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THE INTERPRETATION

OF

A MAGNETIC SURVEY OVER THE WHIN DYKES

-o-O-o-

Being

a Thesis submitted for the degree of Master of Science

in the

University of Durham

by

R. A. Stacey, B.Sc.

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SUMMARY

The purpose of this research has been to investigate the direction of magnetisation of the Whin dykes of north-east England. A fortnight in October, 1960, was spent making detailed magnetic traverses over the Hett, Haydon Bridge and Causey Park dykes using a Proton Magnetometer. The profiles obtained were then scaled, in accordance with a suggestion by Dr. Bott, to the same amplitude and horizontal spread between the mean values of the maximum and the minimum of the anomalies. This effectively eliminates the intensity of magnetisation and the scale of the body causing the anomaly and allows a direct comparison to be made with a similarly scaled set of standard profiles over theoretical models.

Two computer programmes have been written to facilitate the calculations. The first programme gives the horizontal, vertical and total magnetic anomalies at a series of specified points over a two-dimensional body of polygonal cross-section. In the second programme, which has been used to prepare the standard curves, the horizontal and vertical anomalies have been dispensed with and the amplitude and spread of the total anomaly has been scaled in the same way as the observed profiles.

It is shown that the comparison of similarly scaled observed and theoretical profiles can lead to a rapid interpretation of the anomalies - although, in the case of the Whin dykes it is found that the presence of the Whin Sill at depth has complicated the interpretation at a number of localities.

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It is concluded that the Whin Sill may be faulted below the western end of the Hett dyke, suggesting the dyke was intruded after the Sill. The period separating their intrusion cannot have been a long one owing to their similarity in composition and their relation to the mineralization of the area.

There also seems to be a consistent difference between the direction of the total intensity of magnetisation of the Whin Sill and the dykes. Various reasons are put forward to account for this, but the data is insufficient to permit any definite conclusions to be drawn.

CHAPTER I

INTRODUCTION AND GEOLOGY

1. Introduction

This thesis is the result of one year's work in the Geology Department of the Durham Colleges in the University of Durham. The purpose of the research has been to investigate the magnetisation of the Whin dykes of Northumberland and Durham using profiles obtained with a Proton Magnetometer.

Last year, R. M. Smith (1960, unpublished) completed a number of traverses over the Whin Sill where it is affected by the Stublick Fault, which suggested the direction of the total magnetisation was horizontal with a declination of about 180°. This agrees with the palaeomagnetic work on orientated specimens by Creer, Irving and Nairn (1959). However, traverses over the Haydon Bridge dyke by undergraduates (during practical classes in geophysics) suggested that the direction of the total magnetisation was close to that of the present Earth's field. Part of the year's work has been to test the consistency of this contrast and, if possible, to explain it.

It is well known that the number of variables involved in magnetic work makes quantitative interpretation difficult. However, Dr. D. Masson-Smith (1961) has pointed out that it follows from the formulae given in Heiland's "Geophysical

Exploration" (1940) for the horizontal and vertical magnetic anomalies over a semi-infinite slab with a sloping end, that by scaling the amplitude and the horizontal spread of a total anomaly, certain variables could be eliminated, namely the intensity of the total magnetisation and the scale of the body. This is because the anomaly at any given angular position with respect to a body is independent of the size of that body, provided its shape remains the same. He has also produced (1961) a catalogue of curves with their respective scaling factors for a series of directions of total magnetisation over numerous two-dimensional bodies, each with a different depth-width ratio.

The second purpose of this work has been to write computer programmes to calculate the magnetic anomalies over two-dimensional bodies and to carry out the scaling operations. These programmes are then used to interpret the anomalies found over the Whin dykes. The programmes, suitable for Ferranti Pegasus machines, are set out together with explanatory flow diagrams after a discussion of the geology of the area and of the fieldwork completed. The detailed specifications of the programmes are included in the Appendices.

2. Geology

Teall (1884) first drew attention to the petrological similarities between the Great Whin Sill and the dykes at Hett and High Green. Since then, it has been realised that these similarities hold for all the dykes with a strike of about N80°E

between Holy Island in the north, and Teesdale in the south - the same area as that within which the Whin Sill has been found. Holmes and Harwood (1928) have summarized the voluminous early literature on the Whin Sill and the dykes and concluded that the evidence suggests they had a common origin.

The Great Whin Sill consists of a number of concordant quartz-dolerite intrusions within the Lower Carboniferous of north-east England. The dykes fall into four well defined echelons: the first includes the Holy Island dykes in the north; the second includes the High Green and Rothbury dykes; the third stretches from Haltwhistle to Causey Park and the coast; the fourth, and most southerly echelon, includes the long Hett dyke (Figure I). All cut Westphalian rocks, but none appear to cut the Permian over-lying the Coal Measures unconformably. This upper limit to the age of the dykes can only be confirmed indirectly as the Hett dyke does not reach the Permian escarpment and the Ludworth dyke may die out vertically before reaching the base of the Permian. Better evidence of the age of the intrusions is available from their relationship to the North Pennine Orefield. (Dunham, 1948),

Throughout the area the Whin Sill is cut by mineralized veins, many of which have been seen during mining operations to actually pass through the Sill. Similar veins also cut the dykes - among them being those of High Green and Hett. Thus, it can be said with certainty that the mineralization post-dates the Whin Sill and the dykes.

The age of this mineralization, calculated by Moorbath (1959) from lead isotope measurements, is 280 $\pm$ 30 million years -

which agrees with the estimated age of the Upper Carboniferous on the revised Holmes Time-scale (Holmes, 1960). Also, Holmes (1960) suggests the intrusion of the Whin Sill was a late Carboniferous event (280 million years) on the basis of a corrected Helium dating made in 1928 (Holmes and Dubey, 1928).

Direct proof of the upper limit of the age of the Whin Sill has been furnished by the discovery, by Holmes, of a quartz-dolerite pebble, in every way similar to the quartz-dolerite of the Whin Sill, in the Upper Brockram near Appleby. This, together with a consideration of the tectonics of the area, suggests the following sequence of events (Holmes and Harwood, 1928):

5. Upper Brockram and Yellow Sands.
    4. Faulting movements accompanied by the main deposition of the ores.
    3. Intrusion of the Whin Sill and dykes.
    2. Folding movements accompanied by faulting.
1. Westphalian Coal Measures.

Thus, from the geological evidence it would seem reasonable to conclude that the Whin Sill and the dykes were intruded during the late Carboniferous, possibly during the period of denudation preceding the Permian.

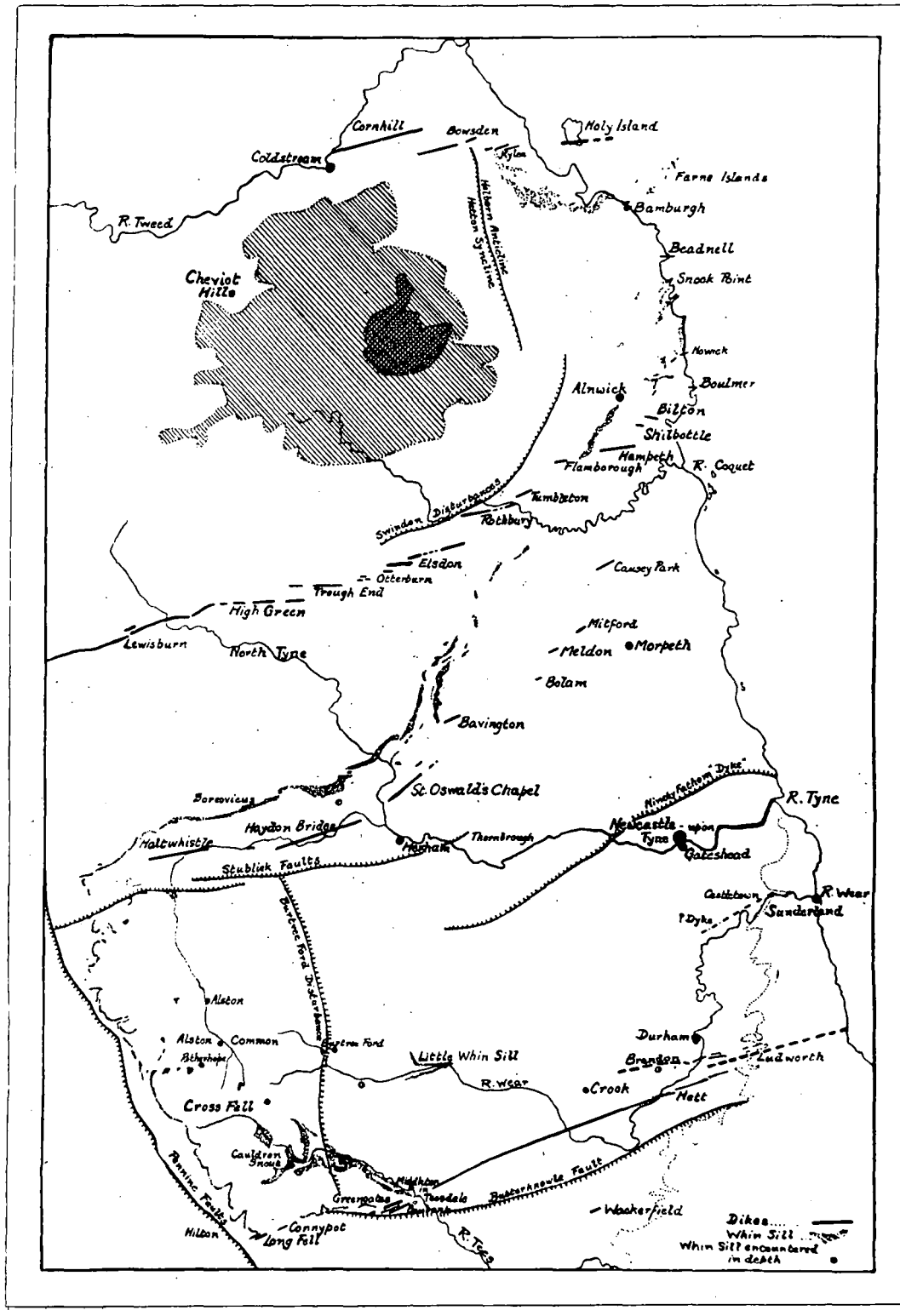


Figure 1. The Whin Sill and its associated Dykes (after Holmes and Harwood, 1928).

CHAPTER II

THE MAGNETIC SURVEY

1. The Instrument

The Littlemore Proton Magnetometer (Type 592) used for the Survey has been designed for measuring the total magnetic field to better than one part in 35,000 over the entire range of field strengths found on the Earth's surface. The components of the field cannot be measured with this instrument, but against this limitation can be set the rapidity of taking readings which is possible because precise orientation or levelling of the detector head is unnecessary. In this model readings may be taken every six seconds.

The magnetometer gave no trouble throughout the survey apart from the batteries becoming exhausted on one occasion, and this was probably due to a short circuit in the worn co-axial cable between the instrument and the bottle. This cable was later replaced. Occasionally, gradients over the dykes appeared to be too steep for the size of bottle used (radius 1", length 4"), and spurious readings resulted. However, it is possible that these gradients were due to some iron object just below the surface.

Owing to the distance necessary between the instrument and the bottle it was found most convenient for two people to work together - one moving the bottle and the other taking the readings.

On occasions when only one person was available it was found that the cable could be shortened to a minimum of ten feet without seriously affecting the readings. Even this, however, was comparitively time consuming.

2. Fieldwork

The majority of the time in the field (about ten days in October, 1960), was spent making detailed traverses across the Hett, Haydon Bridge, St. Oswald's Chapel, and Causey Park dykes. The first being the chief member of the most southerly echelon and the last three being parts of the Haltwhistle system (Holmes and Harwood, 1928).

The dykes would be located approximately from the $\frac{1}{4}$" Geological Survey maps and then one or two extended traverses made over the area to fix its position more accurately. From these positions and the numerous quarries along the dykes it was possible to interpolate the position of the dyke across the 6" Ordnance Survey maps with reasonable confidence. The areas most likely to be free from underground pipes, cables and wire fences etc., were then selected for detailed traverses. When possible an unobstructed line about 300 yards long and perpendicular to the strike of the dyke was chosen. In practice this was rarely possible and several traverses are interrupted by fences and occasionaly by roads. However, this seems permissable as long as the break in the profile is well away from the steep gradients associated with the dyke itself.

Once the anomaly associated with the dyke had been found, readings were taken at about 30 yard intervals until the field had resumed its background value (i.e. little or no change of reading between stations). At this position a base station was established. Readings were then taken at about 20 yard intervals towards the dyke, the station spacing decreasing to a minimum of about one yard as the gradient of the anomaly steepened. The traverse was continued until the background anomaly was regained. The base station was then repeated and any error due to diurnal variation distributed along the traverse according to time.

3. Reduction of Observations

The instrument readings are converted to Oersteds according to the following formula given by the manufacturers:

$$\text{Magnetic field} = \frac{24050}{\text{reading}} \text{ Oersteds}$$

The meter readings can be broken down into the Earth's field x, and a superimposed anomaly dx units:

$$\text{Magnetic field} = \frac{24050}{x + dx} \text{ Oersteds}$$

$$= \frac{24050}{x}\left(1 + \frac{dx}{x}\right)^{-1}$$

Expanding by the Binomial Theorem:

$$\text{Magnetic field} \doteq \frac{24050}{x}\left(1 - \frac{dx}{x} + \left(\frac{dx}{x}\right)^2 \ldots\ldots\ldots\right.$$

For northern England $x \doteq 49{,}000$ gamma, thus neglecting

second and higher powers of dx;

$$\text{Magnetic field} \doteq \frac{24050}{49000} - \frac{24050.dx}{49000.49000}$$

$$\doteq (0.49081 - dx.10^{-5}) \text{ Oersteds}$$

$$\doteq (49.081 - dx) \text{ gamma}$$

Thus, as the instrument reading decreases the field strength increases, and by plotting the meter readings negatively against the distance along the traverse a fair approximation to the anomaly in gamma can be obtained. It is possible to ignore the latitude and longtitude corrections since the traverses were never longer than 300 yards. Similarly, as a single traverse takes only about half an hour it is possible to distribute the diurnal variation throughout the readings.

Before plotting the results the maximum and minimum values of the anomaly are estimated and the difference between these scaled to 500 gamma. Similarly, the mean of the maximum and minimum values of the anomaly is found and the distance between the two positions having this mean value is scaled to 40 yards. The curves are now ready to be compared with the calculated theoretical profiles.

The results are presented in Chapter IV.

Symbols used in Chapter III

B Ferranti Pegasus main store address.

$F = \Sigma \Delta F$ The total field magnetic anomaly, which equals the sum of the effects of each face of the two-dimensional polygonal model.

F_{max}, F_{min}, F_m The maximum, minimum and mean values of the maximum and minimum of the anomaly.

$H = \Sigma \Delta H$ and $Z = \Sigma \Delta Z$ These are the horizontal and vertical components of the total magnetic anomaly.

I and I' The inclination of the Earth's present field and of the total magnetization, J.

J The intensity of the total magnetisation (c.g.s. units).

$J_x = \cos I' . \sin \alpha'$

$J_z = \sin I'$

U Ferranti Pegasus computing store address.

X Ferranti Pegasus accumulator within the computing store.

X_o and X_t The factor required to scale the separation of the mean values of the maximum and the minimum of the observed and theoretical anomalies to the required distance.

X_{max}, X_{min}, X_m The horizontal position corresponding to the maximum, minimum and the mean values of the maximum and minimum of the anomaly.

XST, YST Horizontal and vertical station co-ordinates.

Y_o, Y_t The factor needed to scale the observed and theoretical

anomalies to the required amplitude.

I, II, III .....IX Stages in the computer programmes.

α and α' The strike of the model measured clockwise from the horizontal projections of the Earth's field and the total magnetisation, J.

CHAPTER III

INTERPRETATION OF THE PROFILES

1. Method of Interpretation

The essential information required from the observed profiles is the shape and intensity and direction of the total magnetisation of the body causing the anomaly. The best way of achieving this is by comparing the observed profiles with those calculated for theoretical models.

It is assumed that any two-dimensional body (including dykes) may be defined by an n-sided polygon (Figure 2b,) each face of which will cause a disturbance ΔF in the total magnetic field. The anomaly due to each face in turn is calculated from formulae given by Heiland (1940) for the horizontal and vertical disturbances at a point, due to a semi-infinite slab with a sloping end (Figure 2a). The effect of the complete body at this point is obtained by summing the disturbances due to each face. The formulae which have been employed are:

$$\Delta H = 2J.\sin i \left[J_x \left(\sin i.(\theta_1 - \theta_2) - \cos i.\ln(\frac{r_2}{r_1})\right) + J_z \left(\sin i.\ln(\frac{r_2}{r_1}) + \cos i.(\theta_1 - \theta_2)\right)\right]$$

$$\Delta Z = 2J.\sin i \left[J_x \left(\sin i.\ln(\frac{r_2}{r_1}) + \cos i.(\theta_1 - \theta_2)\right) + J_z \left(\cos i.\ln(\frac{r_2}{r_1}) - \sin i.(\theta_1 - \theta_2)\right)\right]$$

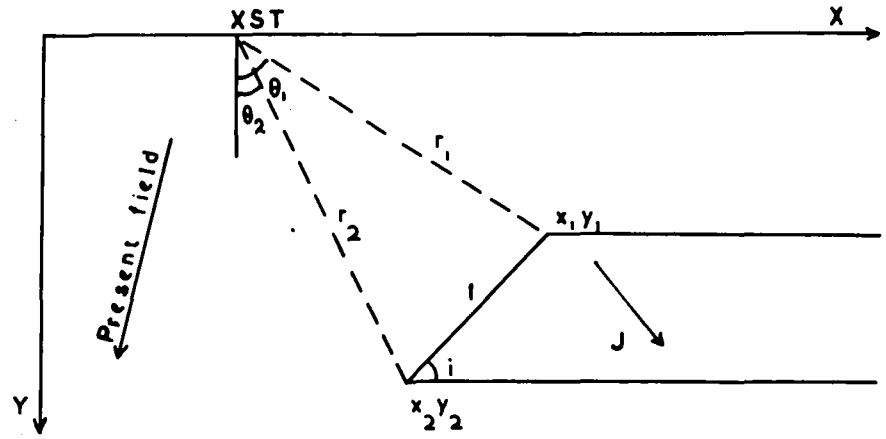


Figure 2a

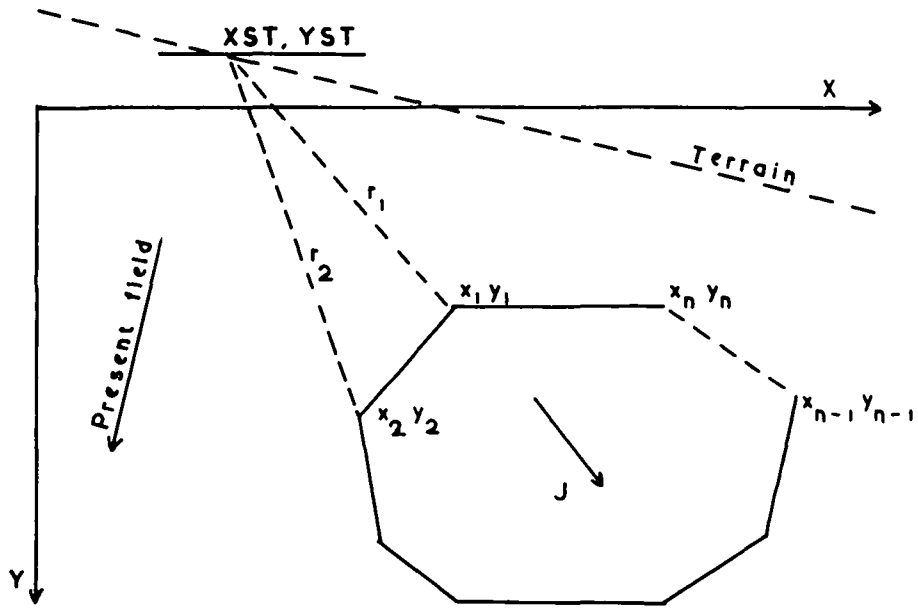


Figure 2b

Diagrams illustrating the method used for computing the anomalies over two-dimensional models.

As the intensity of the Earth's field is large compared with the total anomaly, a good approximation to ΔF is given by the projection of ΔH and ΔZ in the direction of the Earth's field:

$$\Delta F = \Delta H . \cos I . \sin \alpha + \Delta Z . \sin I$$

The depth to the bottom of a body, provided that it is at least ten times the width, has a negligible effect on the anomaly. Thus, assuming a simple dyke of rectangular cross-section and uniform polarization, five variables remain: the depth to the top of the dyke, its width, and α', I' and J. Of these, α' is indeterminate as only the polarization in the plane of the traverse is relevant. However, even with four variables, an unwieldy number of theoretical curves are necessary if a large number of field profiles is involved. Therefore, some simplication is needed.

Drs. Masson-Smith and Bott have pointed out that it follows from the character of magnetic anomalies that J can be effectively eliminated by scaling both the observed and theoretical profiles to the same amplitude:

$$F = J \, . \, f \text{ (depth, width and } I')$$

A further inspection of all magnetic formulae shows that the anomaly at any given angular position is independent of the scale of the body causing it. Thus, by reducing all the observed and theoretical anomalies to a constant width - here the distance between the mean values of the maximum and minimum of the anomaly is kept constant - a dimensional factor of the body can also be eliminated. In the interpretations

which are discussed later it is the width that has been effectively eliminated.

The procedure which has been followed is:

(a) Scale the amplitude of the observed curve to 500 gamma using a desk calculator. The scale factor being Y_o.

(b) Scale the distance between the mean values of the maximum and the minimum of the anomaly to 40 yards - also by means of a desk calculator. The scale factor being X_o.

(c) Prepare a series of standard profiles, all scaled to 500 gamma amplitude and with a spread of 40 yards (Y_t and X_t), for a series of models with different depth - width ratios and inclinations of polarization.

(d) Compare the observed profile (scaled as in (a) and (b)), with the similarly scaled standard curves over theoretical models.

(e) When a theoretical profile has been found matching the observed curve, the inclination of the total magnetisation is obtained immediately and the true dimensions of the body giving rise to the observed anomaly are given by:

$$\text{Depth} = \frac{(\text{Unscaled depth to top of model}).X_t}{X_o}$$

$$\text{Width} = \frac{(\text{Unscaled width of model}).X_t}{X_o}$$

(f) The intensity of the total magnetisation is found in a similar manner:

$$J = \frac{J_{model}.Y_t}{Y_o}$$

Two computer programmes, which are discussed in the next section, have been written to facilitate the calculation of the theoretical profiles. Programme one (2D MAGNETIC MK1) gives the anomalies H, Z, and F at a series of points XST, YST over a two-dimensional body with a polygonal cross-section. Programme two (2D MAGNETIC MK2) gives the total anomaly scaled horizontally and vertically to required limits for a series of points over a simple dyke.

PROGRAMME ONE

1. Introduction

It is assumed that any two-dimensional body can be defined by an n-sided polygon (Figure 2), each face of which will cause a disturbance ΔF in the magnetic field of the Earth. The effect of the whole body at a point is given by $F = \Sigma \Delta F$. The form of an anomaly can be found by calculating H, Z and F ($\Sigma \Delta H$, $\Sigma \Delta Z$ and $\Sigma \Delta F$) for a series of points along a line, perpendicular to the long axis of the body.

R. M. Smith (1960) has produced an Autocode programme for the Ferranti Pegasus computer giving the anomalies H, Z and F at a series of points XST, YST due to a two-dimensional body. The calculations are based on the equations from Heiland for ΔH, ΔZ and ΔF over a single sloping face (see page 14), the effect of all the faces being summed at each point in turn. The Machine Orders programme described here performs the same operations in about a quarter of the time.

2. The Programme

The data is read in as a series of stations followed by the direction of the Earth's field and the co-ordinates and polarisation of the polygonal shaped body. As all the stations and co-ordinates are read in as fractions the scale factor used

is included after the stations so the results may be printed at their normal values.

Cos I.sinα and SinI are calculated first and stored. Following this, the computer takes the co-ordinates and polarization of each face in turn, calculates sin i, cos i, J_x and J_z, and returns the result to the main store. When this has been done for all faces one point is taken at a time and the values ΔH, ΔZ and ΔF due to each face are calculated and summed. On completion of the last face, the results for the point are printed as H, Z and F, together with the co-ordinates XST, YST. When this has been done for all the points, the co-ordinates and polarization of the next body are read in and the process is repeated. This programme can be modified to give XST and F alone if speed is essential.

The programme is explained more fully in the flow diagram which follows and the actual programme, together with explanatory notes, can be found inside the back cover. The operating procedure and the details of the data layout can be found in Appendix A.

Flow Diagram for Programme One:

I. Data tape: B 70.0 Station co-ordinates XST, YST.

B 60.0 α and I for present Earth's field.

B100.0 Co-ordinates x_1y_1 and x_2y_2 and magnetisation J, α', I' for each face.

B 15.6 Factor by which model has been scaled down.

Enter programme:

II. B60 to U5

Form $\sin\alpha . \cos I$ and $\sin I$ and return to B60.

2m = 0

B(100 + 2m) to U5 co-ordinates and magnetisation of first/next face (x_1y_1, x_2y_2, J, α', I', +0)

If $x_1 = 0$ (after last face) jump to III

Calculate $\frac{\sin i}{2}$, $\frac{\cos i}{2}$, J_x, J_z (changing the sign of J and exchanging x_1y_1 and x_2y_2 if y_2 y_1) and return to main store B100 (x_1y_1, x_2y_2, $\frac{\cos i}{2}$, J_z $J_x . \frac{\sin i}{2}$)

2m' = 2m + 1.0

III. 2m = 0

Clear B50 for accumulating ΔH, ΔZ, ΔF at first/next station

3m = 0

B(70 + 2m) to U4

If XST = 0 (after last station) jump to V

B(100 + 3m) to U5

If x_1 = 0 (after last face) jump to IV

Store modifiers

Form $r_1^2 = (x_1 - XST)^2 + (y_1 - YST)^2$

$r_1^2 = (x_2 - XST)^2 + (y_2 - YST)^2$

If $r_1 = r_2$ put U4.7 = 0 and jump

Calculate $-\frac{1}{64\pi}.\ln(\frac{r_1}{r_2})^2$ or if $r_1 > r_2$, change the sign of $\frac{1}{\pi}$ and calculate $+\frac{1}{64\pi}.\ln(\frac{r_2}{r_1})^2$, store in U4.7

Form $\frac{\theta}{\pi}1 = \frac{1}{\pi}\arctan\frac{(x_1 - XST)}{(y_1 - YST)}$

$\frac{\theta}{\pi}2 = \frac{1}{\pi}\arctan\frac{(x_2 - XST)}{(y_2 - YST)}$

$\frac{(\theta_1 - \theta_2)}{32\pi}$

Form $\Delta H = \frac{\sin i}{256\pi}\left[J_x\left(\sin i.(\theta_1 - \theta_2) - \frac{\cos i}{2}.\ln(\frac{r_2}{r_1})^2\right) + J_z\left(\cos i.(\theta_1 - \theta_2) + \frac{\sin i}{2}.\ln(\frac{r_2}{r_1})^2\right)\right]$

$\Delta Z = \frac{\sin i}{256\pi}\left[J_x\left(\frac{\sin i}{2}.\ln(\frac{r_2}{r_1})^2 + \cos i.(\theta_1 - \theta_2)\right) + J_z\left(\frac{\cos i}{2}.\ln(\frac{r_2}{r_1})^2 - \sin i.(\theta_1 - \theta_2)\right)\right]$

$\Delta F = \Delta H.\sin\alpha.\cos I + \Delta Z.\sin I$

Accumulate ΔH, ΔZ, ΔF for each face at a given station

```
    in B50.0, 50.1, and 50.2
  Restore modifiers
  3m' = 3m + 1.0
  IV   B50 to U5
       B70 + 2m to U4
       Unscale and print XST
          do          do  YST
       Unscale and print   H
          do          do   Z
          do          do   F
       2m' = 2m + 0.2
  V    Read in next model
```

PROGRAMME TWO

1. Introduction

This is the programme which has been used whenever possible for the interpretation of the observed profiles. It can only be applied to anomaly profiles with a single maximum and single minimum. The first part is substantially the same as the previous programme without the printing. Instead, the values of H and Z have been dispensed with and the total anomaly has been scaled vertically and horizontally to pre-determined limits as discussed earlier.

2. The Programme

The distance required between the mean values of the anomaly has to be read in on the data tape in addition to the data required for Programme One. The first stage of the programme (II) is as before: the calculation of cos I.sin α, and sin I, and the calculation of sin i, cos i, J_x and J_z for each face. After this, accumulator one is set to minus one (to be referred to later), ($\leq \Delta F$) is calculated for each point (XST, YST) and the results are stored (stage III). After the last point has been so treated accumulator one is masked so that it becomes positive, stage IV is entered, and the gradient between the first two points is determined. This procedure is continued for consecutive values

of XST until a turning value of the gradient is reached. The three points nearest this are then selected. Between these three points, sixteen new points are interpolated and the anomaly at each of these is found by re-entering stage III.

When this has been done, accumulator one (previously positive) is remasked and becomes negative. This causes the programme to enter stage V instead of IV and the maximum or minimum of F for the new stations, (whichever has been encountered first as XST increased), is preserved with its corresponding value of XST. If the second turning point has not been found accumulator one is remasked (now positive) and stage IV is re-entered and the search is continued (the modifiers having been preserved after the first turning point). Again sixteen new stations are interpolated between the three nearest points and F, the total anomaly at each of these is found. By remasking accumulator one stage V is entered and, after deciding which of the two points is the maximum and which is the minimum, the co-ordinates of the peak values are preserved in their respective localities.

The difference between these peak values is then made equal to a constant (500 gamma has been chosen), and the scaling factor (Y_t) is printed and the original values of F along the traverse are multiplied by it.

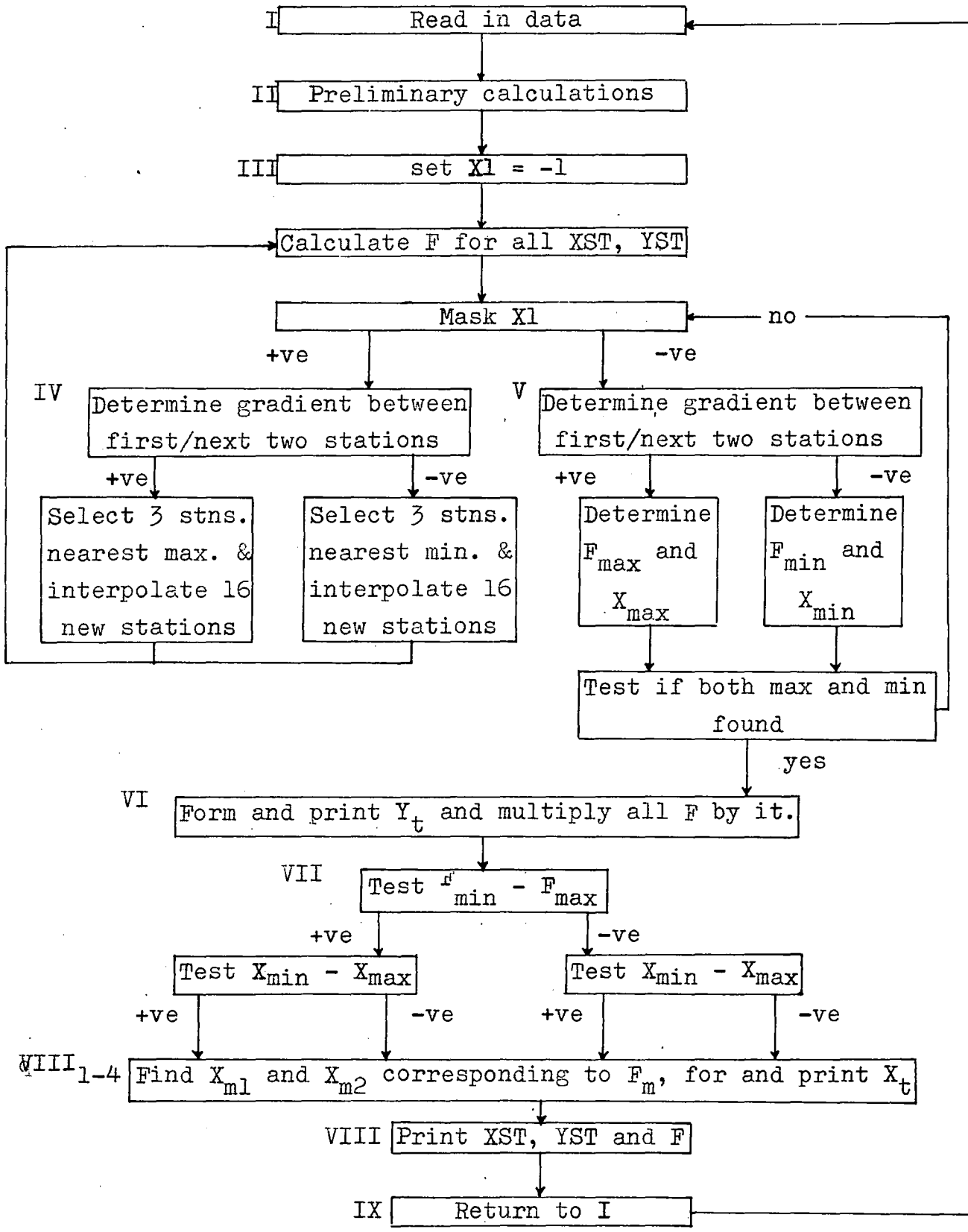
From here reference to the block diagram (page 27) will show the tests made in stage VII and the respective stages VII_{1-4} entered. The programme has been written in this way to

simplify the selection of the mean values of the maximum and the minimum of the anomaly and the subsequent scaling of XST. (The full details may be seen in the print-out of the programmes inside the back cover.) The two values of F nearest F_m and their respective values of XST, are taken and by simple proportion the XST, corresponding to the mean value of F is found. A more accurate method is to interpolate new stations as before and recalculate F around the mean value, but time did not allow this method to be attempted.

The scale factor (X_t) necessary for the required spread of the anomaly is determined and printed. Each of the points in turn is multiplied by this scale factor and then printed together with the corresponding scaled values of F.

To clarify these remarks a block diagram (page 27) has been included and the flow diagram which follows explains the steps more fully. The actual programme together with explanatory notes can be found inside the back cover. The operating procedure and data layout are described in Appendix B,

BLOCK DIAGRAM ILLUSTRATING PROGRAMME TWO



Flow Diagram for Programme Two:

1. Data tape: B100.0 Station co-ordinates XST, YST

B 69.0 α and I for present Earth's field.

B110.0 Co-ordinates x_1y_1 and x_2y_2 and magnetisation J, α', and I' for each face.

B 47.6 and 47.7 Scaling constants.

Enter Programme:

II. Form $\sin\alpha.\cos I$ and $\sin I$ as in programme one.

Form $\frac{\sin i}{2}$, $\frac{\cos i}{2}$, J_x, J_z for each face as in programme one.

III. Clear B70.0 - 79.7 for preserving F.

X1 = -1

2m = 0

3m = 0

B(100 + 2m) to U4 first/next point XST, YST

B(110 + 3m) to U5 co-ordinates and magnetisation of first/next face (x_1y_1, x_2y_2, J α', I, +0)

Store modifiers

Jump if XST $\neq$ 0 (zero after last station)

Mask X1 and preserve in main store

If X1 now negative jump to V

If X1 now positive jumpto IV

2m' = 2m + 0.2

If x_1 = 0 (after last face) jump

Calculate F for each point XST, YST as in programme one

Restore modifiers

Preserve F for each point XST, YST in B70.0 onwards

3m' = 3m + 1.0

IV B17.7 to X3 (zero first time)

Jump if X3 ≠ 0

B100.0 - 109.7 to B80.0 - 89.7

B 70.0 - 79.7 to B90.0 - 99.7

Calculate the gradient of the anomaly between XST_1 and XST_2

Gradient negative: jump

Gradient positive: form differences between consecutive values of F until sign changes (across maximum of anomaly)

Store modifier in B17.7

Interpolate sixteen new points between the three stations nearest the maximum and write these into B100.0 onwards.

Calculate F for each of these new stations.

Repeat above procedure, testing for opposite change in sign (across minimum of anomaly), interpolate sixteen new stations and calculate F as above.

V Calculate gradient of anomaly between first two new stations.

Gradient negative: jump.

Gradient positive: calculate differences as before,

preserving F_{max} and XST_{max}

Test if F_{min} and XST_{min} have been found

If not yet found, jump

If present, jump to VI.

Calculate difference, preserving F_{min} and XST_{min}

Test if F_{max} and XST_{max} have been found

If not yet found, jump.

If present jump to VI.

VI Calculate mean value (F_m) of F_{max} and F_{min}

Calculate Y_t for $|F_{max}| - |F_{min}|$ to equal required amplitude.

Print Y_t.

$Y_t \cdot F_m$

$Y_t \cdot F$ for XST_{o-n}

VII, VIL_{1-4}

Calculate XST_{m1} and XST_{m2} corresponding to F_m.

Calculate X_t for $XST_{m1} - XST_{m2}$ to equal required distance

Print X_t

VIII 2m = 0

3m = 0

B(80 + 2m) to U5 (XST, YST)

If XST = 0 (after last station) jump to IX

Unscale XST and form $XST.X_t$, print.

B(90 + 3m) to U5

Unscale and print F

$2m' = 2m + 0.2$

$3m' = 3m + 0.1$

IX Read in next model.

CHAPTER IV

RESULTS AND CONCLUSIONS

1. Presentation

In the figures that follow the solid and broken lines represent the observed and theoretical profiles respectively. The abbreviations employed in the top left corner are:

H. Hett dyke

H.B. Haydon Bridge dyke

C.P. Causey Park dyke

Together with the number that follows, these give the points on the dykes where the profiles were made (Figure 3). The numbers correspond to those in the first column of Tables I, II and III. Below these are the scale factors for the amplitude and horizontal spread of the field profiles - the corresponding scale factors on the right apply to the theoretical profiles. In the top right corner the unscaled depth-width ratio of the theoretical model is followed by the inclination of its polarization. The figures after this, if any, show the alterations necessary to a simple vertical dyke to obtain a good fit between the profiles. N or S is followed by the unscaled top and bottom widths of the model. The vertical line marks the mid-point of the models.

Tables I, II and III summarize the information gained

from the profiles over the Hett, Haydon Bridge and Causey Park dykes. The first column gives the name and number of the point on the dyke at which the profile was made and the next three give the depth and width of the top of the dyke and any remarks on the interpretation. The last two give the inclination (normally assumed to be north) and intensity of the total magnetisation.

All the profiles are drawn with south to the left and north to the right.

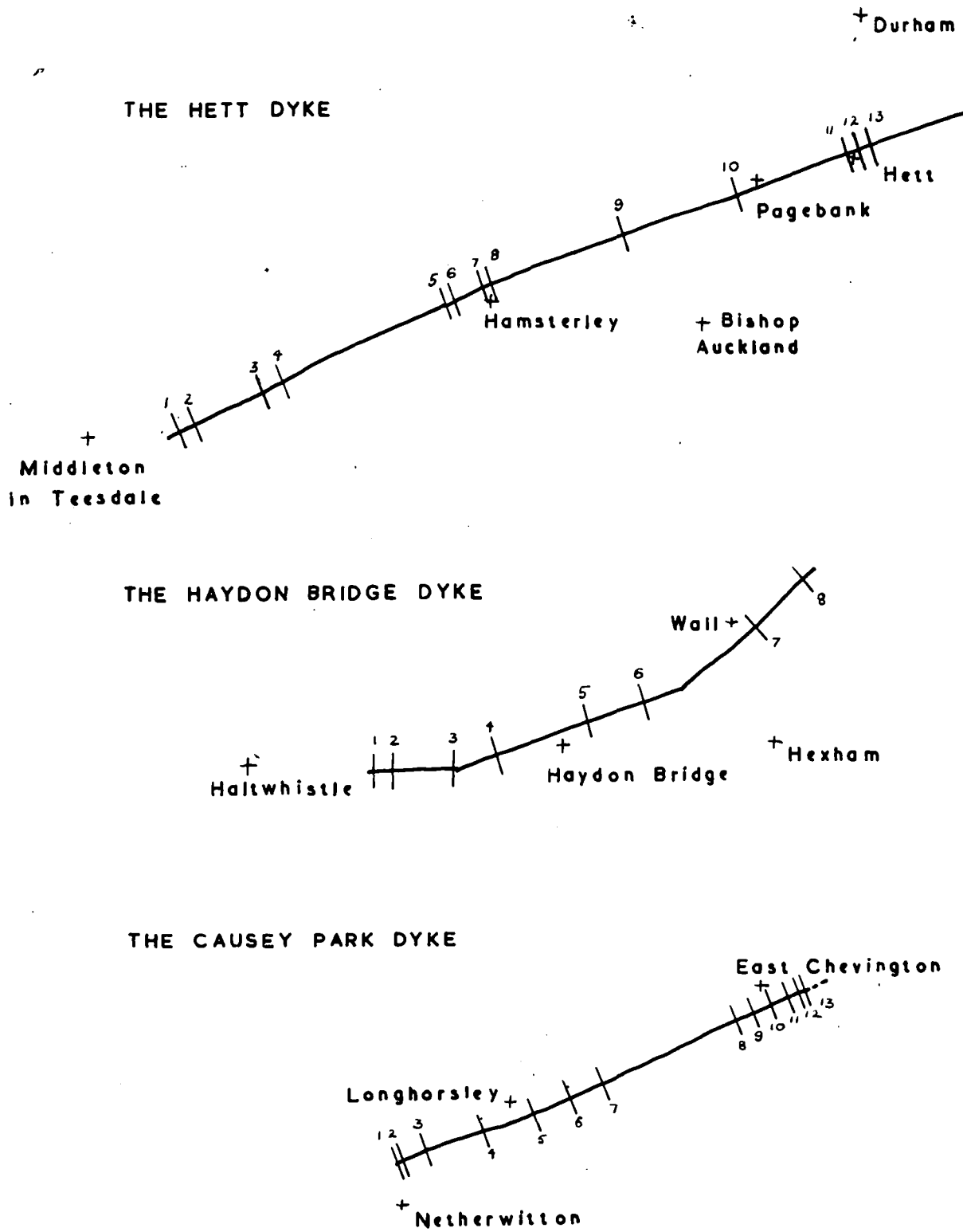


Figure 3. Sketchmap showing the positions of the traverses across the dykes. (Taken from the ¼" Geological Survey Maps.)

H. 1

South Dyke.

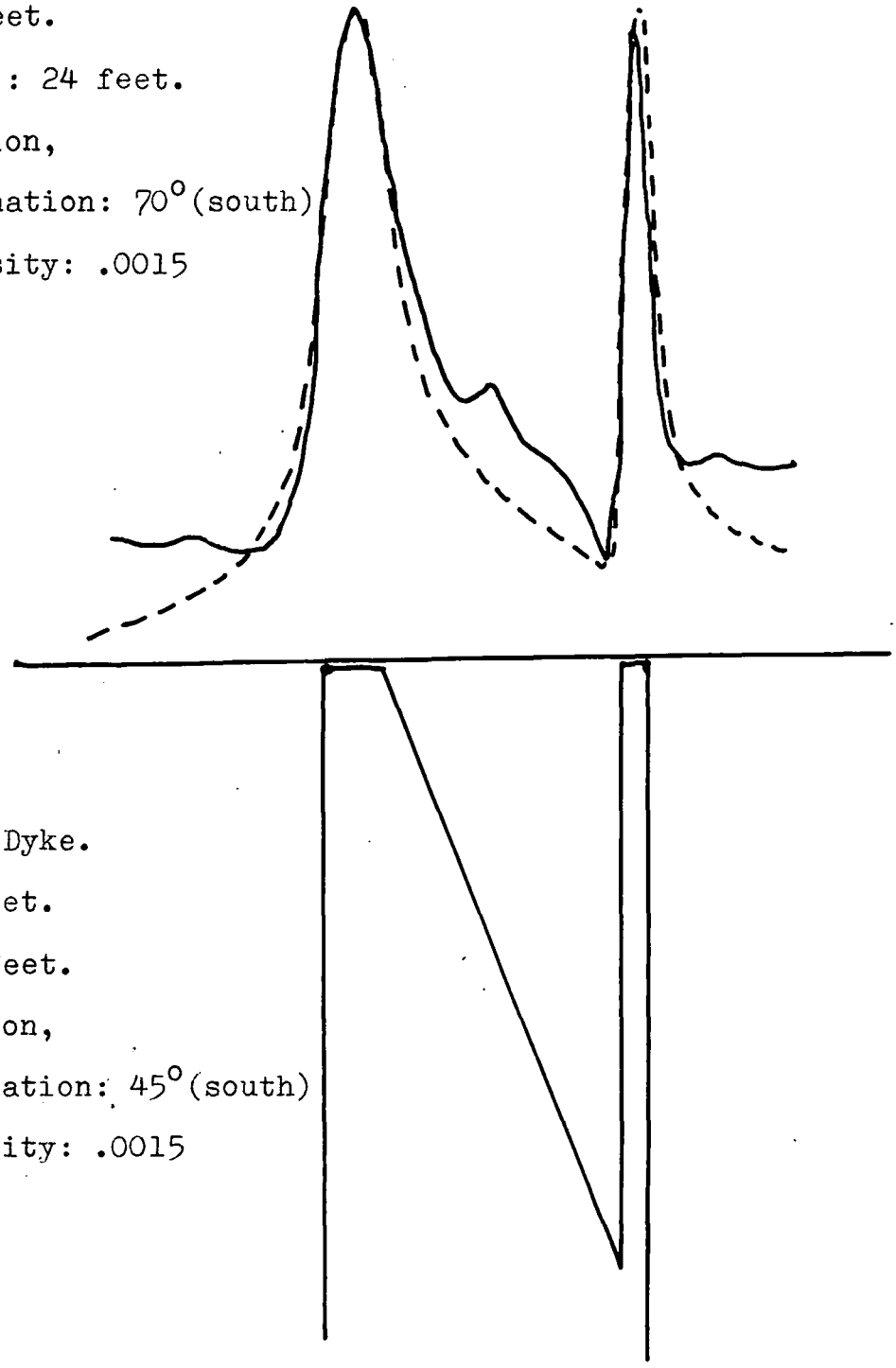
Depth: 9 feet.

Width (top): 24 feet.

Magnetisation,

Inclination: 70° (south)

Intensity: .0015



North Dyke.

Depth: 3 feet.

Width: 12 feet.

Magnetisation,

Inclination: 45° (south)

Intensity: .0015

H. 5

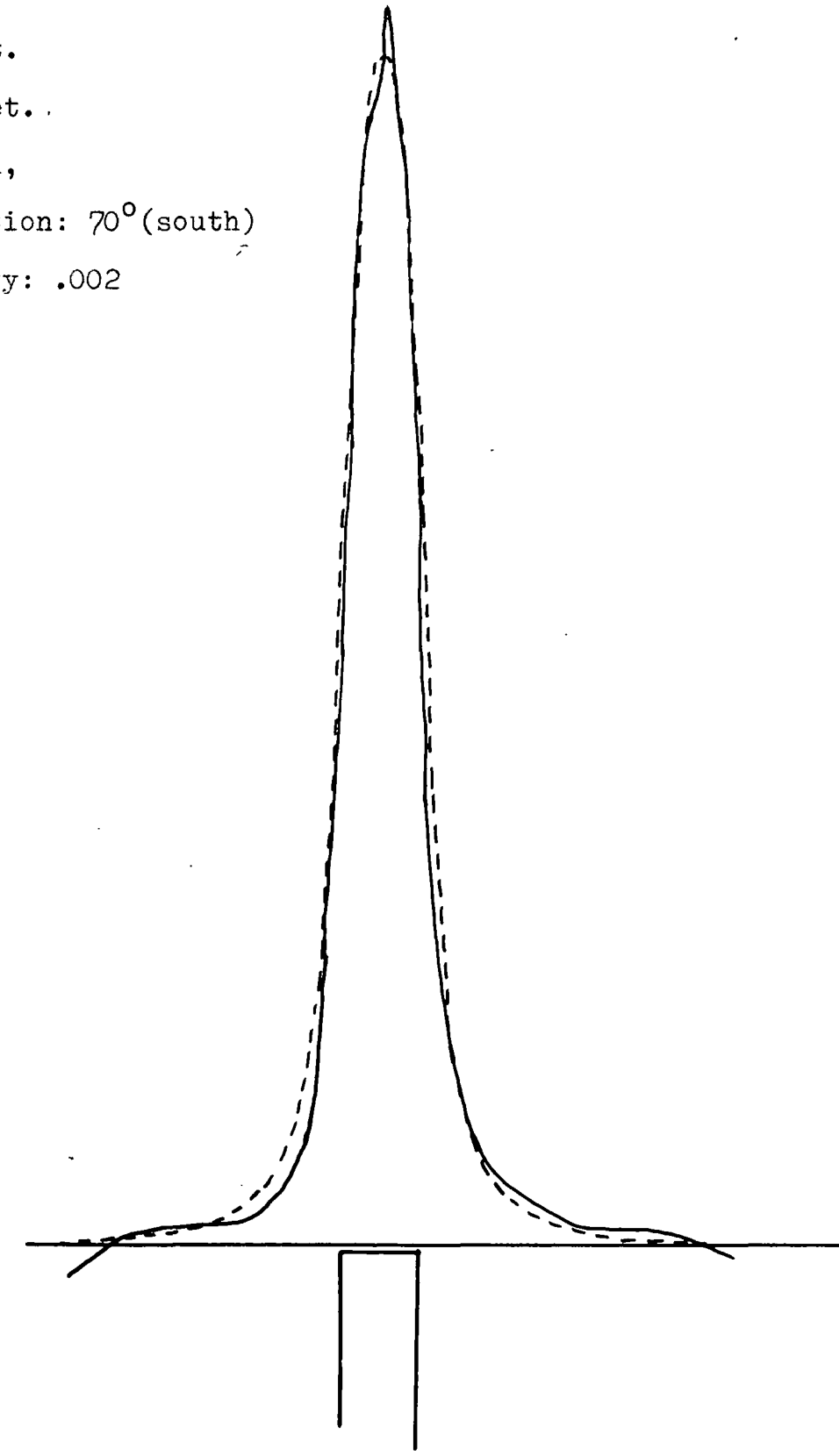
Depth: 9 feet.

Width: 36 feet.

Magnetisation,

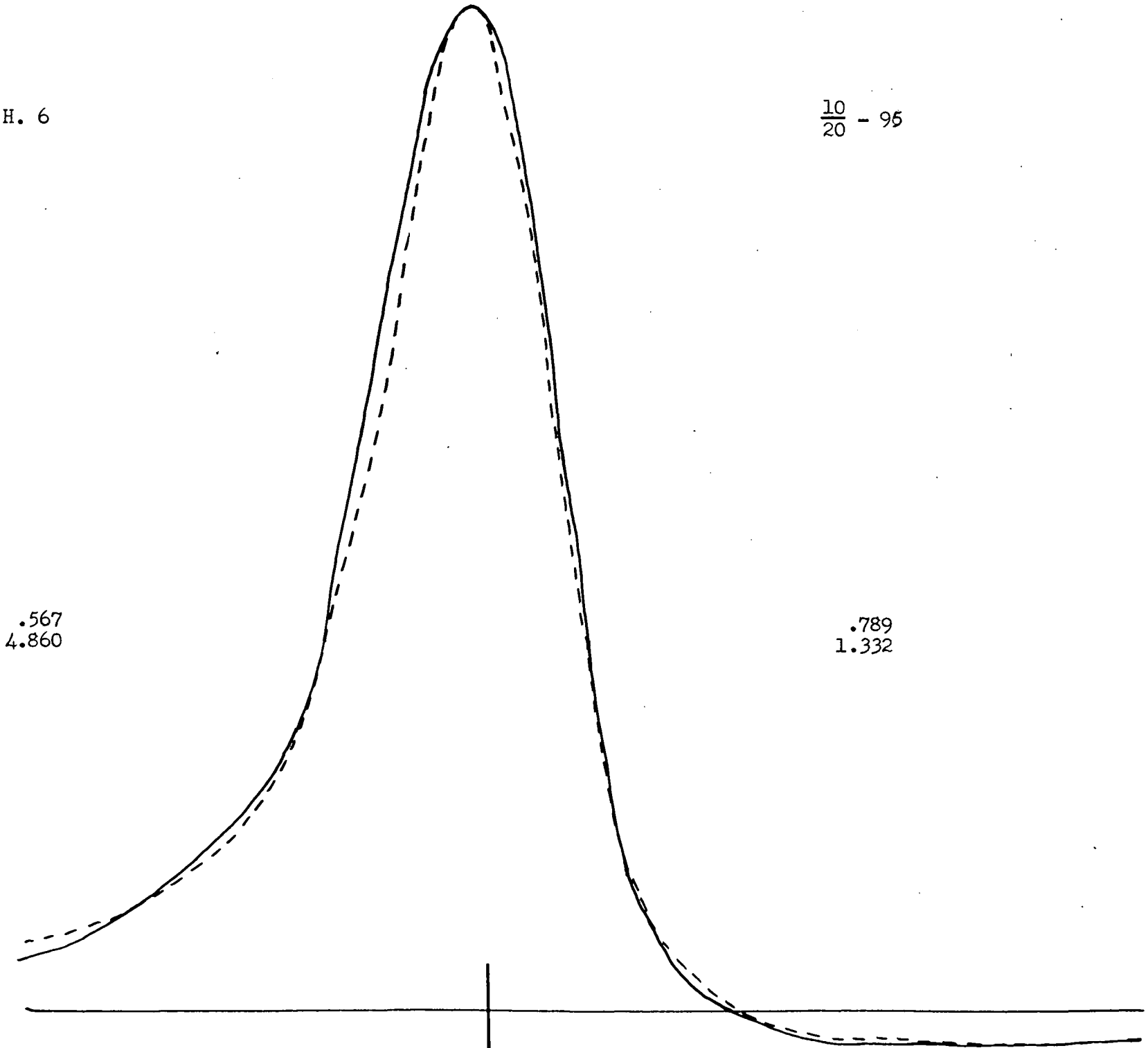
Inclination: 70° (south)

Intensity: .002



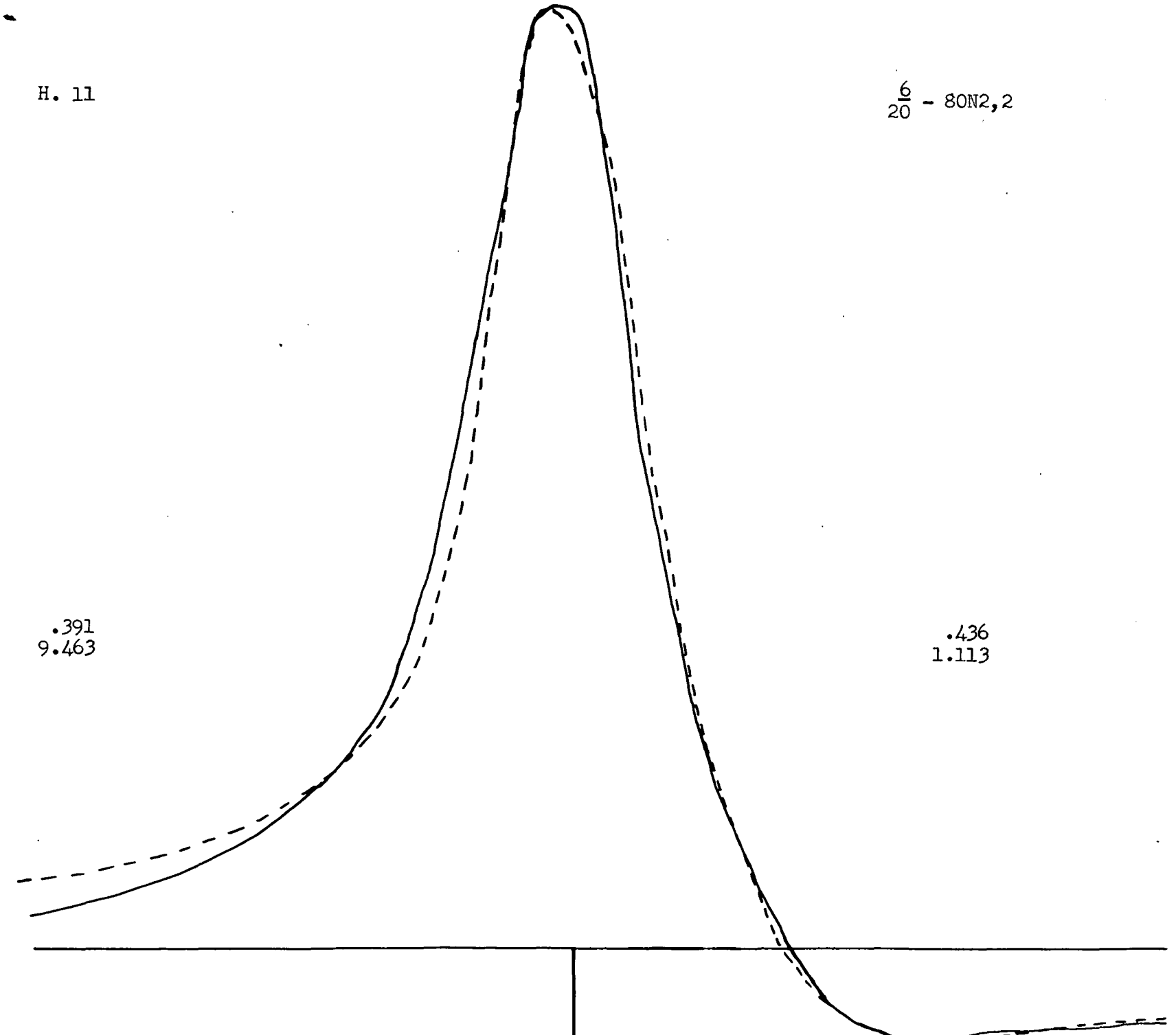
H. 6

$\frac{10}{20}$ - 95



H. 11

$\frac{6}{20}$ - 80N2,2



Locality	Depth (feet)	Width (feet)	Magnetisation		Remarks
			Inclination	Intensity	
1. Eggleston Burn	Probably effected by displacement of the Whin Sill at depth		c.60 (South)		
2. Moorview Quarry			c.60 (South)		
3. Ever Pools			c.60 (South)		
4. High Acton			c.60 (South)		
5. Hamsterley Forest	9	36	70 (South)	.002	Programme One
6. " "	8	16½	85 (South)	.0028	
7. Black Hill Top Farm			c.80		
8. " " " "			c.80		
9. Constantine Farm			c.80		Readings erratic
10. Pagebank					" "
11. Hett	2	14	80	·0022	N20,20
12. "			Double maximum indicates two dykes		
13. "		Two dykes	- see figure	8	Programme One

TABLE I - HETT DYKE

The Hett Dyke

1, General

This dyke is the chief member of the Hett echelon (Holmes and Harwood, 1928). It has been seen underground in Bowburn Colliery, but little is known of its dimensions as it marks the boundary between areas and is rarely cut through. At Tudhoe the dyke has been exposed in the opencast coal workings (since reclaimed), where it is about eleven feet wide. Further west at Whitworth Park opencast site two dykes have been exposed - one eleven feet wide and the other six feet - separated by fifteen feet of cindered coal.

Between Whitworth Park and Hamsterley Forest the course of the dyke is marked by a series of disused quarries. In Hamsterley Forest it is seen cutting the Forestry Commission road south-west of "The Grove". For the next six miles, there is only the indirect evidence of baked shales to indicate its presence until it is seen in the quarry at the back of "Moorview House" on the Eggleston-Stanhope Road. Here it has been intruded along the line of the Knott Fault. It is still on the line of this fault when it is last seen half a mile west in Eggleston Burn.

2. Interpretation of the Profiles

In the east, the country between the Permian escarpment at Quarrington Hill and the railway line one mile west of Bowburn

is built-up and the most easterly traverses were made north of Hett village. Here it appears that two dykes are present (H. 13), the one to the south being the wider, with its northern flank meeting the smaller dyke about 300 feet down. Programme one had to be used in this case owing to the double maxima on the curve. The direction of the magnetisation used for each dyke in the model was different, being south and inclined at 70° for the larger and south, inclined at 45° for the smaller. However, the theoretical profile only gives an approximate fit and this solution is almost certainly incomplete. The profile made 200 yards west (H. 12) still shows a double maximum, but with only a small minimum between, indicating that the dykes are probably running into each other. The next profile, north of the church, shows the type of anomaly expected over a single dyke, but it would appear that the width still increases on the north side (H. 11).

No profiles were made across the reclaimed Tudhoe opencast area as the top of the dyke had been used as an approach road to the workings, with consequent complications to its upper surface. The next profile was attempted north of the River Wear at Pagebank, as near as possible to the Whitworth Park opencast site, but no clear anomaly could be found.

A series of profiles were made half a mile east of the old quarry workings near Constantine Farm, but the readings, although they showed a minimum to the north of the dyke, were too erratic to permit a quantitative interpretation. From here the dyke crosses the River Wear again, and passes north of Hamsterley village. The next profiles were made east of the road at Black

Hill Top Farm. Two profiles were made, one of which defied interpretation, although the direction of the total magnetisation was probably north and inclined at about 80° (H. 7). The second (H. 8) agreed reasonably well with the profile over a model dipping north with its magnetisation north, inclined at 80°. Although there was a quarry 20 yards east of H. 8, it was too overgrown to estimate the width of the dyke.

The same applies to the quarry in Hamsterley Forest east of "The Grove", where a profile across its eastern end gave the direction of magnetisation as south, inclined at 85°, with the width of the dyke five feet less than at Black Hill Top. Across the western end of the quarry the magnetisation has apparently changed to south, inclined at 70° - this has to be dealt with by programme one as no minimum appears in the calculated profile (H. 5).

Further west at High Acton Currick a rough profile was obtained suggesting the direction of magnetisation to be south, inclined at 60°. A similar anomaly was found at Ever Pools although it was not good enough for detailed interpretation. This latter traverse was extended southwards to where Jones (1956) marks the Hett dyke on his map. His evidence for this position being an exposure of altered shale in the Ever Pools overflow channel. The old 1" Geological Survey maps show the dyke following Pennington Ridge, but any exposure there may have been has since been obscured by a Forestry Commision road which follows the same line. The profilesmade at High Acton and just south

of Ever Pools suggest the Survey's position was correct.

Time in the field did not allow any profiles to be made on Eggleston Common, although the open moorland would have allowed long, uninterrupted traverses which would have increased the chances of their complete interpretation.

Several profiles were made between Eggleston Burn and the head of the quarry behind "Moorview House", and all were too short owing to the difficult terrain. However, each one showed a small minimum over the southern edge of the dyke, suggesting the direction of magnetisation to be south and inclined at about 60°. Unfortunately it was not possible to make more traverses further west towards the outcrop of the Whin Sill - with the possibility of locating a feeder to the Sill.

3. Possible Influence of the Whin Sill

The Whin Sill west of Middleton-in-Teesdale is downfaulted to the east a total of 300 feet by the North and South Tees Faults, and, assuming it stays at the same horizon between the Tynebottom and the Single Post Limestones, it is probably present between 300 and 800 feet, depending on the terrain, below the surface outcrop of the Hett dyke.

The throw of the Knott Fault, along which the dyke is intruded at its western end, is 100 feet down to the south bringing the Low Grit Sill against the High Grit Sill in Eggleston Burn. The displacement of the fault decreases eastwards, being 40 feet west of "The Grove" and virtually nil beyond

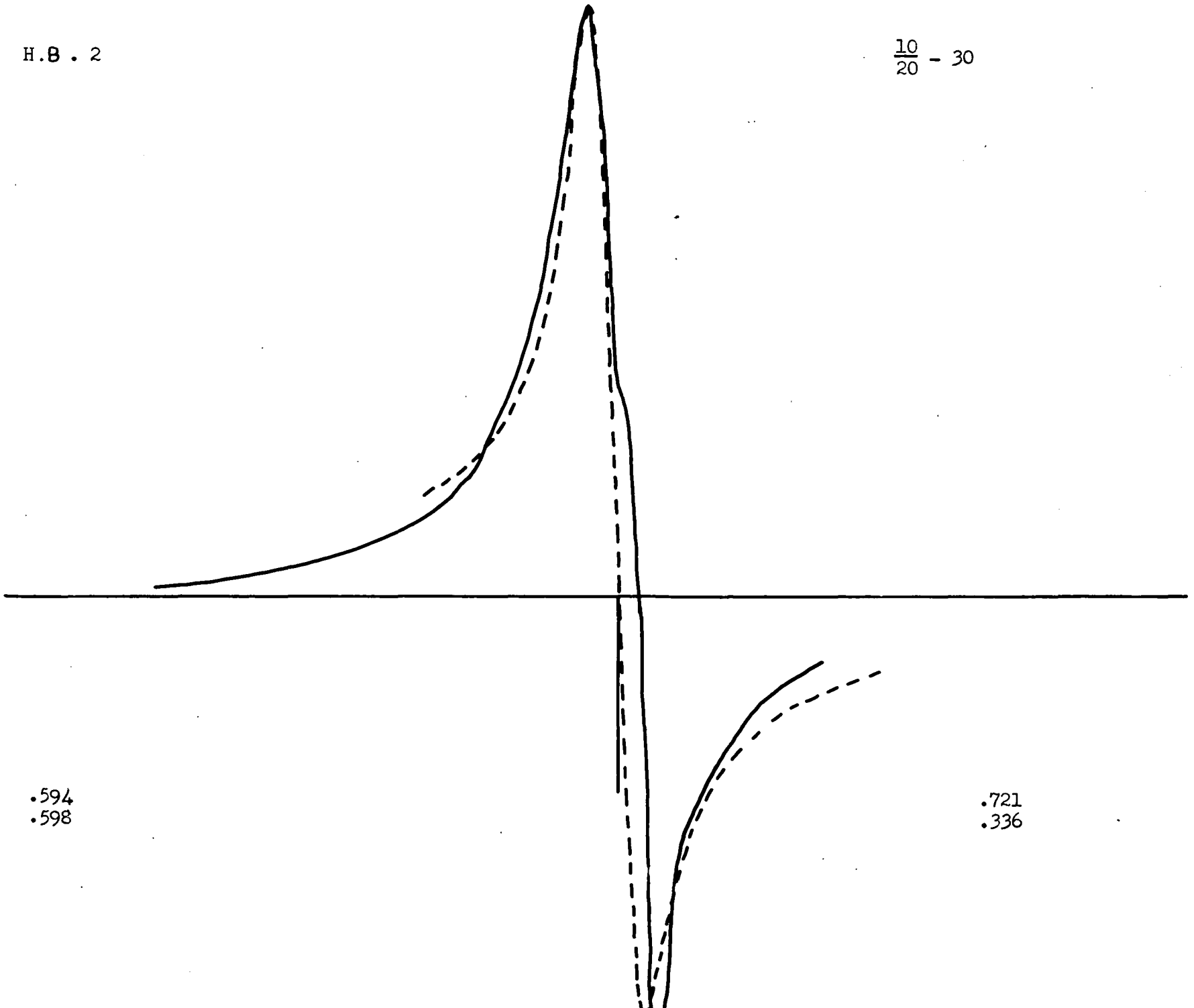
Spurlswood Beck. This is born out by the coal workings in the east where there is very little displacement across the dyke.

The changing displacement of the Whin Sill by the Knott Fault (it does not affect the Sill at outcrop) is clearly demonstrated by the magnetic profiles. The inclination of the total magnetisation of those east of the quarry in Hamsterley Forest is of the same order as that for the Haydon Bridge and Causey Park dykes - whereas the profiles west of the quarry show a consistent change in the inclination.

This change can be simply explained by the influence of the anomaly due to the Whin Sill, assuming it changes its level across the fault - the direction of total magnetisation of the Sill at its outcrop is south and inclined at about 40°.

H.B . 2

$\frac{10}{20}$ - 30

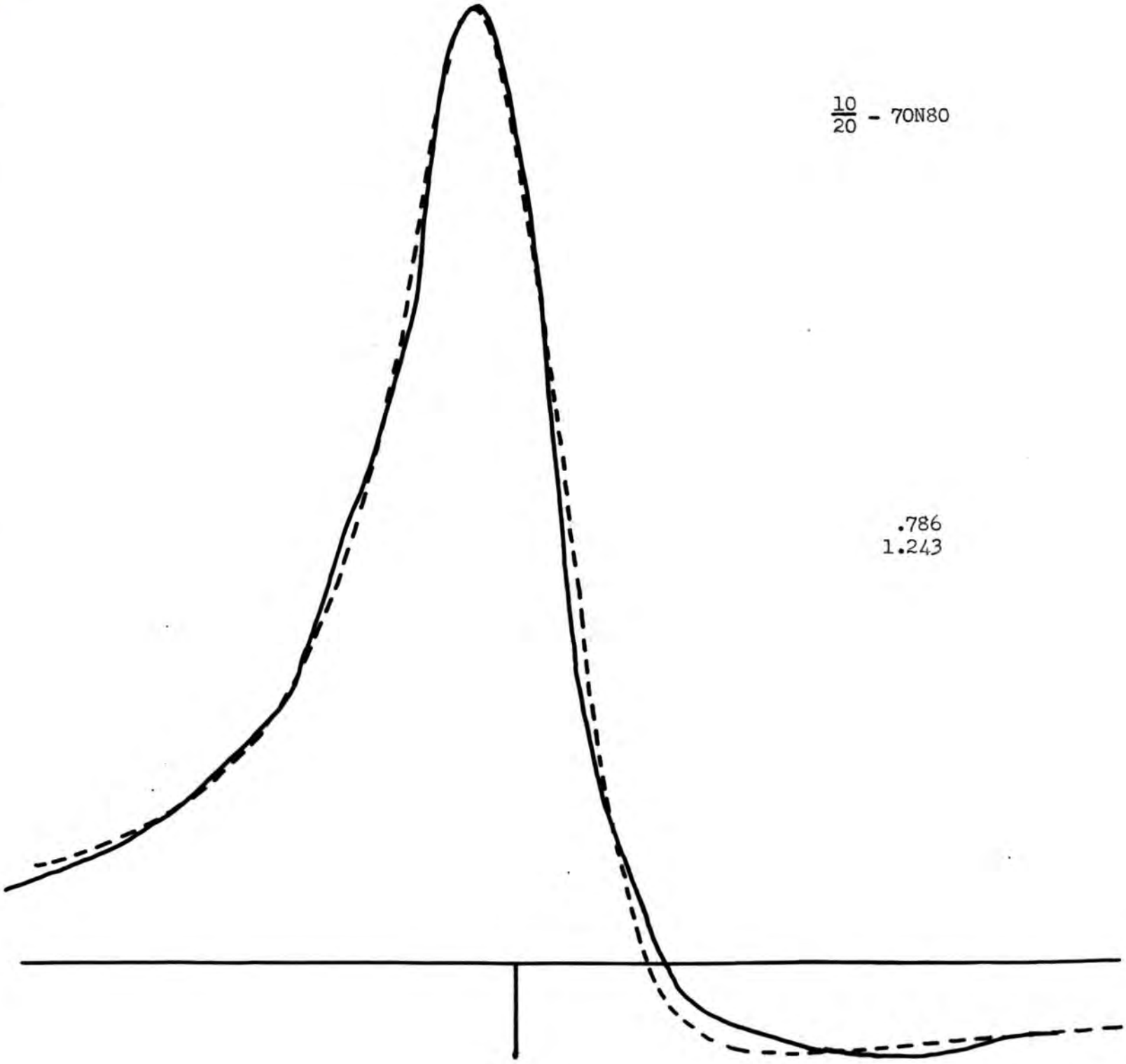


H.B. 3

$\frac{10}{20}$ - 70N80

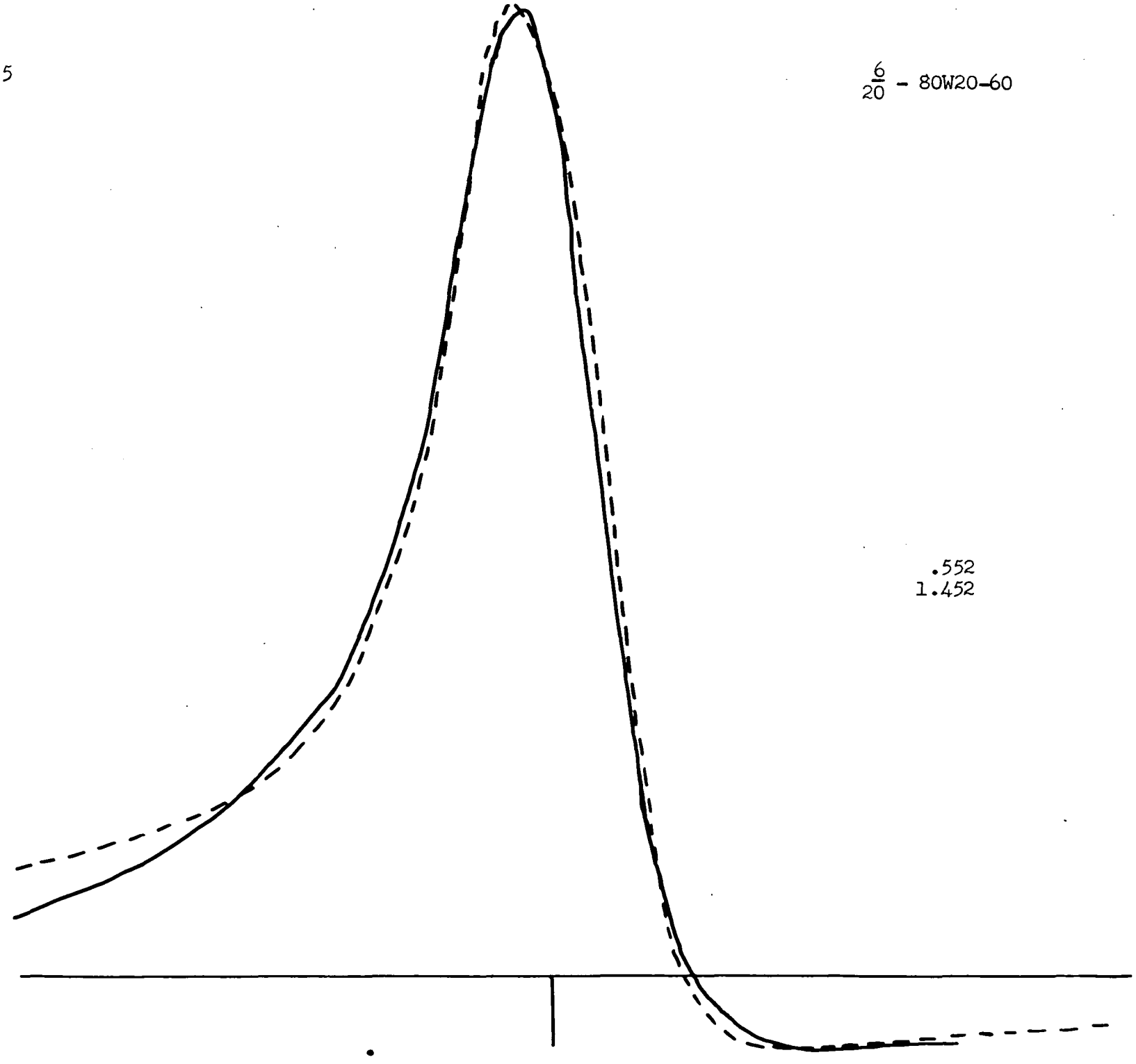
.754
2.855

.786
1.243



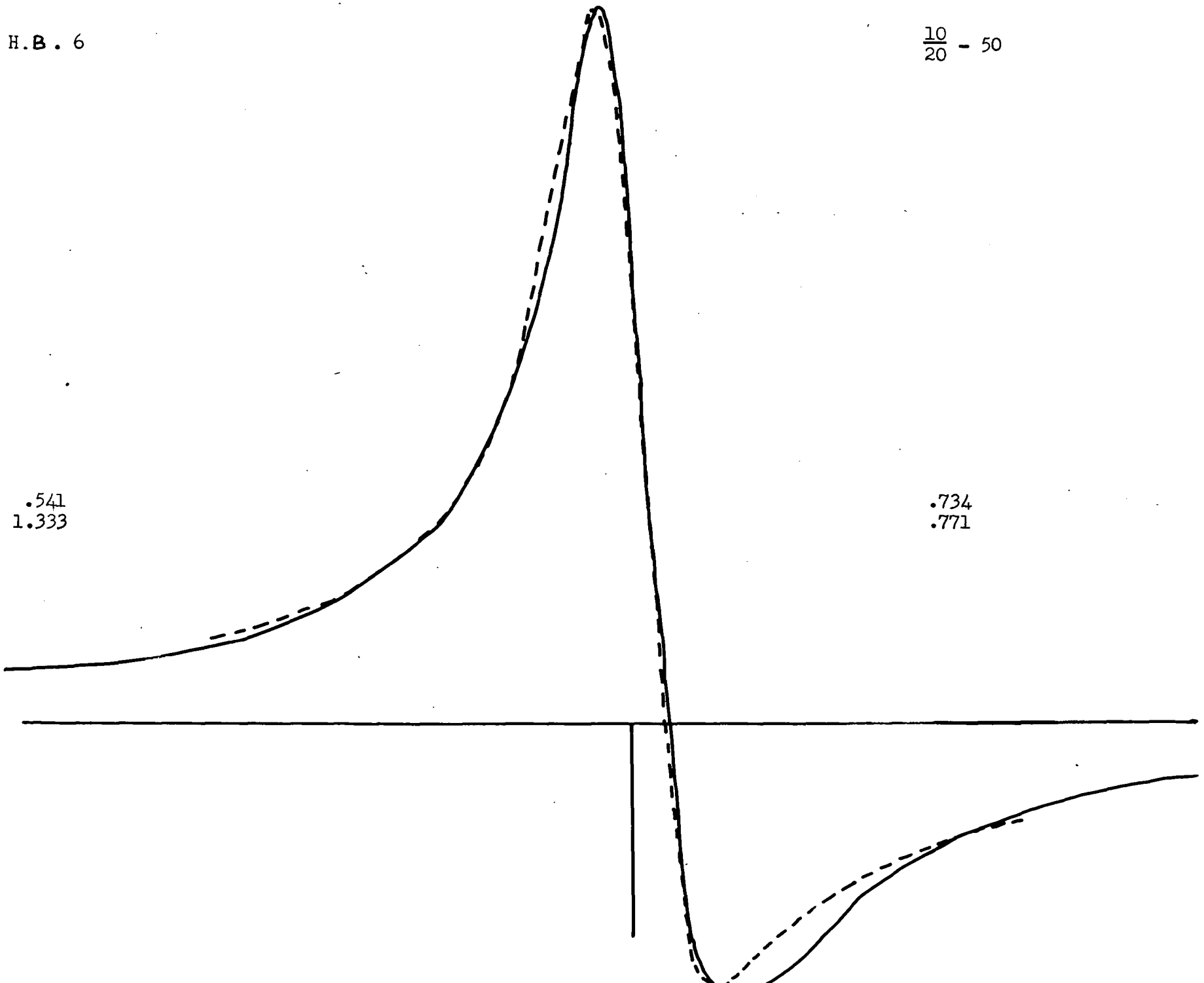
H.B. 5

$\frac{6}{20}$ - 80W20-60



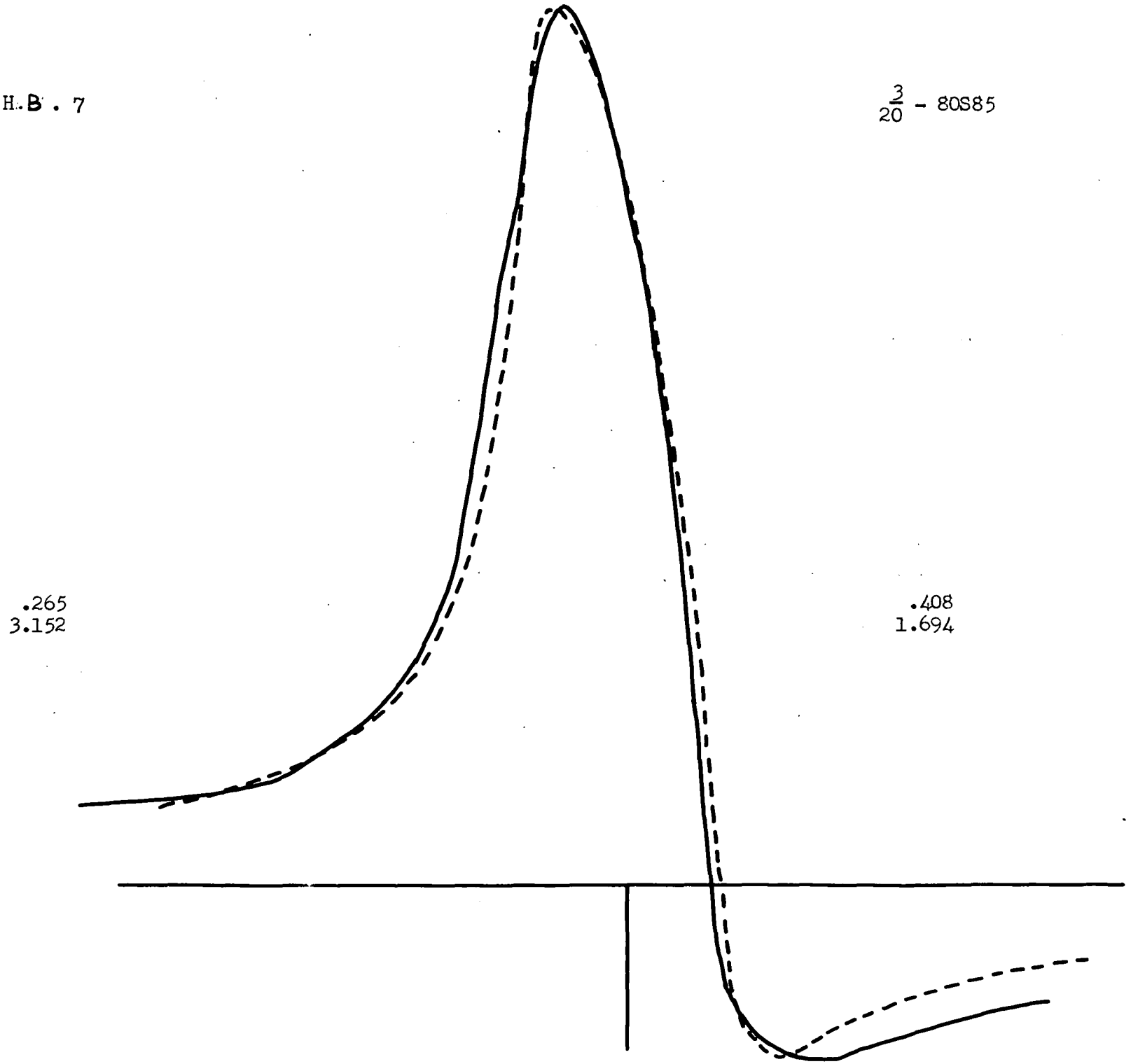
H.B. 6

$\frac{10}{20}$ - 50



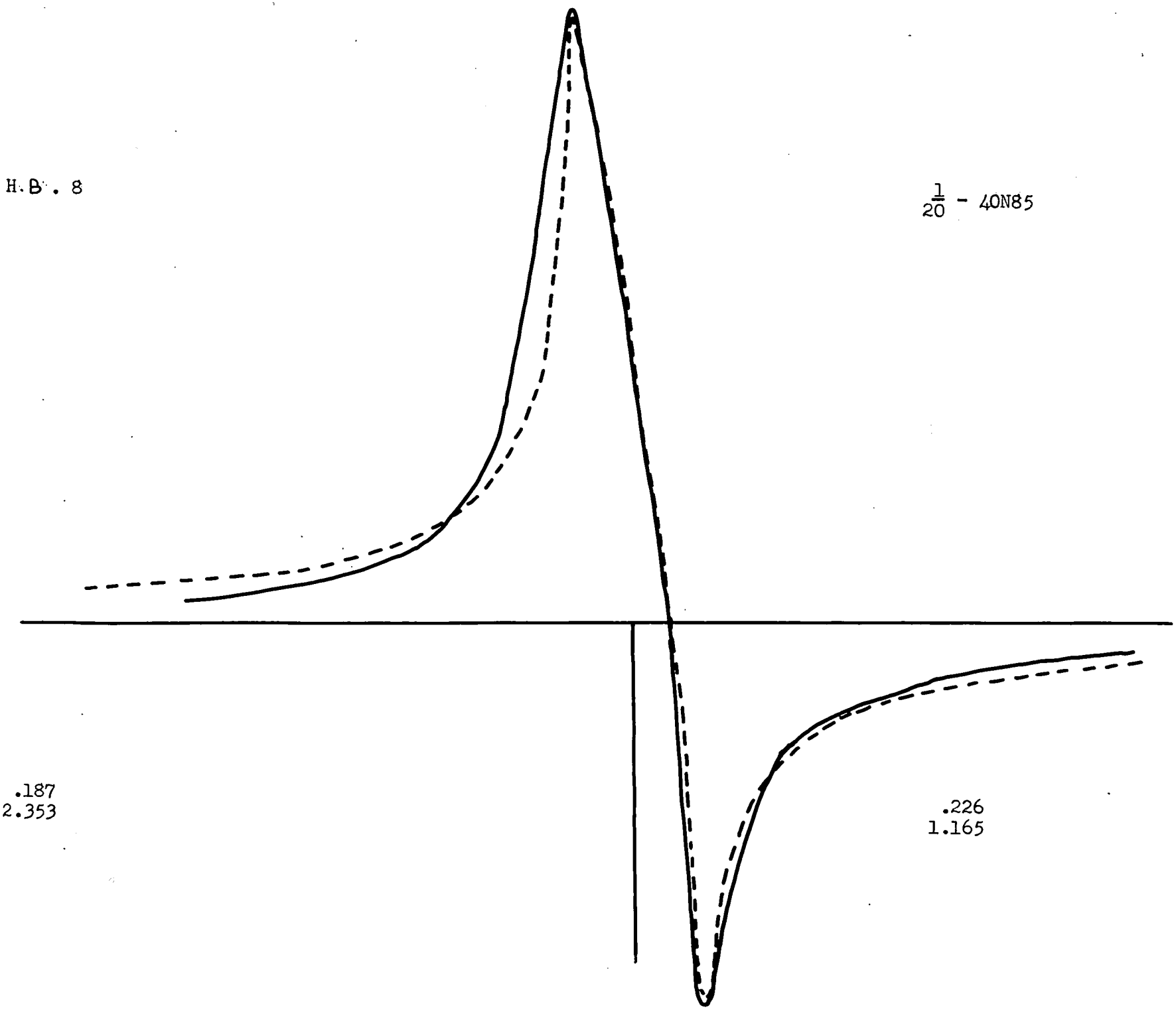
H.B. 7

$\frac{3}{20}$ - 80S85



H.B. 8

$\frac{1}{20}$ - 40N85



.187
2.353

.226
1.165

Locality	Depth (feet)	Width (feet)	Magnetisation		Remarks
			Inclination	Intensity	
1. Haughstrother Wood					Readings erratic
2. Willimontswick Farm	17	34	30	.0024	
3. Allendale	13	26	70	.0021	N80
4. Lipwood Well			No dyke - faulted sill present?		
5. Millhills Quarry	9	29	80	.0022	W20-60
6. Glen Dhu Quarry	17	35	50	.0027	
7. Fallowfield Farm	10	32	80	.0031	S85
8. Cocklaw Dean	1½	30	40	.0024	N85

TABLE II - HAYDON BRIDGE DYKE

The Haydon Bridge Dyke

1. General

The dyke is intruded along the line of a fault which is downthrown to the south bringing the Great Limestone against the Little Limestone in the Haydon Bridge area, suggesting a displacement of about 200 feet. However, the country rocks are virtually non-magnetic and their effect can be ignored. Only the Whin Sill, presumably present at a considerable depth along the entire length of the dyke, can contribute to the anomaly. In the following interpretations its effect has been neglected although a single traverse near Lipwood Well seems to demonstrate its presence.

2. Interpretation of the Profiles

Starting in the west, the first profile was made on the south bank of the River Tyne near Haughstrother Wood where thedyke is seen to be about 30 feet wide in the river cliff. Although the profile fitted the general picture with a minimum on the north side of the dyke, the readings were very erratic - probably due to magnetic debris in the recent river deposits - and no quantitive interpretation was attempted.

The next profile was made east of the road from Willimontswick Farm, the results of which appear in Table II.

The inclination of magnetisation is the lowest found for all three dykes and no explanation can be offered. The Allendale profile was made along a path near the top of the cliff north-west of the suspension bridge and there was some doubt as to whether the top of the dyke could be represented by a two-dimensional model. The results, however, showing the dyke dipping north, fit well with the general picture.

The stretch from Allendale to Haydon Bridge is of interest for no dyke could be proved between Tedcastle Farm and Millhills east of Haydon Bridge. Profiles near the Morralee mine shaft and Tedcastle showed the presence of the dyke but did not permit quantitative treatment. On the north bank of the river, along the expected line of the dyke, an anomaly similar to that expected over a faulted sill was found. Again the river deposits made the readings erratic, but the form of the profile suggests a sill at considerable depth, downfaulted to the south, with no sign of the dyke. The obvious answer would be the Whin Sill, with the direction of magnetisation probably south and inclined at about 40°. This has not yet been checked with theoretical models.

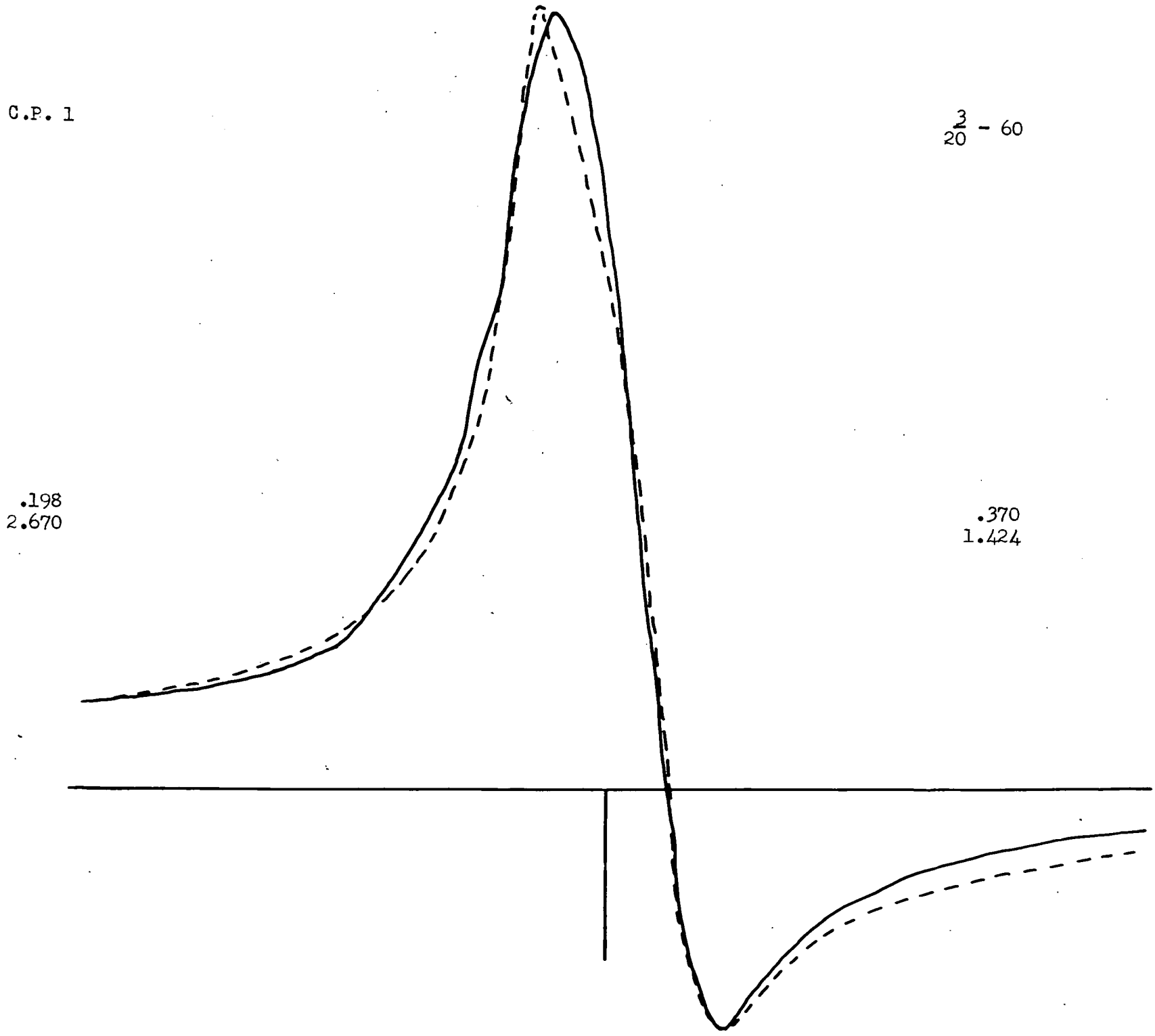
Numerous traverses made between Lipwood Well and West Rattenrow Farm failed to give any indications of the dyke, although a traverse south of Peelwell Farm remained unfinished as the instrument cable snapped. The other side of Haydon Bridge the first profile is about 50 yards west of Millhills Quarry. Here the results fit well with the general picture, although the inclination of the magnetisation seems to have increased and it was found necessary to widen the model downwards until its thickness was

trebled at a depth of 3,000 feet. Millhills Quarry itself is flooded and the width of the dyke cannot be measured.

The next profile is across the long Glen Dhu quarry about half way along it where there has either been little excavation or it has since been filled in with rubble - the calculated cover of 17 feet seems to indicate the latter. The measured width of the quarry is 30 feet, which agrees reasonably well with the calculated value of 35 feet. The low inclination of the magnetisation remains unexplained as the profiles to the west at Millhills and to the east at Fallowfield Farm, suggest an inclination of 80°. The profile at Fallowfield Farm also suggests the dyke is dipping slightly south.

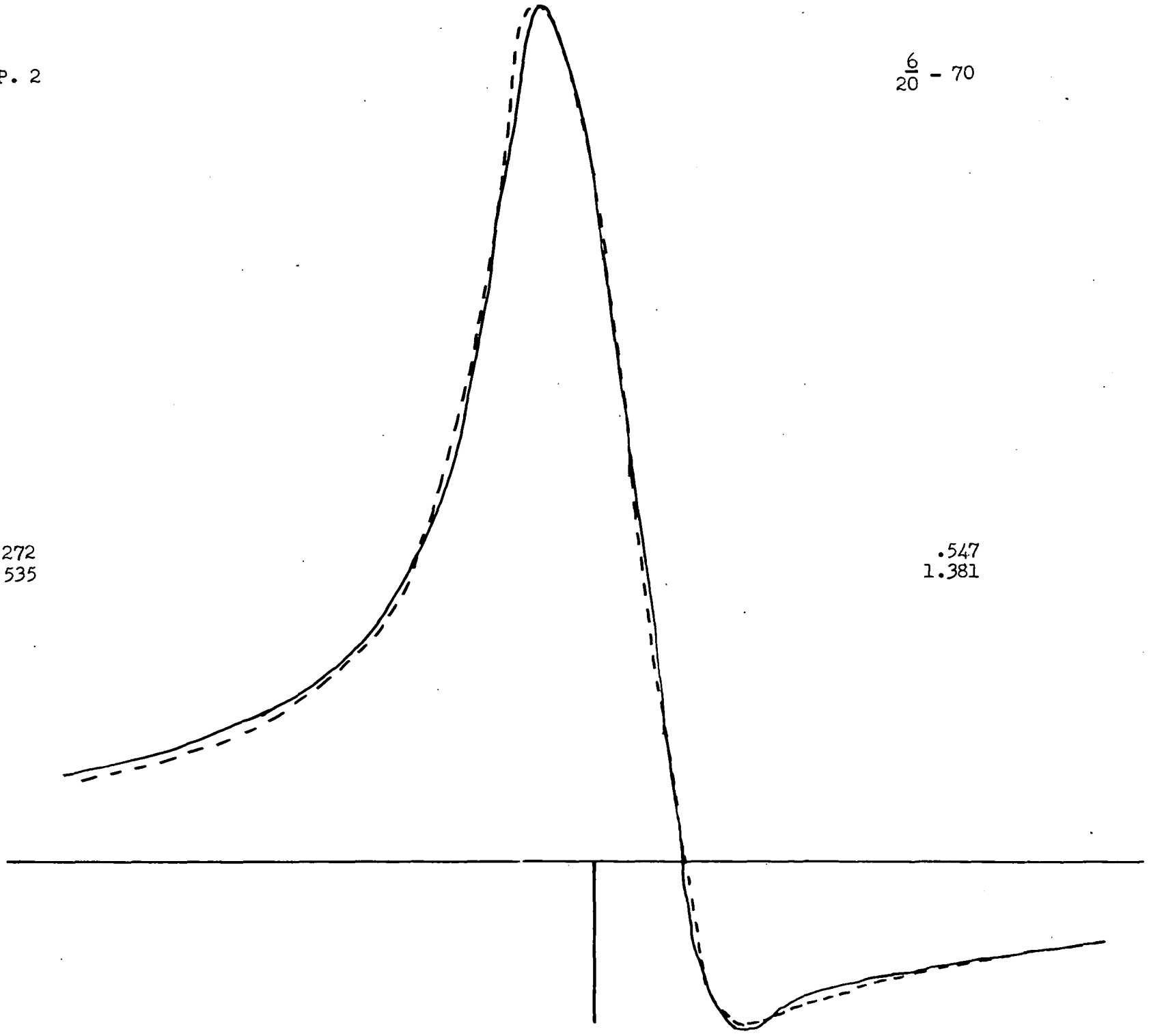
A traverse was started between Hill Head Farm on the B6138 and Brady's Crag to the north but the instrument's batteries were exhausted. This section is equivalent to Holmes and Harwood's (1928) St. Oswald's Chapel dyke. The next successful profile, also across what is known as the St. Oswald's Chapel dyke, was made at Cocklaw Dean, south-west of Keepwick Farm. Here the dyke must be very close to the surface and dipping north, again there is an unaccountable decrease in the inclination of the magnetisation.

The angle for the theoretical models was not altered for these latter profiles although the strike of the dyke changes from N80°E to N45°E. Any such adjustment would have involved a new set of standard curves and time would not permit this.


C.P. 1
$\frac{3}{20}$ - 60
.198
2.670
.370
1.424

C.P. 2

$\frac{6}{20}$ - 70

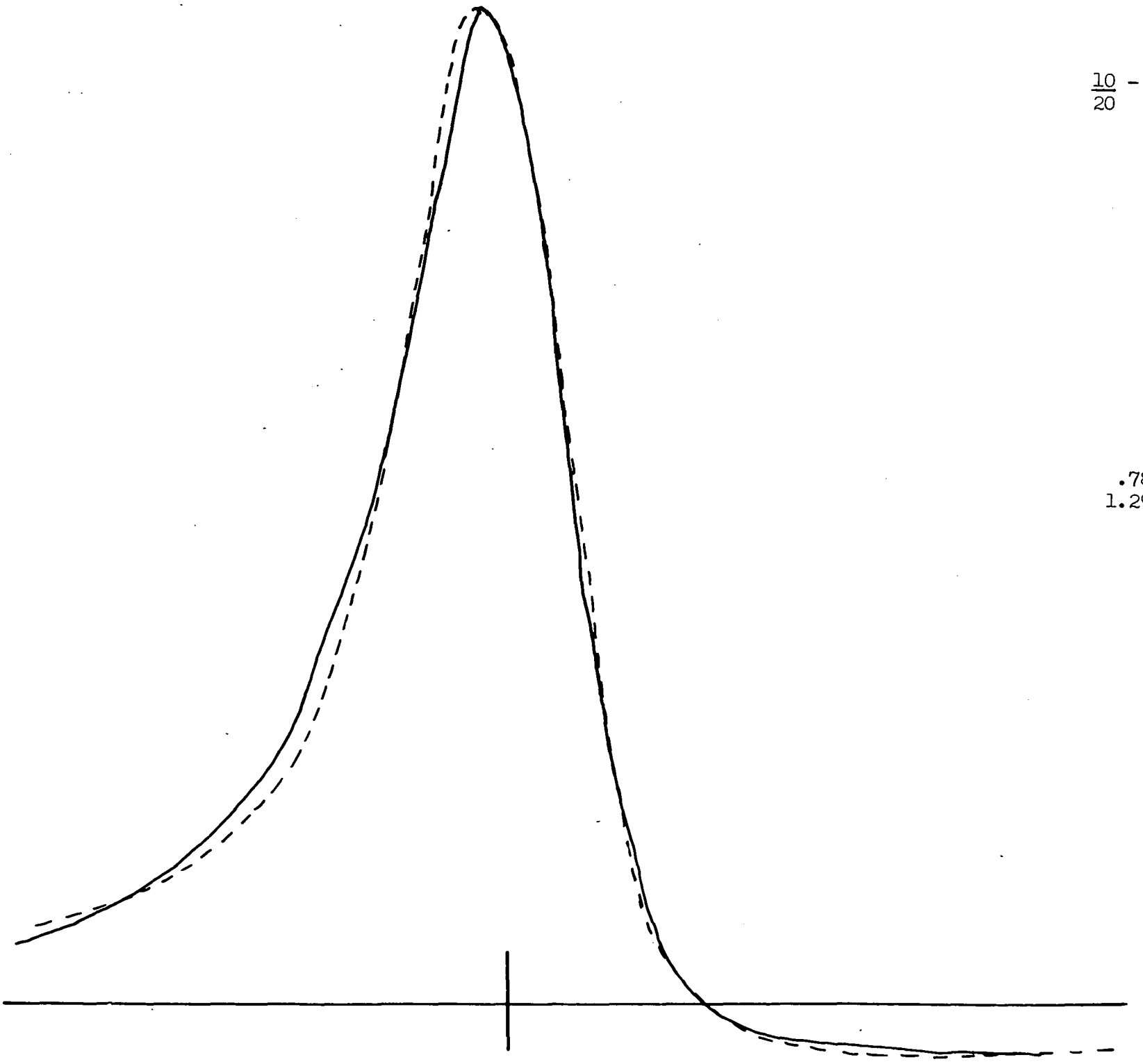


C.P. 5

$\frac{10}{20}$ - 90

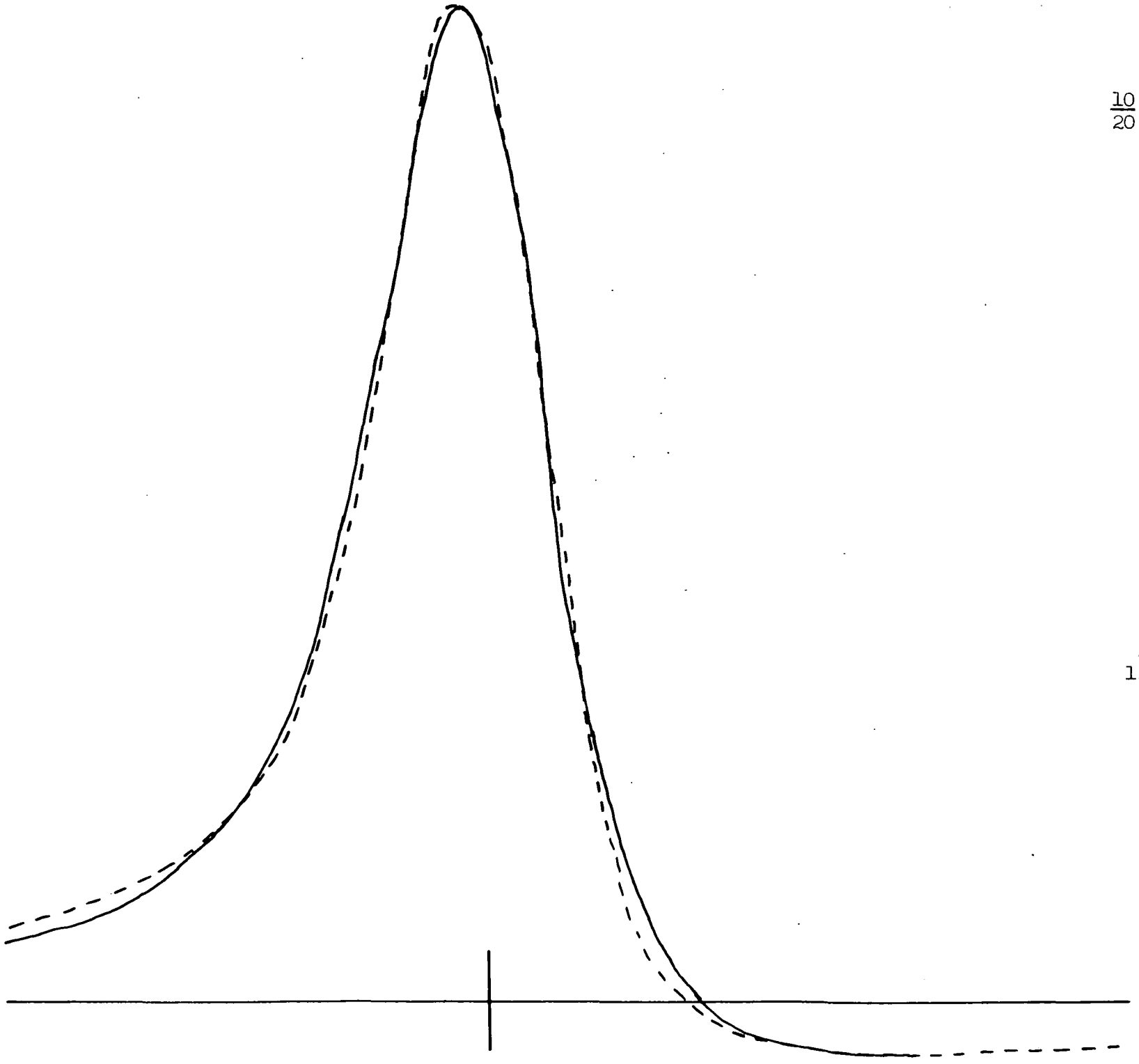
.729
.973

.784
1.291



C.P. 7

$\frac{10}{20}$ - 90



Locality	Depth (feet)	Width (feet)	Magnetization		Remarks
			Inclination	Intensity	
1. Parkhead Farm	5	37	60	.0037	
2. " "	10	33	70	.004	
3. Rayburn Lake					
4. Muckley Farm			c.70		Readings erratic
5. Whemley Burn	51	51	90	.0037	
6. Old Paxton Dean					
7. Causey Park	30	61	90	.0027	
8. Bullock Hall Farm					
9. Whitefield Farm					
10. East Chevington	Probably effected by displacement of the				
11. " "		Whin Sill at depth			
12. " " (sand dunes)					
13. " " (beach)					

TABLE III - CAUSEY PARK DYKE

The Causey Park Dyke

1. General

This dyke is the last seen at the eastern end of the St. Oswald's Chapel echelon. The short dyke at Causey Park shown in figure one has been extended in later maps to the coast on evidence from opencast and underground coal workings. The 6" Geological Survey maps show a continuous dyke intruded along the line of a fault between Parkhead Farm in the west and East Chevington on the coast. In the east where the dyke cuts the Coal Measures, the throw of the fault is given as 275 feet down to the north. Further west the throw is still assumed to be down to the north, but of uncertain amount. At Parkhead Farm there is no mention of any displacement.

As with the two previous dykes, the country rocks are virtually non-magnetic and their effect has been ignored. Only the Whin Sill, assuming it has been displaced or has changed its horizon across the fracture, can have any effect on the anomaly. The possible effect of the Whin Sill has not been considered in the interpretation of the anomalies below although the agreement between the observed and theoretical profiles may be improved if displacement of the Sill is considered.

2. Interpretation of the Profiles

At Parkhead Farm, one and a half miles north of

Netherwitton, the Geological Survey give the width of the dyke seen in the quarry as 25 feet and the overburden as 8 feet. West of this quarry the dyke has not been recorded. From the profile made 25 yards east of the quarry the width is interpreted as 37 feet, the depth of the drift as 5 feet and the inclination of the magnetisation as 60°. A further 100 yards east a second profile gave the width as 33 feet and the depth as 10 feet, with magnetisation inclined at 70° (C.P. 1 and 2, Table III). The intensity of the magnetisation at this end of the dyke seems to be higher than in the east, in fact, higher than found for any of the profiles across the other two dykes.

Profiles made at Rayburn Lake, north-east of Netherwitton, show that the thickness of drift has increased and that the magnetisation can still be interpreted as being inclined at about 70°. Unfortunately, no model could be devised to give a good enough fit to justify a quantitative interpretation. Further east again, at Muckley Butts, the readings were very erratic, but the inclination of the magnetisation as interpreted, appears to be about 70° and the thickness of the drift similar to that at Rayburn Lake. No detailed interpretation was attempted.

At Whemleyburn there appears to be 50 feet of drift over the dyke which is 50 feet wide and the magnetisation seems to have steepened to 90°. Near the western end of the long Causey Park quarries (C.P. 6), the Survey have recorded the dyke dipping south-south-east at 70°. Models dipping in this direction at 70°, 80°

and 85° with varying directions of magnetisation were tried, but none agreed closely with the field profiles.

Between the eastern end of the Causey Park quarries and Bullock Hall Farm, near West Chevington, Poole, Whetton and Taylor (1935) completed a number of profiles using a vertical force magnetometer. Their interpretation of these profiles was based on the assumption that the direction of the dykes' magnetisation was parallel to the Earth's present field and that it did not change. They concluded that at Causey Park the dyke dipped south and was 70 feet wide, then dipped appreciably north half way to Bullock Hall, where it dipped south again and was 57 feet wide - the 6" Geological Survey maps show it to be 63 feet wide.

The total magnetic profiles made during the present survey are, at Causey Park, close to profile 17 made by Poole et al and at Bullock Hall 400 yards east of there traverse one. At the former locality a vertical model 61 feet wide and 30 feet deep with the magnetisation inclined at 90° gave good agreement between the observed and theoretical profiles. The intensity of the magnetisation is considerably lower than that found at the western end of the dyke - but it is close to the average value for the Hett and Haydon Bridge dykes. At Bullock Hall it is possible that the opencast workings, now reclaimed, have altered the configuration of the upper part of the dyke, for no model could be found giving a reasonable agreement with the observed curve.

The probable explanation of the contrast between the interpretation of the earlier vertical force magnetometer results

and those of the present survey is that Poole and his co-workers assumed a constant direction of magnetisation for the dykes. In the present work the dyke has been assumed to be vertical and the direction of magnetisation consequent on this assumption has been calculated. If the theoretical and observed profiles still do not fit, then the anomaly would be calculated over an inclined dyke to obtain a better agreement.

Between Bullock Hall and the coast Anson and Sharp (1960) have produced a contoured map of the rockhead from borehole data. Their results show a drift filled river channel (the "Druridge Valley") running approximately east-west, across which the dyke cuts diagonally. The rockhead, about 100 feet below C.P. 9, is shown to be sloping steeply north. This may be the reason why the calculated profiles did not compare favourably with the field curve at this point. Traverses C.P. 10-13 are on the north side of the buried valley, the depth decreasing from approximately 75 feet to about 40 feet. Unfortunately, no suitable model could be found for these either.

The Whin Sill, presumably present at depth, would certainly complicate the anomalies if it had been displaced or had changed its horizon across the fracture. Unfortunately, time did not permit the preparation of theoretical profiles over such models.

Conclusions

1. General

It has been shown that the comparison of similarly scaled observed and theoretical profiles can lead to the rapid interpretation of total magnetic anomalies over two-dimensional dyke-like bodies. Unfortunately, the presence of the Whin Sill at depth may have complicated the interpretations of some of the anomalies in the present survey. Also, it may have detracted from the accuracy of the results for it is possible that some of the agreements between the observed and theoretical profiles are to a certain extent fortuitous.

Where it can be proved that there has been little or no displacement of the country rocks across the fracture into which the dykes have been intruded the results may be reliable. However, even if the Sill has not been displaced, it may have changed its horizon across the fracture and so these may be suspect too.

2. Contrast in the Magnetisation of the Whin Sill and the Dykes

Whatever the influence of the Sill on the detailed interpretation, the fact remains that the direction of the total magnetisation in the plane of the traverses (roughly north-south) is north and inclined at about 70°, whereas the direction for the Whin Sill is south and inclined at about 40° along its outcrop

(Creer, Irving and Nairn, 1960). This is based on an average Königsberger ratio for the Sill of 2.5 (Manley, 1949).

Four possible explanations of this phenomena are put forward:

(a) The Sill and dykes may be of different ages. The geological evidence points quite definitely to a period of intrusion for the dykes and the Sill during the late Carboniferous, and it is unlikely that the magnetic field of the Earth was changing rapidly enough at the time to cause the difference. Infact, the virtual geomagnetic poles for the whole Carboniferous fall into a well defined group with those for the Permian nearby (Cox and Doell, 1960). Also, the results do not suggest that the dykes are simply reversals of the Whin Sill magnetisation, as the directions are scattered on both sides of the direction of the present field.

(b) Bruckshaw and Robertson (1948) have collected orientated specimens across sections of the Cleveland dyke, which is of Tertiary age, and measured the direction and intensity of the magnetisation for each specimen. The chilled margins gave a consistent direction, but the inner specimens gave apparently haphazard results. The writers suggested the centre of the dyke had been in motion after the magnetite had cooled past its Curie point. This seems unlikely as basalts begin to crystallize at temperatures of the order of 1,000°C.

Assuming this did occur in the Whin dykes, although time has not permitted the collection of orientated specimens, it could explain the differences in the inclination of the magnetisation along the length of each dyke. Further, one would expect the narrowest dyke, in this case the Hett dyke, to have cooled fastest and to hence show the least scatter and the greatest intensity of magnetisation. Unfortunately, the presence of the Whin Sill seems to have complicated the anomalies over the western end of the dyke and so no conclusions can be drawn at this stage.

(c) Recently, the effect of pressure on the direction of magnetisation of a system of ring dykes has been studied (Girdler and Kume, personal communication), and orientated specimens showed a direction of magnetisation tangential to the rings. However, in the case of the Whin dykes, they do not appear to have undergone any great stress during or since their emplacement. Besides this, the average intensity of the magnetisation is about .003 suggesting a major component lies in the plane of the traverses, perpendicular to the dykes.

(d) Strangway (1961) has analysed the magnetisation or orientated specimens collected from Pre-cambrian dykes of different ages in Canada. In all cases the remnant magnetisation, after cleaning, reduced to a small component with varying inclination lying within the plane of the dyke. In trying to account for this he decided it was unlikely that the Earth's field was parallel to the strike of each dyke as

it was intruded. He also notes that the grains possessed a random orientation, with no flow structures apparent. Pressure effect are assumed to be negligible as they cannot account for the varying inclinations of the magnetisation within the plane of the dyke.

He concludes that internal demagnetisation has taken place. In support of this he has proved experimentally that the magnetite present is of two generations - each with its own Curie point. The first, main component, appears to have been unstable in geological time. Whilst the second, although weaker, is preserved today. This earlier generation probably acquired a direction of magnetisation paralled to the Earth's field at that time, whereas the later phase cooled under the influence of the Earth's field and that of the dyke itself. This, Strangway, states, has resulted in it lying in the same plane as the dyke.

Of the four possibilities discussed above the last offers the most promise for it is possible that something similar has occurred during a period of about 280 million years in the Whyn dykes. It would certainly account for their apparently large induced component of magnetisation. To be certain, orientated specimens would have to be collected and the nature of the magnetite in them investigated.

It would also be interesting to carry out a similar study to Strangway's on the Whin Sill, for the direction of magnetisation is again in the plane of intrusion.

3. Relative Ages of the Whin Sill and the Dykes

The best evidence for their relative ages is to be found in the profiles at the western end of the Hett dyke where it is intruded along the lineof the Knott Fault. Here it is possible that the anomaly at the surface is due to a dyke magnetised roughly parallel to the present field and a displaced sill with a direction of magnetisation south and inclined at about 40° (Creer, Irving and Nairn, 1960 and Manley, 1949). Unfortunately, time did not permit a quantitive investigation of this possibility.

Assuming the Sill is at different levels on either side of the fault, two possibilities exist:

(i) The Sill could have been displaced after it had consolidated and the dyke intruded along the fracture formed. If this were so, the period between their emplacement must have been very short as their compositions are almost identical (Holmes and Harwood, 1928).

(ii) The Sill could have encountered a pre-existing fracture across which it simply changed its horizon and at the same time sent up an off-shoot to the surface as the Hett dyke.

Of these two possibilities the first explains better the correlation between the increasing throw of the fault westwards and the accompanying change in the apparent inclination of the magnetisation. It must be emphasised that this remains merely an hypothesis until profiles over suitable models can be prepared.

APPENDIX A

Specification for Programme One

(Suitable for Ferranti Pegasus computers)

Name 2D MAGNETIC MK1

Purpose To calculate the horizontal (H), vertical (Z) and total (F) magnetic anomaly at a series of points (XST, YST) due to a two-dimensional model.

Principle It is assumed that any two-dimensional body may be defined by an n-sided polygon (Figure 2b), each face of which will cause a disturbance ΔF in the total magnetic field. The anomaly due to each face in turn is calculated from formulae given by Heiland ("Geophysical Exploration", Prentice-Hall, New York, 1940) for the horizontal and vertical disturbances at a point, due to a semi-infinite slab with a sloping end (Figure 2a). The effect of the complete body at this point is obtained by summing the disturbances due to each face. The formulae which have been employed are:

$$\Delta H = 2J.\sin i \left[J_x \left\{ \sin i.(\theta_1 - \theta_2) - \cos i.\ln(\frac{r_2}{r_1}) \right\} + J_z \left\{ \sin i.\ln(\frac{r_2}{r_1}) + \cos i.\ (\theta_1 - \theta_2) \right\} \right]$$

$$\Delta Z = 2J.\sin i \left[J_x \left\{ \sin i.\ln(\frac{r_2}{r_1}) + \cos i.\ (\theta_1 - \theta_2) \right\} \right.$$

$$+ \; J_z \left[\cos i.\ln\left(\frac{r_2}{r_1}\right) - \sin i.(\theta_1 - \theta_2)\right]$$

As the intensity of the Earth's field is large compared with the total anomaly, a good approximation to ΔF is given by the projection ΔH and ΔZ in the direction of the Earth's field:

$$\Delta F = \Delta H.\cos I.\sin\alpha + \Delta Z.\sin I$$

I and I' The inclination of the Earth's present field and of the total magnetisation, J.

J The intensity of the total magnetisation in c.g.s. units.

$J_x = \cos I'.\sin\alpha'$

$J_z = \sin I'$

α and α' The strike of the body measured clockwise from the horizontal projections of the Earth's field and the total magnetisation, J.

<u>Preparation of Data Tapes</u>

Station co-ordinates: T70.0

+XST_1 space +YST_1

+XST_2 +YST_2

+ .. ..

+XST_n +YST_n

+0

Earth's field T60.0

+ space +I

Scaling factor for models and stations:

T15.6

+N

Name of Model: N

TEST ONE

Co-ordinates, intensity and direction of magnetisation for each face of the model:

T100.0

$+x_1$ space	$+y_1$ sp.	$+x_2$ sp.	$+y_2$ sp.	+J sp,	$+\alpha$ sp.	+I' sp.	+0
$+x_2$	$+y_z$	$+x_3$	$+y_3$	+J	$+\alpha$	+I'	+0
..	..	..	..	..	..	..	..
$+x_n$	$+y_n$	$+x_1$	$+y_1$	+J	$+\alpha$	+I'	+0

After the first model: J2.0

After each succeeding model: L

After the last model: L

Z

<u>Notes</u>:

(a) All data must be preceded by a plus or minus sign.

(b) Units: station and model co-ordinates must have the same units.

(c) Scaling: $(XST_m, YST_m) - (x_n, y_n) < 1$ The scale factor (N) being read into T15.6 as, say +1000.

(d) The +0's signify that the last station or face has been dealt with and must not be omitted. This means $XST_i \neq 0$ and $x_j \neq 0$.

(e) J is unscaled as it is assumed to less than one.

(f) All angles must lie between $\pm 90^{\circ}$ and be multiplied by 0.01.

(g) Maximum number of stations (XST, YST) is 120.

(h) Maximum number of faces to the polygon is 27.

Operating Instructions

Tape reader one:

Programme - start and run.

77 stop - put in data tape here for XST, YST, H, Z and F.
For XST and F only - run. N.B. Data tape is unaltered.

77 stop - put in data tape - run.

77 stop - after last model.

Results

These appear:

Station co-ordinates		Horizontal	Vertical	Total Anomalies
XST	YST	H	Z	F

Timing

For H, Z and F at one point XST, YST due to one face of the polygonal model: 0.67 secs.

For print out of F and XST alone: 0.42 secs.

Durham University Computing Laboratory Job Number 298.

APPENDIX B

Specification for Programme Two

(Suitable for Ferranti Pegasus Computers)

Name 2D MAGNETIC MK2

Purpose To calculate the total magnetic anomaly (F) at a series of points (XST, YST) over a two-dimensional body. The anomalies and stations being scaled to chosen limits for the direct comparison of curves over simple dyke-forms.

Principle (a) The first half of the programme calculates the total intensity anomaly at a series of points and is based on the same formulae as Mark I. The sign conventions are also the same as in Mark I.

(b) When the total anomaly has been found for the last point, the three stations closest to the first peak value of the curve are picked out and sixteen new points are interpolated between them. The anomaly at each of these is found by re-entering the first part of the programme. The largest or smallest of these new values (which ever is applicable) is taken to be the maximum or minimum of the anomaly. This is repeated for the second peak on the curve. The difference between these values is scaled to 500 gamma

(or some other chosen value) and the value of F at each of the original points is multiplied by the scaling factor. The horizontal spread of the anomaly between the mean values of the maximum and minimum is also made equal to a constant and the XST co-ordinate of each station is multiplied by the scale factor.

Data This is laid out in a similar way to Mark I, only the Main Store localities and the additional constants required for the scaling are different. A typical tape appears:

N

Name

Blank tape

T 100.0

Station co-ordinates XST_n, YST_n as in Mark I.

+0

T 110.0

Co-ordinates and magnetisation of each face as in Mark I.

+0

T 69.0

Direction of the Earth's present field with respect to the body.

T 47.6

+.001 (Factor by which the X co-ordinate of the stations have been scaled for reading in on the data tape).

+40 (Unscaled distance between the mean values of the anomaly).

T 50.7

+100 (Factor for unscaling the face co-ordinates, corresponds to +N in programme one).

J 2.0

T 100.0

Details of the next body.

L

T 100.0

Last body

L

Z

Scaling of data before punching is exactly the same as for Mark I. The maximum number of stations is 40, the maximum number of faces is 10, but care must be taken not to allow a shape having more than one maximum and one minimum.

Operating Instructions

Tape reader: (1) - start and run

77 stop

(2) Data tape - run

77 stop after last body.

Results

These appear:

F scale factor (Y_t) X scale factor (X_t)

Station co-ordinates (XST, YST) Total anomaly (F)

Timing

For one station and one face - 1·88 secs.

Durham University Computing Laboratory Job Number 298.

ACKNOWLEDGMENTS

I wish to thank Professor K. C. Dunham for providing the facilities for this research and Dr. M. H. P. Bott for his close supervision during the past year. My thanks are also due to J. C. Branson for his unstinting help in the field and to the staff of the Durham University Computing Laboratories for their assistance with the programmes. D.S.I.R. have provided an Advanced Course Studentship for the past year.

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Step 1: GM and I's contribution of the m-term I and GM I.

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 2: GM's and I's contribution to the m-term I and GM I.

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 3: GM's and I's contribution to the m-term I and GM I.

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 4: GM's and I's contribution to the m-term I and GM I.

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 5:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 6:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 7:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 8:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 9:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 10:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 11:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 12:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 13:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 14:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 15:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 16:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 17:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 18:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 19:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 20:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 21:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 22:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 23:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 24:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 25:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 26:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 27:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 28:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 29:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 30:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 31:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 32:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 33:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 34:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 35:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 36:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 37:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 38:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 39:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 40:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 41:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 42:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 43:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 44:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 45:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 46:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 47:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 48:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 49:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 50:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 51:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma 1, Step 1 in Note. Thus you divide the two resonances.

$$\Omega_{GM} = \frac{\Omega_{GM}}{\Omega_{GM} + \Omega_{I}}$$

$$\Omega_{I} = \frac{\Omega_{I}}{\Omega_{GM} + \Omega_{I}}$$

Since $\Omega_{GM} + \Omega_{I} = 1$ the sharing has things of a

$$\Omega_{GM} = \frac{\Omega_{GM}}{1}$$

$$\Omega_{I} = \frac{\Omega_{I}}{1}$$

Since GM m is the sum of the two resonances, we have:

$$\Omega_{GM} + \Omega_{I} = 1$$

Step 52:

GM m is a convolution of Ω_{GM} & Ω_{I}, by Lemma

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TRANSLATE FROM BINARY (MARK IV)

13/7/61----25
2D MAGNETIC MK1

XST YST H Z F

B-U	Order	Stage	Function
	T2.0		
2-0	60 572	II	B60 to U5(α and I for the Earth's field)
	5.0 100		
	1.7 121		Convert to radians
	0.3 100		
	28 072		Sin α (R240)
	0.0 060		
	2 072		
	0.4 060		
	5.7 610		U5.7 = sin α
	5.1 100		
	3 172		
	1.7 121		Convert I to radians
	4 610		
	1.0 100		
	28 072		
	0.0 060		Sin I (R240)
3-1	3 172		
	1.1 060		
	5.6 610		U5.6 = sin I
	4 600		
	0 000		
	1.4 100		
	28 072		
	0.0+060		Cos I (R240)
	3 172		
	1.5 060		
	5.7 621		
	5.7 610		U5.7 = sin α.cos I
	60 573		U5 to B60
	2.0 060		
	71 0707.		$\frac{100}{180}$
	44 0306		
4-(2)0	4 072		
	5 172		
	6 272		
	0.2 060		
	0 200		2m = 0
	100 5722		B(100 +2m) to U5 Co-ordinates and magnetisation of first/next face (x_1y_1, x_2y_2, J, α', I', +0)
	5.0 300		
	0.6 361		
	10 172		
	11 272		
	9 072		
	0.1 060		If x_1 = 0 (after last face) jump to III
	5.3 400		
	5.1 403		$y_2 - y_1$
	1.3+462		
	5.2 100		
5-1	5.0 110		
	5.2 310		When $y_2 < y_1$:
	5.1 411		Exchange x_1y_1 and x_2y_2 and put J = -J
	5.3 413		
	5.4 102		
	5.4 110		
	4 402		
	5.0 300		
	5.2 303		
	3 320		
	4 422		$l^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$
	1.7 100		
	27 072		
	0.0 060		l (R200)
	1.7+060		
	1 351		
6-2	1 451		
	6 500		
	5 326		
	5.7 710		U5.7 = $\frac{(x_1 - x_2)}{2.l} = \frac{\cos i}{2}$
	5 426		
	4.7 710		U4.7 = $\frac{(y_1 - y_2)}{2.l} = \frac{\sin i}{2}$
	5.5 100		
	2.7 120		
	2.6 100		
	0 000		
	28 072		
	0.0 060		Sin α' (R240)
	7 372		
	3.0 060		
	71 0707.		$\frac{100}{180}$
	44 0306		
7-3	5.6 100		
	5.6 610		U5.6 = sin α'
	2.7 120		
	4 610		
	3.4 100		
	0 000		
	28 072		Sin I' (R240)
	0.0 060		
	3.4+060		
	5.4 621		
	5.5 610		U5.5 = J.sin I' or J_z
	4 600		
	8 072		
	0.0 060		
	0 000		
	0 000		
8-0	0.2 100		
	0 700		
	28 072		
	0.0+060		Cos I' (R240)
	8 072		
	0.3 060		
	5.6 621		
	5.4 621		
	5.6 610		U5.6 = J.sin α'.cos I' or J_x
	4.7 100		
	5.4 110		U5.4 = $\frac{\sin i}{2}$
	100 5732		U5 to B(100 +2m) (x_1y_1, x_2y_2, $\frac{\sin i}{2}$, J_z, J_x, $\frac{\cos i}{2}$)
	34 201		2m' = 2m + 1.0
	5 172		
	4 072		
	0.2+060		
9-0	19 072		
	0.0 060		
	0 200 ← III		2m = 0
	50 673		Clear B50 for $\Sigma\Delta$H, $\Sigma\Delta$Z, $\Sigma\Delta$F at each XST, YST
	0 300		3m = 0
	70 4722		B(70 + 2m) to U4 First/next point XST, YST
	4.0 4002		
	0.0 460		If XST = 0 (after last station) jump to V
	100 5723		B(100 + 3m) to U5 Results of atage II for first/next face
	5.0 100		
	40 773		Store modifiers
	0.7 161		
	16 072		
	0.0 060		If x_1 = 0(After last face) jump to IV
	0 000		
	11 272		
10-1	4 103		
	5.0 110		U5.0 = (x_1 - XST)
	1 120		
	5.1 100		
	4.1 1032		
	5.1 110		U5.1 = (y_1 - YST)
	1 122		
	6 500		X5 = $r_1^2 = (x_1 - XST)^2 + (y_1 - YST)^2$
	5.2 404		
	5.2 410		U5.2 = (x_2 - XST)
	4 420		
	5.3 400		
	4.1 4032		
	5.3 410		U5.3 = (y_2 - YST)
	4 422		
	6 400		X4 = $r_2^2 = (x_2 - XST)^2 + (y_2 - YST)^2$
11-2	4.7 010		
	5 403		$r_2^2 - r_1^2$
	12 372		
	3.0+460		If $r_1 = r_2$ leave U4.7 = 0
	3.7 200		
	2.4+462		
	4 501		
	4 603		If $r_1 > r_2$ exchange r_1 and r_2 and put $\frac{1}{\pi} = -\frac{1}{\pi}$
	2 202		
	6 526		
	7 600		
	2.7 100		
	35 072		
	0.0 060		$\frac{1}{32}.\ln(\frac{r_1}{r_2})^2$ (R221)
	2.7+060		
	2 621		
12-3	4.7 612		U4.7 = $-\frac{1}{\pi}.\frac{1}{32}.\ln(\frac{r_1}{r_2})^2$
	5.1 700		
	5.0 600		
	3.3 100		
	31 072		
	0.1+060		$\frac{\theta_1}{\pi} = \frac{1}{\pi}.\arctan\frac{(x_1 - XST)}{(y_1 - YST)}$ (R241)
	3.3+060		
	5.0 610		U5.0 = $\frac{\theta_1}{\pi}$
	5.3 700		
	5.2 600		
	13 072		
	0.0 100		
	31 072		
	0.1+060		$\frac{\theta_2}{\pi} = \frac{1}{\pi}.\arctan\frac{(x_2 - XST)}{(y_2 - YST)}$
	40 5746.		$\frac{1}{\pi}$
	14 3777		
13-0	13 072		
	0.1 060		
	5.0 604		X5 = $\frac{\theta_1 - \theta_2}{32\pi}$
	5 651		
	6 500		
	5.4 521		U5.3 = $\frac{\sin i}{2}.\frac{(\theta_1 - \theta_2)}{32\pi}$
	5.3 610		
	5.7 521		U5.2 = $\frac{\cos i}{2}.\frac{(\theta_1 - \theta_2)}{32\pi}$
	5.2 610		
	4.7 500		
	1 551		
	5.7 521		U5.7 = $\frac{\cos i}{2}.\frac{1}{}.\frac{1}{32\pi}.\ln(\frac{r_1}{r_2})^2$
	5.7 610		
	5.4 521		X5 = $\frac{\sin i}{2}.\frac{1}{}.\frac{1}{32\pi}.\ln(\frac{r_1}{r_2})^2$
	6 500		
	14 172		
14-1	5.2 501		
	5.5 520		
	5.3 100		
	5.7 103		X4 = $\Delta H = \frac{\sin i}{256\pi}\Big[J_x\Big(\sin i.(\theta_1 - \theta_2) - \frac{\cos i}{2}.\ln(\frac{r_1}{r_2})^2\Big) + J_z\Big(\cos i.(\theta_1 - \theta_2) + \frac{\sin i}{2}.\ln(\frac{r_1}{r_2})^2\Big)\Big]$
	5.6 122		
	5.4 621		
	4 610		
	5.5 121		
	6 602		X5 = $\Delta Z = \frac{\sin i}{256\pi}\Big[J_x\Big(\frac{\sin i}{2}.\ln(\frac{r_1}{r_2})^2 + \cos i.(\theta_1 - \theta_2)\Big) + J_z\Big(\frac{\cos i}{2}.\ln(\frac{r_1}{r_2})^2 - \sin i.(\theta_1 - \theta_2)\Big)\Big]$
	5.6 522		
	5.4 621		
	6 500		
	60 572		
	5.7 420		X6 = $\Delta F = \Delta H.\sin\alpha.\cos I + \Delta Z.\sin I$
	5.6 522		
	15 272		
15-2	50 572		
	5.0 411		U5.0 = $\Sigma\Delta$H
	5.1 511		U5.1 = $\Sigma\Delta$Z
	5.2 611		U5.2 = $\Sigma\Delta$F
	50 573		
	40 772		Restore modifiers
	34 301		3m' = 3m + 1.0
	9 072		
	0 000		
	10 172		
	0 000		
	0.2+060		
	0 0000.		
	0 000		Scale factor for model
	0 0000.		
	0 000		
16-0	50 572		B50 to U5 (H, Z, F)
	70 4722		B70 to U4 (XST, YST)
	2.6 600		
	4.0 6202		Unscale XST
	8 440		
	2 540		
	0.5 100		
	0 000		
	20 072		
	0.0 060		Print XST (R5)
	16 072		
	0.6 060		
	2.6 600		
	4.1 6202		Unscale YST
	17 172		
	1.1 100		
17-1	20 072		
	0.2+060		Print YST (R5)
	1.1+060		
	1.7 600		
	5.0 620		
	7 440		
	2 540		
	1.5 100		
	20 072		
	0.2+060		Print H (R5)
	1.5+060		
	1.7 600		
	18 272		
	2.0 060		
	0 0231.		
	53 400		Factor required to convert H, Z, F into gamma
18-2	5.1 620		
	2.2 100		
	20 072		
	0.2+060		Print Z (R5)
	2.2+060		
	1.7 600		
	5.2 620		
	2.5 100		
	20 072		
	0.2+060		Print F (R5)
	2.5+060		
	35 201		
	35 201		2m' = 2m + 0.2
	10 172		
	9 072		
	0.1+060		
19-0	0.2 100 ← V		Enter Initial Orders and read in next model
	33 400		
	10 0724		
	0.0 060		
	19 072		
	0.3 060		
	3 172		
	4 272		
	2 072		
	0.0 060		Re-enter programme (stage II)
	0 0000.		
	0 000		
	0 0000.		
	0 000		
	0 0000.		
	0 000		
20 etc.	0 773		Library Subroutines: R5, R200, R221, R240, R241
	2 142		
	22 072		
	127 445		
	0 000		
	0 773		
	0 100		
	0.1 167		
	24 172		
	0 772		
	0.5 110		
	0.5 060		
	0.6 110		
	10 140		
	1 443		
	1.3 6604		
	87 7537.		
	124 1363		
	17 0101		
	1.0 700		
	0.6 262		
	2 202		
	7 702		
	35 301		
	0.7 760		
	0.5 064		
	2 023		
	0.6 064		
	6 701		
	2 023		
	10 3403		
	1.3+060		
	0.0 660		
	21 072		
	12 6314.		
	4.6 3146		
	127 545		
	0.4+461		
	0.3 560		
	0 600		
	6 023		
	9 1101		
	24 173		
	5 300		
	6 200		
	33 600		
	15 141		
	25 172		
	0.7+2662		
	16 310		
	16 610		
	1.3 0606		
	14 640		
	16 610		
	1 720		
	0.7 2672		
	35 601		
	0.7 6662		
	0.4 560		
	12 340		
	5 200		
	0 500		
	0.6 700		
	1.3 266		
	0 0000.		
	0 000		
	0 0000.		
	0 000		
	0 0000.		
	0 000		
	0 0000.		
	0 000		
	0 0000.		
	0 000		
	0 0000.		
	0 000		
	0 0000.		
	0 000		
	0 0000.		
	0 000		
	1.0 010		
	16 110		
	0.0 560		
	1.0 710		
	0.1 621		
	1.2 367		
	0.0 060		
	7 100		
	1.7 460		
	0.0 600		
	1.6 065		
	0.0 220		
	36 055		
	1 741		
	26 072		
	0.0 060		
	6 023		
	0.4 461		
	2 600		
	0.3 660		
	1 742		
	6 023		
	7 100		
	0.3+6666		
	1.0 200		
	0.4+263		
	23 172		
	12 444		
	11 240		
	0.6+463		
	20 072		
	0.6 060		
	0.0 663		
	6 500		
	0.0 710		
	1 653		
	0.7 613		
	0.0 700		
	0.7 524		
	7 600		
	0.7 604		
	1 055		
	0.4+065		
	0.2 663		
	0.7 603		
	0.7 110		
	0.0 000		
	0 000		
	33 603		
	1 650		
	29 172		
	6 621		
	6 500		
	1.1 600		
	1.1 110		
	0.4 100		
	5 621		
	1.0 6011		
	0.4 166		
	1 651		
	5 603		
	1.7 601		
	1.0 562		
	0 600		
	1.1 064		
	6 602		
	0 0000.		
	60 2261		
	127 7745		
	77 3437		
	0 1703.		
	114 2336		
	7.2 5213		
	7 7555		
	64 7407.		
	4.0 6657		
	68 1303		
	27 1067		
	85 2525.		
	42 5252		
	1.1 462		
	6 602		
	0 0000.		
	10 2135		
	127 7777		
	17 7173		
	0 0006.		
	77 0511		
	127 7716		
	18 4457		
	0.0+0626.		
	124 5531		
	7.7 0216		
	75 5546		
	16 7006.		
	59 6002		
	6 400		
	32 700		
	0.2+060		
	6 400		
	7 406		
	33 202		
	0.4 663		
	6 602		
	0.5 763		
	7 702		
	6 703		
	33 172		
	2 253		
	1.2 762		
	7 601		
	1.1 660		
	1.2 7660.		
	4.6 313		
	0.0 601		
	6 326		
	0.0 710		
	7 721		
	30 172		
	6 300		
	1.1 600		
	0.5 500		
	3 621		
	1.0 6015		
	0.5 566		
	0.0 621		
	2 604		
	1.1 110		
	33 203		
	32 406		
	7 702		
	1.3+065		
	7 300		
	1.2+660		
	6 326		
	7 300		
	1.6 301		
	1.7 721		
	32 072		
	0.1 060		
	1.2 7660.		
	4.6 3003		
	21 7541.		
	1.4 6037		
	2.3 5067		
	80 0406		
	0 0000.		
	3.0 3172		
	0 0037.		
	2.3+4655		
	0 3345.		
	42 3713		
	43 7303.		
	25 3347		
	0.5 710		
	7 721		
	0.6 610		
	36 172		
	0.0 520		
	4 055		
	2 542		
	0.0+663		
	46 556		
	0.5 614		
	0.6 621		
	0.7 611		
	0.5 600		
	0 700		
	0.7 625		
	34 072		
	45 2023.		
	4.6 3177		
	21 7541.		
	1.4 6006		
	7 6073.		
	51 1576		
	0.7 710		
	1.0+661		
	1.6+065		
	0.1 600		
	0.6 621		
	0.2 601		
	0.6 621		
	0.3 601		
	0.6 621		
	0.4 601		
	0.5 621		
	0.0 603		
	5 651		
	0.7 603		
	1.7 110		
	1.7 060		

Z
7**