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**PARENTAL INVESTMENT,
COGNITIVE AND NON-COGNITIVE
SKILLS FORMATION AND
ECONOMIC GROWTH**

YUNG-LIN WANG

Department of Economics and Finance
Durham University Business School

A thesis presented for the degree of
Doctor of Philosophy in Economics

2019



Parental investment, cognitive and non-cognitive skills formation and economic growth

Yung-Lin Wang

Abstract

Parenting time helps a child to develop non-cognitive skills, whereas material investment in children along with the state schooling both help developing cognitive skills. Both cognitive and non-cognitive skills contribute to the formation of human capital, translating into labour productivity of an adult worker. The numerical simulations show that following results: (i) parents' stronger preference to material investment in children relative than time investment increases the rates of growth, as well as the higher relative importance of cognitive skills in human capital. However, the higher importance of parenting time decreases the growth rates. (ii) At the certain social weight, physical and human capital may under- or over-accumulate in the same or opposite directions. Laissez-faire economy does coincides with social optimum. (iii) Labour income taxation and subsidies in material investment are positive to implement the efficient allocations. Capital income is taxed or subsidized depending on the relative preference of parenting time and the agent's patience. (iv) Policy simulations reveal that higher labour and capital income tax rates both increases the growth rates. In contrast, public subsidy in material investment has counter effect to the growth rates. In the benchmark calibration, the best welfare-maximizing policy reformation is an increase in government spending on education funded by higher labour income tax rates.

Dedicated to
my parents,
and beloved Chang-Tzu

Declaration

The work in this thesis is based on research carried out at the Department of Economics and Finance in Durham University Business School, UK. I declare that no portion of the work referred to in the thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning.

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Acknowledgements

While I have fully responsibility for this thesis, however it is at least as much a product of years of interaction with and inspiration by a large number of friends and colleagues. For this reason, I wish to have the warmest gratitude for all those persons whose comments, questions, criticism, supports and encouragements to build this thesis over these four years. I also wish to thank those institutions which have supported me during the work. Regrettably but inevitably, the following list of names is incomplete. However I hope those who are missing can forgive me, and still accept my sincere appreciation of their influences on both my work and life.

First and foremost, I want to send my sincerely gratitude to the primary supervisor Prof. Nigar Hashimzade. It is such an great honour to be her first Ph.D. student in Durham University. She has guided me with both consciously and unconsciously. I appreciate all her contributions of time, suggestion, and support to enrich this productive and encouraging Ph.D. journey. The joy and enthusiasm she has for research are contagious and motivational for me, especially during the tough times in the Ph.D. pursuit. Every training in different institutions which she providing broadens my mind. I hope I can accomplish a tiny fraction of what she has already achieved in her own. In addition, I want to thank Dr. Ayse Yazici, the secondary supervisor, for her sharing everything with Ph.D. process and the information of Job Market. I believe that I cannot find a better supervising team.

Furthermore I would like to thank my reviewing committee members in these: Prof. Parantap Basu and Dr. Leslie J. Reinhorn for their time, interest and helpful comments in 9 months review to help determining the context of the thesis. In particularly, Parantap is my 21 and 33 months reviewer as well. His intelligent suggestions clarify the modelling technology in the growth model. He is also the

co-organiser for International Finance in MSc programme. I learn a lots in teaching and research performance as his teaching assistant.

I would also like to thank the members of my oral defence committee, Prof. Tatiana Kirsanova and Dr. Mauro Bambi, for their time and insightful questions. I am grateful to the administrative assistant who keeps us organized and is always ready to help. The discussions in CEGAP PhD workshop and colleagues, Changhyun Park, Jiunn Wang and Tevy Chawwa in the department of economics and finance, give countless helpful views on my thesis.

My knowledge of economics expanded exponentially when I thrust in front of the classroom; so I owe a debt of gratitude to all the students in the MSc programme: Financial Modelling and Business Forecasting (practicals), Econometric Methods (seminars) and International Finance (seminars), undergraduate programme: Quantitative Methods and The World Economy, Academic Skills Programme, dissertation consulting support in Durham University Business School and Durham University International Study Centre. I am grateful that they give me the opportunity to win the prize of ‘Durham Student Employee of the Year Award’ and ‘Durham University Learning and Teaching Award’, which is accredited by the Higher Education Academy (HEA).

Visiting other institutions definitely have important influences to this Ph.D., thank to Dr. Gabriele Gratton hosts me at University of New South Wales (UNSW) and provides a great opportunity to meet Prof. Arghya Ghosh and other faculty of the departments of economics, specially thanks to Dr. Gonzalo Castex and Dr. Geni Dechter for their delightful Ph.D. lectures and helpful opinions on my empirical research considering the application of skills formation.

Moreover, I need to thank Dr. Solmaz Moslehi who gives me a delightful visiting experience at Monash University in 2019 to explore the city of Melbourne, the best place to live, according to The Economist. The other visiting include University College London in the UK, where I meet Prof. James J Heckman who shed the light of skills formation in this thesis; Michigan State University in US, in which I have fun in playing soccer with Prof. Jeffrey Wooldridge and he gives very useful advises on my empirical research.

Parts of my thesis are presented at the following conferences: Monday Macroeconomics/Trade Seminar, Monash University, Australia; CEGAP PhD Workshop, Durham University, United Kingdom; Workshop of the Sydney Macro Reading Group, UNSW, Australia; 12th Meeting of the Portuguese Economic Journal, Universidade de Lisboa, Portugal; Conference of Society for the Advancement of Economic Theory, Academia Sinica, Taiwan; Public Economic Theory conference, Hue, Vietnam; Taiwan Scientific Symposium, Edinburgh, United Kingdom. I am grateful to all the participants of these conference panels and seminars for numerous valuable comments, questions and suggestions. Thanks to the institutions for the financial support to my research. For examples, Durham University Business School Research Fund, ArkLight Travel and Research Fund, Ustinov College Travel Awards, Norman Richardson Postgraduate Research Fund, Centre for Microdata Methods and Practice Fund, and Taiwan Scientific Symposium.

There is a person who provides me with the guidance and the fully support to succeed at every stage of my Ph.D., even before Ph.D., is Prof. Jhy-Yuan Shieh. I appreciate his every advise and suggestion. He presents a template, for what I view as a successful academic researcher. He also provides the opportunity for me to connect with the Taiwanese economist.

There is one remarkable person I want to thank for in the late stage of my Ph.D. is Dr. Thijs van Rens in Warwick University. We first meet in CEGAP conference at Durham University. We then reunite at UNSW and he gives extremely helpful suggestions on the structure of my thesis and the presentation skills. I will continue to look up to him and feel lucky to be able to learn and work with him.

One fundamental part in my Ph.D. is Durham Saints American Football team, which has also provided a great source of motivation to pursue my goals. The team teaches me more than just the title of National Champion. It shows me that hard working always pays off. In the third year, the victory of national championship in premier division is one thing I never forget in my life. Also, I want to thanks the D&D mates for building the fantastic world together with me. My time at Durham is also enriched by Doctoral Society, King's Church Durham and Durham University Taiwanese Student Society.

Thank you to all the wonderful people who have helped me enormously and who I did not mention here. All thanks are due to my parents, who raise me with love and support me in all my pursuits. Thank you dad, Guang-Shuen Wang, for giving me the opportunity and encouragement as much as you can. My mom, Shu-Hsia Chen, who was always in my heart. I hope you could see the person I have become. All of my achievements belong to both of you. Also, thanks to my sister, Yu-Ching Wang, for her all support. I thank my family for all their love and encouragement.

Finally, most of all for my loving, supportive, encouraging, and patient wife Chang-Tzu Wu whose faithful supports during the every stage of this Ph.D. is so appreciated. From starting a master degree in University of Edinburgh to complete a Ph.D. in Durham University and find a lecturer job in Nottingham Trent University is not easy, thank you for always saying 'Take my hand, we will make it, I swear'. Thank you for showing me the greatest love of all. Thank you for being with me all the time, traveling to every corner in the world, and giving all support, so that I can have time to focus on my research and play in the university's team. I never would have been able to get this far without her support. 'You raise my up, to more than I can be'. Thank you for loving me.

June, 2019

Yung-Lin Wang

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Chapter 1

Introduction

The most valuable of all capital is that invested in human beings.

- Marshall (1890)

This thesis composes seven distinct chapters. They are related by their common theme: the economic analysis on the importance of cognitive and non-cognitive skill formation in the production of human capital. Furthermore, we explore the link between the approaches of decentralised economy and centralised economy. Moreover, we study the taxation implication.

Chapter 2 introduces the competitive equilibrium in a three-periods overlapping generations model with altruistic agents, where parenting time helps a child to develop non-cognitive skills, whereas material investment in children helps developing cognitive skills. Both cognitive and non-cognitive skills contribute to the formation of human capital, translating into labour productivity of an adult worker.

This thesis studies the efficient allocations of the model in Chapter 3. It employs a long-lived and far-sighted social planner and incorporates the laissez-faire supported social weight to compare the private discount factor to identify the source of dynamical inefficiency.

Chapter 4 investigates the dynamical inefficiency by applying the role of government in the overlapping-generations model, in which cognitive skills are formed by government spending on education and material investment by parents, whereas parenting time develops children's non-cognitive skills. Both cognitive skills and non-cognitive skills build the formulation of human capital.

Chapter 5 analyses growth- and welfare-maximising taxation. There are three focuses in the chapter. The first focus is revenue-neutral government spending. The second one is growth-maximising rules, which discuss the influences of each policy variables on the growth rates and presents the sensitivity analysis of the parameters. The welfare-maximising rules are studied by taxation reforms in the final focus.

This thesis discusses the future researches in Chapter 6, including the usage of social security, the application of parenting style, the implication of imperfect market and the feasibility of positive psychology. Finally, Chapter 7 concludes.

Chapter 2

Competitive equilibrium

Abstract

This paper examines the importance of parental investment to growth in an OLG model in which skill formation, a combination of cognitive and non-cognitive skills, matters for the accumulation of human capital. We show that the sign of the impact of the relative importance of non-cognitive skills in human capital and the relative preference for parenting time on growth critically depends on the assumption of the altruistic motives behind the choice to devote time to children.

Keywords: Skill formation technology, Time allocation, Human capital, Overlapping generations

2.1 Introduction

This paper examines the importance of parental investment to growth in an overlapping generations (henceforth, OLG) model in which skill formation, that is the combination of cognitive and non-cognitive skills, matters for the accumulation of human capital and in which parents are altruistic.

The importance of parental investment has been the subject of ongoing historical debate (Baker et al., 2002; Baumrind, 1978; Bernal and Keane, 2010), in which the parental investment includes material and time investment (Zhu and Vural, 2013). There are two examples to argue that material investment is a significant factor within two different generations (i.e. the current and the next generation). The first example is material resources, which include financial support, family background and expectations for children (Schneider and Coleman, 1993). The second example is nutrition (Victora et al., 2008). However, psychologists argue that time investment contributes to the development of non-cognitive skills (Baumrind, 1971; Hirschi and Stark, 1969; Holmbeck et al., 1995; Maccoby and Martin, 1983). The existing literature shows that time investment is a prime factor influencing children’s skill formation (Bernal and Keane, 2010; Del Boca and Flinn, 2014).

More recently, Cunha and Heckman (2007) presents a theoretical framework to organise and interpret a large body of empirical evidence on child development. Early environments play a large role in shaping later outcomes (e.g Cunha and Heckman (2008)). In addition, children’s cognitive and non-cognitive outcomes are largely determined early in life (e.g Cunha et al. (2010)). Heckman and Mosso (2014) points out that scaffolding is the crucial role of child-parent/child-mentor relationships (i.e. to track the child closely and encourage them to take feasible next steps forward in their proximal zone of development). In this paper we build on Casarico and Sommacal (2018), but we consider the interaction between parental investment and growth. Prior research has examined skill formation in relation to human capital (e.g Cunha (2013)), but has not accounted for the role of parental investment in cognitive and non-cognitive skills as a source of economic growth.

For the purpose of exploring how skill accumulation impacts future human capital, this paper examines the role of parenting time and material investment in

children within the overlapping generations model. In the economy, agents live for three periods: childhood, parenthood and retirement. In the first period, the child receives material and time investment. In the second period, the agent determines how much labour income to contribute to child and savings, and how much time to devote to parenting time and labour supply. In the third period, the agent retires and consumes all earnings.

The key feature of the model is that it embeds parenting time for the production of human capital. We assume that the child requires a certain time and that time contributes to the formation of human capital. We maintain that parents do not overprovide time to the child, which means parents allocate time between parenting time and working hours. One more assumption made by this paper is that the child also requires material investment, which comes from parents' income. Parental investment, the combination of parenting time and material investment, influences the child's human capital. Also, the delivery of parental investment depends on the level of parental altruism. More specifically, the agent allocates parenting time for higher stock of the next generation's human capital or into labour supply for higher consumption in retirement. However, the agent enjoys parenting time, so the agent has to balance parenting time and consumption in retirement. In addition, we assume the growth rate of the economy depends on human and physical capital. In particular, we abstract from the role of formal schooling on the accumulation of human capital because we want to study the repercussions of parental investment on growth, focusing on skill formation as the main transmission mechanism, an issue which remains largely unexplored in the theoretical literature.

In the framework we show the rates of growth go down when the importance of parenting time in human capital formation is higher. This result is close to the empirical evidence in [Fiorini and Keane \(2014\)](#), which uses data from Longitudinal Study of Australian Children to point out that parenting time is insignificant in non-cognitive skills. Intuitively, an increase in productive (rather than unproductive) time with children is more beneficial human capital accumulation-wise. However, the reduction in the growth happens because when the wage elasticity of labour supply is high enough. The time decrease is offset by a larger increase in unproductive time,

rather than productive parental time, with negative repression on growth (Casarico and Sommacal, 2018).

This paper uses numerical simulations to solve the model. We assume that non-cognitive skills are more malleable than cognitive skills (Cunha et al., 2010) and parenting time is about 15 percent (Haveman and Wolfe, 1994) to setup the parameters. This experiment captures the effect of the relative importance of non-cognitive skills on growth. We find the growth rate declines under higher importance of non-cognitive skills. This result depends on the importance of parenting time in human capital formation. We also find the relative preference of parenting time benefits growth, but its impact on growth also depends on the importance of parenting time in human capital formation. This is because the opportunity cost (i.e. decreases in material investment) increases alongside the increased importance of parenting time in human capital formation.

The next section presents the main methodology: an OLG model that incorporates cognitive and non-cognitive skills into an endogenous growth. Section 3.3 provides numerical solutions to the model and Section 3.4 supplies the numerical simulations. Finally, this paper concludes.

2.2 The model

The model we introduce draws on the OLG endogenous growth model of Casarico and Sommacal (2018) but allows for different assumptions of parental investment, rather than focusing only on time allocation. The canonical OLG endogenous growth model here undertakes the roles of parenting time and material investment in children. The overall size of the population is a scale parameter and normalises to one, which implies population growth rate is zero. This paper therefore only specifies the aggregate values.

There are three periods in this life-cycle model: childhood, parenthood and retirement. In the first period, the child receives material and time investment. In the second period, the agent determines how much labour income to contribute to material investment and savings, and how much time to devote to parenting time

and labour supply. In the third period, the agent retires and consumes all earnings. The agent has perfect foresight about future variables. The generation that works during period t is indexed by t . The model is a discrete-time, one-good economy that begins operation in period 0 and continues over periods $t = 1, 2, \dots$, extending indefinitely into the future.

2.2.1 Human capital production function

We consider the production of cognitive skills and non-cognitive skills as intermediate inputs in production of human capital. This paper assumes that parental investment is endogenous; parents determine how much labour income to contribute to material investment and savings, and how much time to devote to parenting time and labour supply. However, in line with [Cunha and Heckman \(2007\)](#), the parents make no decision about their own human capital accumulation. In particular, we abstract from the role of formal schooling on the accumulation of human capital, because we want to study the repercussions of parental investment on growth, focusing on the skills formation as the main transmission mechanism, an issue which has so far been afforded little attention by scholars.

The production of cognitive skills

According to the empirical studies, the considered importance of the material investment in children in the accumulation of human capital is significant (see [Bloom and Canning \(2000\)](#); [Bloom et al. \(2002\)](#)). The production of cognitive skills uses material investment in children Z_t , which could be intergenerational investment in private education or children's consumption. The following equation describes the technology of cognitive skills production:

$$H_{c,t} = f_1(Z_t) \tag{2.1}$$

where $H_{c,t}$ is the level of cognitive skills. [Del Boca et al. \(2013\)](#) suggests time investment also appears in cognitive skill formation, and [Appendix G](#) provides more details of the appearance of time investment in the accumulation of human capital.

Assumption 1. $f_1(\cdot)$ is a strictly increasing and a strictly concave function which satisfies the Inada conditions; $f_1(0) = 0$.

The production of non-cognitive skills

The technology of non-cognitive skill formation has been documented in [Cunha et al. \(2006\)](#) and [Cunha and Heckman \(2008\)](#), whose models involve parental investment to promote a child's learning process and the accumulation of human capital by developing non-cognitive skills. [Coleman \(1988\)](#) suggests parents also need a close relationship with their children to pass their human capital on to their children. Parental human capital being passed relies on parenting time, so the production of non-cognitive skills is given by

$$H_{nc,t} = f_2(\phi_t, H_t) \tag{2.2}$$

where $H_{nc,t}$ is the level of non-cognitive skills. Variable ϕ_t denotes parenting time. H_t is parents' human capital.

Assumption 2. $f_2(\cdot)$ is a strictly increasing and a strictly concave function that satisfies the Inada conditions; $f_2(0) = 0$.

The production of human capital

We assume human capital H_{t+1} linearly depends on skill formation and parental human capital ([Cameron and Heckman, 2001](#)):

$$H_{t+1} = f(H_{c,t}, H_{nc,t}) \tag{2.3}$$

According to Assumptions 1 and 2, it is straightforward to verify that the production of human capital is a strictly increasing and concave function that satisfies the Inada conditions, and all factors are essential to the production of human capital. In line with [Casarico et al. \(2015\)](#) this is the simplest way to formalise the idea that parental investment matters for human capital accumulation.

2.2.2 Goods production

The economy produces a single, perishable commodity by changing constant-returns-to-scale technology with physical capital and human capital in each period. The production function satisfies neoclassical assumptions, including the perfectly competitive market:

$$Y_t = F(K_t, (H_t, L_t)) \quad (2.4)$$

where Y_t denotes the goods output. F is twice differentiable and positively homogeneous of degree one with positive but diminishing marginal products. K_t is the aggregate level of physical capital at date t . L_t denotes the labour supply.

The following assumption concerning F is made:

Assumption 3. $F(\cdot)$ is a strictly increasing and a strictly concave function that satisfies the Inada conditions; $F(0) = 0$.

2.2.3 Household

Parents are the decision-makers in the household. Without loss of generality, this paper assumes parents do not derive their utility from their own consumption in adult age but enjoy parenting time and material investment in children. Appendix B provides the version of the model where parents derive the utility from their own consumption in adulthood. Therefore, the utility function is given by

$$U_t = u(Z_t, \phi_t, X_{t+1}) \quad (2.5)$$

where X_{t+1} is the consumption in retirement.

Individual budget constraints

A working agent in adulthood distributes labour income $w_t H_t$ among material investment in children Z_t and saving S_t , where w_t is the wage rate and H_t is parents' human capital. In the next period, the agent retires and consumes the return from savings X_{t+1} . We assume that the agent do not consume in adulthood to focus on our main interest which is the impact of parental investment in children on growth.

Therefore, the budget constraints for the agent in adulthood and in retirement are

$$Z_t + S_t = L_t w_t H_t \quad (2.6a)$$

$$X_{t+1} = (1 + r_{t+1}) S_t \quad (2.6b)$$

$$L_t + \phi_t = 1 \quad (2.6c)$$

where r_t is the rental price of physical capital. The budget constraint in adulthood reflects the trade-off between parenting time and labour supply (De La Croix and Michel, 2002). There is no ‘accident of birth’ or no role for initial financial wealth, parenting income, in determining the optimal level of investment because parents can borrow freely in the market to finance their wealth to maximise the level of investment (Heckman and Mosso, 2014). According to our interest in this paper, bequests play no role in this model (Castelló-Climent and Doménech, 2008).

2.2.4 First-order conditions

Firm’s optimisation problem

The representative firm takes the wage rate and rental price of physical capital as given and maximises profits. In the optimum, each input is paid its marginal product and the profit is zero, hence one observes the following form:

$$r_t = \frac{\partial Y_t}{\partial K_t} \equiv F_t^{(1)} \quad \text{and} \quad w_t = \frac{\partial Y_t}{\partial H_t} \equiv F_t^{(2)} \quad (2.7)$$

where r_t is rental price of physical capital and w_t is the wage rate.

Consumer’s optimisation problem

The agent seeks to maximise the utility subject to lifetime constraints. One can form a Lagrange function to have first-order conditions for an interior solution with

respect to Z_t , X_{t+1} , ϕ_t and λ_t :

$$u_t^{(1)} = \lambda_t \quad (2.8a)$$

$$u_t^{(2)} = \lambda_t w_t H_t \quad (2.8b)$$

$$\nu' = \frac{\lambda_t}{1 + r_{t+1}} \quad (2.8c)$$

$$(1 - \phi_t)w_t H_t - Z_t - \frac{X_{t+1}}{(1 + r_{t+1})} = 0 \quad (2.8d)$$

where λ_t is the shadow price. Here $u_t^{(1)} \equiv \partial u / \partial Z_t$, $u_t^{(2)} \equiv \partial u / \partial \phi_t$ and $\nu' \equiv \partial u / \partial X_{t+1}$.

Using the first-order conditions, one can formulate the following equation:

$$\frac{u_t^{(1)}}{\nu'} = 1 + r_{t+1} \quad (2.9)$$

The left hand side (henceforth, LHS) is the marginal rate of substitution (henceforth, MRS) between material investment in children and consumption in retirement. The $MRS_{Z_t, X_{t+1}}$ depends on $1 + r_{t+1}$.

Using the first-order conditions, one can observe the following equation:

$$\frac{u_t^{(1)}}{u_t^{(2)}} = w_t H_t \quad (2.10)$$

The LHS is the MRS between material investment and time investment in children. The MRS_{Z_t, ϕ_t} is determined by labour income.

One can have the following equation by using the first-order conditions:

$$\frac{u_t^{(2)}}{\nu'} = (1 + r_{t+1})w_t H_t \quad (2.11)$$

The LHS is the MRS between parenting and consumption in retirement. The $MRS_{\phi_t, X_{t+1}}$ is $1 + r_{t+1}$ multiplying the labour income.

2.3 Numerical solution

For analytical tractability and further numerical simulations, this paper now adopts specific functional forms for preferences and technology, frequently used in macroeconomic literature.

Human capital production function

The production of cognitive skills is

$$H_{c,t} = BZ_t \tag{2.12}$$

where the variable B is the exogenous productivity.

The production of non-cognitive skills takes the following form:

$$H_{nc,t} = D\phi_t^\gamma H_t \tag{2.13}$$

where D is the efficient factor of the production of human capital with non-cognitive skills. Parameter γ represents the importance of parenting time.

Therefore, the production of human capital is described as the following equation:

$$H_{t+1} = H_{c,t}^{1-v} H_{nc,t}^v \tag{2.14}$$

where parameter v denotes the relative importance of cognitive and non-cognitive skills in the production of human capital. This Cobb-Douglas function describes the idea that cognitive skills and non-cognitive skills both matter to the production of human capital. If the relative importance of non-cognitive skills in the production of human capital is zero, this model reduces to the traditional model which focuses on the impacts of cognitive skills on human capital formation.

Here we note that human capital does not fully depreciate. If we take log form of (2.14), we find $\ln H_{t+1} = (1 - v)\ln H_c + v\ln H_t + v\ln D\phi_t^\gamma$. Then we set the depreciation rate of human capital δ_H as $(1 - v)$ to focus on the skill formation as the main transmission mechanism.

Goods production

The input-output relation is represented by the Cobb-Douglas production function with constant returns to scale. This production function takes this form:

$$Y_t = AK_t^\alpha(L_tH_t)^{1-\alpha} \quad (2.15)$$

where A is an exogenous productivity parameter. Parameter α and $1 - \alpha$ are the elasticity of production in terms of physical capital and human capital, respectively.

Household

Following [Andreoni \(1989\)](#) and [Taylor and Irwin \(2000\)](#), the model assumes warm-glow altruism in the agent's utility. This paper takes log preference to ensure the changes in the interest rate have no effect on the savings rate, like the capital-labour ratio of economy ([Acemoglu, 2009](#)). The adult agent's utility is given by

$$U_t = (1 - \eta)\ln Z_t + \eta\ln\phi_t + \beta\ln X_{t+1}, 0 < \eta < 1 \quad (2.16)$$

where the parameter β is the time discount factor. In line with the setting in [Becker \(1965\)](#) and [De La Croix and Michel \(2002\)](#), parameter η represents the relative preference for parenting time, while $(1-\eta)$ denotes the relative preference for parental investment on material resource. [Appendix B](#) provides the version of the model where parents derive the utility from their own consumption in adulthood.

Firm's optimisation problem

The firm's profit-maximisation problem is as follows:

$$\max_{K_t, H_t} \pi_t = AK_t^\alpha(L_tH_t)^{1-\alpha} - w_tL_tH_t - r_tK_t \quad (2.17)$$

where π_t is the firm's profit.

Rental rate of physical capital is given by

$$r_t = A\alpha K_t^{\alpha-1}(L_tH_t)^{1-\alpha} \quad (2.18)$$

Equation (2.18) states that the interest rate equals the marginal productivity of physical capital.

Real wage rate of human capital is given by

$$w_t = A(1 - \alpha)K_t^\alpha(L_tH_t)^{-\alpha} \quad (2.19)$$

Equation (2.19) requires that the wage per efficiency unit equals the marginal productivity of human capital in efficiency units.

Consumer's optimisation problem

The agent seeks to maximise the utility (2.16) under lifetime budget constraints (2.6a) - (2.6c). First-order conditions for an interior solution are

$$Z_t = \frac{1 - \eta}{1 + \beta} w_t H_t \quad (2.20)$$

$$X_{t+1} = \frac{\beta}{1 + \beta} (1 + r_{t+1}) w_t H_t \quad (2.21)$$

$$\phi_t = \frac{\eta}{1 + \beta} \quad (2.22)$$

Equation (2.20) gives the equilibrium allocation of material investment in children. Also, it confirms that parental income affects the material investment in children directly (Baker et al., 2002; Hout and Dohan, 1996). Equation (2.21) denotes the optimal consumption in retirement. Equation (2.22) reflects that the optimum choices of parenting time which is time-invariant. Equation (2.22) finds that parenting time depends on the relative preference and patience. The relative preference for parenting time increases the parenting time, while patience reduces parenting time.

One can use goods production function and (2.19) to rewrite (2.20) as

$$Z_t = zY_t \quad (2.23)$$

where $z \equiv [(1 - \alpha)(1 - \eta)] / (1 - \eta + \beta)$. z is the ratio of material investment in children to aggregate output.

2.3.1 Equilibrium

Since there is no international trade and no government part, aggregate investment equals aggregate savings. We make the following assumption:

Assumption 4. $K_{t+1} = I_t = S_t$.

This assumption means the current physical capital stock K_t is fully depreciated at the end of the current period, which means $K_{t+1} = I_t$, where I_t is the aggregate investment. The equality states physical capital available in next period $t+1$ equals the savings from the current period.

The system of equations (2.6a)-(2.8c) describes the competitive equilibrium defined as the following:

Definition 1. (Competitive Equilibrium) *For given H_0 and K_0 , a competitive equilibrium is the path $\{Z_t, X_t, \phi_t, k_t, w_t, r_t\}_{t \geq 0}$ that satisfies firm's optimisation conditions (2.7), agent's optimisation conditions (2.8a) - (2.8c); the time constraints (2.6c); the budget constraints (2.6a) and (2.6b) for consumers; and Assumption 4.*

All equations (2.6a)-(2.8c) determining competitive equilibrium are also stated in Appendix C. Given Definition 1, we make the following definition:

Definition 2. (Balanced Growth Path) *The balanced growth path is a competitive equilibrium where the allocation $\{Z_t, X_t, \phi_t, k_t, w_t, r_t, S_t\}$ is time-invariant, denoted by $\{Z^*, X^*, \phi^*, k^*, w^*, r^*, S^*\}$, i.e. the transformation variable remains at the same level.*

Definition 2 yields $k_{t+1} = k_t = k^*$. Appendix C provides the details of the variables in a steady state (e.g. Z^*, X^*, w^*, r^*). It verifies that there exists a unique balanced growth path. Solving Equation k^* yields

$$k^* = \psi^{\frac{1}{1-\alpha v}} \quad (2.24)$$

where $\psi \equiv [A\beta(1-\alpha)]/[\sigma(1+\beta)(1-\phi)^\alpha]$ and $\sigma \equiv [AB(1-\phi)^{1-\alpha}z]^{1-v}D^v\phi^{\gamma v}$. The LHS of the equation characterises exactly the ratio of physical capital to human capital in the intensive form, where K_{t+1} has to be increased as H_{t+1} rises. Thus k^* is fixed in equilibrium.

The rate of growth increases at a constant common rate $\rho^* = K_{t+1}/K_t - 1 = H_{t+1}/H_t - 1$. Appendix C shows the details of the derivation.

$$1 + \rho^* = \left[\frac{A\beta(1-\alpha)}{(1+\beta)(1-\frac{\eta}{1+\beta})^\alpha} \right]^{\frac{\alpha(1-v)}{1-\alpha v}} \left[D^v \left(\frac{\eta}{1+\beta} \right)^{\gamma v} \left[AB \left(1 - \frac{\eta}{1+\beta} \right)^{1-\alpha} \right]^{1-v} \right]^{\frac{1-\alpha}{1-\alpha v}} \quad (2.25)$$

Equation (2.25) is the balanced growth rates.

Equation (2.25) can be rewritten as

$$1 + \rho^* = \xi(\eta, \gamma, v)$$

where the growth rates contain three psychology factors from utility function and human capital production. They are the relative preference between material and parenting time, the importance of parenting time, the relative importance of cognitive and non-cognitive skills in human capital production respectively.

The effects of relative preference for parenting time on growth

To investigate the impact of relative preference for parenting time on growth, we take the log form of equation (2.25). We then have this equation:

$$\frac{1}{(1+\rho^*)} \frac{\partial \ln(1+\rho^*)}{\partial \eta} = \frac{(1-v)}{(1-\alpha v)} \frac{(2\alpha-1)}{(1+\beta-\eta)} + \frac{(1-\alpha)}{(1-\alpha v)} \frac{\gamma v(1+\beta)}{\eta} \geq 0 \quad (2.26)$$

Equation (2.26) finds the impact of the relative preference for parenting time on growth depends on the importance of parenting time in human capital formation. This is because the opportunity cost (i.e. decreases in material investment) increases with the increased importance of parenting time. Equation (2.26) also shows that the effect of η on ρ^* also depends on the relative important of non-cognitive skills in skill formation.

The effects of the importance of parenting time on growth

The influence of the importance of parenting time in human capital formation on growth is described as follows:

$$\frac{1}{(1 + \rho^*)} \frac{\partial \ln(1 + \rho^*)}{\partial \gamma} = \frac{v(1 - \alpha)}{1 - \alpha v} \ln \frac{\eta}{1 + \beta} < 0 \quad (2.27)$$

Equation (2.27) finds the rate of growth decreases when the importance of parenting time in human capital formation is higher. Intuitively, an increase in productive (rather than unproductive) time with children is more beneficial human capital accumulation-wise. However, the reduction in growth happens because, when the wage elasticity of labour supply is high enough, the time decrease is offset by a larger increase in unproductive time, rather than productive parental time, with negative repression on growth (Casarico and Sommacal, 2018).

One notes the outcome would be the opposite in a different calibration: the importance of parenting time and non-cognitive skills have positive effect on growth. Doepke and Zilibotti (2017) discusses parenting style for different values of the model parameters, the equilibrium is characterised by different type of altruism, for instances, strategic altruism. In our model there is only one type of altruism. However, depending on the model parameters, some economies grow faster when there is more formal education or higher cognitive skills, and some other economies grow faster where there is more parenting time or higher non-cognitive skills.

The effects of the relative importance of non-cognitive skills on growth

The impact of the relative importance of non-cognitive skills in human capital on growth is as follows:

$$\frac{1}{(1 + \rho^*)} \frac{\partial \ln(1 + \rho^*)}{\partial v} = \frac{1 - \alpha}{(1 - \alpha v)^2} \left\{ \ln D + \gamma \ln \frac{\eta}{1 + \beta} - (1 - \alpha) \ln AB - (1 - 2\alpha) \ln \frac{1 + \beta - \eta}{1 + \beta} - \alpha \ln \frac{A\beta(1 - \alpha)}{1 + \beta} \right\} \geq 0 \quad (2.28)$$

We find the impact of importance of non-cognitive skills in human capital on growth depends on the importance of parenting time in the formation of human capital. Later we provide the numerical simulations and sensitivity analysis of the parameters to discuss the sign of the LHS in equation (2.28).

2.4 Numerical simulations

Given the rich setting in human capital formation, we cannot provide an analytical solution and must therefore solve the model numerically. To this end, we have to assign a value to the parameters of the model. For examples, we use (2.22) to calculate η , (2.14) to compute B and D and (2.25) to determine A . The benchmark parametrisation is selected so as to obtain the balanced growth rate equals to 2 percent (Basu et al., 2012). Appendix F presents the details of the calibration. Table 2.1 provides a summary of the values assigned to some parameters and of the methodology adopted to set the others.

Physical capital share of production	α	0.33	Empirical researches
Private discount factor	β	0.6095	De La Croix and Doepke (2003)
<i>Calibrated parameters</i>			<i>Targeting</i>
Productivity in the goods production	A	3.8023	growth rates is 2 percent
Productivity in the cognitive skills production	B	1.9374	growth rates is 2 percent
Productivity in the non-cognitive skills production	D	1.9374	growth rates is 2 percent
<i>Discussed parameters</i>			
Importance of parenting time	γ	0.5	
relative preference between material and time investment	η	0.25	Craig (2005), time investment is 0.15
Weight of non-cognitive skills	v	0.75	Cunha and Heckman (2007), human capital production

Table 2.1: Parametrisation

Here the key finding is the reasonable parametrisation. With numerical simulation the relative preference for parenting time is 0.25. This implies parents prefer to invest more material resources in children than time. We also find the relative importance of non-cognitive skills is 0.75. The meaning of the number is that non-cognitive skills have more influence than cognitive skills on the formation of

the child’s human capital. Moreover, we note the return of parenting time on the technology of non-cognitive skills is lower than 1 unit. This means 1 unit of time investment only brings a lower return on the child’s formation of human capital.

As shown in the previous section, the value of the importance of parenting time in human capital formation, the relative importance of non-cognitive skills in human capital formation and the relative preference of parenting time in utility are salient factors with which to determine the relationship between parameters and growth.

Table 2.2 provides the sensitivity analysis of parameters.

Panel A					
The relative preference of parenting time (η)	0.248	0.249	0.25	0.251	0.252
Growth rate (ρ^*)	0.0177	0.0189	0.02	0.0212	0.0223
Panel B					
The relative importance of non-cognitive skills in human capital formation (v)	0.748	0.749	0.75	0.751	0.752
Growth rate (ρ^*)	0.0221	0.0211	0.02	0.019	0.0179
Panel C					
The importance of parenting time in human capital formation (γ)	0.498	0.499	0.5	0.51	0.52
Growth rate (ρ^*)	0.0226	0.0213	0.02	0.0187	0.0175

Table 2.2: Sensitivity analysis

Notes: The top sector (i.e Panel A), in which $\eta = 0.25$ is the standard case, investigates how changes in the relative preference of parenting time affect growth. Panel B, in which $v = 0.75$ is the standard case, studies the effect of the relative importance of non-cognitive skills in human capital formation on the growth. Panel C, in which $\gamma = 0.5$ is the standard case, describes the relationship between the growth and the importance of parenting time in human capital. To generate the values of the growth rates, we calibrate the exogenous productivity A, B and D. For details of calibration see Appendix F. We also target the growth rate at 2 %.

With a numerical simulation of a decentralised economy, there are several remarks to be made. First, the higher importance of parenting time reduces growth rates. This result is described in equation (2.27). Second, greater importance of cognitive skills in human capital production increases growth rates. This result is close to the empirical evidence in Fiorini and Keane (2014), which uses data from Longitudinal Study of Australian Children to point out that parenting time is insignificant in non-cognitive skills. Intuitively, an increase in productive (rather than unproductive) time with children is more beneficial human capital accumulation-wise. However, the reduction in the growth happens because when the wage elasticity of labour supply is high enough, the time decrease is offset by a larger increase in

unproductive time, rather than productive parental time, with negative repression on growth (Casarico and Sommacal, 2018). Finally, if parents have a stronger preference for material investment in children relative to time investment then growth rates are increased.

2.5 Conclusion

In a model in which cognitive and non-cognitive skills affect the human capital accumulation process, we investigate how parenting time and parental investment in material resource influence growth.

We show that the rate of growth increases when the importance of parenting time is lower in the formation of human capital. Equipped with a numerical simulation of a decentralised economy, we can posit the following: first, greater importance of cognitive skills in human capital production increases growth rates; and second, parents having a stronger preference for material investment in children relative to time investment increases growth rates. The reduction in the growth happens because, when the wage elasticity of labour supply is high enough, the time decrease is offset by a larger increase in unproductive time, rather than productive parental time, with growth being negatively repressed. The numerical simulation results also suggest the outcome would be the opposite in a different calibration: parenting time or non-cognitive skills have a positive effect on growth. Doepke and Zilibotti (2017) notes that the equilibrium is characterised by different types of altruism.

We are aware quantitative assessment of parental investment in skill formation would require a large scale, fully fledged OLG model with intragenerational heterogeneity and a more complete description of the process of human capital accumulation than the one adopted in this paper, which, in particular, abstracts from schooling in later periods of life.

The law of motion of human capital, H , is included in the agent's optimization problem by the assumption of altruism. A parent takes her own human capital as given, because human capital is formed during childhood, when agents do not make decisions. So, at time t , parent's human capital H_t is a state variable. Child's human

capital, H_{t+1} , depends on the parent's choice of time allocation and labour income allocation. Specifically, it depends on parenting time and child's consumption, which are among the choice variables of the parent.

I model parental altruism as child's consumption in parent's utility function. This is a form of paternalistic, or impure altruism. In solving optimisation problem, a parent faces a trade-off between child's consumption and own consumption in retirement (funded by savings) because of the budget constraint. This constraint is included in the optimisation problem, and this trade-off is reflected in the first-order conditions. Therefore, in the model parental investments in child's human capital are endogenous (and are accounted for in the first-order conditions), but the human capital of a working parent is exogenous. The latter can be viewed as an intergenerational externality.

An alternative assumption on altruistic motive is non-paternalistic, or pure altruism, where parent's utility depends on the child's utility. In the additive case parent's utility can be rewritten in the form equivalent to the utility of an infinitely lived agent (a dynasty). In this case an intergenerational externality is internalised because of the additional trade-off between parent's time in paid work (and parenting time) and child's future earnings, and between own consumption (funded from savings), child's current consumption, and child's future earnings.

Changing the model so that it includes formation of human capital as a constraint, would require changing the assumption on the form of altruism. In other words, instead of (or in addition to) including parenting time and child's consumption in parent's utility function, I would need to include child's utility in parent's utility function. (Or, alternatively, I could include child's future earnings in parent's utility function, which would be another form of paternalistic altruism.) Indeed, [Casarico and Sommacal \(2018\)](#) compare parental time allocation and implications for taxation under two different assumptions on altruism in an OLG model, paternalistic and non-paternalistic (referred to as “fully altruistic” in their paper).

This would be an interesting setup to consider and compare with the setup in this chapter. Such a comparison is beyond the scope of this chapter, and I leave it for future work.

We think that investigating theoretically the relationship between parental investment in skill formation and growth under full altruism is important since, to the best of our knowledge, we direct empirical research an exist on the importance of parenting time in human capital formation involved in the child's human capital. The result, according to parenting style, may have different implications in terms of the relationship between parental investment in skill formation and growth would guide empirical analysis. Further work is needed to understand the relationship in greater depth, and our paper provides the empirical analysis with an useful theoretical guide.

Chapter 3

Efficient allocations and dynamical efficiency

Abstract

We characterise the efficient allocations of parental investment in an OLG model in which skill formation affects the formation of human capital. The numerical solutions compare the long-run economic outcomes for various values of the private discount factor and social weight. The simulation results reveal that physical and human capital may under- or over-accumulate in the same or opposite directions. There exists a unique laissez-faire supported social weight, at which a laissez-faire economy coincides with the social optimum.

Keywords: Skill formation technology, Time allocation, Dynamic efficiency, Overlapping generations

3.1 Introduction

The paper aims at characterising the efficient allocation of parental investment in an overlapping generations (OLG) model in which skill formation affects the formation of human capital. By skill formation we mean a combination of cognitive and non-cognitive skills.

Differences in the accumulation of human capital are large and mostly accounted for parental investment in cognitive and non-cognitive skills (Bernal and Keane, 2011; Cunha and Heckman, 2007; Cunha et al., 2010; Heckman and Mosso, 2014). Though the existing empirical literature supports the appropriateness of explicitly including skill formation in human capital process and indicates that the impact of parental investment on the accumulation of human capital may be altruism-specific, the theoretical literature focusing on the design of parental investment in the formation of human capital generally ignore these features. For instance, previous contributions in the efficient allocation of skill formation literature (see Casarico et al. (2015)) set up the optimal tax formulas to incorporate type-specific Pigouvian terms which correct for intergenerational externality in human capital accumulation. However, one must question whether the accumulation of skills is efficient. The efficiency issue in the OLG model dates back to Diamond (1965). The present context is more complicated since the model includes physical and human capital as the choice variables. A relevant study, Bishnu (2013), introduces laissez-faire supported social weight. By construction, at the specific social weight, if there is no externality in the economy, the planner's allocations coincide with that of a laissez-faire economy. Within the context of the OLG economy, it has also been shown that the competitive equilibrium either under-accumulates physical and human capital or over-accumulates both. Thus, the result eliminates the possibility of competitive equilibrium deviating from the social optimum in its allocation of physical and human capital in opposite directions.

By contrast, Docquier et al. (2007) considers a three-overlapping-generations model of endogenous growth wherein human capital is the engine of growth. Within the limits of their model, the rationale for the standard pattern of intergenerational transfers, and the working-aged financing young education and old-age pensions is

seriously questioned. The problem is more complex when the rate of growth is endogenous. For instance, human capital is the engine of growth. Then, for a given social rate of time preference, an individual's saving decisions do not generate the appropriate amount of physical capital accumulation. In the case of over-accumulation, the intergenerational arrangements can be used to increase the welfare of all (present and future) generations. The Golden Rule steady state defines the frontier of Pareto efficient solutions. Introducing a social welfare function is useful in discriminating within all the efficient solutions.

To explore the efficiency of the allocation in a competitive equilibrium, this paper employs the social planner to maximise its utility but this is subject to the resource budget constraints. The planner solves a dynamic-planning problem with declining social weights over future generations. Moreover, this paper uses the notion of a laissez-faire supported social weight to compare the allocations in the competitive equilibrium and the social planner's allocations.

The numerical comparisons then proceed by contrasting competitive allocations with those preferred by utilitarian planners with social discount rates. The numerical analysis sheds light on the quantitative relevance of the inclusion of parental investment in skill formation in terms of the optimal values of choosing variables. At the chosen parameter values, this paper finds that physical and human capital can lead to over- or under-accumulation. This finding contrasts with [Bishnu \(2013\)](#) but is similar to [Docquier et al. \(2007\)](#). In addition, the numerical result suggests there is a unique laissez-faire supported social weight, i.e., a unique private discount rate that equals social weight such that a laissez-faire allocation coincides with the social optimum.

The inclusion of parental investment in the process of skill formation and the specific impact of parenting time on the formation of the child's human capital will be the key aspects of our setup. In our model, parents derive utility from parental investment and consumption in the retirement. The direct dependency of parental investment in children represents a warm-glow component.

The way parental altruism is specified is a crucial modelling issue in our model. There are several possible motivations behind parental investment decisions, in-

cluding pure altruism (parents care about the utility of their children) and impure (warm-glow) altruism (parents derive joy from that parental investment delivered to their children). To the best of our knowledge, one close study is [Casarico et al. \(2015\)](#) which nevertheless ignores the efficiency of parental investment in skill formation.

The next section introduces the OLG model. Section 4.3 presents the planning problem and Section 4.4 provides the numerical solutions. In section 4.5, the results of simulation include the dynamic efficiency and laissez-faire analysis. Finally, this paper concludes in section 4.6.

3.2 The model

We consider a three-period OLG model with skill formation, that is combination of cognitive and non-cognitive skills. In the first period, the child receives material and time investment. In the second period, the agent determines how much labour income to contribute to the child and to saving, and how much time to devote to parenting and labour supply. In the third period, the agent retires and consumes all earnings.

3.2.1 The consumer

Parents are the decision-makers in the household. Without loss of generality, this paper assumes parents do not derive their utility from their own consumption in adult age but enjoy parenting time and materially investing in their children. Therefore, the utility function is given by

$$U_t = u(Z_t, \phi_t, X_{t+1}) \tag{3.1}$$

where Z_t is the parental investment in material resource, this can be intergenerational investment in private education or the child's consumption. ϕ_t represents parenting time. X_{t+1} is consumption in retirement.

A working agent distributes labour income $w_t H_t$ among Z_t and savings S_t , where w_t is the wage rate and H_t is the human capital. In the next period, the agent retires

and consumes the return from savings X_{t+1} . Moreover, we normalise the parenting time and labour supply equals 1. Therefore, the budget constraints for the agent in adulthood and in retirement are

$$Z_t + S_t = L_t w_t H_t \quad (3.2a)$$

$$X_{t+1} = (1 + r_{t+1})S_t \quad (3.2b)$$

$$L_t + \phi_t = 1 \quad (3.2c)$$

where r_t is the rental price of physical capital. L_t denotes the labour supply. Equations (3.2a) and (3.2b) are the budget constraints for second and third time period. Equation (3.2c) describes time constraints in adulthood.

3.2.2 Human capital production function

We consider the production of cognitive skills and non-cognitive skills as intermediate inputs in production of human capital.

This paper assumes parental investment is endogenous; parents determine how much labour income to contribute to material investment and savings, and how much time to devote to parenting and labour supply. However, in line with [Cunha and Heckman \(2007\)](#), the parents make no decision on their own human capital accumulation. In particular, we abstract from the role of formal schooling in the accumulation of human capital, because we want to study the repercussions of parental investment on growth, focusing on skill formation as the main transmission mechanism, an issue which remains largely unaddressed in the theoretical literature.

The production of cognitive skills

According to empirical studies, the considered importance of the material investment in children in the accumulation of human capital is significant (see [Bloom and Canning \(2000\)](#); [Bloom et al. \(2002\)](#)). The following equation describes the technology of cognitive skills production:

$$H_{c,t} = f_1(Z_t) \quad (3.3)$$

where $H_{c,t}$ is the level of cognitive skills. We assume that $f_1(\cdot)$ is a strictly increasing and a strictly concave function which satisfies the Inada conditions: $f_1(0) = 0$.

The production of non-cognitive skills

The technology of non-cognitive skill formation involves parental investment to promote the child's accumulation of human capital (see, e.g. [Cunha and Heckman \(2008\)](#); [Cunha et al. \(2006\)](#)). Parents also need a close relationship with their children in order to pass on their human capital (see, e.g. [Coleman \(1988\)](#)), so the production of non-cognitive skills is given by

$$H_{nc,t} = f_2(\phi_t, H_t) \quad (3.4)$$

where $H_{nc,t}$ is the level of non-cognitive skills. This paper assumes that $f_2(\cdot)$ is a strictly increasing and a strictly concave function that satisfies the Inada conditions. We also assume that $f_2(0) = 0$.

The production of human capital

We assume that human capital H_{t+1} linearly depends on skill formation and parental human capital ([Cameron and Heckman, 2001](#)):

$$H_{t+1} = f(H_{c,t}, H_{nc,t}) \quad (3.5)$$

It is straightforward to verify that the production of human capital is a strictly increasing and concave function that satisfies the Inada conditions, and all factors are essential to the production of human capital. In line with [Casarico et al. \(2015\)](#) this is the simplest way to formalise the idea that parental investment matters for human capital accumulation.

3.2.3 Goods production

The economy produces a single, perishable commodity by changing constant-returns-to-scale technology with physical capital and human capital in each period.

The production function satisfies neoclassical assumptions, including that of the perfectly competitive market:

$$Y_t = F(K_t, (H_t, L_t)) \quad (3.6)$$

where Y_t denotes the goods output. F is twice differentiable and positively homogeneous of degree one with positive but diminishing marginal products. K_t is the aggregate level of physical capital at date t . The following assumptions concerning F are made: $F(\cdot)$ is a strictly increasing and a strictly concave function that satisfies the Inada conditions: $F(0) = 0$.

3.3 The planning problem

One important aspect of an OLG models is that the competitive equilibrium need not be Pareto efficient, in contrast to a competitive model with an infinitely lived representative agent where the First Welfare Theorem guarantees Pareto efficiency. The reason behind this property is the so-called double infinity [Shell \(1971\)](#): infinite horizon and infinite number of agents in the economy. This leads to the infinite total value of resources, so that Pareto improvement can be achieved by transferring resources from each young generation to the current old generation. In the context of the model with altruism and human capital production, the competitive equilibrium is not generically Pareto optimal even when the intergenerational externality in human capital production is fully internalised through the non-paternalistic (pure) altruism. Another attribute of the OLG type models is that it is possible that 'over saving' can occur when capital accumulation is added to the model, a situation which could be improved by a social planner forcing households to draw down their capital stocks. Therefore, this section will study the efficient allocations.

There exists a long-lived and far-sighted central planner in this economy. It is therefore possible to improve the welfare of one agent without diminishing the welfare of another agent, which leads to the First Welfare Theorem ([Debreu, 1954](#)). The central planner chooses the allocations of output to maximise the present discounted value of current and future generations. In this economy, the social planner looks at

an exact time period t , and considers whole generations. The social welfare function takes the following form:

$$\sum_{t=0}^{\infty} \Lambda [U_t(Z_t, \phi_t, X_{t+1})] \quad (3.7)$$

where Λ is the social planner's discount factor.

The resource constraints of physical and human capital take the following forms:

$$K_{t+1} = Y_t - Z_t - X_t \quad (3.8a)$$

$$H_{t+1} = (BZ_t)^{1-v} (D\phi_t^\gamma H_t)^v \quad (3.8b)$$

The objective of the social planner is to maximise the social welfare function (3.7) subject to physical and human capital resource constraints (3.8a) and (3.8b). Let us define \mathcal{L}_t as the Lagrange formulation, we then form social planner problem as follows:

$$\begin{aligned} \mathcal{L}_t = & \sum_{t=0}^{\infty} \Lambda_t \left\{ [U_t(Z_t, \phi_t, X_{t+1})] \right. \\ & + \Lambda q_{t+1} [AK_t^\alpha (H_t L_t)^{1-\alpha} - Z_t - X_t - G_t - K_{t+1}] \\ & \left. + \Lambda \mu_{t+1} [(BG_t^\omega Z_t^{1-\omega})^{1-v} (D\phi_t^\gamma H_t) - H_{t+1}] \right\} \end{aligned} \quad (3.9)$$

where q_{t+1} and μ_{t+1} are the shadow prices of physical and human capital.

First-order conditions for an interior solution (assuming it exists):

$$\frac{\partial \mathcal{L}_t}{\partial Z_t} = U_{Z_t} - \Lambda q_{t+1} + \Lambda \mu_{t+1} \frac{H_{t+1}(1-v)(1-\omega)}{Z_t} = 0 \quad (3.10a)$$

$$\frac{\partial \mathcal{L}_t}{\partial X_t} = \frac{\beta}{\Lambda} U_{X_t} - \Lambda q_{t+1} = 0 \quad (3.10b)$$

$$\frac{\partial \mathcal{L}_t}{\partial \phi_t} = U_{\phi_t} - \Lambda q_{t+1}(1-\alpha) \frac{Y_t}{1-\phi_t} + \Lambda \mu_{t+1} v \gamma \frac{H_{t+1}}{\phi_t} = 0 \quad (3.10c)$$

$$\frac{\partial \mathcal{L}_t}{\partial K_t} = \Lambda q_{t+1} F'_{K_t} - q_t = 0 \quad (3.10d)$$

$$\frac{\partial \mathcal{L}_t}{\partial H_t} = \Lambda q_{t+1}(1-\alpha) \frac{Y_t}{H_t} + \Lambda \mu_{t+1} v \frac{H_{t+1}}{H_t} - \mu_t = 0 \quad (3.10e)$$

where β is private discount factor and Λ is the planner's discount factor (i.e. social discount factor). When utilities are bounded, the assumption that Λ is smaller than 1 ensures that objective function is finite. These conditions are necessary and sufficient for optimality of the constant path starting at k_0 , as this path satisfies the transversality condition.

Equation (3.10a) is the optimal allocation of material investment in children. Equation (3.10b) describes the optimal allocation of old generation. Equation (3.10c) reveals that the marginal utility of material investment in children corrected to parenting time is equalised to the marginal utility of the consumption of the old generation, describing the optimal allocation of the old generation. (3.10c) also indicates that marginal productivity of parenting time on the production of human capital corrects to the marginal productivity of parenting time on goods production function. This result shows that the substitution relationship exists between the production of human capital and goods production. Therefore, a parent invests parenting time in children to the level when loss in the labour supply is equal to the gain in the (altruism-factor) discounted future marginal productivity of parenting time arising from children's human-capital accumulation. Equations (3.10d) and (3.10e) are the resource constraints of physical capital and human capital of economy, respectively. Note that, contrary to the standard Diamond (1965) model, this planner's first-order condition does not respect the first-order condition the individual chooses for himself in a decentralised economy.

Using (3.10a) and (3.10b), one can form the following equation:

$$MRS_{Z_t, X_t} \equiv \frac{U_{Z_t}}{U_{X_t}} = \frac{\beta}{\Lambda} \left[1 + \frac{\mu_{t+1} H_{t+1} (1-v)}{q_{t+1} Z_t} \right] \quad (3.11)$$

The left-hand side (henceforth, LHS) is the marginal rate of substitution (henceforth, MRS) between material investment in children and consumption of old generation.

Comparing (3.10a) and (3.10c), one can observe the following equation:

$$MRS_{Z_t, \phi_t} \equiv \frac{U_{Z_t}}{U_{\phi_t}} = \frac{1 + \frac{(1-v)\mu_{t+1}H_{t+1}}{q_{t+1}Z_t}}{F'_{L_t} + \frac{(1-v)\mu_{t+1}H_{t+1}}{q_{t+1}\phi_t}} \quad (3.12)$$

The LHS is the MRS between material investment and time investment in children.

We arrive at the following equation by using (3.10b) and (3.10c),

$$MRS_{\phi_t, X_t} \equiv \frac{U_{\phi_t}}{U_{X_t}} = \frac{\beta}{\Lambda} \left[F'_{L_t} + \frac{\mu_{t+1} H_{t+1} \nu \gamma}{q_{t+1} \phi_t} \right] \quad (3.13)$$

The LHS is the MRS between parenting and the consumption of the old generation.

3.4 Numerical solutions

The purpose of this section is to illustrate the quantitative relevance of including parental investment and skill formation in the human capital production function, in terms of the optimal values of the chosen variables and in terms of the welfare loss caused by setting the OLG model, neglecting its effect on the human capital process and on over-saving. We maintain the three-period structure developed in the theoretical part of the paper.

Human capital production function

The production of cognitive skills is

$$H_{c,t} = B Z_t \quad (3.14)$$

where the parameter B is the efficient factor in cognitive skills production.

The production of non-cognitive skills takes the following form:

$$H_{nc,t} = D \phi_t^\gamma H_t \quad (3.15)$$

where D is the exogenous productivity. Parameter γ represents the importance of parenting time.

We assume that human capital production is conducted by skills formation combining cognitive and non-cognitive skills. Therefore, the production of human capital

is described in the following equation:

$$H_{t+1} = H_{c,t}^{1-v} H_{nc,t}^v \quad (3.16)$$

where parameter v is the relative importance of cognitive and non-cognitive skills in the production of human capital. This Cobb-Douglas function describes the idea that cognitive skills and non-cognitive skills both matter to the production of human capital. If the relative importance of non-cognitive skills in the production of human capital is zero, this model reduces to the traditional model which focuses on the impact of cognitive skills on human capital formation.

Here, we notice that human capital does not fully depreciate. If we take log form of (3.16), we find that $\ln H_{t+1} = (1-v)\ln H_c + v\ln H_t + v\ln D\phi_t^\gamma$. We then set the depreciation rate of human capital δ_H is $(1-v)$ to focus on skill formation as the main transmission mechanism.

Goods production

The input-output relation is represented by the Cobb-Douglas production function with constant returns to scale. This production function is as follows:

$$Y_t = AK_t^\alpha (L_t H_t)^{1-\alpha} \quad (3.17)$$

where A is an exogenous productivity parameter. Parameter α and $1-\alpha$ are the elasticity of production in terms of physical capital and human capital, respectively.

Household

The model assumes warm-glow altruism in the agent's utility. This paper takes log preference to ensure that the changes in the interest rate have no effect on the savings rate, like the capital-labour ratio of economy (Acemoglu, 2009). The adult agent's utility is given by

$$U_t = (1-\eta)\ln Z_t + \eta\ln\phi_t + \beta\ln X_{t+1}, 0 < \eta < 1 \quad (3.18)$$

where the parameter β is the private time discount factor. Parameter η represents the relative preference for parenting time, while $(1-\eta)$ is the relative preference for parental investment in material resource. Appendix B provides the version of the model where parents derive utility from their own consumption in adulthood.

Firm's optimisation problem

The representative firm takes the wage rate and rental price of physical capital as given and maximises its profits π_t . In the optimum, each input is paid its marginal productivity and the profit is zero, hence rental rate of physical capital takes the following form:

$$r_t = A\alpha K_t^{\alpha-1} (L_t H_t)^{1-\alpha} \quad (3.19)$$

The real wage rate of human capital is given by

$$w_t = A(1-\alpha) K_t^\alpha (L_t H_t)^{-\alpha} \quad (3.20)$$

Equation (3.19) states that the interest rate equals the marginal productivity of physical capital. Equation (3.20) requires that the wage per efficiency unit equals the marginal productivity of human capital in efficiency units.

Consumer's optimisation problem

The agent seeks to maximise the utility (3.18) under the life-time budget constraints (3.2a) - (3.2c). First-order conditions for an interior solution are

$$Z_t = \frac{1-\eta}{1+\beta} w_t H_t \quad (3.21a)$$

$$X_{t+1} = \frac{\beta}{1+\beta} (1+r_{t+1}) w_t H_t \quad (3.21b)$$

$$\phi_t = \frac{\eta}{1+\beta} \quad (3.21c)$$

Equation (3.21a) gives the equilibrium allocation of material investment in children. Also, it confirms that parental income affects the material investment in children directly (Baker et al., 2002; Hout and Dohan, 1996). Equation (3.21b) is the op-

timal consumption in retirement. Equation (3.21c) reflects the optimum choices of parenting time. Equation (3.21c) finds that parenting time depends on the relative preference and patience.

3.4.1 Balanced growth properties

Competitive economy

We assume that $K_{t+1} = S_t$. The equality states that physical capital available in next period $t + 1$ equals the savings from the current period. This assumption means the current physical capital stock K_t is fully depreciated at the end of the current period.

In the balanced growth path, the allocations in a competitive equilibrium $\{Z_t, X_t, \phi_t, k_t, w_t, r_t, S_t\}$ are time-invariant, denoted by $\{Z^*, X^*, \phi^*, k^*, w^*, r^*, S^*\}$, i.e. the transformation variable remains at the same level. This application yields $k_{t+1} = k_t = k^*$. It is straightforward to verify that there exists a unique balanced growth path. Solving k^* yields

$$k^* = \psi^{\frac{1}{1-\alpha v}} \quad (3.22)$$

where $\psi \equiv [A\beta(1-\alpha)]/[\sigma(1+\beta)(1-\phi)^\alpha]$. $\sigma \equiv [AB(1-\phi_t)^{1-\alpha}z]^{1-v}D^v\phi^{\gamma v}$. The LHS of the equation characterises exactly the ratio of physical capital to human capital in the intensive form, where K_{t+1} has to be increased as H_{t+1} rises. Thus k^* is fixed in equilibrium.

The rate of growth increases at a constant common rate $\rho^* = K_{t+1}/K_t - 1 = H_{t+1}/H_t - 1$. Appendix C shows the details of the derivation.

$$1 + \rho^* = \left[\frac{A\beta(1-\alpha)}{(1+\beta)(1-\frac{\eta}{1+\beta})^\alpha} \right]^{\frac{\alpha(1-v)}{1-\alpha v}} \left[D^v \left(\frac{\eta}{1+\beta} \right)^{\gamma v} \left[AB \left(1 - \frac{\eta}{1+\beta} \right)^{1-\alpha} \right]^{1-v} \right]^{\frac{1-\alpha}{1-\alpha v}} \quad (3.23)$$

Equation (3.23) is the balanced growth rates.

Centralised economy

We write the social planner problem in Lagrange formulation to yield the following proposition:

Proposition 1. (Sufficient condition for the planner's optimum) *A positive sequence $(Z_t, X_t, \phi_t, k_{t+1})_{t \geq 0}$ satisfying (3.10a)-(3.10e) and the transversality condition $\lim_{t \rightarrow \infty} \Lambda q_t K_t = 0$ and $\lim_{t \rightarrow \infty} \Lambda \mu_t H_t = 0$ is an optimal solution to the planner's problem.*

Proof. See the following derivation.

Using (3.10d), one obtains

$$q_{t+1} = \frac{q_t}{\Lambda F'_{K_t}} = \frac{q_t K_t}{\alpha \Lambda Y_t} \quad (3.24)$$

where $F'_{K_t} = \alpha Y_t / K_t$.

Substituting (3.10e) into (3.10c), one yields

$$\mu_{t+1} = \left[\frac{q_t K_t (1 - \alpha)}{\alpha (1 - \phi_t)} - \frac{\eta}{\phi_t} \right] \frac{\phi_t}{\Lambda v \gamma H_{t+1}} \quad (3.25)$$

One can plug (3.24) and (3.25) back into Equation (3.10a) to yield

$$Z_t = \alpha Y_t \left[\frac{(1 - \eta)}{q_t K_t} - \frac{\eta (1 - v)(1 - \omega)}{q_t K_t v \gamma} + \frac{\phi_t}{1 - \phi_t} \frac{(1 - \alpha)(1 - v)(1 - \omega)}{\alpha v \gamma} \right] \quad (3.26)$$

Substituting (3.24) back into Equation (3.10b), one can observe

$$X_t = \frac{\beta}{\Lambda} \frac{\alpha Y_t}{q_t K_t} \quad (3.27)$$

One can plug (3.26) and (3.27) into K_{t+1} to obtain

$$K_{t+1} = Y_t - \alpha Y_t \left[\frac{(1 - \eta)}{q_t K_t} - \frac{\eta (1 - v)(1 - \omega)}{q_t K_t v \gamma} + \frac{\phi_t}{1 - \phi_t} \frac{(1 - \alpha)(1 - v)(1 - \omega)}{\alpha v \gamma} - \frac{\beta}{\Lambda} \frac{1}{q_t K_t} \right] \quad (3.28)$$

Multiplying (3.24) and (3.28) term by term, one has

$$q_{t+1}K_{t+1} = q_tK_t \left[\frac{(1 - \phi_t)v\gamma - \phi_t(1 - \alpha)(1 - v)}{\alpha\Lambda v\gamma(1 - \phi_t)} \right] + \frac{\Lambda\eta(1 - v) - \Lambda v\gamma(1 - \eta) - \beta v\gamma}{\Lambda^2 v\gamma} \quad (3.29)$$

Since $F'_{K_t} = \alpha Y_t / K_t$, $q_t K_t$ is the solution to linear dynamic Equation (3.29), and the general solution to this equation is

$$q_t K_t = \frac{\alpha(1 - \phi_t)[\Lambda\eta(1 - v) - \Lambda v\gamma(1 - \eta) - \beta v\gamma]}{\Lambda(\alpha\Lambda - 1)(1 - \phi_t)v\gamma + \Lambda\phi_t(1 - \alpha)(1 - v)} + \varepsilon \left[\frac{(1 - \phi_t)v\gamma - \phi_t(1 - \alpha)(1 - v)}{\alpha\Lambda v\gamma(1 - \phi_t)} \right]^t \quad (3.30)$$

with ε as a real constant.

According to De La Croix and Michel (2002), there is a unique solution which satisfies the transversality condition $\lim_{t \rightarrow \infty} \Lambda q_t K_t = 0$ and $\lim_{t \rightarrow \infty} \Lambda \mu_t H_t = 0$: the constant solution. The transversality condition states that the limit of the actual shadow value of the capital stock equals zero. Therefore, this subsection has the following equation:

$$q_t K_t = \frac{\alpha(1 - \phi_t)[\Lambda\eta(1 - v) - \Lambda v\gamma(1 - \eta) - \beta v\gamma]}{\Lambda(\alpha\Lambda - 1)(1 - \phi_t)v\gamma + \Lambda\phi_t(1 - \alpha)(1 - v)} \quad (3.31)$$

Since $q_t K_t = \text{constant}$ satisfies the transversality condition, the solution for Equation (3.10e) should be $\mu_t H_t = \text{constant}$, hence this subsection yields

$$\mu_t H_t = \mu_{t+1} H_{t+1} = q_t K_t \frac{(1 - \alpha)}{\alpha(1 - \Lambda v)} \quad (3.32)$$

We can rewrite (3.10c) to make the allocation of parenting time in equilibrium:

$$\frac{\eta}{\phi_t} = q_t K_t \left[\frac{1 - \alpha}{\alpha} \frac{1}{1 - \phi_t} - \frac{\Lambda v\gamma}{\alpha(1 - \Lambda v)} \frac{1}{\phi_t} \right] \quad (3.33)$$

Finally, using $q_{t+1} Y_t = q_t K_t / \alpha \Lambda$ and (3.10a)-(3.10e), one finds that the parenting time is a time-invariant variable as

$$\phi = \frac{\eta\Lambda(\alpha\Lambda - 1)(1 - \Lambda v) + \Lambda(1 - \alpha)[\Lambda\eta(1 - v) - \Lambda v\gamma(1 - \eta) - \beta v\gamma]}{(1 - \alpha)[- \Lambda(1 - \eta) - \beta][(1 - \Lambda v) + \Lambda\gamma v] + \eta\Lambda(\alpha\Lambda - 1)(1 - \Lambda v) + \eta\Lambda^2(1 - \alpha)(1 - v)} \quad (3.34)$$

Equation (3.34) is the social planner's allocation of parenting time. We find that parenting time (i.e. ϕ_t) is additionally determined by the relative importance between cognitive and non-cognitive skills in human capital production (i.e. v) and the importance of parenting time in the formulation of non-cognitive skills (i.e. γ), rather than just private discount factor (i.e. β) and the altruism level (i.e. η) as reported in (3.21c).

One can use (3.17), (3.31), (3.34) to observe material investment in (3.26) as

$$Z = \mathcal{Z}F(K_t, (H_t, L_t)) \quad (3.35)$$

where $\mathcal{Z} \equiv \left[\Lambda\gamma v(\alpha\Lambda - 1) + \Lambda(1 - \alpha)(1 - v) \frac{\phi}{1 - \phi} \right] \frac{1 - \eta - \frac{\eta(1 - v)}{v\gamma}}{\Lambda\eta(1 - v) - \Lambda v\gamma(1 - \eta) - \beta v\gamma} + \frac{\phi}{1 - \phi} \frac{(1 - \alpha)(1 - v)}{v\gamma}$. Here it is required that the material investment in children is strictly positive.

One can use first order conditions and (3.34) to observe

$$X = \mathcal{X}F(K_t, (H_t, L_t)) \quad (3.36)$$

where $\mathcal{X} \equiv \frac{\Lambda\gamma v(\alpha\Lambda - 1) + \Lambda(1 - \alpha)(1 - v) \frac{\phi}{1 - \phi}}{\Lambda\eta(1 - v) - \Lambda v\gamma(1 - \eta) - \beta v\gamma}$. Equation (3.36) is greater than zero. Consumption in retirement is strictly positive.

One can have the physical capital in $t+1$ in the following equation by plugging (3.10a)-(3.10e) and (3.34) into the resource constraints of physical capital:

$$K_{t+1} = F(K_t, (H_t, L_t))\alpha\Lambda \quad (3.37)$$

Plugging (3.10a)-(3.10e) and (3.34) into H_{t+1} , we obtain

$$H_{t+1} = F(K_t, (H_t, L_t))^{1-v} \vartheta H_t^v \quad (3.38)$$

where ϑ is constant parameter collection, $\vartheta \equiv \langle B\mathcal{Z} \rangle^{1-v} \langle D\phi^\gamma \rangle^v$.

Finally, one can use (3.37) and (3.38) to obtain the physical-human capital ratio in the following form:

$$k_{t+1} = \frac{K_{t+1}}{H_{t+1}} = \Delta \left(\frac{K_t}{H_t} \right)^{\alpha v} \quad (3.39)$$

where $\Delta \equiv \alpha\Lambda A^v (1 - \phi)^{(1-\alpha)v} / \vartheta$. ■

Let us define $\Delta = K_{t+1}/H_{t+1}$, so the efficient ratio of physical capital to human capital is

$$k^* = \Delta^{\frac{1}{1-\alpha v}} \quad (3.40)$$

The dynamic system (3.35)-(3.39) cannot in general be solved explicitly, as the planner's chosen allocation is described by a set of non-linear difference equations (see, e.g. De La Croix and Michel (2002)). In the general case, further insights into the properties of the solutions can be drawn by first considering the steady state. This analysis is conducted in the next subsection. We also list all the solutions of the social planner's problem in Appendix D.

Growth rates

Substituting (3.31) into (3.28), the growth rates in a centralised economy take the following form:

$$1 + \varrho^* = \alpha \Lambda A \left(\frac{K_t}{H_t} \right)^{\alpha-1} (1 - \phi)^{1-\alpha} \quad (3.41)$$

Equation (3.41) is the balanced growth rate, which depends on the productivity parameters in the goods and human capital production, the relative importance of material investment in human capital production, the relative preference for parenting time in individual's utility function and the social discount factor, Λ .

3.4.2 Laissez-faire economy

At the specific social weight, if there is no externality in the economy, the planner's allocations coincide with that of a laissez-faire economy (Bishnu, 2013). Within the context of the OLG economy, it has also been shown that the competitive equilibrium either under- or over-accumulates physical and human capital. Thus, the result eliminates the possibility of competitive equilibrium deviating from the social optimum in its allocation of physical and human capital in opposite directions.

By contrast, Docquier et al. (2007) considers a three-overlapping-generations model of endogenous growth wherein human capital is the engine of growth. Within the limits of their model, the rationale for the standard pattern of intergenerational transfers, and the working-aged financing youth education and old age pensions

comes into question. The problem is more complex when the rate of growth is endogenous. For instance, human capital is the engine of growth. Then, for a given social rate of time preference, an individual's saving decisions do not generate the appropriate amount of physical capital accumulation. In the case of over-accumulation, the intergenerational arrangements can be used to increase the welfare of all (present and future) generations. The Golden Rule steady state defines the frontier of Pareto efficient solutions. Introducing a social welfare function is useful in discriminating among all the efficient solutions. Therefore, this subsection uses the notion of a laissez-faire supported social weight to explore the efficiency of the optimal allocation in a competitive equilibrium.

Lemma 1. *If there exists a laissez-faire supported social weight $\bar{\beta}$, it must be unique.*

To prove this Lemma, this section defines $k_t^{SP} = K_t^{SP}/H_t^{SP}$ as the efficient capital per capita and rewrites k_{SP}^* in (3.40) as a function of Λ . As $\Lambda \in (0, 1)$, then $\lim_{\Lambda \rightarrow 0} k_{SP}^*(\Lambda) = 0$, $\lim_{\Lambda \rightarrow 1} k_{SP}^*(\Lambda) \equiv k_{SP_{max}}^*$ and thus, $k_{SP}^* \in (0, k_{SP_{max}}^*)$. Let us suppose that there exists a laissez-faire supported social weight $\bar{\beta}$. Since k_{SP}^* strictly increases in Λ , it follows that there exists a unique level of k_{SP}^* that corresponds to $\bar{\beta}$. If one denotes by $k_{SP}^*(\bar{\beta})$ the level of k_{SP}^* corresponding to $\bar{\beta}$, then $k_{SP}^*(\bar{\beta}) \in (0, k_{SP_{max}}^*)$.

This last part indicates that, given a social optimum, and thus the level k_{SP}^* , there is a unique $\bar{\beta}$. Given the uniqueness of the result of k_{SP}^* , this paper also establishes the following uniqueness result:

Lemma 2. *If there exists a $\bar{\beta} = \Lambda$ such that $K^{SP}(\bar{\beta}) = K^{CE}$ holds, then $H^{SP}(\bar{\beta}) = H^{CE}$.*

To prove this Lemma, this section defines $k_t^{CE} = K_t^{CE}/H_t^{CE}$ as the efficient capital per capita and rewrite k_{CE}^* in (3.22). The uniqueness result in Lemma 2 follows directly from human capital production, because there is no spillover from human capital and the form for CE is the same as for SP.

To find such laissez-faire supported social weight, the next section simulates the model and provides numerical simulations of the comparison between the discount factor and social weight over generations.

3.5 Numerical simulations

In the presence of dynamic efficiency and thus, it does not guarantee that k_{CE}^* and k_{SP}^* are identical. To make meaningful comparisons, this paper follows [Bishnu \(2013\)](#) and establishes a common point by devising the notion of a 'laissez-faire supported' social weight, denoted by Λ . By construction, at this specific social weight, if there is no externality, the planner's allocation coincides with the laissez-faire allocation. However, in the presence of consumption externalities, at this specific social weight, the laissez-faire allocation may differ from the planner's allocation.

<i>Variables</i>	
Material investment	Z_t
Time investment	ϕ_t
Consumption in retirement	X_{t+1}
Labour supply	L_t
Wage rate	w_t
Saving	S_t
Human capital	H_t
Cognitive skills	$H_{c,t}$
Non-cognitive skills	$H_{nc,t}$
Output	Y_t
Physical capital	K_t
Profit of firms	π_t
<i>Parameters</i>	
Capital share of production	α
Private discount factor	β
Importance of parenting time	γ
Subsidies on material investment in children	θ
Relative preference between material and time investment	η
Weight of non-cognitive skills	ν
Exogenous productivity in the goods production	A
Exogenous productivity in the production of cognitive skills	B
Exogenous productivity in the production of non-cognitive skills	D
Social discount factor	Λ
Laissez-faire supported discount factor	$\bar{\beta}$

Table 3.1: Variables and parameters

<i>Assigned parameters</i>			
Capital share of production	α	0.33	Empirical researches
Private discount factor	β	0.6095	De La Croix and Doepke (2003)
<hr/>			
<i>Calibrated parameters</i>		<i>Targeting</i>	
Importance of parenting time	γ	0.5	
relative preference between material and time investment	η	0.25	Craig (2005), time investment is 0.15
Weight of non-cognitive skills	v	0.75	Cunha and Heckman (2007), human capital production growth rates is 2 percent
Exogenous productivity in the goods production	A	3.8023	growth rates is 2 percent
Exogenous productivity in the production of human capital with cognitive skills	B	1.9374	growth rates is 2 percent
Exogenous productivity in the production of human capital with non-cognitive skills	D	1.9374	growth rates is 2 percent
<hr/>			
<i>Discussed parameters</i>			
Social discount factor	Λ	0.6095	Docquier et al. (2007) and Bishnu (2013)

Table 3.2: Parametrisation

Notes: The assigned parameters are based on empirical research. The parameters in the second section are calibrated to empirical research, while the exogenous productivity are targeted with the growth rate as 2%. To generate the value of the social discount factor, we follow the previous research to set up the number. For details of calibration see Appendix F.

3.5.1 The benchmark values

Given the rich setting in human capital formation, we cannot provide an analytical solution and therefore, we solve the model numerically. To this end, we have to assign a value to the parameters of the model: Table 3.1 summarises all the variables and parameters, and Appendix F presents the details of the calibration. Moreover, in line with Docquier et al. (2007) and Bishnu (2013), the parameter reasonably range from 0.67 (0.5% annual rate of time preference) to 0.9 (2% annual rate). This paper chooses the value of social discount factor to be 0.6095 (i.e. $\Lambda = 1/(1.02)^{25}$). Attributing all of the growth in multifactor productivity to growth in labour-augmenting productivity, A , implies that the growth rates of A is 3.8023 to set growth rates at 2 percent per year. Using a backwards solving method (i.e. swapping endogenous variables and parameters) and assuming $B = D$, we obtain $B = D = 1.9374$. Since this calibration is not exact, the growth rates are not comparable across papers. Table 3.2 provides a summary of the values assigned to some parameters and of the methodology adopted to set the others.

3.5.2 Comparison of competitive equilibrium and social optimum for different values of private discount factor and social weight over generations

This section studies how the equilibrium compares with the social optimum according to β and Λ . Applying the benchmark values and checking robustness, this paper summarises the results in Figure 3.1 and 3.2 to form the following proposition:

Proposition 2. *The levels of physical and human capital in a competitive equilibrium may differ from those in the social optimum. Human and physical capital may accumulate in the opposite directions.*

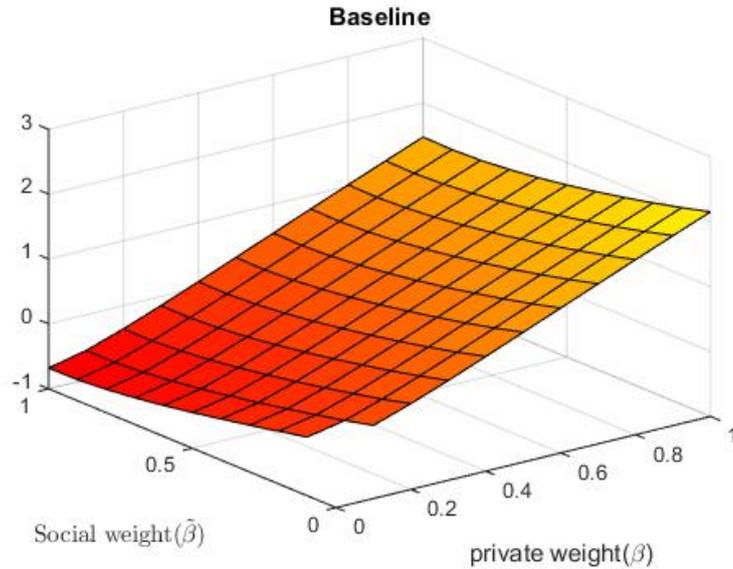


Figure 3.1: The comparison of physical capital in the competitive equilibrium and efficient allocations

Notes: The x-axis represents the values of private discount rate (β) in a competitive economy. The degree of social weight (Λ) in a centralised economy is presented by the y-axis. The values of the z-axis represent the difference between the physical capital in the competitive equilibrium (K^{CE}) and efficient allocations (K^{SP}). If the value in the z-axis is positive, this implies there is over-accumulation of physical capital ($K^{CE} > K^{SP}$).

Proof. There is under- (or over-accumulation) in the competitive allocations of physical and human capital. For instances, physical and human capital are both under-accumulated when β is low (e.g. $\beta = 0.3$) and Λ is high (e.g. $\Lambda = 0.9$). In contrast, there is over-accumulation of physical and human capital by the compet-

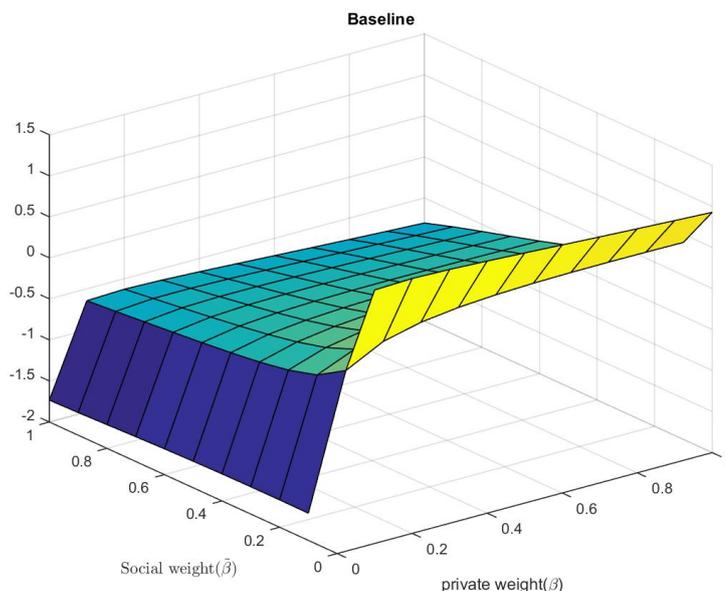


Figure 3.2: The comparison of human capital in the competitive equilibrium and efficient allocations

Notes: The x-axis represents the values of private discount rate (β) in a competitive economy. The degree of social weight (Λ) in centralised economy is presented by the y-axis. The values of the z-axis represent the difference between the human capital in the competitive equilibrium (H^{CE}) and efficient allocations (H^{SP}). If the value in the z-axis is positive, this implies there is over-accumulation of human capital ($H^{CE} > H^{SP}$).

itive economy when β is high and Λ is low: When the $\beta = 0.9$ and $\Lambda = 0.3$, the values of the z-axis in figure 3.1 and 3.2 are positive.

Moreover, we can observe that physical and human capital accumulates in different directions at pairs of low (high) β and Λ . In the first example, human capital is under-accumulated and physical capital is over-accumulated by one economy with low β and low Λ . The other example is that one economy with high β and high Λ under-accumulates human capital and over-accumulates physical capital. ■

The critical thresholds for β and Λ are clearly additionally dependent on the other parameters. For example, on the one hand, as the private rate of time preference decreases, individuals tend to save more. This pushes the interest rate downwards and stimulates investment in human capital. Since both physical and human capital accumulation increase with β as the first-best, both physical and human capital over-accumulate. On the other hand, individuals tend to save less as β is low and

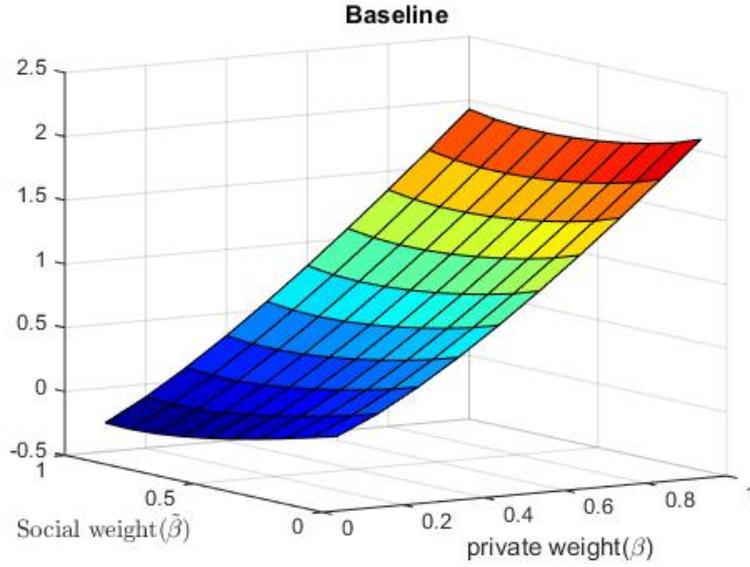


Figure 3.3: The ratio of physical capital to human capital in baseline

Notes: The x-axis represents the values of private discount rate (β) in a competitive economy. The degree of social weight (Λ) in centralised economy is presented by the y-axis. The values of the z-axis represent the difference between the efficient capital ratio in the competitive equilibrium (k^{CE}) and efficient allocations (k^{SP}). If the value in the z-axis is positive, this implies there is over-accumulation of physical capital ($k^{CE} > k^{SP}$).

Λ is high. This means both human and physical capital are under-accumulated. In addition, given Proposition 2, we make the following Proposition:

Proposition 3. *If $\beta = \Lambda$, there are over-accumulation of physical capital and under-accumulation of human capital.*

Proof. On the one hand, when the competitive economy is under low private discount factor (e.g. $\beta = 0.3$) and the social weight is low (e.g. $\Lambda = 0.3$), human capital is under-accumulated and physical capital is over-accumulated. On the other hand, when the private and social weight are both high ($\beta = \Lambda = 0.9$), human capital is under-accumulated and physical capital is over-accumulated. ■

This proposition states a scenario in which the private discount factor β equals the social discount factor Λ and accumulates too much physical capital, while it generates too little human capital.

Moreover, Proposition 2 and 3 implies that both physical and human capital can over- or under- accumulate, one can define $k^{CE} = \frac{K_t^{CE}}{H_t^{CE}}$ and $k^{SP} = \frac{K_t^{SP}}{H_t^{SP}}$ to find

that a scenario in which Laissez-faire economy coincides the social optimum. This paper finds the following proposition:

Proposition 4. *There exists a $\bar{\beta} = \Lambda$ such that $k^{SP}(\Lambda) = k^{CE}$.*

Proof. On the one hand, when β is low and Λ is high, there is under-accumulation of physical and human capital. On the other hand, there is over-accumulation of physical and human capital with high β and low Λ . ■

Proposition 4 specifies the weight at which the allocations in CE and SP are identical in this economy. This result ensures Lemma 2 holds. This result implies that there is no over- or under-accumulation of both types of capital. The implication is supported by Bishnu (2013). The efficient allocation coincides with that of a laissez-faire economy.

3.6 Conclusion

This paper has characterised the optimal level of parental investment in skill formation in an overlapping generations model in which cognitive and non-cognitive skills affect the production of human capital. As far as we know, this is the first model to include parental investment in human capital production function to analyse the efficient allocation.

We postulate that the objective of the social planner is to maximise its utility with material investment in children, the consumption in old age and parental time, which is subject to resource budget constraints. Additionally, this paper considers laissez-faire supported social welfare to compare the competitive equilibrium and efficient allocations.

The findings of numerical simulations are that, at the chosen social weight, the levels of physical and human capital in a competitive equilibrium are different from those in the social optimum. Physical and human capital can under- or over-accumulated in the same or opposite directions depending on the pairs of private and social discount factors. For example, when private discount factor is low and social discount factor is high, there are under-accumulation of physical and human capital. On the other hand, there are over-accumulation of physical and

human capital when the economy is with high private discount factor and low social discount factor. In addition, If private discount factor equals social discount factor, there are over-accumulation of physical capital and under-accumulation of human capital. Therefore, a laissez-faire economy coincides with the social optimum.

Since dynamic efficiency occurs at certain social weights, a central questions arises: how does optimal taxation adjust the dynamic efficiency? The analysis presented in this paper, despite its simplifying assumptions, shows that the existence of a link between parental time devoted to children and child's human capital can be relevant for the design of public policies. It also provides the grounds and motivation for further empirical analysis to improve our knowledge of how alternative parental investment in skill formation can shape the human capital accumulation process.

Chapter 4

Optimal taxation

Abstract

We examine the effects of income taxation and the subsidy on growth in an overlapping generations (OLG) model, in which government spending on education and skill formation enter the human capital production function as complements. With respect to previous contributions, optimal tax formulas correct the intergenerational externality in human capital accumulation. The simulation results find that labour income taxation and subsidies in material investment are positive to implement the efficient allocations. Government should reduce capital and labour income tax rates to subsidise parental investment in material resources.

Keywords: Skill formation technology, Time allocation, Human capital, Overlapping generations, Optimal taxation

4.1 Introduction

In this paper, we examine the effects of income taxation and the subsidy on growth in an overlapping generations (OLG) model, in which government spending on education and the skill formation enter the human capital production function as complements. The skill formation depends on parental investment. By parental investment, we mean both parenting time and material investment (e.g. private spending on education). Parental investment plays a role in the process of skill formation because it affects the quality of the early childhood environment, which in turn affects the growth rate of the economy.

There is a large body of literature in which taxation and growth have been examined (see [Gemmell et al. \(2011\)](#); [Myles \(2000\)](#); [Nijkamp and Poot \(2004\)](#)). Models have also been developed in which growth is driven by human capital accumulation that is related to one or more of the following three inputs: public schooling, the parents' or children's private expenditure on schooling and children's time investments (e.g. [Lee and Gordon \(2005\)](#); [Myles \(2009\)](#)). Nonetheless, according to the psychology and sociology literature, parental investment in skill formation is crucial for children's human capital acquisition. For instance: 1) Subsidising parental investments is more cost-effective in improving the later life outcomes of children such as schooling attainment or earnings ([Cunha and Heckman, 2007](#)); 2) Targeted public investments and targeted transfers restricted to child-related goods that guarantee minimum investment amounts to every child to increase the level of investments received by the children of the least-active parents ([Caucutt and Lochner, 2012](#)); 3) [Del Boca and Flinn \(2014\)](#) shows that unrestricted transfers increase the time parents spend with their children through a wealth effect. Therefore, one must consider how a benevolent government should tax capital in a neoclassical production economy when parents make a material investment in cognitive skills and a time investment in non-cognitive skills. A close study is conducted by [Casarico et al. \(2015\)](#), which suggests that omissions of parenting time from the technology of skill formation can bias the results related to the impact of labour income taxation on growth. Such inconsistency motivates a general question relating to the justification of such a taxation policy. Thus, our aim is to explore the role of optimal taxation in

restoring dynamical inefficiency and we focus purely on distortionary taxation. Prior research has examined public subsidies to day care (e.g. [Blomquist et al. \(2010\)](#); [Casarico and Sommacal \(2012\)](#)), but it has not accounted for the role of parental investment in skill formation on human capital accumulation.

This paper investigates how the tax policy can be used to restore the efficiency. The model examines the role of parenting time, material investment in children and tax policies in a three-period OLG endogenous growth model. Agents live for three periods: childhood, parenthood and retirement. In the first period, the child receives subsidised material investment from the government and time investment. Material investment contains the government spending on education and private material resources, whereas time investment involves parenting time. In the second period, the agent determines how much after-tax labour income to contribute to child and how much to save, as well as how much time to devote to parenting and labour supply. This paper also assumes that parents derive utility from parenting. In the third period, the agent retires and consumes all the after-tax income.

The distinctive feature of our model is that schooling and the quality of the early childhood environment enter the production function of human capital as complements. Because the quality of the early childhood environment depends on parental investment, labour and capital income taxation influence the growth rate not only through the decision to invest in schooling – as is standard – but also through time investment (i.e. through the effect on parenting time).

Our completely analytical characterisation of the solution to the Ramsey problem allows us to show explicitly that optimal taxation depends on the values of parameters. At the chosen parameter values, the optimal labour income tax rates and subsidies in material investment implementing efficient allocations are positive. In addition, the government should reduce capital and labour income tax rates to subsidise more material investment to justify the dynamical efficiency in a competitive equilibrium. Moreover, the sensitivity analysis of income taxation and the subsidies shows the following three scenarios to restore the dynamical efficiency: 1) the agent is more impatient; 2) the labour income tax rate is relatively low; 3) the subsidies to parental investment on material resource is relatively high.

The inclusion of parental investment in the skill formation process on human capital accumulation will be the key aspects of our setup. In our model, parents derive utility from their parenting time, from material investment in child and from the consumption in retirement. The direct dependency of the parents' utility on parental investment represents a warm-glow component. The government is empowered with two nonlinear taxes on labour and capital income. In addition, it can enforce a level of parenting time which is optimally selected. We theoretically characterise the social welfare-maximising policy and also perform a numerical analysis to shed light on the quantitative relevance of the inclusion of parental investment in the human capital production function, both in terms of the optimal values of the policy variables and in terms of the welfare losses caused by setting a policy neglecting its effect on parental investment and on the skill formation process.

The way parental altruism is specified is a crucial modelling issue here. There are several possible motivations behind child care decisions, including pure altruism (parents care about the utility of their children) and impure (warm-glow) altruism (parents derive joy from the level of human capital that child care arrangements deliver to their children). To the best of our knowledge, no direct empirical test exists on the type of altruism involved in the parental decision to devote time to children. However, the evidence on intrafamily income transfers, though not conclusive, rejects the predictions of pure altruism but tends to be consistent with warm-glow altruism (see [Casarico et al. \(2015\)](#); [Casarico and Sommacal \(2012\)](#)). Accordingly, we rely on the warm-glow assumption, which is shared by many papers, on intergenerational transmission of human capital and wealth (e.g. [Glomm and Kaganovich \(2008\)](#)), and it can also be found in optimal taxation literature (see [Cremer and Pestieau \(2006\)](#); [Kopczuk \(2013\)](#)).

In section 2, this paper introduces the main methodology: an overlapping generations model that incorporates non-cognitive skills, parenting time and altruism into an endogenous growth model. Section 3 represents the social optimum and Section 4 studies optimal taxation to restore the efficiency. Section 5 reports the results of the simulation on optimal taxation, after which the paper is concluded in Section 6.

4.2 The model

We develop an OLG model with intragenerational homogeneity and endogenous growth driven by human capital accumulation. Agents have perfect foresight about future variables. They live for three periods: childhood, parenthood and retirement. The population is constant, and the size of each generation is equal to N . For the sake of simplicity, we assume a small open economy.

In the first period, the agent receives parenting time and material investment in schooling, borrowing from the capital market. In the second period, the agent has one child and decides how much to consume and save, and how much time to devote to labour and the child. The child requires a given care time. In the third period, the agent retires and consumes all income.

This paper assumes that parental investment is endogenous; parents determine how much labour income to contribute to material investment and savings, and how much time to devote to parenting time and labour supply. However, in line with [Cunha and Heckman \(2007\)](#), the parents make no decision related to their own human capital accumulation. In particular, we abstract from the role of formal schooling in the accumulation of human capital, because we want to study the repercussions of parental investment for growth, focusing on skill formation as the main transmission mechanism.

4.2.1 Human capital production function

This paper considers the importance of government spending on education G_t and parental investment in the material resource Z_t (e.g. private spending on education) in the production of cognitive skills.

$$H_{c,t} = BG_t^\omega Z_t^{1-\omega} \quad (4.1)$$

where B is the exogenous productivity. Parameter ω determines the relative importance of government spending on education and material investments in the cognitive skills production. This Cobb-Douglas production function captures the interaction between government spending on education and material investments in children.

The production of non-cognitive skills takes the following form:

$$H_{nc,t} = D\phi_t^\gamma H_t \quad (4.2)$$

where D is the exogenous productivity. Parameter γ represents the importance of parenting time. ϕ_t is the parenting time. H_t is the parents' human capital.

The production of human capital is described as follows:

$$H_{t+1} = H_{c,t}^{1-v} H_{nc,t}^v \quad (4.3)$$

where parameter v is the relative importance of cognitive and non-cognitive skills in the production of human capital. Here we note that human capital does not fully depreciate. If we take log form of (4.3), we find that $\ln H_{t+1} = (1-v)\ln H_c + v\ln H_t + v\ln D\phi_t^\gamma$. Then we set the depreciation rate of human capital δ_H is $(1-v)$ to focus on the skill formation as the main transmission mechanism.

4.2.2 Goods production

This production function in the goods sector takes this form:

$$Y_t = AK_t^\alpha (L_t H_t)^{1-\alpha} \quad (4.4)$$

where Y_t and K_t represent the total output and physical capital, respectively. A is the total factor productivity. Parameter α and $1-\alpha$ are the elasticity of production with respect to physical capital and human capital, respectively. L_t determines time spent on supplying labour to the market.

4.2.3 Household

The model assumes warm-glow altruism in the agent's utility. This paper takes log arithmetic preference to ensure that income and substitute effect exactly cancel each other out, so that changes in the interest rate have no effect on the saving rate:

$$U_t = (1-\eta)\ln Z_t + \beta\ln X_{t+1} + \eta\ln\phi_t, 0 < \eta < 1 \quad (4.5)$$

where the parameter β is the psychology discount factor (i.e. patience). X_{t+1} is the consumption in retirement. Parameter η represents the relative preference of parenting time.

A working agent distributes labour income $w_t H_t$ among material investment Z_t and Saving S_t . Moreover, when the agent receives the return from savings and allocates it on consumption X_{t+1} . Therefore, the budget constraints for the agent at adulthood and retirement are

$$(1 - \theta)Z_t + S_t = (1 - \tau_L)(1 - \phi_t)w_t H_t \quad (4.6a)$$

$$X_{t+1} = [1 + (1 - \tau_k)r_{t+1}]S_t \quad (4.6b)$$

$$L_t + \phi_t = 1 \quad (4.6c)$$

where variable θ represents the subsidies to the material investment in children. Variables τ_L and τ_K are labour and capital income tax rates.

4.2.4 Government

The government has at its disposal the following policy instruments: government spending on education, subsidise to parents' material investment in children, taxes on labour income and capital income taxation. In line with [Caucutt and Lochner \(2012\)](#), we assume that the government runs a balanced budget and funds subsidies and education spending from the tax revenues:

$$\theta Z_t + G_t = \tau_L w_t H_t L_t + \tau_K r_t K_t \quad (4.7)$$

This subsection takes parameters τ_L , τ_K and θ as the exogenous policy variables.

4.2.5 First-order conditions

The firm's optimisation problem

The firm determines the demand of physical capital and human capital by maximising its profit π_t with given factor prices of wage and rent, which are determined

under a competitive market. Therefore, the firm's optimisation problem takes the following form:

$$\max_{K_t, H_t} \pi_t = AK_t^\alpha (L_t H_t)^{1-\alpha} - w_t H_t L_t - r_t K_t \quad (4.8)$$

This problem shows the firm sells its goods and pays the rental rate of physical capital and the real wage rate of human capital.

Rental rate of physical capital is given by

$$r_t = A\alpha K_t^{\alpha-1} (L_t H_t)^{1-\alpha} \quad (4.9)$$

Eq. (4.9) states that the interest rate equals the marginal productivity of capital.

The real wage rate per unit of human capital is given by

$$w_t = A(1-\alpha)K_t^\alpha (L_t H_t)^{-\alpha} \quad (4.10)$$

Eq. (4.10) requires that the wage per efficiency unit equals the marginal productivity of aggregate labour in efficiency units.

The agent's optimisation problem

One can form a Lagrange function as follows:

$$\begin{aligned} \mathcal{L}_t = & (1-\eta)\ln Z_t + \beta\ln X_{t+1} + \eta\ln\phi_t \\ & + \lambda_t \left\{ (1-\tau_L)(1-\phi_t)w_t H_t - (1-\theta)Z_t - \frac{X_{t+1}}{[1+(1-\tau_K)r_{t+1}]} \right\} \end{aligned} \quad (4.11)$$

where λ_t is the shadow price of physical capital.

First-order conditions for an interior solution with respect to Z_t , X_{t+1} , ϕ_t takes the following forms:

$$Z_t = \frac{(1-\eta)(1-\tau_L)}{(1+\beta)(1-\theta)} H_t w_t \quad (4.12a)$$

$$X_{t+1} = \frac{\beta[1+(1-\tau_K)r_{t+1}](1-\tau_L)}{1+\beta} H_t w_t \quad (4.12b)$$

$$\phi_t = \frac{\eta}{1+\beta} \quad (4.12c)$$

Eq. (4.12a) gives the equilibrium allocation of material investment in children. Eq. (4.12b) is the optimal consumption in old age. Eq. (4.12c) reflects the optimum choices for parenting time. (4.12c) finds that parenting time only depends on patience and the strength of altruism; the relative preference for parenting time increases parenting time.

One can use (4.9) and (4.10) to rewrite (4.12a) as

$$Z_t = zY_t \quad (4.13)$$

where $z \equiv [(1 - \alpha)(1 - \eta)(1 - \tau_L)]/[(1 - \eta + \beta)(1 - \theta)]$. z represents the proportion of material investment.

Plugging the rental rate of physical capital, the real wage rate of human capital, the equilibrium and (4.13), one can observe

$$G_t = gY_t \quad (4.14)$$

where $g \equiv \alpha\tau_K + (1 - \alpha)[\tau_L - \theta(1 - \eta)(1 - \tau_L)]/(1 - \eta + \beta)(1 - \theta)$, g is the ratio of government spending on education to aggregate output.

Definition 3. *For given H_0 and K_0 , a competitive equilibrium is the path $\{Z_t, X_t, \phi_t, k_t, w_t, r_t, S_t\}_{t \geq 0}$ that satisfies the government balanced constraints (4.7), production of human capital (4.1) - (4.2), the firm's optimisation conditions (4.9) and (4.10) and the agent's optimisation conditions (4.12a) - (4.12c)*

4.2.6 Balanced growth path

We assume the current physical capital stock K_t is fully depreciated at the end of the current period. In a balanced growth path, the transformation variable remains at same level. This yields $k_{t+1} = k_t = k^*$. It is straightforward to verify that there exists a unique steady state. Solving for k^* yields

$$k^* = \psi^{\frac{1}{1-\alpha v}} \quad (4.15)$$

where ψ is the collection of parameters, and $\psi \equiv A\beta(1-\alpha)(1-\tau_L)/\sigma(1+\beta)(1-\phi)^\alpha$. $\sigma \equiv [AB(1-\phi)^{1-\alpha}g^\omega c^{1-\omega}]^{1-v}D^v\phi^{\gamma v}$. The left-hand side of the equation is exactly the ratio of physical capital to human capital in the intensive form, where K_{t+1} has to be increased as H_{t+1} rises. Thus k^* is fixed in equilibrium.

4.2.7 Growth rates

Since the central goal of this section is to analyse the long-run relationship between non-cognitive skills and growth, we assume that the economy has already arrived at a balanced growth path. Therefore we focus on the balanced growth property of this economy. The growth rate of physical capital is as follows:

$$1 + \rho^* = \frac{K_{t+1}}{K_t} = \frac{H_{t+1}}{H_t} = \left[\frac{A\beta(1-\alpha)}{1+\beta(1-\frac{\eta}{1+\beta})^\alpha} \right]^{\frac{\alpha(1-v)}{1-\alpha v}} \left[D^v \left(\frac{\eta}{1+\beta} \right)^{\gamma v} \left[AB \left(1 - \frac{\eta}{1+\beta} \right)^{1-\alpha} \right]^{1-v} \right]^{\frac{1-\alpha}{1-\alpha v}} (1-\tau_L)^{\frac{\alpha(1-v)}{1-\alpha v}} \left\{ \left[\alpha\tau_K + (1-\alpha)\left(\tau_L - \frac{\theta(1-\eta)(1-\tau_L)}{(1-\eta+\beta)(1-\theta)}\right) \right]^\omega \left[\frac{(1-\alpha)(1-\eta)(1-\tau_L)}{(1-\eta+\beta)(1-\theta)} \right]^{1-\omega} \right\}^{\frac{(1-\alpha)(1-v)}{1-\alpha v}} \quad (4.16)$$

Eq. (4.16) is the balanced growth rate, which depend on a standard expression of the productivity parameters in the goods and human capital production, the relative importance of material investment in human capital production, the relative preference for parenting time in an individual's utility function and the private discount factor. It finds that ρ^* has only an ambiguous relationship with policy variables and psychological parameters. Therefore, the later section uses benchmark parameterisation to present the comparative static in this section.

In this model government spending is productive, as it contributes to the production of human capital. Thus, the endogenous growth is partly driven by government spending. Setting $\omega = 0$ shuts down this source of growth.

4.3 The planning problem

4.3.1 Environment

In the centralised version of the model, this section considers a central planner who chooses the allocation of output in order to maximise the present discounted value of current and future generations. In this economy, the social planner looks at the exact time period t and considers whole generations (De La Croix and Michel, 2002). This paper assumes that the central planner's discount factor $\tilde{\beta}$ equals the private discount factor β to focus on our main interest in optimal taxation, so the social welfare function takes the following form:

$$\sum_{t=0}^{\infty} \beta_t [(1 - \eta) \ln Z_t + \ln X_t + \eta \ln \phi_t] \quad (4.17)$$

The resource constraints of physical and human capital are as follows:

$$K_{t+1} = F_t - Z_t - X_t - G_t \quad (4.18a)$$

$$H_{t+1} = (BG_t^\omega Z_t^{1-\omega})^{1-v} (D\phi_t^\gamma H_t)^v \quad (4.18b)$$

where $F_t \equiv F(K_t, (H_t, L_t))$

4.3.2 General equilibrium

Writing social planner problem in Lagrange form to yield the Optimality leads to the maximum of \mathcal{L}_t with respect to Z_t , X_t , ϕ_t , G_t , K_t and H_t . Based on De La Croix and Michel (2002), \mathcal{L}_t equals the sum of the current utilities and the increase in the shadow value of the capital stock: $\beta q_{t+1} K_{t+1} - q_t K_t$ and $\beta \mu_{t+1} H_{t+1} - \mu_t H_t$, and assuming $\beta = \tilde{\beta}$, i.e.

$$\begin{aligned} \mathcal{L}_t = & (1 - \eta) \ln Z_t + \ln X_t + \eta \ln \phi_t \\ & + \beta q_{t+1} [AK_t^\alpha (H_t L_t)^{1-\alpha} - Z_t - X_t - G_t] - q_t K_t \\ & + \beta \mu_{t+1} [(BG_t^\omega Z_t^{1-\omega})^{1-v} (D\phi_t^\gamma H_t)^v] - \mu_t H_t \end{aligned} \quad (4.19)$$

First-order conditions for an interior solution (assuming it exists) are as follows:

$$\frac{\partial \mathcal{L}_t}{\partial Z_t} = \frac{1 - \eta}{Z_t} - \beta q_{t+1} + \beta \mu_{t+1} \frac{H_{t+1}(1 - v)(1 - \omega)}{Z_t} = 0 \quad (4.20a)$$

$$\frac{\partial \mathcal{L}_t}{\partial X_t} = \frac{1}{X_t} - \beta q_{t+1} = 0 \quad (4.20b)$$

$$\frac{\partial \mathcal{L}_t}{\partial \phi_t} = \frac{\eta}{\phi_t} - \beta q_{t+1}(1 - \alpha) \frac{F_t}{1 - \phi_t} + \beta \mu_{t+1} H_{t+1} \frac{v\gamma}{\phi_t} = 0 \quad (4.20c)$$

$$\frac{\partial \mathcal{L}_t}{\partial G_t} = -\beta q_{t+1} + \beta \mu_{t+1}(1 - v)\omega \frac{H_{t+1}}{G_t} = 0 \quad (4.20d)$$

$$\frac{\partial \mathcal{L}_t}{\partial K_t} = \beta q_{t+1} F'_t - q_t = 0 \quad (4.20e)$$

$$\frac{\partial \mathcal{L}_t}{\partial H_t} = \beta q_{t+1}(1 - \alpha) \frac{Y_t}{H_t} + \beta \mu_{t+1} v \frac{H_{t+1}}{H_t} - \mu_t = 0 \quad (4.20f)$$

where β is the planner's discount factor, or social discount factor. When utilities are bounded, the assumption that β is smaller than 1 ensures that objective function is finite (i.e. $\sum_{t=0}^{\infty} \beta^t < \infty$). These conditions are necessary and sufficient for optimality of the constant path starting at k_0 , as this path satisfies the transversality condition.

Eq. (4.20a) is the optimal allocation of material investment in children. Eq. (4.20b) indicates that the consumption of the old generation is equal to the next period of shadow price multiplies discount factor, describing the optimal allocation of the old generation. Eq. (4.20c) reveals that the marginal utility of material investment in children corrected to parenting time is equalised to the marginal utility of the old generation's consumption. Eq. (4.20c) also indicates that marginal productivity of parenting time on the production of human capital corrected to the marginal productivity of parenting time on goods production function. This result indicates that the substitution relationship between the production of human capital and goods production exists, hence a parent invests parenting time in children to the level when loss in the labour supply is equal to the gain in the (altruism-factor) discounted future marginal productivity of parenting time which arises from children's human-capital accumulation. Eq. (4.20d) determines the optimal government spending on education. Eqs. (4.20e) and (4.20f) are the resource constraints of physical capital and human capital, respectively. Note that, contrary to the stan-

dard Diamond (1965) model, this planner's first-order condition does not respect the first-order condition the individual chooses for himself in a decentralised economy.

Using $q_{t+1}Y_t = q_tK_t/\alpha\beta$ and (4.20a)-(4.20f), one finds that the optimal parenting time is a time-invariant variable as

$$\phi = \frac{\eta(\alpha\beta - 1)(1 - \beta v) + \beta(1 - \alpha)[\eta(1 - v) - v\gamma(2 - \eta)]}{(1 - \alpha)(2 - \eta)[\beta v(1 - \gamma) - 1] + \eta[(\alpha\beta - 1)(1 - \beta v) + \beta(1 - \alpha)(1 - v)]} \quad (4.21)$$

One can use (4.20a)-(4.20f) to observe material investment in children as follows:

$$\begin{aligned} Z_t = & F_t \left[\gamma v(\alpha\beta - 1) + (1 - \alpha)(1 - v) \frac{\phi}{1 - \phi} \right] \frac{1 - \eta - \frac{\eta(1-v)(1-\omega)}{v\gamma}}{\eta(1 - v) - v\gamma(2 - \eta)} \\ & + F_t \frac{\phi}{1 - \phi} \frac{(1 - \alpha)(1 - v)(1 - \omega)}{v\gamma} \end{aligned} \quad (4.22)$$

Here, we require that the material investment in children is strictly positive.

One can observe consumption in old age as

$$X_t = F_t \frac{\gamma v(\alpha\beta - 1) + (1 - \alpha)(1 - v) \frac{\phi}{1 - \phi}}{\eta(1 - v) - v\gamma(2 - \eta)} \quad (4.23)$$

This section requires that consumption in old age is strictly positive.

One can use first-order conditions and (4.21) to calculate the optimal government spending on education:

$$G_t = F_t \frac{(1 - v)\omega}{\eta(1 - v) - v\gamma(2 - \eta)} \left\{ \eta(1 - \alpha\beta) - \frac{\phi}{1 - \phi}(1 - \alpha)(2 - \eta) \right\} \quad (4.24)$$

Finally, using the first-order conditions, one can obtain

$$K_{t+1} = F_t \alpha \beta \quad (4.25)$$

With the definition of $k_t \equiv K_t/H_t$, one can make the following definition:

Definition 4. For given H_0 and K_0 , an efficient allocation is the path $\{Z_t, X_t, \phi_t, G_t, k_t, w_t, r_t\}_{t \geq 0}$ that satisfies government budget constraints (4.7), the production of human capital (4.3), the firm's optimisation conditions (4.9) and (4.10) and the agent's optimisation conditions (4.12a) - (4.12c).

4.3.3 Balanced growth properties

In balanced growth path, the transformation variable remains at same level. This yields $k_{t+1} = k_t = k^*$. Rewriting (4.25), one yields

$$k^* = \Gamma^{\frac{1}{1-\alpha v}} \quad (4.26)$$

where $\Gamma \equiv A^v(1-\phi)^{(1-\alpha)v}\alpha\beta$. Equation (4.26) characterises balanced-growth-path physical to human capital ratio in efficient allocations. If the planner assigns more social weight to future generations (a larger $\tilde{\beta}$), there is a larger optimal effective physical-human capital ratio in the long run. However the ratio is smaller when ϕ is higher. The logic is that higher time investment leads to slower accumulation of physical capital. Therefore, the k^* is smaller.

4.3.4 Growth rates

Substituting (4.26) into (4.25) and divided by K_t , the growth rates in efficient allocations take the following form:

$$1 + \varrho = \alpha\beta Ak_t^{\alpha-1}(1-\phi_t)^{1-\alpha} \quad (4.27)$$

Eq. (4.27) is the balanced growth rates, which depend on a standard expression of the productivity parameters in the production of goods and human capital, the relative importance of material investment in human capital production, the importance of parenting time in an agent's utility function and the subjective discount factor. We find that higher time investment decreases the growth rate. The reason is that higher time investment causes lower accumulation of physical capital, so the growth rate declines.

4.4 Optimal taxation

The dynamic optimal taxation problem, referred to as the Ramsey problem, is one of the most fundamental and influential policy problems. In this problem, the

government chooses its tax policy to maximise households' welfare by taking into account the equilibrium reaction of private agents to the tax policy.

Two approaches have been used to solve the problem: the primal and the dual. In the primal, we eliminate taxes and prices, so that the government can be thought of as directly using the quantities as controls. In the dual, the government uses the tax rates or prices as controls. The literature having “fully” solved the Ramsey problem (by full solution, we mean not only the above celebrated qualitative result but also a quantitative solution of the whole optimal path) have used the primal approach. We are not aware of a full solution to a dual problem. This is because, as is widely recognised (Jones et al., 1997), the primal is considerably simpler than the dual.

One important aspect of the OLG model is that the competitive equilibrium need not be efficient, in contrast to the representative agent models where the First Welfare Theorem guarantees Pareto efficiency. The reason behind this is that, with an infinite number of agents in the economy, the total value of resources is infinite, so Pareto improvements can be made by transferring resources from each young generation to the current old generation. Another attribute of the OLG type models is that it is possible that 'over-saving' can occur when capital accumulation is added to the model, a situation that a social planner could improve by forcing households to draw down their capital stocks. Thus, this section will study the efficient allocations.

In this section, we characterise the optimal policy in order to implement the social planner (hereafter, SP) allocation. Comparing competitive-equilibrium allocations and efficient allocations, we have

$$\tau_K = 1 + \frac{1}{A\alpha(1-\phi)^{1-\alpha}\psi^{\frac{\alpha-1}{1-\alpha v}}} \quad (4.28)$$

where $\psi \equiv \frac{A\beta(1-\alpha)(1-\tau_L)}{\sigma(1+\beta)(1-\phi)^\alpha}$, $\sigma \equiv [AB(1-\phi)^{1-\alpha}g^\omega z^{1-\omega}]^{1-v} D^v \phi^{\gamma v}$, $g \equiv \alpha\tau_K + (1-\alpha)[\tau_L - \frac{\theta(1-\eta)(1-\tau_L)}{(1-\eta+\beta)(1-\theta)}]$ and $z \equiv \frac{(1-\alpha)(1-\eta)(1-\tau_L)}{(1-\eta+\beta)(1-\theta)}$. Appendix E details the derivation of (4.28). One must notice that (4.28) is the non-linear solution for τ_K , because there are τ_K , τ_L and θ in the RHS. Being unable to directly control the amount of time that parents devote to their children, the government affects the agents' incentives

to engage in labour market activities in order to influence the time they spend with their children.

Comparing competitive-equilibrium allocations and efficient allocations, we then formulate the following equations:

$$1 - \theta = \frac{1 - \eta}{\left[\frac{1}{A\alpha(1-\phi)^{1-\alpha}\psi^{\frac{\alpha-1}{1-\alpha v}}} + (1 - \tau_K) \right] [1 - \eta + \beta\Delta(1 - v)(1 - \omega)]} \quad (4.29)$$

$$1 - \tau_L = \frac{1}{\left[\frac{1}{A\alpha(1-\phi)^{1-\alpha}\psi^{\frac{\alpha-1}{1-\alpha v}}} + (1 - \tau_K) \right] [1 + \beta\Delta\frac{v\gamma}{\eta}]} \quad (4.30)$$

where $\Delta \equiv \frac{(1-\alpha)}{(1-\beta v)} \frac{(1-\phi)[\eta(1-v) - v\gamma(2-\eta)]}{(\alpha\beta-1)(1-\phi)v\gamma + \phi(1-\alpha)(1-v)}$. Note (4.29) and (4.30) are two non-linear equations for θ and τ_L , and there are still τ_L , τ_K and θ in the RHS. It is impossible to solve (4.29) and (4.30), though this section tries to simplify (4.29) and (4.30). But one can observe the sign of θ and τ_L which depends on the $\eta(1 - v) - v\gamma(2 - \eta)$ and $(\alpha\beta - 1)(1 - \phi)\gamma v + (1 - \alpha)\phi(1 - v)$.

However, using a richer than is possible in a theoretical analysis but means one can rely on numerical analysis to generate results. The next section will use numerical simulation to find the signs of labour and capital income taxation and the subsidies rate to material investment.

4.5 Numerical simulation

Table 4.1 gives the variables and parameters used in this paper.

4.5.1 Benchmark values of parameters

According to Dhont and Heylen (2008), the average tax rates on labour income and capital income in 1995 and 2001 in the US are 0.347 and 0.393, respectively. Parameter θ determines the subsidies rate on material investment in children. Cautt and Lochner (2012) uses data from the Children of the National Longitudinal Survey of Youth to point out the parameter g (i.e. the ratio of government spending on education to output) is approximately 0.072, hence θ is chosen to be 0.5459 subject to the government running balanced budget constraints.

<i>Variables</i>	
Material investment	Z_t
Time investment	ϕ_t
Consumption in retirement	X_{t+1}
Labour supply	L_t
Wage rate	w_t
Saving	S_t
Human capital	H_t
Cognitive skills	$H_{c,t}$
Non-cognitive skills	$H_{nc,t}$
Output	Y_t
Physical capital	K_t
Profit of firms	π_t
Government spending in education	G_t
<i>Parameters</i>	
Capital share of production	α
Private discount factor	β
Relative importance of parenting time	γ
Relative importance of government spending	ω
Subsidies on material investment in children	θ
Relative preference between material and time investment	η
Weight of non-cognitive skills	v
Exogenous productivity in the goods production	A
Exogenous productivity in the production of cognitive skills	B
Exogenous productivity in the production of non-cognitive skills	D
Social discount factor	$\tilde{\beta}$

Table 4.1: Variables and parameters

For the relative importance of material investment in children and government spending on education in the accumulation of cognitive skills, ω , there is no study that calibrates it. Assuming G_t is equally important to Z_t in the accumulation of cognitive skills, this paper sets up ω as 0.5. To this end, we have to assign a value to the parameters of the model. Appendix F presents the details of the calibration. Table 4.2 provides a summary of the values assigned to some parameters and of the methodology adopted to set the others.

4.5.2 Optimal taxation

We compute the optimal policies and present the results in Table 4.3. In the first column, we report the results of the simulation performed in a standard model

<i>Assigned parameters</i>			
Labour income tax rates	τ_L	0.347	Dhont and Heylen (2008)
Capital income tax rates	τ_K	0.393	Dhont and Heylen (2008)
Capital share of production	α	0.33	Empirical researches
Private discount factor	β	0.6095	De La Croix and Doepke (2003)
<i>Calibrated parameters</i>		<i>Targeting</i>	
Weight of government spending on education	ω	0.5	
Importance of parenting time	γ	0.5	
Relative preference between material and time investment	η	0.25	Craig (2005), time investment is 0.15
Weight of non-cognitive skills in the human capital production	v	0.75	Cunha and Heckman (2007), human capital production
Subsidies on material investment in children	θ	0.5459	Caucutt and Lochner (2012)
Productivity in the goods sector	A	5.8229	growth rates is 2 %
Productivity in the cognitive skills production	B	2.0417	growth rates is 2 %
Productivity in the non-cognitive skills production	D	2.0417	growth rates is 2 %

Table 4.2: Parametrisation

		Standard	Quality optimised
Capital income tax rate	τ_K	0.393	0.453
Subsidies on material investment	θ	0.25	0.59
Labour income tax rate	τ_L	0.347	0.3243

Table 4.3: Optimal policies

Notes: (4.28)-(4.30) are the non-linear equations to provide the solutions for τ_L , τ_K , and θ . It is impossible to solve them directly. However we can solve for (4.20c), then we can get to solutions for (4.29) and (4.30). For solving (4.20c), we find the sign depends on $\alpha\tau_K + (1-\alpha)[\tau_L - \frac{\theta(1-\eta)(1-\tau_L)}{(1-\eta+\beta)(1-\theta)}]$. In this case, we have to use τ_L in the standard model and τ_K is 0.453 to solve θ . In the second column, we use $\theta = 0.59$ and τ_K is 0.453 to calculate τ_L .

in which the tax rates and subsidies are entirely exogenous. In the second column, we include the government's policy instruments. The comparison of the optimal policies across the different cases allows us to isolate the roles played by optimal taxation, which are the novelties of our model.

We first highlight that the main difference in results between the standard model and ours is the degree of subsidisation of government spending on education. Also, the marginal income tax rates change, though to a smaller extent. This shows that it is the direct instrument of taxing/subsidising the parental investment which that is mostly affected by the intergenerational externality in the skill transmission.

There are two observations from the numerical simulations. First, we see that τ_K can be positive or negative to justify the over- or under-accumulation in physical

capital. Since $\frac{(1-\alpha)(1-\eta)(1-\tau_L)}{(1-\eta+\beta)(1-\theta)}$ is positive, the sign of τ_K depends on the sign of $\alpha\tau_K + (1-\alpha)(\tau_L - \frac{\theta(1-\eta)(1-\tau_L)}{(1-\eta+\beta)(1-\theta)})$. Second, the baseline and robustness check shows that $\eta(1-v) - v\gamma(2-\eta)$ and $(\alpha\beta - 1)(1-\eta/(1+\beta))\gamma v + [(1-\alpha)(1-v)\eta]/(1+\beta)$ have the same sign. This gives the signs of τ_L and θ as positive. For example, under $\tau_K = 0.453$ and $\tau_L = 0.347$, we can have $\theta = 0.59$. With τ_K as 0.453 and θ as 0.59, the labour income tax rate is 0.3243. If we set τ_K as 0.4, τ_L will be 0.2651 and θ will be 0.5301. This implication means the government should reduce capital income tax rates and labour income tax rates to subsidise more material investment to justify dynamical efficiency in a competitive equilibrium.

4.6 Conclusion

There is a long-standing debate on the effects of labour income taxation. In this paper, we identify a new channel through which income taxation and subsidies affect growth. If parental investment influences the quality of the early childhood environment, and if this environment is an input of the human capital production function, then changes in taxation also affect human capital accumulation through their impact on parental investment choices.

This paper studies the suboptimality of physical and human capital accumulation in a three-period OLG model with consumption externalities and altruism. The purpose of this paper is to study optimal taxation and the allocation of public resources along with material investment and parenting time. The analysis is based on an endogenous growth model with two key features: material investment in children and government spending, and how these affect the accumulation of cognitive skills, while parenting time influences the accumulation of non-cognitive skills. Government spending and subsidies to material investment are financed by labour and capital income taxation.

Optimal taxation is examined in order for the equilibrium to attain the first-best solution. With the assumption that private weight equals social weight, the government should reduce capital income tax rates and labour income tax rates to subsidise greater material investment to justify the dynamical efficiency in a com-

petitive equilibrium. Labour income taxation and subsidies in material investment are positive to implement the dynamical efficiency.

Chapter 5

Growth- and welfare-maximising taxation

Abstract

This paper examines the effects of income taxation and the subsidies on growth and welfare in an OLG model in which parental investment in skill formation plays a role in the production of human capital. Policy simulations reveal that higher labour and capital income tax rates both rise growth rates. In contrast, growth rates are negatively related to the subsidies in material investment. In the benchmark calibration, the best welfare-maximising policy reforms an increase government spending on education funded by higher labour income tax rates.

Keywords: Endogenous growth, Tax structure, Tax rates estimates

5.1 Introduction

In this paper, we examine the effects of income taxation and the subsidies on the growth and welfare in an overlapping generations model in which parental investment in skill formation plays a role in the production of human capital as complements. The quality of the early childhood environment depends on parental investment. By parental investment, we mean parenting time and material investment (e.g. private spending on education). Parental investment plays a role in the process of skill formation because it affects the quality of the early childhood environment, which influences the growth rate of the economy.

Recent growth theories have emphasised the important role of government spending on education (King and Rebelo, 1990; Pecorino, 1993). Analysing tax structure is helpful to understand the growth-maximising taxation (Sørensen, 1994). Dual income systems are introduced in the Nordic countries in the late 1980s and early 1990s, but other countries have moved in a similar direction in pursuit of the more lenient taxation of capital items; see Sørensen (2005) for details of dual income taxation. Crawford and Freedman (2008) proposes the versions of dual income tax schedules as candidates for a future tax system in the UK. Bø et al. (2012) argues that horizontal inequity (henceforth, HI) measurement in the case of a two-rate tax system, such as the dual income tax system, both highlights and pinpoints the concept of HI. Treating public spending in the literature of growth has a long-standing debate. A series of contributions, following an early paper by Barro (1990), treat public spending as a flow (Fiaschi, 1999; Turnovsky, 1996). By contrast, contributions by Dasgupta (1999) treats public spending as a stock. These studies have provided much insight into the determinants of welfare-maximising shares of public spending or investment in infrastructure. Moreover, studies have shown that flow and stock specifications yield qualitatively similar results in some cases. The set of tax rates maximises welfare, subject to the government revenue budget. However, an integral part of human capital accumulation not only depends on government spending on education, but also parental investment in skills formation (Cunha and Heckman, 2008; Heckman and Mosso, 2014). Insofar as distortionary taxes interfere in the private decision to save and invest, they may very well change the accumu-

lation process of capital, and thus alter the growth rates of the economy (Casarico and Sommacal, 2012). Yet, despite how the contribution of government spending on education affects the production of human capital, little theoretical work links government spending on education and parenting time on skill accumulation. This paper helps to fill a gap in the literature by developing a conceptual framework for analysing the impact of government spending on education and parental investment in skill accumulation on long-run growth and social welfare.

This paper develops a three-period OLG endogenous growth model. Agents live for three periods: childhood, parenthood and retirement. In the first period, the child receives subsidised material investment by the government and time investment. Material investment contains government spending on education and private material resources, whereas time investment involves parenting time. In the second period, the agent determines how much after-tax labour income to contribute to child and savings, and how much time to devote to parenting time and labour supply. This paper also assumes that parents derive the utility from parenting. In the third period, the agent retires and consumes all the after-tax income.

This paper first focuses on the effect of income taxation and subsidies on growth. The income tax structure refers to the mix of taxes on the earnings from physical and human capital that satisfies the government's budget constraints. This work follows Rebelo (1991) and Agénor (2008) by presenting a convex model of growth in which human capital is added to a standard neoclassical model. Assuming that both human and physical capital are produced with constant returns to scale in produced inputs, this model results in unending growth. The net effect of income taxation is to reduce the growth rates below its efficient value. The set of tax rates maximises the growth, but is subject to the government revenue constraint. The setting minimises the growth rates distortion. The growth-maximising tax structure is found to depend upon the household sector, relative to the human capital and goods production sector as a whole. Minimising the growth rates distortion will not necessarily maximise welfare due to distortions of the labour-leisure decision and the factor ratios employed in the productive sectors of the economy. By considering the policy implications of the model in a competitive equilibrium, the simulation re-

sults indicate that a rise in subsidised material investment in children can decrease growth rates. On the other hand, raising capital income tax rates and labour income tax rates both benefit growth rates. The simulation of growth-maximising outcomes indicates that the greater relative importance of government spending in cognitive skills strengthens the impact of labour income taxation, subsidies in material investments and capital income taxation on growth rates. In contrast, three scenarios weaken the impacts of labour income taxation, subsidies in material investment and capital income taxation on growth rates: the increasing relative marginal productivity of non-cognitive skills in the production of human capital, parents having stronger preferences toward parenting time and the higher importance of parenting time, respectively.

Moreover, at the chosen parameter values, the simulation results of the social welfare analysis show that funding government spending on education is welfare-improving, even when funded by higher taxes. The best policy reform in the benchmark case is to increase government spending on education by increasing labour income tax rates. The reason is that labour income taxation is the most important source in the ratio of government spending on education to output. However, an increase in the subsidies to material investment by cutting the public spending on education is the most harmful policy for social welfare. The logic is that a higher subsidies rate implies more material investment in children. This increase of material investment means that the consumption in retirement falls. When the decrease in retirement consumption is greater than the increase of material investment, there is a reduction in social welfare.

Section 2 analyses the effect of the tax structure on growth and welfare, Section 3 presents the numerical simulations and Section 4 concludes.

5.2 The model

We develop an OLG model with intragenerational homogeneity and endogenous growth driven by human capital accumulation. In this model, agents have perfect foresight about future variables. They live for three distinct periods: childhood, par-

enthood and retirement. The population is constant, and the size of each generation equals to N . For the sake of simplicity, we assume a small open economy.

In the first period, the agent receives parenting time and material investment in schooling. In the second period, the agent has one child and decides how much to consume and save and how much time to devote to labour, and the child. The child requires a given time period of care. In the third period, the agent retires and consumes all income.

In line with [Cunha and Heckman \(2007\)](#), we assume that the parents make no decision on their own human capital accumulation. In particular, we abstract from the role of formal schooling on the accumulation of human capital, because we want to study the repercussions of parental investment on growth, focusing on the skills formation as the main transmission mechanism, this having been inadequately addressed by existing theoretical work.

5.2.1 Human capital production function

This paper considers the importance of government spending on education G_t and parental investment in the material resource (Z_t , e.g. private spending on education) of cognitive skills:

$$H_{c,t} = BG_t^\omega Z_t^{1-\omega} \quad (5.1)$$

where the variable B is exogenous productivity. Parameter ω determines the relative importance of government spending on education and material investment in the accumulation of cognitive skills.

The production of non-cognitive skills takes the following form:

$$H_{nc,t} = D\phi_t^\gamma H_t \quad (5.2)$$

where D is exogenous productivity. Parameter γ represents the importance of parenting time. ϕ_t is parenting time. H_t is parents' human capital.

The production of human capital is described in the following manner:

$$H_{t+1} = H_{c,t}^{1-v} H_{nc,t}^v \quad (5.3)$$

where parameter v represents the relative importance of cognitive and non-cognitive skills in the production of human capital. Here, we note that the human capital does not fully depreciate. If we take log form of (5.3), we find that $\ln H_{t+1} = (1-v)\ln H_c + v\ln H_t + v\ln D\phi_t^\gamma$. Then we set that the depreciation rate of human capital δ_H is $(1-v)$ to focus on skill formation as the main transmission mechanism.

5.2.2 Goods production

This production function in the goods sector takes the following form:

$$Y_t = AK_t^\alpha (L_t H_t)^{1-\alpha} \quad (5.4)$$

where Y_t and K_t represent the total output and physical capital, respectively. A is the total factor productivity. Parameter α and $1-\alpha$ are the elasticity of production in terms of physical capital and human capital, respectively. L_t determines the time spent on supplying labour to the market.

5.2.3 Household

The model assumes warm-glow altruism in the agent's utility. This paper takes log arithmetic preference to ensure that income and substitute effect exactly cancel each other out, so that changes in the interest rate do not affect on the saving rate. We assume parents do not derive their utility from their own consumption in adult age but enjoy parenting time and material investment in children. Appendix B provides the version of the model where parents derive the utility from their own consumption in adulthood. Therefore, the utility function is given by

$$U_t = (1-\eta)\ln Z_t + \beta\ln X_{t+1} + \eta\ln\phi_t, 0 < \eta < 1 \quad (5.5)$$

where parameter β is the time discount factor (i.e. patience). X_{t+1} is consumption in retirement. Parameter η represents the relative preference for parenting time.

A working agent distributes after-tax labour income $w_t H_t$ among material investment Z_t and savings S_t , where w_t is the real wage rate and H_t is parent's human capital. Moreover, when the agent receives the return from savings and allocates it to consumption X_{t+1} . Therefore, the agent's budget constraints in adulthood and retirement are

$$(1 - \theta)Z_t + S_t = (1 - \tau_L)(1 - \phi_t)w_t H_t \quad (5.6a)$$

$$X_{t+1} = [1 + (1 - \tau_k)r_{t+1}]S_t \quad (5.6b)$$

$$L_t + \phi_t = 1 \quad (5.6c)$$

where variable θ is the subsidies to material investment in children. Variables τ_L and τ_K are the labour and capital income tax rates. Variable r_{t+1} is rental rate of physical capital.

5.2.4 Revenue-neutral government spending

The government has at its disposal the following policy instruments: government spending on education, subsidise to parents' material investment in children, taxes on labour income and capital income taxation. In line with [Caucutt and Lochner \(2012\)](#), we assume that the government runs a balanced budget and funds subsidies and education spending from the tax revenues:

$$\theta Z_t + G_t = \tau_L w_t H_t L_t + \tau_K r_t K_t \quad (5.7)$$

This subsection takes parameters τ_L , τ_K and θ as the exogenous policy variables.

The balanced-growth-path (henceforth, BGP) effects and transitional dynamics associated with revenue-neutral changes in spending shares are straightforward to analyse in the present setting. In particular, increases in labour income tax rate τ_L and capital income tax rate τ_K lead to a higher subsidies to material investment in children. While material investment increases due to the higher subsidies,

the parenting time decreases according to higher tax rates. Therefore, θ has only an ambiguous effect on the BGP growth rates. This result is summarised in the following proposition:

Proposition 5. *With the balanced budget, an increase in the tax rates and the subsidies in material investment has an ambiguous effect on the balanced-growth-path growth rates. If non-cognitive skills do not affect human capital production ($v = 0$), the net effect of revenue-neutral government spending depends only on ω . With $v > 0$, the net effect also depends on v .*

To understand the intuition behind these results, consider the first case where $v = 0$. Human capital only relies on cognitive skills. While government spending on education increases the growth rates, increased government spending on education requires higher labour or capital tax rates, or lower subsidies in material investment. This indicates stock of material investment is lower, and this reduces growth rates. Therefore, the net effect only depends on ω .

The second case, $v > 0$, suggests that the higher fraction of government spending on education (for a given stock of educated labour) increases the marginal product of human capital, which in turn raises BGP growth. However the change is revenue neutral, so the ratio of educated labour-physical capital unambiguously falls. Thus, the positive effect of the increase in the share of spending on infrastructure is accompanied by a lower material investment, which tends to lower the production of human capital and reduce growth rates. In contrast, if non-cognitive skills dominate cognitive skills in skill accumulation, the net effect on output and the growth rates depends on how ‘productive’ the two inputs are in relative terms, that is, on the relative importance of non-cognitive skills in human capital accumulation.

5.2.5 First-order conditions

The firm’s optimisation problem

The firm determines the demand of physical capital and human capital by maximising its profit π_t with given factor prices of wage and rent, which are determined

under a competitive market. The firm's profit-maximisation problem is as follows:

$$\max_{K_t, H_t} \pi_t = AK_t^\alpha (L_t H_t)^{1-\alpha} - w_t L_t H_t - r_t K_t \quad (5.8)$$

The rental rate of physical capital is given by

$$r_t = A\alpha K_t^{\alpha-1} (L_t H_t)^{1-\alpha} \quad (5.9)$$

Eq. (5.9) states that the interest rate equals the marginal productivity of capital.

The real wage rate per unit of human capital is given by

$$w_t = A(1-\alpha)K_t^\alpha (L_t H_t)^{-\alpha} \quad (5.10)$$

Eq. (5.10) requires that the wage per efficiency unit equals the marginal productivity of aggregate labour in efficiency units.

The agent's optimisation problem

One can form a Lagrange function to solve the agent's optimisation problem:

$$\begin{aligned} \mathcal{L}_t = & (1-\eta)\ln Z_t + \beta \ln X_{t+1} + \eta \ln \phi_t \\ & + \lambda_t \left\{ (1-\phi_t)w_t H_t - Z_t - \frac{X_{t+1}}{[1+r_{t+1}]} \right\} \end{aligned} \quad (5.11)$$

where λ_t is the shadow price of physical capital.

First-order conditions for an interior solution with respect to Z_t , X_{t+1} , ϕ_t are

$$Z_t = \frac{(1-\eta)(1-\tau_L)}{(1+\beta)(1-\theta)} H_t w_t \quad (5.12a)$$

$$X_{t+1} = \frac{\beta[1+(1-\tau_K)r_{t+1}](1-\tau_L)}{1+\beta} H_t w_t \quad (5.12b)$$

$$\phi_t = \frac{\eta}{1+\beta} \quad (5.12c)$$

Eq. (5.12a) gives the equilibrium allocation of material investment in children. Eq. (5.12b) is the optimal consumption in old age. Eq. (5.12c) reflects the optimum

choices of parenting time. (5.12b) finds that parenting time only depends on patience and the strength of altruism; the relative preference for parenting time increases parenting time.

One can use (5.9) and (5.10) to rewrite (5.12a) as

$$Z_t = zY_t \quad (5.13)$$

where $z \equiv [(1 - \alpha)(1 - \eta)(1 - \tau_L)] / [(1 - \eta + \beta)(1 - \theta)]$. z represents the proportion of material investment to output.

Plugging the rental rate of physical capital, the real wage rate of human capital, the equilibrium and (5.13), one observes

$$G_t = gY_t \quad (5.14)$$

where $g \equiv \alpha\tau_K + (1 - \alpha)[\tau_L - \theta(1 - \eta)(1 - \tau_L)] / (1 - \eta + \beta)(1 - \theta)$, g is the ratio of government spending on education to aggregate output.

5.2.6 Balanced growth path

We assume the current physical capital stock K_t is fully depreciated at the end of the current period. In a balanced growth path, the transformation variable remains at the same level. This yields $k_{t+1} = k_t = k^*$. It is straightforward to verify that there exists a unique steady state. Solving for k^* yields

$$k^* = \psi^{\frac{1}{1-\alpha v}} \quad (5.15)$$

where ψ is the collection of parameters, and $\psi \equiv A\beta(1 - \alpha)(1 - \tau_L) / \sigma(1 + \beta)(1 - \phi)^\alpha$. $\sigma \equiv [AB(1 - \phi)^{1-\alpha}g^\omega c^{1-\omega}]^{1-v} D^v \phi^{\gamma v}$. The left-hand side of the equation is exactly the ratio of physical capital to human capital in the intensive form, where K_{t+1} has to be increased as H_{t+1} rises. Thus k^* is fixed in equilibrium.

5.2.7 Growth rates

In a balanced growth equilibrium, physical and human capital stocks grow at the same rate:

$$1 + \rho^* = \left[\frac{A\beta(1-\alpha)}{1 + \beta(1 - \frac{\eta}{1+\beta})^\alpha} \right]^{\frac{\alpha(1-v)}{1-\alpha v}} \left[D^v \left(\frac{\eta}{1+\beta} \right)^{\gamma v} \left[AB \left(1 - \frac{\eta}{1+\beta} \right)^{1-\alpha} \right]^{1-v} \right]^{\frac{1-\alpha}{1-\alpha v}} (1-\tau)^{\frac{\alpha(1-v)}{1-\alpha v}} \left\{ \left[\alpha\tau_K + (1-\alpha) \left(\tau - \frac{\theta(1-\eta)(1-\tau)}{(1-\eta+\beta)(1-\theta)} \right) \right]^\omega \left[\frac{(1-\alpha)(1-\eta)(1-\tau)}{(1-\eta+\beta)(1-\theta)} \right]^{1-\omega} \right\}^{\frac{(1-\alpha)(1-v)}{1-\alpha v}} \quad (5.16)$$

Eq. (5.16) is the balanced growth rates, which depends on a standard expression of the productivity parameters in the goods and human capital production, the relative importance of material and time investment to human capital production, the importance of parenting time, the subjective private discount factor and policy variables. The following sections discuss the effects of the policy variables on growth rates and welfare.

5.3 Growth-maximising taxation

This subsection focuses on the effect of the income tax structure on growth. The income tax structure refers to the mix of taxes on physical and human capital earnings which satisfy the government's budget constraints.

To investigate the impacts of τ_L , τ_K and θ on the variables in BGP, one can perform comparative static analysis based on the BGP solutions. According to (5.16), τ_L affects ρ^* through the following equation:

$$\frac{1}{1 + \rho} \frac{\partial \rho^*}{\partial \tau_L} = - \frac{\alpha(1-v)}{(1-\alpha v)(1-\tau_L)} - \frac{(1-\alpha)^2(1-v)(1-\omega)(1-\eta)}{z(1-\alpha v)(1-\eta+\beta)(1-\theta)} + \frac{\omega(1-v)(1-\alpha)^2}{g(1-\alpha v)} \left[1 + \frac{\theta(1-\eta)}{(1-\eta+\beta)(1-\theta)} \right] \quad (5.17)$$

Eq. (5.17) indicates that the effect of τ_L on ρ^* depends on the ratio of material investment to output and the ratio of government spending to output. If $z > g$ (i.e. the ratio of material investment is greater than government spending on education),

there is a positive impact of τ_L on ρ^* . On the other hand, $z < g$ leads to a negative effect of τ_L on ρ^* .

Referring to (5.16), the negative relationship between τ_K and ρ^* can be disentangled as follows:

$$\frac{1}{1 + \rho^*} \frac{\partial \rho^*}{\partial \tau_K} = \frac{\alpha \omega (1 - v)(1 - \alpha)}{g(1 - \alpha v)} \quad (5.18)$$

The influence of τ_K on ρ^* depends on the ratio of government spending on education to output.

According to (5.16), the change in θ should affect ρ^* directly:

$$\frac{1}{1 + \rho^*} \frac{\partial \rho^*}{\partial \theta} = \frac{(1 - \alpha)^2 (1 - v)(1 - \eta)(1 - \tau_L)}{(1 - \alpha v)(1 - \eta + \beta)(1 - \theta)^2} \left(\frac{1 - \omega}{z} - \frac{\omega}{g} \right) \quad (5.19)$$

Eq. (5.4) finds that the effect of θ on ρ^* depends on the last term, $\frac{1 - \omega}{z} - \frac{\omega}{g}$, in which the relative importance between the material investment and government spending on education plays a role in determining the sign of (5.4). For instance, if the material investment is equally important to government spending in cognitive skills technology, the ratio of material investment to output is greater than the ratio of government spending to output. This implies that more material investment than government spending means subsidies for material investment negatively affect the growth rate.

5.4 Welfare-maximising taxation

This section evaluates the predictions of our models for the welfare cost of this tax increase. Our objective is to illustrate the general principle that there are larger welfare effects in endogenous growth models than in the basic and neoclassical model. The traditional solution to the following maximization problem is

$$\begin{aligned} & \text{Max}_{\tau_L, \tau_K, \theta} U_t(Z_t, X_{t+1}, \phi_t) \\ & \text{s. t.} \quad \theta Z_t + G_t = \tau_L w_t H_t L_t + \tau_K r_t K_t \end{aligned} \quad (5.20)$$

This paper uses an alternative method, in which taxation reform is presented by Rebelo (1991) and Agénor (2008) to study the impact of income taxation and subsidies on social welfare.

Two of the biggest problems arising with numerical endogenous growth models are the choice of a human capital technology specification and the calibration of its parameters. In the literature, there is a broad consensus on the production function of consumption and investment goods. There is, however, no real evidence on the choice of the production function of human capital. We opt for a simple specification in which the rate of growth of human capital is a concave function of the time invested in education. In this purpose, we simulate large policy changes and compare the predictions with endogenous growth for the various parameter sets. There are two taxes, labour and capital income taxation, and two types of government spending, subsidies to material investment in children and public spending on education. The next section provides the simulations for the policy tax reforms.

5.5 Numerical simulations

The economic environment depicted above allows us to simulate the transitory and long-run effects of policy changes and other exogenous shocks. This simulation exercise requires calibrating the model, i.e. choosing the values of the parameters and exogenous variables so as to match a series of empirical moments computed on US data. It is often argued that one of the main disadvantages of applied general equilibrium models is the difficulty of calibrating certain parameters. Simulation results are thus followed by sensitivity analysis. This is especially important for the endogenous growth models with human capital since there are no established values on the parameters of the human capital formation technology. Table 5.1 gives the variables and parameters we used in this paper.

5.5.1 Benchmark values of parameters

According to Dhont and Heylen (2008), the average tax rates on labour income and capital income from 1995 to 2001 in the US are 0.347 and 0.393, respectively.

<i>Variables</i>	
Material investment	Z_t
Time investment	ϕ_t
Consumption in retirement	X_{t+1}
Labour supply	L_t
Wage rate	w_t
Saving	S_t
Human capital	H_t
Cognitive skills	$H_{c,t}$
Non-cognitive skills	$H_{nc,t}$
Output	Y_t
Physical capital	K_t
Profit of firms	π_t
Government spending in education	G_t
<i>Parameters</i>	
Capital share of production	α
Private discount factor	β
Relative importance of parenting time	γ
Relative importance of government spending subsidies to material investment in children	ω
Relative preference between material and time investment	θ
Weight of non-cognitive skills	η
Exogenous productivity in the goods production	ν
Exogenous productivity in the production of cognitive skills	A
Exogenous productivity in the production of non-cognitive skills	B
Social discount factor	D
	$\tilde{\beta}$

Table 5.1: Variables and parameters

Parameter θ determines the subsidies rate on material investment in children. [Cautt and Lochner \(2012\)](#) uses data from the Children of the National Longitudinal Survey of Youth to point out the parameter g (the ratio of government spending on education to aggregate output) is approximately 0.072, hence θ is chosen to be 0.5459 by subject to government running balanced budget constraints.

For the relative importance of material investment in children and government spending on education in the accumulation of cognitive skills, ω , there is no study that calibrates this. Assuming G_t is equally important to Z_t in the accumulation of cognitive skills, this paper sets up ω as 0.5. To this end, we have to assign a value to the parameters of the model: [Appendix F](#) presents the details of the calibration. [Table 5.2](#) provides a summary of the values assigned to some parameters and of the methodology adopted to set the others.

<i>Assigned parameters</i>			
Labour income tax rates	τ_L	0.347	Dhont and Heylen (2008)
Capital income tax rates	τ_K	0.393	Dhont and Heylen (2008)
Capital share of production	α	0.33	Empirical researches
Private discount factor	β	0.6095	De La Croix and Doepke (2003)
<i>Calibrated parameters</i>		<i>Targeting</i>	
Weight of government spending on education	ω	0.5	
Importance of parenting time	γ	0.5	
Relative preference between material and time investment	η	0.25	Craig (2005), time investment is 0.15
Weight of non-cognitive skills in the human capital production	v	0.75	Cunha and Heckman (2007), human capital production
Subsidies to material investment in children	θ	0.5459	Caucutt and Lochner (2012)
Productivity in the goods sector	A	5.8229	growth rates is 2 %
Productivity in the cognitive skills production	B	2.0417	growth rates is 2 %
Productivity in the non-cognitive skills production	D	2.0417	growth rates is 2 %

Table 5.2: Parametrisation

5.5.2 Growth-maximisation taxation

Comparative static analysis

This subsection performs comparative static analysis of labour and capital income taxation and subsidies to material investment with regard to the growth rates based on the steady state solutions.

	τ_L	τ_K	θ
ρ^*	+	+	-

Table 5.3: Numerical simulation results of decentralised economy

Notes: The impact of labour income taxation is negative to the growth rate. The higher capital income taxation leads to the higher growth rate. However, the higher subsidies rate to material investment lowers the growth rate.

Table 5.3 shows that g is smaller than z if $\omega = 0.5$. This result implies that the proportion of government spending on education is less than material investment in children. The implication states that the government distributes the taxation to z than g to increase growth rates.

Sensitivity analysis

To test the robustness of the results, this subsection performs a sensitivity analysis on the importance of parenting time γ , the relative importance of non-cognitive

skills in human capital production v , the relative importance of public spending on education in cognitive skills ω , patience β and relative preference for parenting time in utility function η .

Table 5.4 studies the effects of a 10 percent reduction in labour income taxation, capital income taxation and subsidies to material investment in children. The elasticity of growth rate ρ^* to labour income tax rate τ_L is 1.4059. This implies that a 10 percent reduction in τ_L leads to ρ^* decreasing by 14.059 percent. The elasticity of ρ^* to the tax rate of capital income τ_K is 0.5181. A 10 percent reduction in τ_K results in a 5.181 percent decrease in ρ^* . Moreover, Table 5.4 shows that the elasticity of ρ^* to subsidise rate to material investment θ is -1.5888. This means ρ^* is increased by 15.888 percent with a 10 percent lower in θ .

	$\partial\rho^*/\partial\tau_L$	$\partial\rho^*/\partial\tau_K$	$\partial\rho^*/\partial\theta$
$\gamma = 0.5,$ $\eta = 0.25,$ $v = 0.75,$ $\omega = 0.5$	1.4059	0.5181	-1.5888

Table 5.4: Benchmark case

Notes: The numbers states the effects of a 10 percent reduction namely in labour income taxation, capital income taxation and subsidies to material investment on the growth rate.

	$\partial\rho^*/\partial\tau_L$	$\partial\rho^*/\partial\tau_K$	$\partial\rho^*/\partial\theta$
$\omega = 0.1$	-0.1582	0.1238	0.0964
$\omega = 0.2$	0.2882	0.2368	-0.3847
$\omega = 0.3$	0.6960	0.3398	-0.8242
$\omega = 0.4$	1.0678	0.4334	-1.2247
$\omega = 0.5$	1.4059	0.5181	-1.5888
$\omega = 0.6$	1.7125	0.5947	-1.9188
$\omega = 0.7$	1.9898	0.6636	-2.2172
$\omega = 0.8$	2.2397	0.7254	-2.4859
$\omega = 0.9$	2.4641	0.7806	-2.7270

Table 5.5: Sensitivity analysis of the relative importance of government spending in the cognitive skills production

Notes: $\omega = 0.5$ is the standard case. We notice that the higher relative importance of public education in human capital accumulation strengthens the impact of income taxation and subsidies on the growth rate. The only exception is $\omega = 0.1$.

We now discuss how our conclusions from the previous subsection change when we depart from the benchmark of ω , η , γ and v . Table 5.5 - 5.8 present the results of

	$\partial\rho^*/\partial\tau_L$	$\partial\rho^*/\partial\tau_K$	$\partial\rho^*/\partial\theta$
$\eta = 0.1$	1.6825	0.5538	-1.9435
$\eta = 0.2$	1.5307	0.5410	-1.7445
$\eta = 0.3$	1.2728	0.4917	-1.4240
$\eta = 0.4$	1.0053	0.4355	-1.0948
$\eta = 0.5$	0.7516	0.3801	-0.7838
$\eta = 0.6$	0.5175	0.3274	-0.4980
$\eta = 0.7$	0.3045	0.2775	-0.2391
$\eta = 0.8$	0.1136	0.2300	-0.0089
$\eta = 0.9$	-0.0509	0.1822	0.1800

Table 5.6: Sensitivity analysis of the relative preference of parenting time
Notes: $\eta = 0.5$ is the standard case. We notice that the higher relative preference for parenting time in utility weakens the impact of income taxation and subsidies on the growth rate. The only exception is $\eta = 0.1$ and 0.9 .

	$\partial\rho^*/\partial\tau_L$	$\partial\rho^*/\partial\tau_K$	$\partial\rho^*/\partial\theta$
$v = 0.1$	5.9955	2.2097	-6.7755
$v = 0.2$	5.2376	1.9303	-5.9190
$v = 0.3$	4.4925	1.6557	-5.0769
$v = 0.4$	3.7637	1.3871	-4.2534
$v = 0.5$	3.0555	1.1261	-3.4530
$v = 0.6$	2.3723	0.8743	-2.6810
$v = 0.7$	1.7193	0.6337	-1.9430
$v = 0.8$	1.1021	0.4062	-1.2455
$v = 0.9$	0.5268	0.1941	-0.5953

Table 5.7: Sensitivity analysis of the relative importance of non-cognitive skills in the human capital production

Notes: $v = 0.75$ is the standard case. The higher relative importance of non-cognitive skills in human capital accumulation weakens the impact of income taxation and subsidies on the growth rate.

sensitivity analysis. We vary each parameter in turn, setting all the others according to the procedure used in the benchmark case.

For ω , which affects elasticity of substitution between public spending on education and material investment in children, we consider values in the interval (0, 1). This demonstrates that an increasing range of percentage in ω strengthens the positive impact of τ_L , τ_K and θ on ρ^* . The only exception is at $\omega = 0.1$, where τ_L has a negative effect on ρ^* and θ has the opposite effect on ρ^* .

The parameter η clearly plays a role in our analysis because it determines the relative marginal productivity of cognitive skills versus non-cognitive skills in the production of human capital. Parents attaching greater preference toward parenting

	$\partial\rho^*/\partial\tau_L$	$\partial\rho^*/\partial\tau_K$	$\partial\rho^*/\partial\theta$
$\gamma = 0.5$	1.4059	0.5181	-1.5888
$\gamma = 1$	0.7549	0.2782	-0.8531
$\gamma = 1.5$	0.4054	0.1494	-0.4581
$\gamma = 2$	0.2177	0.0802	-0.2460

Table 5.8: Sensitivity analysis of the importance of parenting time
Notes: $\gamma = 0.5$ is the standard case. The higher importance of parenting time in human capital accumulation weakens the impact of income taxation and subsidies on the growth rate.

time strengthens the impacts of τ_L , τ_K and θ on growth rates. However, $\eta = 0.9$ is an exception, in which τ_L has a negative effect on ρ^* and θ has the opposite impact on ρ^* .

For v , the elasticity of substitution between cognitive and non-cognitive skills. An increasing range of percentage in ω weakens the impacts of τ_L , τ_K and θ on ρ^* . The parameter γ is the importance of parenting time in human capital accumulation. Table 5.8 finds that greater importance of parenting time weakens the effects of τ_L , τ_K and θ on ρ^* .

5.5.3 Welfare analysis of taxation reforms

This subsection focuses on analysis of social welfare via revenue-neutral tax reforms. Due to the complex interrelationship between policy variables, changes in taxes and subsidies could result in ambiguous effects in the government budget and complicate the tax reform analysis. To solve this problem, this paper assumes output growth rate is 2 percent and applies the benchmark calibrated parameters listed in Table (4.1) with the assumption of fixed government expenditure. While many revenue-neutral reforms are possible, we focus on the following:

Case 1. An increase in θ is funded by an increase 10 percent of τ_K (i.e. τ_K changes from 0.393 to 0.4323).

Case 2. A 10 percent increase of τ_L (i.e. τ_L changes from 0.347 to 0.3817) funds an increase in θ .

Case 3. g is increased by a 10 percent increases in τ_K .

Case 4. An increase in g is funded by a 10 percent increase in τ_L .

Case 5. An increase in θ is funded by cutting 10% of g .

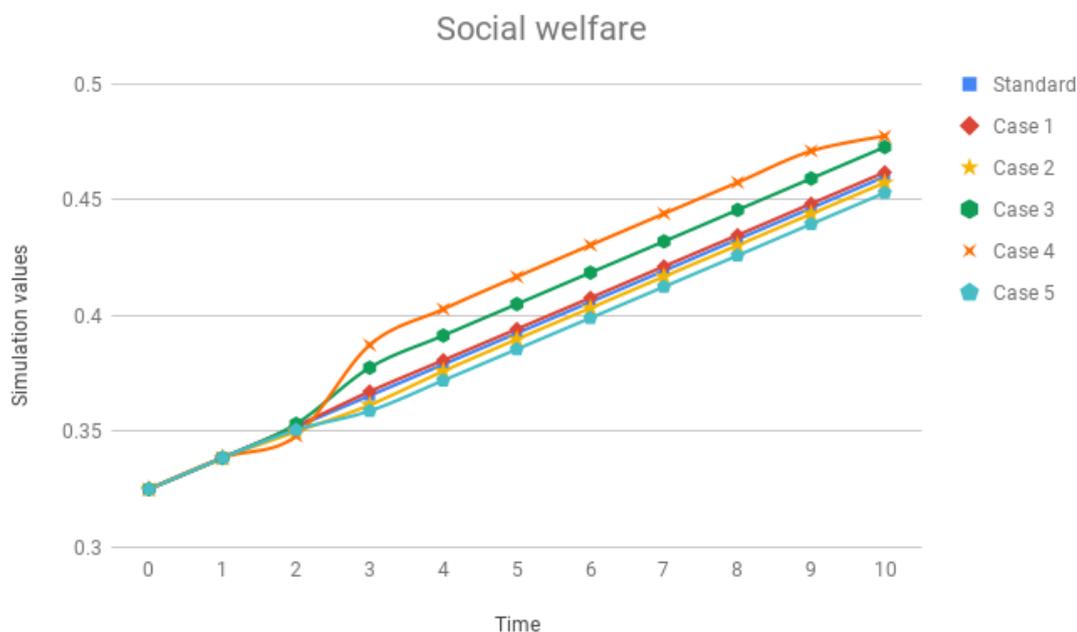


Figure 5.1: Welfare analysis of taxation reforms

Notes: Case 1 is an increase in θ which is funded by an increase 10 percent of τ_K . Case 2 shows a 10 percent increase of τ_L funds an increase in θ . Case 3 represents that g is increased by a 10 percent increases in τ_K . Case 4 describes how an increase in g is funded by a 10 percent increases in τ_L . Case 5 is an increase in θ which is funded by cutting 10% of g .

The results find that funding government spending on education is welfare-improving, even when funded by higher taxes (see Case 3 and 4). At the same time, the best policy change is to fund government spending on education by increasing labour income tax rates. This finding is unsurprising, because τ_L is the most important source in the ratio of government spending on education to output. Rising τ_L gives the largest increase in g . Therefore, increasing τ_L to fund government spending on education brings the largest amount of utility to the agent.

Figure (5.1) indicates that Case 1 and 2 show that the benefits of subsidies to material investment do not outweigh the welfare cost of higher taxes. In addition, case 5 is the most harmful policy to social welfare. This is a reasonable result, because a higher subsidies rate implies more material investment in children. This increase of material investment means that the consumption in retirement falls. When the decrease in retirement consumption is greater than the increase in material investment, there is a reduction in social welfare.

Moreover, we can compare this finding with the results in Table 5.3, which states that growth-maximising taxation subsidises greater material investment. This comparison shows that, a revenue-neutral reform leading to higher social welfare results in a lower growth rate.

5.6 Conclusion

The purpose of this paper is to develop a larger computable model with a complete description of the public finance aggregates and to compare configurations with endogenous growth, which arises in the model because of the accumulation of skills. Since we know little about skills accumulation in the human capital sector, our first goal was to study the robustness of the simulation results for various calibrations of the production function of human capital. We have thus computed three policy parameters: labour income taxation, capital income taxation and subsidies in material investment. The effect on the long-run growth rates is more important when non-cognitive skills affect future wages.

Casarico and Sommacal (2012) shows that the omission of parenting time can bias the impacts of income taxation on the growth rate. Additionally, we show that the impact of income taxation and subsidies on growth depends on the ratio of material investment to output and the ratio of public education to output. We also discuss how the magnitude of this bias depends on several model parameters. The parameters that affect the relative importance of the material investment and the public spending are very important. For instance, if the material investment is equally important to government spending in the cognitive skills technology, the ratio of material investment to output is greater than the ratio of government spending to output. This implies that more material investment than government spending leads to of subsidies to material investment negatively affecting the growth rate.

The simulation results suggest that the higher share of public spending on education in the production of human capital with cognitive skills could lead to labour income taxation and subsidies to material investments in children strongly affect growth. The results also suggest that parents having stronger preference for par-

enting time weakens the effects of the policy variables on growth rates. They also underline that the higher the marginal productivity of non-cognitive skills in human capital production, the weaker the impact of tax cuts on growth.

In general, the simulation of growth-maximising outcomes indicates that the greater relative importance of government spending in cognitive skills strengthens the impacts of income taxation and subsidies on growth rates. In contrast, an increasing relative marginal productivity of non-cognitive skills in the production of human capital weakens the impacts of income taxation and subsidies on growth rates, as well as parents having stronger preferences for parenting time and higher importance of parenting time in human capital accumulation.

The simulation of welfare-maximising results finds that funding government spending on education is welfare-improving, even when it is funded by higher taxes. The best welfare-maximising policy change is to fund government spending on education by increasing labour income tax rates.

Chapter 6

Addendum

6.1 Social security

In Chapter 3, we identify and explain dynamical inefficiency. We can also follow the literature to use social security in the capital accumulation to reflect this issue. This chapter briefly discusses how social security can be introduced to deal with dynamical inefficiency in the OLG model. Therefore pareto-optimal allocation can be decentralised with such transfers (Second Welfare Theorem).

De La Croix and Michel (2002) and Acemoglu (2009) document two types of social security: fully funded system and unfunded (pay-as-you-go, henceforth, PAYGO) system. The difference between these two systems is that the former taxes young generation then returns with interest back to the generation in the next period. But the later taxes young generation and transfers to old generation directly. A PAYGO social security system dates back to Diamond (1977). Samuelson (1975) views the PAYGO pensions as lump-sum transfers that can lead a stationary economy to the golden rule. As is typical presumed, a PAYGO social security discourages aggregate savings. However, when there is dynamical inefficiency, discouraging savings may lead to a Pareto improvement.

Fully funded social security

In a fully funded social security system, the government at date t raises some amount of funds, T_t , from the young. These funds are invested in the only productive

asset of the economy, the capital stock, and the workers receive the returns, given by $R_{t+1}T_t$ when they are old. Thus the individual maximisation problem under a fully funded social security system becomes

$$\begin{aligned} \text{Max}_{Z_t, X_{t+1}, \phi_t} \quad & U_t = \ln Z_t + \beta \ln X_{t+1} + \eta \ln \phi_t \\ \text{s. t.} \quad & Z_t + S_t + T_t = (1 - \phi_t)w_t H_t \\ & X_{t+1} = r_{t+1}(S_t + T_t) \end{aligned} \tag{6.1}$$

We assume savings are the capital in the next period (i.e. $K_{t+1} = S_t$). One can solve the maximum problem to obtain the optimal allocations of Z_t , X_{t+1} , and ϕ_t as follows:

$$\phi_t = \frac{\eta}{1 + \eta + \beta} \tag{6.2a}$$

$$K_{t+1} = \frac{\beta w_t^2 H_t^2 - (1 + \eta + \beta)w_t H_t + T_t}{1 + \eta + \beta} \tag{6.2b}$$

$$Z_t = \frac{(2 + \eta + \beta)(w_t H_t - T_t) - \beta w_t^2 H_t^2}{1 + \eta + \beta} \tag{6.2c}$$

Equation (6.2a) indicates that the fully funded social security does not affect the amount of parenting time, which only depends on η and β . (6.2b) shows that the social security has positive impact to physical capital accumulation. This result also states physical capital accumulates faster with fully funded social security. In contrast, (6.2c) describes the social security has negative effect on material investment in children. This finding leads to under-accumulation of human capital. With the assumption of one-period income, these results are reasonable. The logic is that parents allocate labour income into three parts: material investment in children, saving and social security amount, therefore the new allocation compresses the amount of material investment in children.

Unfunded social security

In line with Acemoglu (2009), the government collects the fund B_t from the young at time t and distributes it to the current old with per capita transfer (i.e. $B_t = (1 + n)B_{t+1}$, implying that takes into account that there are more young than

old because of population growth), but we assume the population growth rates is zero. Therefore the individual maximisation problem becomes

$$\begin{aligned}
& \text{Max}_{Z_t, X_{t+1}, \phi_t} \quad U_t = \ln Z_t + \beta \ln X_{t+1} + \eta \ln \phi_t \\
& \text{s. t.} \quad Z_t + S_t + B_t = (1 - \phi_t)w_t H_t \\
& \quad \quad X_{t+1} = r_{t+1}S_t + B_{t+1}
\end{aligned} \tag{6.3}$$

One can observe the following results by solving the maximum problem:

$$\phi_t = \frac{\eta(w_t H_t - B_t + \frac{B_{t+1}}{r_{t+1}})}{w_t H_t(1 + \eta + \beta)} \tag{6.4a}$$

$$K_{t+1} = \frac{\beta w_t H_t - \beta B_t - (1 + \eta) \frac{B_{t+1}}{r_{t+1}}}{1 + \eta + \beta} \tag{6.4b}$$

$$Z_t = \frac{w_t H_t - (1 + \eta + \eta\beta)B_t + (2\eta - 1) \frac{B_{t+1}}{r_{t+1}}}{1 + \eta + \beta} \tag{6.4c}$$

(6.4a) suggests B_t has negative effect to parenting time, but B_{t+1} has the counter effect. We find that PAYGO can affect parenting time. If the parents receive more social security in the next period, they give more parenting time. However, if the government requests more lump-sum tax in t , the agents decrease parenting time.

For physical capital accumulation, the PAYGO social security discourages savings leading to a Pareto improvement. The reason behind this is social security requires the income reallocation and compresses saving. However, in unfunded social security system, the payback relies on the population growth rates. Applying population into this model could be one of the future researches.

6.2 Parenting style

The development in the early life stage of a child is more important than genes in explaining the later life outcomes (Jablonka and Raz, 2009). In particular, the most important feature affecting the later life outcomes is parenting (i.e. rearing) by parents (Baumrind, 1971; Holmbeck et al., 1995; Maccoby and Martin, 1983). Greater levels of skills promote social inclusion, promote economic and social mobility, economic productivity and well-being (Heckman and Mosso, 2014).

Doepke and Zilibotti (2017) develop a theory of parent-child relations that rationalises the choice between alternative parenting styles. Krueger et al. (2008) also suggest parenting style can affect the heritability of personality. The parents maximise an objective function that combines Beckerian altruism and paternalism towards children. They can affect their children's choices via two channels: either by influencing children's preferences or by imposing direct restrictions on their choice sets. Different parenting styles (authoritarian, authoritative and permissive) emerge as equilibrium outcomes and are affected both by parental preferences and by the socioeconomic environment. Parenting style, in turn, feeds back into the children's welfare and economic success. The theory is consistent with the decline of authoritarian parenting observed in industrialised countries and with the greater prevalence of more permissive parenting in countries characterised by low inequality.

Other related papers in this literature includes Bhatt and Ogaki (2012), which introduces the tough-love model and studies the time-inconsistent decision making and temptation. It suggests an increasing material investment in children will decrease child's patience.

6.3 Imperfect market

Becker and Tomes (1986) argue that there is no role for initial financial wealth, parental income, parental utility, or the magnitude of parental altruism in determining the optimal level of investment, because the parents can borrow freely in the market to finance the wealth-maximising level of investment. Yet, Heckman and Mosso (2014) point out the returns to investments are higher for children as the parents with higher degree of altruism. This is a particular type of market failure due to the accident of birth that induces a correlation of human capital and earnings across generations, even in the absence of financial market imperfections. One possible constraint is the impossibility of borrowing against the child's future earnings (Becker and Tomes, 1986).

Carneiro and Heckman (2003) use NLSY 97 data and IV estimates of the returns to schooling to support the existence of substantial credit constraints. Dahl

and Lochner (2012) use the the evidence from the Earned Income Tax Credit to investigate how credit constraints and family income affect the test scores of children in early adolescence.

Glomm (1997) argues that the representative individual cannot borrow from the future to balance the current investment on child, because of borrowing constraint. Thus, the socioeconomic status can be extended to the discussion of initial wealth and borrowing constraints.

6.4 Positive psychology

Maslow (1954) and Maslow (1970) introduce the research of positive psychology. The application of positive psychology can be used in the human capital formation. For examples, the desire of being aggressive and positive is a self-actualization (Rogers, 1951). Furthermore, the positive emotion ,e.g. happiness and confidence, is meaningful in revolution of human being (Fredrickson, 2000).

Having confidence is positively relative to the life satisfied level and earnings, because a confident person keeps the happy memories and forgets the bad experiences (Seligman, 2002). Resiliency is positively related to leadership (Bandura, 2000). Optimism is considered to involve cognitive, emotional and motivational components (Peterson, 2000). Moreover, Bandura (1977), Bandura and Cervone (1986) and Bandura (1996) mention the efficacy expectation of self-ability is a major deterrent factor of goal-setting, activities choice and willingness to expend effort. If the individual has more self-efficacy, he would raise the inner motivation and enjoy his work. Hackett and Betz (1981) show that self-efficacy is a personal non-cognitive attitude, which drives the person to delivery his best effort to achieve the goal. Luthans (2002) describes self-efficiency as confidence. Therefore, positive psychology could extend the models in this thesis.

Chapter 7

Conclusion

This thesis goes beyond the technology of skill formation to understand the interactions within parenting time, material investments and government spending on education to shape human capital accumulation. The novelty of our approach to the application of skill formation and parenting time is the following. We introduce the contribution of parenting time in the production of non-cognitive skills, and material investment in children along with government spending on education contribute to the production of cognitive skills. We postulate that the objective of the agent is maximising the utility with material investment, parenting time and the consumption in retirement. With this type of preference, the economic agent needs to consider not only the trade-off between parenting time and working time, but also the balance between investing on children and increasing the next period consumption. With the assumption of finite lived agent, there is over- or under-accumulation in the decentralised market. If this is the case, the optimal taxation can be introduced into this economy to restore the inefficiency.

The numerical simulations in Chapter 2 show that the higher importance of parenting time decreases the growth rates. Parents' stronger preference for material investment increases the growth rates, as well as the higher relative importance of cognitive skills in human capital. One notes that the outcome would be the opposite in a different calibration with different type of altruism, for instances, strategic altruism or parenting style.

Chapter 3 uses the simulation results to reveal that, at the certain social weight, the levels of physical and human capital in a competitive equilibrium are different from those in the social optimum. Physical and human capital can under- or over-accumulate in the same or opposite directions depending on the pairs of private and social discount factor. For example, when private discount factor is low and social discount factor is high, there are under-accumulated physical and human capital. On the other hand, there are over-accumulated physical and human capital with high private discount factor and low social discount factor. In addition, If private discount factor equals social discount factor, there are over-accumulation of physical capital and under-accumulation of human capital. Therefore, laissez-faire economy does coincides with social optimum.

The numerical simulation results in Chapter 4 find the positive optimal labour income taxation and subsidies in material investment implement the efficient allocations. Capital income is taxed or subsidised depending on labour income taxation, subsidies to material investment, the relative preference of parenting time and agent's private discount factor.

Chapter 5 applies policy simulations in a calibrated model, in which the human capital counts on cognitive and non-cognitive skills. We find that higher labour and capital income tax rates both rise the growth rates. In contrast, public subsidies in material investment has negative impact on the growth rates. In the benchmark calibration, the best welfare-maximising policy forms with an increase government spending on education funded by higher labour income tax rates. Chapter 6 finds pay-as-you-go social security can affect the decision of allocating the parenting time in children. It suggests the application of parenting style can effect the different values of the parameters in the model.

This thesis contributes to three distinct strands of literature. First, it contributes to the large literature of the production of human capital. Whilst the combination of cognitive skills and non-cognitive skills into production of human capital are well documented in literature, theoretical works lack the convincing evidence for any of specific mechanism to distinguish cognitive skills and non-cognitive skills in analysing a general equilibrium. More specifically, in the model, the agent allocates

the time devoted to parenting is transformed to higher stock of next generation's human capital whereas time into labour supply is translated to higher consumption in retirement. Furthermore, the agent obtains utility from parenting time. This setting is different from the one in [Del Boca et al. \(2013\)](#) in which the parent does not derive utility from parenting time. Therefore this thesis addresses the trade-off between parenting time and the consumption in retirement, and the balance between time and material investment in children. It also opens the door for the study involving intergenerational interaction, such as demonstrate effect ([Jellal and Wolff, 2002](#)), asymmetric information on child's human capital accumulation or effort ([Akabayashi, 2006](#)), tough-love model ([Bhatt and Ogaki, 2012](#)) and parenting style ([Doepke and Zilibotti, 2017](#)).

The second contribution is to the literature in dynamical inefficiency with an overlapping generations model. There is a growing literature evaluating dynamical efficiency with both physical and human capital. For examples, [Boldrin and Montes \(2005\)](#) studies suboptimality of both education and saving decisions. [Docquier et al. \(2007\)](#) stresses the role of intergenerational human capital externalities. [Bishnu \(2013\)](#) contends a lack of empirical backing for human capital externalities and instead advocated the importance of consumption externalities. But this thesis posits a possibility wherein parenting time in non-cognitive skills contributing to the formation of human capital. Compared to the social optimum, both types of capital may under- or over-accumulate, or one type under-accumulates and the other type over-accumulates in the competitive equilibrium, hence laissez-faire economy coincides with social optimum.

Finally, this thesis contributes the literature in growth- and welfare-maximising taxation ([Altig et al., 2001](#); [Conesa et al., 2009](#)). We firstly examines the the standard case of labour income tax, capital income tax, subsidies on material investment in children on the growth rates. In addition, it presents the sensitivity analysis of different parameter values. We then investigate social welfare analysis by providing parametrization at the the aggregate level. We also highlights the interaction within public spending on education, and subsidies to material investment in children when the agent faces labour income tax and capital income tax.

This is the first step toward the theoretical application of personality into human capital accumulation. This study helps explaining how parenting time with impure altruism affects human capital accumulation. A greater knowledge of the mechanisms behind the learning is crucial for the design of more effective policies and interventions. Successful interventions alter parental behavior. Understanding why this happens, how parenting can be incentivised, and through which channels parenting influences child development are crucial tasks for the future studies of importance of parental investment in the production of human capital.

Appendix A

The relationship between the parameters and Big Five

According to American Psychological Association Dictionary, Big Five (i.e. OCEAN) is a taxonomy for personality traits, including (i) *Openness to Experience (Intellect)* is the tendency to be open to new aesthetic, cultural or intellectual experiences. (ii) *Conscientiousness* is the tendency to be organised, responsible and hardworking. (iii) *Extraversion* is an orientation of one's interests and energies toward the outer world of people and things rather than the inner world of subjective experience; characterised by positive affect and sociability. (iv) *Agreeableness* is the tendency to act in a cooperative, unselfish manner. (v) *Neuroticism (Emotional Stability)* is a chronic level of emotional instability and proneness to psychological distress. Emotional stability is predictability and consistency in emotional reactions, with absence of rapid mood changes. These preference parameters are usually applied into behavioral economics (DellaVigna, 2009).

Time Preference is the preference over consumption in different time periods. Survey questions and experiments used to elicit time preference can look at Marshmallow Task: participants (usually, children) will get a marshmallow. The examiner will tell the participants he/she will get one more marshmallow if he/she doesn't eat the current one, and examiner will leave the room. Using this Marshmallow Task to test short-term discounting (Mischel et al., 2011). Example survey question is "How patient are you on a scale from 1 to 10?" (GSOEP, 2008). Empirical

relationships include Conscientiousness, Self-Control, Affective Mindfulness, Elaboration of Consequences, Consideration of Future Consequences (Daly et al., 2009); Extraversion, Time Preference (Dohmen et al., 2010); Agreeableness, Inhibitive Side of Conscientiousness (Anderson et al., 2011).

Risk Aversion is the preference over different states of the risks. Survey questions and experiments used to elicit risk aversion can look at Devil's Task (Slovic, 1966): participant chose ten "switches" in order. Participant is informed that one of these the switches will make participant lose all previous winnings. The number of switches chosen is a measure of risk aversion. Example survey question is "How willing are you to take risks, in general?" (Dohmen et al., 2011). Empirical relationships contain Sensation Seeking (Eckel and Grossman, 2002; Zuckerman, 1994); Openness (Dohmen et al., 2010); Neuroticism, Ambition and Agreeableness (Borghans et al., 2009); Balloon Analogue Risk Task (Lejuez et al., 2003); Neuroticism, Inhibitive Side of Conscientiousness (Anderson et al., 2011).

Preferences for Leisure is the preference over consumption and leisure. Survey questions and experiments used to elicit leisure can look at payments for working: Workers' reservation wage is their preference for leisure (Borghans et al., 2008). Empirical relationships have the inconsistency with psychological measures of leisure preferences (Borghans et al., 2008).

Altruism is an unconditional kindness. *Inequity Aversion* is the value of equality in payoffs. Survey questions and experiments used to elicit altruism and inequality aversion can look at Dictator Game: A "proposer" has the option to transfer part of an endowment to a "responder." The transfer is used as a measure of pure altruism (Fehr and Schmidt, 2006). Empirical relationships are Neuroticism, Agreeableness (Ashton et al., 1998; Bekkers, 2006; Osiński, 2009).

Almlund et al. (2011) reviews German Socio-Economic Panel Study (GSOEP) to comprise preference and personality measures for a representative sample of more than 14,000 individuals. It invests the relationship between preference parameter and Big Five, including how to estimate personality traits, and what are the issues of estimation. It also shows that how personal traits affect performance in many distinct areas of economic and social life.

In concluding, we have the Table A.1 to summarise the relationships between Big Five and the parameters in the models. Almlund et al. (2011) provides the comprehensive results, see Table A.2.

Big Five	The parameters
Openness	v
Conscientiousness	β
Extraversion	β
Agreeableness	η
Neuroticism	η

Table A.1: The relationship between Big Five and the parameters in the model
Notes: The parameter v is the relative importance between cognitive and non-cognitive skills.
 β represents the private discount factor. η determines the relative preference between material and time investment.

Table A.2: Empirical evidence

Author(s)	Main Variables	Data and Methods	Main Result(s)
Baker et al. (2008)	<u>Outcome(s):</u> Family well-being and maternal labour supply <u>Explanatory Variable(s):</u> Child care, child health	<u>Data:</u> National Longitudinal Survey of Children and Youth(NLSCY)	The new childcare program leads to more hostile, less consistent parenting, worse parental health and lower-quality parental relationships.
Bernal and Keane (2010)	<u>Outcome(s):</u> Cognitive development process of children <u>Explanatory Variable(s):</u> Mother's employment, maternal time and childcare decisions	<u>Data:</u> National Longitudinal Survey of Youth 1979 (NLSY79) <u>Method(s):</u> Method of moments	The types of childcare are important. Effect of household income is quantitatively small.
Carneiro et al. (2007)	<u>Outcome(s):</u> schooling attainment, labour market outcomes and social behaviours <u>Explanatory Variable(s):</u> family background and the home learning environment	<u>Data:</u> National Child Development Survey (NCDS)	Both cognitive and non-cognitive skills depend on family background and other characteristics of the home learning environment, and this is likely to be for both genetic and environmental reasons. Social skills may be more malleable than cognitive skills.

Continuation of Table A.1

Cunha et al. (2010)	<p><u>Outcome(s)</u>: multistage production functions for children's cognitive and non-cognitive skills.</p> <p><u>Explanatory Variable(s)</u>: parental environments and investments at different stages of childhood.</p>	<p><u>Data</u>: Children of the NLSY/79 (CNLSY/79)</p> <p><u>Method(s)</u>: nonparametric identification of a general class of production technologies based on nonlinear factor models with endogenous inputs</p>	<p>Substitutability decreases in later stages of the life cycle in the production of cognitive skills. It is roughly constant across the stages of life cycle in the production of non-cognitive skills.</p>
Del Boca et al. (2013)	<p><u>Outcome(s)</u>: cognitive development process of children</p> <p><u>Explanatory Variable(s)</u>: Each parent's time investment and material investment</p>	<p><u>Data</u>: Child Development Supplement of the Panel Study of Income Dynamics</p> <p><u>Method(s)</u>: The Method of Simulated Moments</p>	<p>Parents' time inputs are important for the cognitive development of their children, particularly when the child is young. Money expenditures are less productive.</p>
Fiorini and Keane (2014)	<p><u>Outcome(s)</u>: Cognitive and non-cognitive development process of children</p> <p><u>Explanatory Variable(s)</u>: Each parent's time investment and parenting style</p>	<p><u>Data</u>: Longitudinal Study of Australian Children</p> <p><u>Method(s)</u>: OLS, FE, valued add</p>	<p>Time spent with parents is the most productive input for cognitive skill development. Non-cognitive skills appear insensitive to alternative time allocations. Instead, they are greatly affected by the mother's parenting style.</p>
Heckman et al. (2006)	<p><u>Outcome(s)</u>: labour market outcomes and social behaviour</p> <p><u>Explanatory Variable(s)</u>: Cognitive and non-cognitive factor of children</p>	<p><u>Data</u>: National Longitudinal Survey of Youth 1979 (NLSY79)</p> <p><u>Method(s)</u>: Bayesian Markov chain Monte Carlo methods</p>	<p>Schooling, employment, work experience and the choice of occupation are affected by latent non-cognitive and cognitive skills.</p>

Continuation of Table A.1

<p>Milkie et al. (2004)</p>	<p><u>Outcome(s):</u> Labour market outcomes <u>Explanatory Variable(s):</u> Feeling about time spent with child</p>	<p><u>Data:</u> 2000 General Social Survey (GSS) and 2000 National Survey of Parents</p>	<p>Work hours are strongly related to the feelings. For fathers, those whose youngest child is an adolescent feel more strain than similarly situated mothers. Single parents are not more likely than married parents to feel that they spend insufficient time with children.</p>
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Appendix B

The consumption in adulthood

B.1 The Model

We develop an OLG model with intragenerational homogeneity and endogenous growth driven by human capital accumulation. The agent has perfect foresight about future variables and lives for three periods: childhood, parenthood and retirement. The population is constant, and the size of each generation is equal to N . For the sake of simplicity, we assume a small open economy.

In the first period, the agent receives parenting time and material investment in schooling. In the second period, the agent has one child and decides how much to consume and savings and how much time to devote to labour and parenting time. The child requires a certain care time. In the third period, the agent retires and consumes all income.

This paper assumes the parental investment is endogenous; parents determine how much labour income to contribute to material investment and savings, and how much time to devote to parenting time and labour supply. However, we assume the parents make no decision on their own human capital accumulation. In particular, we abstract from the role of formal schooling on the accumulation of human capital, because we want to study the repercussions of parental investment on growth, focusing on the skills formation as the main transmission mechanism, an issue which is largely unexplored in the theoretical literature.

B.1.1 Household

Parents are the decision maker in the household. The following utility function describes preferences of an agent born in period $t - 1$:

$$U_t = (1 - \eta)\ln Z_t + \eta\ln\phi_t + \ln C_t + \beta\ln X_{t+1}, 0 < \eta < 1 \quad (\text{B.1.1})$$

where C_t and X_{t+1} denote consumption in parenthood/middle age and in retirement/old age, respectively (no consumption takes place during childhood); Z_t stands for the material investment on child (e.g. private education expenditure). The parameter β is the psychology discount factor (i.e. patience). Parameter η represents the relative preference of parenting time.

Assumption 1. U_t is a twice differentiable, strictly increasing, and strictly concave social welfare function.

B.1.2 Human capital production function

This paper considers the importance of government spending on education G_t and parental investment on the material resource in cognitive skills:

$$H_{c,t} = BG_t^\omega Z_t^{1-\omega} \quad (\text{B.1.2})$$

where the variable B is the exogenous productivity. Parameter ω determines the relative importance of government spending on education and material investments in the accumulation of cognitive skills.

The production non-cognitive skills takes the following form:

$$H_{nc,t} = D\phi_t^\gamma H_t \quad (\text{B.1.3})$$

where D is the exogenous productivity. Parameter γ represents the importance of parenting time. ϕ_t is the parenting time. H_t is the parents' human capital.

The production of human capital is described as follows:

$$H_{t+1} = H_{c,t}^{1-v} H_{nc,t}^v \quad (\text{B.1.4})$$

where parameter v is the relative importance of cognitive and non-cognitive skills in the production of human capital. Here we note that the human capital does not fully depreciate. If we take log form of (B.1.11c), we find that $\ln H_{t+1} = (1 - v)\ln H_c + v\ln H_t + v\ln D\phi_t^\gamma$. We set the depreciation rate of human capital δ_H is $(1 - v)$ to focus on the skill formation as the main transmission mechanism.

B.1.3 Goods production

The production function in the goods sector takes the following equation:

$$Y_t = AK_t^\alpha (L_t H_t)^{1-\alpha} \quad (\text{B.1.5})$$

where Y_t and K_t are the total output and physical capital, respectively. A is the total factor productivity. Parameter α and $1 - \alpha$ are the elasticity of production with respect to physical capital and human capital, respectively. L_t determines the time spent on supplying labour to the market. We normalise parenting time and working hours are 1 (i.e. $L_t + \phi_t = 1$).

B.1.4 Government

The government has at its disposal the following policy instruments: government spending on education, subsidy to parents' material investment in children, taxes on labour income and capital income. We assume that the government runs a balanced budget and funds subsidies and education spending from the tax revenues:

$$\theta Z_t + G_t = \tau_L w_t H_t L_t + \tau_K r_t K_t \quad (\text{B.1.6})$$

where θ is subsidy rate to material investment in children. τ_L and τ_K represent labour and capital income tax rates. This subsection takes τ_L , τ_K and θ as exogenous.

B.1.5 Optimisation problems

The firm's optimisation problem

The firm determines the demand of physical capital and human capital by maximising its profit π_t with given factor prices of wage and rent, which are determined under the competitive market:

$$\max_{K_t, H_t} \pi_t = AK_t^\alpha (L_t H_t)^{1-\alpha} - w_t H_t L_t - r_t K_t \quad (\text{B.1.7})$$

The maximisation problem shows the firm sells its goods and pays the rental rate of physical capital and real wage of human capital.

The rental rate of physical capital is formed by

$$r_t = A\alpha K_t^{\alpha-1} (L_t H_t)^{1-\alpha} \quad (\text{B.1.8})$$

Eq. (B.1.8) states that interest rate equals the marginal productivity of capital.

The real wage rate per unit of human capital is given by

$$w_t = A(1-\alpha)K_t^\alpha (L_t H_t)^{-\alpha} \quad (\text{B.1.9})$$

Eq. (B.1.9) requires that the wage per efficiency units equals the marginal productivity of aggregate labour in efficiency units.

Agent's optimisation problem

One can form a Lagrange function as follows:

$$\begin{aligned} \mathcal{L}_t = & (1-\eta)\ln Z_t + \eta\ln\phi_t + \ln C_t + \beta\ln X_{t+1} \\ & + \lambda_t \left\{ (1-\tau_L)(1-\phi_t)w_t H_t - (1-\theta)Z_t - C_t - \frac{X_{t+1}}{[1+(1-\tau_K)r_{t+1}]} \right\} \end{aligned} \quad (\text{B.1.10})$$

First-order conditions for an interior solution (assumes it exists) with respect to

Z_t , C_t , X_{t+1} , ϕ_t and λ_t are

$$Z_t = \frac{1 - \eta}{\lambda_t(1 - \theta)} \quad (\text{B.1.11a})$$

$$C_t = \frac{1}{\lambda_t} \quad (\text{B.1.11b})$$

$$X_{t+1} = \frac{\beta[1 + (1 - \tau_K)r_{t+1}]}{\lambda_t} \quad (\text{B.1.11c})$$

$$\phi_t = \frac{\eta}{\lambda_t[(1 - \tau_L)H_t w_t]} \quad (\text{B.1.11d})$$

$$(1 - \tau_L)(1 - \phi_t)w_t H_t - (1 - \theta)Z_t - \frac{X_{t+1}}{[1 + (1 - \tau_K)r_{t+1}]} = 0 \quad (\text{B.1.11e})$$

Substituting (B.1.11a) - (B.1.11d) into (B.1.11e), one obtains

$$\lambda_t = \frac{2 + \beta}{(1 - \tau_L)H_t w_t} \quad (\text{B.1.12})$$

Equation (B.1.12) indicates the shadow price.

Incorporating equation (B.1.12), equations (B.1.11a) - (B.1.11d) can be rewritten as follows:

$$Z_t = \frac{(1 - \eta)(1 - \tau_L)}{(2 + \beta)(1 - \theta)} H_t w_t \quad (\text{B.1.13a})$$

$$C_t = \frac{(1 - \tau_L)w_t H_t}{2 + \beta} \quad (\text{B.1.13b})$$

$$X_{t+1} = \frac{\beta[1 + (1 - \tau_K)r_{t+1}](1 - \tau_L)}{2 + \beta} H_t w_t \quad (\text{B.1.13c})$$

$$\phi_t = \frac{\eta}{2 + \beta} \quad (\text{B.1.13d})$$

Equation (B.1.13a) gives the equilibrium allocation of material investment in children. (B.1.13a) also confirms that socioeconomic status affects the material investment in children directly (Baker et al., 2002; Hout and Dohan, 1996). (B.1.13a) additionally shows that a reduction in τ_L increases Z_t . Equation (B.1.13c) states that the optimal consumption in retirement. It also describes the negative impacts of τ_L and τ_K to the consumption in retirement. Equation (B.1.13d) reflects the optimum choices of parenting time. It also finds that the level of altruism and patience both benefit parenting time.

One can rewrite (B.1.13a) in the following manner:

$$Z_t = zY_t \quad (\text{B.1.14})$$

where $z \equiv [(1 - \alpha)(1 - \eta)(1 - \tau_L)] / [(2 - \eta + \beta)(1 - \theta)]$.

Observing r_t , w_t , (B.1.13a), (B.1.13c) and (B.1.13d), one yields

$$G_t = gY_t \quad (\text{B.1.15})$$

where $g \equiv \alpha\tau_K + (1 - \alpha)[\tau_L - \frac{\theta(1-\eta)(1-\tau_L)}{(2-\eta+\beta)(1-\theta)}]$.

One rewrites equation (B.1.13c) to have

$$K_{t+1} = \frac{\beta}{2 + \beta} A_t (1 - \alpha)(1 - \tau_L)(1 - \phi_t)^{-\alpha} \left(\frac{K_t}{H_t}\right)^\alpha H_t \quad (\text{B.1.16})$$

One observes human capital production as

$$H_{t+1} = \sigma_t \left(\frac{K_t}{H_t}\right)^{\alpha(1-\nu)} H_t \quad (\text{B.1.17})$$

where $\sigma_t \equiv [A_t B_t (1 - \phi_t)^{1-\alpha} g^\omega z^{1-\omega}]^{1-\nu} D_t^\nu \phi_t^{\gamma\nu}$.

This paper now defines k_t as K_t/H_t , the ration of physical capital to human capital, subjecting to the real wage rate and rental rate of physical capital. One makes the following equation:

$$k_{t+1} = \psi_t k_t^{\alpha\nu} \quad (\text{B.1.18})$$

where $\psi_t \equiv [A_t \beta (1 - \alpha)(1 - \tau_L)] / [\sigma_t (2 + \beta)(1 - \phi_t)^\alpha]$.

Definition 1. (Competitive Equilibrium) *For given H_0 and K_0 , a competitive equilibrium is the path $\{Z_t, C_t, X_t, \phi_t, k_t, w_t, r_t, S_t\}_{t \geq 0}$ that satisfies firm's optimisation conditions in (B.1.8) and (B.1.9), agent's optimisation conditions in (B.1.13a)-(B.1.13d) and the production of human capital in (B.1.2)-(B.1.4).*

B.1.6 Balanced growth properties

Proposition 1. *In the canonical OLG with log preferences and Cobb-Douglas technology, there exists a unique balanced growth path, which physical-human capital ratio k^* is given by (B.1.19).*

In balanced growth path, the allocation in competitive equilibrium $\{Z_t, C_t, X_t, \phi_t, k_t, w_t, r_t, S_t\}$ are time-invariant, denoted by $\{Z^*, C^*, X^*, \phi^*, k^*, w^*, r^*, S^*\}$, i.e. the transformation variable remains at same level. This yields $k_{t+1} = k_t = k^*$. Solving equation (B.1.18) for k^* yields

$$k^* = \psi^{\frac{1}{1-\alpha v}} \quad (\text{B.1.19})$$

The left-hand side of the equation characterises exactly the ratio of physical capital to human capital in the intensive form, where K_{t+1} has to be increased as H_{t+1} rises. Thus k^* is fixed in equilibrium.

B.1.7 Growth rates

This subsection focuses on the balanced growth property of this economy. Substituting (B.1.19) into (B.1.18), the growth rates of physical capital is

$$\begin{aligned} 1 + \rho^* &= \frac{K_{t+1}}{K_t} = \frac{H_{t+1}}{H_t} = \\ & \left[\frac{A\beta(1-\alpha)}{(2+\beta)(1-\frac{\eta}{2+\beta})^\alpha} \right]^{\frac{\alpha(1-v)}{1-\alpha v}} \left[D^v \left(\frac{\eta}{2+\beta} \right)^{\gamma v} \left[AB \left(1 - \frac{\eta}{2+\beta} \right)^{1-\alpha} \right]^{1-v} \right]^{\frac{1-\alpha}{1-\alpha v}} (1-\tau_L)^{\frac{\alpha(1-v)}{1-\alpha v}} \\ & \left\{ \left[\alpha\tau_K + (1-\alpha)(\tau_L - \frac{\theta(1-\eta)(1-\tau_L)}{(2-\eta+\beta)(1-\theta)}) \right]^\omega \left[\frac{(1-\alpha)(1-\eta)(1-\tau_L)}{(2-\eta+\beta)(1-\theta)} \right]^{1-\omega} \right\}^{\frac{(1-\alpha)(1-v)}{1-\alpha v}} \end{aligned} \quad (\text{B.1.20})$$

Equation (B.1.20) is the balanced growth rates.

B.2 Efficient allocations

In building on prior literature, there is dynamically inefficient issue in a competitive equilibrium (Acemoglu, 2009). With limited life time, the agent chooses the

equilibrium allocation of material investment in children, the consumption in retirement and the parenting time. This section considers a central planner who allocates the physical and human capital resources to maximise the present discounted value of current and future generations.

B.2.1 Environment

In the economy, social planner looks at exact time period t , and considers the whole generations (De La Croix and Michel, 2002). The social welfare function takes the following form:

$$\sum_{t=0}^{\infty} \Lambda^t U_t(Z_t, \phi_t, C_t, X_{t+1}) \quad (\text{B.2.21})$$

where Λ^t is the planner's discount factor, i.e. social weight that a planner attaches to the future generations. Following Docquier et al. (2007) and Bishnu (2013), the sum of lifetime utilities over generations is discounted by a factor $\Lambda \in (0, 1)$. When utilities are bounded, Λ is smaller than 1 ensures that objective function is finite (i.e. $\sum_{t=0}^{\infty} \Lambda^t < \infty$). These conditions are necessary and sufficient for optimality of the constant path starting at k_0 , as this path satisfies the transversality conditions.

The social planner's maximisation problems subject to the resource constraints of physical and human capital:

$$K_{t+1} = F_t - Z_t - C_t - X_t - G_t \quad (\text{B.2.22a})$$

$$H_{t+1} = (B_t G_t^\omega Z_t^{1-\omega})^{1-\nu} (D_t \phi_t^\gamma H_t)^\nu \quad (\text{B.2.22b})$$

B.2.2 Planner's optimisation problem

The social planner's maximum problem in Lagrange form yields

$$\begin{aligned} \mathcal{L}_t = & \sum_{t=0}^{\infty} \tilde{\beta}^t \{ [(1-\eta) \ln Z_t + \eta \ln \phi_t + \ln C_t + \beta X_{t+1}] \\ & + \Lambda q_{t+1} [A_t K_t^\alpha (H_t L_t)^{1-\alpha} - Z_t - C_t - X_t - G_t - K_{t+1}] \\ & + \Lambda \mu_{t+1} [(B_t G_t^\omega Z_t^{1-\omega})^{1-\nu} (D_t \phi_t^\gamma H_t)^\nu - H_{t+1}] \} \end{aligned}$$

Optimality leads to the maximum of \mathcal{L}_t with respect to $Z_t, C_t, X_t, \phi_t, G_t, K_t$ and H_t . \mathcal{L}_t equals the sum of the current utilities and the increase in the shadow value of the capital stock: $\Lambda q_{t+1}K_{t+1} - q_tK_t$ and $\Lambda\mu_{t+1}H_{t+1} - \mu_tH_t$. We assume $\Lambda = \beta$, so the social planner's maximum problem is as follows:

$$\begin{aligned}\mathcal{L}_t &= (1 - \eta)\ln Z_t + \ln C_t + \ln X_t + \eta\ln \phi_t \\ &\quad + \Lambda q_{t+1}[A_t K_t^\alpha (H_t L_t)^{1-\alpha} - Z_t - C_t - X_t - G_t] - q_t K_t \\ &\quad + \Lambda \mu_{t+1}[(B_t G_t^\omega Z_t^{1-\omega})^{1-v} (D_t \phi_t^\gamma H_t)^v] - \mu_t H_t\end{aligned}\tag{B.2.23}$$

First-order conditions for an interior solution (assuming it exists) with respect to $\{Z_t, C_t, X_t, \phi_t, G_t, K_t, H_t\}$, respectively:

$$\frac{\partial \mathcal{L}_t}{\partial Z_t} = \frac{1 - \eta}{Z_t} - \Lambda q_{t+1} + \Lambda \mu_{t+1} \frac{H_{t+1}(1 - v)(1 - \omega)}{Z_t} = 0\tag{B.2.24a}$$

$$\frac{\partial \mathcal{L}_t}{\partial C_t} = \frac{1}{C_t} - \Lambda q_{t+1} = 0\tag{B.2.24b}$$

$$\frac{\partial \mathcal{L}_t}{\partial X_t} = \frac{1}{X_t} - \Lambda q_{t+1} = 0\tag{B.2.24c}$$

$$\frac{\partial \mathcal{L}_t}{\partial \phi_t} = \frac{\eta}{\phi_t} - \Lambda q_{t+1}(1 - \alpha) \frac{F_t}{1 - \phi_t} + \Lambda \mu_{t+1} v \gamma \frac{H_{t+1}}{\phi_t} = 0\tag{B.2.24d}$$

$$\frac{\partial \mathcal{L}_t}{\partial G_t} = -\Lambda q_{t+1} + \Lambda \mu_{t+1}(1 - v) \omega \frac{H_{t+1}}{G_t} = 0\tag{B.2.24e}$$

$$\frac{\partial \mathcal{L}_t}{\partial K_t} = \Lambda q_{t+1} F'_t - q_t = 0\tag{B.2.24f}$$

$$\frac{\partial \mathcal{L}_t}{\partial H_t} = \Lambda q_{t+1}(1 - \alpha) \frac{F_t}{H_t} + \Lambda \mu_{t+1} v \frac{H_{t+1}}{H_t} - \mu_t = 0\tag{B.2.24g}$$

These conditions verify the constant quantities, since $\Lambda \in (0, 1)$. Equation (B.2.24a) is the optimal allocation of material investment in children. Equation (B.2.24c) states consumption in adulthood. Equation (B.2.24b) indicates that the consumption in retirement equals the next period of shadow price times discount factor. Equation (B.2.24d) governs that the marginal utility of material investment in children corrected to parenting time equals the marginal utility of consumption in retirement. It also reveals that marginal productivity of parenting time on the production of human capital corrected to the marginal productivity of parenting time on goods production function. This result shows that the substitution relationship between

the production of human capital and goods production. Equation (B.2.24e) determines the optimal spending on education. Equations (B.2.24f) and (B.2.24g) are the resource constraint of physical capital and human capital of economy, respectively. Contrary to the standard Diamond (1965) model, this planner's first-order conditions do not respect the first-order condition the agent chooses for itself in the competitive equilibrium.

Definition 2. (Sufficient condition for the planner's optimum) *A positive sequence $\{Z_t, C_t, X_t, \phi_t, K_{t+1}, H_{t+1}\}_{t \geq 0}$ satisfying (B.2.24a)-(B.2.24g) and the transversality condition $\lim_{t \rightarrow \infty} \Lambda^t q_t K_t = 0$ and $\lim_{t \rightarrow \infty} \Lambda^t \mu_t H_t = 0$ is an optimal solution to the planner's problem.*

Using (B.2.24f), one obtains

$$q_{t+1} = \frac{q_t}{\Lambda F'_t} = \frac{q_t K_t}{\alpha \Lambda F_t} \quad (\text{B.2.25})$$

where $F'_t = \alpha F_t / K_t$.

Substituting (B.2.25) into (B.2.24g) to obtain, one can yield

$$\mu_{t+1} = \left[\frac{q_t K_t (1 - \alpha)}{\alpha (1 - \phi_t)} - \frac{\eta}{\phi_t} \right] \frac{\phi_t}{\Lambda v \gamma H_{t+1}} \quad (\text{B.2.26})$$

One can plug (B.2.25) and (B.2.26) back into Equation (B.2.24a) to formulate

$$Z_t = \alpha F_t \left[\frac{(1 - \eta)}{q_t K_t} - \frac{\eta (1 - v)(1 - \omega)}{q_t K_t v \gamma} + \frac{\phi_t}{1 - \phi_t} \frac{(1 - \alpha)(1 - v)(1 - \omega)}{\alpha v \gamma} \right] \quad (\text{B.2.27})$$

Substituting (B.2.25) back into Equation (B.2.24b) and (B.2.24c), one can observe the optimal consumption in adulthood and retirement:

$$C_t = X_t = \frac{\beta}{\Lambda} \frac{\alpha F_t}{q_t K_t} \quad (\text{B.2.28})$$

We use first-order conditions to calculate the optimal government spending on education as follows:

$$G_t = F_t \frac{(1 - v)\omega}{v \gamma} \left\{ \frac{\phi_t}{1 - \phi_t} (1 - \alpha) - \frac{\alpha \eta}{q_t K_t} \right\} \quad (\text{B.2.29})$$

The government spending on education is required as strictly positive by this section. One can substitute (B.2.27) - (B.2.29) into K_{t+1} to obtain

$$K_{t+1} = F_t - \alpha F_t \left[\frac{(1-\eta)}{q_t K_t} - \frac{\eta(1-v)(1-\omega)}{q_t K_t v \gamma} + \frac{\phi_t}{1-\phi_t} \frac{(1-\alpha)(1-v)(1-\omega)}{\alpha v \gamma} - \frac{2}{q_t K_t} - \frac{\eta \omega}{q_t K_t v \gamma} \right] \quad (\text{B.2.30})$$

Multiplying (B.2.25) and (B.2.30) term by term, one makes

$$q_{t+1} K_{t+1} = q_t K_t \left[\frac{(1-\phi_t)v\gamma - \phi_t(1-\alpha)(1-v)}{\alpha \Lambda v \gamma (1-\phi_t)} \right] + \frac{\eta(1-v) - v\gamma(3-\eta)}{\Lambda v \gamma} \quad (\text{B.2.31})$$

Since $F'_t = \alpha F_t / K_t$, $q_t K_t$ is solution to linear dynamic Eq. (B.2.31), and the general solution of this equation is

$$q_t K_t = \frac{\alpha(1-\phi_t)[\eta(1-v) - v\gamma(3-\eta)]}{(\alpha \Lambda - 1)(1-\phi_t)v\gamma + \Lambda \phi_t(1-\alpha)(1-v)} + \varepsilon \left[\frac{(1-\phi_t)v\gamma - \phi_t(1-\alpha)(1-v)}{\alpha \Lambda v \gamma (1-\phi_t)} \right]^t \quad (\text{B.2.32})$$

with ε is a real constant.

According to De La Croix and Michel (2002), there is a unique solution which satisfies the transversality condition $\lim_{t \rightarrow \infty} \Lambda^t q_t K_t = 0$ and $\lim_{t \rightarrow \infty} \Lambda^t \mu_t H_t = 0$: the constant solution. The transversality condition states that the limit of the actual shadow value of the capital stock equals zero. Therefore, this subsection forms the following equation:

$$q_t K_t = \frac{\alpha(1-\phi_t)[\eta(1-v) - v\gamma(3-\eta)]}{(\alpha \Lambda - 1)(1-\phi_t)v\gamma + \Lambda \phi_t(1-\alpha)(1-v)} \quad (\text{B.2.33})$$

Following De La Croix and Michel (2002), since $q_t K_t = \text{constant}$ satisfies the transversality condition, the solution for Equation (B.2.24g) should be $\mu_t H_t = \text{constant}$, hence this subsection yields

$$\mu_t H_t = \mu_{t+1} H_{t+1} = q_t K_t \frac{(1-\alpha)}{\alpha(1-\Lambda v)} \quad (\text{B.2.34})$$

We rewrite (B.2.24d) to compute the allocation of parenting time in equilibrium:

$$\frac{\eta}{\phi_t} = \frac{1 - \alpha}{\alpha} \frac{q_t K_t}{1 - \phi_t} \left[1 - \frac{\Lambda v \gamma (1 - \phi_t)}{(1 - \Lambda v)} \frac{1}{\phi_t} \right] \quad (\text{B.2.35})$$

Finally, using $q_{t+1} F_t = q_t K_t / \alpha \Lambda$ and (B.2.24a)-(B.2.24g), one can obtain that the parenting time is time-invariant variable as follows:

$$\phi = \frac{\eta(\alpha\Lambda - 1)(1 - \Lambda v) + \Lambda(1 - \alpha)[\eta(1 - v) - v\gamma(3 - \eta)]}{(1 - \alpha)[\Lambda\gamma(1 - v) - 1] + \eta[(\alpha\Lambda - 1)(1 - \Lambda\gamma) + \Lambda(1 - \alpha)(1 - v)]} \quad (\text{B.2.36})$$

Eq. (B.2.36) is the social planner's allocation of parenting time. We require parenting time is strictly positive.

One can see the differences from (B.1.13d). The parenting time (i.e. ϕ_t) is additionally determined by the relative importance between cognitive and non-cognitive skills in human capital production (i.e. v) and the importance of parenting time in the formulation of non-cognitive skills (i.e. γ), rather than just private discount factor (i.e. β) and the altruism level (i.e. η).

Using (B.2.24a)-(B.2.24g) and (B.2.36), one can make the material investment in the following manner:

$$Z_t = \mathcal{Z} F_t \quad (\text{B.2.37})$$

where $\mathcal{Z} \equiv \left[\gamma v (\alpha \Lambda - 1) + (1 - \alpha)(1 - v) \frac{\phi_t}{1 - \phi_t} \right] \frac{1 - \eta - \frac{\eta(1 - v)(1 - \omega)}{v\gamma}}{\eta(1 - v) - v\gamma(3 - \eta)} + \frac{\phi_t}{1 - \phi_t} \frac{(1 - \alpha)(1 - v)(1 - \omega)}{v\gamma}$ and represents the ratio of material investment in children to aggregate output. We require the material investment in children is strictly positive.

One can use the first order conditions and (B.2.36) to observe the consumption in adulthood as follows:

$$C_t = X_t = \mathcal{X} F_t \quad (\text{B.2.38})$$

where $\mathcal{X} \equiv \frac{\gamma v (\alpha \Lambda - 1) + (1 - \alpha)(1 - v) \frac{\phi_t}{1 - \phi_t}}{\eta(1 - v) - v\gamma(3 - \eta)}$ and is the ratio of consumption in adulthood and retirement to aggregate output, respectively. This paper restricts that these two ratios are positive.

Using the first order conditions and (B.2.36), one formulates the government spending in education:

$$G_t = \mathcal{G}F_t \quad (\text{B.2.39})$$

where $\mathcal{G} \equiv \frac{(1-v)\omega}{\eta(1-v)-v\gamma(2-\eta)} \left[\eta(1-\alpha\Lambda) - \frac{\phi_t}{1-\phi_t}(1-\alpha)(2-\eta) \right]$ and is the ratio of government spending in education to aggregate output.

One can rewrite physical and human capital resources by observing (B.2.24a)-(B.2.24g) and (B.2.36)-(B.2.39). In addition, one can obtain the physical-human capital ratio takes the following form:

$$k_{t+1} = \frac{K_{t+1}}{H_{t+1}} = \Delta \left(\frac{K_t}{H_t} \right)^{\alpha v} \quad (\text{B.2.40})$$

where Δ is the collection of parameters.

B.2.3 Balanced growth properties

Let us define $k_t \equiv K_t/H_t$. In balanced growth path, the transformation variable remains at same level. This yields $k_{t+1} = k_t = k^*$. Rewriting (B.2.40), one yields

$$k^* = \Delta^{\frac{1}{1-\alpha v}} \quad (\text{B.2.41})$$

Thus (B.2.41) characterises balanced-growth-path effective physical-human capital ratio in efficient allocations. Moreover, one can calculate the growth rates in efficient allocations by using (B.2.41).

Appendix C

The derivations in the competitive equilibrium

C.1 Derivation of the growth rate

Together with Assumption 4, (2.18) and (2.19), one can rewrite (2.21) to have

$$K_{t+1} = \frac{\beta}{1+\beta} A(1-\alpha)(1-\tau_L)(1-\phi_t)^{-\alpha} \left(\frac{K_t}{H_t}\right)^\alpha H_t \quad (\text{C.1.1})$$

Using (2.20), (2.22) and Assumption 4, one can observe (2.14) as

$$H_{t+1} = \sigma \left(\frac{K_t}{H_t}\right)^{\alpha(1-\nu)} H_t \quad (\text{C.1.2})$$

where $\sigma \equiv [AB(1-\phi)^{1-\alpha}z]^{1-\nu} D^\nu \phi^{\gamma\nu}$.

This Appendix now defines the variables in intensive form. Let $k_t \equiv K_t/H_t$, the ratio of physical capital to human capital, subjecting to real wage rate and rental rate of physical capital. One obtains

$$k_{t+1} = \psi k_t^{\alpha\nu} \quad (\text{C.1.3})$$

where $\psi \equiv [A\beta(1-\alpha)]/[\sigma(1+\beta)(1-\phi)^\alpha]$.

This Appendix then derives $1 + \rho^* = K_{t+1}/K_t$ by using (C.1.2)

$$1 + \rho^* = \frac{K_{t+1}}{K_t} = \frac{\beta}{1 + \beta} A(1 - \alpha)(1 - \phi)^{-\alpha} k_t^{\alpha-1} \quad (\text{C.1.4})$$

In balanced growth path, we form the following equation:

$$1 + \rho^* = \frac{\beta}{1 + \beta} A(1 - \alpha) \left(1 - \frac{\eta}{1 + \beta}\right)^{-\alpha} \psi^{-\frac{1-\alpha}{1-\alpha v}} \quad (\text{C.1.5})$$

Substituting (2.20), (2.21), and (2.22) into $1 + \rho^*$, one can have (2.25).

This Appendix now derives $1 + \rho^* = H_{t+1}/H_t$. From (C.1.1), one can observe

$$1 + \rho^* = \frac{H_{t+1}}{H_t} = [ABz(1 - \phi)^{1-\alpha}]^{1-v} (D\phi^\gamma)^v \left(\frac{K_t}{H_t}\right)^{\alpha(1-v)} \quad (\text{C.1.6})$$

One can plug (2.24) into (C.1.6) to yield (2.25).

Therefore, we have

$$1 + \rho^* = \frac{K_{t+1}}{K_t} = \frac{H_{t+1}}{H_t} \quad (\text{C.1.7})$$

C.2 The transformation variables in steady state

We can rewrite (2.19) and apply (2.24) to make

$$\omega^* = A\alpha \left(\frac{1 + \beta - \eta}{1 + \beta}\right) \psi^{\frac{\alpha}{1-\alpha v}} \quad (\text{C.2.8})$$

One can also rewrite (2.18) and apply (2.24) to form

$$r^* = A\alpha \left(\frac{1 + \beta - \eta}{1 + \beta}\right) \psi^{\frac{\alpha-1}{1-\alpha v}} \quad (\text{C.2.9})$$

Now using (C.2.8) and (C.2.9), we can yield

$$Z^* = \frac{Z_t}{H_t} = \frac{1 - \eta}{1 + \beta} A\alpha \left(\frac{1 + \beta - \eta}{1 + \beta}\right) \psi^{\frac{\alpha}{1-\alpha v}} \quad (\text{C.2.10a})$$

$$X^* = \frac{X_{t+1}}{H_t} = \frac{\beta}{1 + \beta} \left[1 + A\alpha \left(\frac{1 + \beta - \eta}{1 + \beta}\right) \psi^{\frac{\alpha-1}{1-\alpha v}}\right] A\alpha \left(\frac{1 + \beta - \eta}{1 + \beta}\right) \psi^{\frac{\alpha}{1-\alpha v}} \quad (\text{C.2.10b})$$

According to Assumption 4, we can have S^* in steady state.

C.3 All equations in definition 1

The budget and time constraints

The budget constraints for the agent in adulthood and in retirement are

$$Z_t + S_t = L_t w_t H_t \quad (2.6a) \text{ (a)}$$

$$X_{t+1} = (1 + r_{t+1})S_t \quad (2.6b) \text{ (a)}$$

$$L_t + \phi_t = 1 \quad (2.6c) \text{ (a)}$$

Firm's optimisation problem in the general functional forms

Firm's optimisation condition is

$$r_t = \frac{\partial Y_t}{\partial K_t} \equiv F_t^{(1)} \text{ and } w_t = \frac{\partial Y_t}{\partial H_t} \equiv F_t^{(2)} \quad (2.7) \text{ (a)}$$

Consumer's optimisation problem in the general functional forms

First-order conditions for an interior solution are

$$u_t^{(1)} = \lambda_t \quad (2.8a) \text{ (a)}$$

$$u_t^{(2)} = \lambda_t w_t H_t \quad (2.8b) \text{ (a)}$$

$$\nu' = \frac{\lambda_t}{1 + r_{t+1}} \quad (2.8c) \text{ (a)}$$

Firm's optimisation problem in specific functional forms

Rental rate of physical capital is given by

$$r_t = A\alpha K_t^{\alpha-1} (L_t H_t)^{1-\alpha} \quad (2.18) \text{ (a)}$$

Real wage rate of human capital is given by

$$w_t = A(1 - \alpha) K_t^\alpha (L_t H_t)^{-\alpha} \quad (2.19) \text{ (a)}$$

Consumer's optimisation problem in specific functional forms

First-order conditions for an interior solution are

$$Z_t = \frac{1 - \eta}{1 + \beta} w_t H_t \quad (2.20 \text{ (a)})$$

$$X_{t+1} = \frac{\beta}{1 + \beta} (1 + r_{t+1}) w_t H_t \quad (2.21 \text{ (a)})$$

$$\phi_t = \frac{\eta}{1 + \beta} \quad (2.22 \text{ (a)})$$

Assumption 4

$$K_{t+1} = I_t = S_t.$$

Appendix D

The solutions of the social planner's problem

The parenting time is a time-invariant variable:

$$\phi = \frac{\eta\Lambda(\alpha\Lambda - 1)(1 - \Lambda v) + \Lambda(1 - \alpha)[\Lambda\eta(1 - v) - \Lambda v\gamma(1 - \eta) - \beta v\gamma]}{(1 - \alpha)[- \Lambda(1 - \eta) - \beta][(1 - \Lambda v) + \Lambda\gamma v] + \eta\Lambda(\alpha\Lambda - 1)(1 - \Lambda v) + \eta\Lambda^2(1 - \alpha)(1 - v)} \quad (3.34 \text{ (a)})$$

The material investment is as follows:

$$Z = \mathcal{Z}F(K_t, (H_t, L_t)) \quad (3.35 \text{ (a)})$$

where $\mathcal{Z} \equiv \left[\Lambda\gamma v(\alpha\Lambda - 1) + \Lambda(1 - \alpha)(1 - v) \frac{\phi}{1 - \phi} \right] \frac{1 - \eta - \frac{\eta(1 - v)}{v\gamma}}{\Lambda\eta(1 - v) - \Lambda v\gamma(1 - \eta) - \beta v\gamma} + \frac{\phi}{1 - \phi} \frac{(1 - \alpha)(1 - v)}{v\gamma}$.

The consumption in old-age generation takes the following form:

$$X = \mathcal{X}F(K_t, (H_t, L_t)) \quad (3.36 \text{ (a)})$$

where $\mathcal{X} \equiv \frac{\Lambda\gamma v(\alpha\Lambda - 1) + \Lambda(1 - \alpha)(1 - v) \frac{\phi}{1 - \phi}}{\Lambda\eta(1 - v) - \Lambda v\gamma(1 - \eta) - \beta v\gamma}$.

The resource constraints of physical capital is

$$K_{t+1} = F(K_t, (H_t, L_t))\alpha\Lambda \quad (3.37 \text{ (a)})$$

We obtain the resource constraints of human capital as the following:

$$H_{t+1} = F(K_t, (H_t, L_t))^{1-\nu} \vartheta H_t^\nu \quad (3.38 \text{ (a)})$$

where ϑ is constant parameter collection, $\vartheta \equiv \langle BZ \rangle^{1-\nu} \langle D\phi^\gamma \rangle^\nu$.

Finally, one can observe the physical-human capital ratio as

$$k_{t+1} = \frac{K_{t+1}}{H_{t+1}} = \Delta \left(\frac{K_t}{H_t} \right)^{\alpha\nu} \quad (3.39 \text{ (a)})$$

where $\Delta \equiv \alpha \Lambda A^\nu (1 - \phi)^{(1-\alpha)\nu} / \vartheta$.

Appendix E

The derivations in optimal taxation

The purpose of this Appendix is to show the derivations of the optimal tax rates as described in (4.28), (4.29) and (4.30) in Chapter 5. Comparing (4.12a) and (4.20a), we have the relationship between material investment and time investment in the competitive equilibrium:

$$\frac{\eta}{(1-\eta)} \frac{(1-\theta)}{(1-\tau_L)} \frac{Z_t}{\phi_t} = (1-\alpha) \frac{Y_t}{1-\phi_t} \quad (\text{E.0.1})$$

Comparing (4.20c) and (4.20e), the relationship between material investment and time investment in the efficient allocation is

$$\frac{\eta}{\phi_t} - \Lambda q_{t+1} \frac{\eta}{(1-\eta)} \frac{(1-\theta)}{(1-\tau_L)} \frac{Z_t}{\phi_t} + \Lambda \mu_{t+1} H_{t+1} \frac{v\gamma}{\phi_t} = 0 \quad (\text{E.0.2})$$

Rearranging (E.0.2), we formulate

$$\frac{\eta + \Lambda \mu_{t+1} H_{t+1} v\gamma}{\frac{\eta}{(1-\eta)} \frac{(1-\theta)}{(1-\tau_L)}} = \Lambda q_{t+1} Z_t \quad (\text{E.0.3})$$

Now we plug (E.0.3) back to (4.20c) to yield

$$(1-\eta) - \frac{(1-\eta)(1-\tau_L)}{1-\theta} + \Lambda \mu_{t+1} H_{t+1} \left[-\frac{(1-\eta)(1-\tau_L)v\gamma}{\eta(1-\theta)} + (1-v)(1-\omega) \right] = 0 \quad (\text{E.0.4})$$

We can substitute (4.20d) into (4.20c) to have the relationship between material investment and retirement consumption in the efficient allocation:

$$(1 - \eta) - \frac{Z_t}{X_t} + \Lambda\mu_{t+1}H_{t+1}(1 - \nu)(1 - \omega) \quad (\text{E.0.5})$$

Comparing (4.12a) and (4.12b), we have the relationship between material investment and retirement consumption in the competitive-equilibrium allocation:

$$\frac{Z_t}{X_t} = \frac{1 - \eta}{\beta(1 - \theta)[1 + (1 - \tau_K)r_t]} \frac{\lambda_{t-1}}{\lambda_t} \quad (\text{E.0.6})$$

One can obtain the shadow price of the physical capital is as follows:

$$\lambda_t = \frac{1 + \beta}{(1 - \tau_L)H_t w_t} \quad (\text{E.0.7})$$

Substituting (E.0.7) into (E.0.6) and rearranging it, we make

$$\frac{Z_t}{X_t} = \frac{1 - \eta}{\left[\frac{q_{t+1}}{q_t} + \frac{1 - \tau_K}{\Lambda}\right]\beta(1 - \theta)} \quad (\text{E.0.8})$$

Using (4.21), we can rewrite (E.0.8) as

$$\frac{Z_t}{X_t} = \frac{1 - \eta}{\left[\frac{1}{r_t} + (1 - \tau_K)\right]\frac{\beta}{\Lambda}(1 - \theta)} \quad (\text{E.0.9})$$

so that we have the relationship between material investment in children and the consumption of old generation.

Now we substitute (E.0.9) into (E.0.5) to have

$$(1 - \eta) - \frac{(1 - \eta)}{\left[\frac{1}{r_t} + (1 - \tau_K)\right]\frac{\beta}{\Lambda}(1 - \theta)} + \Lambda\mu_{t+1}H_{t+1}(1 - \nu)(1 - \omega) = 0 \quad (\text{E.0.10})$$

Now this Appendix derives the equations to make (3.37) and (3.38). Dividing (4.12b) by (4.12c), we yield

$$\frac{X_t}{\phi_t} = \frac{\beta}{\eta} \left[1 + (1 - \tau_K)\alpha\frac{Y_t}{K_t}\right] \left[(1 - \tau_L)(1 - \alpha)\frac{Y_t}{1 - \phi_t}\right] \frac{Y_{t-1}}{Y_t} \quad (\text{E.0.11})$$

Equation (E.0.11) gives the relationship between parenting time and the consumption of old generation.

Dividing (4.20d) by (4.20e), then using (E.0.11), we form

$$\eta - \frac{\eta}{\left[\frac{1}{r_t} + (1 - \tau_K)\right] \frac{\beta}{\Lambda} (1 - \tau_L)} + \Lambda \mu_{t+1} H_{t+1} v \gamma = 0 \quad (\text{E.0.12})$$

We rearrange the (E.0.4) to have $(1 - \tau_L)$ on the left-hand side:

$$1 - \tau_L = \frac{(1 - \theta)[(1 - \eta) + \Lambda \mu_{t+1} H_{t+1} (1 - v)(1 - \omega)]}{(1 - \eta)[1 + \Lambda \mu_{t+1} H_{t+1} \frac{v\gamma}{\eta}]} \quad (\text{E.0.13})$$

We then plug it into (E.0.12) to yield

$$\eta - \frac{\eta(1 - \alpha)(1 - \eta) \left[1 + \Lambda \mu_{t+1} H_{t+1} \frac{v\gamma}{\eta}\right]}{\left[\frac{1}{r_t} + (1 - \tau_K)\right] \frac{\beta}{\Lambda} (1 - \theta) [(1 - \eta) + \Lambda \mu_{t+1} H_{t+1} (1 - v)(1 - \omega)]} + \Lambda \mu_{t+1} H_{t+1} v \gamma = 0 \quad (\text{E.0.14})$$

Now this Appendix compares the rearrangement of (E.0.11) and (E.0.14) to formulate (4.28). Rearranging (E.0.11) obtains the following equation:

$$1 - \theta = \frac{1 - \eta}{\beta \left[\frac{1}{\Lambda r_t} + (1 - \tau_K) \frac{1}{\Lambda}\right] [(1 - \eta) + \Lambda \mu_{t+1} H_{t+1} (1 - v)(1 - \omega)]} \quad (\text{E.0.15})$$

The rearrangement of (E.0.14) is as follows:

$$1 - \theta = \frac{\eta(1 - \alpha)}{\frac{\beta \left[1 + (1 - \tau_K) r_t\right] \left[(1 - \eta) + \Lambda \mu_{t+1} H_{t+1} (1 - v)(1 - \omega)\right] (\eta + \Lambda \mu_{t+1} H_{t+1} v \gamma)}{(1 - \eta) \left[1 + \Lambda \mu_{t+1} H_{t+1} \frac{v\gamma}{\eta}\right]} \frac{1}{\Lambda r_t}} \quad (\text{E.0.16})$$

we can compare (E.0.15) and (E.0.16) to have (4.28).

One then substitutes (4.28) into (E.0.15) to yield (4.29) and plugs (4.29) into (E.0.14) to find (4.30).

Appendix F

Calibration

In line with empirical evidences, the parameter α is the capital share in goods production and is set to $1/3$ to match the empirical counterpart. This thesis chooses a value of private discount factor that is standard in the literature, $\beta = 1/(1.02^{25}) = 0.6095$, following [De La Croix and Doepke \(2003\)](#).

For parameter γ , which is the share of parenting time in human capital formulation, this section considers the following case.

Case 1. γ is 1, greater or smaller than 1.

The first (second and third) part implies that every unit of parenting time can bring equal (higher and lower) unit of non-cognitive skills times the exogenous productivity to human capital production.

With regard to parameter v , which is the relative importance of cognitive and non-cognitive skills in the production of human capital. This section studies the following case.

Case 2. v is 0.5, greater or smaller than 0.5.

The first (second and third) part implies that non-cognitive skills is equally (more and less) important to cognitive skills in human capital production.

[Heckman et al. \(2006\)](#) shows that the same low-dimensional vector of abilities can explain schooling choices, wages, employment, work experience, choice of occupation and a wide variety of risky behaviors. It demonstrates that cognitive skills and non-cognitive skills are equally important in explaining a variety of aspects of social and economic life. The non-cognitive skill is more malleable than cognitive skill ([Cunha](#)

et al., 2005). It points out that IQ is rank stable after age 10, whereas personality skills are malleable through adolescence and into early adulthood.

Moreover, Cunha and Heckman (2008) use a sample from the Children of the National Longitudinal survey of Youth (CNLSY/79) in the US to suggest that the share of time investment in children in the human capital production is 0.8. This implies that non-cognitive skills are more malleable than cognitive skills, hence the parameter is chosen as 0.75.

For the relative preference between material and time investment, η , this paper analyses the following case.

Case 3. η is 0.5, greater or smaller than 0.5.

The first (second and third) part implies that parents attach equal (higher and lower) utility toward time investment.

The empirical evidence in Haveman and Wolfe (1994) and Knowles (1999) suggest that the parenting time is equivalent to about 15 percent of the parents time endowment, which implies that η is 0.24, as discussed after Equation (2.22):

$$\phi_t = \frac{\eta}{1 + \beta} \quad (2.22 \text{ (a)})$$

so the value of η is chosen as 0.25.

The benchmark parametrisation is selected so as to obtain the balanced growth rate equals to 2 percent (Basu et al., 2012). Attributing all of the growth in multi-factor productivity to growth in labour-augmenting productivity implies that A is 3.8023, see (2.25):

$$1 + \rho^* = \left[\frac{A\beta(1 - \alpha)}{(1 + \beta)(1 - \frac{\eta}{1 + \beta})^\alpha} \right]^{\frac{\alpha(1-v)}{1-\alpha v}} \left[D^v \left(\frac{\eta}{1 + \beta} \right)^{\gamma v} \left[AB \left(1 - \frac{\eta}{1 + \beta} \right)^{1-\alpha} \right]^{1-v} \right]^{\frac{1-\alpha}{1-\alpha v}} \quad (2.25 \text{ (a)})$$

Using a backward solving method (i.e. swapping endogenous variables and parameters) and assuming $B = D$, we obtain $B = D = 1.7249$ in (2.14):

$$H_{t+1} = H_{c,t}^{1-v} H_{nc,t}^v \quad (2.14 \text{ (a)})$$

In addition to choose parameters, this paper also needs to set the initial conditions for the simulations. The overall size of the population is a scale parameter which does not affect the results. In our model, population remains the same size, so the population growth rates is zero. This paper therefore only specifies the aggregate values. The initial distribution of human capital follows a log-normal distribution. This paper provides simulations for different variances of the distribution in order to examine the effects of inefficiency. The initial levels of physical and human capital are chosen, so that the ratio of physical to human capital equals its value in the balanced growth path.

Appendix G

Discussion

G.1 Parenting time appears in both cognitive skills and non-cognitive skills

The production of human capital with cognitive skills is

$$H_{c,t} = BG_t^\omega Z_t^{1-\omega} \phi_t^\gamma \quad (\text{G.1.1})$$

one can see the parenting time appears in cognitive skills accumulation, following [Del Boca et al. \(2013\)](#).

The production of human capital with non-cognitive skills is

$$H_{nc,t} = D\phi_t^\gamma H_t \quad (\text{G.1.2})$$

The production of human capital is described as the following equation:

$$H_{t+1} = (B_t G_t^\omega Z_t^{1-\omega})^{1-\nu} D_t^\nu H_t^\nu \phi_t^\gamma \quad (\text{G.1.3})$$

Here, the issue is one can not distinguish parenting time comes from cognitive skills or non-cognitive skills. Therefore, the setting of the production of human capital in the main text not only can set up two channels but also can tell the parenting time comes from which channels.

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