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**Foreign Reserve Accumulation in China:  
A DSGE Analysis of Main Motivations  
and Their Relative Contributions**

**A Thesis Submitted to School of Economics, Finance and Business,  
in partial fulfilment of the degree of Doctor of Philosophy in  
Accounting and Finance**

**Chuanjie Zhang  
Grey College  
University of Durham  
August 2016**

**To my parents and family**

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Chuanjie Zhang

August 2016

## **DECLARATION**

The content of this doctoral dissertation is based on the research work completed at Durham University Business School, UK. No material contained in the thesis has previously been submitted for a degree in this or any other university.

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*Chuanjie Zhang*

# **Foreign Reserve Accumulation in China: A DSGE Analysis of Main Motivations and Their Relative Contributions**

**Chuanjie Zhang**  
Grey College

**D. Phil (Accounting and Finance)**  
August 2016

## **ABSTRACT**

Over the past decade, the question of why China's emerging market policymakers have accumulated such massive international reserves has been posed and debated by scholars. However, given the existence of several competing explanations, the excess amount and rapid growth of China's foreign exchange reserves remains a puzzle.

In order to identify the primary motivations for the reserve accumulation, this thesis discusses the role of intergenerational transfers, exchange rate policy and the precautionary savings motive. We develop an estimated DSGE model for China, including these three motives, to evaluate various competing explanations for China's foreign exchange reserves. Through analysis of impulse response function, risky steady state and shock decompositions of the model, we pursue the primary motivation for the reserve accumulation.

This thesis uses DSGE techniques to explore and compare the influence of the competing theories. The estimation results suggest that the total economic value of intergenerational transfers in China is bigger than the capital flow caused by the other two motivations. However, from an economic growth perspective, the influence of currency undervaluation plays a more important role.

The thesis findings indicate that the precautionary motivation is the most significant factor in China's accumulation of international reserves. The impact of capital outflows causes more change and decline in China's foreign exchange reserves than any other competing explanation. The precautionary motive is therefore the primary motivation for the current accumulation of foreign exchange reserves by China. The originality of this research, and its contribution to the foreign reserves literature, lies in the use of the DSGE method to analyse the motives for amassing foreign exchange reserves.

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# Chapter 1 Introduction

## 1.1 Motivation and Aims of the Study

In recent years the large amount, and rapid growth, of China's foreign exchange reserves have attracted considerable attention worldwide. Indeed, China's massive reserve holding represents a prominent case of the recent upsurge in reserve hoarding among emerging market economies. In 2006, when its stockpile grew to \$1.066 trillion, China overtook Japan to become the world's biggest holder of foreign exchange reserves. By the end of 2014, China's foreign reserves had reached 3.87 trillion dollars, a remarkable increase compared with the mere 0.167 billion dollars of foreign reserves it held in 1978, cementing even further its position as the world's largest holder of foreign exchange reserves.

However, China's hoarding of foreign exchange reserves begs the question of why the Chinese government decided to accumulate such massive foreign exchange reserves in the first place; in other words, what is the primary motivation for this build-up of reserves? Is the government simply trying to self-insure against foreign exchange market turbulence and potential currency crisis, or is it manipulating the exchange rate to achieve a competitive

advantage? Do other motivations also play a role in China's foreign exchange reserves accumulation?

To pursue the primary motivation for the reserve accumulation, we discuss the role of three main potential motives, namely intergenerational transfers, exchange rate policy and the precautionary motive. We develop an estimated DSGE model for China including these motives to evaluate various competing explanations for the foreign exchange reserves. Although DSGE models are micro-founded models widely applied by central banks as tools of macroeconomic research and policy analysis, they are seldom used in motivation analysis. Therefore, the originality and contribution of this research lies in the use of this new method to analyse the motives behind foreign exchange reserve hoarding. The significance of this approach is to provide a new method to analyse the reasons for China's prodigious amount of foreign exchange reserves, and whether it is sensible for the Chinese government to reduce the current level of international reserves.

## **1.2 Overview**

### **1.2.1 Traditional Theories of International Reserves Accumulation**

Given the massive size of China's international reserves as measured by three main indicators we mentioned below, scholars have been keen to investigate

the motives of the country's policymakers. The debate has led to several competing explanations, but no clear answer. As a result, the excess amount and rapid growth of China's foreign exchange reserves remains a puzzle.

Three main indicators of reserve adequacy, namely import-based measures, money-based measures and debt-based measures of reserve adequacy, suggest that China's international reserves might be excessive. First, import-based indicators of reserve adequacy measure reserves in terms of months of imports, that is, the number of months a country can continue to support its current level of imports if all other inflows and outflows stop. This ratio provides a simple method to scale the level of foreign exchange reserves by the size and openness of the economy. However, for the period from 2001 to 2014, the average value of China's total reserves in months of imports was 17.45. This is much higher than the three months of imports considered by many researchers as the optimal level of foreign reserves.

Money-based indicators of reserve adequacy offer a way of evaluating the potential for resident-based capital flight from the currency. An unstable demand for money or the presence of a weak banking system indicates a greater probability of such capital outflows. In these conditions, the ratio of reserves to base money is a potentially useful indicator. Moreover, the ratio of reserves to broad money is a significant indicator for scaling the optimal level of foreign reserves under fixed exchange rate regimes. In the period from 2001 to 2014, the average value of China's broad money to total reserves ratio was 4.81. This compares with an optimal level between 5 and 10.

In addition, a measure comparing reserves and short-term external debt in terms of all debt repayments to foreigners over the coming year is very helpful to estimate risks associated with adverse developments in international capital markets, such as have been seen in recent years. Such a measure allows a judgement to be made about how quickly a country would pay off its external debts. Empirical work in the Fund implies that a debt-based measure of reserve adequacy is a substantial indicator in countries with significant but uncertain access to capital markets. A smaller ratio of international reserves to short-term debt is associated with a greater incidence and depth of crises. Over the period from 2001 to 2014, the average ratio of short-term external debt to total reserves in China was 16.7%.

The significant increase in China's foreign exchange reserves following the 1997 Asian financial crisis linked the precautionary motive to international reserves hoarding. According to the precautionary motive explanation, the accumulation of international reserves is considered by policy makers as an insurance against sudden capital outflow. Heller (1966) qualified optimal level of reserves by weighting the opportunity cost of holding reserves and the risk of an external disequilibrium. Jeanne and Ranciere (2011) argue that another precautionary reason for foreign exchange reserve accumulation is to sustain domestic absorption in times of a sudden stop in capital flows.

Wyplosz (2005) also believes that precautionary saving is the main driver of China's reserve accumulation. This is a traditional motive for central banks to accumulate international reserves, to meet the need for foreign currency to

protect against potential turbulence on currency markets. Such turbulence occurs when private capital flows suddenly threaten to bring unwelcome changes to a country's exchange rate. Lee (2004) estimates the optimal level of foreign exchange reserves based on option price theory. He assumes that an overall insurance value equal to the amount of short-term external debt is needed for precautionary reasons. This overall insurance level will be met partly through market-based insurance and partly by self-insurance.

Another important motivation for holding foreign exchange reserves cited in recent research is ensuring financial stability. Obstfeld, Shambaugh and Taylor (2010) posit that reserve accumulation is a key tool for managing domestic financial instability. They argue that “a primary reason for a central bank to hold reserves is to protect the domestic banking sector, and domestic credit markets more broadly, while limiting external currency depreciation” (p.3). Joyce and Garcia (2011) also observe that during the recent crisis, countries that had accumulated more foreign exchange reserves experienced less financial and currency instability. They deem that this fact confirms the significance of international liquidity.

Dooley, Folkerts-Landau and Garber (2006) take a different view and follow a modern mercantilist approach to account for hoarding of international reserves as part of a deliberate development strategy. They consider that international reserves act as collateral to encourage foreign direct investment. The mercantilist motive hypothesis regards reserves accumulation as a by-

product of an export-promoting strategy. The reserve accumulation facilitates this strategy by preventing or slowing currency appreciation.

A body of recent empirical research has documented a positive relationship between the real exchange rate and growth. Rodrik (2008) discusses the significant role of an undervalued real exchange rate in stimulating growth. Hausman et al. (2005), Johnson, Ostry and Subramanian (2006) and Bereau et al. (2012) also argue for the importance of a depreciated real exchange rate to accelerate growth, and believe that this exchange rate policy is an efficient tool for economic development.

Using a sample of 93 developed and developing countries, Razin and Collins (1999) construct a fundamentals-based index of real exchange rate overvaluation and find that a depreciated exchange rate favours exports and economic growth, whereas an appreciated exchange rate constrains growth; moreover, they argue that the effect of appreciation is stronger. Rodrik (2008) considers that overvaluation damages growth and undervaluation favours economic growth, but does not find a significant difference in terms of the magnitude of each effect.

Aizenman and Lee (2007) compare the weight of precautionary and mercantilist motives in international reserves accumulation by developing countries. They explicate the importance of mercantilism in international reserves accumulation by distinguishing monetary mercantilism from financial mercantilism. The latter is a modern form, often labelled the export-

led strategy. A common representation of this strategy is that it seeks to boost export growth by maintaining an undervalued exchange rate. Although mercantilist effects are significant for export growth and purchasing power parity, Aizenman and Lee believe that such effects have a smaller impact relative to variables associated with precautionary demand in the determination of the appropriate level of reserves. Their empirical results support the significance of precautionary motives.

Following in the footsteps of Jeanne and Ranciere (2006), Ruiz-Arranz and Zavadjil (2008) explore whether foreign exchange reserves in emerging Asia are excessive when compared with optimal levels predicted by their model. They deem that international reserve holdings by most Asian countries seem not to be above the optimal levels, excluding China. They identify that the costs of sudden stops in emerging Asian countries are greater than the average size of costs derived by Jeanne and Ranciere (2006). The authors conclude that the precautionary motive is a critical driver behind reserve accumulation by Asian countries, and has been reinforced over the last decade.

### **1.2.2 Gaps in the Literature**

Since the Asian financial crisis, models based on the standard economic explanatory variables have underestimated the reserve holdings of China. The unusual accumulation is a sign that factors other than purely precautionary motives might play an important role. Although any or all of the existing conventional explanations may be true to a certain degree (Bonham and

Wiemer, 2013), we consider that the role played by intergenerational transfers in boosting China's savings rate merits further exploration.

Intergenerational transfers within and between Chinese family networks have begun to attract growing interest. Many studies show that intergenerational transfers are a significant driver of high domestic savings and that, due to inadequate domestic financial development, a large portion of such domestic savings overflows into foreign asset investments, leading to persistent foreign reserve accumulations.

Kotlikoff and Summers (1981) show that 80% of US national wealth results from intergenerational transfers, whereas only 20% is accounted for by life-cycle savings. Although the magnitude of this estimation has been disputed by subsequent research (Modigliani, 1988; Gale and Scholtz, 1994; Karagiannaki, 2011), international evidence generally confirms that intergenerational transfers are an important source of wealth accumulation. Given the historically tight family ties and cultural influences in China, it is conceivable that intergenerational transfers are a significant contributor to wealth and thus a critical driver for the growth of Chinese household savings.

According to Wu et al. (2012), Chinese intergenerational transfer arrangements are largely influenced by inflated house prices. In China, the real value of constant quality land has increased by nearly 800% since 2003. Price-to-income ratios are at their highest levels in the coastal areas. In early 2010, average housing prices hovered at approximately 18.5 times average

annual income, and these prices have since risen even higher. Chivakul et al. (2015) show that in 2014, nationwide in China the average house price had risen to about 22 times average annual disposable income in 2013. In these circumstances, it has become extremely difficult, if not impossible, for many young households in China to purchase a satisfactory property without financial support from their parents. Chinese parents who desire to offer this support must save more for the property needs of their offspring.

Yang (2012) argues that when China's ill-functioning financial system fails to channel accumulated savings to investment or consumption, the excess savings end up as foreign reserves. Conversely, the amount earned by the trade surplus that is not consumed or invested must end up being saved.

In short, as a result of factors such as Chinese traditional culture, the undeveloped financial system and excessively high housing prices, Chinese parents may have to save more for their children's education, housing and marriage. As a consequence, intergenerational transfers provide critical support for China's young households to improve their standard of living and general utility. Given their importance to household savings in China, intergenerational transfers should not be overlooked as a driving force behind China's hoarding of foreign reserves.

However, the modelling of intergenerational transfers in China is in its infancy. Studies of Chinese household savings and intergenerational transfers remain relatively scarce. The pioneers in this field are Song, Storesletten and

Zilibotti (2011), who also employ the overlapping generation model to explain, among other things, the relation between parents and children. The present study further explores the effects of domestic savings – explicitly including intergenerational transfers – on China’s foreign reserves accumulation in a Dynamic Stochastic General Equilibrium (DSGE) setting.

An alternative explanation, the mercantilist motive hypothesis, regards reserves accumulation as a by-product of an export promoting strategy. The reserve accumulation facilitates this strategy by preventing or slowing currency appreciation. In the Chinese context, Bonatti and Fracasso (2013) argue that capital controls and an undervalued currency make the country less exposed to sudden stops of capital movements, and so the mercantilist motive is particularly appropriate to account for the reserve accumulation.

In their empirical research, Delatte et al.(2014) formally show that the mercantilist motive rather than the precautionary concern is more consistent with the rapid reserve accumulation in China. Rapetti et al. (2012) also identify a positive relationship between an undervalued exchange rate and growth, and demonstrate a negative relationship between an overvalued exchange rate and growth. They argue that the effect of overvaluation is slightly stronger than that of undervaluation.

However, these views are not supported by many economic models, and a competing strand of economic thought does not predict a positive relationship between an undervalued real exchange rate and growth. According to these

latter theories, the value of a currency should be set at a level that is consistent with both internal and external balances (Krueger, 1983; Edwards, 1989; Williamson, 1990; Berg and Maio, 2010).

This strand of the literature generally suggests that both under- and overvaluation of a currency adversely impact growth by leading to misallocation of resources and eventual declines in real output (Aguirre and Calderon, 2005). Therefore, according to this view, exchange rate deviation from equilibrium levels is associated with macroeconomic disequilibrium, regardless of the direction of the misalignment.

The above contention suggests that there remain gaps in our understanding of the links between real undervaluation, intergenerational transfers and economic growth, and thus challenges the persuasiveness of the traditional theories of international reserve accumulation. This prompts the current paper to dissect the export and growth effects of currency undervaluation with reference to the Chinese case. To pursue the interplays between intergenerational transfers, precautionary savings and China's exchange rate policy, we use DSGE techniques to explore and compare the influence of these competing theories.

### **1.2.3 DSGE as a Modelling Strategy for Analysis of Reserve**

#### **Accumulation**

Over the last decade, Dynamic Stochastic General Equilibrium (DSGE) modelling strategies have become a widely used toolkit for macroeconomic research, each depending on its particular analytic capabilities. There are several key features of DSGE modelling. The term "dynamic" connects to the forward-looking behaviour of households and firms, whereby their behaviour today depends on their expectations and forecasts for the future. The purpose of households and firms is certainly to maximize their utility and profit. "Stochastic" relates to the explication of shocks that households and firms face in the real economy. By making more detailed specification of the stochastic shocks that cause economic fluctuations, it is possible to seek out more clearly the transmission of shocks to the economy. Next, "general" refers to the inclusion of the entire economy. It means that DSGE models are macroeconomic models to define the overall economy. Finally, "equilibrium" refers to the description of optimal behavior of households and firms given their preferences and constraints they face.

Building on microeconomic foundations, DSGE models are applied to interpret and analyse the entire economy. Not only must macroeconomic equality between supply and demand be observed; it is also necessary to ensure that the developments expected by households and firms are consistent with their current and future planned decisions. If the decisions and plans of individual agents are compatible with the macroeconomic conditions, then

this is termed “general equilibrium”. Therefore, the consistent formulation of the interaction between the individual decision makers and the economy as a whole is a substantial characteristic of DSGE models.

The recent DSGE models assume that central bank behaviour is well described by a monetary policy rule. In this kind of formulation, the central banks adjust the nominal interest rate based on several indicators. From a theoretical standpoint, Clarida (2001) uses simple DSGE models to show that if central banks do not adjust their monetary policy instruments sufficiently responsively to changes in inflation, this could destabilize the economy. Similarly, lax fiscal policy may be detrimental to the economy. DSGE models can be used to identify these conditions and to help policy makers avoid detrimental policy decisions more effectively. Micro-founded DSGE models can also help monetary authorities consider uncertainty in their decision-making process.

Because the DSGE models are micro-founded, it seems appropriate and realistic to examine central bank behaviour in such a setting: the central bank may express its preference via a criterion to maximize social well-being; then, it must determine its optimal behaviour taking uncertainties into account. Depending on the circumstances, an aggressive or cautious policy decision may be shown to be advisable. By proposing a consistent analytical framework that satisfies the Lucas Critique, DSGE models are the most appropriate candidate for this type of study.

Escudé (2012) builds a DSGE model for an open economy in which the central bank systematically intervenes in both the foreign exchange markets and domestic currency. He integrates the Taylor rule approach with exchange rate policy to represent how the central bank varies its bond liabilities and foreign exchange reserves to achieve its operational targets: interest rate adjustment and nominal currency depreciation. Adolfson et al. (2011) construct a medium-sized open economy DSGE model to evaluate the influence of the recent recession in the world economy on economic development in Sweden. They apply Ramses using data and Bayesian estimation of parameters to pursue optimal monetary policy projections.

De Carvalho et al. (2015) model a small open economy which receives strong capital inflow from foreign investment. The model parameter is estimated with Bayesian techniques using data from Brazil. In order to compensate for the impact of domestic and international shocks that challenge financial stability in emerging economies, the authors investigate the influence of international reserves management, export-related credit lines and capital control in the DSGE framework to compare the effects of different macroeconomic policy instruments.

The latest generation DSGE models, which have overcome the weaknesses of older models and incorporate the most recent theoretical and econometric advances, are the most sophisticated and realistic tools now available for conducting macroeconomic analysis.

## 1.2.4 Research Questions and Main Findings

China's prodigious amount of foreign exchange reserves raises several questions: What is the primary motivation for this reserve accumulation? How can we compare and evaluate the quantitative importance of different motivations? What are the interplays between intergenerational transfers, precautionary motives, mercantilist motives and China's hoarding of international reserves? Is China's central bank simply trying to counteract the impact of foreign exchange market turbulence and potential currency crisis, or is it manipulating the exchange rate to achieve a competitive advantage?

To address these research questions, we discuss three main motivations for the foreign exchange reserves accumulation: intergenerational transfers, precautionary savings and exchange rate undervaluation. We evaluate policymakers' motivation by building estimated DSGE models for the Chinese economy, using different sets of explanatory variables according to the different motives. The parameters of the model are determined using relevant data collected from the databases of the IMF, World Bank and National Bureau of Statistics of China. DSGE models are micro-founded models widely used to expound the whole economy. However, they are rarely applied to motivation analysis. The contribution of this approach is to provide a new method to analyse the primary reason for China's massive amount of foreign exchange reserves.

First, we shed light on the interplay between intergenerational transfers, household savings, and China's accumulation of foreign exchange reserves. Although China's high domestic savings are a main cause of the growth of its massive foreign exchange reserves, these excessive savings are largely driven by the intergenerational transfers that are prominent in China. It follows that intergenerational transfers should have a significantly positive relation with China's foreign reserve build-up. Our modelling strategy and its estimation demonstrate that intergenerational transfers have a significantly positive impact on foreign reserves accumulation.

Secondly, we analyse the nexus between foreign reserve accumulation and real undervaluation in China from a growth perspective. The examination focuses on the extent to which exchange rate policy has led to reserve accumulation in China. In a general equilibrium setting, our analysis shows that real undervaluation contributes to reserve accumulation in a complex process. Rather than the commonly assumed mercantilist interpretation, undervaluation affects China's reserve build-up mainly through its positive effect on economic growth. In the short run, undervaluation induces a positive response from TFP, which promotes growth. Over the medium term, both exports and imports have a positive response to real undervaluation in the impulse response analysis. In the longer run, undervaluation leads to an increase in income.

Moreover, we use a parameterized DSGE model to quantify the role of precautionary savings in China's economy, which has recently been slowing

down. The quantitative importance of the precautionary savings in the model is equal to 26.11% of gross savings and 30.5% of net foreign assets. The estimation results suggest that precautionary motives play an important role in the economy and are an important driving force behind the accumulation of foreign reserves in China.

Finally, we construct an estimated DSGE model including the above motivations to compare the quantitative importance of these competing explanations. Our modelling strategy and its estimation provide results showing that these main motivations do have a significantly positive impact on foreign reserves accumulation. The estimation results in the model suggest that, in the case of China, intergenerational transfers are relatively more important than the other two motivations. The motivation of intergenerational transfers is an important driving force behind the accumulation of foreign reserves in China.

### **1.3 Organization of the Thesis**

The remainder of this thesis is organized as follows. Chapter 2 comprises an in-depth literature review of DSGE modelling methodology. This chapter introduces key features and advantages of DSGE modelling and provides a review of the historical development and theoretical foundation of DSGE models. We also introduce the methods for estimating and evaluating DSGE models, and describe four methods for solving DSGE models.

Chapter 3 provides an example of the basic structure of DSGE modelling methodology, and describes the DSGE modelling methodology applied later in the thesis. In this chapter, we investigate China's massive international reserves accumulation and develop a new method to include intergenerational transfer behaviour in the DSGE framework, which permits explicit consideration of the influence of intergenerational transfers on Chinese household savings. We then apply Bayesian estimation and evaluation techniques to analyse relevant parameters and model performance. Our estimations show that intergenerational transfers are a significant driver of high domestic savings and persistent foreign reserve accumulations.

Chapter 4 analyses the relationship between real currency undervaluation and China's hoarding of foreign reserves. In a general equilibrium setting, we show that, contrary to the commonly assumed mercantilist interpretation, real undervaluation affects China's reserve build-up mainly through its positive effect on economic growth. In the short run, undervaluation induces a positive response from TFP. Over the medium term, both exports and imports have a positive response to real undervaluation in the impulse response analysis. Model simulation suggests that, in the longer run, the income increase is more important than the other two channels.

Chapter 5 develops a parameterized DSGE model to quantitatively assess the role of the precautionary motive in the accumulation of foreign reserves in China. The model in this chapter offers an innovative approach to exploring the importance of the precautionary motive in foreign reserves holdings. The

impulse response function of net foreign assets indicates a strong correlation between precautionary savings of households and foreign reserves accumulation. A sudden stop in capital flows leads to a significantly lower position of international reserves in the model. These results suggest that the precautionary motive is an important driving force behind the accumulation of foreign reserves in China.

By developing an estimated DSGE model including three potential motives for China's hoarding of reserves, Chapter 6 evaluates various competing explanations for the foreign exchange reserves accumulation. These motivations can be given a structural interpretation in the model as exogenous or endogenous process. Through analysis of impulse response function and shock decompositions of the model, we pursue the primary motivation for the reserve accumulation.

Chapter 7 concludes the thesis and provide avenues for future research.

## **Chapter 2 Literature Review**

### **2.1 Historical development of DSGE models**

As the combination of a short-run Keynesian framework with growth theory, the neoclassical synthesis played a significant role in macroeconomic analyses in the 1960s and 1970s. In the methodology of the neoclassical synthesis, these models represented aggregate supply (AS) and aggregate demand (AD) with the Phillips curve. More and more correlative models were developed with this trend. However, designers and users gradually realized that the models did not perform very well, especially when they were used to forecast or describe certain phenomena. Therefore, the application and development of these models by the academic community declined. In many areas, the models were almost abandoned.

In order to criticize neoclassical synthesis as applied to analyse the influence of economic policy, the Lucas Critique (1976) was proposed. Contrary to the traditional view that private agents never change their behaviour and that individual preference is insignificant, Lucas argued that private agents should adjust their behaviour to optimize their wealth, depending on their rational expectation of the future. In the Lucas Critique, the relationship between endogenous and exogenous variables can be represented in reduced form.

Moreover, on the basis of Lucas (1976) and the research of Lucas and Sargent (1979), Kydland and Prescott (1982) developed the dynamic general equilibrium method. They argued that economic policy makers should take account of both the constraints and the objectives of households and firms.

One surprising development in macroeconomics is the systematic incorporation of the new assumptions about agents into macroeconomic models. These assumptions consider that agents in the economy are utility maximizing, rational, forward-looking and fully informed. This development started with the rational expectations revolution of the 1970s, which taught us that macroeconomic models can be accepted only if agents' expectations are consistent with the underlying model structure. The real business cycle (RBC) theory, proposed by Kydland and Prescott (1982), introduced the idea that macroeconomic models should be "micro-founded" and based on dynamic utility maximization. While RBC models had no place for price rigidities or other inertia, the New Keynesian School systematically introduced rigidities of all kinds into similar micro-founded models.

These developments took place in the ivory towers of academia for several decades, until in recent years the models were implemented empirically such that they have now become tools of analysis in the boardrooms of central banks. The most successful implementations of these developments are to be found in the Dynamic Stochastic General Equilibrium models (DSGE models) that are increasingly used in central banks for policy analysis (see Smets and Wouters 2003,2004; Christiano et al. 2008; Adjemian et al. 2007).

These conclusions are surprising for several reasons. First, while macroeconomic theory enthusiastically embraced the view that agents fully understand the structure of the underlying models in which they operate, other sciences, such as psychology and neurology, increasingly uncovered the cognitive limitations of individuals (Stanovich and West 2000; Damasio 2003; Kahneman 2002; Camerer et al. 2005). We learn from these sciences that agents understand only small fragments of the world in which they live, and instead of maximizing by continuously taking all available information into account, agents use simple rules (heuristics) to guide their behaviour and their forecasts about the future. This raises the question of whether the micro-founded macroeconomic theory that has become the standard is well-grounded scientifically.

A second source of surprise in the development of macroeconomic modelling in general and the DSGE-models in particular is that other branches of economics, such as game theory and experimental economics, have increasingly recognized the need to incorporate the limitations agents face in understanding the world. This has led to models that depart from the rational expectations paradigm (Thaler 1994).

For example, take a linear rational expectation model with one endogenous variable  $y(t)$  and one exogenous variable  $x(t)$ , which represents the economic policy factor. We can assume that the variable  $y(t)$  is a function of the

variable  $x(t)$ . The reduced form (the solution) of this function can be expressed as:

$$y(t) = f(a,b,c) \cdot x(t) \tag{2.1}$$

In (2.1),  $f(a,b,c)$  is a function of a, b and c, which are the economic policy parameters. When these economic policy parameters alter,  $f(a,b,c)$ , which is the parameter of reduced form, also changes. Therefore, Lucas Critique is characterized by the lack of invariance in the reduced form parameter.

In the 1980s and 1990s, many central banks continued to use reduced form statistical models to forecast the economic situation. However, these models could not be used with any degree of confidence to generate forecasts of the results of policy changes. To address the shortcomings of previous macroeconomic models, Lucas (1987) proposed "real business cycle models". Real business cycle (RBC) theory provides pioneering ideas about how it is possible to build quantitative macroeconomic models exclusively from explicit optimizing behaviour at the individual level.

However, RBC models did not receive immediate universal support. When RBC theory was first formulated, cyclical fluctuations were seen as evidence of inefficiencies in the economic system and policy. According to RBC theory, fluctuation is considered as agents' most efficient response to real exogenous shocks. In addition, although the initial work of RBC models was to describe the US economy by applying dynamic general equilibrium under pure and perfect competition, the general equilibrium model under perfect

competition is still not consummate. The unconventional research result of the importance of technology shocks contradicted the common sense that demand and monetary shocks should play a determining role in explaining the business cycle. Furthermore, the disadvantages of small size and high degree of abstraction of RBC models mean that these models are not appropriate to answer all the questions of interest.

In this context, RBC theory was seriously critiqued on several fronts. First, the RBC strand was criticized for its use of pure and perfect competition and rational expectations assumptions from a theoretical viewpoint. According to these assumptions, government fiscal and stabilization policy are inefficient and even detrimental to the entire economy. However, this should not be true in real economies, especially in developed countries.

Second, RBC models were challenged from the theoretical view of point. RBC models assume that labour supplies are generally highly elastic, which is contrary to the results of recent microeconomic research. Moreover, the RBC approach also came in for criticism at the empirical level. RBC models, at least the first-generation models, are unable to reproduce realistic fluctuations in hours worked and real wages.

The disadvantages of real business models meant that the academic community and policy-making institutions were encouraged to be develop more sophisticated and practical models. As RBC theory developed in the 1980s, new Keynesian theory grew to offer micro-foundations for key

Keynesian concepts. Although they began as two independent theories, their integration is an attractive approach for macroeconomists. In this context, the new frameworks reflect a natural synthesis of the new Keynesian and real business cycle approaches. As a result, DSGE (Dynamic Stochastic General Equilibrium) methodologies were gradually proposed.

The macroeconomic models in use today combine the main theoretical basis of RBC models and the theoretical, methodological and empirical advances of the last three decades. According to the results of empirical research, labour and the product market are not perfectly competitive. These results led to the development of RBC models in which pure and perfect competition was replaced by the monopolistic competition assumption. In the definition of DSGE models, the models should contain and dilate the shocks that households and firms confront in the real world. Therefore, DSGE models gradually began to replace the RBC models.

The main shortcoming of first-generation dynamic general equilibrium models is the lack of shock amplification and propagation properties. Some authors believed that poor modelling ability to describe the labour market was the reason for this weakness. In order to eliminate these drawbacks, many authors began to add frictions to the labour market. Danthine and Donaldson (1990), Gomme (1999), and Alexopoulos (2004), advanced the efficiency wage hypothesis, which assumes that wages do not correspond to the marginal productivity of labour. This is a mechanism that may help to propagate the impact of shocks.

In order to correct the shortcomings of first-generation DSGE models, research was also carried out on financial market frictions. Households and firms are subject to budget constraints and technology constraints. Therefore, they have to finance their projects using credit. Hence, endogenous variations in the credit supply constitute a powerful mechanism for amplifying and propagating shocks.

This technological advance led to a change in the view about fluctuation. Although fluctuations are still considered as the equilibrium response of private agents to different shocks, they are no longer seen as the main efficient response. This theoretical shift has made research on general equilibrium models more open to developments in new Keynesian economics.

The combination of advanced RBC models and new Keynesian theory developed sticky price mechanisms into DSGE models. Moreover, the combination of dynamic general equilibrium models and new Keynesian economics constructed the main theoretical framework of new neoclassical synthesis. In the 1990s, several articles demonstrated how to incorporate nominal rigidities into dynamic general equilibrium frameworks. Just as in the new Keynesian literature, the initial emphasis of these models was on nominal price rigidities. The models are essentially monopolistic competition versions of the neoclassical growth model.

As part of these developments, a great deal of research emerged on the new Phillips curve. Econometric estimates based on models with sticky prices or on the new Phillips curve taken in isolation come to the same conclusions. However, an implausible degree of nominal rigidity must be assumed to reproduce the persistence properties of inflation. Based on these findings, Chari et al. (2000) suggested abandoning these models and taking a route not explored in the literature.

Faced with this problem, the literature reviewed the theoretical findings of Ball and Romer (1990), which called for a combination of nominal and real rigidities, particularly on the labour market. In efforts to improve the models, two different methods were used. First, the research summarized by Woodford (2003) was aimed at explaining the shallow slope of the Phillips curve without assuming strong nominal rigidities.

Second, a series of papers explained the persistence of real marginal cost. These researches assume that nominal wages and prices are sticky, and then real wages become less sensitive to shock, again resulting in small changes in real marginal cost. These advances form a bridge between the earlier findings of new Keynesian economics and new neoclassical synthesis models. The latest generation models are medium-sized models that incorporate almost all the theoretical advances described above. These models incorporate many theoretical mechanisms and several shocks, and are generally sufficiently well suited to the data.

Sims (2008) provides higher order approximations method in the DSGE framework to extend Markov-switching multiple-equation models and obtain a more accurate estimate of the marginal data density. Andreasen, Fernandez-Villaverde, and Rubio-Ramrez (2013) extend Sims (2008)'s approach, they provide closed-form expressions for first and second unconditional moments and impulse response functions to derive the approximation of the policy function of any arbitrary order. Fernandez-Villaverde et al.(2016) introduces the latest developments of solution and estimation techniques for DSGE models. They investigate the foundations of current numerical approximation techniques and statistical inference.

Adolfson et al.(2011) build an open economy medium-sized DSGE model to investigate the impact of the recent recession in the world economy on the economic development in Sweden. They believe that DSGE models can be applied in real time policy analysis and provide helpful advice to policymakers to achieve their operational targets. The methodology can also be employed to estimate the initial state of the economy, the policy simulations with historical policy reaction functions and forecasting models.

De Carvalho et al. (2015) model a small open economy which receives strong capital inflow from foreign investment. The parameter of the model is estimated with Bayesian techniques using data from Brazil. They analyze the influence of foreign reserves management, export-related credit lines and capital control in the DSGE model to compare the effects of different macroeconomic policy instruments. They tried to address the concerns in

emerging economies by proposing optimal policy to counteract the impact of domestic and international shocks that challenge financial stability in emerging economies.

These approaches represent that the latest generation DSGE models overcome the weakness of older ones. They incorporate the most recent theoretical and econometric advances, are the most sophisticated and realistic tools now available for conducting macroeconomic analysis.

## **2.2 Advantages of DSGE modelling methodology**

In recent years, DSGE models have been playing an increasingly important role. The defining feature of this class of model is a rigorous microeconomic foundation of macroeconomic relationships. Such models fully and consistently reflect the fact that agents in the economy base their actions to a large extent on expectations about the future. Therefore, shifts in policy can cause changes in behaviour and alter parameters once assumed to be constant.

There are a number of advantages to DSGE models, which mean that they are considered reliable and favourable tools for analysing economic policy, especially monetary policy. First, DSGE models satisfy the principles of precautionary saving and prudence, which are among the objectives of the central banks. As qualified macroeconomic models for economic assessments, DSGE models are able to provide fairly credible reproductions

of the historical trajectories of the variables of interest. Therefore, they can deliver the best analyses of the effects of policy, and forecast future trajectories of certain phenomena. Macroeconomists are able to assess the forecasting performance of a DSGE model in which different policy scenarios lead to changes in the decision rule of private agents. DSGE models are also used to characterize the economic structural distinctions between different countries or areas.

Furthermore, thanks to development in the understanding and control of numerical solution methods, it is now possible to evaluate and estimate relatively large models. The latest-generation DSGE models are compatible with Lucas Critique. Lucas Critique (1976) argues that it is naive to predict the influence of a change in economic policy only depending on relationships observed in historical data. Lucas suggests that if one representative agent wants to predict the effect of a policy, she should model the “microfoundations” that govern individual behaviours, such as preference, technology and budget constraints. Building on microeconomic foundations, DSGE models are applied to interpret and analyse the entire economy. Not only must macroeconomic equality between supply and demand be observed; it is also necessary to ensure that the developments expected by households and firms are consistent with their current and future planned decisions.

Contrary to the previous view that private agents never change their behaviour and that individual preference is insignificant. Lucas (1976) argued that private agents would take into account the change in policy to adjust their

behaviour to optimize their wealth. Following his argument, DSGE models identify that agents in the economy are utility maximizing, rational, forward-looking and fully informed. Their behaviour today depends on their expectations and forecasts for the future. In policy analysis, it is crucial to take into account the way in which expectations would be different.

Moreover, these models fit the data satisfactorily according to the empirical research. Because of the robust consistency of Lucas Critique and expected economic policy variants, DSGE models are valuable analytical tools, particularly for economic policymakers.

Another key advantage is that DSGE models share core assumptions on the behaviour of households and firms. These assumptions mean that DSGE models are more sophisticated in their ability to interpret the economy and address upcoming questions. DSGE models also include an evaluation criterion, set according to the specification of agents' preferences. By using this criterion, it becomes possible to classify different economic policy options and potentially to identify the one that agents would prefer. The model is practical because it takes account of agents with limited or non-existent access to financial markets.

In addition, DSGE models may be used to perform a positive historical analysis. After adjusting the model to suit the data, it is possible to analyse what might have happened had economic policy been different. The DSGE

models that are robust to the Lucas Critique (1976) are needed for such exercises to be well defined.

The recent DSGE models assume that central bank behaviour is well described by a monetary policy rule. In this kind of formulation, the central banks adjust the nominal interest rate based on several indicators. From a theoretical standpoint, Clarida (2001) uses simple DSGE models to show that if central banks do not adjust their monetary policy instruments sufficiently responsively to changes in inflation, this could destabilize the economy. Similarly, lax fiscal policy may be detrimental to the economy. DSGE models can be used to identify these conditions and to help policy makers avoid detrimental policy decisions more effectively. Micro-founded DSGE models can also help monetary authorities consider uncertainty in their decision-making process. Because the DSGE models are micro-founded, it seems appropriate and realistic to examine central banks' behaviour in such a setting: the central bank may express its preference via a criterion that is to maximize social well-being; then, it must determine its optimal behaviour taking these uncertainties into account. Depending on the circumstances, an aggressive or cautious policy decision may be shown to be advisable. By proposing a consistent analytical framework that satisfies the Lucas Critique, DSGE models are the most appropriate candidate for this type of study.

## **2.3 The Limitations of current DSGE Models**

De Grauwe (2010) discusses the scientific foundation of the DSGE models and explicates that they represent two central principles of modern macroeconomics: The first one is that a macroeconomic model should be based on dynamic utility maximization of a representative household. The second one is that expectations should be model-consistent which suggests that agents make forecasts based on the information embedded in the model

According to these two features, DSGE models indicate that individual agents have extraordinary cognitive capabilities and a full understanding of the structure of the fundamental model. One important implication of the behaviour assumption that agents know the underlying model's structure is that all agents are the same. They all use the same information set including the information embedded in the underlying model. Therefore, DSGE models are able to describe the behaviour of all agents in an economy by restricting the analysis to a representative agent. Logical coherence and self-restricted construct of analysis are the main advantages of these models. This framework has intellectual appeal. There is no necessity for ad hoc assumptions about the behaviour and forecasts of individuals, because rational expectations and utility maximization present discipline in modelling the behaviour of individual agents.

However, while logical coherence, intellectual appeal and self-contained framework are all advantages of well-grounded macroeconomic models, they

are not determinants of scientific validity. The scientific validity can be judged only on the models' capacity for making correct empirical predictions that are fit to the empirical data. If they fail to do so, then however coherent and intellectually appealing the models may be, they are not practical tools for policy analysis.

Moreover, there are several limitations to the current DSGE models. The plausibility of the main assumption concerning the behaviour of individual agents is queried by the scientific evidence and actuality. For example, these models suppose that individuals are able to understand the entire economic model and share the information distilled from this model. However, the scientific evidence implies that individual agents are not capable of doing so, and that they rely on rules that use only small parts of the available information. There is now significant evidence that individual agents sustain deep cognitive problems, which limit their capacity to understand and to process the complexity of the information they receive. These cognitive problems drive agents to use much simpler ways to understand the economy and obtain information. Hence, individuals use simple rules, called "heuristics", to guide their behaviour (see Gabaix et al. 2006). They do this not because they are irrational, but rather because they realize that the entire economy is too complicated for them to fully understand. Indeed, it can be said that using heuristics is a rational response of agents who are aware of their limited capacity to understand the world.

Although the informational assumption of rational expectations is implausible, the macroeconomic model based on rational expectations could still be a powerful tool in making empirical predictions. Both Milton Friedman and De Grauwe (2010) emphasize this argument. However, De Grauwe (2010) believes that the main problem is that rational expectations macroeconomic models make systematically wrong predictions. He argues that the degree of inertia in wages and prices is always underestimated by the forward looking individuals in micro-founded macroeconomic models. This empirical failure could have led the profession of macroeconomists to drop the model and to look for another one.

Velupillai (2011) explicates the origin and the development path of DSGE models to criticize the foundations of DSGE modelling, from an explicitly computable and constructive mathematical point of view. He elucidates that DSGE models developed from their origins in the classical Arrow-Debreu General Equilibrium (ADGE), via Scarf's development of Computable General Equilibrium (CGE) theory and its applied version as Applied Computable General Equilibrium model (ACGE), to DSGE dynamization assumption through Recursive Competitive Equilibrium (RCE). He argues that the transmissions of ADGE which is the underlying theoretical basis of DSGE model are not computably and constructively defensible. Therefore, he intends to develop an approach "Beyond DSGE and ABE".

Sims (2006) considers DSGE models to be only story-telling devices and not hard scientific theories. He argues that, "it does not make sense to require

these models to match in fine detail the dynamic behaviour of the accounting constructs and proxy variables that make up our data” (p.2). Tovar (2008) deems that DSGE models still need to prove their ability to fit the empirical data and confirm their usefulness as policy tools, because at their current stage they are not able to satisfy and accomplish all that central banks ask them to do. He argues that there are three limitations to DSGE models: “First, they still need to incorporate relevant transmission mechanisms or sectors of the economy; Secondly, issues remain on how to empirically validate them; and finally, challenges remain on how to effectively communicate their features and implications to policy makers and to the public” (p.1).

Pesaran and Smith (2011) argue that, “while the DSGE approach has provided many theoretical insights, the dominance of this approach and the resistance of some macroeconomists to alternatives has become a straitjacket that restricts empirical and theoretical experimentation, inhibits innovation and reduces the contribution economists can make to policy debates” (p.2). They consider that DSGE models do not focus on the most germane and relative problems for understanding the influence of shocks, informing policy and the whole picture of the economy. They believe that because of this narrow focus, some wider and significant questions might be ignored. This neglect might lead to more confusion in policy discussion and analysis. In addition, they consider that the inter-connections of economic activities of the general differ from the behaviour of the individuals who comprise it. Therefore, they believe that: “Representative agent models lose both the complexity of the interconnections between agents and the compositional

effects which come from aggregation of the behaviour of the interacting individual agents into a system” (p.8)

Pesaran and Smith (2006) argue that the main disadvantage of Vector Autoregressions, VARs, is that they can deal with only a relatively small number of variables. Therefore, Global VAR (GVAR) is proposed to improve the analysis performance of DSGE models. The GVAR approach advanced in Pesaran, Schuermann and Weiner (2004, PSW) provides a simple solution where country specific models in the form of VARX\* structures are estimated relating a vector of domestic variables to their foreign counterparts, and are then consistently combined to form a Global VAR. The high dimensional nature of the model is circumvented at the estimation stage by constructing the country specific foreign variables,  $x_{for}^*$  using predetermined coefficients such as trade weights, and by noting that for relatively small open economies,  $x_{for}^*$  can be treated as weakly exogenous (or forcing) for the long run relationships. The GVAR allows in a transparent way for interdependence at a variety of levels that have long-run relationships consistent with the theory and short run relationships that are consistent with the data.

Because of the limitations and disadvantages of DSGE models at their current stage of development, recent literature has made great efforts to go “beyond the DSGE Straitjacket”. De Grauwe (2010) explicates several of the ways that have been used to repair the shortcomings of “pure” micro-founded macroeconomic models. The main feature of this improvement is the addition of lags into models to create necessary inertia observed in data. In one

approach, consumers are modelled as agents subject to habit formation. This trick allows the researcher to introduce lagged consumption in the utility function and adds welcome inertia. A second popular way to introduce inertia into the model is to invoke Calvo pricing (Calvo, 1983), in which firms are constrained from adjusting prices instantaneously (Christiano et al. 2005). However, Calvo pricing rules are not a perfect methodology to add lags and inertia, because it is hard to explain why rational individuals are willing to accept institutions that limit their freedom to maximize their utility.

De Grauwe (2010) proposes a behaviour model which admits that agents' behaviours are guided by heuristics. He believes that models which incorporate heuristics from the start are able to fit empirical data better than models which assume agents are fully rational and omniscient. The "heuristics" assumptions of behaviour models are more authentic than the assumption of current micro-founded macroeconomic models in the real world. Another advantage of this approach is that one can specify explicitly what kind of heuristics is acceptable.

The ensuing "behavioural model" produces a number of results that distinguish it from the rational expectations models. First, the behavioural model is capable of generating endogenous cycles based on waves of optimism and pessimism. This dynamics is akin to what Keynes called animal spirits. Second, in contrast to the DSGE models, the inertia in output and prices is generated within the model, instead of being "imported". Third, the behavioural model produces a degree of uncertainty about the transmission of

monetary policy shocks that is very different from the uncertainty obtained in DSGE models. In the latter, uncertainty about the effects of monetary policy shocks arises because of the lack of precision in the estimation of the structural parameters of the model.

Pesaran and Smith (2011) attempt to build a more flexible framework in DSGE modelling. They focus on five key elements. The first element is the use of long-run integration where they exist. The second is the use of more flexible short-run dynamics. The third element is the recognition of the wide range of interconnections between heterogeneous agents that exist within any economic system. The fourth is that this wide range of interconnections poses issues of dimensionality, since there will inevitably be a large number of variables and decision making units involved. Therefore, procedures that use theory and the structure of the data to overcome the curse of dimensionality are required. Finally, the wide range of interconnections raises questions about the treatment of shocks. There are, for example, a range of interesting questions about the transmission of shocks. Multi-country VARs, including the GVAR above, have been used to model the international transmission of shocks. The authors believe that a flexible approach would be profitable to macroeconomic modelling, which currently suffers from the narrow focus on the entire economy.

Velupillai (2011) takes a computable approach to macroeconomic modelling in order to go “Beyond DSGE and ABE”. He considers the economy as a Turing machine which can generate recursively enumerable data and make

the computation by itself. If this is the case, what can be inferred about the structure of the economy may only be explored by Turing machine computation. However, this method does not guarantee that a definitive answer will be obtained. Computation in economics becomes epistemologically meaningful only when the economic modellers are using computational metaphors to analyse the data generated by the economy.

This is the natural mode of interaction between the economy and the classical behavioural economist and the computable economist. It is not the natural mode for the CGE theorist, nor for the agent based computational economist. This is why there is a serious epistemological deficit in the practice of the latter two classes of economists. Any attempt to go beyond DSGE and ABE must seek knowledge from numerically meaningful modelling exercises. This is especially so in view of the fact that the economic foundations of computable economics are largely consistent with the basics of classical behavioural economics.

De Grauwe (2009) makes a distinction between top-down and bottom-up macroeconomic models. A “top-down” model is one which individual agents are forward-looking and fully understand the structure of the economy. These agents are capable of representing the whole system in a blueprint that they can store in their mind. “Bottom-up” models are so-called “behavioural models”. They assume that no one can understand the whole picture, and that everyone is able to get only small pieces of the total information and understand a tiny part of the whole picture.

Hayek (1945) argued that no individual can ever hope to understand and to process the full complexity of the world in which he lives. If there were individuals capable of understanding the whole picture, we would not need markets. Therefore, there is no “planner”, who is assumed to be able to understand the entire economy and maximize the welfare of the whole society in socialism.

The ensuing “behavioural model” produces a number of results that distinguish it from the rational expectations models. In the behavioural model there is an additional dimension to uncertainty. This is that the same policy shock can have very different effects depending on what we have called market conditions.

## **2.4 Theoretical foundation: The consensus**

Although there remain divergences of view in macroeconomics, the academic community has achieved some consensus on modern macroeconomic models. First, consistent intertemporal general equilibrium models have become the mainstream of macroeconomic analysis, because these models are able to analyse both short-run fluctuations and long-run growth in a plain, coherent framework. Similarly, microeconomic and macroeconomic analysis is no longer considered to involve fundamentally different principles.

As generally accepted models, recently employed to analyse the short-run effect of policy shifts, DSGE models are often concerned with imperfect

competition in both labour markets and product markets. In this context, among the main reasons why general equilibrium models are welcome and necessary to broad macroeconomic realism are that all equations of the models are derived from mutually consistent foundations, and the specified behaviour of each economic unit makes sense given the environment which is created by the behaviour of the others.

In addition, econometrically structural models are widely accepted as the foundations of quantitative policy (King, 1995), which is now generally agreed to be a significant contribution to macroeconomic theory. A primary goal of theoretical analysis in macroeconomics is to determine the data-generating process implied by one structural model or another, in order to allow consideration of the extent to which the model's predictions match the properties of aggregate time series.

Modern macro-econometric modelling represents a return to the ambitions of the post-war Keynesian modellers in at least two respects. First, the emphasis on the use of estimated structural models for policy analysis contrasts with the preference of many monetarists for drawing inferences about counterfactual policies from reduced form empirical relations. In addition, they fulfil the quest to develop models which are intended to provide a complete quantitative description. These models represent the joint stochastic processes by which a set of aggregate variables evolves.

Nonetheless, modern empirical macroeconomics differs from classic post-war macro-econometric modelling in deeper respects than the mere introduction of new approaches to estimation. In particular, a great deal more attention is given to the grounds for treating an econometric model as "structural" for the purpose of a policy evaluation exercise. Nowadays, specifications that are intended to represent structural relations are derived from explicit decision problems of households or firms; adjustment delays are allowed for, but these are assumed to be constraints that are taken account of by optimizing agents.

Modern macroeconomic modellers also depart from the early post-war literature in taking a more eclectic approach to the estimation of model parameters and testing of model predictions. One reason is that the modern style of structural model, with its deeper behavioural foundations, is not merely a prediction about the statistical properties of one particular type of data: instead, it simultaneously makes claims about many things – both individual behaviour and the behaviour of aggregate, both short-run dynamics and long-run averages – so that many different kinds of data are relevant in principle, both to model parameterization and to judging the model's empirical relevance. As a result, many different approaches to empirical analysis provide complementary perspectives on the quantitative realism of a given model.

Furthermore, it is now widely agreed that it is important to consider future expectations as an endogenous economic parameter. In policy analysis, it is crucial to take into account the way in which expectations would be different.

This was, of course, the point of the celebrated Lucas (1976) critique of traditional methods of econometric policy evaluation. In modern DSGE models with sticky wages and/or prices, the fact that wage and price-setting decisions are made on the basis of rational expectations has important consequences for the nature of the trade-off between inflation and real activity, and for the way in which it makes sense to think about the effects of economic policy.

Moreover, it is now generally accepted that real disturbances are an important source of economic fluctuations: the hypothesis that business fluctuations can largely be attributed to exogenous random variations in monetary policy has few if any remaining adherents. Modern empirical DSGE models include a variety of types of disturbances to technology, preferences, and government policies, and part of the variability in aggregate time series is attributed to each of these types of shocks.

Finally, there is now wide agreement on the effectiveness of monetary policy, especially as a means of inflation control. That central banks can control inflation if they want to (and are allowed to) can no longer be questioned, after the worldwide success of disinflationary policies in the 1980s and 1990s; furthermore, there is consensus that it is reasonable to charge central banks with responsibility for keeping the inflation rate within reasonable bounds.

## 2.5 Estimation and Evaluation Methods

While in the past many econometric techniques have been advanced to analyse DSGE models, over the last decade Bayesian inference techniques, originally proposed by DeJong, Ingram and Whiteman (2000), Schorfheide (2000), and Otrok (2001), have been employed to estimate and evaluate the models. This method is now widely accepted and applied in the literature. Consequently, An and Schorfheide (2007a) and Del Negro and Schorfheide (2010) have developed more elaborate methodology to implement Bayesian inference for DSGE models.

Let  $\theta$  denote the collection of all parameters of the DSGE model described in Section 4.  $p(\theta)$  represents the prior density of the estimated parameter,  $p(\theta|Y)$  denotes posterior probability of the parameter. The prior is combined with the conditional density of the data  $Y$  given in the parameters, denoted by  $p(Y|\theta)$ .

Prior distribution plays an important role in the estimation of DSGE models. It might down weigh regions of the parameter space that are at odds with observations not contained in the estimation sample  $Y$ . It might also add curvature to a likelihood function that is (nearly) flat in some dimensions of the parameter space, and therefore strongly influence the shape of the posterior distribution. While, in principle, priors can be gleaned from personal introspection to reflect strongly held beliefs about the validity of economic theories, in practice most priors are chosen based on some observations.

According to Bayes Theorem of posterior distribution, the posterior density

$p(\theta|Y)$  is related to the prior and the likelihood as follows:

$$p(\theta|Y) = \frac{p(Y|\theta)p(\theta)}{p(Y)}, p(Y|\theta) = L(\theta|Y) = \frac{p(\theta|Y)p(Y)}{p(\theta)} \quad (2.2)$$

$$p(Y) = \int p(Y|\theta)p(\theta)d\theta, \quad (2.3)$$

where  $p(Y)$  is called the marginal likelihood or data density and  $L(\theta|Y)$  is the likelihood of the vector of parameters theta given data  $Y$ .

An and Schorfheide (2007a) describes two algorithms that can be used to generate draws from the posterior distribution of  $\theta$ : Random-Walk Metropolis (RWM) Algorithm and Importance Sampling (IS) Algorithm.

### Random-Walk Metropolis (RWM) Algorithm

1. Use a numerical optimization routine to maximize  $\ln L(\theta|Y) + \ln p(\theta)$ . Denote the posterior mode by  $\tilde{\theta}$ .
2. Let  $\tilde{\Sigma}$  be the inverse of the Hessian computed at the posterior model  $\tilde{\theta}$ .
3. Draw  $\theta^{(0)}$  from  $N(\tilde{\theta}, c_0^2 \tilde{\Sigma})$  or directly specify a starting value.
4. For  $s = 1, \dots, n_{sim}$ , draw  $\mathcal{G}$  from the proposal distribution  $N(\theta^{(s-1)}, c^2 \tilde{\Sigma})$ .

The jump from  $\theta^{(s-1)}$  is accepted ( $\theta^{(s)} = \mathcal{G}$ ) with probability  $\min\{1, r(\theta^{(s-1)}, \mathcal{G}|Y)\}$  and rejected ( $\theta^{(s)} = \theta^{(s-1)}$ ) otherwise. Here

$$r(\theta^{(s-1)}, \mathcal{G}|Y) = \frac{L(\mathcal{G}|Y)p(\mathcal{G})}{L(\theta^{(s-1)}|Y)p(\theta^{(s-1)})}$$

5. Approximate the posterior expected value of a function  $h(\theta)$  by

$$\frac{1}{n_{sim}} \sum_{s=1}^{n_{sim}} h(\theta^{(s)})$$

The RWM algorithm generates a sequence of dependent draws from the posterior distribution of  $\theta$  that can be averaged to approximate posterior moments. Geweke (1999, 2005) reviews regularity conditions that guarantee the convergence of the Markov chain generated by Metropolis–Hastings algorithms to the posterior distribution of interest, and the convergence of  $\frac{1}{n_{sim}} \sum_{s=1}^{n_{sim}} h(\theta^{(s)})$  to the posterior expectations.

DeJong et al. (2000) use an IS algorithm to calculate posterior moments of the parameters of a linearized stochastic growth model. The idea of the algorithm is based on the identity:

$$E(h(\theta) | Y) = \int h(\theta) p(\theta | Y) d\theta = \int \frac{h(\theta) p(\theta | Y)}{q(\theta)} q(\theta) d\theta \quad (2.4)$$

Draws from the posterior density  $p(\theta | Y)$  are replaced by draws from the density  $q(\theta)$  and reweighted by the importance ratio  $p(\theta | Y) / q(\theta)$  to obtain a numerical approximation of the posterior moment of interest.

### Importance Sampling (IS) Algorithm

1. Use a numerical optimization routine to maximize  $\ln L(\theta | Y) + \ln p(\theta)$  and denote the posterior mode by  $\tilde{\theta}$ .
2. Let  $\tilde{\Sigma}$  be the inverse of the Hessian computed at the posterior mode  $\tilde{\theta}$ .
3. Let  $q(\theta)$  be the density of a multivariate  $t$ -distribution with mean  $\tilde{\theta}$ , scale matrix  $c^2 \tilde{\Sigma}$ , and  $\nu$  degrees of freedom.

4. For  $s = 1, \dots, n_{sim}$  generate draws  $\theta^{(s)}$  from  $q(\theta)$ .
5. Compute  $\tilde{\omega}_s = \zeta(\theta^{(s)} | Y) p(\theta^{(s)} / q(\theta^{(s)}))$ ,  $\omega_s = \tilde{\omega}_s / \sum_{s=1}^{n_{sim}} \tilde{\omega}_s$ .
6. Approximate the posterior expected value of a function  $h(\theta)$  by  $\sum_{s=1}^{n_{sim}} \omega(\theta^{(s)}) h(\theta^{(s)})$ .

Schorfheide (2011) believes that it is too difficult to derive moments and quantiles of the posterior distribution analytically in DSGE model applications. Therefore, recent researches have applied Markov Chain Monte Carlo (MCMC) simulation to analyse the estimation of DSGE models.

MCMC algorithms deliver serially correlated sequences  $\{\theta_{(s)}\}_{s=1}^{n_{sim}}$  of draws from the density  $p(\theta | Y)$ . Based on these draws one can approximate the posterior density, its moments and quantiles, and, for instance, construct credible sets. In addition, the sequence  $\{\theta_{(s)}\}_{s=1}^{n_{sim}}$  can be transformed into a sequence  $\{f(\theta_{(s)})\}_{s=1}^{n_{sim}}$  to characterize the posterior distribution of  $f(\theta)$ , where  $f(\theta)$  could be a set of steady states or impulse response functions computed from the DSGE models.

## Estimates of Parameters and Transformations

The output and inflation trade-off faced by a central bank is determined by the New Keynesian Phillips Curve (NKPC), which for values of the target inflation rate near zero can be approximated as follows:

$$\tilde{\pi}_t = \gamma_b \tilde{\pi}_{t-1} + \gamma_f E_t[\tilde{\pi}_{t+1}] + \kappa \tilde{MC}_t \quad , \quad (2.5)$$

$$\text{where: } \gamma_b = \frac{t}{1 + \beta_l}, \gamma_f = \frac{\beta}{1 + \beta_l} \quad , \text{And } \kappa = \frac{(1 - \zeta)(1 - \zeta\beta)}{\zeta(1 + \beta_l)} \quad .$$

$\tilde{x}_t$  denotes percentage deviations from the log-linearization point  $\ln(x_t / x_*)$  and  $MC_t$  abbreviates the marginal cost of producing an additional unit of the intermediate good. Posterior and prior densities for the coefficient on marginal costs  $\kappa$  and lagged inflation  $\gamma_b$  are shown in the top panels of Figure 1. The posterior densities reflect the sample information and turn out to be much more concentrated than the prior densities.

The bottom left panel depicts densities for the percentage loss  $100[1/D_* - 1]$  in output caused by the inability of a fraction of intermediate goods producers to choose their prices optimally.  $D_*$  depends on the steady-state mark-up controlled by  $\lambda$ , as well as the price setting parameters  $\zeta$  and  $l$ : It can be verified that  $D_*$  is bounded below one. This lower bound is attained if prices are flexible ( $\zeta = 0$ ), if all firms that are unable to re-optimize fully index their old prices to inflation ( $l = 1$ ), or if the steady-state inflation rate is zero (meaning that gross inflation  $\pi_* = 1$ ).

Finally, the bottom right panel shows densities for the interest rate coefficient  $(1/\nu(R_* - 1))$  in the log-linearized demand equation for real money balances at the end of period  $t$ :

$$\tilde{M}_{t+1} = -\frac{1}{\nu(R_* - 1)} \tilde{R}_t + \frac{\gamma}{\nu} \tilde{X}_t - \frac{1-\nu}{\nu} E[\tilde{\pi}_{t+1}] + E[\tilde{\chi}_{t+1}] , \quad (2.6)$$

where  $R_*$  is the steady-state nominal interest rate. The (partial) interest-rate elasticity of money demand indirectly affects the welfare costs induced by taxing money balances via inflation.

In a Bayesian framework the posterior densities provide a formal characterization of parameter uncertainty. Point and interval estimates can be derived as solutions to decision problems that entail the minimization of posterior expected losses. The most widely used point estimates are the posterior mean and median, and the so-called highest-posterior density interval is the shortest interval among all (including disconnected) intervals that are  $1-\alpha$  credible, i.e. have posterior probability  $1-\alpha$ .

In summary, the empirical illustration suggests that econometricians have developed a powerful toolkit that enables an elegant econometric analysis of DSGE models. The strengths of the formal econometric analysis are its ability to efficiently extract information about parameters from long-run averages and sample autocovariances of macroeconomic time series, and to account for parameter (and model) uncertainty in inference and decision making.

Researchers have made wide use of these strengths. The Bayesian approach has the additional advantage that it allows the researcher coherently to combine sample information (contained in the likelihood function) with non-sample information represented by prior distributions.

There exist many published papers that to varying degrees follow the template of the empirical analysis presented above, albeit in pursuit of answers to different economic questions. The fact that the computations are now automated in software packages such as DYNARE, and accessible to a large community of empirical macroeconomists, is a reflection of the progress that the literature has made over the past ten years.

In the sections below we study various techniques that can be used to evaluate the model fit. We distinguish the assessment of absolute fit from techniques that aim to determine the fit of a DSGE model relative to some other model. The first approach can be implemented by a posterior predictive model check and has the flavour of a classical hypothesis test. Relative model comparisons, on the other hand, are typically conducted by enlarging the model space and applying Bayesian inference and decision theory to the extended model space.

## **Posterior Predictive Checks**

Predictive checks have been advocated as a tool to assess the absolute fit of a probability model (e.g., Box 1980). A probability model is considered as discredited by the data if one observes an event that is deemed very unlikely by the model. Such model checks are controversial from a Bayesian perspective. Methods that determine whether actual data lie in the tail of a model's data distribution potentially favour alternatives that make unreasonably diffuse predictions. Nevertheless, posterior predictive checks have become a valuable tool in applied Bayesian analysis, although they have

not been used much in the context of DSGE models.

$$p(y_{rep} | y) = \int p(y_{rep} | \theta) p(\theta | y) d\theta \quad (2.7)$$

$p(y_{rep} | y)$  is sampling distribution and  $p(\theta | y)$  is posterior distribution, where  $y_{rep}$  denotes replicated or hypothetical values of  $y$ . We can think of  $y_{rep}$  as values that might have been observed if the conditions generating  $y$  were reproduced. The integral defining the posterior predictive distribution has two parts. The first part gives the probability density of  $y_{rep}$ , given particular values of  $\theta$ . The form of this density is given by the sampling distribution for  $y$ . The second part of the integral is the posterior distribution for  $\theta$ , i.e.  $p(\theta | y) \propto l(\theta; y) p(\theta)$ , where  $l(\theta; y)$  is the likelihood. The posterior predictive distribution thus reflects two kinds of uncertainty: sampling uncertainty about  $y$  given  $\theta$ , and parametric uncertainty about  $\theta$ .

To assess model fit, the posterior predictive distribution can be compared to the observed data. If a model fits the data well, the observed data should be relatively likely under the posterior predictive distribution. On the other hand, large discrepancies between the observed data and the posterior predictive distribution indicate that the model performs poorly.

The use of  $y_{rep}$  to assess model fit is flexible and can be extended considerably. As well as visual inspection of predictive simulations, discrepancy statistics can be computed that help evaluate model fit. These discrepancy statistics can be chosen to capture substantively important features of the data.

## Posterior Odds Comparisons of DSGE Models

The Bayesian framework is naturally geared toward the evaluation of relative model fit. Researchers can place probabilities on competing models and assess alternative specifications based on their posterior odds. An excellent survey on model comparisons based on posterior probabilities can be found in Kass and Raftery (1995). They use a variant of the so-called Bayes factor: the Bayes factor  $B$  is the ratio of the posterior probabilities of two hypotheses  $H_1$  and  $H_2$ . Parameter  $D$  denotes data,  $pr(D|H_1)$  and  $pr(D|H_2)$  represents probability density.

$$\frac{pr(H_1|D)}{pr(H_2|D)} = \frac{pr(D|H_1) pr(H_1)}{pr(D|H_2) pr(H_2)} \quad (2.8)$$

The Bayes factor  $B_{12}$  is:

$$B_{12} = \frac{pr(D|H_1)}{pr(D|H_2)}, \quad (2.9)$$

where  $posterior\_odds = Bayes\_factor \times prior\_odds$ . No matter whether the two hypotheses are distributions with no free parameters, or distributions with unknown parameters,  $B_{12}$  is always the likelihood ratio.

The densities  $pr(D|H_1)$  and  $pr(D|H_2)$  are obtained by integrating over the parameter space:

$$\begin{aligned}
pr(D|H_1) &= \int pr(D|\theta_1, H_1)\pi(\theta_1|H_1)d\theta_1 \\
pr(D|H_2) &= \int pr(D|\theta_2, H_2)\pi(\theta_2|H_2)d\theta_2,
\end{aligned}
\tag{2.10}$$

where  $\theta_1$  and  $\theta_2$  are the parameters under  $H_1$  and  $H_2$ ,  $\pi(\theta_1|H_1)$  and  $\pi(\theta_2|H_2)$  are their prior densities, and  $pr(D|\theta_1, H_1)$  and  $pr(D|\theta_2, H_2)$  are probability density of  $D$  given value of  $\theta$ .

### **Comparison of a DSGE Model with a VAR**

Since the pioneering work of Sims, vector autoregressive (VAR) models have been used extensively by applied researchers, forecasters and policymakers to address a range of economic issues. These models comprise equations explaining a small number of key macroeconomic variables, where each equation includes the same set of explanatory variables, lagged values of all the variables in the system (Liu and Theodoridis 2010). The notion of potential misspecification of a DSGE model can be incorporated in the Bayesian framework by including a more general reference model into the model set. Therefore, VAR is a good choice as a reference model in dynamic macroeconomics. Linearized DSGE models can be understood as restrictions on a VAR representation.

The first step of the comparison is typically to compute posterior probabilities for the DSGE model and the VAR, which can be used to detect the presence of misspecifications. Since the VAR parameter space is generally much larger than the DSGE model parameter space, the specification of a prior

distribution for the VAR parameter requires careful attention. Possible pitfalls are discussed in Sims (2003).

A VAR with a prior that is very diffuse is likely to be rejected even against a mis-specified DSGE model. In a more general context this phenomenon is often called Lindley's paradox. We will subsequently present a procedure that allows us to document how the marginal data density of the DSGE model changes as the cross-coefficient restrictions that the DSGE model imposes on the VAR are relaxed. If the data favour the VAR over the DSGE model then it becomes important to investigate further the deficiencies of the structural model. This can be achieved by comparing the posterior distributions of interesting population characteristics such as impulse response functions obtained from the DSGE model and from a VAR representation that does not dogmatically impose the DSGE model restrictions.

## **2.6 Methods for solving DSGE models**

There are four main linear solution methods of DSGE models: Blanchard and Kahn (1980)'s Method, Sims (2001)'s Method, Klein (2000)'s Method and Uhlig (1999)'s Method. The advantage of Uhlig (1999)'s Method is that he proposes a simpler method for finding log linear approximations. In this paper, we therefore follow Uhlig (1999)'s method to apply log lineared function to solve DSGE models.

Uhlig (1999) proposes a solution based on the method of undetermined coefficients. The method is applied to systems written as:

$$E_t[Ax_{t+1} + Bx_t + Cx_{t-1} + Dz_{t+1} + Gf_t] = 0 \quad (2.11)$$

$$z_{t+1} = Nz_t + v_{t+1}, E_t(v_{t+1}) = 0. \quad (2.12)$$

$$x_t = [y_t, c_t, i_t, k_t]' \quad (2.13)$$

Uhlig (1999) introduced how to log-linearized the equations of nonlinear DSGE models to find the solution of the system. He also discusses the general procedure to solve and analyse dynamic stochastic models, which includes five steps. The first step is calculating first order condition of the model to find necessary equations characterizing the equilibrium. Eq.(2.13) denotes the reduced form of the first order necessary conditions. The second step is selecting parameters and calculating the steady states. The aim of third step is to log-linearized the equations according to first order condition equations of the first step.

According to Uhlig (1999), the principle of log-linearization is:

$$\text{For } x \approx 0, \exp^x \approx 1 + x. \quad (2.14)$$

Therefore, we set  $\hat{x}_t = \log(x_t / \bar{x})$  to be the log-deviation of  $x_t$  from its steady state. Thus,  $\hat{x}_t \times 100\%$  is (approximately) the percentage deviation of  $x_t$  from its steady state  $\bar{x}$ .

$$\text{Thus, } x_t = \bar{x} \exp^{\hat{x}_t} \approx \bar{x}(1 + \hat{x}_t) . \quad (2.15)$$

One will make the equations approximately linear in the log deviation from the steady state. Moreover, Uhlig (1999) applied the recursive equilibrium law of motion to investigate the dynamics of the model. He applied the reduced form of the equations to simplify the system. Eq.(2.15) is the reduced form of the model framework.

The final step of this method is to analyse the solution via impulse response analysis. By tracing out the changes of all variable after the model simulation, one can find the impulse responses to different shocks. Eq.(2.14) represents the expectation and construction of the shocks. To solve DSGE models in this thesis, we will practice this methodology and discuss more details about Uhlig's method in the later sections.

## **Chapter 3**

# **Foreign Reserves Accumulation and Intergenerational Transfers in China**

In this chapter, we employ a dynamic stochastic general equilibrium (DSGE) model to investigate China's massive international reserves accumulation and develop a new method to include intergenerational transfer behaviour in the DSGE framework. The model in this chapter permits explicit consideration of the influence of intergenerational transfers on Chinese household savings. The impulse response functions of key variables to technology and monetary policy shocks are explored in a New Keynesian DSGE setting and the model's calibration is discussed. We then apply Bayesian estimation and evaluation techniques to analyse relevant parameters and model performance. Our estimations show that intergenerational transfers are a significant driver of high domestic savings and that, due to inadequate domestic financial development, a large portion of such domestic savings overflows into foreign asset investments, leading to persistent foreign reserve accumulations.

### **3.1 Introduction**

The massive reserve holdings by China represent a prominent case of the recent upsurge in reserve hoardings among emerging market economies. Recently, researchers have opened a new line of enquiry that links reserve accumulation to underdevelopment of capital markets in developing countries. According to Dominguez (2010), while financial markets in industrial countries have deepened and broadened, markets in many developing countries have remained incomplete. In a country with underdeveloped capital markets, the private sector faces financing constraints and firms' incentives to borrow are distorted. These factors result in underinsurance against future credit constraints and the government's accumulation of reserves may provide a solution to this problem. Through sterilized accumulation of reserves, i.e. the government buys international bonds and in the meantime issues domestic bonds, the government induces the private sector to save and the accumulation of reserves essentially act as a substitute for what would otherwise be private-sector capital outflows.

This research extends this line of reasoning to explore a deeper issue regarding reserve accumulation in China. We argue that, for a better understanding of the links between reserve hoarding and underdevelopment of capital markets, it is necessary and desirable to understand the sources of domestic savings and its relationship with investment. Ultimately, as part of a country's national wealth, foreign reserve holdings are the overflow of the excessive domestic savings over investment.

A high and rising savings rate has been a defining feature of the Chinese economy (Ma and Yang, 2013). In the recent past, China's saving rate was approximately 35% to 40% of its annual GDP in 1980-1999, and then surged from just below 38% in 2000 to 54.3% in 2008 (Ma and Wang, 2010). Since 2000, domestic savings in China have been the highest in the world in terms of share of GDP (Cristadoro and Marconi, 2012). China has surpassed all OECD economies and overtaken Singapore as the top Asian saver (Ma and Yang, 2013).

Although savings growth cuts across the three sectorial components of China's national gross savings – household savings, firm savings, and government savings – household savings stand out as particularly remarkable. Cristadoro and Marconi (2012) note that savings by the Chinese government largely result from tightfisted and underspending budgetary policies and that Chinese firms actually save less than their investment needs. However, relative to disposable income, households save an exceptionally high proportion of their incomes. Kuijs (2005) shows that the Chinese household savings rate is 11.8 percentage points higher than the U.S. rate. Qian (1988) and Kraay (2000) report that in the late 1970s, before the inception of the economic transition, household savings accounted for only 6 to 7% of China's GDP. Yang, Zhang and Zhou (2012) show that the growth of household savings was the highest among the three component sectors during the reform years. Between 1999 and 2007, household savings increased from 16.7 to 22.2% of China's annual GDP, while the share of corporate savings

rose from 14.6 to 18.8% of GDP, and government savings surged from 2.6 to 10.8%. Household savings now represent the largest share of GDP among the three sectors (Lugauer and Mark, 2013). Therefore, the so-called savings glut in China is a phenomenon that is primarily related to household savings.

A growing body of literature has explored the causes behind China's high savings rate, including the high household savings rate (Kraay, 2000; Meng, 2003; Modigliani and Cao, 2004; Horioka and Wan, 2007; Chamon and Prasad, 2010; Ma and Wang, 2010; Wei and Zhang, 2011; Bonham and Wiemer, 2013). Ma and Yi (2010) show that previous research generally relates the high Chinese household savings rate and its dynamics to a combination of factors, including economic growth, income levels, demographic changes, precautionary motives, and institutional changes. Lugauer and Mark (2013) maintain that the high rate of Chinese household savings can be explained by environmental factors, including changing family size, old-age dependency, income uncertainty and income growth.

In terms of theoretical underpinnings, these explanations largely revolve around the conventional theories of life-cycle and permanent income hypotheses, the precautionary savings motive, financial development, and/or habit formation. However, these theories are often found to be difficult to reconcile with the empirical reality of China's high and rising savings rates (Wei and Zhang, 2011). For example, Harbaugh (2004) argues that the conventional life-cycle and permanent income hypotheses do not seem to be supported either by the data or the reality of the Chinese economy.

These hypotheses assume that consumers smooth out consumption over time by saving when their incomes are high and dissaving when their incomes are low, which means that households can choose to consume in youth, save in middle age, and consume in retirement. However, this hypothesized behaviour pattern does not fit China's reality due to the lack of progressive credit markets. In particular, younger consumers might not be able to borrow enough money from banks to finance their education, housing, and plans for marriage and instead must turn to their parents to pay for these expenses. Prior research has also found that the age profile of savings in China is not consistent with the life-cycle theory (Chamon and Prasad, 2010; Cristadoro and Marconi, 2012).

In addition, researchers have disputed other aspects of conventional theories as applied to China's high savings rates, including the effects of the precautionary saving motive, financial development, and habit formation. See, for example, Wei and Zhang (2011), Yang, Zhang and Zhou (2012), Bonham and Wiemer (2013), and Lugauer and Mark (2013), among others.

Although any or all of the existing conventional explanations may be true to a certain degree (Bonham and Wiemer, 2013), we argue that the role that intergenerational transfers play in boosting China's savings rate merits exploration. Intergenerational transfers within and between Chinese family networks have begun to attract growing interest (Cai, Giles and Meng, 2006; Silverstein, Cong and Li, 2006; Meng, 2007; Yin, 2010; Chen, Eggleston and Li, 2011; Wei and Zhang, 2011; Gong, Leigh and Meng, 2012; Song,

Storesletten, Wang and Zilibotti, 2015; Zhu, Whalley and Zhao, 2014). Kotlikoff and Summers (1981) show that 80% of U.S. national wealth results from intergenerational transfers, whereas only 20% is accounted for by life-cycle savings. Barro (1974) discussed that imperfect private markets and a government monopoly in the production of bond leads to a significant and essential chain of intergenerational transfers that connected current to future generations. Because of the uncertainty with respect to individual future tax liabilities and a consideration of the risk of financial market, households will increase the proportion of intergenerational transfers in their wealth. Becker (1974) discussed the importance of social interactions in economic activity. He believes that interactions among family members are operative. Houesholds transfers purchasing power to all other members because he cares about their welfare.

Although the magnitude of this estimation has been disputed by subsequent research (Modigliani, 1988; Gale and Scholtz, 1994; Karagiannaki 2011), international evidence generally confirms that intergenerational transfers are an important source of wealth accumulation. Given the historically tight family ties and cultural influences in China, it is conceivable that intergenerational transfers in China are a significant contributor to wealth and thus a critical driver for growing Chinese household savings.

Chu (1991) shows that intergenerational financial transfers within the Chinese family system have been prevalent since at least the late fourteenth century. Given the historically tight bonds among Chinese family members, Chu

(1991) maintains that transfers within Chinese families have always been abundant both in terms of quantity and variety. In modern times, employing a 2004 survey by the China Panel Study of Family Dynamics (PSFD), Chu and Yu (2010) report that 35.24% of Chinese families had transferred all their assets to a younger generation, 5.78% had transferred part of their assets, and another 25% of families had yet to make the transfers. Those who answered that they had nothing to transfer accounted for only 33.8% of families. These results compare well with similar results from the U.S., in which over one-third of parents give money to their children (Hurd, Smith and Zissimopoulos, 2007). With regard to parental bequests, Yin (2012) finds that Chinese families have strong motivations to transfer wealth. For the country as a whole, over 85% of survey respondents have a bequest motive, which leads to increased savings (or reduces the dissavings) in aging households.

To varying degrees, intergenerational transfers penetrate into the many factors identified in the literature as contributors to China's rising savings rate. Here, the key point is that intergenerational transfers are both the means and the ends of saving. Facing the same institutional environment and the same demographic changes, Chinese families with transfer motives save more than those without that motive.

Ge, Yang and Zhang (2012) find that China's household saving behaviour is affected by the country's population control policies. With reduced fertility resulting from the one-child policy, middle-aged households save more because of the lighter burden of dependent children. This pattern implies that

families might thus provide more transfers to help the next generation. Wu, Gyourko and Deng (2012) believe that Chinese intergenerational transfer arrangements are largely influenced by inflated housing prices. In China, the real value of constant quality land has increased by nearly 800% since 2003. Price-to-income ratios are at their highest levels in the coastal areas. In early 2010, average housing prices hovered at approximately 18.5 times average annual income, and these prices have since risen even higher. In these circumstances, it becomes extremely difficult, if not impossible, for many young households in China to purchase a satisfactory property without financial support from their parents. Chinese parents who desire to offer this support must save more for the property needs of their offspring.

Chen and Yang (2013) estimate the impact of housing prices on urban household savings and find that the rise in housing prices accounts for 45% of the increase in the savings rate based on data from a 2002–2007 survey of urban households in China. Using Chinese provincial panel data from the 1995–2010 period, Wan (2015) finds that inflated housing pricing significantly pushes up the savings rate both in cities and nationwide. Wan (2015) argues that intergenerational transfer behaviour is a plausible reason for this outcome.

Wei and Zhang (2011) suggest that China's one-child policy has resulted in a dramatic increase in the sex ratio imbalance, leading to increased competition in the marriage market. As a result, Chinese parents increase their savings to improve a son's relative attractiveness for marriage. The pressure on savings

spills over to other households and drives wealth accumulation. Wei and Zhang (2011) estimate that this factor might potentially account for approximately half of the actual increase in the household savings rate during the 1990–2007 period. In a recent study, Du and Wei (2013) develop a formal theoretical underpinning for this empirical work.

In short, as a result of factors such as Chinese traditional culture, the undeveloped financial system and excessively high housing prices, Chinese parents may have to save more for their children's education, housing and marriage. As a consequence, intergenerational transfers provide critical support for China's young households to improve their standards of living and general utility.

China's high and rising savings have wider implications. One profound implication concerns external balance. It is well known that a country's external balance ultimately reflects a disparity between domestic savings and investment (Krugman, 1990). Yu (2007) argues that China's high savings rate is one of the main drivers of the growth in the country's current account surplus. By implication, this savings rate in turn would affect the country's net foreign asset position. Yang (2012) shows that when China's ill-functioning financial system fails to channel the increased accumulations in savings to investment or consumption, the excess savings end up as foreign reserves. Conversely, the amount earned by the trade surplus that is not consumed or invested must end up being saved. Lugauer and Mark (2013) also note that China would not have its current account surplus without the support of high

savings from this sector, given that the household sector accounts for the largest share of national savings. Therefore, based on their importance to household savings in China, intergenerational transfers should be a driving force behind China's hoarding of foreign reserves.

To model intergenerational transfers and their effects, Alonso-Carrera, Caballe and Raurich (2007) build an overlapping generations (OLG) model that addresses the importance of parental altruism and show how the process of preference formation affects the bequest motive. These authors describe two types of preference formation: habits and aspirations. Accordingly, habits are based on past consumption, whereas aspirations are determined by the willingness and wealth of parents. They explain that the willingness of parents to leave bequests is reduced by the influence of their habits; however, their aspirations contribute to the likelihood of positive bequests. Therefore, the quantity of the bequest is affected by both habits and aspirations. Lockwood (2012) presents a life-cycle model that captures the behaviour of the bequests factor. In this model, bequests are regarded as an element of a household's consumption path that is applied to maximize expected utility. Lockwood (2012) also discusses the method to calculate bequests according to consumption and personal wealth.

Modelling intergenerational transfers in China is in its infancy. The pioneers in this field are Song, Storesletten and Zilibotti (2011), who also employ the overlapping generation model to explain, among other things, the relation between parents and children. These authors hold that parents transmit

entrepreneurial skills and bequests to their children, and provide them with incentives to avoid opportunistic behavior. By constructing a neoclassical growth model, they show that the build-up of China's large external surplus is the result of financial and contractual imperfections in China's economy because the absence of integrated financial firms in China forces a large portion of domestic savings to be invested abroad. High household savings are thus an important source of China's external surplus and are a major driver of China's growing foreign reserves.

From the perspective of previous research, this study further explores the effects of domestic savings – explicitly including intergenerational transfers – on China's foreign reserves accumulation in a dynamic stochastic general equilibrium (DSGE) setting. Micro-founded DSGE models have been widely adopted by central banks and academic researchers as tools for macroeconomic research and policy analysis. However, DSGE studies of household savings and intergenerational transfers remain relatively rare. This paper intends to make two main contributions in this regard. First, we shed light on the interplay between intergenerational transfers, household savings, and China's accumulation foreign exchange reserves. Although China's high domestic savings are a main cause of the growth of its massive foreign exchange reserves, these excessive savings are largely driven by the intergenerational transfers that are prominent in China. It follows that intergenerational transfers should have a significantly positive relation with China's foreign reserve build-up. Our modeling strategy and its estimation demonstrate that intergenerational transfers have a significantly positive

impact on foreign reserves accumulation. Second, on the basis of this insight, we offer a new explanation for reserve hoarding in China. By combining the overlapping generations (OLG) model, Lockwood's (2012) life-cycle model and recent New Keynesian DSGE models for China, we propose a DSGE model that includes intergenerational transfers and other key macroeconomic elements in China's economic development. The framework follows the approaches developed by Smets and Wouters (2003), Zhang (2009), and Di Giorgio and Nistico (2013), among others.

The remainder of this chapter is organized as follows. In section two and three, we present an example of the basic structure of DSGE models and the framework of the model including intergenerational transfers. Section four shows the consequential first-order conditions engendered by households' and firms' optimization behaviours and presents linearized functions of the general equilibrium. We explore the reactions of inflation and net foreign assets to three different types of shock: monetary policy shock, technology shock and intergenerational transfers shock. We highlight the importance of intergenerational transfers by showing the positive relation between the intergenerational transfer factor and net foreign assets. In the process, Bayesian methods are applied to estimate and evaluate model performance.

## 3.2 The Basic Structure of DSGE Models

I have already expounded in the previous section that one significant feature of DSGE models is the careful consideration of the principle that development of the whole economy is the result of the decisions and behaviour of individual decision makers. Therefore, tenable and rational microeconomic behavioural equations are able to characterize the complicated macroeconomic relationships. With regard to the decision makers, a distinction is generally made between households and enterprises, and central banks and government as the institutions which levy taxes and finance expenditure. In this section, I will expound the main behavioural equations of DSGE models to explicate the basic structure of these models. The baseline DSGE models are expressed in log-linear form to describe their main steady-state relationships. Changes in nominal interest rate and inflation are shown in percentage points.

Typical DSGE models comprise three types of economic decision makers: savers and spenders (households), enterprises, and government (central banks). These decision makers determine the three elements which make up the basic structure of DSGE models. Households make choices about their individual preference, firms decide their production and technology, governments and central banks determine monetary and fiscal policy. Households make decisions on their consumption, their savings, and their labour supply in order to maximize their individual utility throughout their lifetime. Firms consider how to produce goods within the production

boundaries set by their "technological possibilities", how much labour they should employ and how much capital they should own to maximize their profit throughout the period observed. The central bank is usually assumed to desire to keep the rate of inflation close to a target value and to reduce fluctuations in overall capacity utilization. According to DSGE models, decision makers are assumed to be forward-looking. This means that current decisions depend on expectations about future developments. Consequently, households' saving behaviour and firms' investment behaviour hinge on current and expected rates of interest.

### 3.2.1 Households

Households consume, work and save in each period ( $t$ ). There are two kinds of households: savers and spenders. The superscript  $A$  indicates variables associated with savers, while the superscript  $P$  indicates variables related to spenders. Households make decisions about consumption  $C_t$  and labour supply  $L_t$ , and  $C_t = C_t^A + C_t^P$ . The subscript  $t$  represents the time period.

Household decisions can be summarized as (see Pytlarczyk, 2008):

$$C_t = E_t(C_{t+1}) - Ha \times (C_t - C_{t-1}) - \frac{1-Ha}{\sigma} (i_t - E_t(\pi_{t+1})) - \frac{1-Ha}{\sigma} (E_t(\varepsilon_{t+1}^C - \varepsilon_t^C)) \quad (3.1)$$

Equation (3.1) is the Consumption Euler equation. The parameter  $\sigma$  determines the marginal utility of consumption, while  $\varepsilon_t^C$  and  $\varepsilon_t^L$  show the

consumption and labour supply preference shocks.  $i_t$  is the interest rate at time  $t$ .  $\pi_t$  denotes inflation rate. The parameter  $Ha > 0$  is a habit parameter which depends on individual preference.  $Ha$  in equation (1) also implies that the consumption of households today (time  $t$ ) is determined by yesterday's consumption level.

$$L_t = \frac{1}{\mu} (Wag_t - Pri_t - \sigma C_t + \varepsilon_t^L) \quad , \quad (3.2)$$

where  $1/\mu > 0$  in Eq.(3.2) represents the elasticity of labour supply; labour supply depends on the real wage  $Wag_t - Pri_t$  and the marginal utility of consumption  $(-\sigma C_t)$ .  $Wag_t$  denotes nominal wages and  $Pri_t$  represents the price level. The utility  $U_t^A$  is represented by equation (3.3):

$$U_t^A = E_t \sum_{\tau=0}^t \beta^\tau \left[ \frac{(C_\tau^A)^{1-\gamma} - 1}{1-\gamma} + \chi^A \frac{(1-L_\tau^A)^{1-\theta} - 1}{1-\theta} \right] \quad , \quad (3.3)$$

where  $\tau$  is the running time of this utility function.

A representative spender maximizes utility ( $U_t^p$ ) over consumption and leisure at the current period (Sbordone, Tambalotti, Rao and Walsh 2006).

The representative agent assumption in DSGE model means all agents of same type are identical. It was assumed that everyone in the economy had the same preferences, and the same relative income of capital, labour skills, habits and so on, with the suggestion that it was sufficient to model the choice of a single representative agent.

$$U_t^P = \frac{(C_t^P)^{1-\gamma} - 1}{1-\gamma} + \chi^P \frac{(1-L_t^P)^{1-\theta} - 1}{1-\theta}, \quad (3.4)$$

where  $E_t$  is expectation operator conditional on agents' information set at time  $t$ , which contains all dated at  $t$  and before. The discount factor is  $\beta$  ( $0 < \beta < 1$ ).  $\gamma$  and  $\theta$  ( $\gamma > 0$  and  $\theta \geq 0$ ) represent the inverse-elasticity of intertemporal substitution of consumption and leisure respectively for savers. The preference weight that savers place on leisure is  $\chi_t^A$ , while the preference weight that spenders put on leisure is  $\chi_t^P$ .

The decision of savers is subject to the budget constraint:

$$(1 + \tau_t^C)C_t^A + I_t^A + B_t^A \leq (1 - \tau_t^K)r_t K_{t-1}^A + (1 - \tau_t^L)W_t v L_t^A + \delta^T \tau_t^K K_{t-1}^A + B_{t-1}^A R_{t-1} + TR_t^A \quad (3.5)$$

The depreciation rate on capital for tax purposes is  $\delta^T$ , and for economic purposes is  $\delta$ , ( $0 \leq \delta^T, \delta \leq 1$ ).  $I_t^A$  denotes the investment resources and adjustment cost.  $\tau_t^{La}$  is the tax rate on savers' labour income;  $\tau_t^C$  and  $\tau_t^K$  represent the tax rate on savers' consumption and capital income.

A representative saver chooses consumption ( $C_t^A$ ), labour ( $L_t^A$ ), and government bonds ( $B_t^A$ ) to maximize utility ( $U_t^A$ ) over consumption and leisure. ( $K_t^A$ ) represents the quantity of aggregate capital at time  $t$ .  $TR_t^A$  is the allowance that government grants to savers. Government debt at the end of time  $t$  is  $B_t^A$ , which pays  $R_t B_t^A$  at time  $t+1$ . The rental rate of capital is  $r_t$ , and the wage rate is  $W_t$ .

Savers' labour input is weighted by  $\nu$ , which also stands for savers' population weight. Therefore, spenders' labour input is  $(1-\nu)$ . The law of motion of savers for capital is:

$$K_t^A = (1-\delta)K_{t-1}^A + I(I_t^A, I_{t-1}^A) \quad (3.6)$$

The investment function is defined as :

$$I(I_t^A, I_{t-1}^A) \equiv [1 - S(\frac{I_t^A}{I_{t-1}^A})] \times I_t^A \quad (3.7)$$

In a steady state, assume  $S(h) = S'(h) = 0$  and  $s \equiv S'(h) > 0$ , where  $h$  is the output growth rate. The solution method does not require the specification of the investment adjustment cost function  $S(\cdot)$  (see Burnside, Eichenbaum, and Fisher 2004). The decision of spenders is subject to the budget constraint:

$$(1 + \tau_t^C)C_t^P \leq (1 + \tau_t^{LaP})W_t(1-\nu)L_t^P + TR_t^P, \quad (3.8)$$

where  $(1-\nu)$  is the weight of spenders' labour input,  $\tau_t^{LaP}$  denotes the tax rate on spenders' income,  $\tau_t^{LaP} = \tau_t^{La}$ .  $L_t^P$  is spenders' labour and  $TR_t^P$  is transferred by government to spenders.

As spenders cannot save and invest, the optimal consumption level is to consume all disposable income. Sbordone et al. 2006 disagree with the general assumption that spenders' labour decisions are dependent on savers'

labour decisions. They argue that if spenders imitate the decisions of savers, it is hard to clarify the reason why a change in savers' labour income tax rate would not affect spenders' labour income tax rate. Therefore, they believe that "the adoption of an intertemporal optimization problem over consumption and leisure yields a more reasonable result for spenders' labour choice than the copy-cat rule" (p.9).

### 3.2.2 Firms

Each firm produces a miscellaneous good for which it has market power and can, therefore, set the price. A representative firm rents capital from savers and labour from both types of agents to maximize the profit ( $P_t^f$ ):

$$P_t^f = A_t K_{t-1}^\alpha \{h[\nu L_t^A + (1-\nu)L_t^P]\}^{1-\alpha}, \quad (3.9)$$

where  $h$  is the constant rate of labour augmenting technology, the parameter  $\alpha > 0$  represents the share of capital in production, and  $A_t$  is the total factor productivity.

The resulting marginal costs  $\varphi$  can be described by the following equations:

$$\varphi_t = \alpha(K_t^A) + (1-\alpha)(Wag_t - Pr i_t) - A_t \quad (3.10)$$

$$Wag_t = W_t \times L_t \quad (3.11)$$

Eq. (3.10) implies that an increase in real wage  $W_{a,t}$ , and in borrowing capital  $K_t$  will lead to higher marginal costs. On the other hand, the greater the productivity  $A_t$  and the price  $P_{r,t}$  are, the lower the marginal costs will be.

Firms set prices optimally as a mark-up over marginal costs and take into account that prices cannot be completely adjusted to the optimal level each period. Equation (3.11) defines the capital stock of the firms,  $K_t^F$  as:

$$K_t^F = (1 - \delta)K_{t-1}^F + \delta I_t^F + \varepsilon_t^I \quad (3.12)$$

The superscript F represents firms. The rate of depreciation for capital  $K_t^F$  is  $\delta > 0$ . Capital formation depends on investment  $I_t^F$  and can be disturbed by a shock  $\varepsilon_t^I$ :

$$I_t^F = I_{t-1}^F + \frac{1}{\psi} (Q_t^F + \varepsilon_t^I) a \quad (3.13)$$

Investment at time t (today),  $I_t$ , depends on investment adjustment costs and on the shadow price of investment  $Q_t^F$ , which relates the value of investment activity today to that of investment tomorrow:

$$Q_t^F = \frac{K_t^A}{K_t^A + 1 - \delta} E(K_{t+1}^A) + \chi(1 - \delta)E_t(Q_{t+1}) + E_t(\eta_{t+1}) \quad (3.14)$$

The parameter  $\chi$  represents households' subjective time preference rate as stated before, while the variable  $\eta_{t+1}$  represents the stochastic discount factor.

In the economy, each good is produced using labour and capital as factors of production:

$$Y_t = A_t + \alpha K_t^F + (1 - \alpha)L_t \quad (3.15)$$

The higher the productivity  $A_t$ , the higher the output level  $Y_t$ .

### 3.2.3 Macroeconomic supply of and demand for good

The aggregated supply of goods corresponds to the total of consumption and investment demand plus public expenditure  $G_t$ . The parameters  $c$ ,  $i$  and  $g$  are steady-state values.

$$Y_t = cC_t + iI_t + gG_t \quad (3.16)$$

### 3.2.4 Inflation dynamics

Aggregate inflation dynamics derive from firms' price-setting behaviour. The inflation rate is given by a Phillips curve:

$$\pi_t = \chi E_t(\pi_{t+1}) + \kappa \varphi_t + \varepsilon_t^n \quad (3.17)$$

The parameter  $\kappa$  gives the elasticity of inflation to marginal costs. Inflation can also be driven by a cost-induced inflation shock  $\varepsilon_t^n$ .

In each period government chooses  $\{ TR_t^A, TR_t^P, G_t, B_t, \tau_t^K, \tau_t^{La}, \tau_t^{LaP}, \tau_t^C \}$  to satisfy its budget constraint:

$$TR_t^A + TR_t^P + G_t + R_{t-1}B_{t-1} + \delta^T \tau_t^K K_{t-1} = \tau_t^{La} \nu W_t L_t^A + \tau_t^{LaP} (1-\nu) W_t L_t^P + \tau_t^K r_t K_{t-1} + \tau_t^C C_t + B_t,$$

where  $C_t = C_t^A + C_t^P$  is gross consumption,  $K_t = K_t^A, B_t = B_t^A$  because only savers can own capital and government debt, and  $G_t$  is government consumption.

Equilibrium also requires an infinite sequence of fiscal policies to satisfy the intertemporal budget constraint:

$$B_t = E_t \sum_{j=1}^{\infty} h^{j-1} \prod_{i=0}^{j-1} R_{t+i}^{-1} \left[ \begin{array}{l} (1-\alpha)\tau_{t+j}^{La} \nu W_{t+j} L_{t+j}^A + (1-\alpha)\tau_{t+j}^{La} (1-\nu) W_{t+j} L_{t+j}^P + \\ \alpha \tau_{t+j}^K r_{t+j} K_{t+j-1} - \delta^T \tau_{t+j}^K r_{t+j} K_{t+j-1} + C_{t+j} \tau_{t+j}^C - G_{t+j} \\ -TR_{t+j}^A - TR_{t+j}^P \end{array} \right] \quad (3.18)$$

The intertemporal budget constraint implies that  $E_t \lim_{T \rightarrow \infty} \beta^{t+T} u(c_{t+T}) (B^{t+T} / h^{t+T}) = 0$ , and hence the condition for debt is satisfied.

This restriction means that debt cannot forever grow faster than the economy. At time  $t$ , a debt-financed tax cut leads to higher  $B_t$ . The intertemporal budget constraint says that at least one of the fiscal variables on the right hand side must adjust: one or more of the tax rates have to rise, or government consumption or transfers have to fall relative to the values in the path without a tax shock.

Monetary policy is described by an interest rate rule:

$$i_t = \rho i_{t-1} + (1-\rho)(\varpi_{\pi} \pi_t + \varpi_x x_t) + \varepsilon_t^i \quad (3.19)$$

This implies that the central bank wants to stabilize inflation and deviations from long-term potential output  $X_t$  without causing interest rates to fluctuate excessively. Parameter  $\rho$  describes the degree of interest rate variation. If the economy overheats, leading to  $\pi_t > 0$  and  $x_t > 0$ , the central bank will raise the nominal interest rate. The extent to which the interest rate increases is dependent on the interest rate response coefficients of inflation  $\omega_\pi > 1$  and of the output gap  $\omega_x > 0$ . In its most straightforward form, fiscal policy can be defined by:

$$G_t = T_t + m(M_t - M_{t-1} - \text{Pr}i_t) \quad (3.20)$$

The government finances its expenditure  $G_t$  from taxes and the central bank profit  $M_t - M_{t-1} - \text{Pr}i_t$ , where  $M$  represents money stock and  $m$  is a steady-state value. Many paths of fiscal policies can satisfy the intertemporal government budget constraint. Two offsetting policies are considered here: either government consumption or transfer payments would adjust outside of the budget window to address a deteriorating budget after the tax cut. The adjustment magnitude can vary so long as fiscal solvency is maintained.

## 3.3 DSGE model including intergenerational transfers

### 3.3.1 Households

On the basis of Lüth's (2001) general equilibrium setting for intergenerational transfers, we consider an economy that consists of representative agents with intergenerational transfer motives and a maximum life expectancy of two periods. Households decide their first and second period consumption,  $c_s^1$  and  $c_s^2$ . The consumption flows are the total consumption of all existing households in one generation. Superscripts 1 and 2 represent first and second time period of life. The representative households work in the first period of life and retire in the second period. Subscript  $s$  denotes the parents' generation index. That means an agent born in  $s$  will have her children at period  $s+1$ . As a result, at any given point of time, the economy consists of two generations. We assume that both generations receive money and assets from their parents and that both will transfer wealth, including money and property, to their children, for which purpose they save.

The individual receives *per capita* bequests  $B_s$  when they are young and gives *per capita* inheritance  $B_s$  to her children in the second period. Consequently, the expected utility function of a representative agent can be described as:

$$U_s = E_s \left\{ \sum_{s=0}^{\infty} \beta^s \left[ \frac{(c_s^1)^{1-\sigma}}{1-\sigma} + \frac{\ell (c_s^2)^{1-\sigma}}{1-\sigma} + \frac{(B_s)^{1-\gamma}}{1-\gamma} + \frac{(J_s)^{1-\gamma}}{1-\gamma} - \frac{(N_s)^{1+\varphi}}{1+\varphi} \right] \right\} \quad (3.21)$$

According to this formulation, the agent derives utility from first and second time period consumption  $c_s^1$  and  $c_{s+1}^2$ , labor supply  $N_s$ , bequest received from her parents  $B_s$ , and joy brought by her children  $J_s$ . Agents value future consumption less than present because of the uncertainty associated with future events. Therefore, we use parameter  $\ell$  to describe the discount rate between two time periods of live. The parameter  $\sigma$  denotes the elasticity between first period consumption and leisure.  $\gamma$  and  $\varphi$  represent the elasticities among bequeathing, joy and labour supply.

We assume that  $A_{s+1}$  is the overall attention provided by the second generation.  $A_{s+1}$  denotes the fraction of household's children's disposable time not dedicated to work. Household earns a wage per unit of time  $W_s$ , which is taxed at the rate of  $\zeta$ . Consequently, the relation between joy and attention can be stated as:

$$J_s = (1-\zeta)W_s A_{s+1}. \quad (3.22)$$

$$A_s = \vartheta \cdot (1-N_s). \quad (3.23)$$

Households' leisure equals  $(1-N_s)$ , where  $N_s$  denotes labour supply. Therefore, Eq. (3.23) represents the relationship between attention provided by young households and their leisure. Overall attention is a fraction  $\vartheta$  of

leisure that they dedicate to their parents. Eq. (3.22) shows that the joy parents received from their children for attention  $A_{s+1}$  equals to disposable income that young households can earn if they spend same time on work.

$c_t^1$  denotes the aggregate consumption of first generation households at time period  $t$ . Subscript  $t$  present current time period. The dynamic budget constraint of first generation households is:

$$c_t^1 + BO_t / (P_t R_t) = BO_{t-1} / P_t + \zeta (W/P_t) N_t + r_t^c K_t + B_t - d_t, \quad (3.24)$$

where  $B_t$  is intergenerational transfers received from their parents,  $BO_t$  is the coupon bond,  $R_t$  denotes the gross returns on the bond,  $W_t$  denotes the nominal wage rate,  $P_t$  is the consumption price index (CPI), and  $d_t$  represents savings. The consumption inflation rate is represented as  $\Pi_t \equiv P_t / P_{t-1} \Rightarrow \Pi_t = 1 + \pi_t$ , where  $\pi_t$  is the inflation rate.

At the end of the second period, the agent gains interest on her savings at a rate  $r^G$ . Furthermore, she gives bequests to her children and these bequests are shared equally among the agent's  $m_t$  offspring.  $c_t^2$  represents the aggregate consumption of second generation households. Then the second generation budget constraint is therefore given by:

$$c_t^2 + BO_t / (P_t R_t) + J_t = BO_{t-1} / P_t + J_{t-1} + d_t (1 + r^G) - m_t B_t \quad (3.25)$$

### 3.3.2 Firms (Entrepreneurs)

Thanks to Dixit and Stiglitz (1977), which was further developed by Blanchard and Kiyotaki (1987), monopolistic competition can be appropriately incorporated into modern macroeconomic models. Moreover, the Dixit-Stiglitz framework can also be embodied in a general equilibrium setting. In this section, we apply the Dixit-Stiglitz framework to describe the behavior of firms and retailers.

In what follows below,  $Y_t$  is a composite final good that is produced by a representative firm. The final-good producer chooses a continuum of intermediate goods,  $Y_t(i)$ , as production input, indexed by  $i \in [0,1]$ , where each is produced by a unique monopolistically competitive firm. Firms and retailers set prices in a Calvo-staggered manner. The aggregation of production is summarized as follows:

$$\text{A production aggregator } Y_t = \left[ \int_0^1 Y_t(i)^{(1-1/\mu)} di \right]^{(\mu/\mu-1)}, \quad (3.26)$$

where  $1 < \mu < \infty$ ,  $\mu$  is the constant elasticity of substitution between any pair of differentiated goods.  $Y_t(i)$  can be represented by a Cobb-Douglas function:

$$Y_t(i) = Z_t K_t(i)^{1-\alpha} N_t(i)^\alpha \quad (3.27)$$

$i \in [0,1]$  denotes each differentiated good produced by a unique producer.  $\alpha$  represents the elasticity of output with respect to labour supply  $N_t(i)$  in the

production process.  $K_t(i)$  denotes capital at time  $t$ . All firms are assumed to use the same technology and the technology, in turn, is subject to the following path:

$$\ln(Z_t) = (1 - \tau) \ln(\bar{Z}) + \tau \ln(Z_{t-1}) + \varepsilon_t^Z, \quad (3.28)$$

where  $\bar{Z}$  is the mean of  $Z_t$  and  $\varepsilon_t^Z$  denotes the technological shock,  $0 < \kappa < 1$ .

The final good producers are competitive and produce goods according to the production function (6). They choose intermediate goods whose price is  $p_t(i)$  and set the price of the final goods at  $P_t$ . Thus,  $P_t$  is the index for the final goods price. As a consequence, their maximization problem is:

$$\max_{Y_t(i)} [P_t Y_t - \int_0^1 Y_t(i) p_t(i) di] \quad (3.29)$$

The input demand function associated with the intermediate goods is:

$$Y_t(i) = (p_t(i)/P_t)^{-\mu} Y_t \quad (3.30)$$

The price of final goods  $P_t$  is determined by:

$$P_t = \left( \int_0^1 p_t(i)^{1-\mu} di \right)^{\frac{1}{1-\mu}}, \quad (3.31)$$

where  $\mu$  is the parameter of elasticity of substitution between goods, and aggregation of consumption is achieved by the following process:

$$\text{A consumption aggregator } C_t = \left[ \int_0^1 C_t(i)^{(\mu-1/\mu)} di \right]^{(\mu-1/\mu)} \quad (3.32)$$

The behaviours of firms and retailers are consistent with the conventional Calvo-staggered pricing mechanism (Calvo, 1983), as followed in Christiano et al. (2005) and Smets and Wouters (2003), which means that a randomly selected fraction  $(1-\theta)$  of firms adjusts prices, whereas the remaining fraction maintains the prices unchanged as

$$P_t(i) = (1 + \pi_t) P_{t-1}(i) \quad (3.33)$$

Defining the index for the “reset” price in period  $t$  and  $P_t^*(i)$  as the optimal price chosen by the firm to maximize the present value of real profits, the aggregate price index (CPI) is given by:

$$P_t = [\theta(1 + \pi_{t-1}) P_{t-1}^{1-\mu} + (1-\theta)(P_t^*)^{1-\mu}]^{(1/\mu)} \quad (3.34)$$

The relation between the aggregate price index and the optimal price is

$$P_t^*(i)/P_t = (\mu/\mu-1) \frac{E_t \sum_{j=0}^{\infty} \theta^j \beta^j (C_{t+j}/C_t)^{-\sigma} \eta_{t+j} (P_{t+j}/P_t)^\mu Y_{t+j} G_j^{-\mu}}{E_t \sum_{j=0}^{\infty} \theta^j \beta^j (C_{t+j}/C_t)^{-\sigma} \eta_{t+j} (P_{t+j}/P_t)^{\mu-1} Y_{t+j} G_j^{1-\mu}},$$

when  $t=0$ ,  $G_t=1$ .  $G_t = (1 + \pi_1)(1 + \pi_2)\dots(1 + \pi_t)$ , for  $t \geq 1$ .

### 3.3.3 Monetary Policy

The Chinese government determines national monetary policy, which follows a Taylor-type rule (Chen, Funke and Paetz, 2012):

$$R_t = \Phi_1(E\pi_{t+1} - \pi_t) + \Phi_2\pi_t + \Phi_y Y_t + \Phi_r R_{t-1}, \quad (3.35)$$

where  $R_t$  is the nominal interest rate for period  $t$ ; the policy reaction parameters  $\Phi_1$ ,  $\Phi_2$  and  $\Phi_y$  determine the preference of  $v$  with respect to inflation and output gap stabilization from the steady state;  $\Phi_r$  indicates the relation between  $R_t$  and  $R_{t-1}$  and reflects the smoothing of interest rate dynamics. This rule suggests that inflation and the output gap are the main factors influencing the central bank's decisions regarding the nominal interest rate. Moreover, the interest rate depends on the coefficient  $\Phi_r$  or on the previous interest rate. With respect to China, Zhang (2009) argues that following an interest rate rule is preferable to following a money supply rule because it leads to less fluctuation in the economy.

## 3.4 Estimation and Results Analysis

### 3.4.1 First-Order Conditions (FOCs)

We apply Uhlig's (1999) method to calculate the first-order conditions (FOCs) of the model. First, we derive the Lagrangian of Eq.(3.21) based on the first life period budget constraint:

$$L = \max E \sum_{t=0}^{\infty} \beta^t \left\{ \frac{(c_t^1)^{1-\sigma}}{1-\sigma} + \frac{\ell(c_t^2)^{1-\sigma}}{1-\sigma} + \frac{(B_t)^{1-\gamma}}{1-\gamma} + \frac{(J_t)^{1-\gamma}}{1-\gamma} - \frac{(N_t)^{1+\varphi}}{1+\varphi} \right. \\ \left. - \lambda_t (c_t^1 + BO_t/(P_t R_t) - BO_{t-1}/P_t - \zeta(W_t/P_t)N_t - r_t^c K_t - m_t B_t - d_t) \right\} \quad (3.36)$$

The FOC with respect to  $c_t^1$  is:

$$(c_t^1)^{-\sigma} = \lambda_t \quad (3.37)$$

The FOC with respect to  $B_t^1$  is:

$$(B_t)^{-\gamma} = -\lambda_t = -(c_t^1)^{-\sigma} \quad (3.38)$$

The FOC with respect to  $BO_t$  is:

$$-(\lambda_t/P_t R_t) - \beta E[-(\lambda_{t+1}/P_{t+1})] = 0 \\ \Rightarrow \lambda_t = \beta E[(P_t R_t/P_{t+1})\lambda_{t+1}] \quad (3.39)$$

According to Eqs.(3.37) and (3.39), we obtain the equilibrium equations:

$$(c_t^1)^{-\sigma} = \beta E[(P_t R_t/P_{t+1})[c_{t+1}^1]^{-\sigma}] \quad (3.40)$$

$$\Rightarrow \beta E[(P_t R_t/P_{t+1})(c_{t+1}^1/c_t^1)^{-\sigma}] = 1 \quad (3.41)$$

This is the Euler equation with respect to consumption  $c_t^1$ .

The FOC with respect to  $N_t$  (labour) is:

$$\begin{aligned}
-N_t^\varphi + \lambda_t(W_t/P_t) = 0 &\Rightarrow N_t^\varphi = \lambda_t(W_t/P_t), \quad \lambda_t = (c_t^1)^{-\sigma} \\
\Rightarrow N_t^\varphi &= (c_t^1)^{-\sigma} (W_t/P_t)
\end{aligned} \tag{3.42}$$

Eq. (3.22) represents the optimal labour-leisure decision in equilibrium.

Further, we can find the FOCs with respect to investment  $I_t$  and capital stocks  $K_t$  according to the capital stock accumulation equation,  $K_t = (1-\delta)K_{t-1} + I_t$  :

$$\beta E_t (c_{t+1}^1/c_t^1)^{-\sigma} (1-\delta + r_{t+1}^c) = 1 \tag{3.43}$$

We have mentioned the path subject to technology shocks  $Z_t$  and the Cobb-Douglas production function equation (3.27):

$$\ln(Z_t) = (1-\tau)\ln(\bar{Z}) + \tau \ln(Z_{t-1}) + \varepsilon_t^Z \tag{3.44}$$

On the basis of Eqs. (3.42) and (3.27), we have the FOC with respect to  $N_t(i)$  and  $K_t(i)$  as:

$$W_t/P_t = (\alpha r_t^c / (1-\alpha)) (K_t(i)/N_t(i)) \tag{3.45}$$

This formula can be rewritten as:

$$W_t/P_t = \alpha (W_t/\alpha P_t)^\alpha (r_t^c / (1-\alpha))^{1-\alpha} (K_t(i)/N_t(i))^{1-\alpha} \tag{3.46}$$

We set the real marginal cost,  $\eta_t$ , as:

$$\eta_t = (1/Z_t)(W_t/\alpha P_t)^\alpha (r_t^c/(1-\alpha))^{1-\alpha} \quad (3.47)$$

In light of Eqs. (3.46) and (3.47), we can reason out:

$$W_t/P_t = \alpha Z_t \eta_t (K_t(i)/N_t(i))^{1-\alpha} \quad (3.48)$$

The aggregate wage equation is:

$$W_t = \left[ (1-\xi)(W^*)^{1-l} + \xi(W_{t-1})^{1-l} \right]^{1/(1-l)}, \quad (3.49)$$

Furthermore, the Lagrangian of Eq.(3.21) based on the second life period budget constraint can be described as:

$$L = \max E \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} \frac{(c_t^1)^{1-\sigma}}{1-\sigma} + \frac{\ell(c_t^2)^{1-\sigma}}{1-\sigma} + \frac{(B_t)^{1-\gamma}}{1-\gamma} + \frac{(J_t)^{1-\gamma}}{1-\gamma} - \frac{(N_t)^{1+\varphi}}{1+\varphi} \\ -\lambda_t^2 (c_t^2 + BO_t/P_t R_t + J_t - J_{t-1} - BO_{t-1}/P_t - s_t(1+r^G) + B_t) \end{array} \right\} \quad (3.50)$$

The FOC with respect to second life period consumption  $c_t^2$  is:

$$\ell(c_t^2)^{-\sigma} = \lambda_t^2 \quad (3.51)$$

Then, the FOC with respect to  $BO_t$  is:

$$\lambda_t^2 = \beta E \left[ P_t R_t / P_{t+1} \lambda_{t+1}^2 \right] \quad (3.52)$$

Substituting Eq.(3.51) into Eq.(3.52), we have

$$\begin{aligned} \ell(c_t^2)^{-\sigma} &= \beta E \left[ P_t R_t / P_{t+1} \ell(c_{t+1}^2)^{-\sigma} \right] \\ \Rightarrow \beta E \left[ P_t R_t / P_{t+1} (c_{t+1}^2 / c_t^2)^{-\sigma} \right] &= 1 \end{aligned} \quad (3.53)$$

The FOC with respect to  $B_t$  according to the second life period constraint is:

$$(B_t)^{-\gamma} - \lambda_t^2 m_t = 0 \quad \Rightarrow \quad (B_t)^{-\gamma} = \lambda_t^2 m_t \quad \Rightarrow \quad (B_t)^{-\gamma} = \ell(c_t^2)^{-\sigma} m_t \quad (3.54)$$

Then, we can summarize the relevant equations discussed above to obtain the steady state in what follows including state states of the capital accumulation function, the function of monetary policy and the budget constraint of the second life period.

### 3.4.2 Steady States

At the steady state, all variables are constant. We calculate the steady state of equations discussed in the previous sections. The steady state of Euler equation (3.41) is:

$$1 = \beta E \left[ \left( \frac{\bar{P}\bar{R}}{\bar{P}} \right) \left( \frac{\bar{c}^1}{\bar{c}^1} \right)^{-\sigma} \right] \Rightarrow 1 = \beta \bar{R} \Rightarrow \bar{R} = 1/\beta \quad (3.55)$$

The steady state of the optimal labour-leisure decision (3.42) is:

$$\bar{N}^\varphi = \left( \frac{\bar{c}^1}{\bar{c}^1} \right)^{-\sigma} \left( \frac{\bar{W}}{\bar{P}} \right) \quad (3.56)$$

The steady states of technology shocks  $Z_t$  and the Cobb-Douglas production function (3.27) are:

$$\ln(\bar{Z}) = (1-\tau)\ln(\bar{Z}) + \tau\ln(\bar{Z}) = \ln(\bar{Z}) \quad (3.57)$$

$$\bar{Y} = \bar{Z} \left( \bar{K} \right)^{1-\alpha} \left( \bar{N} \right)^\alpha \quad (3.58)$$

The steady state of capital stock equation (3.43) is:

$$\beta E_t \left( \frac{\bar{c}^1}{\bar{c}^1} \right)^\sigma \left( 1 - \delta + r^c \right) = 1 \Rightarrow \beta \left( 1 - \delta + r^c \right) = 1 \Rightarrow r^c = \frac{1}{\beta} + \delta - 1 \quad (3.59)$$

The steady states of aggregate price, wage, capital and inflation are:

$$\bar{P} = \left[ \theta \left( \bar{P} \right)^{1-\mu} + (1-\theta) \left( \bar{P} \right)^{1-\mu} \right]^{1/(1-\mu)} = \left[ \left( \bar{P} \right)^{1-\mu} \right]^{1/(1-\mu)} = \bar{P} \quad (3.60)$$

$$\bar{W} = \left[ (1-\mu) \left( \bar{W} \right)^{1-l} + \mu \left( \bar{W} \right)^{1-l} \right]^{1/(1-l)} = \left[ \left( \bar{W} \right)^{1-l} \right]^{1/(1-l)} = \bar{W} \quad (3.61)$$

$$\bar{K} = (1-\delta)\bar{K} + \delta\bar{I} \Rightarrow \bar{K} = \bar{I} \quad (3.62)$$

$$\Pi \equiv \frac{\bar{P}}{\bar{P}} = 1 \Rightarrow 1 + \bar{\pi} = 1 \Rightarrow \bar{\pi} = 0 \quad (3.63)$$

The steady states of the first and second life period budget constraints are:

$$\bar{c}^1 + \left(\bar{B}O/\bar{P}\right)\left(\left(1/\bar{R}\right) - 1\right) = \zeta\left(\bar{W}/\bar{P}\right)\bar{N} + r^c\bar{K} + \bar{B} - \bar{d} \quad (3.64)$$

$$\bar{c}^2 + \left(\bar{B}O/\bar{P}\right)\left(\left(1/\bar{R}\right) - 1\right) = \bar{d}(1+r^G) - m_t\bar{B} \quad (3.65)$$

Finally, the steady states of joy bought by children and attention are:

$$\bar{J} = (1-\zeta)\bar{W}\bar{A}. \quad (3.66)$$

$$\bar{A} = \vartheta \cdot \left(1 - \bar{N}\right). \quad (3.67)$$

### 3.4.3 Linearized Equations

According to Uhlig (1999) method, linearized functions of the equilibrium equations and first-order conditions can be expressed in the following. First, we have the linearized function of Eq.(3.41):

$$\hat{c}_{t+1}^1 = \hat{c}_t^1 + (1/\sigma)\left(\hat{P}_t - \hat{P}_{t-1} + \hat{R}_t\right) \quad (3.68)$$

The symbol “ ^ ” above a variable denotes the growth of the variable in question. Equation (3.68) is the linearized equation of the Euler equation of first life period consumption,  $c_t^1$ . This condition implies that current consumption is determined by prior consumption, expected future consumption, the inflation rate and the gross returns on bond.

The linearized function of Eq. (3.40) can be stated as:

$$\hat{N}_t = ((-\sigma)/\varphi) \left( \hat{c}_t^1 \right) + (1/\varphi) (\hat{W}_t - \hat{P}_t) \quad (3.69)$$

If we let  $w_t = W_t/P_t$ , then  $\hat{w}_t = \hat{W}_t - \hat{P}_t$ . The relation between the labour supply of firm  $i$ ,  $N_t(i)$ , and the aggregate labour supply,  $N_t$ , is:

$$N_t(i) = (W_t^*(i)G_t/W_t)^{-l} N_t, \quad (3.70)$$

where  $l$  is the elasticity of substitution among differentiated labour services.

When  $t = 0$ ,  $G_t = 1$ , when  $t = 0$ .  $G_t = (1 + \pi_1)(1 + \pi_2) \dots (1 + \pi_t)$ , for  $t \geq 1$ .

On the basis of Eqs. (3.69) and (3.70), we follow Zhang (2009) to rewrite Eq. (3.43) to obtain the linearized equation of real wages:

$$\begin{aligned} & ((1 + \beta)(1 + \varphi l)\xi + (1 - \beta\xi)(1 - \xi)) \cdot \hat{w}_t = (1 + \varphi l)\xi (\hat{w}_{t-1} + \pi_{t-1}) - (1 + \beta)(1 + \varphi l)\xi \pi_t \\ & + (1 + \varphi l)\xi \beta E_t(\hat{w}_{t+1} + \pi_{t+1}) + (1 - \beta\xi)(1 - \xi) \left[ \varphi \hat{N}_t + \sigma(\hat{c}_t^1) \right] \\ \Rightarrow & \hat{w}_t = (1/(1 + \beta)) (\hat{w}_{t-1} + \pi_{t-1}) - \pi_t + (\beta/(1 + \beta)) E_t(\hat{w}_{t+1} + \pi_{t+1}), \end{aligned} \quad (3.71)$$

where  $\xi$  denotes the fraction of households that set their wages according to the last-period wage inflation or to a combination of last-period price and wage inflation rates.

The remaining  $1-\xi$  portion of the households would adjust their wage optimally. This condition indicates that the current nominal wage is a function of inflation, consumption, labour and prior and expected wages. The linearized equation of the FOCs with respect to investment and capital stocks, Eq. (3.23) , is:

$$\hat{c}_{t+1}^1 = \hat{c}_t^1 + (1/\sigma)(1 + \beta\delta - \beta) \hat{r}_{t+1}^c \quad (3.72)$$

This equation is similar to equation (3.48), and, from it, we can derive that rental capital returns depend on the current, prior and expected levels of consumption. We also have the linearized technology function:

$$\hat{Z}_t = (1 - \tau) \hat{Z}_{t-1} + \varepsilon_t \quad (3.73)$$

This is the linearized equation of Eq. (3.44), which expresses the path of the technology shocks. Next, we have:

$$\hat{Y}_t = \hat{Z}_t + (1 - \alpha) \hat{K}_t + \alpha \hat{N}_t \quad (3.74)$$

This is the linearized form of Eq. (3.27), i.e., the Cobb-Douglas production function. Eq. (3.74) shows the linear relation between the three inputs in production, i.e., technology, capital and labour.

Linearized equation of the price index,  $\hat{P}_t$ , is:

$$\pi_t = \hat{P}_t - \hat{P}_{t-1} \quad (3.75)$$

This expression indicates that current inflation is the difference between the current and prior levels of the price index. We also have linearized equation of capital accumulation,  $\hat{K}_t$ :

$$\hat{K}_{t+1} = (1 - \delta)\hat{K}_t + \delta \hat{I}_t \quad (3.76)$$

Eq. (3.76) implies that future stocks of capital are a function of current capital and investment. The depreciation rate,  $\delta$ , affects the present value of changes in investment.

The linearized equation of the real marginal cost of labour,  $\hat{\eta}_t$  (Eq. 3.47), is:

$$\hat{\eta}_t = \alpha \hat{w}_t + (1 - \alpha) \hat{r}_t^c - \hat{Z}_t \quad (3.77)$$

According to Eq. (3.77), the real marginal cost of labour comes from three sources: wages, the rental rate of capital and technology shocks. The elasticities of labour and capital service are the parameters in this equation.

The linearized form of Eq. (3.45) is the equilibrium:

$$\hat{w}_t = \hat{r}_t^c + \hat{K}_t - \hat{N}_t \quad (3.78)$$

The equilibrium indicates that the sum of real wages and real unit labour costs is equal to the sum of capital accumulation and the rental rate of capital. Underlying this equilibrium is the assumption that firms finance their wage bills and decide their demand for labour on the basis of their capital and rental capital income.

The relationship between the aggregate price index,  $P_t$ , and the optimal price,

$P_t^*(i)$ , is:

$$P_t^*(i)/P_t = (\mu/\mu - 1) \frac{E_t \sum_{j=0}^{\infty} \theta^j \beta^j (C_{t+j}/C_t)^{-\sigma} \eta_{t+j} (P_{t+j}/P_t)^\mu Y_{t+j} G_j^{-\mu}}{E_t \sum_{j=0}^{\infty} \theta^j \beta^j (C_{t+j}/C_t)^{-\sigma} \eta_{t+j} (P_{t+j}/P_t)^{\mu-1} Y_{t+j} G_j^{1-\mu}}$$

We already know that when  $t = 0$ ,  $G_t = 1$ .  $G_t = (1 + \pi_1)(1 + \pi_2) \dots (1 + \pi_t)$ , for  $t \geq 1$ .

$\Rightarrow G_j = P_{t+j}/P_t$  Therefore, we have:

$$P_t^*(i)/P_t = (\mu/\mu - 1) \frac{E_t \sum_{j=0}^{\infty} \theta^j \beta^j (C_{t+j}/C_t)^{-\sigma} \eta_{t+j} Y_{t+j}}{E_t \sum_{j=0}^{\infty} \theta^j \beta^j (C_{t+j}/C_t)^{-\sigma} \eta_{t+j} Y_{t+j}} = (\mu/(\mu - 1)) \quad (3.79)$$

We can derive the following linearized Phillips curve from Eq. (3.79):

$$\pi_t = \left( \theta / (1 + \beta\theta^2) \right) \pi_{t-1} + \frac{2 - (1 + \beta)\theta + \beta\theta^2}{1 + \beta\theta^2} E_t \pi_{t+1} + (1 - \theta)(1 - \beta\theta) \hat{\eta}_t \quad (3.80)$$

The inflation rate is a function of the real marginal cost and the prior and expected levels of inflation. As above,  $\beta$  is the discount rate and  $\theta$  denotes the fraction of firms that will adjust their prices according to the last-period price inflation and marginal costs. The linearized function of composite production is in the following form:

$$\hat{Y}_t = \frac{1 + \alpha(\varepsilon - 1)}{\varepsilon} \hat{C}_t + \frac{(1 - \alpha)(\varepsilon - 1)}{\varepsilon} \hat{I}_t \quad (3.81)$$

Eq. (3.81) is derived from Eqs. (3.79) and (3.21). This function indicates that output also depends on the consumption demand of households and on investment. The elasticity of substitution between any pair of differentiated goods, i.e.,  $\varepsilon$ , plays a key role in this equilibrium. The linearized equation of joy bought by children, according to Eqs. (3.22) and (3.23) is:

$$\hat{J}_t = \hat{W}_t - \hat{N}_t \bar{N} / \left( 1 - \bar{N} \right). \quad (3.82)$$

Eq. (3.82) implies that the increase of an agent's joy bought by children is associated with the change of her wage and labour supply. Wage has a positive impact on joy whereas labour supply has a negative impact. Moreover, the linearized function of Eq. (3.82) is:

$$\hat{J}_t = \hat{W}_t + \hat{A}_t. \quad (3.83)$$

The deviation of joy bought by children also depends on the average wage per unit of time and attention provided by children. Then, the linearized function of attention provided by the second generation can be written as:

$$\hat{A}_t = \hat{N}_t \bar{N} / (\bar{N} - 1). \quad (3.84)$$

The linearized equation for second life period consumption is:

$$\hat{c}_{t+1}^2 = \hat{c}_t^2 + (1/\sigma) \left( \hat{P}_t - \hat{P}_{t-1} + \hat{R}_t \right) \quad (3.85)$$

Subtracting E. (3.68) from Eq. (3.85), we have

$$\hat{c}_{t+1}^2 - \hat{c}_{t+1}^1 = \hat{c}_t^2 - \hat{c}_t^1 \Rightarrow \hat{c}_{t+1}^2 - \hat{c}_t^2 = \hat{c}_{t+1}^1 - \hat{c}_t^1 \quad (3.86)$$

Then, we calculate the difference between first and second life period budget constraints, and the result is:

$$\hat{c}_t^2 - \hat{c}_t^1 = d_t (2 + r^G) - (m_t + 1) B_t - \zeta w_t N_t - r_t^C K_t \quad (3.87)$$

Assuming  $\hat{c}_t^2 = \hat{c}_t^1$ , Eq.(3.87) can be rewritten as:

$$B_t = \frac{2+r^G}{m_t+1}d_t - \frac{\zeta}{m_t+1}w_tN_t - \frac{1}{m_t+1}r_t^cK_t \quad (3.88)$$

Thus, the linearized equation of Eq.(3.88) can be stated as:

$$\bar{B} \hat{B}_t = \left( \frac{2+r^G}{m_t+1} \right) \bar{d} \hat{d}_t - \left( \zeta / (m_t+1) \right) \bar{w} \bar{N} \left( \hat{w}_t + \hat{N}_t \right) - \left( 1 / (m_t+1) \right) \bar{r}^c \bar{K} \left( \hat{r}_t^c + \hat{K}_t \right) \quad (3.89)$$

The linearized intergenerational transfer functions imply that deposits have a positive influence on bequests. The current amount of intergenerational transfers is also related to personal income and capital return.

We further assume the function of household savings as:

$$d_{t+1} = (1+r_t^c) / (1+\pi_t) d_t \quad (3.90)$$

Thus, the linearized function of savings in the equilibrium is:

$$\hat{d}_{t+1} = \hat{r}_t^c - \hat{\pi}_t + \hat{d}_t \quad (3.91)$$

This equation posits that the increase in expected household savings depends on the increases in the interest rate, inflation rate and current savings.

Finally, we calculate the first-order condition of joy based on budget constraint of second life period.

$$(J_t)^{-\gamma} = \beta \ell (c_{t+1}^2)^{-\sigma} - \ell (c_t^2)^{-\sigma} \quad (3.92)$$

The linearized function of Eq. (3.92) can be stated as:

$$\hat{J}_t = \sigma / (\gamma(1-\beta)) \hat{c}_{t+1}^2 - \sigma \beta / (\gamma(1-\beta)) \hat{c}_t^2 \quad (3.93)$$

Following the theory of Capiello and Ferrucci (2008), we rebuild the equation of net foreign assets as a key variable in the system. Net foreign assets can be described by the following function:

$$NFA_t = NFA_{t-1} + CA_t + KA_t, \quad (3.94)$$

where  $CA_t$  represents the sum of the current account balance,  $KA_t$  denotes the capital and financial account balance. A nation's current account position is equal to the difference between its savings and its investment. Then, the function of  $CA_t$  can be stated as

$$CA = NFA_t - NFA_{t-1} = \Delta NFA = d_t - I_t \quad (3.95)$$

Because we assume  $KA_t=0$ , the steady state equation of the net foreign assets function is:

$$\hat{NFA}_t = \hat{NFA}_{t-1} + \left( \bar{d} / \bar{NFA} \right) \hat{d}_t - \left( \bar{I} / \bar{NFA} \right) \hat{I}_t \quad (3.96)$$

$\left( \bar{S} / \bar{NFA} \right)$  and  $\left( \bar{I} / \bar{NFA} \right)$  denote the savings to foreign reserves ratio and the import to foreign reserves ratio in equilibrium, respectively.

To determine the steady state value in Eq. (3.96), we rely on the data for China from 2005 to 2012 to calculate the ratio of domestic savings to foreign reserves and investment to foreign reserves. We set the average value of savings, investment and reserves in this time period as the steady state value. Then, Eq. (3.96) can be rewritten as:

$$\hat{NFA}_t = \hat{NFA}_{t-1} + 1.188 \hat{d}_t - 1.051 \hat{I}_t \quad (3.97)$$

### 3.4.4 Parameter Estimates

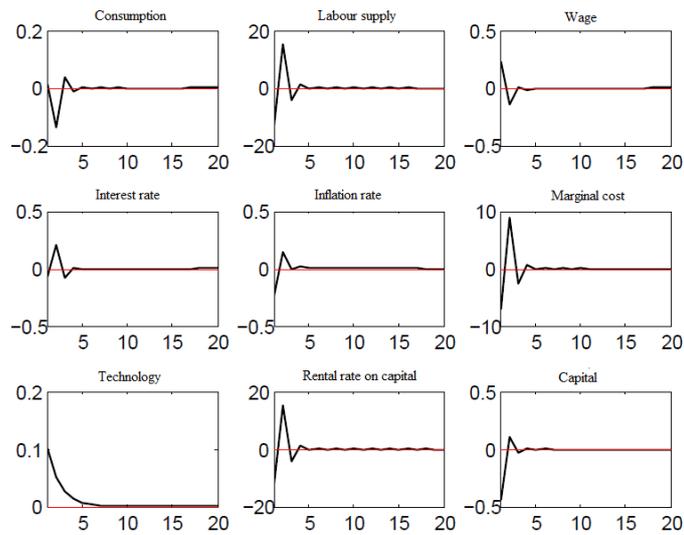
Following Walsh (2003), we set  $\sigma=2$ ,  $\alpha=0.4$  and  $\delta=0.04$ . According to China's total fertility rate from 1993 to 2012, we set the number of children of one couple is 1.63. Therefore, the number of children of one representative agent is  $m_t = 1.63/2 = 0.815$ . China's personal income tax rate is 45%, so  $\zeta=0.45$ . Lacking empirical evidence on attention provided by young agents to their parents, we set  $\vartheta=0.2$ , which means that young agents would like to spend 20% of their leisure time looking after their parents. If  $\alpha=0.4$ , the elasticity of substitution among intermediate goods and final goods can be obtained,  $\mu=4.61$ . The average (annual) nominal interest rate from 1980 to 2015 is 2.073%. Thus, the parameter of the discount rate is  $\beta=1/(1+0.02073)=0.9797 \approx 0.98$ . According to the report of Chinese Academy of Social Sciences, China's average childbearing ages had risen to 28.12 in the period from 2004 to 2014. We therefore consider that the time period of one generation is 28 years and that the interest rate for one life period  $r^G$  is

$(1+0.02073)^{28} = 1.776$ . According to the Labor Contract Law of the People's Republic of China, China has a 44-hour normal working week. Therefore, the steady state of labour  $\bar{N} = (44 \cdot 52) / (24 \cdot 365) = 0.26$ .

The GMM estimation with data for the 1995–2005 period shows that the fraction of firms that adjust their prices according to the last-period prices is  $\theta = 0.84$ . On the basis of Liu (2008), we obtain the elasticity of labour supply,  $\varphi = 6.16$ , the fraction of households that set their wages based on wage inflation,  $\xi = 0.6$ , and the elasticity of substitution between labor services at  $l = 2$ . The estimation of Eq. (41) with data for 1995 to 2005 shows that the elasticity of real money balances  $\gamma$  is equal to 3.13.

### **3.4.5 Simulation and Impulse Response Functions**

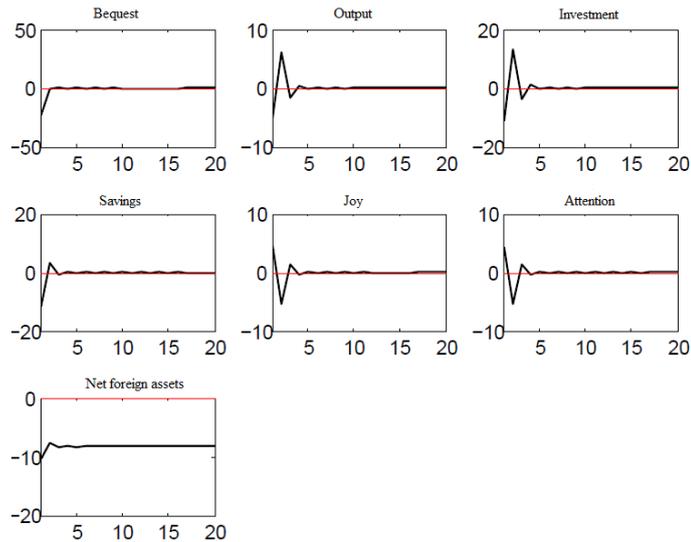
We now explore the reactions of monetary policy, technology and intergenerational transfers to the variables in the model, particularly the intergenerational transfer shocks. First, we present the results of the impulse response functions of every variable to technology shocks. Impulse response functions relate to the reaction of a dynamic system to some exogenous change(s). In our case, the dynamic systems are the linearized models produced in the previous section. According to the dynamic general equilibrium that we have developed above, the model's responses to a technology shock are given in Figures 3.1 and 3.2.



**Figure 3.1 Model Responses to a Positive Technology Shock (1)**

Figure 3.1 shows that the reactions of consumption to a technology shock consist of a standard level response and a negative change in the adjustment speed of responses. Although these estimated results might not be particularly precise, they might be applied to explain and analyse the trend of reactions to different shocks.

An estimated negative response of consumption suggests that a technology shock would cause a decrease in consumption in the short run because households require some time to decide their new consumption demand. Based on the influence of technology shocks, households and firms determine to invest more money and capital in new production.



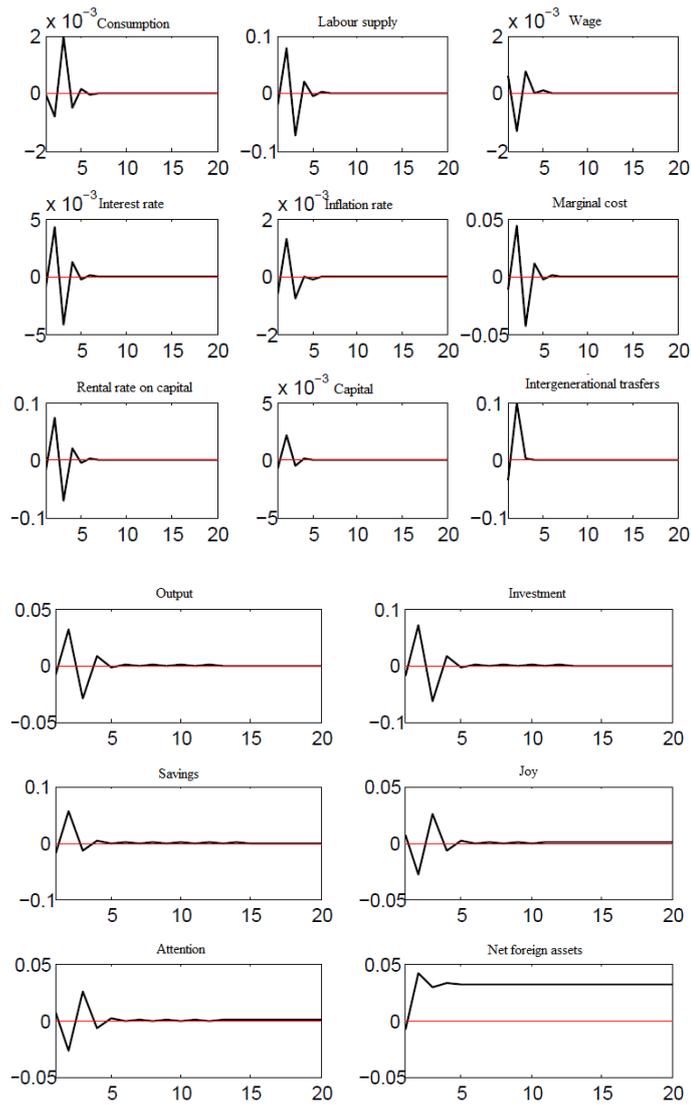
**Figure 3.2 Model Responses to a Positive Technology Shock (2)**

The reactions of attention and joy bought by children to technology shocks are also negative, which suggests that young people spend less time with their parents as technology develops. Moreover, technology shocks lead to a negative response from labour supply and the rental rate of capital in the first period, but bring out positive responses in the next four periods, which implies that demand for both labour and capital declines at the beginning of an industry transformation. However, with the development of a new production industry, firms require more labour and capital. The rental rate of capital increases due to the augmented demand for capital because the reactions of real money balances and the rental rate of capital switch to positive after the first period. The impulse response of technology is positive because the success of invention and innovation will encourage further technological progress.

We also find that the reactions of several variables are related. As noted above, consumption shows a negative response to a positive technology shock.

Moreover, the response of intergenerational transfers to a technology shock is only negative in the initial period and remains stable throughout. The impulse responses of savings are positive over the next five periods but negative in the initial period. This pattern of responses can also be found in the reactions of intergenerational transfers to deposits: in the face of a technology shock, households resolve to reduce their deposits to purchase new technological products at first and then intend to save more because of uncertainty. These responses also conform to the reactions of investment to technology shocks, which are negative in the initial period but positive in the next five periods.

The impulse responses of output are positive after the initial period. The findings correspond to the macroeconomic theory that most technology shocks increase an economy's productivity. Net foreign assets have a negative reaction to a technology shock, suggesting that demand for and supply of foreign technological products augment the process of industry transformation. However, foreign investors are willing to commit money and assets to emerging economies, which have significant potential for technological developments because they consider that these investments are a good opportunity to bring about financial success.



**Figure 3. 3 Model Responses to Intergenerational Transfers**

Figure 3.3 illustrates the impulse responses after an intergenerational transfers shock. We investigate the relationship between intergenerational transfers with a main economic variable to explore the impact of intergenerational transfers on the entire economy.

We can see a clear positive relation between the response of net foreign assets and an intergenerational transfers shock. An increase in intergenerational transfers leads to an increase in net foreign assets and, as a result, positive

productivity output. The intergenerational transfers shock also leads to increased investment, deposits and capital accumulation. In addition, higher intergenerational transfers enable households to reduce their consumption and increase their savings. These results indicate that intergenerational transfers are one reason why China would have accumulated such a large amount of foreign assets. The value of net foreign assets is equivalent to the sum of foreign assets held by monetary authorities and deposit banks, minus their liabilities. Therefore, we can conclude that intergenerational transfers boost savings of households at a micro level and then raise holdings of international reserves on a macro scale.

### **3.4.6 Bayesian Estimation**

As forcefully claimed by An and Schorfheide (2007a) and Fernández-Villaverde et al. (2004), Bayesian estimation and evaluation techniques are now the standard tools for the analysis of DSGE models. Estimation with Bayesian methods is based on the likelihood generated by the DSGE system. In contrast to GMM estimations, which are based on particular equilibrium relations, Bayesian estimation fits the complete and solved DSGE model. The Bayesian estimation is also a bridge between calibration and maximum likelihood. By using the Bayes theorem, the prior density can be combined with the likelihood function to obtain the posterior density. In this case, the posterior distribution avoids peaking at strange points where the likelihood peaks. Moreover, the distribution of priors is also helpful to identify structural parameters and shocks.

We attain the posterior distributions of all the estimated parameters in two steps. First, the posterior mode and an approximate covariance matrix are obtained by numerical optimization on the log posterior density. Second, we apply the Metropolis-Hastings and Markov-Chain Monte Carlo (MCMC) algorithm with 3,000 draws to obtain a sequence from the unknown posterior distribution. A dynare preprocessor for Matlab was employed in computation.

The parameters in Table 3.1 governing the dynamics of the model are estimated. Most pertain to the nominal frictions and elasticity in the model, whereas the remainder denote the exogenous shock processes. Detailed descriptions of the prior distributions for the structural DSGE parameters and the shock parameters are presented in Table 3.1.

**Table 3. 1. Prior and Posterior Distributions of Structural Parameters**

Parameters	Type	Mean	St.Dev	Mode	St.Dev	Conf. Int	Mean
$\alpha$	beta	0.040	0.002	0.3999	0.0018	[0.3814, 0.4245]	0.4047
$\beta$	beta	0.980	0.0002	0.9802	0.0003	[0.9778, 0.9829]	0.9805
$\delta$	beta	0.040	0.003	0.0388	0.0009	[0.0355, 0.0423]	0.0388
$\sigma$	gamma	2.000	0.020	2.0000	0.0026	[1.9751, 2.0256]	1.9974
$\gamma$	gamma	3.100	0.003	3.1000	0.0006	[3.0959, 3.1041]	3.1000
$\varphi$	gamma	6.100	0.003	6.1000	0.0007	[6.0948, 6.1031]	6.0993
$q$	normal	0.500	0.005	0.5000	0.0006	[0.4934, 0.5076]	0.5009
$h$	normal	0.600	0.005	0.6000	0.0007	[0.5906, 0.6119]	0.6012
$\theta$	gamma	0.800	0.003	0.7999	0.0005	[0.7976, 0.8052]	0.8017
$\mu$	gamma	4.600	0.003	4.6000	0.0003	[4.5969, 4.6050]	4.6012
$l$	gamma	2.000	0.003	2.0000	0.0004	[1.9948, 2.0022]	1.9985
$\xi$	gamma	0.600	0.003	0.6000	0.0003	[0.5949, 0.6026]	0.5985
$e$	Inv_gamma	0.010	0.0003	0.0078	0.0003	[0.0071, 0.0095]	0.0084

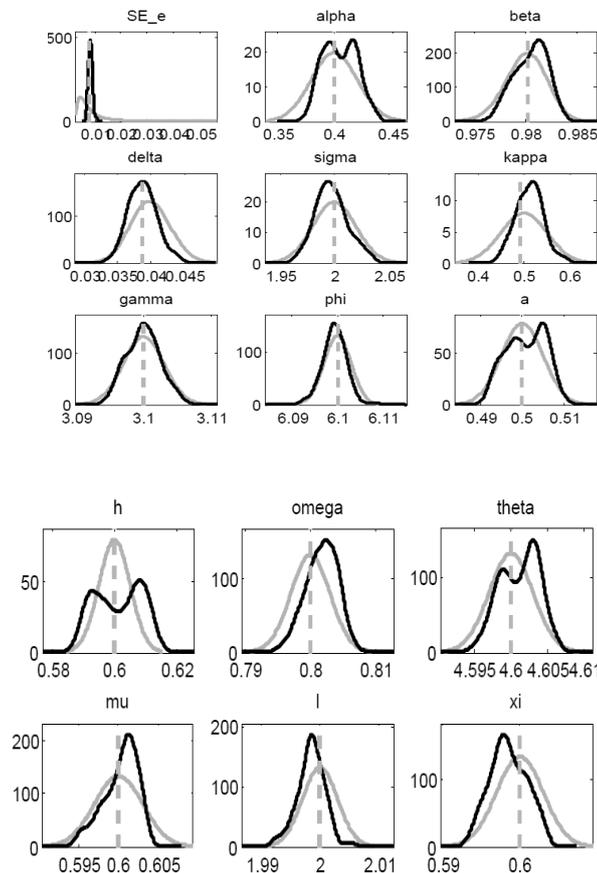
We use the inverse gamma (IG) distribution for the standard deviation of the shocks and set priors with two degrees of freedom. Priors are described by a density function of the form  $p(\phi_\Psi|\Psi)$ , where  $\Psi$  stands for the model,  $\phi_\Psi$  represents the parameters of the model.  $p(\bullet)$  is a probability density function (pdf) such as a normal, gamma, shifted gamma, inverse gamma, beta, generalized beta, or uniform function.

We use the beta distribution for all parameters bounded between 0 and 1, except for  $q$  and  $h$ . For parameters measuring elasticities, such as  $\gamma$ ,  $\sigma$ ,  $\varphi$ ,  $\theta$  and  $\xi$ , we use the gamma distribution. Normal distribution is applied to estimate  $q$  and  $\alpha$ . The elasticity of substitution between any pair of differentiated goods  $\mu$  is estimated to be higher than the priors, suggesting a slower response of substitution among goods. The elasticity of substitution with respect to capital consumption is lower, suggesting a faster response of consumption to the shocks.

The mean of the reaction coefficients to effective consumption and intergenerational transfers is estimated to be higher than its prior distribution. Finally, turning to the exogenous shock variables, the posterior mean of the technology shock parameter  $\tau$  is higher than its prior, with a coefficient of 0.512, pointing to a higher persistence than we previously expected.

Figure 3.4 displays prior and posterior distributions of the parameters and shocks. Overall, all parameters seem to be well identified, as shown in the

outcome that the posterior distributions are either not concentrated on the prior – or are concentrated but with a smaller dispersion – indicating high significance for the estimates. For price stickiness, 80.17% of firms do not adjust prices within one quarter, which implies that prices are re-optimized once every three quarters. The fraction of households setting their wages according to the last-period wage,  $\xi$ , is estimated to be 0.5985, implying a large share of backward-looking firms. The structural parameter  $q$  is estimated to be in the range of 0.4934 to 0.5076, highlighting the importance of intergenerational transfers.

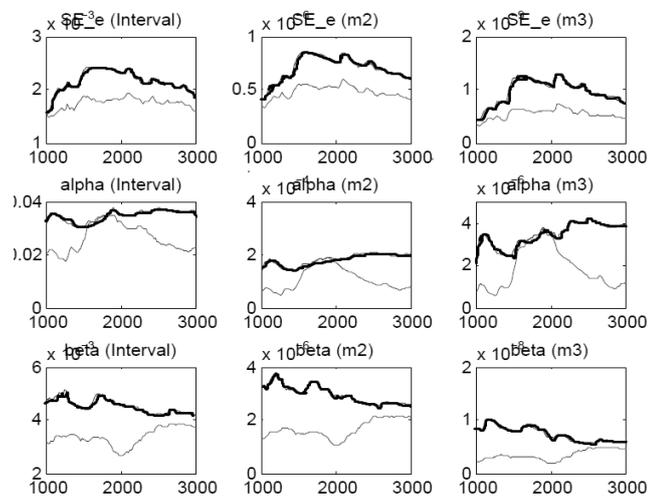


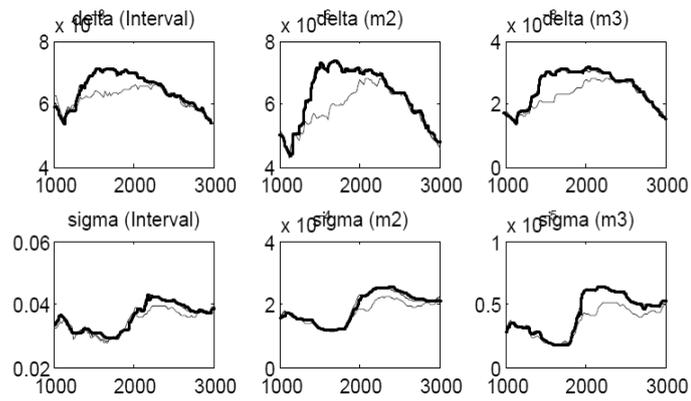
**Figure 3. 4 Prior and Posterior Distributions in Metropolis-Hastings Procedure**

Figure 3.5 presents the Markov-Chain Monte Carlo (MCMC) univariate diagnostics for the parameters and is the main source of feedback to gain

confidence with the results. To diagnose the sensibility and accuracy of the results, attention should be paid to two key points. First, we should note whether the results within any iterations of Metropolis-Hastings simulations, no matter how many they may be, are similar.

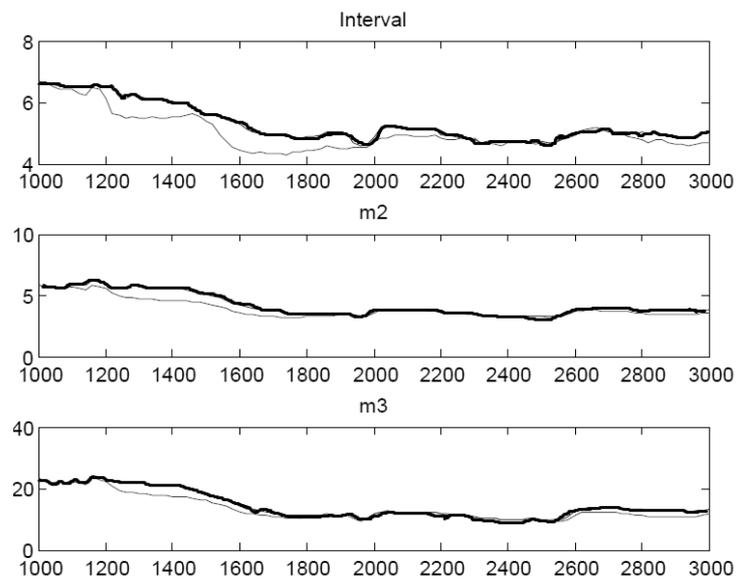
Second, we should observe the distance between various chains. In particular, the two lines on the charts represent measures of parameter vectors within and between chains, respectively. Figure 5 shows that the distance between the two chains is small for most parameters. Moreover, the plotted moments of structural parameters,  $\alpha$ ,  $\beta$ ,  $\delta$  and  $\sigma$ , are relatively stable and converge, which suggests that identification of the priors is reasonable and the results from the chains are sensible.





**Figure 3.5 Markov-Chain Monte Carlo Univariate Diagnostics for Parameters**

Multivariate diagnostics present an aggregate measure based on the eigenvalues of the variance-covariance matrix of individual parameters. The analysis method of multivariate diagnostics is similar to that of the MCMC univariate diagnostics because the results of these two diagnostics are based on the same theory. The plotted moments of multivariate diagnostics are relatively constant and converge, which implies the sensibility of the results.



**Figure 3.6 Multivariate Diagnostics for Parameters**

### 3.5 Conclusions

In this chapter, we develop a DSGE model for the Chinese economy, including key characteristics from the New Keynesian tradition and incorporating intergenerational transfers as a key feature. We estimate the model parameters using Bayesian techniques. The estimation results suggest that intergenerational transfers play an important role in the economy and are an important driving force behind accumulation of foreign reserves in China. The fit of the model is satisfactory as the plotted moments of structural parameters are relatively stable and converge, suggesting that the identification of priors is rational and the results from the chains are sensible. According to the impulse response functions, monetary policy shocks lead to increases in intergenerational transfers, insofar as both consumption and household savings are concerned. A monetary policy shock seems to offer a partial explanation for generous intergenerational transfers in China.

Last but not least, reactions of net foreign assets to an intergenerational transfers shock are significantly positive. Higher intergenerational transfers lead to higher deposits and higher holdings of net foreign assets. In addition to cultural tradition, rising housing prices, income uncertainty and underdevelopment of financial markets have resulted in intergenerational transfers becoming an indispensable source of financing to a large portion of young households in China. Therefore, intergenerational transfers are one of

the major determinants of a household's utility, as well as being fundamental to the high level of household savings in China.

However, a large portion of domestic savings is invested abroad because financial markets are incomplete in China (Song, et al., 2011). The structural imperfection of China's financial markets has brought about increased accumulations of international reserves because the under-development of financial system and incomplete of financial markets in China would induce sizable domestic savings to be invested in foreign assets. Therefore, intergenerational transfers finally convert into massive hoarding of international reserves and this provides a plausible explanation for the phenomenal growth of foreign reserves in China.

On the other hand, China's export promoting strategy is considered as another main driving force behind the country's hoarding of international reserves. We therefore discuss the relation between currency undervaluation and foreign exchange reserve accumulation in the next chapter.

## **Chapter 4**

### **The Nexus between Foreign Reserves Accumulation and Currency Undervaluation**

In this chapter we discuss the nexus between currency undervaluation and foreign reserves in a DSGE framework. We show that, rather than the commonly assumed mercantilist interpretation, real undervaluation affects China's reserve build-up mainly through its positive effect on economic growth, which consists of three channels: TFP growth, trade surpluses and income increases. In the short run, undervaluation induces a positive response from TFP. Over the intermedium run, both exports and imports have a positive response to real undervaluation in the impulse response analysis. In the longer run, undervaluation leads to an increase in income, which has the largest contribution to growth and hence to reserve accumulation in China.

## 4.1 Introduction

At the end of 1978, when economic reforms ushered in a new era of growth and development in China, the country's foreign reserves were a mere 1.6 billion dollars. By the end of the 1980s, this figure had risen by more than 10 times to reach 18 billion dollars. By 1999 China's reserve holdings had increased to 146 billion dollars, and over the next decade they continued to surge, to reach 2399 billion dollars by 2009. Today, China is the largest holder of foreign reserves in the world, with official reserves of around 3.5 trillion dollars.

The rapid and massive accumulation of foreign reserves by China has stoked the debate over some key aspects of the country's international financial policy. Prominent among them is the question of what drives China to stockpile such a massive amount of reserves. While a country may amass foreign reserves for a variety of reasons, and the relative importance of the motives are time varying (Ghosh, Ostry and Tsangarides, 2012), the consensus within existing literature is that "mercantilist" and "precautionary" motives are the main drivers (Aizenman and Lee, 2007, 2008; Bonatti and Fracasso, 2013; Aizenman, Cheung and Ito, 2014; Aizenman, 2015). The mercantilist motive hypothesis regards reserves accumulation as a by-product of an export promoting strategy (Dooley, Folkerts-Landau and Garber, 2006). The reserve accumulation facilitates this strategy by preventing or slowing currency appreciation.

The precautionary argument proposes that holdings of foreign reserves are a self-insurance policy against bad states of nature. To determine the relative importance of these motives, Aizenman and Lee (2005) tested the case of developing countries and found that the variable which captures the mercantilist concern was statistically significant, but its magnitude of impact on reserves accumulation was low. At the same time, the variable related to the precautionary motive was found to be highly significant in explaining reserve accumulation. However, in the Chinese context, Bonatti and Fracasso (2013) argue that capital controls and an undervalued currency make the country less exposed to sudden stops of capital movements, and so the mercantilist motive is particularly appropriate to account for the reserve accumulation. In their empirical research, Delatte et al.(2014) formally show that the mercantilist motive rather than the precautionary concern is more consistent with the rapid reserve accumulation in China.

However, therein lies another big problem. The mercantilist view implies that authorities would use weak and undervalued exchange rate to support the competitiveness and growth of the export sector (Fratzscher, 2012). But whether currency undervaluation can boost export expansion is debatable. While theoretical research is generally lacking, existing empirical evidence as to the effects of undervaluation on exports is inconclusive. At best, Haddad and Pancaro (2010) show that real undervaluation has a positive effect on growth and export expansion in developing countries, but not for long. Therefore, if on the whole the evidence seems insufficient to prove the contention that there is a positive link between an undervalued currency and

exports and growth, this would call into question the mercantilist explanation for the large reserve accumulation in the emerging economies.

In the literature, a body of recent empirical research has documented a positive relationship between the real exchange rate and growth. Rodrik (2008) discusses the significant role of an undervalued real exchange rate in stimulating growth. Hausman et al. (2005), Johnson, Ostry and Subramanian (2006) and Bereau et al. (2012) also argue for the importance of a depreciated real exchange rate to accelerate growth, and believe that this exchange rate policy is an efficient tool for economic development. Using a sample of 93 developed and developing countries, Razin and Collins (1999) construct a fundamentals-based index of real exchange rate overvaluation and find that a depreciated exchange rate favours exports and economic growth, whereas an appreciated exchange rate constrains growth; moreover, they argue that the effect of appreciation is stronger. Rodrik (2008) considers that overvaluation damages growth and undervaluation favours growth, but does not find a significant difference in terms of the magnitude of each effect. Rapetti et al. (2012) also identify a positive relationship between an undervalued exchange rate and growth, and demonstrate a negative relationship between an overvalued exchange rate and growth. They argue that the effect of overvaluation is slightly stronger than that of undervaluation.

However, these views are not supported by many economic models, and a competing strand of economic thought does not predict a positive relationship between an undervalued real exchange rate and growth. According to these

latter theories, the value of a currency should be set at a level that is consistent with both internal and external balances (Krueger, 1983; Edwards, 1989; Williamson, 1990; Berg and Maio, 2010).

This strand of the literature generally suggests that both under- and overvaluation of a currency adversely impact growth by leading to misallocation of resources and eventual declines in real output (Aguirre and Calderon, 2005). Therefore, according to this view, exchange rate deviation from equilibrium levels is associated with macroeconomic disequilibrium, regardless of the direction of the misalignment.

The above contention suggests that there remain gaps in our understanding of the links between real undervaluation and exports and growth, which challenges the persuasiveness of the mercantilist motive for reserve accumulation. This prompts the current paper to dissect the export and growth effects of currency undervaluation with reference to the Chinese case. China is widely regarded by international commentators, policy makers and researchers as having an undervalued currency that has been a significant reason for its economic success in the recent decades. These pundits, practitioners and scholars argue that China has persistently devalued its real exchange rate vis-a-vis its trade partners to stimulate exports and growth (Aguirre and Calderon, 2005; Rodrik, 2008). The logic of this theory is that cheaper prices for China's exported products make them more attractive to foreign consumers, thus fuelling China's trade surplus. As a result, China is able to add rapidly to its foreign exchange reserves.

We investigate the nexus between currency undervaluation and reserve accumulation in a DSGE framework. Most of the previous studies are empirical in nature and seek to detect the econometrical links between the real exchange rate and exports, growth and other major macroeconomic variables. Some recent research has documented the positive association between a depreciated real exchange rate and economic growth (Razin and Collins, 1999; Prasad, 2006; Eichengreen, 2007; Rodrik, 2008, Aflouk and Mazier, 2013). However, lacking a robust theoretical underpinning, it is not yet clear how these empirical associations arise. Levy-Yeyati et al.(2013) call for the development of theoretical models to better our understanding of the empirical insights. The DSGE model developed in this study represents a response to that call.

Our DSGE approach applies a general equilibrium analysis of the relationship between the real exchange rate and other macro variables, particularly exports and growth. Previous studies in this field are dominated by the partial equilibrium analysis. However, impacts of real undervaluation are a wide-ranging and interactive process. Moreover, the exchange rate is only one of the factors that have a bearing on export performance. Other factors, such as wages and labour costs, would also influence the competitiveness of exports. Yang, Chen and Monarch (2009) argue that Chinese wages and labour costs have risen sharply. From 1997 to 2007, China's average real wages experienced a three-fold increase, according to published government data

(e.g., NBSC, 2004, 2007, 2008). This would inevitably offset to some degree the effects of real undervaluation on exports.

Therefore, it is necessary and desirable to conduct an extensive study of a number of economic variables, their interrelations and interdependences, to understand the working of the reserve accumulation and currency undervaluation. Specifically, this DSGE model is related to exchange rate undervaluation, savings, economic growth and net foreign assets. Our concern is to answer the following questions: Does real exchange rate undervaluation continue to have a positive relationship with real output in a theoretical framework? How do exchange rate interventions affect export, import and foreign reserves in a DSGE model? What mechanisms are at play in the theoretical macroeconomic model?

The DSGE modelling will also make it possible to shed light on the mechanism through which interactions between reserve accumulation and real undervaluation occur. Levy-Yeyati et al. (2013) argue that the positive effect on the long-run components of GDP growth appears to come not from export earnings or import substitution, but rather from the expansion of domestic savings, investment and capital accumulation. By contrast, Rodrik (2008) posits that capital accumulation operates exclusively in the tradable goods sector. Wlasiuk (2013) focuses on labour market frictions and inter-sectoral re-allocation to explain the high rate of growth, urbanization and savings observed in many Asian economies.

Mbaye (2013) discusses two theoretical transmission channels to explain a possible positive relationship between real exchange rate depreciation and exports and growth: the “capital accumulation channel” and the “total factor productivity (TFP) growth channel”. The TFP channel highlights the role of real undervaluation in boosting the overall productivity level, which in turn leads to export expansion, while the capital accumulation channel relies on the notion that real undervaluation boosts economic growth by increasing the stock of capital in the economy.

Rapetti (2013) proposes a further mechanism, the “financial globalization channel”, and focuses on foreign capital movements to developing countries to conclude that capital inflows affect economic performance through transitory real exchange rate misalignments.

In this chapter, we attempt to understand the effects of real undervaluation on exports and hence reserve accumulation from a growth perspective. To this end, our DSGE model is applied to explain how exchange rate undervaluation affects domestic savings, wages and TFP growth. The model also explores the relationship between the TFP growth and real output and investigates into how the real exchange rate undervaluation works through this channel to impact on the export-to-import ratio and net foreign assets.

Our analysis shows that real undervaluation contributes to reserve accumulation mainly through the growth effect, which in turn occurs through three channels: the TFP growth, trade surpluses and income (wage) increases.

Model simulation suggests that, in the longer run, the income increase is more important than the other two channels. With the income increase, domestic savings grow consequently. The growing savings become excessive relative to investment, which then overflow to international financial markets leading to China's rapid accumulation of international assets.

The chapter is organized as follows. Following this introductory section, section two presents the structure of the model and describes some new variables in the DSGE framework, such as the real exchange rate and TFP factors. Section three shows the consequential first order conditions (FOCs) engendered by households' and firms' optimization behaviour, and presents the linearized functions of the general equilibrium. Section four estimates and evaluates the parameters, some of which are taken from previous empirical research, and applies Bayesian methods to estimate parameters and evaluate the model's performance. Section five concludes the chapter.

## **4.2 The Model including Currency Undervaluation**

### **4.2.1 Household**

The representative agent assumption in Dynamic Stochastic General Equilibrium model means all agents of same type are identical. It was assumed that everyone in the economy had the same preferences, and the same relative income of capital, labour skills, habits and so on. Therefore, the sum of their choices is equal to the aggregation behaviours in the economy.

The utility function of a patient household who is a representative of parental generation is:

$$U_t = E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \frac{((C_t - hC_{t-1}))^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right] \right\}, \quad (4.1)$$

where  $\beta$  is the discount factor,  $C_t$  denotes the consumption of the household at time  $t$ , and  $\sigma$  and  $\varphi$  are the substitution elasticity of consumption and the elasticity of labour supply, respectively. The consumption index  $C_t$  aggregates the consumption of traded and non-traded goods, which takes the following form:

$$C_t = \left[ (\varpi)^{1/\psi} (C_t^d)^{\frac{\psi-1}{\psi}} + (1-\varpi)^{1/\psi} (C_t^f)^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}}, \quad (4.2)$$

where  $\psi$  denotes the elasticity of substitution between traded and non-traded goods;  $C_t^d$  and  $C_t^f$  are the consumption of domestic and imported goods, respectively.  $\varpi$  represents the fraction of non-traded products in total output, and  $(1-\varpi)$  denotes the fraction of exported products.

The household's real period-by-period budget constraint is given by:

$$C_t + I_t + \frac{BO_t}{P_t R_t} = \frac{BO_{t-1}}{P_t} + \frac{W_t}{P_t} N_t + r_t^C K_t, \quad (4.3)$$

where  $BO_t$  denotes the coupon bond,  $P_t$  is the consumption price index (CPI),  $R_t$  denotes the gross returns on bond,  $W_t$  and  $r_t^C$  denote the nominal wage rate

and rental rate on capital in period  $t$ . The main parameters and variables are collected in Tables 1 and 2.

## 4.2.2 Firms

We apply the Dixit-Stiglitz (1977) framework to describe behaviours of firms and retailers.

$$\text{A production aggregator } Y_t = \left[ \int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \quad (4.4)$$

where  $Y_t$  is a composite final good that is produced by a representative firm.

The final good producer chooses a continuum of intermediate goods,  $Y_t(i)$ , as production input, indexed by  $i \in [0,1]$ , each produced by a unique monopolistically competitive firm. Firms and retailers set prices in a Calvo-staggered manner.  $\theta$  is the elasticity of substitution between goods.

$Y_t$  represents the income or output of the entire economy and can be represented by a Cobb-Douglas production function:

$$Y_t(i) = Z_t K_t(i)^{1-\alpha} N_t(i)^\alpha, \quad (4.5)$$

where  $i \in [0,1]$  denotes each differentiated good produced by a unique producer.  $\alpha$  represents the elasticity of output with respect to labour supply  $N_t(i)$  in the production process.  $K_t(i)$  denotes capital at time  $t$ . The capital

stock accumulation equation in this system is  $K_t(i) = (1-\delta)K_{t-1}(i) + I_t$ .  $Z_t$  represents TFP, which is a key factor affecting total output  $Y_t$ .

The price of final goods  $P_t$  is determined by:

$$P_t = \left( \int_0^1 P_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}, \quad (4.6)$$

where  $\mu$   $\theta$  is the elasticity of substitution between goods, as discussed above.

We consider that a randomly selected fraction  $(1-\mu)$   $(1-\theta)$  of firms adjusts prices, whereas the remaining fraction keeps prices unchanged as

$$P_t(i) = \Pi_t P_{t-1}(i) \Rightarrow P_t(i) = (1+\pi_t)P_{t-1}(i), \text{ where } \Pi_t = (1+\pi_t).$$

Defining the index for the “reset” price in period  $t$ ,  $P_t^*(i)$  is the optimal price chosen by the firm to maximize the present value of real profits, and  $\pi_t$  denotes the inflation rate. Then the aggregate price index is given by:

$$P_t = [\theta(1+\pi_t)P_{t-1}^{1-\theta} + (1-\theta)(P_t^*)^{1-\theta}]^{\frac{1}{1-\theta}} \quad (4.7)$$

The relation between the aggregate price index and the optimal price is:

$$\frac{P_t^*(i)}{P_t} = \left( \frac{\theta}{\theta-1} \right) \frac{E_t \sum_{j=0}^{\infty} \mu^j \beta^j \left( \frac{C_{t+j}}{C_t} \right)^{-\sigma} \eta_{t+j} \left( \frac{P_{t+j}}{P_t} \right)^{\mu} Y_{t+j} G_j^{-\mu}}{E_t \sum_{j=0}^{\infty} \mu^j \beta^j \left( \frac{C_{t+j}}{C_t} \right)^{-\sigma} \eta_{t+j} \left( \frac{P_{t+j}}{P_t} \right)^{\mu-1} Y_{t+j} G_j^{1-\mu}}, \quad (4.8)$$

when  $t=0$ ,  $G_t=1$ .  $G_t = (1+\pi_1)(1+\pi_2)\dots(1+\pi_t)$ , for  $t \geq 1$ .

We assume that currency undervaluation would affect the TFP growth rate and express this relation in the model as:

$$\hat{Z}_t = \tau_1 \hat{Z}_{t-1} + \tau_2 \left( RER_t - \bar{RER}_t \right) + \tau_3 \hat{I}_t + \tau_4 \hat{OPN}_t + \varepsilon_t^Z \quad (4.9)$$

The symbol “ ^ ” above a variable denotes the growth of the variable in question.  $RER_t$  is the real exchange rate, and  $\bar{RER}_t$  is the equilibrium real exchange rate.  $\varepsilon_t^Z$  denotes the shocks of TFP. We let  $\hat{RER}_t = RER_t - \bar{RER}_t$  or  $\hat{RER}_t$  is the deviation from the equilibrium real exchange rate provided by steady state of the real exchange rate.

When  $\hat{RER}_t = RER_t - \bar{RER}_t > 0$ , the real exchange rate is undervalued and if  $\hat{RER}_t < 0$ , it denotes overvaluation of the currency.  $OPN$  is the trade openness index, constructed as  $OPN_t = (EX_t + IM_t)/Y_t$ ;  $EX_t$  and  $IM_t$  denote the exports and imports, respectively.

In Eq. (4.9),  $\tau_1$  is the one period autoregressive parameter of TFP, and  $\tau_2$  measures the effect of exchange rate deviation on growth. If the country intervenes to keep its currency value low, the level of undervaluation  $\hat{RER}_t$  will be positive. We assume that the growth of TFP can be attributed to real exchange rate undervaluation. If the country's currency is overvalued, the appreciation of the exchange rate plays a negative role in TFP growth.  $\tau_3$  and

$\tau_4$ , respectively, are the parameters of investment and the trade openness index representing the impacts of investment and international trade on TFP.

### 4.2.3 Government

Following Chen, Funke and Paetz (2012), we consider an expanded Taylor rule to describe China's monetary policy. The interest rate rule function is:

$$R_t = \phi_R R_{t-1} + \phi_\pi \left( E_t(\pi_{t+1}) - \pi_t \right) + \phi_Y \hat{Y}_t + \varepsilon_t^R, \quad (4.10)$$

where  $R_t$  denotes the actual nominal interest rate, and  $\bar{R}$  is the equilibrium interest rate.  $R_{t-1}$  is the lagged interest rate.  $Y_t$  and  $RER_t$  denote the output gap and the real exchange rate, respectively.  $\phi_R$ ,  $\phi_\pi$  and  $\phi_Y$  are the parameters with respect to the lagged interest rate, inflation deviation, and the output gap.  $\varepsilon_t^R$  is the shock of the interest rate rule.

#### Monetary policy reaction function

$$M_t = \nu_1 M_{t-1} + \nu_2 Y_t + \nu_3 (\pi_t) + \varepsilon_t^m, \quad (4.11)$$

where  $M_t$  is nominal money growth;  $\bar{M}$  is the equilibrium money demand;  $\varepsilon_t^m$  is money growth shock;  $\nu_1$ ,  $\nu_2$ ,  $\nu_3$  are the parameters on a lag of nominal money growth, output and the inflation rate, respectively. Assuming the co-integrating relationship among the logarithms of money, output and inflation, we have  $\bar{M} = \alpha_0 + \alpha_1 \bar{Y} + \alpha_2 \bar{\pi}$ .

#### 4.2.4 Exchange rate policy

According to Mbaye (2013) and Edwards (1989), the influence of the real exchange rate ( $REER$ ) on economic growth can be described by:

$$\hat{Y}_t = \psi_Y \hat{Y}_{t-1} + \psi_{tot} TOT_t + \psi_{open} OPN_t + \psi_I I_t + \psi_{zfp} \hat{Z}_t + \psi_\pi \pi_t + \psi_{rer} \hat{REER}_t + \varepsilon_t^{REER}, \quad (4.12)$$

where  $\hat{Y}_t$  denotes the economy's growth rate,  $TOT_t$  denotes the terms of trade, and  $OPN_t$  is the trade openness index discussed above.  $REER_t$  is the CPI-based real exchange rate, and  $\varepsilon_t^{REER}$  is real exchange rate shock. Terms of trade,  $TOT_t$ , is the value of a country's exports relative to that of its imports; it is proxied in the model as the export price index relative to the import price index.  $\hat{Z}_t$  is the TFP growth rate, while  $\pi_t$  denotes the inflation rate.

The function of the terms of trade,  $TOT_t$ , can be described as:

$$TOT_t = \frac{(P_t Q_t^{ex}) \cdot NER_t}{P_t^F Q_t^{im}}, \quad (4.13)$$

where  $Q_t^{ex}$  and  $Q_t^{im}$  denote the quantity of exports and imports, respectively.

The superscript  $F$  denotes a foreign variable.  $NER_t$  is the nominal exchange rate. We assume that the values of both imports and exports are expressed in the foreign currency.  $P_t$  denotes the average value of exports per product; therefore, it also represents the price index of exports. The value of exports

equals  $EX_t = P_t Q_t^{ex} \cdot NER_t$  and the value of imports is  $IM_t = P_t^F \cdot Q_t^{im}$  with  $P_t^F$  being the price index of the foreign country. So, Eq. (4.13) can also be written as:

$$TOT_t = \frac{P_t Q_t^{ex}}{P_t^F Q_t^{im}} = \frac{P_t Q_t^{ex} (NER_t)}{P_t^F Q_t^{im}} = \frac{EX_t}{IM_t} , \quad (4.14)$$

where  $\frac{EX_t}{IM_t}$  is the ratio of exports to imports in terms of the foreign currency.

We consider this ratio to take an autoregressive form, such that the function of the exports to imports ratio is given by:

$$\frac{EX_t}{IM_t} = \Phi_{em} \frac{EX_{t-1}}{IM_{t-1}} + \rho_{em} , \quad (4.15)$$

where  $\Phi_{em}$  is the parameter on the exports to imports ratio (the terms of trade) one time period ago, and  $\rho_{em}$  is the standard deviation.

Trade openness,  $OPN_t$  is calculated as the ratio of the country's total trade, which equals to the sum of exports  $EX_t$  and imports  $IM_t$ , to the country's gross domestic product . The function of trade openness can be written as:

$$OPN_t = \frac{EX_t + IM_t}{Y_t} . \quad (4.16)$$

The log-linearization of Eq. (4.16) is:

$$\hat{OPN}_t = \frac{\bar{EX}}{\bar{EX} + \bar{IM}} \hat{EX}_t + \frac{\bar{IM}}{\bar{EX} + \bar{IM}} \hat{IM}_t - \hat{Y}_t , \quad (4.17)$$

where  $\bar{EX}$  and  $\bar{IM}$  are the steady states of exports and imports.

We also consider trade openness in an autoregressive framework, and in this light, the function of trade openness is given by:

$$OPN_t = \Phi_{opn} OPN_{t-1} + \rho_{opn} , \quad (4.18)$$

where  $\rho_{opn}$  is the standard deviation of the time series of trade openness, and  $\Phi_{opn}$  is the parameter on trade openness one period earlier.

According to the definition of the real exchange rate,  $RER$  is the relative price between tradables to non-tradables. Empirically, the real exchange rate is commonly provided by the nominal exchange rate times the ratio of domestic price to foreign price. As such, the function of the real exchange rate can take the form:

$$RER_t = \frac{NER_t \cdot P_t}{P_t^F} \quad (4.19)$$

The log-linearized function of Eq. (4.19) is:

$$\hat{RER}_t = \hat{NER}_t + \hat{P}_t - \hat{P}_t^F \quad (4.20)$$

Substituting the real exchange rate function of Eq. (4.19) into Eq. (4.14), we will have:

$$TOT_t = \left( \frac{P_t^I}{P_t^F} \cdot NER_t \right) \cdot \frac{Q_t^{ex}}{Q_t^{im}} = RER_t \cdot \frac{Q_t^{ex}}{Q_t^{im}} \quad (4.21)$$

The log-linearized form of Eq. (4.21) is:

$$\hat{TOT}_t = \hat{RER}_t + \hat{Q}_t^{ex} - \hat{Q}_t^{im} \quad (4.22)$$

From Eq. (4.21) and Eq. (4.22), we note that the variable of terms of trade has a positive relation with the real exchange rate. It is also associated with the difference between the quantities of export and import goods.

Inserting the linearized real exchange rate  $\hat{RER}_t$  into the left hand side of Eq. (4.22), this equation becomes:

$$\hat{Q}_t^{ex} - \hat{Q}_t^{im} = \hat{TOT}_t - \hat{RER}_t \quad (4.23)$$

This function implies that a decrease in the real exchange rate will induce an increase in the difference between exports and imports if the terms of trade remain unchanged. In turn, this result implies that holding the currency lower is helpful for selling a larger quantity of domestic goods to foreign markets.

However, the lower real exchange rate cannot cause an increase in the terms of trade; further, according to Eq. (4,23), it also reduces the ratio of exports to imports. Thus, although the lower real exchange rate leads to an increased volume of sales, undervaluation of the currency may not be profitable overall.

#### 4.2.5 Net foreign assets

Cappiello and Ferrucci (2008) argue that the sum of the current account balance ( $CA$ ), the capital and financial account balance ( $KA$ ), errors and omissions ( $EO$ ) and the change in foreign reserves must be zero. We consider the amount of foreign reserves to be equal to net foreign assets ( $NFA$ ). Therefore, we have:

$$CA_t + KA_t + EO_t - \hat{NFA}_t = 0 \Rightarrow \hat{NFA}_t = CA_t + KA_t + EO_t \quad (4.24)$$

Following Cappiello and Ferrucci (2008), we rebuild the equation of net foreign assets as a key variable in the system. They can be described by the following function:

$$NFA_t = NFA_{t-1} + CA_t + KA_t \quad (4.25)$$

This UB approach means that the fair value of a currency is the level of the exchange rate that is consistent with both the internal and external balance of a country. Most applications of the UB approach consider the internal balance to be a domestic economic process. However, a country's external balance can be broadly represented by its  $CA$  position. Given that a nation's  $CA$  position equals to the difference between its savings and its investment, the balance of payments relation in the  $CA$  approach is:

$$CA = NFA_t - NFA_{t-1} = \Delta NFA = S_t - I_t \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad (4.26)$$

where  $CA$  may be expressed by either changes in net foreign assets  $NFAs$  ( $\Delta NFA$ ) or by excess domestic savings ( $S_t$ ) over domestic investment ( $I_t$ ).

Generally, total aggregate demand ( $AD_t$ ) of an economy can be expressed as the sum of consumption ( $C_t$ ) and investment ( $I_t$ ):

$$AD_t = C_t + I_t \quad (4.27)$$

In the economy, as one man's expenditure is another man's income, total expenditure must be equivalent to total income. In other words, it should be the case that Total income ( $Y_t$ ) = Total expenditure ( $AD_t$ ). Because  $Y_t = AD_t$ , Eq. (4.27) can be written as:

$$Y_t = C_t + I_t \quad (4.28)$$

According to the definition of gross national income ( $GNI$ ),  $GNI$  is the sum of a country's domestic product ( $GDP$ ) plus net income received from overseas. Then the function of national income is :

$$GNI_t = Y_t + \Delta NFA = Y_t + S_t - I_t \quad (4.29)$$

Substituting Eq. (4.28),  $Y_t = C_t + I_t$ , into Eq. (4.29), we obtain:

$$GNI_t = C_t + I_t + S_t - I_t = C_t + S_t \quad (4.30)$$

National income is the sum of wages from employment and self-employment, profits to firms, interest to lenders of capital and rents to owners of land. Omitting profits, interest and rents, Eq. (4.30) can be written as:

$$W_t = C_t + S_t \quad (4.31)$$

In this framework, aggregate wages are equal to the sum of domestic consumption and domestic savings.

### 4.3. Summarizing the Model

#### 4.3.1. Solving first-order conditions (FOCs)

Following Uhlig's (2006) method for solving DSGE models, we first calculate the first-order conditions (FOCs) of the model. The Lagrangian of Eq. (4.1) is derived as follows:

$$L = \max E \sum_{t=0}^{\infty} \beta^t \left[ \frac{(C_t - h \cdot C_{t-1})^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} - \lambda_t (C_t + I_t + \frac{BO_t}{P_t R_t} - \frac{BO_{t-1}}{P_t} - \frac{W_t}{P_t} N_t - r_t^C K_t) \right] \quad (4.32)$$

Differentiating the FOC with respect to  $C_t$  is:

$$\frac{\partial L}{\partial C_t} = (C_t - h \cdot C_{t-1})^{-\sigma} - \lambda_t = 0 \Rightarrow (C_t - h \cdot C_{t-1})^{-\sigma} = \lambda_t \quad (4.33)$$

The FOC differentiating with respect to  $BO_t$  is:

$$\frac{\partial L}{\partial BO_t} = \left( -\lambda_t \frac{1}{P_t R_t} \right) - \beta E \left[ -\frac{1}{P_{t+1}} \lambda_{t+1} \right] = 0 \Rightarrow \lambda_t = \beta E \left[ \frac{P_t R_t}{P_{t+1}} \lambda_{t+1} \right] \quad (4.34)$$

Based on Eqs. (4.33) and (4.34), we obtain the equilibrium equation:

$$\beta E \left[ \frac{P_t R_t}{P_{t+1}} \left( \frac{C_{t+1} - h \cdot C_t}{C_t - h \cdot C_{t-1}} \right)^{-\sigma} \right] = 1 \quad (4.35)$$

This is the Euler equation with respect to consumption  $C_t$ . The FOC with respect to  $\lambda_t$  is:

$$P_t C_t + P_t I_t + \frac{BO_t}{R_t} = BO_{t-1} + W_t N_t + r_t^C K_t P_t \quad (4.36)$$

Replacing  $I_t = K_{t+1} - (1 - \delta)K_t$  in Eq. (4.19), the Lagrangian can be written as:

$$L = \beta^t \left( \frac{(C_t - h \bullet C_{t-1})^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right. \\ \left. - \lambda_t (C_t + K_{t+1} - (1-\delta - r_t^c) K_t) + \frac{BO_t}{P_t R_t} - \frac{BO_{t-1}}{P_t} - \frac{W_t}{P_t} N_t \right) \\ + \beta^{t+1} E_t \left[ \frac{(C_{t+1} - h \bullet C_t)^{1-\sigma}}{1-\sigma} - \frac{N_{t+1}^{1+\varphi}}{1+\varphi} \right. \\ \left. - \lambda_{t+1} (C_{t+1} + K_{t+2} - (1-\delta - r_{t+1}^c) K_{t+1}) + \frac{BO_{t+1}}{P_{t+1} R_{t+1}} - \frac{BO_t}{P_{t+1}} - \frac{W_{t+1}}{P_{t+1}} N_{t+1} \right]$$

The FOC differentiating with respect to  $K_{t+1}$  (capital at time period  $t+1$ ) is:

$$\frac{\partial L}{\partial K_{t+1}} = (-\lambda_t) + \beta E_t (\lambda_{t+1} (1 - \delta + r_{t+1}^c)) = 0 \quad (4.37)$$

$$\Rightarrow \beta E_t \left( (1 - \delta + r_{t+1}^c) (C_{t+1} - h \bullet C_t)^{-\sigma} \right) = (C_t - h \bullet C_{t-1})^{-\sigma} \quad (4.38)$$

$$\Rightarrow \beta (1 - \delta + r_{t+1}^c) E_t \left( \frac{C_t - h \bullet C_{t-1}}{C_{t+1} - h \bullet C_t} \right)^\sigma = 1 \quad (4.39)$$

The FOC with respect to  $N_t$  (labour) is:

$$-N_t^\varphi - \lambda_t \left( -\frac{W_t}{P_t} \right) = 0 \Rightarrow N_t^\varphi = \lambda_t \left( \frac{W_t}{P_t} \right), \quad (4.40)$$

We replace  $\lambda_t = (C_t - h \bullet C_{t-1})^{-\sigma}$  into Eq. (40) to obtain:

$$N_t^\varphi = [(C_t - h \bullet C_{t-1})]^{-\sigma} \left( \frac{W_t}{P_t} \right) \quad (4.41)$$

Moreover, the aggregate wage equation is:

$$W_t = \left[ (1-\xi)(W^*)^{1-l} + \xi(W_{t-1})^{1-l} \right]^{1/(1-l)}, \quad (4.42)$$

where  $W_t^*$  denotes the wage that one can replace in the current time period  $t$ .

In the equation,  $\xi$  denotes the fraction of households that set their wages according to last-period wage inflation or to a combination of the last-period price and the wage inflation rate. The remaining  $(1-\xi)$  of households adjust their wage optimally;  $\pi_t$  represents the inflation rate, i.e.  $P_t = (1+\pi_t)P_{t-1}$ .

Combining the Cobb-Douglas production function  $Y_t(i) = Z_t K_t(i)^{1-\alpha} N_t(i)^\alpha$  and the Lagrangian function in Eq. (19), we have the FOC with respect to  $N_t(i)$  and  $K_t(i)$  as follows:

$$\frac{W_t}{P_t} = \left( \frac{\alpha r_t^c}{1-\alpha} \right) \left( \frac{K_t(i)}{N_t(i)} \right) \quad (4.43)$$

Eq. (4.43) implies that households' wages depend on the ratio of rental capital to labour demand. Further, this formula could be rewritten as:

$$\frac{W_t}{P_t} = \alpha \left( \frac{W_t}{\alpha P_t} \right)^\alpha \left( \frac{r_t^c}{1-\alpha} \right)^{1-\alpha} \left( \frac{K_t(i)}{N_t(i)} \right)^{1-\alpha} \quad (4.44)$$

Now, one can reason out the function of real marginal costs ( $MC_t$ ) as a result of firm  $i$ 's optimization problem:

$$MC_t = \frac{1}{Z_t} \left( \frac{1}{\alpha} \right)^\alpha \left( \frac{W_t}{P_t} \right)^\alpha \left( \frac{r_t^c}{1-\alpha} \right)^{1-\alpha} \quad (4.45)$$

Note that marginal costs are equal for all firms, and index  $i$  can therefore be dropped. Then, Eq. (4.44) can be expressed as:

$$\frac{W_t}{P_t} = \alpha Z_t MC_t \left( \frac{K_t(i)}{N_t(i)} \right)^{1-\alpha} \quad (4.46)$$

Now, we can summarize the equations discussed above to obtain the steady states of the relevant variables. These are the the TFP growth rate function and the growth rate function with respect to the exchange rate:

$$R_t = \phi_R R_{t-1} + \phi_\pi \left( E_t (\pi_{t+1}) - \pi_t \right) + \phi_Y \hat{Y}_t + \varepsilon_t^R$$

$$\hat{Y}_t = \psi_Y \hat{Y}_{t-1} + \psi_{tot} TOT_t + \psi_{open} OPN_t + \psi_i I_t + \psi_{ijp} \hat{Z}_t + \psi_\pi \pi_t + \psi_{rer} \hat{RER}_t + \varepsilon_t^{RER}$$

### 4.3.2. Solving for the steady states

At the steady state, all variables are constant. The superscript “ $\bar{\cdot}$ ” signifies the steady state of the variables. In our model, we first calculate the steady state of the Euler equation, Eq. (4.35), to obtain:

$$\beta E \left[ \frac{\bar{P}\bar{R}}{\bar{P}} \left( \frac{\bar{C}-h\bar{C}}{\bar{C}-h\bar{C}} \right)^{-\sigma} \right] = 1 \Rightarrow \beta \bar{R} = 1 \Rightarrow \bar{R} = \frac{1}{\beta} \quad (4.47)$$

The steady state of the capital stock equation, Eq. (4.39), is:

$$\beta E_t \left( \frac{\bar{C} - h\bar{C}}{\bar{C} - h\bar{C}} \right)^\sigma \left( 1 - \delta + r^c \right) = 1 \Rightarrow \beta \left( 1 - \delta + r^c \right) = 1 \Rightarrow r^c = \frac{1}{\beta} + \delta - 1 \quad (4.48)$$

The steady state of the labour-leisure decision equation, Eq. (4.41), is:

$$\bar{N}^\sigma = \left[ (\bar{C} - h\bar{C}) \right]^{-\sigma} \left( \frac{\bar{W}}{\bar{P}} \right) \quad (4.49)$$

The steady state of the Cobb-Douglas production function is:

$$\bar{Y} = \bar{Z} \left( \bar{K} \right)^{1-\alpha} \left( \bar{N} \right)^\alpha \quad (4.50)$$

The steady states of the capital accumulation and the aggregate wage are:

$$\bar{K} = (1 - \delta)\bar{K} + \bar{I} \Rightarrow \delta\bar{K} = \bar{I} \quad (4.51)$$

$$\bar{W} = \left[ (1 - \mu) \left( \bar{W} \right)^{1-l} + \mu \left( \bar{W} \right)^{1-l} \right]^{1/(1-l)} = \left[ \left( \bar{W} \right)^{1-l} \right]^{1/(1-l)} = \bar{W} \quad (4.52)$$

The steady states of the inflation rate and the aggregate price are:

$$\Pi \equiv \frac{\bar{P}}{\bar{P}} = 1 \Rightarrow 1 + \bar{\pi} = 1 \Rightarrow \bar{\pi} = 0 \quad (4.53)$$

$$\bar{P} = \left[ \theta \left( 1 + \bar{\pi} \right) \left( \bar{P} \right)^{1-\theta} + (1 - \theta) \left( \bar{P} \right)^{1-\theta} \right]^{\frac{1}{1-\theta}} = \left[ \left( \bar{P} \right)^{1-\theta} \right]^{\frac{1}{1-\theta}} = \bar{P} \quad (4.54)$$

The steady state of the monetary policy reaction function is:

$$\bar{M} = \theta_1 \bar{M} + \theta_2 \bar{Y} + \theta_3 \bar{\pi} \quad \Rightarrow \quad \bar{M} = \frac{\theta_2}{(1-\theta_1)} \bar{Y} + \frac{\theta_3}{(1-\theta_1)} \bar{\pi} \quad (4.55)$$

The steady state of the function of net foreign assets and the CA is:

$$\bar{NFA} = \bar{NFA} + \bar{CA} + \bar{KA} \quad \Rightarrow \quad \bar{CA} + \bar{KA} = 0 \quad (4.56)$$

In equilibrium, the sum of the CA and the capital account is equal to zero.

Furthermore, we have:

$$\bar{CA} = \bar{S} - \bar{I} \quad (4.57)$$

The CA position in the equilibrium equals the difference between savings and investment. Finally, the steady states of real marginal costs ( $\bar{MC}_i$ ), ( $\bar{W}_i/P_i$ ) and the long-term exchange rate are, respectively:

$$\bar{MC} = \frac{1}{\bar{Z}} \left( \frac{1}{\alpha} \right)^\alpha \left( \frac{\bar{W}}{\bar{P}} \right)^\alpha \left( \frac{\bar{r}^c}{1-\alpha} \right)^{1-\alpha} \quad (4.58)$$

$$\frac{\bar{W}}{\bar{P}} = \left( \frac{\alpha \bar{r}^c}{1-\alpha} \right) \left( \frac{\bar{K}}{\bar{N}} \right) \quad (4.59)$$

$$\bar{RER} = \psi_N \left( \bar{NFA} / \bar{Y} \right) \quad (4.60)$$

### 4.3.3. Linearized equations

To facilitate interpretation and the solution to the model, we replace the dynamic nonlinear equations with dynamic linear equations. Following Uhlig's method (1999), a variable, for example  $x_t$ , can be represented as:

$$x_t = \bar{x} e^{\hat{x}_t} \quad (4.61)$$

The symbol “^” above a variable denotes the log-deviation of the variable from its equilibrium value.

$$\text{For } x \approx 0, e^x \approx 1+x. \Rightarrow e^{\hat{x}_t} \approx 1+\hat{x}_t \quad (4.62)$$

Substituting Eq. (4.62) into Eq. (4.61) yields:

$$x_t = \bar{x} e^{\hat{x}_t} \approx \bar{x}(1+\hat{x}_t) \quad (4.63)$$

Thus,  $\hat{x}_t \times 100\%$  is (approximately) the percentage deviation of  $x_t$  from its steady state  $\bar{x}$ .

According to Uhlig's (1999) method, the linearized functions of the equilibrium equations in previous sections can be expressed as follows. First, we have the linear version of Eq. (4.35):

$$\hat{C}_t = \frac{\hat{C}_{t+1} + h\hat{C}_{t-1}}{1+h} + \left( \frac{h-1}{(1+h)\sigma} \right) \left( \hat{R}_t + \hat{P}_t - \hat{P}_{t+1} \right) \quad (4.64)$$

Eq. (4.64) is the linearized Euler equation of consumption  $\hat{C}_t$ . It implies that current consumption is determined by prior consumption, expected future consumption, the inflation rate and the gross returns on bonds.  $h$  represents the habit factor. The linearized equation of labour supply  $N_t$  is:

$$\hat{N}_t = \left( \frac{-\sigma}{\varphi} \right) \left( \frac{\hat{C}_t - h\hat{C}_{t-1}}{1-h} \right) + \frac{1}{\varphi} (\hat{W}_t - \hat{P}_t) \quad (4.65)$$

Linearizing the Cobb-Douglas production function of Eq. (4.5), we have the following:

$$\hat{Y}_t = \hat{Z}_t + (1-\alpha)\hat{K}_t + \alpha\hat{N}_t \quad (4.66)$$

Combining the intermediate goods', firm's and household's FOCs, real marginal costs can be linearized as:

$$\hat{MC}_t = \alpha \left( \hat{W}_t - \hat{P}_t \right) + (1-\alpha)r^c - \hat{Z}_t \quad (4.67)$$

According to Eq. (4.67), linearized real marginal costs depend on the deviation of wages, the rental rate of capital and technology shocks in equilibrium. According to the FOC of  $r_t^c$ , the rental rate of capital can be linearized as:

$$\hat{r}_{t+1}^c = \frac{\sigma}{(h-1)(1+\beta\delta-\beta)} \left( (1+h)\hat{C}_t - h\hat{C}_{t-1} - \hat{C}_{t+1} \right) \quad (4.68)$$

From this equation, we can surmise that the rental capital return is derived from the current, prior and expected levels of consumption. To obtain optimal labour demand in equilibrium, we linearize Eq. (4.44) and obtain:

$$\hat{W}_t - \hat{P}_t = \hat{r}_t^c + \hat{K}_t - \hat{N}_t \quad \Rightarrow \quad \hat{N}_t = \hat{r}_t^c + \hat{K}_t - \left( \hat{W}_t - \hat{P}_t \right) \quad (4.69)$$

The equilibrium indicates that optimal labour demand stems from wages, capital accumulation and the rental rate of capital. Underlying this equilibrium is the assumption that firms finance their wage bills and determine their demand for labour on the basis of their capital and rental capital incomes.

Let real wages be  $w_t = \frac{W_t}{P_t}$ ; the linearized real wage is  $\hat{w}_t = \hat{W}_t - \hat{P}_t$ . The relation

between the labour supply of firm  $i$  and the aggregate labour supply  $N_t$  is:

$$N_t(i) = \left\{ \frac{W_t^*(i)v_t}{W_t} \right\}^{-l} N_t, \quad (4.70)$$

where  $l$  is the elasticity of substitution among differentiated labour services.

When  $t=0$ ,  $v_t=1$ , and when  $t=0$ ,  $v_t = (1+\pi_1)(1+\pi_2)\dots(1+\pi_t)$ , for  $t \geq 1$ .

Based on the aggregate wage Eq. (4.42) and the FOC of optimal wages,  $W_t^*$ , the linearized equation of real wages is as follows (see Zhang, 2009):

$$\Omega \cdot \hat{w}_t = \left( \begin{array}{l} (1+\phi l)\xi(\hat{w}_{t-1} + \pi_{t-1}) - (1+\beta)(1+\phi l)\xi\pi_t \\ + (1+\phi l)\xi\beta E_t(\hat{w}_{t+1} + \pi_{t+1}) + (1-\beta\xi)(1-\xi) \left[ \phi \hat{N}_t + \frac{\sigma}{1-h}(\hat{C}_t - \hat{C}_{t-1}) \right] \end{array} \right) \quad (4.71)$$

with,  $\Omega = (1+\beta)(1+\phi l)\xi + (1-\beta\xi)(1-\xi)$ .

The FOC of labour supply  $N_t$  in Eq. (4.54) can be rewritten as:

$$\phi \hat{N}_t + \frac{\sigma}{1-h}(\hat{C}_t - \hat{C}_{t-1}) = \hat{W}_t - \hat{P}_t = \hat{w}_t \quad (4.72)$$

Substituting Eq. (4.72) into Eq. (4.71), we have:

$$\begin{aligned} (1+\beta)(1+\phi l)\xi \cdot \hat{w}_t &= (1+\phi l)\xi(\hat{w}_{t-1} + \pi_{t-1}) - (1+\beta)(1+\phi l)\xi\pi_t + (1+\phi l)\xi\beta E_t(\hat{w}_{t+1} + \pi_{t+1}) \\ \Rightarrow \hat{w}_t &= \frac{1}{1+\beta}(\hat{w}_{t-1} + \pi_{t-1}) - \pi_t + \frac{\beta}{1+\beta} E_t(\hat{w}_{t+1} + \pi_{t+1}) \end{aligned} \quad (4.73)$$

This condition indicates that the current nominal wage is a function of inflation, effective consumption, labour and prior and expected wages.

Linearizing the inflation equation  $P_t = (1+\pi_t)P_{t-1}$ , we have:

$$\hat{P}_t = \hat{\pi}_t + \hat{P}_{t-1} \Rightarrow \hat{\pi}_t = \hat{P}_t - \hat{P}_{t-1} \quad (4.74)$$

This expression indicates that current inflation is the difference between the current and prior levels of the price index. Substituting  $P_{t-1} = \frac{P_t}{1+\pi_t}$  into the aggregate price index, Eq. (4.7), we have:

$$P_t = \left[ \mu \left( \frac{1+\pi_{t-1}}{1+\pi_t} \right) P_t^{1-\theta} + (1-\mu)(P_t^*)^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

$$\Rightarrow P_t^{1-\theta} = \mu \left( \frac{1+\pi_{t-1}}{1+\pi_t} \right) P_t^{1-\theta} + (1-\mu)(P_t^*)^{1-\theta} \quad (4.75)$$

Eq. (4.75) linearized is:

$$\hat{P}_t = \mu \left( \hat{\pi}_{t-1} - \hat{\pi}_t + \hat{P}_t \right) + (1-\mu)\hat{P}_t^* \Rightarrow \hat{P}_t = \frac{\mu}{1-\mu} \left( \hat{\pi}_{t-1} - \hat{\pi}_t \right) + \hat{P}_t^* \quad (4.76)$$

Combining the linearized aggregate price index, Eq. (4.76), and using the optimal price, Eq. (4.8), the New Keynesian Phillips curve is (Christiano et al., 2005, and Zhang, 2009):

$$\pi_t = \frac{\mu}{1+\beta\mu^2} \pi_{t-1} + \frac{2-(1+\beta)\mu+\beta\mu^2}{1+\beta\mu^2} E_t \pi_{t+1} + (1-\theta)(1-\beta\mu) \hat{MC}_t \quad (4.77)$$

From Eq. (4.60), we can infer that the inflation rate is a function of the deviation of marginal costs from the steady state and from prior and expected levels of inflation. In Eq. (4.77),  $\beta$  is the discount rate, and  $\theta$  denotes the fraction of firms that will adjust their prices according to the last-period price

inflation and marginal costs. In equilibrium, the goods market clearing condition requires that  $Y_t = C_t + I_t$ . Then, the steady state of this condition is:

$$\bar{Y} = \bar{C} + \bar{I} \quad (4.78)$$

Linearizing the equation for the goods market clearing condition yields

$$\hat{Y}_t = \left( \bar{C}/\bar{Y} \right) \hat{C}_t + \left( \bar{I}/\bar{Y} \right) \hat{I}_t, \quad (4.79)$$

where  $\left( \bar{I}/\bar{Y} \right)$  is the steady state of investment to GDP ratio, and

$\left( \bar{C}/\bar{Y} \right) = 1 - \left( \bar{I}/\bar{Y} \right)$  based on Eq. (4.78). This function indicates that total output

in the economy also stems from households' consumption and investment.

The linearized equation of the capital accumulation function is:

$$\bar{K} \hat{K}_t = (1 - \delta) \bar{K} \hat{K}_{t-1} + \bar{I} I_t \Rightarrow \hat{K}_t = (1 - \delta) \hat{K}_{t-1} + \left( \bar{I}/\bar{K} \right) I_t \quad (4.80)$$

Replacing  $\bar{I} = \delta \bar{K}$  into Eq. (4.80), we have the linearized capital accumulation function below:

$$\hat{K}_t = (1 - \delta) \hat{K}_{t-1} + \delta I_t \quad (4.81)$$

In linearized Eq. (4.81), the recent capital amount depends on its prior level and investment. The Taylor rule for setting the nominal interest rate in the log-deviation form is written as:

$$\hat{R}_t = \phi_R \hat{R}_{t-1} + \phi_\pi \hat{\pi}_{t+1} + \phi_Y \hat{Y}_t \quad (4.82)$$

Finally, the linear net foreign assets function is:

$$\hat{NFA}_t = \hat{NFA}_{t-1} + \left( \bar{S} / \bar{NFA} \right) \hat{S}_t - \left( \bar{I} / \bar{NFA} \right) \hat{I}_t \quad (4.83)$$

$\left( \bar{S} / \bar{NFA} \right)$  and  $\left( \bar{I} / \bar{NFA} \right)$  denote the savings to foreign reserves ratio and the import to foreign reserves ratio, respectively, in equilibrium.

## 4.4 Parameter Analysis

### 4.4.1 Description of data

The model is estimated using variables of nominal interest rates, real interest rates, the inflation rate, exports, imports, GDP growth and net foreign assets. We expand the TFP function in the DSGE framework on the basis of the approach of Mbaye (2013) and Jiang (2014). The expanded TFP growth function is presented as Eq. (4.9):

$$\hat{Z}_t = \tau_1 \hat{Z}_{t-1} + \tau_2 \left( RER_t - \bar{RER}_t \right) + \tau_3 \hat{I}_t + \tau_4 \hat{OPN}_t + \varepsilon_t^Z \quad (4.84)$$

For model calibration, the parameter of the lagged TFP growth rate is set to be  $\tau_1=0.163$ , and it describes the impact of the autoregressive parameter on current TFP growth. In addition, exchange rate undervaluation parameter  $\tau_2=0.0268$ , investment parameter  $\tau_3=0.0787$ , and trade openness index parameter  $\tau_4=0.374$ . According to Mbaye (2013), the baseline dataset comprises annual data for 38 developing countries and developed economies observed over the 1970–2008 period, and these parameters are calculated from their TFP growth equations.

We rewrite the output growth function to describe how exchange rate undervaluation and the TFP growth channel affect GDP growth in China. Eq. (4.12) represents the relationship between economic growth and the TFP growth channel. To directly explore the relationship between GDP growth and currency undervaluation, we rewrite the equation as:

$$\hat{Y}_t = \psi_Y \hat{Y}_{t-1} + \psi_{tot} TOT_t + \psi_{open} OPN_t + \psi_i I_t + \psi_{rer} \hat{RER}_t + \psi_\pi \pi_t \quad (4.85)$$

We set the parameter of the lagged economic growth rate as  $\psi_Y=0.173$ , terms of trade parameter  $\psi_{tot}=1.178$ , trade openness parameter  $\psi_{open}=1.19$ , investment parameter  $\psi_i=-1.257$ , inflation parameter  $\psi_\pi=-2.076$ , and exchange rate intervention parameter  $\psi_{rer}=3.217$  according to the TFP growth model in Mbaye's (2013) empirical work.

Following Walsh (2003) and He et al. (2007), we set  $\sigma=2$ ,  $\alpha=0.4$  and  $\delta=0.04$ . If  $\alpha=0.4$ , the elasticity of the substitution between intermediate goods and final goods can be obtained at  $\mu=4.61$ . Following Zhang (2009), we set the habit parameter of Chinese households to be  $h=0.61$ . The discount rate is  $\beta=0.98$ .

In the steady state, the nominal interest rate  $\bar{R}=1/\beta=1/0.98=1.0204$ . The GMM estimation with data for the 1995–2005 period shows that the fraction of firms that adjust their prices according to the last-period prices is  $\theta=0.84$ . On the basis of Liu (2008), we obtain the elasticity of labour supply  $\varphi=6.16$ , and the fraction of households who set their wages depending on wage inflation to be  $\xi=0.6$ . The elasticity of the real money balance,  $\gamma$ , is considered to be 3.13.

We follow Mbaye's (2013) hypothesis to estimate the long-run real exchange rate equation and compute the deviation of the market's real effective rate from its equilibrium level. The estimated real exchange rate function can then be described as:

$$\bar{R}ER_t \hat{RER}_t = 0.27\hat{TOT}_t - 0.04(\hat{EX}_t - \hat{IM}_t) + 0.12\left(\frac{\bar{NFA}}{\bar{Y}}\right)(\hat{NFA}_t - \hat{Y}_t) + 0.06\hat{Y}_t \quad (4.86)$$

The real exchange rate  $\hat{RER}_t$  is related to positive developments in terms of trade,  $\hat{TOT}_t$ , changes in the trade balance,  $\hat{EX}_t - \hat{IM}_t$ , improvements in the country's net foreign asset position, and the domestic productivity growth

rate,  $\hat{Y}_t$ . The steady state of the real exchange rate,  $\bar{RER}$ , equals to the nominal exchange rate times the domestic price level divided by the foreign price level. We calculate the average value of the RMB's nominal exchange rate against the US dollar from 2005 to 2012 as the steady state of the nominal exchange rate. The purchasing power parity (PPP) conversion factor is a technique used to determine the relative value of different currencies. It represents the number of units of a country's currency required to buy the same amount of goods and services in the domestic market that a US dollar would buy in the United States. We apply the PPP conversion factor from 2010 to 2014 to calculate the steady state of the real exchange rate, which is.

$$\bar{RER}=7.137*3.52=25.12$$

According to Jiang's (2014) export and import equations, the relation of exports and imports to the exchange rate can be written as:

$$\log EX_t = -6.347412 + 2.189590 \log RER_t \quad (4.87)$$

$$\log IM_t = -5.374627 + 2.026217 \log RER_t \quad (4.88)$$

We substitute  $EX_t = \bar{EX} \cdot e^{\hat{EX}_t}$ ,  $IM_t = \bar{IM} \cdot e^{\hat{IM}_t}$ ,  $RER_t = \bar{RER} \cdot e^{\hat{RER}_t}$  into Eqs. (4.87) and (4.88) and obtain:

$$\log EX_t = \log \left( \bar{EX} \cdot e^{\hat{EX}_t} \right) = \log \left( \bar{EX} \right) + \hat{EX}_t = -6.35 + 2.19 \left( \log \left( \bar{RER} \right) + \hat{RER}_t \right) \quad (4.89)$$

$$\log IM_t = \log \left( \bar{IM} \right) + \hat{IM}_t = -5.37 + 2.03 \left( \log \left( \bar{RER} \right) + \hat{RER}_t \right) \quad (4.90)$$

From Eqs. (4.87) and (4.88), we can obtain:

$$\log \bar{EX} = -6.35 + 2.19 \log \bar{RER} \quad (4.91)$$

$$\log \bar{IM} = -5.37 + 2.03 \log \bar{RER} \quad (4.92)$$

We substitute Eqs. (4.91) and (4.92) into Eqs. (4.89) and (4.90) to obtain:

$$\hat{EX}_t = 2.19 \left( \hat{RER}_t \right) \quad (4.93)$$

$$\hat{IM}_t = 2.03 \left( \hat{RER}_t \right) \quad (4.94)$$

Using data from International Monetary Fund on China's imports and exports, we calculate the average ratios of China's exports and imports to GDP respectively from 2005 to 2012. We consider these average values to be the steady states in the model and obtain  $\bar{EX}/\bar{Y} = 0.319$  and  $\bar{IM}/\bar{Y} = 0.27$ .

Placing these values into the linearized trade openness function, Eq. (4.17), would enable us to rewrite the Eq. (4.17) as:

$$\hat{OPN}_t = 0.5416 \cdot \hat{EX}_t + 0.4584 \cdot \hat{IM}_t - \hat{Y}_t \quad (4.95)$$

To determine the steady state value in Eq. (4.83), we rely on the data for China from 2005 to 2012 to calculate the ratios of domestic savings to foreign

reserves and of investment to foreign reserves. We set the average values of savings, investment and reserves during this time period as the steady state values. Eq. (4.83) thus can be rewritten as:

$$\hat{NFA}_t = \hat{NFA}_{t-1} + 1.188\hat{S}_t - 1.051\hat{I}_t \quad (4.96)$$

#### 4.4.2. Bayesian estimation of the parameters

Bayesian estimation and evaluation techniques are now standard tools for the analysis of DSGE models. Estimation with Bayesian methods is based on the likelihood generated by the DSGE system. In contrast to GMM estimation, which is based on particular equilibrium relations, Bayesian estimation fits the complete and solved DSGE model. It is also a bridge between calibration and maximum likelihood. By using Bayes' theorem, the prior density with the likelihood function can be combined to obtain the posterior density. In this case, the posterior distribution avoids peaking at the same points as the likelihood. Moreover, the distribution of priors is also helpful for identifying structural parameters and shocks.

We follow Schorfheide (2000) and apply a two-step estimation procedure involving calibration and Bayesian Maximum Likelihood methods. First, the posterior mode and an approximate covariance matrix are obtained by numerical optimization on the log posterior density. Second, we apply the Metropolis-Hastings and Markov-Chain Monte Carlo (MCMC) algorithm

with 2,000 draws to obtain a sequence from the unknown posterior distribution. The Dynare processor for Matlab is employed for the computation.

### **Prior and posterior distributions of estimated parameter**

The parameters governing the dynamics of the model are estimated. Most pertain to the nominal frictions and elasticity in the model, while the remainder denote the exogenous shock processes. We also estimate the steady state of real exchange rate  $\bar{RER}$  to explore the fitness of the real exchange rate function in the model. The parameter for *snfa* characterizes the impact of foreign reserves in the estimated real exchange rate function. We present posterior estimates of the main parameters in Table 4.1 below.

**Table 4. 1. Prior and Posterior Distributions of Structural Parameter**

Parameters	Type	Mean	St.Dev	Mode	St.Dev	Conf. Int	Mean
$\alpha$	Beta	0.040	0.002	0.3989	0.0224	[0.3633, 0.4275]	0.3976
$\beta$	Beta	0.980	0.0002	0.9792	0.0020	[0.9764, 0.9826]	0.9793
$\delta$	Beta	0.040	0.003	0.0467	0.0035	[0.0434, 0.0539]	0.0481
$\sigma$	Gamma	2.000	0.020	2.0000	0.0196	[1.9619, 2.0339]	1.9967
$\gamma$	Gamma	3.100	0.003	3.1000	0.0006	[3.0959, 3.1041]	3.1000
$\varphi$	Gamma	6.100	0.003	6.1000	0.0007	[6.0190, 6.1800]	6.1090
$\kappa$	Normal	0.500	0.050	0.5000	0.0491	[0.4277, 0.5748]	0.5017
$\mu$	Gamma	0.840	0.003	0.8412	0.0027	[0.8365, 0.8456]	0.8414
$\theta$	Gamma	4.610	0.003	4.6100	0.0033	[4.6047, 4.6151]	4.6101
$l$	Gamma	2.000	0.003	1.9986	0.0027	[1.9939, 2.0045]	1.9986
$\xi$	Gamma	0.600	0.003	0.5851	0.0003	[0.5918, 0.5992]	0.5953
$e$	Inv_gamma	0.010	0.0003	2.5882	infin	[2.2836, 2.8893]	2.5882
$\bar{RER}$	Gamma	25.12	0.02	25.12	0.0187	[25.091, 25.160]	25.123
$snfa$	Beta	0.052	0.002	0.060	0.0018	[0.0577, 0.0628]	0.0601

We use the beta distribution for parameters bounded between 0 and 1, except for  $\kappa$  and  $h$ . For parameters measuring elasticity, such as  $\gamma$ ,  $\sigma$ ,  $\varphi$ ,  $\theta$  and  $\xi$ , we use the gamma distribution. The normal distribution is applied when estimating  $\kappa$  and  $\alpha$ . The value for the priors and posteriors of the elasticity parameters are very close to one another, suggesting that the value of the elasticity parameters is stable in the framework and well identified. The

elasticity of substitution with respect to the depreciation rate is higher than its priors, suggesting a slower response from depreciation to the shocks.

The mean of the posterior of the parameter on foreign reserves,  $snfa$ , is estimated to be higher than its prior value. Thus, foreign reserves may play a more important role in exchange rate policy than commonly perceived. Turning to the exogenous shock variables, the posterior mean of the technology shock parameter  $\tau$  is higher than its prior, with a coefficient of 2.5882, pointing to a much higher persistence than we expected.

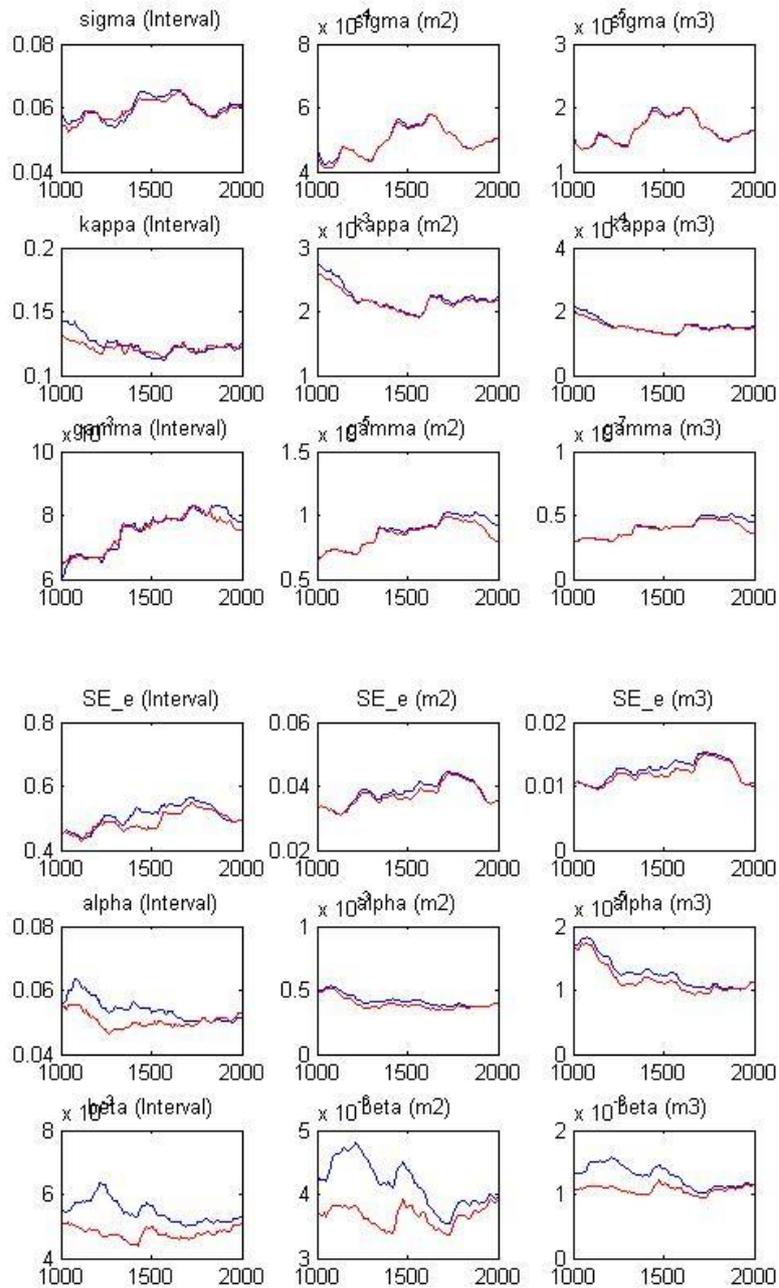
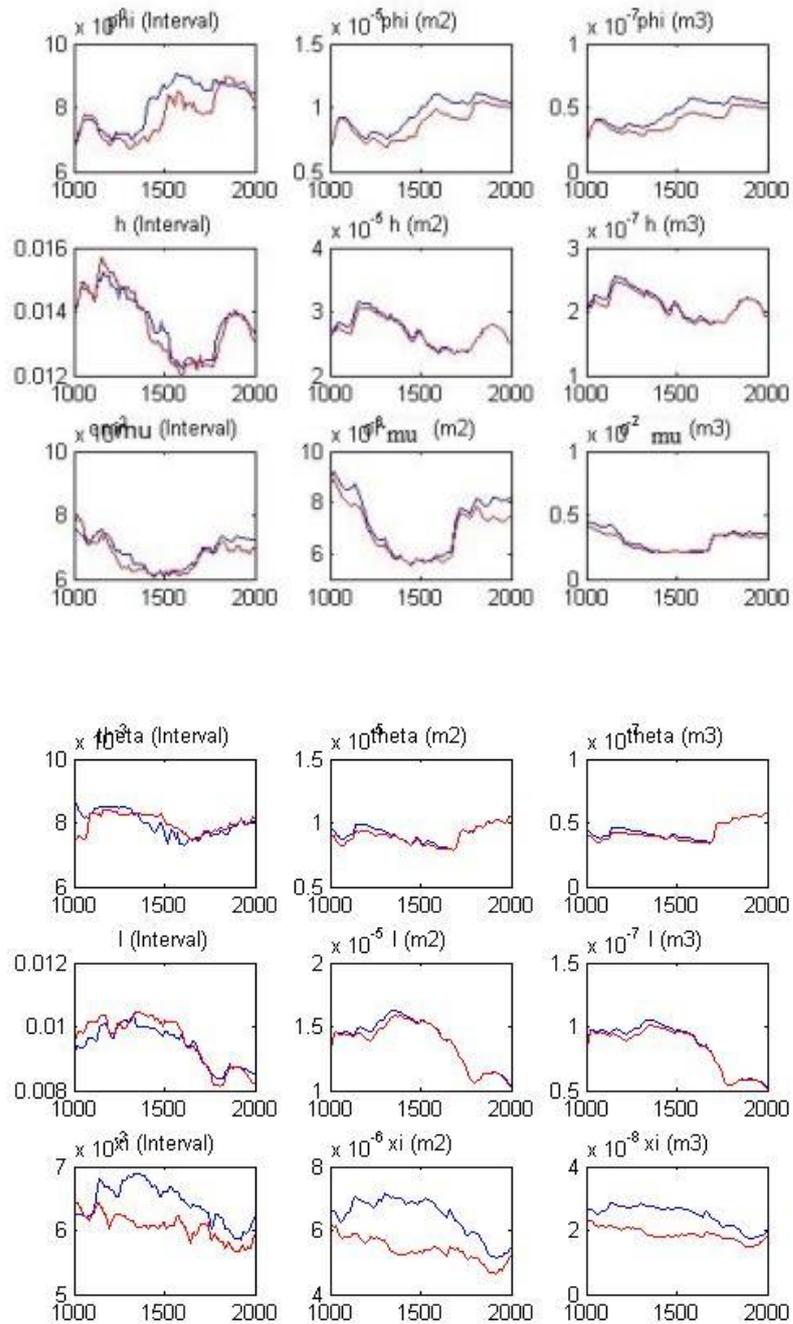


Figure 4.1. 1 Markov-Chain Monte Carlo Univariate Diagnostics for Parameters (1)

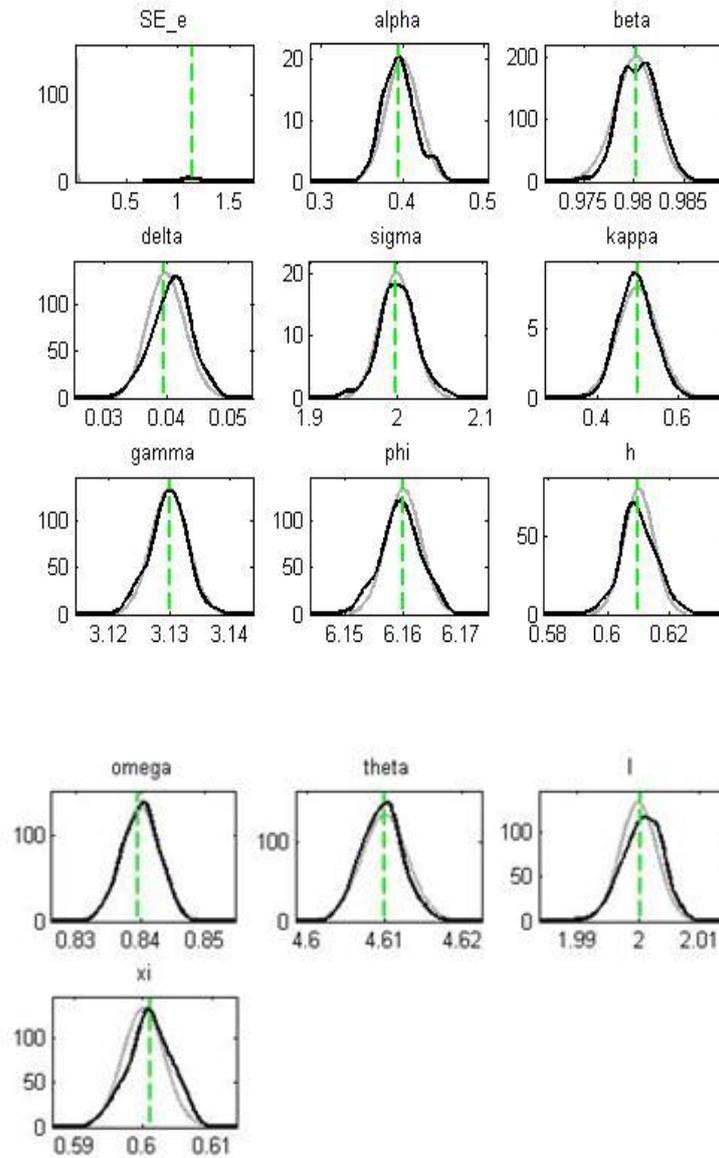


**Figure 4.1. 2 Markov-Chain Monte Carlo Univariate Diagnostics for Parameters (2)**

Figure 4.1.1 and 4.1.2 graphs the Markov-Chain Monte Carlo (MCMC) univariate diagnostics for the parameters. This is the main feedback to increase confidence with the results. To diagnose the degree to which the results are reasonable and accurate, we must focus on two key points. First,

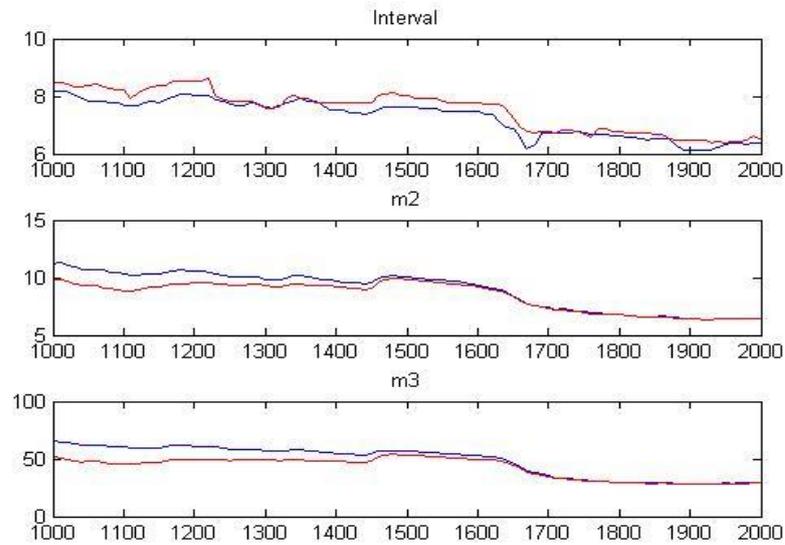
we should note whether the results within any iterations of the Metropolis-Hastings simulations, no matter how many there may be, are similar. Second, we should observe the distance between various chains. More specifically, the two lines on the charts represent measures of parameter vectors within and between chains, respectively. From Figure 1, we note that for most parameters, the distance between the two chains is very close. Moreover, the plotted moments of structural parameters  $\alpha$ ,  $\beta$ ,  $\delta$  and  $\sigma$  are relatively stable and converge, which suggests that identification of the priors is reasonable and the results from the chains are sensible.

Figure 4.2 displays prior and posterior distributions of the parameters and shocks. Overall, all parameters seem to be well identified, as the outcome shows that the posterior distributions are either not centred on the prior or are centred but have a smaller dispersion, indicating high levels of significance. With respect to price stickiness, 84.14 % of the firms do not adjust prices within one year, which implies that prices are re-optimized once every three quarters. The fraction of households setting their wages according to the last-period wage  $\xi$  is estimated to be 59.53%, implying a large share of backward-looking firms.



**Figure 4.2 Prior and Posterior Distributions in the Metropolis-Hastings Procedure**

Multivariate diagnostics present an aggregate measure based on the eigenvalues of the variance-covariance matrix of individual parameters. The analysis method resembles that of the MCMC univariate diagnostics because the results of these two diagnostics are based on the same theory. The plotted moments of multivariate diagnostics are relatively constant and converge implying that the results are reasonable (Figure 4.3).

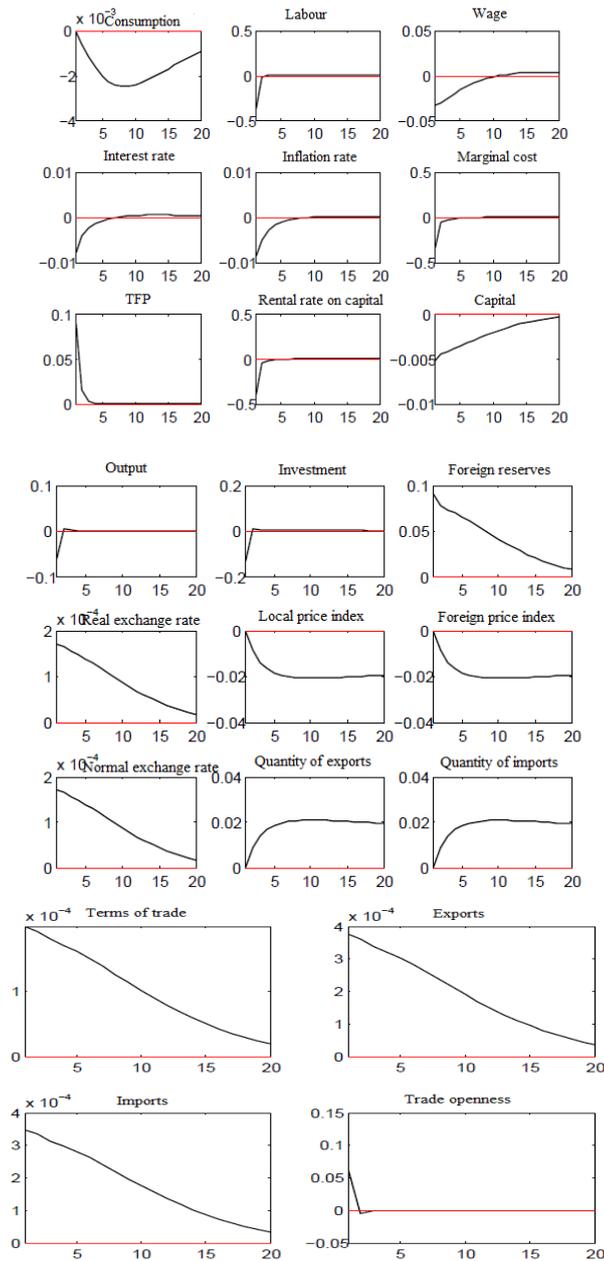


**Figure 4. 3 Multivariate Diagnostics for Parameters**

#### **4.4.3. Impulse response analysis**

Next, we analyse the impulse response functions of the system with a particular emphasis on the effects of RMB undervaluation on growth and on the position of foreign reserves. Our results indicate that the exchange rate undervaluation positively affects the country’s economic growth in three ways: through TFP growth, trade surpluses and wage increases. The dynamic systems are the linearized models derived in the previous sections.

First, we present impulse responses from every variable to TFP growth shocks. Then, we test the impulse response functions in relation to currency undervaluation. Figure 1 below depicts a graphic presentation of responses from the system variables to TFP growth.

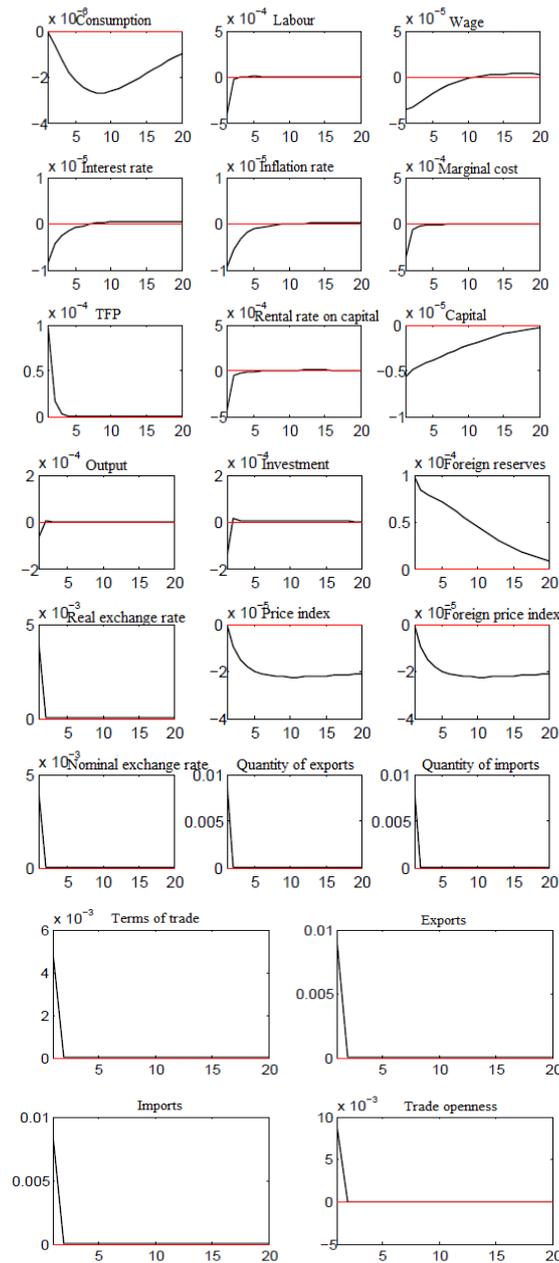


**Figure 4.4** Estimated Model Responses to TFP growth

In Figure 4.4, each row represents responses of the main variables to relevant shocks. We begin by considering the effects of a positive TFP growth shock on the national economy and its external trade. In response to this shock, labour, wages, the inflation rate and the interest rate decrease in the first several time periods but increase over the long run. However, the effects of TFP growth on wages and inflation are positive and last over the long run.

The gross output of the economy has a positive relation with the total productivity shock over the short run. In the long run, the effect of TFP growth on output growth is not obvious, because the result shows that a sudden increase in TFP might improve gross output, but only temporarily. For foreign reserves, the growth of TFP is shown to affect growth of the reserves. While impact effect of TFP growth on foreign reserves is significantly positive, this effect gradually disappears. The positive relation between productivity growth and the increase in China's foreign reserves provides an explanation for China's situation: along with economic growth, the country is able to steadily accumulate external assets.

Figure 4.5 below shows the impulse responses of China's foreign trade to an undervalued RMB exchange rate. The quantity of exports and imports has a long-term positive relation with productivity growth. As the country's economy prospers, both the demand for its exports and the supply of its local products grow. However, increasing TFP growth also boosts the value of exports and imports, although this effect fades over the long run. The impact of TFP growth on exports, imports and terms of trade becomes gradually smaller as time goes on.



**Figure 4. 5 Responses an Undervalued Exchange Rate**

The effects of exchange rate undervaluation to wages, consumption and inflation are similar to that of TFP growth. Currency undervaluation not only causes the depreciation of wages and inflation in the short run but also leads to the appreciation of wages and inflation in the long run. To maintain an undervalued exchange rate, China's central bank must sell RMB to offset upward pressure on the currency. Then, the amount of China's currency in

domestic circulation increases. As a result, inflationary pressures rise, which lead, in turn, to higher living costs for domestic households. Therefore, Chinese enterprises face pressure to increase wages and raise the prices of their products.

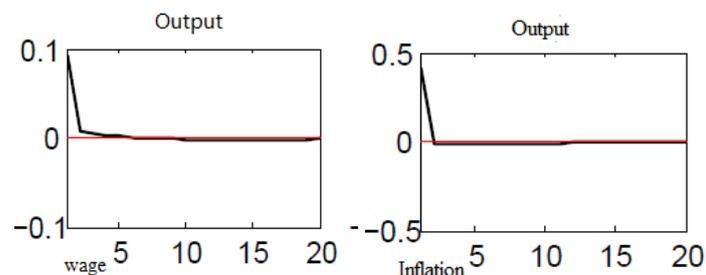
China's exchange rate policy affects the country's holdings of international reserves mainly through three channels: the trade balance (imports and exports), central bank intervention in the exchange market, and foreign direct investment. The response of the central bank is to decrease the short-term interest rate, maintain a stable nominal interest rate in the long run and increase the RMB exchange rate temporarily. The central bank will also purchase dollars and pay out RMB on the foreign exchange market to maintain China's currency at an artificially low value.

The DSGE model result shows that undervaluation shocks lead to a positive response from GDP growth in the first several time periods. This phenomenon suggests that China's exchange rate policy has some effect in promoting the country's growth. It is plausible that an undervalued exchange rate makes China's domestic products cheaper and inward foreign capital investment more productive, thus promoting China's trade and foreign investment. According to the impulse response functions, both exports and imports have positive responses to the exchange rate intervention shock. ( based on the linearized equation of Eq.(4.85) The undervalued domestic currency, which leads to an increase in the RMB exchange rate, generates an

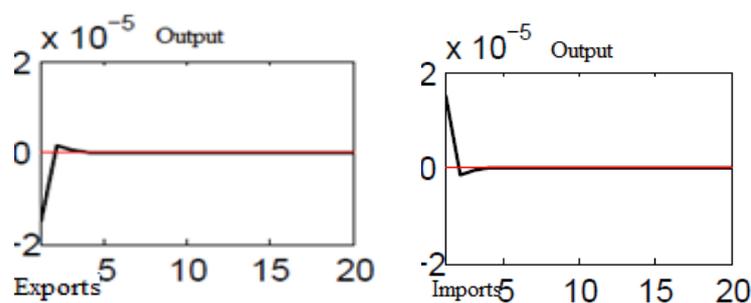
appreciation of foreign currency and stimulates exports and imports. This strategy then leads to improvement in the trade balance.

Thus, the size of China's foreign exchange reserves grows along with the development of the domestic economy and international trade. As such, through growth and the trade balance effects, undervaluation of the Chinese currency has a positive impact on the foreign reserve accumulation in China.

In the domestic economy, undervaluation also causes an increase of wages and inflation, which play a crucial role in China's growth. To maintain a low currency value, China's central bank sells more RMB to buy dollars. As a result, the volume of China's currency in the domestic economy increases, leading to inflationary pressures. Increasing inflation results in a high cost of living for domestic households. In response, workers demand better wages to protect their standard of living. Consequently, Chinese firms face increased labour costs and must raise the prices of their products. Eventually, increasing wages, consumption and inflation lead to the growth of the entire economy. As shown in Figure 6 below, the impulse responses of output to the shocks of wages and inflation are positive.



**Figure 4. 6 Output Responses to Wages and Inflation**



**Figure 4. 7 Output Responses to Exports and Imports**

The positive impact of exchange rate intervention on exports and imports is short term, but this shock has a long-term association with wages and inflation. A lower exchange rate makes China's exports more attractive and competitive in the short run. However, this competitiveness should only be temporary. As a result of increasing wages and costs of living, the competitiveness of exports, will gradually disappear. Therefore, China's exchange rate policy is at best a short-term strategy for fuelling exports. Increasing domestic wages and savings will soon offset the export-promotion effect. But the the country's economy will be stimulated by the rising wages and price.

## **4.5 Conclusions**

This chapter develops a DSGE model to analyse the nexus between foreign reserve accumulation and real undervaluation in China from a growth perspective. The examination focuses on the extent to which—and the mechanisms through which—exchange rate policy has led to reserve

accumulation in China. In a general equilibrium setting, our analysis shows that real undervaluation contributes to reserve accumulation in a complex process. Rather than the commonly assumed mercantilist interpretation, undervaluation affects China's reserve build-up mainly through its positive effect on economic growth. The growth effect is in turn caused by real undervaluation in three ways: Through TFP growth, trade surpluses and income (wage) increases.

In the short run, undervaluation induces a positive response from TFP, which promotes growth. Over the intermedium run, both exports and imports have a positive response to real undervaluation in the impulse response analysis. In the longer run, undervaluation leads to an increase in income. While income increases will cause wages and the price level to rise and hence will offset the trade balance effect to some degree, model results show that the income effect is positively related to China's growth.

To explore the relative contributions of these three channels, we compare output's response to these three variables. In so doing, we find that a 1% TFP growth shock leads to an increase in output of less than 0.01%. A 1% wage shock causes an 8% increase in output and a 1% export shock gives rise to economic growth of less than 0.001%. On the basis of these results, we can conclude that income increase is relatively more important than the other two channels in the longer run.

In all, through the channels discussed, real undervaluation stimulates domestic income growth, and hence promotes domestic business activities. Domestic savings thus grow consequently. This affects the savings-investment relation and, given investment, the growing savings become excessive. Then, the excessive domestic savings overflows to international financial markets leading to China's accumulation of international assets.

## **Chapter 5**

### **Precautionary Demand for Foreign Reserves**

Countries hold foreign reserves to hedge against unexpected adverse shocks arising from the risky world and hence precautionary demand is generally considered as a crucial driving force behind accumulation of foreign exchange reserves in emerging economies. This chapter develops a parameterized DSGE model to quantitatively assess the merit of this hypothesis. Our model captures many of the principal insights from the existing specialized literature on the precautionary motive, deriving an innovative approach to exploring the importance of the precautionary motive in foreign reserves holdings. The impulse response function of net foreign assets indicates a strong correlation between precautionary savings of households and foreign reserves accumulation. A sudden stop in capital flows leads to a significant lower position of international reserves in the model. These results suggest that the precautionary motive is an important driving force behind the accumulation of foreign reserves in China.

## 5.1 Precautionary Motive for International Reserves

Countries hold foreign reserves for many reasons. One ostensible motive is central banks' desire to hold reserves to deal with economic hardships. The notion that foreign reserves are accumulated for precautionary considerations has long been discussed in the literature and is generally undisputed. Heller (1966) considers holding foreign exchange reserves as an insurance against the shocks to the trade balance. In a cost-benefit optimising framework, Heller introduces the method to quantify optimal level of foreign reserves for a country by weighting the adjustment costs, which are caused by external imbalances that cannot be met with reserves against the opportunity cost of holding reserves.

Frenkel and Jovanovic (1981) view international reserves as a buffer stock to accommodate stochastic fluctuations in external transactions. They believe that holding a level of reserves is a decision mainly to cope with the volatility of external transactions. Therefore, they set optimal reserves are a function of the opportunity cost of holding reserves, the cost of adjustment, and the volatility of Wiener increments in the reserve process. Flood and Marion (2002) improve the empirical specifications of Frenkel and Jovanovic (1981)'s model to measuring reserve volatility more precisely.

Lee (2004) estimates the optimal level of foreign exchange reserves based on option price theory. He assumes that an overall insurance value equals to the amount of short term external debt needed for precautionary reasons. He

believes that this overall insurance level will be met partially through market-based insurance and partially by self-insurance. Aizenman and Lee (2007) compare the weightiness of precautionary and mercantilist motives in the hoarding of international reserves by developing countries. They find that, in the determination of the appropriate level of reserves, although mercantilist effects are significant to export growth, they have a smaller impact relative to variables associated with balance of payment crises. Their empirical results support the significance of precautionary motives.

Jeanne and Ranciere (2011) provide a new method to estimate the share of the observed stock of reserves that can be considered as precautionary foreign reserves against sudden stops. They build a model that incorporates the benefit of holding international reserves in sustaining domestic absorption in times of a sudden stop of capital flows. They also calculate the expected costs associated with a sudden stop by estimating a Probit model of the probability of a sudden stop. With the estimated opportunity cost of holding reserves and the average size of capital account reversals, they explore an optimal level of foreign exchange reserves for the emerging economies.

Ruiz-Arranz and Zavadjil (2008) try to explore whether foreign exchange reserves in emerging Asia are excessive when compared with optimal levels predicted by their model. They deem that international reserve holdings by most Asia countries seem not to be above the optimal levels, excluding China. They identify that the costs of sudden stops were greater in emerging Asia countries than the average size that derived by Jeanne and Ranciere (2006).

They conclude that the precautionary motive is a critical driver behind reserve accumulation of Asian countries.

Gonçalves (2007) extends the framework in Jeanne and Ranciere (2006) by incorporating the dollarization of bank deposits into the reserve analysis. They identify a sudden stop in capital flows as an additional element to consider at the time of choosing the optimal level of reserves. They believe that central banks match with their own reserves the equivalent of dollar deposits from non-residents. Households only have a fraction of dollar deposits in their hands and these deposits provide an additional role for government reserve accumulation.

Jeanne (2011) constructs a model of foreign exchange reserves of a small open economy to explore the motive for households of a country to hold foreign assets. The model provides a tractable formula for the economy's target value of assets, including those under the precautionary motive. The model then is applied to explore the relationship between economic development and capital flows, and the long-run impacts of foreign reserves on resorbing global financial imbalances.

In order to investigate the property and structure of precautionary demand for foreign reserves in China, we develop a DSGE model with the precautionary motive, which has been omitted in this class of models by much of the previous literature as intractable. The aim of the current research is to explore the quantitative importance of precautionary motive in holding foreign

reserves in China by estimating the behaviour of precautionary savings of Chinese households and their interaction to other sector of the economy in a general equilibrium setting.

To gauge the link between foreign reserves and precautionary savings, we model the motives of households of a country for holding foreign assets. We consider two types of households: the household with perfect foresight and precautionary households. Perfect foresight households assume a certainty world where their expectations and predictions are all valid. As a result, this type of households does not have a precautionary motive because they do not anticipate future shocks hitting the economy. On the other hand, risk-averse precautionary households are aware the possibility of future shocks and hence reduce their consumption to save in the current period, given incompleteness of insurance markets. In other words, risk-averse households engage in precautionary saving in response to uncertainty regarding future income. In the framework of this setting the representative precautionary household solves a self-insurance problem. The precautionary savings are the change in savings, in both domestic and foreign assets, because of this self-insurance motive, and hence leading to accumulation of foreign reserves of the country in question.

The model is parameterized to capture China's specific characteristics. We develop a dynamic stochastic general equilibrium model based on China's main macroeconomic data for recent years. In the linearization process, the parameters in the steady state and linearized function are estimated by

Bayesian methods. More specifically, we define households' precautionary savings as a fraction of their aggregate consumption and investment. We want to understand the impact of the precautionary motive on net foreign assets and economic growth by investigating the difference between perfect foresight households and precautionary households.

We then present two applications of our framework. First, we look at what the model reveals about the relationship between net foreign assets and the precautionary motive. The difference between the applications indicates the difference between the behaviours of perfect foresight and precautionary households, since only risk-averse households would be aware of the existence of future shocks hitting the economy and hence would delay their spending to accumulate savings for precautionary considerations. We identify that the gap between deterministic steady state and stochastic steady states of international reserves in the model which would denote foreign reserves for precautionary motivation.

Second, we use parameterized data to explore the quantitative importance of precautionary foreign reserves. The model features permanent and temporary shocks to households' consumption, investment and wages. We set technology as a permanent shock, and the change in net foreign assets and precautionary savings as temporary shocks for the perfect foresight household. Assuming precautionary savings represent a permanent share of a risk-averse household's spending, we find a strong and positive correlation

between precautionary savings of households and foreign exchange reserves accumulation in China.

In order to estimating the influence of precautionary motive on foreign reserves , we follow Coeurdacier, Rey and Winant (2011)'s theory to calculate "the risky steady state" in the model. Perfect foresight households assume that the model's prediction is valid, and they therefore do not anticipate the effect of future shocks. Rational agents observe the gap between the current status and the steady state to make optimal choices. The corresponding equilibrium is called the deterministic steady state.

On the other hand, risk-averse agents consider the existence of future shocks hitting the economy. These agents predict the convergence of economic variable to the risky steady state, which is the equilibrium including the impact of future shocks. Coeurdacier, Rey and Winant (2011) define the risky steady state as the point where risk-averse agents take decision to stay at a given date. The realization of shocks is also equal to zero at the date of risk steady state.

To explore the system reaction to sudden stop risks in linearized DSGE model, we follow Uhlig (1999) and Juillard (2012) to apply second order approximation to solve DSGE models. Linearizing model to the second order enables us to take risks and the variance of shocks into consideration while the variance of future shocks remains after taking expectation of linearized

equations. To calculate the deterministic steady state without sudden stop risks, we evaluate our model in first order condition in the dynare software. On the other hand, we can capture the stochastic steady states in the case of a second order approximation of our DSGE model. We identify the gap between deterministic steady state and stochastic steady state of international reserves represents precautionary reserves against the shocks from the external economy.

This chapter is organized as follows. Following this introductory section, sections two and three present the structure of the model and describe some new variables in the DSGE framework, such as precautionary savings of households and precautionary reserves of the central bank. Section four estimates and evaluates the parameters. While some of the parameters are taken from previous empirical research, Bayesian methods are applied to estimate other parameters and the model's performance is evaluated in this light. This section also shows the consequential first order conditions (FOCs) engendered by households' and firms' optimizing behaviours, and presents the linearized functions of the general equilibrium. Section five discusses the impulse response functions to the shocks of net foreign assets and precautionary motivations. We also consider correlation between main macroeconomic variables in the model. Section six concludes the chapter.

## 5.2 DSGE Model Including Precautionary Motive

### 5.2.1 Perfect Foresight Households

In general, we assume the representative household is patient. The representative agent assumption in DSGE model implicated that everyone in the economy had the same preferences, and the same relative income of capital, labour skills, habits and so on. Therefore, the sum of their choices is equal to the aggregation behaviours in the economy. The utility function of a patient household, representative of the parental generation, is:

$$U_t = E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \frac{((C_t - h \cdot C_{t-1}))^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right] \right\}, \quad (5.1)$$

where  $\beta$  is the discount factor;  $C_t$  denotes the consumption of the household at time  $t$ ;  $\sigma$  and  $\varphi$  are the substitution elasticity of consumption and the elasticity of labour supply, respectively. The consumption index  $C_t$  aggregates the consumption of traded and non-traded goods, which takes the following form:

$$C_t = \left[ (\varpi)^{1/\psi} (C_t^d)^{\frac{\psi-1}{\psi}} + (1-\varpi)^{1/\psi} (C_t^f)^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}} \quad (5.2)$$

$\psi$  denotes elasticity of substitution between traded and non-traded goods,  $C_t^d$  and  $C_t^f$  are consumption of domestic and import goods, respectively.  $\varpi$  represents the fraction of non-traded products in total output, then  $(1-\varpi)$  denotes the fraction of imported products.

We further assume there are two types of households in society, i.e the perfect foresight households and risk-averse precautionary households. In order to explore households' precautionary savings, we consider two different budget constraints for two types of these households. Perfect foresight households assume their expectations and predictions are valid; therefore they do not consider precautionary savings in their budget constraint. On the other hand, the risk-averse representative households face uncertainty about the state of the economy in the future. Therefore, precautionary savings occur in their budget constraint due to uncertainty regarding future income.

We consider that the aggregate period-by-period budget constraint of perfect foresight households is:

$$C_t^F + I_t^F + NFA_{t+1} + (BO_t/P_t R_t) = (1+r_t)NFA_t + (W_t/P_t)N_t + r_t^C K_t + (BO_{t-1}/P_t) \quad (5.3)$$

In Eq.(5.3)  $C_t^F$  represents the aggregate consumption of the perfect foresight household;  $NFA_t$  stands for the stock of net foreign assets at the end of period  $t-1$ , and  $r_t$  is the risk-free rate. The difference between aggregate export and import constitutes the trade balance, equal to  $NFA_{t+1} - (1+r_t)NFA_t$ .  $BO_t$  denotes the coupon bond;  $P_t$  is the consumption price index (CPI), and  $I_t^F$  is investment of the perfect foresight household.  $NFA_t$  stands for the stock of net foreign assets at the end of period  $t-1$ ;  $r_t$  is the risk-free rate;  $W_t$  and  $r_t^C$  denote the nominal wage rate and the rental rate on capital in period  $t$ .  $P_t$  is

the consumption price index (CPI);  $w_t$  and  $r_t^C$  denote the nominal wage rate and the rental rate on capital in period  $t$ .

## 5.2.2 Precautionary Households

Differing from the perfect foresight households, precautionary households decide to delay their consumption and save in the current period, due to incompleteness of the capital market and uncertainty regarding future income. Facing an uncertain and risky world, these households predict that their incomes can be affected by negative shocks in the future. To avoid adverse effects of future income fluctuations and to retain a smooth path of consumption, they engage in precautionary savings by consuming and investing less in the current period. Therefore, we consider that

$$C_t^p + I_t^p + NFA_{t+1} + (BO_t/P_t R_t) + S_t^p = (1+r_t)NFA_t + (W_t/P_t)N_t + r_t^C K_t + (BO_{t-1}/P_t), \quad (5.4)$$

Where  $C_t^p$  represents aggregate consumption of precautionary households;  $I_t^p$  represents investment of precautionary households. They set aside precautionary savings  $S_t^p$  in period  $t$ , in case the economy deteriorates in the future.  $r_t$  is the risk-free rate of return.  $R_t = 1+r_t$ ,  $r_t^C$  is the rental rate of capital.

Subtracting  $R_t NFA_t$  from both sides of Eq.(5.4), we have:

$$C_t^p + I_t^p + NFA_{t+1} - R_t NFA_t + (BO_t/P_t R_t) + S_t^p = (W_t/P_t)N_t + r_t^C K_t + (BO_{t-1}/P_t) \quad (5.5)$$

The sum of both sides of Eq.(5) is equal to the aggregate output in the economy. Subtracting the precautionary budget constraint Eq.(5.4) from the perfect foresight budget constraint Eq.(5.3),we have:

$$C_t^P + I_t^P + S_t^P = C_t^F + I_t^F \quad (5.6)$$

Eq.(5.6) describes the relation between the two types of households, which make different choices in their consumption and investment decisions. The model assumes every representative household earns the same income in the current period. Eq.(5.6) expresses the fact that precautionary households decide to consume and invest less in the current time period than do perfect foresight households.

According to the result of Cherif and Hasanov's (2013), they consider precautionary savings of households are about 30% of disposable income. Depending on the data from the IMF Country Report for People's Republic of China and National Bureau of Statistics of China., we calculate the average value of total household disposable income in percent of GDP is 43.47% in the period from 2012 to 2015. Therefore the function of precautionary savings can be considered as:

$$S_t^P = (0.4347Y_t) \cdot rp = C_t^F + I_t^F - (C_t^P + I_t^P) \quad (5.7)'$$

Where  $rp=0.3$  denotes the ratio of precautionary savings of households to disposable income. In the model, precautionary savings are defined as the difference between the consumption and investment of precautionary households and perfect foresight households, due to the precautionary motive.

### 5.2.3 Enterprises

We apply the Dixit-Stiglitz framework to describe the behaviour of firms:

$$\text{A production aggregator } Y_t = \left[ \int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \quad (5.8)$$

where  $Y_t$  is a composite final good that is produced by a representative firm.

The final-good producer chooses a continuum of intermediate goods  $Y_t(i)$  as production input, indexed by  $i \in [0,1]$ , each produced by a monopolistically competitive firm. Firms set prices in a Calvo-staggered manner.  $\theta$  is the elasticity of substitution between goods.  $Y_t$  represents the output of the whole economy, and can be represented by a Cobb-Douglas production function:

$$Y_t(i) = Z_t K_t(i)^{1-\alpha} N_t(i)^\alpha \quad (5.9)$$

where  $i \in [0,1]$  denotes each differentiated good produced by a unique producer.  $\alpha$  represents the elasticity of output with respect to labour supply  $N_t(i)$  in the production process.  $K_t(i)$  denotes capital at time  $t$ . The capital stock accumulation equation in this system is  $K_t(i) = (1-\delta)K_{t-1}(i) + I_t$ . In Eq.

(5.9)  $Z_t$  represents total factor productivity (TFP), which is a key element affecting total output  $Y_t$ .

The price of final goods  $P_t$  is determined by:

$$P_t = \left( \int_0^1 p_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}, \quad (5.10)$$

where  $\mu$  is the elasticity of substitution between the goods mentioned before.

We consider that a randomly selected fraction  $(1-\mu)$  of firms adjusts prices, while the remaining fraction keeps the prices unchanged, so  $P_t(i) = \Pi_t P_{t-1}(i) \Rightarrow P_t(i) = (1+\pi_t)P_{t-1}(i)$ , where  $\Pi_t = (1+\pi_t)$ .

Defining the index for the “reset” price in period  $t$ ,  $P_t^*(i)$ , is the optimal price chosen by the firm in order to maximize the present value of real profits;  $\pi_t$  denotes the inflation rate at time period  $t$ . Then the aggregate price index (CPI) is given by:

$$P_t = [\theta(1+\pi_t)P_{t-1}^{1-\theta} + (1-\theta)(P_t^*)^{1-\theta}]^{\frac{1}{1-\theta}} \quad (5.11)$$

The relation between the  $P_t$  and  $P_t^*(i)$  is:

$$\frac{P_t^*(i)}{P_t} = \left( \frac{\theta}{\theta-1} \right) \frac{E_t \sum_{j=0}^{\infty} \mu^j \beta^j \left( \frac{C_{t+j}}{C_t} \right)^{-\sigma} \eta_{t+j} \left( \frac{P_{t+j}}{P_t} \right)^{\mu} Y_{t+j} G_j^{-\mu}}{E_t \sum_{j=0}^{\infty} \mu^j \beta^j \left( \frac{C_{t+j}}{C_t} \right)^{-\sigma} \eta_{t+j} \left( \frac{P_{t+j}}{P_t} \right)^{\mu-1} Y_{t+j} G_j^{1-\mu}} \quad (5.12)$$

when  $t=0$ ,  $G_t=1$ .  $G_t = (1+\pi_1)(1+\pi_2)\dots(1+\pi_t)$ , for  $t \geq 1$ .

## 5.2.4 Aggregate Output and Net Foreign Assets

In the model, we assume gross domestic product (GDP) is equal to gross output. GDP is the sum of consumption, investment, government spending and net exports:

$$Y_t = C_t + I_t + G_t + (EX_t - IM_t), \quad (5.13)$$

where  $C_t$  is equal to all private consumption, or consumer spending, in the nation's economy;  $G_t$  is the sum of government spending;  $I_t$  is the sum of all the country's investment, including business capital expenditures.  $EX_t$  denotes gross exports, while  $IM_t$  represents gross imports.  $NX_t$  is the nation's total net exports, calculated as total exports minus total imports:

$$NX_t = EX_t - IM_t \quad (5.14)$$

A country's current account  $CA_t$  is equal to the difference between the nation's savings and investment. The current account consists of the balance of trade, net income from abroad and net current transfers. So the current account position can be calculated as:

$$CA_t = (EX_t - IM_t) + NY_t + NCT_t, \quad (5.15)$$

where  $NY_t$  represents net primary income;  $NCT_t$  denotes net current transfers that have taken place over one period of time. In the model we consider the country's financial account balance  $KA_t$  as the sum of net income from abroad and net current transfers:

$$KA_t = NY_t + NCT_t \quad (5.16)$$

A nation's Net Foreign Assets ( $NFA_t$ ) position is the cumulative change in its current account over time. Capiello and Ferrucci (2008) argue that the sum of the current account balance ( $CA_t$ ), the capital and financial account balance ( $KA_t$ ), errors and omissions ( $EO_t$ ) and the change in foreign exchange reserves must be zero. Given this, we consider the amount of foreign exchange reserves to be equal to net foreign assets ( $NFA_t$ ). Therefore, we have

$$CA_t + KA_t + EO_t - \hat{NFA}_t = 0 \Rightarrow \hat{NFA}_t = CA_t + KA_t + EO_t \quad (5.17)$$

$\hat{NFA}_t$  denotes the change in foreign exchange reserves from time period  $t-1$  to  $t$ . Following Capiello and Ferrucci (2008), we rebuild the equation of net

foreign assets as a key variable in the system. Net foreign assets is described by the following expression:

$$NFA_{t+1} = R_t NFA_t + CA_t + KA_t \quad (5.18)$$

Meade (1951), Metzler (1951) and Swan (1963) propose an underlying balance (UB) approach to find the fair value of a currency and characterize the equilibrium in an open economy. This underlying balance (UB) approach states that the fair value of a currency is the level of the exchange rate that is consistent with both the internal and external balance of the country. Most applications of the UB approach consider internal balance as a domestic economic process. On the other hand, a country's external balance can be broadly represented by its current account (CA) position being at equilibrium. Meanwhile, a nation's current account position is equal to the difference between its savings and investment, hence the balance of payments relation in the UB approach is:

$$CA_t + KA_t = NFA_{t+1} - R_t NFA_t = \hat{NFA}_t = S_t - I_t, \quad (5.19)$$

where  $CA_t$  may be expressed by either changes in net foreign assets  $NFAs$  or the excess domestic savings ( $S_t$ ) over domestic investment ( $I_t$ ).

In the model, we define the capital account balance  $KA_t$  as a shock process, which is exogenous to the model. We consider capital flow as a random walk process, the expectation value of capital account is  $E[KA_t] = 0$ .

Substituting Eq. (5.19) into Eq.(5.13), we have:

$$Y_t = C_t + I_t + G_t + \hat{NFA}_t \quad (5.20)$$

Eq.(5.20) shows the positive relation between gross domestic product (GDP) and changes in net foreign assets. For the case of households with precautionary savings, we consider their consumption and investment to be equal to that of the representative households. Therefore, we have:

$$C_t^F = C_t \quad I_t^F = I_t \quad (5.21)$$

Then, the aggregate output function for perfect foresight households is:

$$Y_t = C_t^F + I_t^F + G_t + \hat{NFA}_t \quad (5.22)$$

Substituting Eq.(5.6) into Eq.(5.22), we have the gross domestic product function for precautionary households:

$$Y_t = C_t^P + I_t^P + G_t + \hat{NFA}_t + S_t^P \quad (5.23)$$

Following Cherif and Hasanov (2013), precautionary savings are equal to the difference between income after investment and consumption. The irreversibility of investment and large volatility of permanent income shocks

should produce large precautionary savings. Therefore, the function of precautionary savings can also be written as:

$$S_t^p = Y_t - C_t^p - I_t^p - G_t - \hat{NFA}_t \quad (5.24)$$

### 5.2.5 Government

Based on Chen, Funke and Paetz (2012), we consider an expanded Taylor rule function to describe China's monetary policy. The interest rate rule function is:

$$R_t = \phi_R R_{t-1} + \phi_\pi \left( E_t(\pi_{t+1}) - \pi_t \right) + \phi_Y \hat{Y}_t + \varepsilon_t^R \quad (5.25)$$

where  $R_t$  denotes the actual interest rate and  $\bar{R}_t$  is the equilibrium nominal interest rate.  $R_{t-1}$  is the lagged interest rate.  $Y_t$  and  $RER_t$  denote the output gap and the real exchange rate, respectively.  $\phi_R$ ,  $\phi_\pi$  and  $\phi_Y$  are the parameters with respect to the lagged interest rate, inflation deviation, and output gap, respectively.  $\varepsilon_t^R$  is the shock of the interest rate rule.

### Monetary policy reaction function

$$M_t = v_1 M_{t-1} + v_2 Y_t + v_3 (\pi_t) + \varepsilon_t^m \quad (5.26)$$

where  $M_t$  is nominal money growth;  $\bar{M}$  is the equilibrium of money demand;  $\varepsilon_t^m$  is money growth shock;  $v_1$ ,  $v_2$ ,  $v_3$  are the parameters of nominal money growth, output and the inflation rate, respectively. Assume the co-

integrating relationship between the logarithms of money, output and inflation, we have  $\bar{M} = \alpha_0 + \alpha_1 \bar{Y} + \alpha_2 \bar{\pi}$ .

## 5.3 Summarizing the Model

### 5.3.1. Solving First Order Conditions (FOCs)

We apply Uhlig's (1999) method to calculate the first-order conditions (FOCs) of the model. First, we derive the Lagrangian of utility function Eq.(1) based on budget constraint Eq.(5.3):

$$L = \max E \sum_{t=0}^{\infty} \beta^t \left[ \frac{(C_t - h \cdot C_{t-1})^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} - \lambda_t (C_t + I_t + NFA_{t+1} + (BO_t/P_t R_t) - R_t NFA_t - (W_t/P_t) N_t - r_t^c K_t - (BO_{t-1}/P_t)) \right] \quad (5.27)$$

The FOC differentiating with respect to  $C_t$  is:

$$\frac{\partial L}{\partial C_t} = (C_t - h \cdot C_{t-1})^{-\sigma} - \lambda_t = 0 \Rightarrow (C_t - h \cdot C_{t-1})^{-\sigma} = \lambda_t \quad (5.28)$$

The FOC differentiating with respect to  $BO_t$  is:

$$\frac{\partial L}{\partial BO_t} = \left( -\lambda_t \frac{1}{P_t R_t} \right) - \beta E \left[ -\frac{1}{P_{t+1}} \lambda_{t+1} \right] = 0 \Rightarrow \lambda_t = \beta E \left[ \frac{P_t R_t}{P_{t+1}} \lambda_{t+1} \right] \quad (5.29)$$

Based on Eq.(5.28) and Eq.(5.29), we obtain the equilibrium equation:

$$\beta E \left[ \frac{P_t R_t}{P_{t+1}} \left( \frac{C_{t+1} - h \cdot C_t}{C_t - h \cdot C_{t-1}} \right)^{-\sigma} \right] = 1 \quad (5.30)$$

This is the Euler equation with respect to consumption  $C_t$ .

The FOC differentiating with respect to net foreign assets  $NFA_{t+1}$  is:

$$\frac{\partial L}{\partial NFA_{t+1}} = (-\lambda_t) - \beta E[-R_{t+1} \lambda_{t+1}] = 0 \Rightarrow \lambda_t = \beta E[R_{t+1} \lambda_{t+1}] \quad (5.31)$$

The FOC with respect to  $\lambda_t$  is:

$$P_t C_t + P_t I_t + \frac{BO_t}{R_t} + P_t (NFA_{t+1} - R_t NFA_t) = BO_{t-1} + W_t N_t + r_t^C K_t P_t \quad (5.32)$$

Further, we can find the FOCs with respect to investment  $I_t$  and capital stocks  $K_t$ , according to the capital stock accumulation equation

$$K_{t+1} = (1 - \delta)K_t + I_t.$$

We replace  $I_t = K_{t+1} - (1 - \delta)K_t$  in Eq.(3), then the Lagrangian can be written as:

$$L = \dots + \beta^t \left( \frac{(C_t - h \cdot C_{t-1})^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) - \lambda_t (C_t + K_{t+1} - (1 - \delta - r_t^C)K_t + \frac{BO_t}{P_t R_t} - \frac{BO_{t-1}}{P_t} - \frac{W_t}{P_t} N_t) + \beta^{t+1} E_t \left( \frac{(C_{t+1} - h \cdot C_t)^{1-\sigma}}{1-\sigma} - \frac{N_{t+1}^{1+\varphi}}{1+\varphi} \right) - \lambda_{t+1} (C_{t+1} + K_{t+2} - (1 - \delta - r_{t+1}^C)K_{t+1} + \frac{BO_{t+1}}{P_{t+1} R_{t+1}} - \frac{BO_t}{P_{t+1}} - \frac{W_{t+1}}{P_{t+1}} N_{t+1}) + \dots$$

The FOC differentiating with respect to  $K_{t+1}$  (capital at time period  $t+1$ ) is:

$$\frac{\partial L}{\partial K_{t+1}} = (-\lambda_t) + \beta E_t (\lambda_{t+1} (1 - \delta + r_{t+1}^c)) = 0 \quad (5.33)$$

$$\Rightarrow \beta E_t \left( (1 - \delta + r_{t+1}^c) (C_{t+1} - h \bullet C_t)^{-\sigma} \right) = (C_t - h \bullet C_{t-1})^{-\sigma} \quad (5.34)$$

$$\Rightarrow \beta (1 - \delta + r_{t+1}^c) E_t \left( \frac{C_t - h \bullet C_{t-1}}{C_{t+1} - h \bullet C_t} \right)^\sigma = 1 \quad (5.35)$$

The FOC with respect to  $N_t$  (labour) is:

$$-N_t^\varphi - \lambda_t \left( -\frac{W_t}{P_t} \right) = 0 \Rightarrow N_t^\varphi = \lambda_t \left( \frac{W_t}{P_t} \right), \quad (5.36)$$

We replace  $\lambda_t = (C_t - h \bullet C_{t-1})^{-\sigma}$  into Eq. (5.36), to have:

$$N_t^\varphi = [(C_t - h \bullet C_{t-1})]^{-\sigma} \left( \frac{W_t}{P_t} \right) \quad (5.37)$$

Moreover, the aggregate wage equation is:

$$W_t = \left[ (1 - \xi) (W^*)^{1-l} + \xi (W_{t-1})^{1-l} \right]^{1/(1-l)} \quad (5.38)$$

where  $W_t^*$  denotes the wage that one can replace in the current time period  $t$ .

In the equation,  $\xi$  denotes the fraction of households that set their wages according to the last-period wage inflation or a combination of last-period price and the wage inflation rate. The remaining  $1 - \xi$  portion of households would adjust their wages optimally.  $\pi_t$  represents the inflation rate:

$$P_t = (1 + \pi_t) P_{t-1}.$$

Combining Cobb-Douglas production function  $Y_i(i) = Z_i K_i(i)^{1-\alpha} N_i(i)^\alpha$  and Lagrangian function Eq. (5.25), we have the first order condition with respect to  $N_i(i)$  and  $K_i(i)$  as follows:

$$\frac{W_t}{P_t} = \left( \frac{\alpha r_t^c}{1-\alpha} \right) \left( \frac{K_t(i)}{N_t(i)} \right) \quad (5.39)$$

This equation implies that households' wages depend on the ratio of rental capital and labour demand. This formula can be rewritten as:

$$\frac{W_t}{P_t} = \alpha \left( \frac{W_t}{\alpha P_t} \right)^\alpha \left( \frac{r_t^c}{1-\alpha} \right)^{1-\alpha} \left( \frac{K_t(i)}{N_t(i)} \right)^{1-\alpha} \quad (5.40)$$

Now one can reason out the function of real marginal costs ( $MC_t$ ) as a result of firm  $i$ 's optimization problem:

$$MC_t = \frac{1}{Z_t} \left( \frac{1}{\alpha} \right)^\alpha \left( \frac{W_t}{P_t} \right)^\alpha \left( \frac{r_t^c}{1-\alpha} \right)^{1-\alpha} \quad (5.41)$$

Note that marginal costs are equal for all firms and the index  $i$  can therefore be dropped.

Given the above, Eq. (5.39) can be expressed as:

$$\frac{W_t}{P_t} = \alpha Z_t MC_t \left( \frac{K_t(i)}{N_t(i)} \right)^{1-\alpha} \quad (5.42)$$

### 5.3.2 Solving for the Steady States

At the steady state, all variables are constant. We calculate the steady state of equations discussed in the previous sections. First, the steady state of Euler equation (5.30) is:

$$\beta E \left[ \frac{\bar{P}\bar{R}}{\bar{P}} \left( \frac{\bar{C}-h\bar{C}}{\bar{C}-h\bar{C}} \right)^{-\sigma} \right] = 1 \Rightarrow \beta \bar{R} = 1 \Rightarrow \bar{R} = \frac{1}{\beta} \quad (5.43)$$

The steady state of capital stock equation Eq. (5.35) is:

$$\beta E_t \left( \frac{\bar{C}-h\bar{C}}{\bar{C}-h\bar{C}} \right)^\sigma \left( 1 - \delta + r^c \right) = 1 \Rightarrow \beta \left( 1 - \delta + r^c \right) = 1 \Rightarrow r^c = \frac{1}{\beta} + \delta - 1 \quad (5.44)$$

The steady state of optimal labour-leisure decision Eq.(5.37) is:

$$\bar{N}^\sigma = \left[ (\bar{C}-h\bar{C}) \right]^{-\sigma} \left( \frac{\bar{W}}{\bar{P}} \right) \quad (5.45)$$

The steady state of the Cobb-Douglas production function is:

$$\bar{Y} = \bar{Z} \left( \bar{K} \right)^{1-\alpha} \left( \bar{N} \right)^\alpha \quad (5.46)$$

The steady states of capital accumulation function and aggregate wage are:

$$\bar{K} = (1 - \delta) \bar{K} + \bar{I} \Rightarrow \delta \bar{K} = \bar{I} \quad (5.47)$$

$$\bar{W} = \left[ (1 - \mu) \left( \bar{W} \right)^{1-l} + \mu \left( \bar{W} \right)^{1-l} \right]^{1/(1-l)} = \left[ \left( \bar{W} \right)^{1-l} \right]^{1/(1-l)} = \bar{W} \quad (5.48)$$

The steady states of inflation rate and aggregate price are:

$$\Pi \equiv \frac{\bar{P}}{P} = 1 \Rightarrow 1 + \bar{\pi} = 1 \Rightarrow \bar{\pi} = 0 \quad (5.49)$$

$$\bar{P} = [\theta (1 + \bar{\pi}) (\bar{P})^{1-\theta} + (1-\theta) (\bar{P})^{1-\theta}]^{\frac{1}{1-\theta}} = \left[ (\bar{P})^{1-\theta} \right]^{\frac{1}{1-\theta}} = \bar{P} \quad (5.50)$$

The steady state of monetary policy reaction function is:

$$\bar{M} = \theta_1 \bar{M} + \theta_2 \bar{Y} + \theta_3 \bar{\pi} \Rightarrow \bar{M} = \frac{\theta_2}{(1-\theta_1)} \bar{Y} + \frac{\theta_3}{(1-\theta_1)} \bar{\pi} \quad (5.51)$$

The steady state of the function of net foreign assets (*NFA*) and the current account is:

$$\bar{NFA} = R \bar{NFA} + \bar{CA} + \bar{KA} \Rightarrow \quad (5.52)$$

$$\left(1 - R\right) \bar{NFA} = \bar{CA} + \bar{KA} \quad (5.53)$$

Furthermore, we have:

$$\bar{CA} + \bar{KA} = \bar{S} - \bar{I} \quad (5.54)$$

Current account position in the equilibrium is equal to the difference between savings and investment.

The steady states of real marginal costs ( $MC_t$ ) and ( $W_t/P_t$ ) are respectively:

$$\bar{MC} = \frac{1}{\bar{Z}} \left( \frac{1}{\alpha} \right)^\alpha \left( \frac{\bar{W}}{\bar{P}} \right)^\alpha \left( \frac{\bar{r}^c}{1-\alpha} \right)^{1-\alpha} \quad (5.55)$$

$$\frac{\bar{W}}{\bar{P}} = \left( \frac{\alpha \bar{r}_t^c}{1-\alpha} \right) \left( \frac{\bar{K}}{\bar{N}} \right) \quad (5.56)$$

We calculate the steady state functions for both perfect foresight households and precautionary households. The steady state of aggregate output for perfect foresight households is:

$$\bar{Y} = \bar{C}^F + \bar{I}^F + \bar{G} + \left( \bar{X} - \bar{M} \right) \quad (5.57)$$

According to Eqs. (5.15) and (5.19), the steady state of the current account is:

$$\bar{CA} = \left( \bar{EX} - \bar{IM} \right) = \left( 1 - \bar{R} \right) \bar{NFA} \quad (5.58)$$

Substituting Eq. (5.58) into Eq.(5.57), the perfect foresight households' aggregate output function is:

$$\bar{Y} = \bar{C}^F + \bar{I}^F + \bar{G} + \left( 1 - \bar{R} \right) \bar{NFA} \quad (5.59)$$

Based on the precautionary saving function Eq. (5.7), we calculate the steady state of precautionary saving as follows:

$$\bar{S}^P = \bar{C}^F + \bar{I}^F - \left( \bar{C}^P + \bar{I}^P \right) \quad (5.60)$$

The steady state of aggregate output for precautionary households is:

$$\bar{Y} = \bar{C}^p + \bar{I}^p + \bar{G} + \bar{S}^p + (1 - \bar{R})\bar{NFA} \quad (5.61)$$

### 5.3.3 Linearized Equations

Linearized functions of the equilibrium equations in previous sections can be expressed in what follows. First, we can linearise Eq.(5.30) around the steady state to obtain:

$$\hat{C}_t = \frac{\hat{C}_{t+1} + h\hat{C}_{t-1}}{1+h} + \left( \frac{h-1}{(1+h)\sigma} \right) (\hat{R}_t + \hat{P}_t - \hat{P}_{t+1}) \quad (5.62)$$

Eq.(5.62) is the linearized equation of Euler equation of consumption  $\hat{C}_t$ . This equation implies that current consumption is determined by prior consumption, expected future consumption, the inflation rate and the gross returns on bond.  $h$  represents the habit factor.

The linearized equation of labour supply  $N_t$  is:

$$\hat{N}_t = \left( \frac{-\sigma}{\varphi} \right) \left( \frac{\hat{C}_t - h\hat{C}_{t-1}}{1-h} \right) + \frac{1}{\varphi} (\hat{W}_t - \hat{P}_t) \quad (5.63)$$

Linearizing the Cobb-Douglas production function Eq. (5.5), we have:

$$\hat{Y}_t = \hat{Z}_t + (1-\alpha)\hat{K}_t + \alpha\hat{N}_t \quad (5.64)$$

Combining the intermediate-goods firm's and household's first-order conditions, the real marginal costs function can be linearized as follows:

$$\hat{MC}_t = \alpha\left(\hat{W}_t - \hat{P}_t\right) + (1-\alpha)r^c - \hat{Z}_t \quad (5.65)$$

According to Eq. (5.65), in equilibrium, linearized real marginal costs depend on the deviation of wages, the rental rate of capital and technology shocks.

According to first order condition of  $r_t^c$ , the rental rate of capital can be linearized to the following form:

$$\hat{r}_{t+1}^c = \frac{\sigma}{(h-1)(1+\beta\delta-\beta)}\left((1+h)\hat{C}_t - h\hat{C}_{t-1} - \hat{C}_{t+1}\right) \quad (5.66)$$

From this equation we can derive that rental capital returns come from the current, prior and expected levels of consumption.

To obtain optimal labour demand in equilibrium, we linearize Eq.(5.46) and can obtain:

$$\hat{W}_t - \hat{P}_t = \hat{r}_t^c + \hat{K}_t - \hat{N}_t \quad \Rightarrow \quad \hat{N}_t = \hat{r}_t^c + \hat{K}_t - \left(\hat{W}_t - \hat{P}_t\right) \quad (5.67)$$

The equilibrium indicates that optimal labour demand is determined by wages, capital accumulation and the rental rate of capital. Underlying this

equilibrium is the assumption that firms finance their wage bills and decide their demand for labour on the basis of their capital and incomes from capital rental.

Let real wage be  $w_t = \frac{W_t}{P_t}$ ; the linearized real wage then is  $\hat{w}_t = \hat{W}_t - \hat{P}_t$ . The

relation between labour supply of firm  $i$ ,  $N_t(i)$ , and the aggregate labour supply  $N_t$  is:

$$N_t(i) = \left\{ \frac{W_t^*(i)\nu_t}{W_t} \right\}^{-l} N_t, \quad (5.68)$$

where  $l$  is elasticity of substitution between differentiated labour services.

When  $t=0$ , we have  $\nu_t=1$ , and when  $t \geq 1$ ,  $\nu_t = (1+\pi_1)(1+\pi_2)\dots(1+\pi_t)$ , for  $t \geq 1$ .

Based on the aggregate wage equation Eq. (5.42) and the first order condition of optimal wage  $W_t^*$ , the linearized equation of real wage (Zhang, 2009) is:

$$\Omega \cdot \hat{w}_t = \left( \begin{aligned} & (1+\varphi l)\xi(\hat{w}_{t-1} + \pi_{t-1}) - (1+\beta)(1+\varphi l)\xi\pi_t \\ & + (1+\varphi l)\xi\beta E_t(\hat{w}_{t+1} + \pi_{t+1}) + (1-\beta\xi)(1-\xi) \cdot \left[ \varphi \hat{N}_t + \frac{\sigma}{1-h}(\hat{C}_t - \hat{C}_{t-1}) \right] \end{aligned} \right) \quad (5.69)$$

with  $\Omega = (1+\beta)(1+\varphi l)\xi + (1-\beta\xi)(1-\xi)$ .

The first order condition of labour supply  $N_t$  in Eq.(5.55) can be rewritten as:

$$\varphi \hat{N}_t + \frac{\sigma}{1-h} (\hat{C}_t - \hat{C}_{t-1}) = \hat{W}_t - \hat{P}_t = \hat{w}_t \quad (5.70)$$

Substituting Eq.(5.70) into Eq.(5.69), we get:

$$\begin{aligned} (1+\beta)(1+\varphi l)\xi \cdot \hat{w}_t &= (1+\varphi l)\xi(\hat{w}_{t-1} + \pi_{t-1}) - (1+\beta)(1+\varphi l)\xi\pi_t + (1+\varphi l)\xi\beta E_t(\hat{w}_{t+1} + \pi_{t+1}) \\ \Rightarrow \hat{w}_t &= \frac{1}{1+\beta}(\hat{w}_{t-1} + \pi_{t-1}) - \pi_t + \frac{\beta}{1+\beta} E_t(\hat{w}_{t+1} + \pi_{t+1}) \end{aligned} \quad (5.71)$$

This condition indicates that current nominal wage is a function of inflation, effective consumption, labour, and prior and expected wages.

Linearizing the inflation equation  $P_t = (1+\pi_t)P_{t-1}$ , we have:

$$\hat{P}_t = \hat{\pi}_t + \hat{P}_{t-1} \Rightarrow \hat{\pi}_t = \hat{P}_t - \hat{P}_{t-1} \quad (5.72)$$

This expression indicates that current inflation is the difference between the current and prior levels of price index. Substituting  $P_{t-1} = \frac{P_t}{1+\pi_t}$  into the aggregate price index equation Eq. (5.7), we obtain:

$$\begin{aligned} P_t &= \left[ \mu \left( \frac{1+\pi_{t-1}}{1+\pi_t} \right) P_t^{1-\theta} + (1-\mu)(P_t^*)^{1-\theta} \right]^{\frac{1}{1-\theta}} \\ \Rightarrow P_t^{1-\theta} &= \mu \left( \frac{1+\pi_{t-1}}{1+\pi_t} \right) P_t^{1-\theta} + (1-\mu)(P_t^*)^{1-\theta} \end{aligned} \quad (5.73)$$

Linearization of Eq. (5.73) gets us the following equation:

$$\hat{P}_t = \mu \left( \hat{\pi}_{t-1} - \hat{\pi}_t + \hat{P}_t \right) + (1-\mu) \hat{P}_t^* \Rightarrow \hat{P}_t = \frac{\mu}{1-\mu} \left( \hat{\pi}_{t-1} - \hat{\pi}_t \right) + \hat{P}_t^* \quad (5.74)$$

Combining the linearized aggregate price equation Eq. (5.74) and using the optimal price equation Eq. (10), the New Keynesian Phillips curve (Christiano et al., 2005, and Zhang, 2009) is:

$$\pi_t = \frac{\mu}{1+\beta\mu^2} \pi_{t-1} + \frac{2-(1+\beta)\mu+\beta\mu^2}{1+\beta\mu^2} E_t \pi_{t+1} + (1-\theta)(1-\beta\mu) \hat{MC}_t \quad (5.75)$$

From this, we can infer that the inflation rate is a function of the deviation of marginal costs from steady state, prior and expected levels of inflation. As before,  $\beta$  is the discount rate and  $\theta$  denotes the fraction of firms that will adjust their prices according to the last-period price inflation and marginal costs. The linearized equation of capital accumulation is:

$$\hat{K}_t = (1-\delta) \hat{K}_{t-1} + \frac{\bar{I}}{\bar{K}} I_t \quad (5.76)$$

Plugging  $\bar{I} = \delta \bar{K}$  into Eq.(5.76) in below, we get the linearized capital accumulation function:

$$\hat{K}_t = (1-\delta) \hat{K}_{t-1} + \delta I_t \quad (5.77)$$

The equation implies that recent capital amount depends on its prior level and investment in the linearized equation.

The Taylor rule for setting the nominal interest rate in the log-deviation form is written as:

$$\hat{R}_t = \phi_R \hat{R}_{t-1} + \phi_\pi \hat{\pi}_{t+1} + \phi_Y \hat{Y}_t \quad (5.78)$$

The linearized aggregate output for perfect foresight households is:

$$\hat{Y}_t = \frac{\bar{C}}{\bar{Y}} \hat{C}_t^F + \frac{\bar{I}}{\bar{Y}} \hat{I}_t^F + \frac{\bar{G}}{\bar{Y}} \hat{G}_t + \frac{\bar{NFA}}{\bar{Y}} \hat{NFA}_{t+1} - \frac{\bar{R} \bar{NFA}}{\bar{Y}} \left( \hat{NFA}_t + \hat{R}_t \right) \quad (5.79)$$

$$\Rightarrow \hat{Y}_t = \frac{\bar{W}}{\bar{Y}} \hat{W}_t + \frac{\bar{G}}{\bar{Y}} \hat{G}_t + \frac{\bar{NFA}}{\bar{Y}} \hat{NFA}_{t+1} - \frac{\bar{R} \bar{NFA}}{\bar{Y}} \left( \hat{NFA}_t + \hat{R}_t \right) \quad (5.80)$$

Based on Eq. (5.23) and Eq.(5.80), the linearized aggregate output for precautionary households is:

$$\hat{Y}_t = \frac{\bar{C}}{\bar{Y}} \hat{C}_t^P + \frac{\bar{I}}{\bar{Y}} \hat{I}_t^P + \frac{\bar{S}^P}{\bar{Y}} \hat{S}_t^P + \frac{\bar{G}}{\bar{Y}} \hat{G}_t + \frac{\bar{NFA}}{\bar{Y}} \hat{NFA}_{t+1} - \frac{\bar{R} \bar{NFA}}{\bar{Y}} \left( \hat{NFA}_t + \hat{R}_t \right) \quad (5.81)$$

Finally, the linear net foreign assets function is:

$$\hat{NFA}_{t+1} = \hat{R}_t + \hat{NFA}_t + \left( \frac{\bar{S}}{\bar{NFA}} \right) \hat{S}_t - \left( \frac{\bar{I}}{\bar{NFA}} \right) \hat{I}_t + \left( \frac{\bar{KA}}{\bar{NFA}} \right) \hat{KA}_t, \quad (5.82)$$

where  $\left(\bar{S}/\bar{NFA}\right)$  and  $\left(\bar{I}/\bar{NFA}\right)$  denote the ratio of savings to foreign reserve holdings and the ratio of import to foreign reserves in equilibrium, respectively.  $\left(\bar{KA}/\bar{NFA}\right)$  represent the ratio of capital flows to foreign reserves.

## 5.4 Parameter Analysis

### 5.4.1 Description of Data

The model is estimated using nominal interest rates, real interest rates, the inflation rate, exports, imports, GDP growth and net foreign assets. We expand the function of total factor productivity in the DSGE framework on the basis of Mbaye's (2013) and Jiang's (2014) approaches. The expanded TFP growth function is:

$$\hat{Z}_t = \tau_1 \hat{Z}_{t-1} + \tau_2 \hat{I}_t + \varepsilon_t^Z \quad (5.83)$$

Following the literature, the parameter value of the lagged TFP growth rate is set to be  $\tau_1=0.95$ , which describes the impact of the autoregressive parameter on current TFP growth. The investment parameter is set to be  $\tau_2=0.0787$ .

Following Walsh (2003) and He et al. (2007), we set  $\sigma=2$ ,  $\alpha=0.4$  and  $\delta=0.04$ . If  $\alpha=0.4$ , one can get the elasticity of the substitution among intermediate goods and final goods as  $\mu=4.61$ . The discount rate is  $\beta=0.98$ .

The GMM estimation with data for the period 1995 to 2005 shows that the fraction of firms that adjust their prices according to the last-period prices is  $\theta = 0.84$ . On the basis of Liu (2008), we obtain the elasticity of labour supply  $\varphi = 6.16$ , and the fraction of households who set their wages depending on wage inflation is  $\xi = 0.6$ . The elasticity of real money balance  $\gamma$  is considered to be 3.13.

We calculate the average value of the nominal exchange rate of the Chinese RMB against the US dollar from 2005 to 2012 as the steady state of the exchange value. Purchasing power parity (PPP) conversion factor is a technique used to determine the relative value of different currencies. It represents the number of units of a country's currency required to buy the same amount of goods and services in the domestic market as a US dollar would buy in the United States. We apply the PPP conversion factor from 2010 to 2014 to calculate the steady state of the real exchange rate, which is  $\bar{RER} = 7.137 * 3.52 = 25.12$ .

In order to determine the steady state value in Eq.(85), we use the data for China from 2005 to 2012 to calculate the ratios of domestic savings to foreign reserves and of investment to foreign reserves. We set the average values of savings, investment and reserves in this time period as the steady state values.

$\bar{KA} / \bar{NFA} = 0.044$  represent steady state of the ratio of capital account to foreign reserves Then, Eq.(5.82) can be rewritten as:

$$\hat{NFA}_t = \hat{NFA}_{t-1} + 1.188\hat{S}_{t-1} - 1.051\hat{I}_{t-1} + \hat{R}_{t-1} + 0.044\hat{KA}_t \quad (5.84)$$

Finally, we calculate the parameter values in the linearized aggregate output function using the macroeconomic data for China from 2005 to 2014. We set the average values of household final consumption expenditure of GDP ( $\bar{C}/\bar{Y}$ ), total investment of GDP ( $\bar{I}/\bar{Y}$ ), gross savings of GDP ( $\bar{S}/\bar{Y}$ ), net foreign assets of GDP ( $\bar{NFA}/\bar{Y}$ ) and government final consumption expenditure of GDP ( $\bar{G}/\bar{Y}$ ). The data regarding  $\bar{C}/\bar{Y}$ ,  $\bar{S}/\bar{Y}$ ,  $\bar{NFA}/\bar{Y}$ ,  $\bar{W}/\bar{Y}$  and  $\bar{G}/\bar{Y}$  are collected from the World Bank's national accounts data, while the data regarding  $\bar{I}/\bar{Y}$  are collected from the database of IMF cross country macroeconomic statistics. specifically, we have  $\bar{C}/\bar{Y} = 36.9\% = 0.369$ ,  $\bar{S}/\bar{Y} = 49.94\% = 0.499$ ,  $\bar{NFA}/\bar{Y} = 43.37\% = 0.434$ ,  $\bar{G}/\bar{Y} = 13.47\% = 0.135$  and  $\bar{I}/\bar{Y} = 44.4\% = 0.444$ .

To decide on the parameter value of changes in net foreign assets, we calculate the average value of China's exports and imports in terms of GDP. Average ratio of export of goods and services to GDP from 2005 to 2014 is:

$$\bar{EX}/\bar{Y} = 28.1\% = 0.281 \quad (5.85)$$

Then, the average ratio of import of goods and services to GDP from 2005 to 2014 is:

$$\bar{IM}/\bar{Y} = 23.82\% = 0.238 \quad (5.86)$$

The commercial balance or net exports,  $NX_t$ , is the difference between the monetary value of exports and imports in an economy over a certain period.

The average net export of GDP is:

$$\bar{NX}/\bar{Y} = \bar{EX}/\bar{Y} - \bar{IM}/\bar{Y} = 0.281 - 0.238 = 0.043 \quad (5.87)$$

Based on the average yearly wages of Chinese households and the total labour force of China, we calculate the average yearly wage to GDP ratio,  $\bar{W}/\bar{Y} = 72.76\% = 0.728$ . The average real interest rate from 2005 to 2014 is 1.59%, so we have  $\bar{R} = 1.0159$ . To explore the relative importance of precautionary savings, we designate that  $IF_t$  is foreign direct investment and  $ID_t$  is domestic investment. Then the function of aggregate investment  $I_t$  is:

$$I_t = IF_t + ID_t \quad (5.88)$$

The linearized function of Eq.(5.88) is:

$$\bar{I}I = \bar{IF} IF_t + \bar{ID} ID_t \Rightarrow I = \left(\frac{\bar{IF}}{\bar{I}}\right) IF_t + \left(\frac{\bar{ID}}{\bar{I}}\right) ID_t \quad (5.89)$$

We calculate the average value of foreign direct investment of GDP

$\bar{IF}/\bar{Y} = 0.044$ , and the ratio of total investment of GDP  $\bar{I}/\bar{Y} = 0.444$ , so:

$$\bar{ID}/\bar{Y} = \bar{I}/\bar{Y} - \bar{IF}/\bar{Y} = 0.444 - 0.044 = 0.4 \quad (5.90)$$

Replacing the parameters in the linearized aggregate output for perfect foresight households Eq.(5.79), we have:

$$\hat{Y}_t = 0.369 \hat{C}_t^F + 0.444 \hat{I}_t^F + 0.135 \hat{G}_t + 0.434 \hat{NFA}_{t+1} - 0.44 \left( \hat{NFA}_t + \hat{R}_t \right) \quad (5.91)$$

From Eqs.(5.15), (5.20) and (5.87), we explore the relation between gross domestic product and precautionary savings:

$$Y_t = S_t^p + C_t^p + I_t^p + G_t + NX_t \quad (5.92)$$

We already have the average ratio of yearly household final consumption expenditure to GDP  $\bar{C}/\bar{Y} = 0.369$  , total investment to GDP  $\bar{I}/\bar{Y} = 0.444$  , government final consumption expenditure to GDP  $\bar{G}/\bar{Y} = 0.135$  , and average net exports to GDP  $\bar{NX}/\bar{Y} = 0.043$  , the average ratio of capital flows to foreign reserves  $\bar{KA}/\bar{NFA} = 0.044$  .

However, the sum of consumption and investment for precautionary households should be less than that for perfect foresight households, because that precautionary savings should occur in these households' budget constraint due to uncertainty regarding future income. Precautionary households decide to reduce their consumption and investment in the current period to reserve precautionary savings. In order to evaluate linearized

aggregate output function for precautionary households, we calculate the linearized function of Eq.(5.7) to identify the difference between perfect foresight households and precautionary households. Then we have the linearized function of Eq.(5.7):

$$\hat{C}_i^F + \hat{I}_i^F = (0.4347) \cdot rp \frac{\bar{Y}}{\bar{C}^F + \bar{I}^F} \hat{Y}_i + \frac{\bar{C}^P + \bar{I}^P}{\bar{C}^F + \bar{I}^F} \left( \hat{C}_i^P + \hat{I}_i^P \right) \quad (5.93)$$

We denotes  $rp=0.3$  is the ratio of precautionary savings of households to disposable income. We consider  $\bar{C}/\bar{Y} = \bar{C}^F/\bar{Y} = 0.369$  and  $\bar{I}/\bar{Y} = \bar{I}^F/\bar{Y} = 0.444$ , then we have:

$$\frac{\bar{C}^F + \bar{I}^F}{\bar{Y}} = 0.369 + 0.444 = 0.813 \Rightarrow \bar{C}^F + \bar{I}^F = 0.813\bar{Y} \quad (5.94)$$

According to the steady state of Eq.(5.7), we have:

$$\bar{s}^P = 0.1304\bar{Y} = \left( \bar{C}^F + \bar{I}^F \right) - \left( \bar{C}^P + \bar{I}^P \right) , \quad (5.95)$$

where  $\bar{s}^P$  is the steady state of precautionary savings.

Replacing Eq.(5.94) into Eq.(5.95) we have :

$$0.1304\bar{Y} = 0.813\bar{Y} - \left( \bar{C}^P + \bar{I}^P \right) \Rightarrow \bar{C}^P + \bar{I}^P = 0.6826\bar{Y} \quad (5.96)$$

Then Eq.(5.93) can be rewritten as:

$$\hat{C}_t^F + \hat{I}_t^F = 0.1604 \hat{Y}_t + 0.8396 \left( \hat{C}_t^P + \hat{I}_t^P \right) \quad (5.97)$$

According to Eq.(5.7), we can identify the relation between precautionary savings and output in linearized form:

$$\hat{Y}_t = \hat{S}_t^P \quad (5.98)$$

On the basis of Eq.(5.97) and Eq.(5.98), the linearized function of precautionary savings can be written as:

$$\hat{C}_t^F + \hat{I}_t^F = 0.1604 \hat{S}_t^P + 0.8396 \left( \hat{C}_t^P + \hat{I}_t^P \right) \quad (5.99)$$

$$\Rightarrow \hat{S}_t^P = 6.234 \left( \hat{C}_t^F + \hat{I}_t^F \right) - 5.234 \left( \hat{C}_t^P + \hat{I}_t^P \right) \quad (5.100)$$

We then drive the aggregate output function for precautionary households based on Eqs.(5.91) and (5.100):

$$\hat{Y}_t = 0.3098 \left( \hat{C}_t^P + \hat{I}_t^P \right) + 0.05919 \hat{S}_t^P + 0.075 \hat{I}_t + 0.135 \hat{G}_t + 0.434 \hat{NFA}_{t+1} - 0.44 \left( \hat{NFA}_t + \hat{R}_t \right) \quad (5.101)$$

A sudden stop  $ka_t$  can be represented by the event that the ratio of capital inflows to GDP,  $ka_t = (KA_t - KA_{t-1})/Y_t$ , falls by more than 5 percent relative to the previous year. The economic shock of capital inflows may cause a decrease of aggregate amount of foreign reserves, because reserves can be used to repay external lines of credit that are not rolled over in a sudden stop. This condition could reduce the demand of domestic expenditure and domestic output  $Y_t$ .

Therefore, the exogenous process of the capital account in the model can be written as:

$$KA_t = KA_{t-1} + ka_t * Y_t \quad (5.102)$$

In the Chinese context, it is reasonable to assume that expected values of the shocks to the capital account are  $E[ka_t] = 0$  in most years.  $ka_t$  is stationary deviation of capital account position. When a sudden stop occurs in year  $t$ , we have  $ka_t \leq ka_{t-1} - 5\%$ , where  $ka_t$  denotes a negative shock to the capital account in the DSGE model. We look for sudden stops in China over 1982-2015; and the capital flows data in our sample and the years in which there was a sudden stop are reported in Figure 9. We notice that only recently since 2014 capital outflows from China have somehow grown. The linearized function of the capital account Eq.(5.102) is

$$\hat{KA}_t = \hat{KA}_{t-1} + \left( \frac{\bar{ka}\bar{Y}}{\bar{KA}} \right) \left( \hat{ka}_t + \hat{Y}_t \right) \quad (5.103)$$

According to the capital flows from 1982 to 2015, we calculate the average steady state value of  $\bar{ka}\bar{Y}/\bar{KA}$ , which is the ratio of the changes in the capital account to the current capital account position.

$$\bar{ka}\bar{Y}/\bar{KA}=-0.17. \quad (5.104)$$

### 5.4.2 Bayesian Estimation of the Parameters

Bayesian estimation and evaluation techniques are now the standard tool for analysis of DSGE models. Estimation with Bayesian methods is based on the likelihood generated by the DSGE system. In contrast to GMM estimation, which is based on particular equilibrium relations, Bayesian estimation fits the complete, solved DSGE model. The Bayesian estimation is also a bridge between calibration and maximum likelihood. By using the Bayes theorem, one can combine the prior density with the likelihood function in order to get the posterior density. In this case, the posterior distribution avoids peaking at the same points as the likelihood. Moreover, the distribution of priors is also very helpful to identify structural parameters and shocks. In this case, we apply the method of An and Schorfheide (2007) and Geweke (1999) to estimate both prior and posterior distribution of the parameters.

We follow Schorfheide (2000) and apply a two-step estimation procedure involving calibration and Bayesian Maximum Likelihood methods. First, the posterior mode and an approximate covariance matrix are obtained by numerical optimization on the log posterior density. Second, we apply the

Metropolis-Hastings and Markov-Chain Monte Carlo (MCMC) algorithm with 3,000 draws to obtain a sequence from the unknown posterior distribution. The Dynare processor for Matlab is employed in the computation.

The parameters in Table 5.1 governing the dynamics of the model are estimated. Most pertain to the nominal frictions and elasticity in the model. We also estimate the ratio of the aggregate expenses by precautionary households to that by perfect foresight households  $RPF$ , and the ratio of precautionary savings to households' expenditure  $RSP$ .

**Table 5.1. Prior and Posterior Distributions of Structural Parameters**

Parameters	Type	Prior Mean	St.Dev	St.Dev	Conf. Int	Post. Mean
$\alpha$	Beta	0.040	0.002	0.0224	[0.3633, 0.4275]	0.3976
$\beta$	Beta	0.980	0.0002	0.0020	[0.9786, 0.9848]	0.9816
$\delta$	Beta	0.040	0.003	0.0035	[0.0249, 0.0305]	0.0278
$\sigma$	Gamma	2.000	0.020	0.0196	[1.9738, 2.0345]	2.0028
$\gamma$	Gamma	3.130	0.003	0.0006	[3.0959, 3.1041]	3.1301
$\varphi$	Gamma	6.160	0.003	0.0007	[6.1553, 6.1641]	6.1597
$\kappa$	Normal	0.500	0.050	0.0491	[0.4564, 0.5862]	0.5151
$h$	Normal	0.610	0.005	0.0049	[0.5906, 0.6119]	0.6012
$\mu$	Gamma	0.840	0.003	0.0027	[0.8365, 0.8456]	0.8414
$\theta$	Gamma	4.610	0.003	0.0033	[4.6047, 4.6151]	4.6101
$l$	Gamma	2.000	0.003	0.0027	[1.9939, 2.0045]	1.9986
$\xi$	Gamma	0.600	0.003	0.0003	[0.5918, 0.5992]	0.5953
$e$	Inv_gamma	0.010	0.0003	infinite	[0.1899, 0.2113]	0.2014
$rp$	Beta	0.3	0.003	0.0018	[0.2958, 0.3049]	0.3000

The complete set of estimation results is presented in Table 5.1. We use the beta distribution for parameters bounded between 0 and 1, except for  $\kappa$  and  $h$ . For parameters measuring elasticity, such as  $\gamma$ ,  $\sigma$ ,  $\varphi$ ,  $\theta$  and  $\xi$ , we use the gamma distribution. Normal distribution is applied when estimating  $\kappa$  and  $\alpha$ . The value of priors and posteriors of the elasticity parameters are very close to each other, which suggests that the value of elasticity parameters is very stable in the framework and well identified. The elasticity of substitution with respect to the depreciation rate is higher than its priors, suggesting a slower response of exchange rate depreciation to the shocks.

In below, the graphs in Figure 5.1.1, Figure 5.1.2 and Figure 5.1.3 show the Markov-Chain Monte Carlo (MCMC) univariate diagnostics for the parameters. These are the main sources of feedback to gain confidence in the results. In order to diagnose to what degree the results are reasonable and accurate, attention is to be paid to two key points. First, we should note whether the results within any iterations of Metropolis-Hastings simulations, no matter how many they may be, are similar. Second, we should observe the distance between various chains. More specifically, the two lines in the charts represent measures of parameter vectors within and between chains, respectively. From Figure 1 we note that for most parameters the distance between the two chains is very small. Moreover, the plotted moments of structural parameters  $\alpha$ ,  $\beta$ ,  $\delta$  and  $\sigma$  are relatively stable and converge. This suggests that the identification of the priors and the results from the chains are reasonable.

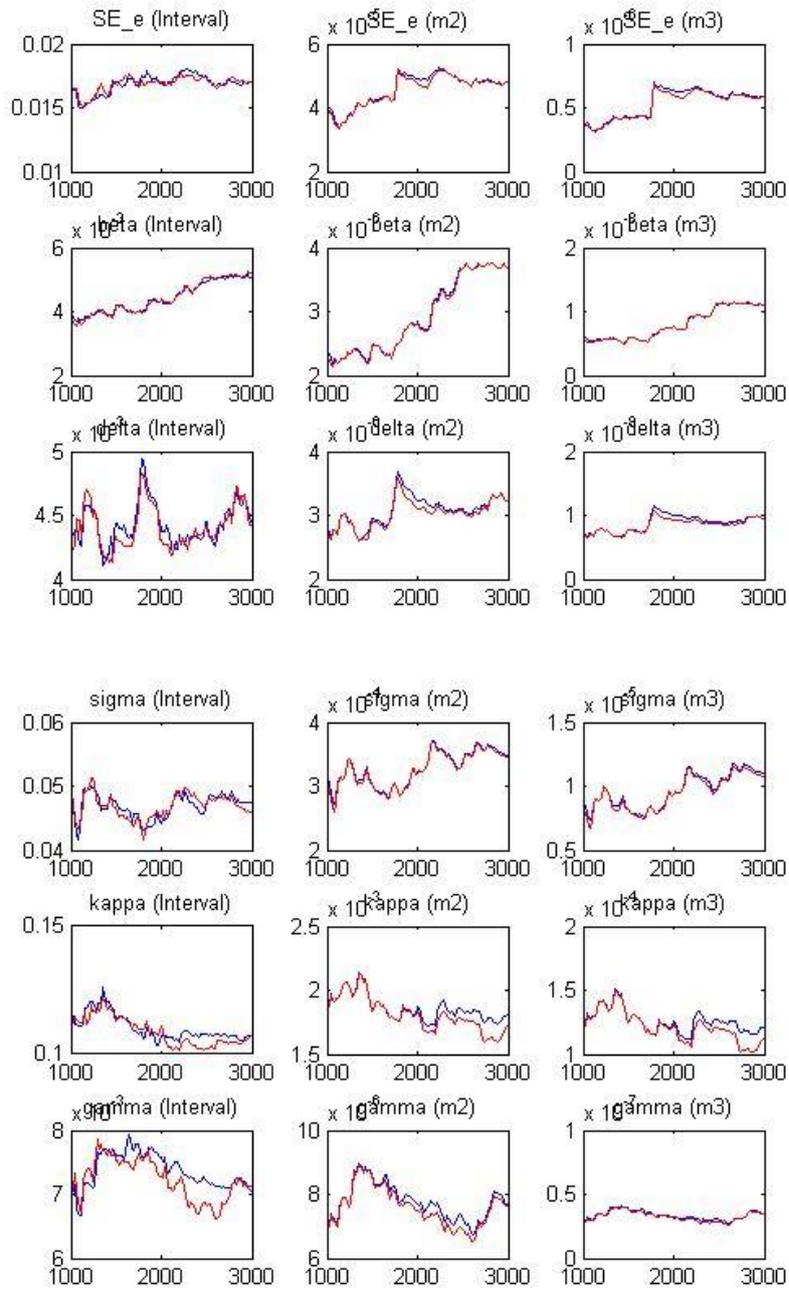


Figure 5. 1. 1 Markov-Chain Monte Carlo Univariate Diagnostics for Parameters (1)

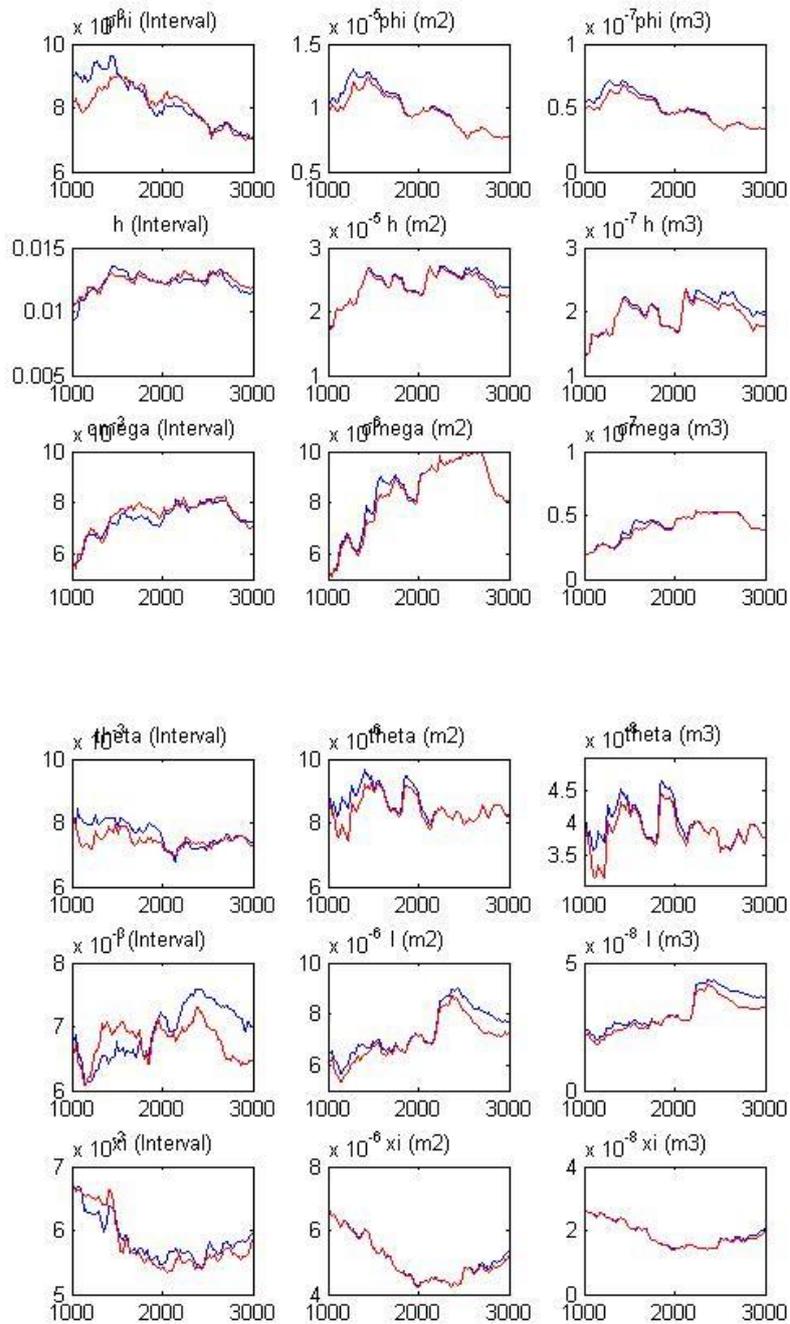
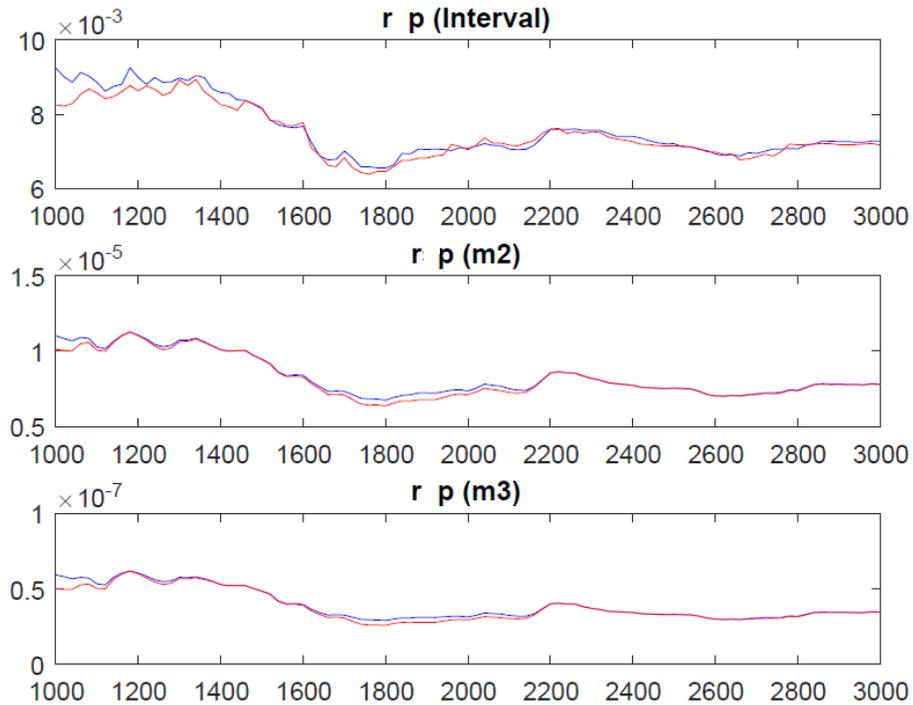
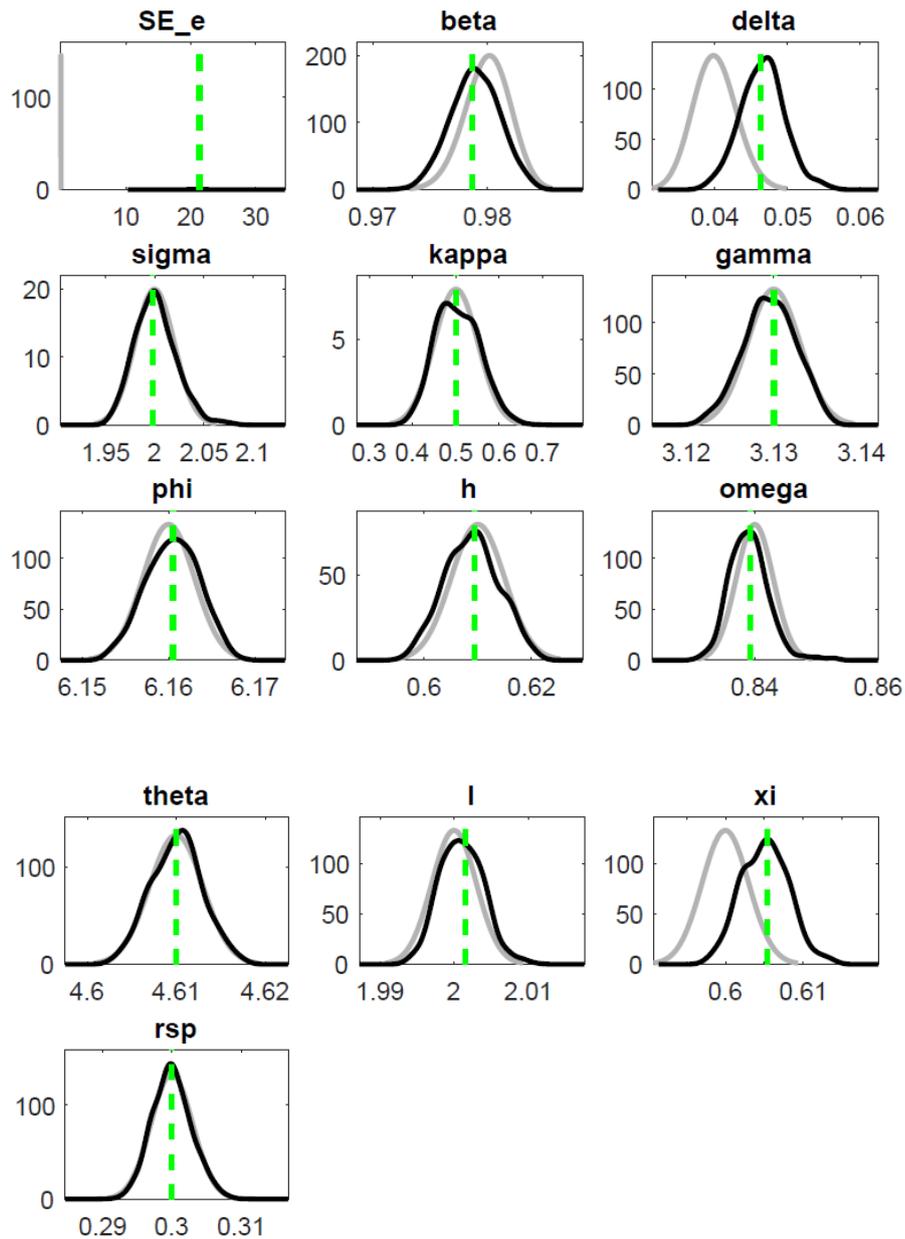


Figure 5. 1.2 Markov-Chain Monte Carlo Univariate Diagnostics for Parameters (2)



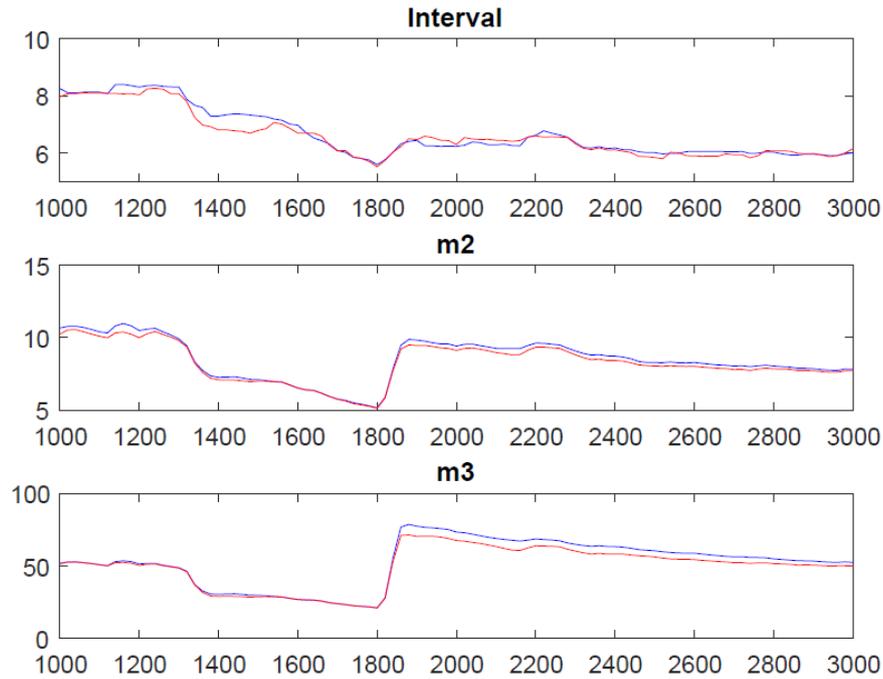
**Figure 5.1.3 Markov-Chain Monte Carlo Univariate Diagnostics for Parameters (3)**

Next, Figure 5.2 displays prior and posterior distributions of the parameters and shocks. Overall, all parameters seem to be well identified, as shown in the outcome that the posterior distributions are either not centred on the priors, or are centred but with a smaller dispersion, indicating high significance of the estimates. For the price stickiness, 84.14 % of firms do not adjust prices within one year. This implies that prices are re-optimized once every three quarters. The fraction of households setting their wages according to the last-period wage  $\xi$  is estimated to be 59.53%, implying a big share of backward-looking firms.



**Figure 5.2 Prior and Posterior Distributions in Metropolis-Hastings Procedure**

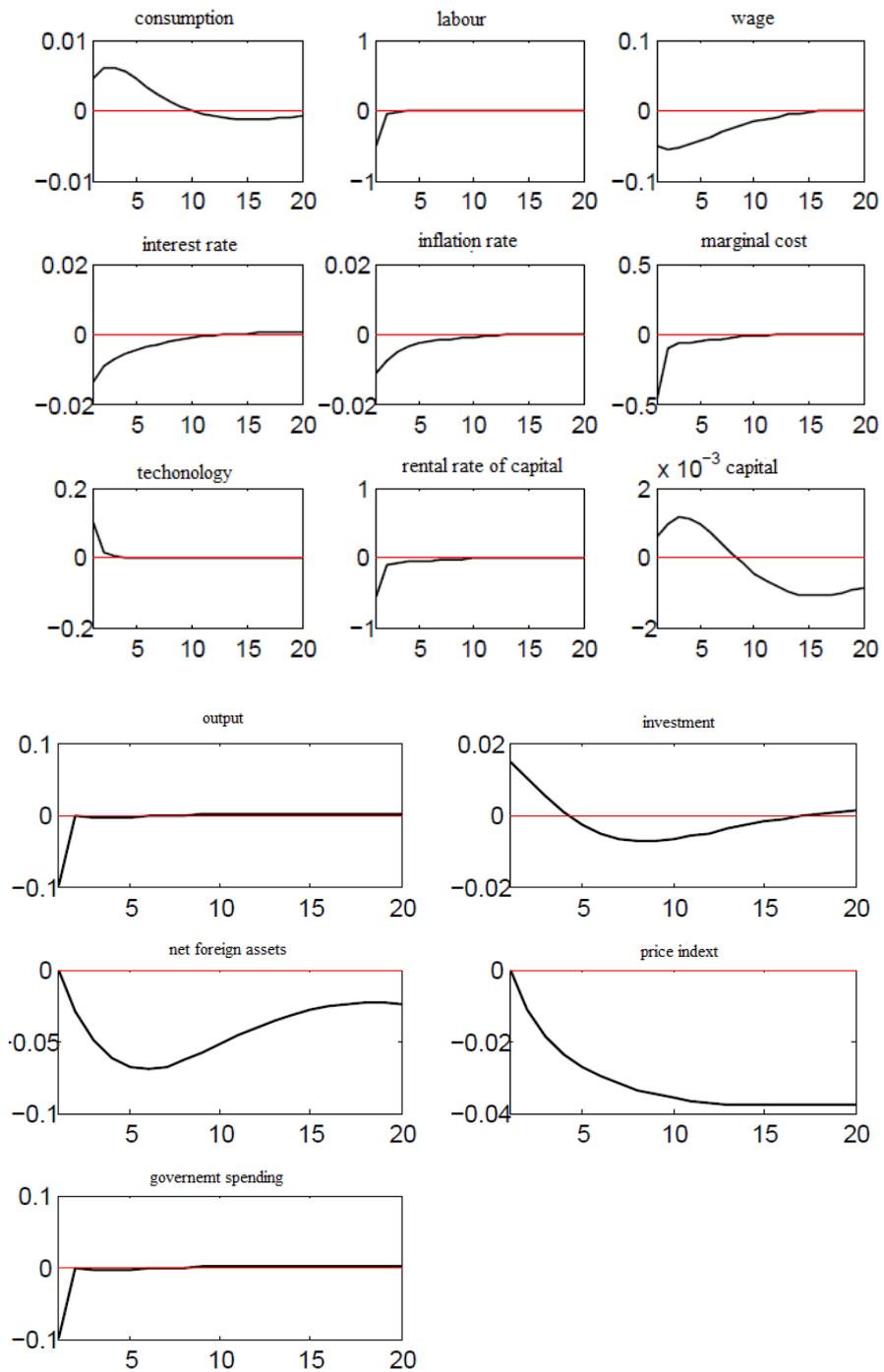
Multivariate diagnostics present an aggregate measure based on the eigenvalues of the variance-covariance matrix of individual parameters. The analysis method of multivariate diagnostics is similar to that of the MCMC univariate diagnostics, because the results of these two diagnostics are based on the same theory. The plotted moments of multivariate diagnostics are relatively constant and converge, implying that the results are reasonable.



**Figure 5.3 Multivariate Diagnostics for Parameters**

## 5.5 Simulation and Impulse Response Functions

We now explore the influences of technology shocks, changes in net foreign assets and precautionary savings in the model, in particular the influence of precautionary savings. First, we present the results of the impulse response functions of every variable to technology shocks. Impulse response functions relate to the reaction of a dynamic system to some exogenous change(s). In our case, the dynamic systems are the linearized models produced in the previous sections. According to the dynamic general equilibrium that we have developed, the model's responses to a technology shock are given in Figure 4.



**Figure 5. 4 Estimated Model Responses to Technology Shock**

Figure 5.4 shows that the reactions of consumption to a technology shock consist of a standard positive level response and a negative change in the adjustment speed of responses. Although these estimated results might not be

precise, they can be applied to explain and analyse the trend of reactions to different shocks.

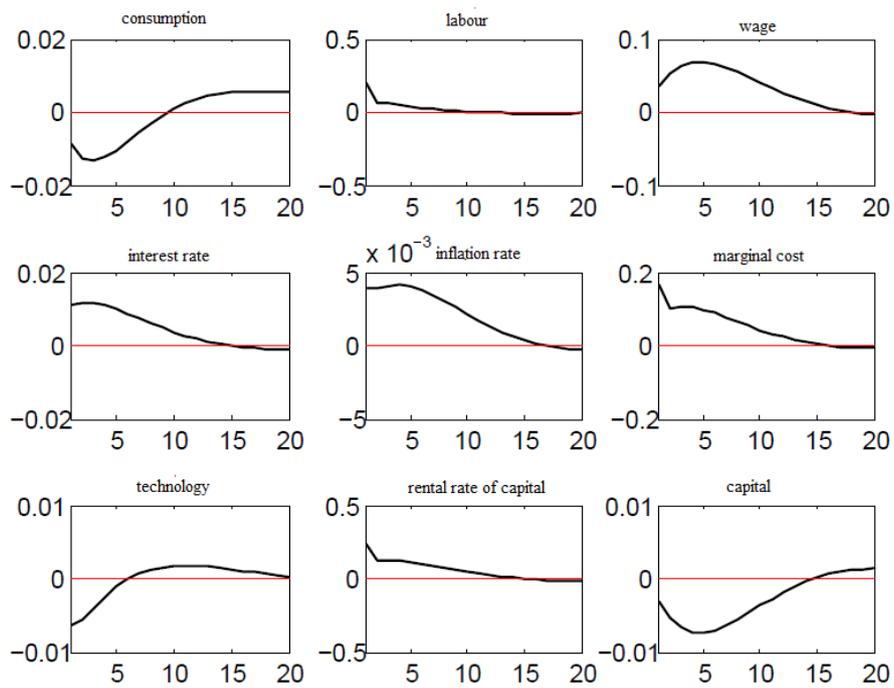
An estimated positive response of consumption in an early time period suggests that a technology shock would cause an increase in consumption in the short run, because more households decide to purchase new technological products to increase their utility. Based on the influence of technology shocks, households and firms determine to invest more money and capital in new production.

Moreover, technology shocks lead to a negative response from labour supply and the rental rate of capital in the first period, but bring out positive responses in the next four periods. This implies that demand for both labour and capital declines at the beginning of an industry transformation; however, with the development of a new production industry, firms require more labour and capital. The augmented demand for capital leads to an increase in the rental rate of capital, because the reactions of real money balances and the rental rate of capital switch to positive after the first period. The impulse response of technology is positive, because the success of invention and innovation will encourage further technological progress.

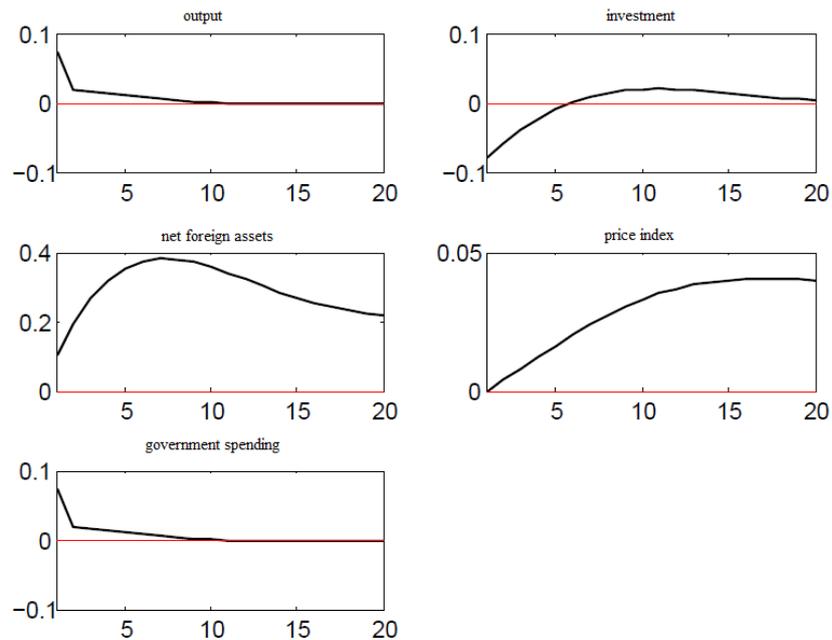
The impulse responses of output are positive after the initial period. The findings correspond to the macroeconomic theory that most technology shocks increase an economy's productivity. Net foreign assets have a negative reaction to a technology shock, suggesting that demand for and

supply of foreign technological products augment the process of industry transformation. However, foreign investors are willing to commit money and assets to emerging economies, which have significant potential for technological development, because they consider that these investments create a good opportunity to bring about financial success.

Figures 5.5 and 5.6 illustrate the impulse responses after a change in net foreign assets. We investigate the relationship between net foreign assets and a main economic variable to explore the impacts of foreign reserves on the general economy.



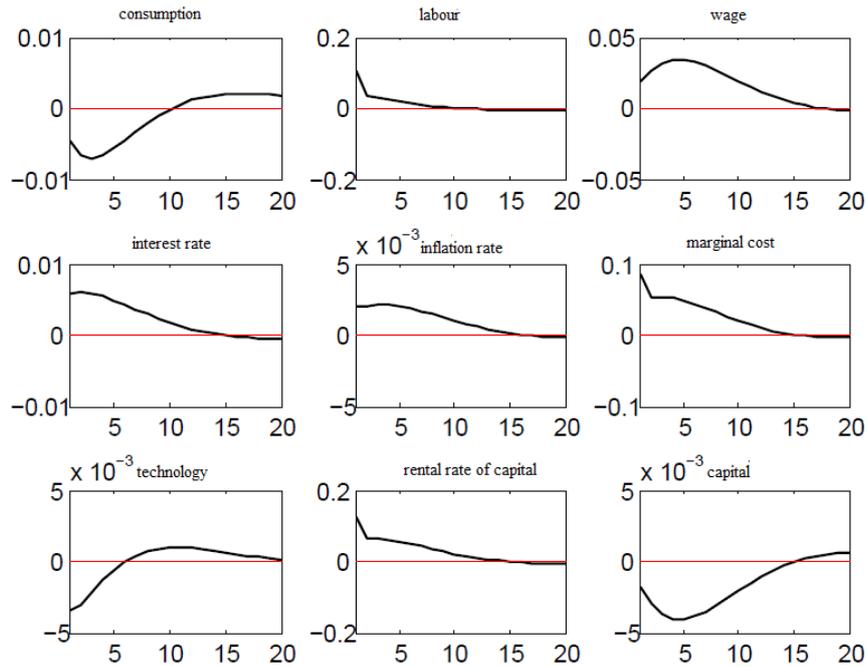
**Figure 5.5 Model Responses to a Change in Net Foreign Assets (1)**



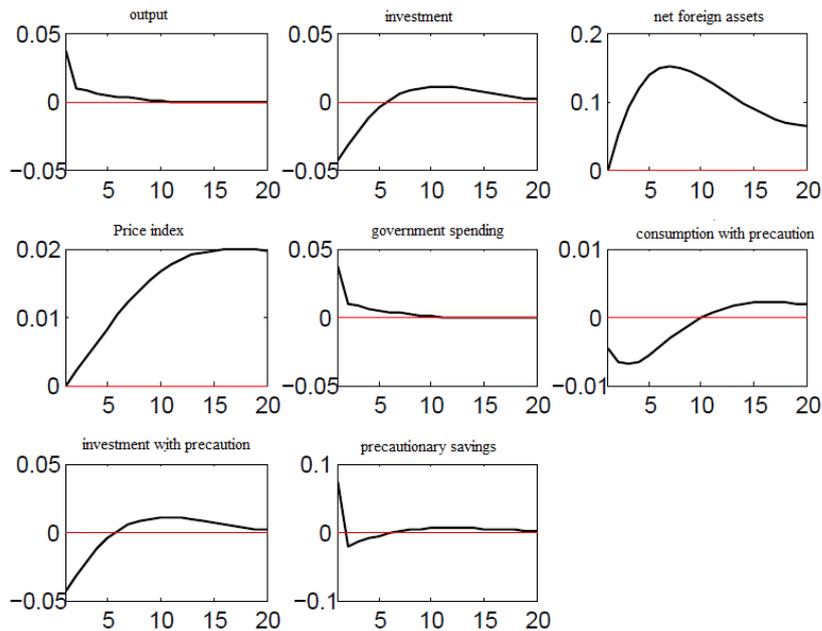
**Figure 5.6 Model Responses to a Change in Net Foreign Assets (2)**

One can see a clear positive relation between the responses of wages and an increase in net foreign assets. An increase in net foreign assets leads to increases in consumption and investments and, as a result, to positive increases in productivity. The contributions of changes in net foreign assets to wages, consumption and inflation are also positive. An increase in foreign reserves causes appreciation of wages, consumption and capital formation in the long run, but also causes the depreciation of investment and capital in the short run. Households reduce their consumption and investment in the current period to accumulate their savings. As a result of inadequate domestic financial development, a large portion of such domestic savings and investments overflows to investments in foreign assets, leading to persistent foreign reserves build-up. In the long run, China's hoarding of foreign reserves has a positive impact on the aggregate output of the economy. The consequent growth of the whole economy raises households' wages and

consumption in the long run, and these play a crucial role in the country's growth. The linearized function in the model represents the behaviour of precautionary households. We then explore the reactions of main macroeconomic factors to a positive shock of precautionary savings in the model of precautionary households.



**Figure 5.7 Model Responses to a Change in Precautionary Savings (1)**



**Figure 5.8 Model Responses to a Change in Precautionary Savings (2)**

According to the impulse response functions, both consumption and investment have negative responses to the shock of precautionary savings. This result is in accordance with the assumption that households decide to reduce their consumption and hence investment to accumulate savings for precautionary purposes. To accommodate the precautionary savings motive, the consumption in the stochastic model is upward tilting and accompanied by a gradual accumulation of a buffer stock of net foreign assets. In initial periods, consumption is lower than in the deterministic case, but as wages increase and the interest income from foreign assets grows, it eventually exceeds consumption in the deterministic case.

Positive changes in precautionary savings also lead to the higher inflation rate and higher wages in the model. Then, according to the model responses, the higher inflation rate leads to higher price index and output. In the domestic economy, changes in precautionary savings cause an increase of wages and inflation, which play a crucial role in China's growth. In order to maintain a low currency value, China's central bank sells more RMB to buy the dollar. As a result, the amount of China's currency in the domestic economy increases, leading to inflationary pressure. The increasing inflation results in high living costs for domestic households. In response, workers demand better wages to protect their living standard. Consequently, Chinese firms face increased labour costs and need to raise the prices of their products. Eventually, increasing wages, consumption and inflation lead to the growth of the whole economy.

The reactions to net foreign assets indicate strong and positive correlation between precautionary savings and foreign reserves accumulation. Quantitatively, in the model economy parameterized to China, an increase by 0.1 percent precautionary savings induces an additional foreign reserves increase by about 0.15 percent of net foreign assets in the initial year. This trend of surpluses gradually decreases thereafter. On the other hand, an 0.1 percent change of precautionary savings leads to 0.05 percent increase of gross output. This suggests that precautionary savings have a positive impact on the growth of the domestic economy.

Following Cherif and Hasanov's (2013) model, we set the ratio of precautionary savings to the households' disposable income be 0.3. On the basis of the parameters values obtained after Bayesian estimation, we calculate that the share of households' precautionary savings in the model is equal to 13.04% of aggregate output. The quantitative importance of the precautionary savings is equal to 26.11% of gross savings and 30.5% of net foreign assets and in the model. This result suggests that precautionary savings are an important driving force behind the accumulation of foreign reserves in China.

Overall, results from the model of precautionary households offer two main findings. First, impulse responses from the parameterized model indicate a significant positive correlation between precautionary savings and net foreign assets. Second, optimal model outcomes for precautionary households indicate that precautionary motives have a positive impact on the growth of

consumption and investment in the long run, while increasing the growth of the domestic economy in the short run.

At last, we explicate how the system reacts to the exogenous process of sudden stop shocks with impulse response functions. We identify expected value of the shocks to capital account  $E[ka_t]=0$  in most years. Capital flows of China are a random walk process in the model, and we report the process of China's capital account over 1998 -2014 in Figure 9 below. When a sudden stop occurs in year  $t$ ,  $ka_t$  denotes a negative shock to the capital account in the model.

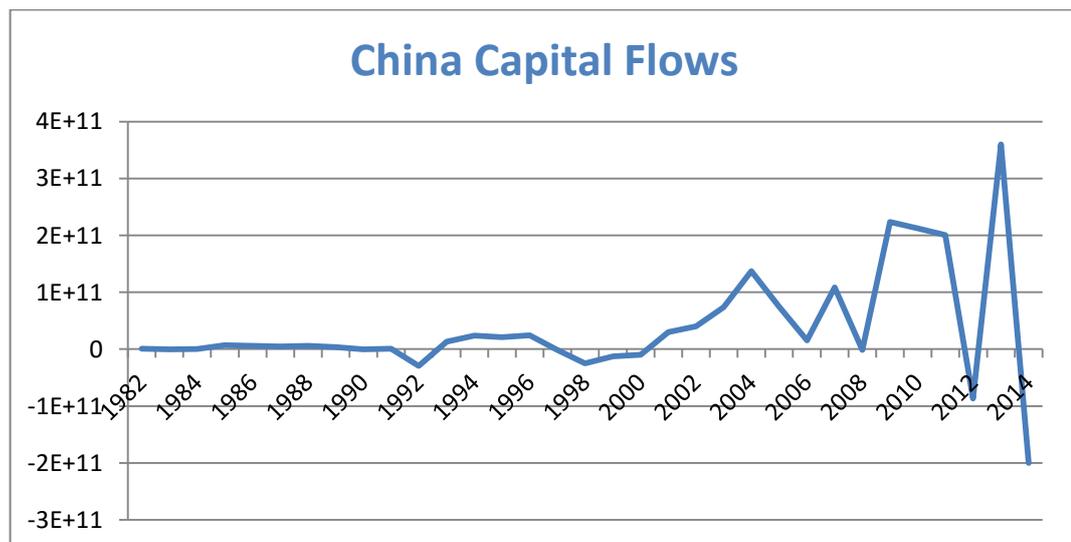


Figure 5. 9 China Capital Flows

As shown in Figure 5.9, a sudden stop in capital flows associated with a fall in output. Capital outflow lead to a long run slowdown of the economy. On the other hand, a sudden stop in capital flows has obvious negative impact on the accumulation of foreign reserves. The decline in net foreign assets is 15% of its initial position at the 40<sup>th</sup> period. This result is in accordance with the

assumption that the Chinese government decide to use large amount of foreign exchange reserves in response to the withdrawal of dollar-denominated deposits from the domestic banking system.

In the first order linearized model, we have the deterministic steady states of net foreign assets is zero. The standard deviation and mean of international reserves is also zero. It means the amount of net foreign assets do not change in a deterministic system.

In second order linear equations, the effect of the shocks depend on the state of the system when the shocks hit. Impulse response functions are the expected future path of the endogenous variables conditional on a shock in period 1 of one standard deviation. They are the results of actual Monte Carlo simulations of future shocks.

We therefore calculate the mean and standard deviation of net foreign assets in second order linear equations. The mean value of the changes of foreign reserves is also zero, but the standard deviation of it changes is 4.7563% in response to a 5% capital outflow of GDP shock in the stochastic system. If we try to explore the reaction of system to a 1% capital outflow of GDP, we will have the standard deviation of foreign reserves is 0.95%. We consider that the gap between deterministic steady state and stochastic steady states of foreign reserves indicates the amount of precautionary foreign reserves against the risk of sudden stop. The amount of precautionary reserves depends on the size of potential capital outflow. We observe a fall in the capital account in 2014,

amounting to more than 3.4 percent of the previous year's output. In this case, the standard deviation of net foreign assets is 3.237%.

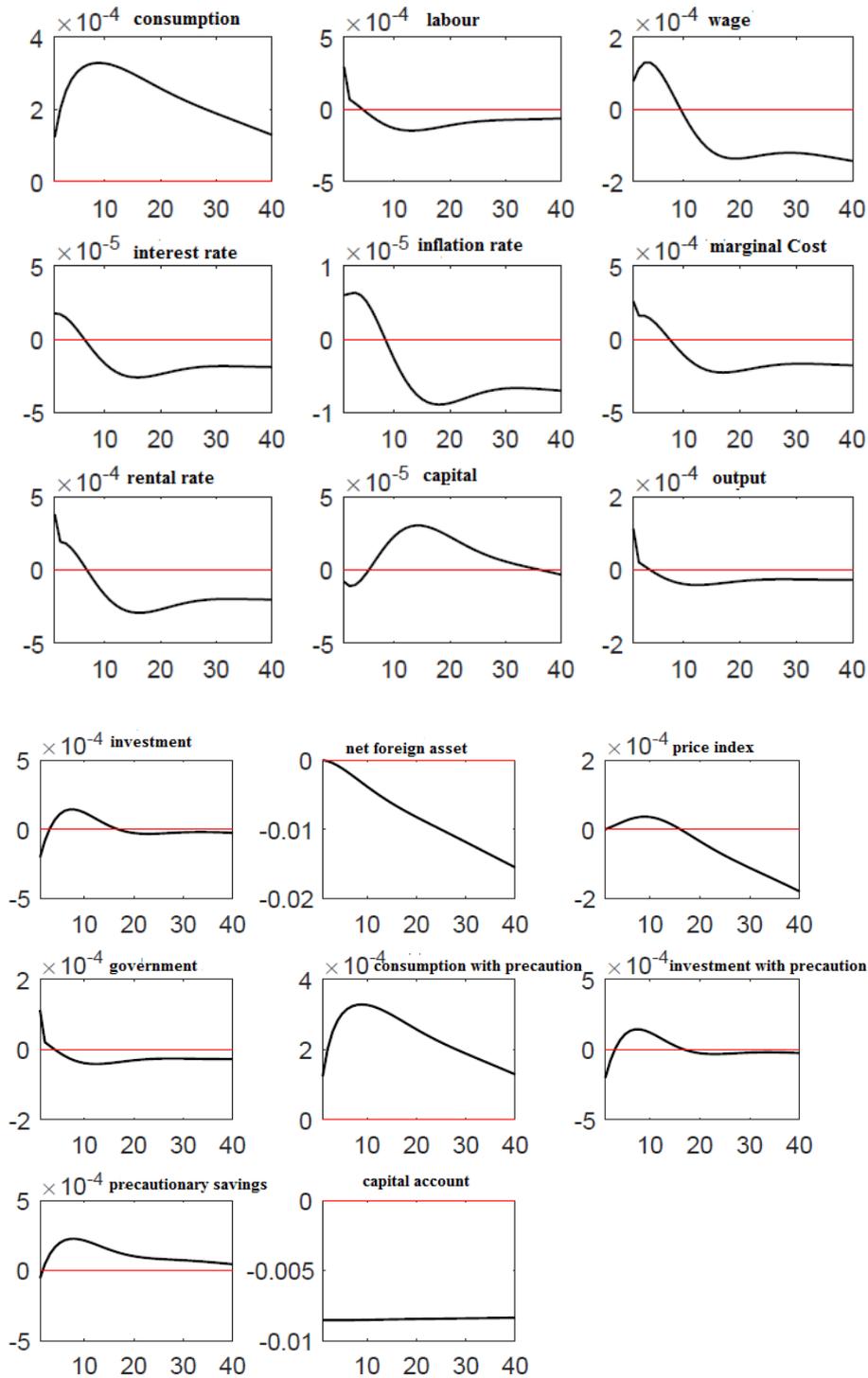


Figure 5.10 Estimated Model Responses to Sudden Stop in Capital Flows

## 5.6 Conclusions

This Chapter uses a parameterized DSGE model to quantify the role of precautionary savings in China's economy, which has recently been slowing down. The model parameters are estimated by Bayesian techniques. The quantitative importance of the precautionary savings is equal to 26.11% of gross savings and 30.5% of net foreign assets and in the model. The estimation results suggest that precautionary motives play an important role in the economy and are an important driving force behind the accumulation of foreign reserves in China.

The importance of the precautionary motive for holding foreign reserves is unquestioned in the literature, whereas quantifying its role in central banks' behaviour of reserve accumulation remains a challenge. This paper develops an innovative approach to measuring the influence of the precautionary motive on foreign reserves build-up in China.

Two types of representative households are considered in a DSGE framework. They are households with perfect foresight and precautionary households. The perfect foresight households assume a certainty world where their expectations and predictions are all valid. They therefore do not have a precautionary motive for holding additional savings for unforeseen income fluctuations. By contrast, precautionary households are aware of the existence of a risky world and anticipate that future shocks would hit the economy. They therefore accumulate additional savings as a buffer stock scheme to

safeguard against future risks that are uninsurable. Precautionary savings are thus the additional amount of the savings of the precautionary households facing uncertainty compared to the savings by perfect foresight households who perceive complete certainty in the economic environment.

When carrying over to the whole economy, the limit behaviour of the perfect foresight households results in the perfect foresight path of the economy. In the DSGE models, the corresponding equilibrium is the deterministic steady state. For the precautionary households, they instead anticipate the convergence of economic variables to some risky steady state. We then solve the DSGE models around the deterministic steady state and the risky steady state, respectively. Using the results from the model solved with the deterministic steady state as the benchmark, we compare the results from the two models to get an empirical gauge of the importance of precautionary demand for foreign reserves which are measured as the difference between the results from the benchmark model and that with the risky steady state.

The estimation results suggest that the precautionary motive is an important driving force behind massive accumulation of foreign reserves in China and play an important role in the economy. The impulse response of net foreign assets to changes in the precautionary motive indicates a positive correlation between the precautionary demand and accumulation of foreign reserves. According to the analysis of impulse response functions, a 0.1 percent change in the precautionary demand leads to a 0.15 percent increase in foreign

reserves in the initial period. Precautionary foreign reserves also have a positive impact on other variables in the model.

For instance, model estimation shows a 0.1 percent increase in precautionary savings and hence the precautionary reserves leads to a 0.05 percent increase of gross output, suggesting that changes in precautionary foreign reserves will affect domestic growth. Furthermore, while the amount of precautionary reserves held by the central bank is a function of risks and shocks, in the Chinese context, they are mainly influenced by potential capital outflows, rather than sudden stops of capital inflows. For a 1% capital outflows in terms of GDP, the standard deviation of foreign reserves will be 0.95%.

Overall, this chapter contributes to the literature by building a parameterized DSGE model with precautionary savings, which are generally absent in previous studies. The estimation results suggest that the precautionary motive has positive impacts on China's hoarding of international reserves, and shed critical lights on the properties of the precautionary for foreign reserves.

## **Chapter 6**

# **The Relative Contribution of Competing Explanations**

In this chapter, we have discussed the role of three fatal motives that play in China's accumulation of international reserves that we have mentioned in the previous chapters. They are intergenerational transfers, mercantilism and precautionary motive. To evaluate competing explanation about the foreign exchange reserves, we develop an estimated DSGE model including above motives for China. These motivations can be given a structural interpretation as exogenous or endogenous process in the model. By the analysis of impulse response function and shock decompositions of the model, we pursue the primary motivation for the reserve accumulation.

### **6.1 Introduction about Competing Explanations**

In recent years the large amount, and rapid growth, of China's foreign exchange reserve has attracted considerable attention worldwide. In 2006, China exceeded Japan to become the world's biggest holder of foreign

exchange reserves. The massive reserve holdings by China represent a prominent case of the recent upsurge in reserve hoardings among emerging market economies, cementing even further its position as the world's largest holder of foreign exchange reserves.

Moreover, three main indicators of reserve adequacy suggest that international reserves might be excessive in China's economy. These indicators are import-based measures of reserve adequacy, money-based measures of reserve adequacy and debt-based measures of reserve adequacy.

Import-based indicators of reserve adequacy depend on reserves in months of imports. This ratio provides a simple method to scale the level of foreign exchange reserves by the size and openness of the economy. Foreign reserves in months of imports have a forthright explanation: the number of months a country can continue to support its current level of imports if all other inflows and outflows stop. Many researchers consider three month of imports is the optimal level of foreign reserves. However, average value of total reserves in months of imports of China is 17.45, in the period from 2001 to 2014. This indicator is much higher than three month of the country's imports.

In addition, Money-based indicators of reserves identify a way of evaluating the potential for resident-based capital flight from the currency. An unstable demand for money or the presence of a weak banking system indicates a greater probability of such capital outflows. In these conditions, the ratio of reserves to base money is therefore a potentially useful indicator. Moreover,

the ratio of reserves to broad money is a significant indicator for scaling the optimal level of foreign reserves under fixed exchange rate regimes. Average value of China's broad money to total reserves ratio is 4.81 in the period from 2001 to 2014.

Moreover, a measure comparing reserves and short-term external debt is very helpful to estimate risks associated with adverse developments in international capital markets in recent years. By comparing the level of reserves to a measure of short-term external debt, one can measure the size of all debt repayments to foreigners over the coming year and make a judgement about how quickly a country would pay off its external debts. Empirical work in the Fund implies that debt-based measure of reserve adequacy is a substantial indicator in countries with significant but uncertain access to capital markets. A smaller international reserves to short-term debt ratio is associated with a greater incidence and depth of crises. The average ratio of short-term external debts to total reserves in China is 16.7% in the period from 2001 to 2014.

However, these circumstances begs the question of why the Chinese government decided to accumulate such massive foreign exchange reserves in the first place; in other words, what is the primary motivation for this reserve accumulation? Is the government trying to self-insure against foreign exchange potential currency crisis, or is it manipulating the exchange rate to achieve a competitive advantage? Does other motivation also play a role in China's hoarding of foreign exchange reserves?

The significant increase in foreign exchange reserves in China following 1997 Asian financial crisis linked precautionary motive to international reserves hoarding. According to the concept of holding international reserves for precautionary reasons, the accumulation of international reserves is considered as an insurance against sudden capital outflow by policy makers. Heller (1966) qualifies optimal level of reserves by weighting opportunity cost of holding reserves and the risk of an external disequilibrium. Jeanne and Ranciere (2006) argue that another precautionary reason of foreign exchange reserve accumulation is to sustain domestic absorption in times of a Sudden Stop in capital flows.

Lee (2004) estimates the optimal level of foreign exchange reserves based on option price theory. He considers the assumption that an overall insurance value equals to the amount of short term external debt is needed for precautionary reasons. He believes that this overall insurance level will be met partially through market-based insurance and partially by self-insurance. Wyplosz (2005) also believes that precautionary saving is the main driver of China's reserve accumulation. This is a traditional motive for central banks to accumulate international reserves, to meet the need for foreign currency to protect against potential turbulence on currency markets. Such turbulence occurs when private capital flows suddenly threaten to bring unwelcome changes to a country's exchange rate.

Following on the steps of Jeanne and Ranciere (2006), Ruiz-Arranz and Zavadjil (2008) try to explore whether foreign exchange reserves in emerging

Asia are excessive when compared with optimal levels predicted by their model. They deem that international reserve holdings by most Asia countries seem not to be above the optimal levels, excluding China. They identify that the costs of sudden stops were greater in emerging Asia countries than the average size that derived by Jeanne and Ranciere (2006). They conclude that the precautionary motive is a critical driver behind reserve accumulation of Asian countries, which has been reinforced over the last decade.

However, since the Asian crisis, models based on the standard economic explanatory variables have underestimated the reserve holdings of China. This unusual accumulation is a sign that factors other than purely precautionary motives might play an important role. Dooley, Folkerts-Landau and Garber (2006) therefore take a different view and follow a modern mercantilist approach to account for hoarding of international reserves as part of a deliberate development strategy. They consider international reserves act as collateral for encouraging foreign direct investment. The mercantilist motive hypothesis regards reserves accumulation as a by-product of an export promoting strategy. The reserve accumulation facilitates this strategy by preventing or slowing currency appreciation.

In the Chinese context, Bonatti and Fracasso (2013) argue that capital controls and an undervalued currency make the country less exposed to sudden stops of capital movements, and so the mercantilist motive is particularly appropriate to account for the reserve accumulation. In their empirical research, Delatte et al. (2014) formally show that the mercantilist

motive rather than the precautionary concern is more consistent with the rapid reserve accumulation in China.

In the literature, a body of recent empirical research has documented a positive relationship between the real exchange rate and growth. Rodrik (2008) discusses the significant role of an undervalued real exchange rate in stimulating growth. Hausman et al. (2005), Johnson, Ostry and Subramanian (2006) and Bereau et al. (2012) also argue for the importance of a depreciated real exchange rate to accelerate growth, and believe that this exchange rate policy is an efficient tool for economic development.

Aizenman and Lee (2007) compare the weightiness of precautionary and mercantilist motives in international reserves accumulation by developing countries. They explicate the importance of mercantilism on international reserves accumulation by distinguishing monetary mercantilism from financial mercantilism. The latter is a modern form, often labelled the export-led strategy. A common representation of this strategy is that it seeks to boost export growth by maintaining an undervalued exchange rate. Although mercantilist effects are significant to export growth and purchasing power parity, they believe that mercantilist effects have a smaller impact relative to variables associated with precautionary demand in the determination of the appropriate level of reserves. Their empirical results support the significance of precautionary motives.

Another important reason for holding foreign exchange reserves is intergenerational transfers. Although any or all of the existing conventional explanations may be true to a certain degree (Bonham and Wiemer, 2013), we consider that the role that intergenerational transfers play in boosting China's savings rate merits exploration. Intergenerational transfers within and between Chinese family networks have begun to attract growing interest. Many research show that intergenerational transfers are a significant driver of high domestic savings and that, due to inadequate domestic financial development, a large portion of such domestic savings overflows into foreign asset investments, leading to persistent foreign reserve accumulations.

Kotlikoff and Summers (1981) show that 80% of U.S. national wealth results from intergenerational transfers, whereas only 20% is accounted for by life-cycle savings. Although the magnitude of this estimation has been disputed by subsequent research, international evidence generally confirms that intergenerational transfers are an important source of wealth accumulation. Given the historically tight family ties and cultural influences in China, it is conceivable that intergenerational transfers are a significant contributor to wealth and thus a critical driver for growing Chinese household savings.

Wu, Gyourko and Deng (2012) believe that Chinese intergenerational transfer arrangements are largely influenced by inflated housing prices. In China, the real value of constant quality land has increased by nearly 800% since 2003. Price-to-income ratios are at their highest levels in the coastal areas. In early 2010, average housing prices hovered at approximately 18.5

times average annual income, and these prices have since risen even higher. Chivakul et al.(2015) argued that China's average nationwide house price in 2014 has risen to about 22 times average annual disposable income in 2013. In these circumstances, it becomes extremely difficult, if not impossible, for many young households in China to purchase a satisfactory property without financial support from their parents. Chinese parents who desire to offer this support must save more for the property needs of their offspring.

In short, as a result of factors such as Chinese traditional culture, the undeveloped financial system and excessively high housing prices, Chinese parents may have to save more for their children's education, housing and marriage. As a consequence, intergenerational transfers provide critical support for China's young households to improve their standards of living and general utility.

China's high and rising savings have wider implications. One profound implication concerns external balance. It is well known that a country's external balance ultimately reflects a disparity between domestic savings and investment (Krugman, 1990). Yu (2007) argues that China's high savings rate is one of the main drivers of the growth in the country's current account surplus. By implication, this savings rate in turn would affect the country's net foreign asset position. Yang (2012) shows that when China's ill-functioning financial system fails to channel the increased accumulations in savings to investment or consumption, the excess savings end up as foreign reserves. Conversely, the amount earned by the trade surplus that is not consumed or

invested must end up being saved. Lugauer and Mark (2013) also note that China would not have its current account surplus without the support of high savings from this sector, given that the household sector accounts for the largest share of national savings. Therefore, based on their importance to household savings in China, intergenerational transfers should be a driving force behind China's hoarding of foreign reserves.

Some authors have already made attempts to elaborate the methodology to compare the influence of the different motivations. Lee (2004) and Aizenman and Lee (2007) estimate existing reserve levels (deflated by the GDP) using two different sets of explanatory variables. The first set attempts to capture the self-insurance motive. It includes a measure of capital account liberalization, crisis dummies, exchange rate volatility, the terms of trade, and openness. The second set concerns the mercantilist motive. It includes export growth and a measure of exchange rate undervaluation.

They find that the self-insurance motive variables are highly significant and in accordance with the theoretical prediction. In particular, capital liberalization emerges as a consistently solid explanatory variable. The variables meant to capture the mercantilist motive are also statistically significant, but their effects are small in comparison with those of the self-insurance variables. Overall, the authors conclude that their "results provide only a limited support for the mercantilist approach" (Aizenman and Lee, 2007, p.4).

In the light of these results, Wyplosz (2007) considers that “recent accumulation of reserves has largely been driven by the recognition that financial globalization calls for a new attitude to deciding on reserve adequacy. In’t Veld et al.(2011) tried to evaluate several competing theories about the recent business cycle by using an estimated DSGE model. They provide a method to calculate the quantitative importance of various shocks. The parameters of the estimated shocks are adjusted to fit the framework of the model and actual macroeconomic data. Therefore, the shocks can be applied to identify the decomposition of the data.

To investigate the primary explanation for China’s hoarding of foreign reserves, we follow In’t Veld et al.(2011)’s method and propose a DSGE model that includes precautionary motive, mercantilist motive and intergenerational transfers arrangements. The impulse response functions of key variables to domestic savings and exchange rate policy are explored in a New Keynesian DSGE setting and the model’s calibration is discussed. The correlation between various key variables is explicated in the impulse response analysis. Bayesian estimation and evaluation techniques are applied to analyse relevant parameters and model performance. On the basis of the estimate historical evolution of shocks, we introduce shock decompositions for intergenerational transfers, exchange rate, precautionary savings, export growth and GDP growth to evaluate the quantitative importance of each explanation.

The chapter is organized as follows. Following this introductory section, section two presents the structure of the model and describes some new variables in the DSGE framework. Section three shows the consequential first order conditions (FOCs) engendered by households' and firms' optimization behaviours, and presents the linearized functions of the general equilibrium. Section four estimates and evaluates the parameters, some of which are taken from previous empirical research. Section five applies Bayesian methods to estimate parameters and evaluate the model's performance. Section six investigates quantitative importance of the shocks. Section seven concludes the chapter.

## **6.2 Model Including Three Motivations**

### **6.2.1 Intergenerational Transfers**

We consider an economy that consists of representative agents with intergenerational transfer motives and a maximum life expectancy of two periods. The representative agent assumption in DSGE model implicated that everyone in the economy had the same preferences, and the same relative income of capital, labour skills, habits and so on. After the first period of life, each individual has  $m_t$  children, where  $\rho$  denotes the parents' generation index. That means an agent born in  $t_\rho$  will have her children at period  $t_{\rho+1}$ . According to the report of Chinese Academy of Social Sciences, China's average childbearing ages had risen to 28.12 in the period from 2004 to 2014.

We consider  $t$  as the time period of one year and  $t_\rho = t$ , then we can have

$$t_{\rho+1} = t + 28.$$

As a result, at any given point of time, the economy consists of two overlapping generations. We assume that both generations receive money and assets from their parents and that both will transfer wealth, including money and property, to their children, for which purpose they save. Households decide their first and second period consumption,  $C_{t_\rho}^1$  and  $C_{t_\rho+1}^2$ . The superscripts 1 and 2 represent the time period. The individual receives *per capita* bequests  $B_t$  when they are young and gives *per capita* inheritance  $B_t$  to her children in the second period. Consequently, the expected utility function of a representative agent can be described as:

$$U_t = E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \frac{(C_{t_\rho}^1)^{1-\sigma}}{1-\sigma} + \frac{\ell (C_{t_\rho+1}^2)^{1-\sigma}}{1-\sigma} + \frac{(B_t)^{1-\gamma}}{1-\gamma} + \frac{(J_t)^{1-\gamma}}{1-\gamma} - \frac{(N_t)^{1+\varphi}}{1+\varphi} \right] \right\} \quad (6.1)$$

According to this formulation, the agent derives utility from first and second life time period consumption  $C_{t_\rho}^1$  and  $C_{t_\rho+1}^2$  labour supply  $N_t$ , bequest received from her parents  $B_t$ , and joy brought by her children  $J_t$ . Agents value future consumption less than present because of the uncertainty associated with future events. Therefore, we use parameter  $\ell$  to describe the discount rate between two time periods of live.  $\ell = (1/R)^{\wedge 28}$ . We have  $R = 1+r$ , where  $r$  is average interest rate. The parameter  $\sigma$  denotes the elasticity between first period consumption and leisure.  $\gamma$  and  $\varphi$  represent the elasticities among

bequeathing, joy and labour supply. Replacing  $t_\rho = t$  and  $t_{\rho+1} = t + 28$  in Eq.(6.1), we can have

$$U_t = E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \frac{(C_t^1)^{1-\sigma}}{1-\sigma} + \frac{\ell(C_{t+28}^2)^{1-\sigma}}{1-\sigma} + \frac{(B_t)^{1-\gamma}}{1-\gamma} + \frac{(J_t)^{1-\gamma}}{1-\gamma} - \frac{(N_t)^{1+\phi}}{1+\phi} \right] \right\} \quad (6.2)$$

We assume that  $A_t$  is the overall attention provided by the second generation.  $A_t$  denotes the fraction of household's children's disposable time not dedicated to work. Household earns a wage per unit of time  $W_t$ . Consequently, the relation between joy and attention can be stated as:

$$J_t = W_t A_t . \quad (6.3)$$

$$A_t = \vartheta \cdot (1 - N_t) . \quad (6.4)$$

The dynamic budget constraint of first generation households is:

$$C_t^1 + BO_t / (P_t R_t) = BO_{t-1} / P_t + (W/P_t) N_t + r_t^C K_t + B_t - d_t , \quad (6.5)$$

where  $B_t$  is intergenerational transfers received from their parents,  $BO_t$  is the coupon bond,  $R_t$  denotes the gross returns on the bond,  $W_t$  denotes the nominal wage rate,  $P_t$  is the consumption price index (CPI), and  $d_t$  represents savings. The main parameters and variables are summarized in Tables 1 and 2. The consumption inflation rate is represented as  $\Pi_t \equiv P_t / P_{t-1} \Rightarrow \Pi_t = 1 + \pi_t$ , where  $\pi_t$  is the inflation rate.

At the end of the second generation period, the agent gains interest on his savings at a rate  $r^G$ ,  $r^G = (1+r)^{\wedge 28}$ . Furthermore, she gives bequests to her children and these bequests are shared equally among the agent's  $m_t$  offspring. The second period budget constraint is therefore given by:

$$C_t^2 + BO_t / (P_t R_t) + J_t = BO_{t-1} / P_t + J_{t-1} + d_t (1+r^G) - m_t B_t \quad (6.6)$$

Then, we calculate the difference between first and second life period budget constraints, and the result is:

$$C_t^2 - C_t^1 = d_t (2+r^G) - (m_t + 1) B_t - w_t N_t - r_t^C K_t \quad (6.7)$$

Assuming  $c_t^2 = c_t^1$ , Eq.(6.6) can be rewritten as:

$$B_t = \frac{2+r^G}{m_t + 1} d_t - \frac{1}{m_t + 1} w_t N_t - \frac{1}{m_t + 1} r_t^C K_t \quad (6.8)$$

## 6.2.2 Perfect Foresight Households

We assume there are two types of households in society, i.e the perfect foresight households and risk-averse precautionary households. In order to explore households' precautionary savings, we consider two different budget constraints for two types of these households. Perfect foresight households assume their expectations and predictions are valid; therefore they do not

consider precautionary savings in their budget constraint. On the other hand, the risk-averse representative households face uncertainty about the state of the economy in the future. Therefore, precautionary savings occur in their budget constraint due to uncertainty regarding future income.

We consider that the aggregate period-by-period budget constraint of perfect foresight households is:

$$C_t^F + I_t^F + NFA_{t+1} + (BO_t/P_t R_t) = (1+r_t)NFA_t + (W_t/P_t)N_t + r_t^C K_t + (BO_{t-1}/P_t) \quad (6.9)$$

In Eq.(6.9)  $C_t^F$  represents the aggregate consumption of the perfect foresight household;  $NFA_t$  stands for the stock of net foreign assets at the end of period  $t-1$ , and  $r_t$  is the risk-free rate. The difference between aggregate export and import constitutes the trade balance, equal to  $NFA_{t+1} - (1+r_t)NFA_t$ .  $BO_t$  denotes the coupon bond;  $P_t$  is the consumption price index (CPI), and  $I_t^F$  is investment of the perfect foresight household.  $NFA_t$  stands for the stock of net foreign assets at the end of period  $t - 1$ ;  $r_t$  is the risk-free rate;  $W_t$  and  $r_t^C$  denote the nominal wage rate and the rental rate on capital in period  $t$ .  $P_t$  is the consumption price index (CPI);  $W_t$  and  $r_t^C$  denote the nominal wage rate and the rental rate on capital in period  $t$ .

### 6.2.3 Precautionary Households

Differing from the perfect foresight households, precautionary households decide to delay their consumption and save in the current period, due to incompleteness of the capital market and uncertainty regarding future income. Facing an uncertain and risky world, these households predict that their incomes can be affected by negative shocks in the future. To avoid adverse effects of future income fluctuations and to retain a smooth path of consumption, they engage in precautionary savings by consuming and investing less in the current period. Therefore, we consider that the aggregate period-by-period budget constraint of precautionary households is:

$$C_t^p + I_t^p + NFA_{t+1} + (BO_t/P_t R_t) + S_t^p = (1+r_t)NFA_t + (W_t/P_t)N_t + r_t^c K_t + (BO_{t-1}/P_t) \quad (6.10)$$

In Eq.(6.10),  $C_t^p$  represents aggregate consumption of precautionary households;  $I_t^p$  represents investment of precautionary households. They set aside precautionary savings  $S_t^p$  in case the economy deteriorates in the future.

$r_t$  is the risk-free rate of return.  $R_t = 1+r_t$ ,  $r_t^c$  is the rental rate of capital.

Subtracting  $R_t NFA_t$  from both sides of Eq.(6.10), we have:

$$C_t^p + I_t^p + NFA_{t+1} - R_t NFA_t + (BO_t/P_t R_t) + S_t^p = (W_t/P_t)N_t + r_t^c K_t + (BO_{t-1}/P_t) \quad (6.11)$$

The sum of both sides of Eq.(6.10) is equal to the aggregate output.

Subtracting the precautionary budget constraint Eq.(6.11) from the perfect foresight budget constraint Eq.(9),we have:

$$C_t^P + I_t^P + S_t^P = C_t^F + I_t^F \quad (6.12)$$

Eq.(6.12) describes the relation between the two types of households, which make different choices in their consumption and investment decisions. The model assumes every representative household earns the same income in the current period. Eq.(6.12) expresses the fact that precautionary households decide to consume and invest less in the current time period than do perfect foresight households. We calculate the difference between the disposable income of precautionary households and perfect foresight households, then Eq.(6.12) can be rewritten as:

$$S_t^P = C_t^F + I_t^F - (C_t^P + I_t^P) \quad (6.13)$$

In the model, precautionary savings are defined as the difference between the consumption and investment of precautionary households and perfect foresight households, due to the precautionary motive.

According to the result of Cherif and Hasanov's (2013), they consider precautionary savings of households are about 30% of disposable income. Depending on the data from the IMF Country Report for People's Republic of China and National Bureau of Statistics of China., we calculate the average value of total household disposable income in percent of GDP is 43.47% in

the period from 2012 to 2015. Therefore the function of precautionary savings can be considered as:

$$S_t^P = (0.4347Y_t) \cdot rp = C_t^F + I_t^F - (C_t^P + I_t^P) \quad (6.14)$$

Where  $rp=0.3$  denotes the ratio of precautionary savings of households to disposable income. In the model, precautionary savings are defined as the difference between the consumption and investment of precautionary households and perfect foresight households, due to the precautionary motive.

#### 6.2.4 Firms (Entrepreneurs)

We apply the Dixit-Stiglitz (1977) framework to describe the behaviour of the firms and retailers.

**A production aggregator**  $Y_t = \left[ \int_0^1 Y_t(i)^{(\theta-1/\theta)} di \right]^{(\theta-1/\theta)}$ , (6.15)

where  $Y_t$  is a composite final good that is produced by a representative firm.

The final-good producer chooses a continuum of intermediate goods  $Y_t(i)$  as production input, indexed by  $i \in [0,1]$ , each produced by a unique monopolistically competitive firm. Firms and retailers set prices in a Calvo-staggered manner.  $\theta$  is the elasticity of substitution between goods.

$Y_t$  represents the income or output of the whole economy, and can be represented by a Cobb-Douglas production function:

$$Y_t(i) = Z_t K_t(i)^{1-\alpha} N_t(i)^\alpha \quad , \quad (6.16)$$

where  $i \in [0,1]$  denotes each differentiated good produced by a unique producer.  $\alpha$  represents the elasticity of output with respect to labour supply  $N_t(i)$  in the production process.  $K_t(i)$  denotes capital at time  $t$ .

The capital stock accumulation equation in this system is  $K_t(i) = (1-\delta)K_{t-1}(i) + I_t$ .  $Z_t$  represents total factor productivity (TFP), which is a key element affecting total output  $Y_t$ . The price of final goods  $P_t$  is determined by:  $P_t = \left( \int_0^1 p_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}$ .

$\mu$  is the elasticity of substitution between goods we have mentioned before. We consider that a randomly selected fraction  $(1-\mu)$  of firms adjusts prices, while the remaining fraction keeps the prices unchanged as  $P_t(i) = \Pi_t P_{t-1}(i)$   
 $\Rightarrow P_t(i) = (1+\pi_t)P_{t-1}(i)$ , where  $\Pi_t = (1+\pi_t)$ .

Defining the index for the “reset” price in period  $t$ ,  $P_t^*(i)$  is the optimal price chosen by the firm in order to maximize the present value of real profits;  $\pi_t$  denotes the inflation rate at time period  $t$ . Then the aggregate price index (CPI) is given by:

$$P_t = [\theta(1+\pi_t)P_{t-1}^{1-\theta} + (1-\theta)(P_t^*)^{1-\theta}]^{\frac{1}{1-\theta}} \quad (6.17)$$

The relation between the aggregate price index and the optimal price is:

$$\frac{P_t^*(i)}{P_t} = \left( \frac{\theta}{\theta-1} \right) \frac{E_t \sum_{j=0}^{\infty} \mu^j \beta^j \left( \frac{C_{t+j}}{C_t} \right)^{-\sigma} \eta_{t+j} \left( \frac{P_{t+j}}{P_t} \right)^{\mu} Y_{t+j} G_j^{-\mu}}{E_t \sum_{j=0}^{\infty} \mu^j \beta^j \left( \frac{C_{t+j}}{C_t} \right)^{-\sigma} \eta_{t+j} \left( \frac{P_{t+j}}{P_t} \right)^{\mu-1} Y_{t+j} G_j^{1-\mu}}, \quad (6.18)$$

when  $t=0$ ,  $G_t=1$ .  $G_t = (1+\pi_1)(1+\pi_2)\dots(1+\pi_t)$ , for  $t \geq 1$ .

Based on the empirical work of Mbaye (2013), we consider that the undervaluation of currency would affect the growth rate of total factor productivity growth rate, and express this circumstance in the model as:

$$\hat{Z}_t = \tau_1 \hat{Z}_{t-1} + \tau_2 \left( RER_t - \bar{RER}_t \right) + \tau_3 \hat{I}_t + \tau_4 \hat{OPN}_t + \varepsilon_t^Z \quad (6.19)$$

The symbol “ $\hat{\phantom{x}}$ ” above a variable denotes the growth of total factor productivity  $Z_t$ .  $RER_t$  is the real exchange rate,  $\bar{RER}_t$  is the real exchange rate in equilibrium.  $\varepsilon_t^Z$  denotes the shock of TFP,  $0 < \kappa < 1$ . We consider

$\hat{RER}_t = RER_t - \bar{RER}_t$  because  $\hat{RER}_t$  is the deviation from the steady state.

So when  $\hat{RER}_t = RER_t - \bar{RER}_t > 0$ , the real exchange rate is undervalued in

the model. When  $\hat{RER}_t < 0$ ,  $\hat{RER}_t$  denotes overvaluation of the currency. OPN

is the trade openness index and is constructed as  $OPN_t = (EX_t + IM_t)/Y_t$ ;  $EX_t$  and

$IM_t$  denote the amount of exports and imports, respectively.

In Eq. (6.19),  $\tau_1$  is the one period autoregressive parameter of TFP and  $\tau_2$  measures the effect of exchange rate deviation on growth. If the country does keep its currency down, the level of undervaluation  $\hat{RER}_t$  will be positive. We assume that growth of the total factor productivity can be attributed to real exchange rate undervaluation.

If the country's currency is overvalued, the appreciation of exchange rate plays a negative role in the TFP growth.  $\tau_3$  and  $\tau_4$  are respectively the parameters of investment and the trade openness index. They represent the impact of investment and international trade on TFP growth.

### 6.2.5 Government

Based on Chen, Funke and Paetz (2012) we consider an expanded Taylor rule function to describe China's monetary policy. The interest rate rule is;

$$R_t = \phi_R R_{t-1} + \phi_\pi \left( E_t(\pi_{t+1}) - \pi_t \right) + \phi_Y \hat{Y}_t + \varepsilon_t^R, \quad (6.20)$$

where  $R_t$  denotes the actual interest rate and  $\bar{R}_t$  is the equilibrium nominal interest rate.  $R_{t-1}$  is the lagged interest rate.  $Y_t$  and  $RER_t$  denote output gap and the real exchange rate, respectively.  $\phi_R$ ,  $\phi_\pi$  and  $\phi_Y$  are the parameters with respect to the lagged interest rate, inflation deviation, and output gap.  $\varepsilon_t^R$  is the shock of the interest rate rule.

## Monetary policy reaction function

$$M_t = \nu_1 M_{t-1} + \nu_2 Y_t + \nu_3 (\pi_t) + \varepsilon_t^m, \quad (6.21)$$

where  $M_t$  is nominal money growth;  $\bar{M}$  is the equilibrium of money demand;  $\varepsilon_t^m$  is money growth shock;  $\nu_1, \nu_2, \nu_3$  are the parameters with lag of nominal money growth, output and the inflation rate. Because of the co-integration relationship of the logarithms of money, output and inflation, we assume  $\bar{M} = \alpha_0 + \alpha_1 \bar{Y} + \alpha_2 \bar{\pi}$ .

## 6.2.6 Exchange rate policy

The influence of the real exchange rate ( $RER$ ) on economic growth can be described by:

$$\hat{Y}_t = \psi_Y \hat{Y}_{t-1} + \psi_{tot} TOT_t + \psi_{open} OPN_t + \psi_i I_t + \psi_{tfp} \hat{Z}_t + \psi_\pi \pi_t + \psi_{rer} \hat{RER}_t + \varepsilon_t^{RER}, \quad (6.22)$$

where  $\hat{Y}_t$  denotes economy growth rate,  $TOT_t$  is terms of trade, and  $OPN_t$  indicates the trade openness index mentioned earlier.  $RER_t$  is the CPI-based real exchange rate.  $\varepsilon_t^{RER}$  is real exchange rate shock. Terms of trade,  $TOT_t$ , is the value of a country's exports relative to that of imports; it is defined in the model as the export price index relative to the import price index.  $\hat{Z}_t$  is the TFP growth rate while  $\pi_t$  denotes the inflation rate.

The function of terms of trade  $TOT_t$  can be described as:

$$TOT_t = \frac{(P_t Q_t^{ex}) \cdot NER_t}{P_t^F Q_t^{im}} , \quad (6.23)$$

where  $Q_t^{ex}$  and  $Q_t^{im}$  denote the quantity of exports and imports respectively. The superscript  $F$  denotes foreign variables.  $NER_t$  is the nominal exchange rate. We assume that the values of both imports and exports are expressed in the foreign currency.  $P_t$  denotes the average value of exports per product; therefore it also represents the price index. The value of export equals to  $EX_t = P_t Q_t^{ex} \cdot NER_t$  and the value of imports is  $IM_t = P_t^F \cdot Q_t^{im}$ , because  $P_t^F$  is the price index of foreign country. Then Eq. (6.23) can be written as:

$$TOT_t = \frac{P_t Q_t^{ex}}{P_t^F Q_t^{im}} = \frac{P_t Q_t^{ex} (NER_t)}{P_t^F Q_t^{im}} = \frac{EX_t}{IM_t} , \quad (6.24)$$

where  $\frac{EX_t}{IM_t}$  is the ratio of exports to imports. We consider the process of exports to imports ratio to take an autoregressive form, and then the function of the exports to imports ratio is given by:

$$\frac{EX_t}{IM_t} = \Phi_{em} \frac{EX_{t-1}}{IM_{t-1}} + \rho_{em} , \quad (6.25)$$

where  $\Phi_{em}$  is the parameter of exports to imports ratio one time period ago.  $\rho_{em}$  is the standard deviation.

Trade openness  $OPN_t$  is calculated as the ratio of the country's total trade, which equals to the sum of exports  $EX_t$  plus imports  $IM_t$ , to the country's gross domestic product (GDP). Then the function of trade openness can be written as:

$$OPN_t = \frac{EX_t + IM_t}{Y_t} \quad (6.26)$$

The log-linearization of Eq. (6.26) is:

$$\hat{OPN}_t = \frac{\bar{EX}}{\bar{EX} + \bar{IM}} \hat{EX}_t + \frac{\bar{IM}}{\bar{EX} + \bar{IM}} \hat{IM}_t - \hat{Y}_t, \quad (6.27)$$

where  $\bar{EX}$  and  $\bar{IM}$  are the steady states of exports and imports.

We also consider trade openness in an autoregressive framework, and in this light the function of trade openness is given by:

$$(6.28)$$

$$OPN_t = \Phi_{opn} OPN_{t-1} + z_{opn},$$

where  $z_{opn}$  is the standard deviation of the time series of trade openness, and  $\Phi_{opn}$  is the parameter of trade openness one period ago.

According to the definition of the real exchange rate ( $RER$ ),  $RER$  is the relative price between tradable goods to the non-tradable goods. Empirically, the real exchange rate is provided by the nominal exchange rate times the ratio of domestic price to the foreign price of the item. So the function of the real exchange rate is:

$$RER_t = (NER_t \cdot P_t) / P_t^F \quad (6.29)$$

The log-linearized function of Eq. (6.29) is:

$$\hat{RER}_t = \hat{NER}_t + \hat{P}_t - \hat{P}_t^F \quad (6.30)$$

Substituting the real exchange rate function of Eq. (6.29) into Eq. (6.24), we can have:

$$TOT_t = (NER_t \cdot P_t) / P_t^F \cdot (Q_t^{ex} / Q_t^{im}) = RER_t \cdot (Q_t^{ex} / Q_t^{im}) \quad (6.31)$$

The log-linearized form of Eq. (31) is:

$$\hat{TOT}_t = \hat{RER}_t + \hat{Q}_t^{ex} - \hat{Q}_t^{im} \quad (6.32)$$

From Eq. (6.31) and Eq. (6.32) we can notice that terms of trade have a positive relation with the real exchange rate. Terms of trade are also associated with the difference between the quantity of export and import goods. Inserting the linearized real exchange rate  $\hat{RER}_t$  into the left hand side of Eq. (6.32), this equation becomes:

$$\hat{Q}_t^{ex} - \hat{Q}_t^{im} = \hat{TOT}_t - \hat{RER}_t \quad (6.33)$$

This function implies that decrease of the real exchange rate will induce an increase in the difference between exports and imports, if terms of trade remain unchanged. This in turn means that holding the currency lower is helpful to sell a larger quantity of domestic goods to foreign markets.

However, the lower real exchange rate not only cannot cause an increase in the terms of trade; according to Eq. (6.33) it also reduces the ratio of exports

to imports. So can conclude that while the lower real exchange rate leads to increased volume of sales, but the undervaluation of the currency might not be profitable overall.

### 6.2.7 Aggregate Output and Net Foreign Assets

In the model, we assume gross domestic product (GDP) is equal to gross output. GDP is the sum of consumption, investment, government spending and net exports:

$$Y_t = C_t + I_t + G_t + (EX_t - IM_t) \quad (6.34)$$

where  $C_t$  is equal to all private consumption, or consumer spending, in the nation's economy;  $G_t$  is the sum of government spending;  $I_t$  is the sum of all the country's investment, including business capital expenditures.  $EX_t$  denotes gross exports, while  $IM_t$  represents gross imports.  $NX_t$  is the nation's total net exports, calculated as total exports minus total imports:

$$NX_t = EX_t - IM_t \quad (6.35)$$

A country's current account  $CA_t$  is equal to the difference between the nation's savings and investment. The current account consists of the balance

of trade, net income from abroad and net current transfers. So the current account position can be calculated as:

$$CA_t = (EX_t - IM_t) + NY_t + NCT_t, \quad (6.36)$$

where  $NY_t$  represents net primary income;  $NCT_t$  denotes net current transfers that have taken place over one period of time. In the model we consider the country's financial account balance  $KA_t$  as the sum of net income from abroad and net current transfers:

$$KA_t = NY_t + NCT_t, \quad (6.37)$$

A nation's Net Foreign Assets ( $NFA_t$ ) position is the cumulative change in its current account over time. Cappiello and Ferrucci (2008) argue that the sum of the current account balance ( $CA_t$ ), the capital and financial account balance ( $KA_t$ ), errors and omissions ( $EO_t$ ) and the change in foreign exchange reserves must be zero. Given this, we consider the amount of foreign exchange reserves to be equal to net foreign assets ( $NFA_t$ ). Therefore, we have:

$$CA_t + KA_t + EO_t - \hat{NFA}_t = 0 \Rightarrow \hat{NFA}_t = CA_t + KA_t + EO_t, \quad (6.38)$$

$\hat{NFA}_t$  denotes the change in foreign exchange reserves from time period  $t-1$  to  $t$ . Following Cappelletto and Ferrucci (2008), we rebuild the equation of net foreign assets as a key variable in the system. Net foreign assets is described by the following expression:

$$NFA_{t+1} = R_t NFA_t + CA_t + KA_t \quad (6.39)$$

Meade (1951), Metzler (1951) and Swan (1963) propose an underlying balance (UB) approach to find the fair value of a currency and characterize the equilibrium in an open economy. This underlying balance (UB) approach states that the fair value of a currency is the level of the exchange rate that is consistent with both the internal and external balance of the country. Most applications of the UB approach consider internal balance as a domestic economic process. On the other hand, a country's external balance can be broadly represented by its current account (CA) position being at equilibrium. Meanwhile, a nation's current account position is equal to the difference between its savings and investment, hence the balance of payments relation in the UB approach is:

$$CA_t + KA_t = NFA_{t+1} - R_t NFA_t = \hat{NFA}_t = S_t - I_t, \quad (6.40)$$

where  $CA_t$  may be expressed by either changes in net foreign assets  $NFAs$  or the excess domestic savings ( $S_t$ ) over domestic investment ( $I_t$ ). In the model, we define the capital account balance  $KA_t$  as a shock process, which is exogenous to the model. We consider capital flow as a random walk process, the expectation value of capital account is  $E[KA_t] = 0$ .

Substituting Eq. (6.40) into Eq.(6.34), we have:

$$Y_t = C_t + I_t + G_t + \hat{NFA}_t \quad (6.41)$$

Eq.(6.41) shows the positive relation between gross domestic product (GDP) and changes in net foreign assets. For the case of households with precautionary savings, we consider their consumption and investment to be equal to that of the representative households. Therefore, we have:

$$C_t^F = C_t \quad I_t^F = I_t \quad (6.42)$$

Then, the aggregate output function for perfect foresight households is:

$$Y_t = C_t^F + I_t^F + G_t + \hat{NFA}_t \quad (6.43)$$

Substituting Eq.(6.11) into Eq.(6.44), we have the gross domestic product function for precautionary households:

$$Y_t = C_t^P + I_t^P + G_t + \hat{NFA}_t + S_t^P \quad (6.44)$$

Following Cherif and Hasanov (2013), precautionary savings are equal to the difference between income after investment and consumption. The irreversibility of investment and large volatility of permanent income shocks should produce large precautionary savings. Therefore, the function of precautionary savings can also be written as:

$$S_t^P = Y_t - C_t^P - I_t^P - G_t - \hat{NFA}_t \quad (6.45)$$

## 6.3 Estimation and Results Analysis

### 6.3.1 First-Order Conditions (FOCs)

First, we derive the Lagrangian of Eq.(6.1) based on the first life period budget constraint:

$$L = \max E \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} \frac{(c_t^1)^{1-\sigma}}{1-\sigma} + \frac{\ell(c_{t+28}^2)^{1-\sigma}}{1-\sigma} + \frac{(B_t)^{1-\gamma}}{1-\gamma} + \frac{(J_t)^{1-\gamma}}{1-\gamma} - \frac{(N_t)^{1+\varphi}}{1+\varphi} \\ -\lambda_t(c_t^1 + BO_t/(P_t R_t) - BO_{t-1}/P_t - \varsigma(W_t/P_t)N_t - r_t^C K_t - B_t - d_t) \end{array} \right\} \quad (6.46)$$

The FOC with respect to  $c_t^1$  is:

$$(c_t^1)^{-\sigma} = \lambda_t \quad (6.47)$$

The FOC with respect to  $B_t^1$  is:

$$(B_t)^{-\gamma} = -\lambda_t = -(c_t^1)^{-\sigma} \quad (6.48)$$

The FOC with respect to  $BO_t$  is:

$$\begin{aligned} -(\lambda_t/P_t R_t) - \beta E[-(\lambda_{t+1}/P_{t+1})] &= 0 \\ \Rightarrow \lambda_t &= \beta E[(P_t R_t/P_{t+1})\lambda_{t+1}] \end{aligned} \quad (6.49)$$

According to Eqs.(6.47) and (6.49), we obtain the equilibrium equations:

$$(c_t^1)^{-\sigma} = \beta E \left[ (P_t R_t / P_{t+1}) [c_{t+1}^1]^{-\sigma} \right] \quad (6.50)$$

$$\Rightarrow \beta E \left[ (P_t R_t / P_{t+1}) (c_{t+1}^1 / c_t^1)^{-\sigma} \right] = 1 \quad (6.51)$$

This is the Euler equation with respect to consumption  $c_t^1$ .

The FOC with respect to  $N_t$  (labour) is:

$$\begin{aligned} -N_t^\varphi + \lambda_t (W_t / P_t) &= 0 \Rightarrow N_t^\varphi = \lambda_t (W_t / P_t), \quad \lambda_t = (c_t^1)^{-\sigma} \\ \Rightarrow N_t^\varphi &= (c_t^1)^{-\sigma} (W_t / P_t) \end{aligned} \quad (6.52)$$

Eq. (6.52) represents the optimal labour-leisure decision in equilibrium.

Further, we can find the FOCs with respect to investment  $I_t$  and capital stocks

$K_t$  according to the capital stock accumulation equation,  $K_t = (1-\delta)K_{t-1} + I_t$  :

$$\beta E_t (c_{t+1}^1 / c_t^1)^{-\sigma} (1 - \delta + r_{t+1}^c) = 1 \quad (6.53)$$

On the basis of Eq. (6.52), the FOC with respect to  $N_t(i)$  and  $K_t(i)$  is:

$$W_t / P_t = (\alpha r_t^c / (1-\alpha)) (K_t(i) / N_t(i)) \quad (6.54)$$

This formula can be rewritten as:

$$W_t/P_t = \alpha (W_t/\alpha P_t)^\alpha (r_t^c/(1-\alpha))^{1-\alpha} (K_t(i)/N_t(i))^{1-\alpha} \quad (6.55)$$

We set the real marginal cost,  $\eta_t$ , as:

$$\eta_t = (1/Z_t)(W_t/\alpha P_t)^\alpha (r_t^c/(1-\alpha))^{1-\alpha} \quad (6.56)$$

In light of Eqs. (6.55) and (6,56), we can reason out:

$$W_t/P_t = \alpha Z_t \eta_t (K_t(i)/N_t(i))^{1-\alpha} \quad (6.57)$$

The aggregate wage equation is:

$$W_t = \left[ (1-\xi)(W^*)^{1-l} + \xi(W_{t-1})^{1-l} \right]^{1/(1-l)}, \quad (6.58)$$

Furthermore, the Lagrangian of Eq.(6.1) based on the second life period budget constraint can be described as:

$$L = \max E \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} \frac{(c_t^1)^{1-\sigma}}{1-\sigma} + \frac{\ell(c_t^2)^{1-\sigma}}{1-\sigma} + \frac{(B_t)^{1-\gamma}}{1-\gamma} + \frac{(J_t)^{1-\gamma}}{1-\gamma} - \frac{(N_t)^{1+\varphi}}{1+\varphi} \\ -\lambda_t^2 (c_t^2 + BO_t/P_t R_t + J_t - J_{t-1} - BO_{t-1}/P_t - s_t(1+r^G) + m_t B_t) \end{array} \right\} \quad (6.59)$$

The FOC with respect to second life period consumption  $c_t^2$  is:

$$\ell(c_t^2)^{-\sigma} = \lambda_t^2 \quad (6.60)$$

Then, the FOC with respect to  $BO_t$  is:

$$\lambda_t^2 = \beta E \left[ P_t R_t / P_{t+1} \lambda_{t+1}^2 \right] \quad (6.61)$$

Substituting Eq.(6.31) into Eq.(6.32), we have

$$\begin{aligned} \ell(c_t^2)^{-\sigma} &= \beta E \left[ P_t R_t / P_{t+1} \ell(c_{t+1}^2)^{-\sigma} \right] \\ \Rightarrow \beta E \left[ P_t R_t / P_{t+1} (c_{t+1}^2 / c_t^2)^{-\sigma} \right] &= 1 \end{aligned} \quad (6.62)$$

The FOC with respect to  $B_t$  according to the second life period constraint is:

$$(B_t)^{-\gamma} - \lambda_t^2 m_t = 0 \Rightarrow (B_t)^{-\gamma} = \lambda_t^2 m_t \Rightarrow (B_t)^{-\gamma} = \ell(c_t^2)^{-\sigma} m_t \quad (6.63)$$

Then, we can summarize the relevant equations discussed above to obtain the steady state in what follows including state states of the capital accumulation function, the function of monetary policy and the budget constraint of the second life period.

### 6.3.2 Steady States

At the steady state, all variables are constant. We calculate the steady state of equations discussed in the previous sections. The steady state of Euler equation (6.51) is:

$$1 = \beta E \left[ \left( \frac{\bar{P} \bar{R}}{\bar{P}} \right) \left( \frac{\bar{c}^1}{\bar{c}^1} \right)^{-\sigma} \right] \Rightarrow 1 = \beta \bar{R} \Rightarrow \bar{R} = 1/\beta \quad (6.64)$$

The steady state of the optimal labour-leisure decision (6.52) is:

$$\bar{N}^{\varphi} = \left( \frac{\bar{c}^1}{\bar{c}^1} \right)^{-\sigma} \left( \frac{\bar{W}}{\bar{P}} \right) \quad (6.65)$$

The steady states of the Cobb-Douglas production function (6.16) are:

$$\bar{Y} = \bar{Z} \left( \frac{\bar{K}}{\bar{K}} \right)^{1-\alpha} \left( \frac{\bar{N}}{\bar{N}} \right)^{\alpha} \quad (6.66)$$

The steady state of capital stock equation (6.53) is:

$$\beta E_t \left( \frac{\bar{c}^1}{\bar{c}^1} \right)^{\sigma} \left( 1 - \delta + r^c \right) = 1 \Rightarrow \beta \left( 1 - \delta + r^c \right) = 1 \Rightarrow r^c = \frac{1}{\beta} + \delta - 1 \quad (6.67)$$

The steady states of aggregate price, wage, capital and inflation are:

$$\bar{P} = \left[ \theta \left( \frac{\bar{P}}{\bar{P}} \right)^{1-\mu} + (1-\theta) \left( \frac{\bar{P}}{\bar{P}} \right)^{1-\mu} \right]^{1/(1-\mu)} = \left[ \left( \frac{\bar{P}}{\bar{P}} \right)^{1-\mu} \right]^{1/(1-\mu)} = \bar{P} \quad (6.68)$$

$$\bar{W} = \left[ (1-\mu) \left( \frac{\bar{W}}{\bar{W}} \right)^{1-l} + \mu \left( \frac{\bar{W}}{\bar{W}} \right)^{1-l} \right]^{1/(1-l)} = \left[ \left( \frac{\bar{W}}{\bar{W}} \right)^{1-l} \right]^{1/(1-l)} = \bar{W} \quad (6.69)$$

$$\bar{K} = (1-\delta) \bar{K} + \delta \bar{I} \Rightarrow \bar{K} = \bar{I} \quad (6.70)$$

$$\Pi \equiv \frac{\bar{P}}{P} = 1 \Rightarrow 1 + \bar{\pi} = 1 \Rightarrow \bar{\pi} = 0 \quad (6.71)$$

The steady states of the first and second life period budget constraints are:

$$\bar{c}^1 + \left( \bar{B}O/\bar{P} \right) \left( \left( 1/\bar{R} \right) - 1 \right) = \zeta \left( \bar{W}/\bar{P} \right) \bar{N} + r^c \bar{K} + \bar{B} - \bar{d} \quad (6.72)$$

$$\bar{c}^2 + \left( \bar{B}O/\bar{P} \right) \left( \left( 1/\bar{R} \right) - 1 \right) = \bar{d} (1 + r^G) - m_t \bar{B} \quad (6.73)$$

Finally, the steady states of joy bought by children and attention are:

$$\bar{J} = (1 - \zeta) \bar{W} \bar{A}. \quad (6.74)$$

$$\bar{A} = \vartheta \cdot \left( 1 - \bar{N} \right). \quad (6.75)$$

The steady state of monetary policy reaction function is:

$$\bar{M} = \theta_1 \bar{M} + \theta_2 \bar{Y} + \theta_3 \bar{\pi} \Rightarrow \bar{M} = (\theta_2 / (1 - \theta_1)) \bar{Y} + (\theta_3 / (1 - \theta_1)) \bar{\pi} \quad (6.76)$$

The steady state of interest rate  $r_t$  is:

$$\bar{R} = 1 + \bar{r} = 1 \Rightarrow \bar{r} = 0 \quad (6.77)$$

The steady state of the function of net foreign assets is:

$$\left( 1 - \bar{R} \right) \bar{NFA} = \bar{CA} + \bar{KA} \quad (6.78)$$

Replacing  $\bar{R} = 1$  into Eq.(6.78) we have,

$$\bar{CA} + \bar{KA} = 0 \quad (6.79)$$

In the equilibrium, the sum of current account and capital account is equal to zero. Furthermore, we calculate the steady states of current account position:

$$\bar{CA} = \bar{S} - \bar{I} \quad (6.80)$$

Current account position in the equilibrium is equal to the difference between savings and investment.

The steady states of real marginal costs ( $MC_t$ ), wage ( $W_t/P_t$ ) and the long-run exchange rate  $RER$  are respectively:

$$\bar{MC} = \frac{1}{\bar{Z}} \left( \frac{1}{\alpha} \right)^\alpha \left( \bar{W}/\bar{P} \right)^\alpha \left( \bar{r}^c / (1 - \alpha) \right)^{1 - \alpha} \quad (6.81)$$

$$\left( \bar{W}/\bar{P} \right) = \left( \alpha \bar{r}^c / (1 - \alpha) \right) \left( \bar{K}/\bar{N} \right) \quad (6.82)$$

$$\bar{RER} = \psi_N \left( \bar{NFA}/\bar{Y} \right) \quad (6.83)$$

We calculate the steady state functions for both perfect foresight households and precautionary households. The steady state of aggregate output for perfect foresight households is:

$$\bar{Y} = \bar{C}^F + \bar{I}^F + \bar{G} + \left( \bar{EX} - \bar{IM} \right) \quad (6.84)$$

Substituting Eq. (6.78) into Eq.(6.84), the perfect foresight households' aggregate output function is:

$$\bar{Y} = \bar{C}^F + \bar{I}^F + \bar{G} + (1 - \bar{R})\bar{NFA} - \bar{KA} \quad (6.85)$$

Based on the precautionary saving function Eq. (6.13), we calculate the steady state of precautionary saving as follows:

$$\bar{S}^P = \bar{C}^F + \bar{I}^F - (\bar{C}^P + \bar{I}^P) \quad (6.86)$$

Therefore, the steady state of output for precautionary households is:

$$\bar{Y} = \bar{C}^P + \bar{I}^P + \bar{G} + \bar{S}^P + (1 - \bar{R})\bar{NFA} \quad (6.87)$$

### 6.3.3 Linearized Equations

To facilitate interpretation and solution of the model, we replace the dynamic nonlinear equations with dynamic linear equations. Following Uhlig's method (1999), we can linearise Eq.(6.51) around the steady state to obtain:

$$\hat{c}_{t+1}^1 = \hat{c}_t^1 + (1/\sigma) \left( \hat{P}_t - \hat{P}_{t-1} + \hat{R}_t \right) \quad (6.88)$$

Equation (6.88) is the linearized equation of the Euler equation of first life period consumption,  $c_t^1$ . This condition implies that current consumption is determined by prior consumption, expected future consumption, the inflation rate and the gross returns on bond.

The linearized function of Eq. (6.52) can be stated as:

$$\hat{N}_t = ((-\sigma)/\varphi) \left( \hat{c}_t^1 \right) + (1/\varphi) (\hat{W}_t - \hat{P}_t) \quad (6.89)$$

If we let  $w_t = W_t/P_t$ , then  $\hat{w}_t = \hat{W}_t - \hat{P}_t$ . The relation between the labour supply of firm  $i$ ,  $N_t(i)$ , and the aggregate labour supply,  $N_t$ , is:

$$N_t(i) = (W_t^*(i)G_t/W_t)^{-l} N_t \quad , \quad (6.90)$$

where  $l$  is the elasticity of substitution among differentiated labour services.

When  $t = 0$ ,  $G_t = 1$ .  $G_t = (1 + \pi_1)(1 + \pi_2) \dots (1 + \pi_t)$ , for  $t \geq 1$ .

On the basis of Eqs. (6.89) and (6.90), we follow Zhang (2009) to obtain the linearized equation of real wages:

$$\begin{aligned} & \left( (1 + \beta)(1 + \varphi l)\xi + (1 - \beta\xi)(1 - \xi) \right) \cdot \hat{w}_t = (1 + \varphi l)\xi(\hat{w}_{t-1} + \pi_{t-1}) - (1 + \beta)(1 + \varphi l)\xi\pi_t \\ & + (1 + \varphi l)\xi\beta E_t(\hat{w}_{t+1} + \pi_{t+1}) + (1 - \beta\xi)(1 - \xi) \left[ \varphi \hat{N}_t + \sigma(c_t^1) \right] \end{aligned}$$

$$\Rightarrow \hat{w}_t = \left( \frac{1}{1+\beta} \right) (\hat{w}_{t-1} + \pi_{t-1}) - \pi_t + \left( \frac{\beta}{1+\beta} \right) E_t(\hat{w}_{t+1} + \pi_{t+1}) , \quad (6.91)$$

where  $\xi$  denotes the fraction of households that set their wages according to the last-period wage inflation or to a combination of last-period price and wage inflation rates. The remaining portion of the households would adjust their wage optimally. This condition indicates that the current nominal wage is a function of inflation, consumption, labour and prior and expected wages.

The linearized equation of the FOCs with respect to investment and capital stocks, Eq. (6.53), is:

$$\hat{c}_{t+1}^1 = \hat{c}_t^1 + (1/\sigma)(1 + \beta\delta - \beta)r_{t+1}^c \quad (6.92)$$

Next, we have:

$$\hat{Y}_t = \hat{Z}_t + (1-\alpha)\hat{K}_t + \alpha\hat{N}_t \quad (6.93)$$

This is the linearized form of Eq. (6.16), i.e., the Cobb-Douglas production function. Eq. (54) shows the linear relation between the three inputs in production, i.e., technology, capital and labour.

Linearized equation of the price index,  $\hat{P}_t$ , is:

$$\pi_t = \hat{P}_t - \hat{P}_{t-1} \quad (6.94)$$

This expression indicates that current inflation is the difference between the current and prior levels of the price index. We also have linearized equation of capital accumulation,  $\hat{K}_t$ :

$$\hat{K}_{t+1} = (1 - \delta)\hat{K}_t + \delta\hat{I}_t \quad (6.95)$$

Eq. (6.94) implies that future stocks of capital are a function of current capital and investment. The depreciation rate,  $\delta$ , affects the present value of changes in investment.

The linearized equation of the real marginal cost of labour,  $\hat{\eta}_t$  is:

$$\hat{\eta}_t = \alpha\hat{w}_t + (1 - \alpha)r_t^c - \hat{Z}_t \quad (6.96)$$

According to Eq. (6.96), the real marginal cost of labour comes from three sources: wages, the rental rate of capital and technology shocks. The elasticities of labour and capital service are the parameters in this equation.

The linearized form of Eq. (6.54) is the equilibrium:

$$\hat{w}_t = r_t^c + \hat{K}_t - \hat{N}_t \quad (6.97)$$

The equilibrium indicates that the sum of real wages and real unit labour costs is equal to the sum of capital accumulation and the rental rate of capital.

Underlying this equilibrium is the assumption that firms finance their wage bills and decide their demand for labour on the basis of their capital and rental capital income.

The relationship between the aggregate price index,  $P_t$ , and the optimal price,  $P_t^*(i)$ , is:

$$P_t^*(i)/P_t = (\mu/\mu-1) \frac{E_t \sum_{j=0}^{\infty} \theta^j \beta^j (C_{t+j}/C_t)^{-\sigma} \eta_{t+j} (P_{t+j}/P_t)^\mu Y_{t+j} G_j^{-\mu}}{E_t \sum_{j=0}^{\infty} \theta^j \beta^j (C_{t+j}/C_t)^{-\sigma} \eta_{t+j} (P_{t+j}/P_t)^{\mu-1} Y_{t+j} G_j^{1-\mu}}$$

We already know that when  $t=0$ ,  $G_t=1$ .  $G_t = (1+\pi_1)(1+\pi_2)\dots(1+\pi_t)$ ,

for  $t \geq 1$ .  $\Rightarrow G_j = P_{t+j}/P_t$

Therefore, we have:

$$P_t^*(i)/P_t = (\mu/\mu-1) \frac{E_t \sum_{j=0}^{\infty} \theta^j \beta^j (C_{t+j}/C_t)^{-\sigma} \eta_{t+j} Y_{t+j}}{E_t \sum_{j=0}^{\infty} \theta^j \beta^j (C_{t+j}/C_t)^{-\sigma} \eta_{t+j} Y_{t+j}} = (\mu/(\mu-1)) \quad (6.98)$$

We can derive the following linearized Phillips curve from (6.97):

$$\pi_t = \left( \theta / (1 + \beta\theta^2) \right) \pi_{t-1} + \frac{2 - (1 + \beta)\theta + \beta\theta^2}{1 + \beta\theta^2} E_t \pi_{t+1} + (1 - \theta)(1 - \beta\theta) \hat{\eta}_t \quad (6.99)$$

The inflation rate is a function of the real marginal cost and the prior and expected levels of inflation. As above,  $\beta$  is the discount rate and  $\theta$  denotes

the fraction of firms that will adjust their prices according to the last-period price inflation and marginal costs.

The linearized equation of joy bought by children, according to Eqs. (6.3) and (6.4) is:

$$\hat{J}_t = \hat{W}_t - \hat{N}_t \bar{N} / (1 - \bar{N}). \quad (6.100)$$

Eq. (6.100) implies that the increase of an agent's joy bought by children is associated with the change of her wage and labour supply. Wage has a positive impact on joy whereas labour supply has a negative impact. Moreover, the linearized function of Eq. (6.3) is:

$$\hat{J}_t = \hat{W}_t + \hat{A}_t. \quad (6.101)$$

The deviation of joy bought by children also depends on the average wage per unit of time and attention provided by children. Then, the linearized function of attention provided by the second generation can be written as:

$$\hat{A}_t = \hat{N}_t \bar{N} / (\bar{N} - 1). \quad (6.102)$$

The linearized equation for second life period consumption is:

$$\hat{c}_{t+1}^2 = \hat{c}_t^2 + (1/\sigma) \left( \hat{P}_t - \hat{P}_{t-1} + \hat{R}_t \right) \quad (6.103)$$

Subtracting E. (6.88) from Eq. (6.103), we have

$$\hat{c}_{t+1}^2 - \hat{c}_{t+1}^1 = \hat{c}_t^2 - \hat{c}_t^1 \Rightarrow \hat{c}_{t+1}^2 - \hat{c}_t^2 = \hat{c}_{t+1}^1 - \hat{c}_t^1 \quad (6.104)$$

Then, we calculate the difference between first and second life period budget constraints, and the result is:

$$c_t^2 - c_t^1 = d_t (2 + r^G) - (m_t + 1) B_t - w_t N_t - r_t^c K_t \quad (6.105)$$

Assuming  $c_t^2 = c_t^1$ , Eq.(6.105) can be rewritten as:

$$B_t = \frac{2 + r^G}{m_t + 1} S_t - \frac{1}{m_t + 1} w_t N_t - \frac{1}{m_t + 1} r_t^c K_t \quad (6.106)$$

Thus, the linearized equation of Eq.(6.106) can be stated as:

$$\bar{B} \hat{B}_t = \left( \frac{2 + r^G}{m_t + 1} \right) \bar{S} \hat{S}_t - (1/(m_t + 1)) \bar{w} \bar{N} \left( \hat{w}_t + \hat{N}_t \right) - (1/(m_t + 1)) \bar{r}^c \bar{K} \left( \hat{r}_t^c + \hat{K}_t \right) \quad (6.107)$$

The linearized intergenerational transfer functions imply that deposits have a positive influence on bequests. The current amount of intergenerational transfers is also related to personal income and capital return.

We further assume the function of household savings as:

$$d_{t+1} = (1 + r_t^c) / (1 + \pi_t) d_t \quad (6.108)$$

Thus, the linearized function of savings in the equilibrium is:

$$\hat{d}_{t+1} = \hat{r}_t^c - \hat{\pi}_t + \hat{d}_t \quad (6.109)$$

This equation posits that the increase in expected household savings depends on the increases in the interest rate, inflation rate and current savings.

Finally, we calculate the first-order condition of joy based on budget constraint of second life period.

$$(J_t)^{-\gamma} = \beta \ell (c_{t+1}^2)^{-\sigma} - \ell (c_t^2)^{-\sigma} \quad (6.110)$$

The linearized function of Eq. (6.110) can be stated as:

$$\hat{J}_t = \sigma / (\gamma(1-\beta)) \hat{c}_t^2 - \sigma \beta / (\gamma(1-\beta)) \hat{c}_{t+1}^2 \quad (6.111)$$

According to first order condition of  $r_t^c$ , the rental rate of capital can be linearized to the following form:

$$\hat{r}_{t+1}^c = \frac{\sigma}{(h-1)(1+\beta\delta-\beta)} \left( (1+h)\hat{C}_t - h\hat{C}_{t-1} - \hat{C}_{t+1} \right) \quad (6.112)$$

From this equation we can derive that rental capital returns come from the current, prior and expected levels of consumption.

The Taylor rule for setting the nominal interest rate in the log-deviation form is written as:

$$\hat{R}_t = \phi_R \hat{R}_{t-1} + \phi_\pi \hat{\pi}_{t+1} + \phi_Y \hat{Y}_t \quad (6.113)$$

We expand the function of total factor productivity in the DSGE framework on the basis of Mbaye's (2013) and Jiang's (2014) approaches. The expanded linearized TFP growth function is:

$$\hat{Z}_t = \tau_1 \hat{Z}_{t-1} + \tau_2 \hat{I}_t + \varepsilon_t^Z \quad (6.114)$$

The linearized aggregate output for perfect foresight households is:

$$\hat{Y}_t = \frac{\bar{C}}{\bar{Y}} \hat{C}_t^F + \frac{\bar{I}}{\bar{Y}} \hat{I}_t^F + \frac{\bar{G}}{\bar{Y}} \hat{G}_t + \frac{\bar{NFA}}{\bar{Y}} \hat{NFA}_{t+1} - \frac{\bar{RNFA}}{\bar{Y}} \left( \hat{NFA}_t + \hat{R}_t \right) \quad (6.115)$$

Based on Eq.(6.115), the linearized aggregate output for precautionary households is:

$$\hat{Y}_t = \frac{\bar{C}}{\bar{Y}} \hat{C}_t^p + \frac{\bar{I}}{\bar{Y}} \hat{I}_t^p + \frac{\bar{S}^p}{\bar{Y}} \hat{S}_t^p + \frac{\bar{G}}{\bar{Y}} \hat{G}_t + \frac{\bar{NFA}}{\bar{Y}} \hat{NFA}_{t+1} - \frac{\bar{RNFA}}{\bar{Y}} \left( \hat{NFA}_t + \hat{R}_t \right) \quad (6.116)$$

Finally, the linear net foreign assets function is:

$$\hat{NFA}_{t+1} = \hat{R}_t + \hat{NFA}_t + \frac{\bar{S}}{\bar{NFA}} \hat{S}_t - \frac{\bar{I}}{\bar{NFA}} \hat{I}_t + \frac{\bar{KA}}{\bar{NFA}} \hat{KA}_t, \quad (6.117)$$

where  $\left( \frac{\bar{S}}{\bar{NFA}} \right)$  and  $\left( \frac{\bar{I}}{\bar{NFA}} \right)$  denote the ratio of savings to foreign reserve holdings and the ratio of import to foreign reserves in equilibrium, respectively.  $\left( \frac{\bar{KA}}{\bar{NFA}} \right)$  represent the ratio of capital flows to foreign reserves.

## 6.4 Parameter Analysis

### 6.4.1 Description of Data

Following the literature, the parameter value of the lagged TFP growth rate is set to be  $\tau_1=0.95$ , which describes the impact of the autoregressive parameter on current TFP growth. The investment parameter is set to be  $\tau_2=0.0787$ . Following Walsh (2003) and He et al. (2007), we set  $\sigma=2$ ,  $\alpha=0.4$  and  $\delta=0.04$ . If  $\alpha=0.4$ , one can get the elasticity of the substitution among intermediate goods and final goods as  $\mu=4.61$ . According to China's total fertility rate from 1993 to 2012, we set the number of children at  $m_t=1.63$ . The average (annual) nominal interest rate from 1980 to 2015 is 2.073%. Thus, the parameter of the discount rate is  $\beta=1/(1+0.02073)=0.9797 \approx 0.98$ . According to the report of Chinese Academy of Social Sciences, China's

average childbearing ages had risen to 28.12 in the period from 2004 to 2014. We therefore consider that the time period of one generation is 28 years and that the interest rate for one life period  $r^G$  is  $(1+0.02073)^{28}=1.776$ . According to the Labour Contract Law of the People's Republic of China, China has a 44-hour normal working week. Therefore, the steady state of labour  $\bar{N}=(44 \cdot 52)/(24 \cdot 365)=0.26$ .

The GMM estimation with data for the period 1995 to 2005 shows that the fraction of firms that adjust their prices according to the last-period prices is  $\theta=0.84$ . On the basis of Liu (2008), we obtain the elasticity of labour supply  $\varphi=6.16$ , and the fraction of households who set their wages depending on wage inflation is  $\xi=0.6$ . The elasticity of real money balance  $\gamma$  is considered to be 3.13.

We calculate the average value of the nominal exchange rate of the Chinese RMB against the US dollar from 2005 to 2012 as the steady state of the exchange value. Purchasing power parity (PPP) conversion factor is a technique used to determine the relative value of different currencies. It represents the number of units of a country's currency required to buy the same amount of goods and services in the domestic market as a US dollar would buy in the United States. We apply the PPP conversion factor from 2010 to 2014 to calculate the steady state of the real exchange rate, which is  $\bar{RER}=7.137 \cdot 3.52=25.12$ .

In order to determine the steady state value in Eq.(5.117), we use the data for China from 2005 to 2012 to calculate the ratios of domestic savings to foreign reserves and of investment to foreign reserves. We set the average values of savings, investment and reserves in this time period as the steady state values.

$\left(\bar{KA}/\bar{NFA}\right)=0.044$  represent steady state of the ratio of capital account to foreign reserves Then, Eq.(6.117) can be rewritten as:

$$\hat{NFA}_t = \hat{NFA}_{t-1} + 1.188\hat{S}_{t-1} - 1.051\hat{I}_{t-1} + \hat{R}_{t-1} + 0.044\hat{KA}_t \quad (6.118)$$

Finally, we calculate the parameter values in the linearized aggregate output function using the macroeconomic data for China from 2005 to 2014. We set the average values of household final consumption expenditure of GDP ( $\bar{C}/\bar{Y}$ ), total investment of GDP ( $\bar{I}/\bar{Y}$ ), gross savings of GDP ( $\bar{S}/\bar{Y}$ ), net foreign assets of GDP ( $\bar{NFA}/\bar{Y}$ ) and government final consumption expenditure of GDP ( $\bar{G}/\bar{Y}$ ). The data regarding  $\bar{C}/\bar{Y}$ ,  $\bar{S}/\bar{Y}$ ,  $\bar{NFA}/\bar{Y}$ ,  $\bar{W}/\bar{Y}$  and  $\bar{G}/\bar{Y}$  are collected from the World Bank's national accounts data, while the data regarding  $\bar{I}/\bar{Y}$  are collected from the database of IMF cross country macroeconomic statistics. specifically, we have  $\bar{C}/\bar{Y} = 36.9\% = 0.369$ ,  $\bar{S}/\bar{Y} = 49.94\% = 0.499$ ,  $\bar{NFA}/\bar{Y} = 43.37\% = 0.434$ ,  $\bar{G}/\bar{Y} = 13.47\% = 0.135$  and  $\bar{I}/\bar{Y} = 44.4\% = 0.444$ .

To decide on the parameter value of changes in net foreign assets, we calculate the average value of China's exports and imports in terms of GDP.

The average ratio of export to GDP from 2005 to 2014 is:

$$\bar{EX}/\bar{Y} = 28.1\% = 0.281 \quad (6.119)$$

Then, the average ratio of import of goods and services to GDP from 2005 to 2014 is:

$$\bar{IM}/\bar{Y} = 23.82\% = 0.238 \quad (6.120)$$

The commercial balance or net exports,  $NX_t$ , is the difference between the monetary value of exports and imports in an economy over a certain period.

The average net export of GDP is:

$$\bar{NX}/\bar{Y} = \bar{EX}/\bar{Y} - \bar{IM}/\bar{Y} = 0.281 - 0.238 = 0.043 \quad (6.121)$$

Based on the average yearly wages of Chinese households and the total labour force of China, we calculate the average yearly wage to GDP ratio,

$\bar{W}/\bar{Y} = 72.76\% = 0.728$ . To explore the relative importance of precautionary

savings, we designate that  $IF_t$  is foreign direct investment and  $ID_t$  is domestic investment. Then the function of aggregate investment  $I_t$  is:

$$I_t = IF_t + ID_t \quad (6.122)$$

The linearized function of Eq.(6.91) is:

$$\bar{I}I = \bar{I}F IF_t + \bar{I}D ID_t \Rightarrow I = \left( \frac{\bar{I}F}{\bar{I}} \right) IF_t + \left( \frac{\bar{I}D}{\bar{I}} \right) ID_t \quad (6.123)$$

We calculate the average value of foreign direct investment of GDP

$\bar{I}F/\bar{Y} = 0.044$ , and the ratio of total investment of GDP  $\bar{I}/\bar{Y} = 0.444$ , so:

$$\bar{I}D/\bar{Y} = \bar{I}/\bar{Y} - \bar{I}F/\bar{Y} = 0.444 - 0.044 = 0.4 \quad (6.124)$$

Replacing the parameters in the linearized aggregate output for perfect foresight households Eq.(6.115), we have:

$$\hat{Y}_t = 0.369 \hat{C}_t^F + 0.444 \hat{I}_t^F + 0.135 \hat{G}_t + 0.434 \hat{NFA}_{t+1} - 0.44 \left( \hat{NFA}_t + \hat{R}_t \right) \quad (6.125)$$

From Eqs.(6.11) and (6,34), we explore the relation between gross domestic product and precautionary savings:

$$Y_t = S_t^p + C_t^p + I_t^p + G_t + NX_t \quad (6.126)$$

We already have the average ratio of yearly household final consumption expenditure to GDP  $\bar{C}/\bar{Y} = 0.369$ , total investment to GDP  $\bar{I}/\bar{Y} = 0.444$ , government final consumption expenditure to GDP  $\bar{G}/\bar{Y} = 0.135$ , and average

net exports to GDP  $\bar{NX}/\bar{Y} = 0.043$ , the average ratio of capital flows to foreign reserves  $\bar{KA}/\bar{NFA} = 0.044$ .

However, the sum of consumption and investment for precautionary households should be less than that for perfect foresight households, because that precautionary savings should occur in these households' budget constraint due to uncertainty regarding future income. Precautionary households decide to reduce their consumption and investment in the current period to reserve precautionary savings. In order to evaluate linearized aggregate output function for precautionary households, we calculate the linearized function of Eq.(13) to identify the difference between perfect foresight households and precautionary households. Then we have the linearized function of Eq.(6.13):

$$\hat{C}_t^F + \hat{I}_t^F = (0.4347) \cdot rp \frac{\bar{Y}}{\bar{C}^F + \bar{I}^F} \hat{Y}_t + \frac{\bar{C}^P + \bar{I}^P}{\bar{C}^F + \bar{I}^F} \left( \hat{C}_t^P + \hat{I}_t^P \right) \quad (6.127)$$

We denotes  $rp = 0.3$  is the ratio of precautionary savings of households to disposable income. We consider  $\bar{C}/\bar{Y} = \bar{C}^F/\bar{Y} = 0.369$  and  $\bar{I}/\bar{Y} = \bar{I}^F/\bar{Y} = 0.444$ , then we have:

$$\left( \bar{C}^F + \bar{I}^F / \bar{Y} \right) = 0.369 + 0.444 = 0.813 \quad \Rightarrow \quad \bar{C}^F + \bar{I}^F = 0.813 \bar{Y} \quad (6.128)$$

According to the steady state of Eq.(6.13), we have is the steady state of precautionary savings:

$$\bar{s}^P = 0.1304\bar{Y} = \left(\bar{C}^F + \bar{I}^F\right) - \left(\bar{C}^P + \bar{I}^P\right), \quad (6.129)$$

where  $\bar{s}^P$  is the steady state of precautionary savings.

Replacing Eq.(6.128) into Eq.(6.129) we have

$$0.1304\bar{Y} = 0.813\bar{Y} - \left(\bar{C}^P + \bar{I}^P\right) \Rightarrow \bar{C}^P + \bar{I}^P = 0.6826\bar{Y} \quad (6.130)$$

Then Eq.(6.127) can be rewritten as:

$$\hat{C}_t^F + \hat{I}_t^F = 0.1604\hat{Y}_t + 0.8396\left(\hat{C}_t^P + \hat{I}_t^P\right) \quad (6.131)$$

According to Eq.(6.13), we can identify the relation between precautionary savings and output in linearized form:

$$\hat{Y}_t = \hat{s}_t^P \quad (6.132)$$

On the basis of Eq.(6.131) and Eq.(6.132), the linearized function of precautionary savings can be written as:

$$\hat{C}_t^F + \hat{I}_t^F = 0.1604 \hat{S}_t^P + 0.8396 \left( \hat{C}_t^P + \hat{I}_t^P \right) \quad (6.133)$$

$$\Rightarrow \hat{S}_t^P = 6.234 \left( \hat{C}_t^F + \hat{I}_t^F \right) - 5.234 \left( \hat{C}_t^P + \hat{I}_t^P \right) \quad (6.134)$$

We then derive the aggregate output function for precautionary households based on Eqs.(6.125) and (6.134):

$$\hat{Y}_t = 0.3098 \left( \hat{C}_t^P + \hat{I}_t^P \right) + 0.05919 \hat{S}_t^P + 0.075 \hat{I}_t + 0.135 \hat{G}_t + 0.434 \hat{NFA}_{t+1} - 0.44 \left( \hat{NFA}_t + \hat{R}_t \right) \quad (6.135)$$

A sudden stop  $ka_t$  can be represented by the event that the ratio of capital inflows to GDP,  $ka_t = (KA_t - KA_{t-1})/Y_t$ , falls by more than 5 percent relative to the previous year. The economic shock of capital inflows may cause a decrease of aggregate amount of foreign reserves, because reserves can be used to repay external lines of credit that are not rolled over in a sudden stop. This condition could reduce the demand of domestic expenditure and domestic output  $Y_t$ .

Therefore, the exogenous process of the capital account in the model can be written as:

$$KA_t = KA_{t-1} + ka_t * Y_t \quad (6.136)$$

In the Chinese context, it is reasonable to assume that expected values of the shocks to the capital account are  $E[ka_t] = 0$  in most years.  $ka_t$  is stationary

deviation of capital account position. When a sudden stop occurs in year  $t$ , we have  $ka_t \leq ka_{t-1} - 5\%$ , where  $ka_t$  denotes a negative shock to the capital account in the DSGE model. We look for sudden stops in China over 1982-2015; and the capital flows data in our sample and the years in which there was a sudden stop are reported in Figure 9. We notice that only recently since 2014 capital outflows from China have somehow grown.

The linearized function of the capital account Eq.(6.136) is

$$\hat{KA}_t = \hat{KA}_{t-1} + \left( \frac{\bar{ka}\bar{Y}}{\bar{KA}} \right) \left( \hat{ka}_t + \hat{Y}_t \right) \quad (6.137)$$

According to the capital flows from 1982 to 2015, we calculate the average steady state value of  $\frac{\bar{ka}\bar{Y}}{\bar{KA}}$ , which is the ratio of the changes in the capital account to the current capital account position.  $\frac{\bar{ka}\bar{Y}}{\bar{KA}} = -0.17$ .

The function of intergenerational transfers EQ.(6.106) can be rewritten as:

$$B_t = 1.44S_t - 0.38w_tN_t - 0.38r_t^cK_t \quad (6.138)$$

The steady state of Eq.(138) is

$$\begin{aligned} \bar{B} &= 1.44\bar{S} - 0.38\bar{W} - 0.38\bar{K} \Rightarrow \frac{\bar{B}}{\bar{Y}} = \left( \frac{1.44\bar{S} - 0.38\bar{W} - 0.38\bar{K}}{\bar{Y}} \right) \\ &\Rightarrow \frac{\bar{B}}{\bar{Y}} = 0.2732 \end{aligned} \quad (6.139)$$

The ratio of intergenerational transfers to GDP is 27.32%.

### **6.4.2 Bayesian Estimation of the Parameters**

Bayesian estimation and evaluation techniques are now the standard tool for analysis of DSGE models. Estimation with Bayesian methods is based on the likelihood generated by the DSGE system. In contrast to GMM estimation, which is based on particular equilibrium relations, Bayesian estimation fits the complete, solved DSGE model. The Bayesian estimation is also a bridge between calibration and maximum likelihood. By using the Bayes theorem, one can combine the prior density with the likelihood function in order to get the posterior density. In this case, the posterior distribution avoids peaking at the same points as the likelihood. Moreover, the distribution of priors is also very helpful to identify structural parameters and shocks. In this case, we apply the method of An and Schorfheide (2007) and Geweke (1999) to estimate both prior and posterior distribution of the parameters.

We follow Schorfheide (2000) and apply a two-step estimation procedure involving calibration and Bayesian Maximum Likelihood methods. First, the posterior mode and an approximate covariance matrix are obtained by numerical optimization on the log posterior density. Second, we apply the Metropolis-Hastings and Markov-Chain Monte Carlo (MCMC) algorithm with 3,000 draws to obtain a sequence from the unknown posterior distribution. The Dynare processor for Matlab is employed in the computation.

### Prior and posterior distributions of estimated parameters

The parameters in Table 6.1 governing the dynamics of the model are estimated. Most pertain to the nominal frictions and elasticity in the model, while the rest denote the exogenous shock processes. Detailed descriptions of the prior distributions for the structural DSGE parameters and the shock parameters are given in Table 6.1. We also estimate the ratio of precautionary savings to households' expenditure  $RSP$

**Table 6.1 Prior and Posterior Distributions of Structural Parameters**

Parameters	Type	Prior Mean	St.Dev	St.Dev	Conf. Int	Post. Mean
$\alpha$	Beta	0.040	0.002	0.0224	[0.3633, 0.4275]	0.3976
$\beta$	Beta	0.980	0.0002	0.0020	[0.9786, 0.9848]	0.9816
$\delta$	Beta	0.040	0.003	0.0035	[0.0249, 0.0305]	0.0278
$\sigma$	Gamma	2.000	0.020	0.0196	[1.9738, 2.0345]	2.0028
$\gamma$	Gamma	3.130	0.003	0.0006	[3.0959, 3.1041]	3.1301
$\varphi$	Gamma	6.160	0.003	0.0007	[6.1553, 6.1641]	6.1597
$\kappa$	Normal	0.500	0.050	0.0491	[0.4564, 0.5862]	0.5151
$h$	Normal	0.610	0.005	0.0049	[0.5906, 0.6119]	0.6012
$\mu$	Gamma	0.840	0.003	0.0027	[0.8365, 0.8456]	0.8414
$\theta$	Gamma	4.610	0.003	0.0033	[4.6047, 4.6151]	4.6101
$l$	Gamma	2.000	0.003	0.0027	[1.9939, 2.0045]	1.9986
$\xi$	Gamma	0.600	0.003	0.0003	[0.5918, 0.5992]	0.5953
$e$	Inv_gamma	0.010	0.0003	infini	[0.1899, 0.2113]	0.2014
$RPF$	Gamma	0.700	0.003	0.0187	[0.6932, 0.7025]	0.6979
$RSP$	Beta	0.3	0.003	0.0018	[0.2956, 0.3051]	0.3006

The complete set of estimation results is presented in Table 6.1. We use the beta distribution for parameters bounded between 0 and 1, except for  $\kappa$  and  $h$ . For parameters measuring elasticity, such as  $\gamma$ ,  $\sigma$ ,  $\varphi$ ,  $\theta$  and  $\xi$ , we use

the gamma distribution. Normal distribution is applied when estimating  $\kappa$  and  $\alpha$ . The value of priors and posteriors of the elasticity parameters are very close to each other, which suggests that the value of elasticity parameters is very stable in the framework and well identified. The elasticity of substitution with respect to the depreciation rate is higher than its priors, suggesting a slower response of exchange rate depreciation to the shocks.

In below, the graphs in Figure 6.1.1, 6.1.2 and 6.1.3 show the Markov-Chain Monte Carlo (MCMC) univariate diagnostics for the parameters. These are the main sources of feedback to gain confidence in the results. In order to diagnose to what degree the results are reasonable and accurate, attention is to be paid to two key points. First, we should note whether the results within any iterations of Metropolis-Hastings simulations, no matter how many they may be, are similar. Second, we should observe the distance between various chains. More specifically, the two lines in the charts represent measures of parameter vectors within and between chains, respectively. From Figure 1 we note that for most parameters the distance between the two chains is very small. Moreover, the plotted moments of structural parameters  $\alpha$ ,  $\beta$ ,  $\delta$  and  $\sigma$  are relatively stable and converge. This suggests that the identification of the priors and the results from the chains are reasonable.

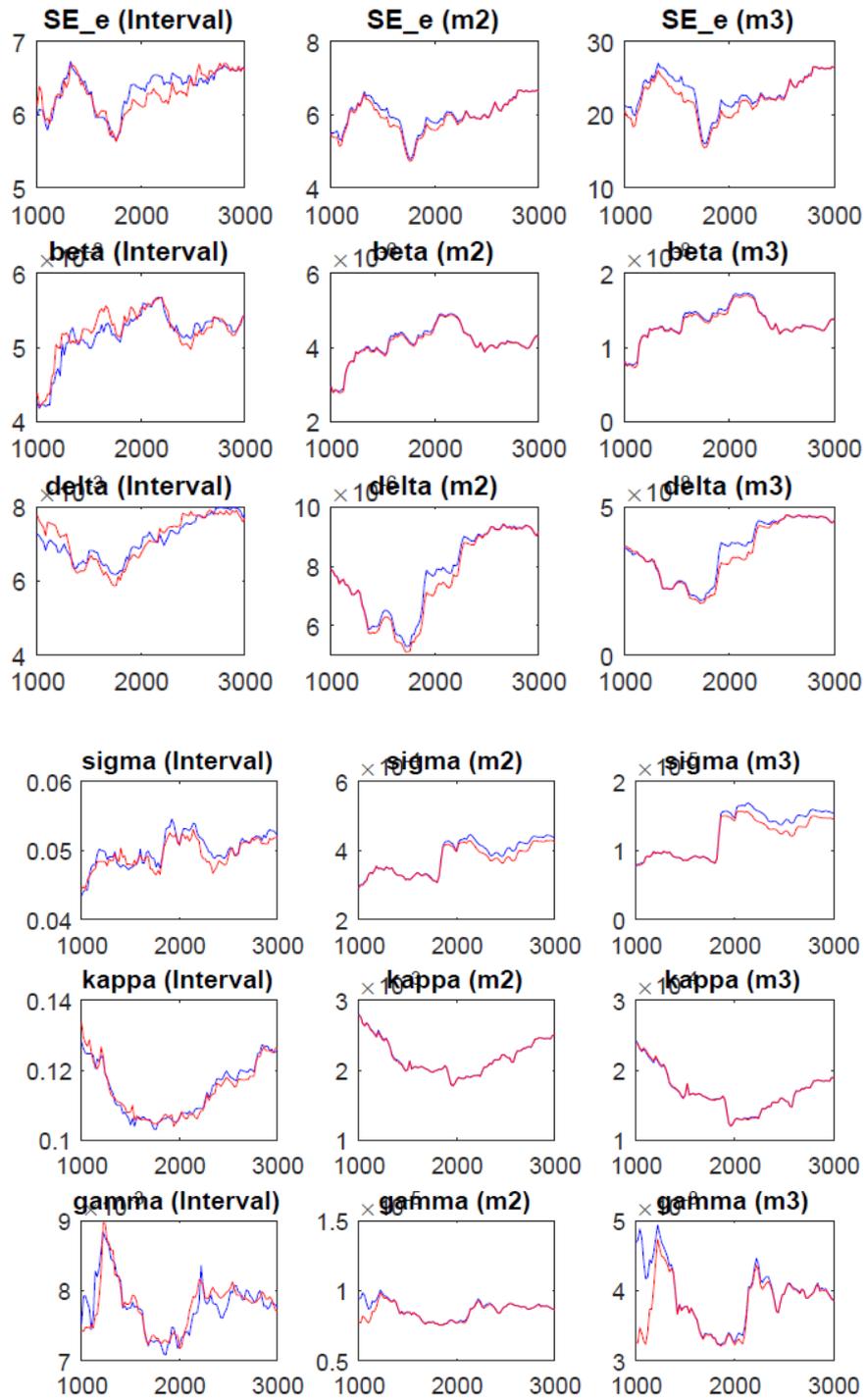


Figure 6.1.1 Markov-Chain Monte Carlo Univariate Diagnostics for Parameters(1)

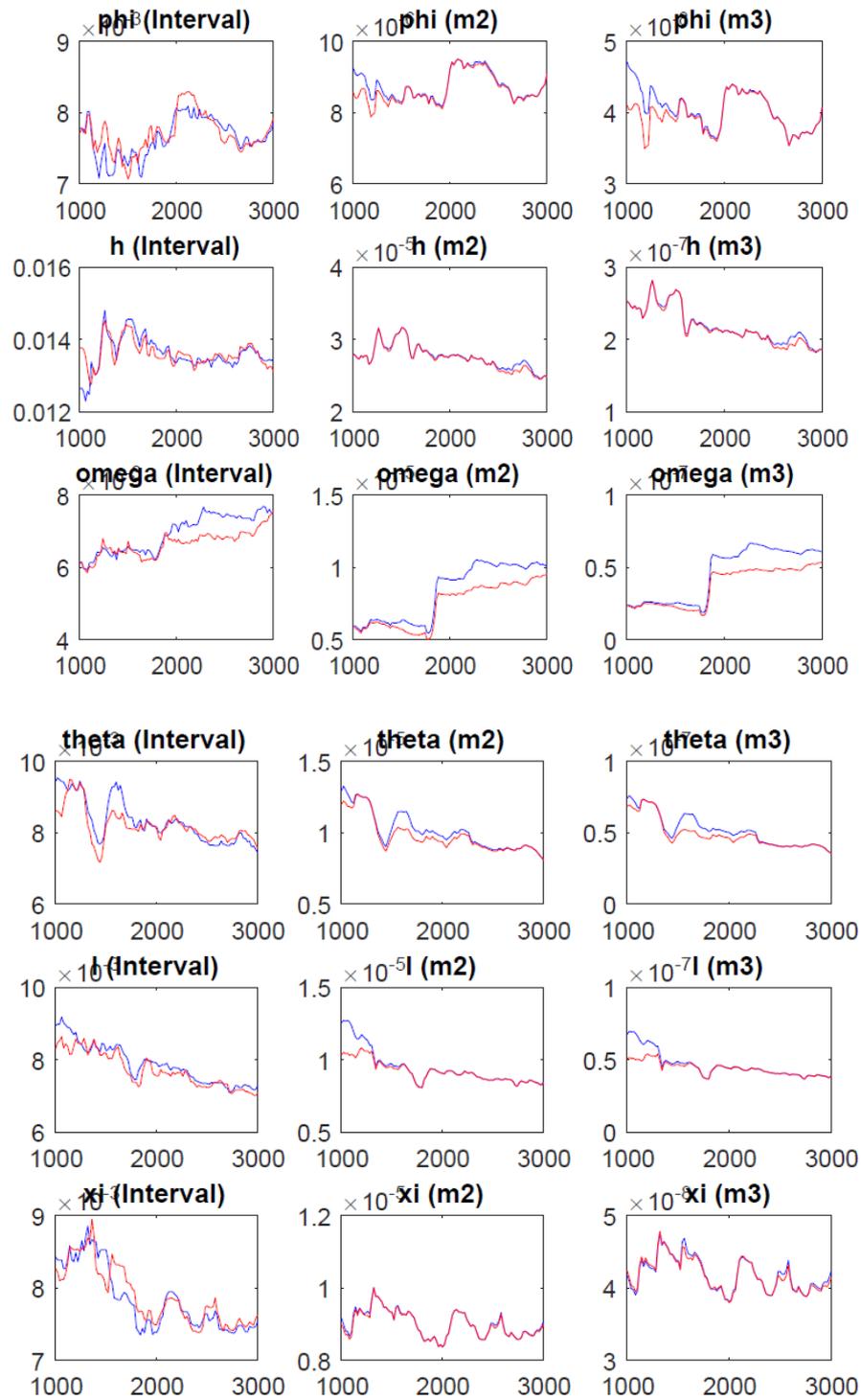
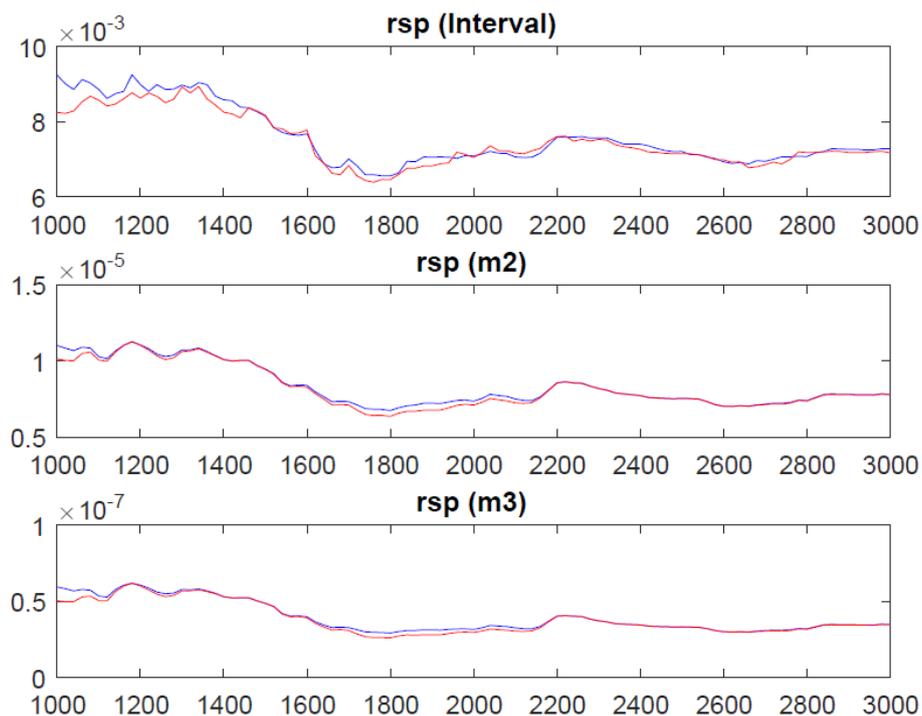
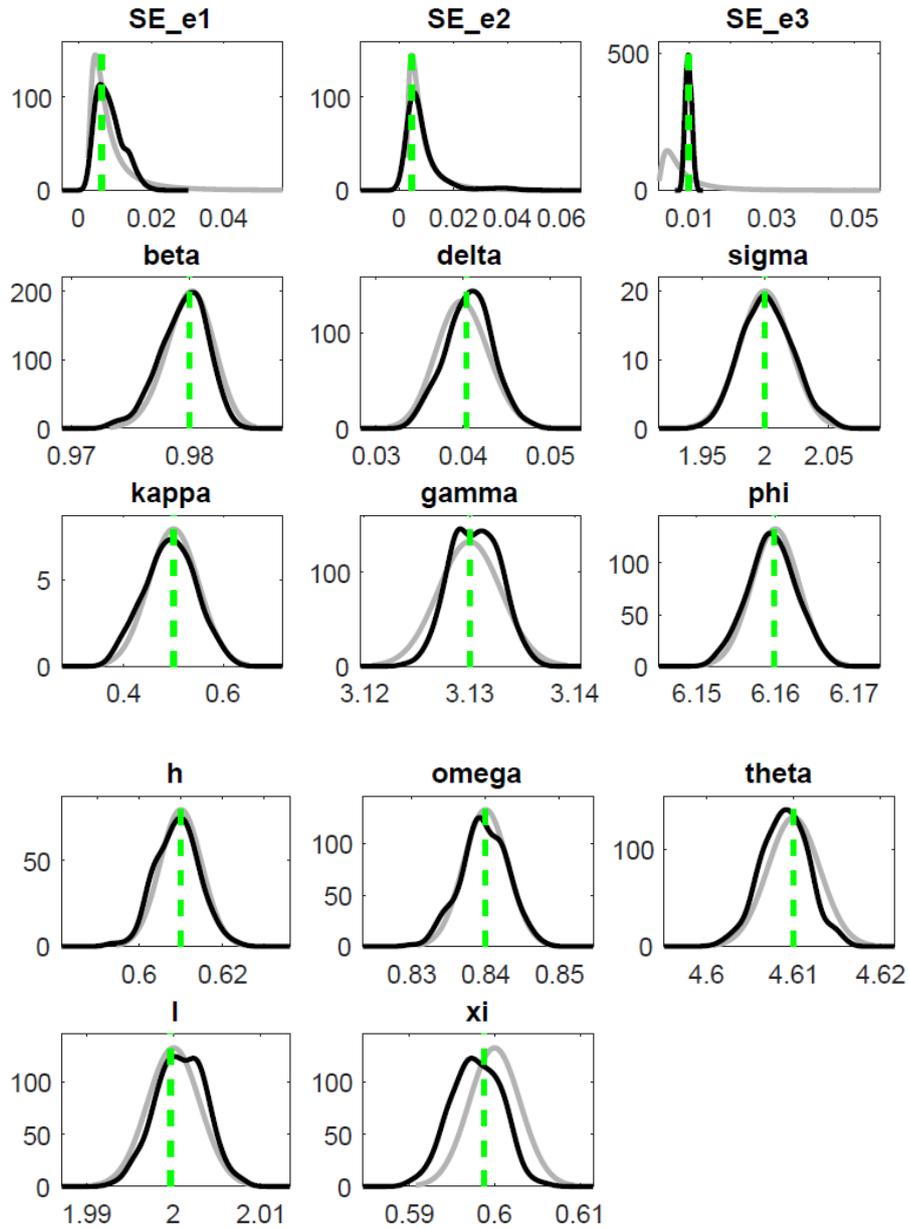


Figure 6.1. 2 Markov-Chain Monte Carlo Univariate Diagnostics for Parameters(2)



**Figure 6.1.3 Markov-Chain Monte Carlo Univariate Diagnostics for Parameters (3)**

Next, figure 6.2 displays prior and posterior distributions of the parameters and shocks. Overall, all parameters seem to be well identified, as shown in the outcome that the posterior distributions are either not centred on the priors, or are centred but with a smaller dispersion, indicating high significance of the estimates. For the price stickiness, 84.14 % of firms do not adjust prices within one year. This implies that prices are re-optimized once every three quarters. The fraction of households setting their wages according to the last-period wage  $\xi$  is estimated to be 59.53%, implying a big share of backward-looking firms.



**Figure 6.2 Prior and Posterior Distributions in Metropolis-Hastings Procedure**

Multivariate diagnostics present an aggregate measure based on the eigenvalues of the variance-covariance matrix of individual parameters. The analysis method of multivariate diagnostics is similar to that of the MCMC univariate diagnostics, because the results of these two diagnostics are based on the same theory. The plotted moments of multivariate diagnostics are relatively constant and converge, implying that the results are reasonable.

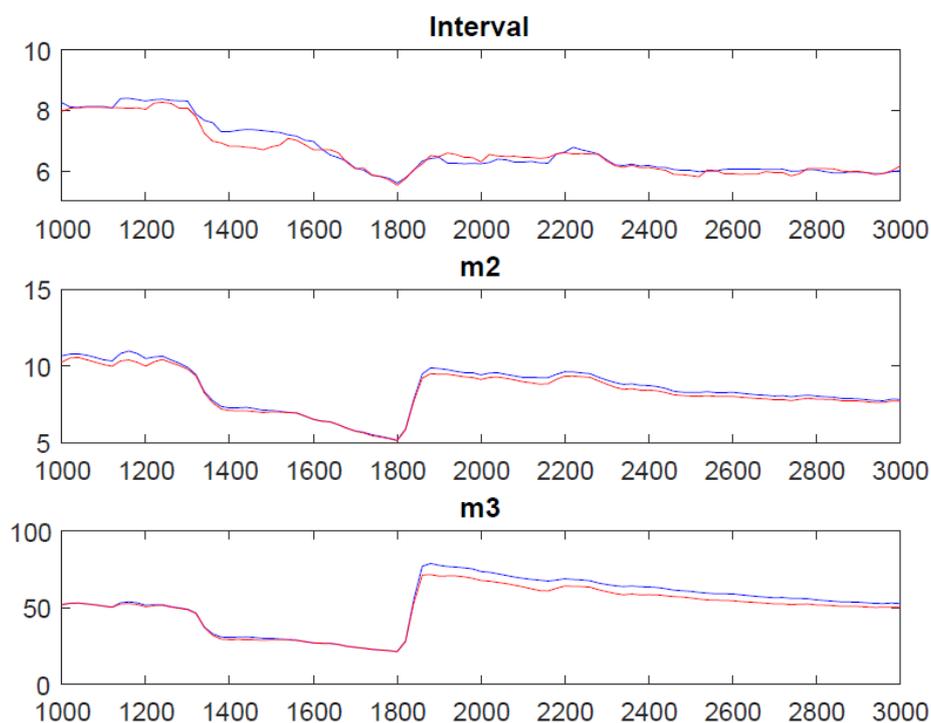
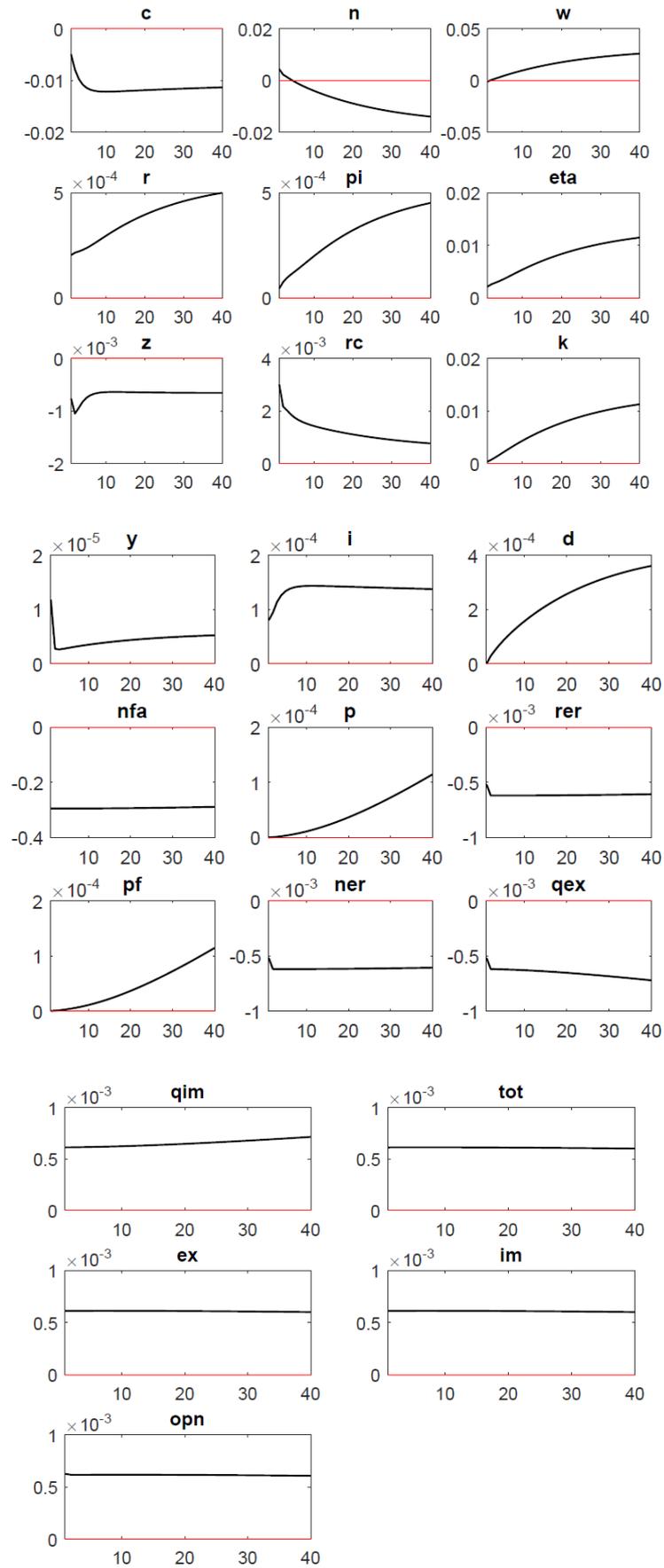


Figure 6.3 Multivariate Diagnostics for Parameters

## 6.5 Simulation and Impulse Response Functions

We now explore the influences of currency devaluation, intergenerational transfers, precautionary savings and capital outflows in the model, in particular the influence of precautionary savings. First, we present the results of the impulse response functions of every variable to currency devaluation. Impulse response functions relate to the reaction of a dynamic system to some exogenous change(s). In our case, the dynamic systems are the linearized models produced in the previous sections. According to the dynamic general equilibrium that we have developed, the model's responses to a 1% currency devaluation shock are given in Figure 4.



**Figure 6.4 Estimated Model Responses to Currency Undervaluation**

Figure 4 shows that the reactions of consumption to currency devaluation consist of a negative change in the adjustment speed of responses. Households decide to reduce their consumption when local currency is depreciating.

The contribution of exchange rate undervaluation to wages and inflation are similar. Currency undervaluation causes the appreciation of wages and inflation in the long run. In order to maintain the undervalued exchange rate, China's central bank needs to sell RMB to offset upward pressure on the currency. Then, the amount of China's currency in domestic circulation increases. As a result, inflationary pressures arise. Rising inflation causes higher living costs for domestic households. Therefore, Chinese enterprises face pressure to increase wages, and raise the prices of their products.

China's exchange rate policy affects the country's holdings of international reserves mainly in three channels: the trade balance (imports and exports), central bank intervention in the exchange market, and foreign direct investment. The response of the central bank is to decrease the short-term interest rate, maintain a stable nominal interest rate in the long run and increase the RMB exchange rate temporarily.

In order to maintain RMB exchange rate against currency devaluation shock, China's central bank will sell foreign currency and buy RMB on the foreign exchange market. Therefore, currency undervaluation shock has a negative impact on net foreign asset position. A primary reason for Chinese central

bank to accumulate reserves is to protect the domestic credit markets and ensuring financial and exchange rate stability, while limiting external currency depreciation.

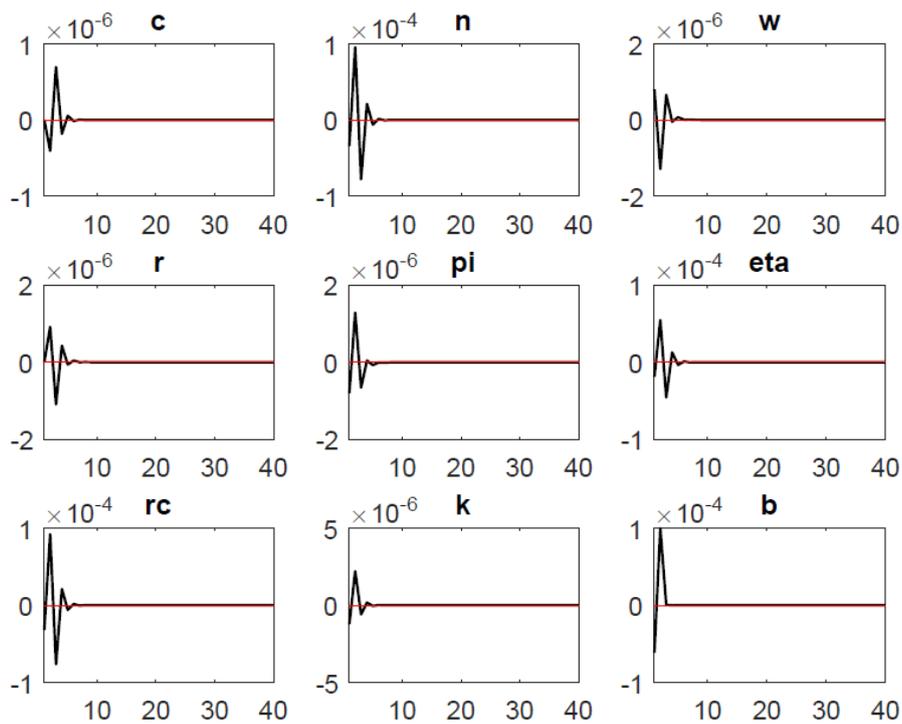
In addition, undervaluation shocks lead to a positive response of GDP growth in the first several time periods. This phenomenon suggests that China's exchange rate policy has some effect promoting the country's growth. Undervalued exchange rate makes China's domestic products and capital cheaper and more alluring to foreign investors, so that more foreign investment will be attracted to China's domestic market.

According to the impulse response functions, both exports and imports have positive responses to the exchange rate intervention shock. Undervaluation of domestic currency, which leads to an increase in the RMB exchange rate, brings about appreciation of foreign currency and stimulates exports and imports. This strategy then leads to improvement in the trade balance.

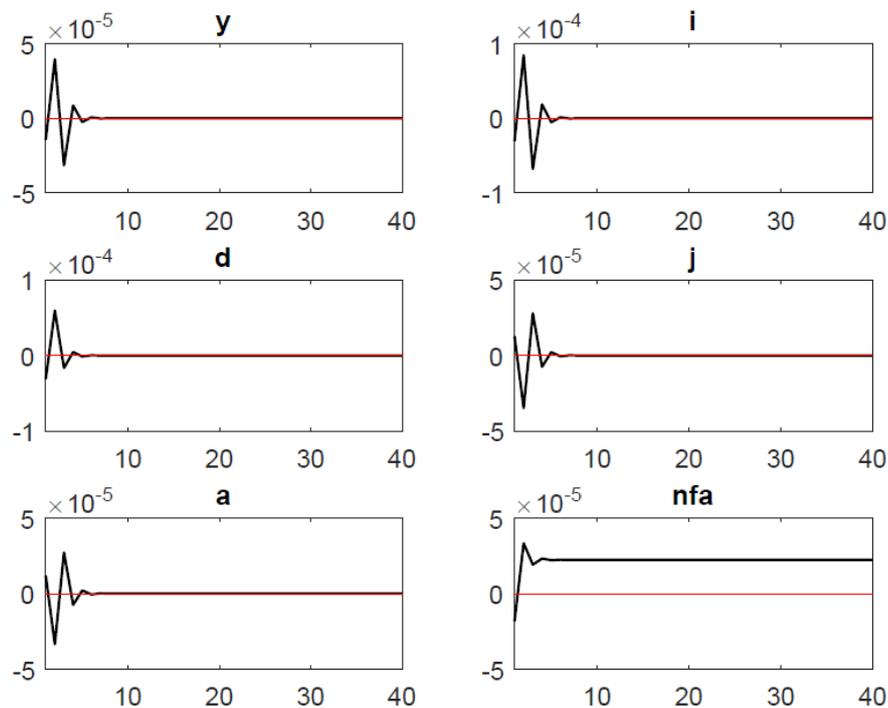
In the domestic economy, undervaluation also causes increase of wages and inflation, which play a crucial role in China's growth. In order to maintain a low currency value, China's central bank sells more RMB to buy the dollar. As a result, the amount of China's currency in the domestic economy increases, leading to inflationary pressure. The increasing inflation results in high living costs for domestic households. In response, workers demand better wages to protect their living standard. Consequently, Chinese firms face increased labour costs and need to raise the prices of their products.

Eventually, increasing wages, consumption and inflation lead to the growth of the whole economy. In the model, an 1 percent currency depreciation induces an additional aggregate output increase by about 1.2 percent and a decrease of net foreign assets by 0.52 percent.

Figures 6.5 and 6.6 illustrate the impulse responses after a change in intergenerational transfers. We investigate the relationship between net foreign assets and a main economic variable in China to explore the impacts of foreign reserves on the general economy.



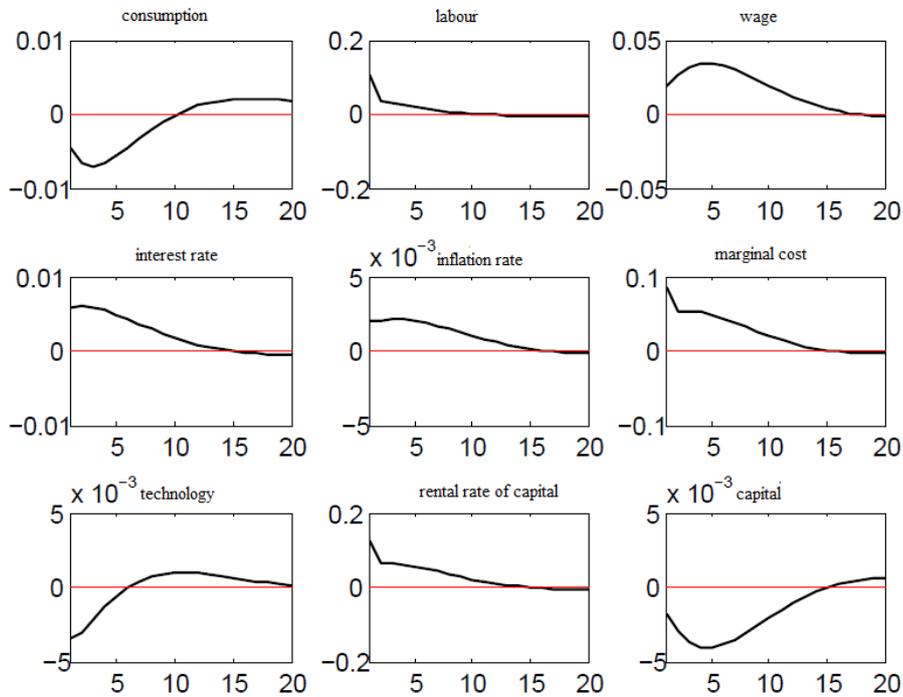
**Figure 6.5 Model Responses to Intergenerational Transfers (1)**



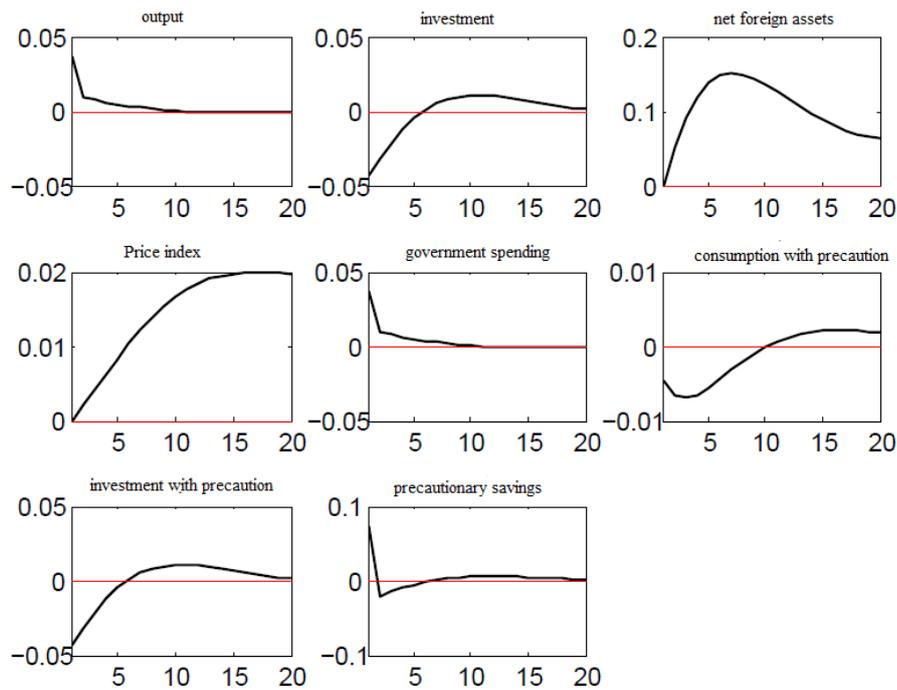
**Figure 6.6 Model Responses to Intergenerational Transfers (2)**

We can see a clear positive relation between the response of net foreign assets and an intergenerational transfers shock. An increase in intergenerational transfers leads to an increase in net foreign assets and, as a result, positive productivity output. The intergenerational transfers shock also leads to increased investment, deposits and capital accumulation. In addition, higher intergenerational transfers enable households to reduce their consumption and increase their savings. These results indicate that intergenerational transfers are one reason why China would have accumulated such a large amount of foreign assets. The value of net foreign assets is equivalent to the sum of foreign assets held by monetary authorities and deposit banks, minus their liabilities. Therefore, we can conclude that intergenerational transfers boost savings of households at a micro level and then raise holdings of international reserves on a macro scale. Quantitatively, in the model economy

parameterized to China, one percent standard deviation of the change of intergenerational transfers leads to 0.279% standard deviation of GDP, and it also causes 1.2% standard deviation of net foreign assets.



**Figure 6.7 Model Responses to a Change in Precautionary Savings (1)**



**Figure 6.8 Model Responses to a Change in Precautionary Savings (2)**

According to the impulse response functions, both consumption and investment have negative responses to the shock of precautionary savings. This result is in accordance with the assumption that households decide to reduce their consumption and hence investment to accumulate savings for precautionary purposes. To accommodate the precautionary savings motive, the consumption in the stochastic model is upward tilting and accompanied by a gradual accumulation of a buffer stock of net foreign assets. In initial periods, consumption is lower than in the deterministic case, but as wages increase and the interest income from foreign assets grow, it eventually exceeds consumption in the deterministic case.

Positive changes in precautionary savings also lead to the higher inflation rate and higher wages in the model. Then, according to the model responses, the higher inflation rate leads to higher price index and output. In the domestic economy, changes in precautionary savings cause an increase of wages and inflation, which play a crucial role in China's growth. In order to maintain a low currency value, China's central bank sells more RMB to buy the dollar. As a result, the amount of China's currency in the domestic economy increases, leading to inflationary pressure. The increasing inflation results in high living costs for domestic households. In response, workers demand better wages to protect their living standard. Consequently, Chinese firms face increased labour costs and need to raise the prices of their products. Eventually, increasing wages, consumption and inflation lead to the growth of the whole economy.

The reactions to net foreign assets indicate strong and positive correlation between precautionary savings and foreign reserves accumulation. Quantitatively, in the model economy parameterized to China, an increase by 0.1 percent precautionary savings induces an additional foreign reserves increase by about 0.15 percent of net foreign assets in the initial year. This trend of surpluses gradually decreases thereafter. On the other hand, an 0.1 percent change of precautionary savings leads to 0.05 percent increase of gross output. This suggests that precautionary savings have a positive impact on the growth of the domestic economy.

Following Cherif and Hasanov's (2013) model, we set the ratio of precautionary savings to the sum of consumption and investment to be 0.3. On the basis of the parameters values obtained after Bayesian estimation, we calculate that the share of households' precautionary savings in the model is equal to 13.04% of aggregate output. This result suggests that precautionary savings are an important driving force behind the accumulation of foreign reserves in China.

Overall, results from the model of precautionary households offer two main findings. First, impulse responses from the parameterized model indicate a significant positive correlation between precautionary savings and net foreign assets. Second, optimal model outcomes for precautionary households indicate that precautionary motives have a positive impact on the growth of consumption and investment in the long run, while increasing the growth of the domestic economy in the short run.

At last, we explicate how the system reacts to the exogenous process of sudden stop shocks with impulse response functions. We identify expected value of the shocks to capital account  $E[ka_t]=0$  in most years. Capital flows of China are a random walk process in the model, and we report the process of China's capital account over 1998 -2014 in Figure 9 below. When a sudden stop occurs in year  $t$ ,  $ka_t$  denotes a negative shock to the capital account in the model.

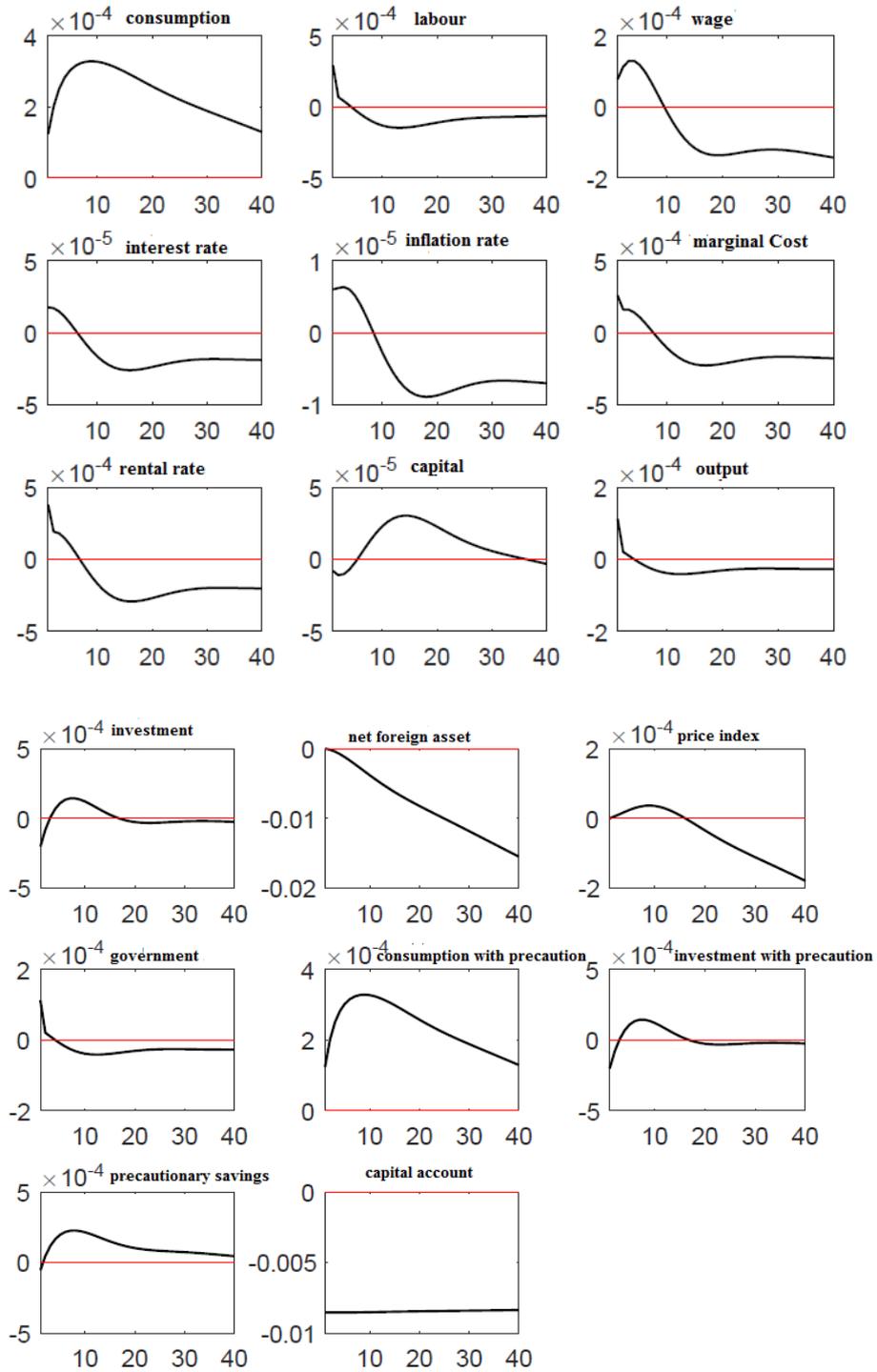
A sudden stop in capital flows associated with a fall in output. Capital outflow lead to a long run slowdown of the economy. On the other hand, a sudden stop in capital flows has obvious negative impact on the accumulation of foreign reserves. The decline in net foreign assets is 15% of its initial position at the 40<sup>th</sup> period. This result is in accordance with the assumption that the Chinese government decide to use large amount of foreign exchange reserves in response to the withdrawal of dollar-denominated deposits from the domestic banking system.

In the first order linearized model, we have the deterministic steady states of net foreign assets is zero. The standard deviation and mean of international reserves is also zero. It means the amount of net foreign assets do not change in a deterministic system.

In second order linear equations, the effect of the shocks depend on the state of the system when the shocks hit. Impulse response functions are the

expected future path of the endogenous variables conditional on a shock in period 1 of one standard deviation. They are the results of actual Monte Carlo simulations of future shocks.

We therefore calculate the mean and standard deviation of net foreign assets in second order linear equations. The mean value of the changes of foreign reserves is also zero, but the standard deviation of it changes is 4.7563% in response to a 5% capital outflow of GDP shock in the stochastic system. If we try to explore the reaction of system to a 1% capital outflow of GDP, we will have the standard deviation of foreign reserves is 0.95%. We consider that the gap between deterministic steady state and stochastic steady states of foreign reserves indicates the amount of precautionary foreign reserves against the risk of sudden stop. The amount of precautionary reserves depends on the size of potential capital outflow. We observe a fall in the capital account in 2014, amounting to more than 3.4 percent of the previous year's output. In this case, the standard deviation of net foreign assets is 3.237%.

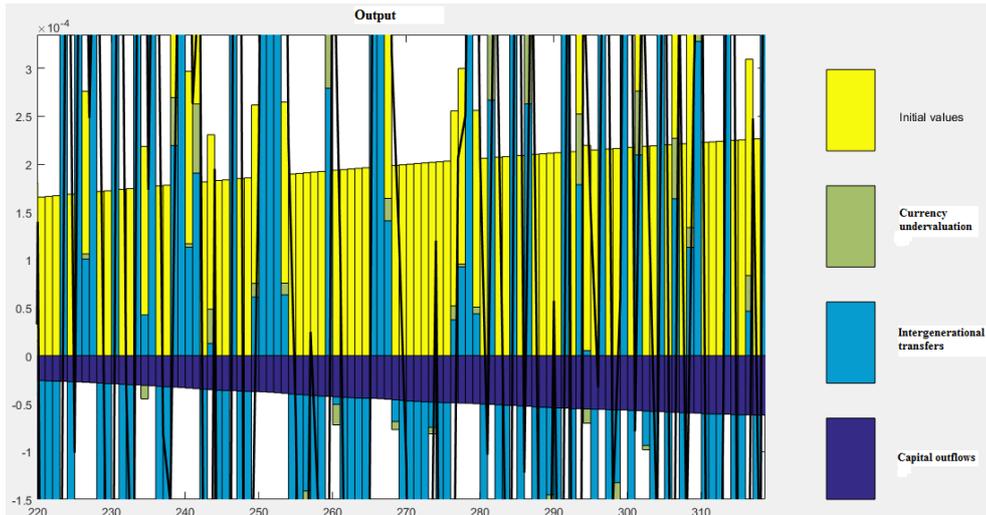


**Figure 6. 9 Model Responses to Sudden Stop in Capital Flows**

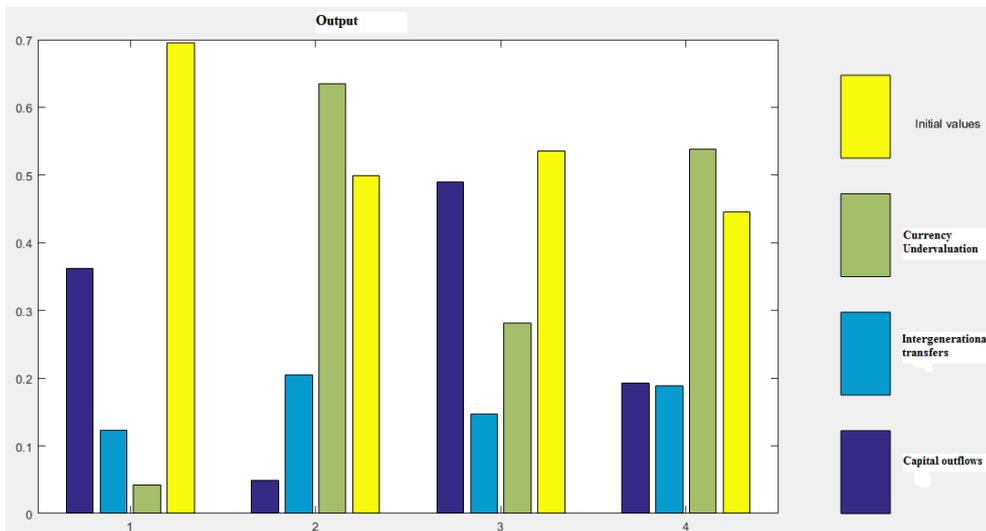
## 6.6 Shock Decompositions

Figure 6.10-6.12 represent how aggregate output have been affected by the selected shocks. Both exchange rate depreciation hypothesis and intergenerational transfers can explain a large fraction of the increase of economy. Capital outflow, however, play a negative role in the economic growth. Net outflows leads to the slowdown in long run GDP growth. The relative contribution of different shocks to the aggregate output has been shown in Figure 6.11 and 6.12. The influence of currency undervaluation is more significant than two other factors.

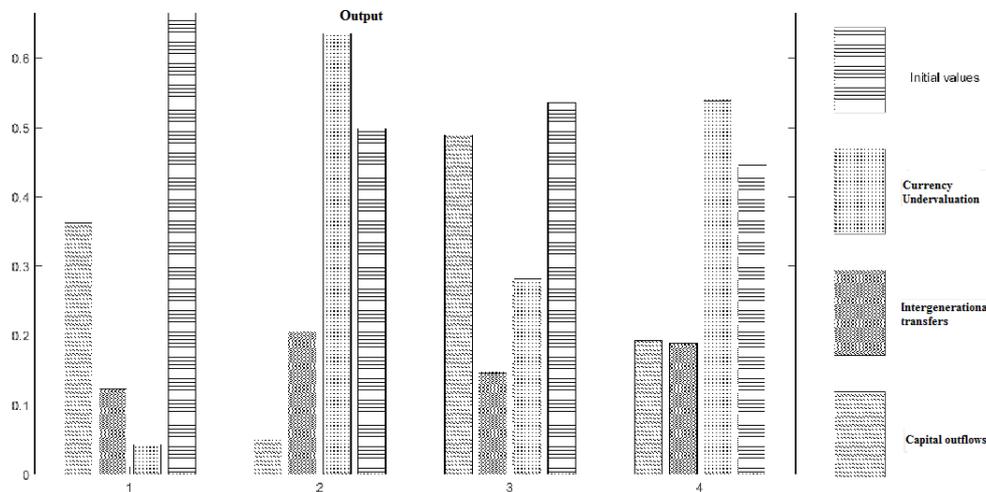
Figure 6.13-6.15 provide shock decompositions for China's hoard of foreign exchange reserves. Capital outflow exerts a strong negative effect on the size of foreign exchange reserves, which dominates the positive impacts of intergenerational transfers and exchange rate depreciation. China's capital flights, starting in 2014 is clearly bringing out the decline in international reserves. These results suggest that large precautionary demand is consistent with high levels of international reserves.



**Figure 6.10 The Contribution of Shocks to Deviations of Output (1)**



**Figure 6.11 The Contribution of Shocks to Deviations of Output (2)**



**Figure 6.12 The Contribution of Shocks to Deviations of Output (3)**

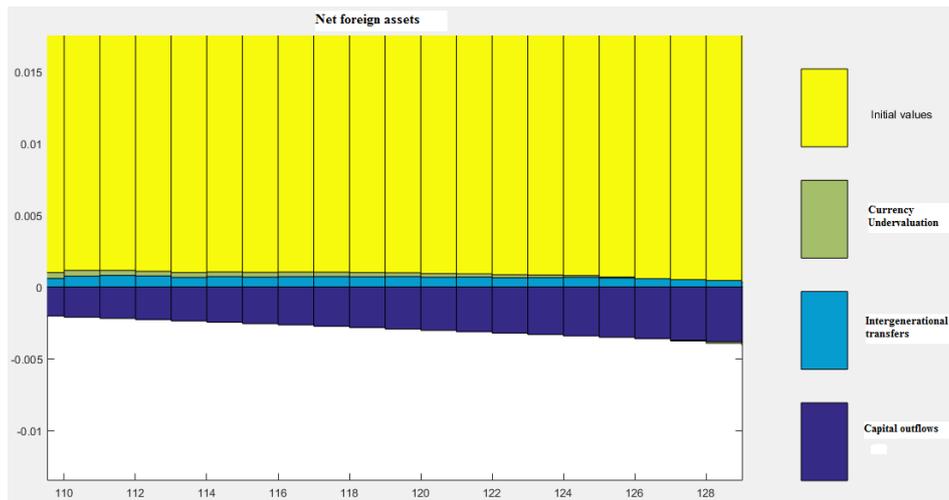


Figure 6.13 The Contribution of Shocks to Deviations of NFA (1)

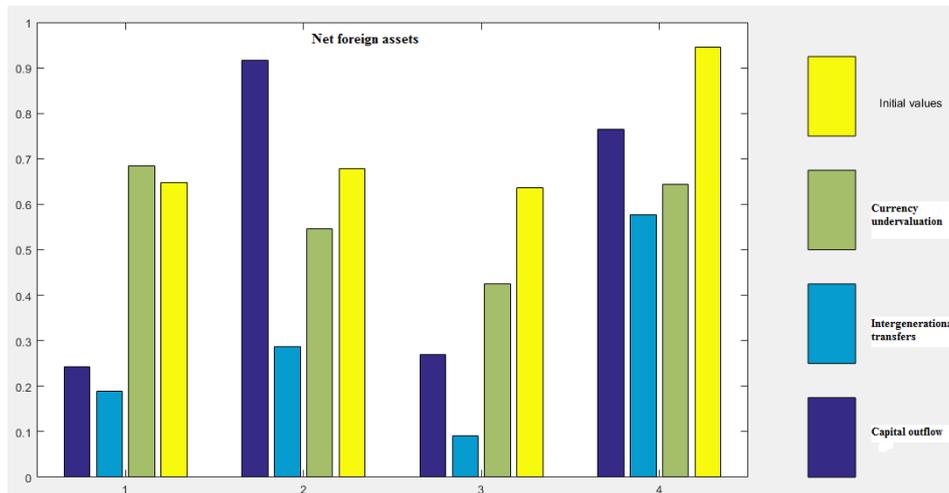


Figure 6.14 The Contribution of Shocks to Deviations of NFA (2)

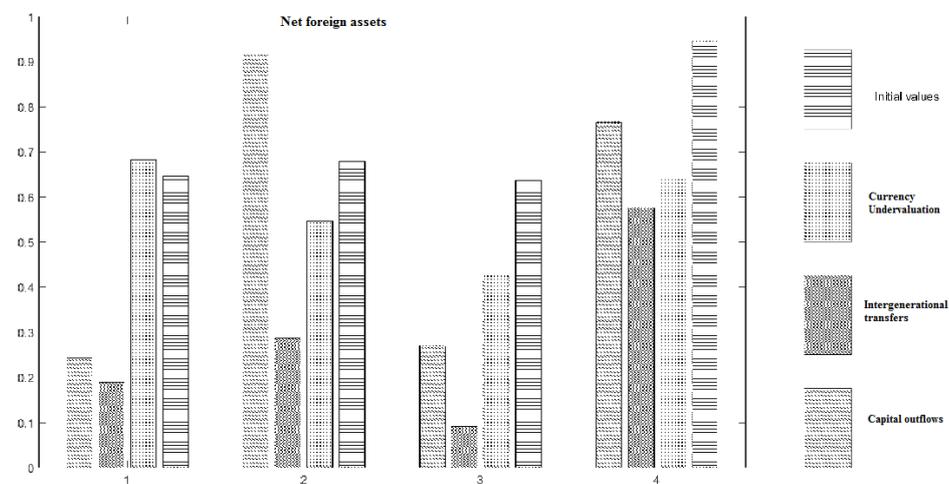


Figure 6.15 The Contribution of Shocks to Deviations of NFA (3)

## 6.7 Conclusions

This Chapter uses a parameterized DSGE model to pursue the primary motivation for China's hoarding of reserve accumulation. We discussed the role of three vital motives that play in China's accumulation of international reserves. They are intergenerational transfers, currency undervaluation and precautionary motive. To compare the quantitative importance of these competing explanations, we develop an estimated DSGE model including above motives for China.

According to the parameters and the calculation of the model, we have the ratio of intergenerational transfers to GDP is 27.32%. Quantitatively, in the model economy parameterized to China, one percent change of intergenerational transfers leads to a 0.279% standard deviation of GDP and it also causes 1.2% standard deviation of net foreign assets. 1% changes of intergenerational transfers in terms of GDP led to 1.02% variation in GDP and 0.449% variation in international reserves.

On the other hand, the quantitative importance of the precautionary savings is equal to 26.11% of gross savings and 30.5% of net foreign assets in the model. Furthermore, while the amount of precautionary reserves held by the central bank is a function of risks and shocks, in the Chinese context, they are mainly influenced by potential capital outflows, rather than sudden stops of capital inflows. For a 1% capital outflows in terms of GDP, the standard deviation of foreign reserves will be 0.95%. Finally, 1% currency

depreciation induces an additional aggregate output increase by about 1.2 % and a decrease of net foreign assets by 0.52 percent.

The estimation results suggest that the total economic value of intergenerational transfers in China is bigger than the capital flow caused by the other two motivations. The ratio of intergenerational transfers to GDP is 27.32%. However, the influence of currency undervaluation plays a more importance role in an economic growth perspective. One percent changes in the exchange rate will bring out more economic growth than same percent changes in international transfers.

In the respect of international reserve accumulation, precautionary motivation is the most significant factor. The results of this chapter represents that the precautionary motive is an important driving force behind massive accumulation of foreign reserves in China and play an important role in the economy. The impulse response of net foreign assets to precautionary motive indicates a positive correlation between the precautionary demand and accumulation of foreign reserves. In the analysis of shock decompositions, capital outflows have a strong negative impact on the accumulation of international reserves, which dominates the positive effect of intergenerational transfers and currency undervaluation. According to the model results, precautionary motive is the primary motivations for China's foreign exchange reserve accumulation.

## **Chapter 7 Conclusions**

### **7.1 Main Findings**

In this thesis, we address the question of why China's central bank decided to accumulate such a massive amount of foreign exchange reserves. We begin by presenting an overview of the traditional theories of international reserves accumulation and the limitations of the current literature. Inspired by the advantages of DSGE modelling methodology, we apply DSGE techniques to analyse the reasons behind the reserve accumulation in China. To investigate the primary motivation for China's accumulation of foreign exchange reserves, we discuss the relative importance of three potential motives for hoarding international reserves, namely intergenerational transfers, precautionary savings and exchange rate undervaluation.

First, we shed light on the interplay between intergenerational transfers and China's hoarding of foreign exchange reserves. Although China's high domestic savings are a main cause of the growth of its massive foreign exchange reserves, these excessive savings are largely driven by the intergenerational transfers that are prominent in China. It follows that

intergenerational transfers should have a significantly positive relation with China's foreign reserve build-up. Given Chinese tradition and the structural imperfection of the country's financial markets, we show that intergenerational transfers are a significant contributor to China's wealth and thus a critical driver for the growth of its international reserves.

In a general equilibrium setting, our analysis shows that real undervaluation contributes to reserve accumulation in a complex process. Contrary to the commonly assumed mercantilist interpretation, undervaluation affects China's reserve build-up mainly through its positive effect on economic growth. We therefore investigate the relation between foreign reserve accumulation and real exchange rate depreciation in China from a growth perspective. In the short run, undervaluation induces a positive response from the total factor productivity channel, which promotes growth. Over the medium term, both exports and imports have a positive response to real undervaluation in the impulse response analysis. In the longer run, undervaluation leads to an increase in income. While income increases will cause wages and the price level to rise and hence will offset the trade balance effect to some degree, model results show that the income effect is positively related to China's growth.

Further, we employ a parameterized DSGE model to quantify the role of precautionary savings in China's economy. We calculate that the share of households' precautionary savings is equal to 13.04% of aggregate output. The quantitative importance of the precautionary savings is equal to 26.11%

of gross savings and 30.5% of net foreign assets in the model. The estimation results suggest that the precautionary motive is an important driving force behind the massive accumulation of foreign reserves in China and plays an important role in the economy. The impulse response of net foreign assets to changes in the precautionary motive indicates a positive correlation between the precautionary demand and accumulation of foreign reserves.

Finally, we follow the method of In't Veld et al. (2011) and propose a DSGE model that includes precautionary motive, mercantilist motive and intergenerational transfer arrangements to pursue the primary explanation for China's hoarding of foreign reserves. Our modelling strategy and its estimation provide results showing that these main motivations do have a significantly positive impact on foreign reserves accumulation.

The results suggest that the total economic value of intergenerational transfers in China is bigger than the capital flow generated by the other two motivations. The ratio of intergenerational transfers to GDP is 27.32%. On the other hand, currency undervaluation plays a more important role in an economic growth perspective.

Overall, the findings indicate that the most significant element driving the accumulation of international reserves in China is the precautionary motivation. In the analysis of risky steady state, 1% of capital outflows in terms of GDP caused 0.95% standard deviation in foreign reserves. The impact of capital outflows causes more change and decline in China's foreign

exchange reserves than does any other competing explanation. Thus the precautionary motive is the primary motivation for China's current foreign exchange reserve accumulation. The contribution of this thesis is to provide an innovative approach to compare the influence of different motives on the build-up of foreign reserves in China.

## **7.2 Future Research**

The results of this thesis raise several questions. First, there are still some gaps in the literature about risky steady state in a DSGE framework. Although linearized form of DSGE models are widely used, most relevant approach tried to solve the risk steady state in nonlinear models. In this thesis, we have investigated the impact of risky steady state in a linearized framework. However, further research is still very necessary to support the new development in this area.

Now that we have provide a solid analysis about the influence of competing theories in China's international reserves accumulation, it would be very interesting to consider how to apply this approach to evaluate China's international reserve strategy. Capital outflow is a serious problem China's economy is facing now. This phenomenon implies the importance of precautionary motivation, which accord with the results of this thesis. The future research may force on the changes of China's exchange rate policy.

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