The Determinants of Long Run Growth and the Role of Competition in Schumpeterian Growth Model

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Chapter 1

INTRODUCTION

Economic growth is arguably the issue of primary concern to economic policy makers. Intertemporal models of capital accumulation known as the neoclassical models are not well suited to address issues regarding long run growth because they yield stationary equilibria. Hence, this thesis is an aspiration to study the determinants of long run growth; in order to explain long run growth rate as an endogenous equilibrium outcome of the behaviour of rational optimizing agents, an outcome that reflects the structural characteristics of the economy, such as technology and preference as well as macroeconomic policy.

In Chapter two, I approach the question on whether long run growth depends on both innovation and physical capital accumulation or whether it depends on either of the two? This inquiry is motivated by the prediction of neoclassical model which argued that capital accumulation has no long run effects on growth, and the conventional macroeconomic policy has no influence on long run growth performance. It is also implied by the early innovation literature such as Aghion and Howit (1992) which shows that long run growth is determined by technological progress independent of physical capital accumulation. Against this predictions, DeLong and Summers (1991) found that countries with the highest growth rates are those that invest highly in machinery and in which the relative price of equiment has fallen more quickly. Mankiw (1995) argued that growth can be accounted for by physical capital accumulation independent of technological progress (see:). But DeLong and Summers (1992) findings stroke a more reconciliatory tone by showing that large difference in growth rates cannot be driven by shifts in equipment investment rate uncorrelated with TFP growth. Chapter two follows Aghion & Howitt (1998) who argued that new technology is capital using and showed a complementary model where both innovation and physical capital accumulation matter for the long run growth of an economy. By using a monopoly market structure, they showed that capital accumulation would raise the equilibrium flow of profits as a result of increasing national income which in turn raises the demand of the monopolist's goods. High profits due to high capital intensity would then enhance the incentive for more innovation; just as more innovation would raise the productivity of capital.

My model extends Aghion & Howitt (1998) by introducing learning into the basic Schumpeterian model to show essentially that economies that focus on technological progress alone without learning adaptation of these technology are more likely to grow less than economies with learning adaptation. Learning extrnalities is defined as the total amount of capital that has been accumulated by all firms which determine the local condition of capital use in the economy. So that , learning adaptation is how each monopolist who produces new technology tailors her production to be more efficient in response to local condition. Consequently, learning adaptation will induce higher R&D by raising the national income and hence the demand for the monopolist good. A higher monopolist profit due to efficient use of capital in response to local condition drives the incentive for more intense R&D. Unlike learning by doing

model of Romer(1986) and Frankel(1962) under perfect competition where learning contributes directly to technological progress in order to obtain long run growth; in my model with monopolist market structure, long run growth depends directly on innovation through R&D but learning has an indirect effect on long run growth by raising the incentive for research.

In Chapter three, I approach the question on whether both human capital and innovation matter for long run ? Also I approach the question om whether horizontal innovation has a neutralizing effect on long run growth? Unlike physical capital, early endogenous growth theory unequivocally argued that human capital has a positive long run effect on growth(see: Lucas (1988). Early R&D growth literatures also argued that innovation has a long run effect on growth(see: Aghion & Howitt (1992), , Grossman and Helpman (1991)). But the prediction of scale effect[1] in the early R&D growth literatrues has questioned the role of innovation in the long run. For instance, the empirical study of Jones (1995a) showed the absence of scale effect in the post war II era. Jones showed that the number of scientist and engineers engaged in R&D in the United State grew from under 200,000 in 1950 to almost 1 million by 1987 yet growth rate in the United state remained constant during this period. To eliminate scale effect, Jones (1995b) developed a semi - endogenous growth model which assumed decreasing returns in technological progress to account for the declining growth in the post war era II, and used labour as an input in research, so that increasing population growth is require to offset the decreasing return in technological progress and thus put the economy on a constant returns. The implication of his model overturned the predictions of early R&D growth literatures, and implied that policies that affect research intensity has no long run growth since long run growth depends on exogenous population growth. Some endogenous literatures such as Arnold (1998) and Blackburn Keith, Victor T.Y. Hung, and Alberto F. Pozzolo (2000) have exploited the semi - endogenous growth model of Jones (1995b) by using human capital implace of labour as an input in research to argue that long run growth rate would depend solely on human capital accumulation. Despite the advantages of these scale invariant models that deny the role of innovation in the long run; the empirical works of Brander and Dowrick (1994), Kelley and Schmidt (1995), and Ahituv (2001) showed that population growth has a negative effect on economic

[1] Scale effect means that variations in the size or scale of the economy, as measured by population, say, affect the size of the long run growth rate.

growth. Arnold (1998) and Blackburn Keith, Victor T.Y. Hung, and Alberto F. Pozzolo (2000) also used human capital as the only input in innovation but R&D uses other kinds of inputs and machinery. DeLong and Summers (1992) findings that that large difference in growth rates cannot be driven by shifts in equipment investment rate uncorrelated with TFP growth provides support for the role of innovation in the long run.

Howitt (1999) in an attempt to restore the policy implications of the early R&D growth model while at the same time eliminate scale effect built a model that combined both horizontal and vertical innovation; and argued that horizontal innovation proliferates vertical innovation. That is, as the economy grows, horizontal innovations neutralizes the scale effect on the incentive to innovate by adding to the number of independent sectors over which research must be spread, and over which manufacturing labour must also be spread. Jones(1999) determined that Howitt's model would only succeed in eliminating scale effect if population growth and horizontal innovation grows at a constant rate. He found that if population growth outgrows horizontal innovation then scale effect will resurface and if horizontal innovation outgrows population growth then horizontal innovation will have negative effect on growth.

Chapter three used a share of output instead of human capital alone as an input in research and introduce human capital in the product market to show that both innovation and human capital matter for long run growth rate. I show that once human capital , population growth, horizontal innovation , physical capital and technological progress are growing at a constant rate; then scale effect will be eliminated. I also show that if horizontal innovation outgrows population growth, the economy will grow by more because horizontal innovations open up new sectors on which vertical innovation could thrive when the existing vertical innovation becomes difficult to innovate on. If on the other hand population growth outgrows horizontal innovation then population growth will have a negative effect on growth. Therefore given the empirical support mentioned above that population growth has negative effect on growth suggest that horizontal innovations have not kept pace with population growth.

Chapter four challenges another benchmark of the basic Schumpeterian model, namely that competition has a negative effect on innovation and growth. Contrary to this prediction, the empirical works of Nickell (1996) and Blundell R., Griffith, R. and Van reennen, J. (1995) showed that competition has a positive effect on innovation and growth. Notably, the basic Schumpeterian model discussed competition as firm entry but the existing theoretical model on innovation that showed positive effect of competition on innovation discussed competition among existing firms with no firm entry (See: Aghion , Harris and Vickers (2001)). The current chapter provides the rationale for exogenous threat of firm entry. I endogenise firm entry and show that the reason why firm entry is represented as an exogenous threat is because in a Nash equilibrium, incumbents who could engage in innovation would have technological advantage over entrants(Otherwise, incumbents could not innovate due to Arrow's effect). Hence, when technological advantage is large, incumbents would raise their R&D effort to deter firm entry. Furthermore I argue that there is an implicit psychological threat that the incumbent feel to make him innovate and escape competition when he has technological advantage. This framework is supported by the empirical work of Goolsbee and Syverson (2004) who examined how incumbent behavior changes in response to exogenous changes in potential entry that otherwise have no effect on current competitive conditions. They found that incumbent airlines cut their fares when an entrant merely annouces thier intention of entry, even before actual entry.

In chapter five, I examine the impact of RJV on economic growth when RJV firms also engage in collusion in the product market. This research is motivated by the empirical work of Sovinsky and Helland (2012) which showed that the incentive to engage in RJV is to collude in the product market. To simply the analysis, I introduce a duopoly market structure where the duopolists are level in terms of technological progess in their sector. Thus they have more incentive to collude in other to avoid Bertrand competition. Then I made a novel contribution by introducing consumption externality under duopoly market structure with level sectors to show that duopolist in a level sector may also collude in the product market in order to internalize consumption externality that no single firm can internalize, in an economy where consumers' utility depends not only on the level of their consumption but also on how their consumption compares to some reference stock widely known as "keeping up with the Joneses." By colluding to internalize the reference stock, the duopolist can operate a dynamic pricing model to encourage habit formation. That is, they reduce their price when the reference stock is low in order to encourage consumption but gradually increase their price as the reference stock increases. This process encourages individuals who would have been detered by high price of a product to learn how to spend more as thier habit towards the product increases. This paper features two key parameters denoted by γ which measures the importance individuals place on the reference stock and the parameter J which measures the level of the reference stock. I found that increase in γ has a negative effect on growth because it raises the shadow cost of dynamic pricing model. But increase in J has a positive effect on growth because it raises the price that the duopolist can charge as habit formation rises. Hence it raises incentive for more innovation.

Chapter 2

LEARNING AND INNOVATION: A COMPLEMENTARY RELATIONSHIP.

2.1 Introduction

From the empirical perspective, the empirical work of Delong and Summers (1991) suggests that countries with the highest growth rates are those that invests *highly* in machinery and in which the relative price of equipment has fallen more quickly; thereby emphasized the role of physical capital in determining growth rate. This position is also supported by other models that argued for the role of capital in the long run. For instance, Mankiw (1995) argued that cross - country variation in growth rate could be explained by capital accumulation independent of technological progress. Jorgnenson (1995) maintained similar position in his study of the U.S economy.

But from the theoretical point of view, early innovation literatures like Aghion and Howitt (1992) emphasized the role of innovation in the long run independent of capital accumulation. Even the standard neoclassical model taught us that physical capital would not have a long run effect on growth due to diminishing return in capital accumulation and that growth depends solely on technological progress. On the other hand, theoretical models that supported the role of physical capital in the long run were mostly learning by doing models such as Romer (1987), Arrow (1962); but these class of learning by doing models predicted that in the long run, growth is determined by steady state level and not the per capita steady state growth rate. Yet the U.S experience showed that per capita growth rate has been constant between the period of 1870 to 2000 at the rate of 1.8 percent per year(See: Maddison (1991)). Hence there is a theoretical gap to account for per capita steady state growth rate that shows the importance of physical capital in the long run.

DeLong and Summers (1992) empirical study stroke a more reconciliatory tone by showing that large difference in growth rates cannot be driven by shifts in equipment investment rate uncorrelated with TFP growth.the incentive for more intense R&D.

Aghion and Howit (1998), then made the first attempt to develop a complementary model of both innovation and capital accumulation. They made a specification in which capital was used as an input in the production of intermediate goods by monopolists who hold patent right for innovation. Hence, they showed that capital accumulation would have an indirect effect on long run growth by raising the equilibrium flow of profits as a result of increasing national income that would raise the demand of monopolist's goods, which would then enhance the incentive to innovate[1]; just as more innovation would raise the productivity of capital.

My present research argues that there is a reason to suspect that capital would have long run effect on growth through learning by doing . For instance, Mowery & Rosenberg (1989) criticized those who regard innovation as the application of upstream scientific knowledge to the downstream activity of new products and new manufacturing process. In light with that criticism, I argue that monopolist who produces new technology tailors her production to be more efficient in response to local condition. Thus I defined learning extrnalities as the total amount of capital that has been accumulated by all firms which determines the local condition of capital use in the economy. So that , learning adaptation is how each monopolist who produces new technology tailors her production to be more efficient in response to local condition. Consequently, learning adaptation will induce higher R&D by raising the national income by more and hence the demand for the monopolist good by more. A higher monopolist profit due to efficient use of capital in response to local condition drives the incentive for more intense R&D. I found that the presence of learning implies that net income will be growing my more than when learning is absent. Finally, I introduce the social planner's framework with learning and technological progresss in order to determine the effects of tax and subsidy policies.

The rest of the paper proceeds as follows: in section 2.2, I introduce the decentralized economy with private monopolist's market structure and shows how learning enters R&D growth model. I also introduce the research sector which is the engine of growth in this model and present the utility function and the dynamic optimization solution for the decentralized economy. In section 2.3, I analyse the steady state of the decentralized economy and show that when learning is present, then long run growth is higher than when learning is absent. In section 2.4, I introduce the social planners economy and solve the social planner's dynamic optimizaton problem. In section 2.5, I determine the effects of tax and subsidy policies. Finally, I offer conclusion.

2.2 Model.

This paper follows Aghion and Howitt (1998) that studied the role of physical capital on long run growth but introduce learning. The setup is the basic Schumpeterian growth framework of Aghion and Howitt (1992) which argued that only the entrant firms do innovation (never the incumbent firms). The equilibrium features a previous entrant who becomes an incumbent and internalizes his obsolesence in calculating his present discounted value due to expected arrival of next entrant that will destroy his monopoly position in the process of creative destruction.

[1] By convention, a monopoly market structure is used in modeling private R&D because private inventors would need profit incentive to innovate. This incentive is secured by patent award which gives them exclusivity to commercialize their innovation.

2.2.1 Product Market.

Output is produced under perfect competition and is used for consumption, research and can be stored in the form of capital. The production of output depends on inputs of different intermediate products. Once idea with a productivity A_i, where $i \in [0,1]$ is generated from research, it is produced into intermediate goods using capital, then the intermediate goods serve as inputs in the production of the final good. Thus the aggregate production function is given as

$$Y_t = C_t + N_t + I_t = L^{1-a} x_t^{1-a} \int_0^1 A_{it}^{1-a} x_{it}^a di, \tag{2.1}$$

where $0 < a < 1$

Equation (2.1) shows that output Y_t can be stored as capital, denoted by gross investment I_t, used for consumption denoted by C_t and for research denoted by N_t. Output is produced using a constant supply of the society's fixed stock of labour L along with the flow of intermediate inputs x_{it} . A_{it} Notice that x_t^{1-a} is outside the intergral sign in equation (2.1); hence I define $x_t = \frac{\int_0^1 x_{it} d_i}{\int_0^1 A_{it} d_i}$ to denote the learning externality expressed in productivity adjusted term[2], that no single firm can internalize. Because learning becomes difficult as innovation becomes difficult, this model does not exhibit explosive growth. Notice that learning externalities are the total amount of capital that has been accumulated by all firms which determines the local condition of capital use in the economy. So that , learning adaptation is how each monopolist who produces new technology tailors her production to be more efficient in response to local condition. .

Each intermediate input is produced using capital that is rented from household by monopolists with patent award

$$x_{it} = K_{it} \tag{2.2}$$

where K_{it} is the amount of capital used as input. Thus the cost to monopolist is $R_t x_{it} = R_t K_{it}$. Where R_t is the rental rate of capital. The monopolist maximize his profit Π_{it}, measured in units of final good through the profit function

$$\Pi_{it} = P_{it} x_{it} - R_t x_{it} \tag{2.3}$$

where P_{it} denotes the price of intermediate good and his revenue equals $P_{it} x_{it}$.

I assume that the final good production sector is perfectly competititve, thus the price of a factor of production equals the value of its marginal product. Firms in the final goods sector maximize profit given by

[2]Notice that I expressed learning in productivity adjusted term and it is outside the integral. Thus following the conventional argument made in innovation literature that innovation becomes more complex as it increases; I assume that learning becomes more difficult as it increases.

$$Y_t - w_t L - \int_0^1 P_{it} x_{it} \tag{2.4}$$

The first order conditions implies that

$$P_{it} = L^{1-a} a A_{it}^{1-a} x_{it}^{a-1} x_t^{1-a} \tag{2.5}$$

and

$$w_t = (1-a)\, L^{-a} \int_0^1 A_{it}^{1-a} x_{it}^{a} di \tag{2.6}$$

where w_t is the equilibrium wage rate .
From equation (2.5) the corresponding demand function implies

$$x_{it} = \left(\frac{a L^{1-a} A_{it}^{1-a} x_t^{1-a}}{P_{it}} \right)^{\frac{1}{1-a}} \tag{2.7}$$

To determine the monopolist's price, substitute equation (2.7) into equation (2.3) to get

$$\Pi_{it} = (P_{it} - R_t) \left(\frac{a L^{1-a} A_{it}^{1-a} x_t^{1-a}}{P_{it}} \right)^{\frac{1}{1-a}} \tag{2.8}$$

which can be rewritten as

$$\Pi_{it} = \left(P_{it}^{\frac{-a}{1-a}} \left(a L^{1-a} A_{it}^{1-a} x_t^{1-a} \right)^{\frac{1}{1-a}} - R_t \left(a L^{1-a} A_{it}^{1-a} x_t^{1-a} \right)^{\frac{1}{1-a}} P_{it}^{-\frac{1}{1-a}} \right) \tag{2.9}$$

Finding the first order condition with respect to P_{it} gives

$$\frac{\partial \Pi_{it}}{\partial P_{it}} = \left(\frac{-a}{1-a} P_{it}^{\frac{-1}{1-a}} \left(a L^{1-a} A_{it}^{1-a} x_t^{1-a} \right)^{\frac{1}{1-a}} + \frac{1}{1-a} R_t \left(a L^{1-a} A_{it}^{1-a} x_t^{1-a} \right)^{\frac{1}{1-a}} P_{it}^{\frac{-2+a}{1-a}} \right) \tag{2.10}$$

solving for P_{it} gives the monopoly price mark up on the marginal cost of production.

$$P_{it} = \frac{R_t}{a} \tag{2.11}$$

By substituting equation (2.11) into equation (2.7) gives the equilibrium quantity chosen by the monopolist to maximize her profit ,

$$x_{it} = \left(\frac{a^2 x_t^{1-a}}{R_t} \right)^{\frac{1}{1-a}} A_{it} L \tag{2.12}$$

The rental rate of capital is determined in the capital market, where supply is K_t and the demand of capital is the sum of all sectors demand thus;

$$\int_0^1 x_{it} d_i = \int_0^1 K_{it} d_i \tag{2.13}$$

Using equation (2.12) the equilibrium quantity can be expressed in terms of capital thus

$$K_t = L \int_0^1 \left(\frac{a^2}{R_t}\right)^{\frac{1}{1-a}} A_{it} dix_t^{\frac{1-a}{1-a}} = \left(\frac{a^2 x_t^{1-a}}{R_t}\right)^{\frac{1}{1-a}} A_t L \tag{2.14}$$

Where $A_t = \int_0^1 A_{it} d_i$ represent the average productivity parameter and K_t denotes the aggregate capital stock . We can then capture the aggregate capital stock in per effective worker unit as

$$k_t = x_t = \frac{K_t}{A_t} \tag{2.15}$$

Hence equation (2.14) can be expressed in capital efficiency unit to determine the rental rate of capital in per effective worker thus

$$R_t = a^2 \tag{2.16}$$

where $L = 1$

Notice that the equilibrium rental rate, R_t is a constant a^2. The reason is because learning offsets diminishing return in capital accumulation. In absence of learning in equation(2.14), the rental rate of capital yields the outcome in Aghion and Howitt (1998) where learning is absent written here as.

$$R_t = a^2 k_t^{a-1} L^{1-a} \tag{2.17}$$

The difference between the rental rate under learning adaption in equation (2.16) and the Aghion and Howitt (1998) version in equation (2.17) is the presence of diminishing return exhibited by Aghion and Howitt's rental rate whereas the rental rate is constant in the present paper. Since learning is the factor that plays the offsetting effect on diminishing return on capital, it underlies the argument that learning adaptation leads to more efficient use of capital in response to local condition.

From the perspective of the owner of capital, the equilibrium rental rate of capital must pay for the interest rate r_t and the depreciation rate δ. Thus the equilibrium rental rate can be expressed as

$$R_t = r_t + \delta \tag{2.18}$$

Equation (2.16) and (2.18) implies that the rate of interest will be given as

$$r_t = a^2 - \delta \tag{2.19}$$

Since

$$x_{it} = A_{it}\left(\frac{K_t}{A_t}\right) = A_{it}k_t \tag{2.20}$$

substituting equation (2.16) and (2.20) into equation (2.3) implies

$$\Pi_{it} = \pi\left(k_t\right) A_{it}L^{1-a} \tag{2.21}$$

where the productivity adjusted profit function is

$$\pi\left(k_t\right) = a(1-a)k_t \tag{2.22}$$

in Aghion and Howitt (1998) where learning is absent, the productivity adjusted profit function is

$$\pi\left(k_t\right) = a(1-a)k_t^a L^{1-a} \tag{2.23}$$

This can easily be derived using equation (2.17)

If you substitute equation (2.19) into equation (2.1) , you get the aggregate production function in capita per efficient unit

$$Y_t = C_t + N_t + I_t = A_t k_t \tag{2.24}$$

where $L = 1$

This aggregate production function shows the economy's GDP because it shows that output equals consumption plus investment in capital accumulation and research.

2.2.2 Innovation

I assume that innovations are either funded publicly[3] or privately and are the source of long run growth. Each time there is investment to innovate within the sectors in the economy, the aim is to improve the quality of the previous innovation. Quality improvement is referred in innovation literatures as vertical innovation(see: Aghion

[3] I assume public funding of innovation for the sake of section 2.6 where I used the social planner's framework.

and Howitt (1992)). Ideas flowing out from innovation are used to produce an intermediate products which is used as an input in the production of the final goods in order to increase the level of output.

There are different research sectors for different intermediate goods. Following the conventional assumption of Poisson rate of arrival of innovation; the Poisson arrival rate of innovation in each sector i is λN_{it}. N_{it} denotes the amount of research effort devoted to sector i. Because the prospective payoff to research is the same in all sectors, the equilibrium research expenditure is the same in all sectors.

Let the Poisson rate of arrival of innovations in equilibrium is given as

$$\dot{A}_t = \lambda N_t \tag{2.25}$$

$\lambda > 0$ is the productivity parameter for R&D.

Following Caballero and Jaffe (1993), the impact of the leading edge technology as innovation arrives is denoted by the extra services it offers represented as $\sigma > 0$. Hence the rate of technological progress at each date yields

$$g_t = \frac{\dot{A}_t}{A_t} = \lambda \acute{n}_t \ \sigma; \tag{2.26}$$

where $\acute{n}_t = \frac{N_t}{A_t}$

In equilibrium, the level of vertical R & D is determined by the zero profit condition ; that is, the marginal cost of an extra unit of vertical R & D equal the marginal expected benefit given as

$$A_t = \lambda V_t. \tag{2.27}$$

Where V_t is the value of a vertical innovation and A_t represents the marginal cost of raising the research intensity N_t . This equation governs the dynamics of the economy over its successive innovation.

The value V_t is determined by the asset equation;

$$V_t = \int_t^{\infty} \Pi_{t\tau} e^{-\int_t^{\tau}(r_s+\lambda \acute{n}_t)ds} d\tau \tag{2.28}$$

This equation says that the expected present value of the future profits during a unit time interval is equal to the flow of profit to be earned by the incumbent before being replaced. The value to the incumbent is thus

$$V_t = \frac{\Pi_t}{r_t + \lambda \acute{n}_t} \tag{2.29}$$

This equation captures the effect of creative destruction on innovation, particularly increase in the rate of arrival of innovation shortens the duration of the monopoly's profit thereby destroying the position of the current incumbent monopolist . The standard model predicts that this effect will kill the incentive to innovate. For instance if inventors have perfect foresight and knows that thier innovation will be destroyed by the next entrant, they will not innovate. Since variables will be growing in thier efficiency unit, the zero profit condition implies .

$$1 = \lambda v_t \tag{2.30}$$

where $v_t = \frac{V_t}{A_t}$

substitute equation (2.29) into (2.30) to determine equilibrium level of research intensity

$$1 = \lambda \frac{a(1-a)k_t L^{1-a}}{r_t + \lambda \acute{n}_t} \tag{2.31}$$

in Aghion and Howitt(1998) where learning is absent, the equilibrium research intensity is

$$1 = \lambda \frac{a(1-a)k_t^a L^{1-a}}{r_t + \lambda \acute{n}_t} \tag{2.32}$$

equation(2.31) determines the equilibrium research intensity as a function of capital intensity k. In Aghion and Howitt (1998) it is denoted as k^a. Becuase of decreasing return in capital accumulaton present in Aghion and Howitt's model, research intensity is lower in their model compared to the present model . The solution will be shown in the steady state analysis.

2.2.3 Household

The utility function is time separable. The aim is to maximize the discounted, infinite stream of utility U, given by

$$\max \int_0^\infty \log(C_t) e^{-\theta t} dt. \tag{2.33}$$

θ is the rate of time preference. C_t denotes aggregate consumption. The utility function is logarithmic.

Let the budget constraint be represented as

$$\dot{K}_t = A_t k_t - \delta K_t - \ C_t - N_t \tag{2.34}$$

Where N_t denotes the share of output used for research. K_t denotes capital and δ is the depreciation rate of capital

The current value Hamiltonian is

$$H = \log(C_t) + \mu.[rK_t + wL - \ C_t - N_t] \tag{2.35}$$

There is only one state variable denoted by K_t and the associated co state variable μ. μ is the shadow value of investment evaluated in current utils in the manufacturing sector.

The necessary first order condition for maximizing the Hamiltonian can then be expressed as follows

$$e^{-\theta t}\frac{1}{A_t\frac{C_t}{A_t}} = \mu \tag{2.36}$$

where $\frac{C_t}{A_t}$ is consumption adjusted productivity

$$-\dot{\mu} = \mu(r_t - \delta) \tag{2.37}$$

where r_t is the interest rate determined in equation (2.18)

the transversality condition can thus be expressed as

$$\lim_{t\to\infty}\left[(k_t.\mu\exp^{-\theta t}\right] = 0 \tag{2.38}$$

Since

$$\frac{\frac{d}{dt}\left(\frac{1}{A_t\frac{C_t}{A_t}}\right)}{\left(\frac{1}{A_t\frac{C_t}{A_t}}\right)} = -\left(\frac{\dot{A}_t}{A_t} + \frac{\dot{c}_t}{c_t}\right)$$

Differentiating equation (2.36) with respect to time and combining with equation (2.37) shows that consumption grows at the rate equal to the difference between the net marginal product of capital, $a^2 - \delta$, and the effective discount rate, $\theta + g_t$ thus

$$\frac{\dot{c}_t}{c_t} = a^2 - \delta - \theta - g_t \tag{2.39}$$

where $g_t = \frac{\dot{A}_t}{A_t}$

the budget constraint will be growing in their efficiency units as

$$\dot{k}_t = k_t - c_t - \acute{n}_t - (\delta + \lambda\acute{n}_t\ \sigma)\,k_t \tag{2.40}$$

2.3 Steady State Growth Analysis

In the steady state, all variables such as capital k, research intensity $\acute{n}$, consumption c will be growing at a constant rate so I shall henceforth drop the time subscript.

To determine research intensity $\acute{n}$, we make use of equation (2.18) and (2.39) to determine r, and then substitute r into equation (2.31) then solve for $\acute{n}$ from equation (2.31) here as

$$\acute{n} = \frac{\lambda a(1-a)kL^{1-a} - (\theta + \delta)}{\lambda + \lambda\ \sigma} \tag{2.41}$$

Finally, the growth rate of the economy is determined by substituting equation (2.41) into the growth equation (2.26) thus

$$g = \frac{\dot{A}}{A} = \sigma \left[\frac{\lambda a(1-a)kL^{1-a} - (\theta + \delta)}{1 +\ \sigma} \right]; \tag{2.42}$$

For comparative statistic, increase in λ, σ, a have a positive effect on growth and increase in θ have a negative effect on growth. Increase in L denotes the presence of scale effect in the standard Schumpeterian model[4].

Proposition 1. *Growth rate is higher under learning compared to when learning is absent.*

To see this, Aghion and Howitt (1998) showed a similar model where learning is absent, which we can easily determine using his model specification in equation (2.32). If you hold the interest rate constant, then the growth rate in absence of learning is

$$\bar{g} = \frac{\dot{A}}{A} = \sigma \left[\lambda a(1-a)k^a L^{1-a} - r \right] \tag{2.43}$$

similarly the growth rate in my model where learning is present can be rewritten holding interest rate constant as

$$g = \frac{\dot{A}}{A} = \sigma \left[\lambda a(1-a)kL^{1-a} - r \right] \tag{2.44}$$

Differentiate g and $\bar{g}$ with respect to k yields respectively

[4]Scale effect is the feature that variation in the size or scale of the economy, measured by population has a positive long run effect on growth .

$$\sigma\lambda a(1-a)L^{1-a} \tag{2.45}$$

and

$$\sigma\lambda a^2(1-a)k^{a-1}L^{1-a}$$

Subtracting $\bar{g}$ from g gives

$$\sigma\lambda a(1-a)L^{1-a}\left(1-ak^{a-1}\right) \tag{2.46}$$

since $R=r+\delta$ and $r=\theta+g-\delta$ equation (2.17) implies that capital per effective worker in absence of learning is

$$k=\left(\frac{\theta+g_t-\delta}{a^2}\right)^{\frac{1}{a-1}} \tag{2.47}$$

then substitute equation (2.47) into equation (2.46) to get

$$\sigma\lambda(1-a)L^{1-a}\left(a-[\theta+g_t-\delta]\right)>0 \tag{2.48}$$

Given the value of the parmeters, it is possible to show that when the factor share of capital a is high , model with learning will outgrow model without learning due to higher capital intensity that raises the demand of the innovator's goods thereby raising incentive for future innovation.

2.4 Welfare.

In this section, I examine the social planner's problem of R&D - based growth model in order to determine the effects of tax and subsidy policies.The social planner invests in R&D, and ideas generated from R&D are used to produce intermediate goods. Intermediate goods are then used as an input in producing the final goods in other to increase the level of output. The main difference between the social planner's framework and the Shumpeterian framework presented above is that here, I assume unlike the Schumpeterian model that the intermediate good sector is perfectly competitive.

By nomalizing $P_{it}=1$ (2.7) you get the demand function for the social planner as

$$x_{it}=a^{\frac{1}{1-a}}A_{it}Lx_t^{\frac{1-a}{1-a}} \tag{2.49}$$

The supply of capital K_t must equal the demand of capital in all sectors demand thus; $\int_0^1 x_{it}d_i=\int_0^1 K_{it}d_i$. Hence in equilibrium, you can write equation (2.49) in terms of capital per efficiency unit thus;

$$\frac{x_t}{A_t}=k_t=a^{\frac{1}{1-a}}Lk_t \tag{2.50}$$

where $k_t = \frac{K_t}{A_t}$

By substituting equation (2.50) into the aggregate production function of equation (2.1) yields

$$Y_t = C_t + N_t + I_t = A_t L a^{\frac{a}{1-a}} k_t, \tag{2.51}$$

This equation says that output depends on technological progress and capital accumulation, where k_t denotes learning.

Since the arrival of new innovation raise the average productivity parameter; The average change in the stock of knowledge is

$$\frac{dA_t}{dt} = \lambda N_t \left(A_t^{\max} - A_t\right) \tag{2.52}$$

where $A_t^{\max}$ denotes the maximum level of technological progress at a given date t

Using the equations 2.50- 2.52 we can work out the social planner's optimization problem. Hence the social planner maximizies the utility of the representative householed by choosing consumption and research expenditure.

The current value Hamiltonian is written as

$$H = \log C_t + \mu \left[A_t L k_t \left(\frac{1}{a} - 1\right) a^{\frac{1}{1-a}} - C_t - N_t \right] + \Psi \left[\lambda N_t \left(A_t^{\max} - A_t\right)\right] \tag{2.53}$$

The utility function comes from equation (2.33). The second equation in the square bracket comes from the combination of equation (2.50)and (2.51) . The third equation comes from equation (2.52)

The necessary first order condition for maximizing utility can then be expressed as follows

$$\frac{\partial H}{\partial C_t} = e^{-\theta t} \frac{1}{C_t} = \mu \tag{2.54}$$

$$\frac{\partial H}{\partial N_t} = \mu = \Psi \left[\lambda \left(A_t^{\max} - A_t\right)\right] \tag{2.55}$$

$$\dot{\Psi} = -\left(\lambda N_t\right) \Psi \tag{2.56}$$

Since output and technological progress will be growing at at constant rate hence I rewrite equation (2.56) thus

$$\dot{\Psi} = -\left(L k_t \left(\frac{1}{a} - 1\right) a^{\frac{1}{1-a}} \right) \left[\lambda \left(A_t^{\max} - A_t\right)\right] \Psi \tag{2.57}$$

the transversality condition can thus be expressed as

$$\lim_{t\to\infty}\left[(W)\mu\exp^{-\theta t}\right]=0 \tag{2.58}$$

where W captures the entire wealth in the economy
Since
$\frac{A_t^{\max}}{A_t}=\sigma+1^5$ from equation (2.27). This implies that $(A_t^{\max}-A_t)$ will grow at the rate of σ . Since $-\frac{\dot{\Psi}}{\Psi}=\frac{\dot{c}}{c}+\theta$ the optimal consumption as

$$\frac{\dot{c}}{c}=\left(kL\left(\frac{1}{a}-1\right)a^{\frac{1}{1-a}}\lambda\sigma-\theta\right) \tag{2.60}$$

2.5 The Effects of Tax and Subsidy Policies

Comparing research intensity between the private monopolist and the social planner shows that research intensity under decentralized economy is smaller than research intensity under the centralized economy. Since $\frac{\dot{c}}{c}=g=\lambda\acute{n}\ \sigma$ the research intensity for the social planner can be determined using equation(2.60) thus

$$\acute{n}=\left(\frac{kL\left(\frac{1-a}{a}\right)a^{\frac{1}{1-a}}\lambda\sigma-\theta}{\lambda\sigma}\right) \tag{2.61}$$

and the research intensity equation for the private agent is taken from equation (2.42) thus

$$\acute{n}=\lambda\frac{a(1-a)kL-(\theta+\delta)(1-s)}{(1-s)\left[\lambda\sigma+\lambda\right]} \tag{2.62}$$

where s denotes the rate of subsidy

Looking at the denomenator, the social discount rate is given as $\lambda\sigma$ which is less than the private discount rate given as $[\lambda\sigma+\lambda]$ thus there is more research in the

[5] The average change in the stock of knowledge is $A_t^{\max}-A_t$. This implies that
$\frac{dA_t}{dt}=\lambda\acute{n}_t\left(A_t^{\max}-A_t\right)$
let $\varsigma_t=\frac{A_t^{\max}}{A_t}$; using equation (24), the evolution of ς_t takes the form
$\frac{1}{\varsigma}\left(\frac{d\varsigma_t}{dt}\right)=\lambda\acute{n}_t\sigma-\lambda\acute{n}_t\left(\varsigma-1\right)$
By setting $\frac{1}{\varsigma}\left(\frac{d\varsigma_t}{dt}\right)=0$ we have

$$\varsigma=\sigma+1 \tag{2.59}$$

social planner's economy than in the decentralized economy. Differentiating research of the private agent with respect to research subsidy yields

$$\frac{d\acute{n}}{ds} = \frac{\lambda a(1-a)kL}{(1-s)} > 0$$

This equation shows that research subsidy has a positive effect on research intensity.

2.5.1 Subsidies to capital accumulation .

Turning to capital accumulation the social planner's rate of return is $a - \delta$ which is greater than the private monopolist rate of return $a^2 - \delta$ which suggests that capital investment should be subsidized. For instance if one compares the rate of return in a monopolistic economy and the social planner's economy we get

$$[1 - T_s]a^2 = a \tag{2.63}$$

where T_s is the tax rate, so that

$$T_s = \frac{(a-1)}{a} < 0 \tag{2.64}$$

Thus optimal tax policy is a negative tax.

2.6 Conclusion

This paper extends the Schumpeterian framework of Aghion and Howitt (1998) in which new technology is capital using . I showed that there an be other reasons why capital will have a long run effect on growth. Particulary, I argue that innovation is not simply the application of upstream scientific knowledge to the downstream activity of new products and new manufacturing process. Rather an innovators who produces new technology tailors her production to be more efficient in response to local condition. Hence learning adaptation viewed as how each innovator who produces new technology tailors her production to be more efficient in response to local condition can show a different role played by capital on long run growth. I found that the presence of learning implies that net income will be growing by more than when learning is absent. Hence long run growth rate is higher when learning is present than when learning is absent. Finally, I introduce the social planner's framework with learning and technological progresss in order to determine the effects of tax and subsidy policies.

Chapter 3

A SCALE INVARIANT MODEL WITHOUT DILUTION EFFECT OF HORIZONTAL INNOVATION.

3.1 Introduction

This paper takes the broadest view so far to study the relationship between physical capital accumulation, human capital accumulation, innovation(vertical and horizontal) and population growth in order to address the issues ranging from the absence of scale effect, the impact of physical capital in long run growth, the relationship between human capital and innovation, and the relationship between population growth and horizontal innovation.

Early endogenous growth theory unequivocally argued that human capital has a positive long run effect on growth(see: Lucas (1988). Early R&D growth literatures also argued that innovation has a long run effect on growth(see: Aghion & Howitt (1992), , Grossman and Helpman (1991)). But the prediction of scale effect in the early R&D growth literatrues has questioned the role of innovation in the long run. For instance, the empirical study of Jones (1995a) showed the absence of scale effect in the post war II era. Jones showed that the number of scientist and engineers engaged in R&D in the United State grew from under 200,000 in 1950 to almost 1 million by 1987 yet growth rate in the United state remained constant during this period. To eliminate scale effect, Jones (1995b) developed a semi - endogenous growth model which assumed decreasing returns in technological progress to account for the declining growth in the post war era II, and used labour as an input in research, so that increasing population growth is require to offset the decreasing return in technological progress and thus put the economy on a constant returns. The implication of his model overturned the predictions of early R&D growth literatures, and implied that policies that affect research intensity has no long run growth since long run growth depends on exogenous population growth. Some endogenous literatures such as Arnold (1998) and Blackburn Keith, Victor T.Y. Hung, and Alberto F. Pozzolo (2000) have exploited the semi - endogenous growth model of Jones (1995b) by using human capital implace of labour as an input in research to argue that long run growth rate would depend solely on human capital accumulation. Despite the advantages of these scale invariant models that deny the role of innovation in the long run; the empirical works of Brander and Dowrick (1994), Kelley and Schmidt (1995), and Ahituv (2001) showed that population growth has a negative effect on economic growth. Arnold (1998) and Blackburn Keith, Victor T.Y. Hung, and Alberto F. Pozzolo (2000) also used human capital as the only input in innovation but R&D uses other kinds of inputs and machinery. DeLong and Summers (1992) findings that that large difference in growth rates cannot be driven by shifts in equipment investment rate uncorrelated with TFP growth provides support for the role of innovation in the long run.

Howitt (1999) in an attempt to restore the policy implications of the early R&D growth model while at the same time eliminate scale effect built a model that combined both horizontal and vertical innovation; and argued that horizontal innovation proliferates vertical innovation. That is, as the economy grows, horizontal innovations neutralizes the scale effect on the incentive to innovate by adding to the number of independent sectors over which research must be spread, and over which manufacturing labour must also be spread. Jones(1999) determined that Howitt's model would only succeed in eliminating scale effect if population growth and horizontal innovation grows at a constant rate. He found that if population growth outgrows horizontal innovation then scale effect will resurface and if horizontal innovation outgrows population growth then horizontal innovation will have negative effect on growth.

Chapter three used a share of output instead of human capital alone as an input in research and introduce human capital in the product market to show that both innovation and human capital matter for long run growth rate. I show that once human capital , population growth, horizontal innovation , physical capital and technological progress are growing at a constant rate; then scale effect will be eliminated. I also show that if horizontal innovation outgrows population growth, the economy will grow by more because horizontal innovations open up new sectors on which vertical innovation could thrive when the existing vertical innovation becomes difficult to innovate on. If on the other hand population growth outgrows horizontal innovation then population growth will have a negative effect on growth. Therefore given the empirical support mentioned above that population growth has negative effect on growth suggest that horizontal innovations have not kept pace with population growth.

The rest of the paper proceeds as follows as: In section 3.2, I introduce the decentralized economy with private monopolist's market structure and showe how population growth, physical and human capital enters R&D growth model. Then, I introduce human capital accumulation production function and solve the household dynamic optimization for the decentralized economy. In section 3.3, I work out the steady state analysis and in section 3.4 I offer conclusion.

3.2 Model

In this paper, I built a scale invariant model where a share of output is used for research . The intermediate good's sector uses physical capital as an input; and the final good's sector uses human capital and intermediate goods as input. Like chapter two(except for the concept of learning), the present model follows a Schumpeterian framework but introduce horizontal innovation and population growth.

3.2.1 Product Market

Output is produced under perfect competition and used for consumption, research and can be stored in the form of capital . The production of output depends on inputs of different intermediate products. Once idea with a productivity parameter A_i is generated from research, it is produced into intermediate goods using physical capital by the firm with exclusive access to the technology offered by patent protection. By virtue of that exclusivity, such a firm becomes a monopolist in his industry. Then, the intermediate goods serve as input in the production of the final good. Thus the aggregate production function is given as

$$Y_t = C_t + N_t + I_t = \left((1-u) \frac{L_t}{Q_t} h_t \right)^{1-a} A_t^{1-a} \int_0^{Q_t} x_{it}^a di = \left((1-u) H_t \right)^{1-a} A_t^{1-a} \int_0^{Q_t} x_{it}^a di \tag{3.1}$$

where $0 < a < 1$

This equation shows that output Y_t can be stored as capital, denoted by gross investment I_t, used for consumption denoted by C_t and for research denoted by N_t. x_{it} is the flow of intermediate input $i \in [0, Q_t]$. A_t is the productivity attached to the latest quality improvement of the existing sectors . Unlike chapter two where I considered fixed population, here $L_t = e^{nt}$, where n_t denotes exogenous population growth rate. I assume that horizontal innovation is an exogenous serendipitious process which occurs for the emergence of new sectors. Let Q_t denote the number of intermediate goods . So that $\frac{L_t}{Q_t}$ represents the assumption that the number of sectors Q_t must grow at the same rate as the number of people L_t which will tend to a constant φ[1]. This condition is usually used to eliminate scale effect in models with proliferation argument (see: Howit (1999), Jones (1999), Aghion and Howit (1999)). Let $(1-u)$ denote the fraction of time devoted to the production by human capital in production of output; where $\frac{L_t}{Q_t} h_t = H_t$, so that we can consider human capital H_t, as the number of workers per sector $\frac{L_t}{Q_t}$, multiplied by the human capital of the typical worker, h_t. This assumption implies that the quantity of workers $\frac{L_t}{Q_t}$ and the

[1] To see how this is derived, lets assume that horizontal innovation arrives at the poisson rate ϱ. Where $\dot{Q} = \varrho L_t$ and $\dot{L}_t = n L_t$. Thus the ratio $\varphi \equiv \frac{L_t}{Q_t}$ implies $\dot{\varphi} = n\varphi_t - \mu\varphi_t^2$. By setting $\dot{\varphi} = 0$, you get $\varphi \equiv \frac{n}{\varrho}$

quality of workers h_t are perfect substitute, so that we can think of h_t as human capital embodied labour supply.

The supply of physical capital K_t must equal the demand of physical capital which is the sum of all sector's demand $\int_0^{Q_t} x_{it}d_i = \int_0^{Q_t} K_{it}d_i$. The average productivity $A_t = \int_0^1 A_{it}d_i$ represent productivity of already existing sectors. Intermediate inputs across all sectors are produced using physical capital by a monopolist firms with access to the latest technology according to

$$x_t = (1 - v)\, K_t \tag{3.2}$$

where $(1 - v)$ is the fraction of time devoted by physical capital in the production of intermediate goods. K_t is the amount of physical capital used as input. Thus

the economy wide cost to monopolists is $R_t x_t = R_t K_t$ Where R_t is the rental rate of physical capital .

The monopolists maximize their profit Π_t, measured in units of final good through the profit function

$$\Pi_t = P_t x_t - R_t x_t \tag{3.3}$$

where P_t denotes the price of intermediate goods and their revenues equal $P_t x_t$. I assumed that the final good production sectors are perfectly competititve, so that P_t is the marginal product of the intermediate inputs in producing the final goods. Therefore the marginal revenue $P_t x_t$ will imply

$$H_t^{1-a} a A_t x_t^a, \tag{3.4}$$

hence the profit to monopoly firms would be

$$\Pi_t = H_t^{1-a} a A_t^{1-a} x_t^a, -R_t x_t \tag{3.5}$$

The equilibrium demand function expressed in terms of the ratio of effective physical capital to human capital ratio can be written as

$$\frac{(1 - v)\, K_t}{A_t\,(1 - u)\, H_t} = \left(\frac{a^2}{R_t}\right)^{\frac{1}{1-a}} \tag{3.6}$$

the equilirium rental rate and wage rate in per efficiency unit of physical capital to human capital ratio are respectively written as

$$R_t = a^2 \left(\frac{(1 - v)\, K_t}{A_t\,(1 - u)\, H_t}\right)^{a-1} = a^2 \left(\frac{(1 - v)\, K_t}{A_t\,(1 - u)\, h_t \varphi}\right)^{a-1} \tag{3.7}$$

and

$$\omega = \frac{w_t}{A_t} = (1-a)\left(\frac{(1-v)\,K_t}{A_t\,(1-u)\,H_t}\right)^a = (1-a)\left(\frac{(1-v)\,K_t}{A_t\,(1-u)\,\varphi h_t}\right)^a \tag{3.8}$$

substituting equation (3.7) into equation (3.5) to determine the profit function as

$$\Pi_t = \pi\,(k_t)\,A_t\,(1-u)\,H_t \tag{3.9}$$

where the productivity adjusted profit function is

$$\pi\,(k_t) = a(1-a)\left(\frac{(1-v)\,K_t}{A_t\,(1-u)\,H_t}\right)^a \tag{3.10}$$

If you substitute equation (3.6) into equation (3.1) , then the production function for output can be expressed in efficiency unit as

$$y_t = (1-u)\,H_t\left(\frac{(1-v)\,K_t}{A_t\,(1-u)\,H_t}\right)^a \tag{3.11}$$

where $y_t = \frac{Y_t}{A_t}$

By substituting equation(3.11) into equation (3.8) we can determine the wage rate which depends on output as follows

$$\omega_t = (1-a)\left(\frac{y_t}{(1-u)\,H_t}\right)^a \tag{3.12}$$

equation(3.12) provides a working equation which helps in solving the solution of this model as we will see later.

Notation 2. *The equilirium in this model is a senario where output Y_t, human capital h_t, labour supply L_t, horizontal innovation Q_t, research effort N_t, physical capital K_t and vertical innovation A_t are all growing at the same constant rate.*

3.2.2 Vertical Innovation

Quality improvement is referred in innovation literature as vertical innovation(see: Aghion and Howit (1992)). Ideas flowing out from innovation are used to produce intermediate products which are used as inputs in the production of the final goods in order to increase the level of output.

There are different research sectors for intermediate goods. Following the conventional assumption of Poisson rate of arrival of innovation; the Poisson arrival rate of innovation in each sector i is λN_{it} .N_{it} denotes the amount of research effort devoted to sector i. Because the prospective payoff to research is the same in all sectors, the equilibrium research expenditure is the same in all sectors.

Let the Poisson rate of arrival of innovations be given as

$$\dot{A}_t = \lambda N_t \tag{3.13}$$

$\lambda > 0$ is the productivity parameter for R&D.

Following Caballero and Jaffe (1993), the impact of the leading edge technology as innovation arrives is denoted by the extra services it offers represented as $\sigma > 0$. Since knowlege spillover circulate without cost, knowledge grows at a rate proportional to the aggregate rate of innovations. Therefore the rate of technological progress at each date yields

$$g_t = \frac{\dot{A}_t}{A_t} = \lambda \acute{n}_t \ \sigma; \tag{3.14}$$

where $\acute{n}_t = \frac{N_t}{A_t(1-u)H_t}$

In equilibrium, the level of vertical R & D is determined by the zero profit condition ; that is, the marginal cost of an extra unit of vertical R & D equal the marginal expected benefit given as

$$A_t = \lambda V_t. \tag{3.15}$$

Where V_t is the value of a vertical innovation and A_t represents the marginal cost of raising the research intensity N_t . This equation governs the dynamics of the economy over its successive innovation.

The value V_t is determined by the asset equation;

$$rV_t = \Pi_t - \lambda \acute{n}_t V_t \tag{3.16}$$

this equation says that the expected present value of the future profits during a unit time interval is equal to the flow of profit to be earned by the incumbent before being replaced. This equation assumes Arrow effect: which is the assumption that the incumbent does not innovate because he view further innovation as a negative expected value. The value to the leader is thus

$$V_t = \frac{\Pi_t}{r + \lambda \acute{n}_t} \tag{3.17}$$

This equation captures the effect of creative destruction on innovation, particularly increase in the rate of arrival of innovation shortens the duration of the monopoly's profit thereby destroying the position of the incumbent monopolist . Thus it is standard to predict that this effect will kill the incentive for innovation. For instance if inventors have perfect foresight and knows that thier innovation will be destroyed by the next entrant, they will not innovate.

In the steady state , the zero profit condition in equation (3.15) can be divided by A_t to get $1 = \lambda\tilde{v}$, where $\tilde{v} = \frac{V_t}{A}$ is the productivity - adjusted value of a vertical innovation. Furthermore I assume that R&D expenditure are subsidized at a rate proportional to s, so that the zero profit condition implies

$$1 - s = \lambda\tilde{v} \tag{3.18}$$

Where s is the subsidy rate.

Using equation (3.9) ,(3.17) and (3.18) we can express the steady state equilibrium level of research intensity where research effort N, is growing at a constant rate with human capital , horizontal innovation, vertical innovation and population growth. Thus the research intensity equation will imply

$$1 - s \;= \lambda\frac{a(1-a)\frac{y}{(1-u)H}}{r + \lambda\acute{n}} \;=\; \lambda\frac{a(1-a)\left(\frac{(1-v)K}{A(1-u)h\varphi}\right)^{a}}{r + \lambda\acute{n}} \tag{3.19}$$

where $\varphi \equiv \frac{n}{\varrho}$

Proposition 3. *If horizontal innovation and population growth do not grow at a constant rate; then increase in the growth rate of horizontal innnovation has a positive effect on growth* $\frac{\partial\breve{n}}{\partial\varrho} > 0$ *while increase in population growth has negative effect on growth.* $\frac{\partial\breve{n}}{\partial n} < 0$

The intuition behind the proposition $\frac{\partial\breve{n}}{\partial\varrho} > 0$ is that increase in horizontal innovation parameter leads to the creation of new sectors on which vertical innovation can build on when vertical innovation on the existing sectors become difficult to improve on; which is contrary to the predictions of Jones (1999), Howitt (1999) and Aghion and Howitt (1999) that horizontal innovation would have a neutralizing effect on vertical innovation. For $\frac{\partial\breve{n}}{\partial n} < 0$, this is simply the dilution effect of population growth on capital accumulation and explaines why the present model is scale invariant.

Substitute the steady state interest rate derived below from equation (3.42) as $\theta + \varepsilon g_n$ to determine the steady state research intensity as

$$\acute{n}_t = \frac{\lambda a(1-a)\left(\frac{(1-v)K}{A(1-u)h\varphi}\right)^{a} - \theta\,(\,1-s)}{[\varepsilon\lambda\sigma + \lambda]\,(\,1-s)} \tag{3.20}$$

Finally substitute $\acute{n}$ into the growth equation (3.14) to yield

$$g_n = \frac{\dot{A}_t}{A_t} = \left(\frac{\lambda\left(a(1-a)\frac{y}{(1-u)H}\right) - \theta\,(\,1-s)}{[\varepsilon\sigma + 1]\,(\,1-s)}\right)\sigma\;; \tag{3.21}$$

.

This model used a share of output instead of human capital alone as an input in research and introduce human capital in the product market to show that both innovation and human capital matter for long run growth rate. I show that once human capital , population growth, horizontal innovation , physical capital and technological progress are growing at a constant rate; then scale effect will be eliminated. I also show that if horizontal innovation outgrows population growth, the economy will grow by more because horizontal innovations open up new sectors on which vertical innovation could thrive when the existing vertical innovation becomes difficult to innovate on. If on the other hand population growth outgrows horizontal innovation then population growth will have a negative effect on growth. Therefore given the empirical support in Brander and Dowrick (1994), Kelley and Schmidt (1995) and Ahituv (2001) that population growth has negative effect on growth suggest that horizontal innovations have not kept pace with population growth.

3.2.3 Human Capital Accumulation

The flow of human capital can be written as

$$\dot{H} = \xi \left((uH_t)^{\beta} . (vK_t)^{1-\beta} \right) \tag{3.22}$$

where $0 < \beta < 1$

where v is the fraction of time that physical capital is used in the production of human capital and u denotes the fraction of time that human capital is used in the production of human capital. ξ denotes the productivity parameter of human capital accumulation.

The growth rate of human capital can be written as

$$\frac{\dot{H}}{H} = \xi u^{\beta} . v^{1-\beta} \left(\frac{K_t}{H_t} \right)^{1-\beta} \tag{3.23}$$

3.2.4 Household

The utility function is time separable. The aim is to maximize the discounted, infinite stream of utility U, given by

$$\max \int_0^{\infty} U(C_t) e^{-(\theta - n)t} dt. \tag{3.24}$$

θ is the rate of time preference,

The instantanous utility can be represented as

$$U(\ C_t) = \frac{C_t^{1-\varepsilon}}{1-\varepsilon} \tag{3.25}$$

C_t denotes aggregate consumption. ε denotes the elasticity of marginal utility
Let the budget constraint be represented as

$$\dot{K}_t = w_t\,(1-u)\,H_t + r_t\,(1-v)\,K_t - nK_t - \delta K_t - \ C_t - N_t \tag{3.26}$$

Where N_t denotes the share of output used for research. w_t is the wage rate and r_t is the rate of returns

The current value Hamiltonian is

$$\bar{H} = U(C_t)+\mu[w_t\,(1-u)\,H_t+r_t\,(1-v)\,K_t-nK_t-\delta K_t-\ C_t-N_t] +\psi\left[\xi\left((uH_t)^{\beta}\,(vK_t)^{1-\beta}\right)\right] \tag{3.27}$$

There are two state variable denoted by physical capital and human capital and the associated co state variable μ and ψ respectively.

The necessary first order condition is

$$(A_t c_t)^{-\varepsilon} = \mu \tag{3.28}$$

where $c_t = \frac{C_t}{A_t}$.

$$\dot{\mu} = \mu\theta - [(1-v)\,r_t\ \ -\delta]\,\mu - \psi\left[\xi\left((uH_t)^{\beta}\,v^{1-\beta}\,(1-\beta)\,K_t^{-\beta}\right)\right] \tag{3.29}$$

$$\dot{\psi} = \psi\theta - [w_t\,(1-u)]\,\mu - \psi\left[\xi\left(u^{\beta}\beta H_t^{\beta-1}\,(vK_t)^{1-\beta}\right)\right] \tag{3.30}$$

$$\frac{\partial H}{\partial v} = \mu r K_t = \psi\left[\xi\left((uH_t)^{\beta}\,(K_t)^{1-\beta}\,(1-\beta)\,v^{-\beta}\right)\right] \tag{3.31}$$

where

$$\frac{\psi}{\mu} = \frac{rK_t}{\left[\xi\left((uH_t)^{\beta}\,(K_t)^{1-\beta}\,(1-\beta)\,v^{-\beta}\right)\right]} = \frac{r}{\left[\xi\left((uH_t)^{\beta}\,(K_t)^{-\beta}\,(1-\beta)\,v^{-\beta}\right)\right]} \tag{3.32}$$

$$\frac{\partial \bar{H}}{\partial u} = \mu w H_t = \psi\left[\xi\left(H_t^{\beta}\beta u^{\beta-1}\,(vK_t)^{1-\beta}\right)\right] \tag{3.33}$$

where

$$\frac{\mu}{\psi} = \frac{\left[\xi\left(H_t^{\beta}\beta u^{\beta-1}\,(vK_t)^{1-\beta}\right)\right]}{wH} = \frac{\left[\xi\left(H_t^{\beta-1}\beta u^{\beta-1}\,(vK_t)^{1-\beta}\right)\right]}{w} \tag{3.34}$$

$$\frac{\dot{\mu}}{\mu} = \theta - r + \delta \tag{3.35}$$

$$\frac{\dot{\psi}}{\psi} = \theta - \xi \left(H_t^{\beta-1} \beta u^{\beta-1} \left(vK_t\right)^{1-\beta} \right) + \delta = \theta - \xi \left(\beta \left(\frac{vK_t}{uH_t} \right)^{1-\beta} \right) \tag{3.36}$$

By equating equation(3.32) and (3.34) yields

$$\frac{K_t}{H_t} = \frac{w\left(1-\beta\right)u}{r\beta v} \tag{3.37}$$

the transversality condition can thus be expressed as

$$\lim_{t\to\infty} \left[(K_t)\mu \exp^{-\theta t} \right] = 0 \tag{3.38}$$

$$\lim_{t\to\infty} \left[(H_t)\psi \exp^{-\theta t} \right] = 0 \tag{3.39}$$

Since

$$\frac{\frac{d}{dt}\left(A_t c_t\right)^{-\varepsilon}}{\left(A_t c_t\right)^{-\varepsilon}} = -\varepsilon \left(\frac{\dot{A}_t}{A_t} + \frac{\dot{c}_t}{c_t} \right)$$

then the equation that determines consumption will imply that variables will be growing in their efficiency units according to the growth of innovation g_n is

$$\left(\frac{\dot{c}_t}{c_t} \right) = \left(\frac{a^2 \left(\frac{(1-v)K_t}{A_t(1-u)H_t} \right)^{a-1} - \delta - \theta}{\varepsilon} - g_n \right) \tag{3.40}$$

set $\left(\frac{\dot{c}_t}{c_t} \right) = 0$ so that the steady state interest rate gives

$$r = a^2 \left(\frac{(1-v)\,K_t}{A_t\,(1-u)\,H_t} \right)^{a-1} - \delta = \theta + \varepsilon g_n \tag{3.41}$$

This equation shows the steady state interest rate used to discount monopolist's profit

The budget constraint will be growing at the rate of innovation thus

$$\dot{k}_t = (1-u)\,H_t \left(\frac{(1-v)\,k_t}{A_t\,(1-u)\,H_t} \right)^a - c_t - \frac{N_t}{A_t} - (\delta + g_n + n+)\,k \tag{3.42}$$

Note that all variables expressed in the budget constriant are all represented in their efficiency unit.

Using equation (3.23), (3.37) and (3.12) the growth rate of human capital in per capita terms can be written as

$$g_h = \frac{\dot{h}}{h} = \xi u \left(\frac{(1-a)\left(\frac{y}{(1-u)H}\right)^a (1-\beta)}{r\beta} \right)^{1-\beta} \tag{3.43}$$

3.3 Steady State Growth Analysis.

Since the contribution of physical and human capital in both physical capital accumulation and human capital accumulation will be equal in the steady state, therefore the growth rate of human capital will equal the growth rate of innovation thus;

$$\left(\frac{\lambda\left(a(1-a)\frac{y}{(1-u)H}\right) - (\theta+n)(1-s)}{[\varepsilon\sigma+1](1-s)} \right)\sigma = \xi u \left(\frac{(1-a)\left(\frac{y_t}{(1-u)H_t}\right)^a (1-\beta)}{r\beta} \right)^{1-\beta} \tag{3.44}$$

where $\frac{y_t}{H_t} = (1-u)\left(\frac{\theta+\varepsilon g_n+\delta}{a^2}\right)^{\frac{a}{a-1}}$ can be determined using equation (3.7) and equation (3.41)

This equation shows the steady state equilibrium condition for human capital accumulation and for innovation. Hence both innovation and human capital matters for long run growth rate.

3.4 Conclusion.

This paper introduces physical and human capital accumulation into R&D growth model in a scenario where population is growing. Unlike Keith, Hung, & Pozzolo (2000) which argued that long run growth may be determined by human capital alone, here; I show that long run growth depends on both human capital accumulation and innovation. The reason why Arnorld and Blackburn et.al' s result are not robust is because they assumed that research sector uses only human capital input. Such assumption is hardly convincing because R&D uses a whole lot of input like physical capital, human capital etc.

Finally, this paper presented a scale invariant model where horizontal innovation does not have dilution effect on vertical innovation; rather it enhances vertical innovation by introducing new sectors from which vertical innovation can flourish.

Chapter 4

The Effects of Competition as Psychological Threat on Innovation and Growth.

4.1 Introduction.

Is competition good or bad for innovation and growth? The standard Schumpeterian literatures (see: Aghion and Howit (1992) , Grossman and Helpman (1991)) argued that the incentive to perform research by entrant firms depends on the monopoly rent earned by the incumbent monopolist who has access and exclusive right to appropriate from the leading edge innovation; often secured by patent. The limit of appropriation of the present discounted profit occurs when entrant firms develop superior technology that leapfrog the incumbent through the process of creative destruction. Thus Schumpeterian literature concluded unequivocally that intense competition would disincentivize the entrants from doing innovation for fear of being leapfrogged by the next entrant and thus have negative effect on innovation and growth.

But this prediction about the effects of competition on innovation runs contrary to the empirical works of Nickell (1996) and Blundell et al. (1995) which showed a positive relationship between product market competition and productivity growth within a firm or industry. Thier research has motivated theoretical model of R&D where product market competition has positive effect on innovation. For instance, Aghion , Harris and Vickers (2001) developed a model with no entry where only the incumbents compete in a Bertrand duopoly equilibrium with cost reducing view of innovation, where the value of each firm depends on technological gap across firms and not on the productivity level of innovation. Their framework eliminate leapfroging effect(which is the reason why standard Schumpeterian model predicted a negative effect of competition on innovation). Thus they argued that some amount of product market competition will lead to more innovation when firms are neck neck as the leader tries to escape competition to avoid his profit being dissipated by other firms. But competition by incumbent firms alone in a duopoly setting does not highlight an economic structure with firm entry. For instance, Nicoletti and Scarpetta (2003) showed that the high cost of entry and lower degree of turnover accounted for the lower growth rate of Europe compared to United state. So it would be interesting to study how cost of entry determine the nature of competition and innovation.

The current chapter provides the rationale for exogenous threat of firm entry. I endogenise firm entry and show that the reason why firm entry is represented as an exogenous threat is because in a Nash equilibrium, incumbents who could engage in innovation would have technological advantage over entrants(Otherwise, incumbents could not innovate due to Arrow's effect). Hence, when technological advantage is large, incumbents would raise their R&D effort to deter firm entry. Furthermore I argue that there is an implicit psychological threat that the incumbent feel to make him innovate and escape competition when he has technological advantage. This framework is supported by the empirical work of Goolsbee and Syverson (2004) who examined how incumbent behavior changes in response to exogenous changes in potential entry that otherwise have no effect on current competitive conditions. They found that incumbent airlines cut their fares when an entrant merely annouces thier intention of entry, even before actual entry.

The rest of the paper proceeds as follows: in section 4.2 I introduce innovation by incumbent and entrants and show that incumbent have technological advantage over entrants. In section 4.3 I determine the equilibrium outcome which shows that only incumbents innovate in equilibrium. Then I argue that there is an implicit pyschological threat that makes the incumbent to innovate when he has technological advantage. Finally I offer conclusion.

4.2 Model

The setup in this paper is built to challenge the basic Schumpenterian growth framework of Aghion and Howitt (1992) which argued that only the entrant firms do innovation (and never the incumbent firm).

Their model predicted a continual leapfrogging of leadership position within an industry as the incumbent is replaced at the time of the next quality improvement by an entrant competitor, who is subsequently replaced by another entrant. Hence their model predicted that intense competition will disuade firms from doing innovation which leads to negative effect on innovation, as continual leapfrogging reduces the size of profit that the leader(a monopolist) gains. Because thier model runs contrary to the real world experience where quality improvements are carried out mostly by existing incumbents firms; and where competition has a positive effect on growth; the present model is built to capture this real world experience.

The present model considers an economy with industries indexed by $j \in (0, 1)$. The final goods sector is perfectly competitive and is produced using a continuum of different intermediate goods from each industry. The total endowment of labour in the economy is denoted by $L_t = e^{n_t}$ which is supplied inelastically and n_t is exogenous population growth. In each intermediate input industry, firms can devote R&D resources to improve the quality of of industry's intermediate input. By improving on the current best-quality intermediate input produced in an industry, a successful R&D firm earns monopoly profits from selling its leading-edge intermediate input to final good producers. The production process implicity imposes that only highest quality intermediate good input will be used in the production of the final goods. Over time, as the quality of intermediate inputs used in final good production rises, workers become more productive, and thus R&D fuels per capita consumption growth.

4.2.1 The Consumer Sector

I assume that economy is populated by identical individuals who supply labour in exchange for wage and recieve interest income on assets, buys goods for consumption and save by accumulating assets.

Each consumer maximizes a familiar expression for utility

$$U(C_t) \equiv \int_0^\infty \left(\frac{C_t^{1-\varepsilon} - 1}{1-\varepsilon} \right) e^{-\rho t} dt \tag{4.1}$$

C_t is the consumer's final good consumption at time t, $\rho > 0$ is the subjective discount rate, and $\varepsilon > 0$ is the constant elasticity of marginal utility with respect to consumption. Maximizing the utility subject to the consumer's intertemporal budget constraint yields the usual intertemporal consumer optimization condition;

$$\frac{\dot{C}_t}{C_t} = \frac{r_t - \rho}{\varepsilon} \tag{4.2}$$

where r_t is the equilibrium interest rate at time t.

4.2.2 Product Markets

The production of output depends on inputs of different intermediate products that are produced by firms with access to leading edge innovation. Let the production function for final good be denoted by

$$Y_t = A \int_0^1 (q_{jt} x_{jt})^a \, dj, \tag{4.3}$$

where $0 < a < 1$,

x_{jt} denotes the quantity of intermediate input of type j at time t and q_{jt} denotes the quality of the intermediate input used in the production process. The parameter A is the overall measure of productivity. 'The model starts off with growth engine that will depend on two forms of process innovation that leads to quality improvement namely: (1) Incremental innovation by incumbents and (2) innovation by entrants via creative destruction. But in equilibrium the model will show that only the incumbents will undertake innovation. For the time being, let q_{jt} be the quality of intermediate input j at time t. When innovation occurs via incremental innovation by an incumbent, the quality ladder for each intermediate good can be denoted as

$$q_{jt} = \phi^{z_j} \tag{4.4}$$

where $\phi > 1$ and z_j denotes the number of incremental innovations on the intermediate good j.

Similarly if innovation occurs via creative destruction by an entrant the quality ladder for each intermediate good can be denoted as

$$q_{jt} = \theta^{z_j} \tag{4.5}$$

where $\theta > \phi$ and z_j denotes the number of quality improvement by entrant in industry j. The assumption that $\theta > \phi$ captures the intuition that innovation by entrants are more radical than innovation by incumbents. Empirical evidence for this assumption can be found in the work of Akcigit and Kerr (2010) who showed from the US Census of Manufacturers that large firms engage more in exploitative R&D, while small firms do exploratory R&D (defined similarly to the notions of incremental and radical R&D here).

By taking into account the aggregate quality index $\sigma_t \equiv \int_0^1 q_{jt}^{\frac{a}{1-a}} dj$, the production function of equation (4.3) can be re-written as

$$Y_t = A\sigma_t^{1-a} (x_t)^a \, , \tag{4.6}$$

Each intermediate input is produced using one unit of physical capital

$$x_{jt} = K_{jt} \tag{4.7}$$

where K_j is the amount of physical capital used as input in industry j .

Any firm with access to the leading edge innovation becomes the monopolist and produces the highest quality intermediate good. The final goods sector is perfectly competitive so the price of each input equal its marginal product; $aA\sigma_t^{1-a} (x_{jt})^{a-1}$. Hence the profit maximization problem in sector j is ;

$$\Pi_{jt} = aA\sigma_t^{1-a} (x_{jt})^a - R_t x_{jt} \tag{4.8}$$

the cost to the monopolist is $R_t x_j$; where R is the a given rental rate. The equilibrium demand function is the sum of all sectors' demands $\int_0^1 x_j = \int_0^1 K_j$ which must equal the supply of capital, thus the equilibrium demand can be written in terms of capital as

$$K_t = \left(\frac{a^2}{R_t} \right)^{\frac{1}{1-a}} A^{\frac{1}{1-a}} \sigma_t \tag{4.9}$$

the equilirium rental rate of capital in per efficiency unit is

$$R_t = a^2 \left(\frac{K_t}{\sigma_t}\right)^{a-1} A \tag{4.10}$$

equation (4.8) and (4.10) allows us to determine the equilibrium profit flow as

$$\Pi_t = \pi\left(k_t\right)\sigma_t \tag{4.11}$$

where the productivity adjusted profit function per effective spillover is

$$\pi\left(k_t\right) = A^{\frac{1}{1-a}} a(1-a)\left(\frac{k_t}{\sigma_t}\right)^a \tag{4.12}$$

4.2.3 Innovation

In each industry j, there are two types of firm that can engage in R&D namely the incumbent and the entrants. Both the incumbent and the entrants make their R&D expenditure decisions simultaneously and independently and are free to adjust their expenditures at any point in time. This model allows for free entry by entrants into the R&D race and all follower firms has the same R&D technology. There is perfect competition among entrants in each industry so the the R&D expenditures of each entrant will be negligible. Both entrants and the incumbent in the economy uses labour $L_t = e^{nt}$ as an input in R&D; where n_t denotes exogenous population growth.

The market clearing condition for the larbour market implies

$$N_t l_t = L_m + L_e \tag{4.13}$$

where L_m denotes the flow of resources devoted to R&D by incumbents; while L_e denotes the flow of resources devoted to R&D by entrants

The aggregate quality index can be expressed in terms of contribution due to incumbents and entrants as follows

$$\sigma_t = \sigma_e + \sigma_m \tag{4.14}$$

where σ_e denotes the size of aggregate quality index due to entrants innovation, while σ_m denotes the size of aggregate quality index due to incumbents innovation.

Let the instantenous probability of R&D success by entrants be defined as

$$I_e = \frac{L_e}{\sigma_e} = \frac{N_t l_e}{\sigma_e} \tag{4.15}$$

so that increase in N_t will raise the number of entrants engage in R&D . Since this model assumes free entry $N = +\infty$. $\sigma_e \equiv \int_0^1 \theta^{z_j \frac{a}{1-a}} dj$ denotes size of aggregate quality index due to entrants innovation. Notice that the flow of resources L_e is deflated by the size of this aggregate quality index σ_e in order to capture the specific nature of complexity implicit in entrants innovation which will be different from the nature of complexity implicit in incumbents innovation. Sagestrom (1998) was the first to introduce complexity argument in R&D growth literature which captures the intuition that growth in economy which raises the quality index σ_t raises the complexity of innovation over time. But his model introduced complexity argument on entrant's innovation which is the only engine of growth in that model. Sagestrom and Zolnierek(1998) introduced complexity argument in a model with both incumbent and entrants innovation but assumed that both incumbent and entrants faced the same type of complexity. My present model differs from thiers by making the complexity argument specific to the nature of innovation.

Because leaders are already on the technology frontier, it is easier for them to advance the frontier than entrant firms. Hence the instantenous probability of R&D success by incumbents is defined as

$$I_m = \frac{L_m}{\sigma_m} \tag{4.16}$$

where $\sigma_m = \int_0^1 \phi^{\frac{a}{1-a} z_j} dj$ denotes the size of aggregate quality index due to incumbents innovation. Similarly the flow of resources L_m is deflated by a share of this aggregate quality index σ_m in order to capture the specific nature of complexity implicit in incumbent innovation. I assume that $\theta > \phi$, so that the complexity argument implies that incumbents has technological advantage over entrants. Alternatively we can interprete the specific nature of complexity between entrants and incumbents as follows; as innovation grows which raises the quality index σ, it is more difficult to engage in radical innovation θ^{z_j} than incremental innovation ϕ^{z_j}. Schumpeterian R&D literature is clear on the issue that only entrants engage in R&D in equilibrium due to Arrow effect[1] except if the incumbent has technological advantage. Sagestrom and Zolnierek(1998) studied a model with both entrants and incumbent faced with the same type of complexity, but introduced exogenous parameter to indicate incumbent's technological advantage over entrants. By making complexity argument specific to the nature of innovation, the present model endogenised technological advantage of the incumbent.

4.3 The balance Growth equilibrium

[1]see: Aghion & Howit (1992)

In this section I analyze the balanced growth equilibrium properties of the model. When per-capita consumption grows over time at a constant rate. Equation(4.2) implies that the market interest rate r must be constant over time. Let $V_{m(z)}$ represents the expected discounted profits earned by a leader that sells a quality z intermediate input. Likewise, let $V_{e(z)}$ represents the expected discounted profits earned by an entrant when the state-of-the-art quality in its industry is z. To maximize expected discounted profits, both incumbents and entrants must solve stochastic optimal control problems where the state variable z in each industry j is a Poisson jump process with intensity $I_m + I_e$ and V^d is the magnitude which show the value that either the incumbent or the entrant can earn when the state of the art quality in its industry is higher than z.

The relevant Hamilton-Jacobi-Bellman equation for each incumbent is

$$rV_{m(z)} = \Pi_{(z)} + I_m \left[V^d_{m(z)} - V_{m(z)}\right] - L_{m(z)} - I_e V_{m(z)}$$

This equation shows that the incumbent earns the profit flow $\Pi_{(z)}$ and incurs the R&D costs $L_{m(z)}$ today. With instantaneous probability I_m, the incumbent innovates and learns how to produce an intermediate input higher than quality z . The equation states that the maximized expected returns on an incumbent firm's stock must equal the return on an equal-sized investment in a riskless bond.

Similarly the relevant Hamilton-Jacobi-Bellman equation for entrants is

$$rV_{e(z)} = I_e \left[V^d_{m(z)} - V_{e(z)}\right] - L_{e(z)} - I_m V_{e(z)} \tag{4.17}$$

followers incur the R&D costs $L_{e(z)}$ today but earns no profit flow. With instantaneous probability I_e, the entrants innovate become a leader, and learns how to produce an intermediate input higher than quality z . This equation states that the maximized expected return on an entrant firm's stock must equal the return on an equal-sized investment in a riskless bond. Since this model assumes free entry $N = +\infty$. Thus, the individual contribution of any particular follower firm i to the aggregate innovation rate of all entrants is negligible, and

$$V_{e(z)} = 0 \tag{4.18}$$

Free entry condition implies that the net return for entrants R&D is

$$I_e V^d_{m(z)} - L_{e(z)} = 0 \tag{4.19}$$

This condition says that the net return from entrants is zero.

Likewise the net return for the incumbent is

$$I_m \left[V^d_{m(z)} - V_{m(z)}\right] - L_{m(z)} - I_e V_{m(z)} \tag{4.20}$$

Just for the moment if we assume that both entrants and the incumbent are equally good at conducting research, that is; if $I_e = I_m > 0$ and $L_{e(z)} = L_{m(z)}$, then the free entry condition in equation (4.19) will imply that the incumbent's net return is negative since the net return for entrants is zero. If the incumbent takes entrants R&D expenditure as given, an increase in the incumbent's R&D outlay raises the total R&D effort thereby lowers the incumbent's net return. If entrants also undertake a given amount of R&D, the leader's best response is to shut down research. This is called the Arrow's effect and is the reason why incumbents dont engage in R&D in the standard Schumpeterian model.

In order to encourage incumbent's R&D, the present model assumes that incumbents have cost advantage over entrants. That is $I_e = \frac{L_e}{\sigma_e} \neq I_m = \frac{L_m}{\sigma_m}$ because of the assumption that $\theta > \phi$, which implies that the growth rate of innovation which raises the quality index σ has specific complexity impact depending on the nature of the innovation. That is, it is more difficult to engage in radical innovation θ^{z_j} by entrants than incremental innovation ϕ^{z_j} by incumbent. Under this assumption, the free entry condition (4.19) will still imply that entrants net return is zero. But $I_m V^d_{m(z)} - L_{m(z)} > 0$ instead of zero. Hence the incumbent is now encourage to engage in R&D until he drives the entrants outside the market.

Henceforth, I shall focus attention to the scenario where entrants do not innovate, i.e $I_e = 0$, Hence the no arbitrage equation for the incumbent can be rewritten as

$$rV_{m(z)} = \Pi_{(z)} + I_m \left[V^d_{m(z)} - V_{m(z)}\right] - L_{m(z)} \tag{4.21}$$

By differentiating $V_{m(z)}$ with respect to I_m yields the first order condition

$$\left[V^d_{m(z)} - V_{m(z)}\right] = \sigma_m = \frac{L_m}{I_m} \tag{4.22}$$

notice that this equation shows that the marginal cost σ_m is equated to the incremental value $\left[V^d_{m(z)} - V_{m(z)}\right]$ rather than the full present value $V_{m(z)}$ since the innovating incumbent does not value the expropriation of his own monopoly profit. Finally substitute equation (4.22) into equation(4.21) to determine $V_{m(z)}$ *as*

$$V_{m(z)} = \frac{\Pi_{(z)}}{r} \tag{4.23}$$

4.3.1 The Effects of Competition on the incumbent

It is important to remind the readers that incumbent is able to deter firm entry in this model because he has technological advantage over entrants. Furthermore, I argue that there is an implicit psychological threat that the incumbent feel to make him innovate and escape competition when he has technological advantage. Let the incumbent's e pschological threat of firm entry be modelled as exogenous parameter χ. Equation(4.23) shows the full expected value of the incumbent with no real potential of actual entry. The incumbent at the technological frontier at time $t > 0$ would face no exogenous threat of entry because σ is positive, hence entrants as well as the incumbents face difficulty to innovate and the incumbent has technological advantage to deter entrants. Therefore the incumbent earns the profit $\Pi_{(z)} = \pi(k)\,\sigma_m$. But at time $t = 0$, the profit earned by the incumbent is $\Pi_{(z)} = \pi(K)$, because $\sigma = 0$ and no difficulty is faced by any firm to innovate. But the incumbent earns this profit only at a probability $(1 - \chi)$ which is the probability of no firm entry. Otherwise Arrow effect implies that incumbets wont innovate and entrant would leapfrog the incumbent. Using this intuition it is plausible to interprete the first order condition in equation (4.22) in the light of an incremental value derived from the time the incumbent has no technological advantage but faces no entry to the time he has technologocal advantage but deter entry. Thus equation (4.22) can be written as ;

$$\frac{\pi(k)\,\sigma_m}{r} - \frac{(1-\chi)\,\pi(k)}{r} = \sigma_m \tag{4.24}$$

Finally the steady state interest when t is constant is

$$r = \pi(k)\left[1 - (1-\chi)\,\sigma_m^{-1}\right] \tag{4.25}$$

Diffrentiate r with respect to χ shows the effect of threat of entry as

$$\frac{\partial r}{\partial \chi} = \frac{\pi(k)}{\sigma_m} > 0 \tag{4.26}$$

Increase in χ has a positive effect on growth because incumbents with technological advantage over entrants innovate in order to escape firm entry.

Finally long run growth is determined by exogenous population growth which can be derived by differentiating equation (4.16) over time as

$$\frac{\dot{I}_m}{I_m} = \frac{\dot{L}_m}{L_m} - \frac{\dot{\sigma}_m}{\sigma_m} = 0 \tag{4.27}$$

where $\lambda_t = \int_0^1 z_j dj$, $\lambda_t = I_m$ and $\bar{\phi} = \log\phi$

$$I_m = \frac{n(1-a)}{a\bar{\phi}} \tag{4.28}$$

where $n = \frac{\dot{L}_m}{L_m}$ is exogenous population growth.

4.4 Conclusion

This paper examine the effect of threat of competition on growth in a context where incumbents innovate because they face psychological threat of entry by new firms. I found that increase in the threat of competition has positive effect on the balanced growth innovation rate. Finally the model predicts that long run growth will depend on exogenous population growth so the model like Sagestrom (1998), Jones (1995b) is scale invariant.

Chapter 5

COMPARISON UTILITY FUNCTION AND REASEARCH JOINT VENTURE.

5.1 Introduction.

In 2010, the European commision extended the scope of *Block Exemption of R&D Agreement thus;*

> With a view to facilitating innovation in Europe, the Commission has considerably extended the scope of the R&D Block Exemption Regulation, which now not only covers R&D activities carried out jointly but also so-called 'paid-for research' agreements where one party finances the R&D activities carried out by the other party. In addition, the new Regulation gives parties more scope to jointly exploit the R&D results.

Block exemption of R&D agreement provides an organising mode for R&D among firms active in the same market and it is often lauded to prevent duplication of research which according to Jones (1995) leads to decreasing return in knowledge spillover. Cozzi and Tarola (2006) showed that duplication of research was due to information transmission lag; and that the motive to reduce it is an incentive for RJV. The theoretical works of Brander and Spencer (1983); Spence (1984); Katz (1986); Kamien, Muller, and Zang (1992) all argued in favour of Research Joint Venture(RJV), that it will have positive welfare effects.

One key refutation of RJV is that it could lead to cooperative firms in R&D colluding in the product market. Hence there is a trade off between enhanced innovation and reduced competition.

The empirical work of Sovinsky and Helland (2012) examined the question on whether product market collusion is an incentive for *Research joint Venture* by looking at how changes in the antitrust policy affects collusive benefit without affecting Research Joint Ventures. They came up with the conclusion as follows;

> we find the decision to join a RJV is impacted by the policy change. We also find the magnitude is significant: the policy change resulted in an

> average drop in the probability of joining a RJV of 34% among telecommunications firms, 33% among computer and semiconductor manufacturers, and 27% among petroleum refining firms. Our results are consistent with research joint ventures serving a collusive function.

Other empirical literatures on the subject includes Snyder and Vonortas (2005) who showed that multiproject contact can enhance explicit collusion by serving to bundle markets, which reduces the heterogeneity of firms private information thereby making collusive agreements more efficient. Duso T. , Röller L-H. and Seldeslachts J (2014) examined on whether firms engaging in RJV experience a fall in market share as a result of less competition and found that RJV leads to product market collusion.

From the theoretical point of view, Industrial organizational literatures such as Cabral (2000) and Martin (1995) argued that RJV could lead to participating firms owning common assets, hence has shared interest and this provides a punishment device for non collusive memeber. Cooper and Ross (2009) argued that RJV could lead to collusion because firms that participate in several market could punish other firms who deviate from collusion through price war in markets where they are both active. Miyagiwa (2009); Motchenkova and Rus (2011) aruges that RJV may reduce cost asymmetries among firms thereby making product market agreements more stable

The aim of this paper is to provide a macroeconomic framework which examines the impact of RJV on economic growth when RJV firms also engage in collusion in the product market. To simply the analysis, I introduce a duopoly market structure where the duopolists are level in terms of technological progess in their sector. Thus they have more incentive to collude in other to avoid Bertrand competition. Then I made a novel contribution by introducing consumption externality under duopoly market structure with level sectors to show that duopolist in a level sector may also collude in the product market in order to internalize consumption externality that no single firm can internalize, in an economy where consumers' utility depends not only on the level of their consumption but also on how their consumption compares to some reference stock widely known as "keeping up with the Joneses." By colluding to internalize the reference stock, the duopolist can operate a dynamic pricing model to encourage habit formation. That is, they reduce their price when the reference stock is low in order to encourage consumption but gradually increase their price as the reference stock increases. This process encourages individuals who would have been detered by high price of a product to learn how to spend more as thier habit towards the product increases. This paper features two key parameters denoted by γ which measures the importance individuals place on the reference stock and the parameter J which measures the level of the reference stock. I found that increase in γ has a negative effect on growth because it raises the shadow cost of dynamic pricing model. But increase in J has a positive effect on growth because it raises the price that the duopolist can charge as habit formation rises. Hence it raises incentive for more innovation.

Other related literatures include; Turnovsky and Liu (2005) who introduced consumption externality into a neoclassical technology but emphasised on characterizing the equilibrium and efficiency issues. Ljungqvist and Uhlig (2000) and Dupor and Liu (2003) showed that consumption externality is a major cause of inefficiency as its competitive equilibria breaks even with pareto optimality. Outside growth literatures, consumption externality has been introduced in asset pricing literatures to explain equity premium puzzle (see: Abel (1990), Constantinides (1990), Gali (1994), and Campbell and Cochrane (1995) among others). Easterlin (1995) empirical study shows that unilateral income growth does not increase happiness which supports consumption externality models. Clark and Oswald (1996) offer empirical evidence to support that well being is dependent on income comparison. Akerlof and Yellen (1990) happiness depend on relative well being (see also: Carroll 1998 and Frank 1985). Campbell and Cochrane (1999) follow similar framework to study savings and consumption behaviour .

The rest of the paper proceeds as follows: in section 5.2, I present the utility function with non time separable preference as well as the reference stock. Then I show how RJV internalize the evolution of the referenc stock in order to maximize profit by encouraging habit formation. In section 5.3 I show the impact of the reference stock on the steady state. Finally I offer conclusion.

5.2 Model

This paper features a duopoly in a level sector where the two firms collude in order to avoid price competition and to enable them internalize the evolution of the reference stock. Internalizing the evolution of the reference stock implies that they operate a dynamic pricing model in order to encourage habit formation.

5.2.1 Utility function

Given L identical households where population is constant through time. I assume that household's utility depends not only on the absolute level of consumption but also on a reference stock. My utility function follows Carroll , Overland and Weil (1997) who has earlier considered an interdependent preference using the conventional AK production function.

The representative household maximizes the discounted, infinite stream of utility U, given by

$$U = \max \int_t^{\infty} U(C_t, Z_t) e^{-\theta t} dt. \tag{5.1}$$

θ is the rate of time preference, where C_t denotes aggregate goods consumption of goods and Z_t represents the reference stock.

The instantenous utility for each household is given by

$$U(C_t, z_t) = \int_0^1 \left(\frac{C_{jt}\ Z_{jt}^{-\gamma}}{1-v} \right)^{1-v} dj, \tag{5.2}$$

where $j \in (0, 1)$ denotes the sectors.

Notice that this equation can be rewritten as $\int_0^1 \left(\frac{C_{jt}^{1-\gamma} \left(\frac{(C_{jt})}{Z_{jt}} \right)^{\gamma}}{1-v} \right)^{1-v} dj$, which shows more clearly that utility does not depend only on absolute level of consumption but also on consumption relative to the reference stock. Let $0 < \gamma < 1$. I assume that $v = \frac{1}{1-\gamma}$, which implies that $v > 1$[1]. v determines the elasticity of marginal utility; and γ determines the importance placed on the reference stock and thus characterises the effects of the reference stock on the wellbeing of an individual. Here it exerts negative externality on the individual absolute level of consumption. For instance, the standard isoelastic utility will imply a rapid decline in the marginal utility when $v > 1$ in response to increase in individual's consumption level . Hence an individual with $v > 1$ will not be able to consume relative to the reference stock unless there is a non time separable preference which exerts a negative exerternality on individual's absolute consumption level. In general the interaction between v and γ determine the intertemporal elasticity of substitution which may vary depending on the time horizon considered in equation (5.3). If the time horizon is zero then habit does not evolve but remains fixed and the intertemporal elasticity of substitution would be $\frac{1}{v}$. If on the other hand, the time horizon goes to infinity, then habit fully adjusts to change in consumption. Setting $C_{jt} = Z_{jt}$ implies a long run intertemporal elasticity of substitution equal to $\frac{1}{v+\gamma(1-v)}$. Thus the intertemporal elasticity of substitution will be greater in the case of comparison utility function where $0 < \gamma < 1$ than the case of standard isoelastic utility function when $\gamma = 0$ which implies $\frac{1}{v} < 1$.

Proof. since elasticity of intertemporal substitution(EIS) is the reciprocal of the proportionate change in the magnitude of the elasticity of marginal utility with respect to change in consumpution. Given that relative risk aversion $RRA = -\frac{U_{cc}c^{\cdot} + U_{cz}z^{\cdot}}{U_c}$. Since$U_c^{\cdot} = U_{cc}c^{\cdot} + U_{cz}z^{\cdot}$ and $\frac{U_{cc}}{U_c} = \frac{-v}{c_i}$, $\frac{U_{cz}}{U_c} = \frac{\gamma(v-1)}{z_i}$. Then $EIS = \frac{1}{v+\gamma(1-v)}$. If $\gamma = 0$ then $EIS = \frac{1}{v}$. For $0 < \gamma < 1$ and $v > 1$ then $\frac{1}{v+\gamma(1-v)} > \frac{1}{v}$ □

Each C_j is the sum of two goods produced by a duopolist in sector j

[1] $v > 1$ is empirically supported by Guvenen(2006)

$$C_{jt} = C_{Aj} + C_{Bj} \tag{5.3}$$

where $i = A, B$

5.2.2 The Reference Stock

The reference stock is defined as the average level of past consumption of others. An individual who consumes relative to the reference stock ignores the effect that his present consumption induces on his future utility through its effect on the average consumption. So the household assumes that the reference stock will be constant if he raises his net utility.

Let the reference stock for each household be

$$Z_{jt} = J \int_{-\infty}^{t} e^{Jt} C_t dt \tag{5.4}$$

where C_t is the average level of consumption in the economy. $J > 0$ is a parameter that captures the relative weight of consumption at different times . If J is small then less weight is placed on past consumption and an individual who consumes relative to the references stock will consume more today under the assumption of $v > 1$, hence the level of the reference stock is low. But if J is large, more weight is placed on past consumption and an individual who consumes relative to the reference stock will consume less today, hence the level of the reference stock is high . Therefore the parameter J shows the influence of current consumption in determining future reference stock.

Differentiating equation (5.4) over time yields the evolution of the reference stock;

$$\dot{Z}_{jt} = J \left(C_t - Z_{jt} \right) \tag{5.5}$$

Individuals maximize their static utility by spreading their expenditure E_t equally across all product lines j. The primal problem is to
maximize

$$U(C_t, Z_t) \tag{5.6}$$

subject to

$$E_t = \int_0^1 \rho_{jt} C_{jt} dj \tag{5.7}$$

where E_t is the expenditure and ρ_{jt} represents price of good C_{jt}. Solving for c_{jt} yields the marshallian demand function for c_{jt} units of good j (see: Appendix A)

$$c_{jt} = \frac{E_t \rho_{jt}^{-\frac{1}{v}} Z_{jt}^{\frac{\gamma(v-1)}{v}} \quad A_{jt}^{\frac{1-v}{v}}}{\int_0^1 \left(\frac{\rho_{jt} Z_{jt}^{\gamma}}{A_{jt}}\right)^{\frac{v-1}{v}} dj} \tag{5.8}$$

where $c_{jt} = \frac{C_{jt}}{A_{jt}}$ represents demand good j adjusted by the technological level , the aggregate equilibrium price index is given by (See also: Appendix A)

$$P_t = \left[\int_0^1 \left(\frac{\rho_{jt} Z_{jt}^{\gamma}}{A_{jt}}\right)^{\frac{v-1}{v}} dj\right]^{\frac{v}{v-1}} \tag{5.9}$$

Finally the condition for equilibrium in the final goods market implies

$$E_t = \int_0^1 \rho_{jt} c_{jt} dj = \rho_t^{\frac{v-1}{v}} Z_t^{\frac{\gamma(v-1)}{v}} A_t^{\frac{1-v}{v}} \left[P_t\right]^{-\left(\frac{v-1}{v}\right)} E_t = P_t c_t \tag{5.10}$$

The optimization problem for the household can be written by substituting equation (5.10) into equation (5.1) thus.

$$\max \int_0^\infty \frac{E}{[P_t]} e^{-\theta t} dt. \tag{5.11}$$

expressed in log form as

$$\max \int_0^\infty \log E_t - \log\left[P_t\right] e^{-\theta t} dt. \tag{5.12}$$

subject to an intertemporal budget constraint.

$$\dot{\alpha} = w_t L + r_t.\alpha - \ E_t \tag{5.13}$$

w_t is the wage rate and L is the society's fixed labour supply which is supplied inelastically. r_t is the rate of returns and α denotes assets.

By maximization, spending grows according to

$$\frac{\dot{E}_t}{E_t} = r_t - \theta \tag{5.14}$$

This is the conventional Euler equation which holds for every household. We can normalize price to ensure that nominal spending stays constant through time by setting $E_t = 1$, so that equation(5.14) implies

$$r_t = \theta \tag{5.15}$$

5.2.3 Production Function

Each product j has the potency of being produced into differentiated qualities due to vertical improvement in that product. Utility for vertically differentiated goods will generate demand and hence we can study innovation that generate quality advancement. Let the quality A_{jt} of any given product in industry j be marked by the

generation m of that product. I assume that each new generation provides σtimes better services than the previous generation. Thus firms in sector j uses labour as the only input, according to a constant returns production function, and takes the wage rate as given. Therefore one unit of labour currently employed by sector j generates an output flow equal to

$$A_{jt} = A_{jt} = \sigma^{m_{jt}} \tag{5.16}$$

where $\sigma > 1$ is a parameter that measures the size of innovation.

5.2.4 Equilibrium in Level Sectors

In this model, I consider a duopolist in a level sector who collude as to maximize thier joint profit and share the proceeds. One reason for duopolist in a level sector to collude is to avoid price competition. Because this will imply that equilibrium price will fall to a unit cost and each firm will earn zero profit. The present model argues later that there can be other incentive for a duopolist to collude in the product market. In equilibrium all sectors will act the same so I shall hencefort drop subscript j in order to consider equilibrium conditions.

Equilibrium profit maximization implies

$$\pi_t = (\rho_t - R_t)\, c_t \tag{5.17}$$

By substituting c_t from equation (5.8) into equation (5.17) yields the profit function

$$\pi_t = Z_t^{\frac{\gamma(v-1)}{v}} A_t^{\frac{1-v}{v}} [P_t]^{-(\frac{v-1}{v})} E_t(\rho_t^{\frac{v-1}{v}} - R_t\rho_t^{-\frac{1}{v}}) \tag{5.18}$$

5.2.5 Innovation.

The research sector comprises of a duopoly with the cost function

$$n_{it} \tag{5.19}$$

where $i = A, B$ and $n_t = \frac{N_t}{A_t}$ is the share of output adjusted productivity used in research effort

with the total cost function given as

$$n_t = n_A + n_B \tag{5.20}$$

I assume that firms collude in the research sector so that they can collude in the product market in order to internalize the evolution of the reference stock and encourage habit formation. This specification captures the intuition in Sovinsky and Helland (2012) whose empirical result found that collusion in the product market is often an incentive for research joint venture.

The growth rate of technological progress is written as

$$g_t = \frac{\dot{A}_t}{A_t} = \lambda n_t \sigma \tag{5.21}$$

where $\left[1 - \frac{1}{A_t}\right] = \sigma$ is the size of innovation and λ is the productive parameter for innovation.

5.2.6 Other Incentive for collusion in the Product Market.

Consumption externality model has been studied under both AK model and Romer type endogenous growth model which employed perfect competitive market structure. But much of the contribution has focused on the effects on transitional dynamics [see: Carroll , Overland and Weil. (1997), Alvarez-Cuadrado et.al (2004)]. Here I introduce consumption externality in order to explain its long run growth rate effect as an endogenous equilibrium outcome of the behaviour under imperfect market structure. I assume that each individual duopolist cannot internalize the reference stock, hence there is an incentive to collude in order to internalize the reference stock. By colluding to internalize the reference stock, the duopolist can encourage habit formation.

Hence, I allow collusive firms to exploit the impact of consumption externality on the demand behaviour of household and share the profit. Thus the present value of profits that can be appropriated by collusive firms is therefore subject to equation (5.5) since they internalize the impact of consumption externality on the behaviour of household when choosing their price . The optimization behavior of collusive firms is thus formulated as follows:

$$V_t = \max \int_t^{\infty} \pi_t e^{-\int_t^{\tau}(r_\omega)d\omega} d\tau \tag{5.22}$$

V_t is the value of innovation for collusive duopolist which is the present profit π flow. r_ω is the instantanous rate of interest at date ω. Thus using equation (5.8) the evolution of the reference stock can be rewritten as

$$\dot{Z}_t = J\left(\rho_t^{\frac{-1}{v}} Z_t^{\frac{\gamma(v-1)}{v}} A_t^{\frac{1-v}{v}} \left[P_t\right]^{-\left(\frac{v-1}{v}\right)} E_t - Z_t\right) \tag{5.23}$$

The current value Hamiltonian is therefore

$$H = Z_t^{\frac{\gamma(v-1)}{v}} A_t^{\frac{1-v}{v}} [P_t]^{-(\frac{v-1}{v})} E_t(\rho_t^{\frac{v-1}{v}} - R_t\rho_t^{-\frac{1}{v}}) + \Omega\left(J\left(\rho_t^{\frac{-1}{v}} Z_t^{\frac{\gamma(v-1)}{v}} A_t^{\frac{1-v}{v}} [P_t]^{-(\frac{v-1}{v})} E_t - Z_t\right)\right) \tag{5.24}$$

The state variable is the reference stock Z_t and the control variable is price ρ_t. Ω is the shadow price.

The first order condition is given as

$$\frac{\partial H}{\partial \rho_t} = \frac{v-1}{v}\rho_t^{-\frac{1}{v}} + R_t\frac{1}{v}\rho_t^{\frac{-(1+v)}{v}} - \Omega J\frac{1}{v}\rho_t^{\frac{-(1+v)}{v}} = 0 \tag{5.25}$$

Solving for ρ_t gives the optimal pricing formula as

$$\rho_t = \frac{1}{v-1}(\Omega J - R_t) \tag{5.26}$$

And the shadow value is

$$\Omega^{\cdot} = (r_t + J)\,\Omega - \left[\frac{\gamma(v-1)}{v} Z_t^{\frac{\gamma(v-1)-v}{v}} A_t^{\frac{1-v}{v}} [P_t]^{-(\frac{v-1}{v})} E\left(\rho_t^{\frac{v-1}{v}} - R_t\rho^{-\frac{1}{v}} + \Omega J\rho_t^{\frac{-1}{v}}\right)\right] \tag{5.27}$$

Solving for Ω implies

$$\Omega = \frac{\left[\frac{\gamma(v-1)}{v} Z_t^{\frac{\gamma(v-1)-v}{v}} A_t^{\frac{1-v}{v}} [P_t]^{-(\frac{v-1}{v})} E_t\left(\rho_t^{\frac{v-1}{v}} - R_t\rho_t^{-\frac{1}{v}} + \Omega J\rho_t^{\frac{-1}{v}}\right)\right]}{(r_t + J)} \tag{5.28}$$

Since $v > 1$ and $\gamma > 0$; it implies that $\Omega. > 0$

Notice that the last equation in the bracket in equation (5.28) implies

$$\left(\rho_t^{\frac{v-1}{v}} - R_t\rho_t^{-\frac{1}{v}} + \Omega J\rho_t^{\frac{-1}{v}}\right) = \rho_t v\left(\frac{1}{v}\rho_t^{\frac{-1}{v}} - R_t\frac{1}{v}\rho_t^{\frac{-1-v}{v}} + \Omega\frac{1}{v}J\rho_t^{\frac{-1-v}{v}}\right) = v\rho_t^{\frac{v-1}{v}} \tag{5.29}$$

so that we can rewrite equation (5.28) as

$$\Omega = \frac{\left[\gamma(v-1) Z_t^{\frac{\gamma(v-1)-v}{v}} A_t^{\frac{1-v}{v}} [P_t]^{-(\frac{v-1}{v})} E_t\rho_t^{\frac{v-1}{v}}\right]}{(r_t + J)} \tag{5.30}$$

Equation (5.2s) shows the dynamic pricing model rewritten as

$$\rho_t = \frac{1}{v-1}(\frac{\Omega J}{R_t} - 1)R_t \tag{5.31}$$

The term $(\frac{\Omega J}{R_t}-1)$ shows an additional cost that raise the cost of producing innovation which makes this equation deviate from the standard price markup on marginal cost given as $\rho_t = \frac{1}{v-1}R_t$, where R_t is the rental rate for producing innovation and the mark up is $\frac{1}{v-1}$ which is constant overtime. In the present setting, the markup is time varying and depends on the evolution of Ω.

Given $E = 1$, substitute equation (5.31) into equation (5.8) to get

$$c_t = \frac{1}{\rho} = \frac{v-1}{(\Omega J - R_t)} = \frac{v-1}{(\frac{\Omega J}{R_t}-1)R_t} \tag{5.32}$$

By substituting equation (5.32) into equation (5.17) yields

$$\pi = \left[1 - \frac{R_t\,(v-1)}{(\Omega J - R_t)}\right] = \left[1 - \frac{(v-1)}{(\frac{\Omega J}{R_t}-1)}\right] \tag{5.33}$$

The equilibrium aggregate price index implies in equation (5.9) implies that

$$P_t = \left(\frac{\rho_t Z_t^{\gamma}}{A_t}\right) \tag{5.34}$$

Since

$$A_t^{\frac{1-v}{v}}\,[P_t]^{-(\frac{v-1}{v})}\,E_t\rho_t^{\frac{v-1}{v}} = (Z_t^{\gamma})^{\frac{1-v}{v}} \tag{5.35}$$

where $E_t = 1$
we can rewrite equation (5.30) to

$$\Omega = \frac{\left[\gamma(v-1)\frac{1}{z_t}\right]}{(r_t + J)} \tag{5.36}$$

From equation (5.32) R_t can be written as

$$R_t = \frac{(1-v)}{c_t} + \Omega J \tag{5.37}$$

Substitute R_t into the profit function so that

$$\pi_{(t)} = [1 - (1-v) - (\Omega J)\,c_t] \tag{5.38}$$

Finally substitute equation (5.36) into equation (5.38) to get

$$\pi_{(t)} = \left[v - \left(\left[\frac{\gamma(v-1)}{(r_t + J)} \frac{c_t}{Z_t} \right] J \right) \right] \tag{5.39}$$

The equilibrium profit function derived here shows that the profit function depends on $\frac{c_t}{Z_t}$ ratio. Notice that if you set $\gamma = 0$ or $J = 0$, then the profit function collapses to the case of time separable preference where the profit function is denoted by $\pi_t = v$. Therefore the profit funtion above captures the internalization of the reference stock for habit formation.

5.3 The balance Growth equilibrium

In this section we analyze the balanced growth equilibrium properties of the model where per-capita consumption and the reference stock grows over time at a constant rate. So henceforth I drop subscript t. Equation(5.15) implies that the market interest rate r must be constant over time. Let V_m represents the expected discounted profits earned the duopolists who sells a quality z innovation. To maximize expected discounted profits, the duopolists must solve stochastic optimal control problems where the state variable m is a Poisson jump process with intensity $n = n_A + n_B$ and V_m^d is the magnitude which show the value the duopolists can earn when the state of the art quality in its industry is higher than m.

Since the duopolist engage in research joint venture the relevant Hamilton-Jacobi-Bellman equation for RJV is

$$rV_m = \pi_m + n_m \left[V_m^d - V_m \right] - N_m \tag{5.40}$$

This equation shows that the duopolists earn the joint profit flow π_m and incur the R&D costs N_m today. With instantaneous probability n_m, they learns how to produce the next quality innovation higher than quality m . This equation states that the maximized expected returns on the duopolists' stock must equal the return on an equal-sized investment in a riskless bond.

By differentiating V_m with respect to n_m yields the first order condition

$$\left[V_m^d - V_m \right] = A \tag{5.41}$$

this equation shows that the marginal cost A is equated to the incremental value $\left[V_m^d - V_m \right]$

Finally substitute equation (5.41) into equation(5.40) to determine V_m *as*

$$V_m = \frac{\pi_m}{r} \tag{5.42}$$

with $V_m^d = AV_m$, substitute equation (5.42) into equation (5.41) to determine r as

$$r = \pi_m \left[1 - \frac{1}{A}\right] \tag{5.43}$$

In the long run, the reference stock will be growing at the rate of technological progress thus

$$\dot{z} = J\left(c - z\right) - zg \tag{5.44}$$

Hence the steady state ratio $\frac{c}{z}$ is

$$\frac{c}{z} = \frac{J + g}{J} \tag{5.45}$$

Substitute equation (5.15), (5.39) and (5.45) into equation (5.43) to arrive at

$$\theta = \left[v - \left(\frac{\gamma(v-1)(J + \lambda n_m \sigma)}{(\theta + J)}\right)\right]\sigma \tag{5.46}$$

where $\left[1 - \frac{1}{A}\right] = \sigma$ and $g = \lambda n_m \sigma$
once we determine research intensity n_m it is straightforward to calculate long run growth using equation (5.21) hence

$$n_m = \frac{\sigma v\theta + \sigma vJ - \theta^2 - J\theta - \gamma\sigma J(v-1)}{\gamma\sigma^2\lambda(v-1)} \tag{5.47}$$

Finally the growth rate of innovation can be written in terms of parameters as

$$g = \left(\frac{\sigma v\theta + \sigma vJ - \theta^2 - J\theta - \gamma\sigma J(v-1)}{\gamma\sigma(v-1)}\right) \tag{5.48}$$

By differentiating the growth rate with respect to J yields

$$\frac{\partial g}{\partial J} = \left(\frac{\sigma v - \theta - \gamma\sigma(v-1)}{\gamma\sigma(v-1)}\right) > 0 \tag{5.49}$$

By differentiating the growth rate with respect to γ yields

$$\frac{\partial g}{\partial \gamma} = \left(\frac{-\sigma J(v-1)\gamma\sigma(v-1) - \sigma(v-1)}{\gamma\sigma(v-1)}\right) < 0 \tag{5.50}$$

This paper features two key parameters denoted by γ which measures the importance individuals place on the reference stock and the parameter J which measures the level of the reference stock. I found that increase in γ has a negative effect on growth because it raises the shadow cost of dynamic pricing model. But increase in J has a positive effect on growth because it raises the price that the duopolist can charge as habit formation rises. Hence it raises incentive for more innovation.

5.4 Conclusion

This paper introduced non time separable preference into R&D growth model to examine the effect of market collusion in RJV. I showed that firms may collude in order to encourage habit formation and maximize profit. On the one hand colluding in the product market comes with an associated cost due to shadow cost of dynamic optimal pricing. But on the other hand, I showed that when market collusion encourage habit formation, the collusive firms will raise profit, which in turn raises incentive for innovation.

Chapter 6
EPILOGUE

Now that we have come to the end of the inquiry made in this long thesis, it is now important to reflect on the journey and what we have learnt so far. Firstly, the obvious contradition between theories and empirics; on what are the determinants of long run growth have to some extent been resolved. For instance, this thesis showed that both physical capital and human capital matter for long run growth rate(see: chapter two and three).

For physical capital, this thesis shows that when learning is introduced into R&D growth model, it enhances the growth rate of income thereby raising demand for monopolist's goods, which raise incentive for innovation; where long run growth is determined by innovation. Although Aghion & Howitt (1998) have earlier identified that physical capital could affect long run growth through this indirect channel by using Schumpeterian framework with monopoly market structure; my work extends their model by showing that growth rate is higher when learning is introduced.

With respect to human capital, this thesis shows that human capital introduces another engine for growth different from innovation. And both human capital and innovation matter for long run growth rate. The thesis also shows that it is possible to build a scale invariant model where horizontal innovation does not have a neutralizing effect on vertical innovation . For instance I show that if horizontal innovation outgrows population growth, the economy will grow by more because horizontal innovations open up new sectors on which vertical innovation could thrive when the existing vertical innovation becomes difficult to innovate on. If on the other hand population growth outgrows horizontal innovation then population growth will have a negative effect on growth. Therefore given the empirical support in that population growth has negative effect on growth suggest that horizontal innovations have not kept pace with population growth.

The third contribution made by this thesis is to reconcile theory with empirics on the impact of competition on growth. I argue that there is an implicit psychological threat that the incumbents feels to make him innovate and escape competition when he has technological advantage. I then show that the threat of firm entry has a positive effect on growth.

Finally, this paper provides a macroeconomic framework which examines the impact of RJV on economic growth when RJV firms also engage in collusion in the product market. I found that when collusion in the product market encourages market formation, it raises incentive to innovate and has positive effect on growth. But the cost of operating a dynamic pricing model for encouraging habit formation has negative effect on growth.

Appendix A

The first order condition when maximizing $U(C_t, z_t)$ with respect to C_j is

$$U(C_t, z_t) = \int_0^1 \left(\frac{C_j \ Z_j^{-\gamma}}{1-v} \right)^{1-v} dj, \tag{6.1}$$

subject to

$$E = \int_0^1 \rho_j C_j dj \tag{6.2}$$

can be written as

$$\mu \rho_j = c_j^{-v} Z_j^{\gamma(v-1)} \ A_j^{1-v} \tag{6.3}$$

where $C_j = A_j \frac{C_j}{A_j}$ represents demand of good j,

Integrating both sides of this equation over all j's yields

$$\mu = \frac{1}{P_D} \tag{6.4}$$

which when combined with equation above first order condition yields equation(5.8) in the paper

To determine the equilibrium aggregate price, multiply equation(5.8) by ρ_j to yield

$$\rho_j c_j = \frac{E \rho_j^{\frac{v-1}{v}} Z_j^{\frac{\gamma(v-1)}{v}} \ A_j^{\frac{1-v}{v}}}{\int_0^1 \left(\frac{\rho_{(j)} Z_j^{\gamma}}{A_j} \right)^{\frac{v-1}{v}} dj} \tag{6.5}$$

Integrating both sides of this equation over all j's yields equation (5.9) in the paper

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