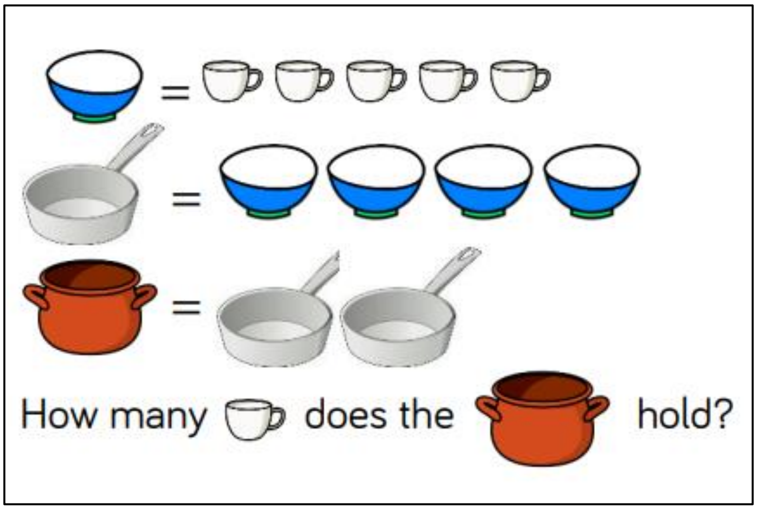


Figure 26: *An LG problem within the Year 2 sample materials (WRM, 2020)*

To solve this problem, pupils must work step-by-step, first working out that if there are 5 cups in one bowl and four bowls in one pan, there must be 5 x 4 = 20 cups in one pan. Since there are 2 pans in one large pot, there must be 20 x 2 = 40 cups in one large pot. For Year 2 pupils, it is unlikely they will identify the starting point to solve this problem immediately. Furthermore, many children in Year 2 may not make the link with multiplication to determine the correct calculations needed for solving. Therefore, the level of cognitive effort required to solve the problem is likely to be high.

The Year 4 TfM materials contained the greatest number of opportunities for solving non-routine LG problems, with a total of 11 for the SUB and FA materials combined. However, this is significantly lower than the number of opportunities for solving FAP problems. In addition, there was a difference between the number of opportunities for solving LG problems between the SUB and FA materials, with 8 in the SUB materials compared with 3 in the FA materials.

In the Year 6 materials, there was a combined total of 7 opportunities for pupils to solve LG problems in the TfM sample materials, with a similar amount provided in the SUB and FA materials (3 and 4, respectfully). However, the problems tended to require a greater number of calculations and sustained engagement with the problem to solve them. An example of an LG problem is given in Figure 27, which is taken from a lesson on mean in the statistics topic.

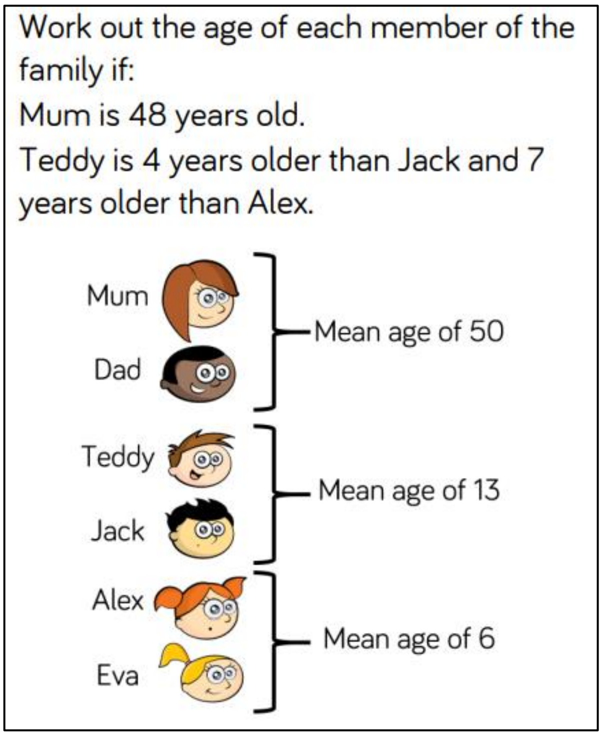


Figure 187: *An LG problem within the Year 6 sample materials (WRM, 2023)*

In this problem, cognitive demand placed on the solver is increased due to the number of conditions and interdependent data. To solve this problem, students must draw on their knowledge of calculating the mean value to work backwards and find the ages of each family member. Using the first clue that Mum is 48 years old, they may recognise that Dad must be 52 so that their ages sum to double the mean value. From here, there are various methods students may use to find the ages of the remaining family members. For example, students may use a bar model to find the ages of Teddy and Jack where the mean value of their ages is equal to 13 and the difference between their ages is 4, as shown in Figure 28.

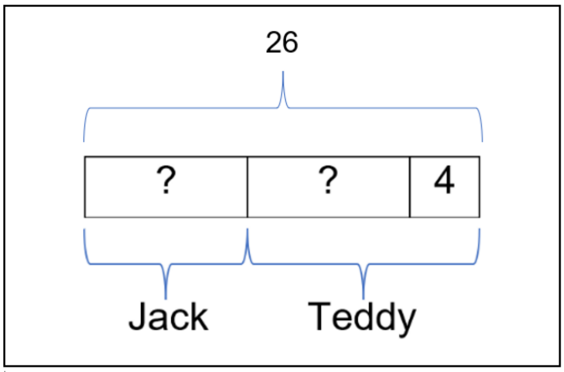


Figure 198: *A possible solution method for a Year 6 LG problem using a bar model*

Using the bar model, students may recognise that they can find Jack's age by subtracting 4 from 26 and halving the answer:

26 – 4 = 22

22 ÷ 2 = 11

Jack is 11 years old.

Teddy is 4 years older.

11 + 4 = 15

Alternatively, pupils could use algebra to find the ages of Teddy and Jack using the mean value of their ages and the clue that Teddy is 4 years older than Jack:

Where J is Jack's age,

2J + 4 = 26

$2J = 22$

$J = 11$

We know that Teddy is 4 years older than Jack, so where T is Teddy's age,

$T = J + 4$

$T = 11 + 4$

$T = 15$

Working deductively using their chosen solution method, students can work in a step-by-step manner to find the ages of the remaining family members.

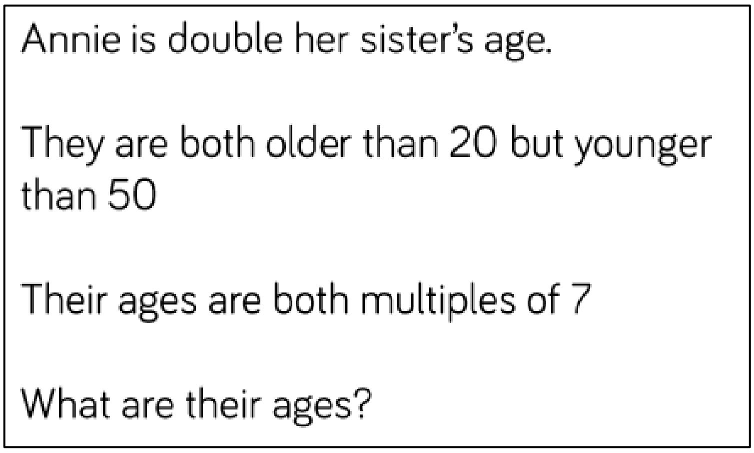
As with FAP problem types, there were a greater number of LG GATs than LG NRPs. Two LG GATs are shown in Figures 29 and 30.

> I am thinking of two numbers.
> The sum of the numbers in 17.
> The product of the numbers is 72.
> What are my secret numbers?

Figure 20: *A GA logic problem within the Year 4 sample materials (WRM, 2023)*

Figure 29 is taken from the Year 4 FA materials in a lesson based on the 9 times tables. In accordance with the analysis framework (appendix D), this problem was categorised as a GAT due to the number of conditions and interdependency of the data provided, coupled with the fact that the question did not follow from an example demonstrating how it should be solved. However, some students are likely to recognise that one of the secret numbers will be 9, since this links with earlier fluency and reasoning tasks and the lesson title (9 times tables). Once this has been determined, students may quickly determine that the other number is 8 using inverse operations (17 – 9 = 8 and 72 ÷ 9 = 8). Therefore, while this may constitute a problem for some children, it is likely that many children would immediately identify the starting point for solving, rendering this task straightforward.

Figure 30 is another LG GAT taken from the Year 6 sample materials in a lesson with the objective of finding multiples.


Annie is double her sister's age.

They are both older than 20 but younger than 50

Their ages are both multiples of 7

What are their ages?

Figure 21: *An LG GAT within the Year 6 sample materials (WRM, 2023)*

Similar to the Year 4 LG GAT given in Figure 29, students are likely to identify the starting point for this task by writing out the multiples of 7 between 20 and 50:

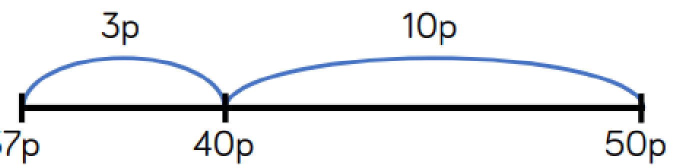
21, 28, 35, 42, 49

At this point, most Year 6 students would be likely to recognise that 42 is double 21, therefore Annie must be 42 and her sister must be 21.

5.2.5 Word problems

Although there were some examples of non-routine W problems within the WRM TfM materials, the majority of worded questions were straightforward as they followed directly from an example of how to solve it or required repetition of a procedure/strategy from earlier fluency tasks. Therefore, many of these tasks could not be categorised as GATs or NRPs and only provided practice in using a known calculation. An example of some worded STs is given in Figure 31, which follows directly from an example of how to calculate the answers.

Mo buys a chocolate bar for 37p. He pays with a 50p coin. How much change will he receive?



Mo will receive ____ p change.

Use a number line to solve the problems.

- Ron has £1. He buys a lollipop for 55p. How much change will he receive?
- Whitney has £5. She spends £3 and 60p. How much change will she receive?

Figure 31: *Examples of STs labelled as 'problems' (WRM, 2023)*

Although these tasks were straightforward in nature, WRM refer to them as 'problems' within the task instructions, which could create confusion for teachers in selecting NRPs from the materials.

In all the sample TfM materials analysed, there were examples of worded tasks which required more than one step to find an answer, however, in following guidance from van Zanten & van den Heuvel-Panhuizen (2018) regarding the accurate use of the analysis framework, multi-step worded questions were categorised as routine tasks unless specific indicators of GATs or NRPs within the framework were evident. In the Year 2 materials, there were no examples of W problems, however, there were some examples of W GATs, most of which were multi-step problems with interdependent data. Figure 32, which is taken from a Year 2 lesson on adding 2-digit numbers, is an example of this kind of grey-area task.

Annie has 12 marbles.

Ron has 13 marbles more than Annie.

How many marbles do they have altogether?

Figure 222: *A multi-step, W GAT within the Year 2 sample materials (WRM, 2033)*

To obtain the answer to this worded question, children must recognise that a calculation is needed to find out how many marbles Ron has, before calculating Annie and Ron's combined total. Some Year 2 students would be likely to simply add the two given values and give an answer of 25 marbles for their combined total, however, some children are likely to determine the starting point without significant cognitive struggle, at which point only simple calculations are needed to find the answer (12 + 13 = 25, 12 + 25 = 37 marbles altogether).

The W GATs within the Year 4 materials were of a very similar structure to the Year 2 example shown in Figure 32, being multistep and requiring children to recognise the significance of the word 'more' to ascertain the starting point for solving. The question shown in Figure 33 gives a hint given at the end of the question which encourages children to consider what the first step is:

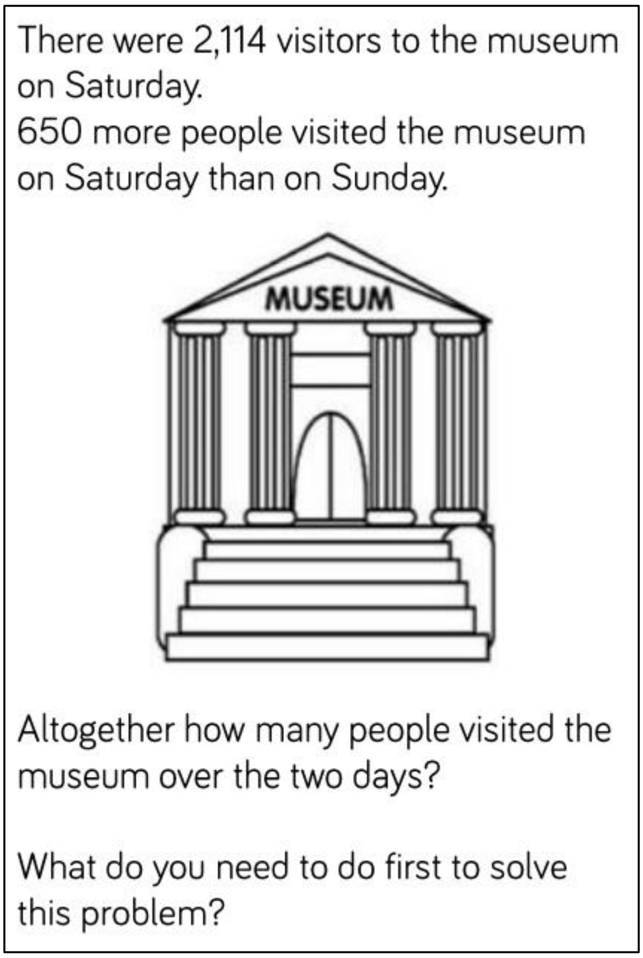


Figure 233: *A multi-step, W GAT within the Year 4 sample materials (WRM, 2023)*

To find the answer, the student must first perform a subtraction calculation to work out the number of visitors on Sunday. However, children may misunderstand the use of the word ‘more’ and add 650 to 2114, rather than subtracting (since 650 more people visited the museum on Saturday *than on Sunday*). Once misconceptions surrounding language have been addressed, the calculations required are straightforward (2114 – 650 = 1464 visitors on Sunday. 1464 + 2114 = 3578).

Several similar worded questions within the Year 4 materials were classified as ST due to their use of a prescribed method for solving (often using bar models). Figure 34 is an example of this type of question with the bar model representation provided underneath, from a lesson based on multiplying 3-digit numbers by 1-digit numbers.

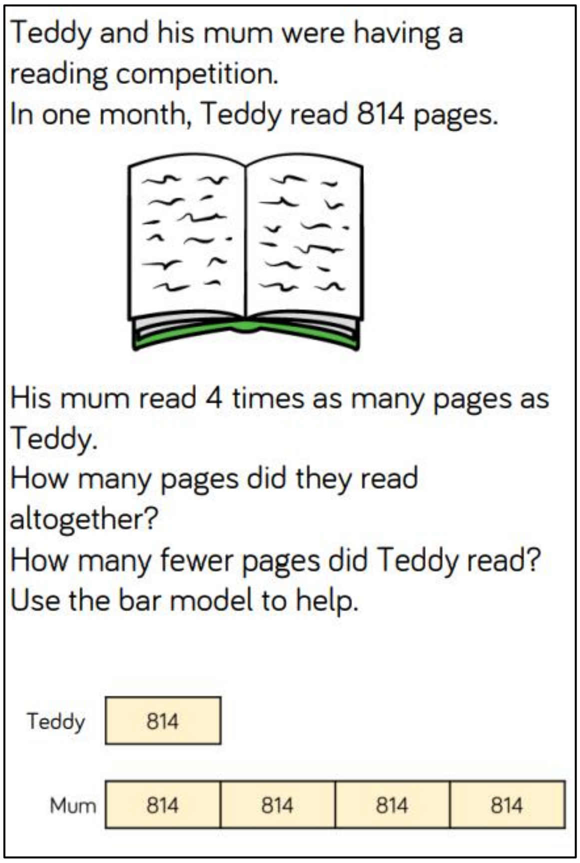


Figure 34: *A W ST with prescribed use of bar model within the Year 4 sample materials (WRM, 2023)*

The use of a prescribed strategy, especially considering the bar model is provided with the data already entered within each ‘bar’, creates a clear starting point for children in finding the total value shown in the bar models and finding the difference, therefore reducing the requirement for cognitive struggle and removing the potential for creative solution strategies.

However, a small number of questions within the Year 4 materials could be categorised as NRPs; with one example found in the SUB materials and one in the FA materials. The W NRP from the FA materials is given in Figure 35, which is taken from a lesson on converting metres to kilometres within a measurement (length and perimeter) topic.

> Dexter and Rosie walk 15 kilometres altogether for charity.
> Rosie walks double the distance that Dexter walks.
> How far does Dexter walk?

Figure 245: *A W NRP within the Year 4 sample materials (WRM, 2023)*

The problem in Figure 35 is likely to present difficulties to Year 4 children in terms of finding a suitable starting point, especially considering that the distance Rosie walked is only given in terms of proportion to the distance walked by Dexter, therefore children have minimal data to calculate with. Since algebra is not taught until Year 6 in accordance with the National Curriculum in England (DfE, 2013), children are unlikely to use an algebraic procedure. Some children may recognise that the whole must be split into three equal parts, with Dexter's distance being one-third and Rosie's distance being double this (two-thirds), while other children may use a bar model representation to support their calculations.

The majority of W NRPs were found in the Year 6 sample materials, with 4 and 3 examples in the SUB and FA materials respectively. An example of a W NRP is given in Figure 36, which is taken from the Year 6 FA materials for a lesson based on missing values and percentages.

A golf club has 200 members.

58% of the members are male.
50% of the female members are children.

(a) How many male members are in the golf club?
(b) How many female children are in the golf club?

Figure 256: *A W NRP within the Year 6 sample materials (WRM, 2023)*

While part A is an ST relating directly to earlier fluency and reasoning questions, part B requires pupils to overcome several difficulties with the data provided, particularly that the second percentage figure represents the proportion of *female* members, rather than the proportion of *all* members (as with the first percentage figure provided). Many pupils are likely to calculate 50% of 200 and give an answer of 100 female children, while pupils who have understood the proportions correctly may not have a clear starting point at their disposal, although they may recognise that the answer they gave in part a provides some support with this as they can deduct 58% of 200 (116) from 200 to find the total number of female (adult and child) members (200 – 116 = 84). However, there are other solution strategies available: while some pupils may begin with the method described above and then find 50% of 84 (42), others may deduce that if 58% of the members are male, then 42% must be female, and since 50% of these are children, the percentage of female children must be 21%: 21% of 200 = 42. Therefore, this problem may offer pupils opportunities to demonstrate flexibility in their solution strategies and allow them to use skills in deduction to solve the problem.

There were also examples of W GATs within the Year 6 sample materials, most of which were found in the materials for the ‘fractions’ topic. An example of a W GAT is given in Figure 37, taken from a lesson on multiplying fractions by integers.

There are 9 lamp posts on a road. There is $4\frac{3}{8}$ of a metre between each lamp post.

What is the distance between the first and last lamp post?

Figure 267: *A W GAT within the Year 6 sample materials (WRM, 2023)*

Although this task did not meet the number of indicators needed to be classified as an NRP, many children could face difficulty in finding a solution due to the figures given. Although there are 9 lampposts, multiplying this value by the distance between each lamppost would give an incorrect answer since it is the distance from the first lamppost to the ninth which is required (or 8 widths). Without drawing a picture or diagram, most children are likely to use the incorrect solution strategy, therefore there is potential for cognitive struggle, as well as the use of problem-solving strategies.

Although many of the W GATs and W NRPs contained references to constructs that will be familiar to most students (such as reading/walking competitions and toys such as marbles), there were few problems which simulated the kind of problems children may encounter in real-life scenarios.

5.2.6 Diagram/visual problems

There were very few examples of DV NRPs within the WRM materials, with no examples found in the SUB or FA Year 2 materials and only 1 example in the Year 4 materials (within the FA materials). However, there were more opportunities to solve DV NRPs within the Year 6 materials, with 6 DV NRPs in the SUB materials, although there was only half this amount in the FA materials. The sole example found in the Year 4 materials is given in Figure 38, which is taken from a lesson on perimeter of rectilinear shapes.

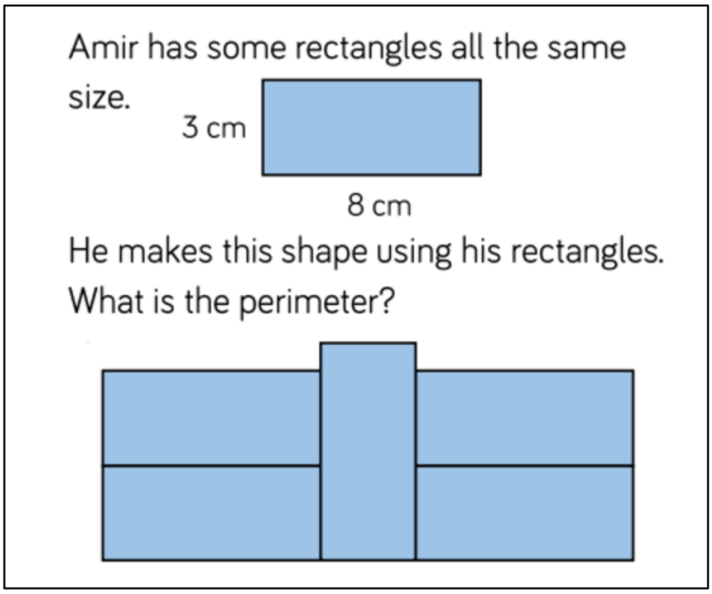


Figure 278: *A DV NRP within the Year 4 sample materials (WRM, 2023)*

In this problem, children may start by labelling the sides where the entire length/width of a rectangle makes an outer edge of the new shape. However, to calculate the size of the edges relating to the middle rectangle, they must work deductively to find the values of the parts of the sides which are on the outermost edge of the new shape. In addition, children must have a way of systematically recording which sides have already been added to give the perimeter of the new shape, so that sides are not repeated.

The majority of DV Ps and GATs in the Year 6 sample materials were found within the topic of shape, as with Figure 39, which is from a lesson based on finding angles in a triangle:

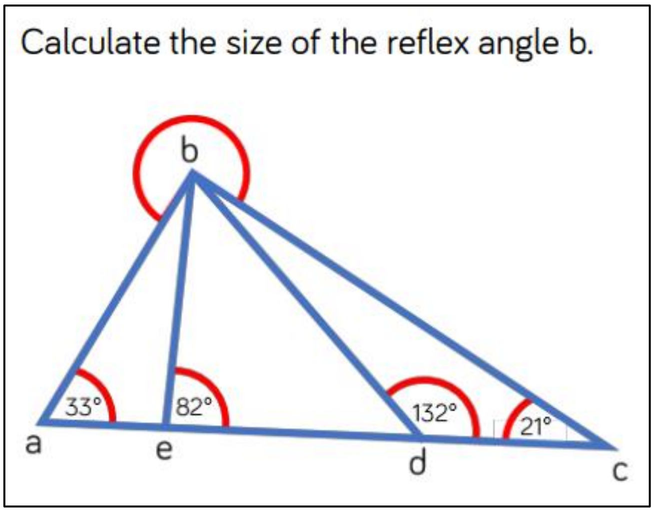


Figure 39: *A DV NRP within the Year 6 sample materials (WRM, 2023)*

To solve this problem, pupils could draw on a range of different rules (such as interior angles of a triangle sum to 180°, angles on a straight line sum to 180°, and angles around a point sum to 360°) to calculate the values of the missing angles step-by-step. A common solution method may involve working deductively to find the value of all angles in each individual triangle, however, this would give many opportunities for errors due to the number of calculations required. Alternatively, some pupils may recognise that the three triangles make one new triangle with angles at abc (with angles provided at D and E acting as extraneous information). Since only the interior angle at b is unknown, we can deduct the angles at a and c from 180° to find the missing interior angle of the triangle (180° - 33° - 21° = 126°), before deducting this answer from 360° to find the reflex angle at b (360° - 126° = 234°). Considering the different possible approaches and opportunities for deductive reasoning, the level of cognitive demand required to solve this problem is likely to be fairly high and many children may struggle to identify the most efficient starting point. There is an opportunity for more creative problem-solving approaches, as well as discussion surrounding efficient methods for solving the problem.

A further example of a DV NRP is taken from a lesson based on the area of triangles from the Year 6 perimeter, area and volume topic, which is shown in Figure 40:

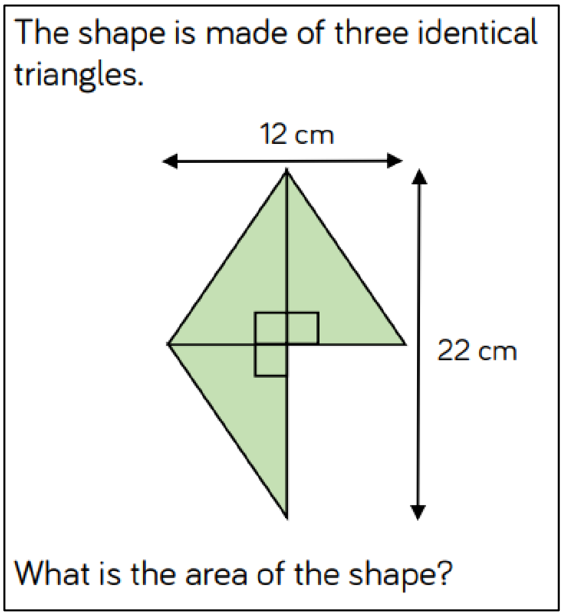


Figure 40: *A DV NRP within the Year 6 sample materials (WRM, 2023)*

There are several different ways in which pupils may approach this problem, drawing on their prior knowledge of calculating the area of a triangle. Some possible solution strategies are shown below:

Finding the value of 4 right-angled triangles and then finding ¾ to calculate the area of the irregular green shape:

$\frac{(12 \times 22)}{2}$ = 132cm x ¾ = 99

or

Halving the length of the height and width first then finding half of their product:

6x11 = 66

66 ÷ 2 = 33

If one triangle has an area of 33, three identical triangles will have an area of 99cm.

This problem is likely to be problematic for Year 6 pupils as the starting point for solving is not clear in the task itself. Additionally, there are opportunities for pupils to demonstrate flexible and creative approaches in the process of solving the problem.

5.2.7 Finding rules and describing patterns

The White Rose Maths TfM materials offered only a very small selection of RP NRPs, ranging from no examples in the Year 2 SUB materials to only 2 examples in the Year 6 SUB materials. This means that children in Years 2, 4 or 6 would have a maximum of 2 opportunities in the school year to solve these kinds of problems.

In the Year 2 FA materials, the only example of a finding rules and describing patterns problem was taken from a lesson on drawing 2D shapes in the topic of shape, which is shown in Figure 41:

> Using geoboards, how many different rectangles can you make?
>
> What's the same about the rectangles? What's different?
>
> Has your friend made any different rectangles?

Figure 281: *An RP NRP within the Year 2 sample materials (WRM, 2023)*

By creating a number of rectangles with various lengths and widths and exploring the similarities and differences between the rectangles they have made, children may begin to form generalisations about rectangles using inductive reasoning (eg. opposite sides of rectangles are equal). In addition, some children may recognise that a square is a special type of rectangle as the opposite sides are equal in length *and* all sides are equal in length. This may lead children to form a rule that a square is a special kind of rectangle, but a rectangle is not always a square, although this is likely to be heavily dependent on how this task is presented and the quality of teacher questioning during the activity.

A similar example is taken from the Year 4 sample materials within the shape topic, which is shown in Figure 42:

Always, Sometimes, Never

When all the sides of a rectangle are odd numbers, the perimeter is even.
Prove it.

Figure 42: *An RP NRP within the Year 4 sample materials (WRM, 2023)*

By exploring various examples of rectangles with odd side-lengths, pupils may surmise that since the perimeter is always even with the specific examples they have used, all rectangles with odd side-lengths have an even perimeter. This could lead to children making links to prior knowledge that two odd numbers sum to an even number and two even numbers sum to an even number, therefore 4 odd numbers must also sum to an even number. Alternatively, children may argue that since the opposite sides of a rectangle are always equal, the perimeter can be found by multiplying the sum of the height and width of the rectangle by 2, and all multiples of 2 are even. In this way, pupils have opportunities to use deductive and inductive reasoning and prove that their conclusion is mathematically sound.

Most of the other RP NRPs found in the TfM materials were also found within topics of shape or area and perimeter. For example, Figure 43 asks children to use knowledge of angles in a triangle to find the sum of interior angles in a quadrilateral. By splitting quadrilateral shapes into 2 triangles, pupils may move from the knowledge that interior angles of a triangle sum to 180°, to the rule that interior angles of a quadrilateral shape will sum to double that figure (360).

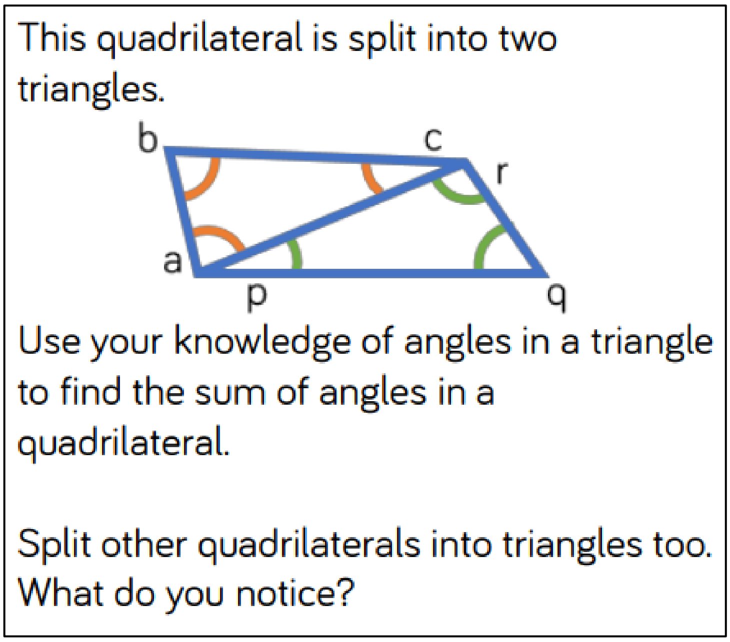


Figure 293: *An RP NRP within the Year 6 sample materials (WRM, 2023)*

5.3 Conclusions

This chapter presented the results from the quantitative and qualitative analysis of the TfM-aligned resources. Quantitative content analysis of the WRM materials revealed that the majority of tasks were straightforward, with few opportunities for pupils to engage in NRPS activities, and tasks which link more closely with problem-solving were also poorly represented. This is a significant finding considering the importance of NRPS in mathematics education (as described in Chapter 2) and it is also hugely important in answering the first research question, concerning the level of opportunity for NRPS within the WRM TfM materials. GATs, which may be problematic for some students within a particular year group, appeared more frequently than NRPs within the sample materials, and although some of these could provide opportunities for teachers to adjust the task to make it an NRP for students, there is potential for teachers to confuse GATs for NRPs when selecting tasks for mathematics lessons.

Qualitative analysis allowed for exploration of the nature of the problems provided within the sample materials, which demonstrated that the majority of NRPs within the sample were FAPs, with significantly fewer opportunities for children to solve LG problems and very few opportunities for pupils to engage with other types of problems. Examples of NRPs and GATs have been explored and discussed with a focus on the opportunity for children to develop problem-solving skills through a range of different problem-solving experiences.

The next chapter draws on data obtained via interviews with a head teacher and subject leader for mathematics within one primary school in North East England to explore their opinions about opportunities for NRPS within WRM materials.

Chapter 6: Results - Perceptions about opportunities for NRPS within WRM TfM materials

This chapter presents the results from semi-structured interviews with a headteacher and subject leader for mathematics within one primary school, which were focused on answering the second research question: *What are the perceptions school senior leaders about problem-solving opportunities within the WRM TfM materials?* Thematic analysis and coding of the interview data allowed for patterns to be identified and this led to the conceptualisation of several themes based on the data and the research questions. These themes were sorted into 5 categories: understandings of TfM, motivation to adopt a TfM approach, choice of scheme, understandings of problem-solving, and problem-solving in School A. The following sections represent these categories, which are further divided into the themes which emerged in relation to each category. The chapter concludes with a brief summary of the findings linked with the second research question. For clarity, the head teacher at School A will be referred to as 'HT' throughout this chapter, while the subject leader for mathematics at School A will be referred to as 'SLM'.

6.1 Understandings of TfM

The HT and SLM in School A had fairly homogenous understandings of TfM and summarised the approach in a similar way during the interviews, although neither one of the participants was able to provide a comprehensive definition. The prevalent themes related to understandings of TfM were depth and small steps/success for all which are described in the following sections.

6.1.1 Depth

Both participants referred to depth of understanding in their definitions of TfM, despite differences in their understandings of the approach. The HT referred to TfM in terms of outcomes for pupils.

> *Teaching for Mastery means showing real depth of understanding…I think the best and true mastery is when they can be really competent and show someone else how to do it, I think that's mastery. – HT*

On the other hand, the SLM's response focused on the features of TfM-aligned instruction, particularly with regard to the structure of a TfM-aligned mathematics lesson whereby fluency tasks precede more difficult reasoning and problem-solving tasks.

> *That means getting to grips with the basic skills that a child needs and taking the learning deeper. It starts with the fluency aspect at the beginning of the lesson, so it's really having that strength of knowledge before they start to reason and problem-solve, so we can take the learning as deep as possible. - SLM*

The SLM's response also alluded to the need for pupils to acquire 'basic skills' and 'strength of knowledge' before they progress to reasoning and problem-solving tasks. When asked to clarify what she meant by 'taking the learning deeper', the SLM responded,

> *It's about whether they can apply what they've learned in different ways, like 'write your answer in a sentence', all of that comes on board here. I think, over time, that has really linked in with the SATs papers and reasoning papers that have been brought out with those sorts of aspects. – SLM*

This response suggests that the SLM links depth of learning with the ability to apply their mathematical understanding to reasoning and problem-solving tasks, which may require different ways of forming an answer (such as in a sentence rather than numerically). She also appears to see this as a means of preparing pupils for aspects of SATs mathematics reasoning papers.

6.1.2 Small steps and success for all

Although they did not form part of the participants' definitions of TfM, their responses to further questioning demonstrated that both participants perceived small steps as a fundamental feature of a TfM approach.

> *With Mastery, it's broken down into really manageable steps and you're always building on the previous learning, which I really do like. It means that all the children move through the topics at the same pace, so everyone's kept together. – SLM*

The SLM believed that small, manageable steps which build logically on prior learning were key to ensuring children could progress at the same pace. Similarly, the HT indicated that small steps in learning allowed all children to experience success in mathematics lessons.

> *Now that we're using Mastery, I think we're better at setting up a lot of skills in smaller steps and allowing kids to consolidate those skills and apply them. That helps all the children to make the same progress, so all children have the chance to succeed in maths. – HT*

However, when asked if all children make the same progress during lessons, the HT admitted that this was not always possible, citing additional difficulties for some children.

> *Some children might not be able to access the maths without a lot of support. That's the reality of it. If it's a SEN child or they're working significantly below age-related [expectations], you have to put a lot of structures in place, because they can't access it on their own. – HT*

This response indicates that even with small steps, ensuring the same rate of progress for all children within a class is problematic, particularly for children with SEN or for those who have significant gaps in their mathematical knowledge.

Neither of the participants referred to the 5 big ideas of Mastery as outlined by the NCETM (2023), although they did refer to some elements of this model such as small steps (coherence) and fluency. Variation (procedural or conceptual) and mathematical thinking were not mentioned by either participant during the interviews and representations and structure were referred to only briefly in terms of the overuse of concrete representations. This suggests that the participants held incomplete understandings of TfM despite having incorporated what they deem to be a TfM approach to mathematics in their school for over 4 years.

6.2 Motivation to adopt a TfM approach

Two themes emerged during interviews relating to the motivations behind the implementation of a TfM approach in School A, which were outside influences and negative attitudes towards differentiation. These themes are outlined in the following sections.

6.2.1 Outside influences

Both interview participants identified outside influences as the most influential factor in the decision to adopt a TfM approach in School A. The HT indicated that this approach was imposed on schools, rather than being presented as a choice.

> *…definitely outside influences, government imposition really. HT*

The HT also cited pressure from Ofsted as a motivating factor for instigating change in School A with regard to TfM.

> *…it depends how much force they put on you. That force tends to come through Ofsted really. We just have to tick the right boxes – HT*

While the HT perceived that there was an obligation for the school to roll out a TfM approach to mathematics, the SLM viewed herself as a key instigator of change as subject leader for mathematics, after she was encouraged to adopt the approach during subject leadership meetings.

> *It was quite a bit of influence from me really… from going to subject leadership meetings, we were really being encouraged at the time to go down one route or another, which were White Rose and Inspire. Most schools were going for one or another. - SLM*

The SLM's response indicates that external pressure from government representatives at subject leader meetings, coupled with a form of peer pressure as more and more schools began to implement the approach, were the most influential factors in her decision to adopt the approach in School A. However, the SLM also suggested that she was amenable to adopting the TfM approach even without factoring in outside influences.

> *I wanted to push the children even further and have that deeper understanding and be able to use mathematical language. – SLM*

It would seem that the participants experienced different levels of perceived pressure with regard to the implementation of TfM, with the SLM having had greater access to information regarding the approach through subject leadership meetings. Since the SLM was in favour of the TfM approach as a means of engendering a deeper mathematical understanding and promoting the use of mathematical vocabulary, it is possible that some of the perceived pressure to adopt a TfM approach from the HT's perspective may have come from the SLM herself, as subject leader for mathematics.

6.2.2 Negative attitudes towards differentiation

While there were differences between the HT and SLM in terms of their attitude towards the TfM approach (with the HT perceiving the reform as one imposed by the government, rather than a means of improving standards in mathematics, as perceived by the SLM), both participants indicated unfavourable views about the use of differentiation by task in previous teaching practices (used prior to rolling out the TfM approach in School A).

> *Differentiation for me has always been a race to the bottom, not the top…if you constantly pander to people's lack of ability, it's human nature that they will find the easy route. – HT*

> *I really agree with always teaching to the highest level in the class and the others have to kind of 'up their game'. They're clinging on in there, but with the right support, they do manage to do it. If you teach to the lower-level kids, are you really ever going to see that progress being made? – SLM*

Both participants associated differentiation by task with lower levels of progress for pupils and suggested that this led to a lack of challenge for pupils accessing lower-level tasks.

6.3 Choice of TfM materials (WRM)

Interview responses from participants in School A indicated three key reasons behind the school's decision to adopt the WRM scheme as a representative of the TfM approach: free access, flexibility within the scheme and the fact that it could be used as a temporary measure which could be quickly changed if necessary. These themes are described in the following sections.

6.3.1 Free access

Both interview participants from School A identified the fact that access to WRM materials was (initially) free as a crucial factor in the decision to adopt this scheme rather than other TfM-aligned schemes on the market.

> *The fact that they were free at the time. Well, I think some of it still is free, but it was all completely free to start with and that was a big influence…we needed something and there was nothing else freely available at the time. – HT*

> *The resources were open and made free to everyone, so there wasn't any massive investment initially. We trialled them first and decided that's what we were going to stick with. – SLM*

The HT's response suggests that other TfM-aligned schemes would have been explored as an option if they had been freely available to schools, suggesting that cost was the overarching factor in the decision to use WRM materials as a means of implementing a TfM approach in School A. Additionally, the fact that the materials were free made it relatively easy to trial the materials, as noted by the SLM.

6.3.2 Flexibility

The HT and SLM referred to flexibility as an influential factor in their decision to follow the WRM TfM scheme. While other TfM schemes available came in the form of physical, printed textbooks, WRM tasks and activities took the form of online resources which could be adapted according to the teacher's preferences.

> *We looked at the Singapore maths, but we didn't feel it was right for this school and it didn't have the flexibility that White Rose does. I don't think many schools would want a totally rigid approach. – HT*

> *…it didn't need to be followed exactly as it was set out by White Rose, it could be adapted because it was all online, and I think we have made it very bespoke for our school. I know when we looked through the initial planning documents, we would use our teacher judgements and think, 'no, that's not appropriate - that won't work for that class', and you could pick out the parts that you want to use and just print them out. – SLM*

These responses indicate that the staff in School A value the opportunity to use their professional judgement in the selection of mathematical tasks and activities, rather than follow a particular set of materials faithfully. The fact that WRM were available online meant that individual tasks could be selected and printed while other tasks could be disregarded, whereas this approach would be unsuitable for schemes based on physical textbooks.

6.3.3 A temporary measure

Linking with the earlier references to flexibility, both participants viewed the use of WRM materials as a temporary measure to be used until the next policy change. They expressed concern that such shifts in education policy could result in the need for primary schools to abandon TfM altogether and suggested that implementing a future policy change would be more problematic if they were following a particular TfM-aligned scheme rigidly.

> *I do fear, just because of the nature of education, that the over-differentiators are probably just over the horizon to criticize and pull holes in this [Mastery]. Unfortunately, that's the nature of education. They reckon if you stay the same and stick to one thing that you believe in, you'll be right twice in your career. That is awful, but it's true. – HT*

> *I don't think it's ever wise to follow a scheme rigidly because… it could all change tomorrow and policies and guidance change so quickly. You have to be able to adapt quickly, but if you're following a particular scheme to the letter then it's going to be much harder when change comes, and you have to leave all that behind. - SLM*

The SLM also argued that policy changes disrupt pupils' learning where this necessitates abandoning the rigid use of a particular scheme.

> *It would be much more upheaval for the children as well because they get used to a set way of doing things. I mean, it was a big change when we started doing Mastery as well. The children have to adapt, just like we do! – SLM*

This response suggests that educational change can actually hold back pupil learning as they adjust to the changes brought about by new policy initiatives, especially where this entails a shift from a rigid scheme, and this may discourage teachers from fully embracing educational change.

6.4 Understandings of problem-solving

School A was chosen using purposive sampling based largely on the decision of the leadership team to focus specifically on NRPS in their SDP. Therefore, there was an expectation that interviews with the HT and SLM could provide valuable information about opportunities for NRPS within a TfM approach to mathematics. However, when asked to define NRPS, both participants struggled to provide a clear explanation.

> *Non-routine problem? Can you explain that to me? Is it worded problems? Or shape problems and things like that? - HT*

> *Oh, it means a lot of different things, doesn't it? Erm, I think it's really all about identifying what type of problem we are addressing here, how we are going to tackle it. - SLM*

The lack of understanding of NRPS demonstrated by the participants in School A was a particularly surprising finding, not only considering the high status of NRPS in the National Curriculum (DfE, 2013) but also in light of their own decision to focus on NRPS in the SDP. However, while the participants expressed confusion surrounding the term 'non-routine' in NRPS, they identified multiple approaches and cognitive challenge as key elements of problem-solving (when no reference to the 'non-routine' component was made). These two themes are outlined in the following sections.

6.4.1 Multiple approaches

In defining problem-solving, the SLM identified the feature of having multiple possible approaches to solve it as a feature.

> *There's different ways to tackle it isn't there? It might be trial and error to solve it, where we might work systematically. – SLM*

The SLM also believed that a key feature of problem-solving was the open-endedness of the task, a feature she associated with having multiple possible approaches.

For me, problem-solving is very open-ended. It could be explored in any way; you could come to the answer in a different way. It might be where you say 'talk to your partner' about it, to decide how you are going to tackle this problem to come up with a solution. Whereas I think a more worded maths problem or multistep problem is more clear-cut. –SLM

The SLM recognised that most problems have multiple possible solutions that can be used in solving and suggests that this presents difficulties for children, requiring the teacher to implement supportive structures such as peer discussion. She also acknowledged that some word problems (whether single or multi-step) can be 'clear-cut' (possibly in reference to routine word problems) and differentiates this from 'open-ended' problems.

6.4.2 Cognitive challenge

When asked what problem-solving meant to them, both participants suggested that a key feature was the level of cognitive challenge for pupils as they solve mathematical problems:

…no matter how effectively we feel we are teaching maths, children repeatedly find it [problem-solving] the most difficult thing to do. So any analysis of children's performance on tests always shows up problem-solving, and that can kind of disillusion you sometimes because you can think, 'where do we go with this? – HT

It is a challenge, and you are going to have to put your mind to it and try hard…The children need to give a lot of time and effort to solving it. – SLM

The HT's response suggested a negative attitude towards the level of challenge inherent in problem-solving activities, as he appears to show some frustration that children find problem-solving difficult 'no matter how effectively we feel we are teaching maths'. His answer also indicates that there is some confusion surrounding the effective teaching of problem-solving skills to improve problem-solving performance in pupils. In contrast, the SLM, whilst acknowledging the sustained effort required in solving mathematical problems, viewed this as a necessary, rather than negative, aspect of problem-solving.

6.5 Problem-solving in School A

Previous sections have highlighted that the participants in School A had a very limited understanding of NRPS. This raised the question of why they would have decided to focus on this element of mathematics education in the SDP. When asked to explain the main reason for this decision, there were conflicting opinions.

> *Our children are extremely numerate, they like maths and achieve very highly. But they are derailed when they can't see what the problem is asking them to do. And it's the same problem in maths in all schools. Instead of teaching them the answer, we need to teach them the skills. - HT*

> *I think the reasons are because it develops that resilience, that 'I am going to give this a go' Having that ethos of its okay to get something wrong and it's not the end of the world if that happens. It's about them wanting and accepting a challenge, and that develops a growth mindset. - SLM*

For the HT, the key aim is for children to develop the problem-solving skills required to be successful in solving NRPs, while the SLM's main focus is on the development of a growth mindset, where children are encouraged to believe they can improve their mathematical performance through perseverance and seeking a greater challenge.

Due to School A placing an increased focus on NRPS skills by including this target in their School Development Plan, it was expected that the compatibility of this with the TfM programme already in use within the school would have already been carefully considered. However, due to the disruption caused by Covid 19, School A was in the very early stages of developing NRPS at the time of the interviews, which may have contributed to the contrasting viewpoints between participants and may also have affected their ability to answer the questions thoroughly. Despite this drawback, a number of themes did emerge through analysis of the data gained from the participants, including routine problems, modelling and growth mindset/problem-solving. Patterns and contradictions within these themes are illustrated through interview quotations in subsequent sections.

6.5.1 Routine problems

When asked what problem-solving looked like in his school, the HT indicated that children have limited experiences with problem-solving in School A, which occur mainly at the end of a mathematics topic and are largely routine, although he refers to the situation having improved slightly since the introduction of a TfM approach.

> *What I fear it looks like is all problems are the same type given at the end of a topic of work. I think that's become less so over the last 5 years, but that's what I fear it is. I don't think that is the case exclusively, but it can be like that. - HT*

In contrast, The SLM believed that there were many opportunities for problem-solving at the end of a mathematics lesson, but her response suggests that these are routine problems rather than NRPs.

> *There's lots of opportunities for them [pupils] to apply the skills they've learned earlier in the fluency aspect of the lesson to a problem that's set out in a different way, so they can be more flexible with how they're thinking about the maths. – SLM*

The SLM's description of applying learning from earlier fluency tasks to 'a problem that's set out in a different way' seems to correspond more closely to routine problems, which are used to allow pupils to practise mathematical procedures and skills which have been taught previously to tasks which may be presented in a different way (such as worded questions).

These responses suggest an over-emphasis on routine problems in School A, despite the focus on NRPS in the SDP and the requirement for pupils to engage in NRPS in the National Curriculum (DfE, 2013).

6.5.2 Modelling

Linking with the earlier theme highlighted above (routine problems), participants from School A both believed that problem-solving must be modelled by the teacher if children are to be given the opportunity to succeed in problem-solving activities.

> *It's really all about modelling how to solve a problem and explaining, right, I'm going to use trial and error here…before they have a go themselves.*

Or sometimes it starts with a discussion with the class, and we have a chat about how we are going to get IN to this problem. The teacher needs to lead the way because the children need to be guided to get on board with that. Some of them can find that quite frustrating as they want to get straight into it independently, but I think most can see the benefit of that as well. – SLM [emphasis added]

The SLM suggests that modelling of the solution strategy is used even where children want to attempt the problem independently, causing them to become 'frustrated'. Alternatively, whole-class discussion is used to find the starting point for solving the problem, which would reduce the cognitive challenge significantly and allow all children to access the problem.

The HT also stated that problem-solving in School A is characterised by the modelling of the solution strategy by the teacher.

… kids need to be given opportunities to enquire, but that has to be alongside the structured teaching of skills, or they can't solve the problem and they just flounder. They become demotivated and make no progress because it is so open. As it stands at the moment, the teacher has to model it a lot. I'd like to change that, so that instead of modelling different problems, we teach them which suite of skills matches that type of problem. I think it needs a coordinated, concerted approach, which we haven't had. – HT

This response highlights the difficulties for teachers in dealing with higher levels of cognitive struggle inherent in problem-solving activities and suggests that prolonged struggle may cause pupils to become demotivated unless the teacher models how to solve the problem. However, the HT believed that a more effective way of supporting pupils in the development of problem-solving skills may lie in the teaching of heuristics linked explicitly with the different problem types that pupils may encounter, as well as metacognitive strategies to monitor their progress during problem-solving activities.

We need to teach them in a metacognitive way, when it is this type of problem, you need these types of tools and skills. That would signify what method to use with different problems and these would be signposted and constantly rehearsed so they could be identified easily. Really this is the

> *key thing on the School Development Plan, although it has been derailed this year, for all sorts of reasons including Covid. This is a long-term target. – HT*

It is clear from the responses that the key objectives within the SDP for NRPS had not begun to be implemented at the time of this study, with barriers such as the Covid-19 pandemic having prevented greater progress in this area.

6.5.3 Growth Mindset and problem-solving

Participants from School A had divergent views about whether *all* pupils' problem-solving abilities could be developed. The SLM spoke of the importance of the classroom ethos in developing a growth mindset in her pupils.

> *…it's about having that ethos in the classroom….making sure they're not going to quit at the first hurdle. Having that ethos of 'it's okay to get something wrong' … It's about them wanting a challenge, and that develops a growth mindset. – SLM*

In contrast, the HT did not believe that all children could solve mathematical problems, demonstrating a fixed mindset about problem-solving ability.

> *…you can actually feel like - this is terrible for an educator to admit to, really - you feel like the only ones who can solve maths problems like that are the ones that are considered natural mathematicians. Which is wrong because that in itself is me limiting my aspirations for other kids. – HT*

The HT also referred frequently to pupils as being 'maths-ready' or 'non-maths ready' to describe their ability to solve problems (or not) and spoke of a small group of children who 'innately' know how to solve mathematical problems, discussing these children using the analogy of 'naturally gifted sportsmen'.

> *I see some kids who are naturally gifted sportsmen. They have a sense of balance, vision, special understanding which is innate…I see that with kids in mathematics and they appear to be the ones who can readily take on a problem in mathematics and solve them. They just have an understanding of what's required and innately know how to solve the problem. - HT*

6.6 Opinions of NRPs within WRM materials

Although the participants from School A were not able to provide an accurate definition of NRPS, they did demonstrate some knowledge about problem-solving in terms of such features as multiple possible approaches and higher levels of cognitive demand, therefore they were able to give some insights about the quality of the problems within the WRM materials. However, in general, they did not demonstrate particularly strong opinions when asked what they thought about the problems provided within WRM materials during the interview, although the SLM had more positive views.

> *They're alright. They're not the answer to everything but they're probably better than a lot of others that we have access to. – HT*

> *I like them in general - they're nice and varied. – SLM*

The HT's response seems to indicate a general dissatisfaction with problems within TfM-aligned materials, however, this may be due to budget constraints limiting access to higher-quality resources. At the time that the interviews took place, School A had recently purchased the SUB materials from WRM, having previously used the FA materials only. The SLM declared a preference for the SUB materials over the FA materials, although her response was more strongly focused on an increased number of fluency tasks within the SUB materials.

> *[the FA materials] definitely didn't go deep enough for the three aspects, but I think the new subscription materials are better in all respects, there's definitely more fluency. – SLM*

Although the SLM described the problems within the WRM materials as being 'varied', only two problem types were referred to during the interview. These problem types were FAP problems and real-life word problems, which are outlined in the following sections.

<u>6.6.1 Finding all Possibilities</u>

The main problem type referred to during the interview with the SLM was FAP problems, although the HT did not specifically mention any particular types of problems.

There are problems where the children have to find all the possible answers. There's a lot of problems like that in White Rose… the children get that experience of working systematically, so they might say to me, 'I'm going to start with the smallest answer' and they check that they haven't missed out any of the answers. – SLM

The SLM discussed the value of FAP problems in providing opportunities for pupils to work systematically, sometimes by starting with the lowest value, to ensure they have found all possible answers to the question and can check that they have not missed any values out.

<u>6.6.2 Real-life problems</u>

In addition to FAP problems, the SLM identified word problems with real-life scenarios as another problem type which she thought were particularly prominent within the WRM materials.

…everyday scenarios as well. Like, we've been looking at decimals and you've got children with amounts of money that want to buy such-and-such. Our children can relate to that. Whereas I think with some questions you just think, how on earth are the children going to get their heads around that? Because they don't understand all of the things being mentioned in that problem. So yes, they [problems within WRM materials] are quite successful in a lot of ways. – SLM

The SLM states that word problems which link more closely to real-life experiences of primary children are more accessible to them, with familiar scenarios helping them to understand what is required. Her statement also suggests that such tasks are largely routine, with a clear starting point that pupils can quickly identify.

<u>6.6.3 Supplementing WRM Resources</u>

When asked if WRM offer sufficient opportunities for NRPS, the participants from School A held differing opinions.

Yes, I do actually. There are a lot from NRICH to give a bigger menu to choose from, but I think many of them aren't accessible to children who are not natural mathematicians. – HT

> *No. Not just as a stand-alone. You have to incorporate other resources… It's all about finding them and also writing them yourself - what you think will work well for the children in your class and the learning that's been going on in the classroom. – SLM*

The HT's response suggests that problems within WRM materials require lower levels of cognitive demand to solve them in comparison with materials provided on the NRICH website, while the SLM indicated that teachers select particular tasks based on their knowledge of the pupils in the class.

Despite juxtaposed opinions about the number of opportunities for NRPS within the WRM materials, both participants agreed that WRM materials were not used exclusively in School A and were supplemented with additional problems from different sources.

> *You have to incorporate other resources. Although there are really good examples of problem-solving questions, there are loads of other resources out there to take that learning even deeper. – SLM*

> *Some of the stuff is a bit dry and you need more stuff. I think it lacks open-ended problems. – HT*

Some of these answers seem to contradict earlier responses, which may be due to misinterpretations of 'non-routine' problem-solving. The SLM also indicated that WRM materials lacked fluency tasks rather than NRPs.

> *You do need abundance of the fluency questions and the same for the reasoning and problem-solving. Maybe you don't need loads more for those two aspects but still more than they provide you with… I was very aware of this when we started to use it. It's good to use as a lead in but it's not the be-all-and-end-all. – SLM*

This response may indicate that teachers in School A use supplementary resources for NRPs less frequently than they do for fluency tasks, which raises further doubt about whether pupils in School A are provided with sufficient opportunities for NRPS.

In terms of accessing additional resources, only two sources were mentioned explicitly, these being the NRICH website and the National Numeracy Strategies documents published by the DfE over 20 years ago.

6.7 Conclusions

Participants from School A were responsible for the decision to focus on developing NRPS skills in the SDP, after having adopted a TfM approach to mathematics instruction over four years before interviews were conducted. However, neither participant was able to articulate what NRPS means and showed confusion when asked to define it. Furthermore, they also had limited and divergent understandings of TfM, with the HT viewing it as an outcome rather than a pedagogical approach (as expressed by the SLM). This finding is consistent with the recent research conducted by Simpson & Wang (2023), which called attention to a lack of a mutual understanding of TfM in the education world and suggested that this may seriously hamper teachers' attempts at implementing TfM effectively in the classroom.

Participants from School A had different experiences in terms of the amount of perceived pressure to adopt a TfM approach, with the HT describing government imposition and pressure from Ofsted to implement the changes in policy, and the SLM perceiving greater autonomy in the decision-making process, despite citing pressure from other professionals during subject-leadership meetings. The SLM believed that the TfM approach would benefit pupils in School A by engendering a deeper understanding of mathematical concepts, as well as greater proficiency with mathematical vocabulary. Both participants viewed the TfM approach as a means of dispensing with differentiation of tasks based on prior ability, which was perceived as a positive change. However, aspirations for NRPS do not appear to have been a motivating factor for either participant in adopting a TfM approach in School A.

The decision to follow the WRM scheme was largely based on the fact that this was the only TfM-aligned scheme that was freely available to schools, with other influential factors including flexibility for teachers in adapting the resources and cutting out unwanted activities, since resources are provided in the form of electronic worksheets. A flexible approach was seen as being preferable to a more rigid approach since it would be less upheaval to make changes when future policy changes are rolled out. Again, there was no mention of NRPS in participants' explanations of the reasoning behind choosing WRM as a TfM-aligned scheme.

The lack of a clear and mutual rationale for the decision to focus on NRPS in the SDP suggests a knee-jerk reaction to policy initiatives imposed by external sources.

In justifying their decision, the HT referred to the need for pupils (who achieve well in other aspects of mathematics) to develop problem-solving skills through the use of heuristics linked with particular problem types, while the SLM aspired to develop a growth mindset in pupils. However, despite the importance accorded to NRPS within the SDP, both participants struggled to define NRPS during interviews.

The descriptions of the situation in School A with regards to problem-solving suggest that mathematics instruction is largely characterised by fairly homogenous routine problems which allow pupils to practise procedures and techniques they have learned in previous fluency tasks. In addition, interview responses suggest that problem-solving activities are heavily modelled by the teacher so that all pupils can immediately access the problem, even where this causes frustration for pupils who want to attempt it independently. Furthermore, the belief that only ‘natural mathematicians’ can solve mathematical problems (or a fixed mindset) expressed by the HT could be seen as a significant barrier to developing NRPS skills in School A.

Finally, participants in School A believed that WRM materials provided sufficient opportunity for pupils to engage in NRPS, despite their lack of understanding about NRPS and their inability to define it during interviews. Their explanations of NRPS in School A also suggested a lack of more challenging problems in normal mathematics instruction in School A, as well as routine supplementation of WRM materials with other resources (although this may be mainly for fluency tasks rather than for NRPs). Although the HT did not refer to any particular problem types within the WRM materials, the SLM spoke of frequent opportunities for children to engage in FAP problems, as well as W problems based on real-life scenarios.

This chapter has explored the perceptions of decision-makers for mathematics in one school regarding opportunities for NRPS within a TfM approach. Although the interview responses indicate a lack of understanding about NRPS, participants from School A believe that WRM provides sufficient opportunities for pupils to engage in NRPS during mathematics lessons, although, as outlined in Chapter 5, this is not actually the case. Having reported the results relating to the two stages of this study in Chapters 5 and 6, the next chapter discusses the findings in relation to the current research base.

Chapter 7: Discussion

This study centred around the fitness-for-purpose of popular TfM-aligned materials, with a particular focus on opportunities for NRPS. A combination of quantitative and qualitative methods was employed to engender a comprehensive understanding in addressing three overarching research questions:

1) What opportunities for NRPS exist within the WRM TfM materials?
2) What are the perceptions of school senior managers in England about problem-solving opportunities within the WRM TfM materials?

This chapter draws the findings for each research question and discusses these in terms of their significance within the current knowledge base. Following this, some limitations of the study are outlined, key implications are discussed, and a conclusion is presented. The chapter concludes with some suggestions and directions for future research.

7.1 What opportunities for NRPS exist within a TfM approach?

7.1.1 A balanced diet?

TfM has become the prevailing approach underpinning mathematics instruction in UK primary schools (Boylan et al., 2019; Marks et al., 2023; NCETM, 2022), despite wide variation in how this approach has been implemented in different settings (Blausten et al., 2020) and divergent interpretations of the approach by educators (Simpson & Wang, 2023). A number of TfM-aligned schemes, textbooks and online resources are now available for schools to choose from (Boylan et al., 2018), and more than half of UK primary schools surveyed by Marks et al., (2023) reported using one such scheme as the main source of activities for mathematics instruction. However, there is a gap in the research literature surrounding the degree to which the use of these resources fulfils key aims of the National Curriculum (DfE, 2013), such as NRPS. Research over many decades has highlighted NRPS as the ultimate aim of mathematics education (Borthwick, 2019; Foster, 2015; Liljedahl & Cai, 2021; Liljedahl et al., 2016; Schoenfeld, 2014), and in order for children to be prepared for solving mathematical problems in adulthood, a *substantial proportion* of the tasks they are exposed to should be non-routine problems (Burkhardt & Schoenfeld, 2019).

This research used one of the resources mentioned above: WRM materials represent one of the most popular TfM-aligned schemes, and these are used (to some extent) by approximately 80% of English primary schools (Marks et al., 2023; WRM [personal correspondence], 2022). A framework amended from van Zanten & van den Heuvel-Panhuizen (2018) was used to analyse opportunities for NRPS within these materials through quantitative and qualitative content analysis. The findings of the quantitative content analysis revealed that the proportion of NRPs within the WRM materials is surprisingly low, with less than 3% of the tasks classified as NRPs in each subscription type and year group. This finding corresponds with previous studies which have found proportions of NRPs within curricular materials to be between 0-5% (Kolovou, 2011; van Zanten & van den Heuvel Panhuizen, 2018; Zhu & Fan, 2006). If WRM materials were used exclusively, this would provide less than 1 opportunity per week for NRPS if only the FA materials were used. Where SUB materials were purchased and used in addition to the free materials, this would still provide less than 2 opportunities per week (on average) for pupils to engage in NRPS. However, these averages are dependent upon *all* of the NRPs available being selected for use in mathematics lessons by the teacher (this is discussed in the next section), and since NRPs were not evenly spread within the materials, children may go for long periods without engaging in NRPS.

Qualitative content analysis allowed for the exploration of the nature of NRPs within the WRM materials. In particular, the distribution of different problem types was investigated to better understand the range of different NRPS experiences that pupils are likely to be exposed to through the WRM materials. The findings demonstrated that the majority of NRPs were FAP problems, which represented approximately 68% of the collective materials. LG problems formed the second most popular problem type, making up 15% of all NRPs within the WRM sample materials. However, W, DV, and RP problems made up a much smaller proportion of the materials, representing between 4-7% of NRPs within the sample. In terms of opportunity within mathematics lessons, Year 2 pupils would have no opportunities to engage in W or DV problems during the school year where the WRM materials (FA and SUB) were used exclusively. Year 4 children would have a maximum of 5 opportunities in the school year to engage in problems other than FAP and LG if all the NRPs from the FA and SUB materials were selected for use in mathematics lessons, and Year 6 children would have between 3-9 opportunities to solve W, DV or RP problems.

Although the use of FAP problems in the classroom is beneficial in providing opportunities for pupils to work systematically, a skill associated with the ability to structure effective strategies in the face of different problem types (Barmby et al., 2014; Woodham, 2018), the very low level of opportunity to solve other types of problem is concerning. Oftsed (2021) outlined the need for pupils to be able to identify different problem types as well as effective strategies for solving these types of problems in order to develop problem-solving proficiency. However, it would seem that exclusive use of the WRM materials would lead to pupils' experiences with NRPs being disproportionately oriented to FAP problems during their school career. Furthermore, without supplementation from other sources, children may experience extended periods without encountering the same problem type again, reducing opportunities for them to become familiar with different types of problems and effective strategies for solving them.

Chapter 5 provided an in-depth exploration of specific examples of the represented problem types within the sample materials, and this evidenced some high-quality problems offered within the WRM materials which provided opportunities for higher levels of cognitive activation and creative approaches. However, the infrequent rate at which these appear in the materials means that exclusive use of WRM resources cannot be expected to provide the 'balanced diet' recommended in the research literature (Burkhardt & Schoenfeld, 2019, p.37). Consequently, this raises questions regarding the fitness-for-purpose of TfM-aligned resources in supporting children to develop crucial problem-solving skills as well as knowledge of effective strategies for different problem types.

7.1.2 Grey areas and misleading labels

The results from the quantitative content analysis revealed that there were more examples of GATs than NRPs within the sample WRM materials. With GATs, while the solution pathway is not completely straightforward, the cognitive demand required to find an answer and the opportunity for creative approaches are limited compared to NRPs (van Zanten & van den Heuvel-Panhuizen, 2018). While these kinds of tasks are important in their own right to allow for practising mathematical procedures in a variety of different contexts (English & Gainsburg, 2015), GATs cannot replace NRPs or allow pupils to reap the many benefits associated with NRPS (Drury, 2019). While there is the potential for teachers to adapt GATs to increase cognitive demand and allow greater opportunity for creativity, research suggests that prospective teachers struggle to reformulate tasks in this manner (Leavy & Hourigan, 2019). Although it is not clear in the research literature whether the same is true of qualified teachers, the lack of understanding of NRPS problem-solving demonstrated by the HT and SLM in School A would suggest a similar situation with qualified and experienced teachers.

Furthermore, since GATs share some characteristics with NRPs, such as an increased level of challenge compared to STs and a lack of a straightforward route to a solution, this may cause confusion for teachers in distinguishing between them. Adding to the confusion, GATs were provided under a subheading of ‘reasoning and problem-solving’ within the WRM FA materials, and on many occasions, STs were labelled as ‘problems’ within the task instructions (eg, ‘solve these problems using the same method’). Baumanns & Rott (2022) argued that this kind of indiscriminate use of the term ‘problem’ within curricular materials culminates in difficulties for teachers in discriminating between NRPs and other task types, and this is exacerbated by a lack of precise guidance from policymakers surrounding what constitutes an NRP (Xenofonotos & Andrews, 2014).

7.1.3 ‘Scaffolding away’ the challenge

Chapter 2 drew upon the research literature to position cognitive challenge as a crucial component of NRPS, engendering high levels of thinking and reasoning and creating opportunities for pupils to persevere to find a solution (DfE, 2013; English & Gainsburg, 2015). Furthermore, research suggests that higher levels of challenge in mathematics lessons leads to improved mathematical achievement (Li et al., 2020). However, teachers must walk a fine line in terms of the level of support they provide for their pupils during more challenging mathematical activities (Dingman et al., 2019; Lester, 2013). Scaffolding refers to the supportive structures implemented by teachers to allow pupils to succeed in more challenging tasks (Wood et al., 1976, as cited by Bakker et al., 2015). However, too little scaffolding results in frustration and unproductive exploration in pupils and too much scaffolding reduces cognitive demand and constrains pupils’ opportunities to persevere in seeking a solution.

Empirical studies in China have found that many challenges are ‘scaffolded away’ in mathematics lessons (Zhou et al., 2023), and this may also be the case in English schools following a TfM approach due to the prevalence of worked examples in the WRM materials. Whilst worked examples can be useful in drawing pupils’ attention to effective problem-solving strategies (Barbieri et al., 2023; Henderson et al., 2022) and improving pupils’ performance in solving *routine* problems (Adeniji & Baker, 2023), overuse of worked examples can be detrimental in reducing pupils’ opportunities to engage in NRPS by scaffolding away the challenge involved in problem-solving, and research suggests that using worked examples does not improve problem-solving performance (Adeniji & Baker, 2023).

Furthermore, interview participants following a TfM approach in School A suggested that typical teaching practices involved high levels of modelling, which they believed was important to ensure pupils were not left to ‘flounder’ during more challenging activities. Dale & Scherrer (2015) suggested that the inclination to intervene and reduce the level of struggle for pupils was a common culture of Western teaching, and the findings from this study corroborate this standpoint. High levels of modelling are yet another example of NRPs being scaffolded away to render the problem relatively straightforward, and while Dale & Scherrer (2015) suggested that too little scaffolding can lead to frustration for pupils, findings from this study suggest that too *much* scaffolding through modelling can have a similar effect: the SLM reported that students often become frustrated when the teacher models the problem, as they ‘want to get straight into it independently’.

The HT in School A appeared to recognise a need to reduce modelling in mathematics lessons, suggesting that teaching heuristics linked with different problem types alongside metacognitive skills may benefit pupils in their development of problem-solving skills. Although the research suggests that explicit teaching of heuristics in isolation is insufficient to develop pupils’ problem-solving ability (Schoenfeld, 1985, Sweller, Clark & Kirschner, 2010), many studies have linked high levels of metacognition with problem-solving proficiency (Quigley et al., 2018), however, this is dependent upon pupils being exposed to a wide range of different NRPS experiences.

7.2 What are the perceptions of teachers in England about problem-solving opportunities within the WRM TfM materials?

7.2.1 Why WRM?

The findings from semi-structured interviews revealed that the motivation behind the decision to adopt a TfM approach to mathematics instruction in School A was based largely on the perceived need to ‘tick a box’ in response to outside pressures from government agencies such as Ofsted. However, both participants in School A reported dissatisfaction with previous practices which involved high levels of differentiation by task. Instead, TfM promotes differentiation by support (or scaffolding), and this may have been an additional motivating factor for the shift to TfM. With many TfM-aligned resources available, the choice of WRM materials was not based on any judgements about the opportunity for NRPS, but more general, practical factors such as the resources being freely available and easily accessible online, and the fact that they could be easily adapted at teachers’ discretion.

Although School A reported using WRM as the main source of mathematical tasks, common practice involved replacing or supplementing tasks within the materials with activities retrieved from other sources such as old National Strategy resources (DfES, 2004) or the NRICH website (University of Cambridge, 2023), although there may be greater supplementation of straightforward fluency tasks than for NRPs. School A’s unwillingness to follow one particular scheme or textbook resource faithfully provides more evidence of an anti-textbook ethos in England, as described by Oates (2014), with the SLM in School A demonstrating deprecating views about a ‘rigid approach’ and describing a school culture whereby teachers have high levels of autonomy in using their professional judgement, based on their particular pupils, in their selection of mathematical tasks.

While participants did not have particularly strong opinions about WRM materials, the reported need to supplement problems within WRM materials with those from other sources may suggest some dissatisfaction with these materials. Furthermore, the later decision to focus on NRPS within the SDP provides more evidence that opportunities for NRPS within the WRM materials were deemed insufficient to develop problem-solving proficiency in pupils.

7.2.2 "Can you explain that to me?"

Chapman (2015) highlighted the wide range of knowledge teachers need to be able to teach problem-solving effectively, and the selection of appropriate problems by teachers is dependent upon their having a clear understanding of what constitutes an NRP (Chapman, 2015; van Zanten & van den Heuvel-Panhuizen; Xenofontos & Andrews, 2014). The current evidence base demonstrates that pre-service teachers typically struggle to identify fundamental features of NRPs (Baumanns & Rott, 2022; Kohar et al., 2019; Leavy & Hourigan, 2019; Walsh, 2016; Xenofontos & Andrews, 2014), however, few studies have explored the level of understanding in qualified teachers. The findings from this study suggest that the lack of understanding about NRPS may be more widespread than previously believed, since participants with substantial experience (in their relative roles as subject leader for mathematics and head teacher) had difficulty in defining NRPS, despite this being a key area for development on the SDP for School A. Furthermore, when asked to describe what NRPS looks like in School A, both participants described the use of *routine* problems, demonstrating a mismatch between their beliefs about teaching practices in School A and the actual experiences being offered to pupils.

The implications of this lack of understanding about NRPS are many. Firstly, as Barmby et al. (2014) warned, this lack of understanding seems to have led to an over-reliance on routine problems and high levels of modelling by teachers in School A. Secondly, participants believed they were providing NRPS opportunities when this does not seem to be the case from their explanation of normal teaching practices in School A. Furthermore, this lack of understanding about NRPS undoubtedly constrains their ability to evaluate curricular materials in terms of the opportunities they offer for NRPS (Leavy & Hourigan, 2019). This may explain conflicting opinions about the level of opportunity for NRPS within the WRM materials in participants from School A.

The finding that experienced educators lack understanding of NRPS is particularly noteworthy considering that NRPS is a core objective in mandatory legislation (DfE, 2013). The response given by the HT when asked to define NRPS, 'Can you explain that to me?' lends greater weight to Xenofontos & Andrews' (2014) argument that curriculum writers should provide a full explication of mathematical NRPs and NRPS to ensure a mutual understanding in the education world. Without this, many teachers are likely to assume that they are providing opportunities for NRPS despite the fact that many of the tasks they utilise may not actually constitute a problem at all, or they may structure NRPS opportunities in such a way that reduces the level of challenge and render the problem relatively straightforward, which could significantly impede pupils' mathematics learning.

7.2.3 A fixed mindset?

Another surprising finding from this study involved the apparent 'fixed mindset' of the HT in School A, particularly considering that this is in direct contrast to the growth mindset promoted by TfM. The HT believed that 'problems' within WRM materials were more accessible to pupils, while more challenging problems from other sources were deemed to be suitable only for pupils who were deemed 'natural mathematicians' who 'innately know how to solve [a] problem'. Although this study did not explore whether there were any negative effects of this on pupils' perceptions about their own mathematical abilities, this finding is worrying considering that pupils are more likely to hold fixed mindset attitudes about their mathematical ability than for other subject areas (Jonsson Beach, Korp & Erlandson, 2012). If pupils adopt the idea that mathematical intelligence *cannot* be improved with effort and perseverance, this could severely impact their attitudes towards NRPS and, subsequently, their mathematical achievement (Yeager & Dweck, 2012).

Additionally, the HT's response demonstrates that his 'fixed mindset' beliefs about pupils' abilities influenced the kinds of tasks he deemed appropriate for mathematics lessons. While the research evidence on growth mindset is fairly limited to date and is largely based on students' mindsets rather than educators' (see, for example, Rattan, Savani, Chugh & Dweck, 2015; Sun, 2018), the impact of teachers' mindset beliefs on their choices of curricular materials and selection of tasks is an area worthy of further exploration: if educators with 'fixed mindset' beliefs about pupils' abilities select less challenging tasks and fewer NRPs for mathematics lessons, this could reduce pupils' opportunities to develop problem-solving proficiency and also have negative implications for their mathematical achievement.

7.3 Methodological Lessons and Limitations

Throughout this study, efforts have been made to ensure that the methods implemented are appropriate and reporting is accurate. While some significant results and implications can be drawn from this research, there are some limitations which should be acknowledged. While some of these limitations have been discussed in Chapter 5, this section describes some additional limitations which should also be addressed.

First, there are some potential shortcomings relating to the analysis framework utilised in quantitative content analysis: in the original framework (van Zanten & van den Heuvel-Panhuizen, 2018 [see appendix C]) indicators of NRPs were designed in the context of materials aimed at children between the ages of 8 and 11 (van Zanten & Van den Heuvel-Panhuizen, 2018). Therefore, instrument validity may be lower for the Year 2 materials (which are comprised of tasks aimed at children aged between the ages of 6 and 7). However, since the number of non-routine problems in the Year 2 FA materials is similar to that for higher year groups, it is reasonable to assume that the framework was appropriate for materials aimed at younger children. Additionally, while the original analysis framework had undergone reliability checks by an independent expert during the course of van Zanten & van den Heuvel-Panhuizen's (2018) study, there was no scope to apply the same measures to the amended framework in this study. However, since no changes were made to the indicators for NRPs, GATs or STs, there can be a reasonable level of confidence in the reliability of the new framework with regard to these task types. Indicators for all other task types were drawn from the research literature and the author's own experience with mathematical tasks as a practising primary teacher.

Other limitations of this study relate to the sample used for content analysis. While the materials analysed represented the most up-to-date versions available on the WRM website at the time that content analysis was carried out, new schemes were published by WRM after this had taken place, therefore no inferences can be drawn about the opportunity for NRPS in the new resources. Additionally, whilst the sample used for content analysis in this study comprised materials for three of the six year-groups, the findings regarding the opportunities for NRPS may not represent the level of opportunity in the materials for other year groups. Also, considering that participants in School A reported supplementing WRM tasks with activities from other sources, content analysis of the WRM materials cannot fully describe the level of opportunity for pupils to engage in NRPS in School A. Rather, the results of content analysis outlined the opportunities for NRPS where WRM materials were used exclusively. However, the case study involving interviews with educators in School A allowed for a better understanding of the level of opportunity for NRPS in their school setting. Finally, although WRM represents one of the most popular resources used in primary schools in England, these may not be a representative sample of all TfM-aligned resources available to schools. Therefore, the findings could differ if other TfM-aligned mathematics resources had been included in this study.

Additionally, previous sections have highlighted the difficulties in categorising tasks (particularly problem tasks) due to the nature of problem-solving being relative to the solver's characteristics, prior knowledge and experience (Zhu & Fan, 2006; van Zanten & van & den Heuvel-Panhuizen, 2018). While every effort has been made to replicate the decision-making process in the categorisation of tasks as outlined by van Zanten & van & den Heuvel-Panhuizen's (2018) using their analysis framework (see appendix C), the classification of tasks is ultimately based on the interpretations of the researcher. This reduces the extent to which conclusions can be drawn from the results and represents an additional limitation to this study.

This study took place during the Covid-19 pandemic, which caused severe disruption to normal school management. This had 'derailed' key actions of the SDP in School A so that attempts to increase opportunity for NRPS and plan for more effective teaching of problem-solving skills had not yet been put into action at the time when interviews took place. Whilst it is important to acknowledge the fact that under normal conditions the situation may have been slightly different, it is unlikely to have been so different as to completely negate the importance of the findings set out.

7.4 Key Implications

7.4.1 Developing a mutual understanding

As demonstrated in Chapter 2, the importance of NRPS is incontrovertible (Borthwick, 2019; Foster, 2015; Liljedahl & Cai, 2021; Liljedahl et al., 2016;

Schoenfeld, 2014), and the benefits of providing pupils with regular opportunities to engage in NRPS are wide-ranging and profound (Burkhardt & Schoenfeld, 2019; Hiebert et al., 2007; Li et al., 2020; Woodward et al., 2012). However, teachers invariably find this the most difficult aspect of mathematics education to understand and implement successfully in the classroom (Chapman, 2015). The roll-out of the TfM initiative as an interpretation of SSHK pedagogy represented a significant change in mathematics education in England, and due to the emphasis on NRPS in SSHK curricula, coupled with increased opportunities for teachers to access subject-specific CPD, there was the potential to improve the teaching of NRPS. However, the findings from this study indicate that many years after the introduction of the reform in England, teachers still lack an understanding of both TfM *and* NRPS. This problem could be attributed to the lack of unambiguous guidance about both constructs from curriculum writers and policymakers, and a lack of emphasis on NRPS by the DfE and NCETM. Additionally, this issue may be exacerbated by the fact that evaluations and progress reports by the NCETM do not address the impact of the TfM reform on problem-solving performance, which is rarely mentioned in such documents.

The low levels of understanding of these constructs substantiate recommendations in the research literature which call for properly researched explications of these constructs from policymakers and curriculum writers, to establish a mutual understanding in the education world (Simpson & Wang, 2023; Xenofontos & Wang, 2014). The findings from the case study in School A add weight to arguments in the research literature that a lack of understanding about TfM and NRPS has a profound impact on teachers' capacity to implement these successfully in the mathematics classroom (Barmby et al., 2014; Chapman, 2015; Donnell & Gettinger, 2015; Owens & Nolan, 2022; Simpson & Wang, 2023). For example, participants in School A could not provide an accurate and comprehensive definition for either TfM or NRPS and also reported teaching practices which were not conducive to either construct. If teachers were provided with clear, all-encompassing guidance, preferably alongside targeted CPD and access to demonstrations of effective procedures, this would place them in a better position to implement both TfM and NRPS effectively in the mathematics classroom.

7.4.2 Drawbacks of a 'pick and mix' approach

It could be argued that a 'pick and mix' approach to policy-borrowing at different levels of implementation of the TfM reform has severely hindered efforts to increase mathematical achievement in pupils. At the highest level, while policymakers largely ignored contextual and cultural factors, these could be important contributing factors in SSHK's pupils' high mathematical attainment.

At the school level, there is evidence of surface-level implementation of a TfM approach to 'fit in' with previous teaching practices, which involves selecting those components which are deemed most important or fit more closely with teachers' beliefs about their professional practice. However, due to a lack of unambiguous guidance from policymakers regarding the crucial components of TFM, this has led to wide variation in the way it has been implemented in different schools, as well as a lack of understanding about the key factors of a TfM approach in teachers. For example, the research suggests that high value is placed on variation in curricular materials in China as part of a TfM approach, and variation has been linked with increased problem-solving proficiency in Chinese pupils, however, this element of TfM was barely mentioned by the participants in School A.

At the teaching level, there is a culture of a 'pick and mix' approach to task selection by English teachers when planning mathematics lessons (Marks et al., 2023), and this was also the case in School A. This may have culminated from an anti-textbook ethos in England. Alternatively, teachers may be reluctant to adopt a rigid implementation of one TfM-aligned textbook due to the possibility that another initiative will soon replace TfM, as suggested by participants in School A. However, these kinds of practices mean that TfM-aligned materials are rarely used with fidelity (due to omitting and supplementing tasks from other sources), and therefore the potential for variation in the sequencing of tasks within the materials to impact mathematical achievement or improve problem-solving performance is negated. Consequently, more research is needed to ascertain the true impact of teaching with variation in England, particularly with regard to the development of problem-solving skills. If positive impacts are proven, teachers should have access to CPD focused on variation so that teachers can implement this component of TfM to the greatest effect in the classroom.

7.4.3 Improving NRPS in English Schools

The findings from this study demonstrated a severe lack of opportunity for NRPS in the TfM-aligned materials, as well as a lack of variety in terms of the different types of problems provided. These findings highlight the need for comprehensive evaluations of educational reforms in the future *prior to* its wide-scale roll-out at great cost to the English taxpayer. Furthermore, as Bolden & Tymms (2020) point out, this rushing through of reform without consideration for crucial components of education (such as NRPS) is a symptom of the English political system, whereby reforms are brought about with the election of each political party every 5 years, who attempt to demonstrate they can make significant improvements during their short term in office. This study agrees with the argument set out by Bolden & Tymms (2020) that the onus for decisions about educational reform should be transferred to a non-political body, so that policy change is not tied to the short terms of political parties. This would provide more time to trial particular initiatives and ensure that important components of education, such as NRPS, can be fulfilled in the context of the reform. Furthermore, this could also encourage educators to fully embrace new reform, as it would provide reassurance that it will not be replaced by another reform in a short period.

Worryingly, findings from this study suggest that teachers may believe they are providing NRPS opportunities for their pupils even when this is not actually the case, due to misconceptions surrounding NRPS as opposed to routine problems. Furthermore, even where teachers have access to NRPs, the way in which they structure pupils’ experiences with NRPs may significantly reduce the extent to which problem-solving skills can be developed. For example, interview findings from this study suggest that a lack of knowledge about NRPS and effective pedagogy to develop problem-solving skills could lead to an over-emphasis on modelling and worked examples in mathematics lessons. All of this implies that CPD focused on NRPS is desperately needed to ensure pupils receive sufficient opportunities for NRPS in mathematics lessons. This CPD should focus on identifying appropriate NRPs from curricular materials so that teachers are better able to evaluate textbooks and schemes in terms of the opportunities provided for NRPS. To this end, the analysis framework utilised in this study could provide a means of supporting teachers to become proficient in identifying NRPs and distinguishing them from

GATs and STs. Additionally, CPD should highlight effective pedagogy for implementing NRPS in the primary mathematics classroom so that challenges are not ‘scaffolded away’ and pupils have opportunities to experience productive struggle. Finally, providers of Initial Teacher Training (ITT) and Continuing Professional Development (CPD) should give careful attention to these aspects of mathematics pedagogy in their programmes.

7.6 Conclusion

This study used a mixed-methods design to investigate opportunities for NRPS within a TfM approach to primary mathematics. Whilst prior studies have investigated opportunities for NRPS in mathematics textbooks, few of these studies have been conducted in England and none have focused on NRPS within TfM-aligned curricular materials in England. Additionally, no previous studies have incorporated qualitative methods alongside content analysis to explore teachers’ perceptions of NRPS within a TfM approach. Therefore, this study has contributed to the current research base.

The findings suggest that the introduction of the TfM approach in England has done little to improve the situation in terms of effective teaching of NRPS: there are few opportunities for NRPS within the WRM materials and English teachers in this study had a limited understanding of the construct, with their explanations of normal teaching practices suggesting an overemphasis on routine problems and practices which function to ‘scaffold away’ challenges. Therefore, it can be argued that there is much to be done in terms of developing a mutual understanding of NRPS and how it can be successfully implemented in primary mathematics lessons to support pupils in the development of vital problem-solving skills.

This study also adds to the current research evidence which suggests that raising educational outcomes through large-scale policy-borrowing is incredibly problematic: many years after the reform was initially rolled out, there continues to be divergent understandings of what TfM actually entails, and some aspects of TfM do not appear to be well-understood in England. For example, participants in School A did not associate the TfM approach with variation, despite research in China suggesting a strong link between teaching with variation and mathematical achievement and problem-solving proficiency.

Where teachers do not have a deep understanding of the components of a reform which have the greatest impact on pupil learning, this can negate any potential for improved pupil outcomes. For example, practices which lack fidelity to one scheme and involve a ‘pick and mix’ approach to task selection are not conducive to teaching with variation (especially where teachers lack understanding of the construct), therefore pupils are unlikely to reap the benefits associated with this aspect of TfM. This suggests a kind of paradox for teachers: using the WRM materials with fidelity results in few opportunities for NRPS due to low proportions of NRPs, while supplementation from other resources negates the potential for teaching with variation to increase problem-solving proficiency in pupils. Therefore, there is a desperate need for evidence-based clarification on the impact of teaching with variation in an English context.

Furthermore, whilst contextual and cultural factors are often disregarded by policymakers, these may have had a significant impact on the successful implementation of the TfM reform. Firstly, an anti-textbook culture in England continues to be prevalent in England, with teachers valuing their professional autonomy in the selection of tasks and teachers in School A showing disfavour towards a ‘rigid’ approach. This has limited the faithfulness of TfM in England to SSHK pedagogy, which involves high fidelity to government-approved mathematics textbooks. Secondly, while increased planning time, regular opportunities for CPD and observing mathematics lessons, and collaboration and discussion with peers appear to represent important aspects of the TfM approach in SSHK, there have been no changes to school structures to allow teachers to access such opportunities in their normal teaching routines. Finally, high mathematical attainment in China could be attributed to cultural factors such as a strong work ethic in pupils and high levels of parental involvement to prepare their children for high-stakes tests. The current evidence base suggests that the transfer of pedagogy alone has not culminated in the high mathematical outcomes anticipated by policymakers, therefore future policy initiatives should take greater consideration of the cultural factors which may contribute significantly to PISA outcomes.

7.7 Future research

The findings from this study have highlighted several avenues for future research. For example, while this study found low proportions of NRPs within the popular WRM materials, this may not be representative of other TfM-aligned resources. An extension of this study could explore the opportunities for NRPS within other TfM-aligned textbooks and resources. Specifically, a comparative study of TfM-aligned textbooks (particularly those with government approval) and curricular resources could be beneficial in informing decisions by schools regarding their choice of scheme or resource for mathematics lessons, particularly where large investment is needed. Additionally, action research based on supporting teachers to develop their understanding of NRPS through subject-specific CPD could investigate the impact of this on teachers' ability to select appropriate, high-quality NRPs from curricular materials, or reformulate tasks to create NRPs. Where this is successful, this CPD could be rolled out in other schools to support teachers' professional development in this area, as well as their ability to provide regular opportunities for NRPS in mathematics lessons.

The findings from interviews in this study suggest that teachers may 'scaffold away' more challenging activities and reduce pupils' opportunities for higher cognitive activation and creative approaches. Dale & Scherrer (2015) suggested an alternative to these kinds of practices involved a 'goldilocks discourse', whereby teachers use probing questions that support higher levels of reflection to provide 'just right' levels of scaffolding. Future research could explore the impact of this approach to scaffolding on pupils' problem-solving proficiency and provide an exemplification of effective scaffolding structures. Furthermore, an unanticipated finding from this study was that the head teacher at School A perceived that mathematical ability could not be improved, with only 'natural mathematicians' being able to solve problems, demonstrating a 'fixed mindset'. Although studies have explored the impact of a fixed versus growth mindset in pupils on their subsequent attainment, no research (that the author is aware of) has explored the impact of educators holding a fixed mindset on their teaching practices or their pupils' beliefs about their mathematical ability, and this could be an additional avenue for exploration.

Creativity is a crucial component of NRPS in mathematics, however, teachers in School A did not make any mention of creativity in their descriptions of NRPS during interviews, while the Chinese expert appeared to place higher value on creativity in mathematics education. While this finding is based on only 2 participants from one school and may not reflect the views of other teachers, an extension of this study could incorporate a larger sample to investigate teachers' understandings and perspectives about creativity in mathematics and how they implement opportunities for creative approaches in the mathematics classroom.

Finally, variation appears to be viewed as an important element of TfM in SSHK, and teaching with variation has been linked with increased problem-solving proficiency in China. Future research could explore the effect of teaching with variation in England context, in order to understand whether this element of TfM has the potential to support English pupils in the development of problem-solving skills. Additionally, since the vast majority of primary schools appear to use a 'pick and mix' approach to task selection from different sources rather than using textbooks with fidelity and teaching with variation, future research could adopt an experimental approach to identify whether following a textbook scheme with fidelity results in higher gains in terms of pupils' problem-solving proficiency than a 'business as normal' model involving high levels of omitting and supplementing tasks from a variety of sources.

Appendices

Appendix A: Participant information sheet

Participant Information Sheet

Date:

Project title: Opportunities for non-routine problem solving within a Teaching for Mastery approach for primary mathematics

You are invited to take part in a study that I am conducting as part of a Masters dissertation (Opportunities for non-routine problem solving within a Teaching for Mastery approach for primary mathematics) at Durham University. This research project is supervised by Dr David Bolden, d.s.bolden@durham.ac.uk, from the School of Education Ethics Sub-Committee of Durham University.

Before you decide whether to agree to take part it is important for you to understand the purpose of the research and what is involved as a participant. Please read the following information carefully. Please get in contact if there is anything that is not clear or if you would like more information.

The rights and responsibilities of anyone taking part in Durham University research are set out in our 'Participants Charter':
https://www.dur.ac.uk/research.innovation/governance/ethics/considerations/people/charter/

What is the purpose of the study?
The aim of this study is to…

- Analyse the opportunities for non-routine problem solving in a Teaching for Mastery approach to mathematics education.
- Evaluate whether Teaching for Mastery materials represent an accurate interpretation of East Asian teaching practices.
- Investigate perceptions of senior leadership team members about opportunities for non-routine problem solving in Teaching for Mastery materials.
- The study will be completed by 30th November 2021.

Why have I been invited to take part?
You have been invited due to …

- Your knowledge of mathematics instruction in your school and role in decision making in the subject of mathematics.

Do I have to take part?
Your participation is voluntary and you do not have to agree to take part. If you do agree to take part, you can withdraw at any time, without giving a reason. Your rights in relation to withdrawing any data that is identifiable to you are explained in the accompanying Privacy Notice.

What will happen to me if I take part?
If you agree to take part in the study, you will…

- *Take part in two interviews conducted in your usual place of work, lasting no longer than 30 minutes per interview session.*
- *You can choose not to answer any question if you wish.*

Are there any potential risks involved?

- There are minimal risks involved in the study but there may be inconvenience caused to you by the time required to participate in interviews. Interview sessions will not exceed 30 minutes and will take place at a time most convenient to you.

- Potential benefits of the study include increased knowledge of Teaching for Mastery approach and opportunities for non-routine problem solving incorporated in White Rose materials.

Will my data be kept confidential?
All information obtained during the study will be kept confidential. If the data is published it will be entirely anonymous and will not be identifiable as yours. If you give permission, direct quotes from your interview responses may be used but will be anonymised.
Full details are included in the accompanying Privacy Notice.

What will happen to the results of the project?

- No personal data will be shared, however anonymised (i.e. not identifiable) data may be used in publications, reports, presentations, web pages and other research outputs. At the end of the project, anonymised data may be archived and shared with others for legitimate research purposes.

All research data and records needed to validate the research findings will be stored for [10] years after the end of the project.

Durham University is committed to sharing the results of its world-class research for public benefit. As part of this commitment the University has established an online repository for all Durham University Higher Degree theses which provides access to the full text of freely available theses. The study in which you are invited to participate will be written up as a thesis. On successful submission of the thesis, it will be deposited both in print and online in the University archives, to facilitate its use in future research. The thesis will be published open access

Who do I contact if I have any questions or concerns about this study?
If you have any further questions or concerns about this study, please speak to the researcher or their supervisor. If you remain unhappy or wish to make a formal complaint, please submit a complaint via the University's Complaints Process.

Thank you for reading this information and considering taking part in this study.

Appendix B: Interview participant consent form

Consent Form

Project title: Opportunities for non-routine problem solving within a Teaching for Mastery approach for primary mathematics

This form is to confirm that you understand what the purposes of the project, what is involved and that you are happy to take part. Please initial each box to indicate your agreement:

I confirm that I have read and understand the information sheet dated [dd/mm/yy] and the privacy notice for the above project.	Yes/No
I have had sufficient time to consider the information and ask any questions I might have, and I am satisfied with the answers I have been given.	Yes/No
I understand who will have access to personal data provided, how the data will be stored and what will happen to the data at the end of the project.	Yes/No
I agree to take part in the above project.	Yes/No
I understand that my participation is voluntary and that I am free to withdraw at any time without giving a reason.	Yes/No

Participant's Signature________________________________ Date______________

(NAME IN BLOCK LETTERS)____

Researcher's Signature_____________________ Date_____________

(NAME IN BLOCK LETTERS)____

Notes

- I understand that anonymised (i.e. not identifiable) versions of my data may be archived and shared with others for legitimate research purposes.

  Please choose one of the following two options
  - EITHER I give permission for my words to be quoted in publications, reports, and other research outputs.
  - OR I do **not** give permission for my words to be quoted in publications, reports, and other research outputs.

Appendix C: van Zanten & van den Heuvel-Panhuizen's, (2018) analysis framework

Category	Indicators and decision rules
Non-routine problems	The task meets *two or three* of the following features: The unknown has to meet three or more conditions The data provided are interdependent The data are provided in another order than needed for solving the task In case the task has multiple correct solutions: All possible correct solutions have to be given *or* The total number of all possible correct solutions has to be given
Gray-area tasks	The task meets *one* of the following features: The unknown has to meet three or more conditions The data provided are interdependent The data are provided in another order than needed for solving the task In case the task has multiple correct solutions: One possible correct solution has to be given *or* Some but not all possible correct solutions have to be given
Straightforward tasks	The task is solvable by straightforward calculation *or* The task is offered after an explanation or an example which demonstrates how it can be solved

Appendix D: Revised analysis framework (Adapted from van Zanten & van den Heuvel-Panhuizen's, [2018] analysis framework)

Table 3: Amended analysis framework

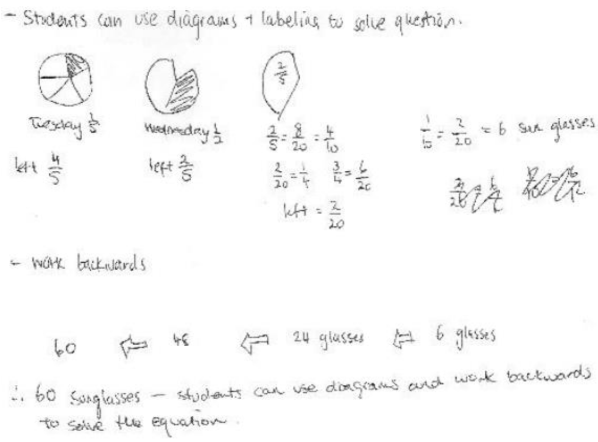
(Amended from van Zanten & van den Heuvel-Panhuizen, 2018)

NRPs	**The task meets two or three of the following features:** The unknown has to meet three or more conditions. The data provided are interdependent. The data are provided in another order than needed for solving the task. **In case the task has multiple correct solutions:** All possible correct solutions have to be given. **or** The total number of all possible correct solutions has to be given.
GATs	**The task meets one of the following features:** The unknown has to meet three or more conditions. The data provided are interdependent. The data are provided in another order than needed for solving the task. **In case the task has multiple correct solutions:** One possible correct solution has to be given. **or** Some but not all possible correct solutions have to be given.
Straightforward tasks	The task is solvable by straightforward calculation, or the task is offered after an explanation or an example which demonstrates how it can be solved.
Reasoning tasks	Require pupils to explain their solution method. or Require pupils to diagnose misconceptions (e.g., explain Annie's mistake). or Require pupils to justify assertions. (e.g., explain how they know an answer is correct) Or Require pupils to say what is the same and what is different about two different mathematical procedures. Or Require pupils to give an opinion (e.g., which solution method is easiest/more efficient?).
Representing tasks	Require pupils to represent a mathematical concept in another way, such as representing a given value on a place value chart using counters.

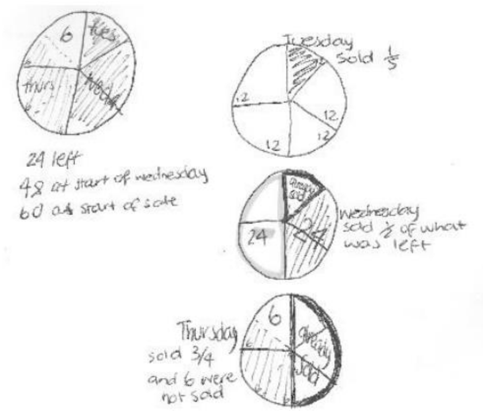
Mathematical investigations	Can result in a wide range of different approaches and outcomes. Allows pupils freedom to pose their own mathematical problems.
Problem-posing	An activity involving the generation of a new problem, or a reformulation of a particular given problem.

Appendix E: The 'fashion warehouse problem' and prospective teachers' solution methods (Berenger, 2018, p.167)

The fashion warehouse was having a sale on sunglasses. On Tuesday, it sold 1/5 of its sunglasses. On Wednesday, it sold 1/2 of what was left. On Thursday, it sold 3/4 of what was left from Wednesday. If 6 sunglasses were not sold, how many sunglasses did the Fashion warehouse have when the sale started? (Berenger, 2018, p.164)



Solution 1: Work backwards



Solution 2: Pictures/diagrams

Adding the denominators and multiplying by 6 then subtracting
~~Multiplying the denominators~~ final amount left which was 6
~~Adding the fractions~~ If you add up the denominators of the three fractions, 4 + 2 + 5, you get 11. If you multiply by the 6 sunglasses left at the end of the sale you get 66 and then you subtract 6 which was ~~wh~~ how many were left. 66 - 6 = 60. There were 60 sunglasses at the start.

Solution 3: creative manipulation of fractions

Appendix F: Interview transcripts

Head teacher (School A)

Could you tell me about your teaching philosophy?

I believe in rigour. I believe in kids being able to assess themselves against set criteria. I believe in that criteria being set and considered carefully by teachers who also have a clarity of thought about what they're doing.

What does Teaching for Mastery mean to you?

Teaching for Mastery means showing real depth of understanding. In the past, we used to kind of clatter through a load of stuff which was very much surface knowledge. I think the best and true mastery is when they can be really competent and show someone else how to do it, I think that's mastery.

Can you tell me what influenced your decision to incorporate a Teaching for Mastery approach in mathematics in your school?

Essentially…Outside influences really in terms of that kind of model that was shown to us, which was less differentiation, and rather than a group class going up a staircase, that model where they showed everyone gets in the lift and goes up together? That's what made deepest sense to me. But yes, definitely outside influences, government imposition really.

Were you approached about Teaching for Mastery?

I don't know, it just felt that everything in maths teaching was moving that way. But to be honest I feel that's the best way to go anyway. We want all the kids to have that depth of understanding, not just the highers. Differentiation for me has always been a race to the bottom, not the top. It sets a load of…I'm not assuming for one moment that everyone in the class is as skilled as each other, but I think if you constantly pander to people's lack of ability, it's human nature that you will find the easy route. Now that we're using Mastery, I think we're better at setting up a lot of skills in smaller steps and allowing kids to consolidate those skills and apply them. That helps all the children to make the same progress, so all children have the chance to succeed in maths.

Do children make the same progress during mathematics lessons?

Some children might not be able to access the maths without a lot of support. That's the reality of it. If it's a SEN child or they're working significantly below age-related, you have to put a lot of structures in place, because they can't access it on their own.

Can you tell me what influenced your decision to incorporate White Rose materials specifically in your school?

The fact that they were free at the time. Well, I think some of it still is free, but it was all completely free to start with and that was a big influence. Erm…we needed something and there was nothing else freely available at the time. And they had an element of… just easily accessible really. Because I do fear, just because of the nature of education, that the over-differentiators are probably just over the horizon to criticize and pull holes in this. Unfortunately, that's the nature of education. They reckon if you stay the same and stick to one thing that you believe in, you'll be right twice in your career. That is awful, but it's true. It depends how much force they put on you. That force tends to come through Ofsted really.

Were there any other programmes that you looked at or considered for Mastery?

We looked at the Singapore maths, but we didn't feel it was right for this school and it didn't have the flexibility that White Rose does. I don't think many schools would want a totally rigid approach.

Are White Rose Maths materials used exclusively as mathematics resources in your school?

No.

Why is that?

Some of the stuff is a bit dry and you need more stuff. I think it lacks open-ended problems.

Can you tell me what problem-solving means to you?

Problem-solving to me is a problem. It is a problem in that no matter how effectively we feel we are teaching maths, children repeatedly find it the most difficult thing to do. So, any analysis of children's performance on tests always shows up problem-solving, and that can kind of disillusion you sometimes because you can think, 'where do we go with this?' And you can actually feel like - this is terrible for an educator to admit to, really - you feel like the only ones who can solve maths problems like that are the ones that are considered natural mathematicians. Which is wrong because that in itself is me limiting my aspirations for other kids. But I see some kids who are naturally gifted sportsmen. They have a sense of balance, vision, special understanding which is innate, and I think, 'wow I can do great things with you'. I see that with kids in mathematics and they appear to be the ones who can readily take on a problem in mathematics and solve them. They just have an understanding of what's required and innately know how to solve the problem.

What does *non-routine* problem-solving mean to you?

Non-routine problem? Can you explain that to me? Is it worded problems? Or shape problems and things like that?

As opposed to routine problem solving - as it is on your School Development Plan - what does non-routine problem-solving, specifically, mean to you?

What is in my school development plan is what I see as the route to accessing the brains of those maths-ready kids for those non-maths-ready kids. To use an analogy of sport again, if I teach a PE lesson, I will use a more able child as a demo so others can see how they stand, how they move and the skills they use, so the other kids can see and emulate that.

Okay, so what does problem-solving look like in your school?

What I fear it looks like is all problems are the same type given at the end of a topic of work. I think that's become less so over the last 5 years, but that's what I fear it is. I don't think that is the case exclusively, but it can be like that. But there is a place for that as well. There is a place for practising problems of a particular type to kind of ingrain the skills required to solve that problem.

So, for you, non-routine problem-solving is something you can show others?

Yes. Kids need to be given opportunities to enquire, but that has to be alongside the structured teaching of skills, or they can't solve the problem and they just flounder. They become demotivated and make no progress because it is so open. As it stands at the moment, the teacher has to model it a lot. I'd like to change that, so that instead of modelling different problems, we teach them which suite of skills matches that type of problem. I think it needs a coordinated, concerted approach, which we haven't had.

So, for you, it's about teaching the skills?

Yes, and being able to recognise the type of problem. Really this is one of the key things on the school development plan which has been derailed this year, unfortunately, for all sorts of reasons including covid. This is a long-term route, and I don't know if anyone has the answer.

What were your reasons for including non-routine problem-solving in the school development plan?

Our children are extremely numerate, they like maths and achieve very highly. But they are derailed when they can't see what the problem is asking them to do. And it's the same problem in maths in all schools. Instead of teaching them the answer, we need to teach them the skills. We need to teach them in a metacognitive way, when it is this type of problem, you need these types of tools and skills. That would signify what method to use with different problems and these would be signposted and constantly rehearsed so they could be identified easily. Really this is the key thing on the School Development Plan, although it has been derailed this year, for all sorts of reasons including Covid. This is a long-term target.

Do you think White Rose Maths materials offer sufficient opportunities for non-routine problem-solving in maths?

Yes, I do actually. There are a lot from NRICH to give a bigger menu to choose from, but I think many of them aren't accessible to children who are not natural mathematicians.

What do you think of the problems in White Rose Maths materials?

They're alright. They're not the answer to everything but they're probably better than a lot of others that we have access to.

<u>Subject Leader for Mathematics (School A)</u>

Could you tell me about your teaching philosophy?

In general, I believe in trying to achieve a child's full potential in all subject areas and looking for different ways to engage different groups of learners and encourage them. Also, pushing them, never Accepting anything that is second best, always asking for more from them, but in a constructive way with praise and encouragement so they work even harder.

What does Teaching for Mastery mean to you?

That means getting to grips with the basic skills that a child needs and taking the learning deeper. It starts with the fluency aspect at the beginning of the lesson, so it's really having that strength of knowledge before they start to reason and problem-solve, so we can take the learning as deep as possible.

Could you tell me what you mean by 'taking the learning deeper'?

It's about whether they can apply what they've learned in different ways, like 'write your answer in a sentence', all of that comes on board here. I think, over time, that has really linked in with the SATs papers and reasoning papers that have been brought out with those sorts of aspects. And that's pushing the children, I think. I really agree with always teaching to the highest level in the class and the others have to kind of 'up their game'. They're clinging on in there, but with the right support, they do manage to do it. If you teach to the lower-level kids, are you really ever going to see that progress being made?

Can you tell me what influenced your decision to incorporate a Teaching for Mastery approach in mathematics in your school?

We are a really high-achieving school, we've proven that through our SATs results in KS1 and KS2. There is a lot of value added for each of the children because they come in with a very low baseline. I think it's really about encouraging those children to aspire to do the best they can. Mastery is the way forward for that because it offers that depth of learning. With Mastery, it's broken down into really manageable steps and you're always building on the previous learning, which I really do like. It means that all the children move through the topics at the same pace, so everyone's kept together.

Can you tell me what influenced your decision to use White Rose materials (specifically) in your school?

The resources were open and made free to everyone, so there wasn't any massive investment initially. We trialled them first and decided that's what we were going to stick with. And it didn't need to be followed exactly as it was set out by White Rose, it could be adapted because it was all online, and I think we have made it very bespoke for our school. I know when we looked through the initial planning documents, we would use our teacher judgements and think, 'no, that's not appropriate - that won't work for that class', and you could pick out the parts that you want to use and just print them out.

Are White Rose Maths materials used exclusively as mathematics resources in your school?

No

Why is that?

Well, they're not always appropriate, they don't always match in with our school policies. I think it's really about just finding the best fit for your class. I don't think it's ever wise to follow a scheme rigidly because, if you think about it, it could all change tomorrow and policies and guidance change so quickly. You have to be able to adapt quickly, but if you're following a particular scheme to the letter then it's going to be much harder when change comes, and you have to leave all that behind. It would be much more upheaval for the children as well because they get used to a set way of doing things. I mean, it was a big change when we started doing Mastery as well. The children have to adapt, just like we do! You have to incorporate other resources. Although there are really good examples of problem-solving questions, there are loads of other resources out there to take that learning even deeper.

Are there any other reasons why you don't use White Rose Maths resources exclusively?

I don't think you would get that depth of learning, that's all, because there are loads of other resources out there to take that learning even deeper.

Can you tell me what problem-solving means to you?

There's different ways to tackle it isn't there? It might be trial and error to solve it, where we might work systematically. For me, problem-solving is very open-ended. It could be explored in any way, you could come to the answer in a different way. It might be where you say 'talk to your partner' about it to decide how you are going to tackle this problem to come up with a solution. Whereas I think a more worded maths problem or multi-step problem is more clear-cut. It is a challenge, and you are going to have to put your mind to it and try hard. It's not clear cut as with other exercises. They need to give time and effort to it. And really just exploring what you've been asked to do. The children need to give a lot of time and effort to solving it.

What does non-routine problem-solving mean to you?

Oh, it means a lot of different things, doesn't it? Erm, I think it's really all about identifying what type of problem we are addressing here, how we are going to tackle it. That kind of thing.

What does problem-solving look like in your school?

There's lots of opportunities for them to apply the skills they've learned earlier in the fluency aspect of the lesson to a problem that's set out in a different way, so they can be more flexible with how they're thinking about the maths. It's really all about modelling how to solve a problem and explaining, right, I'm going to use trial and error here, or I'm going to work systematically, before they have a go themselves. Or sometimes it starts with a discussion with the class, and we have a chat about how we are going to get IN to this problem. The teacher needs to lead the way because the children need to be guided to get on board with that. Some of them can find that quite frustrating as they want to get straight into it independently, but I think most can see the benefit of that as well.

What were your reasons for focusing on non-routine problem-solving in the school development plan?

I think the reasons are because it develops that resilience, that ‘I am going to give this a go’ Having that ethos of its okay to get something wrong and it’s not the end of the world if that happens. It’s about them wanting and accepting a challenge, and that develops a growth mindset. It’s about having that ethos in the classroom and just making sure they’re not going to quit at the first hurdle. But you have to pave the way for that as well. I think that begins at the beginning of the term in September. I always say to them no try, don’t quit. They’ll say to me, erm…I’m not sure if I have it right but I’ll give it a go. You need to create that atmosphere that they can’t always get everything right.

Do you think White Rose Maths materials offer sufficient opportunities for non-routine problem-solving in maths?

No. Not just as a stand-alone. You have to incorporate other resources. That’s important. It’s all about finding them and also writing them yourself - what you think will work well for the children in your class and the learning that’s been going on in the classroom. Plus, the free sheets definitely didn’t go deep enough for the three aspects, but I think the new subscription materials are better in all respects, there’s definitely more fluency. I don’t think…although I like the structure, the way it’s set out in blocks and the way it builds on previous learning, I don’t think any of the three aspects of White Rose Maths really go into enough depth. There aren’t enough fluency questions there. They are okay for a lead-in, but we all look for more questions and further resources to back that up. I mean, four or five questions for varied fluency is not enough. You do need abundance of the fluency questions and the same for the reasoning and problem-solving. Maybe you don’t need loads more for those 2 aspects but still, more than they provide you with, you know? I was very aware of this when we started to use it. It’s good to use as a lead in but it’s not the be-all-and-end-all.

What influenced your decision to use White Rose Maths materials in school?

It was quite a bit of influence from me really. I wanted to push the children even further and have that deeper understanding and be able to use mathematical language. And from going to subject leadership meetings, we were really being encouraged at the time to go down one route or another, which were White Rose and Inspire. Most schools were going for one or another. White Rose even changed through lockdown as well. They produced resources that we could share over Twitter initially, for parents to access. Their resources were open and made free for everybody as well. Then later when schools were offering virtual lessons, we were using their PowerPoints through Zoom, with video resources to play, so they had something to share with children. You forget, over time, why you went down that route.

What do you think of the problems in White Rose materials?

I like them in general - they're nice and varied. There are problems where the children have to find all the possible answers. There's a lot of problems like that in White Rose and then the children get that experience of working systematically, so they might say to me, 'I'm going to start with the smallest answer' and they check that they haven't missed out any of the answers. And everyday scenarios as well. Like, we've been looking at decimals and you've got children with amounts of money that want to buy such-and-such. Our children can relate to that. Whereas I think with some questions you just think, how on earth are the children going to get their heads around that? Because they don't understand all of the things being mentioned in that problem. So yes, they are quite successful in a lot of ways.

Is there anything you want to add?

I think it is about finding and choosing from a range of resources and not being afraid to dip into different things.

Appendix G: A summary of my experiences in a school in Shenzhen, China

In March 2019, I took part in a teaching placement opportunity in Shenzhen, China led by the School of Education at Durham University. This involved delivering English, mathematics and science lessons to three Year 6 classes with a fellow trainee teacher and observing mathematics lessons delivered by the Chinese teachers. This appendix provides a summary of my experiences during this placement.

On the first day of the placement, I was able to meet with the head teacher at the school, who described himself as an expert in TfM and problem-solving. With the help of a translator from the English department, I was able to ask him some questions about his perspective on problem-solving within a TfM approach as practised in China. He explained that problem-solving is the most important focus in SSHK pedagogy, and pupils in his school engaged in a range of different problem-solving activities on a regular basis. He then showed some NRPs from a mathematics textbook, and there appeared to be many different types of problems including many LG, RP and DV problems. He offered to give a presentation about TfM and the role of problem-solving before we returned to England.

In subsequent conversations with mathematics teachers within the school, they explained that pupils have a mathematics homework textbook containing different problems to attempt at home. In examining one of these textbooks (with some help with translation from a Chinese teacher), I found the level of challenge to be very high, and the teacher explained that the pupils spend time discussing their different solution strategies when handing in their homework so that they understand that it is not enough to find an answer, they must find efficient and creative ways of working.

In further conversations with mathematics teachers in China, there were several differences in terms of normal school routines between England and China. One teacher described her usual working day as delivering lessons during the morning and spending the afternoon watching other lessons, accessing CPD, planning future lessons or delivering targeted mathematics interventions for small groups. The Chinese teachers also appeared shocked that teachers in England teach all subjects in the curriculum, rather than having one specialist subject.

One teacher explicitly stated that English teachers should avoid copying teaching practices in China, stating that in China, “Children don’t play, they only work”. She explained that in one particular class of Year 6 pupils (of 47 pupils), only 3 did not receive regular tuition for Chinese, English and mathematics after school, and she saw this as a key reason for high attainment. She asked a few children in the class to explain how they usually study during the week, and all stated that they spent at least three hours studying and doing homework after school, in addition to any after-school tuition. Furthermore, children did not go outside to play during break-times and stayed in the classroom for almost the entire school day. All of the children I spoke with had high aspirations for their future careers and had a strong work ethic which they believed would allow them to reach their career goals.

In the final week of the placement, the head teacher gave a presentation about TfM and problem-solving. PowerPoint presentation slides were used to exemplify the different kinds of problems that he believed were crucial to developing problem-solving skills, and he perceived this endeavour to be a crucial component of a TfM approach. Other aspects of TfM that were outlined were always described in relation to its use in developing problem-solving skills, such as conceptual and procedural variation to develop a conceptual understanding which can be drawn on in problem-solving situations. When offered the opportunity to ask questions about the presentation, I explained that many of the problems within TfM-aligned resources in England linked directly with earlier fluency tasks, and asked if this was the case in China. He responded that this would not allow pupils to develop a range of different problem-solving skills, but that these kinds of practices would *sometimes* be useful.

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