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### *Financial Frictions and Macroeconomic Fluctuations*

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# Financial Frictions and Macroeconomic Fluctuations

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# Abstract

In this thesis, I assess the propagation power of financial rigidities, related to firm and bank financial health, and their impact on the fluctuations of external finance premium (*efp*) and on business cycles. The thesis is organized in four chapters.

The *introductory chapter* is a review of the literature related to the role of firm and bank financial health in general equilibrium models. The emphasis is on the most recent papers that are related to my thesis.

In the *second chapter*, I empirically assess the relative significance of financial health of non-financial and financial sector for the external finance premium (*efp*) based on US and limited UK data using unrestricted vector autoregression (VAR). I also evaluate to what extent fundamental shocks like the total factor productivity (*tfp*), investment specific technology (*ist*) and monetary policy drive *efp* and output. The result that emerges from this analysis is that *efp* and output are primarily driven by the *tfp* shocks and second by monetary policy shocks. In addition, financial sector net worth, rather than non-financial sector net worth, drives *efp* and output when fundamental shocks are absent.

In the *third chapter*, I set up a general equilibrium framework with two financial rigidities on firm and bank level to investigate the propagation of the fundamental shocks on *efp* and other macro and financial variables. Finan-

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cial frictions are set-up as leverage constraints on borrower and lender in this framework (named *FAGK*).

To evaluate the propagation of shocks in each model I compare the second moments and impulse responses to those from a financial accelerator (*FA*) and a bank friction model (*GK*). Two main results come out. First, baseline *FAGK* model yields greater volatility of *efp* and of real variables, compared to *FA* or *GK* when driven by shocks of the same size. Second, the dynamics of overall *efp* are dominated by the fluctuations of the lender premium which is propagated by the bank constraint in *FAGK* model. In the second part of the chapter, I evaluate how changes in the severity of frictions or in the conduct of monetary policy may lead to greater propagating power of shocks in the model economy. The main conclusions are that, negligible policy response to output gap and a more persistent policy rate may contribute to greater propagating power of financial frictions.

In *chapter 4*, I estimate the baseline *FAGK* model and two single-friction models, *FA* and *GK*, employing Bayesian estimation method with quarterly data spanning 1955-2014. I assess the business cycle properties of each estimated model-economy and compare them to actual data. The baseline model can outperform the other two models, *FA* and *GK*, in describing the economy for the period.

To assess the stability of parameters I estimate the baseline *FAGK* model in two sub-samples, a *tranquil* sub-period, 1985-2004, and a recession period, 2005-2014. Next, I analyse the factors behind the increase in volatility during the recession period. Quantitative analysis based on *counterfactual* exercises leads me to conclude that the decline in investment adjustment cost and the increase in dispersion of returns across firms have shaped the business cycle properties of *efp* and of most indicators during the recession.

# Declaration

I confirm that the material contained in this thesis has not been submitted for any degree or qualification in this or any other institution. I declare that this thesis is solely based on my own research and all sources are fully acknowledged.

Bledar Hoda

## Statement of Copyright

The copyright of this thesis rests with the author. No quotation from it should be published without the author's prior written consent and information derived from it should be acknowledged.

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*To...*

*My Parents Asqeri and Stoli,*

*and*

*My Wife Edarjola*

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# Chapter 1

## Financial Frictions in General

### Equilibrium Models: Introductory

#### Review

Standard real business cycle models and new Keynesian models have become very popular tools for analysing the propagation of technology shocks and the role of monetary policy in recent decades. It is a common feature of these models to assume that the financial market is a perfectly competitive market. The assumption is based on the hypothesis of the irrelevance of firm capital structure (Modigliani and Miller (1958)). The hypothesis implies that firms can easily borrow at the risk free rate as the amount of debt they accumulate relative to their own equity does not matter for their cost of borrowing. In new Keynesian (NK) and real business cycle models this principle is articulated as an arbitrage condition implying that the required return on capital is risk free rate.

Over the course of the following decades it has been evident that the Modigliani-Miller hypothesis does not hold in financial markets. Neither is the arbitrage condition in NK models consistent with the data. In the long-run

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firms pay a borrowing premium above the risk free rate on loans. In the short-run this premium is volatile and negatively correlated with the financial health of borrowers. The borrowing premium is also negatively correlated with output gap. It goes up in times of recession or in periods of low economic growth and declines during booms.

Indeed, in the most recent crisis the premium of external borrowing above risk free rate jumped to unprecedented levels. At the same time investment and output dropped to trigger the greatest recession in post-war time. The negative correlation between output (or investment) and the premium on external financing (hereby *efp*) suggest that *efp* amplifies the business cycles and the latter potentially affects *efp*. Such strong relationship between borrowing premium above risk free rate and output growth has been a rough indicator of the severity of financial rigidity (friction) in the economy. At the core of these financial frictions are the pro-cyclical behaviour of borrower and lender financial health. The strong positive correlation of firm and bank net worth with output amplifies the severity of financial frictions giving rise to more volatile cost of external finance.

The empirical literature triggered by the recent crisis supports the link between the premium on cost of external finance and key macroeconomic variables. Empirical evidence indicates that fluctuations on *efp* can predict economic aggregates by suggesting credit tightening (Gilchrist et al. (2009b)). In a separate study Gilchrist et al. (2009a) find that corporate bond spreads in credit-risk portfolios are good indicators of economic activity. Further findings suggest, a considerable portion of the ability of an index of credit spread to predict economic activity is due to excess bond premium (Gilchrist and Zakrajšek (2012)). In a similar line of research, Faust et al. (2011) find improved accuracy in forecasting real economic variables using credit spreads based on corporate bond portfolios. The basis for this link is the counter cyclical be-

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haviour of credit flows (Covas and Den Haan (2011)). If these credit flows respond to fluctuations in the price of credit that is partly due to *efp*, then they are a key channel for the transmission of the *efp* impact on business cycles. Earlier studies have reported similar findings for different time periods in US (Gertler and Lown (2000), Mody and Taylor (2003)).

The above selected studies quantify the relationship between credit spreads and the real sector. The evidence points to the hypothesis that variations of premium on external borrowing lead to financial frictions that further amplify the business cycle fluctuations.

In my thesis I analyse how the degree of the financial rigidity that arises due to borrower and lender financial health relate to *efp*. I assess the propagation power of financial frictions and the relative impact they have on the dynamics of *efp* and of business cycles.

In this introductory chapter, I review the literature related to the role of firm and bank financial health in general equilibrium models. Initial literature has focused on the effects of the pro-cyclical firm leverage driven by fundamental shocks. Post-recession literature focuses on frictions arising due to counter-cyclical behaviour of lender net worth. The most recent studies that are related to my work consider both. I emphasize how my work distinguishes from other similar double-friction studies that bring the dynamics of lender and borrower financial health in a single general equilibrium framework.

The remaining part of the thesis is organised in three chapters.

In chapter 2, I empirically analyse three fundamental shocks, total factor productivity (*tfp*), investment specific technology (*ist*) and monetary policy shocks. These shocks compete to explain the fluctuations in external finance premium and output. The result that comes out is that *tfp* is the main driver of *efp* and output fluctuations. Monetary policy shocks also have a significant

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impact. In the absence of these shocks, the exogenous innovations to financial sector net worth, rather than firm net worth, explain a significant fraction of *efp* and output fluctuations. These results set the stage to set up a general equilibrium model to further investigate these relationships.

In the 3<sup>rd</sup> chapter I propose a framework with two layers of financial friction. The first layer relies on a financial accelerator (*FA*) mechanism building on the information asymmetry between the borrower and the lender. The second layer builds on the moral hazard problem in bank-depositor relationship to give rise to a bank friction (*GK*). The two mechanisms, the *FA* and the *GK*, introduce financial frictions as leverage constraints on firm and bank, respectively. Firm and bank financial health are critical for the severity of the two frictions in my set up named the *FAGK* model. The model features other nominal and real frictions typical for a new Keynesian framework. I evaluate the propagation role of endogenous fluctuations in firm and bank net worth to explain the fluctuations of external finance premium *efp*, and potentially of output.

To evaluate the propagation of shocks in my model I compare the impulse response functions against two single friction models, a financial accelerator model and a bank friction model. I conclude that the framework with constraints on bank and firm balance sheets delivers significantly greater response of *efp* and macro variables to standard shocks. The model with double financial frictions has stronger propagation power compared to single friction models. Impulse responses show that the constraint on bank balance sheet is the dominant driver of total external finance premium.

Next I assess how the interaction of both frictions might have contributed to propagation of fundamental shocks to financial variables, particularly *efp*, during the recent crisis. The fundamental shocks I consider here are neutral

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technology shock, investment specific technology shock and monetary policy shock ( $R^n$ ). To do so I evaluate how changes in the behavior of the agents present in this model can explain sudden increases in fluctuations of  $efp$  and real macro indicators. Two results stand out. Changes in monetary policy response to output gap and in investment adjustment costs ( $IAC$ ) are critical for the propagation of shocks. Compared to single friction models, double friction mechanism propagates shocks of the same size by a greater magnitude when monetary policy is sluggish and less responsive to output gaps. Second, changes in investment adjustment costs lead to greater volatility of asset prices, ( $Q$ ), investment ( $I$ ) and of firm and bank net worth. Following these changes standard shocks of the same size have the potential to generate much greater volatility in external cost of financing in a model implied economy when both frictions are present.

In chapter 4, I evaluate the performance of the three model frameworks, the financial accelerator model ( $FA$ ), the bank friction model ( $GK$ ) and the baseline model with both frictions ( $FAGK$ ), using Bayesian estimation method.

First, the three models are estimated based on observable series covering the full sample of data available, 1955-2014. Business cycle properties of the model-implied economy from each framework are compared to those from actual data.  $FAGK$  can outperform the other two models,  $FA$  and  $GK$ , in describing the economy for the relevant period. In particular, it describes better the properties of the series not included in estimation.

Next, the baseline model  $FAGK$  is estimated to evaluate the stability of parameters in two sub-samples, 1985-2004 and 2005-2014. I assess the quantitative effects of changes in potential candidate parameters for the business cycle properties of the model-implied economy in 2005-2014. Main candidate parameters that come out from estimation are the decline in investment adjustment

costs *IAC* parameter, the increase in dispersion of returns across borrowing firms and the rise of monitoring costs. Results that come out from the *counter-factual* exercises suggest that changes in the former two parameters, lower *IAC* and higher dispersion of returns across borrowing firms, have shaped the business cycle properties of most variables in the model implied economy during the recession.

## 1.1 Bank and Firm Financial Health in General Equilibrium Models

In this section I review the literature that brings in the impact of firm and bank financial health on *external finance premium* in general equilibrium models.

### 1.1.1 Balance Sheet Channel

Early literature on implications of frictions present in borrower-lender relationships focuses on the equilibrium that leads to full credit rationing due to adverse selection and moral hazard (Stiglitz and Weiss (1981)). In the equilibrium with no credit rationing, lenders discount expected returns on basis of risks that the asymmetric information problem creates. Therefore, risk is accounted for in the inevitable deadweight losses arising due to asymmetric information, called *agency costs*. Agency costs have been a key concept for modeling frictions arising in borrower-lender relationship. These costs are allocated either into monitoring or in screening the borrower. The optimal debt contract with costly state verification (CSV) of borrower upon his default assumes monitoring costs (Townsend (1979)). In these type of contracts, lower borrower net worth aggravates the agency problem as higher leverage is asso-

ciated with lower ability to repay. The inability of borrower to repay involves costly monitoring by the lender to liquidate upon default. To account for this agency cost, that is monitoring and state verification of borrower, the lenders charge higher premiums on the whole pool of borrowers.

The alternative agency cost that lenders can incur is the screening cost of the pool of borrowers. Lenders chase high quality borrowers to keep monitoring costs and the number of defaults in their loan portfolio to a minimum. To minimize agency costs lenders keep a balance between screening and monitoring. [Gilchrist et al. \(1994\)](#) and [Gertler and Gilchrist \(1994\)](#) noticed a "flight-to-quality" in lending, as the share of credit flowing to small firms had declined. Small firms are identified as borrowers with potentially high monitoring costs. Screening based only on the size of firms keeps the screening (agency) costs at negligible levels. Identifying high quality borrowers (in terms of ability to repay) with firm size means lower total agency costs by lenders. Higher agency costs imply lenders will pass these costs on borrowers through a higher premium on loans above risk free rate.

Agency costs are inversely related to net worth of the borrowing firm. Lower net worth signals to a lender a higher probability of incurring monitoring costs. [Bernanke and Gertler \(1994\)](#) make use of this inverse relationship to model the impact of adverse shocks on external finance premium. A small decline in borrower net worth raises agency costs which in the following period give rise to higher borrowing premium. The original impact of an adverse shock is amplified as higher premium leads to lower demand for loans, lower investment and output. [Bernanke et al. \(1999\)](#) (hereby *BGG*) demonstrated in a partial equilibrium model how the propagation by procyclical borrower net worth of exogenous shocks depends critically on the size of agency cost.

The propagation mechanism of shocks arising due to credit market frictions has been an appealing feature of research in the late '90s following the publication of two studies by Carlstrom and Fuerst (1997) and Bernanke et al. (1999)<sup>1</sup>.

Agency costs are a rough measure of the financial friction related to borrower financial health present in a model economy. The more severe the friction, that is higher agency costs, implies that low net worth firms pay higher premium on external finance. Higher  $efp$  is associated with lower credit flows and larger declines in investment and output. This chain of effects adds the existing layers of amplification that nominal and real frictions generate in a new Keynesian framework.

Severity of agency costs is critical for the magnitude of  $efp$  and the size of financial frictions due to firm balance sheet health. It comprises an elasticity measure of the  $efp$  to borrowers' leverage. The financial accelerator mechanism amplifies the effects of adverse technology and monetary policy shocks by driving down investment and output. Lower output drives borrower net worth downward giving rise to second round effects. Explicit time-varying agency costs can also account for  $efp$  volatility due to the severity of asymmetric information problem, implying positive relationship with  $efp$  (Gilchrist and Leahy (2002)). The motivation for time-variation assumes deteriorating information asymmetry gives rise to rents associated with cost of monitoring. Results by Levin et al. (2004) featuring a partial equilibrium version of BGG with a panel of 900 US non-financial firms for the period 1997-2003 are strong empirical evidence of time variation in the marginal cost of bankruptcy.

Agency costs are not the only way to incorporate this financial friction in new Keynesian models. Kiyotaki and Moore (1997) set-up a framework em-

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<sup>1</sup> Carlstrom and Fuerst (1997) framework is similar to EA of Bernanke et al. (1999). Kiyotaki and Moore (1997) proposed a slightly different set up where the borrower net worth has the role of collateral.

phasizing the collateral function of borrower net worth. Thus asset prices and net worth feed into each other exacerbating the impact of the initial shock on firms' net worth, hence on their capacity to borrow, leading to higher borrowing premium or limited borrowing capacity. The inability of borrowers to raise new equity adds persistence. Other studies like [Iacoviello \(2005\)](#) and [Monacelli \(2009\)](#) have highlighted the significance of housing wealth and consumer durable goods respectively for the severity of financial frictions.

Typically in these studies authors emphasize two key facts, the procyclical financial health of borrowers and magnitude of agency costs. The former captures borrower balance sheet effects, hence the name *balance sheet channel*. The second, agency costs, capture informational frictions in borrower lender relationships. A full list of papers that make use of the extra layer of friction coming from the financial accelerator set up is impossible and beyond the scope of my thesis<sup>2</sup>.

An interesting dimension along which this literature expanded is the introduction of exogenous shocks to the financial contract or on the borrower net worth. Exogenous shocks to net worth of the borrower may seem a solution to obtain larger variations of agency costs and of the premium in calibrated models. These extra shocks have become more common in estimated models. Estimation of dynamic stochastic equilibrium models (*dsge*) has made a technical necessity the introduction of additional shocks. These additional shocks have also become statistically important in driving macroeconomic fluctuations. Shocks to agency (monitoring) costs aim at explaining the higher magnitude of fluctuations in real and financial variables. Varying agency costs during financial stress periods has been empirically supported by [Levin et al. \(2004\)](#).

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<sup>2</sup> [Christensen and Dib \(2008\)](#), [De Graeve \(2008\)](#), [Nolan and Thoenissen \(2009\)](#), [Queijo von Heiden \(2009\)](#), [Gerali et al. \(2010\)](#), [De Fiore and Tristani \(2013\)](#), [Fernández-Villaverde \(2010\)](#) among many others.

Similarly, a strand of literature applies shocks to borrower net worth to address the requirement of matching the number of shocks with that of observable variables in estimation. [Nolan and Thoenissen \(2009\)](#) motivate financial shocks as exogenous variations on the efficiency of contractual relations between borrower and lenders. [Gilchrist et al. \(2009b\)](#) introduce shocks to net worth and the external finance premium itself. Beyond the technical necessity for extra shocks, the authors find that a large share of fluctuations is due to these financial (or net worth) shocks. [Nolan and Thoenissen \(2009\)](#) lend close to half of volatility in output and investment to shocks to borrower net worth. [Gilchrist et al. \(2009b\)](#) estimate that a financial (*efp*) shock depresses the level of output and of investment by 15 % and 75 % respectively. In this strand of literature shocks to financial variables or friction mechanisms, rather than fundamental shocks, have gained importance for business cycles <sup>3</sup>.

There are two dimensions where my framework is different from this stream of literature. First, adding exogenous shocks to endogenous variables like net worth shocks in *dsge* models does not determine the fundamental source that drives external finance premium. The approach leaves a gap by not addressing the more fundamental question of how these adverse shocks to financial variables themselves took place.

Second and more important, this stream of literature is silent regarding the health of financial sector. The recent crisis emphasized that financial health of banks can be crucial for their ability to intermediate funds. The financial health of lender may propagate exogenous shocks through higher premium on external finance. Low net worth of financial intermediary may lower its ability to absorb funds from the saving agents or to monitor the borrower. This channel is commonly referred as *the lending channel*. In the next sections

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<sup>3</sup> Investment-specific shock, initially introduced by [Greenwood et al. \(1997\)](#) has become common among this set of technology shocks.

I review the literature incorporating this channel and its potential to explain features not captured in the literature above.

### 1.1.2 Bank Lending Channel

Banks' ability to intermediate funds does depend on their financial soundness. The better financial health of banks contributes to the ability of banks to monitor borrowers or to raise risk free funds from saving households. When low net worth firms are adversely hit, banks may also see their financial health deteriorate and struggle to access the external funds. Access to deposits will depend on how reliable banks are to households. The financial soundness of banks boosts their reliability and their ability to reach on their savings. Banks that lend in risky projects, or have non-diversified loan portfolios, expose households deposits to risk. Higher capital is a layer of protection against the risk in the loan portfolio of the bank. Alternatively, low net worth intermediaries may have to raise the premium they charge on loans to accumulate higher levels of net worth.

Accessibility to the pool of depositors is critical for the economy. Inability of banks to raise funds and supply financing may squeeze small firms out of the market. Empirical evidence using bank level US data indicate that banks with low capital squeeze lending more upon an adverse monetary policy shock (Kishan and Opiela (2000), Van Den Heuvel (2007b), Van Den Heuvel (2007a)). The findings emphasize how important the financial health of intermediaries is for the functioning of the credit supply channel. This friction arising due to loan supply by intermediaries is not related to firm financial health or to any features of a new Keynesian model.

The loan supply channel has gained more attention after the recent crisis. The erosion of bank capital that followed after a drastic decline in asset prices

in US led to sharp spike in cost of external borrowing and further decline in investment. The propagation of shocks by intermediary capital in an imperfectly competitive banking sector triggers a fall in output and investment in the study by [Gerali et al. \(2010\)](#). [Meh and Moran \(2010\)](#), [Dib \(2010a\)](#) introduce frictions due to bank's inability to meet specific capital targets. The threshold level of net worth is motivated by a regulatory requirement. Bank capital channel propagates effects of technology shocks to output, investment and inflation. Capital adequacy ratios depend on returns on bank equity and bank deposits. Shocks to bank capital affect real sector through bank balance sheet size ([Aguiar and Drumond \(2007\)](#), [Angeloni and Faia \(2013\)](#), [Dib \(2010a\)](#)). The models impose shocks to financial variables, bank net worth or capital quality, to replicate the sharp increase in volatility seen in external finance premium. They do not address why these financial variables became so volatile in the first place. This stream of literature does not emphasize the role of bank net worth as a mitigating buffer for the cost of external finance on the supply side.

The recent crises emphasized the risk premium's critical role for real activity, rather than the squeeze in credit amount ([Adrian et al. \(2012\)](#)). In that light [Dib \(2010b\)](#) had introduced a moral hazard problem in interbank market and an adverse selection problem in bank capital market to emphasize liquidity risk on *efp*. There is also a different stream of papers that looks at the role of cost of external finance, but they look at the liquidity premium effects on *efp* ([Angeloni and Faia \(2013\)](#), [Dib \(2010a\)](#)). In these models, there is no amplification by the procyclical net worth of intermediaries on real sector through the cost of external finance.

A framework that builds a connection between the endogenously evolving lender net worth and *efp* is proposed by [Gertler and Kiyotaki \(2010\)](#) (GK)<sup>4</sup>. They propose an agency problem between intermediaries and depositors giv-

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<sup>4</sup> A similar mechanism is present in [Gertler and Karadi \(2011\)](#).

ing rise to moral hazard risks. The moral hazard problem arises between banker that may abscond after collecting deposits and households. Depositors require banker to maintain the value of the bank net worth above a threshold as a guarantee for them not to withdraw deposits. Due to such threat, banker has the incentive to keep the value of the bank above that threshold value for depositors not to liquidate the bank.

The moral hazard problem gives rise to a mechanism where fluctuating net worth drives the dynamics of lending rate relative to the risk free rate. The mechanism works as follows. Bank value is equivalent to the discounted stream of future profits. Adverse shocks will lower bank profits and therefore drive down bank value. When bank's net worth declines, banker faces the risk of a bank run. As management faces the risk of being liquidated by depositors, the bank has to raise the premium on loans to build up higher levels of net worth. This wedge between the required return on loans and the risk free rate is net of monitoring costs or any other costs to cover default losses. Propagation of the fundamental shocks due to lower firm net worth or any agency costs capturing information asymmetry between banks and firms is absent in these frameworks.

The mechanism has similar appealing features as the FA model had initially. The models recognize the role of *bank net worth* for the variations in the premium at which external finance can be obtained by a potential borrower. To explain the unprecedented increase in external finance premium and the associated decline in real variables the authors appeal to capital quality shocks. The purpose is to simulate the financial crisis and contribute by evaluating policy alternatives.

### 1.1.3 Review of Recent Literature

The literature that addresses the implications of financial conditions of borrowers and lenders has boomed in the recent years. There is a long list of works that replicate the standard models with friction at firm or bank level and focus on estimation of parameters for different economies. In this section I focus on a selected list of studies that contribute in the literature with new models or that are closely related to my work. A short list of studies are related to models with a friction at firm level or with a friction arising due to credit constrained bank. I save more space for the studies with constraints arising at both, firm and bank level.

In the financial accelerator literature, my works is related to [Fuerst et al. \(2016\)](#) who modify the BGG framework to allow for the sharing of aggregate risk by the lender and the entrepreneur. Unlike in BGG where the return to the lender does not depend on realization of aggregate risk, in their contract the average return to the lender varies with macroeconomic conditions. In my framework I let the lender share the idiosyncratic risk. Both lender and borrower share some of the risk.

Similar to [Fuerst et al. \(2016\)](#) I conclude that the financial accelerator is weak compared to the bank friction. That firm friction is weak is also consistent with the results of [Suh and Walker \(2016\)](#). They estimate three different models with credit constraint firms and find that financial frictions at firm level are not able to explain large fluctuations seen during the financial crisis.

A notable contribution is the paper by [Christiano et al. \(2010\)](#). At firm level, they replicate the financial accelerator mechanism and let banks issue working loans to make bank's net worth fluctuate. Their object of research is not a comparison of the relative significance of bank and firm friction for the external finance premium. Banks are designed as entities that own a technology to con-

vert labor and capital into deposit accounts, and short term security accounts. These liabilities are then offered to firms as debt loans and working capital loans. The latter transmit the impact of firm default on bank financial health. A direct exposure of bank net worth to asset price fluctuations does not take place. The model is estimated with 16 shocks. They add two financial shocks, one on firm net worth and the other on the idiosyncratic project returns of the borrowing firm. They find the latter, named *risk shock*, important for the dynamics of the model and in particular for the correlation of credit flows with output.

Finally, in the financial accelerator literature, my work in chapter 4.1 is related to the work by Galvao et al. (2016) who estimate a time varying DSGE model only with a financial accelerator and find that volatility of financial friction shock has changed during 2007-2011 compared to 1985-2006.

More recent bank channel models emphasize the collateral value of bank net worth. Among these studies there is a strong support to the role of financial sector leverage for the risk premium or macroeconomic stability.

Nuno and Thomas (2017) analyze the dynamics of bank leverage subject to aggregate and idiosyncratic shocks on bank asset returns. In their model banks borrow in the form of short-term collateralized risky debt and own the firms. Due to a moral hazard problem on the part of the banks, leverage is endogenously evolving. They conclude that idiosyncratic shocks, rather than aggregate technology ones, help replicate the procyclicality of bank leverage and the size of its volatility. The similarity with my framework is that they make banks subject to both aggregate and idiosyncratic shocks while I evaluate the impact of sudden a change in the long-run value of idiosyncratic shock.

My framework is related to the work of Boissay et al. (2013) with two frictions in the bank-depositor relationship and in the interbank market. Banks

are credit constrained as they can divert the non-collateralized funds (against a diversion cost). Interbank market arises due to private information on banker skills as a borrower giving rise to asymmetric information. The difference is that they evaluate the probability of endogenously arising banking crisis.

My paper is related to [He and Krishnamurthy \(2013\)](#) who analyze the dynamics of risk premia during crisis in asset markets where the investor (borrower) is a financial intermediary. In continuous time model the authors evaluate the probability of change in risk premium in equilibrium rather than in a loglinearized model around the steady state. In a similar framework, [Brunnermeier and Sannikov \(2014\)](#) analyse the links between intermediaries' financing positions to risk premia while [Phelan \(2016\)](#) investigates implications of financial sector leverage for macroeconomic instability and welfare<sup>5</sup>.

The recent literature with two-sided (double) frictions has blossomed in the recent years. While most double friction models have some similarities in common with my work there are two papers, [Kühl \(2017\)](#) and [Rannenberg \(2016\)](#), that share many features with my model framework. I dedicate more space in this section to those two studies and discuss the remaining studies shortly.

My work is closely related to the work of [Kühl \(2017\)](#). The similar features are that both banks and the small firms with loans on their balance sheet are financially constrained by assuming a limited enforcement problem as in [Gertler and Kiyotaki \(2010\)](#). Similar to my framework, banks hold a combination of state contingent assets - 100% of the equity securities of large firms- and non-state contingent loans issued to smaller firms.

My work differs from his as in my framework firms can default (as in a [Bernanke et al. \(1999\)](#) model), while he abstracts from loan defaults in his

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<sup>5</sup> Continuous-time models with frictions at intermediary-level allow the authors to set up highly non-linear relationship between the financial health of intermediary and the risk premium ([He and Krishnamurthy \(2013\)](#), [Brunnermeier and Sannikov \(2014\)](#) and [Phelan \(2016\)](#)).

model. With some modifications as in [Fuerst et al. \(2016\)](#), this feature allows me to pass some losses to the banks. In [Kühl \(2017\)](#) banks suffer losses only from equity securities. In his framework only (small) firms that take a loan are financially constrained and accumulate net worth.

Another difference is that the objective of [Kühl \(2017\)](#) work is to investigate the implications of state contingency of bank assets by changing the equilibrium composition of bank balance sheets. He analyses how bank net worth evolution depends on the composition of bank assets (in equilibrium) and how this composition matters for the transmission of shocks and for the procyclical behaviour of bank leverage. His key result is that amplification of shocks depends on bank balance sheet composition. In my framework, I compare the propagation of shocks in a two-sided (double) friction model relative to the financial accelerator model and the simple GK model. I do not explore the implications of different compositions of bank balance sheet. Another difference is that I estimate the model in two subsamples to evaluate how certain friction parameters may have changed during the recent recession in US.

The second paper that my work is related to is [Rannenberg \(2016\)](#). In his work both banks and firms are constrained as in my framework (GK and BGG respectively). In a similar way [Rannenberg \(2016\)](#) models firms as in [Bernanke et al. \(1999\)](#) and adds a set-up borrowed from [Gertler and Karadi \(2011\)](#) to model banks. He analyzes the dynamics of cyclical behaviour of bank and firm leverage as well as the dynamics of external finance premium. He makes a horse-race of (exogenous) firm and bank net worth shocks. I differ from his paper in that I include two types of financing for the firm which become two assets on bank's balance sheet. In addition, Rannenberg includes within

period working loans which pay no interest and are of a negligible amount to make bank leverage procyclical<sup>6</sup>.

A second difference is that, these studies rely on capital quality shocks or exogenous net worth shocks to simulate the fluctuations on bank or firm net worth during the recent crisis. I assess the propagation of standard technology and policy shocks as driving forces of the model through the interaction of the two frictions. Finally, a critical part of my work which is different from [Rannenberg \(2016\)](#) is that I estimate the parameters for two different sub-samples, 1985-2004 and 2005-2014, which allows me to identify shifts in deep parameters to explain the dynamics during the recent crisis.

To bring in a role for the bank net worth [Zhang \(2009\)](#) modifies the *BGG* model by introducing fixed lending rate contract (instead of state-contingent). In his framework both borrowers and lenders share aggregate systemic risk. The capital stock has to be maintained to keep the required return on deposits and on equity at low levels. Though there is loan supply channel in his framework, the bank is only exposed to the friction due to agency costs. Bank balance sheet in his model is not exposed to changes in asset prices which are the main cause triggering the loan supply channel in US.

Finally, a common feature between the above papers, [Zhang \(2009\)](#), [Rannenberg \(2016\)](#) and [Fuerst et al. \(2016\)](#), and in my framework is that we use predetermined contractual loan rate which allows for some the aggregate risk to be shared between the banker and the firm. [Bernanke et al. \(1999\)](#) assume the entrepreneurs will absorb all the aggregate risk, as lenders get risk free rate by issuing state contingent (not fixed) contract.

My work is related to [Hirakata et al. \(2011\)](#) and [Hirakata et al. \(2013\)](#) who employ a costly state verification contract (financial accelerator mechanism) to

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<sup>6</sup> While the author mentions loans within the period which are critical for the procyclical behaviour of bank leverage, they are not explained in the model section.

model frictions at firm and bank level. Their work is different first because they rely on exogenous net worth shocks, and second because their objective is to explain fluctuations in output and investment and to analyze optimal Taylor rules, respectively<sup>7</sup>.

My thesis is related to [Hristov and Hülsewig \(2017\)](#), [Bonciani and van Roye \(2015\)](#) and [Iacoviello \(2015\)](#) who set up two-sided friction models. The objective of [Hristov and Hülsewig \(2017\)](#) is to replicate the procyclicality of bank profitability, counter-cyclicality of firm default rates and credit spreads to monetary policy shocks. Their FA mechanism allows banks to share the losses. [Bonciani and van Roye \(2015\)](#) set up a double moral hazard framework with a [Kiyotaki and Moore \(1997\)](#) (hereby *KM*) type constraint on firms and a monopolistic banking sector. Their model is different as they investigate the effects of uncertainty shocks on economic activity in the euro area. [Iacoviello \(2015\)](#) builds a framework with credit-constrained banks, firms and households in the form of collateral constraints as in *KM*. He evaluates the impact of financial shocks for the business cycles. [Clerc et al. \(2015\)](#) also bring three credit constraint agents, households, banks and firms, in a general equilibrium framework. Unlike in my thesis, they analyse the implications of default for capital regulation.

My work has a common feature with other papers that consider two-sided financial frictions allowing for interaction between constrained borrowers (or firms or households) and financial shocks arising in interbank market as in [Gertler and Kiyotaki \(2015\)](#), [Dib \(2010a\)](#), [Ajello \(2016\)](#). A series of other papers evaluate the interaction between frictions due to credit constraints of borrower and intermediary at a more theoretical level ([Rohan \(2016\)](#), [Sandri and Valencia \(2013\)](#), [Zeng \(2013\)](#)).

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<sup>7</sup> Another difference of my framework is that the wholesale producer finances the purchase of capital stock with standard debt and by selling equity stake securities.

Finally, my work is related to [Verona et al. \(2017\)](#) who consider two types of financing, bonds and loans. While the loan market works as in the financial accelerator of BGG, the bond market is slightly different from the equity securities market used in my framework. The objective of their work is to evaluate interest rate rules in the presence of financial frictions.

None of the studies addresses the relative significance of financial and non-financial sectors in propagating fundamental shocks. Where lender net worth is modeled, in one stream of literature, the authors are interested on policy implications of exogenous fluctuations in (lender) net worth. Alternatively, most studies compare propagation of exogenous firm and bank net worth shocks rather than propagation of fundamental, *tfp*, *ist* and monetary shocks.

## 1.2 Concluding Remarks

There is a broad consensus among economists that Modigliani-Miller hypothesis of irrelevant capital structure is only a textbook case. The assumption of perfectly competitive markets inherent in new Keynesian models is a baseline simplification from which general equilibrium modeling is increasingly departing. Since the '90s frictions arising in credit markets have been shown to produce a better representation of data moments. The recent crisis in US emphasized a strong role for financial markets as well. The latter models have been useful in evaluating policy implications, particularly macro-prudential measures aiming at smoothing frictions in the financial market. The critical interaction between non-financial sector and financial sector in propagating the fundamental shocks has only recently gained momentum.

In this introductory chapter I review how the literature has treated the role of firm and bank financial health in amplifying the impact of standard

fundamental shocks. That countercyclical fluctuation of firm net worth leads to procyclical behavior of borrowing premium by firms has been identified since the 90s. A financial accelerator mechanism in a partial equilibrium model has become a popular tool to illustrate how endogenous movements in firm net worth lead to higher premium on external borrowing.

I review to what extent the current literature has explored the significance of firm financial health and of bank financial health for the cost of external financing. I refer with the term *the balance sheet channel* the modeling of the asymmetric information between the borrower and the lender in *dsge* models. In this literature, the dynamics in the firm balance sheet, particularly the fluctuations of firm net worth, are critical for the fluctuations of external finance premium.

Frictions arising due to countercyclical bank net worth have come to focus only after the recent recession. Declining lender net worth drives the premium above the risk free rate for any type of firm. Therefore, easy access to household savings by the lender will depend on his ability to maintain sound financial health. I refer to this mechanism as the *the bank lending channel*. In the last section of this chapter I reviewed to what extent the significance of these two frictions for the external finance has been treated in a single general equilibrium framework.

In the next chapter, I make an empirical investigation of the impact of firm and bank net worth for the fluctuations of external finance premium. I focus mainly in US where I get a rich set of financial and nonfinancial sector balance sheet data. I also empirically analyse the implications of fundamental shocks, like *tfp*, *ist* and monetary policy shocks, for the business cycle properties of the cost of external finance and the macro variables in US. A similar empirical inquiry for UK economy is based on a limited set of data.

## Chapter 2

# Significance of Bank and Firm

# Financial Health for the Borrowing

# Premium

In this chapter I make an empirical assessment of the determinants of the cost of external borrowing. My work expands in two directions. First, is the assessment of the relative impact of the financial health of two main sectors of the economy, financial and non-financial sector, on cost of external borrowing. Second, is the analysis of the potential of fundamental technology and monetary policy innovations to drive the financial variables and output.

This chapter is organised in five sections. The *first* and *second sections* are dedicated to data construction and methodology.

The empirical exercises and respective results are reported in the *third* and *fourth sections*. The issue addressed in *third section* is the relative importance of the financial health of lenders relative to borrowers on the cost of external borrowing. The tests to address it look at the extent that innovations to net worth of each sector, non-financial and financial sector, can explain fluctuations in cost of external borrowing. The main result that comes out is that bank net

worth has become a significant determinant of external finance premium and of business cycles after the 80s with the start of the *Great Moderation* period. Results based on UK series are in line with those based on US data. Robustness checks do not change this result.

The tests above assumed that variations of net worth of each sector are solely due exogenous innovations of their own. One issue that still remains is that the net worth of either sector, financial or non-financial, can be driven by fundamental technology and monetary policy shocks.

In the *fourth section* I analyse how primary technology and policy variables interact with net worth to drive the variations of the external financing premium and output. I run a 6-variable VAR with three additional series of total factor productivity (*tfp*), investment specific technology (*ist*) and nominal interest rate ( $R^n$ ).

The result that emerges is that total factor productivity and monetary policy shocks, rather than net worth shocks, explain the variations in external finance premium fluctuations. Similarly, these shocks explain a great fraction of output fluctuations. I make a short summary of those results in the *fifth section*.

## 2.1 Data

I collect data that serve as proxy for three key indicators under analysis, *external finance premium*, *financial health of firms* and *financial health of banks*, for US and UK.

*External Finance Premium* in US. A quick review of literature suggests there are several measures used as proxy for the premium on external financing. A typical reference is the study by [Bernanke et al. \(1999\)](#). They consider the historical average of the difference between the prime loan rate and the

six-month T-bill as an average for the steady state premium. This measure has the disadvantage that does not reflect market behaviour of interest rates. [Christiano et al. \(2010\)](#) consider the difference between Baa and Aaa yields on corporate bonds as measure of the premium. The advantage of these series is that they are market based and are easily available. Similar to [Christiano et al. \(2010\)](#) I make use of the available measures based on Moody's corporate bond yields. I obtain two measures of premium on external financing<sup>1</sup>.

- The first variable, named *efpBA*, is the difference of *Moody's Baa-rated corporate bond yield* less the *Aaa-rated corporate bond yield*.
- The second proxy, called *efpB*, is the difference between the *Moody's Baa-rated corporate bond yield* and *Fed's risk free rate*.

These two measures are easily available and can be replicated over time. Moody's corporate bond yields are highly liquid borrowing instruments based on trade of corporate bonds that are most active in the market. I use both measures as proxy of the cost of obtaining external financing.

*Net Worth of non-financial sector.* There are slightly different measures of firm net worth used in literature. I find slightly different measures used in three works, [Jermann and Quadrini \(2006\)](#), [Fuentes-Albero \(2014\)](#) and [Rannenberg \(2016\)](#). The common thread is that they all originate from the Flow of Funds accounts provided by the Board of Governors of the Federal Reserve System. I follow [Fuentes-Albero \(2014\)](#) and define the net worth of non-financial sector as the *real per capita net worth of business sectors: non-financial corporate business plus non-financial non-corporate business sector*. Net worth of each sector is defined as tangible assets minus credit market instruments (market value). Tangible assets are related to the physical capital of the firms (including inventories), but not financial capital. Net worth is evaluated at current

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<sup>1</sup> [Gilchrist et al. \(2009b\)](#) calculate the average credit spread constructed from corporate bond data. These data are only provided by *Lehman/Warga* and *Merril Lynch*.

market prices, though deflated by GDP deflator. The purpose is to let changes in the value of collateral due to asset prices be reflected in net worth in line with the theoretical assumption of general equilibrium models.

**Net Worth of financial sector.** I use the series from the Financial Account of the United States (table Z1) of the Federal Reserve Board to get a market value for the net worth of Financial Sector. I obtain the series '*Financial business; corporate equities (liability)*' (code LM793164105.Q from table Z1/L108). I deflate the nominal net worth series using the GDP deflator provided by Bureau of Economic Analysis.

**Technology shocks.** I construct series for two technology shocks. **Total factor productivity** series is defined as the change in *tfp* series provided by Federal Reserve Bank of San Francisco website. To get a measure of **investment specific technology** (*ist*) I consider the ratio of *investment price* to overall *price index*<sup>2</sup>. I can obtain these data starting from first quarter of 1950.

### UK data

**External Finance Premium.** I obtain a proxy for the average lending rate and a proxy for the risk free rate from *Datastream* database. The proxy for external finance premium is the difference between the two.

- The lending rate is average interest rate on consumer loans provided by *Oxford Economics* (code name:UKXRLND.R). The series is available from first quarter of 1980<sup>3</sup>.
- The risk free rate is measured as the *interest rate on 6-month sterling certificates* by Thompson Reuters (code name: LDNCD6M). The series is

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<sup>2</sup> Consumption deflator and total consumer price index series are similar for US so the relative price of investment will be the same using either index.

<sup>3</sup> The lending rate for consumer loans is the best proxy at this length under the assumption that lending rate to consumers and to firms have a strong correlation.

a market based rate closely following the fixed interest rate of Bank of England.

**Net Worth.** To obtain net worth series for financial sector in UK, I make use of two equity indices of financial and banking sector, respectively, constructed by *Datastream* (Datastream code names: FINANUK and BANKSUK). Similarly, to obtain net worth series for non-financial sector I use the equity index constructed by Datastream for non financial firms listed in UK (Datastream code name: TOTLIUK).

**GDP.** Gross domestic product at constant prices is obtained from the Office for National Statistics, UK.

### **Cyclical Components**

I filter the data using one of the *band-pass filters* to extract fluctuations in series that are relevant to behaviour in the series consistent with the business cycles in the economy. Band pass filters are combinations of moving average (MA) filters, which in turn are defined by polynomials in the lag operator. These filters are designed to eliminate both high and low frequency movements in the data. I filter the series with the [Christiano and Fitzgerald \(2003\)](#) (*CF*) filter by selecting the range 6 to 32 quarters. *CF* filters out low frequency (of less than 6 quarters) noise and very high frequency (more than 32 quarters) fluctuations. As a cross check I also filter the data with Hodrick and Prescott (*HP*) filter. The two filters generate series with very similar second moments. The reason for preferring the *CF* filter is that, following [Canova \(2007\)](#), other filters leave or exaggerate the portion of variability that is present at high frequency and that is not relevant for business cycle properties.

## 2.2 Methodology

To run empirical tests I make use of vector autoregressive (VAR) models ([Amisano and Giannini \(1997\)](#)).  $x_t$  is a set of  $n$  economic variables expressed as a vector of  $n$  stochastic processes jointly covariance stationary and possessing a finite order ( $p$ ) autoregressive representation. In matrix form it evolves as a  $n$ -variate dynamic simultaneous equations model.

$$A_0 x_t = A_1 x_{t-1} + \dots + A_p x_{t-p} + \epsilon_t \quad (2.1)$$

where,  $A_0$  is a  $n \times n$  matrix of coefficients with diagonal elements *normalised* to 1 and  $x_t$  is a vector of  $n$  variables by  $T$  observations. The non-diagonal elements of  $A_0$  capture the contemporaneous impact of variables in matrix  $x_t$  on each other.  $A_p$  is the matrix of coefficients on the  $p^{\text{th}}$  lagged variables and  $\epsilon_t$  is the vector of  $n$  *structural shocks* such that

$$\begin{aligned} E(\epsilon_t) &= 0 \\ E[\epsilon_t \epsilon_t'] &= \Sigma \\ E[\epsilon_t \epsilon_s'] &= 0 \quad \forall t \neq s \end{aligned}$$

where  $\Sigma$  is a diagonal matrix with elements  $\sigma_i^2$  for  $i = 1$  to  $n$ .

Assume  $A$  is a  $n \times n$  invertible matrix that pre-multiplies both sides of equation 2.1 to induce a transformation of disturbances  $\epsilon_t$ :

$$AA_0 x_t = AA_1 x_{t-1} + \dots + AA_p x_{t-p} + A\epsilon_t \quad (2.2)$$

Setting  $A = A_0^{-1}$ , assuming it exists, yields a reduced form representation of VAR,

$$x_t = \Phi_1 x_{t-1} + \dots + \Phi_p x_{t-p} + A_0^{-1} \epsilon_t \quad (2.3)$$

where,  $\Phi_p = A_0^{-1}A_p$  is the coefficient vector on the  $p^{\text{th}}$  lagged variable.  $e_t = A_0^{-1}\epsilon_t$  are the reduced form (forecast) errors with  $E(e_t) = 0$  and a non-diagonal variance-covariance matrix  $\Omega$  which now captures the contemporaneous effects of shocks on the variables.

- **Structural VAR: The A-B Model**

The vector  $A\epsilon_t$  can further be written as a product of

- a matrix B, and
- $n$  independent orthonormalised disturbances  $u_t$  with zero mean and covariance equal to the unit matrix  $I_n$  ( $E_t[u_t u_t'] = I_n$ ).

$$A\epsilon_t = Bu_t \tag{2.4}$$

Matrices A and B can be chosen to capture the contemporaneous interactions among the  $x_t$  along with the standard deviations of the structural shocks.

In the next section I use the structural formulation of VAR (SVAR) to evaluate to what extent shocks to *net worth of banks* and on *net worth of firms* explain the fluctuations on a third variable of interest. Although the SVAR model allows me to obtain non-recursive orthogonalization of the errors terms,  $\epsilon_t$ , initially I make use of the simplicity of recursive formulation to obtain a lower triangular A matrix and a diagonal B matrix<sup>4</sup>. Next, I add further restrictions on the lower triangular A matrix to define contemporaneous effects of one net worth shock on the other net worth variable consistent with my interest. These additional restrictions on A matrix will be defined in each subsection separately.

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<sup>4</sup> In a *normalised* system the diagonal elements of A will be *normalised* to one while the diagonal elements of B matrix will contain the size of the variances of each structural innovation.

## 2.3 Empirical Results from a 3-variable VAR

In this section I analyze the impact that variations in firm and bank net worth have for the fluctuations of the premium of external financing and for the business cycles.

I run a three-variable vector-auto-regression (VAR) for the set of observed variables  $x'_t = [nwF_t, nwB_t, efp_t]$ , where  $efp_t$  is either  $efpB_t$  or  $efpBA_t$ . As the variables are filtered there is only a vector of constants  $\alpha$  but no deterministic trend (eq. 2.5).

$$x_t = \alpha + \sum_{j=1}^T \Phi_j x_{t-j} + A\epsilon_t \quad (2.5)$$

where,  $\alpha$  is a vector  $k \times 1$  and  $\Phi_j$  is a matrix  $k \times k$  of coefficients, and  $x_t = [nwF_t, nwB_t, efp_t]$  is a vector of three variables, *firm net worth*, *bank net worth* and *external finance premium* respectively.  $T$  is the time lag of the VAR and the matrix  $A\epsilon'_t = A[\epsilon_t^{nwF}, \epsilon_t^{nwB}, \epsilon_t^{efp}]$  is the set of observed residuals obtained from the unrestricted VAR. I use a lag of  $T = 4$  across all VARs.

I define a 3-variable structural VAR (S-VAR) whereby I give equal weights to the innovations of the first two variables to affect the third one. In this S-VAR model both *firm net worth* and *bank net worth* are considered to vary only due to exogenous innovations of their own while both affect the *efp* variable. It ensures equal weights of each *net worth* shock on the *efp* variable. To set up the S-VAR I define matrices A and B as in equation 2.6.

$$A\epsilon_t = Bu_t \quad (2.6)$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ NA & NA & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} NA & 0 & 0 \\ 0 & NA & 0 \\ 0 & 0 & NA \end{bmatrix}$$

where  $u_t$  is the unobserved orthonormal structural innovation ( $[u_t, u_t'] = I$ ). The assumption of orthonormal innovations implies B is a diagonal matrix. The diagonal elements of B to be estimated are the sizes of unobserved orthonormal innovations driving the VAR. A non-zero  $A(i, j) = NA$  (for  $i \neq j$ ) element implies the innovations of the  $j^{\text{th}}$  variable have an impact on the fluctuations of  $i^{\text{th}}$  variable.

Definitions of matrices A and B in the structural VAR (S-VAR) above assume that exogenous innovations of *firm net worth* and of *bank net worth* do not affect each other. Based on ordering of the variables this definition of matrices A and B gives the innovations on *firm net worth* and *bank net worth* an equal chance to explain the fluctuations of *external finance premium*.

I run two alternative tests to check the robustness of results. The first is a Cholesky decomposition of shocks given the current ordering of the variables (named Cholesky VAR). For a Cholesky VAR the element  $A(2, 1) = NA$  of matrix is non-zero (eq. 2.7). When the  $A(2, 1)$  coefficient is not restricted to zero the innovations of the 1<sup>st</sup> variable are allowed to have an effect on fluctuations of the 2<sup>nd</sup> variable. In a separate subsection, I will check how results would differ when a *Cholesky* VAR is estimated.

$$A\epsilon_t = Bu_t \tag{2.7}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ NA & 1 & 0 \\ NA & NA & 1 \end{bmatrix}$$

The second robustness test is a Cholesky decomposition of shocks with reversed order of net worth variables. I run a similar VAR as in equation 2.7, but the order of variables now is  $x'_t = [nwB_t, nwF_t, efp_t]$  so that bank net worth affects firm net worth in addition to their individual effect on the third variable.

### 2.3.1 How strong is the Impact of Firm and Bank Net Worth on the Premium?

The focus of these empirical tests is the period 1985-20014. Given the long series of data in US it allows me to compare fluctuations in *efp* series due to innovations on *firm net worth* or *bank net worth*.

I run the tests for the two measures of external finance premium, *efpBA* and *efpB*, as defined earlier in section 2.1. I will use an index of *forecast error variance decomposition* (FEVD) to compare the variance decomposition of *efp* variable due to innovations on each *net worth* series. Define by index  $\Pi_{efp}^i$  the average *forecast error variance decomposition* of the *efp* variable due to shock *i* for lags 1-12, where *i* stands for either *firm net worth* (*nwF*) or *bank net worth* (*nwB*).

#### Results from a S-VAR in US

Table 2.1 shows the fraction of forecast errors of borrowing premium *efpBA* explained by an exogenous innovation on *firm net worth* and on *bank net worth* for US. Column [1] shows that 47% of fluctuations of *efpBA* is mainly explained by innovations of *bank net worth* for the full sample 1950-2014.

To identify during which periods this effect is stronger I split the full sample into two sub-periods and run the same S-VAR with data from each sub-period. I split the full sample in a Great Moderation period (1985-2014) and a sub-period before that (1950-1984) (column [2] of table 2.1). Then I split the

### 2.3. Empirical Results from a 3-variable VAR

**Table 2.1:** The 12 lag average forecast error variance decomposition (FEVD) of  $efpBA$ ,  $\pi_{efp}^j$ , due to innovations on net worth of  $j = nwF, nwB$ .

Lags 1-12		1950-2014	1950-1984	1950-1997	1950-2001
		[1]	[2]	[3]	[4]
US	$\Pi_{efpBA}^{nwF}$	2	16	7	3
US	$\Pi_{efpBA}^{nwB}$	47	19	23	33
		1985-2014	1998-2014	2002-2014	
US	$\Pi_{efpBA}^{nwF}$		10	11	6
US	$\Pi_{efpBA}^{nwB}$		58	72	69

Structural VAR (S-VAR) with matrices A and B as in equation 2.6. VAR order: nwF, nwB, efpBA; neither net worth affects the other, both affect the premium.

full sample at an arbitrary selected year (1997) and further into the year 2002 right before the start of the recession (columns [3] and [4] of table 2.1)<sup>5</sup>. For each case I report the average *forecast error variance decomposition* index,  $\Pi_{efp}^i$ , calculated from the two sub-sample estimations. In lower (upper) part of the table I report the results from the recent (earlier) sup-period. The choice of years to split the sample indicates my interest in the Great Moderation period and in the recession period.

Columns [2] to [4] in upper panel of table 2.1 show results from the S-VAR tests for the first sub-sample period. In lower panel I show results of the remaining sample period. A bird-eye view of the table in columns [2] to [4] indicates the impact of *bank net worth* innovations on external finance premium has become stronger since the end of 80s or the beginning of 90s. These results are displayed in lower part of table 2.2. This period corresponds with the Great Moderation period in US economy (Stock and Watson (2002)).

<sup>5</sup> Ideally the sample for the crisis period would be 2007-2014. The problem with such a sample is that the time series are not stationary in such a short sample due to large drops in all three variables in 2007 which take several quarters to bounce back.

### 2.3. Empirical Results from a 3-variable VAR

Table 2.2 shows the variance decomposition of forecast errors of the other premium measure, the  $efpB$  series, due to innovations on the same two net worth series,  $nwF$  and  $nwB$ . Definitions of matrices A and B are the same.

Table 2.2: The 12 lag average forecast error variance decomposition (FEVD) of  $efpB$ ,  $\Pi_{efp}^j$ , due to innovations on net worth of  $j = nwF, nwB$ .

		Lags 1-12	1950-2014	1950-1984	1950-1997	1950-2001
			[1]	[2]	[3]	[4]
US	$\Pi_{efpB}^{nwF}$		4	13	12	18
US	$\Pi_{efpB}^{nwB}$		5	11	2	2
			1985-2014	1998-2014	2002-2014	
US	$\Pi_{efpB}^{nwF}$			11	7	17
US	$\Pi_{efpB}^{nwB}$			7	49	77

Structural VAR (S-VAR) with matrices A and B as in equation 2.6. VAR order:  $nwF, nwB, efpB$ ; neither net worth affects the other, both affect the premium.

For the full sample period 1950-2014 in column [1], the variations in the premium series,  $efpB$ , can be explained by the exogenous innovations of its own<sup>6</sup>. In the more recent periods, starting from the end of '90s, around 50-70% of fluctuations in the premium series  $efpB$  have been driven by exogenous innovations on *bank net worth* (lower panel of columns [3] and [4])<sup>7</sup>. The results echo the findings above. One difference from previous results is that the impact of *bank net worth* on  $efp$  becomes stronger slightly later during the Great Moderation period and therefore does not show up in the full sample S-VAR.

<sup>6</sup> As this is a 3-variable VAR where *firm net worth* and *bank net worth* explain 4 and 5 % of  $efpB$  fluctuations respectively (column [1]), the remaining 91% of variations are explained by innovations on  $efpB$ .

<sup>7</sup> It is possible that the strong monetary policy stance undertaken during Volcker rule of Federal Reserve is dominating the behavior of series  $efpB$  during 1985-2014.

### Robustness check

I check how robust these results are to the ordering of the VAR variables. I now re-run the VARs with Cholesky ordering to check the robustness of the results for both *efp* measures, *efpBA* and *efpB*. For that I redefine matrices A as in equation 2.8 while the diagonal B matrix remains the same. The difference from the S-VAR case shown earlier is that I set no restriction on the element  $A(2,1) = NA$ . It implies that, now I let innovations on first variable to have a contemporary effect on the second one.

$$A\epsilon_t = B u_t \tag{2.8}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ NA & 1 & 0 \\ NA & NA & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} NA & 0 & 0 \\ 0 & NA & 0 \\ 0 & 0 & NA \end{bmatrix}$$

This standard Cholesky decomposition of forecast errors assumes that now the innovations on *firm net worth* matter for the variation of *bank net worth*. This is in addition to their individual effects on the third variable, *efp*. This seems a reasonable ordering given that it is the firms' assets that are first exposed to fundamental exogenous shocks in the economy. In any financial system firm liabilities are mostly bank assets. Both firm liabilities and net worth will depend on quality of firm assets. Any adverse shock deteriorating the value of firm assets will affect both, the firm net worth and its ability to repay its liabilities. Hence will affect bank assets and its financial health (net worth).

### 2.3. Empirical Results from a 3-variable VAR

In each column of table 2.3 I report an index  $\Pi_{efp}^i$ . It is the 12-quarter average forecast error variance decomposition of  $efp$  due to shock 'i', where 'i' stands for either firm net worth [ $nwF$ ] or bank net worth [ $nwB$ ]. In column (i) I report the same S-VAR results shown in column [3] (lower panel) of tables 2.1 and 2.2 for the period 1998-2014<sup>8</sup>. In column (ii) I report the VAR results for the same period but with Cholesky ordering.

As a final robustness check, in column (iii) I report the VAR results for the same period but now with the *reversed order* of the variables as  $x_t' = [nwB_t, nwF_t, efp_t]$ . In what I name the *reverse order VAR* I switch the ordering of variables where *bank net worth* is the first and *firm net worth* is the second variable. This ordering assumes innovations on *bank net worth* are now allowed to explain variations in *firm net worth* while both affect  $efp$  variable.

**Table 2.3:** The 12 lag average forecast error variance decomposition (FEVD) of  $efpBA$  and  $efpB$ ,  $\Pi_{efpBA}^i$  and  $\Pi_{efpB}^i$  respectively, due to innovations on net worth  $i = nwF, nwB$  (US).

	Lags 1-12	[i] S-VAR	[ii] Cholesky VAR	[iii] Reverse order VAR
$efpBA$	$\Pi_{efpBA}^{nwF}$	11	10	11
$efpBA$	$\Pi_{efpBA}^{nwB}$	<b>72</b>	<b>71</b>	<b>69</b>
$efpB$	$\Pi_{efpB}^{nwF}$	7	4	7
$efpB$	$\Pi_{efpB}^{nwB}$	<b>49</b>	<b>46</b>	<b>44</b>

(\*) Order: [i] SVAR: neither net worth affects the other, both affect  $efp$  variable;

[ii] Cholesky VAR:  $nwF, nwB, efp$ -variable; [iii] Reverse Cholesky VAR:  $nwB, nwF, efp$ -variable.

(\*\*) IRFs in figures A.1 to A.3 of appendix A.

The results are very similar across the three columns. Columns (ii) and (iii) suggests that innovations on *bank net worth* are important for the variations in the  $efp$  variable,  $efpBA$  or  $efpB$ , and do not depend on ordering, S-VAR or Cholesky VAR. The impulse responses from the VAR results of table 2.3 are

<sup>8</sup> I choose a shorter sample period for the robustness tests to match the period when bank net worth becomes significant in the VAR results with  $efpB$  as a proxy for premium (column [ii] in lower panel of table 2.2). *Bank net worth* has become important for external finance premium proxy,  $efpB$ , only after this period

## 2.3. Empirical Results from a 3-variable VAR

shown in figures A.1 to A.3 of appendix A. Each figure shows the responses of the two measures of external finance premium,  $efpBA$  and  $efpB$ , following innovations on *firm* and *bank net worth*.

< Figures A.1 to A.3 here >

### Is bank net worth more significant in UK?

The data for UK are of a shorter time span. I can collect data on net worth based on stock indices from *Datastream* as proxy for net worth<sup>9</sup>. These data are different from balance sheet data collected in the case of US as described earlier in section 2.1.

In table 2.4 I report average forecast error variance decomposition (FEVD) of one borrowing spread measure for UK,  $efp$  due to innovations on *net worth* based on UK data.

**Table 2.4:** The 12 lag average forecast error variance decomposition (FEVD) of  $efp$ ,  $\Pi_{efp}^i$ , due to innovations on net worth  $i = nwF, nwB$  (UK).

		Lags 1-12	[i] S-VAR	[ii] Cholesky VAR	[iii] Reverse order VAR
[A]					
$efp$	$\Pi_{efp}^{nwF}$		11	12	7
$efp$	$\Pi_{efp}^{nwB}$		54	34	39
[B]					
$efp$	$\Pi_{efp}^{nwF}$		14	9	8
$efp$	$\Pi_{efp}^{nwB}$		44	24	25

(\*) Order: [i] SVAR: neither net worth affects the other, both affect  $efp$  variable;

[ii] Cholesky VAR:  $nwF, nwB, efp$  variable; [iii] Reverse Cholesky VAR:  $nwB, nwF, efp$  variable.

Bank net worth for UK is an equity index of banking sector in panel [A] and of financial sector in panel [B].

Source: *Datastream* 1985-2014.

<sup>9</sup>US and UK are two economies that have well developed financial markets that allow for a considerable share of market based financing for corporate firms to take place and for equity securities to be traded in the financial markets.

I use two different series of bank net worth. One is an equity index of banking sector in UK (panel [A]) and the other is an equity index of financial sector in UK (panel [B]). Data are obtained from *Datastream* for the period 1985-2014. The results are similar to those in US. Exogenous innovations of bank net worth are significant for the variations in *external finance premium*. Impulse response functions (IRFs) A.5 in appendix indicate that external finance premium declines following a positive innovation on bank net worth.

< Figure A.5 here >

One possible argument to motivate these latter results is the largely developed market-based financial system in US and partially in UK. In market-based financial systems firms can obtain financing by selling equity stakes to financial institutions. While these equity securities can also be bought by households and other entities banks and other financial entities buy the majority of them. The amount that may go to households and non-financial institutions is negligibly small.

### 2.3.2 Do fluctuations in *net worth* have real effects?

In this part I explore whether the innovations on *firm net worth* and *bank net worth* explain the variations in output gap. I run similar S-VARs as in previous section but now instead of a series for the external finance premium I test how the exogenous innovations on bank net worth and firm net worth affect the fluctuations of *per-capita-GDP*.  $Y$  is *per-capita-GDP* deviations from the *potential GDP per capita* ( $\bar{Y}$ ). The latter is approximated by the trend of the HP filter. The rationale for replacing the external finance premium with output gap is to check for robustness of the previous results. If the bank net worth effect on external finance premium is strong and robust then it must have an impact,

### 2.3. Empirical Results from a 3-variable VAR

through credit flows, on output gap fluctuations as well. Results are reported in table 2.5.

**Table 2.5:** The 12 lag average forecast error variance decomposition (FEVD) of *GDP per capita*,  $\Pi_Y^i$ , due to innovations on net worth  $i = nwF, nwB$ .

		Lags 1-12	1950-2014	1950-1984	1950-1997	1950-2001
			[1]	[2]	[3]	[4]
Y (US)	$\Pi_Y^{nwF}$		1	10	1	0
Y (US)	$\Pi_Y^{nwB}$		23	15	15	19
			1985-2014	1998-2014	2002-2014	
Y (US)	$\Pi_Y^{nwF}$			10	15	23
Y (US)	$\Pi_Y^{nwB}$			47	51	41

Structural VAR (S-VAR) with matrices A and B as in equation 2.6. VAR order: nwF, nwB, Y; neither net worth affects the other, both affect the output.

The results resonate with the previous ones regarding the role of *bank net worth* for the fluctuations of the *efp*. Innovations on *bank net worth* explain the variations of output during the Great Moderation period. Results support the hypothesis that the significance of *bank net worth* relative to *firm net worth* on *efp* measures, as seen in previous tables, is strong and robust enough to show up as an effect on the variations of output in the US economy.

For the sake of comparison I follow the same robustness tests with output gap as with *efp*.  $Y_t$  is the variable of interest (measured as *GDP per capita* less potential output). Results are reported in table 2.6. In column (i) I list the same S-VAR results shown in column [3] of tables 2.5. In columns (ii) and (iii) I report the Cholesky VAR results for both cases when innovations of *firm net worth* affect *bank net worth* and vice versa.

The results from these alternative VAR tests confirm the same findings from the structural VAR. *Bank net worth* is more significant for the dynamics of US business cycles. Impulse response functions (IRFs) in figure A.4 of appendix

### 2.3. Empirical Results from a 3-variable VAR

**Table 2.6:** The 12 lag average forecast error variance decomposition (FEVD) of  $Y$ ,  $\Pi_Y^i$ , due to innovations on net worth  $i = nwF, nwB$  (US).

	Lags 1-12	[i] S-VAR	[ii] Cholesky VAR	[iii] Reverse order VAR
Y (US)	$\Pi_Y^{nwF}$	15	6	15
Y (US)	$\Pi_Y^{nwB}$	51	46	51

(\*) Order: [i] SVAR: neither net worth affects the other, both affect Y variable;

[ii] Cholesky VAR: nwF, nwB, Y variable; [iii] Reverse Cholesky VAR: nwB, nwF, Y variable.

(\*\*) IRFs in figure A.4 of appendix A.

indicate that positive innovations on *bank net worth* give rise to higher output gap<sup>10</sup>. Occasionally, depending on the ordering of the variables, output gap responds positively to innovations on *firm net worth*. The magnitude of their impact measured by the forecast error variance decomposition shown here in table 2.6 indicates that the impact of *bank net worth* is strong and robust for any ordering in the VARs.

< Figure A.4 here >

#### Is there any real effect of firm and bank net worth in UK?

I report the average forecast error variance decomposition (FEVD) of output gap,  $Y$ , due to innovations on *net worth* based on UK data in table 2.7. Results are similar to those in the case of US. It is exogenous innovations of bank net worth that are significant for the variations in output in UK.

Impulse response functions (IRFs) in figure A.6 on page 54 of appendix indicate that output goes up upon a positive innovation on *bank net worth*. IRFs indicate a slightly positive impact of output due to innovations on *firm net worth* as well. Output response to firm net worth is not strong as shown by

<sup>10</sup>In the IR graphs in appendix, innovations on *bank net worth* take the name 'shock 2' in the S-VAR and Cholesky VAR. With reverse ordering of variables in *reversed Cholesky VAR*, innovations on *bank net worth* get the name 'shock 1'.

### 2.3. Empirical Results from a 3-variable VAR

**Table 2.7:** The 12 lag average forecast error variance decomposition (FEVD) of  $Y$ ,  $\Pi_Y^i$ , due to innovations on net worth  $i = nwF, nwB$  (UK).

		Lags 1-12	[i] S-VAR	[ii] Cholesky VAR	[iii] Reverse order VAR
[A]					
Y (UK)	$\Pi_Y^{nwF}$		17	14	10
Y (UK)	$\Pi_Y^{nwB}$		47	27	31
[B]					
Y (UK)	$\Pi_Y^{nwF}$		25	11	16
Y (UK)	$\Pi_Y^{nwB}$		52	33	28

(\*) Order: [i] SVAR: neither net worth affects the other, both affect Y variable;

[ii] Cholesky VAR: nwF, nwB, Y variable; [iii] Reverse Cholesky VAR: nwB, nwF, Y variable.

Bank net worth for UK is an equity index of banking sector in panel [A] and of financial sector in panel [B].

Source: *Datastream* 1985-2014.

FEVD results in table 2.7 above. The share of output variations explained by innovations on *firm net worth* is less than 20%.

< Figure A.6 here >

*Firm* and *bank net worth* are endogenous variables in the economy driven by technology innovations and possibly exogenous policy and other shocks. The positive correlation of *net worth* series with output and investment (table A.1 in appendix) is a potential signal that fundamental shocks have strong impact on all these variables (table A.1 in appendix). To assess how productivity or policy shocks affect fluctuations of the two *net worth* variables, in the next subsection I add technology and policy shocks to the list of variables in the current VAR.

< Table A.1 here >

## 2.4 What are the fundamental drivers of net worth and the *efp*?

Fluctuations in *bank net worth* and *firm net worth* variables are not solely due to exogenous innovations of their own. They also respond to key fundamental shocks. In this section I analyse how the results from the previous section change when (proxy for) key technology or policy shocks are present in a VAR. I ask "what fraction of the fluctuations of *efp* do innovations in fundamental shocks and in both *net worth* variables explain in the VAR system?". To do so I add three additional variables to the previous 3-variable VAR system. I add a proxy for *total factor productivity* ( $tfp_t$ ), one for *investment specific technology* ( $ist_t$ ) and one for the Fed's *nominal interest rate* ( $R_t^n$ ). This is a 6-variable VAR system that identifies the extent to which technology or policy shocks are critical for *external finance premium* and for *net worth* variables. *Bank net worth* and *firm net worth* can still have an impact on the variations in *external finance premium*.

I run the 6-variable vector-auto-regression (VAR) with data from the period 1985-2014. The selected sample is based on results in previous section that bank net worth had become significant for *efp* in US.

$$x_t = \alpha + \sum_{j=1}^T \Phi_j x_{t-j} + A \epsilon_t \quad (2.9)$$

$$x_t' = [tfp_t, ist_t, R_t^n, nwF_t, nwB_t, efp_t]$$

where  $x_t$  is now a matrix of six variables.  $Tfp_t$  is total factor productivity shock,  $ist_t$  is investment technology shock and  $R_t^n$  is Fed's nominal interest rate. The three variables from previous section are *firm net worth*, *bank net worth* and a proxy for *external finance premium*. The matrix  $A \epsilon_t' = A[\epsilon_t^{tfp}, \epsilon_t^{ist}, \epsilon_t^{Rn}, \epsilon_t^{nwF}, \epsilon_t^{nwB}, \epsilon_t^{efp}]$  is the set of observed residuals obtained from

the unrestricted VAR and the remaining coefficients have the same meaning as before. The number of lags is  $T = 4$  again.

### 2.4.1 Definition of Structural VAR

#### Definition of S-VAR 1

A and B matrices for a structural VAR (S-VAR) are defined in equation 2.10.  $u_t$  is the vector of unobserved orthonormal structural innovation such that  $[u_t, u_t'] = I$ . Matrix B is defined as a 6-by-6 diagonal matrix. Its non-zero (diagonal) elements, denoted 'NA', imply that the size of the exogenous innovations of each variable will be determined by the data. Definition of matrix B will remain the same in all the 6-variable S-VAR tests of this section. For the structural VAR (S-VAR 1) I will explain in detail the definition of A matrix.

$$A\epsilon_t = Bu_t \tag{2.10}$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ NA & NA & NA & 1 & 0 & 0 \\ NA & NA & NA & 0 & 1 & 0 \\ NA & NA & NA & NA & NA & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} NA & 0 & 0 & 0 & 0 & 0 \\ 0 & NA & 0 & 0 & 0 & 0 \\ 0 & 0 & NA & 0 & 0 & 0 \\ 0 & 0 & 0 & NA & 0 & 0 \\ 0 & 0 & 0 & 0 & NA & 0 \\ 0 & 0 & 0 & 0 & 0 & NA \end{bmatrix}$$

## 2.4. What are the fundamental drivers of net worth and the *efp*?

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All  $A(i,i) = 1$  elements for  $i = 1$  to 6 assume that each variable in the vector of variables  $x'_t$  responds to its own exogenous orthonormal innovation. All non-diagonal elements of the first three rows of  $A$  are zero (all  $A(i,j)$  elements for  $i \neq j$  and  $i,j = 1,2,3$ ). This definition assumes that  $tfp_t$ ,  $ist_t$  and  $R_t^n$  respond only to their own exogenous innovations. The motivation is to replicate the definition of these shocks in general equilibrium models<sup>11</sup>. The current set-up gives an equal chance to the innovations on these fundamental variables to affect the net worth and *efp* variables.

A non-zero element  $A(i,j)$ , for  $i \neq j$  and denoted 'NA', assumes that the innovation of the  $j^{\text{th}}$  variable is allowed to have a contemporaneous impact on the fluctuations of  $i^{\text{th}}$  variable. In rows 4 to 5, 'NA' elements imply that innovations on  $tfp_t$ ,  $ist_t$  and  $R_t^n$  are allowed to contemporaneously affect the two net worth variables,  $nwF_t$  and  $nwB_t$ . In row 6 the 'NA' elements imply that innovations on all the other 5 variables are allowed to contemporaneously affect the last variable of  $x'_t$ , that is *efp*. Note that element  $A(4,5)=0$  implies that innovations on the 4<sup>th</sup> variable, *firm net worth*, has a zero contemporaneous effect on the 5<sup>th</sup> variable, *bank net worth*.

### Definition of S-VAR 2

I define a second structural VAR model which I name it S – VAR2. Matrix  $B$  is the same. Here I do not restrict innovations on *firm net worth* to have a zero contemporaneous effect on *bank net worth*. Instead I let element  $A(4,5) = NA \neq 0$  assuming that innovation on *firm net worth* have a contemporaneous

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<sup>11</sup>Reporting results from VARs where the three shocks affect each other would overload this section with results that do not add much and that goes beyond the scope of this analysis.

2.4. What are the fundamental drivers of net worth and the *efp*?

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effect on *bank net worth* (eq. 2.11). All the other elements of A matrix remain unchanged.

$$A\epsilon_t = Bu_t \quad (2.11)$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ NA & NA & NA & 1 & 0 & 0 \\ NA & NA & NA & NA & 1 & 0 \\ NA & NA & NA & NA & NA & 1 \end{bmatrix}$$

**Definition of S-VAR 3**

Finally I define a third structural VAR named S – VAR3 where matrices A and B are the same as above but the order of variable changes such that  $nwF_t$  and  $nwB_t$  switch places as shown in equation 2.12.

$$x'_t = [tfp_t, ist_t, R_t^n, nwB_t, nwF_t, efp_t] \quad (2.12)$$

The definition of A matrix is the same as in equation 2.11. The re-ordering of two net worth series assumes that innovations on *bank net worth* have a contemporaneous effect on *firm net worth*.

Finally in each of these three S-VAR models I replace the external finance premium proxy variable, *efp*, with the output gap variable, Y. The motivation is to compare the impact of fundamental shocks in explaining external finance premium to their impact on output. An impact that is similar in size would be a robustness check for the role of external finance premium.

2.4. What are the fundamental drivers of net worth and the *efp*?

2.4.2 Which fundamental shocks drives *efp* in a 6-variable S-VAR?

I run the tests for the two measures of external finance premium, *efpBA* and *efpB*, as defined earlier in section 2.1. I use the same index of 12 lag average forecast error variance decomposition (FEVD) of an *efp* variable,  $\Pi_{efp}^i$ , due to shock *i* where *i* stands for either *tfp*, *ist*, *Rn*, firm net worth (*nwF*) or bank net worth (*nwB*)<sup>12</sup>.

**Table 2.8:** The 12 lag average forecast error variance decomposition (FEVD) of *efp* ( $\Pi_{efpBA}^i$  and  $\Pi_{efpB}^i$  respectively) due to innovations on *i* = *tfp*, *ist*, *Rn*, *nwF* and *nwB* for the period 1985-2014.

		Lags 1-12	[i] S-VAR 1	[ii] S-VAR 2	[iii] S-VAR 3
<b>[A] :</b>		$x_t' = [tfp_t, ist_t, R_t^n, nwF_t, nwB_t, efpBA_t]$			
<i>efpBA</i>	$\Pi_{efpBA}^{tfp}$		<b>55</b>	<b>61</b>	<b>61</b>
<i>efpBA</i>	$\Pi_{efpBA}^{ist}$		7	8	8
<i>efpBA</i>	$\Pi_{efpBA}^{Rn}$		9	10	10
<i>efpBA</i>	$\Pi_{efpBA}^{nwF}$		10	4	8
<i>efpBA</i>	$\Pi_{efpBA}^{nwB}$		12	10	6
<b>[B] :</b>		$x_t' = [tfp_t, ist_t, R_t^n, nwF_t, nwB_t, efpB_t]$			
<i>efpB</i>	$\Pi_{efpB}^{tfp}$		<b>25</b>	<b>27</b>	<b>27</b>
<i>efpB</i>	$\Pi_{efpB}^{ist}$		6	7	7
<i>efpB</i>	$\Pi_{efpB}^{Rn}$		<b>41</b>	<b>44</b>	<b>44</b>
<i>efpB</i>	$\Pi_{efpB}^{nwF}$		10	5	10
<i>efpB</i>	$\Pi_{efpB}^{nwB}$		13	12	7

(\*) [i] SVAR 1: neither net worth affects the other, both affect *efp* variable; [ii] SVAR 2: Shocks on firm net worth affect bank net worth; [iii] SVAR 2: bank net worth shocks affect firm net worth.

The numbers do not add up to 100 column-wise (for each panel) as I have not included the percent of variations explained by own shocks of each variable.

In table 2.8 I report results with US data series for the period 1985-2014 for the two different proxy series of external finance premium.

- In panel [A] the vector of variables is  $x_t' = [tfp_t, ist_t, R_t^n, nwF_t, nwB_t, efpBA_t]$ .
- In panel [B] the vector of variables is  $x_t' = [tfp_t, ist_t, R_t^n, nwF_t, nwB_t, efpB_t]$ .

<sup>12</sup>I do not report FEVD due to exogenous *efp* shock. Hence column-wise the sum of each column is not 100.

## 2.4. What are the fundamental drivers of net worth and the *efp*?

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In each column I report the percentage of variations in *efp* variables explained by variations of the other 5 variables of vector  $x_t'$ . The results in panel [A] indicate that *tfp* innovations explain around 55-60% of variations of the variable, *efpBA*. The fraction of variations of *efpBA* explained by innovations in other variables is negligibly small.

In panel [B] the VAR results with the second proxy series, *efpB*, indicate that innovations on *policy rate*, *Rn*, and on *tfp* explain around 40% and 25%, respectively, of fluctuations of *efpB*. The results do not depend on the ordering of the two wealth variables, firm and bank net worth.

In a 3-variable VAR in previous section, bank net worth was explaining both *efp* fluctuations. From these tests I learn that, as expected, *tfp* takes over in explaining the variations in premium measures. Investment specific technology shocks do not seem to matter for the *efp* while policy shocks have a strong impact only on the second premium measure, *efpB*.

### 2.4.3 Impact of fundamental shocks for *Output* fluctuations

In a separate test I repeat the above exercise with the vector of variables  $x_t' = [tfp_t, ist_t, R_t^n, nwF_t, nwB_t, Y_t]$  where  $Y_t$  is the output variable replacing the *efp* variable. I ask '*what fraction of output fluctuations do these three exogenous innovations explain?*'. If the results from the previous two VARs are robust then, given the strong counter-cyclical nature of external finance premium shown earlier in table A.1 (see appendix), then the same policy shocks should have similar impact on output variations. Therefore, the aim is to verify to what extent the above VAR results are consistent with this argument of counter-cyclical external finance premium.

In table 2.9 I show that a fraction of around 20-25% of output fluctuations are explained by innovations of *policy rate* and a similar share by *tfp*. While

## 2.4. What are the fundamental drivers of net worth and the *efp*?

**Table 2.9:** The 12 lag average forecast error variance decomposition (FEVD) of *Output* ( $\Pi_Y^i$ ) due to innovations on *tfp*, *ist*, *Rn* and *net worth*  $i = tfp, ist, Rn, nwF, nwB$  for the period 1985-2014.

Lags 1-12		[i] S-VAR 1	[ii] S-VAR 2	[iii] S-VAR 3
[C] : $x_t' = [tfp_t, ist_t, R_t^p, nwF_t, nwB_t, Y_t]$				
Y	$\Pi_Y^{tfp}$	<b>25</b>	<b>24</b>	<b>24</b>
Y	$\Pi_Y^{ist}$	6	7	7
Y	$\Pi_Y^{Rn}$	<b>23</b>	<b>22</b>	<b>22</b>
Y	$\Pi_Y^{nwF}$	7	5	7
Y	$\Pi_Y^{nwB}$	3	3	2

(\*) [i] SVAR 1: neither net worth affects the other, both affect *efp* variable; [ii] SVAR 2: Shocks to *firm net worth* affect *bank net worth*; [iii] SVAR 2: *bank net worth* shocks affect *firm net worth*.

The numbers do not add up to 100 column-wise (for each panel) as I have not included the percent of variations explained by own shocks of each variable.

negligibly small fraction can be explained by other variables, the remaining variations are explained by own shocks<sup>13</sup>. The significance of *policy* and *tfp* shocks for *output* and for *efpB* variations is consistent with a counter-cyclical external finance premium.

### 2.4.4 What drives *net worth*?

What explains the variations in net worth of firm and bank in these same S-VARs? In this subsection I report share of *net worth* fluctuations due to fundamental and own shocks from the same S-VARs reported in section 2.4.2.

In tables 2.10 and 2.11 I report the share of fluctuations of net worth variables, *firm net worth* and *bank net worth* respectively, that are explained by variation in *tfp*, *ist*, *Rn* and in both *net worth* variables. These results are obtained from the same S-VARs that delivered the results reported in table 2.8 with matrices A and B defined in equations 2.10, 2.11 and 2.12. Note that in the upper

<sup>13</sup>The sum of numbers in each column do not add to 100. In these tables, I have not included the fraction of output variations explained by own shocks. In table 2.9 exogenous output shock explains around 30% of output fluctuations.

## 2.4. What are the fundamental drivers of net worth and the *efp*?

panel the vector of variables is  $x'_t = [tfp_t, ist_t, R_t^n, nwF_t, nwB_t, efpBA_t]$  while in lower panel I replace the proxy for the premium,  $efpBA_t$ , with  $efpB_t$ .

**Table 2.10:** The 12 lag average forecast error variance decomposition (FEVD) of *firm net worth*,  $nwF$ , ( $\Pi_{nwB}^i$ ) due to innovations on *tfp*, *ist*, *Rn* and *net worth*  $i = tfp, ist, Rn, nwF, nwB$  for the period 1985-2014.

		Lags 1-12	[i] S-VAR 1	[ii] S-VAR 2	[iii] S-VAR 3
[A] :		$x'_t = [tfp_t, ist_t, R_t^n, nwF_t, nwB_t, efpBA_t]$			
nwF	$\Pi_{nwF}^{tfp}$		<b>27</b>	<b>32</b>	<b>32</b>
nwF	$\Pi_{nwF}^{ist}$		9	10	10
nwF	$\Pi_{nwF}^{Rn}$		<b>22</b>	<b>26</b>	<b>26</b>
nwF	$\Pi_{nwF}^{nwF}$		<b>33</b>	<b>24</b>	<b>28</b>
nwF	$\Pi_{nwF}^{nwB}$		8	7	4
[B] :		$x'_t = [tfp_t, ist_t, R_t^n, nwF_t, nwB_t, efpB_t]$			
nwF	$\Pi_{nwF}^{tfp}$		11	13	13
nwF	$\Pi_{nwF}^{ist}$		9	10	10
nwF	$\Pi_{nwF}^{Rn}$		<b>26</b>	<b>29</b>	<b>29</b>
nwF	$\Pi_{nwF}^{nwF}$		<b>30</b>	<b>24</b>	<b>29</b>
nwF	$\Pi_{nwF}^{nwB}$		<b>23</b>	<b>23</b>	<b>18</b>

(\*) [i] SVAR 1: neither net worth affects the other, both affect *efp* variable; [ii] SVAR 2: Shocks to *firm net worth* affect *bank net worth*; [iii] SVAR 3: *bank net worth* shocks affect *firm net worth*.

The numbers do not add up to 100 column-wise (for each panel) as I have not included the percent of variations explained by shocks to the last variable.

The result indicate that *policy shock* explains around 25-30% of *firm net worth* fluctuations in both VAR tests, independent of whether  $efpBA_t$  or  $efpB_t$  is included in VAR (panels [A] and [B] of table 2.10). Innovations on  $tfp_t$  explain around 27-32% of *firm net worth* fluctuations only when  $efpBA_t$  is used in the VAR system (panel [A])<sup>14</sup>.

In table 2.11 I show the fraction of FEVD of *bank net worth* explained by each shock. Innovations on *tfp* and *policy rate* each explain around 27-30% of *bank net worth* fluctuations in the S-VAR with *efpBA* proxy for the premium (panel [A] of table 2.11). In panel [B], with *efpB* as proxy for the premium, none of

<sup>14</sup>A large fraction of net worth variations in tables 2.10 is explained by own (net worth) shocks as shown. Fraction of FEVD explained by *efp* shock are not shown.

## 2.5. Conclusions

the shocks is significant for the bank net worth. Its fluctuations are driven by exogenous shocks of its own (75-80%).

**Table 2.11:** The 12 lag average forecast error variance decomposition (FEVD) of *bank net worth*,  $nwB$ , ( $\Pi_{nwB}^i$ ) due to innovations on *tfp*, *ist*, *Rn* and *net worth i* = *tfp*, *ist*, *Rn*, *nwF*, *nwB* for the period 1985-2014.

		Lags 1-12	[i] S-VAR 1	[ii] S-VAR 2	[iii] S-VAR 3
[A] :		$x_t' = [tfp_t, ist_t, R_t^n, nwF_t, nwB_t, efpBA_t]$			
nwB	$\Pi_{nwB}^{tfp}$		<b>29</b>	<b>30</b>	<b>30</b>
nwB	$\Pi_{nwB}^{ist}$		9	9	9
nwB	$\Pi_{nwB}^{Rn}$		<b>27</b>	<b>28</b>	<b>28</b>
nwB	$\Pi_{nwB}^{nwF}$		2	6	1
nwB	$\Pi_{nwB}^{nwB}$		<b>32</b>	<b>24</b>	<b>29</b>
[B] :		$x_t' = [tfp_t, ist_t, R_t^n, nwF_t, nwB_t, efpB_t]$			
nwB	$\Pi_{nwB}^{tfp}$		3	3	3
nwB	$\Pi_{nwB}^{ist}$		2	2	2
nwB	$\Pi_{nwB}^{Rn}$		7	7	7
nwB	$\Pi_{nwB}^{nwF}$		4	8	4
nwB	$\Pi_{nwB}^{nwB}$		<b>80</b>	<b>75</b>	<b>80</b>

(\*) [i] SVAR 1: neither net worth affects the other, both affect *efp* variable; [ii] SVAR 2: Shocks to *firm net worth* affect *bank net worth*; [iii] SVAR 2: *bank net worth* shocks affect *firm net worth*.

The numbers do not add up to 100 column-wise (for each panel) as I have not included the percent of variations explained by shocks to the last variable, which are negligibly low.

Earlier in table 2.1 I reported strong and significant impact of *bank net worth* on the *borrowing premium* starting from the end of '80s. To reconcile the current results with those in table 2.1 I can argue that *tfp* and *policy* shocks are key drivers of *efp*. Net worth fluctuations have potentially become critical transmitters of these two shocks in US since the start of the Great Moderation period<sup>15</sup>.

## 2.5 Conclusions

There are three main fundamental drivers of business cycles that I analyse in this chapter, the total factor productivity (*tfp*), the investment specific technol-

<sup>15</sup>In one case, in panel [B] of table 2.11, the results indicate that shocks other than *tfp* and policy rate may be affecting the bank net worth.

ogy (*ist*) and monetary policy. The shocks on these variables are employed in a horse-race to explain fluctuations of financial and output variables in a vector autoregressive model. Two main results emerge from this analysis.

First, total factor productivity (*tfp*) shocks and monetary policy shocks are key drivers of the external finance premium and output fluctuations. Investment specific technology shock is not important.

Second, the fluctuations in the financial sector net worth are also critical factor for the dynamics of external finance premium. In the absence of fundamental shocks, it is the exogenous innovations to financial sector net worth, rather than firm net worth, that explain a large share of *efp* and business cycle fluctuations.

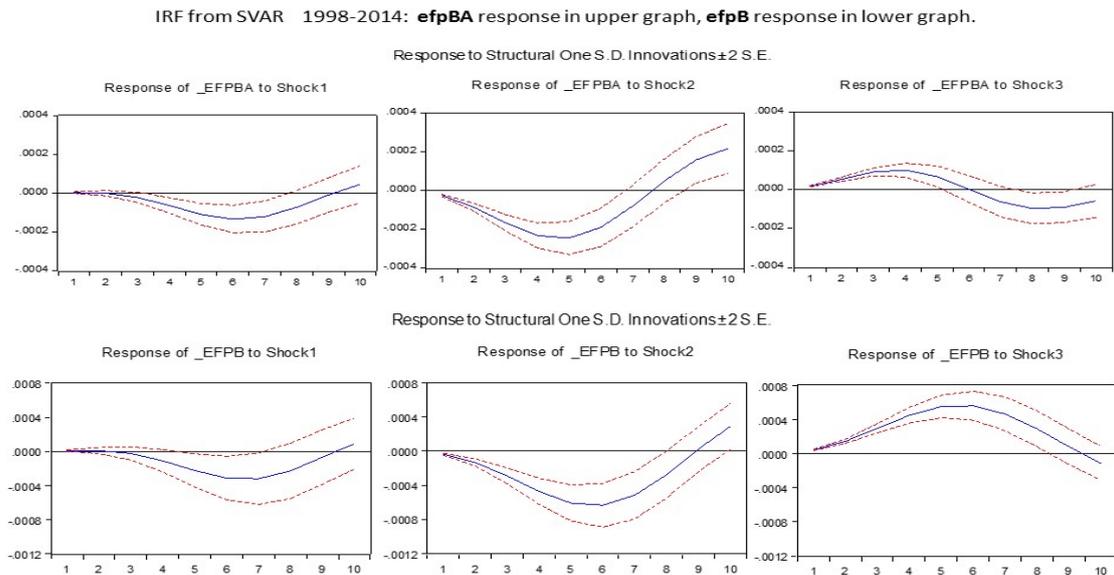
In the next chapter I lay out a *dsg*e model with two-sided financial frictions that will shed light on the propagation of these fundamental shocks.

# Appendix A

## A.1 Tables and Figures

### Net Worth and volatility of EFP in US

Figure A.1: S-VAR: Impulse Responses (IR) of *efpBA* and *efpB* due to innovations on *firm net worth*[=shock 1], *bank net worth*[=shock.2] and own (*efp*) [=shock.3] (US).



## A.1. Tables and Figures

Figure A.2: Cholesky VAR: Impulse Responses (IR) of *efpBA* and *efpB* due to innovations on *firm net worth*[=shock 1], *bank net worth*[=shock.2] and own (*efp*) [=shock.3] (US).

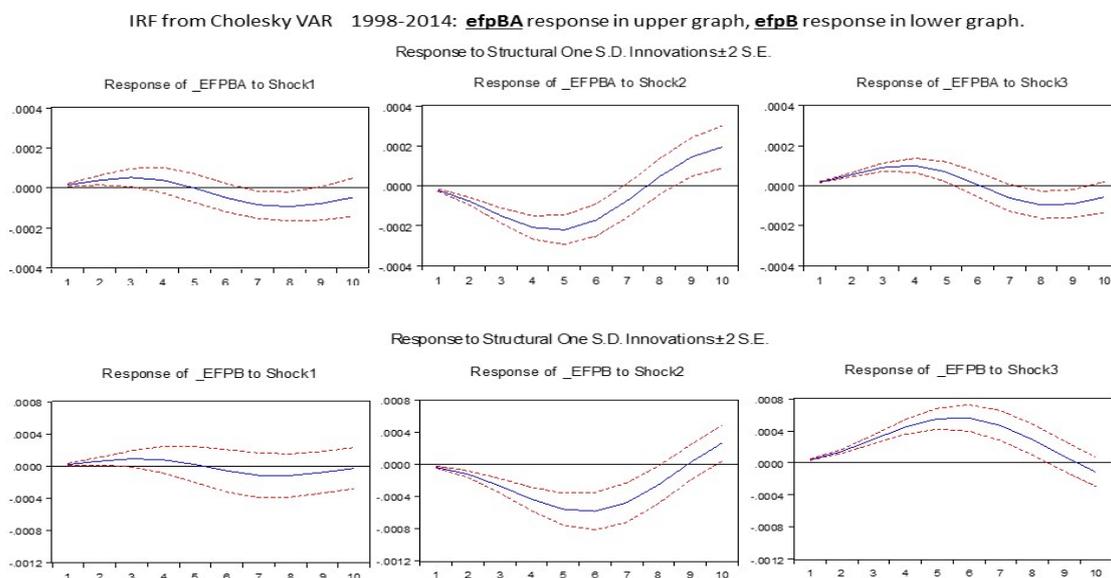
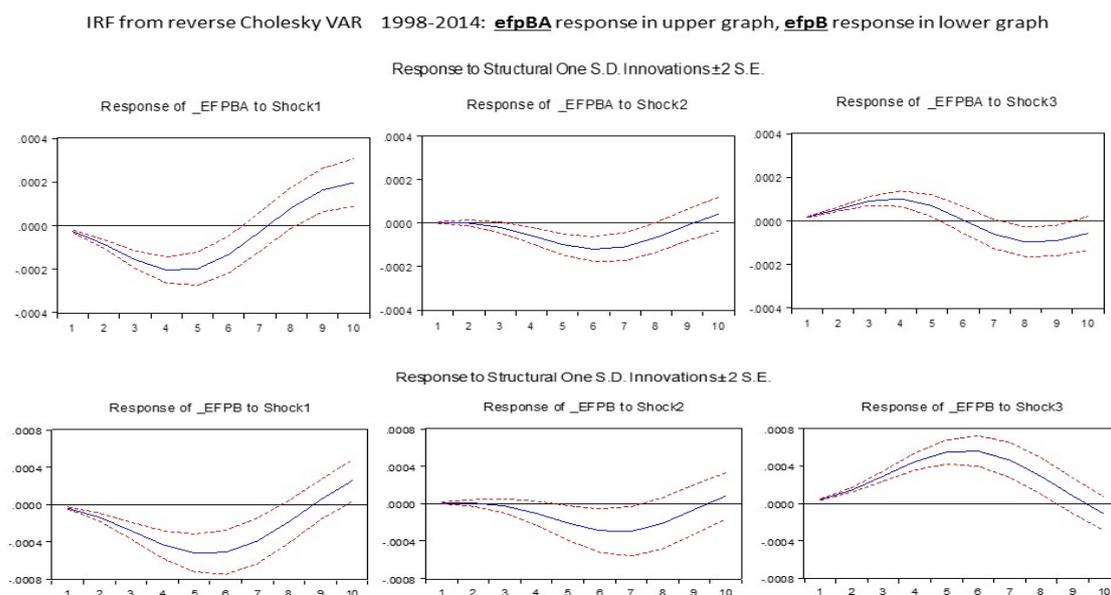


Figure A.3: Reverse order Cholesky VAR: Impulse Responses (IR) of *efpBA* and *efpB* due to innovations on *bank net worth*[=shock 1], *firm net worth*[=shock.2] and own (*efp*) [=shock.3] (US).



## A.1. Tables and Figures

Figure A.4: S-VAR, Cholesky VAR & reverse order Cholesky VAR: Impulse Responses (IR) of  $Y$  due to innovations on *firm net worth* [=shock 1], *bank net worth* [=shock.2] and own ( $Y$ ) [=shock.3] (US).

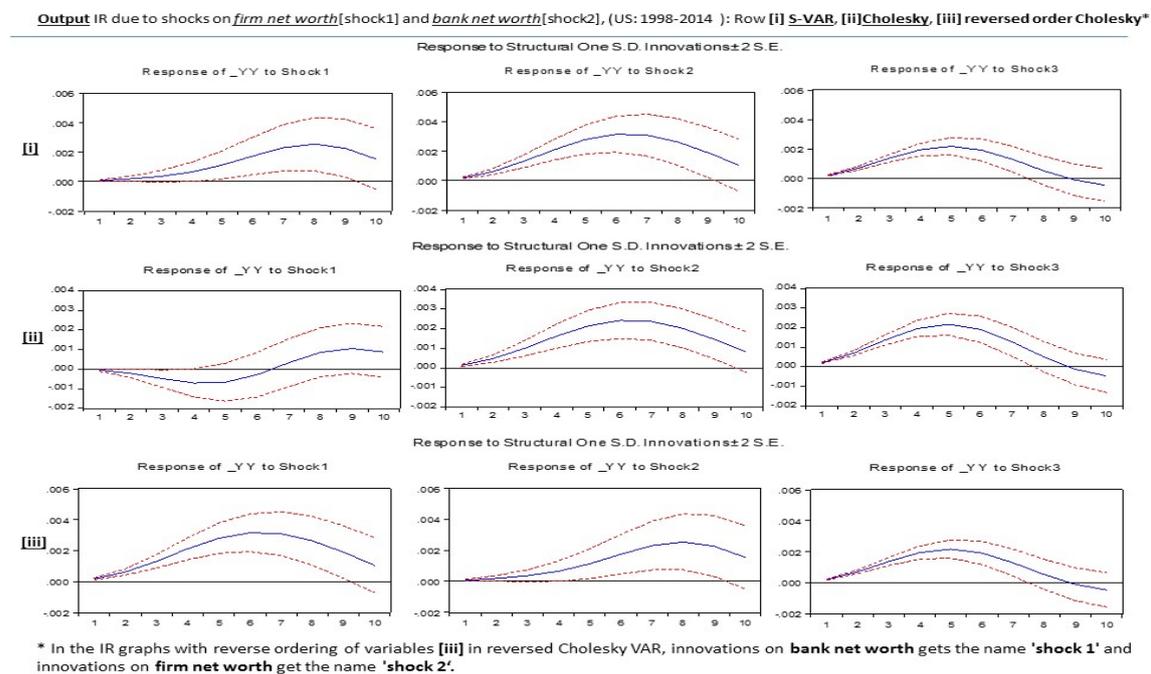
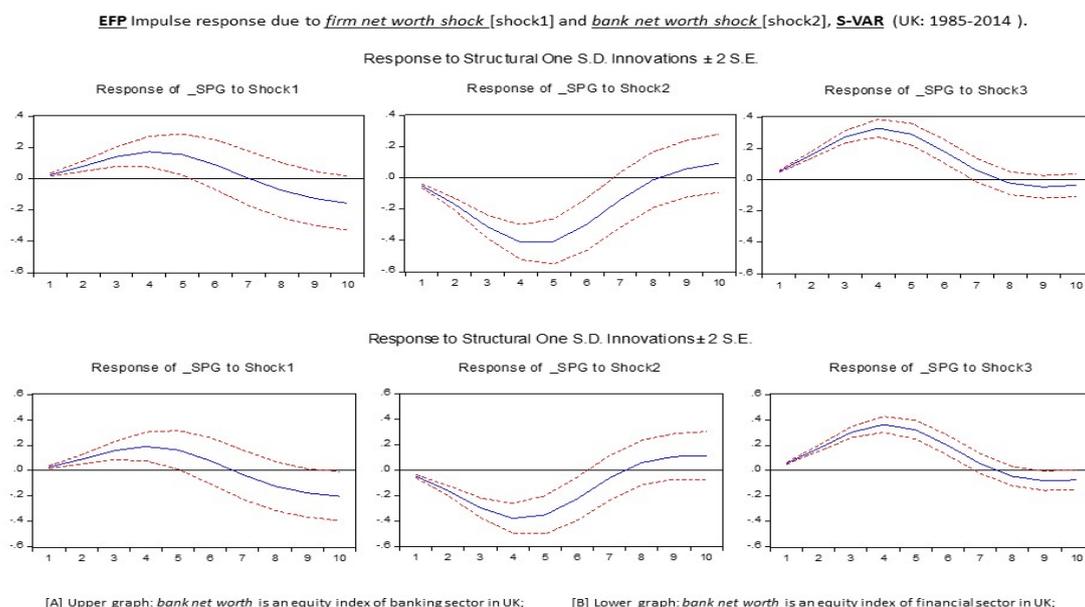
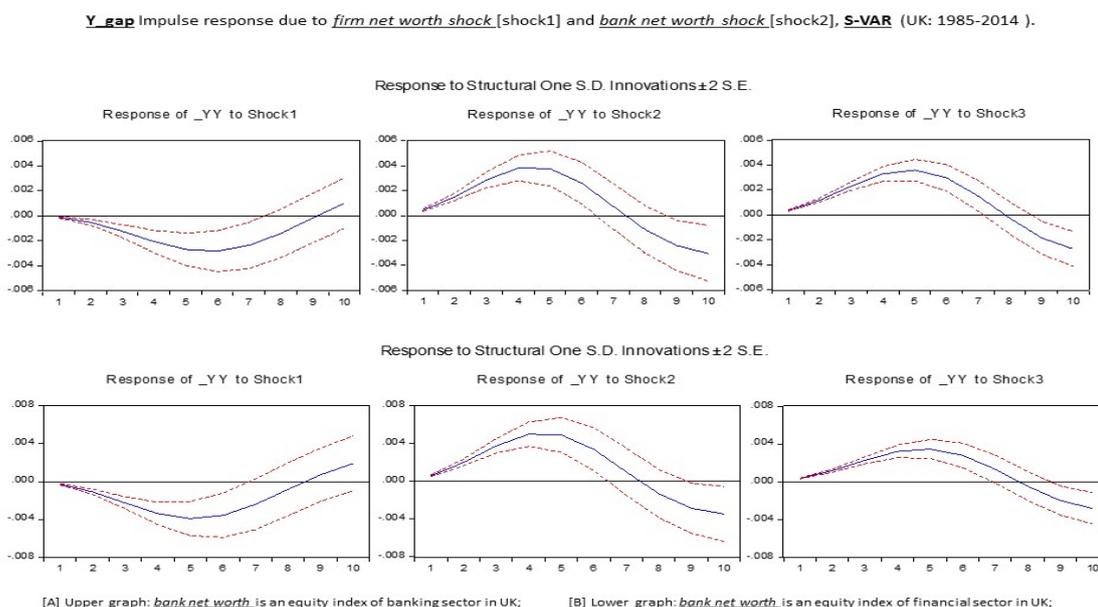


Figure A.5: S-VAR: Impulse Responses (IR) of *efp* due to innovations on *firm net worth* [=shock 1] and *bank net worth* [=shock.2] and own (*efp*) [=shock.3] (UK).



## A.1. Tables and Figures

**Figure A.6: S-VAR: Impulse Responses (IR) of Y due to innovations on *firm net worth* [=shock 1], *bank net worth* [=shock.2] and *own (efp)* [=shock.3] (UK).**



**Table A.1: Correlation with output ( $\% \rho_{x_t, Y}$ ) data (\*)**

	[a]'50-2014	[b]'80-2014	[c]'05-2014
Y	1	1	1
C	0.85	0.86	0.92
I	0.90	0.88	0.95
H	0.88	0.88	0.91
W	0.85	0.25	-0.08
$\Pi$	0.20	0.32	0.42
Q	—	—	—
Rn	0.37	0.42	0.77
Moody's BAA yield	-0.30	-0.35	-0.61
Firm Net Worth (FoF)	0.33	0.34	0.93
Bank Net Worth (FoF)	0.46	0.45	0.73
Firm Net Worth (DS)	0.41	0.61	0.81
Bank Net Worth (DS)	0.40	0.42	0.82
efpBA	-0.54	-0.40	-0.44
efpA	-0.54	-0.55	-0.78
efpB	-0.60	-0.57	-0.79

(\*) Correlations ( $\rho_{x_t, Y}$ ) multiplied by 100 to express in % terms.

(\*\*) All indicators are in real terms deflated by GDP deflator, except *efp*.

(\*\*\*) (FoF): Flow of Funds Accounts; (DS) Datastream proxy series based on equity indices.

## A.2 Data

### US Data

I use US data from NIPA-BEA, CPS-BLS, the FRED database, and the Flow of Funds Accounts from the Federal Reserve Board. Though data for US are available starting from 1947, the earliest, I use data for the period 1980-2015, as the longest time span for which I can construct data for Euro area.

1. *Non-financial sector net worth: Tangible Assets minus Credit Market Liabilities.* Tangible Assets obtained from the Flow of Funds Account (FFA) of the Federal Reserve Board, sum of 'Nonfinancial Assets of Non-financial Corporate Business', series ID FL102010005.Q from Table B.102 and 'Nonfinancial Assets of Nonfinancial Corporate Business', series ID FL112010005.Q from Table B.103. Credit Market Liabilities obtained from the Flow of Funds Account (FFA) of the Federal Reserve Board, sum of 'Credit Market Instruments of Nonfinancial Noncorporate Business', series ID FL114104005.Q from Table B.102 and 'Credit Market Instruments of Nonfinancial Corporate Business', series ID FL114102005.Q from Table B.103; I deflate the nominal net worth series using the GDP deflator provided by NIPA Table 1.1.4.
2. *Financial sector net worth: Financial business; corporate equities; liability.* I use the series from the Financial Account of the United States (table Z1) of the Federal Reserve Board to get a market value for the net worth of Financial Sector. I obtain the series '*Financial business; corporate equities (liability)*', coded LM793164105.Q from table Z1/L108. (Before August 2015 the table was named Z1/L107 and the series name FL793164105.Q). I deflate the nominal net worth series using the GDP deflator provided by NIPA Table 1.1.4.

3. *External Finance Premium*. Moody's Baa-rated corporate bond yield minus the Aaa-rated corporate bond yield. Alternatively, the spread is calculated as primary lending rate minus 3 months T-bill rate. Source: Board of Governors of the Federal Reserve System (H15).
4. *Total factor productivity (tfp)*. I obtain the total factor productivity series from Federal Reserve Bank of San Francisco website. The series is provided in percent change at an annual rate ( $=400 * \text{change in natural log}$ ). To obtain the original series I convert the original series with base 1947Q1=100. (I use the reverse formula  $\exp(\text{dtfp}/400) * \text{tfp}(-1)$ ).
5. *Investment specific technology (ist)*. (i) I replicate Pakko (2002) file replicated into quarterly; (ii) alternatively, I obtain the series for investment specific technology as a ratio of Investment deflator to GDP deflator.

To deflate level series (net worth) I use the GDP price deflator (Source: Bureau of Economic Analysis).

# Chapter 3

## Financial Frictions and Monetary Policy

### 3.1 Introduction

My focus in this chapter is to evaluate the propagation of shocks in a general equilibrium model with two-sided financial frictions. The two frictions arise as credit constraints on the financial intermediary and on the entrepreneur.

The main contribution in this framework is the interaction of the two frictions in propagating basic fundamental technology and monetary policy shocks. There are two questions I address in this chapter. First, *what are the determinants of the external finance premium in an economy with credit constraint borrower and lender?* Second, *when do the dynamics of their net worth become so volatile that amplified fundamental technology and policy shocks trigger large fluctuations in  $efp$  and business cycles as observed in recent crisis?*

The framework features two critical players, the entrepreneur who borrows to finance wholesale production and the banker who collects deposits to finance the entrepreneur. Financial frictions in the framework arise due to con-

straints on the capacity of both borrower and lender to expand their balance sheets. To set up frictions at firm level I employ a *financial accelerator (FA)* as in [Bernanke et al. \(1999\)](#). To introduce a *bank friction* mechanism I set up a moral hazard problem as in [Gertler and Kiyotaki \(2010\)](#) that gives rise to lender risk premia.

An additional feature of the framework is that banks hold in their assets both loans and equity securities of the borrowing entrepreneur. This feature is consistent with the increasing trend across financial businesses to expand their assets through purchase of corporate equity securities. The motivation is that the price of these equity securities fluctuates procyclically with the state of the economy and is reflected immediately in the net worth value of financial intermediaries. The rest of the players, households, capital producers, retailers and government behave as in a simple new Keynesian framework. In the remaining part of this chapter I name this framework *FAGK*.

Two single friction models are derived as special cases of the proposed double friction framework, a simple financial accelerator (*FA*) model and a bank friction model of [Gertler and Kiyotaki \(2010\)](#) (hereby *GK*). I calibrate the parameters in the three models to match first moments of data during pre-crisis period. The three models are driven by primary technology and policy shocks of the same size.

To assess the propagation of technology and monetary policy shocks by the two frictions, the impulse responses and second moments from the baseline *FAGK* model are compared to those from *FA* and *GK* models. I also assess how the severity of frictions and changes in monetary policy rule can exacerbate amplification of fundamental technology and monetary shocks. The last section concludes.

## 3.2 Model

The model in this section is a standard new Keynesian (NK) framework in which I incorporate a financial accelerator block and a moral hazard problem defining the relationship between the financial intermediary and the saving household. In the framework there are two key players, banks and wholesale firms (entrepreneurs). The bank signs a financing contract with the entrepreneur whereby he issues standard debt loans to the entrepreneur and purchases equity stakes of the firm for the same price the firm buys the physical capital. The wholesale entrepreneur endowed with his own net worth obtains financing from the bank to finance the purchase of physical capital and engage in wholesale production.

The above financial contract distinguishes my framework from the simple financial accelerator *FA* model or from a simple *GK* model with bank friction. To make this distinction clear I construct a simple stylized balance sheet of the borrowing entrepreneur in table 3.1. The framework differs from the simple *FA* model which has no role for equity securities, where  $Q_t S_t = 0$ .

Table 3.1: Borrowing Entrepreneur's Balance Sheet

Assets	Liabilities
$Q_t K_t$	$L_t$
	$Q_t S_t$
	$N_t^E$

The other difference from the *FA* framework is the presence of a credit constraint financial intermediary. The banker is a lender to the entrepreneur and a borrower from the household. The credit constraint arises from the moral hazard problem that the banker may abscond with some of the borrowed deposits. The household requires the banker first to maintain the value of the bank above a certain threshold as a proportion of total funds (assets), and sec-

### 3.2. Model

ond to contribute his own net worth on total funds. In this regard the terms of the relationship between household and the banker are borrowed from [Gertler and Kiyotaki \(2010\)](#). The balance sheet of the financial intermediary in table 3.2 indicates that entrepreneurs liabilities become the banker's assets.

Table 3.2: Bank Balance Sheet

Assets	Liabilities and Net Worth
$L_t$	$B_t$
$Q_t S_t$	
	$N_t^B$

The key difference from the GK model is that the bank finances the firm project by issuing standard debt loans and by buying equity securities. In the simple GK model the banker buys the physical capital  $Q_t K_t$  by issuing equity securities  $S_t$  for the same price of capital  $Q_t$ . The banker in a GK model literally owns the firm undertaking the investment project.

The remaining agents, households, capital producers, monopolistic retailers and government are borrowed from a standard new Keynesian framework. A simplified structure of the relationships between the agents is shown in figure 3.1. Note that the first financial friction arises in the relationship between the banker and household, and the second in the relationship between banker and entrepreneur.

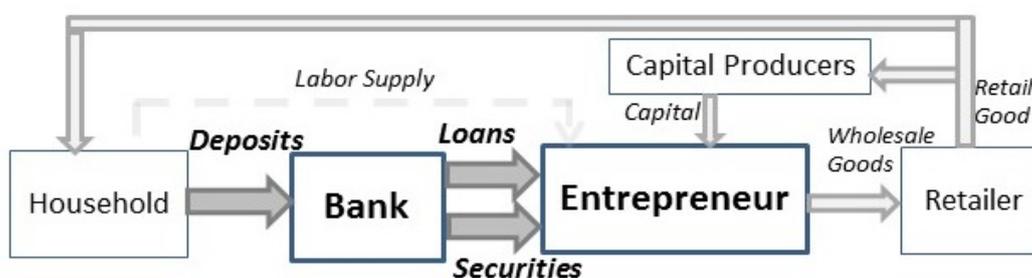


Figure 3.1: A flowchart of agents.

Next follows a detailed description of each agent's problem.

### 3.2.1 Households

Household ( $i$ ) chooses between working and spending leisure time. Let  $H_{i,t}$  be the proportion of time the household spends at work. Household's single-period utility is given by the equation:

$$U_{(C_{i,t}, H_{i,t})} = \left[ \frac{(C_{i,t} - \chi \bar{C}_{t-1})^{1-\sigma_c}}{1-\sigma_c} - \delta_t^h \frac{H_{i,t}^{1+\rho^h}}{1+\rho^h} \right] b_t^u \quad (3.1)$$

where,  $\beta \in (0, 1)$  is the time discount parameter,  $0 < \sigma_c$  is the coefficient of relative risk aversion of households (the inverse of the intertemporal elasticity of substitution) and  $\chi \in (0, 1)$  is a coefficient of external consumption habit formation.  $0 \leq \rho^h$  is the inverse of the Frisch elasticity of labor (the inverse of the elasticity of work effort with respect to the real wage).  $\delta_t^h$  is a labor supply shifter and  $b_t^u$  is a preference parameter as in [Smets and Wouters \(2007\)](#) that enriches the dynamics of the stochastic discount factor<sup>1</sup>.

The household utility is an increasing function of consumption  $C_{i,t}$  relative to external habit,  $\chi \bar{C}_{t-1}$ , and a decreasing function of hours worked,  $H_{i,t}$ .  $\bar{C}_{t-1}$  is average household consumption of previous period. Both internal (see [Christiano et al. \(2005\)](#)) and external habit ([Smets and Wouters \(2003\)](#)) formation models are used in literature. [Dennis \(2009\)](#) shows that to a log-linear approximation those differences make little difference for business cycle behavior. The utility function satisfies:

$$U'_{C_t} > 0, \quad U''_{C_t C_t} < 0, \quad U'_{H_t} < 0, \quad U''_{H_t H_t} < 0, \quad (3.2)$$

<sup>1</sup> I allow also for a shock in labor supply affecting the intratemporal trade-off between consumption and leisure to enrich the dynamics of the model in the absence of wage stickiness. In this chapter I lay out the general form of the framework. As will be clear later, some of these shocks will only be relevant for chapter 4. The processes for non-zero shocks relevant to this chapter are summarized in section 3.2.8.

At any time  $t$ , the working household saves on real deposits,  $B_{i,t}$ , that are expected to pay ex-post inflation-adjusted return,  $R_{t+1} = \frac{R_t^n}{\pi_{t+1}}$ <sup>2</sup>. He also pays taxes,  $T_{i,t}$ . The household earns a competitive wage  $w_t = \frac{W_{i,t}}{P_t}$  from his labour,  $H_{i,t}$ , supplied in flexible labour market. He also earns profits,  $\Pi_{i,t}^m$ , due to ownership interests in banking business (B), wholesale production (E), capital production (CP) and retail business (R) respectively ( $m \in (B, E, CP, R)$ ). His budget constraint follows:

$$C_{i,t} + B_{i,t} - R_t B_{i,t-1} + T_{i,t} = w_t H_{i,t} + \Pi_{i,t}^B + \Pi_{i,t}^E + \Pi_{i,t}^{CP} + \Pi_{i,t}^R \quad (3.3)$$

The household will optimize his consumption and savings in deposits to maximize his utility  $U$ . The optimal path of consumption and optimal labor supply are determined by equations (3.4) and (3.5).

$$1 = E_t R_{t+1} \left( \beta \frac{U_{C_{i,t+1}}}{U_{C_{i,t}}} \right) \quad (3.4)$$

$$w_t = - \frac{U_{H_{i,t}}}{U_{C_{i,t}}} \quad (3.5)$$

where,  $\beta \frac{U_{C_{i,t+1}}}{U_{C_{i,t}}} = \Lambda_{i,t,t+1}$  is the real stochastic discount factor. The Euler equation (3.4) describes household's optimal consumption-saving decision. Labor supply equation (3.5) implies that household will supply labor up to a point where his real wage equates the marginal rate of substitution between consumption and labor.

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<sup>2</sup> Government bonds paying risk free rate can be considered an alternative instrument, though they make no difference here.

### 3.2.2 Retailer

A continuum measure of retailing firms indexed by  $m$  operate in a monopolistic retail market. Retailer  $m$  purchases wholesale goods from entrepreneurs at the nominal price  $P_t^w$ . Each retailer differentiates them into retail goods  $Y_{m,t}$  and resells them at its own retail price  $P_{m,t}$ . Individual retail goods are transformed into aggregate output by means of a [Dixit and Stiglitz \(1977\)](#) aggregator.

$$Y_t = \left( \int_0^1 Y_{m,t}^{\frac{1}{1+\lambda^m}} dm \right)^{1+\lambda^m} \quad (3.6)$$

where,  $1 + \lambda^m = \frac{\epsilon}{\epsilon-1}$  is the retailer's steady state price mark-up and  $\epsilon$  denotes the elasticity of substitution among differentiated goods.

Retailers face a two stage problem. In the first stage the representative retailer chooses  $Y_{m,t}$  to maximize revenues  $\int_0^1 P_{m,t} Y_{m,t} d(m)$ . The maximization problem of the representative retailing firm thus delivers the *demand function* of the  $m^{\text{th}}$  good

$$Y_{m,t} = \left( \frac{P_{m,t}}{P_t} \right)^{-\frac{1+\lambda^m}{\lambda^m}} Y_t \quad (3.7)$$

where  $P_t$  is the average price in the economy. By substituting the *demand function* equation in the composite goods aggregator (equation 3.6), I obtain the aggregate price index,  $P_t$ .

$$P_t = \left( \int_0^1 P_{m,t}^{\frac{-1}{\lambda^m}} dm \right)^{-\lambda^m} \quad (3.8)$$

Retailers face a pricing problem in a sticky price framework as in [Calvo \(1983\)](#). Every period the retailer is allowed to re-optimize his price to  $P_{m,t}^*$  with a constant probability  $1 - \xi_p$ , where  $\xi_p \in (0, 1)$ . This price remains for  $\tau$  periods with probability  $\xi_p^\tau$ . Retailers who do not re-optimize, instead adjust

their prices for next period based on past inflation according to an indexation rule,  $P_{m,t} = P_{t-1}\pi_{t-1}^{\gamma_p}$ . The parameter  $\gamma_p$  captures the degree of partial price indexation to past inflation for those retailers that adjust prices. This set up is a general formulation that nests the less stringent case when no indexation to past inflation takes place ( $\gamma_p = 0$ , and prices adjust to current inflation only)<sup>3</sup>.

The nominal marginal cost to retailer is  $MC_t = P_t^w = P_t mc_t$ , where I define the real marginal cost,  $mc_t$ , as relative price of wholesale good to average retail price  $mc_t = \frac{P_t^w}{P_t}$ . When re-optimization is possible retailer  $m$  will choose to re-set the price  $P_{m,t}^*$  so that he maximizes his discounted stream of profits:

$$\max_{P_{m,t}^*} E_t \int_{\tau=0}^{\infty} \xi_p^\tau D_{t,t+\tau} [(P_{m,t}^* - MC_{t+\tau}) Y_{m,t+\tau}] \quad (3.9)$$

subject to the demand function equation (3.7).  $D_{t,t+\tau}$  is households nominal stochastic discount factor over the interval  $(t, t + \tau)$ . The solution delivers an equation for the relative re-setting price of all retailers that re-optimize.

$$p_t^* = \frac{P_t^*}{P_t} = (1 + \lambda^m) \frac{E_t \int_{k=0}^{\infty} \xi_p^k D_{t,t+k} (mc_{t+k} Z_{t,t+k} Y_{m,t+k})}{E_t \int_{k=0}^{\infty} \xi_p^k D_{t,t+k} (Z_{t-1,t+k-1}^{\gamma_p} Y_{m,t+k})} \quad (3.10)$$

where  $mc_{t+k} = \frac{P_t^w}{P_t}$  and  $Z_{t,t+k} = \prod_{i=1}^k \pi_{t+i}$ . Given the portion of firms that re-set the price and those that index it following the same indexation rule, the aggregate price index is a weighted average of the re-set price  $P_t^*$  and of the indexed price.

$$P_t = \left[ (1 - \xi_p) P_t^* \bar{\lambda}^{-1} + \xi_p (P_{t-1} \pi_{t-1}^{\gamma_p}) \bar{\lambda}^{-1} \right]^{-\lambda^m} \quad (3.11)$$

where  $P_{t-1} \pi_{t-1}^{\gamma_p}$  stands for the price of the retailers who are not re-optimizing, but indexing instead.  $\gamma_p \in (0, 1)$  is a parameter of price indexation to previous period inflation. Non-zero steady-state inflation and indexing of prices to cur-

<sup>3</sup> It nests the case with zero steady-state inflation.

rent and past inflation gives rise to price dispersion  $\Delta_{t+1}$  across all the price setting retailing firms.

$$\Delta_{t+1} = \tilde{\pi}_{t+1}^{\epsilon} \xi_p \Delta_t + (1 - \xi_p) p_{t+1}^{*-\epsilon} \quad (3.12)$$

where,  $\tilde{\pi}_t = \frac{\pi_t}{\pi_{t-1}^{\gamma_p}}$ .

### 3.2.3 Capital Producer

Capital producers are perfectly competitive firms. I borrow the framework from [Christiano et al. \(2005\)](#). The  $n^{\text{th}}$  capital producer purchases final goods,  $I_{n,t}$ , and used capital  $(1 - \delta)K_{n,t}$  from wholesale entrepreneurs to produce new capital.  $I_{n,t}$  is the gross investment of the  $n^{\text{th}}$  capital producer. The existing (partially depreciated) capital is an input for the production of new capital,  $K_{n,t+1}$ . Capital producer transforms the used capital into new capital for a negligible rent, so that the marginal rate of transformation is one. Transformation of final good is subject to an adjustment cost that takes the functional form  $S\left(\frac{I_t}{I_{t-1}}\right) = \Phi_x\left(\frac{I_t}{I_{t-1}} - 1\right)^2$ . The representative capital producer chooses the level of investment such that he maximizes the sum of the stream of discounted future profits  $\Pi_{n,t}^{\text{CP}}$ . At any time  $t$  capital producers profits  $\Pi_{n,t}^{\text{CP}}$  are defined as:

$$\Pi_{n,t}^{\text{CP}} = \frac{P_t^k}{P_t} \Upsilon_t [1 - S(I_{n,t}/I_{n,t-1})] I_{n,t} - I_{n,t} \quad (3.13)$$

Profit equation of capital producer assumes that one unit of time  $t$  investment delivers  $\Upsilon_t(1 - S(\cdot))I_{n,t}$  units of physical capital.  $\Upsilon_t$  is the investment-specific technology shock as defined by [Greenwood et al. \(1997\)](#). *IST* shock introduces exogenous time variation in the price of capital through an AR(1) process as in [3.83](#)<sup>4</sup>. A positive *IST* shock is an exogenous boost to the marginal

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<sup>4</sup> In section [3.2.8](#) I list the evolution of shocks that drive the model in this chapter.

efficiency of investment (Justiniano et al. (2011)). Alternatively, an endogenous source of time variation is introduced through investment adjustment costs (*IAC*) in the production of capital. *IAC* has the advantage that it disciplines the behaviour of investment in line with that observed in data independent of any alternative shocks. Time variation contributes to the volatility of asset prices, and through that transmitted into fluctuations in the net worth of banks and entrepreneurs. A convenient property of this convex functional form of *IAC* is that in the deterministic steady state they disappear ( $S(1) = S'(1) = 0$ ).

The relative price of capital in terms of final good (retail) prices is  $Q_t = \frac{p_t^k}{p_t}$ . The solution to capital producer's problem delivers an equation for the evolution of the price of capital  $Q_t$ .

$$1 = Q_t \Upsilon_t (1 - S(X_{n,t}) - X_{n,t} S'(X_{n,t})) + E_t \left[ \Lambda_{n,t,t+1} Q_{t+1} \Upsilon_{t+1} X_{n,t+1}^2 S'(X_{n,t+1}) \right] \quad (3.14)$$

where  $X_{n,t} = \frac{I_{n,t}}{I_{n,t-1}}$  and  $\Lambda_{n,t,t+1}$  is the household's stochastic discount factor in real terms. Equation 3.14 determines a supply schedule for capital. The demand for capital is determined by the entrepreneur's problem (eq. 3.18). The new capital,  $K_{n,t}$ , is sold back to entrepreneurs at the same price,  $Q_t$  that the used capital was bought for to be used in the next production cycle. The total capital employed in the following period is the partly depreciated capital from the previous period,  $(1 - \delta)K_{n,t-1}$ , plus the newly produced investment  $I_{n,t}$ . The newly produced investment is subject to adjustment costs and to an exogenously evolving investment specific shock,  $\Upsilon_t$ .

$$K_{n,t} = (1 - \delta)K_{n,t-1} + \left[ 1 - S\left(\frac{I_{n,t}}{I_{n,t-1}}\right) \right] I_{n,t} \Upsilon_t \quad (3.15)$$

### 3.2.4 Wholesale Firms (Entrepreneurs)

At any time  $t$  only a small fraction of the households are entrepreneurs who are involved in wholesale production. Entrepreneurs outsource the capital production and accumulation process to a capital producer. At the end of period  $t$  and beginning of period  $t+1$  the  $j^{\text{th}}$  entrepreneur buys new physical capital  $K_{j,t}$  from capital producer for price  $Q_t$ . He hires labour  $H_{j,t+1}$  from households and employs a constant returns to scale (CRS) production technology subject to economy-wide productivity factor  $A_{t+1}$  to produce the wholesale good  $Y_{j,t+1}^w$ .

$$Y_{j,t+1}^w = (A_{t+1}H_{j,t+1})^\alpha K_{j,t}^{1-\alpha} \quad (3.16)$$

At the end of period  $t+1$  the entrepreneur sells the wholesale good to retailers for a competitive price  $P_{t+1}^w$ . He also sells the depleted capital  $(1 - \delta)K_{j,t}$  back to capital producer at a new price  $Q_{t+1}$ . He therefore derives two kinds of revenues:

1. income from sale of produced wholesale goods  $\frac{P_{t+1}^w}{P_{t+1}} Y_{j,t+1}^w$  to retailer, and
2. income from resale of depleted capital  $Q_{t+1}(1 - \delta)K_{j,t}$  to capital producer.

$P_{t+1}^w$  is the nominal sale price of wholesale output to retailer. Therefore,  $\frac{P_{t+1}^w}{P_{t+1}}$  is the sale price in real terms.

The entrepreneur takes the evolution of technology  $A_{t+1}$  as an exogenous process. Given the price of capital,  $Q_t$ , the entrepreneur will demand factors of production, labour  $H_{j,t+1}$  and capital  $K_{j,t}$ , up to the point where,

- the marginal return on labor will equal the real wage, or the household's marginal rate of substitution between consumption and labor (3.5 of section 3.2.1), and

- the marginal return on capital is equal to the expected required return,  $E_t R_{t+1}^k$ , determined in his financial contract.

$$w_{t+1} = \alpha \frac{P_{t+1}^w Y_{j,t+1}^w}{P_{t+1} H_{j,t+1}} \quad (3.17)$$

$$E_t R_{t+1}^k = \frac{(1 - \alpha) \frac{P_{t+1}^w Y_{j,t+1}^w}{K_{j,t}} + Q_{t+1}(1 - \delta)}{Q_t} \quad (3.18)$$

Next, I will elaborate in detail the financial contract that determines the return on capital,  $E_t R_{t+1}^k$ , that the entrepreneur is expected to earn in order to remain in business. As it will be clear, the commitments of the entrepreneur to his lender and the commitments of the banker to his depositors will drive,  $E_t R_{t+1}^k$ , to be greater than the risk free return,  $R_{t+1}$ , of the household.

#### 3.2.5 Financial Contract

In this section I will refer to the firm producing wholesale good with the terms *entrepreneur* and *firm* interchangeably. For the firm to engage in wholesale production a total amount of financial capital  $Q_t K_{j,t}$  is required, where  $K_{j,t}$  is total physical capital investment and  $Q_t$  is its price.

I assume there are banks in this stylized environment that can provide financing. Upon the request of the entrepreneur for additional financial endowments the bank proposes a financing contract to the entrepreneur<sup>5</sup>.

1. The bank will offer a standard debt loan for the amount  $L_t$ . The design of debt contract will be detailed in a separate subsection.
2. The bank will also purchase equity securities  $S_t$  that are equivalent to the units of physical capital the firm (or entrepreneur) is going to buy with

<sup>5</sup> The details how banks obtain funds and how they operate is delayed until the next section.

this extra financing. As a result of arbitrage the price the banker pays to purchase firm's equity securities is  $Q_t$ . This is the same price that the firm pays to purchase physical capital<sup>6</sup>.

With this financing contract the  $j^{\text{th}}$  entrepreneur's own endowment at the end of period  $t$  is net worth  $N_{j,t}^E$  and his balance sheet looks as in equation 3.19.

$$Q_t K_{j,t} = N_{j,t}^E + L_{j,t} + Q_t S_{j,t} = N_{j,t}^E + A_{j,t}^B \quad (3.19)$$

where  $A_{j,t}^B = L_{j,t} + Q_t S_{j,t}$  is the total amount of financing the bank invests which make up for 100% of total bank assets (equation 3.38 in the next section). I have assumed for simplicity that the shares of each instrument in the total assets of the bank are kept constant,  $\kappa = \frac{Q_t S_{j,t}}{A_{j,t}^B}$  and  $1 - \kappa = \frac{L_{j,t}}{A_{j,t}^B}$ , and are exogenously determined. In aggregate this assumption would be equivalent to the case where the bank finances a constant  $\kappa$  fraction of firms with equity securities and the remaining  $1 - \kappa$  fraction of firms with standard debt loans. Similar assumptions are made in other papers like [Kühl \(2017\)](#) and [Verona et al. \(2017\)](#).

For the sake of comparison, this framework would be equivalent to a simple *FA* model in case  $S_{j,t} = 0$  and the financial intermediary would not be credit constraint, hence accumulate no net worth. Next I detail the design of the debt contract between the bank and the firm.

### The Debt Contract

Typical literature related to debt financing and agency costs build on the work of [Townsend \(1979\)](#) and [Gale and Hellwig \(1985\)](#). The most relevant to this

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<sup>6</sup>There is no frictions to drive the price of securities above or below  $Q_t$ .

study are the papers by [Bernanke et al. \(1999\)](#) and [Carlstrom and Fuerst \(1997\)](#). I follow the financial accelerator proposed by the former to set up a debt contract between the bank and the entrepreneur.

The design of the standard debt contract assumes that the expected return from the project investment,  $E_t R_{t+1}^k$ , is subject to an aggregate productivity shock (equation 3.18) and an *i.i.d* idiosyncratic shock  $\psi_{t+1}^j$ . The idiosyncratic shock is drawn from a log-normal distribution with mean  $-0.5A_{\psi}^2$  and standard deviation  $A_{\psi}$  and expected value of one<sup>7</sup>.

$$E_t \psi_{t+1}^j = 1 \quad (3.20)$$

The *i.i.d* property of the idiosyncratic shock  $\psi_{t+1}^j$  (zero covariance with  $R_{t+1}^k$ ) guarantees that the expected gross capital return  $E_t \psi_{t+1}^j R_{t+1}^k$  of any  $j^{\text{th}}$  entrepreneur is the same and equal to  $E_t R_{t+1}^k$  (equation 3.18) for any level of total project cost,  $Q_t K_{j,t}$ . It is the actual ex-post return  $\psi_{t+1}^j R_{t+1}^k$  that is different across entrepreneurs.

Next, I summarize the timing of the events as follows:

- End of period  $t$ 
  1. The  $j^{\text{th}}$  entrepreneur obtains loan amount  $L_{j,t}$  to lever up the purchase of physical capital  $K_{j,t}$  for production in the following period. In addition to issuing a loan, the banker purchases firms' equity securities  $S_t$  at the price  $Q_t$ .
- Beginning of period  $t+1$ 
  1. The idiosyncratic shock  $\psi_{t+1}$  realizes.
  2. Aggregate shock to productivity realizes.

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<sup>7</sup> These values guarantee that expected value is unity.

3. The entrepreneur either pays off the loan or declares default.
4. In case of no default, the firm's net worth,  $N_{j,t}^E$ , for the next period realizes after paying all the financial obligations.
5. In case of default, the bank verifies the state of the gross capital investment,  $\psi_{j,t+1} R_{t+1}^k Q_t K_{j,t}$  of defaulting entrepreneurs. To do so the bank incurs monitoring cost which amounts a percentage  $\mu$  of the gross capital investment and seizes them.

The terms of the debt contract assume the entrepreneur is required to pay a contractual loan rate  $Z_{j,t}^L$  to the bank. A threshold value  $\bar{\psi}_{t+1}^j$  of the idiosyncratic shock exists such that actual income from production is just sufficient to pay the loan amount,  $L_t$ , and its interest  $Z_{j,t}^L$ .

$$\bar{\psi}_{t+1}^j R_{t+1}^k Q_t K_{j,t} = Z_{j,t}^L L_{j,t} \quad (3.21)$$

The entrepreneur declares default if the realized value is below threshold  $\psi_{t+1}^j < \bar{\psi}_{t+1}^j$ . The realized  $\psi_{t+1}^j$  can only be observed by the entrepreneur. The bank will incur monitoring costs  $\mu$ , in percent of the seized gross assets, to truly evaluate the state of the assets. The banks seizes the remaining assets  $(1 - \mu)\psi_{t+1}^j R_{t+1}^k Q_t K_{j,t}$  net of monitoring costs<sup>8</sup>. When the entrepreneur does not claim default, he pays the loan and delivers the remaining revenues to himself and to the owner of equity securities, which happen to be the bank. When defaulting, the entrepreneur does not deliver any profits to the holder of the securities  $S_{j,t}$  and the payoff to the firm is zero. I summarize the payoffs of the entrepreneur and lender in table 3.3 followed by detailed description for each outcome of this payoff table.

<sup>8</sup> Knowing the bank will make verification, there is zero probability that entrepreneur will lie about the state of his assets being lower than  $Z_{j,t}^L L_{j,t}$  when it is not. The reason is that the banker will be seizing assets of value greater than  $Z_{j,t}^L L_{j,t}$ .

Table 3.3: Payoff Table

State	No Default: $(\psi_{t+1}^j \geq \bar{\psi}_{t+1}^j)$	Default: $(\psi_{t+1}^j < \bar{\psi}_{t+1}^j)$
Firm Payoff	$(\psi_{t+1}^j - \bar{\psi}_{t+1}^j) R_{t+1}^k K_{j,t} Q_t (1 - X_{j,t}^{eq})$	o (zero)
Bank Payoff	$(\psi_{t+1}^j - \bar{\psi}_{t+1}^j) R_{t+1}^k K_{j,t} Q_t X_{j,t}^{eq} + \bar{\psi}_{t+1}^j Q_t K_{j,t} R_{t+1}^k$	$(1 - \mu) \psi_{t+1}^j Q_t K_{j,t} R_{t+1}^k$

where, I denote by  $X_{j,t}^{eq}$  the ratio of equity securities bought by the bank to the sum of firm net worth and equity securities.

$$X_{j,t}^{eq} = \frac{Q_t S_{j,t}}{Q_t S_{j,t} + N_{j,t}^E} = \frac{\kappa(\phi_{j,t}^E - 1)}{\kappa(\phi_{j,t}^E - 1) + 1} \quad (3.22)$$

$\phi_{j,t}^E = \frac{Q_t K_{j,t}}{N_{j,t}^E}$  is the ratio of total physical capital purchased by entrepreneur to his own net worth.

The payoff structure implies that gross payoffs to the bank and the firm both depend on the distribution of idiosyncratic shock  $[\psi_{t+1}^j]$ .

- In case of no default,  $(\psi_{t+1}^j > \bar{\psi}_{t+1}^j)$  the entrepreneur pays first the loan and its interest,  $\bar{\psi}_{t+1}^j R_{t+1}^k Q_t K_{j,t} = Z_{j,t}^L L_{j,t}$ . He then delivers the share  $[1 - X_t^{eq}]$  of the remaining gross revenues,  $(\psi_{t+1}^j - \bar{\psi}_{t+1}^j) R_{t+1}^k K_{j,t} Q_t$ , to the equity security holding bank.  $X_t^{eq}$  is defined in equation 3.22.
- In case of no default, gross payoff to the entrepreneurs is the share  $[X_t^{eq}]$  times the remaining gross profits,  $(\psi_{t+1}^j - \bar{\psi}_{t+1}^j) R_{t+1}^k K_{j,t} Q_t$ , after paying the loan.
- In case of default, the debt-holding bank seizes all the assets following the realization of idiosyncratic shock. The entrepreneur earns zero payoff. So does the bank owning equity securities.

For comparison, note that for  $X_{j,t}^{eq} = 0$  this payoff table becomes identical to that in a simple financial accelerator model of [Bernanke et al. \(1999\)](#). The opportunity cost of funds to the lender would be different from risk free rate

even in the case when  $\chi_{j,t}^{eq} = 0$ . The latter, the lender's opportunity cost, is determined by whether the lender is credit constrained or not. If banker were not credit constraint he would not need to accumulate net worth and hence his opportunity cost would be the household's deposit rate (risk free rate).

The payoffs refer to payments following one particular realization of  $\psi_{t+1}^j$ . I define the expected net payoff to the entrepreneur and bank across all states of nature of idiosyncratic shock  $\psi_{t+1}^j$ .

1. Based on payoff table [3.3 on the previous page](#), the value of the firm,  $V_{j,t+1}^E$  across all states nature of  $\psi_{t+1}^j$  is:

$$V_{j,t+1}^E = E_t R_{t+1}^k K_{j,t} Q_t \left( \int_{\bar{\psi}_{t+1}^j}^{\infty} (\psi_{t+1}^j - \bar{\psi}_{t+1}^j) dF_{\psi^j} \right) (1 - \chi_{j,t}^{eq}) \quad (3.23)$$

2. Summing across all states of  $\psi_{t+1}^j$  the total payoff on equity portfolio will deliver  $R_{t+1}^S Q_t S_{j,t}$ , which is equivalent to the proportion,  $\chi_{j,t}^{eq}$ , of total payoff from project after paying the loan.  $R_{t+1}^S$  is the average return on *the portfolio of equity securities* held by bank.

$$E_t R_{t+1}^k K_{j,t} Q_t \left( \int_{\bar{\psi}_{t+1}^j}^{\infty} (\psi_{t+1}^j - \bar{\psi}_{t+1}^j) dF_{\psi^j} \right) \chi_{j,t}^{eq} = R_{t+1}^S Q_t S_t \quad (3.24)$$

3. The total payoff on *the loan portfolio* will deliver  $R_{t+1}^L L_t$  equivalent to the fixed payoff from non-defaulting firms plus the payoff from sale of assets of defaulting firms across all states of nature of  $\psi_{t+1}^j$ .  $R_{t+1}^L$  is the average return on loan portfolio held by bank.

$$E_t \left[ Q_t K_{j,t} R_{t+1}^k \int_{\bar{\psi}_{t+1}^j}^{\infty} \bar{\psi}_{t+1}^j dF_{\psi^j} + (1 - \mu) Q_t K_{j,t} R_{t+1}^k \int_0^{\bar{\psi}_{t+1}^j} \psi_{t+1}^j dF_{\psi^j} \right] = R_{t+1}^L L_t \quad (3.25)$$

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The returns  $R_{t+1}^L$  and  $R_{t+1}^S$  are endogenously determined as will be made clear in the bank's problem. Finally for reference in the following sections I define three terms.

$\Gamma_{\bar{\Psi}_{t+1}}^j$  is the payoff in percent of gross return  $R_{t+1}^k Q_t K_{j,t}$  that goes to the debt holder across all states of  $\psi_{t+1}^j$ , but before the accrual of the monitoring costs.

$$\Gamma_{\bar{\Psi}_{t+1}}^j = \bar{\Psi}_{t+1}^j \int_{\bar{\Psi}_{t+1}^j}^{\infty} dF_{\psi^j} + \int_0^{\bar{\Psi}_{t+1}^j} \psi_{t+1}^j dF_{\psi^j} \quad (3.26)$$

$G_{\bar{\Psi}_{t+1}}^j$  is the payoff in percent of gross return  $R_{t+1}^k Q_t K_{j,t}$  that is seized from defaulting firms, before monitoring costs are subtracted.

$$G_{\bar{\Psi}_{t+1}}^j = \int_0^{\bar{\Psi}_{t+1}^j} \psi_{t+1}^j dF_{\psi^j} \quad (3.27)$$

$p_{\bar{\Psi}_{t+1}}^j$  is the probability of firm default.

$$p_{(\bar{\Psi}_{t+1})}^j = \int_0^{\bar{\Psi}_{t+1}^j} dF_{\psi^j} \quad (3.28)$$

The three are related among them according to the following equation:

$$\Gamma_{\bar{\Psi}_{t+1}}^j = (1 - p_{\bar{\Psi}_{t+1}}^j) \bar{\Psi}_{t+1}^j + G_{\bar{\Psi}_{t+1}}^j \quad (3.29)$$

Finally, I make use of definitions 3.26 through 3.28 to write the expected total return on bank assets, which are equity securities and loan portfolios. The expected total income on bank assets at the end of time  $t+1$  is the sum of income in equations 3.24 and 3.25.

$$E_t R_{t+1}^k Q_t K_{j,t} \left( \Gamma_{\bar{\Psi}_{t+1}}^j - \mu G_{\bar{\Psi}_{t+1}}^j \right) + E_t R_{t+1}^k Q_t K_{j,t} \left( 1 - \Gamma_{\bar{\Psi}_{t+1}}^j \right) X_{j,t}^{eq} = E_t R_{t+1}^L L_{j,t} + R_{t+1}^S Q_t S_{j,t} \quad (3.30)$$

A. *Lender's One-Period Participation Constraint*

In a simple financial accelerator model the bank's required return would be equivalent to risk free rate. In this set up the required return on total assets  $R_{t+1}^B$  is determined by the requirement on the bank to generate a minimum value for the bank  $V_{i,t}^B$ . For a risk averse banker to agree to sign a contract to finance the entrepreneur the value of the bank from participating in financing the firm,  $V_{i,t}^B$ , should be greater than the alternative from not participating in financing, but rather investing the funds in risk-free assets, denoted  $V_{i,t}^{B_0}$ . Therefore, I define a **Minimum Bank Value Constraint**,

$$V_{i,t}^B > V_{i,t}^{B_0} \quad (3.31)$$

where,

- $V_{i,t}^B$  is the discounted stream of future gross returns when (i) bank total assets are invested in loans and equity securities earning a gross return (payoff) shown in equation 3.30, and (ii) deposits pays risk free rates on deposits to households.
- $V_{i,t}^{B_0}$  is the discounted stream of future returns when bank total assets are invested in risk free instruments, earning a gross return  $A_{i,t}^B R_{t+1}$  in any  $t+1$ , less the payments made to depositor

**Proposition I.**

For the **Minimum Bank Value Constraint** in equation 3.31 to hold the following **Lender One-Period Participation Constraint**:

$$E_t R_{t+1}^k Q_t K_{j,t} \left( \Gamma_{\Psi_{t+1}}^j - \mu G_{\Psi_{t+1}}^j \right) + E_t R_{t+1}^k Q_t K_{j,t} \left( 1 - \Gamma_{\Psi_{t+1}}^j \right) X_{j,t}^{eq} \geq E_t R_{t+1}^B A_{j,t}^B \quad (3.32)$$

is an equivalent sufficient condition when these minimal restrictions are satisfied:

- the bank **Incentive Constraint**  $V_{i,t}^B \geq \Theta_B A_{i,t}^B$ ,
- the bank is endowed with a fixed initial net worth that does not depend on whether he invests in risk free assets or finances the firm.
- the constraint  $\Theta_B A_{i,t}^B > N_{i,t}^B$  holds for the value of  $\Theta_B$ ,

where  $R_{t+1}^B$  is the required return on bank assets, or alternatively, the opportunity cost of funds for the bank.

The full proof of **Proposition I** is postponed in section [B.1.8](#) in appendix [B.1](#). A critical condition to the proof is that constraint  $\Theta_B A_{i,t}^B > N_{i,t}^B$  holds, as the first two are satisfied by construction. This last condition is equivalent to saying that the amount of assets the banker can abscond with should be greater than the value of his own net worth  $\Theta_B A_{i,t}^B > N_{i,t}^B$ . Such a constraint holds intuitively as there is no benefit for the banker to abscond with an amount which is less than the value of net worth he already owns. Therefore such an assumption is quite reasonable and easily satisfied.

Eventually, as will be shown later in section [3.2.6](#),  $R_{t+1}^B$  will be greater than the risk free rate, due to the moral hazard problem between the banker and depositors. Therefore, as long as  $R_{t+1}^B > R_{t+1}$  the minimum bank value constraint in equation [3.31](#) will always hold.

### B. Entrepreneur's Value Function

The lay out of the financial contract (payoff table [3.3](#)) allows me to write the **Firm Value Function** obtained in equation [\(3.23\)](#) in terms of  $\Gamma_{\Psi_{t+1}}^j$  (equation [3.26](#)).

$$V_{j,t+1}^E = E_t R_{t+1}^k K_{j,t} Q_t \left( 1 - \Gamma_{\Psi_{t+1}}^j \right) (1 - X_{j,t}^{eq}) \quad (3.33)$$

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Unlike in a financial accelerator model the entrepreneur shares the net payoff (after the loan payment) with the holder of equity securities. The entrepreneur will choose the optimal level of physical capital,  $K_{j,t}$  that he employs into the project and the threshold level of idiosyncratic shock,  $\bar{\psi}_{t+1}^j$ , below which he will declare default so that he optimizes his value function:

$$\max_{(\bar{\psi}_{t+1}^j, K_{j,t})} V_{j,t+1}^E$$

subject to the **lender's one period participation constraint** given by equation 3.32.

where  $X_{j,t}^{eq}$  is the share of revenues going to the bank after the firm pays the the loan and depends critically on the parameter  $\kappa = \frac{Q_t S_{j,t}}{Q_t S_{j,t} + L_{j,t}}$ . Compared to a simple BGG model the value function of the firm includes the extra term  $(1 - X_{j,t}^{eq})$ . Also, the one period participation constraint (eq. 3.30) includes the extra term capturing the return on securities  $R_{t+1}^k K_{j,t} Q_t (1 - \Gamma_{\bar{\psi}_{t+1}^j}) X_{j,t}^{eq}$  which is linearly related to  $X_{j,t}^{eq}$ , and therefore to the contract instrument parameter  $\kappa$ . For  $X_{j,t}^{eq} = 0$  all the equations and the following optimal conditions converge to BGG.

An increase in  $\kappa$  lowers the share  $1 - X_{j,t}^{eq}$  of net profits (after the loan repayment) going to entrepreneur, while it raises the share of profits going to the holder of equity securities (bank). The latter effect makes the participation constraint of the bank (equation 3.30) less binding. The result is a weaker financial accelerator effect and a smaller feedback of borrower premium to adverse shocks.

The first order condition with respect to  $\bar{\psi}_{t+1}^j$  delivers an equation relating the Lagrange multiplier,  $\lambda_{j,t+1}^E$ , to a schedule of threshold values  $\bar{\psi}_{t+1}^j$ , for any

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combination of shares of equity securities and loans the bank issues to the entrepreneur.

$$E_t \lambda_{j,t+1}^E = E_t \frac{\Gamma'_{\Psi_{t+1}^j} (1 - X_{j,t}^{eq})}{\Gamma'_{\Psi_{t+1}^j} (1 - X_{j,t}^{eq}) - \mu G'_{\Psi_{t+1}^j}} \quad (3.34)$$

When there is no equity purchase by banks,  $\kappa = 0$  but only loan financing ( $X_{j,t}^{eq} = 0$ ) equation 3.34 becomes identical to the one in a simple financial accelerator model.

The optimal condition with respect to the entrepreneur's choice of the amount of physical capital,  $K_{j,t}$ , maps a relationship between the borrower's premium,  $E_t \frac{R_{t+1}^k}{R_{t+1}^B}$  and the Lagrange multiplier, for any combination of shares of equity securities and loans ( $\kappa$ ) put into the project.

$$E_t \frac{R_{t+1}^k}{R_{t+1}^B} = E_t \frac{\lambda_{j,t+1}^E}{(1 - \Gamma_{\Psi_{t+1}^j}) (1 - X_{j,t}^{eq} - \phi_{j,t}^E X_{\phi_{j,t}}^{eq'}) + \lambda_{j,t+1}^E \left[ \Gamma_{\Psi_{t+1}^j} - \mu G_{\Psi_{t+1}^j} + (1 - \Gamma_{\Psi_{t+1}^j}) (X_{j,t}^{eq} + \phi_{j,t}^E X_{\phi_{j,t}}^{eq'}) \right]} \quad (3.35)$$

where  $\lambda_{j,t+1}^E$  is the Lagrange multiplier linked to the participation constraint and  $E_t \frac{R_{t+1}^k}{R_{t+1}^B}$  is the premium the borrowing firm is charged on his financing. While the first order condition above is more difficult to interpret than in the *FA* model, it is useful to think along two lines. The easiest way to interpret the impact of  $X_{j,t}^{eq}$  in this equation is to start from the case when  $X_{j,t}^{eq} = 0$ . The above expression collapses to the same first order condition as in the simple *FA* model (see section 3.3.1 for a detailed representation of *FA* model). As one starts to raise  $\kappa$  the share of net profits, after paying the loan, that goes to the lender,  $X_{j,t}^{eq}$ , increases. This increase in  $X_{j,t}^{eq}$  makes the Lagrange multiplier less binding and therefore weakens the financial accelerator mechanism<sup>9</sup>.

<sup>9</sup> Higher  $X_{j,t}^{eq}$  raises the share of equity stake portfolio in bank balance sheet. By construction the average return on equity stake portfolio that the bank will earn will be greater than the average return on loan portfolio.

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Equation (3.35) relates the threshold level  $\bar{\psi}_{t+1}^j$  to the borrower's premium  $E_t \frac{R_{t+1}^k}{R_{t+1}^B}$ . In the simple FA model this first order condition holds with  $\kappa = 0$  and  $X_{j,t}^{eq} = 0$ .

Finally, the first order condition with respect to the Lagrange multiplier delivers an equation that maps borrower's premium  $E_t \frac{R_{t+1}^k}{R_{t+1}^B}$  on firm leverage  $\phi_{j,t}^E$ .

$$E_t R_{t+1}^B = E_t R_{t+1}^k \frac{\phi_{j,t}^E}{\phi_{j,t}^E - 1} \left( \Gamma_{\bar{\psi}_{t+1}^j} - \mu G_{\bar{\psi}_{t+1}^j} + (1 - \Gamma_{\bar{\psi}_{t+1}^j}) X_{j,t}^{eq} \right) \quad (3.36)$$

$\phi_{j,t}^E = \frac{Q_t K_{j,t}}{N_{j,t}^E}$  is the ratio of total physical capital purchased by entrepreneur to his own net worth. The equation sets a (positive) relationship between the leverage of the entrepreneur,  $\phi_{j,t}^E$ , and the premium on external borrowing,  $\frac{R_{t+1}^k}{R_{t+1}^B}$ , he is going to face everything else remaining the same. A higher ratio of financial capital to borrower's own net worth,  $\frac{Q_t K_t}{N_t^E}$ , drives up the borrower's (firm) external finance premium. Also a higher share of equity financing relative to debt financing the bank decides to hold on its balance sheet lowers the borrower's premium in equilibrium (in steady state). The latter feature is relevant to firms that are able to raise equity financing.

#### C. *Aggregation of Entrepreneurs Value Function*

To ensure entrepreneurs do not accumulate sufficient net worth to become self-financed I assume each survives with a probability  $\sigma^E < 1$  and exits with probability  $1 - \sigma^E$ . At the end of  $t$  a fraction  $1 - \sigma^E$  of entrepreneurs exit. The end of period aggregate value of those not exiting is  $\sigma^E V_{t+1}^E$ . The end of period aggregate value of exiting entrepreneurs is  $(1 - \sigma^E) V_{t+1}^E$ .

To keep the number of entrepreneurs constant I assume the same number of entrepreneurs  $1 - \sigma^E$  enter the market every period. The exiting entrepreneurs

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transfer a fraction  $\frac{\xi_E}{1-\sigma^E}$  of their own aggregate wealth  $(1 - \sigma^E)V_{t+1}^E$  to new entrants.  $\xi^E \in (0,1)$  is an exogenous parameter. The aggregate value (or net worth) of new entrepreneurs is now the transfer percentage  $\frac{\xi_E}{1-\sigma^E}$  times the aggregate net worth of the exiting entrepreneurs,  $(1 - \sigma^E)V_{t+1}^E$ , which is equivalent to  $\xi_E V_{t+1}^E$ .

The total aggregate net worth held by all the entrepreneurs that will operate in the following period is the sum of new entrepreneurs' net worth,  $\xi_E V_{t+1}^E$ , plus that of existing entrepreneurs,  $\sigma^E V_{t+1}^E$ .

$$N_{t+1}^E = (\sigma^E + \xi_E)E_t V_{t+1}^E \quad (3.37)$$

where the aggregate value of entrepreneurs before exiting,  $V_{t+1}^E$ , is given by equation 3.33<sup>10</sup>. The exiting entrepreneurs retire and consume the remaining part of their wealth,  $\Pi_{t+1}^E = (1 - \sigma^E) \left(1 - \frac{\xi_E}{1-\sigma^E}\right)$ .

#### 3.2.6 Banks

Banks intermediate savings of households into production activity. In this framework I adopt the framework of [Gertler and Kiyotaki \(2010\)](#) to introduce a moral hazard problem between bankers and households. The key implication is that the opportunity cost of financing the wholesale firm will be  $R_t^B > R_t$  as will be shown in this section. I summarize the banker's problem and provide the details in appendix [B.1](#).

A small fraction of households are bankers who remain banker next period with an exogenous probability  $\sigma_B < 1$ . The banker  $i$  is endowed with net worth  $N_{i,t}^B$  and collects deposits  $B_{i,t}$  from other households. He can invest these funds into standard debt loan  $L_{i,t}$  to wholesale producer. In addition, he can purchase equity securities,  $S_{i,t}$ , of the wholesale firm for the price of the

<sup>10</sup>Equation 3.33 after dropping the subscript  $j$ .

same price  $Q_t$  that the firm buys physical capital  $K_t$ . The bank balance sheet equation in real terms is:

$$N_{i,t}^B + B_{i,t} = A_{i,t}^B = L_{i,t} + Q_t S_{i,t} \quad (3.38)$$

where  $A_{i,t}^B$  stands for the total assets of the banker. As a follow up, when bank issues no loans ( $L_{i,t} = 0$ ) the bank balance sheet is identical to the one in bank model with moral hazard of [Gertler and Kiyotaki \(2010\)](#). The banker accrues income and expenses from his balance sheet. At the end of time  $t+1$  the banker:

- earns average return  $R_{t+1}^s$  on his portfolio of equity securities;
- earns average return  $R_{t+1}^L$  on loan portfolio net of any costs associated with potential entrepreneurial bankruptcy;
- pays risk free rate  $R_{t+1}$  to households' deposits from previous period.

I define the required return  $R_{t+1}^B$  on bank assets is a weighted average of the returns on the portfolio of loans and securities.

$$R_{t+1}^B = R_{t+1}^L \frac{L_{i,t}}{A_{i,t}^B} + R_{t+1}^s \frac{Q_t S_{i,t}}{A_{i,t}^B} \quad (3.39)$$

The household makes deposits only if the banker has no incentive to divert assets. That ensures the bank never defaults and hence the deposits become a risk free instrument paying only risk-free rate in equilibrium. Banker's net worth at the end of  $t+1$  is gross return on his assets less the interest paid on deposits to the households.

$$N_{i,t+1}^B = R_{t+1}^L L_{i,t} + R_{t+1}^s Q_t S_{i,t} - R_{t+1} B_{i,t} \quad (3.40)$$

The banker's value function is the expected discounted terminal wealth from financing activity:

$$V_{i,t}^B = E_t \sum_{\tau=1}^{\infty} (1 - \sigma_B) \sigma_B^{\tau-1} \Lambda_{t,t+\tau} N_{i,t+\tau}^B \quad (3.41)$$

where,  $\Lambda_{t,t+\tau}$  is the real stochastic discount factor (SDF) of household, and  $(1 - \sigma_B) \sigma_B^{\tau-1}$  is the probability of surviving until the  $\tau^{\text{th}}$  - 1 period therefore exiting in  $\tau^{\text{th}}$  period.

The critical role of financial intermediary net worth arises from the agency problem between the bankers and households (as borrowed by [Gertler and Kiyotaki \(2010\)](#)). The banker can transfer a fraction  $\Theta^B$  of funds,  $A_{i,t}^B$  supplied by depositors to his own account. If he absconds the bank defaults. The banker loses future stream of positive bank net worth while the households claim the remaining share of assets  $(1 - \Theta^B)A_{i,t}^B$ . Households will limit the amount of deposits they supply, and the expansion of banker's assets, to ensure the banker maintains the value of the bank,  $V_{i,t}^B$ , above the threshold value  $\Theta^B A_{i,t}^B$ . This relationship determines the incentive constraint (IC) of the banker.

$$V_{i,t}^B \geq \Theta^B A_{i,t}^B \quad (3.42)$$

As long as bank value is greater than this threshold value, the banker has the incentive to keep the bank running rather than abscond with a fraction  $\Theta^B$  of funds. The constraint provides a one-to-one mapping of bank asset size and the average excess return on assets,  $R_{t+1}^B - R_{t+1}$  that the banker should generate, for any  $\Theta^B$ . The banker will aim to maximize his value function (eq. 3.41) subject to the above constraint (eq. 3.42).

To solve the problem I conjecture that the value of the bank is homogenous to degree one in net worth. Given the linear relationship in the balance sheet

equation 3.38, then  $V_{i,t}^B$  should be homogenous of degree one in  $N_{i,t}^B$  and also in  $L_{i,t}$  and  $S_{i,t}$ . Then I can write the bank value function as:

$$V_{i,t}^B(L_{i,t}, N_{i,t}, S_{i,t}) = \mu_{i,t}^L L_{i,t} + \mu_{i,t}^S Q_t S_{i,t} + v_{i,t}^B N_{i,t}^B \quad (3.43)$$

where  $\mu_{i,t}^L$ ,  $\mu_{i,t}^S$  and  $v_{i,t}^B$  are the marginal values of holding each asset<sup>11</sup>. The banker's problem is now to maximize the value function of the bank,  $V_{i,t}^B$ , in equation 3.43 subject to the bank incentive equation  $V_{i,t}^B \geq \Theta^B A_{i,t}^B$  (equation 3.42).

The solution to the banker's problem delivers an equation relating the bank leverage,  $\phi_{i,t}^B = \frac{A_{i,t}^B}{N_{i,t}^B}$ , to the weighted average of the two excess returns on assets  $\mu_{i,t}^L$  and  $\mu_{i,t}^S$ .

$$\phi_{i,t}^B = \frac{v_{i,t}^B}{\Theta^B - ((1 - \kappa)\mu_{i,t}^L + \kappa\mu_{i,t}^S)} \quad (3.44)$$

where,  $v_{i,t}^B$  is the discounted value of marginal (risk free) return on banker's net worth, and  $\mu_{i,t}^L$  and  $\mu_{i,t}^S$  are the discounted future excess returns on banker's assets relative to deposit rate, which after some derivation take the form given in the next three equations<sup>12</sup>.

$$v_{i,t}^B = E_t \Lambda_{t,t+1} \Omega_{i,t+1} R_{t+1} \quad (3.45)$$

$$\mu_{i,t}^S = E_t \Lambda_{t,t+1} \Omega_{i,t+1} (R_{t+1}^S - R_{t+1}) \quad (3.46)$$

$$\mu_{i,t}^L = E_t \Lambda_{t,t+1} \Omega_{i,t+1} (R_{t+1}^L - R_{t+1}) \quad (3.47)$$

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<sup>11</sup>Less the cost of financing them. Their functional form will be derived should the conjecture be right.

<sup>12</sup> $\kappa = \frac{Q_t S_{i,t}}{Q_t S_{i,t} + L_{i,t}}$  is the ratio of equity securities on total bank assets.

where,  $\Omega_{i,t+1} = (1 - \sigma_B) + \sigma_B \Theta_B \phi_{i,t+1}^B$  is positively related to leverage and inversely related to net worth. As banker's constraint is tighter during recessions, then a unit value of net worth is more valuable in bad times. Hence  $\Omega_{i,t+1}$  is countercyclical. As household's  $\Lambda_{t,t+1}$  is also countercyclical, the banker's excess returns are discounted by an *augmented discount factor* which is households stochastic discount factor,  $\Lambda_{t,t+1}$ , multiplied by  $\Omega_{i,t+1}$ . [Gertler and Kiyotaki \(2010\)](#) coin the terms '*stochastic shadow marginal value*' of net worth in order to highlight the highly valued net worth by a highly leveraged banker. The increased volatility of stochastic discount factor reduces the excess value of the banker's assets.

*Why does the lender premium arise in this framework?* The banker's choice of leverage is a choice made regarding the expected average required excess return on his assets  $E_t(R_{t+1}^B - R_{t+1})$ . The latter is a weighted average of excess returns on each particular financing instrument,  $E_t(R_{t+1}^L - R_{t+1})$  and  $E_t(R_{t+1}^S - R_{t+1})$ . Equation 3.44 leads to two critical relationships.

- the critical threshold parameter  $\Theta^B$  set by households is **inversely** related to the banker's leverage,  $\phi_{i,t}^B$ .
- The banker's leverage,  $\phi_{i,t}^B$ , is **positively** related to a weighted average of the discounted future excess returns on banker's assets,  $\mu_{i,t}^L$  and  $\mu_{i,t}^S$ , as shown in equations 3.46 and 3.47. Adverse shocks in the economy lower bank's net worth. Pro-cyclical bank leverage then drives the premiums  $E_t(R_{t+1}^L - R_{t+1})$  and  $E_t(R_{t+1}^S - R_{t+1})$  up.

For a certain level of threshold parameter set by households, the banker will have to deliver excess returns on his assets such that he maintains the value of the bank,  $V_t^B$ , above that threshold.

Equations 3.44 and 3.45, 3.46, 3.47 determine  $\phi_{i,t}^B$  and  $\mu_{i,t}^L$ ,  $\mu_{i,t}^S$ ,  $v_{i,t}^B$  as a function of  $\Theta_B$ ,  $R_{t+1}^L - R_{t+1}$ ,  $R_{t+1}^S - R_{t+1}$  and  $R_{t+1}$ .

- *Aggregation of Bank Value Function*

As stated at the beginning of this section, a banker lives with probability  $\sigma_B < 1$ . The aggregate net worth of surviving bankers to be carried next period is  $\sigma_B N_{i,t}^B$ .

Each period the fraction  $1 - \sigma_B$  of entrepreneurs exit the banking market to eventually become households. To keep the number of bankers constant I assume the same number of new bankers enter the market, which make up for a fraction  $1 - \sigma_B$  of bankers that exited.

Exiting bankers  $(1 - \sigma^B)$  transfer an amount of net wealth to the new entrants which is equal to an exogenous fraction  $\frac{\xi_B}{1 - \sigma_B}$  of the aggregate gross return before paying depositors,  $(1 - \sigma^B) R_{t+1}^B A_{i,t}^B$ <sup>13</sup>. Therefore, the net wealth transferred to the new entrepreneurs is now  $\xi_B R_{t+1}^B A_{i,t}^B$ . The fraction  $1 - \frac{\xi}{1 - \sigma_B}$  times their aggregate end-of-period gross return  $(1 - \sigma_B) R_{t+1}^B A_{i,t}^B$  is consumed.

Evolution of aggregate bank net worth at the beginning of next period  $t+1$  is the sum of the net worth of surviving bankers plus that of new bankers.

$$N_{t+1}^B = \sigma_B \left( R_{t+1}^L L_t + R_{t+1}^S Q_t S_t - R_{t+1} B_t \right) + \xi_B R_{t+1}^B A_t^B \quad (3.48)$$

where, I have substituted equation 3.40 for  $N_{i,t}^B$ .

#### 3.2.7 Government and Monetary Policy Rule

The government has a minimal role of collecting taxes  $T_t$  and spending them on public goods. It maintains a balanced budget.

$$G_t = T_t \quad (3.49)$$

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<sup>13</sup>Any assumption about the transfer to the new bankers that helps pinpoint the steady state banker leverage ratio would work.

The process for government spending  $G_t$  is given by  $G_t = g_y Y_t$  with government spending-to-output ratio,  $g_y$ , being constant. In addition, government delegates to a monetary authority the task of running the monetary policy. The latter does so following a Taylor type interest rate rule with the short-term nominal interest rate responding to deviations of inflation and output from steady state values.

$$\frac{R_{n,t}}{\bar{R}_n} = \left( \frac{R_{n,t-1}}{\bar{R}_n} \right)^{\rho_R} \left[ \left( \frac{Y_t}{\bar{Y}} \right)^{\theta_y} \left( \frac{\pi_{t+1}}{\bar{\pi}} \right)^{\theta_\pi} \right]^{1-\rho_R} e^{\epsilon_t^R} \quad (3.50)$$

with  $\rho_R \in (0, 1)$ ,  $0 < \theta_\pi, \theta_y$ .

### 3.2.8 Aggregate Resource Constraint and Market Clearing Condition

The goods market clearing condition gives an aggregate resource constraint for the whole economy.

$$Y_t = C_t + I_t + G_t + \mu G_{\bar{\psi}_{t+1}} R_t^k Q_{t-1} K_{t-1} \quad (3.51)$$

where  $\mu G_{\bar{\psi}_{t+1}} R_t^k Q_{t-1} K_{t-1}$  is the amount of agency costs incurred by banks monitoring the entrepreneur.

Similarly, the consolidated credit market clearing condition implies that the total physical capital purchased by the entrepreneur is financed by the net worth of entrepreneur  $N_t^E$ , net worth of the bank  $N_t^B$  and total deposits of household  $B_t$ .

$$Q_t K_t = B_t + N_t^B + N_t^E \quad (3.52)$$

To derive the economy-wide resource constraint equation 3.51 for the whole economy I aggregate the budget constraint of household, the government balanced budget as well as the profit equations of wholesale producer, bank, capital producer and retailer. I refer to section B.1.9 of appendix for a detailed derivation the economy-wide resource constraint equation 3.51.

The credit market clearing equation 3.52 follows by the consolidation of the balance sheet equations of entrepreneur (eq. 3.19) and the banker (eq.3.38).

### 3.2.9 Competitive Equilibrium

A competitive equilibrium of the model economy consists of sequences of allocations

$\{H_t, \bar{\Psi}_t, \lambda_t^E, N_t^E, Y_t^w, K_t, v_t^B, \mu_t^L, \mu_t^S, \Omega_t, N_t^B, L_t, S_t, I_t, \Delta_t, \pi_t, J_t, J_t, Y_t, C_t, B_t\}_{t=0}^\infty$  of market prices  $\{R_{t+1}, R_{t+1}^k, w_t, R_{t+1}^B, R_t^L, R_t^S, Q_t\}_{t=0}^\infty$  and of exogenous processes  $\{A_t, G_t, \epsilon_t^R, \Upsilon_t\}_{t=0}^\infty$  such that the allocation solves the problem of household, entrepreneur, banker, capital producer and of retailer at equilibrium prices and that markets clear. The equilibrium conditions that must satisfy are:

$$1 = E_t \left( \beta \frac{U_{C_{t+1}}}{U_{C_t}} \right) R_{t+1} \quad (3.53)$$

$$w_t = - \frac{U_{H_t}}{U_{C_t}} \quad (3.54)$$

$$E_t \frac{R_{t+1}^k}{R_{t+1}^B} = E_t \frac{\lambda_{j,t+1}^E}{(1 - \Gamma_{\bar{\Psi}_{t+1}}^j) (1 - X_{j,t}^{eq} - \phi_{j,t}^E X_{\phi_{j,t}}^{eq'}) + \lambda_{j,t+1}^E \left[ \Gamma_{\bar{\Psi}_{t+1}}^j - \mu G_{\bar{\Psi}_{t+1}}^j + (1 - \Gamma_{\bar{\Psi}_{t+1}}^j) (X_{j,t}^{eq} + \phi_{j,t}^E X_{\phi_{j,t}}^{eq'}) \right]} \quad (3.55)$$

$$E_t R_{t+1}^B = E_t R_{t+1}^k \frac{\phi_t^E}{\phi_t^E - 1} \left( \Gamma_{\bar{\Psi}_{t+1}} - \mu G_{\bar{\Psi}_{t+1}} + (1 - \Gamma_{\bar{\Psi}_{t+1}}) X_t^{eq} \right) \quad (3.56)$$

$$E_t \lambda_{t+1}^E = E_t \frac{\Gamma'_{\bar{\Psi}_{t+1}} (1 - X_t^{eq})}{\Gamma'_{\bar{\Psi}_{t+1}} (1 - X_t^{eq}) - \mu G'_{\bar{\Psi}_{t+1}}} \quad (3.57)$$

$$N_t^E = (\sigma^E + \xi_E) E_t R_t^k Q_{t-1} K_{t-1} \left( 1 - \Gamma_{\bar{\Psi}_t} \right) (1 - X_{t-1}^{eq}) \quad (3.58)$$

$$Y_t^w = (A_t H_t)^\alpha K_{t-1}^{1-\alpha} \quad (3.59)$$

$$w_t = \alpha \frac{P_t^w Y_t^w}{P_t H_t} \quad (3.60)$$

$$R_t^k = \frac{(1 - \alpha) \frac{P_t^w Y_t^w}{P_t K_{t-1}} + Q_t(1 - \delta)}{Q_{t-1}} \quad (3.61)$$

$$\phi_t^B = \frac{\nu_t^B}{\Theta_B - \mu_t^L(1 - \kappa_t) - \mu_t^S \kappa_t} \quad (3.62)$$

$$\nu_t^B = E_t \Lambda_{t,t+1} \Omega_{t+1} R_{t+1} \quad (3.63)$$

$$\mu_t^L = E_t \Lambda_{t,t+1} \Omega_{t+1} (R_{t+1}^L - R_{t+1}) \quad (3.64)$$

$$\mu_t^S = E_t \Lambda_{t,t+1} \Omega_{t+1} (R_{t+1}^S - R_{t+1}) \quad (3.65)$$

$$\Omega_{t+1} = 1 - \sigma_B + \sigma_B \Theta_B \phi_{t+1}^B \quad (3.66)$$

$$N_{t+1}^B = \sigma_B (R_{t+1}^L L_t + R_{t+1}^S Q_t S_t - R_{t+1} B_t) + \xi_B R_{t+1}^B A_t^B \quad (3.67)$$

$$R_t^B = R_t^L(1 - \kappa) + R_t^S \kappa \quad (3.68)$$

$$R_{t+1}^S = E_t R_{t+1}^k \frac{Q_t K_t}{Q_t S_t} \left(1 - \Gamma_{\Psi_{t+1}}\right) X_t^{eq} \quad (3.69)$$

$$L_t = (1 - \kappa)(Q_t K_t - N_t^E) \quad (3.70)$$

$$S_t = \kappa(Q_t K_t - N_t^E)/Q_t \quad (3.71)$$

$$1 = Q_t \gamma_t \left(1 - S(I_t/I_{t-1}) - \frac{I_t}{I_{t-1}} S'(I_t/I_{t-1})\right) + E_t [\Lambda_{t,t+1} Q_{t+1} \gamma_{t+1} (I_{t+1}/I_t)^2 S'(I_{t+1}/I_t)] \quad (3.72)$$

$$K_t = (1 - \delta)K_{t-1} + (1 - S(I_t/I_{t-1}))I_t \gamma_t \quad (3.73)$$

$$J_t = \frac{1}{1 - 1/\epsilon} m c_t u_{C_t} Y_t + \xi_p \beta E_t \left(\frac{\pi_{t+1}}{\pi_t^{\gamma_p}}\right)^\epsilon J_{t+1} \quad (3.74)$$

$$J_t = u_{C_t} Y_t + \xi_p \beta E_t \tilde{\pi}_{t+1}^{\epsilon-1} J_{t+1} \quad (3.75)$$

$$1 = \xi_p \left(\frac{\pi_t}{\pi_{t-1}^{\gamma_p}}\right)^{\epsilon-1} + (1 - \xi_p) \left(\frac{J_t}{J_t}\right)^{1-\epsilon} \quad (3.76)$$

$$Y_t = (1 - c) \frac{Y_t^w}{\Delta_t} \quad (3.77)$$

$$\Delta_t = \left( \frac{\pi_t}{\pi_{t-1}^{\gamma_p}} \right)^\epsilon \xi_p \Delta_{t-1} + (1 - \xi_p) \left( \frac{J_t}{JJ_t} \right)^{-\epsilon} \quad (3.78)$$

$$Y_t = C_t + I_t + G_t + \mu G_{\bar{\psi}_t} R_t^k Q_{t-1} K_{t-1} \quad (3.79)$$

$$Q_t K_t = B_t + N_t^B + N_t^E \quad (3.80)$$

The model is driven by four fundamental shocks:

$$\ln A_t = (1 - \rho_A) \ln(\bar{A}) + \rho_A \ln A_{t-1} + \sigma^A \epsilon_t^A \quad (3.81)$$

$$\ln G_t = (1 - \rho_G) \ln(\bar{G}) + \rho_G \ln G_{t-1} + \sigma^m \epsilon_t^G \quad (3.82)$$

$$\ln \epsilon_t^R = \rho_m \ln \epsilon_{t-1}^R + \sigma^m \epsilon_t^m \quad (3.83)$$

$$\ln \Upsilon_t = (1 - \rho_\Upsilon) \ln(\bar{\Upsilon}) + \rho_\Upsilon \ln \Upsilon_{t-1} + \sigma^m \epsilon_t^\Upsilon \quad (3.84)$$

$A_t$  is total factor productivity,  $G_t$  is government expenditures and  $\Upsilon_t$  is investment specific technology and where  $\sigma^i$  is the size of exogenous shock  $i$ ,  $\epsilon_t^i \sim N(0, 1)$  and  $\rho^i \in (0, 1)$  for  $i = A, G, m, \Upsilon$ . I allow for the persistence of monetary shock parameter to be greater or equal to zero,  $0 \leq \rho_m < 1$ .

The system has 13 *macroeconomic* variables, 15 *financial* variables and 4 *exogenous* shocks following an AR(1) process.

The set of *macroeconomic* variables, hours of work  $H_t$ , wholesale output  $Y_t^w$ , capital stock  $K_t$ , investment  $I_t$ , price dispersion  $\Delta_t$ , inflation  $\pi_t$ , two auxiliary variables  $JJ_t$  and  $J_t$  determining the price set by retailers  $p_t^* = \frac{J_t}{JJ_t}$ , final output  $Y_t$ , consumption  $C_t$ , risk free rate  $R_t$ , real wage  $w_t$  and asset prices  $Q_t$  is determined by the 13 equations (3.53, 3.54, 3.59, 3.60, 3.61, 3.72, 3.73, 3.74, 3.75, 3.76, 3.77, 3.78, 3.79).

The set of *financial* variables, the threshold level of idiosyncratic shock  $\bar{\psi}_t$ , return on firm project  $R_{t+1}^k$ , Lagrangean multiplier  $\lambda_t^E$ , firm net worth  $N_t^E$ , required return on bank assets  $R_{t+1}^B$ , the discounted value of marginal (risk free) return on banker's net worth  $v_t^B$ , the discounted future excess returns

### 3.3. Two special cases of FAGK Model

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on banker's assets relative to deposit rate  $\mu_t^L$  and  $\mu_t^S$ , shadow marginal value of net worth  $\Omega_t$ , bank net worth  $N_t^B$ , bank loans  $L_t$ , equity securities held on bank balance sheets  $S_t$ , bank deposits  $B_t$ , average return on securities portfolio  $R_t^S$  and average return on loan portfolio  $R_t^L$  is determined by the other 15 equations (3.55, 3.56, 3.57, 3.58, 3.62, 3.63, 3.64, 3.65, 3.66, 3.67, 3.68, 3.69, 3.70, 3.71 and 3.80).

The evolution of the *exogenous* shocks is shown by the last 4 equations (3.81, 3.82, 3.83 and 3.84)

## 3.3 Two special cases of FAGK Model

In this section, I lay out two special cases of my baseline FAGK model, a bank friction (GK) model as in Gertler and Kiyotaki (2010) and a simple financial accelerator (FA) model as in Bernanke et al. (1999).

The bank friction, GK, model is a special case of my *baseline framework*, FAGK. The key difference GK model from my baseline framework (FAGK) is the absence of standard debt loans. In the absence of the debt loan, banks finance the purchase of physical capital  $Q_t K_t$  via his own net worth and the households deposits by purchasing of 100 % of firm equity stakes. The entrepreneur does not accumulate any net worth.

In the FA model, I rule out moral hazard risk on the side of banks and let firms obtain only loans besides their own net worth. The standard debt loans allow for introduction of the financial accelerator of Bernanke et al. (1999).

In the absence of both, the financial accelerator mechanism and the banker moral hazard, neither bank nor firm accumulate net worth. The purchase of physical capital,  $Q_t K_t$ , is financed 100% by household deposits. There is no financial constraint on any agent to drive the premium  $R_{t+1}^k - R_{t+1}$  above

zero. The expected return from investing in production of wholesale goods is equivalent to risk free rate, hence the new Keynesian arbitrage condition.

$$1 = E_t R_{t+1}^k \left( \beta \frac{U_{C_{t+1}}}{U_{C_t}} \right) \quad (3.85)$$

In the next section, I describe the two special cases of my framework, the financial accelerator (FA) model and the bank friction (GK) model and provide relevant key equations.

### 3.3.1 Financial Accelerator Model

In this model, the banker offers short term deposit opportunities to households and pools them into loans to firm. There is no moral hazard problem between him and depositors. The banker does not need to accumulate any net worth and does not purchase any firm equity stakes. The model simplifies into a financial accelerator model identical to that of [Bernanke et al. \(1999\)](#).

The banker monitors the entrepreneurs that declare default and seizes their assets upon default. The entrepreneur finances the purchase of physical capital  $Q_t K_t$  through his net worth and the loan.

$$Q_t K_{j,t} = N_{j,t}^E + L_{j,t} \quad (3.86)$$

The firm value function is simply the return from investment less the payoff portion ( $\Gamma_{\bar{\Psi}_{t+1}}$ ) paid to the bank to cover the cost of standard debt loan.

$$V_{j,t+1}^E = E_t R_{t+1}^k \left( 1 - \Gamma_{\bar{\Psi}_{t+1}}^j \right) Q_t K_{j,t} \quad (3.87)$$

The banker earns payoff share from the loan contract ( $\Gamma_{\bar{\Psi}_{t+1}}$ ) less the cost of monitoring ( $\mu G_{\bar{\Psi}_{t+1}}$ ). The contract he signs with the entrepreneurs requires

### 3.3. Two special cases of FAGK Model

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only that the opportunity cost of funds lent to entrepreneurs is equal no less than risk free rate.

$$E_t R_{t+1}^k Q_t K_{j,t} \left( \Gamma_{\bar{\psi}_{t+1}^j} - \mu G_{\bar{\psi}_{t+1}^j} \right) \geq E_t L_{j,t} R_{t+1} \quad (3.88)$$

In the simple financial accelerator model, there is no equity securities purchased by the banker. The total assets are now equal to  $L_t = A_t^B$ . In addition, as bankers are not credit constrained, they accumulate no equity hence deposits,  $D_t$ , are equal to total assets,  $L_t$ . Second, the opportunity cost of funds is not a required return on bank assets,  $E_t R_{t+1}^B$  but instead the households risk free interest rate,  $R_{t+1}$ .

The only external finance premium relevant in this framework is the borrower premium,  $E_t(R_{t+1}^k - R_{t+1})$ , that banker will charge on loans to cover the agency costs. Entrepreneur optimizes the employed capital, hence loan amount, and obtains a one-to-one relationship between borrower premium and threshold value of the idiosyncratic shock.

$$E_t \frac{R_{t+1}^k}{R_{t+1}} = E_t \frac{\lambda_{j,t+1}^E}{(1 - \Gamma_{\bar{\psi}_{t+1}^j}) + \lambda_{j,t+1}^E \left[ \Gamma_{\bar{\psi}_{t+1}^j} - \mu G_{\bar{\psi}_{t+1}^j} \right]} \quad (3.89)$$

where,  $\lambda_{j,t+1}^E(\bar{\psi}_{t+1}^j)$  is a one-to-one mapping of  $\bar{\psi}_{t+1}^j$ . Equation 3.89 relates the financing premium with the level of idiosyncratic shock  $\bar{\psi}_{t+1}^j$ . An adverse shock pushes up the threshold level of  $\bar{\psi}_{t+1}^j$  that borrowers choose to default. As a result more borrowers default for everything else the same.

Optimizing with respect to the multiplier delivers an inverse relationship between the premium and firm leverage.

$$1 = E_t \frac{R_{t+1}^k}{R_{t+1}} \frac{\phi_{j,t}^E}{\phi_{j,t}^E - 1} \left( \Gamma_{\bar{\psi}_{t+1}^j} - \mu G_{\bar{\psi}_{t+1}^j} \right) \quad (3.90)$$

In this equation the borrower budget constraint sets up a critical relationship between the procyclical firm leverage and the borrowing premium over the risk free rate. Firm leverage is  $\phi_{j,t}^E = \frac{Q_t K_{j,t}}{Q_t K_{j,t} - L_{j,t}}$ . An adverse shock will lower bank net worth,  $Q_t K_{j,t} - L_{j,t}$  and therefore drive up leverage. Higher leverage, hence a lower ratio  $\frac{\phi_{j,t}^E}{\phi_{j,t}^E - 1}$ , will drive up the borrower premium  $E_t \frac{R_{t+1}^k}{R_{t+1}}$ <sup>14</sup>. Equations 3.90 and 3.89 are identical to the first order conditions 3.36 and 3.35 in baseline model with  $X_{j,t}^{eq} = 0$ ,  $L_{j,t} = A_{j,t}^B$  and  $E_t R_{t+1}^B = R_{t+1}$

Finally the aggregate resource constraint is the same as in the baseline model, *FAGK*.

### 3.3.2 Bank Friction (GK) Model

In the special case with only a bank friction (GK), banks own the firms by financing 100 % of entrepreneurs project with equity securities,  $S_t = K_t$ . This model is the same as the [Gertler and Kiyotaki \(2010\)](#) model. The banker invests his net worth and deposits into the project (equivalent of equation 3.38).

$$N_{i,t}^B + B_{i,t} = Q_t S_{i,t} = Q_t K_{i,t} \quad (3.91)$$

The banker still has the incentive to accumulate net worth and keep the value of bank above a certain threshold. Doing so, he alleviates the moral hazard problem by ensuring depositors he has no incentive to run away with their money. The bank's net income is return from project less cost of deposits on which the bank pays a risk free rate. The analogue of equation 3.40 from baseline *FAGK* model is now:

$$N_{i,t+1}^B = R_{t+1}^k Q_t S_{i,t} - R_{t+1} B_{i,t} \quad (3.92)$$

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<sup>14</sup>The decline of the ratio  $\frac{\phi_{j,t}^E}{\phi_{j,t}^E - 1}$  dominates the dynamics of the right hand in equation 3.90.

### 3.4. Parameter Calibration

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The banks value function is the same discounted stream of future bank net worths.

$$V_{i,t}^B = E_t \sum_{\tau=1}^{\infty} (1 - \sigma_B) \sigma_B^{\tau-1} \Lambda_{i,t+\tau} N_{i,t+\tau}^B \quad (3.93)$$

where,  $\Lambda_{i,t+\tau}$  is the real stochastic discount factor (SDF) of household, and  $(1 - \sigma_B) \sigma_B^{\tau-1}$  is the probability of surviving until the  $\tau^{\text{th}} - 1$  period therefore exiting in  $\tau^{\text{th}}$  period. The banker is under pressure from depositors to keep the value of the bank above a similar threshold ratio in terms of its assets as in equation 3.42.

$$V_{i,t}^B \geq \Theta_B A_{i,t}^B \quad (3.94)$$

Given the investment opportunity by owning 100 % of firm's securities, the banker's strategy is to maximize the value of his bank subject to incentive constraint set by the depositors.

## 3.4 Parameter Calibration

### 3.4.1 Data Set

I use US data from NIPA-BEA, the Bureau of Labour Statistics (BLS) for real sector and from Flow of Funds Accounts of the Federal Reserve Board<sup>15</sup>. I construct the seven key macroeconomic variables used in [Smets and Wouters \(2007\)](#) following their guidelines. Construction of these key macroeconomic variables is similar across the literature. I provide the details in appendix. My particular interest is on measures of external finance premium and net worth of bank and of entrepreneur. I focus only on these variables here.

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<sup>15</sup>NIPA-BEA are the National Income and Product Accounts of Bureau of Economic Analysis.

In my theoretical framework I define two measures of external finance premium and their total sum as follows.

- The *lender premium* is the difference  $R_t^B - R_t$  where  $R_t^B$  is the average required return on bank assets and  $R_t$  is risk free rate.
- The borrower premium is the difference  $R_t^k - R_t^B$  where  $R_t^k$  is return on capital.
- The *total premium* is the sum of the two spreads above, that is  $R_t^k - R_t$ .

#### **Proxy for external finance premium**

To get empirical counterparts to total external finance premium and its two components I look at Moody's corporate bond yields relative to risk free rate. Moody's BAA and AAA rated corporate bonds provide measures of return to two types of borrowers.

AAA rated borrowers are the ones with very low or no borrower risk. The spread they are charged relative to risk free rate takes into account the lenders cost of transforming its liabilities into longer term assets. I interpret that spread as the *lender premium* required by lenders to meet their net worth targets.

The additional spread charged to BAA-rated corporate borrowers on top of AAA yield is only due to higher borrower risk, hence *borrower premium*. The rationale for such a definition is that any premium that is due to other factors, not related to the borrower ability to repay, would raise the yield of the AAA rated borrowers as well <sup>16</sup>. The definition for the three premiums is summarized.

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<sup>16</sup>A natural question would then arise whether the the additional higher effect on BAA borrowers is due to liquidity or due to borrower quality. One can still reconcile the greater effect of absence of liquidity in interbank market on BAA rated borrowers with their lower ability to pay. I abstract from those considerations as I do not have an interbank market in this framework.

### 3.4. Parameter Calibration

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1. *efpB*, the difference between Moody's BAA rated corporate bond yield and Fed rate as a proxy for the *total premium* (or *total spread*  $R_t^k - R_t$  in the model).
2. *efpA*, the difference between Moody's AAA rated corporate bond yield and Fed rate, a proxy for the *lender risk premium* (*lender spread*  $R_t^B - R_t$  in the model).
3. *efpBA*, the difference between Moody's BAA and AAA rated corporate bond yields as a proxy for the *borrower risk premium* (*borrower spread*  $R_t^k - R_t^B$  in the model).

Finally a statistical motivation for the definition of *efpBA* as the *borrower premium* is that the standard deviation of this spread and of firm net worth (based on Flow of Funds data) during the last recession jump up by a similar magnitude when compared to the more tranquil period before the recession<sup>17</sup>.

One particular concern with these measures is that their difference relative to Fed rate may contain noise due to other risks. Liquidity risk premium related to short term instruments is a particular concern raised in the literature. To address such concern, I consider the spreads of Moody's BAA and AAA rated bond yields relative to Government 10 year constant maturity bond yield, named *efpBG total* and *efpAG* respectively. The latter two series are alternative proxy variables for the *total premium* and *lender premium*, respectively. I list the second moments of both sets of measures when comparing with second moments.

#### Net Worth

Data for the net worth of entrepreneur can be found in the Flow of Funds account of Federal Reserve. I define non-financial sector net worth as tangible

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<sup>17</sup>I show in table [B.10 on page 171](#) of appendix B that the ratio of standard deviation for both these two indicators during the recession period jumps by 2.8 times (column[d] of the table).

### 3.4. Parameter Calibration

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assets minus credit market liabilities of non-farm corporate and non-corporate businesses in US. This measure matches the definition of *firm net worth* in the theoretical model. I exclude non-tangible assets from calculating net worth as they are not accounted for in the model.

To obtain a series for *bank net worth* I use the market value of corporate equities on liability side of financial businesses from the the US financial account (table Z1) of the FRB <sup>18</sup>.

An alternative measure of financial and non-financial sector wealth is provided by the stock index measure for each sector. [Christiano et al. \(2010\)](#) include stock market index as a proxy for net worth of banks. *Datastream* provides stock index measures of financial sector and non-financial sector. Volatility of these series is of a similar scale as the volatility of the series obtained from the Flow of Funds tables. For comparative purposes, I report moments from both sets of data, the Flow of Funds and *Datastream*. For the sake of space I describe all the series in detail in Appendix [B.1.11](#).

#### 3.4.2 Baseline *FAGK* Model

I calibrate the baseline model to match first moments of key US data variables for the period 1985-2005. The second moments of key indicators for the samples 1970-2005 and the sample 1955-2005, nesting the first one, are significantly higher than in 1985-2005. I focus on the latter period of low macroeconomic volatility, named the Great Moderation. I split the parameters in two groups. I refer to all the parameters that are relevant only to a simple new Keynesian framework without any financial frictions with the term *NK parameters*. I refer

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<sup>18</sup>I do not use the data from Federal Deposit Insurance Corporation (FDIC) balance sheets of commercial banks in US. They are a small fraction of the financial sector that was hit by the recent crisis in US. In addition, the model definition of a bank takes into account that bank also buy firm securities, which commercial banks do not.

### 3.4. Parameter Calibration

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with the term *financial friction parameters* to all those parameters that are only in the financial accelerator model and in the bank friction model, but not in a simple NK model.

#### **New Keynesian Parameters**

The first group is the set of parameters typical to standard New Keynesian (NK) features of the model, namely  $\beta$ ,  $\delta$ ,  $\alpha$ ,  $g_y$  and  $\xi_p$ ,  $\epsilon$ ,  $\gamma_p$ . I report the NK parameters in table B.5 in appendix B.

< Table B.5 here >

Some of the remaining parameters are set to match data first moments. The value for the household discount parameter,  $\beta$ , is set at 0.9966 so that nominal risk free interest rate in annual terms matches the observed average after the 1980s. The real yield is in the range 1.4-1.7 %. I pick a value close to the lower end of this range as this is more relevant for the most recent recession I am focusing on. Depreciation rate,  $\delta$ , is set at 0.025 implying an annual depreciation rate of 10 %. The exogenous government spending to gross output ratio is set close to historical average of  $g_y = 0.2$ . The steady state inflation rate is set to match 2.4 percent on annual basis.

I set the values for the remaining *NK parameters* following the works of [Christiano et al. \(2005\)](#), [Smets and Wouters \(2007\)](#), with minor changes to match some of the second moments of the data. Share of capital stock on output  $1 - \alpha$  is set at 0.3, considering a steady state share of capital income of roughly  $1/3$ . The inter-temporal elasticity of substitution  $1/\sigma_c$  is set at  $1/1.4$ , similar to the mode estimate of 1.39 reported by [Smets and Wouters \(2007\)](#). I have to set two parameters, labor supply shifter  $\delta_t^h$  and the inverse of the Frisch elasticity of labor  $\rho^h$ , to obtain a target for steady state hours of work. I set the inverse of the Frisch elasticity of labor  $\rho^h$  at 0.5. [Smets and Wouters](#)

(2007) estimate of inverse of the Frisch elasticity of labor at 1.92 when wages are sticky, but at 0.25 when he allows for flexible wage. Then I set the steady state hours of work,  $\bar{H}$  to 0.35 and let the labor supply shifter set to match this target. I obtain a value for the labor supply shifter  $\delta_t^h$  equal to 36.65<sup>19</sup>.

The Calvo parameter,  $\xi_p$ , and the coefficient on price indexation,  $\gamma_p$  are set at 0.75 and 0.5 respectively. The Calvo parameter implies an average contract duration of 3.3 quarters (Smets and Wouters (2007) and Levine et al. (2005))<sup>20</sup>. This is the average of the interval in literature and is close to the estimate of Smets and Wouters (2007)<sup>21</sup>. The parameter for the elasticity of substitution among varieties,  $\epsilon$ , is set at 7.

I have added two standard frictions to the NK set up. Both the investment adjustment cost (IAC),  $S'' = \phi_x$ , and the consumption habit formation parameter,  $\chi$ , introduce hump-shaped response functions to investment and consumption respectively. I choose a value of 0.65 for the habit parameter and a value of 2 for the IAC parameter. These values are in the range of those estimates reported by Smets and Wouters (2007) and Christiano et al. (2005)<sup>22</sup>. The latter suggests a slow response of investment to changes in the value of capital.

I set the parameters of the Taylor rule close to values initially proposed by Taylor (1999) and generally applied in this literature. The smoothing parameter,  $\rho_R$ , is set at 0.8. Inflation and output feedback parameters,  $\theta_\pi$  and  $\theta_Y$  are set at 1.5 and 0.5/4 respectively, given the quarterly frequency of the model.

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<sup>19</sup>Model results are not sensitive to small changes in the value of inverse of Frisch elasticity of labor  $\rho^h$ .

<sup>20</sup>Relevant cited studies that make empirical estimates are [Bils and Klenow \(2004\)](#) and [Nakamura and Steinsson \(2008\)](#). The former finds a low value based on evidence on micro data, while the more recent study reports a value closer to the estimate of the macro econometric models referred to in this study.

<sup>21</sup>The literature offers a wide range of values for the parameter for price indexation going as low as 0.11 (see [Levine et al. \(2005\)](#)).

<sup>22</sup>[Smets and Wouters \(2007\)](#) report a slightly higher estimated mean of 5.7 for the investment adjustment costs parameter

### 3.4. Parameter Calibration

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These values are very close to estimates  $\rho_R = 0.80$ ,  $\theta_Y = 0.1$ , and  $\theta_P = 1.5$  by [Clarida et al. \(1998\)](#) for the post-1979 period.

Finally, I calibrate the standard errors of the shocks. Initially I set the size of neutral technology shock, public expenditure shock and investment specific shock plainly at 1%. The size of monetary shock relative to technology shock is set at around 0.15% based on estimated (approximate) relative values from several papers papers ([Smets and Wouters \(2007\)](#) and [Christiano et al. \(2010\)](#)). Then I slightly adjusted the size of neutral technology shock so that, with minor changes of its ratio to the size of monetary shock, it leads to a standard deviation of output similar to that observed in the data during the period of interest (1985-2005).

I set the persistence of all shocks at 0.9 except for the persistence of exogenous monetary shock,  $\rho_m$  set at 0. Taylor rule persistence parameter,  $\rho_R$ , set at 0.8 introduces persistence of monetary.

The set of parameters relevant to the New Keynesian block of the framework are kept the same in the simple *NK*, in *GK*, in *FA* and in this baseline framework *FAGK* with both frictions (table [B.5 on page 168](#)).

#### **Financial Friction Parameters**

The remaining parameters are related to financial frictions. I first set firm and bank survival probabilities,  $\sigma^E$  and  $\sigma^B$ , at 0.985 and 0.977 implying average tenure of 16.6 years and 10.8 years respectively. The former is similar to values used by [Bernanke et al. \(1999\)](#) and implies survival of firms similar to the median tenure of 16 years reported by [Levin et al. \(2004\)](#). The lower bank survival rate is similar the tenure of around 10.8 years in [Gertler and Karadi \(2011\)](#). I report in table [B.6](#) of appendix the steady state values of financial

### 3.4. Parameter Calibration

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variables that I use as a target to calibrate *financial friction parameters*. For each financial variable I report the respective calibrated parameter<sup>23</sup>.

< Table B.6 here >

Remaining financial friction parameters are calibrated so that I match as close as possible the premium and leverage targets of the firm and banks. The standard deviation of the idiosyncratic shock is calibrated at 0.306 and is slightly higher than the value 0.28 set by [Bernanke et al. \(1999\)](#). This value was conditioned by the target for the firm leverage. I targeted the ratio of total firm assets to total assets less loans at 1.84, matching the respective ratio of tangible assets over tangible assets less credit market liabilities (hereby *CML*) of non-financial businesses from Flow of Funds table of FRB. I report the calibrated values of *financial friction parameters* in table B.7 in appendix B.

< Table B.7 here >

The bank friction parameter, that is the fraction of bank assets that can be diverted,  $\Theta^B$ , is set at 0.76, which is a higher value than the calibrated value of [Gertler and Karadi \(2011\)](#). The parameters  $\Theta^B$  and the bank exit rate  $\xi^B = 0.00023$  were calibrated to meet specific equilibrium targets for *bank leverage* and *lender premium*, that is steady state values of  $\phi^B$  and  $R^B - R$ .

Equilibrium level for *bank leverage*, which is steady state value of  $\phi^B$  is set at 6.4. This is the ratio of total bank assets to net worth. Potential measure of this ratio for financial businesses comes from the Financial Accounts of Federal Reserve Board (Tables Z.1). Total assets over equity liabilities ratio for financial businesses is above 20. Such a measure includes liabilities other than credit market debt which are not accounted for in the model. An alternative measure is the ratio of the sum of '*credit market liabilities*' and '*equity liabilities*'

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<sup>23</sup>In some cases two parameters may be involved simultaneously in setting two parameters.

### 3.4. Parameter Calibration

to the latter which is around 4.4 for the period 1985-2014<sup>24</sup>. Given different results, I tend to set a target of 6.4 which is above 4 as considered by [Gertler and Karadi \(2011\)](#). I settle with this target as the results are not sensitive to different values of steady state leverage in a similar interval<sup>25</sup>.

To set a target for lender premium  $R^B - R$  at equilibrium I target the difference between Moody's AAA corporate bond yields relative to Government 10 year Treasury bond yield of 1.2 % (table 3.4)<sup>26</sup>. I motivate the use of Moody's AAA rated corporate bond yield to obtain lender risk premium since this yield has very low borrower risk.

Table 3.4: Actual external finance premium measures based on US data.

		1985-'06	1985-'14
Total premium	BAA yield (less) Fed rate	3.5 %	3.9 %
Lender premium	AAA yield (less) Fed rate	2.6 %	2.9 %
Total premium	BAA yield (less) 10-y Gov.B rate	2.1 %	2.3 %
Lender premium	AAA yield (less) 10-y Gov.B rate	1.2 %	1.3 %
Borrower premium	BAA yield (less) AAA yield	0.9 %	1.0 %

I target a steady state of 0.81 % for the *borrower premium* to proxy the average difference between BAA and AAA corporate bond yields. I calibrate the monitoring cost at 0.11 to achieve that target. The monitoring cost parameter is close to value in [Bernanke et al. \(1999\)](#) and well within the mid-range of calibration values suggested in literature. Empirical estimates are in the range 0-0.45 ([Alderson and Betker \(1995\)](#), [Levin et al. \(2004\)](#)).

The value for the average quarterly rate of firm default is set 0.75% as in [Bernanke et al. \(1999\)](#) and in line with empirical evidence for default rates of

<sup>24</sup>Asset to net worth ratio of commercial bank balance sheet data of *Federal Deposit and Insurance Scheme* (FDIC) is around 10. Such a measure may not correspond to the leverage measured in the model since the latter includes all financial institutions that are able to purchase securities of the corporate business.

<sup>25</sup>The bank leverage steady state value of 6.4 allows me some flexibility to also achieve the other targets.

<sup>26</sup>Spreads of BAA and AAA corporate bond yields relative to Fed rate for the period 1985-2006 are 3.5 % and 2.6 % respectively. I choose a total premium of around 2% commonly chosen in literature. I check that my results are not sensitive to the steady state premium.

### 3.4. Parameter Calibration

US bonds that claim an average of 3 % for the period 1971 - 2005(Altman and Pasternack (2006)). Following this calibration, the implied fraction of aggregate output lost in monitoring costs is 0.23% <sup>27</sup>.

To get a rough measure of the share of equity finance on banks assets,  $\frac{Q_t S_t}{Q_t S_t + L_t}$ , I look at the ratio of *corporate equities plus debt securities over debt securities* held on the asset side of the balance sheet of financial businesses.

$$\text{share of equitiy finance by banks} = \frac{\text{corporate.equities(+)debt.securities}}{\text{debt.securities}}$$

The data are collected from the Financial Account of the US (Z.1, table L108). The share of firm equity security purchases on financial businesses assets is on an upward trend in US (see figure B.1 in appendix).

< Figure B.1 here >

The ratio fluctuates in the range of 30-45 % during the period 1985-2005, with an average of 34% (table 3.5). I consider an average of 0.33 as a baseline measure, which fits better in the financial businesses balance sheet.

**Table 3.5:** Financing Structure of non financial businesses based on data from Financial Accounts in US(\*).

Firm or Bank	Ratio	1970 -1985	1986-2014
Financial	$equity / (equity + debt (A) )$	23%	34%
Nonfinancial	$equity / (equity + debt (L) )$	22%	48%

(\* Source: Federal Reserve Board Financial Accounts, Table Z1.L108, Domestic Financial Sectors.

*Equity* stands for corporate equity on the asset side of financial businesses balance sheet, while (A) and (L) show from which side of the balance sheet of that entity it comes from.

Following this calibration, the steady state ratios of consumption and investment to output are 0.55 and 0.25 respectively. The great ratios are different for the new Keynesian model. The reason is that capital to wholesale output

<sup>27</sup>A table relating financial targets and the relevant parameters is provided in table B.6 of appendix B.

### 3.4. Parameter Calibration

ratio,  $\frac{K}{Y^w}$ , that is consistent with zero external finance premium at equilibrium is higher compared to the same ratio when total premium is 2%. This ratio further determines the ratio of investment, consumption and capital stock to final output. Across models with steady state total  $efp = 2\%$  these ratios are the same. They are different in NK model.

Table 3.6: Steady States of Macro variables.

Variable	Description	NK	1.GK	2.FA	3.FA-GK
$\frac{C}{Y}$	Average Consumption to GDP ratio	0.51	0.57	0.57	0.57
$\frac{I}{Y}$	Average Investment to GDP ratio	0.29	0.23	0.22	0.22
$\frac{K}{Y}$	Average Capital Stock to GDP ratio	11.8	9.00	9.00	9.00
$\pi$	Average inflation rate	2.4%	2.4%	2.4%	2.4%
R	Average real interest rate	1.4%	1.4%	1.4%	1.4%
$H_t$	Working hours	0.35	0.35	0.35	0.35

#### 3.4.3 Financial Accelerator (FA) Model

I keep most of the parameters from my baseline *FAGK* model. I report only the ones I have changed in the *FA* model. By construction this model has no equity finance, hence  $\kappa = 0$ . The focus of re-calibrating parameters relevant to firm problem is the external finance premium and firm leverage (asset to net worth ratio). The firm ratio of physical capital to net worth is targeted at 1.83, very close to the ratio in baseline model. The total  $efp$  is targeted at 2.0% which is very close to the target for the total spread in baseline *FAGK* model. To obtain these targets I calibrate monitoring cost,  $\mu = 0.12$  compared to 0.11 in *FAGK*. I also recalibrate  $\xi_E = 0.0033$  to a slightly different value (see table B.7 in appendix for a comparison of calibrated parameters in different models).

### 3.4.4 Bank Friction (GK) Model

I keep the bank incentive parameter,  $\Theta^B$ , at 0.75, very close to the value of 0.76 set in the baseline *FAGK* framework (table B.7 in appendix). This is the share of assets the banker can run away with. The main change is to raise the *lender premium* to 2% in equilibrium, similar to the target for the overall premium in my baseline *FAGK* model. For that I set the bank survival probability,  $\sigma^B$  to 0.971. I also adjust the share of wealth transferred to new bankers,  $\xi^B$ , to 0.0016 so that banker leverage in equilibrium is in a reasonable interval. With these changes bank leverage is around 4.1 and matches the same leverage that is common to the value as in [Gertler and Karadi \(2011\)](#). Bank tenure is around 8.6 years, slightly less than the tenure of 10.8 years assumed in my baseline model. The equilibrium targets of leverage and tenure of bank are the closest values I can get after having to raise the *lender premium* <sup>28</sup>.

The dynamics of the models are driven by the same four shocks, neutral technology shock (*tfp*), monetary shock (*MP*), public expenditures shock (*G*) and investment specific technology shock (*IST*). The magnitude of exogenous shocks and the persistence are the same across the models.

## 3.5 Moment Comparison

In this section I study the quantitative predictions of the baseline model by examining the results obtained from a numerical simulation of the model economy calibrated to US data. I evaluate the propagation of shocks in the baseline model *FAGK* with double moral hazard and compare them to the two one-

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<sup>28</sup>[Gertler and Karadi \(2011\)](#) set a steady state target of 1% (annualized) for the total *efp* while I set a target of 2% in order to keep it at the same value as in the *FAGK* model.

sided financial friction models, *FA* and *GK*. The second moments are obtained from the three calibrated models following stochastic simulation at order 1<sup>29</sup>.

#### 3.5.1 The Propagation of Shocks in a Double Friction Framework

To assess the propagation power of the models I compare the impulse response functions (*IRFs*) following the *tfp* and the monetary shock for the three friction models in graph 3.2 (page 107). Impulse responses are obtained from the nonlinear (exact) model and show the deviation of any variable  $\hat{X}_t$  following a *tfp* or monetary policy shock.  $\hat{X}_t$  is calculated as percentage changes of that variable from the steady state  $\hat{X}_t = \frac{X_t - X}{X}$  where  $X$  is the steady state of variable  $X_t$ . Compared to *FA* or *GK*, *FAGK* model with both frictions yields a stronger response of price of capital,  $Q$ , following a negative *tfp* shock or a positive monetary shock (panels (a) and (b) of figure 3.2 respectively). As a result both, firm and bank net worth, and hence leverage, respond by a greater magnitude<sup>30</sup>.

I motivate the stronger response in *FAGK* model with the tightening of both constraints at the same time upon a decline in price of capital. Total spread (*efp*) responds much stronger following either a *tfp* or a monetary shock. The greater response of total spread and of Tobin's  $Q$  trigger a stronger response of investment and output.

A second observation is that impulse response of total spread *efp* is dominated by the response of lender spread in *FAGK* model. Upon a 1.1 % *tfp*

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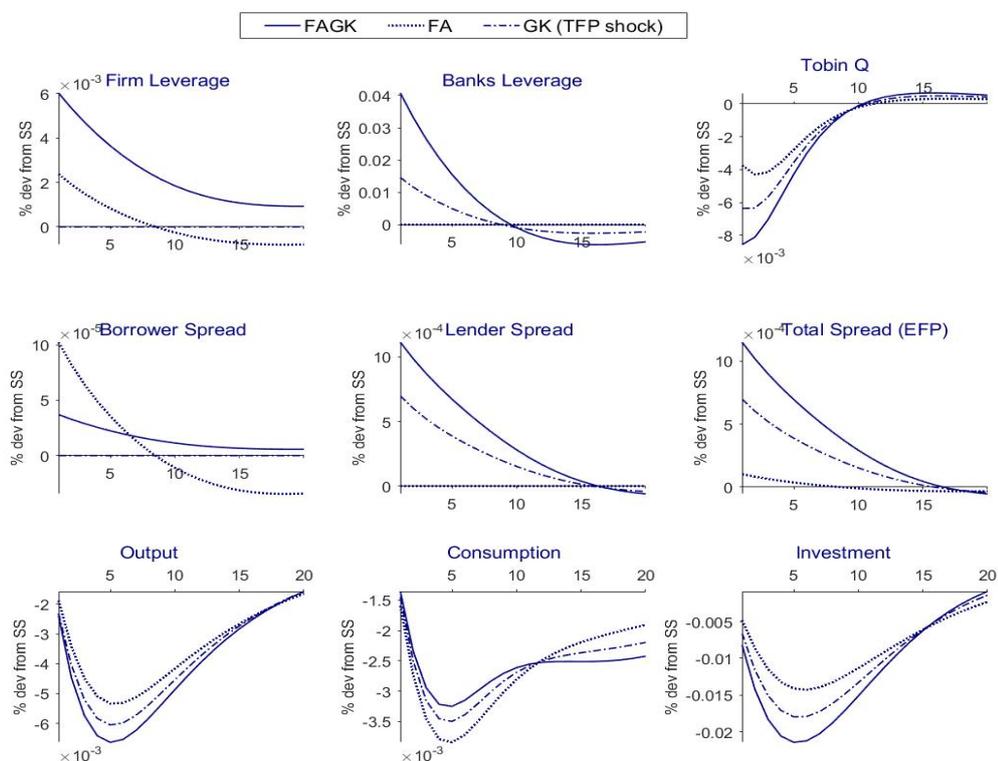
<sup>29</sup>Up to order 1, the exact model and the log-linearized one yield the same second moments and impulse responses.

<sup>30</sup>IRFs of firm related variables, like *firm leverage* and *borrower spread* are zero in the *GK* model. Similarly, IRFs of bank related variables, like *bank leverage* and *lender spread* are zero in the *FA* model. Monetary policy rule reacts to deviations of output from flexible price output.

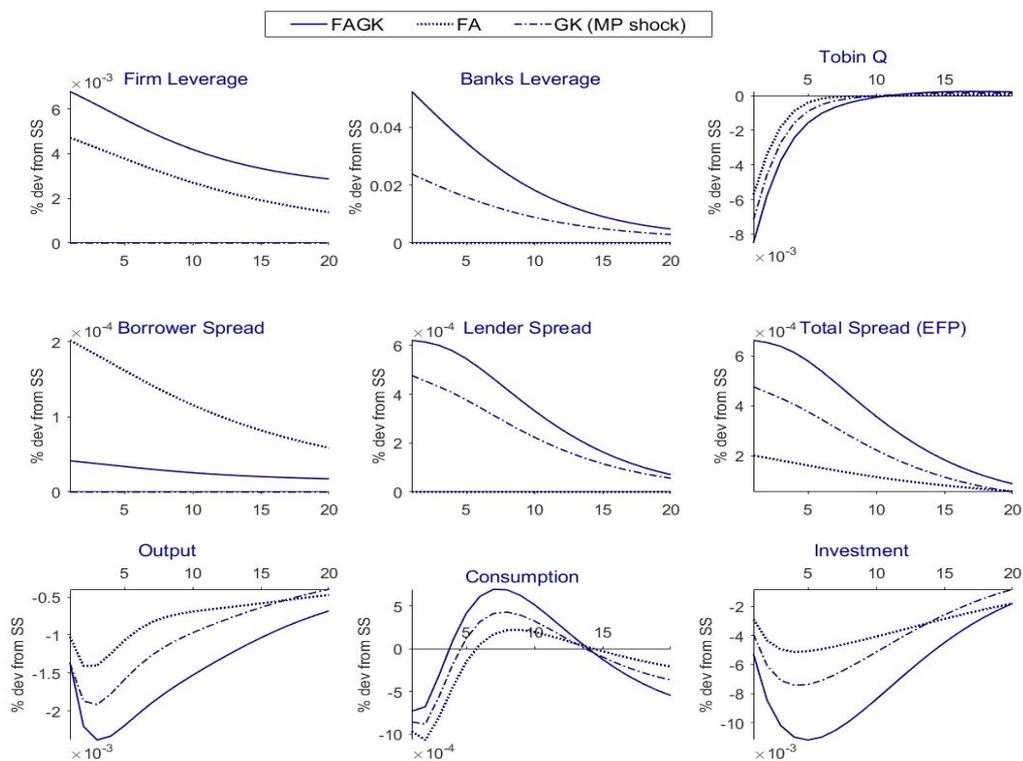
### 3.5. Moment Comparison

Figure 3.2: IRFs due to (-1.1%) TFP and to (+0.15%) MP shock: GK, FA and FAGK models.

(a) IRFs upon neutral productivity (*tfp*) shock.



(b) IRFs upon monetary (*mp*) shock.



### 3.5. Moment Comparison

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shock the lender's premium goes up by around 0.1 percentage points, while borrower's premium only by a negligible 0.01 percentage points. Total spread goes up by a similar magnitude of 0.1%. Similarly, the response of *lender premium* dominates the behavior of *total premium* following a (+0.15%) monetary shock.

Fluctuations in financial variables are driven by neutral technology (*tfp*) and monetary policy (MP). Variations in real variables are mainly driven by *tfp* shock. Both shocks explain 90% of fluctuations in the baseline *FAGK* model (table B.9, page 170). This result is also consistent with empirical results that monetary policy had some effect on financial variable but not on real ones<sup>31</sup>.

< Table B.9 here >

To quantify the differences in propagation power of each model I compare the business cycle properties of model-implied variables generated by stochastic simulation to the moments of data. I compare moments of Hodrick-Prescott (*HP*) filtered model-implied variables to the moments of one-sided *HP* filtered data series for the period 1985-2004<sup>32</sup>. Moments from the new Keynesian framework with no financial friction are listed as a benchmark for key macroeconomic variables (tables B.10 and B.11, page 172).

< Table B.10 here >

< Table B.11 here >

Comparison of second moment and of impulse responses leads to similar conclusions. First, the model with both frictions *FAGK* is able to produce

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<sup>31</sup>Public expenditure and investment specific technology (IST) shock account for less than 10 % of fluctuations. Impulse responses shown in graph B.2 of appendix.

<sup>32</sup>Moments of two-sided *HP* filtered and one-sided *HP* filtered data series are almost the same. For estimation purposes later, one-sided filtered data are preferable so I use the same series throughout the thesis.

### 3.5. Moment Comparison

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significantly greater volatility of model-implied variables relative to models with single financial friction, *FA* and *GK*. The standard deviations 1.74 and 3.69 of price of capital,  $Q_t$ , and of investment,  $I_t$ , respectively, are significantly higher in *FAGK* model than the 3.10 and 1.43 obtained in *GK* model. The *FA* model delivers even lower standard deviations for most variables. The standard deviations of firm and bank net worth and of total spread (*EFP*) are also higher in the *FAGK* model.

Second, lender's constraint leads to larger fluctuations on his own premium (spread  $EPF_a$ ), relative to that of borrower (spread  $EPF_b$ ). The standard deviation of borrower and lender spreads are 0.01 and 0.18, respectively, compared to standard deviation of 0.19 for total spread (table [B.10](#)). The bank friction is clearly dominating the effect of the financial accelerator in the model with both frictions, *FAGK*.

I can motivate the greater volatility of *efp* in the baseline *FAGK* model with the procyclical behaviour of the two net worth variables. Positive correlation of constraints delivers positive covariance of borrower and lender spread. The second round effects are stronger as a result, propagating the initial shocks by a greater magnitude as seen in impulse responses in figure [3.2](#).

To illustrate the positive correlation of the two constraints I check the variance-covariance matrix of total spread with the borrower and lender spread (table [B.12](#)). It shows covariance between borrower and lender spreads is  $0.12/10^6$  compared to the variance of total spread around  $3.51/10^6$ .

< Tables [B.12](#) here >

For convenience I normalize these numbers by the variance of total *efp* (table [B.13](#)). Around 92% of the volatility of total spread *EFP* is due to variance of lender's spread. The remaining volatility of total spread is due to the covariance between the spreads of the two agents, firm and bank. This observation

supports also the second result that bank constraint dominates the behaviour of *total premium*.

< Tables [B.13](#) here >

Finally, all the three models with friction deliver correlations with output consistent with the data. While difficult to compare the magnitudes, the only observation is that *FAGK* model with two leverage constraints and the *GK* model mimic much better the positive correlation of net worth with output. They replicate better the counter-cyclical behavior of the total spread.

#### 3.5.2 Simulating the Financial Crisis

In this section I assess how shifts in certain parameters have the potential to propagate shocks to an extent that I can mimic the increases in volatility of *efp* and other key variables during the recent recession. To this end I select few model parameters that are best related to characteristic features that marked the recent financial episode. In addition, I assess the implications of shifts in policy rule parameters.

##### Motivation for Shifts in Parameters

Two typical features of recent crisis are the large spike in external finance premium and the drastic fall in asset prices. I analyze the propagation of shocks subject to shifts in parameters that define the model-implied dynamics of *efp* and of asset prices,  $Q_t$ . The candidate parameters for this exercise are the financial friction parameters,  $\mu$  and  $\Theta^B$ , and investment adjustment cost parameter (*IAC*). These parameters are related to the dynamics of variables that

jumped during the recession. It is not unusual to think that these parameters may change overtime, as I will briefly motivate.

The second set of parameters I investigate are the ones in the policy rule. Analyzing the change in the dynamics of *efp* and of key macro variables subject to shifts in policy parameters is of interest to any policymaker.

The motivation for an upward shift in monitoring costs,  $\mu$ , is that several studies rely on shocks on monitoring cost to explain the variation in the external financing premium (Nolan and Thoenissen (2009) and Fuentes-Albero (2014)). There is also some empirical evidence for time varying monitoring costs (Levin et al. (2004)).

I motivate the reduction in  $\Theta_B$  with a more deregulated financial activity and less stringent regulatory framework on banking in US. Deregulation implies that banks have lower criteria to satisfy in terms of their capital requirements. The immediate implication of lower bank incentive constraint is that banks increase their leverage, which is consistent with lower  $\Theta^B$ .

A shift in *IAC* parameter can be motivated by the unusual circumstances of the failure of a banks in August 2007<sup>33</sup>. Following the crash of major investment banks in US, firms may review their practices of doing business. That may include how quickly they respond to adverse shocks by adjusting the decision-making process of cutting down or delaying investments. The *IAC* parameter is one of the few parameters added in an ad-hoc manner to obtain hump-shaped impulse responses of investment to monetary policy shocks that fit with impulse responses based on empirical analysis. The rationale is that firms take time to make decisions whether to invest or not and to implement the decision. I motivate the change in *IAC* parameter by assuming that these time lags may change in unusual circumstances.

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<sup>33</sup>The failure of two Bear Stearns hedge funds.

The motivation for an upward shift in the policy response to inflation,  $\theta_p$ , is the increased trend by Fed emphasizing the focus on inflation. A stronger response to inflation or a weaker response to output,  $\theta_Y$ , follows when policy maker becomes more inflation-target oriented. This scenario would correspond to a case when newly appointed board members care more for inflation. Decision making process would be tilted to reflect their positions. The weaker feedback to output is consistent with market concerns towards the end of Greenspan term. I assume increased persistence of policy rate at the same time that I lower the response to output.

Episodes of Fed changing the way it reacts to inflation and to output gap not unheard. The most well-known case is the looser reaction to inflation in the 1970s (Burns-Miller) and the tightening in the response to inflation after 1985 (Volcker's era). In more recent case, there has been a concern regarding the Fed's less aggressive stance on (positive) output gaps prior to the crisis and towards the end of Greenspan era (Taylor (2007)).

#### Shift in Friction parameters

I raise the monitoring cost parameter  $\mu$  by 50% in *FAGK* and *FA* models<sup>34</sup>. The upward shift raises the volatility of the borrower's premium in the *FA* and *FAGK* models by 1.62 and 1.64 respectively (table B.14). In either model there is no significant increase in standard deviation of macroeconomic variables. IRFs also show that upon an increase in monitoring cost, there is little difference in the behavior of other indicators, except for a greater response by the borrower's premium (figure B.3). In the baseline *FAGK* model, the impulse response of total spread is dominated by the response of lender's spread as shown earlier. The latter does not change, so the impact on other macro vari-

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<sup>34</sup>In *FAGK* model monitoring cost goes up from  $\mu' = 0.113$  to  $\mu' = 0.17$ . In *FA* model it goes up from  $\mu' = 0.12$  to  $\mu' = 0.18$ .

ables is not significant as well. In FA model only with firm friction, shift in  $\mu$  has a significant impact on  $efp$  but no implications for key macro variables.

I can infer that shift in the size of monitoring costs alone can hardly explain the jump in fluctuations observed recently in the data across most indicators.

< Table B.14 here >

< Figure B.3 here >

Next, I evaluate the impact of a decline in  $\Theta^B$  by 50%<sup>35</sup>. Volatility of lender net worth goes up by 13% and 14% in the *FAGK* and *GK* models, respectively (table B.15 in appendix). Similarly, volatility of lender's spread goes up by around 26 % and 38% in *FAGK* and *GK* models, respectively. IRFs in figure B.4 in appendix show that the increase in volatility is due to the stronger response of bank leverage following an exogenous monetary shock. The stronger response of the total spread triggers slightly stronger responses of investment and output. Comparison of standard deviations with the new parameter value shows the change in volatility of macro variables is negligible (table B.15). Impulse responses following a *tfp* shock are not different with the new  $\Theta^B$  parameter.

I conclude that I can reproduce the increase in volatility of total spread  $efp$  by lowering the bank friction parameter. Impulse responses of investment and output also decline further. There is no change in the response of asset prices and on the volatility of other macroeconomic variables, which is a feature of the recent financial crisis.

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<sup>35</sup>In *FAGK*  $\Theta^B = 0.76$  goes down to  $\Theta^B = 0.38$ . In the *GK* model  $\Theta^B = 0.75$  goes down to  $\Theta^B = 0.375$ . I re-calibrate the model so that no change in bank leverage takes place in steady state. In the case when bank leverage is allowed to go up, the results are similar. The difference is the size of change in volatility of lender and borrower spread is twice as high. But there is no significant change in the volatility of other key variables. The shift in leverage implies that for the same one unit of net worth, banker incurs more liabilities and levers up his balance sheet. The downside of high leverage is that a unit decline of asset value bites a larger amount of banker net worth. For the same elasticity of the external finance premium to leverage, greater steady state leverage implies stronger reaction by  $efp$  upon the same shock size.

< Table B.15 here >

< Figure B.4 here >

I assess the impact of lowering investment adjustment cost (*IAC*) from  $\phi_X = 2$  to  $\phi'_X = 1$  on *efp* and output. By construction, the investment adjustment cost function is introduced to get hump shaped responses of investment to monetary and neutral productivity shocks. For the same decline in asset prices,  $Q_t$ , upon a positive monetary shock or a negative neutral productivity shock the response of investment, and consequently of output, will be stronger (figure B.5). As a result, the standard deviation of investment and output will go up in the *FAGK* model by 32% and 22% respectively (table B.16). I obtain similar values in the *GK* and *FA* models. The total spread and other financial and macroeconomic variables do not seem to change their response upon such a shift.

< Table B.16 here >

< Figure B.5 here >

#### **Impact of Weaker Policy**

I assume a stronger feedback to inflation by changing  $\theta_P = 1.5$  to  $\theta'_P = 1.8$ . The change in volatility of all variables is negligible (table B.17). Impulse responses of all variables are the same with the new parameter value (figure B.6)<sup>36</sup>.

< Table B.17 here >

< Figure B.6 here >

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<sup>36</sup>The shift in feedback to inflation and a greater interest rate persistence has a small upward impact on the volatility of key variables.

### 3.5. Moment Comparison

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I analyze how the volatility of *efp* and other variables changes upon a shift in policy feedback to output gap from  $\theta_Y = 0.125$  to  $\theta'_Y = 0.025$ . Volatility of all measures of external finance premium and most key variables goes up by around 5-10% in the *FAGK* model (table B.18). In all three other models, *GK*, *FA* and *NK* models, the impact of weaker policy feedback to output alone is negligible for most variables, except for inflation.

< Table B.18 here >

I assess the impact of the same shift in feedback to output and a slightly more persistent policy rule at the same time. I choose to slightly raise the persistence parameter from  $\rho_R = 0.8$  to  $\rho'_R = 0.88$  with  $\theta_Y = 0.025$ . The simulation corresponds to a situation when monetary policy makers care about over-reactive policy and target only inflation. In the baseline *FAGK* model, standard deviation of *efp* goes up by around 35% (table B.19). The surge in the volatility of other financial and macroeconomic indicators is on average in the range of 20-70%<sup>37</sup>. The propagation effect comes from the bank friction. The *GK* model delivers higher volatility following a monetary shock. The ratios of volatility to the baseline *GK* parametrization are slightly smaller.

< Table B.19 here >

When policy feedback to output gap is low there is slightly greater fall in price of capital,  $Q_t$ , following an exogenous monetary shock (figures B.7 and B.8). Decline in  $Q_t$  erodes some of the net worth of lenders and borrowers, which exercises further effect on investment and asset prices. The propagation that both constraints set on borrower's capacity to purchase capital exercises greater second round effects on price of capital and on investment. The missing countercyclical policy response to output gap, due to low  $\theta_Y$ , allows second

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<sup>37</sup>Except standard deviation of inflation which increases by 127 %.

round effect to take place. Again lender friction dominates the behaviour of total *efp*.

< Figure B.7 and B.8 here >

Volatility of financial and macroeconomic variables in single friction models does increase as well. The magnitude is on average about half of that delivered by the double moral hazard framework *FAGK*. In the absence of counter cyclical policy, financial health of lenders and borrowers deteriorates (improves) faster upon an adverse (positive) shock. In the *FAGK*, the demand as well as supply for funds shrink at the same time. This eventually limits the amount of capital the entrepreneur can purchase. The second round effects that kick in are amplified by a greater magnitude. Each constraint amplifies further the already propagated shock by the constraint of the other agent.

These results are robust for any reasonable change in friction parameters leading to different steady state values of the external finance premium.

There is some anecdotal evidence that might lend support to these simulations. Several economists have argued that Fed might have contributed to the recent housing bubble by keeping the interest rates too low for a long time before the financial crisis ([Taylor \(2007\)](#), [Kahn \(2008\)](#)). This period would correspond to the end of Greenspan era, 2000-2006, when there was a boom in housing market associated with mild consumer price inflation. The criticism referred to Feds unwillingness to respond to output gaps driven by housing bubble. With low feedback to output, absence of high inflation would correspond to more persistent monetary policy instrument. In such an environment the presence of frictions would propagate immediately any exogenous policy shocks on both macro and financial variables.

## 3.6 Conclusions

A key indicator of the recent crisis has been the sharp increase in yields of most credit instruments, raising the cost of external finance. To assess the propagation effect of the borrower and lender financial health I employed a financial accelerator mechanism and a moral hazard problem between bank and depositor in the new Keynesian framework. The aim was to evaluate the propagation strength of financial frictions when both lender and borrower are constrained following technology and monetary shocks.

Two main results emerged from calibration exercise. First, relative to financial accelerator and bank friction models, the model with double frictions delivers significantly greater fluctuations of external finance premium. As both constraints tighten at the same time, both supply and demand for funds face constraints during a downturn. For the same magnitude of shocks, fluctuations of total external finance premium (*efp*) are significantly larger than in either FA or GK model. The stronger impact that the two constraints have on the price of capital delivers significantly greater fluctuations on investment and output.

Second, in the model with both frictions it is the bank friction that dominates in driving the external finance premium. It is the lender's constraint that has a greater impact on propagating fundamental technology and monetary policy shocks. The additional effect of the borrower constraint amplifies the response of asset prices leading to stronger second round effects on both agents' financial health.

In a framework with both firm and bank constraints binding, simulation results showed that shifts in monitoring cost or bank friction parameter, do lead to slightly greater volatility of borrower or lender risk premia, respectively.

Also, shifts in investment adjustment costs parameter may raise the volatility of investment and output only. Reasonable changes in the magnitudes of each friction parameter can deliver higher volatility only on a few variables. Individual friction parameters have limited impact on the dynamics of the model as a whole.

Finally, I find that evolution in the conduct of monetary policy can contribute to greater volatility of external financing premium and of other financial and real variables. Absence of countercyclical monetary policy can reinforce the propagation of exogenous monetary shocks at a much greater scale when policy maker is sluggish to respond. The same shocks lead to significantly larger fluctuations in financial and real variables.

# Appendix B

## B.1 Model Derivation

### B.1.1 Households

The household maximizes the utility function <sup>1</sup>:

$$U_{(C_t, H_t)} = \left[ \frac{(C_t - \chi C_{t-1})^{1-\sigma_c}}{1-\sigma_c} - \delta^h \frac{H_t^{1+\rho^h}}{1+\rho^h} \right] b_t^u$$

subject to his budget constraint:

$$w_t H_t + \Pi_t^B + \Pi_t^E + \Pi_t^{CP} + \Pi_t^R = C_t + B_t - R_{t-1} B_{t-1} + T_t$$

where  $1 - H_t$  is the amount of labour he supplies in labour market.

The shock  $b_t^u$  represents a general shock to preferences that affects the intertemporal rate substitution of households (preference shock) (Smets and Wouters (2003)).

In this current chapter  $b_t^u = 1$ .

The Lagrange is:

$$\max_{C_t, l_t, B_t} E_t \sum_{s=0}^{\infty} \beta^s \left[ U(C_{t+s}, l_{t+s}) + \lambda_{t+s}^h (w_{t+s} H_{t+s} + \Pi_{t+s}^{\text{own}} - C_{t+s} - (B_{t+s} - R_{t+s} B_{t+s-1}) - T_{t+s}) \right]$$

where  $\Pi_{t+s}^{\text{own}} = \Pi_{t+s}^R + \Pi_{t+s}^B + \Pi_{t+s}^E + \Pi_{t+s}^{CP}$  is revenue from ownership on respective firms.

The FOCs of the household problem are:

<sup>1</sup> In the next subsections, the subscript 'j' refers to the agent in the respective section. This way I save the usage of different subscripts which given the burden of different symbol can make the reading difficult. In the description of the model in chapter 3 I use different subscripts for each agent.

$$C_t: \quad 0 = U'_{C_t} + \lambda_t^h(-1)$$

$$H_t: \quad 0 = U'_{H_t} + \lambda_t^h w_t$$

$$B_t: \quad 0 = -\lambda_t^h + \beta E_t \lambda_{t+1}^h R_t$$

Use the first two FOCs to get a labor supply equation (MRS equals the real wage), and Euler equation:

**Labor supply Equation:**

$$w_t = -\frac{U_{H_t}}{U_{C_t}} = \frac{U_{C_t}}{U_{H_t}} \quad (\text{B.1})$$

**Consumption Euler Equation:**

$$1 = E_t (R_t \Lambda_{t,t+1}) \quad (\text{B.2})$$

I am solving the household problem in real terms. It is straightforward to see that budget constraint is the equivalent of the budget constraint expressed in nominal terms:

$$P_t C_t + B_{n,t} = R_{n,t-1} B_{n,t-1} + P_t w_t H_t + P_t \Pi_t^R + P_t \Pi_t^B - P_t T_t$$

$$C_t + \frac{B_{n,t}}{P_t} = \frac{R_{n,t-1}}{P_t/P_{t-1}} \frac{B_{n,t-1}}{P_{t-1}} + w_t H_t + \Pi_t^R + \Pi_t^B - T_t$$

$$C_t + B_t = \frac{R_{n,t-1}}{\pi_t} B_{t-1} + w_t H_t + \text{prof}_t^B + \text{prof}_{n,t}^R - T_t$$

**Functional Forms**

$$U_{(C_t, H_t)} = \left[ \frac{(C_t - \chi C_{t-1})^{1-\sigma_c}}{1-\sigma_c} - \delta^h \frac{H_t^{1+\rho^h}}{1+\rho^h} \right] b_t^u \quad (\text{B.3})$$

$$U'_{C_t} = b_t^u (C_t - \chi C_{t-1})^{-\sigma_c} - \beta^o \chi E_t b_{t+1}^u (C_{t+1} - \chi C_t)^{-\sigma_c} \quad (\text{B.4})$$

$$U'_{H_t} = -U'_{L_t} = - \left[ \delta^h H_t^{\rho^h} \right] b_t^U \quad (\text{B.5})$$

where  $l_t^h = 1 - H_t$

$$U'_{H_t} = -U'_{l_t^h} = - \left[ \delta^h H_t^{\rho^h} \right] b_t^U \quad (\text{B.6})$$

The utility function satisfies :

$$U'_{C_t} > 0, U'_{H_t} < 0, U''_{CC} < 0, U''_{HH} \leq 0 \quad (\text{B.7})$$

### B.1.2 Retailer's Problem

Retailer faces a two stage problem. In the first stage, the  $j^{\text{th}}$  retailer produces the final retail good  $Y_t$ , using wholesale good  $Y_{j,t}$ , and the production technology:

$$Y_t = \left( \int_0^1 Y_{j,t}^{\frac{1}{1+\lambda^m}} dj \right)^{1+\lambda^m} \quad (\text{B.8})$$

where,  $\lambda^m = \frac{1}{\epsilon-1}$  is the retailers mark up and  $\epsilon$  is the elasticity of substitution between any two varieties.

The retailer buys inputs  $Y_{j,t}$  and produces the final good in order to maximize profits subject to constraint B.8. Alternatively, the firm tries to minimize expenditure given the production constraint. I can write the Lagrangean of the retailer as:

$$L = P_t \left( \int_0^1 Y_{j,t}^{\frac{1}{1+\lambda^m}} dj \right)^{1+\lambda^m} - \int_0^1 P_{j,t} Y_{j,t} d(j) \quad (\text{B.9})$$

after making use of equation B.8. Optimal choice of  $Y_{j,t}$  solves first order condition  $\frac{\partial L}{\partial Y_{j,t}} = 0$ , that is:

$$P_{j,t} = P_t \frac{\partial Y_t}{\partial Y_{j,t}}$$

Using

$$\frac{\partial Y_t}{\partial Y_{j,t}} = (1 + \lambda^m) \left( \int_0^1 Y_{j,t}^{\frac{1}{1+\lambda^m}} dj \right)^{1+\lambda^m-1} \frac{1}{1+\lambda^m} Y_{j,t}^{-\frac{\lambda^m}{1+\lambda^m}} = \left( \frac{Y_t}{Y_{j,t}} \right)^{\frac{\lambda^m}{1+\lambda^m}}$$

Obtain the ratio 
$$\frac{P_{j,t}}{P_{i,t}} = \left( \frac{Y_{j,t}}{Y_{i,t}} \right)^{-\frac{\lambda^m}{1+\lambda^m}}$$

$$P_{j,t} Y_{j,t} = (Y_{j,t})^{\frac{1}{1+\lambda^m}} P_{i,t} Y_{i,t}^{\frac{\lambda^m}{1+\lambda^m}}$$

Integrate

$$\int_0^1 P_{j,t} Y_{j,t} dj = \int_0^1 (Y_{j,t})^{\frac{1}{1+\lambda^m}} dj P_{i,t} Y_{i,t}^{\frac{\lambda^m}{1+\lambda^m}}$$

Zero-profit condition 
$$\int_0^1 P_{j,t} Y_{j,t} dj = P_t Y_t$$

$$P_t Y_t = Y_t^{\frac{1}{1+\lambda^m}} P_{i,t} Y_{i,t}^{\frac{\lambda^m}{1+\lambda^m}}$$

$$P_t = Y_t^{-\frac{\lambda^m}{1+\lambda^m}} P_{i,t} Y_{i,t}^{\frac{\lambda^m}{1+\lambda^m}}$$

Obtain a demand function for  $Y_{j,t}$ . The retailer will produce:

$$Y_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\frac{1+\lambda^m}{\lambda^m}} Y_t \tag{B.10}$$

Plug the **demand function** for  $Y_{j,t}$  (eq. [B.10](#)) into **production technology** equation (eq. [B.8](#))

$$Y_t = \left( \int_0^1 \left[ \left( \frac{P_{j,t}}{P_t} \right)^{-\frac{1+\lambda^m}{\lambda^m}} Y_t \right]^{\frac{1}{1+\lambda^m}} d(j) \right)^{1+\lambda^m} \tag{B.11}$$

$$Y_t = Y_t \left( \int_0^1 \left[ \frac{P_{j,t}}{P_t} \right]^{\frac{-1}{\lambda^m}} dj \right)^{1+\lambda^m} \tag{B.12}$$

to obtain an equation for aggregate price index  $P_t$  ( $Y_t$  on both sides drops):

$$P_t = \left( \int_0^1 P_{j,t}^{\frac{-1}{\lambda^m}} dj \right)^{-\lambda^m} \tag{B.13}$$

## I. Price Setting

Following Calvo (1983) every firm faces a constant probability,  $1 - \xi_p$ , of re-optimizing its price to  $P_{j,t}^*$  any given period, whereas the non-reoptimizing firms index their prices for next period to past inflation  $\pi_{t-1}$  according to indexation rule  $P_{j,t} = P_{t-1}\pi_{t-1}^{\gamma_p}$ . For zero steady state inflation,  $\pi = 1$  or no indexation  $\gamma_p = 0$  price are held fixed  $P_{j,t} = P_{t-1}$ . Aggregate price evolve as:

$$P_t = \left[ (1 - \xi_p) P_t^{*\frac{-1}{\lambda^m}} + \xi_p (P_{t-1}\pi_{t-1}^{\gamma_p})^{\frac{-1}{\lambda^m}} \right]^{-\lambda^m} \quad (\text{B.14})$$

where  $P_{j,t} = P_{t-1}\pi_{t-1}^{\gamma_p}$  stands for the price of the retailers who are not re-optimizing, but indexing instead.

A share  $1 - \xi_p$  of retailers choose their price  $P_t^*$  at time  $t$  that maximizes the present value of future expected nominal profits while the rest of retailers index their price. After re-optimization with probability  $1 - \xi_p$  the retailer will not re-optimize next period and with probability  $(1 - \xi_p)^\tau$  the retailer will not re-optimize for  $\tau$  period from now. The real marginal cost to retailer is the price of wholesale good relative to retail price  $mc_t = \frac{P_t^w}{P_t}$  while the nominal marginal cost is  $MC_t = P_t mc_t = P_t^w$ . For each retailer  $j$  the objective is to choose the re-set price  $P_{j,t}^*$  that maximizes his discounted stream of profits:

$$\max_{P_{j,t}^*} E_t \int_{k=0}^{\infty} \xi_p^k D_{t,t+k} [(P_{j,t}^* - MC_{t+k}) Y_{j,t+k}] \quad (\text{B.15})$$

subject to demand function eq.B.10:  $Y_{j,t+k} = \left( \frac{P_{j,t}^*}{P_{t+k}} \right)^{-\frac{1+\lambda^m}{\lambda^m}} Y_{t+k}$ ,

where  $MC_{t+k} = P_{t+k} mc_{t+k}$  is the nominal marginal cost and  $D_{t,t+k} = \beta^k \frac{\Lambda_{t,t+k}}{P_{t+k}/P_t}$  is the marginal value of a dollar to the household, i.e. the nominal stochastic discount factor over the interval  $(t, t+k)$ .

With prices being indexed thereafter the demand function at any time  $t + k$  becomes:

$$Y_{j,t+k} = \left( \frac{P_{j,t}^*}{\bar{P}_{t+k}} \left( \frac{P_{t+k-1}}{\bar{P}_{t-1}} \right)^{\gamma_p} \right)^{-\frac{1+\lambda^m}{\lambda^m}} Y_{t+k}$$

or otherwise

$$Y_{j,t+k} = \left( \frac{P_{j,t}^*}{\bar{P}_{t+k}} Z_{t-1,t+k-1}^{\gamma_p} \right)^{-\frac{1+\lambda^m}{\lambda^m}} Y_{t+k}$$

where I denote

$$Z_{t-1,t+k-1} = \frac{P_{t+k-1}}{P_{t-1}} = \pi_t \pi_{t+1} \dots \pi_{t+k-1} = \prod_{i=1}^k \pi_{t+i-1} \quad (\text{B.16})$$

All firms that re-optimize their price at time  $t$  choose the same price  $P_{j,t} = P_t^*$

to maximize (after substituting the demand function and re-arranging):

$$\max_{P_t^*} E_t \int_{k=0}^{\infty} \xi_p^k D_{t,t+k} \left[ \left( P_t^* \left( \frac{P_{t+k-1}}{\bar{P}_{t-1}} \right)^{\gamma_p} - MC_{t+k} \right) \left( \frac{P_{j,t}^*}{\bar{P}_{t+k}} \left( \frac{P_{t+k-1}}{\bar{P}_{t-1}} \right)^{\gamma_p} \right)^{-\frac{1+\lambda^m}{\lambda^m}} Y_{t+k} \right]$$

$$\max_{P_t^*} E_t \int_{k=0}^{\infty} \xi_p^k D_{t,t+k} \left[ \left( P_t^* Z_{t-1,t+k-1}^{\gamma_p} - MC_{t+k} \right) \left( \frac{P_{j,t}^*}{\bar{P}_{t+k}} Z_{t-1,t+k-1}^{\gamma_p} \right)^{-\frac{1+\lambda^m}{\lambda^m}} Y_{t+k} \right]$$

$$\max_{P_t^*} E_t \int_{k=0}^{\infty} \xi_p^k D_{t,t+k} \left[ \left( P_t^* Z_{t-1,t+k-1}^{\gamma_p} - MC_{t+k} \right) \left( \frac{P_t^*}{\bar{P}_{t+k}} Z_{t-1,t+k-1}^{\gamma_p} \right)^{-\frac{1+\lambda^m}{\lambda^m}} Y_{t+k} \right]$$

$$\max_{P_t^*} E_t \int_{k=0}^{\infty} \xi_p^k D_{t,t+k} \left[ P_{t+k} \left( \frac{P_t^*}{\bar{P}_{t+k}} Z_{t-1,t+k-1}^{\gamma_p} \right)^{\frac{-1}{\lambda^m}} - MC_{t+k} \left( \frac{P_t^*}{\bar{P}_{t+k}} Z_{t-1,t+k-1}^{\gamma_p} \right)^{-\frac{1+\lambda^m}{\lambda^m}} Y_{t+k} \right]$$

This is a result of the indexing of prices by  $P_{\text{non},t+1} = P_t \pi_t^{\gamma_p}$  each consecutive period where  $P_{\text{non},t+1}$  is the price next period of those retailers who do not re-optimize. Price re-optimized at time  $t$  and indexed for  $k$  period (at time  $t + k$ ) becomes  $P_t^* \left( \frac{P_{t+k-1}}{\bar{P}_{t-1}} \right)^{\gamma_p}$  The FOC is:

$$0 = E_t \int_{k=0}^{\infty} \xi_p^k D_{t,t+k} \left[ \left( Z_{t-1,t+k-1}^{\gamma_p} - (1 + \lambda^m) \frac{1}{\bar{P}_t^*} MC_{t+k} \right) \left( \frac{P_t^*}{\bar{P}_{t+k}} Z_{t-1,t+k-1}^{\gamma_p} \right)^{-\frac{1+\lambda^m}{\lambda^m}} Y_{t+k} \right]$$

$$0 = E_t \int_{k=0}^{\infty} \xi_p^k D_{t,t+k} \left[ \left( Z_{t-1,t+k-1}^{\gamma_p} - (1 + \lambda^m) \frac{1}{p_t^*} mc_{t+k} P_{t+k} \right) Y_{j,t+k} \right]$$

where small letter  $mc_{t+k}$  is marginal cost in real terms. It follows that:

$$P_t^* = (1 + \lambda^m) \frac{E_t \int_{k=0}^{\infty} \xi_p^k D_{t,t+k} (mc_{t+k} P_{t+k} Y_{j,t+k})}{E_t \int_{k=0}^{\infty} \xi_p^k D_{t,t+k} (Z_{t-1,t+k-1}^{\gamma_p} Y_{j,t+k})}$$

After dividing both sides by  $P_t$ .

$$p_t^* = \frac{P_t^*}{P_t} = (1 + \lambda^m) \frac{E_t \int_{k=0}^{\infty} \xi_p^k D_{t,t+k} (mc_{t+k} \frac{P_{t+k}}{P_t} Y_{j,t+k})}{E_t \int_{k=0}^{\infty} \xi_p^k D_{t,t+k} (Z_{t-1,t+k-1}^{\gamma_p} Y_{j,t+k})}$$

Recall from B.16 that  $\frac{P_{t+k}}{P_t} = \frac{P_{t+k}}{P_t} = \pi_{t+1} \pi_{t+2} \dots \pi_{t+k} = \prod_{i=1}^k \pi_{t+i} = Z_{t,t+k}$

$$p_t^* = \frac{P_t^*}{P_t} = (1 + \lambda^m) \frac{E_t \int_{k=0}^{\infty} \xi_p^k D_{t,t+k} (mc_{t+k} Z_{t,t+k} Y_{j,t+k})}{E_t \int_{k=0}^{\infty} \xi_p^k D_{t,t+k} (Z_{t-1,t+k-1}^{\gamma_p} Y_{j,t+k})} = (1 + \lambda^m) \frac{J_t}{\bar{J}_t} \quad (\text{B.17})$$

## II. Recursive Derivation

Denote with  $\bar{J}_t$  the numerator and  $\bar{\bar{J}}_t$  the denominator in equation B.17.

Denote:  $Z_{t,t+k} = \frac{1}{X_{t,t+k}} = \frac{P_{t+k}}{P_t} = \pi_{t+1} \dots \pi_{t+k} = \prod_{i=1}^k \pi_{t+i} \quad \forall k > 0, \&$   
 $X_{t,t} = 1.$

### (a) Recursive derivation for the numerator

$$\bar{J}_t = \int_{k=0}^{\infty} \xi_p^k D_{t,t+k} Y_{j,t+k} mc_{t+k} Z_{t,t+k}$$

Plug:  $Y_{t+k} \left( \frac{P_t^*/P_t}{P_{t,t+k}/P_t} Z_{t-1,t+k-1}^{\gamma_p} \right)^{-\epsilon}$  instead of  $Y_{j,t+k}$

or otherwise:  $Y_{t+k} \left( \frac{P_t^*}{Z_{t,t+k}} Z_{t-1,t+k-1}^{\gamma_p} \right)^{-\epsilon}$  instead of  $Y_{j,t+k}$

$$\bar{J}_t = \int_{k=0}^{\infty} \xi_p^k D_{t,t+k} \left[ Y_{t+k} \left( \frac{P_t^*}{Z_{t,t+k}} Z_{t-1,t+k-1}^{\gamma_p} \right)^{-\epsilon} mc_{t+k} Z_{t,t+k} \right]$$

$$\bar{J}_t = Y_t (p_t^*)^{-\epsilon} mc_t + \int_{k=1}^{\infty} \xi_p^k D_{t,t+k} \left[ Y_{t+k} \left( \frac{P_t^*}{Z_{t,t+k}} Z_{t-1,t+k-1}^{\gamma_p} \right)^{-\epsilon} mc_{t+k} Z_{t,t+k} \right]$$

## B.1. Model Derivation

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where  $(p_t^*)^{-\epsilon} = \left(\frac{p_t^*}{Z_{t,t+0}} Z_{t-1,t+0-1}^{\gamma_p}\right)^{-\epsilon}$

$$\bar{J}_t = Y_t (p_t^*)^{-\epsilon} m c_t + \xi_p D_{t,t+1} \left(\frac{Z_{t-1,t}^{\gamma_p}}{Z_{t,t+1}}\right)^{-\epsilon} Z_{t,t+1} \dots$$

$$\dots \int_{k=1}^{\infty} \xi_p^{k-1} D_{t+1,t+1+k-1} \left[ Y_{t+1+k-1} \left(\frac{p_t^*}{Z_{t+1,t+1+(k-1)}} Z_{t,t+k-1}^{\gamma_p}\right)^{-\epsilon} m c_{t+1+(k-1)} Z_{t+1,t+1+(k-1)} \right]$$

Plug  $s = k - 1 = 0$ ,  $Z_{t-1,t} = \pi_t$ , or  $Z_{t,t+1} = \pi_{t+1}$ ,  $D_{t,t+1} = \frac{\Lambda_{t,t+1}}{\pi_{t+1}}$ .

$$\bar{J}_t = Y_t (p_t^*)^{-\epsilon} m c_t + \xi_p \frac{\Lambda_{t,t+1}}{\pi_{t+1}} \underbrace{\left(\frac{\pi_t^{\gamma_p}}{\pi_{t+1}}\right)^{-\epsilon}}_{\tilde{\pi}_{t+1}^\epsilon} \pi_{t+1} \dots$$

$$\dots \left(\frac{p_t^*}{p_{t+1}^*}\right)^{-\epsilon} \int_{s=0}^{\infty} \xi_p^s D_{t+1,t+1+s} \left[ \underbrace{Y_{t+1+s} \left(\frac{p_{t+1}^*}{Z_{t+1,t+1+s}} Z_{t,t+s}^{\gamma_p}\right)^{-\epsilon} m c_{t+1+s} Z_{t+1,t+1+s}}_{Y_{j,t+1+s}} \right]$$

$$\bar{J}_t = Y_t (p_t^*)^{-\epsilon} m c_t + \xi_p \Lambda_{t,t+1} \tilde{\pi}_{t+1}^\epsilon \left(\frac{p_t^*}{p_{t+1}^*}\right)^{-\epsilon} \underbrace{\int_{s=0}^{\infty} \xi_p^s D_{t+1,t+1+s} [Y_{j,t+1+s} m c_{t+1+s} Z_{t+1,t+1+s}]}_{\bar{J}_{t+1}}$$

Replace  $\beta \frac{u_{C_{t+1}}}{u_{C_t}} = \Lambda_{t,t+1}$

$$\bar{J}_t = Y_t (p_t^*)^{-\epsilon} m c_t + \xi_p \beta \frac{u_{C_{t+1}}}{u_{C_t}} \tilde{\pi}_{t+1}^\epsilon \left(\frac{p_t^*}{p_{t+1}^*}\right)^{-\epsilon} \bar{J}_{t+1}$$

$$\underbrace{\bar{J}_t (p_t^*)^\epsilon u_{C_t}}_{J_t} = Y_t u_{C_t} m c_t + \xi_p \tilde{\pi}_{t+1}^\epsilon \beta \underbrace{u_{C_{t+1}} (p_{t+1}^*)^\epsilon}_{J_{t+1}} \bar{J}_{t+1}$$

$$J_t = m c_t u_{C_t} Y_t + \xi_p \beta E_t \tilde{\pi}_{t+1}^\epsilon J_{t+1} \quad (\text{B.18})$$

$$J_t = m c_t u_{C_t} Y_t + \xi_p \beta E_t \tilde{\pi}_{t+1}^{\frac{1+\lambda^m}{\lambda^m}} J_{t+1} \quad (\text{B.19})$$

### (b) Recursive derivation for the denominator

## B.1. Model Derivation

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$$\bar{J}_t = \int_{k=0}^{\infty} \xi_p^k D_{t,t+k} Z_{t-1,t+k-1}^{\gamma_p} Y_{j,t+k}$$

Plug:  $Y_{t+k} \left( \frac{p_t^*/P_t}{p_{t,t+k}/P_t} Z_{t-1,t+k-1}^{\gamma_p} \right)^{-\epsilon}$  instead of  $Y_{j,t+k}$

or otherwise:  $Y_{t+k} (p_t^*/Z_{t,t+k} Z_{t-1,t+k-1}^{\gamma_p})^{-\epsilon}$  instead of  $Y_{j,t+k}$

Plug:  $Y_{t+k} (p_t^*/Z_{t,t+k} Z_{t-1,t+k-1}^{\gamma_p})^{-\epsilon}$  instead of  $Y_{j,t+k}$

$$\bar{J}_t = \underbrace{\xi_p^0 D_{t,t+0}}_1 Y_{j,t+0} + \int_{k=1}^{\infty} \xi_p^k D_{t,t+k} Z_{t-1,t+k-1}^{\gamma_p} \left[ Y_{t+k} (p_t^*/Z_{t,t+k} Z_{t-1,t+k-1}^{\gamma_p})^{-\epsilon} \right]$$

$$\bar{J}_t = Y_{j,t} + \xi_p D_{t,t+1} Z_{t-1,t}^{\gamma_p} \left( \frac{Z_{t-1,t}^{\gamma_p}}{Z_{t,t+1}} \right)^{-\epsilon} \dots$$

$$\dots \int_{k=1}^{\infty} \xi_p^{k-1} D_{t+1,t+1+(k-1)} Z_{t,t+k-1}^{\gamma_p} \left[ Y_{t+1+k-1} \left( p_t^* \frac{Z_{t,t+k-1}^{\gamma_p}}{Z_{t+1,t+1+k-1}} \right)^{-\epsilon} \right]$$

Plug:

$$D_{t,t+1} = \frac{\Lambda_{t,t+1}}{\pi_{t+1}},$$

$$Z_{t-1,t} = \pi_t \quad () Z_{t,t+1} = \pi_{t+1},$$

$$s = k - 1 = 0.$$

Multiply and divide by :  $p_{t+1}^*$ .

$$\bar{J}_t = Y_{j,t} + \xi_p \frac{\Lambda_{t,t+1}}{\pi_{t+1}} \pi_t^{\gamma_p} \left( \frac{\pi_t^{\gamma_p}}{\pi_{t+1}} \right)^{-\epsilon} \left( \frac{p_t^*}{p_{t+1}^*} \right)^{-\epsilon} \int_{s=0}^{\infty} \xi_p^s D_{t+1,t+1+s} Z_{t,t+s}^{\gamma_p} \left[ Y_{t+1+s} \left( \frac{p_{t+1}^* Z_{t,t+s}^{\gamma_p}}{Z_{t+1,t+1+s}} \right)^{-\epsilon} \right]$$

Plug back:  $Y_{j,t+1+s} = Y_{t+1+s} \left( \frac{p_{t+1}^*}{Z_{t+1,t+1+s}} Z_{t,t+s}^{\gamma_p} \right)^{-\epsilon}$ .

$$\bar{J}_t = Y_{j,t} + \xi_p \Lambda_{t,t+1} \left( \frac{\pi_t^{\gamma_p}}{\pi_{t+1}} \right)^{1-\epsilon} \left( \frac{p_t^*}{p_{t+1}^*} \right)^{-\epsilon} \underbrace{\int_{s=0}^{\infty} \xi_p^s D_{t+1,t+1+s} Z_{t,t+s}^{\gamma_p} Y_{j,t+1+s}}_{\bar{J}_{t+1}}$$

Replace:  $\tilde{\pi}_{t+1} = \frac{\pi_t^{\gamma_p}}{\pi_{t+1}}$

$$\beta \frac{u_{C_{t+1}}}{u_{C_t}} = \Lambda_{t,t+1} \quad Y_{j,t} = Y_t \left( \frac{p_t^*}{p_{t+0}} Z_{t,t+0}^{\gamma_p} \right)^{-\epsilon} = Y_t p_t^{*\epsilon}$$

$$\bar{J}_t = Y_t p_t^{*\epsilon} + \xi_p \beta \frac{u_{C_{t+1}}}{u_{C_t}} (\tilde{\pi}_{t+1})^{1-\epsilon} \left( \frac{p_t^*}{p_{t+1}^*} \right)^{-\epsilon} \bar{J}_{t+1}$$

$$\underbrace{\bar{J}_t u_{C_t} p_t^{*\epsilon}}_{J_t} = Y_t u_{C_t} + \xi_p \tilde{\pi}_{t+1}^{1-\epsilon} \beta \underbrace{u_{C_{t+1}} (p_{t+1}^*)^\epsilon}_{J_{t+1}} \bar{J}_{t+1}$$

$$J_t = Y_t u_{C_t} + \xi_p \tilde{\pi}_{t+1}^{1-\epsilon} \beta J_{t+1}$$

Finally,  $p_{i,t}^* = \frac{\bar{J}_t}{\bar{J}_t} = \frac{J_t / u_{C_t} p_t^{*\epsilon}}{J_t / u_{C_t} p_t^{*\epsilon}} = \frac{J_t}{J_t}$

since,  $\bar{J}_t = \frac{J_t}{u_{C_t} p_t^{*\epsilon}}$  and  $\bar{J}_t = \frac{J_t}{u_{C_t} p_t^{*\epsilon}}$

$$J_t = Y_t u_{C_t} + \xi_p \beta \tilde{\pi}_{t+1}^{\epsilon-1} J_{t+1} \tag{B.20}$$

$$J_t = Y_t u_{C_t} + \xi_p \beta \tilde{\pi}_{t+1}^{\frac{1}{\lambda^m}} J_{t+1} \tag{B.21}$$

### III. Evolution of Aggregate Prices

In a continuum of firms or goods the general aggregation for the price index is :

$$P_t = \left( \int_0^1 P_{j,t}^{-\frac{1}{\lambda^m}} dj \right)^{-\lambda^m}$$

Since the probability of re-optimizing is  $1 - \xi_p$  it implies that:

$$P_t = \left( \int_0^{\xi_p} (P_{j,t})^{-\frac{1}{\lambda^m}} dj + \int_{\xi_p}^1 (P_{j,t}^*)^{-\frac{1}{\lambda^m}} dj \right)^{-\lambda^m}$$

At any time period  $1 - \xi_p$  share of firms re-optimize their price  $P_{\text{reset},t} = P_t^*$  while  $\xi_p$  portion of firms index their price it to past inflation  $P_{\text{index},t} = P_{t-1} \pi_{t-1}^{\gamma_p}$ .

The above equation can be further written as:

$$P_t = \left[ (1 - \xi_p) P_t^{*\epsilon} + \xi_p (P_{t-1} \pi_{t-1}^{\gamma_p})^\epsilon \right]^{-\frac{1}{\lambda^m}} \tag{B.22}$$

where  $P_{\text{indexer},t} = P_{t-1}\pi_{t-1}^{\gamma_p}$  stands for the price of the retailers who are not re-optimizing, but indexing instead. After dividing by  $P_t$  it is re-written as:

$$1 = \xi_p \left( \frac{\pi_{t-1}^{\gamma_p}}{\pi_t} \right)^{-\frac{1}{\lambda^m}} + (1 - \xi_p) p_t^*{}^{-\frac{1}{\lambda^m}} \quad (\text{B.23})$$

Making use of equation ??, the above equation relates inflation rate to aggregate variables only:

$$p_t^* = \left[ \frac{1 - \xi_p (\tilde{\pi}_t)^{\frac{1}{\lambda^m}}}{1 - \xi_p} \right]^{-\lambda^m} = (1 + \lambda^m) \frac{J_t}{JJ_t} \quad (\text{B.24})$$

where  $\tilde{\pi}_t = \frac{\pi_t}{\pi_{t-1}^{\gamma_p}}$

$$1 = \xi_p (\tilde{\pi}_t)^{\frac{1}{\lambda^m}} + (1 - \xi_p) p_t^*{}^{-\frac{1}{\lambda^m}} \quad (\text{B.25})$$

## IV. Inefficiency of Price Dispersion

Given three equations:

a) eq. B.10:  $Y_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\frac{1+\lambda^m}{\lambda^m}} Y_t$

b) eq. 3.16:  $Y_{j,t}^w = (A_t H_{j,t})^\alpha K_{j,t-1}^{1-\alpha} = A_t H_{j,t} \frac{K_{j,t-1}}{Y_{j,t}^w} \frac{1-\alpha}{\alpha}$

c) eq. :  $Y_{j,t} = (1 - c) Y_{j,t}^w$  where  $c = \frac{1}{\epsilon}$

Show that :  $Y_t = \frac{1-c}{\Delta_t} Y_t^w$

Proof:

Start by integrating eq. B.10 across all retailers:

a):  $\int_0^1 Y_{j,t} dj = Y_t \underbrace{\int_0^1 \left( \frac{P_{j,t}}{P_t} \right)^{-\frac{1+\lambda^m}{\lambda^m}} dj}_{\text{denote: } \Delta_t}$

and integrate over  $j$ .

Plug c):  $\int_0^1 (1 - c) Y_{j,t}^w dj = Y_t \Delta_t$

$$\begin{aligned} \text{Plug b): } \quad (1-c)A_t \int_0^1 H_{j,t} \frac{K_{j,t-1}}{Y_{j,t}^w} \frac{1-\alpha}{\alpha} dj &= Y_t \Delta_t \\ (1-c)A_t \frac{K_{t-1}}{Y_t^w} \frac{1-\alpha}{\alpha} \int_0^1 H_{j,t} dj &= Y_t \Delta_t \end{aligned}$$

$$\begin{aligned} \text{Note that: } \quad \frac{K_{j,t-1}}{Y_{j,t}^w} &= \frac{K_{t-1}}{Y_t^w} \quad \forall j \text{ and } \int_0^1 H_{j,t} dj = H_t \\ (1-c)A_t \frac{K_{t-1}}{Y_t^w} \frac{1-\alpha}{\alpha} H_t &= Y_t \Delta_t \end{aligned}$$

$$Y_t = \frac{1-c}{\Delta_t} Y_t^w \quad (\text{B.26})$$

## V. Deriving Expression for Evolution of Price Dispersion $\Delta_{t+1}$

From integration over  $j$  of retailers demand equation eq. B.10 :

$$\text{a): } \quad \int_0^1 Y_{j,t} dj = Y_t \underbrace{\int_0^1 \left( \frac{P_{j,t}}{P_t} \right)^{-\frac{1+\lambda^m}{\lambda^m}} dj}_{\text{denote: } \Delta_t}$$

I obtained (shifted now at time  $t+1$ ):

$$\begin{aligned} \Delta_{t+1} &= \int_0^1 \left( \frac{P_{t+1}(m)}{P_{t+1}} \right)^{-\frac{1+\lambda^m}{\lambda^m}} \\ \Delta_{t+1} &= \int_0^{\xi_p} \left( \frac{P_{t+1}(m)}{P_{t+1}} \right)^{-\frac{1+\lambda^m}{\lambda^m}} + \int_{\xi_p}^1 \left( \frac{P_{*t+1}(m)}{P_{t+1}} \right)^{-\frac{1+\lambda^m}{\lambda^m}} \\ \Delta_{t+1} &= \int_0^{\xi_p} \left( \frac{P_t(m) \pi_t^{\gamma_p} P_t}{P_t P_{t+1}} \right)^{-\frac{1+\lambda^m}{\lambda^m}} + (1-\xi_p) (p_{*t+1})^{-\frac{1+\lambda^m}{\lambda^m}} \\ \Delta_{t+1} &= \left( \frac{P_t \pi_t^{\gamma_p}}{P_{t+1}} \right)^{-\epsilon} \int_0^{\xi_p} \left( \frac{P_t(m)}{P_t} \right)^{-\frac{1+\lambda^m}{\lambda^m}} + (1-\xi_p) (p_{*t+1})^{-\frac{1+\lambda^m}{\lambda^m}} \\ \Delta_{t+1} &= \left( \frac{\pi_{t+1}}{\pi_t^{\gamma_p}} \right)^{\frac{1+\lambda^m}{\lambda^m}} \xi_p \Delta_t + (1-\xi_p) (p_{*t+1})^{-\frac{1+\lambda^m}{\lambda^m}} \\ \Delta_{t+1} &= (\tilde{\pi}_{t+1})^{\frac{1+\lambda^m}{\lambda^m}} \xi_p \Delta_t + (1-\xi_p) (p_{*t+1})^{-\frac{1+\lambda^m}{\lambda^m}} \end{aligned}$$

Plug eq. B.17

$$\Delta_{t+1} = (\tilde{\pi}_{t+1})^{\frac{1+\lambda^m}{\lambda^m}} \xi_p \Delta_t + (1-\xi_p) \left( (1+\lambda^m) \frac{J_t}{JJ_{tt}} \right)^{-\frac{1+\lambda^m}{\lambda^m}} \quad (\text{B.27})$$

This is the end of derivations related to **Retailer's Problem**.

### B.1.3 Capital Producer Problem

Capital producers are perfectly competitive firms. The capital producer engages in two activities:

- produces new investment goods using retail consumption goods  $I_t$  as an input and purchased at unit price. The new investment good is sold back to wholesale producer for the relative price  $Q_t$ ,
- replenishes the depreciated capital stock of wholesale firms purchased at price  $Q_t$  and reselling it for  $Q_t$ .

Production of investment good is subject to:

- IAC (Investment adjustment costs) as a function of the growth rate of investment  $(1 - S_{(X_t)})$ . CEE emphasize that this functional form can generate a hump-shaped response of aggregate investment to MP shock consistent with implications of a VAR.
- investment specific technology shock  $\Upsilon_t$ .

Capital producers problem:

$$\text{Max}_{I_t} \sum_{\tau=0}^{\infty} \Lambda_{t,t+\tau} [Q_{t+\tau} \Upsilon_{t+\tau} (1 - S(I_{t+\tau}/I_{t-1+\tau})) I_{t+\tau} - I_{t+\tau}] \quad (\text{B.28})$$

where  $\Lambda_{t,t+\tau}$  is the real SDF. Profit maximizing conditions imply:

$$0 = Q_t \Upsilon_t \left( 1 - S(X_t) + 0 - I_t \frac{\partial S(X_t)}{\partial I_t} \right) - 1 + E_t \left[ \Lambda_{t,t+1} Q_{t+1} \Upsilon_{t+1} \frac{\partial S(X_{t+1})}{\partial I_t} - 0 \right] \quad (\text{B.29})$$

where  $X_t = \frac{I_t}{I_{t-1}}$ . The capital accumulation process is:

$$1 = Q_t \gamma_t (1 - S(X_t) - X_t S'(X_t)) + E_t \left[ \Lambda_{t,t+1} Q_{t+1} \gamma_{t+1} X_{t+1}^2 S'(X_{t+1}) \right] \quad (\text{B.30})$$

Certain functional forms of adjustment costs allow for these costs to disappear in the long run [Smets and Wouters \(2007\)](#). An adjustment cost function of the form:

$$S(X_t) = \phi_x (X_t - 1)^2 \quad (\text{B.31})$$

results in no adjustment costs along the balanced growth path implying  $X_t = 1$ ,  $S(1) = S'(1) = 0$ . The property leaves the steady state of the model unchanged.

#### B.1.4 Wholesale Firm Problem

Representative wholesale firm will aim to maximize:

$$E_t \sum_{si=0}^{\infty} \Lambda_{t,t+si} \left( \frac{P_{t+si}^w}{P_{t+si}} Y_{j,t+si}^w + Q_{t+si} (1 - \delta) K_{j,t-1+si} - w_{t+si} H_{j,t+si} - Q_{t-1+si} R_{t+si}^k K_{j,t-1+si} \right) \quad (\text{B.32})$$

s.t. production technology eq.(3.16):  $Y_{j,t+1}^w = (A_{j,t+1} H_{j,t+1})^\alpha K_{j,t}^{1-\alpha}$

where  $w_t$  and  $\frac{P_t^w}{P_t}$  are real wage and the selling price of wholesale output in real terms at time  $t$  respectively.

Labor demand:

$$w_{t+1} = \alpha \frac{P_{t+1}^w}{P_{t+1}} \frac{Y_{j,t+1}^w}{H_{j,t+1}} \quad (\text{B.33})$$

Capital demand:

$$R_{t+1}^k = \frac{(1 - \alpha) \frac{P_{t+1}^w}{P_{t+1}} \frac{Y_{j,t+1}^w}{K_{j,t}} + Q_{t+1}(1 - \delta)}{Q_t} \quad (\text{B.34})$$

### B.1.5 Financial Contract

#### A. Borrower's Financial Contract:

I have denoted by  $X_t^{eq}$  the ratio of equity securities to the sum of firm net worth and equity securities.

$$X_{j,t}^{eq} = \frac{Q_t S_{j,t}}{Q_t S_{j,t} + N_{j,t}^E} = \frac{\kappa_t(\phi_{j,t} - 1)}{\kappa_t(\phi_{j,t} - 1) + 1} \quad (\text{B.35})$$

where  $\kappa_t = \kappa = \frac{Q_t S_{j,t}}{Q_t S_{j,t} + L_t}$  since  $\kappa_t$  is assumed a fixed parameter.

The cut-off point of the idiosyncratic shock should satisfy equation: (3.21).

Following that the payoff matrix from the

$$\bar{\Psi}_{t+1}^j = \frac{Z_{j,t}^L L_{j,t}}{R_{t+1}^k Q_t K_{j,t}} \quad (\text{B.36})$$

Table B.1: Payoff Table

State	No Default: $(\psi_{t+1}^j \geq \bar{\Psi}_{t+1}^j)$	Default: $(\psi_{t+1}^j < \bar{\Psi}_{t+1}^j)$
Firm Payoff	$(\psi_{t+1}^j - \bar{\Psi}_{t+1}^j) R_{t+1}^k K_{j,t} Q_t (1 - X_t^{eq})$	0
Bank Payoff	$\bar{\Psi}_{t+1}^j Q_t K_{j,t} R_{t+1}^k + (\psi_{t+1}^j - \bar{\Psi}_{t+1}^j) R_{t+1}^k K_{j,t} Q_t X_t^{eq}$	$(1 - \mu) \psi_{t+1}^j Q_t K_{j,t} R_{t+1}^k$

The payoff structure implies that

Following this pay-off table the gross payoffs to the bank and the firm both depend on the distribution of idiosyncratic shock  $[\psi_{t+1}^j]$ .

- In case of no default, the firm pays first the loan and the remaining profit is shared between the firm and the holder of equity securities based on shares  $[1 - X_{j,t}^{eq}]$  and  $[X_{j,t}^{eq}]$  respectively.
- In case of no default, the bank earns a fixed payoff based on the loan contract and the share of remaining profits as a security holder.

- In case of default, the firm earns zero payoff while the debt-holding bank seizes all the assets following the realization of idiosyncratic shock. Bank as a security-holder earns zero payoff as well.

Now I define the payoffs to the entrepreneur, to security holder (bank) and to debt-holder (bank again).

1. The **bank payoff on equity securities** will deliver an average return  $R_{t+1}^S$  on equity portfolio  $Q_t S_t$ :

$$E_t R_{t+1}^k K_{j,t} Q_t \chi_{j,t}^{eq} \left( \int_{\bar{\psi}_{t+1}^j}^{\infty} (\psi_{t+1}^j - \bar{\psi}_{t+1}^j) dF_{\psi^j} \right) = R_{t+1}^S Q_t S_t \quad (B.37)$$

2. The **bank payoff on loan portfolio** will deliver an average return  $R_{t+1}^L$  on loan portfolio  $L_t$  :

$$E_t \left[ Q_t K_{j,t} R_{t+1}^k \int_{\bar{\psi}_{t+1}^j}^{\infty} \bar{\psi}_{t+1}^j dF_{\psi^j} + (1 - \mu) Q_t K_{j,t} R_{t+1}^k \int_0^{\bar{\psi}_{t+1}^j} \psi_{t+1}^j dF_{\psi^j} \right] = R_{t+1}^L L_t \quad (B.38)$$

3. The **entrepreneurs payoff** from the contract (from table [B.1 on the preceding page](#)) is:

$$V_{j,t+1}^E = E_t R_{t+1}^k K_{j,t} Q_t (1 - \chi_{j,t}^{eq}) \left( \int_{\bar{\psi}_{t+1}^j}^{\infty} (\psi_{t+1}^j - \bar{\psi}_{t+1}^j) dF_{\psi^j} \right) \quad (B.39)$$

$$\text{Given } E_t (\psi_{t+1}^j) = 1 = \int_0^{\bar{\psi}_{t+1}^j} \psi_{t+1}^j dF_{\psi^j} + \int_{\bar{\psi}_{t+1}^j}^{\infty} \bar{\psi}_{t+1}^j dF_{\psi^j}$$

I define  $\Gamma_{\bar{\psi}_{t+1}^j}$ ,  $G_{\bar{\psi}_{t+1}^j}$  and probability of default on loan  $p_{\bar{\psi}_{t+1}^j}$  :

$$\Gamma_{\bar{\psi}_{t+1}^j} = \bar{\psi}_{t+1}^j \int_{\bar{\psi}_{t+1}^j}^{\infty} dF_{\psi^j} + \int_0^{\bar{\psi}_{t+1}^j} \psi_{t+1}^j dF_{\psi^j} \quad (B.40)$$

$$G_{\bar{\Psi}_{t+1}^j} = \int_0^{\bar{\Psi}_{t+1}^j} \psi_{t+1}^j dF_{\psi^j} \quad (\text{B.41})$$

$$p_{(\bar{\Psi}_{t+1}^j)} = \int_0^{\bar{\Psi}_{t+1}^j} dF_{\psi^j} \quad (\text{B.42})$$

The three are related as in following equation:

$$\Gamma_{\bar{\Psi}_{t+1}^j} = \left(1 - p_{\bar{\Psi}_{t+1}^j}\right) \bar{\Psi}_{t+1}^j + G_{\bar{\Psi}_{t+1}^j} \quad (\text{B.43})$$

### B. Lender's One-Period Participation Constraint

Making use of the notations above I rewrite equations B.37 and B.38 :

- The average return on loan portfolio  $R_{t+1}^L L_{j,t}$  will be equal to the payoff to bank from loan contract (equation B.44);
- The average return on securities  $R_{t+1}^S Q_t S_{vt}$  will be equal to the payoff earned from securities for all possible realizations of  $\psi_{t+1}^j$  (equation B.45).

$$E_t R_{t+1}^L L_{j,t} = E_t \left[ \Gamma_{\bar{\Psi}_{t+1}^j} - \mu G_{\bar{\Psi}_{t+1}^j} \right] Q_t K_{j,t} R_{t+1}^k \quad (\text{B.44})$$

$$E_t R_{t+1}^k Q_t K_{j,t} \left(1 - \Gamma_{\bar{\Psi}_{t+1}^j}\right) X_{j,t}^{eq} = R_{t+1}^S Q_t S_{j,t} \quad (\text{B.45})$$

Bank payoff should satisfy the **one period participation constraint** (eq. ?? on page ??):

$$E_t \left[ R_{t+1}^L L_{j,t} + R_{t+1}^S Q_t S_{j,t} \right] \geq E_t A_{j,t}^B R_{t+1}^B \quad (\text{B.46})$$

where  $R_{t+1}^L L_{j,t}$  and  $R_{t+1}^S Q_t S_{j,t}$  are given by equations B.44 and B.45 respectively. I plug the latter and obtain

$$\mathbb{E}_t \left[ R_{t+1}^k Q_t K_{j,t} \left( \Gamma_{\Psi_{t+1}^j} - \mu G_{\Psi_{t+1}^j} \right) + R_{t+1}^k K_{j,t} Q_t X_{j,t}^{eq} \left( 1 - \Gamma_{\Psi_{t+1}^j} \right) \right] \geq \mathbb{E}_t A_{j,t}^B R_{t+1}^B \quad (\text{B.47})$$

In terms of entrepreneurs leverage  $\phi_{j,t}^E$  the bank **one period constraint** constraint is:

$$\mathbb{E}_t \left[ R_{t+1}^k \phi_{j,t}^E \left( \Gamma_{\Psi_{t+1}^j} - \mu G_{\Psi_{t+1}^j} \right) + R_{t+1}^k \phi_{j,t}^E X_{j,t}^{eq} \left( 1 - \Gamma_{\Psi_{t+1}^j} \right) \right] \geq \mathbb{E}_t R_{t+1}^B (\phi_{j,t}^E - 1) \quad (\text{B.48})$$

where  $\phi_{j,t}^E = \frac{Q_t K_{j,t}}{N_{j,t}^E}$  is firm leverage .

### C. Firm's Value Function

I re-write the **Firm Value Function** (equation B.39) in terms of definitions in equations B.40 through B.43.

$$V_{j,t+1}^E = \mathbb{E}_t R_{t+1}^k K_{j,t} Q_t \left( 1 - X_{j,t}^{eq} \right) \left( 1 - \Gamma_{\Psi_{t+1}^j} \right) \quad (\text{B.49})$$

In terms of firm leverage  $\phi_{j,t}^E$  the entrepreneur maximizes his value function :

$$\frac{V_{j,t+1}^E}{N_{j,t}^E} = \mathbb{E}_t \left[ R_{t+1}^k \phi_{j,t}^E \left( 1 - X_{j,t}^{eq} \right) \left( 1 - \Gamma_{\Psi_{t+1}^j} \right) \right] \quad (\text{B.50})$$

subject to bank **one period constraint** constraint equation B.48 <sup>2</sup>.

---

<sup>2</sup> Since  $N_{j,t}^E$  is the net worth from the previous period, maximization of  $V_{j,t+1}^E$  and  $\frac{V_{j,t+1}^E}{N_{j,t}^E}$  is equivalent. Alternatively I can write the Lagrangean in terms of  $Q_t K_{j,t}$  and then divide both the firm value function and the bank participation constraint by net worth of firm.

### The Lagrangean and FOC

I write the Lagrangian equation :

$$\begin{aligned} \ell = & \mathbb{E}_t \left[ R_{t+1}^k K_{j,t} Q_t \left( 1 - X_{j,t}^{eq} \right) \left( 1 - \Gamma_{\bar{\Psi}_{t+1}^j} \right) \right] + \dots \\ & \dots + \lambda_{j,t+1}^E \left[ R_{t+1}^k Q_t K_{j,t} \left( \Gamma_{\bar{\Psi}_{t+1}^j} - \mu G_{\bar{\Psi}_{t+1}^j} \right) + R_{t+1}^k K_{j,t} Q_t X_{j,t}^{eq} \left( 1 - \Gamma_{\bar{\Psi}_{t+1}^j} \right) - A_{j,t}^B R_{t+1}^B \right] \end{aligned}$$

After dividing the value function and the bank **one period constraint** by  $N_t^E$  and writing in terms of  $\phi_{j,t}^E$  I get:

$$\begin{aligned} \ell = & \mathbb{E}_t \left[ R_{t+1}^k \phi_{j,t}^E \left( 1 - \Gamma_{\bar{\Psi}_{t+1}^j} \right) \left( 1 - X_{j,t}^{eq} \right) \right] + \dots \\ & \dots + \lambda_{j,t+1}^E \left[ R_{t+1}^k \phi_{j,t}^E \left( \Gamma_{\bar{\Psi}_{t+1}^j} - \mu G_{\bar{\Psi}_{t+1}^j} \right) + R_{t+1}^k \phi_{j,t}^E X_{j,t}^{eq} \left( 1 - \Gamma_{\bar{\Psi}_{t+1}^j} \right) - (\phi_{j,t}^E - 1) R_{t+1}^B \right] \end{aligned}$$

The FOCs are:

**foc 1:**  $\bar{\Psi}_{t+1}^j$ :

$$\begin{aligned} 0 = & \mathbb{E}_t \left( 1 - X_{j,t}^{eq} \right) \left( -\Gamma'_{\bar{\Psi}_{t+1}^j} \right) + \lambda_{j,t+1}^E \left[ \left( \Gamma'_{\bar{\Psi}_{t+1}^j} - \mu G'_{\bar{\Psi}_{t+1}^j} \right) + \left( -\Gamma'_{\bar{\Psi}_{t+1}^j} \right) X_{j,t}^{eq} \right] \\ & \mathbb{E}_t \lambda_{j,t+1}^E = \mathbb{E}_t \frac{\Gamma'_{\bar{\Psi}_{t+1}^j} \left( 1 - X_{j,t}^{eq} \right)}{\Gamma'_{\bar{\Psi}_{t+1}^j} \left( 1 - X_{j,t}^{eq} \right) - \mu G'_{\bar{\Psi}_{t+1}^j}} \end{aligned} \quad (\text{B.51})$$

**foc 2:**  $\phi_{j,t+1}^E$ :

$$\begin{aligned} 0 = & \mathbb{E}_t \left[ R_{t+1}^k \left( 1 - \Gamma_{\bar{\Psi}_{t+1}^j} \right) \left( 1 - X_{j,t}^{eq} - \phi_{j,t}^E X_{\phi_{j,t}}^{eq'} \right) \right] + \\ & + \lambda_{j,t+1}^E \left[ R_{t+1}^k \left( \Gamma_{\bar{\Psi}_{t+1}^j} - \mu G_{\bar{\Psi}_{t+1}^j} \right) + R_{t+1}^k \left( 1 - \Gamma_{\bar{\Psi}_{t+1}^j} \right) \left( X_{j,t}^{eq} + \phi_{j,t}^E X_{\phi_{j,t}}^{eq'} \right) - R_{t+1}^B \right] \end{aligned}$$

Given the  $\lambda_{j,t+1}^E$  the second FOC gives a relationships between the premium and the threshold level of idiosyncratic shock.

$$\mathbb{E}_t \frac{R_{t+1}^k}{R_{t+1}^B} = \mathbb{E}_t \frac{\lambda_{j,t+1}^E}{\left( 1 - \Gamma_{\bar{\Psi}_{t+1}^j} \right) \left( 1 - X_{j,t}^{eq} - \phi_{j,t}^E X_{\phi_{j,t}}^{eq'} \right) + \lambda_{j,t+1}^E \left[ \Gamma_{\bar{\Psi}_{t+1}^j} - \mu G_{\bar{\Psi}_{t+1}^j} + \left( 1 - \Gamma_{\bar{\Psi}_{t+1}^j} \right) \left( X_{j,t}^{eq} + \phi_{j,t}^E X_{\phi_{j,t}}^{eq'} \right) \right]} \quad (\text{B.52})$$

**foc3:**  $\lambda_{t+1}^E$ :

$$0 = \left[ R_{t+1}^k \phi_{j,t}^E \left( \Gamma_{\bar{\psi}_{t+1}}^j - \mu G_{\bar{\psi}_{t+1}}^j \right) + R_{t+1}^k \phi_{j,t}^E \left( 1 - \Gamma_{\bar{\psi}_{t+1}}^j \right) X_t^{eq} - (\phi_{j,t}^E - 1) R_{t+1}^B \right]$$

and after rearranging I obtain:

$$E_t R_{t+1}^B = E_t R_{t+1}^k \frac{\phi_{j,t}^E}{\phi_{j,t}^E - 1} \left( \Gamma_{\bar{\psi}_{t+1}}^j - \mu G_{\bar{\psi}_{t+1}}^j + (1 - \Gamma_{\bar{\psi}_{t+1}}^j) X_t^{eq} \right) \quad (B.53)$$

Aggregation involves the assumption that entrepreneurs exit with probability  $1 - \sigma^E$ <sup>3</sup>. They transfer a proportion  $\xi^E/(1 - \sigma^E)$  to new entrants and consume the rest of their accumulated wealth. Aggregate net worth will be net worth of surviving entrepreneurs  $\sigma^E V_{t+1}^E$  plus the fraction  $\xi^E/(1 - \sigma^E)$  of the wealth of exiting ones  $(1 - \sigma^E) V_{t+1}^E$  transferred to new entrants.

$$N_t^E = (\sigma^E + \xi_E) V_t^E \quad (B.54)$$

The remaining equity value of exiting firms is consumed:

$$\Pi_t^E = (1 - \sigma^E - \xi_E) V_t^E \quad (B.55)$$

Aggregating across wholesale producers yields the output function:

$$Y_t^w = (A_t H_t)^\alpha K_{t-1}^{1-\alpha} \quad (B.56)$$

From foc3 I can obtain an expression for the average return  $R_t^B$ .

$$E_t R_{t+1}^B = E_t R_{t+1}^k \frac{\phi_t^E}{\phi_t^E - 1} \left( \Gamma_{\bar{\psi}_{t+1}} - \mu G_{\bar{\psi}_{t+1}} + (1 - \Gamma_{\bar{\psi}_{t+1}}) X_t^{eq} \right) \quad (B.57)$$

<sup>3</sup> Such assumption avoids accumulation of capital by firms so they become self financed.

Finally, a small share of the rent on capital is lost on monitoring of the wholesale firms business by the bank:

$$\text{Monitoring Cost} = E_t \mu G_{\bar{\Psi}_{t+1}} R_{t+1}^k Q_t K_t \quad (\text{B.58})$$

### B.1.6 Bank Problem

This section is similar to [Gertler and Karadi \(2011\)](#) (hereby GK2011). Having one more financing instrument makes up for few differences. In this Appendix I will:

- (a) set up a one-to-one relationship between leverage and excess return on assets for any fixed moral hazard parameter  $\Theta_B$ ,
- (b) derive expression for these excess returns, and
- (c) show that banks' choice on leverage and excess returns does not depend on bank specific factors, therefore allowing for aggregation.

The value of the bank is homogenous to degree one in net worth and given the linear relationship in the balance sheet equation [3.40](#), then  $V_{i,t}^B$  should be homogenous of degree one in  $B_{i,t}$ ,  $A_{i,t}^B$  as well as  $L_{i,t}$  and  $S_{i,t}$ .

$$V_{i,t}^B (L_{i,t}, B_{i,t}, S_{i,t}) = v_{i,t}^L L_{i,t} + v_{i,t}^S Q_t S_{i,t} - v_{i,t}^B B_{i,t} \quad (\text{B.59})$$

where  $Q_t S_{i,t} = \kappa A_{i,t}$  and  $L_{i,t} = (1 - \kappa) A_{i,t}^B$ .  $v_{i,t}^L$ ,  $v_{i,t}^S$  and  $v_{i,t}^B$  are the marginal values of respective asset as of end of period. Using balance sheet equation [3.38](#) the value function is:

$$V_{i,t}^B (L_{i,t}, N_{i,t}, S_{i,t}) = \mu_{i,t}^L L_{i,t} + \mu_{i,t}^S Q_t S_{i,t} + v_{i,t}^B N_{i,t}^B \quad (\text{B.60})$$

Banker maximizes  $V_{i,t}^B (L_{i,t}, B_{i,t}, S_{i,t})$  subject to:

- the incentive constraint eq. 3.42:  $V_{i,t}^B \geq \Theta A_{i,t}$

- equation B.60:  $V_{i,t}^B = \mu_{i,t}^L L_{i,t} + \mu_{i,t}^S Q_t S_{i,t} + v_{i,t}^B N_{i,t}^B$

The Lagrangean  $\ell = V_{i,t}^B + \lambda_t^B (V_{i,t}^B - \Theta_B(A_{i,t}^B))$  becomes:

$$\ell = \left( \mu_{i,t}^L \phi_{i,t}^B (1 - \kappa) + \mu_{i,t}^S \phi_{i,t}^B \kappa + v_{i,t}^B \right) (1 + \lambda_{i,t}^B) - \lambda_{i,t}^B \Theta_B \phi_{i,t}^B \quad (\text{B.61})$$

$\lambda_{i,t}^B > 0$  if the constraint (IC) binds, or  $\lambda_{i,t}^B = 0$  if it does not.

First order conditions with respect ( $\phi_{i,t}$ ,  $\kappa$  and  $\lambda_{i,t}$  KT1 and KT2) when IC constraint binds are:

foc 1.  $\phi_{i,t}^B$ :

$$0 = (1 + \lambda_{i,t}^B) (\mu_{i,t}^L (1 - \kappa) + (\mu_{i,t}^L \kappa)) - \lambda_{i,t}^B \Theta_B \quad (\text{B.62})$$

foc 2.  $\lambda_{i,t}$ :

$$0 = \mu_{i,t}^L \phi_{i,t}^B (1 - \kappa) + \mu_{i,t}^S \phi_{i,t}^B \kappa + v_{i,t}^B - \Theta_B \phi_{i,t}^B \quad (\text{B.63})$$

KT2:

$$\lambda_t^B \geq 0 \quad (\text{B.64})$$

My interest is when the IC binds  $\lambda_{i,t}^B > 0$ , the KT1 condition B.63 leads to an expression relating leverage to bank premium:

$$\phi_{i,t}^B = \frac{v_{i,t}^B}{\Theta_B - \mu_{i,t}^L (1 - \kappa) - \kappa \mu_{i,t}^S} \quad (\text{B.65})$$

where  $\phi_{i,t}^B = \frac{\Lambda_{i,t}^B}{N_{i,t}^B}$ . For non-binding IC, i.e.  $\lambda_{i,t}^B = 0$ , then the 1<sup>st</sup> FOC implies  $\mu_{i,t}^L = 0$ . Hence,

$$(1 - \kappa)\mu_{i,t}^L = \max \left[ 0, \Theta_B - \frac{\nu_{i,t}^B}{\phi_{i,t}^B} - \kappa\mu_{i,t}^S \right] \quad (\text{B.66})$$

Now I need to relate the  $\mu_{i,t}^L$  and  $\mu_{i,t}^S$  to respective returns of  $L_{i,t}$  and  $S_{i,t}$  financing instruments. In terms of leverage equation B.60 is:

$$V_{i,t}^B = \left( \mu_{i,t}^L (1 - \kappa)\phi_{i,t}^B + \kappa\mu_{i,t}^S\phi_{i,t}^B + \nu_{i,t}^B \right) N_{i,t}^B \quad (\text{B.67})$$

where  $\mu_{i,t}^L = \nu_{i,t}^L - \nu_{i,t}^B$  is the excess value of all loans over cost of deposits  $R_{t+1}$

$\mu_{i,t}^S = \nu_{i,t}^S - \nu_{i,t}^B$  is the excess value of S-type financing above risk free return  $R_{t+1}$ .

Rewrite equation 3.41:

$$V_{i,t}^B = E_t \sum_{\tau=1}^{\infty} (1 - \sigma_B) \sigma_B^{\tau-1} \Lambda_{t,t+\tau} N_{i,t+\tau}^B$$

in recursive form (shown in subsection B.1.7)

$$V_{i,t}^B = E_t \Lambda_{t,t+1} \left( (1 - \sigma_B) N_{i,t+1}^B + \sigma_B V_{i,t+1}^B \right) \quad (\text{B.68})$$

where  $N_{i,t+1}^B$  is defined as in the income statement equation 3.40.

Plug equation B.67 shifted at  $t+1$  into equation B.68 to get:

$$V_{i,t}^B = E_t \Lambda_{t,t+1} \Omega_{i,t+1} N_{i,t+1}^B \quad (\text{B.69})$$

where,

$$\Omega_{i,t+1} = (1 - \sigma_B) + \sigma_B \left( (1 - \kappa)\mu_{i,t+1}^L \phi_{i,t+1}^B + \kappa\mu_{i,t+1}^S \phi_{i,t+1}^B + \nu_{i,t+1}^B \right)$$

is the shadow value of a unit of net worth and can be re-written making use of equation (B.65) (when IC binds):

$$\Omega_{i,t+1} = (1 - \sigma_B) + \sigma_B \Theta_B \phi_{i,t+1}^B \quad (\text{B.70})$$

or equivalently :

$$V_{i,t}^B = E_t \Lambda_{t,t+1} \left[ (1 - \sigma_B) + \sigma_B \left( (1 - \kappa) \mu_{i,t+1}^L \phi_{i,t+1}^B + \kappa \mu_{i,t+1}^s \phi_{i,t+1}^B + v_{i,t+1}^B \right) \right] N_{i,t+1}^B$$

Plug equation 3.40:  $N_{i,t+1}^B = (R_{t+1}^L - R_{t+1}) L_{i,t} + (R_{t+1}^s - R_{t+1}) Q_t S_{i,t} + R_{t+1} N_{i,t}^B$  into equation B.69 above yields:

$$V_{i,t}^B = E_t \Lambda_{t,t+1} \Omega_{i,t+1} \left[ (R_{t+1}^L - R_{t+1}) L_{i,t} + (R_{t+1}^s - R_{t+1}) Q_t S_{i,t} + R_{t+1} N_{i,t}^B \right] \quad (\text{B.71})$$

Finally, plugging Equation B.60; ( $v_{i,t}^B = \mu_{i,t}^L L_{i,t} + \mu_{i,t}^s Q_t S_{i,t} + v_{i,t}^B N_{i,t}^B$ )

into equation (B.71) leads to three expressions for  $\mu_{i,t}^L$ ,  $\mu_{i,t}^s$  and  $v_{i,t}^B$ :

$$\mu_{i,t}^L = (v_{i,t}^L - v_{i,t}^B) = E_t \Lambda_{t,t+1} \Omega_{i,t+1} (R_{t+1}^L - R_{t+1}) \quad (\text{B.72})$$

$$\mu_{i,t}^s = (v_{i,t}^s - v_{i,t}^B) = E_t \Lambda_{t,t+1} \Omega_{i,t+1} (R_{t+1}^s - R_{t+1}) \quad (\text{B.73})$$

$$v_{i,t}^B = E_t \Lambda_{t,t+1} \Omega_{i,t+1} R_{t+1} \quad (\text{B.74})$$

Equations B.65 and B.72, B.73, B.74 complete the banker's solution and determine  $\phi_{i,t}^B$  and  $\mu_{i,t}^L$ ,  $\mu_{i,t}^s$ ,  $v_{i,t}^B$  as a function of  $\Theta_B$ ,  $R_{t+1}^B - R_{t+1}$ ,  $R_{t+1}^s - R_{t+1}$  and  $R_{t+1}$ , which do not depend on bank specific factors. They are either exogenous to banks or depend on economy wide variables. Therefore, they are common across all banks.

Since all banks chose the same leverage ratio and observe the same excess returns on their assets  $\phi_t^B$  and  $\mu_t^L, \mu_t^s, \nu_t^B$ , the bank balance sheet and leverage can be aggregated across banks.

$$N_t^B + B_t = A_t^B = L_t + Q_t S_t \quad (\text{B.75})$$

$$\phi_t^B = \frac{A_t}{N_t^B} = \frac{Q_t K_t - N_t^E}{N_t^B} \quad (\text{B.76})$$

where  $N_t^E$  is firms' net worth and  $Q_t K_t$  total project cost. Aggregate bank net worth  $N_t^B$  evolves as the sum of:

- net worth of old banks surviving from last period,  $N_{o,t+1}^B = \sigma_B N_{t+1}^B$ ,
- and the net worth of new ones  $N_{new,t+1}^B = \xi_B A_t^B R_{t+1}^B$ , since on aggregate new banks receive a transfer of  $\frac{\xi_B}{1-\sigma_B}$  from absconding ones<sup>4</sup>.

Aggregate net worth of banks is:

$$N_{t+1}^B = N_{new,t+1}^B + N_{o,t+1}^B$$

where  $\sigma_B$  is the probability of the bank not absconding (surviving) next period. Evolution of net worth of banks not absconding at time ' $t+1$ ' is obtained using equation B.88:

$$N_{o,t+1}^B = \sigma_B (R_{t+1}^L L_t + R_{t+1}^s Q_t S_t - R_{t+1} B_t)$$

In addition a transfer  $\frac{\xi_b}{1-\sigma_b}$  of those  $1 - \sigma_b$  absconding transfer some net worth to new banks:

---

<sup>4</sup> After canceling out  $1 - \sigma_B$  portion of exiting banks

$$N_{new,t+1}^B = \xi_B R_{t+1}^B A_t^B$$

Then aggregate net worth is:

$$N_{t+1}^B = \sigma_B \left( R_{t+1}^L L_t + R_{t+1}^S Q_t S_t - R_{t+1} B_t \right) + \xi_B R_{t+1}^B A_t^B \quad (B.77)$$

Intuitively, aggregate consumption of exiting banks is<sup>5</sup>:

$$\Pi_{t+1}^B = (1 - \sigma_B)(R_{t+1}^B A_t^B - R_{t+1} B_t) - \xi_B R_{t+1}^B A_t^B \quad (B.78)$$

For the sake of reference, the average return on securities  $R_{t+1}^S$  is defined as:

$$R_{t+1}^S Q_t S_t = E_t R_{t+1}^k Q_t K_t \left( 1 - \Gamma_{\bar{\Psi}_{t+1}} \right) X_t^{eq} \quad (B.79)$$

For the sake of reference I define the average return  $R_{t+1}^B$  on total bank assets  $A_t^B$  as:

$$R_{t+1}^B = R_{t+1}^L \frac{L_t}{A_t^B} + R_{t+1}^S \frac{Q_t S_t}{A_t^B} \quad (B.80)$$

---

<sup>5</sup> Since  $\frac{\xi}{1-\sigma_B}$  portion of net worth is transferred by a fraction  $1 - \sigma_B$  of exiting bankers to new ones, then  $1 - \frac{\xi}{1-\sigma_B}$  of the fraction  $1 - \sigma_B$  of exiting bankers' net worth is consumed.

### B.1.7 Banker's Value Recursion

This subsection is the recursive derivation of equation B.68 starting with 3.41:

$$V_{i,t}^B = E_t \sum_{\tau=1}^{\infty} (1 - \sigma_B) \sigma_B^{\tau-1} \Lambda_{t,t+\tau} N_{i,t+\tau}^B \quad (\text{B.81})$$

$$V_{i,t}^B = (1 - \sigma_B) \left[ \sigma_B^{1-1} \Lambda_{t,t+1} N_{i,t+1}^B + \Lambda_{t,t+1} \left( \sigma_B^{2-1} \frac{\Lambda_{t,t+2}}{\Lambda_{t,t+1}} N_{i,t+2}^B + \sigma_B^{3-1} \frac{\Lambda_{t,t+3}}{\Lambda_{t,t+1}} N_{i,t+3}^B + \dots \right) \right] \quad (\text{B.82})$$

$$V_{i,t}^B = (1 - \sigma_B) \Lambda_{t,t+1} N_{i,t+1}^B + \sigma_B \Lambda_{t,t+1} (1 - \sigma_B) \left( \Lambda_{t+1,t+2} N_{i,t+2}^B + \sigma_B^1 \Lambda_{t+1,t+3} N_{i,t+3}^B + \dots \right) \quad (\text{B.83})$$

$$V_{i,t}^B = (1 - \sigma_B) \Lambda_{t,t+1} N_{i,t+1}^B + \sigma_B \Lambda_{t,t+1} \underbrace{\left( E_t \sum_{\tau=1}^{\infty} (1 - \sigma_B) \sigma_B^{\tau-1} \Lambda_{t+1,t+1+\tau} N_{i,t+1+\tau}^B \right)}_{V_{i,t+1}^B} \quad (\text{B.84})$$

$$V_{i,t}^B = (1 - \sigma_B) \Lambda_{t,t+1} N_{i,t+1}^B + \sigma_B \Lambda_{t,t+1} V_{i,t+1}^B \quad (\text{B.85})$$

which is eq. B.68

### B.1.8 Bank Participation Constraint

For a risk averse banker to agree to sign a contract with the entrepreneur in financing the wholesale firm as in section 3.2.4 the following **Participation Constraint** should be satisfied.

$$V_{i,t}^B > V_{i,t}^{B_0} \quad (\text{B.86})$$

where,

$V_{i,t}^B$  : is the value of the bank when the bank finances the entrepreneur and earn a gross return  $A_{i,t}^B R_{t+1}^B$  defined by eq. 3.39:

$$R_{t+1}^B A_{i,t}^B = R_{t+1}^L L_{i,t} + R_{t+1}^s Q_t S_{i,t}$$

and

$V_{i,t}^{B_0}$  : is the value of the bank when bank total assets are invested in risk free instruments and earn a gross return a gross return  $A_{j,t}^B R_{t+1}$ .

The equation for the value of the bank is eq. 3.41:

$$V_{i,t}^B = E_t \sum_{\tau=1}^{\infty} (1 - \sigma_B) \sigma_B^{\tau-1} \Lambda_{t,t+\tau} N_{i,t+\tau}^B \quad (\text{B.87})$$

where  $N_{i,t+1}^B$  is defined as:

$$N_{i,t+1}^B = R_{t+1}^B A_{i,t}^B - R_{t+1} B_{i,t} = (R_{t+1}^L - R_{t+1}) L_{i,t} + (R_{t+1}^s - R_{j,t+1}) Q_t S_{j,t} + R_{t+1} N_{i,t}^B \quad (\text{B.88})$$

while

$$N_{i,t+1}^{B_0} = R_{t+1} A_{i,t}^B - R_{t+1} B_{i,t} = R_{t+1} N_{i,t}^{B_0} \quad (\text{B.89})$$

*Assumption 1.*

No matter what strategy the banker follows, whether

(a) issuing standard debt loans and buying equity stakes or (b) investing assets into risk free instruments,

will not make any difference regarding the initial net worth the banker is endowed with, that is:

$$N_{i,t}^B = N_{i,t}^{B_0} \quad (\text{B.90})$$

where  $N_{i,t}^{B_0}$  is the net worth when the banker invests all assets in risk free instruments.

**Proposition 1.** *Given that the the following constraints are satisfied:*

- the bank **Incentive Constraint**  $V_{i,t}^B \geq \Theta_B A_{i,t}^B$  in Bank's problem (eq. 3.42, subsection 3.2.6),
- the constraint  $\Theta_B > \frac{1}{\phi_{i,t}^B}$  for the value of  $\Theta_B$ ,
- the **Assumption 1** about initial net worth,

then the one-period bank **Participation Constraint** in Wholesale Firms problem (eq. ??, subsection 3.2.4)

$$E_t R_{t+1}^k Q_t K_{j,t} \left( \Gamma_{(\bar{\Psi}_{t+1}^j)} - \mu G_{(\bar{\Psi}_{t+1}^j)} \right) + R_{t+1}^k (1 - G_{(\bar{\Psi}_{t+1}^j)}) Q_t S_{j,t} \geq E_t A_{j,t}^B R_{t+1}^B$$

is an equivalent sufficient condition that guarantee that the constraint B.86 is fulfilled.

*Proof:*

$V_{i,t}^{B_0}$  is the value of the bank when all assets are invested in risk free assets, therefore  $R_{t+1}^L = R_{t+1}$  and  $R_{t+1}^S = R_{t+1}$

Then eq. B.88 becomes:

$$N_{i,t+1}^{B_0} = R_{t+1} N_{i,t}^{B_0} \quad (\text{B.91})$$

Solving eq. B.91 forward

$$N_{i,t+\tau}^{B_0} = \prod_{h=1}^{\tau} R_{t+h} N_{i,t}^{B_0} \quad (\text{B.92})$$

Then to obtain the value of bank when all assets are invested in risk free instruments  $V_{i,t}^{B_0}$  plug eq. B.92 into eq. B.87:

$$V_{i,t}^{B_0} = E_t \sum_{\tau=1}^{\infty} (1 - \sigma_B) \sigma_B^{\tau-1} \Lambda_{t,t+\tau} \underbrace{\prod_{h=1}^{\tau} R_{t+h} N_{i,t}^{B_0}}_{N_{i,t+\tau}^{B_0}} \quad (\text{B.93})$$

Note that once  $(1 - \sigma_B)$  and  $N_{i,t}^{B_0}$  come in front of the summation, eq. B.93 in expanded form becomes:

$$V_{i,t}^{B_0} = (1 - \sigma_B) N_{i,t}^{B_0} \left[ \sigma_B^0 \Lambda_{t,t+1} R_{t+1} + \sigma_B^1 \Lambda_{t,t+2} R_{t+1} R_{t+2} + \sigma_B^2 \Lambda_{t,t+3} R_{t+1} R_{t+2} R_{t+3} \dots \right] \quad (\text{B.94})$$

Noting that as  $\Lambda_{t,t+j} = \Lambda_{t,t+1} \Lambda_{t+1,t+2} \dots \Lambda_{t+j-1,t+j}$ , the above equation condenses into:

$$V_{i,t}^{B_0} = (1 - \sigma_B) N_{i,t}^{B_0} \underbrace{\sum_{\tau=1}^{\infty} \sigma_B^{\tau-1}}_{1/(1-\sigma_B)} \quad (\text{B.95})$$

which implies

$$V_{i,t}^{B_0} = N_{i,t}^{B_0} \quad (\text{B.96})$$

I have shown that the value of the bank  $i$  when assets are invested in risk free instruments  $V_{i,t}^{B_0}$  is the initial net worth  $N_{i,t}^{B_0}$ .

Now, I discuss the value of the bank when banker finances the wholesale firm  $V_{i,t}^B$ . Note that the Banks' Problem in section 3.2 is built under the assumption that the bank **Incentive Constraint 3.42**

$$V_{i,t}^B \geq \Theta_B A_{i,t}^B$$

imposed by the depositors binds with equality as shown in Appendix C.1.

That is

$$V_{i,t}^B = \Theta_B A_{i,t}^B \quad (\text{B.97})$$

where  $A_{i,t}^B = \phi_{i,t}^B N_{i,t}^B$  with  $\phi_{i,t}^B$  denoting bank leverage. By plugging B.97 and the bank leverage definition into eq. B.86, the proof of the bank **Participation Constraint** eq. B.86 boils down to showing that

$$\Theta_B \phi_{i,t}^B N_{i,t}^B > N_{i,t}^{B_0} \quad (\text{B.98})$$

Making use of *Assumption 1* (eq. B.90) above that  $N_{i,t}^B = N_{i,t}^{B_0}$ , simplifies the participation constraint B.98 into:

$$\Theta_B > \frac{1}{\phi_{i,t}^B} \quad (\text{B.99})$$

or alternatively, plugging back the leverage definition  $A_{i,t}^B = \phi_{i,t}^B N_{i,t}^B$ , I obtain

$$\Theta_B A_{i,t}^B > N_{i,t}^B \quad (\text{B.100})$$

which is quite a rational constraint. Such a constraint will already hold under the bank **Incentive Constraint** in the Bank's Problem that depositors impose by setting a value for  $\Theta_B$  that already satisfies eq. B.99, since depositors can NOT accept a value of  $\Theta_B$  that implies bank assets  $\Theta_B A_{i,t}^B$  be less than the value of net worth  $N_{i,t}^B$  at any time. Assume they do. Then some depositors would be losing their deposits since the bank would not be able to pay back some of the deposits given that bank value less than net worth is too small to reimburse all deposits. The point behind such a bank **Incentive Constraint** is to guaranteed that the value of the bank is kept above that minimal threshold.

Hence that constraint in eq. B.99 holds is guaranteed by the Bank **Incentive Constraint** already imposed by depositors in the Bank's Problem, for values of  $\Theta_B$  satisfying that particular constraint. The value of  $\Theta_B$  has to be calibrated at a value greater than  $\frac{1}{\phi_{i,t}^B}$  should one expect to hit targets of positive external finance premiums  $R_{t+1}^L - R_{t+1}$  and  $R_{t+1}^s - R_{t+1}$ .

Finally, the value of the bank when it invests its assets into loans and equity stake  $V_{i,t}^B$  is given by plugging equation B.88 into B.87, which say that the value of the bank is a discounted stream of the sum of future of *excess returns on assets* and *marginal return on net worth*. That each of these streams of future returns is equal to the payoff the bank receives from participating in the financing contract with the wholesale firm (under the Wholesale Firm's Problem) in each single period is guaranteed by: (i) the one-period bank **Participation Constraint** in Wholesale Firms Problem (eq. ??, subsection 3.2.4):

$$E_t R_{t+1}^k Q_t K_{j,t} \left( \Gamma_{(\bar{\Psi}_{t+1}^j)} - \mu G_{(\bar{\Psi}_{t+1}^j)} \right) + R_{t+1}^k (1 - G_{(\bar{\Psi}_{t+1}^j)}) Q_t S_{j,t} \geq E_t A_{j,t}^B R_{t+1}^B$$

that already holds with equality under the wholesale producer's optimal conditions, where  $A_{i,t}^B R_{t+1}^B$  is defined by eq. 3.39:

$$R_{t+1}^B A_{i,t}^B = R_{t+1}^L L_{i,t} + R_{t+1}^s Q_t S_{i,t}$$

Therefore, given that the three conditions (a,b,c) in *Proposition 1* hold:

then the one-period bank **Participation Constraint** in equation ?? in **Wholesale Firms** problem (subsection 3.2.4)

$$E_t R_{t+1}^k Q_t K_{j,t} \left( \Gamma_{(\bar{\Psi}_{t+1}^j)} - \mu G_{(\bar{\Psi}_{t+1}^j)} \right) + R_{t+1}^k (1 - G_{(\bar{\Psi}_{t+1}^j)}) Q_t S_{j,t} \geq E_t A_{j,t}^B R_{t+1}^B$$

is an equivalent sufficient condition that guarantees that the constraint B.86

$$V_{i,t}^B > V_{i,t}^{B_0}$$

is satisfied with strict inequality.

### B.1.9 Aggregate Resource Constraint

To aggregate the balance sheet of agents and their flow of funds into consolidated national accounts I use the following equations:

Household budget constraint:

$$1. \text{ eq. 3.3: } C_t + B_t - R_t B_{t-1} + T_t = w_t H_t + \Pi_t^B + \Pi_t^E + \Pi_t^{CP} + \Pi_t^R$$

**Bank:** The bank balance sheet is given by eq. 3.38:  $N_t^B + B_t = A_t^B = L_t + Q_t S_t$ , also provided in table B.2 below.

Table B.2: Bank Balance Sheet

Assets	Liabilities and Net Worth
$L_t$	$B_t$
$Q_t S_t$	$N_t^B$

To obtain the funds flow I make use of the following equations from the section on banks.

$$\text{eq. B.77: } N_{t+1}^B = \sigma_B (R_{t+1}^B A_t^B - R_{t+1} B_t) + \xi_B R_{t+1}^B A_t^B$$

$$\text{eq. B.78: } \Pi_{t+1}^B = (1 - \sigma_B)(R_{t+1}^B A_t^B - R_{t+1} B_t) - \xi_B R_{t+1}^B A_t^B$$

Adding up eq. B.77 and B.78 I obtain the the flow of funds equation for the banker (shifted 1 period backwards):

$$\Pi_t^B + N_t^B = R_t^B A_{t-1}^B - R_t B_{t-1} \quad (\text{B.101})$$

**Wholesale Firm :** The balance sheet of the wholesale firm is given by equation 3.19:

$$Q_t K_{j,t} = N_{j,t}^E + L_{j,t} + Q_t S_{j,t} \text{ (table B.3).}$$

**Table B.3:** Wholesale Firm Balance Sheet

Assets	Liabilities and Net Worth
$Q_t K_t$	$L_t$
	$Q_t S_t$
	$N_t^E$

I obtain an equation for the rental income of wholesale firm after paying for wages from equations B.32, B.33 and B.34 in Appendix ?? :

$$R_t^k Q_{t-1} K_{t-1} = \frac{P_t^w}{P_t} Y_t^w + (1 - \delta) Q_t K_{t-1} - w_t H_t \quad (\text{B.102})$$

Table B.1 in subsection B.1.4 shows how this income is split into payoffs to agents involved in the contract and into deadweight monitoring costs.

#### The Entrepreneurs Flow of Funds

$$E_t R_{t+1}^k Q_t K_t = E_t \left[ \underbrace{R_{t+1}^k Q_t K_t (\Gamma_{\bar{\Psi}_{t+1}} - \mu G_{\bar{\Psi}_{t+1}})}_{\text{Bank-share}} + \underbrace{R_{t+1}^s Q_t S_t + R_{t+1}^k Q_t K_t (1 - \Gamma_{\bar{\Psi}_{t+1}}) X_t^{\text{eq}}}_{V_t^E} + \underbrace{\mu G_{(\bar{\Psi}_{t+1})} R_{t+1}^k Q_t K_t}_{\text{Monitoring-Cost}} \right]$$

I make use of (a) the binding bank participation constraint equation *holding with equality* B.46, (b) the wholesale firm net worth equation B.54 and (c) consumption equation B.55 : (a) eq. B.46  $E_t R_{t+1}^k Q_t K_t (\Gamma_{\bar{\Psi}_{t+1}} - \mu G_{\bar{\Psi}_{t+1}}) +$

$$R_{t+1}^s Q_t S_t = E_t A_t^B R_{t+1}^B \quad \text{where } A_t^B R_{t+1}^B = L_t R_{t+1}^L + Q_t S_t R_{t+1}^S$$

$$(b) \text{ eq. B.54: } N_t^E = (\sigma^E + \xi_E) E_t R_{t+1}^k \left[ Q_t K_t (1 - \Gamma_{\bar{\Psi}_{t+1}}) X_t^{\text{eq}} \right]$$

$$(c) \text{ eq. B.55: } \Pi_t^E = (1 - \sigma^E - \xi_E) E_t R_{t+1}^k \left[ Q_t K_t (1 - \Gamma_{\bar{\Psi}_{t+1}}) X_t^{\text{eq}} \right]$$

$$\text{Equations (b) + (c) give the value function } V_t^E = \Pi_t^E + N_t^E$$

I substitute the equation (a), (b),(c) above in the **Flow of Funds**. Shifting one period backwards, the aggregate flow of funds equation of wholesale firm becomes:

$$R_t^k Q_{t-1} K_{t-1} = R_t^B A_{t-1}^B + \Pi_t^E + N_t^E + \mu G_{(\bar{\Psi}_t)} R_t^k Q_{t-1} K_{t-1} \quad (\text{B.103})$$

where Monitoring Cost =  $\mu G_{(\bar{\psi}_t)} R_t^k Q_{t-1} K_{t-1}$  is deadweight cost of eq. B.58. Finally I re-write equation B.103 by plugging  $A_{t-1}^B + N_t^E = Q_{t-1} K_{t-1}$ :

$$\Pi_t^E = R_t^k (A_{t-1}^B + N_t^E) - R_t^B A_{t-1}^B - N_t^E - \mu G_{(\bar{\psi}_t)} R_t^k Q_{t-1} K_{t-1} \quad (\text{B.104})$$

Total revenues of the firm from sales of output and resale of capital (equation B.32) are equal to  $R_t^k Q_t K_t = R_t^k (A_{t-1}^B + N_{t-1}^E)$ :

$$R_t^k (A_{t-1}^B + N_{t-1}^E) = \frac{P_t^w}{P_t} Y_t^w + (1 - \delta) Q_t K_{t-1} - w_t H_t \quad (\text{B.105})$$

**Capital Producer:** To obtain e cash flow for capital producer I make use of equations:

$$\text{eq. 3.13: } \Pi_t^{\text{CP}} = Q_t \Upsilon_t (1 - S(X_t)) I_t - I_t$$

$$\text{eq. 3.15: } K_t = (1 - \delta) K_{t-1} + (1 - S(X_t)) I_t \Upsilon_t$$

I subtract eq.3.15 multiplied with  $Q_t$  from eq. 3.13, that is  $\Pi_t^{\text{CP}} - Q_t K_t$  to obtain the resource equation for capital producer (after re-arranging the terms):

$$\Pi_t^{\text{CP}} = Q_t K_t - Q_t (1 - \delta) K_{t-1} - I_t \quad (\text{B.106})$$

**Retailer profits.:** The retailer buys wholesale output  $Y_t^w$  for (nominal) wholesale price  $P_t^w$  and sells final output  $Y_t$  at selling price  $P_t$ . Net profits of retailer in real terms are:

$$\Pi_t^R = Y_t - \frac{P_t^w}{P_t} Y_t^w \quad (\text{B.107})$$

**Government Resource Constraint.** eq. 3.49

$$0 = T_t - G_t$$

**Aggregate Resource Constraint:**

I aggregate the balance sheets of wholesale firms and banks to get a consolidated balance sheet equations of both agents as shown in table B.4.

Table B.4: Consolidated Balance Sheet of Bank and Wholesale Firm

Assets	Liabilities and Net Worth
$Q_t K_t$	$B_t$
	$N_t^B$
	$N_t^E$

Finally, I sum up across equations B.101, B.105, B.104, B.106, B.107, 3.49 and 3.3, and make use of consolidated balance sheet in table B.4 to obtain the national accounting identity. For the sake of space I denote bank monitoring costs with  $\mu_{MC}$ :

$$\mu_{MC} = \mu G_{(\bar{\psi}_t)} R_t^k Q_{t-1} K_{t-1} \quad (\text{B.108})$$

Given the

- household budget constraint (eq. 3.3)  $C_t = Y_t - I_t - G_t - \mu G_{\psi_t} R_t^k Q_{t-1} K_{t-1}$ ,
- the entrepreneurs profit flow (eq. B.104)  $\Pi_t^E = R_t^k (A_{t-1}^B + N_t^E) - R_t^B A_{t-1}^B - N_t^E - \mu G_{(\bar{\psi}_t)} R_t^k Q_{t-1} K_{t-1}$ ,
- the balance sheet equation that return from project should equal the total return shared between entrepreneur and the bank (eq. B.105)  $R_t^k (A_{t-1}^B + N_{t-1}^E) = \frac{P_t^w}{P_t} Y_t^w + (1 - \delta) Q_t K_{t-1} - w_t H_t$ ,
- the bank profit flow (dividend to his household eq. B.101)  $\Pi_t^B + N_t^B = R_t^B A_{t-1}^B - R_t B_{t-1} = R_t^L L_{t-1} + R_t^S Q_{t-1} S_{t-1} - R_t B_{t-1}$

## B.1. Model Derivation

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- the capital producer profit flow (B.106)  $\Pi_t^{\text{CP}} = Q_t K_t - Q_t(1 - \delta)K_{t-1} - I_t$ ,
- retailer real profit flow (B.107)  $\Pi_t^{\text{R}} = Y_t - \frac{P_t^w}{P_t} Y_t^w$ ,
- government balanced budget (eq. 3.49)  $0 = G_t - T_t$ ,

the markets will clear to yield the aggregate resource constraint.

--- Agg.Uses --- = ----- Agg.Resources -----

$$\begin{aligned} \Pi_t^{\text{B}} &= R_t^{\text{B}} A_{t-1}^{\text{B}} - R_t B_{t-1} - N_t^{\text{B}} \\ R_t^{\text{k}}(A_{t-1}^{\text{B}} + N_{t-1}^{\text{E}}) &= \frac{P_t^w}{P_t} Y_t^w + (1 - \delta)Q_t K_{t-1} - w_t H_t \\ \Pi_t^{\text{E}} &= R_t^{\text{k}}(A_{t-1}^{\text{B}} + N_t^{\text{E}}) - R_t^{\text{B}} A_{t-1}^{\text{B}} - N_t^{\text{E}} - \mu_{\text{MC}} \\ \Pi_t^{\text{CP}} &= Q_t K_t - Q_t(1 - \delta)K_{t-1} - I_t \\ \Pi_t^{\text{R}} &= Y_t - \frac{P_t^w}{P_t} Y_t^w \\ 0 &= T_t - G_t \\ C_t &= w_t H_t + \Pi_t^{\text{B}} + \Pi_t^{\text{E}} + \Pi_t^{\text{CP}} + \Pi_t^{\text{R}} - B_t + R_t B_{t-1} - T_t \end{aligned}$$

--- Agg.Uses --- = ----- Agg.Resources -----

$$C_t = Y_t - \mu_{\text{MC}} - I_t - G_t + \underbrace{Q_t K_t - N_t^{\text{B}} - N_t^{\text{E}} - B_t}_0$$

----- = -----

$$C_t = Y_t - I_t - G_t - \mu_{\text{G}\psi_t} R_t^{\text{k}} Q_{t-1} K_{t-1}$$

where I have denoted  $\mu_{\text{G}(\bar{\psi}_t)} R_t^{\text{k}} Q_{t-1} K_{t-1} = \mu_{\text{MC}}$ . To obtain the final national accounting identity line above I make use of bank balance sheet table B.2 and the bank and wholesale firm consolidated balance sheet table B.4.

**Legend of Variables.**

- $w_t$  = Real wage  
 $H_t$  = Hours worked  
 $\Pi_t^R$  = Retailer profits (real)  
 $\Pi_{C_t}^B$  = Banker profit (consumption)  
 $\Pi_{C_t}^E$  = Entrepreneur profit (consumption)  
 $C_t$  = Household consumption  
 $B_t$  = Current period stock of deposits  
 $R_t$  = Risk free real return  
 $\text{tax}_t$  = Household taxes paid to Government  
 $L_t$  = Standard debt loan to wholesale firm  
 $S_t$  = Equity stakes of wholesale firm bought by the bank  
 $Q_t$  = Price of one unit of equity or physical capital  
 $R_t^k$  = Expected return on wholesale producing firm investment  
 $R_t^s$  = Expected return on equity stake  
 $R_t^L$  = Average return on standard loans  
 $R_t^B$  = Weighted average return on bank assets (L and S)  
 $N_t^B$  = Accumulated Bank net worth  
 $N_t^E$  = Accumulated Entrepreneur net worth  
 $K_t$  = Physical capital invested in project by Entrepreneur  
 $1 - \Gamma_{\psi_t}$  = Share of project return paid to owner of each unit of physical capital  
 $\Gamma_{\psi_t} - \mu G_{\psi_t}$  = Banker's pay-off from loan contract net of monitoring cost  
 $\mu G_{\psi_t}$  = Portion of project return spent for monitoring costs (deadweight cost)  
 $I_t$  = Investment  
 $\Upsilon_t$  = Investment Specific Technology affecting investment production  
 $S(X_t)$  = Investment Adjustment Costs, where  $X_t = \frac{I_t}{X_{t-1}}$   
 $G_t$  = Government expenditure  
 $\kappa$  = Share of equity stake finance on total assets of the bank.



### B.1.10 Steady State Equations

#### A. Steady State of LogNormal Distribution Equations

$$F_{\psi}^{ss} = 0.75\% \quad (\text{B.109})$$

$$Z_{\psi} = \text{Ncdf}^{-1}(F_{\psi}^{ss}) \quad (\text{B.110})$$

$$\psi_t = e^{Z_{\psi}\sigma - 0.5\sigma^2} \quad (\text{B.111})$$

$$G_{\psi} = \text{Ncdf}(Z_{\psi} - \sigma) = 1 - \text{Ncdf}(\sigma - Z_{\psi}) \quad (\text{B.112})$$

$$\Gamma_{\psi} = \psi(1 - F_{\psi}) + G_{\psi} \quad (\text{B.113})$$

$$G'_{\psi} = \frac{\text{Npdf}(Z_{\psi})}{\sigma} = \frac{\text{Npdf}(Z_{\psi} - \sigma)}{\psi\sigma} = \psi F'_{\psi} \quad (\text{B.114})$$

since  $\frac{\text{Npdf}(Z_{\psi} - \sigma)}{\psi} = \text{Npdf}(Z_{\psi})$

$$\Gamma'_{\psi} = 1 - F_{\psi} \quad (\text{B.115})$$

#### B. Steady State of Model Equation

Steady State hours of work

$$H = 0.35;$$

Steady State total external finance premium

$$\text{efp}^{ss} = 2\%$$

Steady State public expenditures to final output ratio

$$g^Y = 0.2;$$

1. Risk free discount rate

$$R = \frac{1}{\beta} = \frac{1}{\Lambda} \quad (\text{B.116})$$

2. Definition

$$\tilde{\pi} = \pi^{1-\gamma_p} \quad (\text{B.117})$$

3. The price that retailing firms reset in next period

$$p^* = \frac{J}{JJ} = \left( \frac{1 - \xi_p \pi^{(1-\gamma_p)(\epsilon-1)}}{1 - \xi_p} \right)^{1/(1-\epsilon)} \quad (\text{B.118})$$

4. Real marginal cost to Retailer

$$mc = (1 - 1/\epsilon) \frac{1 - \xi_p \beta \pi^{(1-\gamma_p)\epsilon}}{1 - \xi_p \beta \pi^{(1-\gamma_p)(\epsilon-1)}} p^* \quad (\text{B.119})$$

5. Wholesale price in real terms

$$\frac{p^w}{P} = mc \quad (\text{B.120})$$

6. Price Dispersion

$$\Delta = \frac{1 - \xi_p}{1 - \xi_p \pi^{(1-\gamma_p)\epsilon}} p^{*\epsilon} \quad (\text{B.121})$$

7. Return on Capital

$$R^k = (\epsilon p^{ss} + 1)^{1/4} * R \quad (\text{B.122})$$

8. Capital to wholesale output Ratio

$$\frac{K}{Y^w} = (R^k - (1 - \delta))(1 - \alpha) \frac{P^w}{P} \quad (\text{B.123})$$

9. Investment to output ratio

$$\frac{I}{Y} = \delta \frac{K}{Y^w} \frac{\Delta}{1 - 1/\epsilon} \quad (\text{B.124})$$

10. Agency costs to output ratio

$$\mu^Y = \mu G_\psi R^k \frac{K}{Y^w} \frac{\Delta}{1 - 1/\epsilon} \quad (\text{B.125})$$

10. Consumption to output ratio

$$CY = \frac{C}{Y} = 1 - \frac{I}{Y} - g^Y - \mu^Y \quad (\text{B.126})$$

12. Wholesale Production

$$Y^w = (AH) \left( \frac{K}{Y^w} \right)^{\frac{1-\alpha}{\alpha}} \quad (\text{B.127})$$

13. Real wage

$$w = \alpha \frac{P^w}{P} \frac{Y^w}{H} \quad (\text{B.128})$$

14. Final (retail) output

$$Y = (1 - 1/\epsilon) \frac{Y^w}{\Delta} \quad (\text{B.129})$$

15. Capital Stock

$$K = \frac{K}{Y^w} * Y^w \quad (\text{B.130})$$

16. Investment

$$I = \frac{I}{Y} * Y \quad (\text{B.131})$$

17. Government expenditure

$$G = g^y * Y \quad (\text{B.132})$$

18. Consumption

$$C = \frac{C}{Y} * Y \quad (\text{B.133})$$

19. Labor supply shifter

$$\delta^h = \alpha \frac{P^w}{P} \frac{\Delta}{1 - 1/\epsilon} \frac{1 - \beta^x \chi}{(1 - \chi)^{\sigma^c}} \frac{C^{1 - \sigma^c}}{CY} H^{-1 - \sigma^h} \quad (\text{B.134})$$

20 Household Utility Function

$$U_{(C,H)} = \frac{(C - \chi \bar{C})^{1 - \sigma^c}}{1 - \sigma^c} - \delta^h \frac{H^{1 + \rho^h}}{1 + \rho^h} \quad (\text{B.135})$$

21. Marginal Utility w.r.t consumption

$$U'_C = (C - \chi \bar{C})^{-\sigma^c} \quad (\text{B.136})$$

22. Marginal Utility w.r.t hours worked

$$U'_H = -U'_L = -\delta^h H^{\rho^h} \quad (\text{B.137})$$

23. Wholesale Firm Net Worth Evolution

$$N^E = \frac{(\sigma^E + \xi_E) R^k QK (1 - \Gamma_{\Psi}^h) - \kappa * QK}{1 - \kappa} \quad (\text{B.138})$$

24. Wholesale Firm Asset to Net Worth Ratio

$$\phi^E = \frac{QK}{N^E} \quad (\text{B.139})$$

25. The Ratio of equity securities to total capital less loans

$$\chi^{eq} = \frac{QS}{QK - L} \quad (\text{B.140})$$

26. Wholesale Firm: FOC w.r.t  $\bar{\psi}$

$$\lambda^E = \frac{\Gamma'_{\bar{\psi}}(1 - X^{eq})}{\Gamma'_{\bar{\psi}}(1 - X^{eq}) - \mu G'_{\bar{\psi}}} \quad (\text{B.141})$$

27. Borrower premium: FOC w.r.t choice of Capital Stock K

$$\rho_{\bar{\psi}} = \frac{\lambda^E}{(1 - \Gamma_{\bar{\psi}})(1 - X^{eq} - \phi^E X_{\phi}^{eq'}) + \lambda^E [\Gamma_{\bar{\psi}} - \mu G_{\bar{\psi}} + (1 - \Gamma_{\bar{\psi}})(X^{eq} + \phi^E X_{\phi}^{eq'})]} \quad (\text{B.142})$$

28. Return on equity securities to the bank

$$R^S = R^k \frac{QK}{QS} (1 - \Gamma_{\bar{\psi}}) X^{eq} \quad (\text{B.143})$$

29. Opportunity cost of Bank Funds (Required return on Bank Assets)

$$R^B = \frac{R^k}{\rho_{\bar{\psi}}} \quad (\text{B.144})$$

30. Contractual Loan Rate

$$Z^L = \bar{\psi} R^k \frac{QK}{(1 - \kappa)(QK - N^E)} \quad (\text{B.145})$$

31. Bank Assets

$$A^B = QK - N^E \quad (\text{B.146})$$

31. Bank Asset: Loans

$$L = (1 - \kappa)(QK - N^E) \quad (\text{B.147})$$

32. Bank Asset: Equity Securities

$$S = \kappa A^B / Q \quad (\text{B.148})$$

33. Average return on Loan Portfolio

$$R^L = \frac{R^B A^B - R^S QS}{L} \quad (\text{B.149})$$

34. Bank Net Worth Evolution

$$N^B = \frac{(\sigma_B + \xi_B)(R^L L + R^S QS) - \sigma_B R(QK - N^E)}{1 - \sigma_B R} \quad (\text{B.150})$$

35. Bank Asset to Net Worth Ratio

$$\phi^B = \frac{A^B}{N^B} \quad (\text{B.151})$$

36. Shadow value of a unit of bank net worth

$$\Omega = 1 - \sigma_B + \sigma_B \Theta_B \phi^B \quad (\text{B.152})$$

37. Excess marginal value to bank of an extra unit of loan

$$\mu^L = \Omega \left( \frac{R^L}{R} - 1 \right) \quad (\text{B.153})$$

38. Excess marginal value to bank of an extra unit of equity security

$$\mu^s = \Omega \left( \frac{R^s}{R} - 1 \right) \quad (\text{B.154})$$

39. Marginal value of a unit of bank net worth

$$\nu^B = \Omega \quad (\text{B.155})$$

40. Household deposits

$$B = A^B - N^B \quad (\text{B.156})$$

### B.1.11 Data

I use US data from NIPA-BEA, CPS-BLS, the FRED database, and the Flow of Funds Accounts from the Federal Reserve Board. Though data for US are available starting from 1947, the earliest, I use data for the period 1980-2015, as the longest time span for which I can construct data for Euro area.

- ***Non-financial sector net worth: Tangible Assets minus Credit Market Liabilities.*** Tangible Assets obtained from the Flow of Funds Account (FFA) of the Federal Reserve Board, sum of 'Nonfinancial Assets of Non-financial Corporate Business', series ID FL102010005.Q from Table B.102 and 'Nonfinancial Assets of Nonfinancial NonCorporate Business', series ID FL112010005.Q from Table B.103. Credit Market Liabilities obtained from the Flow of Funds Account (FFA) of the Federal Reserve Board, sum of 'Credit Market Instruments of Nonfinancial Noncorporate Business', series ID FL114104005.Q from Table B.102 and 'Credit Market Instruments of Nonfinancial Corporate Business', series ID FL114102005.Q from Table B.103; I deflate the nominal net worth series using the GDP deflator provided by NIPA Table 1.1.4.
- ***Financial sector net worth: Financial business; corporate equities; liability.*** I use the series from the Financial Account of the United States (table Z1) of the Federal Reserve Board to get a market value for the net worth of Financial Sector. I obtain the series '*Financial business; corporate equities (liability)*', coded LM793164105.Q from table Z1/L108. (Before August 2015 the table was named Z1/L107 and the series name FL793164105.Q). I deflate the nominal net worth series using the GDP deflator provided by NIPA Table 1.1.4.
- ***External Finance Premium.*** Moody's Baa-rated corporate bond yield minus the Aaa-rated corporate bond yield. Alternatively, the spread is

calculated as primary lending rate minus 3 months T-bill rate. Source: Board of Governors of the Federal Reserve System (H15).

- Real per capita GDP
- Real per capita consumption, defined as nondurable consumption and services
- Real per capita gross private investment
- *Hours worked*. The (log of) average weekly hours worked (BLS series PRS85006023) divided by 100 and multiplied by the ratio of civilian population over 16 (CE16OV) to a population index. As the hours worked is an index with 1992=100, the population index is the ratio of population each quarter divided by the population in the third quarter of 1992.
- *Labor share*. Labor share is defined as the ratio of total compensation of employees (NIPA Table 2.1) corrected by the size of the non-farm business sector to the gross value added by the non-farm business sector. Inflation is defined as the log of the GDP deflator (the price index for gross value added by the nonfarm business sector (NIPA Table 1.3.4)).

### External Finance Premium

To get an approximate measure of increase in volatility of external finance premium I look at the following spreads.

1. *External Finance Premium EFP<sub>b</sub>*, the overall premium, as the difference between Moody's BAA rated corporate bond yield and Fed rate,
2. *EFP<sub>a</sub>*, the difference between Moody's AAA rated corporate bond yield and Fed rate,
3. *EFP<sub>ba</sub>*, Moody's Baa-rated corporate bond yield minus the Aaa-rated corporate bond yield. Alternatively, the spread is calculated as primary lending rate minus 3 months T-bill rate.

The first one is an approximate for the overall external finance premium. The second one is an approximate for the premium not related to the quality of borrower, therefore pertaining to *lenders moral hazard* premium. The third one, their difference is considered a proxy for *borrower quality* premium. Since the difference relative to Fed rate may include, liquidity risk premium related to short term instruments, I calculated the first and second spreads relative to Government 10 year Treasury constant maturity bond return, and name them *EFP<sub>bg</sub>* and *EFP<sub>ag</sub>*.

1. *EFP<sub>bg</sub>*, the overall premium, as the difference between Moody's BAA rated corporate bond yield and Government 10 year Treasury constant maturity bond yield,
2. *EFP<sub>ag</sub>*, the difference between Moody's AAA rated corporate bond yield and Government 10 year Treasury constant maturity bond yield.

The *borrower quality* premium is the same. The ratios of volatilities during crisis relative to before 2006 are similar whichever set of data I use.

## B.1. Model Derivation

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Source: Board of Governors of the Federal Reserve System (H15).

## B.2 Tables and Figures

Table B.5: Parameters relevant to NK equations in all Models, *GK*, *FA* and *FAGK*.

Variable	Description	Target Value
$\beta$	Discount rate	0.9966
$\delta$	Depreciation rate	0.025
$\alpha$	Labor share on output	0.7
$\chi$	Degree of habit formation	0.65
$\phi_x$	Investment adjustment cost	2
$\xi_p$	Calvo parameter (1 - prob of re-optimizing)	0.75
$\epsilon$	Elasticity of substitution among differentiated goods	7
$\gamma_p$	Degree of price indexation to past inflation	0.5
$\sigma_c$	Inverse of inter-temporal elasticity of substitution	1.4
$\rho^h$	Inverse of the Frisch elasticity of labor	0.5
$\rho_R$	Policy rate smoothing parameter	0.8
$\theta_\pi$	MP reaction to inflation expectation	1.5
$\theta_Y$	MP reaction to output gap	0.5/4
$\theta_{\Delta\pi}$	MP reaction to changes in inflation	0
Shocks		
$\rho_A$	Persistence of neutral productivity shock	0.90
$\rho_G$	Persistence of public spending shock	0.90
$\rho_m$	Persistence of MP shock	0
$\rho_{IS}$	Persistence of IST shock	0.90
$\sigma_A$	St.deviation of productivity shock	$1.1 * 10^{-2}$
$\sigma_G$	St.deviation of public spending shock	$1.0 * 10^{-2}$
$\sigma_m$	St.deviation of MP shock	$1.0 * 10^{-2}$
$\sigma_{IS}$	St.deviation of IST shock	$0.15 * 10^{-2}$

## B.2. Tables and Figures

**Table B.6:** Steady State Targets of non-financial and financial sector and the relevant parameters calibrated to match the respective target

Target Variable		GK	FA	FA-GK	Parameter*
Firm tenure		–	16.6 yrs	16.6 yrs	$\sigma^E$
Firm Asset/(N.Worth+Equity)	$\frac{QK}{N^E+QS}$	–	1.843	1.841	$A_{\Psi}$
Bank tenure		8.6 yrs	–	10.8 yrs	$\sigma^B$
Bank Asset/N.Worth	$\frac{QS+L}{N^B}$	4.1	–	6.4	$\xi^B$
Borrower risk premia	$R^K - R^B$	2.0 %	–	0.81 %	$\mu$
Lender risk premia	$R^B - R$	–	2.0%	1.18 %	$\Theta^B$
Overall premium	$R^K - R$	2.0%	2.0%	2.0%	$\Theta^B, \mu$
	$R^L - R, R^S - R$	–	–	0.0047	$\xi^E$

\*Values of calibrated parameters are reported in the following table B.7

**Table B.7:** Calibrated values of firm and bank friction parameters.

Description	GK	FA	FA-GK
$\sigma^E$ Survival probability of entrepreneur	–	0.985	0.985
$A_{\Psi}$ Standard deviation of the idiosyncratic shock	–	0.30	0.30
$\sigma^B$ Survival probability of banks	0.971	–	0.977
$\xi^B$ Wealth share transferred to new banks	0.0016	–	0.00023
$\mu$ Monitoring costs parameter	–	0.12	0.11
$\Theta^B$ Share of bank assets bankers can run away with	0.75	–	0.76
$\xi^E$ Wealth share transferred to new entrepreneurs	–	0.0033	0.0047
$F_{\Psi}$ Probability of default in any quarter	–	0.75%	0.75%
$\mu_{\gamma}$ Deadweight cost due to monitoring in % of GDP	–	0.25 %	0.23 %

**Table B.8:** Steady State Targets of Macro variables.

Variable	Description	NK	1.GK	2.FA	3.FA-GK
$\frac{C}{Y}$	Average Consumption to GDP ratio	0.51	0.57	0.57	0.57
$\frac{I}{Y}$	Average Investment to GDP ratio	0.29	0.23	0.22	0.22
$\frac{K}{Y}$	Average Capital Stock to GDP ratio	11.8	9.00	9.00	9.00
$\pi$	Average inflation rate	2.4%	2.4%	2.4%	2.4%
$R$	Average real interest rate	1.4%	1.4%	1.4%	1.4%
$H_t$	Working hours	0.35	0.35	0.35	0.35

**Table B.9:** Variance Decomposition of FA-GK model: neutral technology (TFP), government expenditures (G), monetary policy (MP), investment-specific technology (IST) shocks.

Variance Decomposition				
	TFP	G	MP	IST
Y	81	3	10	5
C	71	2	18	9
I	69	1	18	12
H	69	5	18	8
W	69	0	29	1
$\Pi$	44	0	53	3
Q	52	1	30	16
R	22	0	75	3
Rn	22	0	72	6
BAA[ $R_{t+1}^K$ ]	28	2	68	2
NW	46	1	38	15
NB	36	1	50	13
NW (FoF)	–	–	–	–
NB (FoF)	–	–	–	–
<i>efpBA</i> [spreadKB]	29	1	48	15
<i>efpA</i> [spreadRB]	69	3	26	2
<i>efpB</i> [spread]	36	3	27	2

**Figure B.1:** Trend in share of corporate equities held by US banks on their assets.

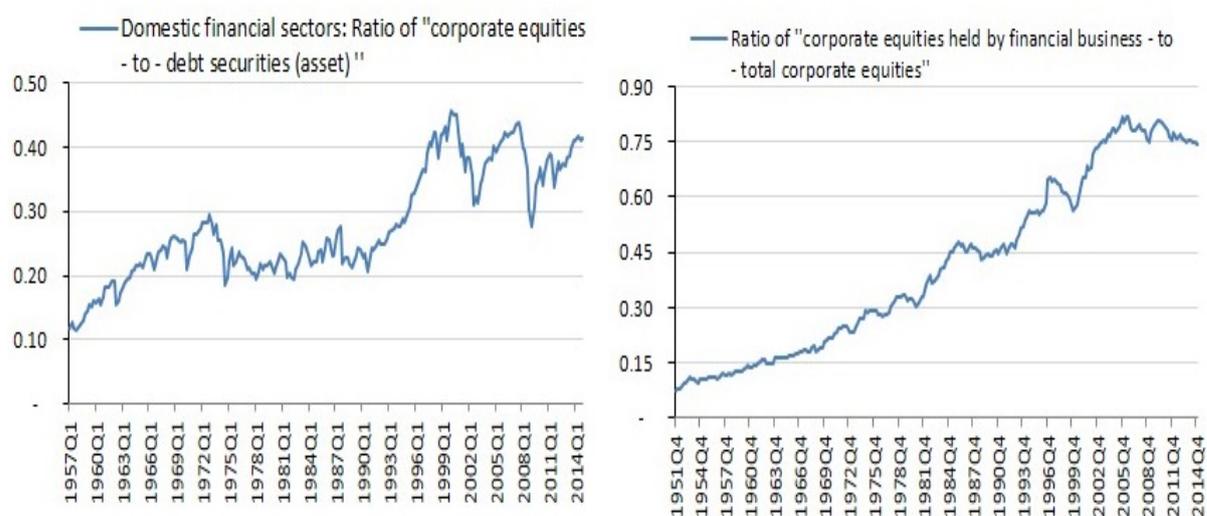


Table B.10: Standard deviation ( $\% \sigma_{x_i}$ ) of key variables: data and calibrated models(\*).

	'75-2004 [a]	'85-2004 [b]	'05-2014 [c]	[d]	NK	GK	FA	FA-GK
	$\% \sigma_{\text{data}}$			$\frac{[c]}{[b]}$	$\% \sigma_{\text{calibrated.model}}$			
Y	1.37	0.98	1.16	1.2	0.84	0.89	0.77	0.98
C	0.84	0.73	0.93	1.3	0.53	0.47	0.52	0.44
I	4.18	3.22	4.17	1.3	2.56	3.10	2.33	3.69
H	1.17	0.92	1.34	1.5	1.15	1.10	1.09	1.14
W	1.00	1.11	0.71	0.6	0.90	0.85	0.87	0.81
$\Pi$	0.21	0.12	0.17	1.4	0.19	0.18	0.18	0.18
Q	–	–	–	–	1.02	1.43	1.20	1.74
R	0.23	0.20	0.27	1.4	0.30	0.29	0.29	0.29
Rn	0.29	0.24	0.28	1.2	0.16	0.16	0.16	0.16
BAA[R <sub>t+1</sub> <sup>k</sup> ]	0.77	0.49	0.84	1.7	–	0.33	0.29	0.38
NW (FoF)	2.21	2.48	6.92	2.8	–	–	1.96	2.81
NB (FoF)	10.8	11.6	12.8	1.1	–	5.25	–	10.4
NW (DS)	9.77	10.3	11.4	1.1	–	–	–	–
NB (DS)	10.74	11.4	15.2	1.3	–	–	–	–
<i>efpBA</i>	0.06	0.04	0.10	2.8	–	–	–	0.01
<i>efpA</i>	0.23	0.20	1.17	5.9	–	–	–	0.18
<i>efpB</i>	0.25	0.21	0.71	3.3	–	0.12	0.04	0.19
<i>efpBA</i>	0.06	0.04	0.10	2.8	–	–	–	–
<i>efpAG</i>	0.05	0.05	0.17	3.4	–	–	–	–
<i>efpBG</i>	0.09	0.07	0.22	3.1	–	–	–	–

(\*) Except for ratios, the values are standard deviations ( $\sigma_{x_i}$ ) multiplied by 100.

**Table B.11:** Correlation with output ( $\% \rho_{x_i,y}$ ) : data and calibrated models(\*)

	'75-2004 [a]	'85-2004 [b] $\% \rho_{data}$	'05-2014 [c]	NK	GK	FA	FA-GK
				$\% \rho_{calibrated.model}$			
Y	1	1	1	1	1	1	1
C	0.85	0.91	0.93	0.59	0.70	0.77	0.64
I	0.95	0.94	0.96	0.91	0.93	0.92	0.95
H	0.93	0.86	0.91	0.30	0.32	0.19	0.42
W	0.21	0.24	-0.30	0.69	0.77	0.77	0.77
$\Pi$	0.27	0.29	0.59	0.10	0.18	0.14	0.20
Q	-	-	-	0.16	0.31	0.20	0.35
R	0.22	0.25	0.50	-0.34	-0.34	-0.37	-0.32
Rn	0.36	0.42	0.77	-0.41	-0.32	-0.41	-0.22
BAA[R <sub>t+1</sub> <sup>K</sup> ]	-0.30	-0.35	-0.61	-	-0.48	-0.36	-0.52
NW (FoF)	0.08	0.51	0.87	-	-	0.23	0.47
NB (FoF)	0.38	0.38	0.58	-	0.33	-	0.39
NW (DS)	0.50	0.57	0.93	-	-	0.23	0.47
NB (DS)	0.36	0.31	0.90	-	0.33	-	0.39
<i>efpBA</i>	-0.74	-0.68	-0.47	-	-	-	-0.48
<i>efpA</i>	-0.55	-0.45	-0.76	-	-	-	-0.68
<i>efpB</i>	-0.68	-0.53	-0.76	-	-0.63	-0.14	-0.67
<i>efpBA</i>	-0.74	-0.68	-0.47	-	-	-	-
<i>efpAG</i>	-0.45	-0.29	-0.39	-	-	-	-
<i>efpBG</i>	-0.73	-0.54	-0.45	-	-	-	-

(\*) Except for ratios, the values are correlations ( $\rho_{x_i,y}$ ) multiplied by 100 to express in % terms.

**Table B.12:** Variance-Covariance Matrix for *External Finance Premium* ( $\times 10^6$ ).

Data Cov. matrix			
	efpB	efpA	efpBA
efpB [spread]	4.5	-	-
efpA [spreadRB]	4.1	3.9	-
efpBA [spreadKB]	0.35	0.22	0.13
Model Cov. matrix			
	efpB	efpA	efpBA
efpB [spread]	3.51	-	-
efpA [spreadRB]	3.38	3.26	-
efpBA [spreadKB]	0.13	0.12	0.01

(\*) All numbers multiplied by  $10^6$ .

Table B.13: Variance-Covariance Matrix for *External Finance Premium* ( $\times 10^6$ ).

Data Cov. matrix (in % of VAR(spread) )			
efpB [spread]	100	–	–
efpA [spreadRB]	92	76	–
efpBA [spreadKB]	8	5	3
Model Cov matrix (in % of VAR(spread) )			
efpB [spread]	100	–	–
efpA [spreadRB]	96	93	–
efpBA [spreadKB]	4	3	0.2

Table B.14: Change in volatility relative to the baseline model due to shift in monitoring costs  $\mu$  by 50 %.

Standard Deviations							
	[b]'85-2005	[c]'06-2014	$\frac{c}{b}$	NK	GK	FA	FA-GK
	% $\sigma_{data}$			$\frac{\% \sigma'_{x_i}}{\% \sigma_{x_i}}$ (model)			
Y	0.98	1.16	1.2	–	–	0.99	1.02
C	0.73	0.93	1.3	–	–	1.00	0.98
I	3.22	4.17	1.3	–	–	0.98	1.05
H	0.92	1.34	1.5	–	–	1.00	1.02
W	1.11	0.71	0.6	–	–	1.00	0.98
$\Pi$	0.12	0.17	1.4	–	–	1.00	1.01
Q	–	–	–	–	–	1.08	1.10
R	0.20	0.27	1.4	–	–	1.00	1.01
Rn	0.24	0.28	1.2	–	–	1.00	1.01
BAA[R <sub>t+1</sub> <sup>K</sup> ]	0.49	0.84	1.7	–	–	1.06	1.10
NW (FoF)	2.48	6.92	2.8	–	–	1.10	1.11
NB (FoF)	11.6	12.8	1.1	–	–	–	1.64
NW (DS)	10.3	11.4	1.1	–	–	1.10	1.11
NB (DS)	11.4	15.2	1.3	–	–	–	1.64
efpBA	0.04	0.10	2.8	–	–	–	1.64
efpA	0.20	1.17	5.9	–	–	–	1.16
efpB	0.21	0.71	3.3	–	–	1.62	1.18
efpBA	0.04	0.10	2.8	–	–	–	–
efpAG	0.05	0.17	3.4	–	–	–	–
efpBG	0.07	0.22	3.1	–	–	–	–

**Table B.15:** Change in volatility relative to the baseline model due to downward shift in bank incentive constraint  $\Theta^B$  by 50 %.

Standard Deviations							
	[b]'85-2005	[c]'06-2014	$\frac{c}{b}$	NK	GK	FA	FA-GK
	% $\sigma_{data}$			$\frac{\% \sigma'_{x_i}}{\% \sigma_{x_i}}(\text{model})$			
Y	0.98	1.16	1.2	–	1.00	–	1.01
C	0.73	0.93	1.3	–	0.99	–	1.00
I	3.22	4.17	1.3	–	0.99	–	1.01
H	0.92	1.34	1.5	–	1.00	–	1.01
W	1.11	0.71	0.6	–	1.00	–	1.00
$\Pi$	0.12	0.17	1.4	–	1.01	–	1.01
Q	–	–	–	–	1.11	–	1.11
R	0.20	0.27	1.4	–	1.01	–	1.01
Rn	0.24	0.28	1.2	–	0.99	–	1.00
BAA[ $R_{t+1}^K$ ]	0.49	0.84	1.7	–	1.06	–	1.14
NW (FoF)	2.48	6.92	2.8	–	–	–	1.13
NB (FoF)	11.6	12.8	1.1	–	1.14	–	1.12
NW (DS)	10.3	11.4	1.1	–	–	–	1.13
NB (DS)	11.4	15.2	1.3	–	1.14	–	1.12
<i>efpBA</i>	0.04	0.10	2.8	–	–	–	1.14
<i>efpA</i>	0.20	1.17	5.9	–	–	–	1.26
<i>efpB</i>	0.21	0.71	3.3	–	1.38	–	1.26
<i>efpBA</i>	0.04	0.10	2.8	–	–	–	–
<i>efpAG</i>	0.05	0.17	3.4	–	–	–	–
<i>efpBG</i>	0.07	0.22	3.1	–	–	–	–

**Table B.16:** Change in volatility relative to the baseline model due to a decline in investment adjustment cost ( $IAC$ ) from  $\phi_X = 2$  to  $\phi_X = 1$ .

Standard Deviations							
	[b]'85-2005	[c]'06-2014	$\frac{c}{b}$	NK	GK	FA	FA-GK
	% $\sigma_{data}$			$\frac{\% \sigma'_{x_i}}{\% \sigma_{x_i}}(\text{model})$			
Y	0.98	1.16	1.2	1.13	1.21	1.17	1.22
C	0.73	0.93	1.3	1.07	0.93	0.94	0.93
I	3.22	4.17	1.3	1.09	1.34	1.32	1.32
H	0.92	1.34	1.5	1.08	1.03	0.99	1.06
W	1.11	0.71	0.6	1.09	1.03	1.02	1.04
$\Pi$	0.12	0.17	1.4	1.00	0.97	0.95	0.99
Q	–	–	–	0.73	0.96	0.99	0.93
R	0.20	0.27	1.4	0.99	0.99	0.98	1.00
Rn	0.24	0.28	1.2	0.96	0.95	0.94	0.96
BAA[ $R_{t+1}^K$ ]	0.49	0.84	1.7	–	1.02	0.99	1.03
NW (FoF)	2.48	6.92	2.8	–	–	0.99	0.93
NB (FoF)	11.6	12.8	1.1	–	0.97	–	0.95
NW (DS)	10.3	11.4	1.1	–	–	0.99	0.93
NB (DS)	11.4	15.2	1.3	–	0.97	–	0.95
<i>efpBA</i>	0.04	0.10	2.8	–	–	–	0.95
<i>efpA</i>	0.20	1.17	5.9	–	–	–	1.07
<i>efpB</i>	0.21	0.71	3.3	–	1.06	1.00	1.06
<i>efpBA</i>	0.04	0.10	2.8	–	–	–	–
<i>efpAG</i>	0.05	0.17	3.4	–	–	–	–
<i>efpBG</i>	0.07	0.22	3.1	–	–	–	–

**Table B.17:** Change in volatility due to shift in policy feedback to inflation: the ratio of standard deviation (STD) of variable  $X_{i,t}$  when  $\theta_P = 1.80$  to the STD of the same  $X_{i,t}$  when  $\theta_P = 1.5$ .

Standard Deviations							
	[b]'85-2005	[c]'06-2014	$\frac{c}{b}$	NK	GK	FA	FA-GK
	% $\sigma_{data}$			$\frac{\% \sigma'_{x_i}}{\% \sigma_{x_i}}(\text{model})$			
Y	0.98	1.16	1.2	0.95	1.01	1.01	1.01
C	0.73	0.93	1.3	1.12	0.99	1.00	0.98
I	3.22	4.17	1.3	0.83	1.00	1.01	1.00
H	0.92	1.34	1.5	1.03	0.98	0.98	0.98
W	1.11	0.71	0.6	1.03	0.97	0.98	0.98
$\Pi$	0.12	0.17	1.4	0.95	0.90	0.90	0.91
Q	–	–	–	0.71	1.00	1.00	1.00
R	0.20	0.27	1.4	0.96	0.95	0.95	0.95
Rn	0.24	0.28	1.2	1.05	1.03	1.02	1.03
BAA[ $R_{t+1}^K$ ]	0.49	0.84	1.7	–	0.96	0.95	0.98
NW (FoF)	2.48	6.92	2.8	–	–	1.00	1.00
NB (FoF)	11.6	12.8	1.1	–	0.98	–	0.99
NW (DS)	10.3	11.4	1.1	–	–	1.00	1.00
NB (DS)	11.4	15.2	1.3	–	0.98	–	0.99
<i>efpBA</i>	0.04	0.10	2.8	–	–	–	1.00
<i>efpA</i>	0.20	1.17	5.9	–	–	–	1.01
<i>efpB</i>	0.21	0.71	3.3	–	1.00	0.98	1.01
<i>efpBA</i>	0.04	0.10	2.8	–	–	–	–
<i>efpAG</i>	0.05	0.17	3.4	–	–	–	–
<i>efpBG</i>	0.07	0.22	3.1	–	–	–	–

**Table B.18:** Change in volatility due to shift in policy feedback to output: The ratio of standard deviation (STD) of variable  $X_{i,t}$  when  $\theta'_Y = 0.025$  to STD of the same  $X_{i,t}$  when  $\theta_Y = 0.125$ .

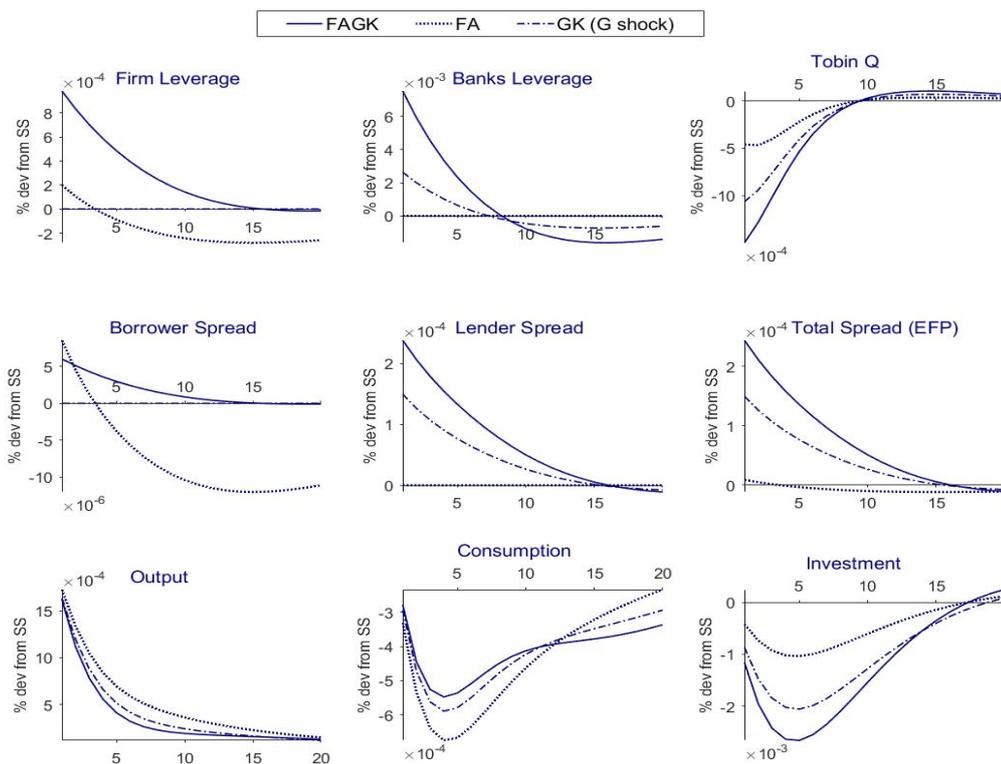
Standard Deviations							
	[b]'85-2005	[c]'06-2014	$\frac{c}{b}$	NK	GK	FA	FA-GK
	% $\sigma_{data}$			$\frac{\% \sigma'_{x_i}}{\% \sigma_{x_i}}(\text{model})$			
Y	0.98	1.16	1.2	0.95	1.01	1.00	1.03
C	0.73	0.93	1.3	1.08	1.02	1.00	1.06
I	3.22	4.17	1.3	0.83	1.02	0.99	1.05
H	0.92	1.34	1.5	1.10	1.08	1.06	1.09
W	1.11	0.71	0.6	1.06	1.05	1.02	1.08
$\Pi$	0.12	0.17	1.4	1.28	1.32	1.28	1.37
Q	–	–	–	0.68	0.99	0.97	1.04
R	0.20	0.27	1.4	1.12	1.17	1.14	1.21
Rn	0.24	0.28	1.2	1.12	1.09	1.08	1.09
BAA[ $R^K_{t+1}$ ]	0.49	0.84	1.7	–	1.12	1.10	1.16
NW (FoF)	2.48	6.92	2.8	–	–	0.95	1.05
NB (FoF)	11.6	12.8	1.1	–	1.01	–	1.09
NW (DS)	10.3	11.4	1.1	–	–	0.95	1.05
NB (DS)	11.4	15.2	1.3	–	1.01	–	1.09
<i>efpBA</i>	0.04	0.10	2.8	–	–	–	1.07
<i>efpA</i>	0.20	1.17	5.9	–	–	–	1.05
<i>efpB</i>	0.21	0.71	3.3	–	1.02	0.97	1.05
<i>efpBA</i>	0.04	0.10	2.8	–	–	–	–
<i>efpAG</i>	0.05	0.17	3.4	–	–	–	–
<i>efpBG</i>	0.07	0.22	3.1	–	–	–	–

**Table B.19:** Change in volatility relative to the baseline model due to shift in policy feedback to output from  $\theta_Y = 0.125$  to  $\theta'_Y = 0.025$  with higher persistence up from  $\rho_R = 0.80$  to  $\rho'_R = 0.88$ .

Standard Deviations							
	[b]'85-2005	[c]'06-2014	$\frac{c}{b}$	NK	GK	FA	FA-GK
	% $\sigma_{data}$			$\frac{\% \sigma'_{x_i}}{\% \sigma_{x_i}}(\text{model})$			
Y	0.98	1.16	1.2	1.00	1.12	1.05	1.18
C	0.73	0.93	1.3	1.17	1.17	1.09	1.29
I	3.22	4.17	1.3	0.86	1.16	1.07	1.28
H	0.92	1.34	1.5	1.19	1.29	1.18	1.39
W	1.11	0.71	0.6	1.39	1.40	1.34	1.46
$\Pi$	0.12	0.17	1.4	1.83	2.07	1.92	2.27
Q	–	–	–	0.80	1.30	1.20	1.40
R	0.20	0.27	1.4	1.49	1.66	1.57	1.79
Rn	0.24	0.28	1.2	0.97	1.00	0.97	1.03
BAA[R $^K_{t+1}$ ]	0.49	0.84	1.7	–	1.63	1.57	1.69
NW (FoF)	2.48	6.92	2.8	–	–	1.24	1.46
NB (FoF)	11.6	12.8	1.1	–	1.50	–	1.63
NW (DS)	10.3	11.4	1.1	–	–	1.24	1.46
NB (DS)	11.4	15.2	1.3	–	1.50	–	1.63
<i>efpBA</i>	0.04	0.10	2.8	–	–	–	1.56
<i>efpA</i>	0.20	1.17	5.9	–	–	–	1.34
<i>efpB</i>	0.21	0.71	3.3	–	1.33	1.31	1.35
<i>efpBA</i>	0.04	0.10	2.8	–	–	–	–
<i>efpAG</i>	0.05	0.17	3.4	–	–	–	–
<i>efpBG</i>	0.07	0.22	3.1	–	–	–	–

Figure B.2: IRFs due to (-1.0%) IST and (+1.0%) G shock: GK, FA and FAGK models.

(a) IRFs upon Government Spending (G) shock.



(b) IRFs upon Investment Specific Technology (IST) shock.

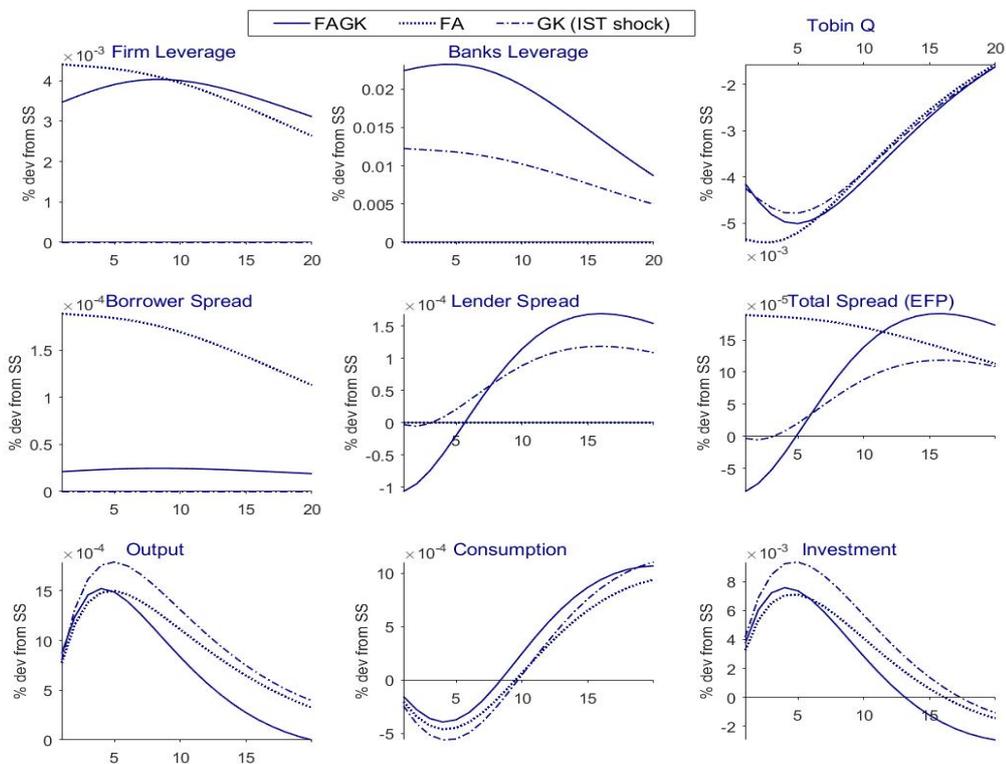
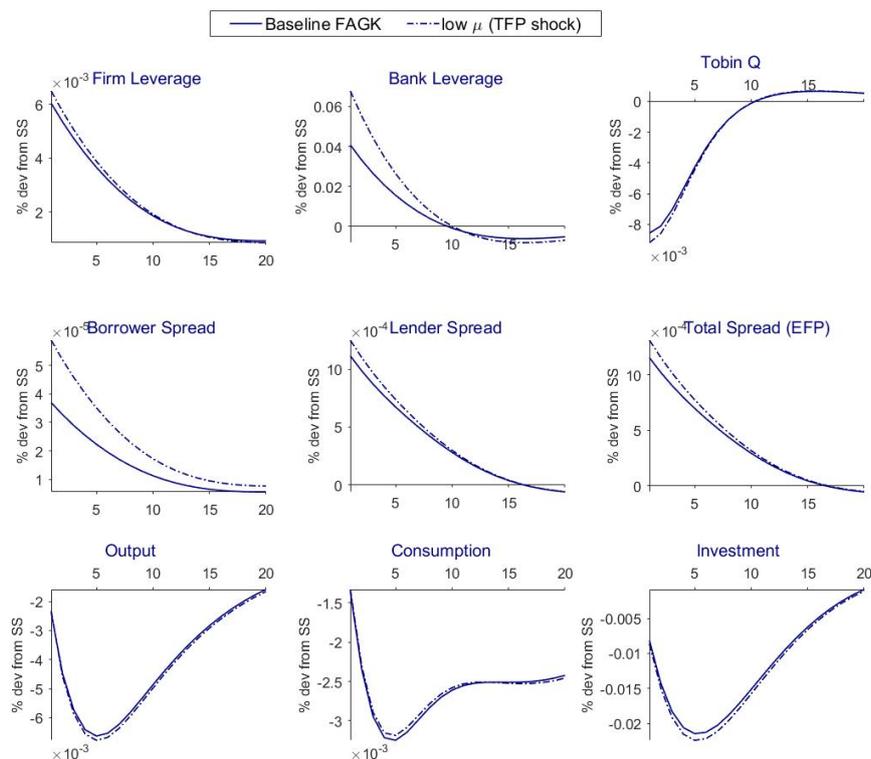


Figure B.3: IRFs after a shift in monitoring cost  $\mu$ : Baseline  $\mu = 0.11$ , higher  $\mu' = 0.17$ . FAGK model only.

(a) IRFs upon TFP shock.



(b) IRFs upon MP shock.

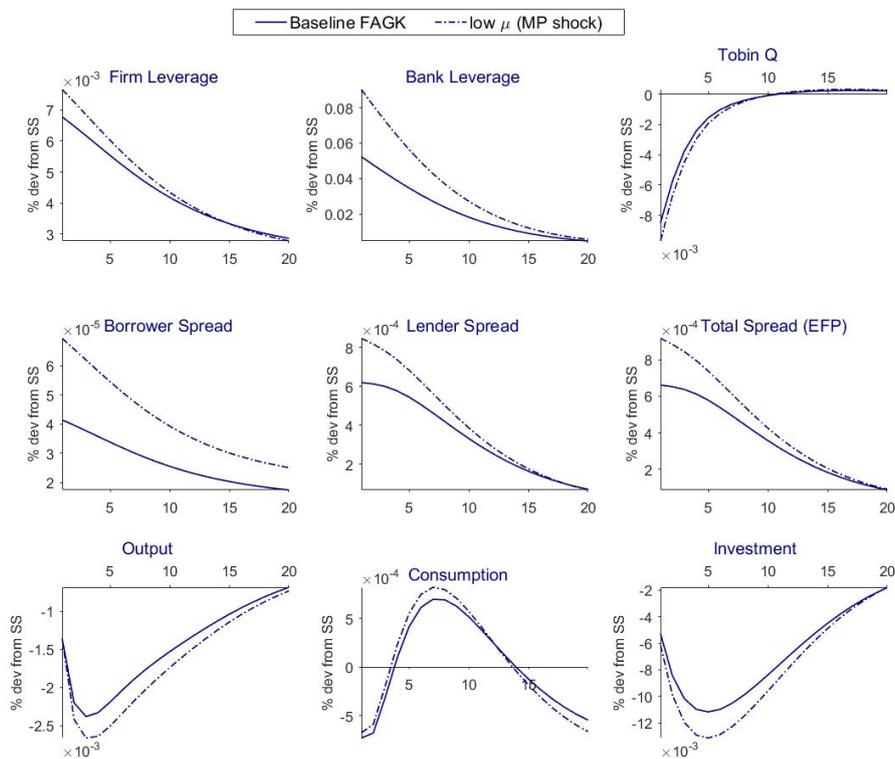
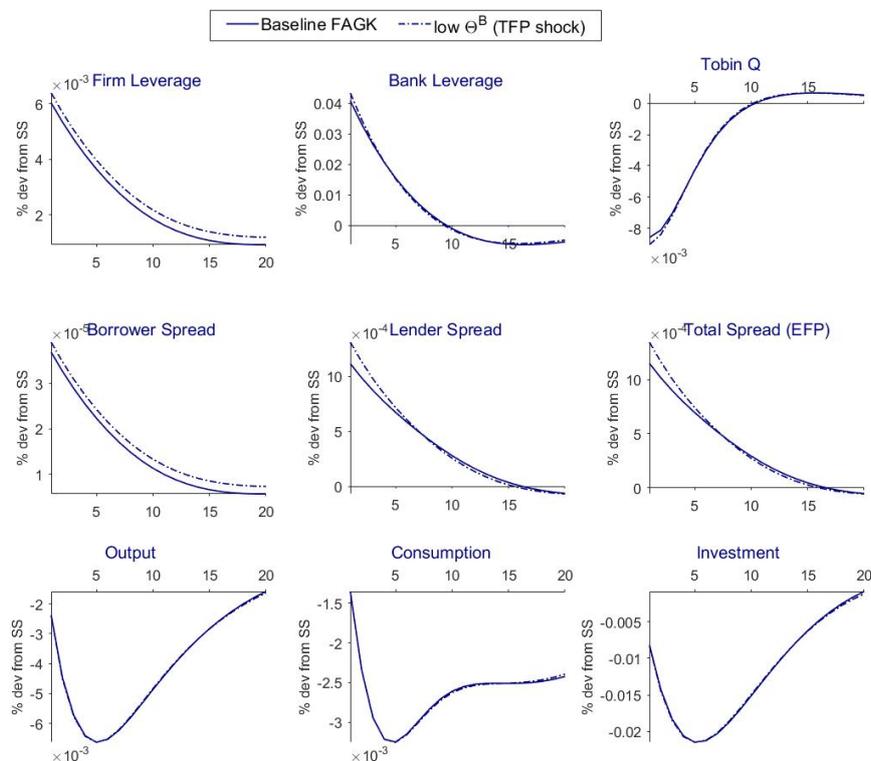


Figure B.4: IRFs after a shift in bank friction parameter  $\Theta_B$ : Baseline  $\Theta_B = 0.76$ , lower  $\Theta'_B = 0.54$ . FAGK model only.

(a) IRFs upon TFP shock.



(b) IRFs upon MP shock.

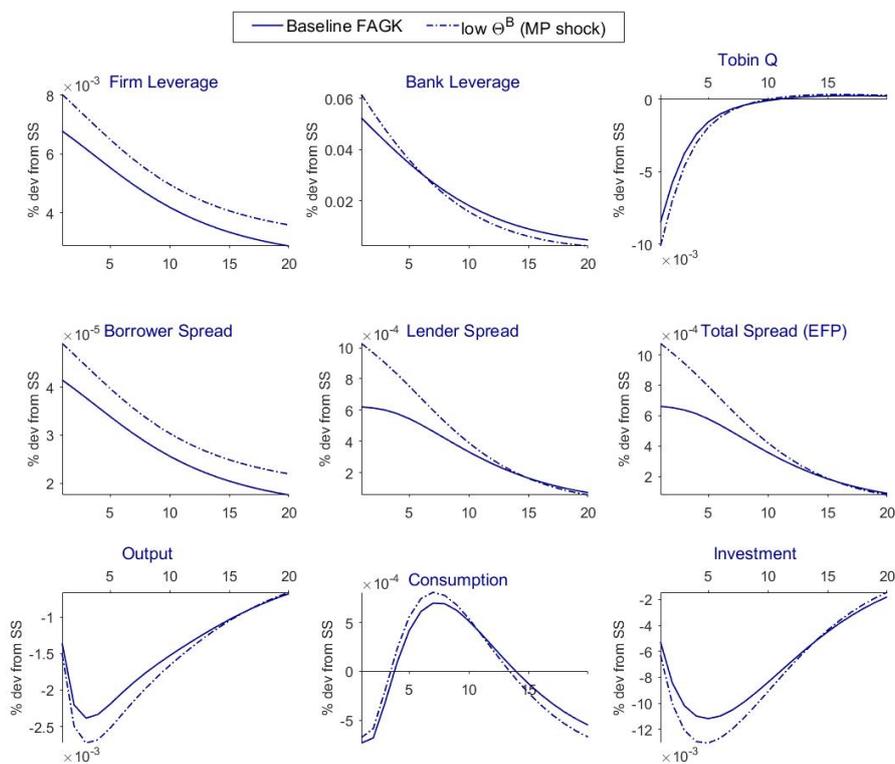


Figure B.5: IRFs after a decline in investment adjustment cost (IAC) from  $\phi_X = 2$  to  $\phi_X = 1$ .

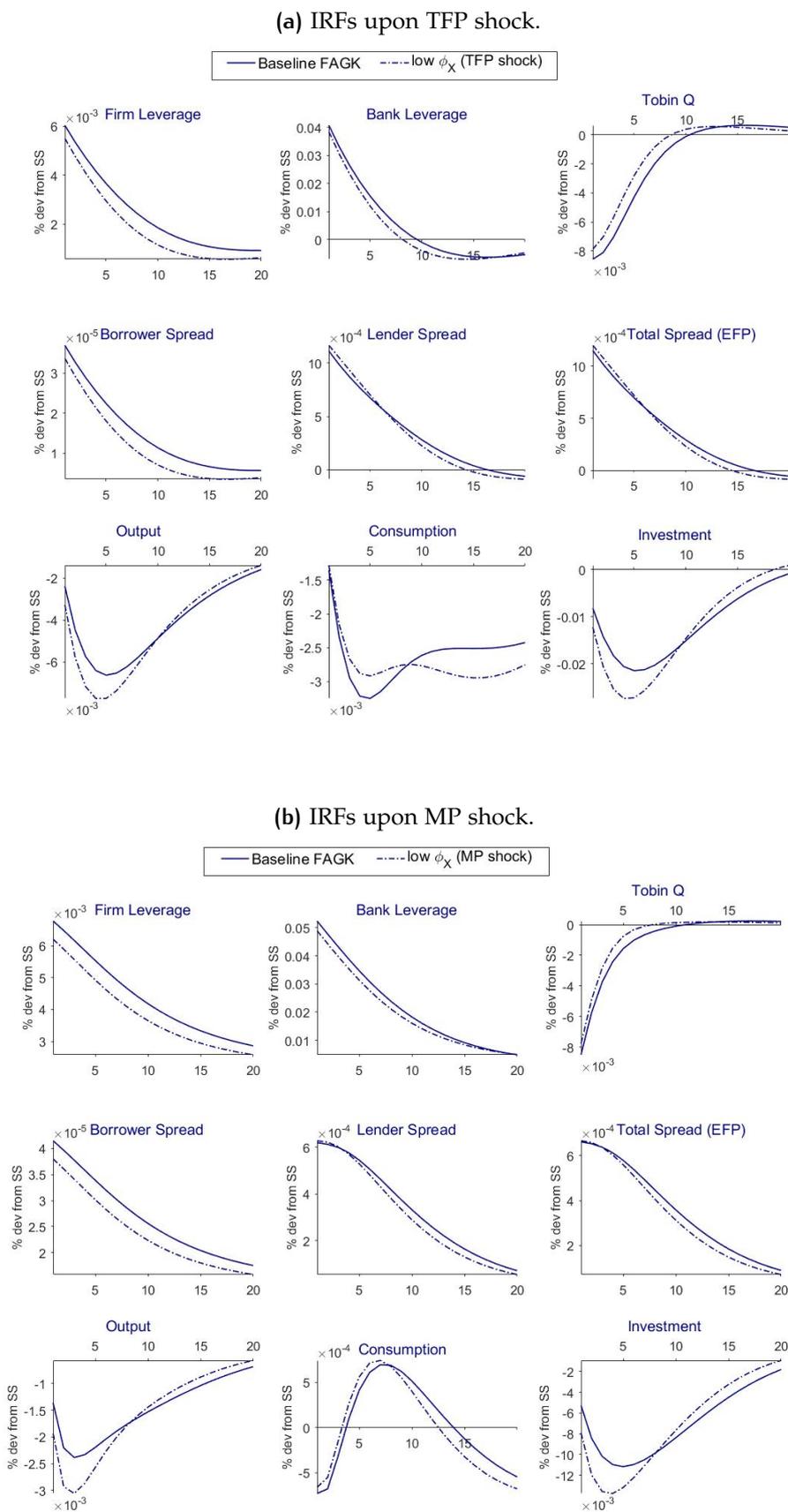
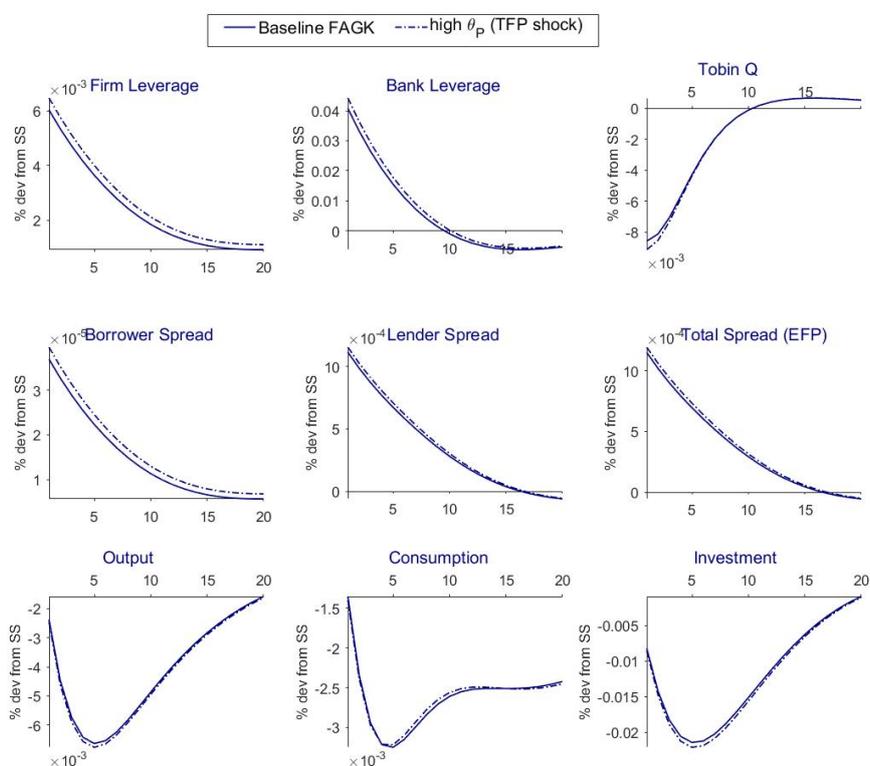


Figure B.6: IRFs after a shift in policy feedback to inflation  $\theta_P$ : Baseline  $\theta_P = 1.5$ , higher  $\theta'_P = 1.8$ .

(a) IRFs upon TFP shock.



(b) IRFs upon MP shock.

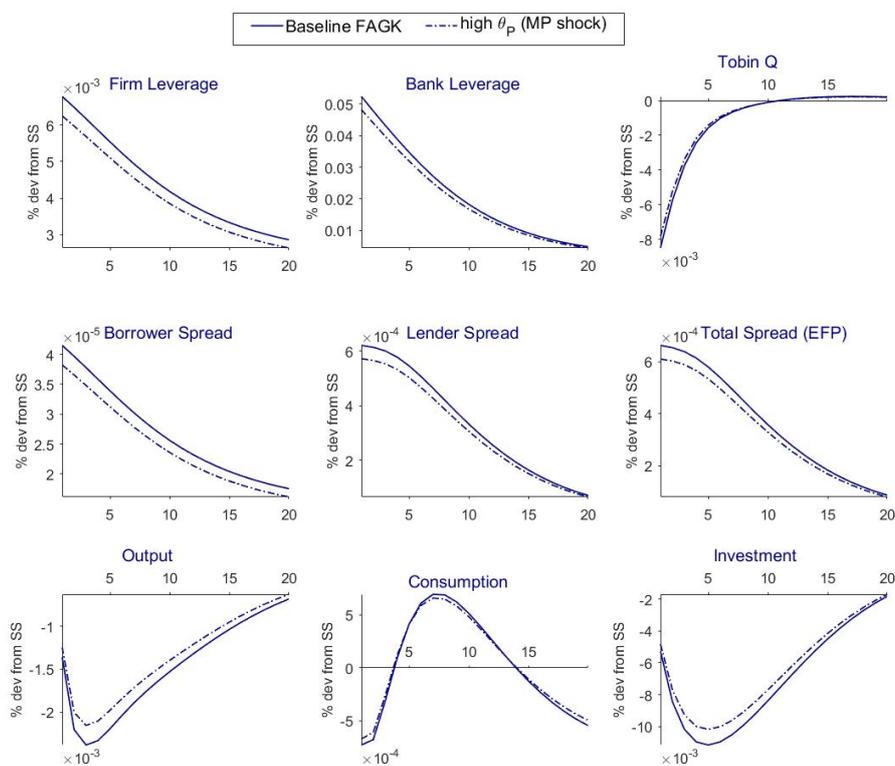
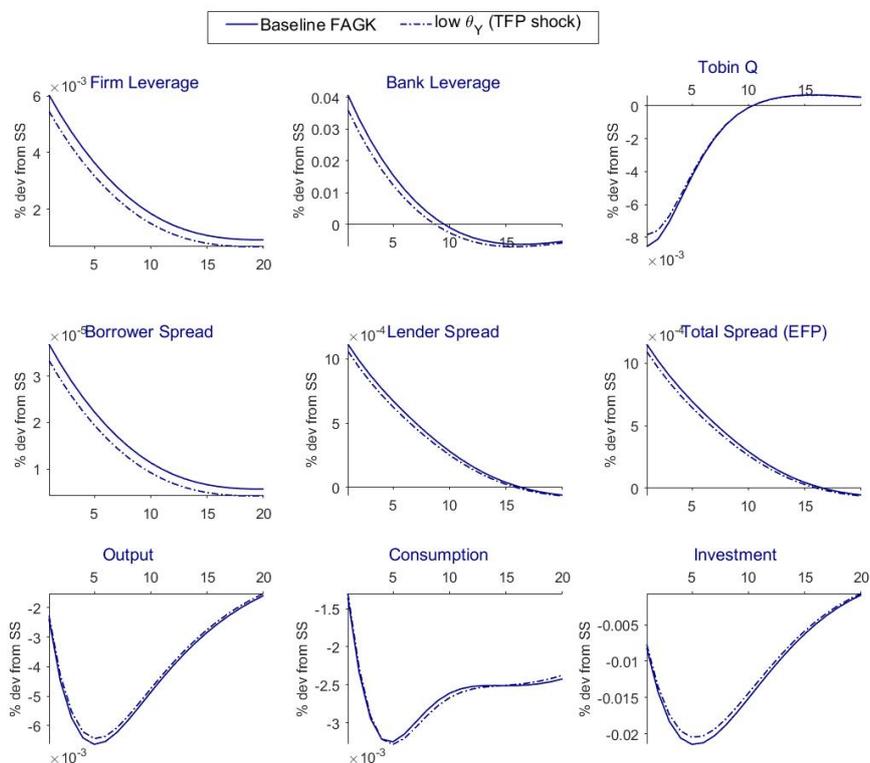


Figure B.7: IRFs after a shift in policy feedback to output  $\theta_Y$ : Baseline  $\theta_Y = 0.125$ , lower  $\theta'_Y = 0.025$ .

(a) IRFs upon TFP shock.



(b) IRFs upon MP shock.

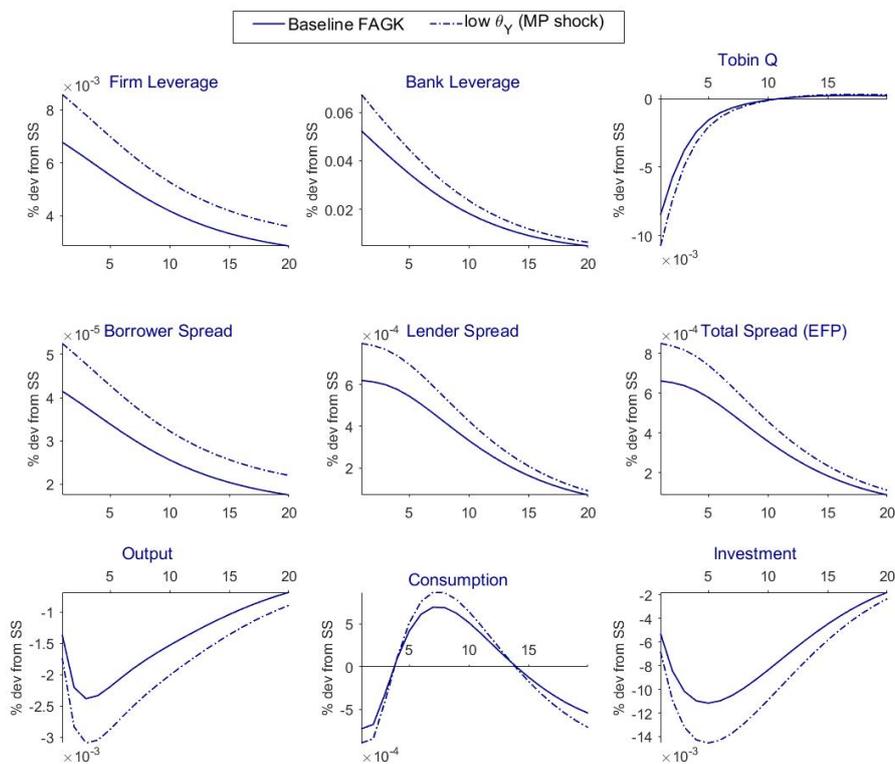
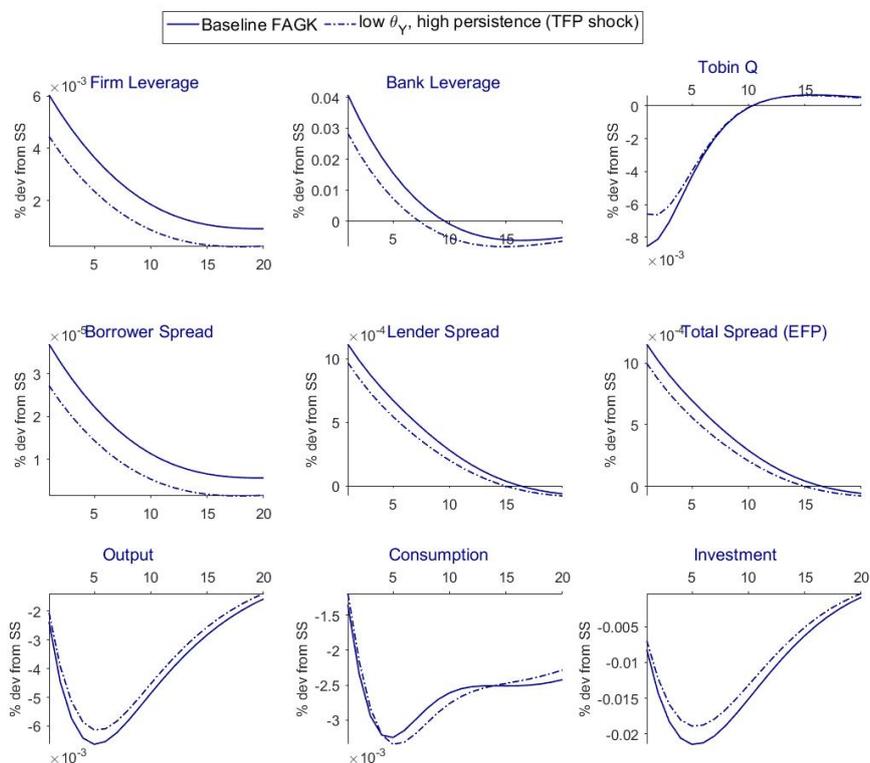
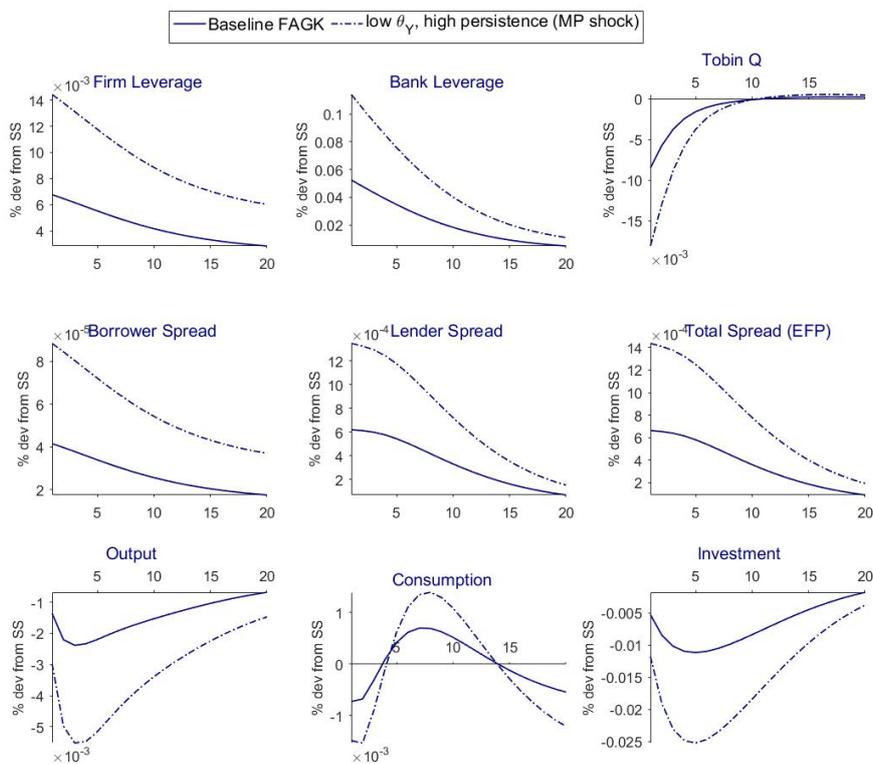


Figure B.8: IRFs after a shift in policy feedback to output  $\theta_Y$ : Baseline  $\theta_Y = 0.125$  and  $\rho_R = 0.80$ , lower  $\theta'_Y = 0.025$  and  $\rho_R = 0.88$ .

(a) IRFs upon TFP shock.



(b) IRFs upon MP shock.



# Chapter 4

## Assessing the Propagation Dynamics during the US Recession

### 4.1 Introduction

In this chapter I assess how the propagation of shocks changed during the recent financial crisis in US. To highlight the recession I compare the business cycle properties of actual financial and macroeconomic data during recession relative to the properties of the sample data before the year 2005. Then, the strategy is to estimate the deep parameters of the model during each subsample and assess the implications for the propagation of shocks. The objective is to shed light on the underlying factors that might have changed and led to the greater volatility of cost of external financing and of macro-financial indicators during the last recession.

The chapter will proceed with a summary of business cycle properties of each sample period, the Great Moderation and the *Great Recession*. In the following section, I describe the methodology of estimation, the data and the exogenous shocks driving the competing models at hand. I make a horse race of the financial accelerator model (*FA*), the bank friction model (*GK*) and of

my two sided-friction baseline model (*FAGK*). My aim is to assess the business cycle properties of the model-implied economy that each of three models yields based on estimated parameters. I conclude that my baseline model can outperform the other two models, *FA* and *GK*, in describing the economy for the full period of available data, 1955-2014.

In the following section I evaluate the stability of parameters for the baseline model *FAGK* in the two selected sub-samples of interest, 1985-2004 and 2005-2014. The Great Moderation period 1985-2004 is selected as a *tranquil* period of reference. The results suggest that the posterior estimates of several parameters change from the *tranquil* period to the *recession* period.

To quantitatively assess the implications of the change in posterior estimates of parameters for the business cycle properties of the model-implied economy I run *counterfactual* exercises. The exercises help identify the extent to which changes in the parameters shape the business cycle properties of the model variables.

From this assessment I conclude that the changes in two parameters, the decline in investment adjustment costs *IAC* parameter and the increase in dispersion of returns across borrowing firms are critical for the propagation of shocks during the Great Recession. I motivate why slight changes in estimates of some of the parameters are not significant for the business cycle features of the model economy during the recession. In the last section of the chapter I conclude.

## 4.2 Stylized Facts

In this section I summarize the business cycle statistics for the recession period and compare them to the preceding Great Moderation period.

I select the sub-period 1985-2004 as the sample that represents the *tranquil* period for the business cycle properties of the US economy. I motivate the choice of the starting point of the *tranquil* period based on reference from [Stock and Watson \(2002\)](#). The *tranquil* period ends with the start of the recent recession. This selection is also motivated by the higher volatility of macroeconomic and financial variables for the US economy. Then I define the **recession** period, 2005-2014, as the period when the volatility of key macroeconomic and financial data spiked<sup>1</sup>.

The Great Moderation is the best time period for the post-war US economy in terms of the stability of macroeconomic indicators. To emphasize the change in business cycle statistics during the recession period I compare the standard deviations of data series in the recession period to the second moments of the Great Moderation period. In columns [1] and [2] of table [C.1](#) I show the standard deviations of macro-financial variables for the two respective sub-samples. In column [3] I show the ratio of the standard deviation in recession period relative to that in the *tranquil* period.

< Table [C.1](#) here >

The volatility of most series went up significantly during recession. The standard deviation of macroeconomic indicators like output, consumption, inflation and nominal interest rate ( $Y, C, \Pi, R^n$ ) went up by around 20-25% higher during the recession period. Investment,  $I$ , and of hours of work,  $H$ , became even more volatile. Their standard deviation went up by 69% and 54% respectively.

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<sup>1</sup> For the sake of business cycle statistics of data series the choice of the year 2005 as the start of the recession sample period does not make much difference. Choosing the year 2006 I obtain almost the same business cycle statistics (or slightly higher volatility). The motivation for the longer sub-sample to define the recession period is that any shorter period makes the observable series included in estimation less stationary. The large V-shape in macro variables and the sharp spike in financial variables during the recession sub-sample makes these series less stationary when the series are short.

The increase in the volatility of financial variables is similar in magnitude to the increase in volatility of investment. Standard deviations of Moody's BAA corporate bond yield deflated by GDP deflator,  $BAA_{real}$ , of total external finance premium (hereby  $efpB$ ) and of lender premium (hereby  $efpA$ ), went up by around 40-50%. An exception is the standard deviation of borrower premium (hereby  $efpBA$ ). The standard deviation of this spread and that of firm net worth spike by more than three times the standard deviation in *tranquil* period. Such a high correlation in these two variables motivates why I relate the observable  $efpBA$  spread to the 'borrower premium' in the model framework.

In the same table C.1, in column [6], I show how the correlation with output of each variable changed during the recession period<sup>2</sup>. The main picture is that for those variables that already had a strong correlation with output, mainly macro variables, the change (in correlation) is small. For those variables that had a weaker (positive or negative) correlation with output, mainly financial variables, that correlation became much stronger during recession. The countercyclical (procyclical) variables became more countercyclical (procyclical) during the recession<sup>3</sup>.

## 4.3 Bayesian Estimation

I use standard Bayesian estimation techniques to estimate the deep parameters of the models. The estimation procedure for all the models went through two steps. The first step is maximization of the posterior distribution in order to maximize the posterior mode. The process combines the prior distribution of structural parameters that I estimate with the likelihood of the data. To evaluate the likelihood function I employ the Kalman filter by assuming normally

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<sup>2</sup> Column [6] has the difference in correlation between the two periods.

<sup>3</sup> One reason for the "stronger correlation" during recession is also the short number of observations included in the recession period compared to the *tranquil* period.

distributed i.i.d errors. To do so I employ at least two optimization algorithms to compute the mode of the posterior<sup>4</sup>.

In the second step, I use random walk Metropolis-Hastings (MH) algorithm to obtain draws from the posterior distribution. I run two Markov chains of 300,000 replications each to obtain diagnostic statistics regarding the convergence within and between chains (Brooks and Gelman (1998)). 25% of draws are discarded in order to minimize dependence (of the chain) on starting values. The data series are filtered through a one-sided HP filter to obtain business cycle properties (Pfeifer (2013)). I exclude the initial 16 data points of each observable series from the estimation of the likelihood function. Ideally, the likelihood function will not depend on the erratic behaviour of initial data points of filtered series.

#### 4.3.1 Data

In this subsection I discuss the observable series used in estimation. In all the models I include the same seven macroeconomic time series which are similar to the data set in Smets and Wouters (2007). In addition, I include 2 financial time series in the baseline, *FAGK*, model but only 1 financial time series in the one-sided friction models, *FA* and *GK*. The total number of time series (and exogenous shocks) used in estimation of the *FAGK* model is 9. The total number of time series used in *FA* and *GK* estimation is 8.

The set of 7 macroeconomic quarterly US time series I construct for the time period 1955-2014 is listed below<sup>5</sup>:

---

<sup>4</sup> Due to the large number of equations simple algorithms like the the Chris Sims algorithm can not find the mode of the posterior. Instead I employ algorithms 8 and 9 of Dynare. These algorithms are (informally) widely suggested by Dynare authors for models with potentially more than one local mode.

<sup>5</sup> While some data series can be observed for longer time periods, starting from 1947, the time period for which I can observe all the time series used is 1955-2014. In addition it is a common practice in literature to skip some of the data points at the beginning of the series as they are

- log of real per capita GDP,
- log of real per capita consumption, defined as non-durable consumption and services,
- log of real per capita gross private investment,
- log of weekly hours worked,
- log of labor share,
- log of GDP deflator,
- federal funds rate,
- borrower's premium: Moody's Baa-rated corporate bond yield minus the Aaa-rated corporate bond yield,
- lender's premium: Moody's Aaa-rated corporate bond yield minus the Fed rate.

The construction of these time series follows the same definitions as in [Smets and Wouters \(2007\)](#) and are common across the literature. For a detailed description I refer to appendix [B.1.11](#) (appendix of chapter 3). I save the space in this section to motivate the inclusion of financial variables that are the focus of this work.

The rationale for the inclusion of two financial observables (and exogenous shocks) in *FAGK* is twofold. First, I can track the dynamics of two external finance premiums, borrower and lender premium. I have two observable series, one for each premium, and two model counterparts that match them. By including the observable series for each premium I can identify the propagation of shocks due to financial health of the borrowing firms and of the lending bank.

Bayesian estimation requires that the number of shocks match at least the number of observables. This brings me to the the second argument why I include 2 additional observables and shocks in *FAGK*. In the baseline model I not considered as good representatives of the full series (see for example [Smets and Wouters \(2007\)](#) or [Christiano et al. \(2010\)](#)).

bring two shocks on the financial friction parameters, monitoring cost ( $\mu$ ) and bank incentive constraint ( $\Theta^B$ ). By including two premiums, I can match the shock on friction parameters with the respective observable premium.

As will be clear later in subsection 4.3.3, the 8<sup>th</sup> observable series included in estimation of *FA* and *GK* models is the total premium *efpB*. By construction, I have one friction parameter on which I can introduce the 8<sup>th</sup> shock in these models, either  $\mu$  or  $\Theta^B$  but not both. I dwell more on single-friction models in subsection 4.3.3.

Next, I define the 8<sup>th</sup> and the 9<sup>th</sup> variables that I include in estimation of *FAGK*. The two financial premiums are:

- *efpBA*, the spread between Moody's BAA-rated corporate bond yields relative to Aaa-rated yields, and
- *efpA*, the spread between Moody's Aaa-rated corporate bond yield and the Fed's risk free rate.

The first spread, *efpBA*, is assumed to vary only due to the financial health of the firms with a lower rating, therefore with a worse financial health. In the theoretical framework this spread captures the borrowing entrepreneur's premium.

The model counterpart of the series *efpBA* is the difference between the expected return of entrepreneur's project,  $E_t R_{t+1}^K$ , less the expected required return on bank assets,  $E_t R_{t+1}^B$ . It is the extra return a bank will charge on a borrower accounting for the agency costs on lender-borrower relationship provided that the borrower may default with a certain nonzero probability. This premium accounts only for the additional risk related to the quality of the borrower to whom the lender agrees to lend.

The second financial variable, *efpA*, is much closely related to the premium charged by the lenders on firms with very good financial health and negligible or no risk of default. AAA-rated firms have a sound financial health, hence fluctuations of the premium firms pay to raise debt is closely related to turbulence in the financial markets. That is a higher premium on firms rated AAA could be due to a higher required return on assets of all the lenders across the market. In practice this is the premium lenders require on top of what they pay on liabilities in order to transform their short term liabilities into long term assets. Therefore, in my theoretical framework, this spread can best capture the minimum premium that the lenders require from a typical healthy firm that has negligible or no risk of default.

The matching premium in the model that corresponds to the observable series *efpA* is the spread between bank's required return on assets,  $E_t R_{t+1}^B$ , and risk free rate,  $R_t$ . This spread is the average return a financial intermediary is required to earn on its assets to keep the net worth at a certain level that it convinces its depositors not to withdraw their funds (deposits). With higher net worth the lender will build net worth to a level that it can address its moral hazard problem. Intuitively, it is net of any extra returns it needs to generate on its assets to cover for the losses due to borrower default or the monitoring costs.

The premium *efpA* would be equivalent to the premium charged on a safe government bond. Interestingly the premium on corporate bonds rated AAA based on Moody's rating system and long-term government bond is highly correlated (98 %).

Ultimately, no proxy for the unobservable external finance premium is perfect. Alternative measures through literature do not serve the scope of my study. During the last recession the spread between prime lending rate and

the 6-month Treasury bill rate used by [Bernanke et al. \(1999\)](#) did not show any volatility. Constructing a corporate credit spread like in [Gilchrist et al. \(2009b\)](#) goes beyond the scope of this thesis<sup>6</sup>. The advantage of using the spreads based on Moody's rating is that they match as closely as possible the borrower and lender premium and are updated and can be replicated over time.

Finally, I track the business cycle properties of two net worth series, lender and borrower net worth. I construct two sets of net worth series. The first set of firm and bank net worth series is constructed based on Flow of Funds data. The second set is constructed based on two stock price indices from *Datastream*<sup>7</sup>. Net worth series are not included as observables in estimation, so I refer these series in appendix [C.2](#).

I use one-sided filtered Hodrick-Prescott (*HP*) filtered data as suggested by [Pfeifer \(2013\)](#) to filter the series. I make use of the one-sided HP filter code provided online by [Meyer-Gohde \(2010\)](#) to filter the series.

#### 4.3.2 Exogenous Shocks

The number of exogenous shock processes that drive the models is the same with the number of observable series I use in each model.

I have 9 observables in my estimation, of which 7 are macroeconomic variables and 2 are financial variables. Estimation requires that I have (at least) as many shocks as the number of observables included in estimation. To match the 7 macroeconomic observables I define shocks on the new Keynesian equations of the model that introduce exogenous process in the following variables:

1. total factor productivity,  $A_t$ ,
2. government expenditures,  $G_t$ ,

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<sup>6</sup>The authors construct a corporate credit spread index using security-level data for individual firms.

<sup>7</sup>Similar approach with stock price index is followed [Christiano et al. \(2010\)](#).

3. monetary policy,  $\epsilon_t^R$ ,
4. investment specific technology,  $\Upsilon_t$ ,
5. retail price mark-up,  $\Lambda_t^M$ ,
6. labor supply shift,  $\delta_t^h$ ,
7. household preferences,  $b_t^u$ ,

The two financial variables are related to two key financial frictions inherent in the theoretical framework. The steady state values of the firm and bank friction parameters,  $\mu$  and  $\Theta^B$ , determine the steady state borrower and lender premium,  $efpBA$  and  $efpA$ , respectively. I introduce two shocks on these two friction parameters.

- A shock in the cost accrued by the banks in monitoring the borrowing firms,  $\mu_t$ , and
- a shock in the bank friction parameter determining the minimum bank value relative to total assets that depositors impose on the banks,  $\Theta_t^B$ .

I assume the fluctuations in the cost of monitoring can trigger fluctuations of the borrower risk premium captured by the spread between Moody's BAA and AAA rated corporate bond yields as a proxy ( $efpBA$ ). At times of distress the cost banks incur to monitor firms may deteriorate and therefore drive up the financial premium on lending.

In equilibrium, the premium banks will charge on any borrower with zero probability of default is a function of steady state value of  $\Theta^B$ . An exogenous shock on  $\Theta_t^B$  drives up the lender premium. A clear motivation for varying  $\Theta_t^B$  is the varying attitude of depositors towards the lenders. Fluctuations in bank friction,  $\Theta_t^B$ , will imply higher volatility of the premium that the bank

expects on its assets to accumulate sufficient net worth for the supplier of funds (households) not to liquidate the bank.

### Normalization of Shocks.

I follow [Justiniano et al. \(2011\)](#) to normalize some of the exogenous shocks. I normalize the price mark-up shock  $\Lambda_t^M$ , the labor supply shock  $\delta_t^h$ , the firm friction (monitoring cost) shock  $\mu_t$  and the bank friction shock  $\Theta_t^B$ . Normalization consists in writing the variables as a product of the steady state value of that variable and a variable with mean unity in steady state but which follows an autoregressive process AR(1). For any normalized variable  $\tilde{X}_t$  I re-write:

$$\tilde{X}_t = X * \hat{X}_t \quad (4.1)$$

In writing the exogenous process I only use the normalized variable with unit steady state  $\hat{X}_t$ . This process simplifies the choice of priors for the size of the shock  $\sigma^i$  for each exogenous process. I summarize the processes for the exogenous shocks in a list of equations.

$$\hat{A}_t = \rho_A \hat{A}_{t-1} + \sigma^A \epsilon_t^A \quad (4.2)$$

$$\hat{G}_t = \rho_G \hat{G}_{t-1} + \sigma^G \epsilon_t^G \quad (4.3)$$

$$\hat{\epsilon}_t^m = \rho_m \hat{\epsilon}_{t-1}^m + \sigma^m \epsilon_t^{\epsilon^m} \quad (4.4)$$

$$\hat{\Upsilon}_t = \rho_\Upsilon \hat{\Upsilon}_{t-1} + \sigma^\Upsilon \epsilon_t^\Upsilon \quad (4.5)$$

$$\hat{\Lambda}_t^M = \rho_\Lambda \hat{\Lambda}_{t-1}^M + \sigma^{\Lambda^M} \epsilon_t^{\Lambda^M} \quad (4.6)$$

$$\widehat{\delta}_t^h = \rho_H \widehat{\delta}_{t-1}^h + \sigma^{\delta^h} \epsilon_t^{\delta^h} \quad (4.7)$$

$$\widehat{b}_t^u = \rho_b \widehat{b}_{t-1}^u + \sigma^{b^u} \epsilon_t^{b^u} \quad (4.8)$$

$$\widehat{\mu}_t = \rho_\mu \widehat{\mu}_{t-1} + \sigma^\mu \epsilon_t^\mu \quad (4.9)$$

$$\widehat{\Theta}_t^B = \rho_{\Theta^B} \widehat{\Theta}_{t-1}^B + \sigma^{\Theta^B} \epsilon_t^{\Theta^B} \quad (4.10)$$

$\epsilon_t^i$  is normally distributed i.i.d with  $N(0,1)$  while  $\sigma^i$  is the size of the exogenous shock for  $i = A, G, M, \gamma, \Lambda^M, \delta^h, b^u, \mu, \Theta^B$ . Here I assume  $\rho_m = 0$ . The persistence of policy rate is captured by  $\rho_R \neq 0$  in the Taylor rule in equation 3.50 in section 3.2.7<sup>8</sup>.

### 4.3.3 Estimation of the FA and the GK Model

I estimate two single-friction models, the financial accelerator model (*FA*) and the bank friction model (*GK*), separately in this section. For each model estimation I use 8 observables and 8 shocks. The 7 macroeconomic time series and the relevant 7 shock processes are the same as in the baseline model. Here I define the 8<sup>th</sup> observable series.

The 8<sup>th</sup> observable in both single-friction models is *the total spread between Moody's BAA-rated corporate bond yield and the Fed's risk free rate (efpB)*. I use the same set of 8 observables in each single-friction model. The only few differences during the estimation of each single-friction model are listed below.

<sup>8</sup> I allow a more general form with  $1 > \rho_M \geq 0$ . The odds ratio rejected  $0 < \rho_M$ . Results are not shown.

- In the FA model there is no steady state of the bank friction parameter  $\Theta^B$  as well as no persistence and standard deviation parameters of an exogenous shock,  $\rho_{\Theta^B}$  and  $\sigma^{\Theta^B}$ , on this friction. The exogenous processes are the set of equations 4.2 through 4.8 (page 196) related to seven macroeconomic observables, plus the 8<sup>th</sup> equation (eq. 4.9) which is an exogenous process for the monitoring cost .
- Similarly, in the GK model the steady state of monitoring cost  $\mu$  and the relevant AR parameters  $\rho_{\mu}$  and  $\sigma^{\mu}$  are also absent. The shock processes for the GK model are equations 4.2 through 4.8 (page 196) and the exogenous process for the bank friction parameter, which is equation 4.10.

The quarterly observable time series have the same length as in the baseline scenario, 1955q1-2014q4, in both models. The priors for the common parameters are the same as in the baseline model (table C.2 in appendix C.1).

## 4.4 Parameter Estimates

For the sake of reference throughout this section I refer to my framework *FAGK* model with both frictions as *the baseline model*. The two terms, *FAGK* and *the baseline model* are used interchangeably. These terms distinguish my framework from the two single friction models, *FA* and *GK*.

### 4.4.1 Prior distribution

In this section I discuss the choice of priors. I will refer with the term *NK parameters* to all the parameters that are relevant only to a simple new Keynesian framework without any financial frictions. I refer with the term *Financial Fric-*

*tion parameters* to all those parameters that are only in the financial accelerator model and in the bank friction model, but not in a simple NK model.

I give more space to parameters on financial frictions block and the literature related to them and mention very briefly the priors of parameters related to new Keynesian framework. The priors are the same for all the three models estimated, *FA*, *GK* and *FAGK* (with exceptions on *FA* and *GK* mentioned in 4.3.3). I report the prior means and standard errors of all the estimated parameters in table C.2 in appendix C.

< Table C.2 here >

#### **Priors on Parameters relevant to new Keynesian block**

- Fixed NK Parameters

It is common in literature to fix some parameters that are not identified in the model (see for example [Smets and Wouters \(2007\)](#)). The argument is that these parameters are either not identified in the model or they determine the steady-state values of either macroeconomic or financial ratios. The public spending ratio,  $\frac{G}{Y}$ , and weekly hours of work,  $H$  are not identified in the model. I fix them at 0.20 and 0.35 respectively. The parameter  $\Pi_{ss}$  is fixed at 1.0084 which implies an average steady state inflation rate of 3.4% on annual basis. This is the average inflation for the full period 1955-2014. The household discount rate is fixed at 0.99 implying a risk free rate of 4% annually in real terms. The public spending ratio, and the discount rate are common values across the literature and consistent with the actual data for the US economy. Inflation rate is calculated on basis of data for the period 1955-2014.

I fix the Calvo parameter at 0.75. This value implies a contract duration of 3.3 quarters. This is a well documented parameter value across models of

general equilibrium and empirical ones. For estimates in general equilibrium models I refer to [Smets and Wouters \(2007\)](#) and [Levine et al. \(2005\)](#). For empirical studies that estimate a Calvo parameter value based on empirical data I refer to [Nakamura and Steinsson \(2008\)](#). [Nakamura and Steinsson \(2008\)](#) report a value closer to the estimate of the macro econometric models referred to in this study based on evidence on micro data<sup>9</sup>.

- Priors for NK Parameters

To estimate the posterior mode, I draw on standard values from the existing studies of [Smets and Wouters \(2007\)](#) and [Christiano et al. \(2010\)](#) as a rough guide. In general, the priors were set slightly looser initially to allow for the data to speak freely. Eventually, I modify most priors along the estimation process until I get reasonable likelihood functions. The prior values that I describe here (see table [C.2](#)) are the final prior values with which I ultimately draw from the posterior distribution.

I assume a beta distribution for the prior mean of the labor share in the production function and the habit formation of household. For both these parameters, the prior mean is 0.7 with standard error 0.1. The prior for investment adjustment cost parameter follows a Gaussian distribution with mean 4 and standard error 1.5. I choose a gamma distribution for the elasticity of substitution among goods with mean 7 and standard error 1. The prior mean implies a price markup at the steady state of around 0.167. The intertemporal elasticity of substitution parameter is normally distributed with mean 1 and standard deviation 0.375. The prior for the inverse of labor Frisch-elasticity is set at 1 with standard deviation of 2<sup>10</sup>.

<sup>9</sup> For empirical studies who report lower values of Calvo parameter see [Bils and Klenow \(2004\)](#).

<sup>10</sup> Labor supply shifter is calculated as function of the inverse of labor Frisch-elasticity,  $\rho^h$ , and the steady state hours of work.

The degree of price indexation follow a beta distribution located at a mean 0.4 with standard errors of 0.2. This value is slightly lower than the value of 0.5 with standard error 0.1 set in [Smets and Wouters \(2007\)](#) following the findings of [Bils and Klenow \(2004\)](#). The values for price indexation in the exiting studies vary in a wide range going as low as 0.11 (see [Levine et al. \(2005\)](#)). The wide standard deviation of this prior in my estimation allows for the data to determine the likelihood function within a reasonably wide range.

The response of monetary policy rate to inflation deviations,  $\theta^\pi$ , is normally distributed with mean 1.5 and standard error 0.15. I choose a gamma distribution for the feedback to output gap,  $\theta^Y$ , to ensure that the estimated mean is positive. I set a diffuse prior mean of 0.125 close to the zero boundary with standard error of 0.10. This type of distribution avoids estimates below zero. [Christiano et al. \(2010\)](#) set these priors at 1.75 and 0.25 respectively with same standard error of 0.1. [Smets and Wouters \(2007\)](#) set prior means at 1.5 and 0.125, and standard errors 0.25 and 0.05 respectively. My priors are in the same range with priors from these studies and close to values initially proposed by [Taylor \(1999\)](#).

I set a Beta prior mean of 0.75 with standard error 0.2 for the persistence of monetary policy,  $\rho_R$ . This is a looser prior compared to 0.75 with standard error 0.10 in [Smets and Wouters \(2007\)](#).

#### **Priors on Financial Friction Parameters**

- Fixed Financial Friction Parameters

There are few parameters from the block of financial frictions that can not be identified in estimation. I fix the survival probability of firms and banks,

$\sigma_E$  and  $\sigma_B$  at 0.985 and 0.977<sup>11</sup>. The firm survival probability parameter is pinned down so that in steady state firms survive for a period of around 17 years. Firm survival time is in the same range of 15-17 years assumed in the literature (Bernanke et al. (1999), Christiano et al. (2010)) and consistent with empirical estimates of Levin et al. (2004)<sup>12</sup>. The tenure of 10.8 for banks is similar to the tenure of around 10 years assumed in Gertler and Karadi (2011).

The parameter indicating the firms' dispersion  $A_{\overline{\psi}}$  in the financial accelerator block is fixed at 0.306 for both models, *FAGK* and *FA*. The prior mean for this parameter is obtained from the calibration phase and is similar to the value 0.28 assumed in Bernanke et al. (1999)<sup>13</sup>.

I also fix the average percentage of wealth that exiting firms and banks transfer to the newly entering firms and bankers,  $\xi_E$  and  $\xi_B$ , at 0.003 and 0.00023 respectively. The primary role of these parameters is to ensure that the firms do not accumulate enough net worth to grow out of debt. In addition, in the baseline framework *FAGK* the transfer rate,  $\xi_E$ , helps calibrate the steady state interest rates on debt loans above the risk free rate at a reasonable positive value. These values are usually fixed during the calibration stage of the model (with minor adjustments)<sup>14</sup>.

- Priors for Financial Friction Parameters

I assume a beta distribution for the firm default probability in the financial accelerator block of the *FAGK* model (and in *FA*). I set the prior for the steady state default probability of the firm,  $F_{\overline{\psi}}$ , at 0.75% on quarterly basis with a

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<sup>11</sup>These values are similar to those in the calibrated model from chapter 3.

<sup>12</sup>Levin et al. (2004) report a similar tenure for the median firm based on data for the sample period 1997-2003 coming from a panel of 900 firms.

<sup>13</sup>The value is calibrated to match a ratio of total assets to assets less loans of around 1.81 at steady state.

<sup>14</sup>The average return on loan portfolio less risk free rate,  $RL - R$ , is set at a reasonably positive value in steady state. I calibrate the amount of net worth that exiting firms transfer to newly entering firms to ensure that this spread is positive.

standard error of 0.2%. The prior mean at this value is standard in financial accelerator model (Bernanke et al. (1999), Carlstrom and Fuerst (1997)). There is an empirical evidence by Altman and Pasternack (2006) for default rates of US bonds that claim an average of 3 % for the period 1971 - 2005. It is the same with the value calibrated initially by Bernanke et al. (1999), so allows for some comparisons with earlier literature.

The prior on marginal bankruptcy cost,  $\mu$ , is an informative one that has to lie in the range 0-1. A prior mean of 0.15 and standard error of 0.10 is chosen to allow for an 95% interval that captures most possible values from micro evidence mentioned in Chapter 2 (Altman (1984), Alderson and Betker (1995), Levin et al. (2004))<sup>15</sup>. Finally the bank friction parameter,  $\Theta_B$  follows a beta distribution with prior mean 0.54 and standard error 0.15. This value is higher than the value of 0.37 calibrated in the Gertler and Karadi (2011) model. The prior mean in my framework is calibrated to get a reasonable leverage ratio for the bank. The standard error allows for a wide range of values for each respective parameter and let the data determine the degree of financial friction.

Finally, following Smets and Wouters (2007) I assume the standard errors of all the shocks follow inverse-gamma distribution with mean 0.10 and infinite degrees of freedom. Following this study the persistence parameters of those shocks follow a beta distribution with mean 0.5 and standard deviation 0.2.

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<sup>15</sup>Several studies shed light on the size of the bankruptcy costs for firms based on micro-evidence. Altman (1984), finds out that bankruptcy costs are less than 20% of the firm's value prior to bankruptcy, in a small sample of only 26 firms. Similar findings by Alderson and Betker (1995) analyze a larger sample of 201 firms that filed for bankruptcy during 1982-1993 and conclude that liquidation costs are on average 36.5 %. Levin et al. (2004) estimate a partial equilibrium model with financial accelerator using panel data for the period 1997-2004. They estimate a variable bankruptcy parameter which varies from close to zero to as high as 0.45.

### 4.4.2 Posterior Estimates

I report the posterior mode, mean and the 90% credible intervals obtained by Metropolis Algorithm for the three models, the *FA*, *GK* and the baseline model with two frictions, in tables C.3 in appendix C.

< Table C.3 here >

I will refer to the values reported in table C.3 throughout this section.

#### Estimates of NK Parameters

The mean estimates of parameters relevant to the new Keynesian part of the model are close to the estimates seen in the literature in all the three models, the *FA*, *GK* and the baseline *FAGK* model. A quick observation is that the persistence and the size of the exogenous shocks is similar across the three models. The magnitude of total factor productivity (TFP) persistence shock is 0.8 while persistence of other shocks is lower. One exception is the size of the persistence of labor supply shock which is close to 0.99. A potential reason for such a high persistence of the labor supply shock may be the assumption of flexible wages in the wage market. With flexible wages assumed in the model the burden of generating persistence on the hours of work and on wages falls on the exogenous labour supply shock. To replicate the behavior in real wage series or in hours of work the model estimate for the persistence of labour supply shock turns up very high.

For the remaining parameters, I will only refer to the posterior means of the baseline model with two frictions, as the posterior estimates of all parameters from the other two models, *FA* and *GK*, are in the same range.

There is a set of parameter estimates which are within the range of estimates in most studies. The posterior mean estimate for labor share of 0.76, the

posterior elasticity of substitution among goods of around 6.25, the posterior intertemporal elasticity of substitution in consumption of 1.9 and the posterior mean for inverse of Frisch elasticity of labor of 0.84 are all very similar to estimates in the literature<sup>16</sup>.

A second subset of parameters estimates are determined more by the likelihood of the data. The posterior mean for the adjustment cost parameter (*IAC*) of 1.14 is smaller than the prior mean of 4. The posterior mean of habit parameter in consumption at 0.40 is slightly lower than the prior 0.7 and lower than reported means in DSGE estimation across the studies ([Smets and Wouters \(2007\)](#), by [Justiniano et al. \(2011\)](#)). I can motivate the lower posterior estimate *IAC* and habit formation parameters due to the presence of additional financial frictions which may induce extra persistence in the dynamics of the model. As one introduces extra frictions in general equilibrium models, the burden of generating extra persistence in the variables is split among many more parameters. In this line of argument it is reasonable to get lower posterior estimates of the habit formation parameter.

The posterior mean estimates for monetary policy rule parameters are also slightly different from those reported in prominent papers. The posterior mean of the policy rate smoothing parameter,  $\rho_R = 0.62$ , and posterior of the response to inflation,  $\theta_p = 1.57$  are closer to the calibrated values in literature. There is a stream of papers, like [Smets and Wouters \(2007\)](#) and [Justiniano et al. \(2011\)](#), that report posterior estimates for policy rate smoothing parameters and inflation feedback parameter around values of 0.8 and 2.0 respectively. Yet my estimated means are within the range of values seen in other studies ([Christensen and Dib \(2008\)](#), [Christiano et al. \(2010\)](#)).

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<sup>16</sup>I compare my estimates to those reported in at least three articles, by [Smets and Wouters \(2007\)](#), by [Justiniano et al. \(2011\)](#) and the more recent one by [Fuentes-Alberro \(2014\)](#). The latter also includes financial accelerator block.

The more interesting posterior estimate I get is the feedback to output which is very high. The posterior mean estimate for the policy feedback to output of  $\theta_y = 0.24$  is higher than the estimates reported by [Smets and Wouters \(2007\)](#) and [Justiniano et al. \(2011\)](#), which vary around 0.15. This feature of stronger feedback to output and weaker reaction to inflation could be seen in estimated general equilibrium models with financial frictions ([Fuentes-Albero \(2014\)](#), [Christensen and Dib \(2008\)](#), [Christiano et al. \(2010\)](#)).

[Christensen and Dib \(2008\)](#) estimate a new Keynesian model with and without financial accelerator block via maximum likelihood method. They obtain a higher posterior estimate of 0.295 (0.0690) for the output response parameter in the model with financial accelerator compared to a posterior estimate of 0.138 (0.065) in the basic new Keynesian model.

Similarly, [Christiano et al. \(2010\)](#) estimate a model with financial accelerator and a liquidity (or banking) friction, though they include feedback to change in inflation and to credit growth in their Taylor rule. They obtain posterior estimates of 0.307 and 0.321 for the output feedback parameter in Euro area and US, respectively. The higher estimate of output response in models with financial frictions seems a common feature, though not in all of them (see [Gilchrist et al. \(2009b\)](#), or [De Graeve \(2008\)](#) for estimates lower than the calibrated values in the literature).

To reconcile this stronger feedback to output in models with financial frictions and lower estimate in simple new Keynesian models I can appeal to the extra propagation of shocks that is introduced by the added financial frictions in the model. As the financial friction may propagate shocks by a greater magnitude or more persistently than other frictions there is a greater burden on the policy rate to stabilize output. The greater burden on policy drives the posterior estimate of policy response to output up.

That the financial frictions, and their respective propagation role, are superior to other frictions in the basic new Keynesian models has been well established in the DSGE literature and supported by data (Christensen and Dib (2008), Gilchrist et al. (2009b), De Graeve (2008), Queijo von Heidiken (2009)). There is a challenging argument that arises due to the fact that not all financial friction models come up with such high estimates, as mentioned in this paragraph, but that is beyond the scope of this study.

#### Estimates of Financial Friction Parameters

The second set of estimates at the focus of this chapter are the posterior estimates of the financial friction parameters. These parameters are model specific (see table C.3 on page 226). In particular, in the *FAGK* and *FA* models I obtain estimates for:

- monitoring cost  $\mu$ ,
- the default rate of firms  $F_{\bar{\psi}}$ .

The above parameter estimates are not relevant to the *GK* model. The posterior mean estimate of the monitoring cost parameter in *FA* and *FAGK* models are similar, at  $\mu = 0.07$  and  $\mu = 0.04$ , respectively. The upper and lower bounds of the credible interval obtained via MH algorithm for the *FAGK* model are  $[0.06, 0.02]$ . For the *FA* the 90% credible interval is  $[0.10, 0.04]$ . These estimates indicate to a certain extent the financial accelerator friction is slightly less important in the *FAGK* model with both frictions. A potential explanation is that the presence of a bank friction in the baseline *FAGK* model downgrades the role of monitoring costs in generating any extra friction.

The posterior mean estimates for the quarterly default rate of firms is very similar in the two models,  $F_{\bar{\psi}} = 0.61\%$  and  $F_{\bar{\psi}} = 0.59\%$  in *FAGK* and *FA* models,

respectively. This value is close to the prior mean and the data confirm the empirical finding mentioned earlier (Altman and Pasternack (2006)) and close to the 0.75% assumed in calibrated models.

In the *FAGK* and *GK* models I obtain estimates for:

- bank friction parameter  $\Theta^B$ .

The posterior mean for the bank friction parameter is the same,  $\Theta^B = 0.78$ , in both *GK* and *FAGK* models. It is similar to the calibrated value of 0.75 in previous chapter 3 although I set a lower prior of 0.54.

#### 4.4.3 Matching Business Cycle Facts

In this section I evaluate the model fit by comparing business cycle properties of the model with those from the data. The estimation of baseline model *FAGK* maximizes the likelihood function for 9 observable series while the *FA* and *GK* are estimated with 8 observables.

One implications of the estimation with different number of observations is that I can not compare the three models using Bayesian model comparison. Instead, I study the absolute model fit by using the posterior distribution. I compare the second moments from the actual data to the model-implied empirical statistics.

To obtain these empirical statistics, I set the model framework to generate samples of the same length as the data, after a burn-in of 100 observation points. I summarize the standard deviations and correlations for key macroeconomic and financial variables relative to those of output in tables C.4 through C.9 of appendix C<sup>17</sup>.

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<sup>17</sup>Tables C.4, C.5 for moments from *FAGK* model; tables C.6, C.7 for moments from *GK* model; tables C.8, C.9 for moments from *FA* model.

< Tables C.4, through C.9 here >

The business cycle properties of the model-implied real sector variables are broadly consistent with the data according to all three models. I first summarize these dimensions of the business cycle properties along which the three models could have done better.

- Model-implied correlations of *inflation* and *nominal interest rate* with all external finance premium measures are all in the neighborhood of zero or positive. Data statistics show strong negative correlation for these pairs (I refer to columns [Π] and [R<sup>n</sup>] in tables C.5, C.7 and C.9). Similarly model-implied correlations of *inflation* and *nominal interest rate* with agents' *net worth* series are not consistent with the data (the same columns and same tables).
- To a similar extent *FAGK* and *GK* fail to capture the strong negative correlation of *real wage* and *hours of work* with the financial variables (columns [H] and [W] in tables C.5 and C.7).
- The models *FAGK* and *FA* fail to replicate the strong positive correlation of *nominal interest rate* with *real consumption* (column [C] in tables C.5 and C.9).
- Finally, a less problematic issue is that all models slightly over-predict the volatility of *working hours* but under-predict that of *wages* (columns [W] and [H] in tables C.4, C.6 and C.8). A potential reason could be the assumption of flexible wages in the model, but that is beyond the scope of this work.

Next I evaluate where the baseline model, *FAGK*, outperforms the single friction models.

- The baseline *FAGK* model does better in capturing the *strong positive correlation of lender net worth with three key macro indicators,  $Y_t$ ,  $C_t$  and  $I_t$*  (row [NB (FoF)] in table C.5).

The *GK* model yields correlations of *lender net worth* with these three macro indicators in the neighborhood of zero (row [NB (FoF)] in table C.7). The *FA* model is silent about *lender net worth* (same row in table C.9).

- As a result of the above point, the baseline *FAGK* model does better in replicating *the strong negative correlation of total premium  $efpB$  with  $Y_t$ ,  $C_t$  and  $I_t$*  (row [*efpB* total] in table C.5).

Single friction models, *FA* and *GK*, replicate a weak (negative) correlation of total *efpB* with  $Y_t$  and  $I_t$ , but an almost zero or positive correlation with  $C_t$  (row [*efpB* total] in tables C.7 and C.9).

Do the *FA* or *GK* perform better in any dimension? Compared to the other two models, the *GK* model replicates well the positive correlation of nominal interest with consumption but at the cost of failing the positive correlation with real wages. The *FA* model does perform better in capturing the negative correlation of the financial variables, *efpB* and firm net worth with real wage.

The model economy incorporating both financial frictions, *FAGK*, does perform as good as the single friction models in replicating the business cycle properties of key macro variables. It does perform much better in terms of *replicating the strong negative correlation of the external finance premium(s) with output, consumption and investment*. In addition, it is by construction more informative in terms of the properties of each agent, borrower and lender, financial premium. In the following sections I will proceed with my baseline model, *FAGK*, that incorporates two financial frictions.

## 4.5 A Quantitative Analysis: Assessing the main Drivers of *efp* Fluctuations

In this section I study the dynamics of the cost of external finance during two sub-periods, before and during the crisis. I first define the sub-samples and report estimates from each estimation. I evaluate how the model replicates the business cycle properties of actual data series for the relevant sub-period.

Next, I summarize the estimated parameters that remain stable and put my emphasis on those parameters that change from one sub-sample estimation to another. The ultimate objective of this section is to evaluate the potential candidates that have driven the sharp shift in business cycle properties of the model-implied economy during the recession sub-period.

The critical time period in which I focus is the recent crisis. I define the cut-off observation that splits the recession sub-sample from the previous period by setting the observation in quarter 2005:01 as the start of the most recent sub-sample. The time frame includes the onset of the recession followed by the unfolding of financial meltdown. I coin the term *recession* sub-sample to refer to this period.

To identify the magnitude of the financial turmoil I compare the data statistics during the financial crisis relative to the sub-period 1985-2004 when the volatility of the macroeconomic and financial data is relatively stable. I coin the term *tranquil* period of macroeconomic performance in the United States for this sub-period. It is popularly known in the literature as the "Great Moderation". The empirical evidence of relatively low volatility of macroeconomic variables after the mid 1980s has been documented in several papers like [Kim and Nelson \(1999\)](#), [McConnell and Perez-Quiros \(2000\)](#) and [Stock and Watson \(2002\)](#).

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I estimate the same baseline *FAGK* model with the same priors over the sub-periods 1985-2004 and 2005-2014<sup>18</sup>.

##### 4.5.1 Posterior Estimates from Sub-sample Estimation

I summarize the estimated parameters that do not differ from one sub-period to the other. I report the posterior mode and mean estimates of parameters for the *tranquil* and the *recession* sub-samples in table C.10 in appendix.

< Tables C.10 here >

Labor share in the wholesaler's production function, habit formation parameter, risk aversion parameter of household, the elasticity of substitution among goods (determining the steady state mark up on prices) and the degree of price indexation of the monopolistic retailer are the parameters that change very little during the *recession* sub-sample.

Similarly, the posterior estimates for most persistence parameters change very little from one sub-sample to the other. An eyeball view of the 90% credible interval from the posterior distribution for most shock persistence parameters indicates that they remain very stable during the recession. Sizes of estimated posterior means of exogenous shocks are also very similar in the two sub-sample estimations as well. There are few exceptions to this general evaluation.

First, persistence of neutral technology shock,  $\rho_A$  is slightly lower, but the magnitude of the shock,  $\sigma^\gamma$ , is higher during *recession* period. Second, investment specific technology shock  $\rho_\gamma$ , is slightly more persistent, but the magnitude of the shock,  $\sigma^\gamma$ , is lower during *recession*. Simulation of the model with

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<sup>18</sup>I re-estimate the models by defining the cut-off quarter to be 2004:01 and in a separate estimation I set the cut-off observation in 2006:01. The posterior estimates of the means of the parameters are similar across the three sets of estimation hence I conclude that the choice of sub-samples at least within this range is not critical for the results.

#### 4.5. A Quantitative Analysis: Assessing the main Drivers of *efp* Fluctuations

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these two different sets of estimated values shows that the net effect on the standard deviation of most variables of interest is negligible.

Third, the magnitude of labor supply shock,  $\sigma_H$ , is slightly higher during the *recession* sub-sample. I will return to this parameter in the following paragraphs.

Next, I identify the changes in the posterior estimates of deep parameters whose posterior estimates see significant shifts from one estimation to the other. I motivate why the shift in some of these parameter values from one sub-sample to another are not critical for the business cycle properties of the model-implied variables during recession period.

- *The posterior estimate of the inverse of the Frisch elasticity of labor,  $\rho^h$ , and of the labor supply shock,  $\sigma_H$ , are higher in the magnitude during the recession period.*

I can motivate the increase of the inverse of labor Frisch elasticity,  $\rho^h$ , with the fact that the upward shift in the magnitude of the labor supply shock,  $\sigma_H$ , has an offsetting impact for the dynamics of the households labor supply. As it will be illustrated later, the joint impact of greater magnitude of these two parameters, labor supply shock  $\sigma_H$  and the inverse of Frisch elasticity  $\rho^h$ , is almost zero for the business cycle properties of model-implied macro and financial variables. I will elaborate this issue in detail in subsection [4.5.2 on page 217](#).

- *The posterior mean of the policy feedback to output,  $\theta_Y$ , is slightly lower during the recession sample estimation.*

The posterior mode, mean and the credible interval of the feedback policy to output gap,  $\theta_Y$ , indicates that the policy reaction to output gaps could have

been weaker at the onset of and during the recession. A weak policy response to output gaps and high persistence of policy may lead to large volatility in economy as seen in chapter two. On this occasion, it is difficult to attribute large impact on the shift on this parameter as the Fed took additional non-conventional measures during this period. As the Fed's policy rate hit the lower zero bound during the recession, it switched to non-conventional liquidity and monetary targeting. These policies undermine any conclusion reached based on a simple Taylor rule of Fed. I report in the next section that the change in volatility of model-implied variables that can be attributed to the shift in posterior  $\theta_Y$  is negligible.

- *The posterior estimate for investment adjustment cost (IAC) parameter,  $\Phi_X$  is lower during the recession sample.*

The lower estimate of  $\Phi_X$  reduces the time lags that takes for firms to re-size their total physical capital involved in production following a(n) (adverse) shock. A reasonable argument is that it captures the quick reaction of firms in shrinking their capital investments following the first crisis signals in mid 2007. I analyse quantitatively the impact of this lower parameter estimate.

- *The posterior estimates for firm parameters,  $\mu$  and  $A_{\Psi}$ , are larger during the recession sample.  $\mu$  and  $A_{\Psi}$  are the steady state monitoring cost and the measure of dispersion of borrowing firms returns respectively.*

A critical result is that the posterior estimate for the monitoring cost parameter,  $\mu$ , and the estimate of the dispersion of firms revenues (or profits),  $A_{\Psi}$ , went up during the *recession* sample. The upper bound of the credible interval of the posterior distribution for both parameters shifted up.

In the next section, I will focus on assessing the extent to which the increase in volatility of model-implied variables can be attributed to the potential candidates identified in this section. I am aware that the current formulation of

#### 4.5. A Quantitative Analysis: Assessing the main Drivers of *efp* Fluctuations

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the policy rule does not take into account the additional measures that Fed took during the crisis. Also, changes in estimates of parameters related to labor supply are a probably a consequence of the absence of friction in wage market and that their impact cancels out. Finally, the posterior estimates for the bank friction parameter,  $\Theta^B$ , in the two sub-sample estimations are very similar. The credible intervals indicate that this parameter is relatively stable before and during the crisis. Therefore, my focus in the next subsection will be on investment adjustment costs,  $\Phi_X$ , and the firm friction parameters,  $\mu$  and  $A_{\Psi}$ .

##### 4.5.2 What are the drivers of *efp* fluctuations?

In this section I analyze the contribution of potential candidates, like changes in severity of financial rigidities at firm level and changes in the adjustment cost of firm investment in driving the shifts in business cycles properties of the model-implied series during the recession.

##### Simulation Method

To assess the contribution of each potential candidate I run *counterfactual* exercises similar to [Smets and Wouters \(2007\)](#) and [Arias et al. \(2007\)](#). I proceed in five steps.

1. Simulate the model economy with the estimated parameter set from the sample period 1985-2004.
2. Simulate the model economy with the estimated parameter set from the sample period 2005-2014.
3. Compute the ratio of standard deviations between the two samples.

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4. Simulate the model economy using the estimated parameter vector of the period 1985-2004, overwritten by the *counterfactual parameter* selected from the estimated parameter vector of the period 2005-2014.
5. Compute the ratio of standard deviations obtained from the *counterfactual* exercise (from step 4) relative to the standard deviations obtained with the estimated parameter vector of the period 1985-2004 (from step 1).

I report the results for steps 1 to 3 in table C.11 of appendix (columns [4] and [5]). They are the standard deviations of model-implied variables obtained by simulating the model economy with the vector of estimated posterior means of parameters from each respective sub-sample, *tranquil* and *recession* (as shown in steps 1 to 3).

< Table C.11 here >

For comparative purposes I report in columns [1] and [2] the standard deviation of the cyclical component of filtered data series before and after the recession. I use the Hodrick-Prescott (HP) filtering method to extract the cyclical component of each series. Their ratio in third column is a rough measure of the behaviour of each variable during the recession relative to normal times.

The simulation with the posterior means replicates main dynamics of the model-implied economy during the recession similar that in the data. This result is of no surprise as most variables listed in this table are used as observables during the estimation. Clearly the estimated posterior means of the *recession* sample, replicate the higher volatility of most observable series. The model predicts the higher volatility of bank net worth during the recession though it under-predicts the volatility of firm net worth.

### Simulation Results

In the remaining part of this section I study the results from the *counterfactual* exercises, step 4 and 5. In the first two columns of table C.12 on page 234 I report the ratios of standard deviation of variables during *recession* relative to the *tranquil* period for both, actual filtered data and model-implied variables. They are copies of 3<sup>rd</sup> and 6<sup>th</sup> columns of previous table C.11.

< Table C.12 here >

In columns [1] through [6] of table C.12 I report the ratio of standard deviation of a model-implied economy with the 1985-2004 parameter vector plus the *counterfactual* parameter of 2005-2014 relative to standard deviation of the variable in *tranquil* sub-sample. These results are obtained by repeating steps 4 and 5 from the list of exercises in the previous subsection. The ratio reported in each of the columns [1] to [6] of C.12 is an approximate measure of the increase in volatility of model-implied variables that can be attributed to changes in the estimate of each parameter for the *recession* period.

- **Counterfactual 1.** *In counterfactual 1, I study the relative contribution of a lower estimated posterior mean of investment adjustment cost (IAC) during the recession period.*

The results from this exercise are shown in column [1] of table C.12. A lower *IAC* during the recent sub-sample accounts for 129% of the model-implied increase in cyclical volatility of investment. It also accounts for 41% increase in model-implied cyclical volatility of output and 32 % increase in working hours volatility. Lower (*IAC*) accounts for about 15% increase in the cyclical volatility of the lender premium. The latter is mirrored in a similar increase in volatility of model-implied total *efp*. The impact of lower *IAC* on business cycle properties of other model-implied variables is negligible.

#### 4.5. A Quantitative Analysis: Assessing the main Drivers of *efp* Fluctuations

The common motivation for the presence of *IAC* dynamic stochastic general equilibrium models (hereby *dsgе*) has been to introduce inertia in investment and generate a hump-shaped response to monetary shocks in new Keynesian models (i.e. [Christiano et al. \(2005\)](#)). A lower *IAC* parameter implies greater sensitivity of current investment to the shadow value of installed capital.

I can relate the lower estimate of *IAC* during the *recession* sub-sample to the borrowing firms' speed of adjusting to adverse circumstances. Borrowing firms delay investment for the same fall in price of capital '*q*'.

A similar behaviour is that of firms during the recession in US. As news of a failing large bank, like Lehman Brothers or Bear Stearns hedge funds, spreads out the borrowing firms' momentum of cutting down on investment may gain pace even though the magnitude of the fall in '*q*' is not unusually large. That momentum, when is not explicitly modelled, could be captured by the lower value of an *IAC* parameter that delivers similar dynamics in the model for small decline in asset prices. The *counterfactual* exercise showed that indeed model-implied investment can become 129% more volatile with the lower *IAC* even though the volatility of shadow price of capital barely changes (*q* is 4% less volatile). In short, it takes much less time for firms to delay a new investment or cut on existing investment than to actually materialize it when facing rare adverse circumstances.

- **Counterfactual 2.** I analyze the effect of an increase in the standard deviation of idiosyncratic shock,  $A_{\overline{\Psi}}$ , in counterfactual 2. The results from this exercise are shown in column [2] of table [C.12](#).

There are two implications of a fall in  $A_{\overline{\Psi}}$ . First, from a single firm's perspective [Bernanke et al. \(1999\)](#) interpret  $A_{\overline{\Psi}}$  as a measure of uncertainty on the firm's return on project. Alternatively,  $A_{\overline{\Psi}}$  is a measure of the dispersion

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of project return across firms. The greater dispersion of firms profits, implies the default threshold of borrowing firms follows a sharper increase upon an adverse shock and bank net worth declines faster.

Second, in setting up the model I assume the survival of the firm and the transfer to the newly entering firms are kept fixed through all periods, a common feature in these models. With these two parameters fixed, the lower posterior estimate (sub-sample 1985-2004) of the dispersion parameter,  $\Lambda_{\psi}$ , matches the fact that the borrowing firms were highly leveraged before recession.

A higher estimate of  $\Lambda_{\psi}$  accounts for a sharper decline in model-implied bank net worth forcing banks to cut back on lending. The standard deviations of bank net worth and price of capital,  $q$ , go up by 13% and 23% respectively. As a result, it accounts for a 13% and 15% increase in volatility of model-implied lender premium and total premium *efpB*. Finally, it accounts for a 47% increase in volatility of model-implied borrower premium. The latter has a negligible weight on the volatility of total premium *efpB*.

This result accords well with findings from empirical studies and general equilibrium models (Kehrig (2011), Bloom et al. (2013), Christiano et al. (2014) and Arellano et al. (2010))<sup>19</sup>.

- **Counterfactual 3.** *In counterfactual 3 I evaluate the impact of an estimated shift in the mean of cost of monitoring,  $\mu$ , from 0.07 to 0.11 during the recent recession. The results are shown in column [3] of table C.12.*

<sup>19</sup>Kehrig (2011) calculates a measure of dispersion of plant level TFP for the period 1972-2009 which shows a significant jump during the last recession (figure 1). Bloom et al. (2013) results also reject the hypothesis that uncertainty (measured by dispersion) is driven by the TFP shocks themselves. The latter result is critical to rule out the possibility that dispersion across firms' returns be a function of TFP movements. A significant role for the idiosyncratic shock during the recession is reported in few recent studies (Christiano et al. (2014) and Arellano et al. (2010)).

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This is the cost banks incur when firms declare default and is expressed as percentage of seized assets of defaulting firms. The greater magnitude of  $\mu$  is proportionally linked to the borrower's premium as these costs motivate the charging of an extra premium in a typical financial accelerator framework. The higher monitoring costs can account for 67% increase in model-implied borrower premium volatility. It accounts for greater volatility of bank net worth by 17% but has negligible effect on other variables.

- **Counterfactual 4.** *I report the impact of the estimated looser monetary policy feedback to output gaps at the onset and during the recent crisis in counterfactual 4. Results are shown in column [4] of table C.12.*

Estimated posterior mean for  $\theta_Y$  declined from 0.14 to 0.11 in table C.10. The impact of lower  $\theta_Y$  on the volatility of most macroeconomic and financial indicators is negligible. The design of policy feedback here does not take into account alternative instruments that were undertaken by the FED after the third quarter of 2007 like monetary measures in terms of quantitative easing. Analyzing the policy effects based on the interest rate rule can not account for a full evaluation of policies during this period. Therefore, I do not elaborate further on this parameter.

- **Counterfactual 5.** *in counterfactual 5 I report the joint effect of changes in two parameters related to labor supply,  $\rho^h$  and  $\sigma_H$ . Results are shown in column [5] of table C.12.*

The parameter  $\rho^h$  is the labor supply shifter in household's utility function while  $\sigma_H$  is the magnitude of labor supply shock. The posterior estimates for these two parameters have been significantly larger during the recession subsample (from table C.10). Their joint effect is negligible across most variables of

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interest, except for real wages. I have motivated the change in estimated values of these two parameters with the fact that I assumed flexible wage setting in the model. To account for higher volatility of wages during the recession the model re-evaluates these two parameters to match the higher wage volatility. Due to the limited impact on other variables other than real wages I do not consider these two parameters as primary drivers of the increase in volatility during the recession.

- **Counterfactual 6.** *For illustrative purposes my final exercise in counterfactual 6 is to quickly illustrate the impact of the lower adjustment cost in downturns and of larger dispersion of firms' returns (column [6] of table C.12).*

From the above exercises I distinguish two key factors that had a significant effect in shaping the behaviour of the business cycle properties of key financial and macro indicators identified with the recession. The first, lower investment adjustment costs, refer to the speed at which borrowing firms adjust their investments upon usual shocks under rare adverse circumstances. For the same decline in the price of capital, the decline in investment, and therefore output, is of greater magnitude upon the same size of shocks. The second, higher standard deviation of idiosyncratic shock, relates to the greater dispersion of borrowing firms' return on capital. It drives up the cost of external financing, due to both greater borrower spread and lender spread.

The shifts in estimated values of these two parameter can replicate the business cycle properties of most model-implied macro and economic financial variables during the recession period in US. The higher *monitoring costs* have had only limited effects on the borrower's premium.

## 4.6 Concluding Remarks

In this chapter I have estimated three models with nominal, real and financial frictions for the period 1955-2014. I conclude that the baseline model *FAGK* with two financial frictions, a financial accelerator and a bank friction, best captures the business cycle properties of key macro and financial indicators.

Estimation of the model in two sub-periods showed that shock sizes and their persistence have not changed significantly to account for the increase in the amplitude of the cyclical volatility in 2005-2014. Instead, the quicker adjustment of firms' investments towards usual shocks, *IAC*, and the higher dispersion of borrowing firms profits are critical for the evolution of events in the model economy during recession.

The lower investment adjustment cost, *IAC*, implied a faster shrink of investment by the borrowing firms upon usual shocks of similar size as before. A potential explanation for the lower *IAC* is a shift in the attitude of borrowing firms towards new investment due to unusual circumstances captured by a lower *IAC* parameter in the model.

The greater dispersion of borrowing firms returns implies that the defaulting firms trigger larger losses on the bank's balance sheet. As lender net worth declines the (lender) premium they charge on new loans spikes up.

I can reconcile the sequence of events in the model implied economy with actual outcomes during the recent crisis. A rare event during the recent recession is the looming news of a bankruptcy of a major investment bank in US, like Lehman Brothers. It is after this episode that firms cut on investment which was followed by declining output. As highly leveraged firms relied on the high growth rate to turn positive profits, they failed to meet their financial obligations which triggered a chain effect through the economy. The latter

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triggered huge losses on banks' net worth and drove the premium on new external financing to unprecedented high.

It is commonly agreed in literature that the crisis worsened due to the involvement of banks in financing the firms through equity securities (Gertler and Kiyotaki (2010)). In my framework the presence of banks allows for firms losses to be transmitted on banks' balance sheets, while the presence of financial accelerator block allows to replicate the impact of higher dispersion of firm revenues on lender balance sheets.

Finally, the large spike in external finance premium during the recent recession in US is largely attributed to the spike in lenders' premium. The sharp increase in monitoring costs can drive up the borrower's premium but has less effect on total external finance premium.

# Appendix C

## C.1 Tables and Figures

Table C.1: Standard deviations in absolute value of actual observables and model-implied series using Hodrick-Prescott filter.

Series $i$	Standard Deviation [ $\sigma^i$ ]			Correlation [ $\rho^i$ ]		
	1985-2004 [1]	2005-2014 [2]	$\frac{[2]}{[1]}$ [3]	1985-2004 [4]	2005-2014 [5]	Change [5] – [4] [6]
Y	1.17	1.45	1.25	1	1	–
C	0.83	1.00	1.20	0.86	0.92	0.06
I	3.59	6.06	1.69	0.88	0.95	0.07
H	1.08	1.66	1.54	0.88	0.91	0.04
W	1.26	0.90	0.71	0.25	-0.08	-0.33
$\Pi$	0.16	0.20	1.23	0.32	0.42	0.09
$R^n$	0.27	0.32	1.16	0.42	0.77	0.36
EFPba	0.03	0.10	3.20	-0.40	-0.44	-0.04
EFPa	0.23	0.32	1.39	-0.55	-0.78	-0.23
EFP total	0.24	0.37	1.53	-0.57	-0.79	-0.22
BAA real	0.68	0.96	1.40	-0.35	-0.61	-0.26
NW (FoF)	2.74	9.88	3.60	0.34	0.93	0.58
NB (FoF)	13.6	15.2	1.11	0.45	0.73	0.28
NW (DS)	12.8	11.0	0.86	0.61	0.81	0.20
NB (DS)	14.5	22.6	1.56	0.42	0.82	0.40

'EFPba' stands for the spread between Moody's BAA and AAA corporate bond yields.

'EFPb' stands for the spread between Moody's AAA corporate bond yield and Fed rate.

'EFP' stands for the total spread between Moody's BAA corporate bond yield and Fed rate.

'BAA real' stands for the spread between Moody's BAA corporate bond yield and Fed rate.

'NW' and 'NB' stand for Firm and Bank net worth respectively.

'FoF' stands for Flow of Funds data; 'DS' stands for *Datastream* data.

Table C.2: Priors

Parameter	Description	Density	FAGK		GK		FA		95 % CI
			Mean	St.Dev	Mean	St.Dev	Mean	St.Dev	
$\beta$	Discount Rate	Fixed	0.99	–	0.99	–	0.99	–	
$\delta$	Depreciation Rate	Fixed	0.025	–	0.025	–	0.025	–	
$\alpha$	Labor share on output	Beta	0.7	0.1	0.7	0.1	0.7	0.1	
$\chi$	Degree of habit formation	Beta	0.7	0.1	0.7	0.1	0.7	0.1	
$\phi_x$	Investment adjustment cost	Gaussian	4	1.5	4	1.5	4	1.5	
$\epsilon$	Elasticity of substitution among goods	Gamma	7	1	7	1	7	1	
$\sigma_c$	Inter-temporal elasticity of substitution	Gaussian	1	0.375	1	0.375	1	0.375	
$\rho^h$	Inverse of labor Frisch-elasticity	Gamma	1	2	1	2	1	2	
$\frac{G}{Y}$	Public Spending Share	Fixed	0.20	–	0.20	–	0.20	–	
$H_{s,s}$	Average number of hours of work	Fixed	0.35	–	0.35	–	0.35	–	
$\xi_p$	Calvo parameter (1-prob of re-optimizing)	Fixed	0.75	–	0.75	–	0.75	–	
$\gamma_p$	Degree of price indexation to past inflation	Beta	0.4	0.2	0.4	0.2	0.4	0.2	
$\Pi_{ss}$	Steady State Inflation	Fixed	1.0084	–	1.0084	–	1.0084	–	
$\rho_R$	Policy rate smoothing parameter	Beta	0.75	0.15	0.75	0.15	0.75	0.15	
$\theta_\pi$	MP reaction to inflation expectation	Gaussian	1.5	0.15	1.5	0.15	1.5	0.15	
$\theta_Y$	MP reaction to output gap	Gaussian	0.5/4	0.10	0.5/4	0.10	0.5/4	0.10	
$\mu^*$	Monitoring costs parameter	Beta	0.15	0.10	–	–	0.15	0.10	
$A_{\Psi}^*$	Standard deviation of idiosyncratic shock	Fixed	0.306	–	–	–	0.306	–	
$F_{\Psi}^*$	Probability of Default	Beta	0.75%	0.2%	–	–	0.75%	0.2%	
$\sigma^{E*}$	Survival probability of entrepreneur	Fixed	0.985	–	–	–	0.985	–	
$\xi^{E*}$	Wealth transfer to new entrepreneurs (%)	Fixed	0.003	–	–	–	0.003	–	
$\Theta^{B+}$	Share of assets divertible by bankers	Beta	0.54	0.15	0.54	0.15	–	–	
$\sigma^{B+}$	Survival probability of banks	Fixed	0.977	–	0.977	–	–	–	
$\xi^{B+}$	Wealth transfer to to new banks	Fixed	0.0002	–	0.0002	–	–	–	
$\rho_i$	Persistence of exogenous shocks	Beta	0.5	0.2	0.5	0.2	0.5	0.2	
$\sigma_i$	Magnitude of Exogenous shocks	Inv.Gamma	0.10	inf	0.10	inf	0.10	inf	

(\*) These parameters do not enter in the GK model with bank friction.

(+) These parameters do not enter in the Financial Accelerator Model.

Table C.3: Posterior Distribution

Parameter	FA Model			GK Model			FAGK Model		
	Mode	Mean	90% CI	Mode	Mean	90% CI	Mode	Mean	90% CI
$\alpha$	0.79	0.80	[0.84, 0.77]	0.73	0.73	[0.77, 0.69]	0.73	0.76	[0.80, 0.70]
$\chi$	0.44	0.47	[0.54, 0.39]	0.34	0.36	[0.43, 0.29]	0.36	0.40	[0.47, 0.32]
$\phi_x$	0.76	1.30	[2.03, 0.63]	0.72	0.91	[1.35, 0.44]	0.71	1.14	[1.75, 0.51]
$\epsilon$	5.69	6.27	[7.73, 4.81]	5.60	6.19	[7.70, 4.66]	5.55	6.25	[7.75, 4.71]
$\sigma_c$	1.79	1.81	[2.18, 1.43]	2.20	2.24	[2.66, 1.82]	1.93	1.91	[2.26, 1.55]
$\rho^h$	0.73	0.73	[0.93, 0.53]	0.84	0.84	[1.05, 0.63]	0.87	0.84	[1.05, 0.63]
$\gamma_p$	0.09	0.35	[0.66, 0.04]	0.15	0.42	[0.69, 0.06]	0.11	0.38	[0.66, 0.06]
$\rho_R$	0.62	0.63	[0.67, 0.59]	0.62	0.62	[0.67, 0.58]	0.62	0.62	[0.66, 0.58]
$\theta_\pi$	1.62	1.60	[1.75, 1.44]	1.62	1.58	[1.73, 1.43]	1.62	1.57	[1.72, 1.42]
$\theta_Y$	0.13	0.19	[0.29, 0.08]	0.23	0.25	[0.34, 0.16]	0.22	0.24	[0.32, 0.15]
$\Theta_B$	-	-	[-, -]	0.81	0.79	[0.91, 0.66]	0.80	0.78	[0.91, 0.66]
$\mu$	0.08	0.07	[0.10, 0.04]	-	-	[-, -]	0.04	0.04	[0.06, 0.02]
$F_{\Psi}$	0.56%	0.59%	[0.85%, 0.32%]	-	-	[-, -]	0.60%	0.61%	[0.87%, 0.33%]
$\rho_A$	0.82	0.83	[0.87, 0.78]	0.82	0.82	[0.87, 0.78]	0.81	0.81	[0.86, 0.77]
$\rho_G$	0.78	0.76	[0.84, 0.69]	0.77	0.76	[0.84, 0.67]	0.74	0.74	[0.81, 0.66]
$\rho_M$	-	-	-, -	-	-	-, -	-	-	-, -
$\rho_Y$	0.31	0.30	[0.40, 0.19]	0.32	0.30	[0.43, 0.17]	0.30	0.23	[0.37, 0.08]
$\rho_{MS}$	0.56	0.36	[0.61, 0.10]	0.50	0.29	[0.53, 0.07]	0.53	0.32	[0.56, 0.08]
$\rho_{bu}$	0.68	0.68	[0.74, 0.61]	0.65	0.65	[0.72, 0.58]	0.67	0.66	[0.72, 0.59]
$\rho_H$	0.99	0.99	[0.999, 0.998]	0.99	0.99	[0.999, 0.998]	0.99	0.99	[0.999, 0.998]
$\rho_\mu$	0.76	0.76	[0.80, 0.73]	-	-	[-, -]	0.76	0.77	[0.84, 0.72]
$\rho_{\Theta_B}$	-	-	[-, -]	0.75	0.75	[0.79, 0.72]	0.72	0.72	[0.76, 0.68]
$100 * \sigma_A$	0.67	0.66	[0.73, 0.61]	0.70	0.70	[0.80, 0.66]	0.73	0.71	[0.78, 0.64]
$100 * \sigma_G$	2.38	2.43	[2.61, 2.23]	2.18	2.20	[2.38, 2.02]	2.29	2.34	[2.54, 2.14]
$100 * \sigma_M$	0.20	0.20	[0.21, 0.18]	0.20	0.20	[0.22, 0.18]	0.20	0.20	[0.21, 0.18]
$100 * \sigma_Y$	2.13	3.62	[5.51, 1.74]	2.15	2.75	[4.10, 1.32]	2.12	3.53	[5.59, 1.59]
$100 * \sigma_{MS}$	10.7	15.8	[22.8, 9.5]	11.4	16.7	[23.5, 9.5]	10.8	16.5	[23.4, 9.1]
$100 * \sigma_{bu}$	0.72	0.76	[0.86, 0.68]	0.71	0.72	[0.80, 0.65]	0.71	0.73	[0.80, 0.66]
$100 * \sigma_H$	1.64	1.73	[1.97, 1.46]	1.72	1.79	[2.03, 1.53]	1.62	1.67	[1.90, 1.45]
$100 * \sigma_\mu$	47.1	63.5	[95.4, 33.7]	-	-	[-, -]	76.5	91.2	[134.1, 43.4]
$100 * \sigma_{\Theta_B}$	-	-	[-, -]	0.62	0.63	[0.72, 0.54]	0.54	0.56	[0.63, 0.48]

**Table C.4:** Baseline *FAGK* model: Standard deviations relative to St.deviation of seven key Macro Variables: HP filtered data (1955-2014) vs Model implied statistics.

Series	Data	Model	Data	Model	Data	Model								
	Y	Y	C	C	I	I	H	H	W	W	$\Pi$	$\Pi$	$R^n$	$R^n$
Y	1	1	1.70	1.65	0.34	0.31	1.16	0.87	0.67	1.08	6.43	4.07	4.61	3.44
C	0.59	0.61	1	1	0.20	0.18	0.68	0.53	0.39	0.65	3.78	2.47	2.71	2.09
I	2.91	3.28	4.95	5.41	1	1	3.38	2.84	1.95	3.54	18.7	13.3	13.4	11.3
H	0.86	1.15	1.46	1.90	0.30	0.35	1	1	0.58	1.25	5.53	4.70	3.96	3.97
W	1.49	0.93	2.53	1.53	0.51	0.28	1.73	0.80	1	1	9.58	3.77	6.86	3.19
$\Pi$	0.16	0.25	0.26	0.41	0.05	0.07	0.18	0.21	0.10	0.27	1	1	0.72	0.85
$R^n$	0.22	0.29	0.37	0.48	0.07	0.09	0.25	0.25	0.15	0.31	1.40	1.18	1	1
efpBA	0.04	0.05	0.06	0.08	0.01	0.01	0.04	0.04	0.02	0.05	0.24	0.19	0.17	0.16
efpA	0.18	0.24	0.31	0.40	0.06	0.07	0.21	0.21	0.12	0.26	1.17	0.99	0.84	0.84
efpB total	0.20	0.26	0.33	0.43	0.07	0.08	0.23	0.23	0.13	0.28	1.27	1.07	0.91	0.90
BAA real	0.52	0.40	0.89	0.67	0.18	0.12	0.61	0.35	0.35	0.44	3.37	1.65	2.49	1.39
NW (FoF)	2.83	2.74	4.82	4.51	0.97	0.83	3.29	2.37	1.90	2.95	18.2	11.1	13.1	9.42
NB (FoF)	8.37	4.93	14.3	8.13	2.88	1.50	9.74	4.27	5.62	5.32	53.8	20.1	38.6	17.0
NB (DS)	8.33	–	14.2	–	2.86	–	9.69	–	5.59	–	53.6	–	38.4	–
NW (DS)	6.19	–	10.5	–	2.13	–	7.20	–	4.16	–	39.8	–	28.5	–

Model empirical moments are drawn from the posterior distribution.

In the text I refer to each column as [Y] instead of 'Y' or [C] instead of 'C'.

Table C.5: Baseline FAGK model: Correlations with seven key Macro Variables: HP filtered data (1955-2014) vs Model implied statistics.

Series	Data	Model												
	Y	Y	C	C	I	I	H	H	W	W	$\Pi$	$\Pi$	$R^n$	$R^n$
Y	1	1												
C	0.85	0.50	1	1										
I	0.90	0.76	0.74	0.28	1	1								
H	0.88	0.78	0.78	0.22	0.87	0.62	1	1						
W	0.85	0.39	0.81	0.49	0.75	0.25	0.74	0.20	1	1				
$\Pi$	0.20	0.32	0.24	0.15	0.15	0.11	0.30	0.45	0.18	0.12	1	1		
$R^n$	0.37	0.17	0.34	-0.21	0.34	0.04	0.51	0.43	0.21	-0.25	0.55	0.54	1	1
efpBA	-0.54	-0.11	-0.45	0.08	-0.57	-0.16	-0.46	-0.12	-0.46	-0.06	-0.12	0.04	-0.14	0.02
efpA	-0.54	-0.45	-0.49	-0.13	-0.52	-0.66	-0.66	-0.29	-0.40	-0.14	-0.52	0.10	-0.91	0.14
efpB total	-0.60	-0.43	-0.54	-0.11	-0.59	-0.64	-0.69	-0.29	-0.46	0.24	-0.51	0.10	-0.87	0.13
BAA real	-0.33	-0.32	-0.34	-0.33	-0.32	-0.48	-0.35	-0.09	-0.31	0.18	-0.61	0.12	-0.27	0.66
NW (FoF)	0.33	0.40	0.33	0.40	0.47	0.46	0.48	0.04	0.13	0.24	0.26	-0.30	0.46	-0.61
NB (FoF)	0.46	0.34	0.35	0.33	0.52	0.44	0.38	0.001	0.43	0.18	-0.16	-0.29	0.01	-0.64
NB (DS)	0.41	-	0.37	-	0.51	-	0.41	-	0.33	-	-0.13	-	0.01	-
NW (DS)	0.41	-	0.46	-	0.43	-	0.41	-	0.36	-	-0.08	-	0.12	-

In the text I refer to each column as [Y] instead of 'Y' or [C] instead of 'C'.

**Table C.6:** GK model: Standard deviations relative to St.deviation of seven key Macro Variables: HP filtered data (1955-2014) vs Model implied statistics.

Series	Data	Model	Data	Model	Data	Model								
	Y	Y	C	C	I	I	H	H	W	W	$\Pi$	$\Pi$	R <sup>n</sup>	R <sup>n</sup>
Y	1	1	1.70	1.59	0.34	0.26	1.16	0.74	0.67	1.10	6.43	4.66	4.61	3.57
C	0.59	0.63	1	1	0.20	0.17	0.68	0.47	0.39	0.69	3.78	2.94	2.71	2.25
I	2.91	3.78	4.95	5.99	1	1	3.38	2.79	1.95	4.16	18.7	17.6	13.4	13.5
H	0.86	1.35	1.46	2.15	0.30	0.36	1	1	0.58	1.49	5.53	6.31	3.96	4.84
W	1.49	0.91	2.53	1.44	0.51	0.24	1.73	0.67	1	1	9.58	4.23	6.86	3.25
$\Pi$	0.16	0.21	0.26	0.34	0.05	0.06	0.18	0.16	0.10	0.24	1	1	0.72	0.77
R <sup>n</sup>	0.22	0.28	1.70	0.44	0.07	0.07	0.25	0.21	0.15	0.31	1.40	1.30	1	1
efpBA	0.04	–	0.06	–	0.01	–	0.04	–	0.02	–	0.24	–	0.17	–
efpA	0.18	–	0.31	–	0.06	–	0.21	–	0.12	–	1.17	–	0.84	–
efpB total	0.20	0.26	0.33	0.41	0.07	0.07	0.23	0.19	0.13	0.28	1.27	1.19	0.91	0.92
BAA real	0.52	0.42	0.89	0.66	0.18	0.11	0.61	0.31	0.35	0.46	3.37	1.94	2.49	1.49
NW (FoF)	2.83	–	4.82	–	0.97	–	3.29	–	1.90	–	18.2	–	13.1	–
NB (FoF)	8.37	3.95	14.3	6.26	2.88	1.05	9.74	2.92	5.62	4.35	53.8	18.4	38.6	14.1
NB (DS)	8.33	–	14.2	–	2.86	–	9.69	–	5.59	–	53.6	–	38.4	–
NW (DS)	6.19	–	10.5	–	2.13	–	7.20	–	4.16	–	39.8	–	28.5	–

In the text I refer to each column as [Y] instead of 'Y' or [C] instead of 'C'.

Table C.7: GK model: Correlations with seven key Macro Variables: HP filtered data (1955-2014) vs Model implied statistics.

Series	Data	Model	Data	Model										
	Y	Y	C	C	I	I	H	H	W	W	$\Pi$	$\Pi$	R <sup>n</sup>	R <sup>n</sup>
Y	1	1												
C	0.85	0.54	1	1										
I	0.90	0.70	0.74	0.29	1	1								
H	0.88	0.83	0.78	0.46	0.87	0.56	1	1						
W	0.85	0.15	0.81	-0.07	0.75	0.20	0.74	0.09	1	1				
$\Pi$	0.20	0.25	0.24	0.32	0.15	0.21	0.30	0.41	0.18	0.05	1	1		
R <sup>n</sup>	0.37	0.36	0.34	0.18	0.34	0.18	0.51	0.40	0.21	-0.12	0.55	0.43	1	1
efpBA	-0.54	-	-0.45	-	-0.57	-	-0.46	-	-0.46	-	-0.12	-	-0.14	-
efpA	-0.54	-	-0.49	-	-0.52	-	-0.66	-	-0.40	-	-0.52	-	-0.91	-
efpB total	-0.60	-0.20	-0.54	0.21	-0.59	-0.55	-0.69	-0.09	-0.46	-0.19	-0.51	0.04	-0.87	0.18
BAA real	-0.33	-0.03	-0.34	0.09	-0.32	-0.36	-0.35	0.01	-0.31	-0.25	-0.61	0.01	-0.27	0.68
NW (FoF)	0.33	-	0.33	-	0.47	-	0.48	-	0.13	-	0.26	-	0.46	-
NB (FoF)	0.46	0.06	0.35	0.11	0.52	0.18	0.38	-0.06	0.43	-0.13	-0.16	-0.21	0.01	-0.70
NB (DS)	0.41	-	0.37	-	0.51	-	0.41	-	0.33	-	-0.13	-	0.01	-
NW (DS)	0.41	-	0.46	-	0.43	-	0.41	-	0.36	-	-0.08	-	0.12	-

In the text I refer to each column as [Y] instead of 'Y' or [C] instead of 'C'.

**Table C.8:** FA model: Standard deviations relative to St.deviation of seven key Macro Variables: HP filtered data (1955-2014) vs Model implied statistics.

Series	Data	Model	Data	Model	Data	Model								
	Y	Y	C	C	I	I	H	H	W	W	$\Pi$	$\Pi$	R <sup>n</sup>	R <sup>n</sup>
Y	1	1	1.70	1.37	0.34	0.34	1.16	0.88	0.67	1.02	6.43	3.88	4.61	3.52
C	0.59	0.73	1	1	0.20	0.25	0.68	0.65	0.39	0.75	3.78	2.83	2.71	2.57
I	2.91	2.92	4.95	3.99	1	1	3.38	2.58	1.95	2.98	18.7	11.3	13.4	10.3
H	0.86	1.13	1.46	1.55	0.30	0.39	1	1	0.58	1.16	5.53	4.39	3.96	3.99
W	1.49	0.98	2.53	1.34	0.51	0.34	1.73	0.87	1	1	9.58	3.80	6.86	3.45
$\Pi$	0.16	0.26	0.26	0.35	0.05	0.09	0.18	0.23	0.10	0.26	1	1	0.72	0.91
R <sup>n</sup>	0.22	0.28	1.70	0.39	0.07	0.10	0.25	0.25	0.15	0.29	1.40	1.10	1	1
efpBA	0.04	–	0.06	–	0.01	–	0.04	–	0.02	–	0.24	–	0.17	–
efpA	0.18	–	0.31	–	0.06	–	0.21	–	0.12	–	1.17	–	0.84	–
efpB total	0.20	0.22	0.33	0.30	0.07	0.08	0.23	0.19	0.13	0.22	1.27	0.85	0.91	0.77
BAA real	0.52	0.36	0.89	0.49	0.18	0.12	0.61	0.31	0.35	0.36	3.37	1.38	2.49	1.26
NW (FoF)	2.83	2.01	4.82	2.75	0.97	0.69	3.29	1.77	1.90	2.05	18.2	7.79	13.1	7.07
NB (FoF)	8.37	–	14.3	–	2.88	–	9.74	–	5.62	–	53.8	–	38.6	–
NB (DS)	8.33	–	14.2	–	2.86	–	9.69	–	5.59	–	53.6	–	38.4	–
NW (DS)	6.19	–	10.5	–	2.13	–	7.20	–	4.16	–	39.8	–	28.5	–

In the text I refer to each column as [Y] instead of 'Y' or [C] instead of 'C'.

Table C.9: FA model: Correlations with seven key Macro Variables: HP filtered data (1955-2014) vs Model implied statistics.

Series	Data	Model	Data	Model										
	Y	Y	C	C	I	I	H	H	W	W	$\Pi$	$\Pi$	R <sup>n</sup>	R <sup>n</sup>
Y	1	1												
C	0.85	0.61	1	1										
I	0.90	0.65	0.74	0.35	1	1								
H	0.88	0.76	0.78	0.27	0.87	0.57	1	1						
W	0.85	0.33	0.81	0.56	0.75	0.12	0.74	0.04	1	1				
$\Pi$	0.20	0.30	0.24	0.19	0.15	0.07	0.30	0.43	0.18	0.08	1	1		
R <sup>n</sup>	0.37	0.11	0.34	-0.23	0.34	0.02	0.51	0.40	0.21	-0.39	0.55	0.50	1	1
efpBA	-0.54	-	-0.45	-	-0.57	-	-0.46	-	-0.46	-	-0.12	-	-0.14	-
efpA	-0.54	-	-0.49	-	-0.52	-	-0.66	-	-0.40	-	-0.52	-	-0.91	-
efpB total	-0.60	-0.15	-0.54	0.03	-0.59	-0.22	-0.69	-0.12	-0.46	-0.07	-0.51	0.06	-0.87	0.07
BAA real	-0.33	-0.17	-0.34	-0.31	-0.32	-0.21	-0.35	0.01	-0.31	-0.43	-0.61	0.03	-0.27	0.68
NW (FoF)	0.33	0.21	0.33	0.35	0.47	0.16	0.48	-0.12	0.13	0.29	0.26	-0.34	0.46	-0.67
NB (FoF)	0.46	-	0.35	-	0.52	-	0.38	-	0.43	-	-0.16	-	0.01	-
NB (DS)	0.41	-	0.37	-	0.51	-	0.41	-	0.33	-	-0.13	-	0.01	-
NW (DS)	0.41	-	0.46	-	0.43	-	0.41	-	0.36	-	-0.08	-	0.12	-

In the text I refer to each column as [Y] instead of 'Y' or [C] instead of 'C'.

Table C.10: Posterior Distribution for the two sub-samples

Parameter	1985-2004			2005-2014			Full sample		
	Mode	Mean	90% CI	Mode	Mean	90% CI	Mode	Mean	90% CI
$\alpha$	0.82	0.82	[0.88, 0.77]	0.78	0.78	[0.84, 0.72]	0.73	0.76	[0.80, 0.70]
$\chi$	0.47	0.47	[0.56, 0.37]	0.49	0.52	[0.64, 0.40]	0.36	0.40	[0.47, 0.32]
$\phi_x$	3.41	3.83	[5.22, 2.38]	0.91	1.10	[1.83, 0.34]	0.71	1.14	[1.75, 0.51]
$\epsilon$	6.36	6.65	[8.20, 5.10]	6.44	6.63	[8.18, 5.11]	5.55	6.25	[7.75, 4.71]
$\sigma_c$	1.51	1.49	[1.91, 1.08]	1.42	1.17	[1.68, 0.66]	1.93	1.91	[2.26, 1.55]
$\rho^h$	0.59	0.67	[0.97, 0.35]	1.46	1.68	[2.35, 0.96]	0.87	0.84	[1.05, 0.63]
$\gamma_p$	0.06	0.14	[0.27, 0.01]	0.08	0.19	[0.35, 0.01]	0.11	0.38	[0.66, 0.06]
$\rho_R$	0.70	0.71	[0.76, 0.65]	0.75	0.72	[0.79, 0.65]	0.62	0.62	[0.66, 0.58]
$\theta_\pi$	1.69	1.68	[1.88, 1.47]	1.59	1.60	[1.82, 1.38]	1.62	1.57	[1.72, 1.42]
$\theta_\gamma$	0.09	0.14	[0.26, 0.01]	0.06	0.11	[0.22, 0.00]	0.22	0.24	[0.32, 0.15]
$\mu$	0.07	0.07	[0.11, 0.02]	0.13	0.11	[0.17, 0.05]	0.04	0.04	[0.06, 0.02]
$\Theta_B$	0.75	0.73	[0.88, 0.58]	0.67	0.65	[0.85, 0.47]	0.80	0.78	[0.91, 0.66]
$A_{\bar{\psi}}$	0.19	0.19	[0.22, 0.17]	0.24	0.25	[0.30, 0.19]	-	-	[-, -]
$F_{\bar{\psi}}$	0.71%	0.72%	[1.04%, 0.40%]	0.70%	0.71%	[1.02%, 0.41%]	0.60%	0.61%	[0.87%, 0.33%]
$\rho_A$	0.83	0.81	[0.93, 0.69]	0.61	0.65	[0.85, 0.46]	0.81	0.81	[0.86, 0.77]
$\rho_G$	0.71	0.70	[0.83, 0.56]	0.63	0.64	[0.81, 0.47]	0.74	0.74	[0.81, 0.66]
$\rho_M$	-	-	- , -	-	-	- , -	-	-	- , -
$\rho_\gamma$	0.03	0.05	[0.09, 0.01]	0.18	0.25	[0.43, 0.07]	0.30	0.23	[0.37, 0.08]
$\rho_{MS}$	0.74	0.65	[0.84, 0.47]	0.18	0.22	[0.39, 0.04]	0.53	0.32	[0.56, 0.08]
$\rho_{bu}$	0.68	0.67	[0.76, 0.57]	0.68	0.69	[0.81, 0.58]	0.67	0.66	[0.72, 0.59]
$\rho_H$	0.99	0.98	[0.998, 0.97]	0.99	0.98	[0.999, 0.93]	0.99	0.99	[0.999, 0.998]
$\rho_\mu$	0.77	0.76	[0.88, 0.65]	0.79	0.76	[0.88, 0.64]	0.76	0.77	[0.84, 0.72]
$\rho_{\Theta_B}$	0.81	0.81	[0.87, 0.75]	0.84	0.83	[0.89, 0.78]	0.72	0.72	[0.76, 0.68]
$100 * \sigma_A$	0.47	0.48	[0.54, 0.41]	0.60	0.62	[0.74, 0.50]	0.73	0.71	[0.78, 0.64]
$100 * \sigma_G$	1.55	1.58	[1.79, 1.37]	1.56	1.64	[1.94, 1.33]	2.29	2.34	[2.54, 2.14]
$100 * \sigma_M$	0.14	0.14	[0.15, 0.12]	0.16	0.17	[0.20, 0.13]	0.20	0.20	[0.21, 0.18]
$100 * \sigma_\gamma$	8.8	10.1	[13.9, 6.3]	1.83	2.27	[3.77, 0.72]	2.12	3.53	[5.59, 1.59]
$100 * \sigma_{MS}$	6.8	8.3	[11.6, 5.1]	16.1	18.1	[24.4, 12.1]	10.8	16.5	[23.4, 9.1]
$100 * \sigma_{bu}$	0.63	0.66	[0.78, 0.55]	0.76	0.82	[1.05, 0.59]	0.71	0.73	[0.80, 0.66]
$100 * \sigma_H$	1.12	1.14	[1.35, 0.92]	1.97	1.96	[2.43, 1.47]	1.62	1.67	[1.90, 1.45]
$100 * \sigma_\mu$	28.5	36.4	[59.4, 15.0]	32.8	43.8	[70.3, 18.5]	76.5	91.2	[134.1, 43.4]
$100 * \sigma_{\Theta_B}$	0.52	0.53	[0.66, 0.40]	0.53	0.57	[0.73, 0.41]	0.54	0.56	[0.63, 0.48]

**Table C.11:** Standard deviations in absolute value of actual observables and model-implied series using Hodrick-Prescott filter.

Series	Data			Model			RATIO
	1985-2004 [1]	2005-2014 [2]	$\frac{2005-2014}{1985-2004}$ [3] $\left(\frac{[2]}{[1]}\right)$	1985-2004 [4]	2005-2014 [5]	$\frac{2005-2014}{1985-2004}$ [6] $\left(\frac{[5]}{[4]}\right)$	
Y	1.17	1.45	1.25	0.64	0.96	1.48	
C	0.83	1.00	1.20	0.59	0.78	1.31	
I	3.59	6.06	1.69	2.13	4.47	2.09	
H	1.08	1.66	1.54	0.87	1.30	1.50	
W	1.26	0.90	0.71	0.86	1.65	1.91	
$\Pi$	0.16	0.20	1.23	0.19	0.26	1.38	
$R^n$	0.27	0.32	1.16	0.20	0.20	0.99	
efpBA	0.03	0.10	3.20	0.02	0.06	3.09	
efpA	0.23	0.32	1.39	0.20	0.22	1.12	
efpB total	0.24	0.37	1.53	0.20	0.24	1.17	
BAA real	0.68	0.96	1.40	0.31	0.41	1.32	
NW (FoF)	2.74	9.88	3.60	3.40	2.88	0.85	
NB (FoF)	13.6	15.2	1.11	6.16	6.90	1.12	
NB (DS)	14.5	22.6	1.56	–	–	–	
NW (DS)	12.8	11.0	0.86	–	–	–	

**Table C.12:** Change in Standard deviations attributed to each Counterfactual coming from the last sub-sample estimation.

Series	$\frac{2005-2014}{1985-2004}$		$\Delta$ STD attributed to each Counterfactual					[1] & [2] [6]
	Data	Model	$\Phi_X$ [1]	$A_{\psi}$ [2]	$\mu$ [3]	$\theta_Y$ [4]	$\rho^h, \sigma_H$ [5]	
Y	1.25	1.48	1.41	1.02	1.01	1.00	1.05	1.43
C	1.20	1.31	0.94	0.99	1.00	1.00	1.13	0.93
I	1.69	2.09	2.29	1.07	1.02	1.00	1.01	2.36
H	1.54	1.50	1.32	1.01	1.01	1.01	1.02	1.34
W	0.71	1.91	1.07	1.00	1.00	1.03	1.46	1.07
$\Pi$	1.23	1.38	0.97	1.00	1.00	1.03	1.16	0.97
Q	–	0.86	0.96	1.13	1.04	0.99	1.05	1.07
Rr	–	1.38	1.01	1.00	1.00	1.01	1.14	1.01
$R^n$	1.16	0.99	1.01	1.01	1.01	0.96	1.10	1.01
efpBA	3.20	3.09	1.00	1.47	1.67	1.00	1.00	1.47
efpA	1.39	1.12	1.15	1.13	1.02	1.00	1.01	1.31
efpB total	1.53	1.17	1.15	1.15	1.05	1.00	1.01	1.33
BAA real	1.40	1.32	1.09	1.09	1.03	1.00	1.07	1.20
NW (FoF)	3.60	0.85	0.93	0.94	1.05	0.99	1.03	0.94
NB (FoF)	1.11	1.12	0.97	1.23	1.17	1.00	1.01	1.23
NB (DS)	1.56	–	–	–	–	–	–	–
NW (DS)	0.86	–	–	–	–	–	–	–

## C.2 Financial Net Worth Data

- **Net Worth Variables.**

In the baseline framework I have both agents, firm and bank, facing a financial friction related to their respective financial health. While I do not include net worth variables in the list of observable series for estimation, I closely follow how their business cycle properties match that of the model implied economy. I keep track of two data series related to net worth of firm and bank. I summarize below the definitions of firm and bank net worth series that I follow.

- The model considers the purchase of tangible assets by firms with financing from banks and its own net worth. A standard measure in literature matching this definition is the difference between tangible assets and credit market liabilities from the Flow of Funds account. This measure is similar to the readily available net worth series of non-financial non-farm net worth series provided in the Flow of Funds tables. Their growth rates are highly correlated (86 %).
- The theoretical definition of bank in my model is that of an entity that accepts deposits from households and issues standard loans, but can also hold corporate equities in its balance sheet. I use corporate equities on liability sides of financial business balance sheet from the Flow of Funds (Table L108) as a proxy for bank net worth. The measure captures all financial businesses, including investment banks and commercial banks. There are other financial entities included in this measure which do not accept household deposits, therefore not qualify based on the theoretical definition. The weight of other entities not relevant to the definition in my framework is negligible to affect the properties of the series.

I do not refer to a commercial bank in my theoretical models (see [Christiano et al. \(2010\)](#), [Rannenberg \(2016\)](#) for such definition). Limiting my definition of net worth only to commercial banks would leave a gap between theoretical and actual definition of net worth. Banks that can hold firm equity on the asset side of their balance sheets are critical for the second financial friction related to bank financial health. These kind of banks are financial institutions other than commercial banks that issue only loans. These intermediaries hold the bulk of corporate equities on their asset side of their balance sheets. Therefore, I follow the second moments of the net worth of all financial institutions and compare it to the theoretical moments of the net worth variable from the model.

As alternative measures I follow stock price indices. [Christiano et al. \(2010\)](#) use the Dow Jones Wilshire 5000 index but that does not distinguish between firm and bank net worth. *Datastream* database provides sector based indices. I consider stock indices relevant financial and nonfinancial sector provided by *Datastream* as alternative measures that take into account the firm and bank market value respectively.

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