Structural properties of flat sandwich panels

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"Structural Properties of Flat Sandwich Panels"

by

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SYNOPSIS

This thesis covers the preliminary work of a long term project to construct domes from flat triangular sandwich panels which are of identical size and shape. The text includes a brief survey of the historical development of sandwich panels with descriptions of the different combinations of materials which have been used and their applications.

Simple analytical methods are used to predict the bending and torsional stiffness of a sandwich beam consisting of a core of low elastic modulus contained between thin faces of relatively high modulus.

A series of experiments on beams with plywood faces and a polyurethane foam core show good agreement with the theory. Simple strut tests confirm that with panels of the proportions used behaviour under end load was similar to that predicted.

NOTATION

A	=	$1 + 2F$
a	=	$\frac{f}{2}$
B	=	$\frac{b}{t}$
b	=	Width of beam
F	=	$\frac{f}{t}$
f	=	Thickness of faces
H	=	Total depth of sandwich
G_c	=	Shear modulus of core material
G_f	=	Shear modulus of face material
I_f E_f	=	Elastic modulus of face material
L	=	$\frac{t}{2} + a$
Q	=	$\frac{G_f}{G_c} \frac{F}{B^2}$
T	=	Torque
W	=	Hanger load in simple bending
t	=	thickness of core
u, v, w	=	Displacements in X, Y, Z directions
V	=	Strain energy
X, Y, Z	=	Coordinates
x, y, z	=	Coordinates in faces
α, β	=	Dimensionless parameters
γ_{XY}, etc.	=	Shear strains in core

ϵ_X, etc. = Direct strains

θ = Angle of twist per unit length

τ_c = Shear stress in core

μ = Poisson's ratio

DIAGRAMS

INTRODUCTION

For several years there has been a growing interest in the applications of lightweight sandwich construction particularly in the U.S.A., Great Britain and Holland. Much of the early work was done in the United States Forest Products Laboratory as a result of efforts to use timber efficiently as a structural medium.

However, the potential of lightweight sandwich panels was exploited mainly by the aircraft industry where there was an obvious use for this type of construction. It is only in comparatively recent years that the building industry with the aid of advanced architectural concepts has been able to find use for sandwich panels as structural members rather than as cladding materials.

The advantages of sandwich construction are numerous:-

Through efficient structural design each material can be stressed to its practical limit thus eliminating waste which occurs in non-composite structures.

The efficient use of material helps in the conservation of natural resources.

The wide variety of materials which can be employed ensures that particular combinations of materials can be used to obtain the mechanical, structural and insulation properties required for a particular set of circumstances. These properties are not yet available in any one material.

Rapid advances have been made in the last decade mainly through recent improvements in fabricating techniques plus the availability of a wide variety of suitable facing and core materials.

The most popular materials at present seem to be plastics, in different forms, for faces and polyurethane for cores, combining lightness with good insulation properties.

These two materials are by no means the only useful combination. Aluminium and other lightweight alloys have been used successfully as skins with honeycomb metallic cores in the aircraft industry. However the permutations of material combinations are enormous. Totally dissimilar materials can now be bonded together easily to give specific panel properties.

This thesis deals with two materials, birch plywood and polyurethane foam which have been bonded together to form flat sandwich panels.

An attempt has been made to predict the behaviour of these panels in bending and torsion using simple theoretical methods. The values for the different elastic constants for the two materials were found by separately testing each material on standard laboratory equipment.

All tests were carried out on the panels within the elastic limit of the separate materials and the theoretical analysis only applies to these conditions. The analysis does not cater for panels subject to large deflections or in conditions where significant creep occurs.

Definition

A structural sandwich panel can be described as a composite construction of alternate layers of dissimilar elements bonded rigidly to each other so as to use the properties of each to give specific structural advantages to the whole assembly.

The dissimilar elements of structure employed, which may or may not be of the same material, each have one of two functions:

(i) the faces, which form the outside layers of the sandwich, are of high density material with high stiffness and membrane strength.

(ii) the core, which forms the central element of the sandwich, is of low density, strength and stiffness.

The function of the core is to separate, support and restrain the facings so as to prevent elastic instability of the facings individually and the assembly as a whole when stressed.

Shear is transferred between the faces by means of the sandwich core which must have a shear rigidity sufficiently large to prevent shearing deformations cancelling the advantage gained through increased flexural stiffness of the panel.

The shearing stiffness is always smaller than that of a homogeneous material of the same flexural stiffness and because of the light core shearing deformations cannot be disregarded in stability and stiffness calculations.

CHAPTER I

HISTORICAL DEVELOPMENT OF SANDWICH PANELS

Aeroplane builders and designers have been the first to efficiently exploit the properties of sandwich construction although one of the earliest examples recorded was during the construction of the Britannia Tubular Bridge in 1846(1) which carries the railway across the Menai Straits in North Wales. Compression panels were constructed using thin malleable iron sheets riveted to each side of a wooden core.

From about 1920 onwards aeroplanes were constructed using sandwich components for pontoons and fuselage in America and Germany. In 1938 the de Havilland Albatross had a sandwich fuselage while in France a plane had been built using sandwich elements in the wings.

The classic example of these early applications was the Mosquito Bomber built during World War II which had a plywood-balsa sandwich monocoque fuselage. The wings were also of sandwich construction with balsa core and three ply birch for the faces. Later in the war smoother surfaces were required for both fuselages and wings because of rapid increase in aeroplane speeds. This meant that even more interest was shown in the development of sandwich construction which was continued after the war when the rapid growth in size of both civil and military planes required a great reduction in airframe weight because of slow development of engine power.

Wooden cores and faces were no longer used after the end of the war. The first non-wooden aeroplane sandwich was used on the Martin Matador which was a ground to air missile built in America. The core was made of phenolic impregnated cotton fabric with metal faces. Later when synthetic adhesives were further developed the core was made from a honeycomb structure of aluminium foil which could now be reliably bonded to the metal faces[2].

Radar was another development of the war which generated great interest in sandwich materials. The stiff dome-like shields were made from non-metal faced cellular rubber honeycombs and foamed plastics[3].

Further developments in the field of radar such as the Ballistic Missile Early Warning System, which has highly sophisticated delicate instrumentation requiring protection from the weather, have produced radomes which are one of the most spectacular uses of sandwich construction. These domes must be light and modular in design, the latter for economy in production and erection. The main requirement is that the structure be transparent to electromagnetic radiation. Plastic materials in sandwich type construction are ideal for such purposes.

Extensive work has been done in Canada on radomes. Research work there has concentrated mainly on the use of plastic foams as structural materials because it is known that low density foams are practically transparent to radio waves. Polystyrene and polyurethane foams have given most encouraging results and polyurethane also gives the required mechanical strength for domes of large diameter.

The faces have been made from glass cloth glued to the foam with epoxy resin giving protection from weathering and accidental damage. Tongue and groove joints have been found to be most satisfactory in joining the panels in the Canadian domes.

The greatest plastics dome built so far is the 140 ft. diameter structure built in America as part of the Ballistic Missile Early Warning system. This dome is built from panels which are of a honeycomb sandwich construction. The basic skin thickness is 0.042 in. and the honeycomb consists of kraft paper 6 in. thick. The honeycomb core presents a reduced electrical obstruction and is excellent structurally leading to a highly economical solution.

The building industry has generally lagged behind the aircraft industry because of the different conditions which exist. Economic advantages of other types of construction have in the past outweighed the most important advantage of sandwich construction which is its weight:strength ratio. This is probably because early designs using sandwich type construction have not exploited its most desirable attributes to the fullest extent.

Aerodynamically smooth surfaces are not required in building and the prerequisite dielectric qualities which make so many sandwich panels useful in radar are also missing.

There are however advantages in sandwich construction used in the building industry shown by research and development programmes in recent years. The major advantage is the great versatility of sandwich construction exemplified by the many variations in component materials that may be employed. Specialist properties such as heat resistance, weather resistance, etc. can be built into the sandwich by careful selection of component materials[4].

Another advantage is simplification of construction by the reduction in the number of components which are used for one single purpose only, e.g. roofing felts and insulation.

Large size sandwich panels speed up erection, reduce on-site labour and probably require fewer skilled craftsmen than conventional constructions. Because the panels are shop manufactured greater efficiency can be obtained both in quality control and use of materials.

The first sandwich panel used in the building industry was produced in 1933 for a house in Long Island, America. It was called "Cemesto Board"[5] and consisted of cement-asbestos core and fibreboard faces. In World War II as an answer to the need for low cost housing the "Cemesto House" was developed in America and many were built.

From 1944 the Forest Products Laboratory[4] in America became a major centre for theoretical and experimental work on sandwich construction and in 1947 a test house was constructed to investigate the long term behaviour of sandwich panels. The results have been favourable and the structure has retained its strength.

Plastics have been used in sandwich construction since the fifties. The faces are generally made from glass reinforced plastics and the cores from foamed plastic. Now insitu foaming techniques have helped the factory production of panels.

The first project to arouse interest in building was the Monsanto "House of the Future"[6]. This structure was built in 1956 and consisted of four curved wings cantilevered from a central core. The basic unit was an 8 ft. x 16 ft. prefabricated shell made as a laminated sandwich panel with a 4 in. honeycomb core. The faces were made from glass fibre reinforced polyester plastics and the panels were bonded with a fire resistant polyester resin having chemical and water resistant characteristics and good resistance to heat distortion. Periodic on-site tests have showed that the structural performance of the house was good and no evidence of structural weakness could be detected. In order to utilise the properties of the plastic panels to their greatest extent the house was of unorthodox design breaking away from the traditional architecture. From architectural and structural points of view the house was a great success but on a cost basis such an all-plastics structure in 1956 could not compete with traditional techniques even taking into account all the advantages offered by plastic sandwich panels.

As a result of the Monsanto House project various designs for plastic sandwich structures were put forward by architects and engineers. A research group at M.I.T. in America have been working since 1954 on the structural use of plastic sandwich panels and this work led to the design and construction of a school with Hyperbolic paraboloid sandwich umbrella roofs[7]. This approach enabled a complete cost evaluation to be made and it was claimed that technically and economically this project could compete with traditional forms of construction provided there were several schools to be built enabling factory production for industrialised building.

One of the many designs produced in the mid-fifties for plastic sandwich construction was the experimental French all-plastics house built in 1956 for the Salon des Arts Managers de Paris[3]. It was designed by Yonel Schein, Yver Magnant and R.A. Coulon. This structure is an excellent example of a prefabricated panelised system. It consisted of a circular core of eight prefabricated segments covered by a roof constructed of eight units overhanging at the perimeter and jointed together at the centre to a hollow column which collected the rainwater from the whole roof area.

The main feature of the design is its flexibility. Two, three or four rooms can be added to the central core according to the needs of the occupants. The floor consists of strong, light plastic sandwich units and the wall panels have a foam core giving the required stiffness and thermal insulation. The interior partitions are made in light glass reinforced polyester sections, including the built-in furniture in the bedrooms, kitchen and bathroom. The windows of clear acrylics are built into the wall units and form an integral part of the load bearing elements. The whole house weighs 1800 lb. and has 6,000 cu. ft. of useful volume.

During the industrial exhibition in Berlin in 1957 the Owopor house[3] was constructed using prefabricated segments. The units consisted of Styropor foam core 2 in. thick having outer facings in glass reinforced plastics and inner facings of plywood.

In 1958 a German Architect, Rudolph Doernach, displayed at the Stuttgart Plastics Exhibition[3] a house using doubly curved segments. The units consisted of a plastic foam core with aluminium facing. The structure was supported at four corners only and was meant to be a weekend cottage which could easily be enlarged by linking two or more units together.

Other examples of plastics sandwich panel construction have been built in Italy (G.R.P. facings, saturated paper honeycomb core), Brussels (American Pavilion, G.R.P. facings, metal honeycomb core), and in Russia where an all-plastics house has been built in Leningrad[3].

All these examples of plastics sandwich panel construction have been experimental, built either for research projects or as exhibition buildings. Since the sixties sandwich panels have been commercially used in certain areas. Mass production has removed cost restriction. However two important constraints remain on engineers who wish to use sandwich construction[(1)]. These are fire resistance and durability. Attempts have been made to solve these problems by using cheap mineral cores and glass reinformced plastic facings for structural cladding and roofing. Claims have been made that this combination of materials gives a satisfactory solution to the problem.

Holiday homes and chalets have attracted the attention of designers wishing to exploit lightweight sandwich construction and several designs are now on the market. Most of the designs are still of a mixed system of building in which the framework is constructed of timber or steel with plastics sandwich panels used semi-structurally as infilling. Two Japanese houses are well known and are mass produced in Germany under licence. The walls of these houses have a polystyrene core and the ceilings and roof units are of hard vinyl chloride sheets with ribs.

In Britain, Mickleover Transport Ltd. have developed a special prefabricated building used for relay stations on the signalling system of British Railways Eastern Region. The main advantages of this building are that it could be erected within a few hours, does not need painting and requires no maintenance. The buildings are composed of three basic types of unit; a corner unit and side units of two different spans. A unit consists of a wall and roof in one shell of double curvature. The outer facings of the sandwich are of laminated polyester reinforced with glass fibre and $^1/_8$ in. thick with a smooth face from the mould. The core is $^3/_4$ in. thick of phenolic foam to give thermal insulation and fire resistance while the inner facing is similar to the outer facing but formulated to give a low surface flame spread. The units are bolted together with stiffening flanges of solid polyester. Substations for the South of Scotland Electricity Board have been built using these plastics sandwich structures. The same firm have built a two storey telephone exchange block in Birmingham using the same technique as the relay buildings. Also the British Antarctic Survey used this type of building with great success.

As more and more use is being made of sandwich construction further experimental and theoretical work is being done to design the sandwichs more rationally. Most applications of sandwich construction have been shown to be feasible technically as well as economically although improved theoretical analysis must mean there will be an even wider scope for sandwich applications.

CHAPTER 2

MATERIALS AND CONSTRUCTION OF PANELS

2.1. Materials

In order to keep the parameters as few as possible only one combination of core and face materials was used, i.e. marine plywood for the faces and polyurethane foam for the core. The plywood was standard marine Birch nominally 1.5 mm thick supplied in 52" x 52" sheets. This was the limiting factor on the maximum size of panel which could be produced. The core material was a rigid polyurethane foam manufactured by I.C.I. Agricultural Division at Billingham as an insulating material for use in the building industry. The foam is marketed in the form of a laminate with protective cardboard faces. In the manufacturing process the polyurethane is foamed onto one cardboard face and the second face is glued to the foam after it has set. These faces are difficult to remove without damaging the polyurethane core and it was decided to construct the sandwich panels by gueing the plywood to the cardboard. Manufacturing processes are being developed so that the polyurethane can be foamed directly onto a variety of different face materials thus increasing the bond between face and core.

The foam was supplied in sheets 8' x 4' with cardboard faces 0.6 mm thick. Panels were made from three different thicknesses of core material, i.e. nominal 1 in., $^3/_4$ in., and $^1/_2$ in.

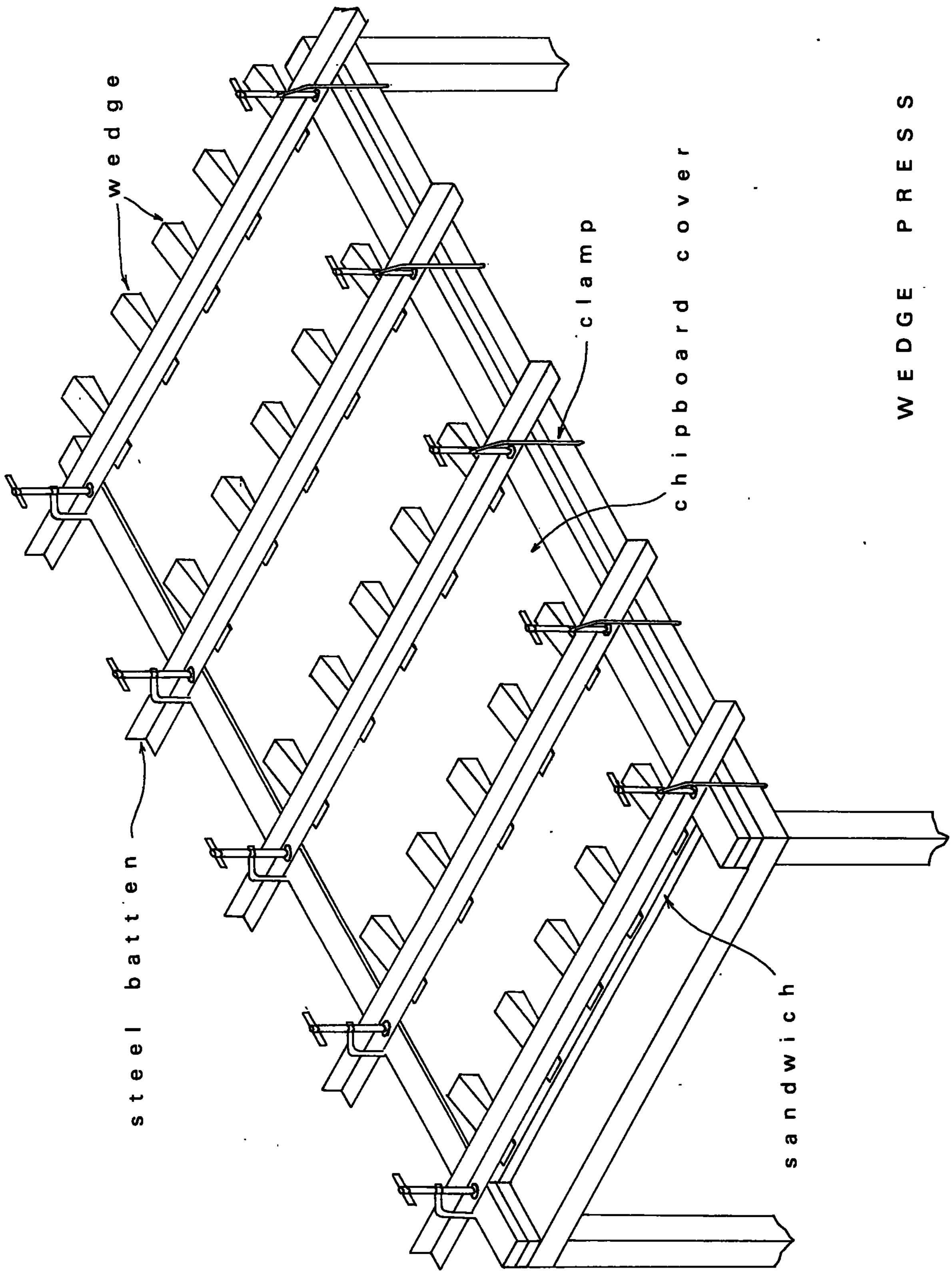
'Mouldrite' UF 232, a urea formaldehyde synthetic resin, was used to glue the faces and core together. A powder hardener gave a pot life of 30 minutes and full strength in 48 hours. The panels could be handled about 3 hours after manufacture but tests were not made until the glue had achieved full strength.

2.2. Construction of Panels

Several methods of assembling the panels were tried. Peel tests showed that the most consistent bond was achieved by using a wedge press (Fig. 1). The Mouldrite glue was spread by hand to a thickness of about .005 in. on both plywood and cardboard and the panel was then assembled and placed in the wedge press. A 1 in. thick piece of chipboard was placed over the panel to distribute the load evenly. The load was applied by inserting wedges between the chipboard and cross battens which were firmly clamped to the base table. The panels were left in the press until the 3 hour setting time had been reached. They were then removed and stored until at least 48 hours had elapsed giving the glue time to reach full strength. Checks were made on core thickness before and after construction to see if the pressure exerted by the press had any effect on the core. No significant difference in thickness was detected after the panel was assembled. All panels were made as uniformly as possible. The grain of the outside laminates of the plywood were made to run in the same direction for both the top and bottom faces. The faces were always placed in the same manner on to the core with the grain of the plywood parallel to the warp in the protective cardboard cover.

In order to standardise the strength of the glue the constituents were weighed accurately on a chemical balance each time a

FIG. 1.


wedge
steel batten
clamp
chipboard cover
sandwich
WEDGE PRESS

panel was made. A ratio of 5 parts of Mouldrite to 1 part of powder hardener gave a reasonable pot life, long enough to glue and position both faces of the panel in the wedge press.

2.3. Materials Testing

Tension and compression tests were carried out on the two main constituents of the sandwich panel, i.e. the plywood faces and the polyurethane core. Shear tests were also done using a method similar to the A.S.T.M. method for shear testing materials. The cardboard protective faces were tested in tension only.

2.4. Plywood Faces

Tension

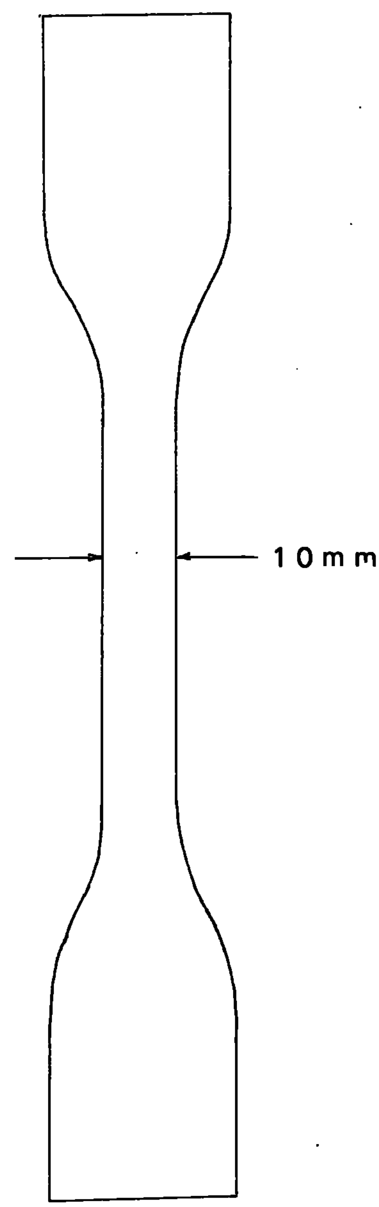
A Hounsfield 'E' Type tensometer was used to test the plywood in simple tension. Specimens 100 mm long and 10 mm wide (Fig. 2) were stamped out using a special cutting tool. The ends of the specimens were drilled in order to fit into the jaws of the tensometer. Care had to be taken to ensure that no damage or distortion occurred because small defects in the specimen significantly altered the results.

All tensile specimens were loaded at the same rate giving an extension of 1.5 mm/min. A full scale reading of 250 Kg was used on the load-extension chart. Even with a 16:1 magnification of extension it was considered that the automatic recorder was not accurate enough in measuring the extension. Also there was no accurate method of assessing the gauge length of the specimen. Two methods were used to measure the extension both of which gave very similar results.

FIG. 2.

Used for plywood and cardboard

in 'E' Type tensometer



STANDARD TENSILE SPECIMEN

Method I (Mechanical)

A Hounsfield extensometer was used on the first group of tensile specimens. It had a gauge length of two inches and worked on a lever principle. The extension was measured in units of .0001 in. and was displayed on a dial gauge. The gauge was attached to the centre line of the specimen by gripping screws which deformed the specimens slightly. This was thought to be the reason for rather low values for the modulus of elasticity calculated from a load extension plot. A mean value of 1.2×10^{10} N/m^2 was obtained for this group of specimens.

Method II (Electrical

Electrical Resistance strain gauges were attached to both faces of the second group of tensile specimens. These electrical strain gauges were of 30 mm gauge length and had a gauge factor of 2.01. From direct plots of load against micro-strain the modulus of elasticity calculated was found to vary between 1.37 and 1.43 x 10^{10} N/m^2 with a mean value of 1.4×10^{10} N/m^2. This value was used in all further calculations and is in the direction of the grain of the wood in the outside faces of the plywood.

Each of the first two groups consisted of 10 specimens. Throughout the period of panel manufacture frequent tensile tests were made on the plywood and it was found that no significant deviation occurred in the results obtained. The plywood was stored in reasonably stable temperature and atmospheric conditions in order to eliminate errors caused by the physical properties of the wood changing.

Tensile tests were made on plywood specimens with the grain of the outer laminates at right angles to the tensile force. Values for the modulus of elasticity in this direction were less than the values in the direction of the grain by a factor of 0.6. Independent tests were carried out by G.M. Parton of the University of Durham and a factor of 0.7 was obtained.

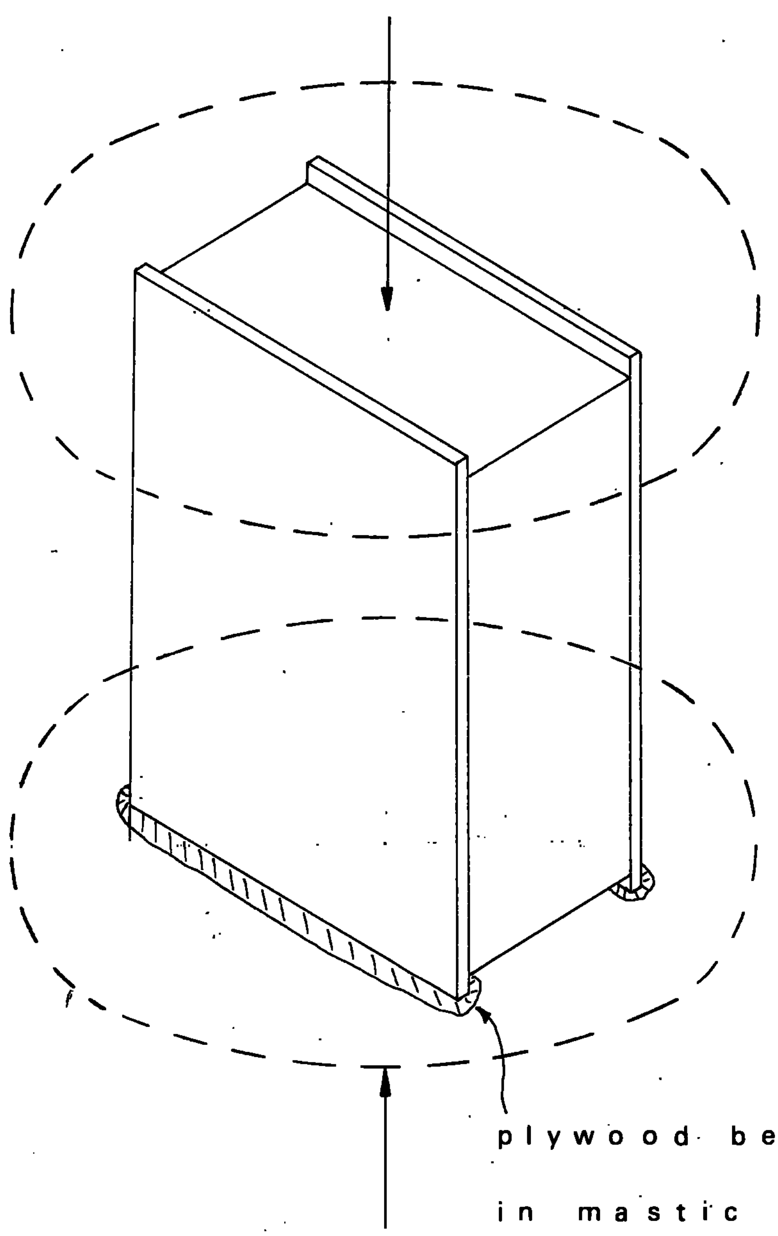
In order to obtain a value for Poisson's ratio in tension electrical strain gauge rosettes were fastened to the faces of the specimens. The maximum size of gauge length of these rosettes which could be used on the tension specimen was only 10 mm so that possibly only localised effects could be measured; however the mean value of $\mu = 0.27$ obtained seemed to be a reasonable result. The value of Poisson's ratio was obtained from a direct plot of longitudinal micro-strain against lateral micro-strain for the specimen which was loaded in line with the grain.

2.5. Compression

The plywood faces were tested in compression using the 50 ton Denison machine (Fig. 3). The plywood was loaded in line with the grain in the outer laminates.

The specimens used were cut from a one inch thick sandwich panel and were 2" x 2" square. The core was not removed from between the faces so that when placed in the machine the core exerted a certain amount of lateral restraint preventing buckling of the faces. The loads were not increased sufficiently so that lateral buckling could be visibly detected. The bearing areas of the plywood were bedded on a mastic filler to try and ensure that the

FIG. 3.



DENNISON COMPRSSION SPECIMEN

load was applied uniformly across both faces. Electrical strain gauges were used to measure both axial and transverse strains. The gauges were 30 mm long and were stuck to both faces of the specimen.

The average value obtained for E_{fc} was 1.3×10^{10} N/m^2. The value of Poisson's ratio was 0.104 obtained from direct plots of lateral micro-strain v longitudinal microstrain. This low figure compared with the result in tension is probably attributable to the lateral constraint due to the shortness of the specimen.

2.6. Shear

The value of shear modulus for the plywood faces used in both bending and torsion calculations was 5.5×10^9 N/m^2. This is an approximate value obtained from the formula

$$G_f = \frac{E}{2(1 + \mu)}$$

where $E = 1.4 \times 10^{10}$ N/m^2

and $\mu = 0.27$ obtained from a direct plot of lateral micro-strain for an orthotropic test on a tensile specimen.

The Plywood is not isotropic and experimental values of shear modulus vary depending on which axis the specimen is tested. The elastic moduli E_1 and E_2 have different values as shown in the tensile tests; thus the Poisson's ratio μ_1 and μ_2 are different. The Maxwell-Betti reciprocal theorem demonstrates that $\mu_1 E_2 = \mu_2 E_1$ giving two possible values of shear modulus from the above formula.

The elasticity matrix for an isotropic plate takes the form

$$\frac{E}{1-\mu^2} \begin{matrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & G \end{matrix}$$

where the shear stresses are not affected by normal strains.

However the shear terms in the orthotropic case are only zero if E_1 and E_2 are measured on the principle axes of orthotropy. When related to axes other than the principle axes the elasticity matrix D becomes D^1.

where $D^1 = T \quad D \quad T^T$

$$\text{and } T = \begin{matrix} \cos^2\alpha & \sin^2\alpha & -2\sin\alpha\cos\alpha \\ \sin^2\alpha & \cos^2\alpha & 2\sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha & -\sin\alpha\cos\alpha & \cos^2\alpha - \sin^2\alpha \end{matrix}$$

The multiplication of the above matrices eliminates all zero values in the elasticity matrix demonstrating that there is no one single value for Shear Modulus.

However this does not affect the bending results for beams, where G_f is not used, and it is demonstrated in the torsion discussion (Ref. 4.8.) that the approximate shear modulus value is adequate in predicting torsional stiffness for twisted panels.

2.7. Cardboard Protective Covering to Polyurethane Core

Owing to the difficulty of separating the protective cardboard face from the polyurethane core without damaging the core the composite panels were assembled with the plywood facings glued to the cardboard. It was obviously necessary to attempt to get values for the elastic moduli of the cardboard. The cardboard was carefully peeled from the polyurethane and made into tensile specimens, similar in dimensions to the plywood specimens, which were tested in the 'E' Type tensometer. The values of E obtained were of the order of 10^7 N/m^2 which meant that the cardboard had a negligible effect on the theoretical flexural stiffness.

2.8. Polyurethane Core

Tensile and shear tests were made on the core to establish whether the core contributed to the bending stiffness of the panel and to find the shear modulus.

Tension

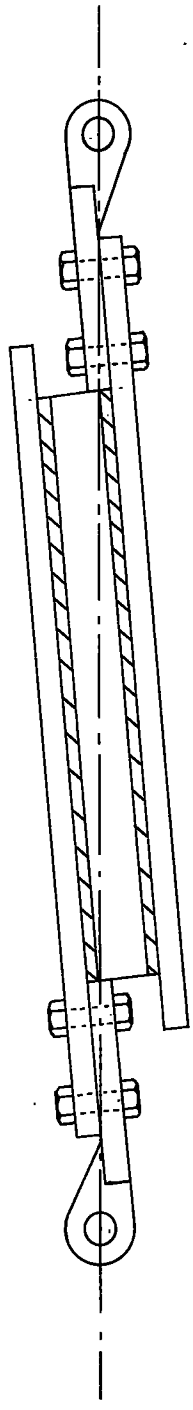
The 'E' Type tensometer was used to test the core in simple tension (Fig. 5). The cardboard faces were carefully stripped from the polyurethane core material and specimens 5 cm sq. and 1 in. nominal thickness were made. Araldite was used to glue the specimens to flat plattens which could be attached to the 'E' type tensometer. The load was applied at a constant strain rate of 1.5 mm/min. Values for E_c obtained for all three directions were of the order of 10^7 N/m^2. The core can be considered not to make any significant contribution to the bending stiffness.

Shear

Shear tests on the core were done using the 'E' type tensometer with special attachments. The equipment is similar to that used in the ASTM method (Fig. 4). The specimens were 1 in. nominal thickness, 6 in. long and 2 in. wide. The polyurethane was stripped of its cardboard faces and bonded to $^1/_8$ in. thick plywood which was screwed to the loading plates as shown. Deformations were measured on the automatic recorder and the specimens were tested to destruction.

FIG. 4.

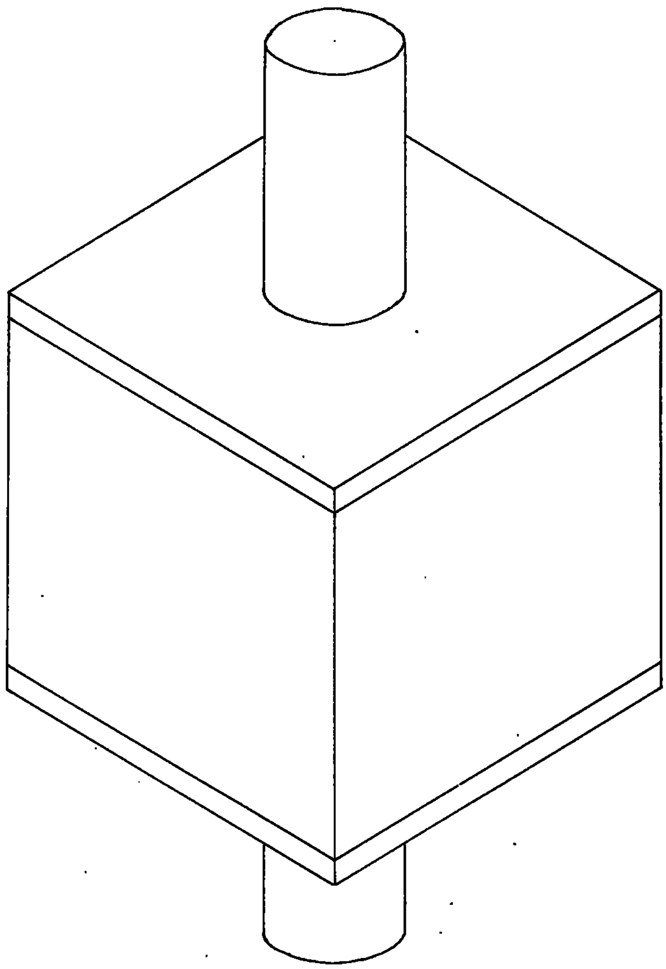
Used in 'E' Type



SHEAR SPECIMEN

FIG. 5.

U s e d i n 'E' T y p e



T E N S I L E S P E C I M E N (p o l y u r e t h a n e)

The results of the shear tests showed that the polyurethane core had an average value of 2.0×10^6 N/m^2.

Torsion

Torsion tests were carried out on the core for various widths and thicknesses. The apparatus is described in (4.7.) and the results summarised in Table 4.2.

CHAPTER III

3.1. Behaviour of Panels in Bending

Simple elastic theory is used in an attempt to predict the bending stiffness of a sandwich beam constructed as described in Chapter II.

3.2. Assumptions made in the theory of elastic Sandwich Panels

1. The core is assumed to be homogeneous.
2. The core and faces are assumed to be elastic and isotropic.
3. The elastic modulus of the core in the plane of the plate are assumed to be zero.
4. Plane sections are assumed to remain plane after bending (in pure bending only).

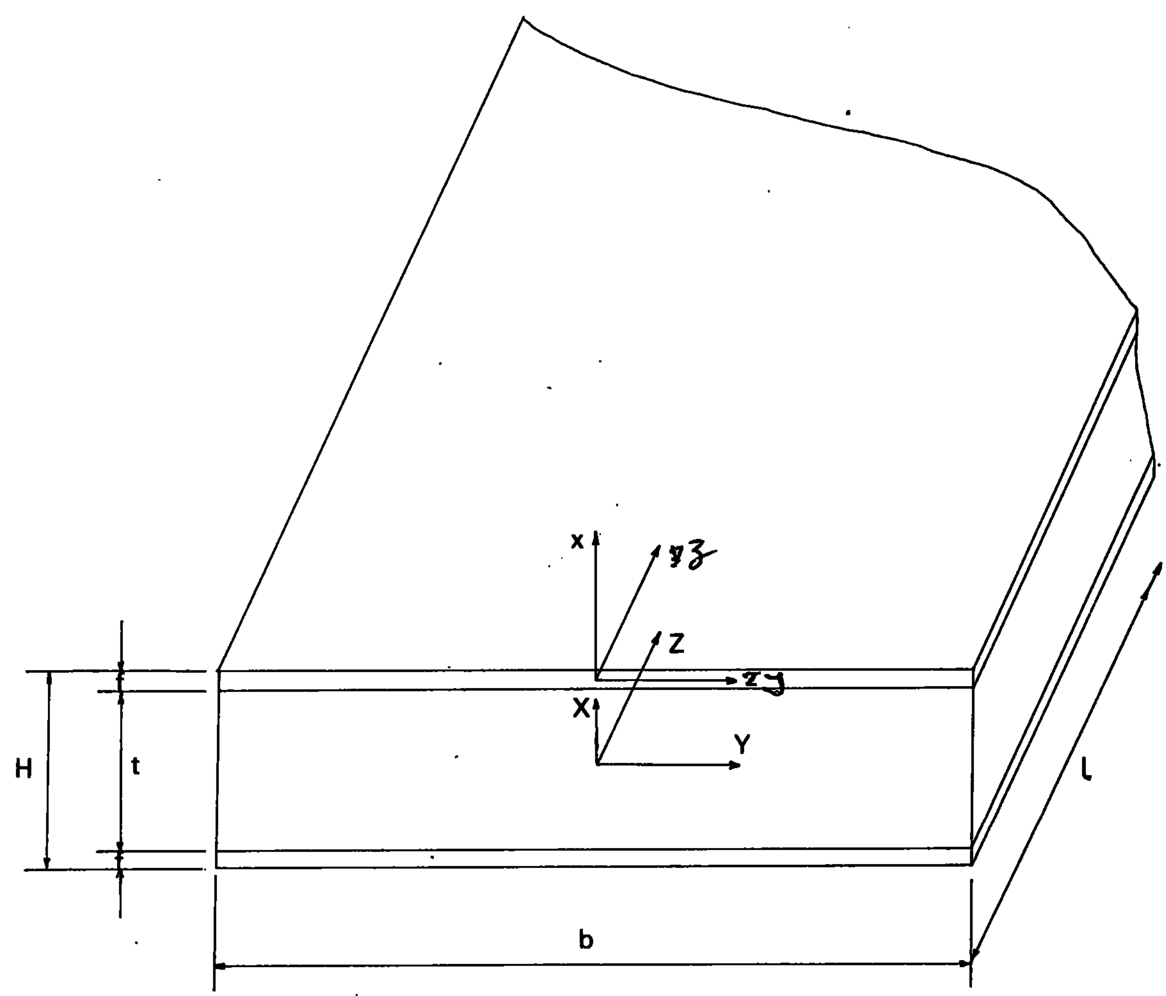
It will be shown subsequently that the flexural stiffness of the faces about their own middle surface is negligible. Most of the strain energy in the faces of a deformed panel is extensional and the strain may be assumed constant across the thickness of the face. The flexural strain energy is negligible if the face thickness is small compared to the core, i.e. less than 1 to 10

3.3. Simple Theory

Dimensions and coordinates are shown in Fig. 6.

From the previous assumptions (Ref. 3.2.) the shear strains in the faces can be neglected but the shear strain in the core cannot be neglected because the shear modulus of the core is so small.

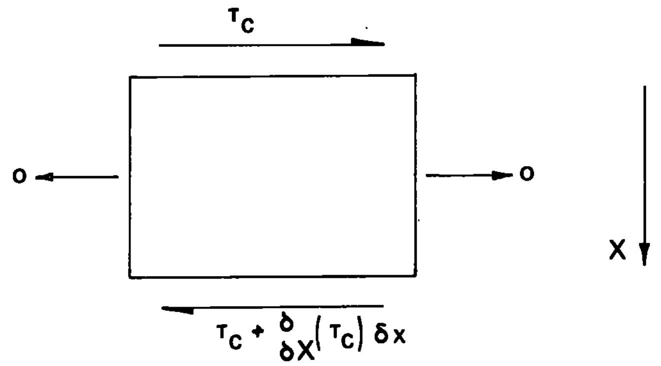
FIG. 6.



DIMENSIONS AND COORDINATES

Considering equilibrium in the core in the X direction.

$(\sigma_Z)_c = 0$



For equilibrium $\frac{\delta}{\delta X}(\tau_c) = 0$

τ_c is independent of X at any position of Z or Y.

Effects of Positive BM and SF

The displacements of a simply supported beam due to bending and shear can be separated conceptually into u_b and u_s.

Consider first u_b:

From geometrical considerations

$$\frac{1}{R} = \frac{d^2 u_b}{dZ^2}$$

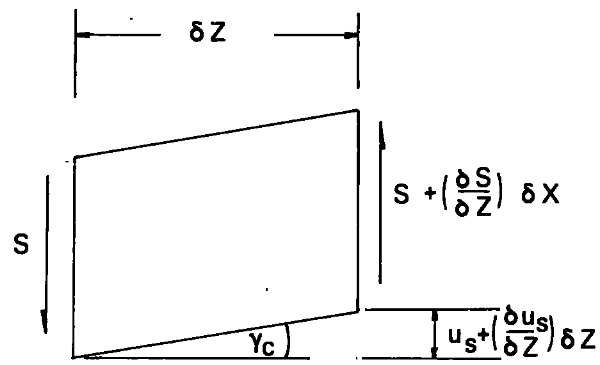
And for elastic faces $M = \frac{E_f}{R} I_f$

Where $I_f = 2\left(\frac{b f^3}{12} + \left(\frac{t + f}{2}\right)^2 fb\right)$

$$\frac{b f}{2}(t + f)^2$$

So that $M = E_f \frac{b f}{2}(t + f)^2 \frac{d^2 u_b}{dZ^2}$

Consider u_s: (ignoring shear deflection in the faces)



Shear strain $\gamma_c = \frac{\delta u_s}{\delta Z}$

Shear stress $_c = \frac{S}{bt} = G_c \cdot \gamma_c = G_c \frac{du_s}{dZ}$

Shear force $S = b.t.\ G_c \frac{du_s}{dZ}$

Now $\frac{dS}{dZ} = b.t.\ G_c \frac{d^2 u_s}{dZ^2}$

Combining deflections for both bending and shear

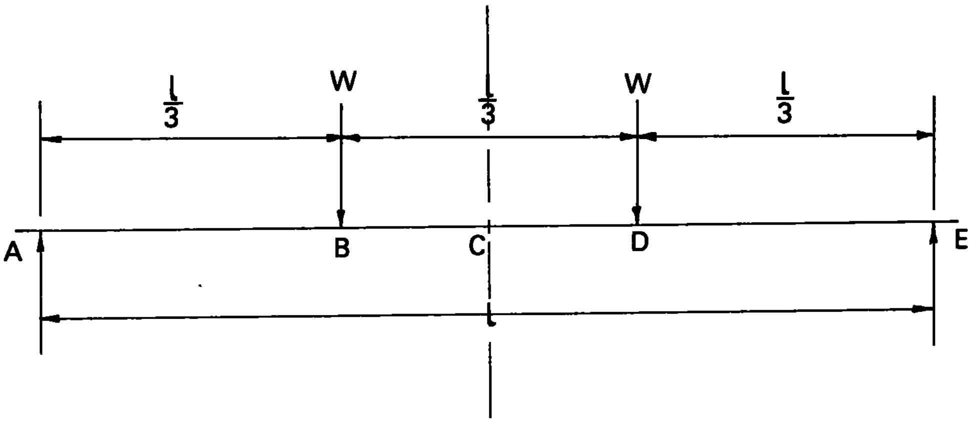
$$u_b + u_s = u$$

$$\frac{d^2 u_b}{dZ^2} + \frac{d^2 u_s}{dZ^2} = \frac{d^2 u}{dZ^2}$$

$$\frac{d^2 u}{dZ^2} = \frac{2M}{E_f.bf.\ (t+f)^2} + \frac{1}{bt.G_c} \cdot \frac{dS}{dZ} \qquad (3.1.)$$

3.4.

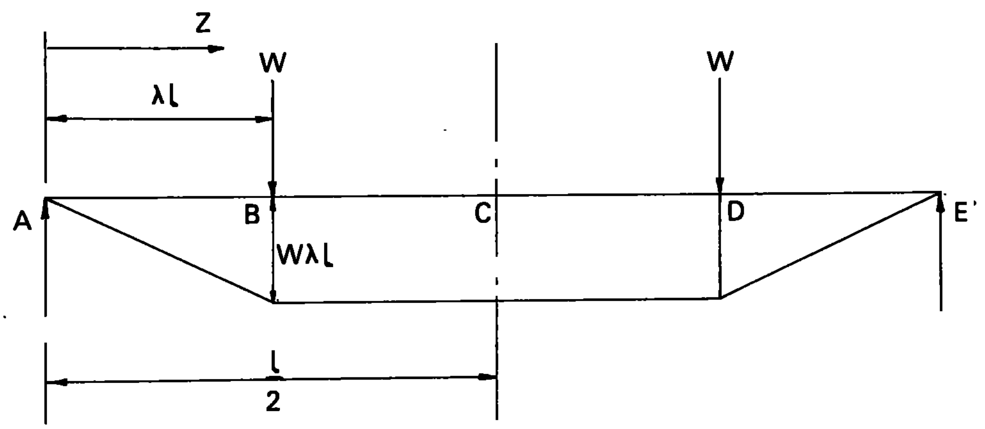
In order to attempt to isolate the separate types of deflection the sandwich panels were loaded in four point bending as shown in the diagram below.



Across the centre span of the panel between the applied loads at B and D the bending moment is constant and the shear force is zero. By measuring the central deflection of the panel relative to the points B and D the deflection due to bending only can be found. (In practice, as will be shown later, localised stress distributions, caused by the method of load application, affected the pure bending deflection and a gauge length shorter than B D was used).

By measuring total central deflection relative to points A and E the deflection due to bending and shear can be found. It is impossible to measure the shear deflection directly. The only method of obtaining a value for the shear deflection is to subtract the bending deflection from the total deflection.

3.5. General Theory for Two Point Bending



Consider B D which is subject to bending only

u_b is central deflection of C relative to B and D

$$u_b = \frac{1}{EI}\left(W\lambda l \left(\frac{1}{2}-\lambda l\right)\left(\frac{1}{2}-\lambda l\right)\frac{1}{2}\right)$$

$$u_b = \frac{1}{EI}\ \frac{W\lambda l^3}{2}\left(\frac{1}{4}+\lambda^2-\lambda\right) \qquad (3.2.)$$

Consider Complete Panel AE subject to bending and shear deflections

u is central deflection of C relative to A and E

$$\frac{d^2u}{dZ^2} = -\frac{WZ}{E_f I} + \Big|\frac{W}{E_f I}(z-\lambda l) + \frac{1}{btG_c}\frac{dS}{dZ}$$

$$\frac{du}{dZ} = = \frac{WZ^2}{2E_f I} + \Big|\frac{W}{2E_f I}(z-\lambda l)\ \frac{S}{btG_c} + A - \Big|\left(\frac{S}{btG_c}\right) \quad Z = \lambda l$$

When $Z = \frac{1}{2}$, $\quad \frac{du}{dZ} = 0$

$$o = \frac{Wl^2}{8E_fI} + \frac{Wl^2}{8E_fI}(1 - 2\lambda) + A$$

$$A = \frac{W\lambda l^2}{4E_fI}$$

$$u = \frac{WZ^3}{6E_fI} + \left| \frac{W(Z - \lambda l)^3}{6E_fI} + \frac{W\lambda l^2 Z}{4E_fI} + \frac{SZ}{btG_c} \right.$$

$$\left. - \left(\frac{S(Z - \lambda l)}{btG_c} \right) \right|_{Z = \lambda l}$$

When $z = \frac{l}{2}$

$$u = \frac{Wl^3}{48E_fI} + \frac{Wl^3}{48E_fI}(1 - 2\lambda)^3 + \frac{W\lambda l^3}{8E_fI} + \frac{Sl}{2btG_c}$$

$$- \frac{Sl}{2btG_c}(1 - 2\lambda)$$

$$u = \frac{Wl^3}{48E_fI} - 1 + (1 - 2\lambda)^3 + 6\lambda + \frac{Sl\lambda}{btG_c}$$

$$u = \frac{Wl^3}{12E_fI}\lambda^2(2\lambda + 3) + \frac{Sl\lambda}{btG_c} \qquad (3.3.)$$

Using the very simple theory above with values E_f and G_c obtained from tests described in Chapter 2, deflections of panels can be predicted and compared with experimental deflections measured on composite panels.

3.6. Experimental Procedure

Simple four point bending tests were carried out on panels with three different core thicknesses, nominal 1 in., 0.75 in. and 0.5 in. (actually 0.96 in. = 24.4 mm, 0.74 in. = 18.8 mm, 0.45 in. = 11.4 mm).

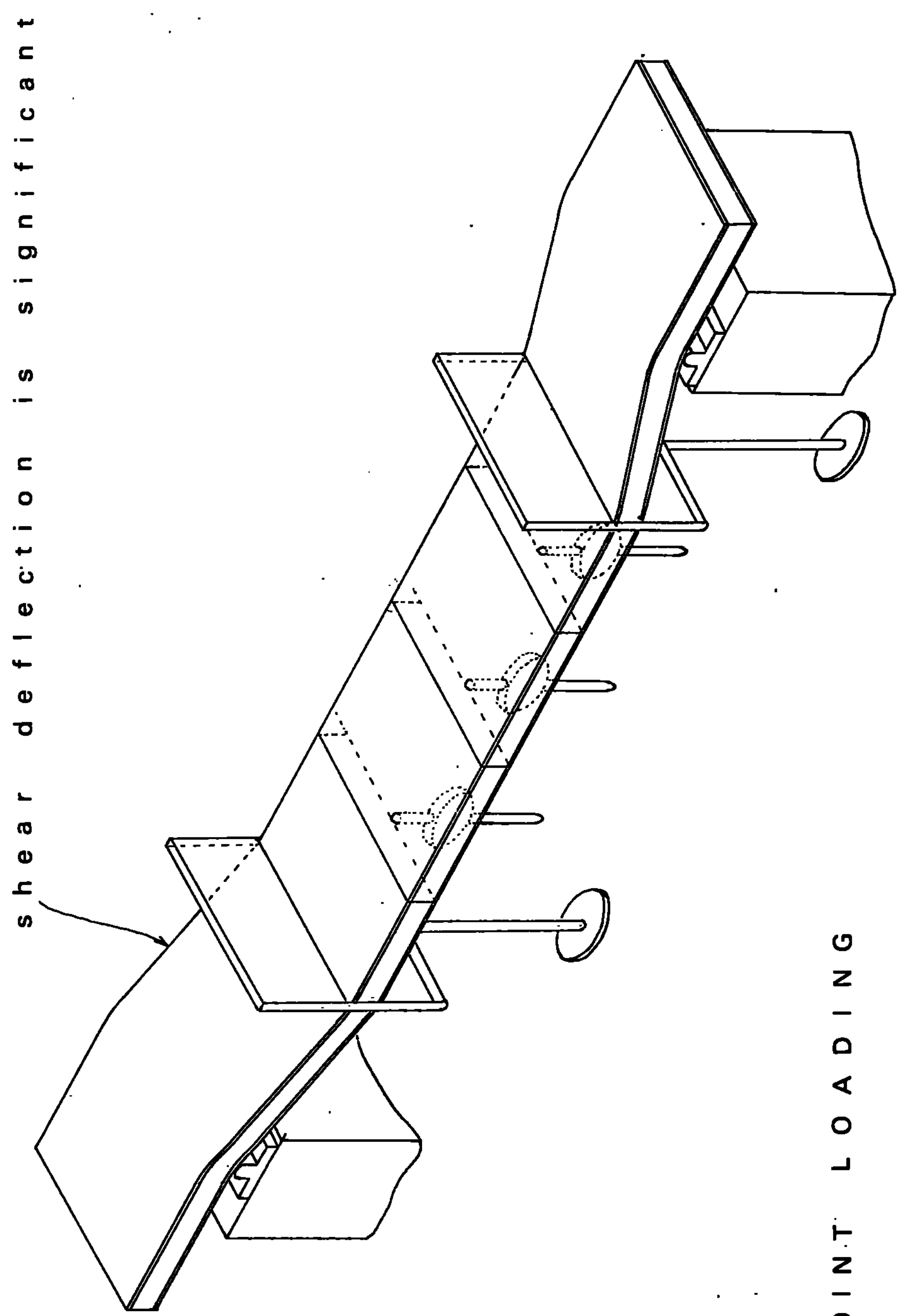
Most of the panels tested were 6 in. wide (15.2 mm) and the span between supports was kept constant. However tests were made on panels of constant depth but varying width to test for anticlastic effects. Also tests were made for varying spans but with the distance between the load hangers kept constant.

When completed the beams were simply supported as in Fig. 7 and loaded by the addition of weights to the two hangers. In order to obtain uniformity of load across the width of the beam the hangers were constructed so that they were very stiff in the direction of their length (i.e. across the width of the panel). It was thought later that the stiff hangers might have distributed the load less evenly than expected because of their tendency to inhibit natural anticlastic curvature, but little or no anticlastic bending was observed in the panels, so that this cannot have been significant.

The knife edges of the load hangers were made from wood and were semi circular in shape so that the outer faces of the panels were not damaged when the loads were applied.

Tests were carried out initially to see if local deformations occurred under the hanger loads. No visible deformation could be seen and no relative movement between the faces could be detected.

FIG. 7.



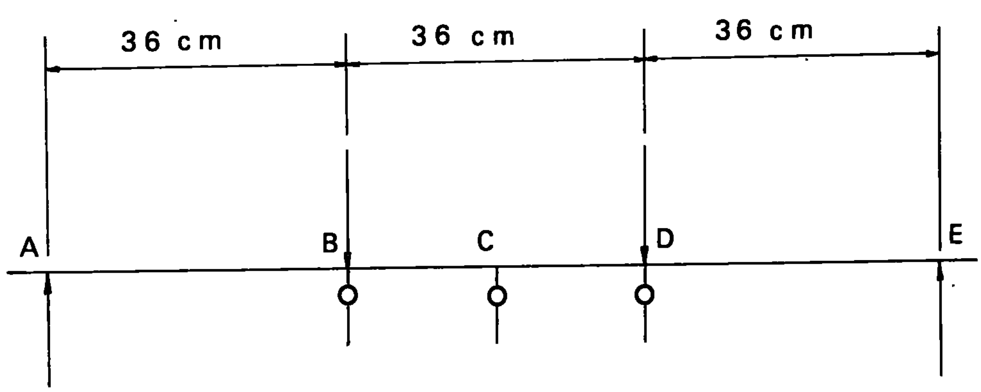
TWO POINT LOADING

Attempts were made to check the reduction in core thickness during the bending test by means of dial gauges on both the top and bottom faces of the panel. The gauges were located as closely as possible to the points where the loads were applied.

For each increment of load readings were taken of the vertical displacement of the centre of the lower skin of each beam and also of the displacement under the hanger loads which were at the third points of the span of the beam. Dial gauges were used to measure the displacement.

All bending deflections and strains were measured within the elastic limit of the component materials of the panels. Tests were made to find the maximum load which could be applied to the panels before they started to creep significantly. No attempt has been made to analyse the non-linear behaviour of the panels under creep conditions.

Initial Arrangement



Hanger loads at third points.

Dial gauges at centre and under hanger loads.

Using hanger loads applied in positions as shown above central deflection readings relative to points B and D were plotted against hanger load for a nominal 1" deep beam 6" wide.

From the simple theory for the pure bending section between B and D the central deflection was calculated to be 0.507×10^{-5} m/N relative to B and D.

Several experimental beams were loaded and graphs were plotted of central deflection against hanger load. The mean value of central deflection relative to B and D for unit hanger load was

$$0.71 \times 10^{-5} \text{ m/N}$$

Theoretical and experimental values were also obtained for total central deflection in bending and shear for the central deflection at C relative to the supports.

The values are as follows:

$$\text{Theoretical } u = 8.54 \times 10^{-5} \text{ m/N}$$

$$\text{Experimental } u = 7.37 \times 10^{-5} \text{ m/N}$$

The difference between theoretical and experimental values for deflection is significant in the pure bending case. However the difference is within the limits of experimental error for the bending and shear case and also the theoretical value is greater than the experimental value.

At this point beams were tested with similar dimensions and loading arrangements to the one above except that electrical resistance strain gauges were used to record the strain in the outer fibres of the plywood faces. It was hoped that the experimental and theoretical strains would be related to show that the increased deflection at C in bending only was due to localised shear strains near to hanger loads.

Graphs were plotted of Micro-strain against hanger load for both 1" and $^3/_4$" deep panels.

Using the simple theory previously described and discounting the anticlastic effect, values of strain at the centre span per unit hanger load were calculated.

$$\text{Strain} = \frac{Mx}{E_f I_f} \qquad x = \frac{t}{2} + f$$

Strain at Centre Span Per Unit Hanger Load

Nominal Panel Thickness	Theoretical	Experimental
1"	$3.85 \times 10^{-6}/N$	$3.70 \times 10^{-6}/N$
$^3/_4$"	$4.87 \times 10^{-6}/N$	$4.84 \times 10^{-6}/N$

The fibre strains are assumed to be due to bending only since the shear strains in the face are negligible compared with those in the core. Since the fibre strains agree the extra deflection in the beam section subject to bending only must be a shear deflection in the core caused by local shear stress distributions due to the fact that the bending moment was not applied in a pure form.

The loading arrangement was changed and the weight hangers were both moved 6 inches further away from the centre of the beam. Under the new system of loading the central deflection was still measured relative to the third points on the beam which were no longer in the regions affected by localised stress distributions from the weight hangers.

3.7. Simply Supported Panels in Bending Only

The comparison of results was much more favourable with the new hanger positions.

Theoretical central deflection per unit hanger load

$$0.292 \times 10^{-5} \text{ m/N}$$

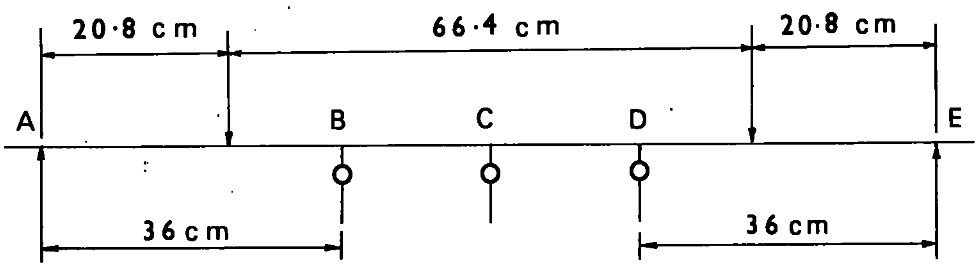
Mean experimental deflection per unit hanger load

$$0.273 \times 10^{-5} \text{ m/N}$$

These results show that for a nominal 1" thick beam the experimental deflection obtained differed from the predicted theoretical deflection by less than 10%

Panels of $^3/_4$" and $\frac{1}{2}$" nominal thickness but with the same length and breadth were tested under the new loading system shown below.

Revised Loading System



Comparison of Central Deflection per Unit Hanger Load relative to Points B and D

Panel Thickness (Nom.)	Theoretical Deflection	Experimental Deflection
1"	0.292×10^{-5} m/N	0.273×10^{-5} m/N
$^3/_4$"	0.473×10^{-5} m/N	0.434×10^{-5} m/N
$\frac{1}{2}$"	0.975×10^{-5} m/N	0.797×10^{-5} m/N

In all three cases the experimental deflection is less than the theoretical result. The simple theory for panels subject to bending only obviously under-estimates the stiffness of the panel the thinner the panel the greater is the discrepency.

Using equations derived by H.W. March and C.B. Smith a closer approximation to the central deflection in pure bending can be achieved.

March and Smith set up stress functions in the faces and core of a sandwich panel and adjusted them so that the proper conditions at the junctions of the facings and core were justified. This theory assumes that the core is constraining the face not to deform laterally.

$$u_b = \frac{Ml^2}{72D} \qquad (3.4.)$$

For four point loading system

$$M = 20.8 \times 10^{-2}\ W\ Nm$$

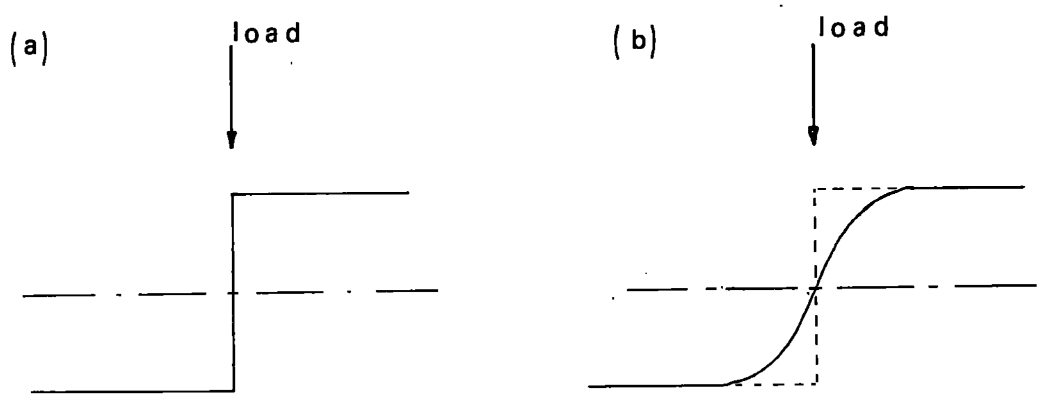
$$D = \frac{E_f\, f\, (t + f)^2}{2\gamma_f}$$

Where $\gamma_f = (1 - \mu_f)$ μ_f denotes Poisson's Ratio for the facings

Panel Thickness (Nom.)	Theoretical Deflection (March)	Experimental Deflection
1"	0.272×10^{-5} m/N	0.273×10^{-5} m/N
$^3/_4$"	0.447×10^{-5} m/N	0.434×10^{-5} m/N
$\frac{1}{2}$"	0.904×10^{-5} m/N	0.797×10^{-5} m/N

It was thought that the extra stiffness of the panels was due to factors initially ignored in making the working assumptions. In fact the assumptions made about the face stiffness, the bending stiffness of the core and the bending stiffness of the cardboard covering still hold true.

However the face stiffness does affect shear stress distribution in the core under concentrated loads such as the line loads used in this case. In theory the increase in shear stress in the core occurs as shown in diagram (a). In practice the shear stress transmitted by the faces, under the concentrated load, causes an increase in shear stress as shown in (b) with a corresponding finite curve profile under the load. This causes an increase in stiffness which is more apparent in the thinner beams.



shear stress profiles

The only factor which cannot be determined is the effect of the glue which may increase the effective face thickness thus increasing the 2nd moment of area of the faces about the centre of the core.

One inch thick panels were tested for constant span but varying width. No signficant difference in stiffness per unit width could be detected from the results for widths varying between 75 mm and 200 mm.

3.8. Simply Supported Panels in Bending Including the Effects of Shear

The same loading arrangement was used as for the panel subject to bending moment only. The central deflection was taken as the total central deflection relative to the supports.

From the simple theory:

$$u = \frac{Wl^3\lambda^2}{12E_fI}(2\lambda + 3) + \frac{Sl\lambda}{btG_c}$$

Comparing experimental and theoretical values for central deflection per unit hanger load it was again apparent that the simple theory underestimates the flexural strength of the panel. Although not to such a great extent as in the bending only.

Panel Thickness (Nom.)	Theoretical Deflection	Experimental Deflection
1"	3.93×10^{-5} m/N	3.7×10^{-5} m/N
3/4"	5.49×10^{-5} m/N	5.47×10^{-5} m/N
½"	10.10×10^{-5} m/N	9.25×10^{-5} m/N

C.B. Norris (U.S. Forest Prod. Lab.) produced formulae for the shear strength of a beam by setting up stress functions in the facings and core and adjusting them to yield proper values at the boundaries. The method was used for a centrally loaded beam assuming hinges, at the centre of span, in the faces.

The formula obtained for the shear stress in the core of the sandwich construction with equal facings was:

$$\tau = \frac{P}{2b} \frac{f^2 + tf + \frac{\rho t^2}{4}}{\frac{4f^3}{3} + 2f^2t + ft^2 + \frac{\rho t^2}{6}}$$

Where $\rho = \frac{E_c (1 - \mu_{ab} \mu_{ba})}{E_f (1 - \mu_{ab} \mu_{ba})}$ and P = central load

By assuming ρ so small that it can be neglected and also the faces are sufficiently thin that $\frac{4}{3} f^3$ may be replaced by f^3 then

$$\tau = \frac{P}{b(H + t)} \qquad (3.6.)$$

Where H is the total thickness of the sandwich

Norris found that the above approximations are quite satisfactory for most sandwich constructions and may be used when the facings are unequal. It also may be used for other types of loading with reasonable accuracy.

Comparison of Theoretical and Experimental Deflections using approximate shear equation by Norris and Bending Stiffness by March.

Panel Thickness	Theoretical Deflection	Experimental Deflection
1"	3.67×10^{-5} m/N	3.7×10^{-5} m/N
3/4"	5.49×10^{-5} m/N	5.47×10^{-5} m/N
½"	10.15×10^{-5} m/N	9.24×10^{-5} m/N

3.9. Discussion

The results of the calculations using the simple expressions (3.2) and (3.3) indicate that the original assumptions made in the bending theory are reasonable. It appears to be hardly worth using the fuller analysis of Norris and March.

It will be noted that the shear modulus of the faces does not affect the result of the bending calculations. The Poisson's ratio used in the fuller analysis was found from an orthotropic tensile test the axis of which is the same as the longitudinal axis of the beams.

CHAPTER IV

PANELS LOADED AS COLUMNS

4.1. Sandwich panels loaded as columns may fail in one of three ways:

4.1.1. (a) The faces under compression may become unstable if not sufficiently supported by the core. If the face is not perfectly flat, the amplitude of the irregularities will grow as the load is applied thus subjecting the core to tensile and shear stresses. These stresses could cause failure of the core before failure would occur in the faces. If this occurs the core is not adequate for the purpose and a different core material should be chosen.

(b) Due to effects similar to those described in (a) local buckling in the faces may become of sufficient amplitude to cause local buckling failure to occur. It is difficult to separate these two effects but they are different, in the sense that (b) effects are due to faces of too small a local stiffness, due to either (i) being too thin or (ii) of a material which is not of high enough modulus, or (iii) a thin material which is not flat enough initially. No general investigation of these phenomena is undertaken here. The purpose of this brief study was restricted to an enquiry into whether the panels used in this project would display local failure phenomena before they failed due to the gross effects in the following paragraphs.

4.1.2. If the column is not straight as a whole or is eccentrically loaded it deflects as soon as load is applied and the deflection increases as the load increases. The faces are subjected to axial stresses due to bending as well as axial stresses due to end-load and the panel will fail in a way accepted as normal for a strut.

4.1.3. Shear stress in the core of a column of this type may also cause failure. The shear stresses are induced by the deflections and the load increase. The transverse shear load is the load on the column multiplied by the slope of the curve that the column assumes under the load.

In the experiments with sandwich panels described later the failure was of the third type, i.e. shear failure of the core. This usually occurred simultaneously with the failure of the cardboard interface between the core and the faces.

4.2. Theory

Normal methods have been used to produce an Euler curve for the columns tested. The experimental curve is compared with a modified theoretical curve. The critical load computed in the normal fashion has been modified to include shear.

It has been assumed that the deflected shape of the strut can be expressed in the form

$$y = a \sin \frac{\pi x}{l}$$

Then $$\frac{dy}{dx} = \frac{a\pi}{l} \cos \frac{\pi x}{l}$$

Now $$\frac{dy}{dx}\left(1 - \frac{P}{G_c A_s}\right) = \frac{P}{EI} \frac{al}{\pi} \cos \frac{\pi x}{l}$$

$$1 - \frac{P}{G_c A_s} = \frac{P}{EI} \frac{l^2}{\pi^2}$$

$$P = \frac{1}{\frac{l^2}{\pi^2 EI} + \frac{1}{G_c A_s}}$$

A_s = bt (Assuming core only takes shear)

A_s = b (t + f) (Obtained by Norris as in bending case)

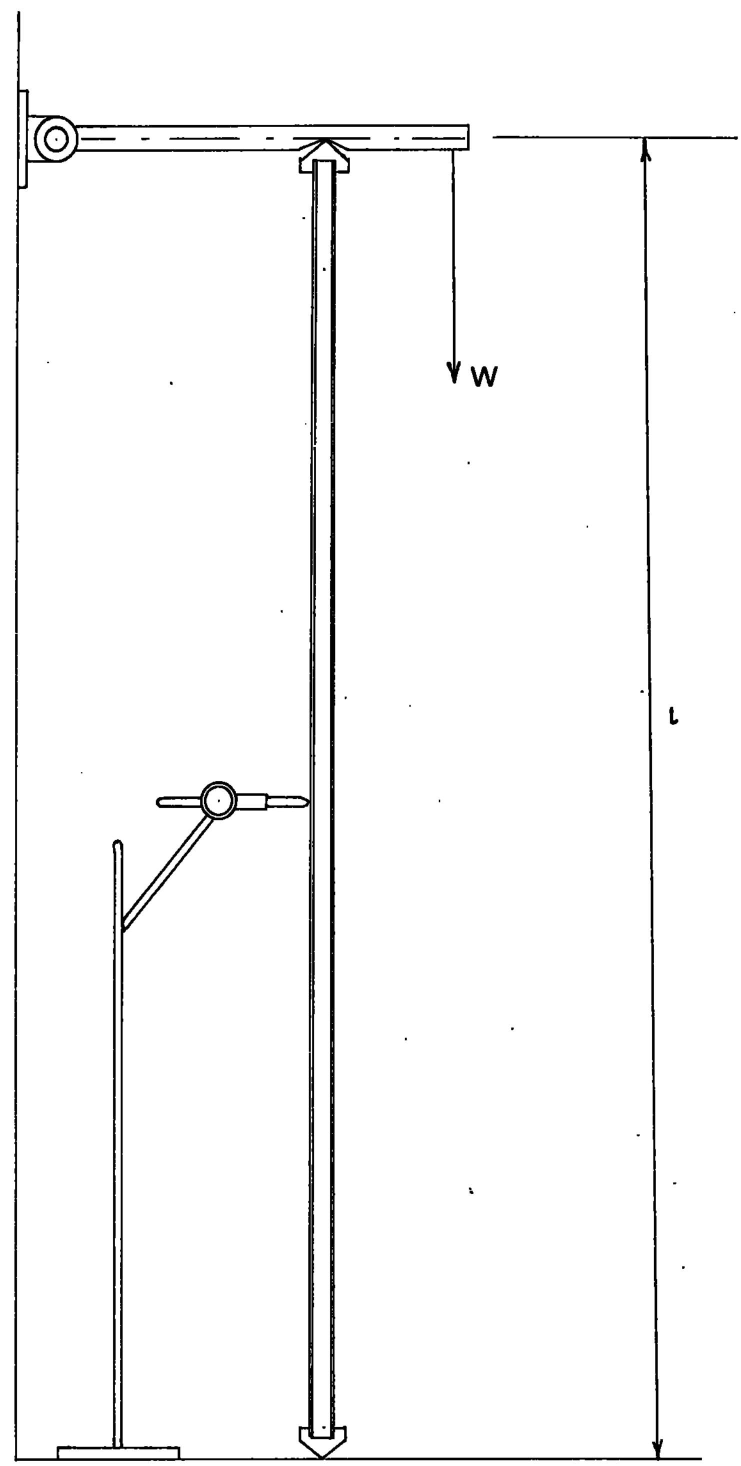
4.3. Panels Loaded as Columns

Experimental Results

Sandwich panels were loaded as columns using apparatus as shown in Fig. 8. It was necessary to use a lever arm in order to obtain loads large enough to cause the columns to buckle. No attempt was made to cause failure by local instability of the faces i.e. only long columns were tested.

The maximum length of the columns was limited to the size of the plywood sheets from which the panels were made. Nominal 1" and 3/4" thick panels did not behave as long columns with the maximum length available so that only ½" thick panels were tested.

FIG. 8.



STRUT LOADING MECHANISM

The $\frac{1}{2}$" thick panels were cut into column strips 1" wide and tested to failure for several different lengths. In every case the grain of the outside plywood laminates ran parallel to the length of the column. All the columns tested had simulated pin jointed ends.

Hardwood blocks with vee shaped bases were used to seat the ends of the column to simulate the pin joint. The lever arm had notches into which the top of the column was located while the bottom of the column rested on a flat horizontal surface.

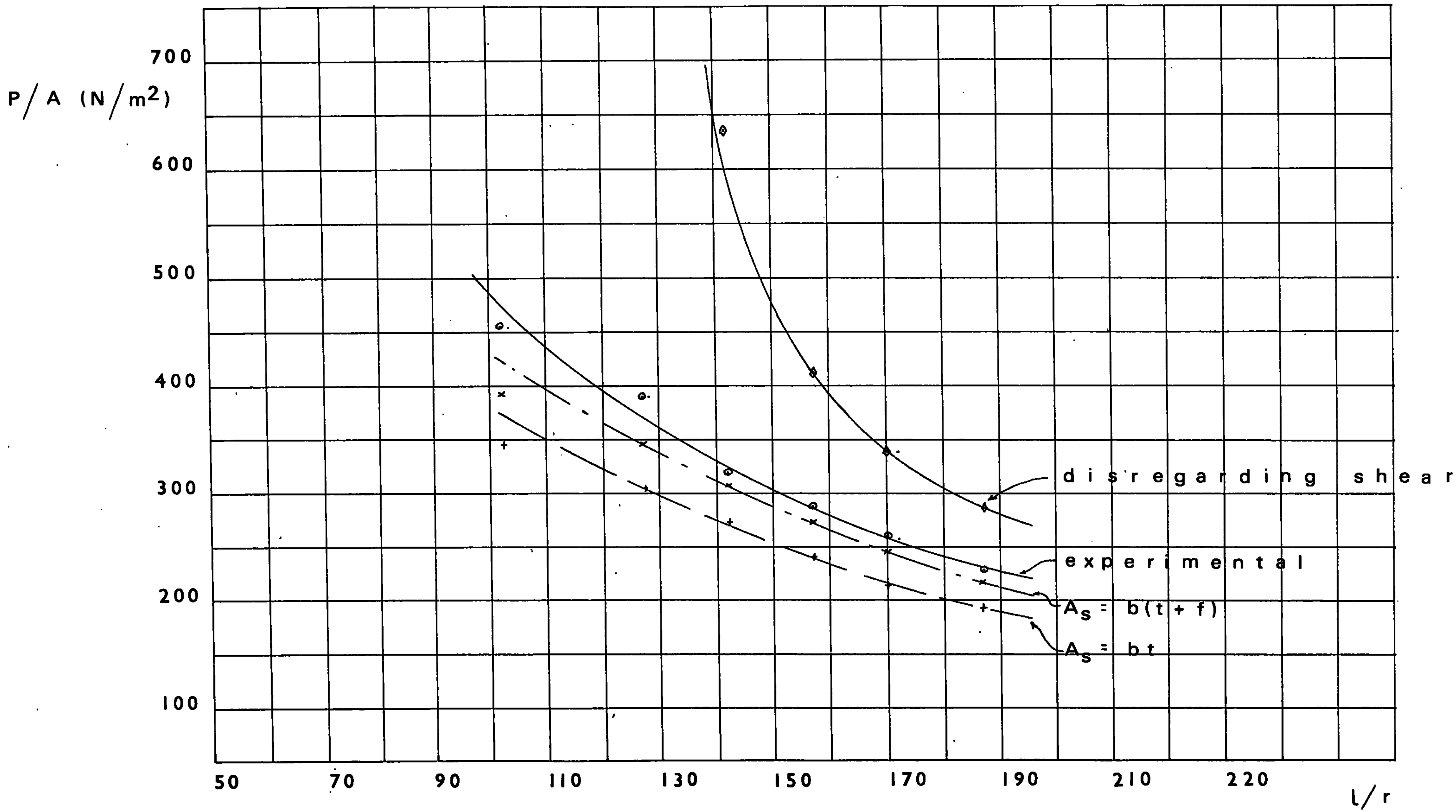
The load was applied to the column by means of hanger weights suspended from the lever arm. Care was taken to see that the column was vertical and that the load was applied symmetrically. The central deflection of the column was measured by means of a dial gauge which was zeroed after the column was set up with the self weight of the lever arm in position.

An attempt has been made to show that the sandwich columns act in a similar way to that predicted by Euler except that a correction must be made for induced shear deflections. A comparison of experimental and theoretical Euler curves is shown in Fig. 9.

Discussion

It can be seen from the Euler curve that the simple theory using A_s = bt under-estimates the strength of a column and the expression derived by Norris gives a closer approximation to the experimental results. The fact that the experimental results are higher than the Norris results may be due to the pin jointed ends not acting properly as pin joints thus giving extra stiffness to the column.

FIG. 9.



EULER CURVE FOR SANDWICH STRUT

It can be seen that if the shear is neglected the stiffness of the beam is grossly inaccurate.

The shear modulus for the faces is again not used as in the simple bending case.

Face wrinkling will not occur with the materials and geometric shapes used in these panels. A description of face wrinkling criteria is given in Plantema, Chapter 2 (II).

CHAPTER V

5.1. Torsion

A strain energy method is used in an attempt to predict the torsional stiffness of a sandwich beam constructed as described in Chapter II. The method is based on the well known theory of torsion for prismatical bars, due to Saint Venant. This theory shows that in bars of non-circular cross-section, warping of the cross-sections plays a dominant part in determining the stresses and the torsional stiffness.

5.2. Geometry of deformation and assumptions

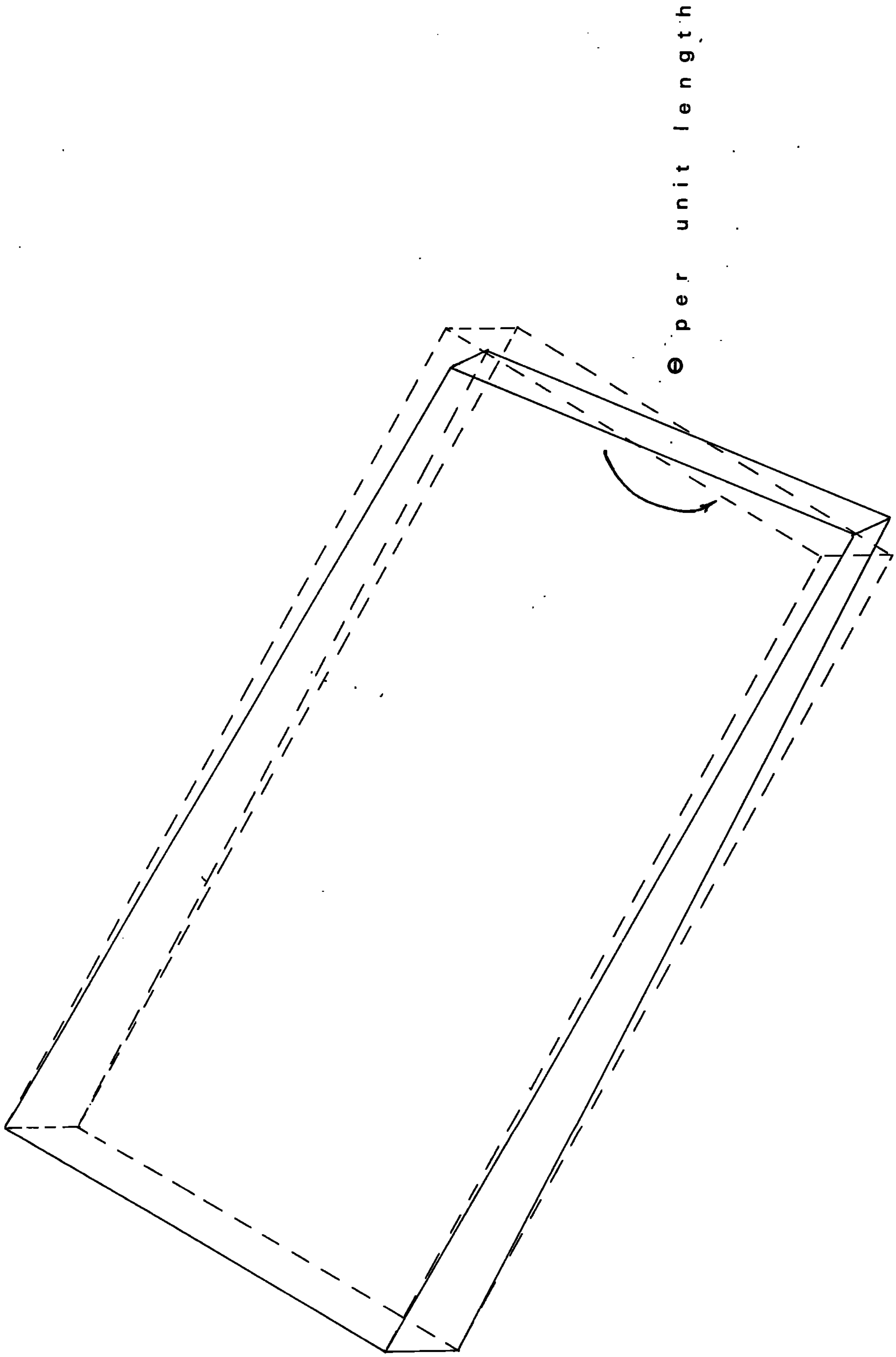
The co-ordinates and dimensions of the beam and the general form of the deformation are shown in Figs. (10) and (11).

It will be assumed, as in the earlier chapters, that the elastic stiffness of the material of the faces is very much greater than that of the core material. In addition the following assumptions will be made (after the classical theory of torsion).

1. The length of the beam is large compared with its other dimensions.
2. The direct stresses on the XYZ planes are zero.

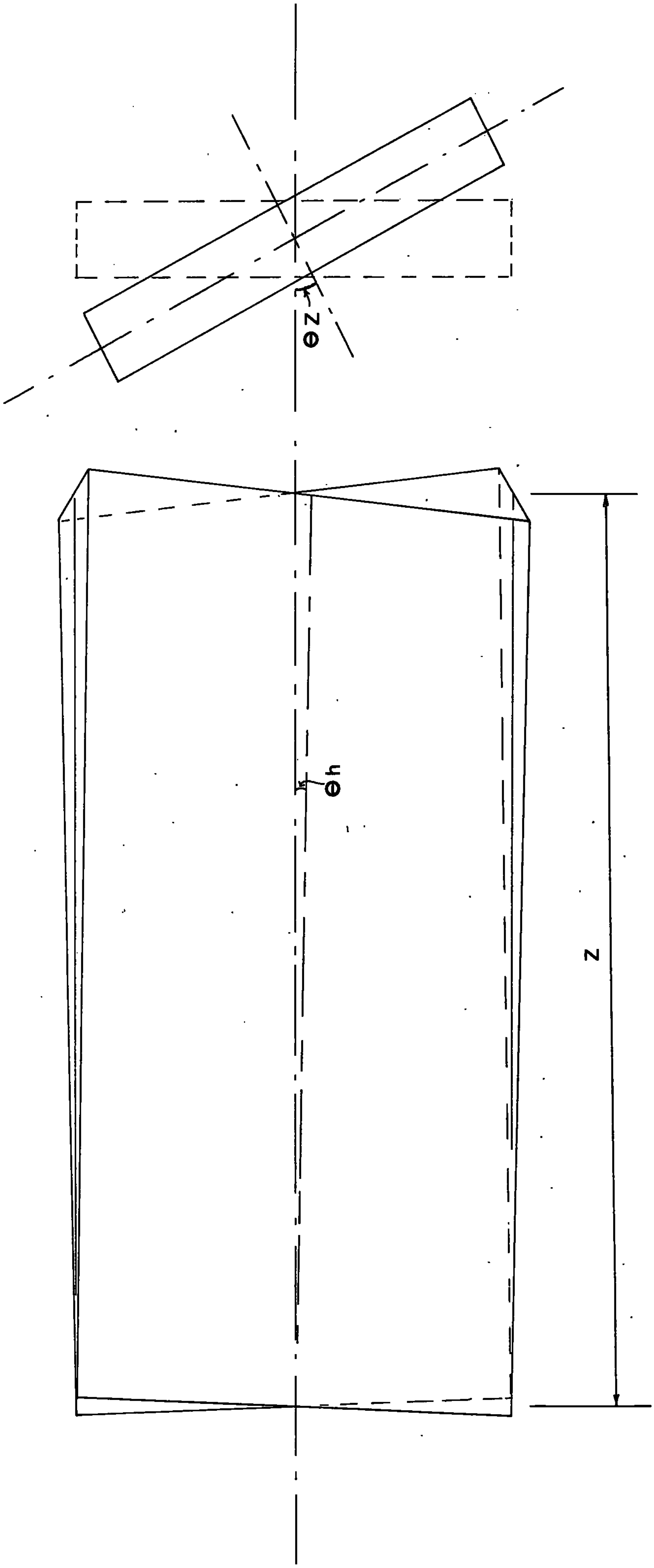
A consequence of (1) is that the differential coefficients with respect to Z of all the strain components are zero (i.e. strain does not vary with depth). Also it follows from (2) that, if the beam is Hookean throughout (although not, of course homogeneous), the direct strains ε_X, ε_Y and ε_Z are everywhere zero. Therefore the six conditions of compatability[8] reduce to two:-

FIG. 10.



DEFORMATION TORSION

FIG. 11.



RIGID ROTATION OF FACES

$$0 = \frac{\partial}{\partial X}\left[-\frac{\partial}{\partial X}\gamma_{YZ} + \frac{\partial}{\partial Y}\gamma_{ZX}\right]$$

$$0 = \frac{\partial}{\partial Y}\left[-\frac{\partial}{\partial Y}\gamma_{ZX} + \frac{\partial}{\partial X}\gamma_{YZ}\right]$$

The expression in brackets is therefore constant throughout the beam. (Strictly, the interface between core and faces should be thought of as a thin layer of transition from the properties of one material to those of the other. In this way the strain components become continuous and differentiable through the whole cross-section).

It can be shown that the expression in brackets is:

$$2\frac{\partial}{\partial Z}\Omega_Z$$

where Ω_Z is the rotation of any line in the beam about the Z axis. The fact that it is constant means that cross-sections are subject to undeformed rotation. The general nature of the deformation is therefore largely determined. The displacements in the XY planes are:

$$u = -\theta ZY, \quad v = \theta ZX$$

It remains only to find the axial displacement w.

5.3. Crude Analysis

In that the shear stiffness of the faces is very much greater than that of the core, it is worth working out the stiffness of the assembly on the assumption that the faces deform just as they would if the core were absent, and that this deformation is imposed upon the core.

The solution of the torsion problem for a homogeneous beam of rectangular cross-section is well known. For the thin plate, whose cross-section is a long thin rectangle the solution is particularly simple and is given to a good approximation by the stress function:

$$\phi = G \theta (a^2 - x^2)$$

where the thickness of the plate is 2a

The Torque is

$$T_f = \frac{G_f \theta f^3 b}{3}$$

and the axial displacement (the warping of the cross-sections) is

$$w = \theta xy$$

When such a plate is one of the faces of a sandwich beam, it will be twisted not about its own central axis but about the axis of the whole assembly. So in addition to its deformation it will experience a rigid body rotation, Fig. (11). The resulting axial displacement of points on the centre plane of the face will be:

$- \theta hY$ for +ve X (upper face)

$+ \theta hY$ for -ve X (lower face)

Consider now the whole assembly.

The displacements are:

$$u = - \theta ZY, \quad v = \theta ZX$$

in the faces and in the core.

In the upper face

$$w = - \theta hY + \theta xy$$

the first term arising from the rigid rotation, the second from the warping. At the interfaces between the faces and the core ($x = -a$ at the upper face, $x = +a$ at the lower face) the values of w are:

$$w = \mp \left(f + \frac{t}{2}\right) \theta Y$$

If w is assumed to be linearly distributed through the thickness of the core, we obtain for w in the core:

$$2 = - \left(\frac{2f}{t} + 1\right) \theta Y X \qquad (5.1.)$$

The shear strains in the core can now be calculated.

$$\gamma_{XY} = \frac{\partial w}{\partial X} + \frac{\partial u}{\partial Y} = 0$$

$$\gamma_{YZ} = \frac{\partial w}{\partial Y} + \frac{\partial v}{\partial Z} = - 2 \frac{f}{t} \theta X$$

$$\gamma_{ZX} = \frac{\partial u}{\partial Z} + \frac{\partial w}{\partial X} = - 2 \left(\frac{f}{t} + 1\right) \theta Y$$

Strictly γ_{YZ} and γ_{XY} must be zero at the free edge of the core (shear stress cannot cross an unloaded boundary). But according to the expression above γ_{XY} is zero and γ_{YZ} is very small everywhere so the expressions are not far off the mark on this point.

The torsional resistance of the core can now be found by strain energy. In the absence of direct stresses the strain energy per unit volume is:

$$V_o = \frac{G}{2} \left(\gamma_{XY}^2 + \gamma_{YZ}^2 + \gamma_{ZX}^2\right)$$

The strain energy of the core per unit length is:

$$V_c = \frac{G_c}{2}\int_{-\frac{b}{2}}^{\frac{b}{2}}\int_{-\frac{t}{2}}^{\frac{t}{2}} \left(\gamma_{YZ}^{\;2} + \gamma_{ZX}^{\;2}\right) dX\, dY$$

$$V_c = \frac{G_c}{2}\int_{-\frac{b}{2}}^{\frac{b}{2}}\int_{-\frac{t}{2}}^{\frac{t}{2}} \left[-2\frac{f}{t}\theta X^{\;2} + \left(-2\left(\frac{f}{t} + 1\right)\theta Y\right)^2 \right] dX\, dY$$

$$V_c = \frac{G_c\theta^2}{2}\int_{-\frac{b}{2}}^{\frac{b}{2}}\int_{-\frac{t}{2}}^{\frac{t}{2}} \left(4\, F\, X^2 + Y^2\, (2 + 2F)^2\right) dX\, dY$$

$$V_c = \frac{G_c\theta^2}{6}\, bt \left(t^2F^2 + b^2\,(1 + F)^2\right)$$

where $F = \frac{f}{t}$

The contribution of the core to the torque is therefore

$$T_c = \frac{2\, V_c}{\theta} = \frac{G_c\,\theta\, b^3 t}{3}\left[(1 + F)^2 + \frac{F^2}{B^2}\right]$$

where $B = \frac{b}{t}$

So the torsional stiffness of the whole sandwich beam is

$$\frac{T}{\theta} = \frac{G_c t\, b^3}{3}\left[(1 + F)^2 + \frac{F^2}{B^2} + 2\frac{G_f}{G_c}\frac{F^3}{B^2}\right] \qquad (5.2.)$$

Orders of magnitude of terms in (2)

$$F \sim 10^{-1} \qquad B \sim 1 - 10$$

$$\text{So} \quad \frac{F^2}{B^2} \sim 10^{-3} \qquad \frac{F^3}{B^2} \sim 10^{-4}$$

Clearly the second term in the square bracket is negligible and the third term is only significant if $\frac{G_f}{G_c}$ is at least of the order of 10^3.

It is interesting to note that the third term in the bracket is the direct contribution of the faces to the torsional stiffness. For the combination of material used in this investigation, this third term is very small indeed. So while the faces have been assumed to dictate the deformation, they absorb very little of the torsional strain energy.

The general validity of this simple mechanism of deformation can be critically examined if we compare the theoretical stiffness of a sandwich and of a homogeneous rectangular beam with measured values. Expression (5.2.) can be written approximately

$$\frac{T}{\theta} \simeq \frac{G_c \, t \, b^3}{3} (1 + F)^2 \qquad (5.3.)$$

The stiffness of a homogeneous bar of rectangular cross-section is

$$\frac{T}{\theta} = k \, G \, bt^3 \qquad (5.4.)$$

where k depends on the ratio $\frac{b}{t}$ and is given numerically in Table 4.1.

Table 5.1.

$\frac{b}{2}$	1.0	1.2	1.5	2.0	2.5	3	4	5	10	
k	.141	.166	.196	.229	.249	.263	.281	.291	.312	.333

Comparing 5.3 and 5.4 it can be seen that:

(a) Whereas the stiffness of a homogeneous beam, for example the core of the sandwich beam without the faces, is proportional to the width b, the estimated stiffness of the sandwich is proportional to the width <u>cubed</u> (a result not unlike that of the incorrect Navier theory for non-circular homogeneous sections).

(b) the stiffness effect of the faces is somewhat greater than $\frac{b}{t}^{2}$.

In an experiment which will be described in more detail later, the results were as follows:

(i)	Stiffness of bare core material 24.4 mm (.96 in.) thick and 76.2 mm (3 in.) wide:	0.56 Nm/rad/m
(ii)	Stiffness of double width:	1.1 Nm/rad/m
(iii)	Stiffness of sandwich 76.2 mm wide:	8.0 Nm/rad/m
(iv)	Stiffness of double width:	61 Nm/rad/m

The ratio of (ii) to (i) is close to 2, i.e. proportional to width; and the ratio of (iv) to (iii) is quite close to 8, i.e. proportional to width cubed.

A series of experiments, described later, confirmed this result generally; but it showed that equation (5.2.) over-estimates the stiffness, significantly but not greatly, at larger values of the face to core thickness ratio F.

In an attempt to reduce the discrepancy a fuller analysis was developed on the following lines.

5.4. Fuller Analysis

In the crude analysis in-plane shear strains of the central planes of the faces were suppressed. The value of w at the X-positive interface was

$$w = -(f + \frac{t}{2})\,\Theta\, Y$$

The effect of the stiffness of the core would be to reduce this; also its own tendency to warp might affect the deformation of the faces. Those effects would be expected to lead to something like

$$w = -(\alpha + \beta \frac{|Y|}{b})(f + \frac{t}{2})\,\Theta\, Y \qquad (5.5.)$$

at the interface, where α and β are parameters yet to be determined. The crude analysis was for $\alpha = 1$ and $\beta = 0$. Considering Y as +ve from now on the w for the upper face is at the interface $x = -a$

$$w_1 = -\Theta y \,(\alpha + \frac{\beta}{b} y)(f + \frac{t}{2})$$

If the warping through the face is still Θxy, then for positive values of Y, the axial displacements will be

In the top face:

$$u = \Theta z\, y$$

$$v = \Theta z\,(h + x)$$

$$w = \Theta y \left[x + \frac{f}{2} - \frac{At}{2} (\alpha + \frac{\beta}{b} y) \right]$$

In the core:

$$u = -\theta Z Y$$

$$v = \theta Z X$$

$$w = -\theta A X Y \left(\alpha + \frac{\beta}{b} Y\right)$$

where $A = 1 + 2F$

The corresponding shear strains are for +ve Y,

In the top face:

$$\gamma_{xy} = 0$$

$$\gamma_{yz} = \theta \left[2x + \frac{At}{2} \left(1 - \alpha - \frac{2\beta}{b} y\right) \right]$$

$$\gamma_{zx} = 0$$

In the core:

$$\gamma_{XY} = 0$$

$$\gamma_{YZ} = -\theta X \left[(A\alpha - 1) + 2A \frac{\beta}{b} Y \right]$$

$$\gamma_{ZX} = -\theta Y \left[(A\alpha + 1) + A \frac{\beta}{b} Y \right]$$

These sets of strains separately satisfy the conditions of compatability.

The strain energy can now be evaluated in the same way as before. In the core it is, per unit length,

$$V_c = 2 G_c \theta^2 \int_0^{\frac{b}{2}} \int_0^c \left[\left((A\alpha - 1) + 2A \frac{\beta}{b} Y \right)^2 X^2 + \left((A\alpha + 1) + A \frac{\beta}{b} Y \right)^2 Y^2 \right] dX \, dY$$

$$V_c = 2\,G_c\theta^2 \int_o^{\frac{b}{2}} \left[\left((A\alpha - 1) + 2A\,\frac{\beta}{b}\,Y \right)^2 \frac{c^3}{3} \right.$$

$$\left. + \left((A\alpha + 1) - A\,\frac{\beta}{b}\,Y \right)^2 Y^2 c \right] dY$$

$$V_c = G_c t\theta^2 \;\frac{b^3}{24} \left[\left((A\alpha + 1)^2 + \frac{3A}{4}\,(A\alpha + 1)\,\beta + \frac{3A^2}{20}\,\beta^2 \right) \right.$$

$$\left. + \frac{1}{B^2} \left((A\alpha - 1)^2 + A\,(A\alpha - 1)\,\beta + \frac{A^2}{3}\,\beta^2 \right) \right]$$

In the faces the strain energy per unit length is

$$V_f = 2\,G_f\theta^2 \int_o^{\frac{b}{2}} \int_{-a}^{a} \left[2x + Ac\,(1 - \alpha - 2\,\frac{\beta}{b}\,y) \right]^2 dx\,dy$$

$$V_f = 2\,G_f\theta^2 \int_o^{\frac{b}{2}} \int_{-a}^{a} \left[4x^2 + 4\,Ac\,(1 - \alpha - 2\,\frac{\beta y}{b})\,x \right.$$

$$\left. + A^2c^2\,(1 - \alpha - 2\,\frac{\beta y}{b})^2 \right] dx\,dy$$

$$V_f = 2\,G_f\theta^2 \int_o^{\frac{b}{2}} 2\left[\frac{4a^3}{3} + A^2c^2a \left((1 - \alpha) - 2\,\frac{\beta}{b}\,y \right)^2 \right] dy$$

$$V_f = 2\,G_f\theta^2 \int_o^{\frac{b}{2}} 2\left[\frac{4a^3}{3} + A^2c^2a \left((1 - \alpha)^2 - 4\,(1 - \alpha) \right. \right.$$

$$\left. \left. + \frac{\beta}{b}\,y + \frac{4}{3}\,\frac{\beta^2}{b^2}\,y^3 \right) \right] dy$$

$$V_f = 2\,G_f\theta^2 \left[\frac{4a^3b}{3} + A^2c^2a \left((1-\alpha)^2 b - (1-\alpha)\,\beta b + \frac{\beta^2 b}{3} \right) \right]$$

$$V_f = G_f\theta^2 \; f \; \frac{bt^2}{4} \left[\frac{4}{3} F^2 + A^2 \left((1-\alpha)^2 - (1-\alpha)\,\beta + \frac{\beta^2}{3} \right) \right]$$

The Total Strain Energy per Unit Length is

$$V = V_c + V_f$$

$$V = G_c\theta^2 \frac{tb^3}{24} \left[(A\alpha + 1)^2 + \frac{3A}{4}(A\alpha + 1) + \frac{3A^2}{20}\beta^2 + \frac{1}{B^2} \left((A\alpha - 1)^2 + A(A\alpha - 1)\,\beta + \frac{A^2}{3}\beta^2 \right) \right]$$

$$+ \; G_f\theta^2 \; f \; \frac{bt^2}{4} \left[\frac{4}{3} F^2 + A^2 \left((1-\alpha)^2 - (1-\alpha)\,\beta + \frac{\beta^2}{3} \right) \right]$$

$$= G_c\theta^2 \frac{tb^3}{24} \left[\Big(\text{CORE} \Big) + 6\,Q \Big(\text{FACES} \Big) \right]$$

Where $Q = \dfrac{G_f}{G_c} \dfrac{F}{B^2}$

The stiffness is

$$\frac{T}{\theta} = \frac{2\,V}{\theta^2} = G_c \frac{t\,b^3}{12} \left[\Big(\text{CORE} \Big) + 6\,Q \Big(\text{FACES} \Big) \right]$$

Now the total strain energy must be a minimum with respect to the unknown parameters α and β

$$\frac{\partial V}{\partial \alpha} = \frac{\partial V}{\partial \beta} = 0$$

$$\frac{\partial V}{\partial \alpha} = 0$$

$$0 = 2A\,(A\alpha + 1) + 3\,\frac{A^2\beta}{4} + \frac{1}{B^2}\Big((2A\,(2\alpha - 1) + A^2\beta\Big)$$

$$+ 6QA\Big(-2\,(1 - \alpha) + \beta)\Big)$$

or $\quad 0 = A_1 + A_2\,\alpha + A_3\,\beta$

where $A_1 = 2\,A - \frac{2A}{B^2} - 12\;QA^2 = 2A\,(\,1 - \frac{1}{B^2} - 6QA\,)$

$$A_2 = 2\,A^2 + \frac{2A^2}{B^2} + 12\;QA^2 = 2A^2\,(\,1 + \frac{1}{B^2} + 6Q\,)$$

$$A_3 = \frac{3\,A^2}{4} + \frac{A^2}{B^2} + 6\;QA^2 = A^2\;(\,\frac{3}{4} + \frac{1}{B^2} + 6Q\,)$$

$$\frac{\partial V}{\partial \beta} = 0$$

$$0 = \frac{3A}{4}\,(A\beta + 1) + \frac{3A^2}{10}\,\beta + \frac{1}{B^2}\Big(A\,(A\alpha - 1) + \frac{2A^2}{3}\,\beta\Big)$$

$$+ 6\,Q\,A^2\Big(-(1 - \alpha) + 2\,\frac{\beta}{3}\Big)$$

or $\quad 0 = B_1 + B_2\,\alpha + B_3\,\beta$

$$\text{where } B_1 = \frac{3A}{4} - \frac{A}{B^2} - 6\,Q\,A^2 = A\left(\frac{3}{4} - \frac{1}{B^2} - 6\,QA\right)$$

$$B_2 = \frac{3A^2}{4} + \frac{A^2}{B^2} + 6\,Q\,A^2 = A^2\left(\frac{3}{4} + \frac{1}{B^2} + 6\,Q\right)$$

$$B_3 = \frac{3A^2}{10} + \frac{2A^2}{3B^2} + 4\,Q\,A^2 = A^2\left(\frac{3}{10} + \frac{2}{3B^2} + 4\,Q\right)$$

$$0 = A_1 + A_2\alpha + A_3\beta$$

$$0 = B_1 + B_2\alpha + B_3\beta$$

$$\alpha = \frac{-A_1B_3 + A_3B_1}{A_2B_3 - A_3B_2} \qquad \beta = \frac{A_1B_2 - A_2B_1}{A_2B_3 - A_3B_2}$$

$$\frac{T}{\theta} = \frac{G_c t b^3}{12}\left[\left((A\alpha + 1)^2 + \frac{3A}{4}(A\alpha + 1)\beta + \frac{3A^2}{20}\beta\right)\right.$$

$$+ \frac{1}{B^2}\left((A\alpha - 1)^2 + A(A\alpha - 1)\beta + \frac{A^2}{3}\beta\right)$$

$$+ 6\,Q\left(\frac{4}{3}F^2 + A^2\left((1 - \alpha)^2 - (1 - \alpha)\beta\right.\right.$$

$$\left.\left.\left. + \frac{\beta^2}{3}\right)\right)\right] \qquad (5.6.)$$

The results obtained by putting $\beta = 0$ are only marginally inferior to (5.6). The expressions then become

$$\alpha = \frac{1 - \frac{1}{6QA}\left(1 - \frac{1}{B^2}\right)}{1 + \frac{1}{6Q}\left(1 + \frac{1}{B^2}\right)}$$

$$\frac{T}{\theta} = \frac{G_c t b^3}{12}\left[(A\alpha + 1)^2 + \left(\frac{(A\alpha - 1)}{B}\right)^2 + 6\,Q \left(\frac{4}{3} F^2 + A^2 (1 - \alpha)^2\right)\right] \qquad (5.7.)$$

In (5.6.) and (5.7.), as in (5.2.), the second term in the main bracket is negligible; the third term, although signficant, is fairly small compared with the first.

5.5. Experimental Procedure

Torsion tests were carried out using three thicknesses of polyurethane core, nominal 1 in., .75 in., and .5 in. (actually .96 in. = 24.4 mm, .74 in. = 18.8 mm, .45 in. = 11.4 mm). The thickness of the plywood was the same on all the sandwich specimens, a nominal 1.5 mm (in fact 1.63 mm).

There were therefore three ratios of $\frac{f}{t}$ and for each of these the ratio of $\frac{b}{t}$ was varied by varying the width in each series of specimens.

5.6. Property Values

The values of shear modulus for the core and faces were determined as described in (2.6.) and (2.8.).

For the torsion calculations the values used for shear Modulus were

Core	2×10^6 N/m^2
Face	5×10^9 N/m^2

5.7. Torsion Tests

The torsion tests were carried out on two standard torsion testing machines which had modified 'chucks' to accept the rectangular cross-section of the specimen. An upper limit of 0.45 m was imposed on the length of the specimens by the design of the larger machine so it was decided to restrict the width to about 0.15 m or less for most of the specimens. This was done in order to be in accordance with the assumption that the length is "large compared with the other dimensions". However, a few wider specimens were tested to see what happened.

Because of the nature of the specimens, some difficulty was experienced in measuring the angle of twist accurately. No standard devices were available for measuring the angle of twist for this type of specimen so that an attempt was made to manufacture a semi-circular scale which was to be clamped to the specimen. This proved too heavy and inaccurate and was discarded.

Several trial specimens were tested, without recording angle of twist, in order to determine the loading range which could be applied for certain widths of panel. It was seen by placing a straight edge on the face of a specimen at right angles to the axis, about which the torque was applied, that the surface remained straight even for large angles of twist, as predicted in the theory.

A base was then machined, for use with a precision clinometer (Fig. 12) so that the angle of inclination of a line could be measured relative to the horizontal. The clinometer was held by hand next to the specimen and supported so that its self-weight did not affect the readings. The angle of twist was found for a range of torque readings by calculating the difference in clinometer readings for a gauge length of 180 mm.

The relation between torque and twist was found to be a linear up to quite large angles (several degrees even for the stiffer beams). The results of the torsion tests are summarised in Fig 13 where the stiffness $\frac{T}{\theta}$ in Nm/rad/m are plotted against beam width for the three core thicknesses. The curves shown are the theoretical values.

In view of the nature of the materials the agreement between theory and experiment is good for widths up to 0.15 m. Only a few experiments were conducted beyond this width, but the agreement can be seen in Fig. 13 to be deteriorating for the thin beams.

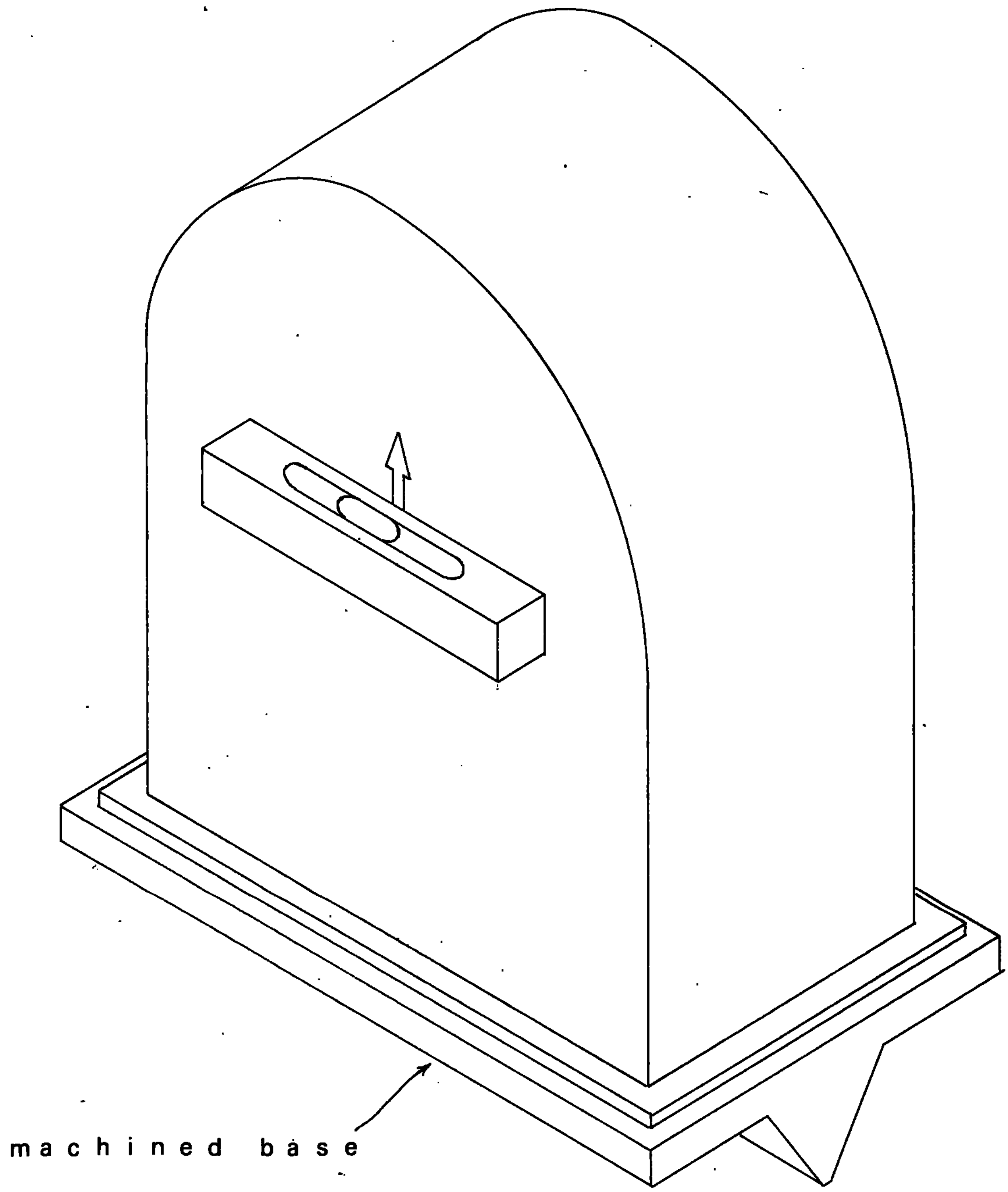
Table (5.2.) gives details of the experiment referred to in section 4.3. Crude Analysis.

Table (5.2.)

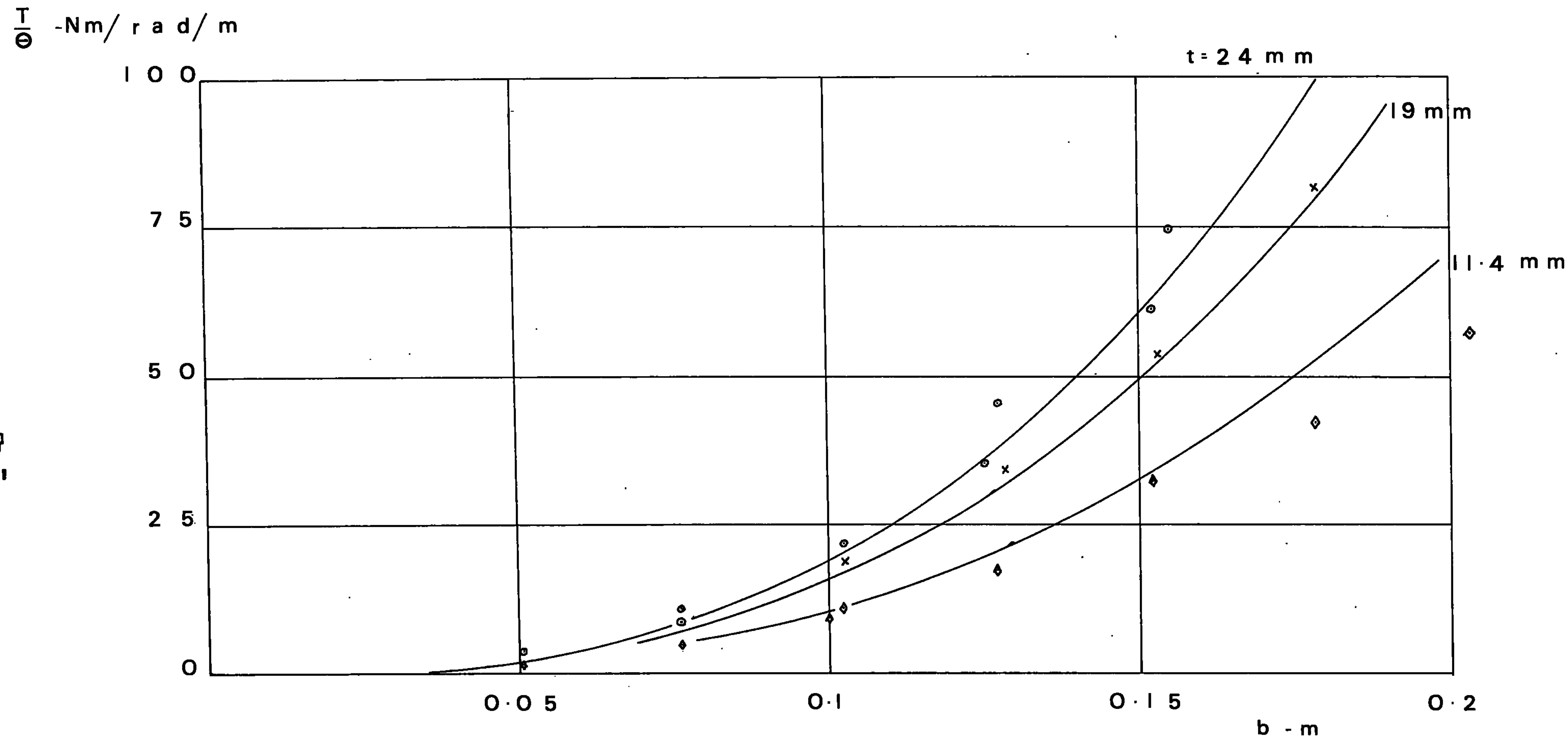
Stiffness $\frac{T}{\theta}$ Nm/rad/m Core thickness 24.4 mm.

	Bare Core		Sandwich	
Width mm.	76	152	76	142
Theoretical	0.65	1.16	9.5	63.5
Experimental	0.56	1.1	8.0	61

angles of inclination measured to an accuracy of 20 secs.



CLINOMETER AND BASE



VARIATION OF TORSIONAL STIFFNESS AND WIDTH FOR THREE THICKNESSES

FIG. 13.

5.8. Discussion

The principal feature of the theory appears to be confirmed by the experiments: viz. that the torsional stiffness of a sandwich beam is approximately proportional to the width cubed and directly proportional to the thickness, unlike the homogeneous rectangular beam whose stiffness is roughly proportional to the cube of the smaller dimension i.e. thickness.

The approximate value of shear modulus for the faces is used in the torsion theory.

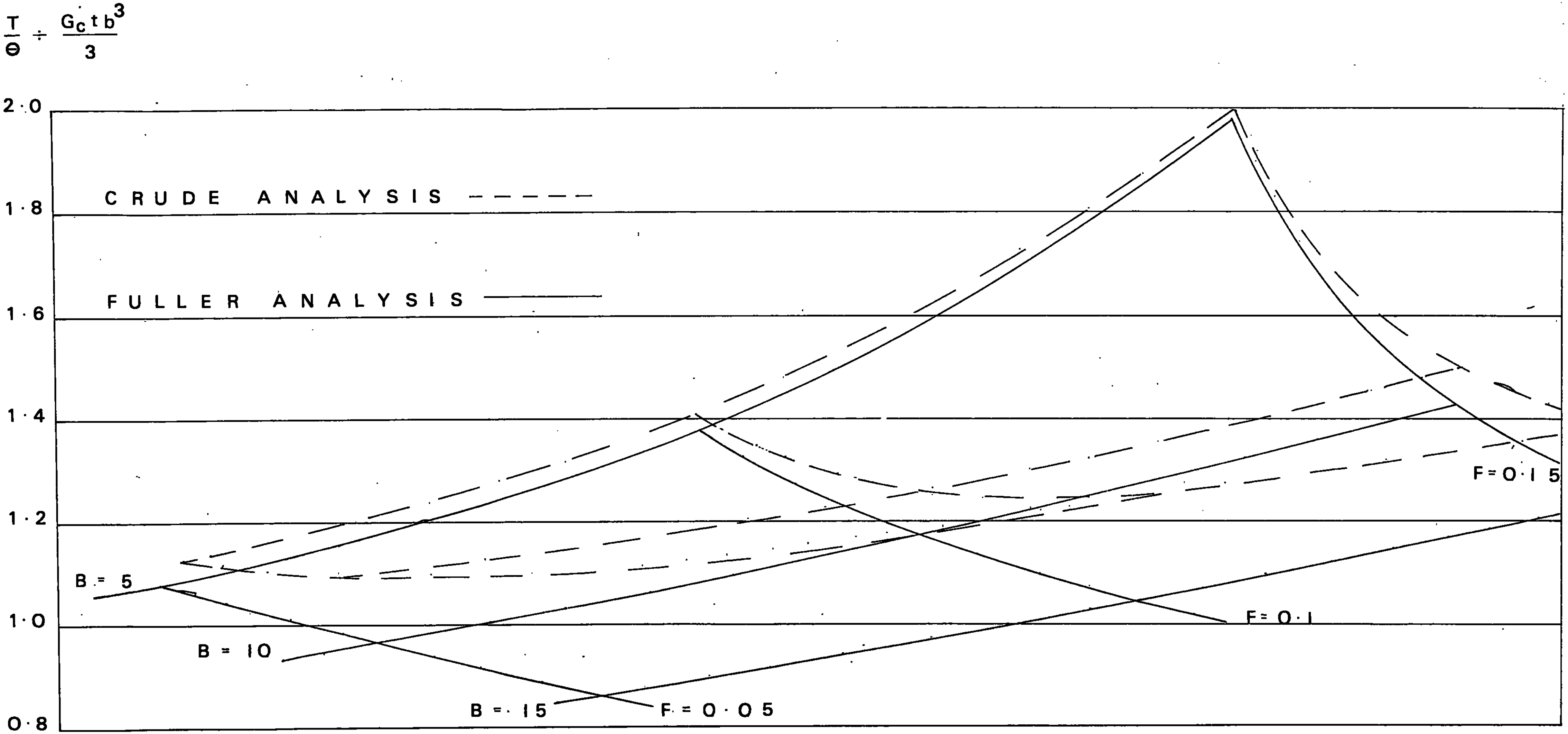
In the very simple expression (4.2.) $\frac{G_f}{G_c}$ is of order 10^3 which makes the expression $2\frac{G_f}{G_c}\frac{F^3}{B^2}$ only just significant so that an approximate value of G_f can be used without great loss in accuracy.

In expression (4.7.) the term in which G_f occurs accounts for only about 20% of the total value of the square bracket and only because $\frac{G_f}{G_c}$ is of order 10^3. Average values for G_f are therefore acceptable and do not give significant errors.

The results of the calculations show that the simple expressions (4.7.) give results which are so close to those of (4.6.) that it is not worth going to the considerable additional labour of using (4.6.). Furthermore, unless the elastic constants of the materials are known to an accuracy closer than about ± 10% it is hardly worth going further than the very simple expression (4.2.). That can be written to a close approximation.

$$\frac{T}{\theta} = \frac{G_c tb^3}{3}\left[(1+F)^2 + 2\frac{G_f}{G_c}\frac{F^3}{B^2}\right]$$

The term in the square bracket is compared with values given by the fuller analysis, for a range of values of F and B, in Fig. 14 for the ratio of shear moduli used in this investigation.



STIFFNESS SCALING FACTOR FOR MODULUS RATIO

$\frac{G_f}{G_c} = 2.5 \times 10^3$

FIG. 14.

CHAPTER VI

DISCUSSION

6.1. Bending

The comparison of experimental and theoretical results from the simple four point bending tests indicate that the simple bending theory is viable. The individual elements of the sandwich behave in a manner very similar to that predicted in the initial assumptions of the bending theory, i.e. most of the strain energy in the faces of a deformed panel is extensional and the shear strain energy is absorbed mainly by the core.

An improvement can be made on the simple theory by accepting the theory of March, Smith and Norris which assumes that the core restrains the face against lateral strain, even in a comparatively narrow beam.

Even with the improved theory good results can only be obtained for panels with a high core/face ratio. The discrepancy between theoretical and experimental values for deflection increased when the thickness of the panel decreased.

The results from the initial loading arrangement show that concentrated loads have a considerable effect on bending stiffness. In the section of the beam subject to bending only, the central deflection is increased by a component of deflection due to shear transfer effects in the region of the concentrated load. The faces take the form of a finite radius under the concentrated load between the area of panel affected by shear loads and that area theoretically unaffected by shear. This transition curve in the faces causes the deflection of the centre of the panel relative to the load points to be greater than would be predicted by assuming constant curvature between the load points. The phenomenon has a greater effect on the parity of theory and fact in panels with higher core/face thickness ratios, though the net magnitude of the effect is greater in thinner panels due to smaller core/face thickness ratios giving larger shear deformations.

An important factor arising from the shear deflection results is that the 'effective' core thickness is at least (t + f). According to Norris it can be as high as (t + 3f) a value which gives good agreement with the results obtained in this investigation.

The effective core thickness is important for mathematical model simulation techniques being developed by G.M. Parton, et. al., though it is more important to note that the model only works well when the face thickness is small compared to the core thickness anyway.

6.2. Panels Subject to End Load

Comparison of theoretical and experimental results show that the Euler crippling load of a panel can be satisfactorily predicted when the effect of shear deformation in the core is taken into account.

No attempt was made to study the effects of local instability causing wrinkling of the faces. The length of the shortest column tested was greater than the length at which local face buckling could be expected in panels of the geometry used. Face wrinkling is unlikely to be a problem in projects using similar panels, and, in current tests, this phenomenon was never seen to occur.

Ultimate failure of the panels was by core failure. The same type of failure occurred in panels loaded as simply supported beams. In both cases the deflections were abnormally large and core failure was followed immediately by failure of the cardboard interface. In torsion the ultimate failure in the core occurred at large deflections and was associated with a lateral tearing of the faces at the end clamps.

6.3. Panels Subject to Torsion

The expression developed to predict the torsional stiffness of a sandwich panel gives results which compare well with experimental observation even when it is used in its simplest form. The theory holds up remarkably well even when the panel dimensions become nearer to those describing a plate rather than a beam.

The difference between the theory and that derived by St. Venant for a homogeneous beam is that the stiffness is approximately proportional to the width cubed and directly proportional to the thickness for a sandwich panel, whereas the stiffness of a homogeneous beam is roughly proportional to the cube of the smaller dimension.

A significant point in sandwich panel torsion is the way in which the deformation is controlled by the stiff faces but the main shear stiffness of the panel is provided by the core. Table (4.2.) compares stiffness values of bare core and sandwich panels of similar dimensions which confirms the difference in behaviour between the core and the sandwich.

The shear modulus of the faces is not an important parameter in the torsion equations. This is rather fortunate because a realistic value for plywood is very difficult to obtain.

It was noted during the torsion experiments that the deformed shape of the panels had straight line sections along and at right angles to the axis of torsion for small deflections. The similarity between the deformed shape and the hyperbolic paraboloid form may be of some use in further work on sandwich plates.

6.4. General

In all aspects of sandwich panel loading the shear modulus of the faces is not of importance so long as the faces are two to three orders of magnitude stiffer than the core.

The investigation carried out on sandwich panels and their components would seem to give a reasonable foundation for further work on a computer model of sandwich plates, dependent on the principle that the deformed shape is dictated by the stiff faces and the approximately linear cross panel compliance of the core.

In general the work has given a useful insight into the use and analysis of sandwich plates with plywood faces and foam polymer cores, and of similar proportions to those used in the investigation.

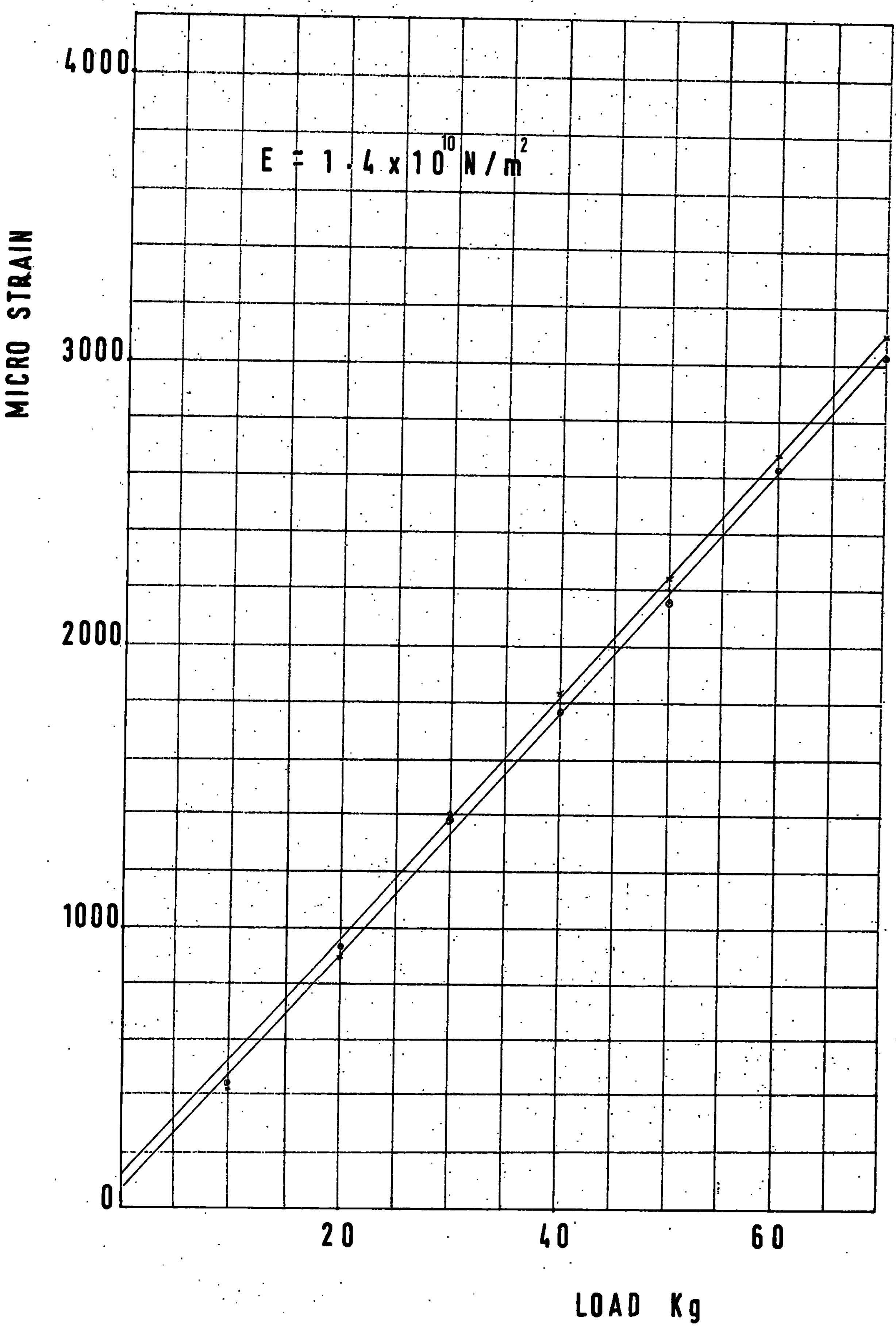
APPENDIX

GRAPHICAL SUMMARY OF EXPERIMENTAL RESULTS

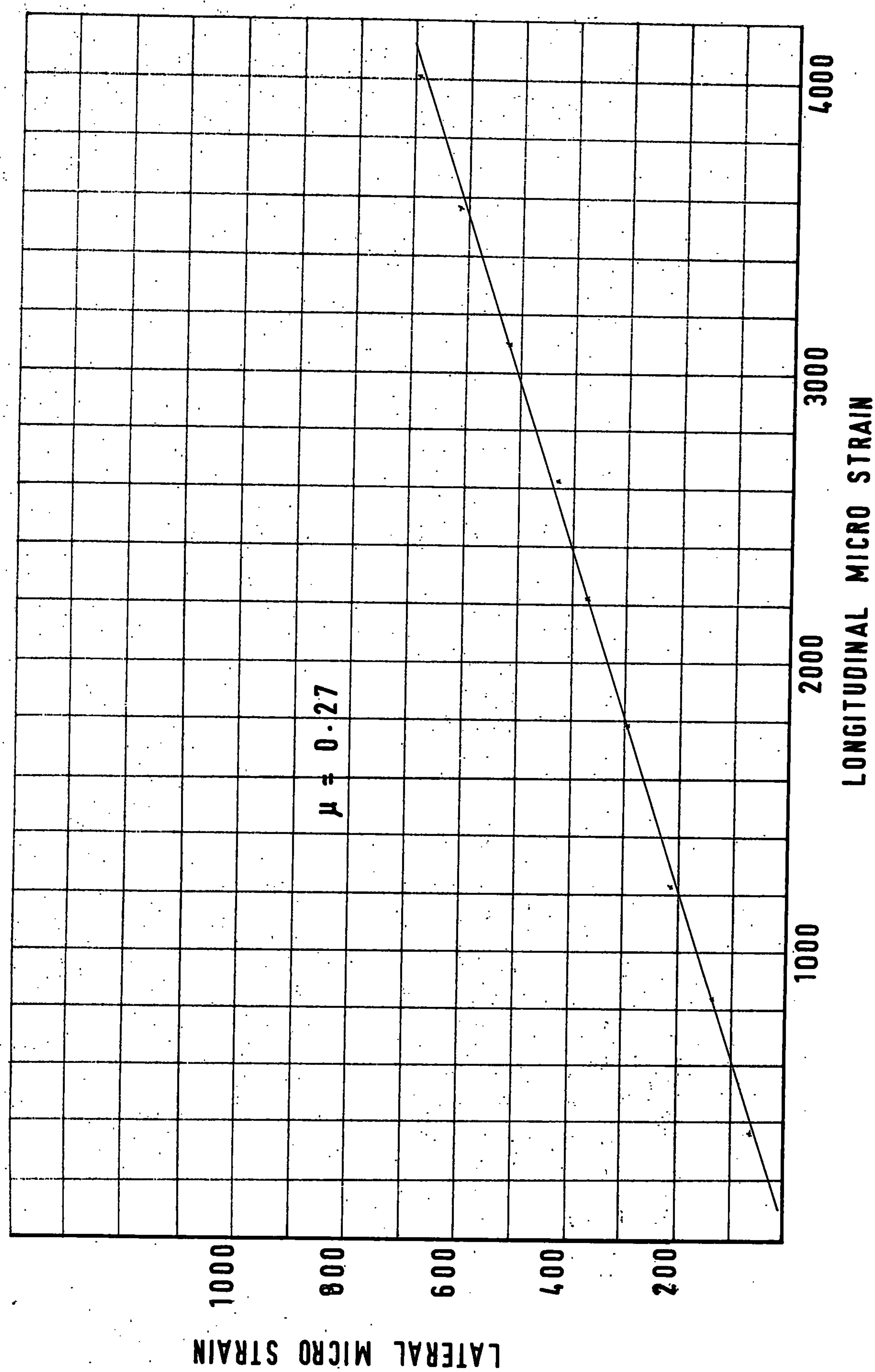
Note: This Appendix includes a summary of the large amount of graphical data obtained during the course of work

E TYPE TENSOMETER TEST

PLYWOOD FACES WITH STRAIN GAUGES ATTACHED

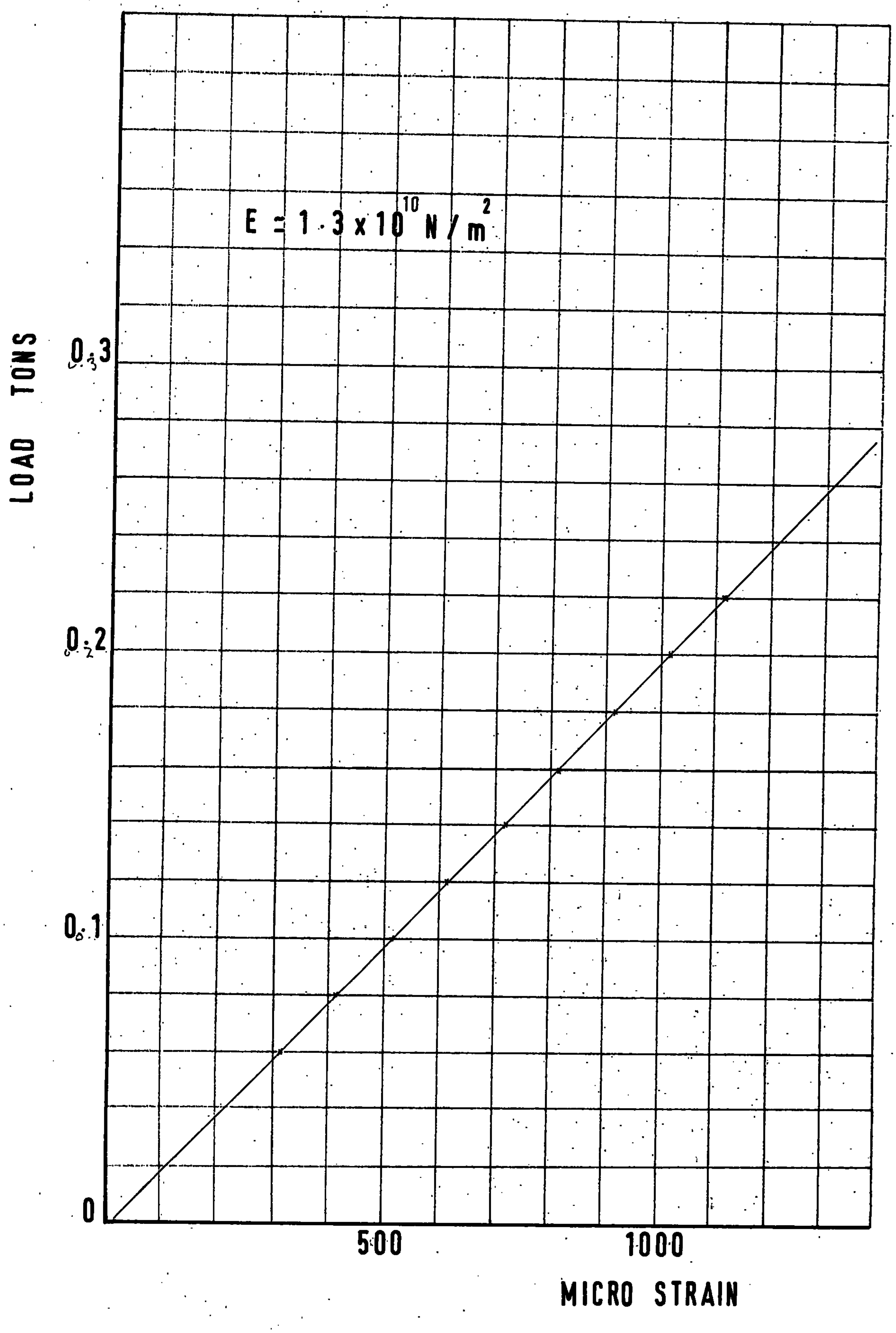


'E' TYPE TENSOMETER TEST

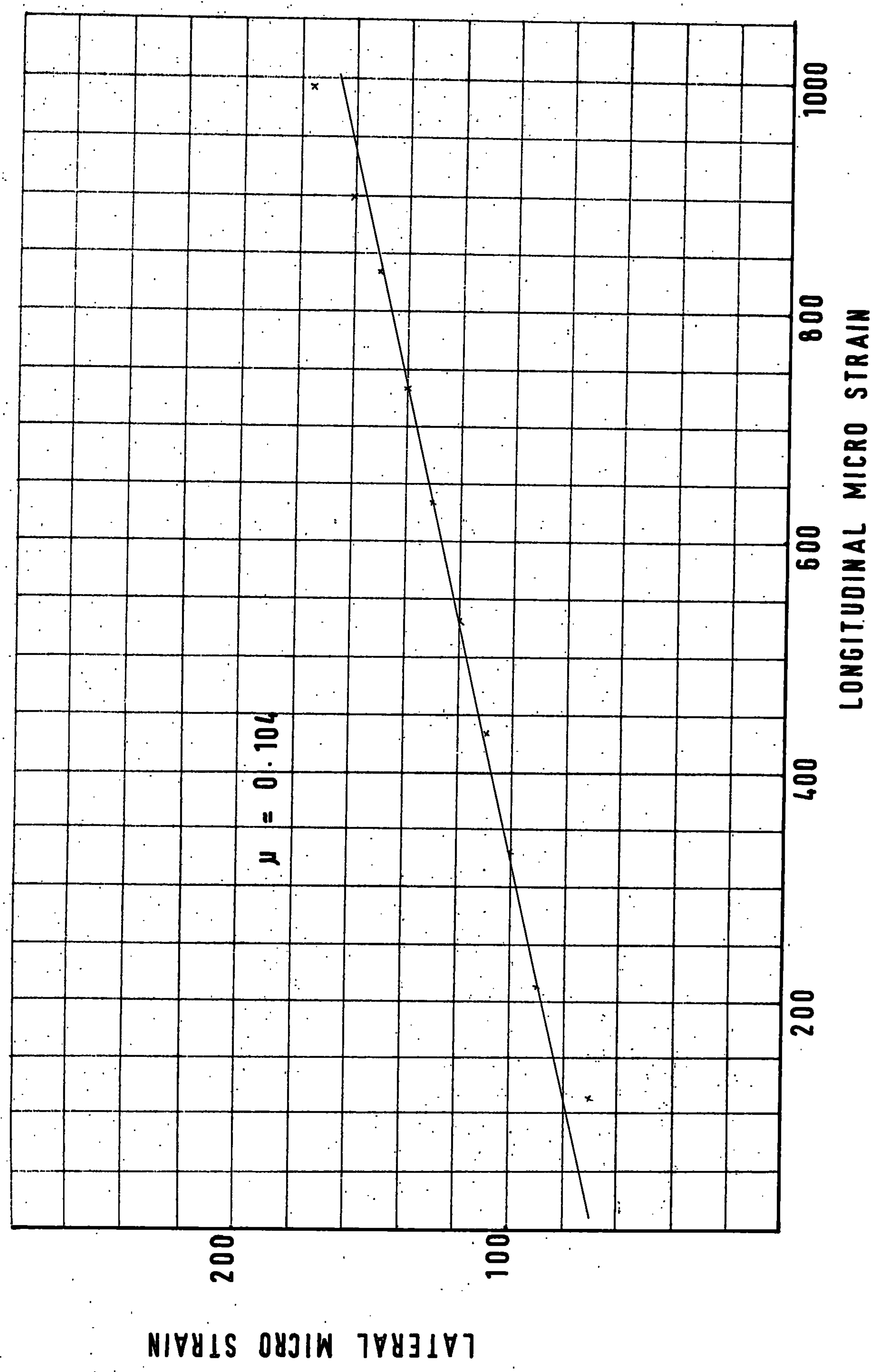


DENNISON COMPRESSION TEST

PLYWOOD FACES WITH STRAIN GAUGES ATTACHED

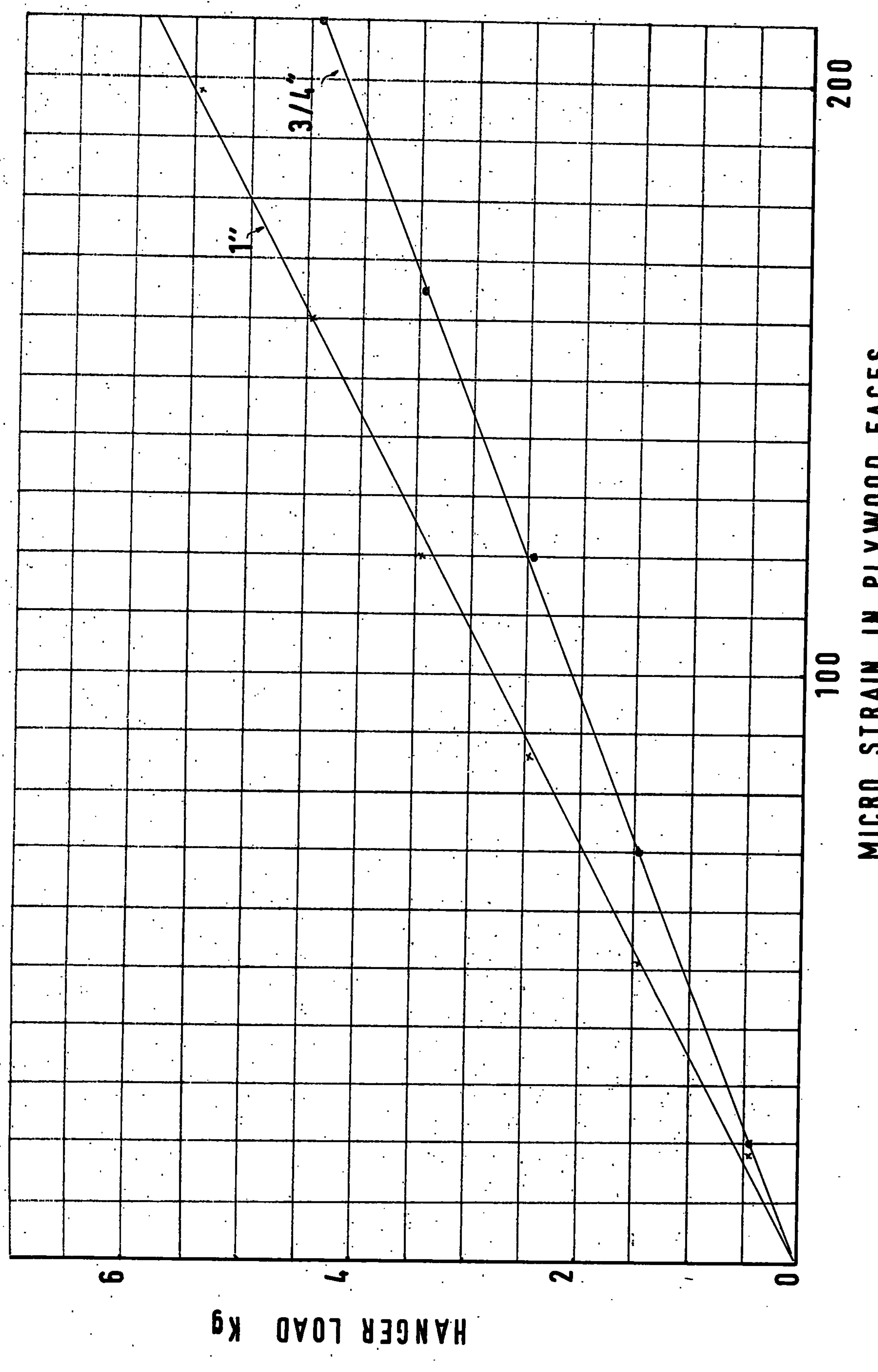


DENNISON COMPRESSION TEST



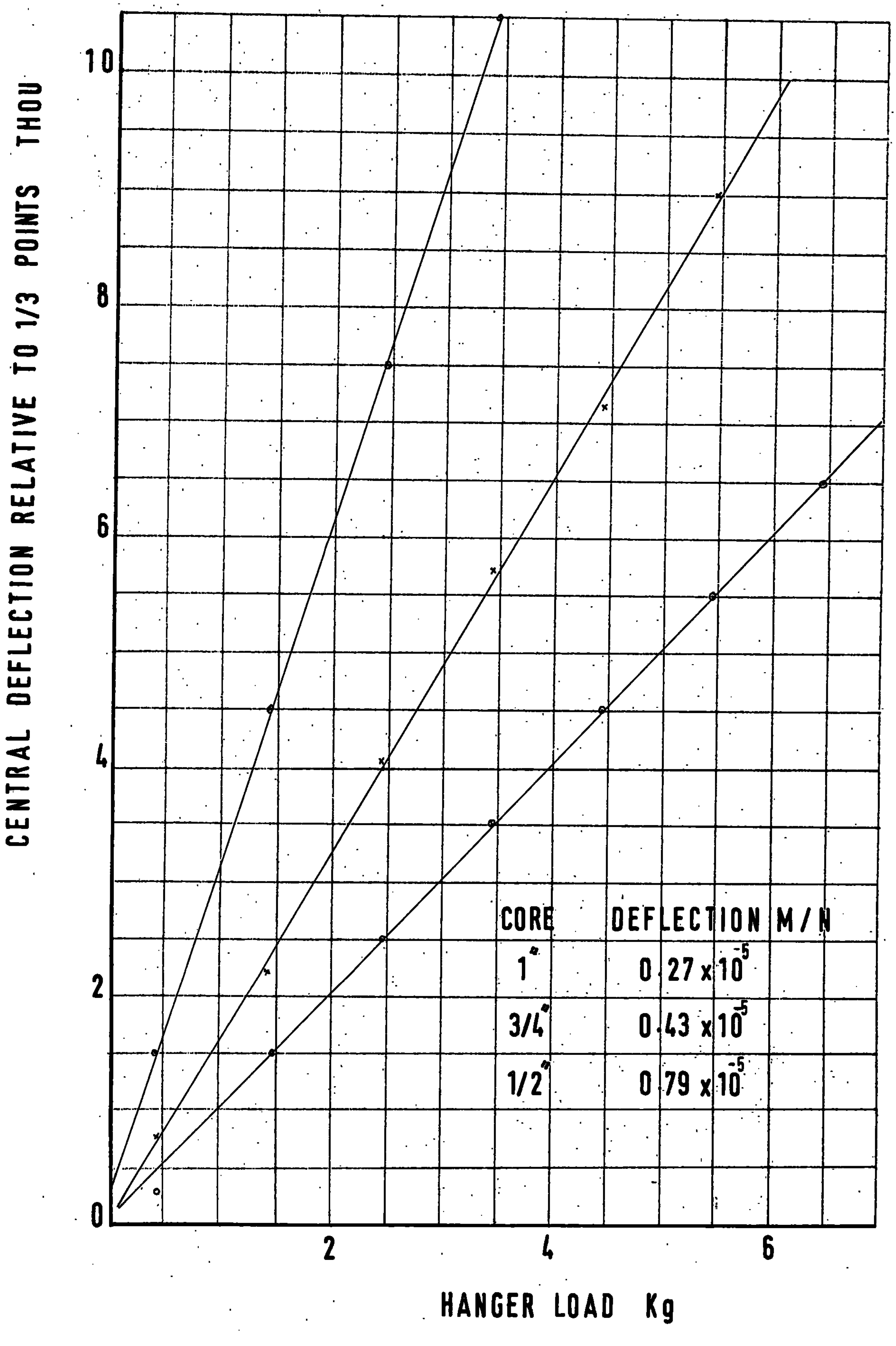
FOUR POINT BENDING

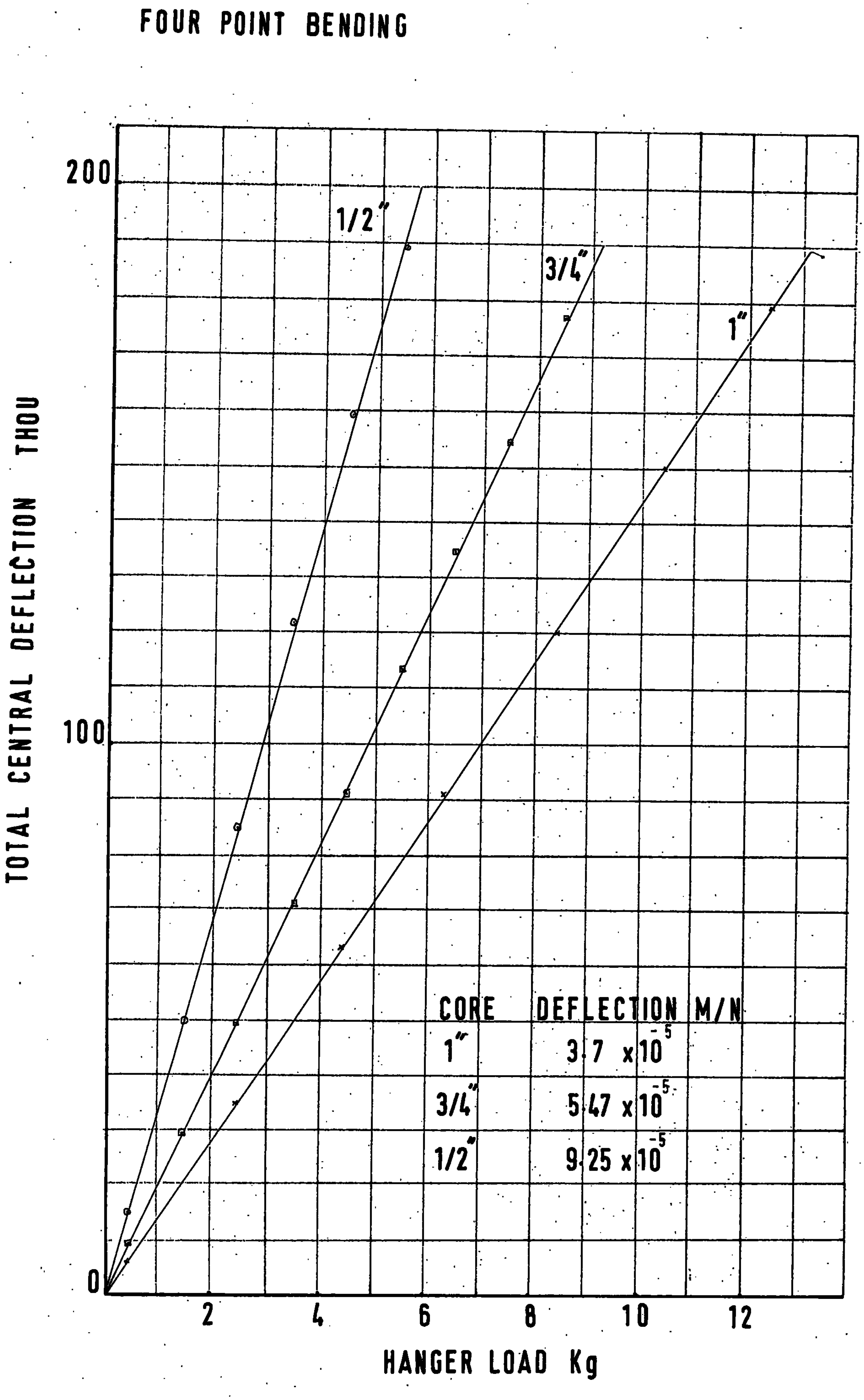
STRAIN GAUGES AT CENTRE SPAN



FOUR POINT BENDING

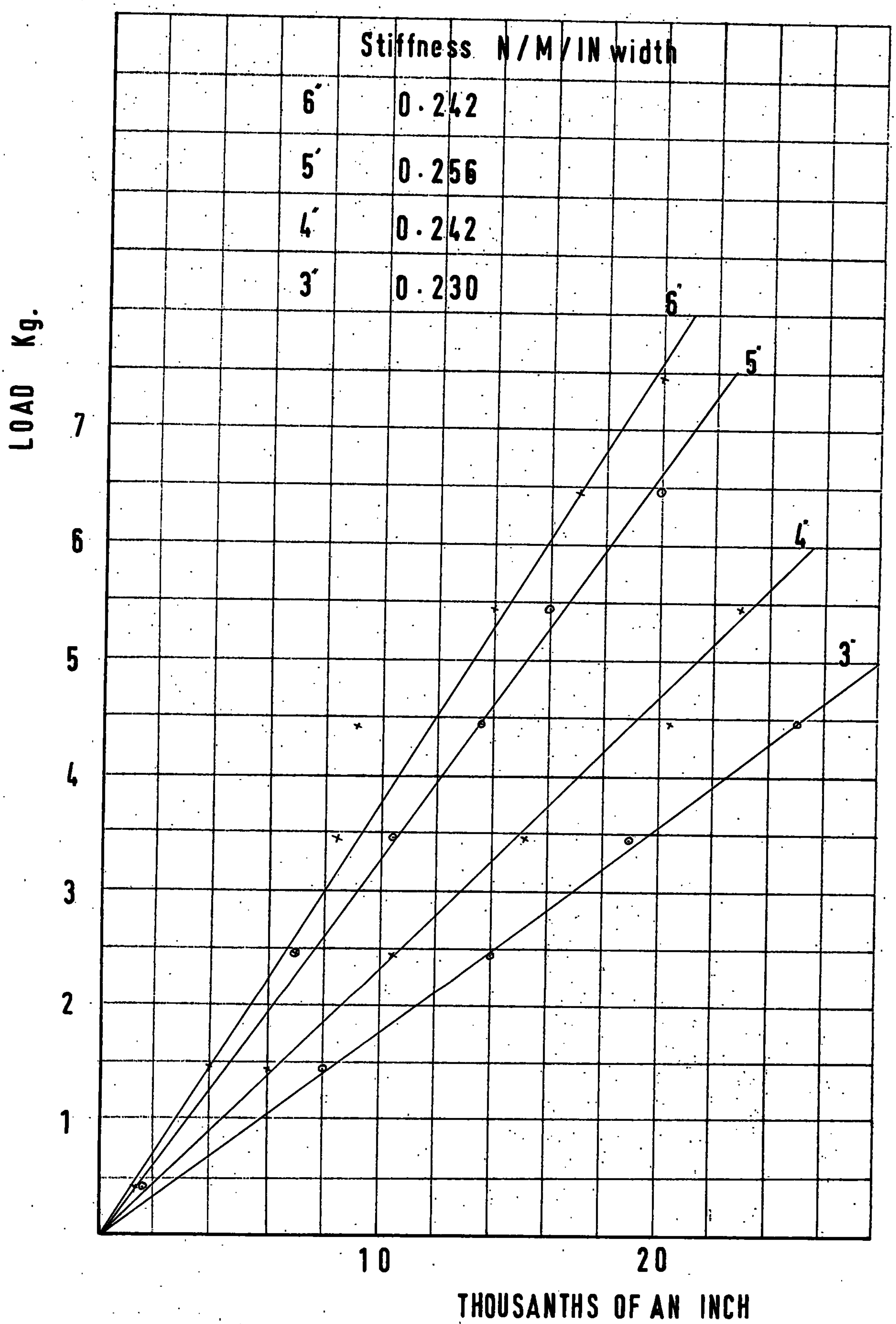
PURE BENDING ONLY





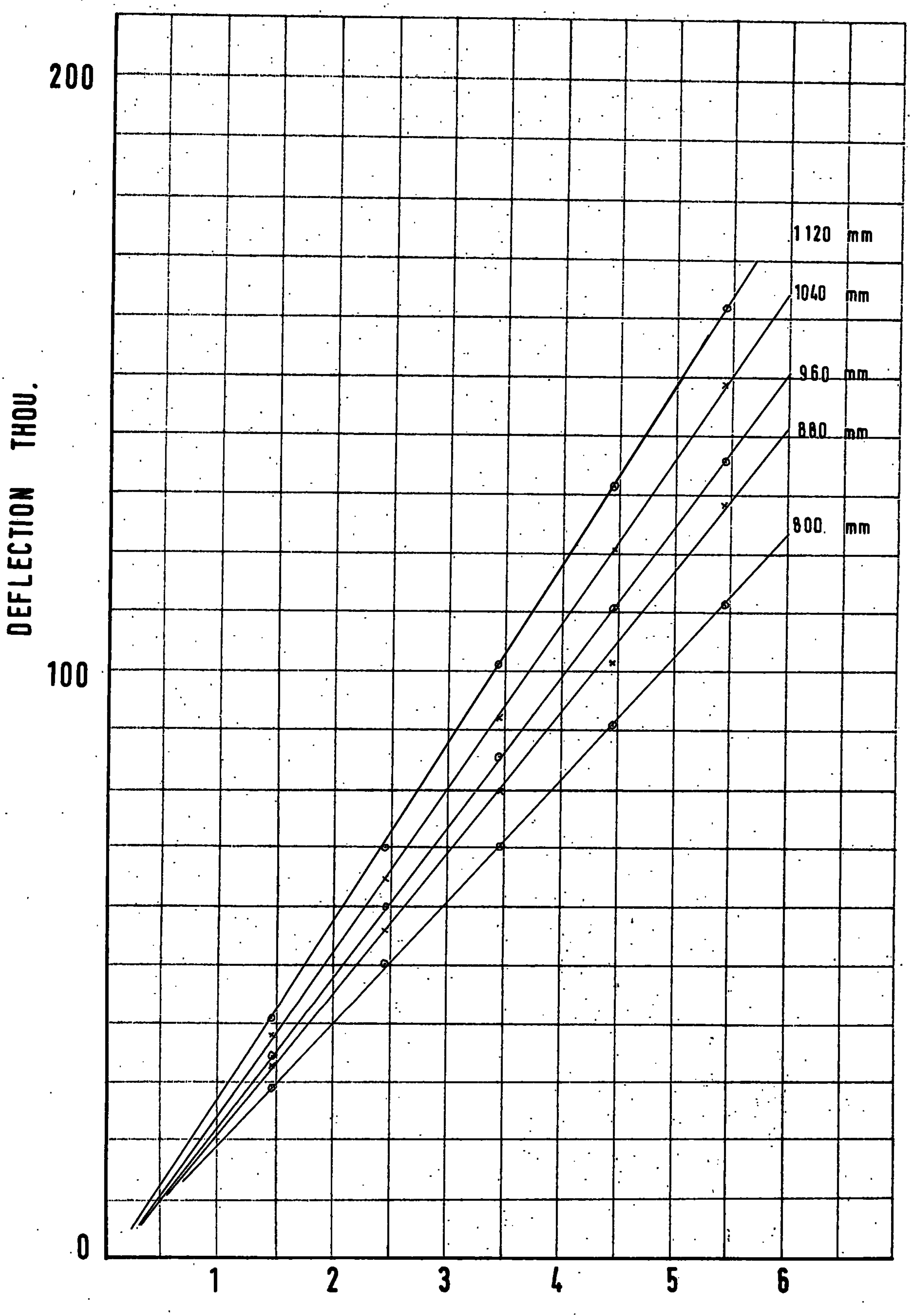
FOUR POINT BENDING
TOTAL CENTRAL DEFLECTION THOU
200
100
0
1/2"
3/4"
1"
CORE DEFLECTION M/N
1" 3.7 x10^-5
3/4" 5.47 x10^-5
1/2" 9.25 x10^-5
2
4
6
8
10
12
HANGER LOAD Kg

HANGER LOAD V CENTRAL DEFLECTION



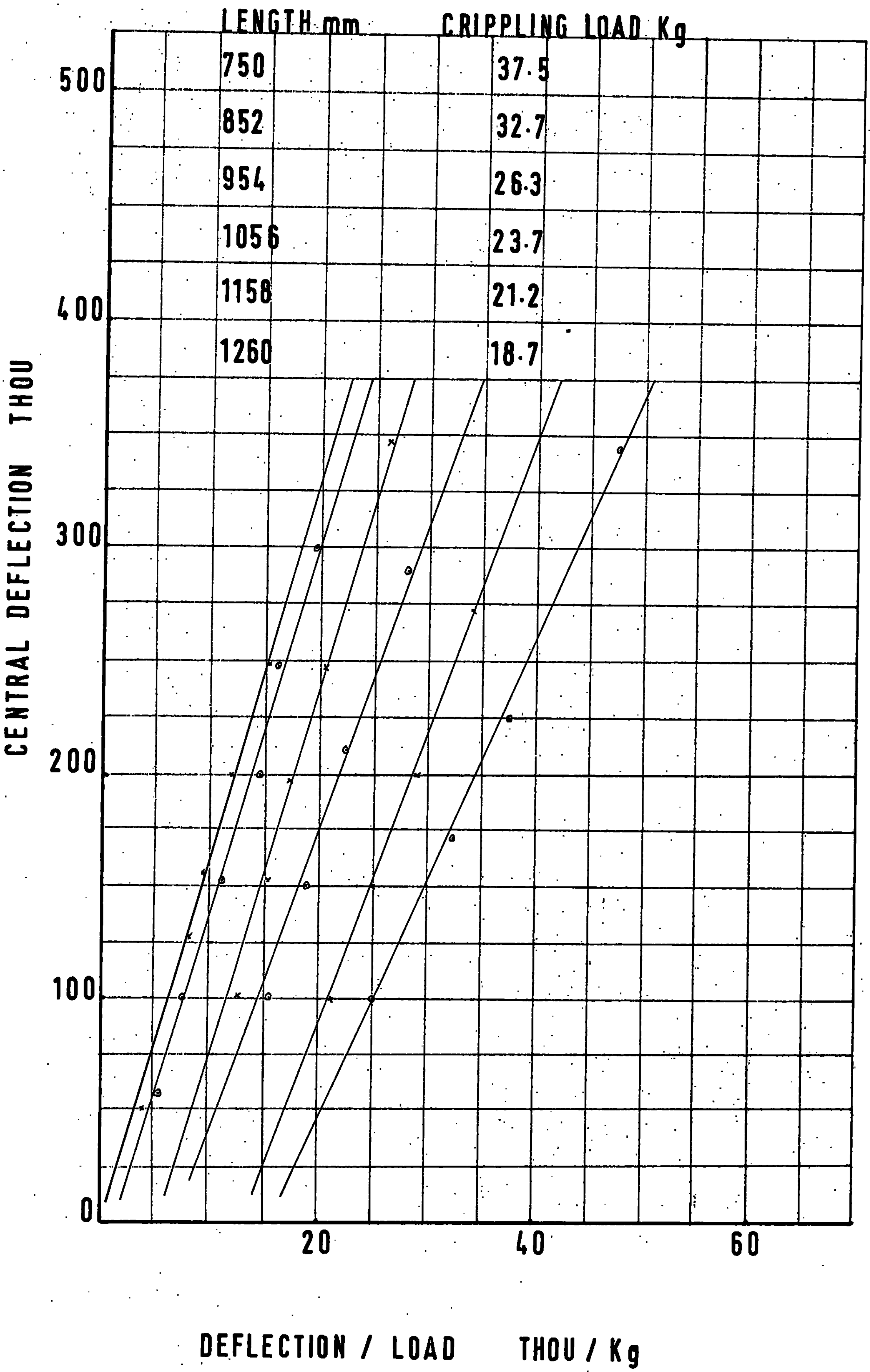
ANTICLASTIC EFFECT 1/2 IN BEAM

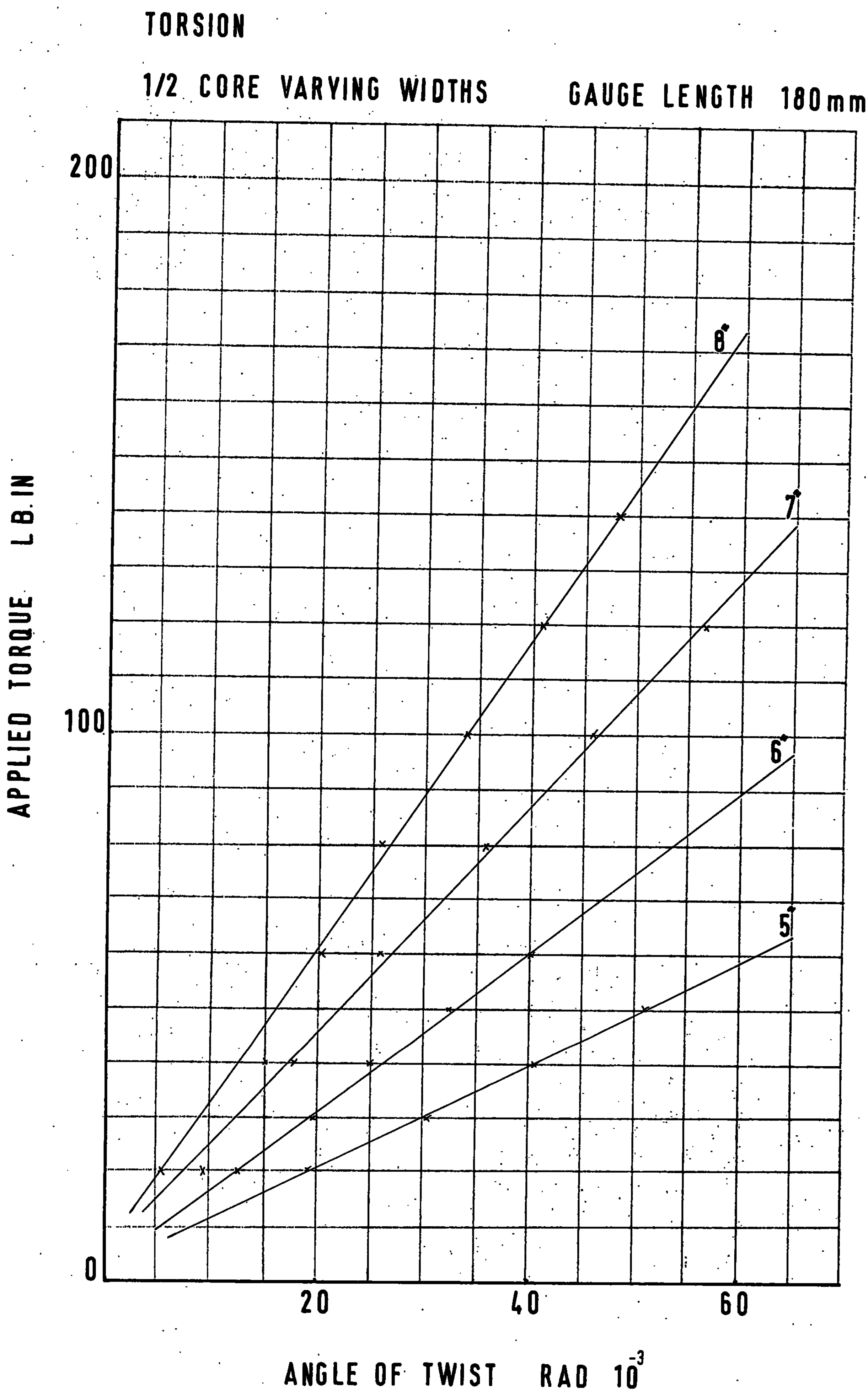
HANGER LOAD V TOTAL CENTRAL DEFLECTION

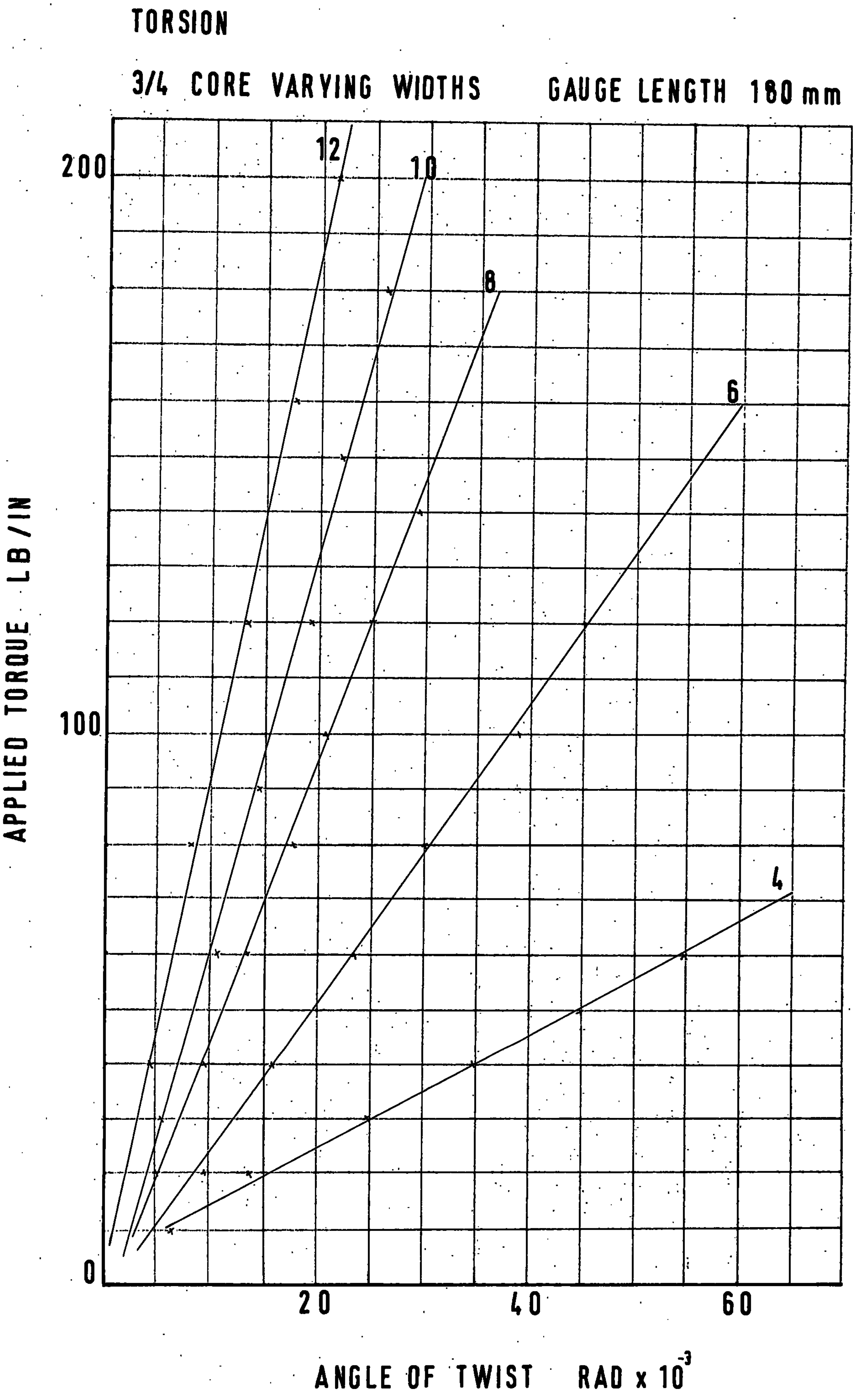


VARYING SPANS 1/2 IN BEAM

BEAMS SUBJECT TO END LOAD

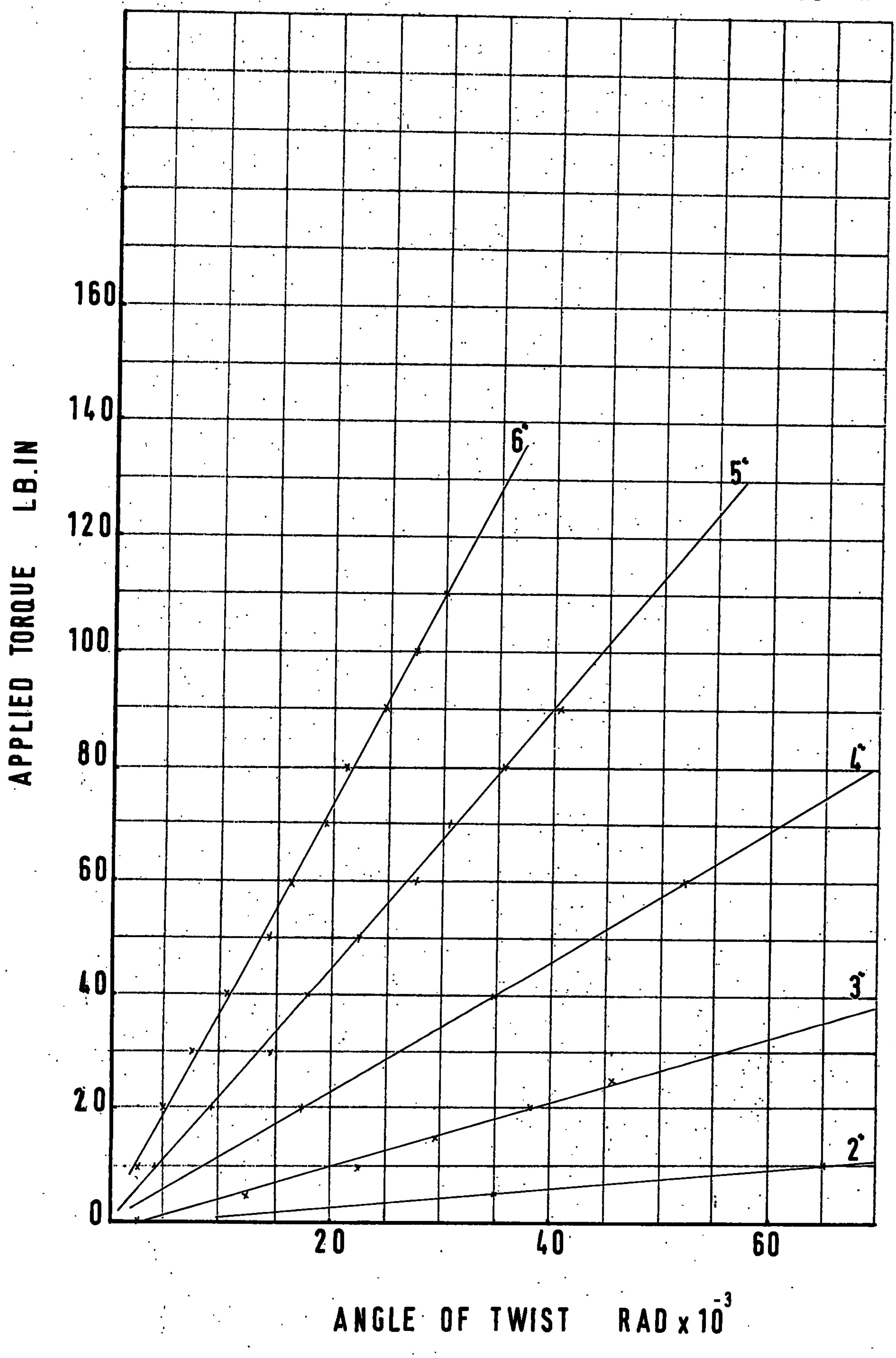



TORSION
1/2 CORE VARYING WIDTHS
GAUGE LENGTH 180mm
APPLIED TORQUE LB.IN
ANGLE OF TWIST RAD 10^-3
200
100
0
20
40
60
8"
7"
6"
5"


TORSION
3/4 CORE VARYING WIDTHS GAUGE LENGTH 180 mm
12
10
8
6
4
200
100
0
20
40
60
APPLIED TORQUE LB/IN
ANGLE OF TWIST RAD x 10^-3

TORSION

1" CORE VARYING WIDTHS GAUGE LENGTH 180 mm



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